SOME LOW-SPEED WIND-TUNNEL EXPERIMENTS ON A SHARP-EDGED DELTA WING OF ASPECT RATIO 1, WITH AND WITHOUT YAW

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some low-speed wind-tunnel experiments on a sharp-edged delta wing of aspect ratio 1, with and without yaw

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SUMMARY

An uncambered, sharp-edged delta wing of aspect ratio 1 was tested in the 1.8 m x 1.25 m (octagonal) low-speed wind-tunnel of the Department of Aerospace Engineering at the Delft University of Technology. Balance measurements were carried out to measure the forces and moments at several angles of attack and yaw. The results are correlated with surface oil-flow patterns and with the behaviour of leading-edge vortices above the wing. Comparisons are made with the results of other investigations known from literature. In addition, theoretical methods that predict sideslip derivatives are discussed and their results are compared with those of the experiments.
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1. LIST OF SYMBOLS

A  aspect ratio, \( \frac{b^2}{s} \)

b  wing span

c  local chord \( \frac{b}{2} \)

c  aerodynamic mean chord, \( \frac{2}{S} \int c^2 \, dy \)

c  centerline chord

c  tip chord

c  drag coefficient, \( \frac{\text{drag}}{\frac{1}{2} \rho V^2 S} \)

c  rolling-moment coefficient, \( \frac{\text{rolling moment}}{\frac{1}{2} \rho V^2 S b} \)

c  lift coefficient, \( \frac{\text{lift}}{\frac{1}{2} \rho V^2 S} \)

c  = \frac{\partial C_l}{\partial \beta}

c  pitching-moment coefficient about 0.25 c, \( \frac{\text{pitching moment}}{\frac{1}{2} \rho V^2 S c} \)

c  yawing-moment coefficient, \( \frac{\text{yawing moment}}{\frac{1}{2} \rho V^2 S b} \)

c  = \frac{\partial C_n}{\partial \beta}

C  normal-force coefficient

C  pressure coefficient

C  resultant-force coefficient

T  tangential-force coefficient

C  lateral-force coefficient

C  = \frac{\partial C_Y}{\partial \beta}

c  distance from vortex-axis to trailing-edge, leeward side

c  distance from vortex-axis to trailing-edge, windward side

Re  Reynolds number

s  wing local semi-span
\[
b_{2}
\]

\[S \quad \text{wing area} = 2 \int_{0}^{c} dy \]

\[t \quad \text{thickness} \]

\[v \quad \text{free stream velocity} \]

\[x, y \quad \text{co-ordinates of body axes system, origin at apex (fig. 3)} \]

\[x_{ac}, y_{ac} \quad \text{co-ordinates of aerodynamic center} \]

\[x_{p}, y_{p} \quad \text{co-ordinates of center of pressure} \]

\[x_{s}, y_{s}, z_{s} \quad \text{co-ordinates of stability axes system, origin at 0.50 c}_{o} = 0.25 c \text{ (fig. 3)} \]

\[\alpha \quad \text{angle of attack} \]

\[\beta \quad \text{angle of sideslip} \]

\[\eta = \frac{y}{s} \]

\[\rho \quad \text{wing taper ratio} = \frac{c_{t}}{c_{o}} \]
2. INTRODUCTION

Slender delta wings with sharp leading edges are used for supersonic aircraft in order to minimise aerodynamic drag during supersonic cruise and because of their favourable low-speed characteristics. At positive angles of attack additional lift is induced by the leading-edge vortices which makes it possible to operate at high angles of attack as required during take-off, landing and manoeuvres. In symmetric flight the core of a leading-edge vortex may undergo a sudden expansion at very high angles of attack. In asymmetric flight this phenomenon, known as vortex breakdown, occurs at a lower angle of attack. The aerodynamic forces and moments are reduced significantly by vortex breakdown.

The presence of leading-edge vortices and the possibility of vortex breakdown complicate the development of methods which predict the characteristics of slender delta wings with sharp leading edges. Non-linear methods are available which, for symmetric flight and at angles of attack low enough to avoid vortex breakdown, quite accurately predict lift and drag and to a lesser extent the pitching moment. These methods have been originated by, among others, Polhamus, Garner and Lehrian, Sacks, Neilson and Goodwin, Mangler and Smith, and Nangia and Hancock (refs. 1 to 5). An overview and a comparison of their theoretical results with experimental data is given by Parker in ref. 6.

To predict the characteristics of a delta wing aircraft in its most critical flight phase, that is during approach and landing, it is necessary to know also the influence of sideslip and ground effect. As far as the influence of sideslip is concerned, for small angles linear methods are available which accurately predict the sideslip derivatives. These methods start from wings with round leading edges and assume that the flow is attached. As a consequence the predicted derivatives vary linearly with lift, which is true for small angles only. For larger angles, the formation of leading-edge vortices causes the sideslip derivatives to vary non-linearly with lift.
Until now, the author has not found a method that predicts the sideslip derivatives for larger angles. In an attempt to support the development of a suitable mathematical model of the airflow which should be the basis for such a method, the Department of Aerospace Engineering of the Delft University of Technology has started a series of wind-tunnel investigations on a delta wing model of aspect ratio 1 having sharp leading edges. This report describes the introductory experiments carried out to gain measuring experience and some insight into the changes in location and nature of the vortices due to sideslip. The influence of these changes on the forces, moments and sideslip derivatives is investigated. The experimental results are compared with those of other investigations and with the theoretical results of some linear methods.
3. APPARATUS AND TESTS

3.1. Wind-tunnel

The tests were carried out in the low-speed wind-tunnel of the Department of Aerospace Engineering at the Delft University of Technology. The wind-tunnel has an octagonal test section, 1.25 m high x 1.80 m wide (fig. 1).

3.2. Model

The model is a flat plate delta wing with symmetrically chamfered, sharp edges. It is made for the most part of wood, the edges are of aluminium. The principal data are (fig. 2):

\[
\begin{align*}
A &= 1 \\
C_o &= 1 \text{ m} \\
b &= 0.5 \text{ m} \\
\tilde{c} &= 0.667 \text{ m} \\
t/C_o &= 0.03 \text{ m}
\end{align*}
\]

The model was suspended through struts from the six-component balance system. The main strut was located at the 0.50 \( C_o \) position (moment center, \( A \) in fig. 2). The movable strut was positioned at 0.85 \( C_o \) (B in fig. 2).

3.3. Balance measurements

At a wind velocity of 80 m/sec, which corresponds to a Reynolds number of \( 3.57 \times 10^6 \) based on \( \tilde{c} \) (5.36 \( \times 10^6 \) based on \( C_o \)), six-component measurements were performed at angles of attack from -4 to 20 deg. in steps of 4 deg., and at yaw angles from -5 to 20 deg. (for the detailed program, see table I). During each run the angle of yaw was varied while the angle of attack was kept constant.
Forces and moments are quoted relative to the stability axes system illustrated in fig. 3 with the 0.50 \( c_0 \) (= 0.25 \( c \)) point as origin. Corrections were applied for aerodynamic interference between struts and model, and for elastic deformation of the struts.

The angle of attack, drag and pitching-moment of the unyawed wing have been corrected for lift-effect by a method given by Berndt in ref. 7. No further corrections were made for tunnel wall effects because no accurate methods were found for delta wings with leading-edge flow separation.

The scale effect has been investigated at 8 deg. angle of attack at 3 different wind speeds and at different yaw angles, see table I. As might be expected for this sharp-edged configuration, the Reynolds number appeared to have no significant effect on the magnitude of the forces and moments.

3.4. Flow visualization tests

The surface oil-flow technique was used to visualize the flow over the surface of the wing. Photographs were taken at the angles of attack and yaw mentioned in table I. The flow about the model was explored with the help of a tuftrod which was inserted through the wind-tunnel wall. In addition, the wind-tunnel air humidity was increased artificially through which at a sufficiently low pressure and temperature condensation took place in the core of concentrated vortices. Photographs taken of the condensation trails thus formed were used to determine the location of the vortex core and breakdown point.
4. UNYAWED WING

4.1. General

The aerodynamic feature which distinguishes sharp-edged delta wings from other wings is the vortex layer separation from the leading edge. As shown in fig. 4, the boundary layer separates already at small angles of attack at the sharp leading edges (primary-separation lines). The position of the primary-separation lines is fixed which results in the advantage that the Reynolds number has no important influence on the magnitude of the forces and moments. At the suction side of the wing, the separated boundary layer forms two coiled vortex layers which roll up to cores of high vorticity. These so-called leading-edge or primary vortices are constantly fed by flow separating from the leading edges, so that they grow more or less linearly in size and in strength from apex to trailing edge. This results in a more or less conical vortex flow. The strong vortices induce on the wing a strong outward flow in spanwise direction. The boundary layer thus formed will separate again (secondary-separation line) as a result of an adverse pressure gradient caused by the presence of a small vortex situated near the leading edge. This secondary vortex has a direction of rotation which is opposite to that of the primary one. Near the trailing edge the primary and secondary vortices deflect towards the free-stream direction. The vortices are no longer conical there due to this trailing-edge effect.

4.2. Flow observations

4.2.1. Upper-surface flow pattern

By using the oil-flow technique it was possible to visualise the symmetric flow pattern over the upper-surface (suction side) of the delta wing. This pattern, sketched diagrammatically in fig. 5, is in general characterized by the following regions separated by rays from the apex:
(i) a region in the middle with attached streamwise flow
(ii) a region between attachment line and secondary-separation line where vortex induced air flows outwards in spanwise direction. The projection of the vortex axis onto the wing surface is indicated by a so-called peak-suction line
(iii) a region between secondary-separation line and leading edge where the airflow is influenced by the secondary vortex which rotates into a direction opposite to that of the primary vortex.

4.2.2. Influence of the angle of attack on the flow above the wing

The upper-surface oil-flow patterns at 4,12 and 20 deg. have been photographed. As illustrated in fig. 6, from each photograph a diagram was reproduced showing the position of the characteristic flow pattern lines. The diagrams for 4,12 and 20 deg. are shown in fig. 7.

An increment of the angle of attack leads to an extension of the region between attachment line and secondary-separation line. This is a consequence of the fact that strength and size of the leading-edge vortices increase with angle of attack, as has been measured by Harvey, ref. 9. In addition, the attachment line, peak-suction line and secondary-separation line move towards the centerline. This must be a result of an inward movement of the vortices. The attachment lines for both halves of the wing coincide at the centerline at about $\alpha = 16$ deg.

The above is also illustrated in fig. 8, where the spanwise position of the characteristic oil-flow pattern lines has been plotted versus angle of attack. This figure holds good from apex to about $0.85 c_o$ (the beginning of the chamfered trailing edge). Over this part of the wing the lines are straight, which may indicate that the vortex flow is conical there.

4.3. Estimated static-pressure distribution

Since in the present model no pressure tubing was installed it was
impossible to measure pressure distributions. Therefore, in order to examine what pressure distribution causes the measured forces and moments this pressure distribution is estimated as follows. In ref. 11 Lemai re found a relation between the upper-surface flow pattern and the spanwise distribution. This relation is illustrated in fig. 9. The pressure is constant in the central streamwise-flow region. Between attachment line and secondary-separation line the pressure drops to have its minimum value at the peak-suction line, that is directly under the vortex axis. Outboard of the peak-suction line an adverse pressure gradient is encountered by the cross flow which forces this flow to separate at the secondary-separation line. From there up to the leading edge the pressure remains almost constant.

The influence of the angle of attack on the spanwise pressure distribution can be derived from experimental results of refs. 11 and 12. With the help of these results and of fig. 8 it has been possible to estimate the spanwise pressure-distributions to be expected at 4, 12 and 20 deg. (fig. 10).

The pressures at the lower surface of the model have not been drawn because the pressure coefficients at this surface are small compared to those at the upper surface.

4.4. Forces and moments

4.4.1. Lift, drag and pitching moment

Results of balance measurements are given in table II. Lift, drag and pitching-moment coefficients have been plotted versus angle of attack in figs. 11 to 13.

The lift coefficient curve is non-linear as a result of the presence of the leading-edge vortices. At an angle of 20 deg. \( C_L = 0.77 \) about 30 percent of the lift is generated by these vortices.

Owing to the increase of the vortex strength with angle of attack span-
wise pressure distributions will be induced as illustrated in fig. 10. In figs. 11 to 13 the coefficients are compared with those for flat-plate delta wing 8A of ref. 10. This wing has also an aspect ratio of 1, but its thickness ratio \( t/c_0 \) is 0.01 instead of 0.03 for the wing tested here. The differences between the coefficients for both wings increase with angle of attack and are possibly due to the differences in thickness. The normal- and tangential-force coefficients \( C_N \) and \( C_T' \), and \( \arctan(C_N/C_T) \) - the angle between the resultant force on the wing and its projection onto the wing surface - are presented in table II. \( C_T' \) (positive backwards) forms the balance between the backwards directed component of the profile drag and the forward directed suction component induced by the vortices on the chamfered leading edges. \( C_T \) has its largest positive value in the region \(-4 < \alpha < 4 \text{ deg.}\); the influence of profile drag dominates there. With increasing angle of attack \( C_T \) decreases as a result of the increasing suction forces. At 16 deg. and beyond, this coefficient is negative, hence the tangential force is directed forwards. From 4 to 20 deg. the \( \arctan(C_N/C_T) \) is almost 90 deg., which means that the resultant force is almost perpendicular to the wing when leading-edge vortices are formed.

4.4.2. Centre of pressure

The position of the centre of pressure for a given angle of attack was calculated from the following equation:

\[
\frac{x_p - x_0}{c} = - \frac{C_m}{C_N}
\]

where \( x_0 = 0.25 \ c \)

Results are given in table II and illustrated in fig. 14; the centre of pressure moves forward (towards the apex) with increasing angle of attack.
4.4.3. **Aerodynamic centre**

The position of the aerodynamic centre also depends on the angle of attack. Its chordwise position was calculated from the following equations, derived in ref. 13:

a. for \( \alpha_{C_N}=0 \approx \alpha_{C_L}=0 \) : 
\[
\frac{x_{ac}-x_o}{c} = - \frac{dC_m}{dC_N}, \text{ and}
\]

b. for \( \alpha \neq \alpha_{C_L}=0 \) : 
\[
\frac{x_{ac}-x_o}{c} = - \frac{1}{C_N} (C_m - \dot{C}_{mac})
\]

where \( C_{mac} = (C_m)_{C_N=0} \)

Results are given in table II and fig. 14. When the angle of attack increases from 0 to 20 deg. the aerodynamic centre moves forward only from \(.43 \overline{c} \) to \(.39 \overline{c} \) (or: from \(.62 \overline{c} \) to \(.59 \overline{c} \)).
5. YAWED WING

5.1. General

When the delta wing model is yawed the flow about it will change due to:

(i) displacements of the leading-edge vortices in a spanwise direction
    and in a direction normal to the wing surface
(ii) changes in the nature of the leading-edge vortices.

5.2. Flow observations

5.2.1. Vortex displacements with sideslip

To investigate the influence of sideslip on the height of the vortices
above the wing the location of the cores were visualized by increasing
the wind-tunnel air humidity (condensation trails) and by using a tuftrod.
The vertical distance between vortex axis and wing was estimated at the
trailing edge for 4 and 12 deg. angle of attack. The results are given
in fig. 15.
The surface oil-flow patterns of the yawed wing at 4,12 and 20 deg.
angle of attack have been reproduced diagrammatically from photographs,
in figs. 16 to 18. These diagrams make it possible to determine the
influence of sideslip on the spanwise position of the attachment, peak-
suction and secondary-separation lines. This was done in fig. 19 for
chordwise station 0.80 c_o.

Figs. 15 and 19 help to give an idea about the displacements of the
vortices. The following conclusions can be drawn:

a. The vortex above the windward side of the wing draws nearer to the
   upper surface when the angle of sideslip is increased (angle of
   attack constant). Its spanwise position remains almost unaltered.
   Only at 4 deg. angle of attack an outward displacement was observed
(see fig. 19a). This may be due to the influence of the chamfered leading edge.

b. The vortex above the leeward side of the wing increases its distance normal to the upper surface with angle of sideslip. At the same time it moves towards the leeward edge and will eventually be outside the wing at angles of sideslip greater than 16 deg.

The above is consistent with the results of refs. 9 and 14.

5.2.2. Changes in the nature of the vortices due to sideslip

By using a tuftrod and by increasing the wind-tunnel air humidity some insight could also be obtained of the influence of sideslip on the nature of the leading-edge vortices. Clearly visible condensation trails are formed as soon as the pressure and temperature in the core are sufficiently low. Harvey's experiments (ref. 9) show that this is the case if a vortex is strong and concentrated.

At the upper surface of the delta wing the following was observed:

a. The vortex at the windward side remained clearly visible, strong and concentrated at all angles of sideslip considered, while the one at the leeward side became invisible already at a small angle of sideslip. This is consistent with the results of ref. 9 where it was found that the strength of the vortex at the windward and leeward side, respectively, increases and decreases with increasing angle of sideslip.

b. At large angles of sideslip and attack vortex breakdown took place above the windward side of the wing. This is shown in the photograph of fig. 20. Upstream of the breakdown point the condensation trail was clearly visible, downstream of it the trail expanded in a radial direction and became rapidly invisible. This is due to the fact that the vortex becomes turbulent as soon as the vortex core breaks down. At zero sideslip both leading-edge vortices break down symmetrically.
at about the same chordwise station. In this case the breakdown point reaches the trailing edge at an angle of attack of about 36 deg. (see ref. 9, fig. 37).

Yawing results in an important upstream movement of the breakdown point which now reaches the trailing edge at a far lower angle of attack: at 20 deg. already when the delta wing is yawed 11 deg. (fig. 21). The breakdown points at 20 deg. angle of attack are also indicated in the oil-flow pattern diagrams of fig. 18 by a $\Theta$ sign. It can be seen in fig. 18c that the radial expansion of the vortex downstream of the breakdown point leads to an extension of the region between attachment and secondary-separation line. Experiments carried out by Lambourne and Bryer (ref. 8) have shown that this results from a reduction of the adverse pressure-gradient experienced by the cross flow under the vortex, and from a delay in separation due to increased turbulence.

5.3. Estimated pressure distribution

Pressure measurements carried out on yawed delta wings of aspect ratio 1 have been reported in refs. 10 and 15. Fig. 22 shows the spanwise pressure distribution measured in ref. 10 on a biconvex delta wing (wing C) at 20.48 deg. angle of attack and at angles of sideslip of 0 and 5 deg. Fig. 23 gives the results of ref. 15 for a delta wing having a flat upper surface. The angle of attack is 26.4 deg. and the angles of sideslip are 5 and 15 deg. Vortex breakdown took place at 15 deg. slip angle.

From these figures the following conclusions can be drawn:

a. The vortex-induced peak suction above the windward side of the wing increases with angle of sideslip. This is in accordance with the fact that an increasing angle of sideslip leads to:

i. an increase in vortex strength

ii. a reduction of the distance between vortex axis and wing upper-surface.
According to ref. 15, the peak suction has approximately the same magnitude from apex to 0.50 \( c_0 \), further aft they gradually reduce to zero at the trailing edge.
As soon as vortex breakdown occurs, the pressure drops strongly downstream of the breakdown point (fig. 23).

b. The peak suction above the leeward side of the wing decreases when the angle of sideslip is increased. This is due to:

i. a decrease of the vortex strength
ii. an increase of the distance from vortex axis to wing upper-surface.

In addition, the vortex will move outside the wing at angles of sideslip larger than about 16 deg.

The pressure peaks gradually reduce from apex to trailing edge (fig. 23).

As mentioned in 4.3 and illustrated in fig. 9, a relation exists between the spanwise pressure distribution and the characteristic oil-flow pattern lines. Based on fig. 19, it is possible to reproduce the qualitative spanwise pressure distributions for various angles of sideslip by utilizing the above-mentioned conclusions. This was done in figs. 24 to 26 for 4, 12 and 20 deg. angle of attack, respectively. A strong pressure drop can be seen in fig. 26 above the windward half of the wing at slip angles larger than 11 deg. This is due to vortex breakdown. It is, however, possible that the pressure there is also reduced by the fact that the vortex above the rear half of the wing will deflect into a direction of the free tunnel stream before it has reached the trailing edge.

5.4. Forces and moments

5.4.1. Lift, drag and pitching moment

Lift, drag and pitching-moment coefficients of the yawed wing are given
in table III and are plotted in figs. 27 to 29.

Up to and including 12 deg. angle of attack the lift and drag coefficients show no important changes with sideslip. This may imply that the decrease in suction above the leeward half of the wing is compensated by the increase in suction above the other half. The pitching moment decreases at slipangles larger that 10 deg. This is thought to be a result of the fact that the vortices deflect into free stream direction even ahead of the trailing edge. As a consequence the suction on the rear part of the wing will decrease and the centre of pressure will shift forward (fig. 30). At 20 deg. angle of attack the coefficients decrease strongly when the slipangle is larger than 11 deg. This is mainly a result of vortex breakdown. The breakdown point moves upstream with increasing angle of sideslip (and attack), thereby extending the region with reduced pressure coefficients into the direction of the apex. Consequently, lift and drag decrease with increasing sideslip, while the pitching moment obtains a less negative value.

5.4.2. Centre of pressure

At 4, 12 and 20 deg. angle of attack the position of the centre of pressure was calculated for several angles of sideslip. The following equations were used:

\[
\frac{x_p - x_0}{c} = -\frac{C_m}{C_N} \quad (x_0 = 0.25 c)
\]

\[
y_p = \frac{2(C_L \cos \alpha - C_n \sin \alpha)}{b/2} \quad \frac{1}{C_N}
\]

Results are given in table IV and illustrated in fig. 30.

The lateral displacement of the centre of pressure is a result of the asymmetric spanwise pressure distributions of the yawed wing (figs. 24 to 26). The lateral displacement is largest for 4 deg. angle of attack. For this angle the distance from the greatest suction peak to the root chord appears to be the largest.
5.4.3. Lateral force, rolling and yawing moment

5.4.3.1. General

Lateral force, rolling- and yawing-moment coefficients are given in table III and have been plotted versus angle of sideslip in figs. 31, 34 and 36. At small angles of attack strong non-linearities are visible, especially in the range $-5 < \beta < 5$ deg. This is probably caused by tail-strut-wake interference effects because such non-linearities did not show up during recent measurements performed on a delta wing of aspect ratio 1 which was suspended through a central strut only. These measurements were carried out at the Department of Aerospace Engineering (ref. 16). The derivatives of the coefficients due to sideslip are given in table V and have been plotted versus angle of attack and lift coefficient $C_L$ in figs. 32, 35 and 37.

5.4.3.2. Lateral force

At zero deg. angle of attack the lateral force of the yawed delta wing corresponds to the $y$-component of the profile drag. As illustrated in fig. 31, the lateral force is then negative for a positive angle of sideslip which results in a negative sideslip derivative $C_{y\beta}$ (fig. 32). At 4 deg. angle of attack and beyond, the lateral force forms the resultant in $y$-direction of the profile-drag component and of the suction forces acting on the chamfered edges, see fig. 33. The influence of the suction forces makes the lateral force positive for a positive angle of sideslip, so that $C_{y\beta}$ is then positive as well. The lateral force changes linearly up to about 10 deg. angle of sideslip, at larger angles the relation becomes non-linear due to the influence of leading-edge vortex displacements. Vortex breakdown takes place above the wing at 20 deg. angle of attack when the angle of sideslip is larger than 11 deg. Its influence on the lateral force is small (fig. 31).
5.4.3.3. Rolling moment

The rolling moment of the yawed wing is generated by the asymmetric vortex-induced pressure distribution above the wing. As can be seen in fig. 34, this moment is very small at zero deg. angle of attack. At larger angles the strong suction above the windward half of the wing generates a negative moment with positive sideslip, so that \( C_{\beta} \) is negative. The rolling moment increases linearly with angle of sideslip up to about 10 deg., at larger angles this moment is slightly affected by vortex displacements. Strong irregularities appear as soon as the vortex breaks down above the wing, particularly at 20 deg. angle of attack.

\( C_{\beta} \) has been drawn in fig. 35 and is compared there with the sideslip derivative of flat-plate (\( t/c_0 = 0.01 \)) delta wing model 8A of ref. 10. The rolling-moment derivative for the wing tested here is slightly larger than that of thinner wing 8A. This is explained by the fact that the leading-edge vortices above the tested wing are situated farther outboard due to the influence of the chamfered edges. Through this, the suctions induced by them act at a greater moment arm.

5.4.3.4. Yawing moment

The yawing moment due to sideslip, plotted in fig. 36, is the balance between a contribution from the suction component, induced by the vortices on the chamfered leading edges, and one from the drag component.

At \(-4,0\) and 4 deg. angle of attack there is a negative, destabilizing yawing moment with positive sideslip, mainly as a result of the position of the profile drag resultant relative to the \( z_s \)-axis. \( C_{n\beta} \) is thus negative, see fig. 37.

At angles of attack of 7 deg. and beyond, the yawing moment becomes positive and stabilizing due to the influence of the drag and the suction on the chamfered leading edges. \( C_{n\beta} \) is then positive.

Compared to the \( C_y - \beta \) and \( C_{\lambda} - \beta \) curves, the \( C_n - \beta \) appears to be least influenced by vortex displacement and vortex breakdown.
6. **COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS**

In appendix A the linear theoretical methods are discussed of Gronau (ref. 18), Gersten and Hummel (ref. 23), and the USAF Stability and Control Handbook (ref. 24). With help of these methods the sideslip derivatives of delta wings can be predicted. The methods are based on theories which start from wings having round leading edges and which assume that the flow remains attached. As a consequence, they are only valid for angles where the leading-edges vortices are absent or very weak, that is for angles of attack and sideslip smaller than 4 deg. ($C_L < 0.11$).

The sideslip derivatives for the present delta wing calculated with the help of the three abovementioned methods are given in table V and are plotted together with the experimentally determined derivatives in figs. 39 to 41.

For small angles ($\alpha < 4$ deg.) the following conclusions can be drawn:
- the differences between the experimental and the theoretical $C_{y\beta}$ are lowest ($\approx 0.01$) when the method of ref. 24 is used.
- $C_{y\beta}$ is quite well predicted by the methods of refs. 18 and 24.
- the methods of refs. 18, 23 and 24 all predict a $C_{n\beta}$ which is about .01 more positive than the experimental one.

At larger angles, the theoretical and experimental sideslip derivatives deviate due to the influence of leading-edge vortices.
7. CONCLUSIONS

a. The experimental results agree with those of other measurements.
b. At small angles \( (\alpha < 4 \text{ deg.}) \) the sideslip derivatives determined by linear theories show only small differences (of the order of .01) with those determined experimentally.

At larger angles, the differences increase.
c. As far as the model is concerned its, cross-sectional shape and thickness ratio are thought to have an important influence on the location of the vortices and consequently on the forces and moments. This implies, among others, that chamfered leading edges are to be avoided for future models. To avoid tailstrut-wake interference effects, the model is to be suspended through one strut only.

In order to understand what causes the measured forces and moments it is worthwhile to provide a future delta wing model with pressure tubes.
8. REFERENCES

1. E.C. Polhamus  
   A concept of the vortex lift of sharp-edged delta wings based on a leading-edge suction analogy.  

2. H.C. Garner  
   D.E. Lehrian  
   Non-linear theory of steady forces on wings with leading-edge flow separation.  
   ARC R&M 3375, 1963.

3. A.H. Sacks  
   J.M. Neilson  
   F.K. Goodwin  
   A theory for the low speed aerodynamics of straight and swept wings with flow separation.  
   Vidya rept. 91, 1963.

4. K.W. Mangler  
   J.H.B. Smith  
   Calculation of the flow past slender delta wings with leading-edge separation.  
   RAE Farnborough, Rept. 66970, 1966.

5. R.K. Nangia  
   G.J. Hancock  
   A theoretical investigation for delta wings with leading edge separation at low speeds.  
   ARC CP No. 1086, 1970.

6. A.G. Parker  
   Aerodynamic characteristics of slender wings with sharp leading edges - A review.  

7. S.B. Berndt  
   Wind tunnel interference due to lift for delta wings of small aspect ratio.  

8. N.C. Lambourne  
   D.W. Bryer  
   Some measurements in the vortex flow generated by a sharp leading edge having
65 degrees sweep.
ARC CP NO. 477, 1960.

9. J.K. Harvey
Some measurements on a yawed slender delta wing with leading-edge separation.
ARC R&M No. 3160, 1958.

10. D.H. Peckham
Low-speed wind-tunnel tests on a series of uncambered slender pointing wings with sharp edges.

11. D.A. Lemaire
Some observations of the low-speed flow over a sharp edged delta wing of unit aspect ratio.
ARL'A 126, Melbourne, 1965.

12. D.J. Marsden
The flow over delta wings at low speeds with leading edge separation.
R.W. Simpson
W.J. Rainbird

13. O.H. Gerlach
Lecture notes on stability and control.

14. J.D. Bird
Tuft-grid surveys at low speeds for delta wings.

15. D. Hummel
Untersuchungen über das Aufplatzen der Wirbel an schlanken Deltaflügeln.
DLR FB 64-12, 1964.
<table>
<thead>
<tr>
<th></th>
<th>Author</th>
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<tbody>
<tr>
<td></td>
<td>Delft University of Technology, Department of Aerospace Engineering, Delft, 1976.</td>
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<tr>
<td>17</td>
<td>J. Weissinger</td>
<td>Der Schiebende Tragflügel bei gesunder Strömung.</td>
</tr>
<tr>
<td>18</td>
<td>K.H. Gronau</td>
<td>Theoretische und experimentelle Untersuchungen an schiebenden Flügeln, insbesondere von Pfeil- und Deltaflügeln.</td>
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<tr>
<td></td>
<td>Jahrbuch 1956 der WGL, pp. 133-150.</td>
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</tr>
<tr>
<td>19</td>
<td>M.M. Munk</td>
<td>The aerodynamic forces on airship hulls.</td>
</tr>
<tr>
<td>20</td>
<td>R.T. Jones</td>
<td>Properties of low-aspect-ratio pointed wings at speeds below and above the speed of sound.</td>
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<tr>
<td>21</td>
<td>H. Ribner</td>
<td>The stability derivatives of low-aspect-ratio triangular wings at subsonic and supersonic speeds.</td>
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<tr>
<td></td>
<td>NACA TN 1423 (1947)</td>
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<tr>
<td>22</td>
<td>T. Nonweiler</td>
<td>Theoretical stability derivatives of a highly swept delta wing and slender body combination.</td>
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</table>
23. K. Gersten
D. Hummel

Untersuchungen über den Eindfluss der Vorderkantenform auf die aerodynamischen Beiwerten schiebenden Pfeil- und Deltaflügel von kleinen Seitenverhältnis.

24. D.E. Ellison et al


25. T.A. Toll
M.J. Queijo

Approximate relations and charts for low-speed stability derivatives of swept wings.

26. M.J. Queijo

Theoretical span load distributions and rolling moments for sideslipping wings of arbitrary plan form in incompressible flow.

27. E.C. Polhamus
W.C. Sleeman

The rolling moment due to sideslip of swept wings at subsonic and transonic speeds.
NACA TN D-209 (1960)

28. W. Wohlhart
D. Thomas

Static longitudinal and lateral stability characteristics at low speeds of 45° swept-back-midwing models having wings with an aspect ratio of 2, 4 or 6.
NACA TN 4077 (1957).

29. A. Goodman
J. Brewer

Investigation at low speeds of the effect of aspect-ratio and sweep on static yawing stability derivatives of untapered wings.
NACA TN 1669 (1948).
30. A. Goodman
   D.F. Thomas

   Effects of wing position and fuselage size on the low-speed static and rolling stability characteristics of a delta wing model.

31. W. Letko

   Experimental determination at subsonic speeds of the oscillatory and static lateral stability derivatives of a series of delta wings with leading-edge sweep from $30^\circ$ to $86.5^\circ$.
   NACA RM L57A30 (1957).

32. C. Mc. Cormack
    W. Walling

   Aerodynamic study of a wing-fuselage combination employing a wing swept-back $63^\circ$. Investigation of a large-scale model at low speed.
   NACA RM A8D02 (1949).
Appendix A: THEORETICAL LINEAR METHODS TO PREDICT THE DERIVATIVES DUE TO SIDESLIP

A.1. Extended lifting-line theory

Weissinger (ref. 17) modified Prandtl's lifting-line theory and developed a vortex model for straight wings shown in fig. 38a. Gronau (ref. 18) extended this theory and applied it to swept wings and delta wings with round leading edges. The vortex model he used is illustrated in fig. 38b. The bound portion of each individual horse shoe vortex is parallel to the wing centre line and its centre lies on the quarter-chord line. The free vortices behind the trailing edge are assumed not to be influences by sideslip but always to lay parallel to the centre line. This assumption together with the fact that the flow is thought to be attached implies that his theory can only be applied to a delta wing with sharp leading edges when its angle of attack and sideslip are small.

Gronau determines first the spanwise circulation distribution

\[ \gamma = \gamma_s + \gamma_\beta \cdot \beta \]

where \( \gamma_s \) is the symmetrical distribution at zero sideslip, and \( \gamma_\beta \) its partial derivative due to sideslip.

Then he calculates with the help of his vortex model the spanwise lift distribution.

The rolling moment is obtained by the integration of the lift distribution:

\[
\text{rolling moment} = -q \int_{-b/2}^{b/2} c_L \cdot c \cdot y \, dy
\]

For its derivative due to sideslip the following expression is derived:

\[ C_{L\beta} = - (I_1 + I_2 + I_3 + I_4 + I_5) \frac{C_L}{2} \]
$I_1$ to $I_5$ denote the contribution of wing taper ratio, tip shape, sweep angle, neutral point displacement and the position of the bound lifting-line, respectively. In ref. 19, table 6, Gronau gives $I_1$ to $I_5$ as computed for delta wings with different aspect ratios. For a delta wing of aspect ratio 1,

$$
I_1 = -0.71 \\
I_2 = 1.13 \\
I_3 = 1.03 \\
I_4 = -0.18 \\
I_5 = -0.04
$$

so that $C_{\ell \beta} = -0.615 C_L$ (per rad)

As can be seen in table V and fig. 40, the theoretical $C_{\ell \beta}$ is in good agreement with the experimental one up to about $C_L = 0.11$ ($\alpha = 4$ deg.). At larger $C_L$, the theoretical and experimental derivatives deviate due to the influence of the leading-edge vortices.

The yawing moment is calculated by the integration of the asymmetrical distribution of induced drag:

$$
yawing \ moment = q \int_{-b/2}^{b/2} c_d \cdot c \cdot \gamma \ dy
$$

The influence of profile drag is neglected.

For the derivative due to sideslip Gronau derives the equation:

$$
C_{n\beta} = -\frac{C_L^2}{A} \left[ \frac{C_{\ell \beta}}{C_L} \frac{1}{R} + \left( \sqrt{1 + \frac{4}{A^2}} \int_{-1}^{1} f \cdot \gamma_s \cdot \gamma_{\beta} \cdot \eta \ d\eta \right) \frac{2}{R^2} \right]
$$

where $R = \frac{C_L}{A}$ and $f = \frac{2b}{k \ c_{L_{\infty}}}$

$k = \text{local chordlength}$

$c_{L_{\infty}} = \text{the increase of the local lift for a wing with infinite span}$
For a delta wing of aspect ratio 1, $C_{\alpha \beta} = 0.490 \ C_L^2$ (per rad).
The theoretical $C_{\alpha \beta} - C_L$ curve lies above the experimental one, see fig. 41 and table V. Below $C_L = 0.11 \ (\alpha = 4 \ \text{deg.})$, the differences between both curves are small ($\approx 0.01$); they may be a consequence of the fact that this theory neglects the influence of profile drag.

A.2. Slender-body theory

This theory, originally developed by Munk (ref. 19), has been applied by Jones (ref. 20) to slender wings, and by Ribner and Nonweiler (ref. 21 and 22) to wings subjected to sideslip.
Gersten and Hummel extended the slender-body theory in ref. 23 to predict sideslip derivatives of uncambered flat-plate delta wings having sharp leading edges. Their vortex model, shown in fig. 38c, is based on the assumption that the flow about a sideslipping delta wing is attached and is thought to be corresponding with the flow field about the yawed wing on which a constant lateral velocity parallel to the wing $Y$-axis is added. This constant lateral velocity does not influence the bound vortices of the wing because the velocity at the upper- and lower surface is changed to the same degree. The free vortices which flow from the trailing edge should leave this in a direction more or less parallel to that of the free stream. This is, however, neglected in the vortex model used by Gersten and Hummel; the free vortices are assumed to be parallel to the zero-sideslip flow direction.
This simplification and the assumption that the flow is attached implies that also this theory is only valid for small angles of attack and sideslip.

With the help of the slender-body theory the local pressure-coefficients are calculated and through these the rolling moment of the wing. For its derivative due to sideslip it was derived that

$$C_{\alpha \beta} = - \frac{\pi \alpha}{3} \left( \frac{1 + 2 \lambda}{1 + \lambda} \right) \ (\text{per rad})$$
For the delta wing used in the present investigation,

\[ C_{n\beta} = -0.693 \, C_L \text{ (per rad)} \]

Fig. 40 illustrates that the theoretical \( C_{n\beta} - C_L \) curve lies below the experimental curve. Discrepancies are small up to \( C_L = 0.11 \) (\( \alpha = 4 \) deg.), but increase with larger \( C_L \).

The yawing moment is determined by the relation \( C_n = C_{n\beta} \cdot \alpha \). From this it follows that

\[ C_{n\beta} = C_{n\beta} \cdot \alpha \]

\[ = - \frac{1}{3} \pi \, \alpha^2 \left( \frac{1+2\lambda}{1+\lambda} \right) \text{ (per rad)} \]

For the delta wing used in the present investigation, \( C_{n\beta} = 0.458 \, C_L^2 \) (per rad). The theoretical \( C_{n\beta} - C_L \) curve lies, as can be seen in Fig. 41, above the one determined experimentally. Up to \( C_L = 0.11 \) (\( \alpha = 4 \) deg.), the discrepancies are small and probably due to thickness influences (flat-plate wing assumed by theory).

Another assumption made by Gersten and Hummel is that the pressure differences at the leading edges of an unyawed flat-plate wing are infinitely large and that consequently the air flows around the leading edges at infinitely large speeds. When the wing is yawed a finite lateral speed is added; this will however have no influence on the flow around the leading edges. Thus, the suction forces of the yawed wing are of the same magnitude as those of the unyawed wing. From this results that \( C_Y = 0 \) and \( C_{Y\beta} = 0 \).

Due to the influence of thickness the above theoretical assumption does not hold for the tested delta wing which has a thickness ratio \( t/c_o = 0.03 \). \( C_{Y\beta} \) is not equal to zero (figure 39 and table V). It is, however, in good accordance with the test results of flat-plate delta wing 8A (\( t/c_o = 0.01 \)) of ref. 10.
(ref. 24)  

A.3.1. Sideslip derivative $C_{Y\beta}$  

The Datcom method for $C_{Y\beta}$ was taken from ref. 25 where a strip theory and lifting-line theory is applied to constant-chord swept wings in sideslip to determine approximate relations for sideslip derivatives. This method is also only valid when the flow is attached. The following expression is given for the lateral-force derivative:  

$$C_{Y\beta} = C_L^2 \left[ \frac{6tg\Lambda sin \Lambda}{A(\Lambda+4cos \Lambda)} \right] \text{ (per rad)}$$  

where $\Lambda$ is the quarter-chord line sweepangle. For the tested delta wing, $\Lambda = 71.6 \deg$, so that $C_{Y\beta} = 2.400 C_L^2 \text{ (per rad)}$.  

As can be seen in fig. 39 and tabel V, the differences between the theoretical and experimental $C_{Y\beta}-C_L$ curves are small up to $C_L = 0.11$ ($\alpha = 4 \deg$). At larger $C_L$, the differences increase rapidly.  

A.3.2. Sideslip derivative $C_{L\beta}$  

The Datcom method for this derivative is a combination of the methods used by ref. 21 for low aspect-ratio wings and used by ref. 17, 26 and 27 for high aspect-ratio wings. For intermediate aspect ratios the experimental data of ref. 28 to 33 are used as a guide in constructing a faired curve between the slender-body values of ref. 21 and the high aspect ratio values of ref. 17, 26 and 27. The method is valid for sideslip angles between -5 and 5 deg., and for low angles of attack.  

The following equation is given for the rolling-moment derivative due to sideslip for a delta wing of aspect ratio 1:
\[ C_{\beta} = 57.3 \ C_L \left[ \left( \frac{C_{\beta}}{C_L} \right)_{\Lambda_c/2} \cdot K_{M_A} + \left( \frac{C_{\beta}}{C_L} \right)_A \right] \text{ (per rad)} \]

where

\[ \left( \frac{C_{\beta}}{C_L} \right)_{\Lambda_c/2} \]

is the wing-sweep contribution

\[ K_{M_A} \]

is the compressibility correction to the sweep contribution

\[ \left( \frac{C_{\beta}}{C_L} \right)_A \]

is the aspect-ratio contribution

For the tested delta wing it follows that:

\[ \left( \frac{C_{\beta}}{C_L} \right)_{\Lambda_c/2} = -0.0048 \]

\[ K_{M_A} \approx 1.0 \]

\[ \left( \frac{C_{\beta}}{C_L} \right)_A = -0.0056 \]

so that \[ C_{\beta} = -0.0104 \ C_L \text{ (per deg)} \]

\[ = -0.596 \ C_L \text{ (per rad)} \]

Up to \( C_L = 0.11 \) (\( \alpha = 4 \text{ deg.} \)), the theoretical \( C_{\beta} \) corresponds very well with the experimental one (fig. 40 and tabel V).
A.3.3. Sideslip derivative $C_{n\beta}$

The Datcom method is based on the theory of ref. 25 where the yawing-moment derivative is given as

$$
C_{n\beta} = C_L^2 \left[ \frac{1}{4\pi A} - \frac{\tan A}{\pi A(1+4\cos A)} \left( \cos A - \frac{A}{2} - \frac{A^2}{8\cos A} + 6 \frac{x}{c} \frac{\sin A}{A} \right) \right]
$$

(per rad)

$A$ is the quarter-chord line sweepangle

and $\bar{x}$ is the longitudinal distance between the center of gravity and the wing aerodynamic center.

For the delta wing tested here:

$$
C_{n\beta} = (0.32372 - 0.04188 \frac{x}{c}) C_L^2 \quad \text{(per rad)}
$$

The $C_{n\beta} - C_L$ curve thus determined lies, as can be seen in tabel V and fig. 41, above the experimentally obtained curve, even below a $C_L = 0.11$ ($\alpha = 4$ deg.). However, the discrepancies are small there.
Table I: Summary of test conditions

<table>
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<tr>
<th></th>
<th>α(deg.)</th>
<th>β(deg.)</th>
<th>V(m/sec)</th>
<th>Re x 10^6 (based on c)</th>
<th>Remarks</th>
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<td>Balance measurements</td>
<td>-4 to 20 (steps of 4)</td>
<td>-5 to 20</td>
<td>80.3</td>
<td>3.57</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0,10 and 20</td>
<td>80.3</td>
<td>3.57</td>
<td>investigation scale effect</td>
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<td></td>
<td></td>
<td></td>
<td>62.6</td>
<td>2.78</td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td>39.6</td>
<td>1.76</td>
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<td>Flow visualization tests</td>
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<td>0 to 21</td>
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<td>oil-flow method</td>
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<tr>
<td></td>
<td>16, 20 and 24</td>
<td>8 to 21</td>
<td>88.1</td>
<td>3.92</td>
<td>tuft rod, increased humidity</td>
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Table II: Unyawed-wing coefficients

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<tr>
<th>α (deg)</th>
<th>$C_L$</th>
<th>$C_D$</th>
<th>$C_N^*$</th>
<th>$C_T^*$</th>
<th>$\arctan\left(\frac{C_N}{C_T}\right)$ (deg)</th>
<th>$C_m$</th>
<th>$\frac{x_d}{c}$</th>
<th>$\frac{x_{ac}}{c}$</th>
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*) $C_N = C_L \cdot \cos \alpha + C_D \cdot \sin \alpha$

$C_T = C_D \cdot \cos \alpha - C_L \cdot \sin \alpha$
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<th>$C_D$</th>
<th>$C_M$</th>
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### Table III (continued)

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Table IV: Centre of pressure co-ordinates, yawed wing.

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Table V: Experimental and theoretical derivatives due to sideslip.

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Fig 1: General arrangement of model set-up
Tunnel dimensions in m.

Cross section

1.80

1.25

2.60

Side view
Fig. 2: Delta wing model (dimensions in m). A and B are the positions of the central hinge and tail strut hinge.
$O_X, O_Y$ body axes
$0_sX_s, 0_sY_s, 0_sZ_s$ stability axes

Fig. 3: Positive sense of various quantities in the body and stability axes systems
Fig. 4: Separated flow above the wing at a moderate angle of attack.
Fig. 5 : Upper-surface flow pattern at moderate angle of attack.
Fig. 6: Diagrammatical presentation of the upper-surface oil-flow pattern ($\alpha = 12$ deg).
Fig. 7: Upper-surface flow patterns at different angle of attack.
Fig. 8  The spanwise position of the flow pattern lines.
Fig. 9: The relation between the upper-surface flow pattern and its spanwise pressure distribution.
Fig. 10: The estimated spanwise pressure distribution at the upper-surface of the wing.
Fig. 11: Lift coefficient ($\beta = 0$ deg)
Fig. 12: Drag coefficient (β = 0 deg)
Fig. 13: Pitching-moment coefficient (β = 0 deg)
Fig. 14: Influence of angle of attack on the position of the aerodynamic center and the center of pressure
Fig. 15: Variation of distance between vortex-axes and trailing-edge with sideslip.
Fig. 16: Upper-surface flow patterns at 4 deg angle of attack.
Fig. 18: Upper-surface flow patterns at 20 deg angle of attack.

- a. $\beta = 20$ deg
- b. $\beta = 16$ deg
- c. $\beta = 21$ deg

$\odot$: vortex core breakdown location.
Fig 19: Influence of sideslip on the position of the characteristic oil-flow pattern lines.

symbols: • spanwise position peak-suction line
        △ spanwise position sec. separation line
        ○ spanwise position attachment line
Fig. 20: Vortex-core breakdown at 80% C₀
(α = 24 deg., β = 9 deg.)
Fig. 21: The position of the breakdown point above the windward side of the wing as function of sideslip.
Fig. 22: Spanwise pressure distribution at 0.50 c_o (wing C, ref. 10)
Fig. 24: Estimated spanwise pressure distribution at chordwise station 0.85 $c_p$ for $\alpha = 4$ deg.
Fig. 25: Estimated spanwise pressure distribution at chordwise station 0.85 \( c_o \) for \( \alpha = 12 \text{ deg} \).
Fig 26: Estimated spanwise pressure distribution at chordwise station 0.85 $c_0$ for $\alpha = 20$ deg.
Fig. 27: Lift coefficient as function of sideslip.
Fig. 28: Drag coefficient as function of sideslip.
Fig. 29: Pitching moment coefficient as a function of sideslip.
Fig. 30: Influence of sideslip on the position of the center of pressure.
Fig. 31: Lateral-force coefficient as function of sideslip.
Fig. 32: $C_{Y\beta}$ as function of $\alpha$, and $C_L$. 
Fig. 33: Lateral forces $Y_1$ and $Y_2$, induced by the asymmetric spanwise pressure distribution on the chamfered edges of the yawed wing.
Fig. 34: Rolling-moment coefficient as function of sideslip
Fig. 35: $C_{L_\beta}$ as function of $\alpha$, and $C_L$, compared with results of ref 10.
Fig. 36: Yawing-moment coefficient as function of sideslip.
Fig. 37: $C_n$ as function of $\alpha$, and $C_L$. 
a. Weissinger (ref. 17)
Lifting-Line Theory

b. Gronau (ref. 18)
Extended Lifting-Line Theory

c. Gersten and Hummel (ref. 23)
Slender-body Theory

Fig. 38: Vortex models.
Fig. 39: $C_{Y_\beta}$ : Comparison of theoretical and experimental results
Fig. 40: $C_{1\beta}$: Comparison of theoretical and experimental results.

Fig. 41: $C_{n\beta}$: Comparison of theoretical and experimental results.