PARTITIONED STRATEGIES FOR OPTIMIZATION IN FSI

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Abstract. In this paper the possibility of the optimization of coupled problems in partitioned approaches is discussed. As a special focus, surface coupled problems of fluid-structure interaction are considered. Well established methods of optimization are analyzed for usage in the context of coupled problems and in particular for a solution through partitioned approaches. The main benefits expected from choosing a partitioned solution strategy as basis for the optimization are: a high flexibility in the usage of different solvers and therefore different approaches for the single-field problems as well as the possibility to apply well tested and sophisticated methods for the modeling of complex problems.

1 INTRODUCTION

Most realistic applications in structure optimization involve several disciplines simultaneously. Prominent examples in the civil engineering context are large membrane structures such as tents. These structures are highly optimized with respect to self-weight, but at the same time extremely susceptible towards wind loadings. An example for such a single-field optimization is the analysis of the membrane roofing of the central station in Dresden. Here the membrane roof is a wide spanned structure and was subject to optimization through form finding (Fig.1, left side). However, the analysis of wind loadings on the roof, here as a first step conducted with static pseudo wind loading, shows unfortunate strong deformation and wrinkling behavior (Fig.1, right side).

So for optimizing the design of such light constructions, not only the load carrying behavior, but also their dynamic behavior due to wind effects need to be considered.

The scope of this research project is to develop a tool for the optimization of structures within multi-field problems, with a special focus on surface coupled fluid-structure interaction. For the analysis of the multi-field problem a partitioned, fully implicit coupling scheme is chosen: the fluid and the structure are modeled separately, exchanging the necessary information at their boundaries. The structure is described via the finite element method, using a Lagrangian description, whereas the fluid description is based on
the Navier-Stokes equations. As optimization methods, different approaches are analyzed towards adaptability for a partitioned analysis.

2 FLUID-STRUCTURE INTERACTION (FSI)

Typically, different physical fields are interacting simultaneously in technical-scientific problems. Usually, it is the task of an engineer to analyze the coherence between these fields and to reduce the problem to simpler models by finding appropriate assumptions. However, in the case of strongly coupled physical fields, simplifications imply the risk of neglecting essential effects resulting from these interactions.

For the solution of coupled problems, different strategies are suitable:

In a so-called monolithical approach all physical fields are solved in one single numerical model. Due to the realization of these approaches by experts from one specific field, many monolithical approaches analyze this field by a complex model while a simplified model is used to simulate the other fields. Over the last few years, interdisciplinary formulations have been developed integrating sophisticated models for all fields involved.

However, with respect to most real-world applications, coupled simulations need to consider multiple factors and be able to appropriately cover complex problems. To fulfill these requirements, here a partitioned approach is chosen: both physical fields are separately modeled and numerically solved [9, 13, 17]. One of the advantages of a partitioned approach is that without significant changes, specialized, well tested and highly adapted simulation codes can be applied in both fields. The coupling of the fields is ensured by the
transfer of corresponding values at the common interface. Moreover, the discretization of
the interface in both fields needs not necessarily to coincide ("non-matching grids") and
thus both fields can be solved using the most advantageous discretization.

With the presented approach, problems of fluid-structure interaction (FSI) describing
the fluid induced deformation of structures and the flow around this structure are modeled
as multi-physics problem. The involved fields are identified as: the fluid field, describ-
ing the flow around the interface and thereby constituting loads on the interface; and
the structural field, solving the structural response on the applied loadings and thereby
changing the boundaries of the fluid calculation. Therefore, in the procedure of a FSI sim-
ulation, the fluid and the structural simulation are computed in a staggered algorithm,
shown in Fig. 2. The interface between the two physical fields is resembled by the surface
part of the structure which is wetted by the fluid. This states a surface coupled problem.

The following subsections provide details about the solution methods for the individual
physical fields and how to establish a coupling simulation.

2.1 Computational Structural Analysis (CSD)

The structural problem is characterized by the governing equations for an elastic body
with large deformations (and the corresponding boundary and initial conditions): equilib-
rium (1), material law (3), and kinematics (4). The acceleration vector vanishes and the
first equation reduces to the static equilibrium condition (2), if only stationary problems
are considered.

\[
\rho_0 \cdot \ddot{u} = \text{DIV}(F \cdot S) + \rho_0 \cdot b
\]

(1)

\[
0 = \text{DIV}(F \cdot S) + \rho_0 \cdot b
\]

(2)

\[
S = C \cdot E
\]

(3)

\[
E = \frac{1}{2} (F^T \cdot F - I)
\]

(4)

Where \( \rho_0 \) is the density in the reference configuration, \( S \) the second Piola-Kirchhoff
stress tensor, \( b \) the body forces, \( F \) the deformation gradient, \( E \) the Green-Lagrange strain
tensor, and \( C \) the elasticity tensor. These equations are solved with the finite element
method. For this purpose the Lagrangian description is used, i. e. the nodes of the
corresponding mesh are fixed to the material points.

The structures of interest in the scope of form optimization are thin-walled and flexible
and are therefore described by either membrane or shell theory. The computation of shell
problems is done with so-called 7-parameter solid shell elements, which assume a linear
strain distribution over the thickness [3, 15]. These models are able to represent three-
dimensional effects, while pertaining the efficiency of a two dimensional formulation due
to a pre-integration of the structural stiffness matrix across the thickness. The specified
shell model is implemented in the in-house research finite element code CARAT under
consideration of well-known methods like ANS, EAS and DSG to avoid locking effects.
In addition to the aforementioned element CARAT provides other efficient and robust
structural finite element types. For the simulation of membrane structures triangular or
quadrilateral membrane elements with three degrees of freedom per node are used.

The non-linear equations during the statical analysis are solved by the Newton-Raphson
method. Several state-of-the-art time integration algorithms, including the generalized-α
method, exist to solve dynamical problems in the code.

2.2 Computational Fluid Analysis (CFD)

The viscous fluid flow is described by the governing Navier-Stokes equations which state
the conservation of mass and momentum. In the scope of this paper, an incompressible
fluid with constant properties is assumed. The governing equations are:

\[
\rho \left( \frac{\partial U_j}{\partial t} + U_i \frac{\partial U_j}{\partial x_i} \right) = -\frac{\partial P}{\partial x_j} + \mu \frac{\partial^2 U_j}{\partial x_i \partial x_i} \\
\frac{\partial U_i}{\partial x_i} = 0
\]  

(5)  

(6)

with \( U_j \) as the velocity component in the \( j \)-direction, \( x_i \) as the Cartesian coordinate in
the \( i \)-direction, \( P \) as the pressure, \( \mu \) as the dynamic viscosity, and \( \rho \) as the density of the
fluid. Adequate boundary conditions are describing wall, inflow, outflow, and symmetry
type boundaries. For cases of moving grids, such as occurring for the deformation of
a FSI interface during a coupled simulation, the arbitrary Lagrangian Eulerian (ALE)
approach is applied. The adaptation of the CFD grid to the updated boundary conditions
is performed by solving a diffusion problem.

In the present work, the CFD calculations are performed by the commercial Compu-
tational Fluid Dynamics Package CFX-5. CFX-5 solves the 3D Navier-Stokes equations
on structured and unstructured grids for compressible and incompressible flows. For the
simulation of turbulent flows several advanced turbulence models are available, includ-
ing Reynolds Averaged Navier-Stokes Models (RANS), the Shear-Stress-Transport (SST)
Model, Large Eddy Simulations (LES), and Detached Eddy Simulations (DES). An ad-
vanced multigrid solver capable of parallelization is applied for efficient computing.

2.3 Bringing together CSD and CFD

The simplest form of a partitioned algorithm is the explicit coupling: the coupling
quantities are only once exchanged every time-step. However, explicit coupling schemes
provide only limited numerical stability for the simulation, especially in problems involving incompressible fluids, large deformations and very light and flexible structures [8, 9].

Here an implicit coupling scheme is applied in which the coupling quantities are exchanged within one time-step, until a kinematic equilibrium for this time-step is reached (Fig. 2) [19]. Thereby, a high numerical stability and accuracy can be reached. However, the use of simple implicit coupling schemes does not guaranty unlimited stability. Furthermore, implicit coupling schemes are likely to lead to very extensive computational effort. By applying under-relaxation techniques on the coupling quantities as a method of stabilization, the stability as well as the efficiency of a coupled computation can be improved without modifying the physics of the problem.

\[
V_{i+1} = V_i + r_{i+1} \cdot \Delta V_{i+1} \quad \text{with} \quad \Delta V_{i+1} = \tilde{V}_{i+1} - V_i \tag{7}
\]

\(V\) is a variable the under-relaxation is applied to, \(\tilde{V}\) is the non-under-relaxed variable, \(r\) is the under-relaxation-factor, and \(i\) is the number of the coupled iteration step within one time-step. Small under-relaxation factors provide a good behavior of the coupled simulation towards stability for the price of a high computational effort. Therefore a reasonable choice for the parameter \(r\) is required. Following the work of Mok and Wall [22] in this approach the Aitken method is applied.

Despite the fact of two separate representations of the same structure in both the fluid- and the structural code, a common geometric model is necessary. This model is created...
and updated on the structural side. Any changes of the geometry, e.g. deformations or modifications in the scope of optimization are included in the this central geometric model. The geometric representation in the fluid model is updated accordingly.

In the case of individually discretized surface meshes on the interface in the fluid- and the structural domain, the data transfer between these non-coinciding meshes needs additional attention [12]. In the current approach for non-matching grids, interface loads and displacements are interpolated using the coupling library MpCCI.

2.4 Examples of coupled computations

In the following, two short examples are presented to show the performance of the presented FSI-approach:
The behavior of an aluminium half-cylinder under wind loading is analyzed in a quasi 2D-setup. The aluminium structure has a thickness of 1 cm, a radius of 5 m and is modeled by enhanced shell elements. The fluid simulation uses a RANS-Turbulence model and the coupling algorithm is a steady-state implicit solution. Figure 3 shows the pressure distribution and the deformation of the cylinder in the reference state and under the effect of wind loading [6].

Figure 3: Comparison: Pressure distribution around the undeformed half-cylinder and the deformed half-cylinder due to wind loading.

In order to show the ability of the approach to simulate complex systems, the wind-membrane interaction of a four-point tent structure is analyzed. The geometry of the structure itself is the result of an optimization process, here the form finding procedure [4]. For the structural simulation, membrane elements without any bending stiffness are
applied, for the fluid simulation the SST-Turbulence model is included. Figure 4 presents the deformation and pressure loading of the structure for its most susceptible orientation towards the wind [27].

Figure 4: Deformation of and pressure distribution on the four-point tent structure.
3 SHAPE OPTIMIZATION OF SINGLE-FIELD PROBLEMS

Shape optimization is a very complex task combining most diverse aspects of mechanics, mathematics and finite element technology. Before approaching the topic of optimization in coupled problems, here different optimization strategies for single field problems are presented which can be applied to the fields of computational structural Mechanics (CSD), computational fluid dynamics (CFD) and the special area of form finding of membrane structures [16, 23].

3.1 Shape optimization in Computational Structural Mechanics

CAGD-based shape optimization

State of the art in the field of structural mechanics is the separation of the geometrical model and the calculation model. The geometrical model is based on CAD or CAGD and the structural model is the discretized version of the CAD/CAGD model. The design variables for the optimization process are the geometry parameters of the CAGD-model. Thus, high numerical efficiency is reached because of a low number of optimization variables.

As an example for a CAGD based shape optimization the shell structure depicted in Fig. 5 is chosen. The geometry is modeled by the use of Bezier Patches, whereas not more than 10 variables are needed as optimization variables, making use of the structural symmetry. The result shows the optimum shape within the possible domain for maximum stiffness [1, 5, 16, 23].

CAGD-free shape optimization

A disadvantage of optimization based on CAGD-models is that by the choice of the geometric design parameters also a restriction of the design space and possible solutions is made. These restrictions can be overcome, if the nodal coordinates of the discretized version of the model are used as optimization parameters. Thus all possible solutions can be achieved. Furthermore, ceasing the conscious choice of the right optimization parameters, modeling and setup are simpler and faster. However, parameter-free optimization
leads to a very high number of parameters. In the case of numerically noisy results leading
to irregular and mesh dependent results, the resulting shape must be carefully controlled
by appropriate methods [10]. Figure 6 shows the results of a parameter free optimization
towards the maximization of stiffness of a circular plate under constant area load with
and without shape control.

Figure 6: Parameter-free optimization without (left) and with (right) shape control

Semi-Analytical Sensitivity Analysis

The computation of sensitivities is necessary for gradient based optimization strategies
like steepest descent or conjugate gradient methods. Here the derivative of the objective
with respect to the design variables is denoted as sensitivity. There are many methods
available to compute the gradient information, e.g. analytical, semi-analytical and nu-
merical approaches.

As an example the derivative of the displacement field with respect to a design variable $s$
is considered which is necessary for nearly all objective functions:

$$\frac{d\mathbf{u}}{ds} = -K^{-1} \left( \frac{\partial K}{\partial \mathbf{u}} \right).$$

(8)

For the semi-analytical sensitivity analysis the analytical derivative of the stiffness ma-
trix is substituted by a finite difference approximation. First order methods are frequently
used due to their efficiency. This yields to:

$$\frac{d\mathbf{u}}{ds} \approx -K^{-1} \left( \frac{K(x_i + \Delta s) - K}{\Delta s} \mathbf{u} \right).$$

(9)

Obviously the perturbation parameter $\Delta s$ is small but finite. Therefore truncation
errors occur due to neglecting higher order terms in the taylor series expansion. These
approximation errors have serious influence on the accuracy of the sensitivity analysis,
c.f. [2]. The source of the accuracy problems in the semi-analytical sensitivity analysis is
the so called rigid body condition for the derivative of the stiffness matrix:

$$\mathbf{\phi}^T \frac{\partial K}{\partial s} \mathbf{\phi} = 0 \quad \mathbf{\phi}^T \frac{\Delta K}{\Delta s} \mathbf{\phi} \neq 0$$

(10)
Only the analytical derivative satisfies the rigid body condition exactly. The semi-
analytical derivative introduces errors which disturb the accuracy of the result. For several
types of finite elements this sensitivity error affects the gradient information dramatically.
The basic goal of the proposed correction method is to derive a modified stiffness matrix
derivative which satisfies the rigid body condition, eq. (10). The general idea is to modify
the stiffness matrix derivative by another matrix based on the dyadic product of element
rotation vectors. For the 3-d case this yields to the formulation:
\[
\phi^T_r \left( \frac{\Delta K}{\Delta s} + a^{jk} \phi^r_j \phi^k_r \right) \phi^l_r = 0, \quad \forall i, j, k, l \in \{1..n_r\}, \quad \forall i, j, k, l \in \{1..n_r\}.
\]
(11)

with the number of rigid body rotation vectors \(n_r\) and the perturbation \(\Delta s\). In general
this formulation contains a system of equations which has to be solved. The size depends
on the number of dimensions and the orthogonality conditions between the rigid body
rotation vectors. Finally a modified approximation of the stiffness matrix derivative is
obtained which fulfills the rigid body condition:
\[
\phi^T_i \Delta K^* \phi^j_i = 0 \quad \forall i, j \in \{1..n_r\}
\]
(12)
For a detailed derivation of the proposed correction method is referred to [7].

Form Finding

Membrane structures use the material best since they act by definition in pure mem-
brane action and the inefficient bending effects are therefore not present. This special
mechanical property can be exploited for the design of optimal and lightweight struc-
tures. Due to this special load carrying behavior the internal forces are directly coupled
to the shape of equilibrium. In many practical applications the membrane structures must
be stabilized by prestress which is already acting on the undeformed configuration which
in turn is defined by this given stresses.

The art of finding the corresponding shapes to given stress distributions and bound-
ary conditions is called form finding. Mathematically spoken, this task to determine
equilibrium shapes is an inverse problem with the respective difficulties. This results in
singular expressions during the numerical solution of the equilibrium conditions which are
the governing equations of form finding. To overcome this problem, several form finding
methods were developed. Within this contribution, a regularization by a homotopy mapp-
ing is successfully used. This approach is called the updated reference strategy (URS)
which was initially developed for the computation of minimal surfaces [4] and more re-
cently extended to more general situations with almost arbitrary stress distributions [26]
which leads to a high flexibility in the definition of membrane shapes. In the latest devel-
opments it was enriched by an adaptive shape finding scheme to ensure convergence even
in physically problematic combinations of stress states and boundaries.
The basic idea of the URS is the modification of the originally singular virtual work expression \( \delta W_\sigma = 0 \) by adding a stabilizing part \( \delta W_S \) (in terms of \( PK2 \) stresses \( S \) rather than Cauchy stresses \( \sigma \)). These are formulated in such a way that they fade out as the solution is approached and the original, unmodified solution is received. The complete weak form of the stabilized form finding scheme (URS) which is solved by the isoparametric finite element method states as follows:

\[
\delta W_\lambda = \lambda \delta W_\sigma + (1 - \lambda) \delta W_S \\
= \lambda \left[ \int_A \det F (\sigma_{\text{pre}} : F^{-T}) : \delta F \, dA \right] \\
+ (1 - \lambda) \left[ \int_A (F : S_{\text{pre}}) : \delta F \, dA \right] = 0
\]  

(13)

This means that the applied shape parametrization is given by three displacements per node which results in a huge number of variables. The obviously more efficient way to use only a few design variables based on a CAD model is not suitable in the context of membrane form finding. At closer inspection of the shapes of equilibrium it turns out that they are free form surfaces. These mechanically defined shapes can therefore not adequately be described by analytical expressions of standard CAD models. An example for a computed minimal surface by applying the URS is shown in Fig. 7 which clarifies the complexity of the shapes under consideration.

3.2 Shape optimization in Computational Fluid Dynamics

Shape optimization in fluid dynamics aims at minimizing e.g. drag or lift coefficients of a certain structure within the fluid flow or the pressure distribution on a surface by changing the shape of the structure.

As in structural optimization, using a CAD/CAGD-based model for the geometry description of the structure again restricts the possible design space. Therefore, with the aim of optimizing free form structures, CAD/CAGD-based parametrization suffers from limitations.
So the main attention is currently in the development of tools able to optimize free forms without geometric restrictions, like the incomplete-gradient adjoint formulation [25]. This approach does not depend on a certain geometry representation, all grid points of the fluid boundary can be used as design parameters. Also an approximate solution is used to describe the adjoint formulation, so that the numerical effort is, compared to other approaches, relatively low.

The solution approaches are mostly based on gradient methods, whereas the adjoint sensitivity analysis is the most widely used [14, 21]. Problems appear in fluid mesh deformations and distortion of the surface mesh, smoothing functions are used as possible solution approaches.

4 SHAPE OPTIMIZATION OF COUPLED PROBLEMS

Within the scope of optimization of fluid-structure interaction problems all the above mentioned optimization types for single-field problems have to be taken into account. In a simplified manner the optimization problem of coupled systems can be stated as follows:

\[
\begin{align*}
  f(p, u, v) & \rightarrow \min; \quad p \in \mathbb{R}^S; u \in \mathbb{R}^n; v \in \mathbb{R}^m; \\
  g(p, u, v) & = 0 \\
  h(p, u, v) & = 0
\end{align*}
\]  

(14)

with \( f \) as the objective function of the coupled system and \( g \) and \( h \) as the governing equations of the structural and the fluid part. For simplicity side constraints (equality and inequality constraints) are not mentioned. The variable \( p \) describes the optimization parameters and the variables \( u \) and \( v \) describe the state variables of structure (displacements) and fluid (pressure and velocity). For the optimization gradient-methods are used, based on the fulfillment of the Karush-Kuhn-Tucker conditions (KKT). After the necessary linearization of the objective function and the governing equations it follows:

\[
\begin{align*}
  \tilde{f} = f_0 + f_p \Delta p + f_u \Delta u + f_v \Delta v & \rightarrow \min \\
  g_0 + g_p \Delta p + g_u \Delta u + g_v \Delta v & = 0; \quad (\cdot)_x = \frac{\partial (\cdot)}{\partial x} \\
  h_0 + h_p \Delta p + h_u \Delta u + h_v \Delta v & = 0;
\end{align*}
\]  

(15)

The partial derivatives of the nonlinear transient structure equations with respect to the optimization variables \( p \) (geometry of the structure) are well-known. Current research is in the field of determination of the partial derivatives with respect to \( v \) and \( u \).

The gradient based solution approaches are based on the direct and the adjoint sensitivity analysis. The direct method is used for problems with high number of objective functions and small number of variables. The adjoint method is used for problems with small number of objective functions and high number of variables.
The Lagrange function $L$ is given by

$$L = f + \lambda^T g + \mu^T h \rightarrow \text{min}$$

with $\lambda$ and $\mu$ as Lagrange-multipliers, which can be identified as adjoint variables. By use of the KKT-constraint the solution can be determined by variation of $L$, leading to:

$$\delta L = \left[ f_p + \lambda^T g_p + \mu^T h_p \right] \delta p + \left[ f_u + \lambda^T g_u + \mu^T h_u \right] \delta u$$
$$+ \left[ f_v + \lambda^T g_v + \mu^T h_v \right] \delta v + \delta \lambda^T g + \delta \mu^T h = 0$$

The adjoint variables can then be found by a sequential solution strategy applied onto the following highly nonlinear set of equations:

$$
\begin{pmatrix}
g_u^T & h_u^T \\
g_v^T & h_v^T
\end{pmatrix}
\begin{pmatrix}
\lambda \\
\mu
\end{pmatrix} =
\begin{pmatrix}
f_u \\
f_v
\end{pmatrix}
$$

As algorithms for the optimization, robust gradient and simple Newton Methods are suitable e.g. Gradient methods (steepest descent with simple step-size control). With these algorithms the authors made very good experiences to problems with strongly non-convex optimization.

Up to now, SQP-Methods did not prove to be very robust. Especially obtaining accurate local second order derivatives is difficult, especially in the case of additional inequality constraints. However, an attempt applying SQP-Methods of the Quasi-Newton family (like BFGS, DFP etc) will be analyzed.

The main benefits expected from choosing a partitioned solution strategy as basis for the optimization are: high flexibility in the usage of different solvers and therefore different approaches for the single-field problems as well as the possibility to apply well tested and sophisticated methods for the modeling of complex problems.

5 CONCLUSIONS

This paper presents the work in progress towards an optimization approach for the optimization of coupled problems concerning fluid-structure interaction. For the solution of coupled problems, a partitioned implicit approach including stability enhancement by under relaxation techniques is suggested. The performance and reliability of the partitioned approach is demonstrated in short examples.

The implementation of optimization strategies into the partitioned approach is subject to recent research. Difficulties concerning the computation of sensitivities, especially for the fluid domain, are to be expected and need to be overcome. Additional attention is required for the algorithms, controlling the deformation of the CFD-mesh. Due to the ALE-formulation used, the CFD-Mesh has to be adapted in order to incorporate the deformation of the interface while conserving a reasonable good mesh quality. Here, a method based on the force density approach is developed.
As a first application, the developed optimization approach will be applied to quasi-stationary problems, whose behavior in the steady state is independent from the initial configuration.

Still in an early state of development, the suggested promises a software tool for the optimization of coupled fluid-structure interaction problems. The advantages of this tool are due to its partitioned solution strategy both its flexibility and its ability to model complex real-world problems.

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