Theory and Numerical Analysis of Single and Multi-Element Nozzle Propellers

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SUMMARY

This paper presents the theoretical and numerical analysis that has been developed to predict the operating performance of nozzle, or ducted, propellers in axisymmetric flow. Surface vorticity techniques are used to model the nozzle (which may be multi-sectioned) and hub, together with lifting line theory for the propeller. The lifting line theory contains corrections for both three-dimensional and finite blade effects. A radial variation in axial inflow velocity is accommodated in the theory to allow systematic calculations of device performance in the wakefield behind a ships' afterbody. Performance prediction at bollard condition is also accommodated by using blade tip speed as a non-dimensionalising parameter instead of the advance velocity.

Considerable familiarity with previous nozzle propeller theory is assumed in Section 2, where a brief description of the entire flow model is presented. In Section 3 a detailed though not exhaustive summary is given of the numerical techniques employed, and reference [2] is a complementary report containing the computer program operating instructions and sample input/output listings.

Presented in Section 4 is a brief summary of the computed output performance parameters together with a comparison between the predicted characteristics and results obtained from model tests. Final conclusions and recommendations for further study are presented in Section 5.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section 1 - Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Section 2 - Theoretical Flow Model</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>3</td>
</tr>
<tr>
<td>2.2 Nozzle and hub</td>
<td>4</td>
</tr>
<tr>
<td>2.3 Slipstream</td>
<td>5</td>
</tr>
<tr>
<td>2.4 Propeller</td>
<td>6</td>
</tr>
<tr>
<td>2.4.1 Blade element theory</td>
<td>6</td>
</tr>
<tr>
<td>2.4.2 Lifting line theory</td>
<td>8</td>
</tr>
<tr>
<td>2.4.3 Cavitation</td>
<td>10</td>
</tr>
<tr>
<td>2.3.4 Slipstream strength</td>
<td>10</td>
</tr>
<tr>
<td>Section 3 - Numerical Techniques</td>
<td>12</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>12</td>
</tr>
<tr>
<td>3.2 Nozzle and hub pivotal point selection</td>
<td>13</td>
</tr>
<tr>
<td>3.3 Nozzle and hub coupling</td>
<td>15</td>
</tr>
<tr>
<td>3.4 Slipstream and body coupling</td>
<td>18</td>
</tr>
<tr>
<td>3.5 Propeller performance</td>
<td>19</td>
</tr>
<tr>
<td>3.6 Final output</td>
<td>20</td>
</tr>
<tr>
<td>Section 4 - Assessment of results</td>
<td>22</td>
</tr>
<tr>
<td>4.1 Open propeller performance</td>
<td>22</td>
</tr>
<tr>
<td>4.2 Nozzle and hub</td>
<td>22</td>
</tr>
<tr>
<td>4.3 Nozzle propeller performance</td>
<td>25</td>
</tr>
<tr>
<td>Section 5 - Conclusions</td>
<td>28</td>
</tr>
<tr>
<td>Section 6 - References</td>
<td>30</td>
</tr>
<tr>
<td>List of figures</td>
<td>32</td>
</tr>
</tbody>
</table>
SECTION 1

INTRODUCTION

This paper has been written as a summary of the theoretical model adopted by the author for the fluid dynamic performance analysis of nozzle propellers operating within an axisymmetric flow field. The majority of this work was undertaken in the late 1960’s and early 1970’s, with more recent additions to accommodate multi-element nozzle sections (slatted/slotted nozzles). A complete description of this work has not previously been published, and would probably have remained so had not a new interest in these devices recently emerged.

The theoretical model consists, essentially, of three parts: A surface vortex method is used to model both nozzle and hub, lifting line theory and thin aerofoil theory are combined with empirical corrections to predict the performance of the finite bladed propeller, and the slipstream is represented by a series of concentric semi-infinite vortex cylinders of constant strength and diameter originating from the propeller plane. Since the theoretical models of the nozzle, hub and slipstream are axisymmetric, the results of the propeller lifting line calculations are circumferentially averaged in order to determine the strength of the shed vorticity in the slipstream.

The model adopted is an extension of the actuator disc model described in reference [1] in which the actuator disc theory has been replaced by propeller lifting line theory. At the outset, a theoretical model was sought that was capable of accurately predicting the surface pressure distributions of both nozzle and hub together with a detailed flow analysis through the propeller and its slipstream for a full range of relevant advance velocities, including bollard condition. The combination of surface vorticity techniques for the nozzle and hub and lifting line theory for the propeller has allowed these objectives to be realised, albeit with some limitations. These are discussed within this paper and suggestions for improvements in the overall approach are contained in the concluding remarks.

As has been noted, the velocity field within which the propulsive system is situated is axisymmetric. This is not simply a limitation imposed by the adopted surface vorticity theory, which is real nevertheless, but stems from
the desire to produce a computational method that uses very little processing time -- a goal that is extremely important when a series of parametric design studies have to be undertaken. With an axisymmetric flow, the effects of the shed vorticity in the slipstream may be calculated from a closed form analytic solution rather than by a step by step integration along individual helical vortex sheets. Although the flowfield is axisymmetric, the adopted propeller theory does permit a radial variation in axial velocity into the propeller plane. This is particularly important in ship propulsion where the propulsor is inevitably located in the wake produced by a ships’ hull.

In the following sections is a description of the theoretical model adopted, a summary of the special numerical techniques, and a presentation of some of the results obtained from the computer program. A detailed description of the operation of the computer program is presented in the complementary report of reference [2]. In Section 2 of this paper is a detailed description of the theoretical model which is followed, in Section 3, by a summary of the adopted numerical techniques. The additional complications introduced by the extension of the basic model to multi-element aerofoil sections influence only the numerical techniques and these are described, as required, in Section 3. In Section 4 is a presentation and discussion of some results obtained from the computer program. And finally, in Section 5, are some concluding remarks and recommendations for further improvements in the method.
SECTION 2

THEORETICAL FLOW MODEL

2.1 Introduction

A brief description of the flow model and general approach to the calculation of nozzle propeller performance is given here in order to relate the analysis of the following sections to the overall method of approach. The model consists of an isolated ducted propeller in an inviscid axisymmetric flow at zero angle of attack. This is illustrated in Figures 1 and 2.

Both nozzle and hub are represented by continuous surface vorticity distributions, \( \gamma(s) \), and the propeller is modelled by lifting line theory with corrections for three-dimensional and finite blade effects. Both axial and radial velocities are considered to be continuous through the propeller plane. Since the blade loading and associated blade bound vorticity vary between one radial station and the next, constant diameter vortex cylinders are assumed to be shed between each radial station. Thus the propeller slipstream consists of a series of concentric vortex tubes of which the outermost is considered to be shed from the propeller blade tips. The strengths of these vortex cylinders are determined from the circumferentially averaged radial variation in propeller blade thrust.

The computer calculation proceeds as follows (see Figure 3): The geometry of the nozzle, hub and propeller is assumed known and may be input in either non-dimensional or dimensional form. The advance coefficient is selected and, if cavitation data is required, both propeller rotational speed and shaft centreline immersion depth are input. A small velocity increment is added to the axisymmetric approach flow to start off the calculation, and the performance of the propeller in this combined flowfield is determined. From these calculations the strengths of the trailing slipstream vortex cylinders are calculated, and hence the radial and axial interference velocities induced by the slipstream on the nozzle and hub surfaces. Once the nozzle and hub bound circulation has been determined, the induced axial velocity in the propeller plane is calculated and compared with the previous value. If the difference in velocities between iterations is less than one percent at each radial station then the calculation is considered to have converged. This
usually requires between three and six complete iterations of the program.

2.2 Nozzle and hub

Both nozzle and hub are represented by continuous surface distributions of ring vorticity, \( \gamma(s) \), which separate the outer flow from regions of motionless fluid inside the body surfaces. The solution is obtained by stating the Dirichlet boundary condition of zero tangential velocity for the interior surfaces of both nozzle and hub. This formulation leads to a set of equations in the form of a Fredholm integral equation of the second kind. As can be seen from Figures 1 and 2, this amounts to a summation of all the induced velocity components at each point, \( P^* \), around the body surfaces. This sum is then equated to zero:

\[
\frac{\gamma(s)}{2} + \oint \gamma(s').K(s^*,s').ds' + V_A \cos \beta + V_w = 0 \quad (2.1)
\]

The first and second terms of this equation represent the velocities induced by the nozzle and hub, and the third term represents the influence of the advance velocity \( V_A \). And \( \beta \) is the slope of the body profile at \( P^* \) as shown in Figure 1. The final term, \( V_w \), represents the induced velocity at \( P^* \) due cylindrical vortex slipstream.

\( K(s^*,s') \) is a coupling coefficient which relates a unit strength ring vortex at \( P' \) to its induced velocity tangential to the body profile at \( P^* \), and follows directly from the Biot-Savart Law (see Figure 4):

\[
K(s^*,s') = -\frac{1}{4\pi r'} \int_0^{2\pi} \frac{\{(1-r\cos \beta) - x \cos \theta \sin \beta \}.d\theta}{(x^2 + r^2 + 1 - 2r\cos \theta)^{3/2}} \quad (2.2)
\]

where \( r = \frac{r^*}{r'} \), and \( x = \frac{x^* - x'}{r'} \) \quad (2.3)

It is more convenient in the solution to replace the integral (2.2) by the axial and radial components of velocity it represents. Following the notation of Figure 3:

\[
K(s^*,s') = W_x \cos \beta - W_r \sin \beta \quad (2.4)
\]

where \( W_x \) and \( W_r \) can be expressed in terms of complete elliptic integrals of the first and second kinds as follows:

\[
W_x = -\frac{1}{2\pi r'} \frac{1}{\sqrt{x^2 + (x+1)^2}} \left\{ K(k) - \left[ 1 + \frac{2(r-1)}{x^2 + (x-1)^2} \right] E(k) \right\} \quad (2.5)
\]
\[ W_R = \frac{1}{2\pi r} \frac{1}{\sqrt{x^2 + (r+1)^2}} \left( K(k) - \left[ 1 + \frac{2r}{x^2 + (r-1)^2} \right] E(k) \right) \] (2.6)

where \( K(k) \) and \( E(k) \) are complete elliptic integrals, and the elliptic coefficient, \( k \), is given by

\[ k^2 = \frac{4r}{x^2 + (r+1)^2} \] (2.7)

It is evident from the above equations that as \( r \to 1 \) and \( k \to 0 \) the integral \( K(k) \) becomes infinite. This means that the axial velocity induced by a ring vortex upon itself cannot be evaluated using equation (2.5). The radial induced velocity is of course zero, and an alternative expression for the axial velocity may be derived by considering the ring vortex to have a finite core diameter. This expression may be written as:

\[ W' = \frac{1}{4\pi r'} \left( \ln \left( \frac{8\pi r'}{\delta s} \right) - \frac{1}{4} \right) \] (2.8)

Ultimately, by the introduction of trapezoidal integration around the body surfaces, equation (2.1) becomes just one of a set of simultaneous linear equations in the unknown surface vorticity \( \gamma(s) \). The solution of this system of equations yields surface velocities and hence surface pressure distributions directly.

The methods of pivotal point selection are presented in Section 3, and the slipstream induced velocities are dealt with in the next section.

2.3 Slipstream

Since propeller blade loading and bound vorticity vary radially, a vortex cylinder is assumed to be shed between each radial station across the propeller disc. Thus the propeller slipstream consists of a series of concentric vortex cylinders which remain constant in diameter and strength from their point of origin at the propeller plane to infinity downstream. For convenience of calculation the propeller plane is located at the origin in the \( x \)-direction and both nozzle and hub are positioned relative to this plane, as shown in Figure 2. Initially, therefore, we may consider the velocity induced by a semi-infinite vortex tube of unit strength and radius \( r' \). From reference [3], the radial and axial components of velocity induced at location \( P^* \) by a vortex tube of this type are given by:
\[ v_R = \frac{1}{\pi k^2} \cdot \frac{2}{\sqrt{\alpha^2 + (r+1)}^2} \cdot \left\{ E(k) - \left[ 1 - \frac{k^2}{2} \right] K(k) \right\} \quad (2.9) \]

\[ v_x = \frac{1}{2\pi} \left\{ A + \frac{x}{\sqrt{\alpha^2 + (r+1)}^2} \cdot \left[ K(k) - \frac{(r-1)}{(r+1)} \cdot \Pi(\alpha^2, k) \right] \right\} \quad (2.10) \]

where

\[
A = \begin{cases} 
\pi & \text{if } r^2 < 1 \\
\pi/2 & \text{if } r^2 = 1 \\
0 & \text{if } r^2 > 1 
\end{cases} \quad (2.11)
\]

\( \Pi(\alpha^2, k) \) is a complete elliptic integral of the third kind and the parameter \( \alpha^2 \) is defined as

\[ \alpha^2 = \frac{4r}{(r+1)^2}, \quad \alpha^2 \neq 1 \quad (2.12) \]

Resolution of equations (2.9) and (2.10) tangential to the body surface at point \( P \) then yields the influence of one vortex cylinder. Radial integration then yields the complete induced velocity, \( v'_w \), due to the entire slipstream:

\[ v_W = \int_{r_h}^{r_t} \eta(r). (v_x \cos \beta - v_r \sin \beta).dr \quad (2.13) \]

where \( r_h \) and \( r_t \) are propeller hub and tip radii, respectively. Calculation of the strength of this shed vorticity, \( \eta(r) \), follows in the next Section which concerns the propeller theory.

2.4 Propeller

The theoretical model for the propeller is based on lifting line theory, thin aerofoil theory for the blade elements, together with empirical corrections for three-dimensional and finite blade effects. The strength of the propeller slipstream is determined by circumferentially averaging the calculated propeller thrust.

2.4.1 Blade element theory

According to thin aerofoil (see reference [4]) the velocity at any point on the foil surface is made up of three components, as illustrated in Figure 5:

(i) the velocity components resulting from the displacement effect of the thickness distribution at zero angle of attack, \( (v/v_0) \);
(ii) the velocity increment resulting from the mean line at its' design angle of attack \( \alpha_{1d} \), \( (\Delta v/V_0) \);

(iii) the velocity increment occurring on the thickness distribution at angles of attack \( \Delta \alpha \), \( (\Delta v_a/V_0) \).

These velocity components are superimposed in order to obtain the resulting velocity distribution:

\[
v_r/v_0 = v/V_0 \pm \Delta v/V_0 \pm \Delta v_a/V_0
\]

where the positive signs refer to the suction side and the negative signs to the pressure side, for positive camber and angles of attack larger than the design angle of attack.

The corresponding pressure, \( p \), on the foil surface may be calculated using Bernoulli's theorem:

\[
\Delta p/q = (v_r/V_0)^2 - 1
\]

where \( \Delta p/q \) is the non-dimensional pressure drop over the foil section, and \( q \) is the local dynamic head.

Using thin aerofoil theory, it is possible to design both meanline and thickness distributions to obtain specific loading distributions along the aerofoil chord. The loading of the section being simply the difference in velocity between pressure and suction surfaces. For marine propeller blades, the most commonly selected camber distribution is that specified by the NACA a = 0.8 meanline, for which the loading is constant from the leading edge to 0.8 chord and then falls linearly to zero at the trailing edge. A major reason for selecting this meanline profile is that viscous flow corrections are small. For the thickness distribution, experience has shown that the NACA 16 series exhibit the most practicable characteristics for marine propellers.

With this selected meanline and thickness distribution, it is a simple matter to show that the design angle of attack, \( \alpha_{1d} \), zero lift angle of attack, \( \alpha_0 \), and camber-chord ratio, \( f/c \), are all proportional to the design lift coefficient, \( C_{L_d} \), and can be calculated directly from thin aerofoil theory as follows:

\[
f/c = 0.0679 C_{L_d}, \quad \alpha_1 = 0.02688 C_{L_d}, \quad \text{etc.}
\]

It is also easy to show that the maximum pressure drop for cambered foil sections at the design angle of attack is given by:

\[
\Delta p_{max}/q = (1 + 1.142 t/c \pm 0.278 C_{L_d})^2 - 1
\]

At other angles of attack, the maximum pressure drop is given
approximately by the equation:

$$\Delta p_{\text{max}} / q = (1 + (1.142 \pm 0.8 C_{\text{L}}) \frac{t}{c} \pm 0.25 C_{\text{L}})^2 - 1$$  \hspace{1cm} (2.18)

where the positive signs refer to the suction side and the negative signs to the pressure side, and $C_{\text{L}}$ is the section lift coefficient.

2.4.2 Lifting line theory

Lifting line theory rests on the simplifying assumption that the propeller blades are each replaced by a vortex line of radially varying strength. This radial variation in blade bound vorticity gives rise to the shedding of free vortex sheets, the direction of which must coincide with the direction of the resulting relative velocity in the slipstream. Thus the free vortex sheets will form helical surfaces which are pushed astern by the local axial velocities, rotate around the slipstream axis in the direction of the propeller rotation, and eventually roll up and diffuse into the surrounding flowfield.

For lightly loaded propellers ($C_{\tau} < 0.1$) it is possible to neglect the local velocities induced by the free vorticity in the slipstream, and the vortex sheets will thus form helical surfaces of constant pitch given by:

$$P_i = \pi D \tan \beta$$  \hspace{1cm} (2.19)

where $\beta$ corresponds to the advance angle of the propeller blade sections (see Figure 6).

For moderately loaded propellers ($0.1 \leq C_{\tau} < 2.0$) the influence of the slipstream induced velocities can no longer be neglected. However, the effects of slipstream contraction and pressure gradient within the slipstream can be regarded to cancel out. Fortunately the vast majority of marine propellers fall into this category, including propellers operating within a nozzle. And for this type of propeller it is found that the pitch of free vortex sheets coincides with the pitch of the relative velocities at the bound vortices or propeller plane:

$$P_i = \pi D \tan \beta_i$$  \hspace{1cm} (2.20)

Thus, for a moderately loaded propeller the free vortex system has constant pitch in the axial direction, i.e. a vortex element generated at a particular radius, $r$, will remain at this radius within the slipstream. However, the pitch of these vortex sheets may vary in the radial direction, depending on the magnitude of the locally induced velocities.

One further simplification is that it is possible to show that the
induced velocities at one particular blade bound vortex generated by the others are zero for evenly spaced propeller blades. In consequence, only the induced velocities generated by the free vortex sheets need be determined. Unfortunately, the pitch of the free vortex sheets depends on the induced velocities at the bound vortex and an iterative solution must be adopted. One such iterative solution is the induction factor of Lerbs [5], in which the induced velocities are calculated using the Biot-Savart law. The iterative solution adopted in this paper is semi-empirical in approach:

First of all, an initial guess is made at the magnitude of the induced axial velocity at the propeller plane and hence the inlet flow angle $\beta_i$, as shown in Figure 6. Since the blade pitch angle, $\alpha_i$, and the design angle of attack, $\alpha_{id}$, are known from the blade geometry, the effective angle of attack, $\alpha_e$, may be calculated from:

$$
\alpha_e = \alpha_n - \alpha_{id} - \beta_i
$$

(2.21)

The lift coefficient of the section is then given by

$$
C_{Lp} = 0.8 \ C_{L1} + 6.1 \ \alpha_e
$$

(2.22)

The propeller design equation for optimum homogeneous flow propellers, viz:

$$
C_{Lp} = 4 \pi x \frac{k}{Z} \sin \beta_i \ tan (\beta_i \ - \ \beta)
$$

(2.23)

yields a second expression involving the section lift coefficient which may then be used to assess the accuracy of $C_{Lp}$ as evaluated from equation (2.22). In equation (2.23) $x$ is the non-dimensional propeller radius $(r/R)$, $Z$ is the number of propeller blades, and $k$ is an empirical correction factor allowing for the effect of a finite number of blades, i.e. similar in effect to the Goldstein factor. If the two lift coefficients differ in magnitude more than a prescribed amount, the induced axial velocity is readjusted and another iteration is begun.

For a nozzle propeller, there is an additional axial velocity induced by the nozzle which is added to the axial velocities induced by the propeller in the plane of the propeller, as shown in Figure 6. The justification for this (see Reference [6]) is that the effect of the nozzle-induced velocity is limited to the immediate vicinity of the propeller. And whilst it must be taken into account in determining the blade circulation, the propeller-induced velocities should be calculated on the assumption that the pitch angle of the free vortex sheets, $\beta_i$, remains largely uneffected by the nozzle-induced
velocity.

Finally, then, the drag coefficient for the blade section is calculated from the empirical formula:

\[ C_D = 0.0035 + 0.017t/c + 0.0001z^2 + 6.2832 \left( 1 - \frac{0.75t/c}{0.057 + t/c} \right) \sin^2 \alpha \]  

(2.24)

2.4.3 Cavitation

Simply stated, if the minimum pressure on a blade surface falls below the vapour pressure, \( p_v \), of the surrounding fluid, cavitation phenomena may become detectable. In other words, cavitation can only be avoided if

\[ p_{\text{min}} > p_v \]  

(2.25)

Using Bernoulli's equation for the flow around a foil section, equation (2.25) may be rewritten as:

\[ \frac{(p_\infty - p_v)}{q} > \frac{\Delta p_{\text{max}}}{q} \]  

(2.26)

where \( \Delta p_{\text{max}}/q \) may be calculated from equation (2.18). The expression on the left is called the cavitation number, \( \sigma \), and is usually determined for a propeller blade in the vertical upright position. At each radius, \( r \), the local blade cavitation number, \( \sigma_x \), is given by the expression:

\[ \sigma_x = \frac{(p_a + \rho g(h - r)) - p_v}{q_o} \]  

(2.27)

where \( p_a \) is atmospheric pressure, \( g \) is the acceleration due to gravity, \( h \) is the propeller centerline immersion, and \( q_o \) is dynamic head of the undisturbed flow relative to the propeller blade section (see Figure 6).

2.4.4 Slipstream strength

In order to calculate the velocities induced by the propeller slipstream on the surface of both nozzle and hub it is necessary to circumferentially average the effect of the free vortex sheets in the slipstream. The reason for this is simply that the theoretical model for the hub and nozzle is limited to axisymmetric flow and, in consequence, there is nothing to be gained by integrating the effect of each vortex sheet independently. And since the propeller thrust is already known at each radius, the most direct approach to the calculation of slipstream strength is via Bernoulli's equation (see Figure 7):

Region A: \[ p_o + \frac{1}{2} \rho V_o^2 = p + \frac{1}{2} \rho V^2 \]  

(2.28)

Region B: \[ p_o + \frac{1}{2} \rho V_i^2 = p + \Delta p + \frac{1}{2} \rho V^2 \]  

(2.29)
Subtracting and rearranging yields the following expression for the velocity within the streamtube:

\[ \frac{1}{2} \rho V_i^2 = \Delta p + \frac{1}{2} \rho V_o^2 \]  \hspace{1cm} (2.30)

For the outer vortex tube, which corresponds to the tip radius of the propeller, the outer velocity \( V_o \) is identical to the advance velocity \( V_A \). The static pressure increment at the propeller disc, \( \Delta p \), is calculated directly from the propeller thrust:

\[ \Delta p \pi r^2 = \Delta T_p \]  \hspace{1cm} (2.31)

Finally, the vortex strength of the cylinder is given directly as the difference between the velocities on the inner and outer surfaces:

\[ \eta = V_i - V_o \]  \hspace{1cm} (2.32)
SECTION 3

NUMERICAL TECHNIQUES

3.1 Introduction

A brief description of the operation of the computer program is given here in order to relate the numerical techniques of the following sections to the overall design philosophy adopted.

The geometries of nozzle, hub and propeller are assumed known at the outset. In order to design a complete nozzle-propeller system from scratch, recourse must be made to either standard open water curves or to a simple computer program in order to produce the initial geometry. It is then possible to use the direct method, described herein, to make a systematic parametric study of all the geometric and flow variables.

The calculation method proceeds as shown in Figure 3: Both nozzle and hub profile geometry are input and the specified number of pivotal points around each profile are selected by interpolation and smoothing methods described below. The propeller geometry is also input in this initial section and both nozzle and hub are placed in the flowfield at locations specified by the relevant input parameters. Induced velocity coefficients relating each ring vortex around the body profiles are then calculated and stored in a square matrix. This matrix is ill-conditioned, and the subsequent rearrangement and modification are described below.

Next, the interaction coefficients between the propeller slipstream and the body pivotal points are calculated together with the axial velocity coefficients induced by the nozzle and hub in way of the propeller plane. Both sets of coefficients are then stored. A guess is then made at the magnitude of the nozzle induced velocity at the propeller plane, and the performance of the propeller calculated. This yields the strength of the vortex cylinders which represent the propeller slipstream and hence a Fredholm Integral Equation of the second kind is formulated from the prescribed surface boundary conditions. A solution of this system of linear equations, which is achieved by Gaussian Elimination, produces the surface vorticity distributions directly and an immediate calculation may then be made of the induced axial velocities in the propeller plane. A comparison of these velocities, at each radial blade location, with that of the previous iteration then forms the basis of the
convergence criterion.

The final output contains both nozzle and hub surface pressure distributions, propeller inflow velocity components, propeller performance characteristics at each radial location, slipstream characteristics and, finally, the overall performance of the propulsive system. A check on propeller blade cavitation margins may also be undertaken if required by the input data.

One particular feature of the computer program is perhaps worth mentioning: It is possible to select any nozzle/hub/propeller permutation through the input data, i.e. a calculation may be made for an open propeller, or an isolated hub, or a nozzle with a controllable pitch propeller at several different pitch settings, etc. A full description of these options is given in reference [2].

The following sections describe the operation of each process in the sequence described above, and Section 4 gives an indication of the range and accuracy of the computed output.

3.2 Nozzle and hub pivotal point selection

In order to improve the accuracy of the overall solution, and to simplify the task of pivotal point selection, it is convenient to apply a transformation so that the pivotal points are equally spaced around the body surfaces.

Let

\[ \int_{s_{\min}}^{s_{\max}} \gamma(s) \, ds = \int_{\theta_{\min}}^{\theta_{\max}} \gamma(\theta) \, d\theta \]

\[ 0 \leq s \leq S \]

\[ 0 \leq \theta \leq 2\pi \]

(3.1)

The 2M pivotal points, corresponding to a range of 0 to 2\pi in \theta, are arranged in equal intervals of \theta around the body profiles as illustrated in Figure 8, i.e.

\[ \Delta \theta = \text{constant} = 2\pi/2M \]

\[ \theta_i = (2i - 1)\pi/2M \quad 1 \leq i \leq 2M \]

(3.2)

As can be seen from Figure 8 the nozzle profile accounts for 2M pivotal points and the hub is modelled by a further M pivotal points with the same spacing \Delta \theta.

For a multi-element nozzle section it is no longer feasible to maintain the same pivotal point spacing, \Delta \theta, on the separate nozzle elements nor on the hub profile since the relative chord lengths may differ considerably. (This same observation may also be made in regard to the nozzle and hub chord
lengths, but normally the distance between them is large enough to avoid any undue numerical problems). To overcome this problem, the most effective expedient is to divide out the total available pivotal points (defined by the maximum specified array dimensions) amongst the number of elements in proportion to their respective chord lengths, as shown in Figure 9. This simply results in a different value of M for each nozzle element and, by use of the step length defined by equation (3.2), a value of Δθ that is specific to each nozzle element.

As shown in Figure 8 the above transformation represents the projection from points equally spaced around a circle onto the profile chord, and results in a concentration of points near the leading and trailing edges where surface curvature is large. Should further bunching of points be required, a second specification may be adopted, viz:

\[
\frac{x_i}{c} = \frac{1}{2} \pm \frac{1}{2\sqrt{1 + m^2 \tan^2 \theta_i}} \quad \text{where} \quad m = \frac{1}{1 + \frac{i}{M} (W^p - 1)}
\]

(3.3)

These equations represent the projection from points equally spaced around an ellipse, with chord as minor axis, onto the chord. From the chordal point a projection is then made back onto the body profile. If a series of ellipses are taken such that the major axis of the ellipse is a function of the chordal position then any desired distribution of chordal points may be obtained. Thus the parameter W above is the ratio of major to minor axis for the point \(i = 1\), and \(p\) is an index \(\geq 1\). The parameter \(c\) is a non-dimensionalising length, usually hub or nozzle chord length.

A further device used in reference [7] to bunch the points at the trailing edge only is given by:

\[
\left\{ \frac{x_i}{c} \right\}_1 = \frac{x_i}{c} \cdot \left\{ 1 + (1 - n) \cdot \left( 1 - \frac{x_i}{c} \right) \cdot \frac{x_i}{c} \right\}
\]

(3.4)

As \(n\) is reduced below unity the bunching of the chordal points increases.

The raw input data for the hub are input directly in \((x,r)\) coordinates, whereas the nozzle data are defined in terms of \((x,y)\) coordinates. The nozzle radial coordinates are subsequently obtained by adding the \(y\) coordinates to a suitable reference radius. From these data points the required pivotal point axial locations are selected by the transformation given by equation (3.2), and the radial coordinates are then obtained by five point Lagrangian interpolation. The interpolation process contains some smoothing but an
additional smoothing process is employed since the final solution may be sensitive to profile shape. This additional smoothing cycle selects each pivotal point coordinate in turn together with two adjacent coordinates on each side; the smoothed point is then obtained from a fitted least squares parabola. The improved point being given by:

\[
\bar{f}_0 = f_0 + \frac{3}{35} \left( -f_2 + 4f_1 - 6f_0 + 4f_{-1} - f_{-2} \right)
\]  

(3.5)

and the body profiles are looped several times until sufficiently smooth. A few points near the leading and trailing edges are frozen to prevent translation of the section.

The derivatives at the profile points with respect to the transformation variable \( \theta \) are found from a derivative of the Lagrange interpolation polynomial using five points:

\[
f_0' = \frac{1}{12H} \left( f_{-2} - 8f_1 + 8f_{-1} - f_2 \right)
\]  

(3.6)

where \( H \) is the step length.

All the above operations are performed with profile coordinates that are non-dimensionalised with respect to either hub chord or nozzle chord length depending on which profile is being processed. For multi-element nozzle sections each element is processed separately and the non-dimensionalising parameter is the axial chord length of the element itself; it is only at the end of the pivotal point selection, smoothing and differentiation processes that the element is redefined in terms of the overall nozzle chord length. A computer listing of the final smoothed points is produced in these coordinates if required.

The final profile manipulation process is the location of both hub and nozzle in their required positions in the flowfield and non-dimensionalisation with respect to propeller radius. For the nozzle a radial clearance between the propeller tip and inner surface is also included, as illustrated in Figure 10. A computer listing of these points is produced if required.

The next process uses these selected pivotal points to determine the coupling coefficients of each ring vortex at all points around the body surfaces.

### 3.3 Nozzle and hub coupling

At each point around the body profiles a Fredholm integral equation of the second kind is produced corresponding to equation (2.1) above. By
affecting the transformation into equal steps and noting that
\[ \cos \beta = \frac{-dx}{ds}, \quad \sin \beta = \frac{dr}{ds} \quad (3.7) \]
equation (2.1) becomes
\[
- \frac{\gamma(\theta)}{2} + \oint \left\{ W_x(-dx/d\theta) - W_r(dr/d\theta) \right\} \gamma(\theta') \, d\theta'
\]
\[
= - V_A(-dx/d\theta) - \int_{r_t}^{r_h} \left\{ V_x(-dx/d\theta) - V_r(dr/d\theta) \right\} \eta(r) \, dr \quad (3.8)
\]

The axial and radial velocities induced by a unit strength ring vortex, \( W_x \) and \( W_r \), are computed via the rapid approximation methods of Hastings [8] except where the ring vortex and the point of interest coincide. In this case the radial velocity component is zero and the axial component is given by equation (2.8):
\[
W'_x = \frac{1}{4\pi r} \left\{ \ln \left( \frac{8Mr'}{ds/d\theta} \right) - \frac{1}{4} \right\} \quad (3.9)
\]

By summing around the nozzle and hub surfaces, as shown in Figure 8, a system of simultaneous linear equations is obtained in the unknown surface vorticity \( \gamma(\theta) \). These equations may readily be written in double suffix notation as follows:
\[
A_{ij} \gamma_j = B_i \quad 1 \leq i \leq 3M \quad 1 \leq j \leq 3M \quad (3.10)
\]
where the summation limits are those corresponding to a single-element nozzle and a hub.

The left hand side coupling matrix \( A_{3M,3M} \) accounts for the nozzle-hub interaction effects and can be conveniently divided into four sections depending on the location of the ring vortex and the point of interest:
\[
A_{3M,3M} = \begin{bmatrix}
A_{1,1} & A_{1,2} & \ldots & A_{1,M} & A_{1,M+1} & \ldots & A_{1,3M} \\
A_{2,1} & HUB & ON & HUB & \ldots & HUB & ON \\
A_{M,1} & \ldots & A_{M,M} & \ldots & \ldots & \ldots & \ldots \\
A_{M+1,1} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
A_{3M,1} & A_{3M,M} & A_{3M,3M}
\end{bmatrix} \quad (3.11)
\]
The lower right hand quadrant represents the coupling between the nozzle ring vortices and the nozzle pivotal points, and it is in this section of the matrix that particular errors may be found to occur. These errors were first discussed by Wilkinson [7] in his study of two-dimensional aerofoils and cascades and they are produced by an overestimation of the velocity at point \( k \) by a vortex ring located at point \( 3M - k \) (where \( k > M \)), particularly for slender aerofoil sections. A correction for this error is derived from the condition of irrotationality around the interior surface of the nozzle, viz:

\[
A_{k,3M-k} = - \sum_{i=M+1}^{3M} A_{i,3M-i} \quad i \neq k \quad (3.12)
\]

This modification to the coupling matrix is known simply as the "correction of the back diagonal".

The system of equations (3.10), even after correction of the back diagonal, cannot be solved directly since there are an infinite number of solutions possible. A unique solution is obtained by specifying the Kutta condition at the nozzle trailing edge, as shown in Figure 8:

\[
\gamma(\theta)_{M+1} = -\gamma(\theta)_{3M} \quad (3.13)
\]

This boundary condition has been discussed at length in reference [9], and is accomplished by subtracting column \( 3M \) from column \( M+1 \) in the coefficient matrix. This permits the elimination of the last row and column from the matrix. Final solution of the \( 3M-1 \) by \( 3M-1 \) matrix is then accomplished via the direct method of Gaussian elimination and back substitution. During the elimination process the matrix \( A \) is transformed into its' upper triangular form and, for the iterative overall solution required, it is convenient to fill the remaining lower triangle with the multipliers derived during the first iteration. In subsequent iterations these multipliers and associated row interchange flags permit rapid solution by back substitution only, since only the right hand side changes during the iterative cycle (see reference [10]).

For multi-element nozzle sections the same principles are followed, the major differences being twofold:

(i) The back diagonal correction must be applied to those parts of the coupling matrix that involve the interaction between a ring vortex on one nozzle element and a pivotal point on the same nozzle element, and

(ii) a Kutta condition must be applied to each nozzle element to ensure 'smooth' flow from their respective trailing edges. This is
accomplished, as with a single-element nozzle, by subtraction of the appropriate inner surface columns from the corresponding outer surface columns in the coefficient matrix A. This process leaves not only redundant columns embedded in the overall coefficient matrix but also leaves an equivalent number of redundant rows, all of which must be removed before attempting a solution by Gaussian elimination.

The additional housekeeping exercises demanded by a multi-element nozzle section are small, and no problems have as yet emerged with the approach adopted above. In the computer program a maximum of four elements are permitted by the specified storage requirements, although the housekeeping sections of the program have been written to accommodate any number of elements.

3.4 Slipstream and body coupling

In the previous section it is evident that a solution of the system of equations (2.1) is dependent upon a known right hand side column matrix \( B_{3M} \). In practise, for a nozzle-propeller, an iterative cycle is used based on an initial guess of the strength of the shed vortex density within the slipstream. Rather than guess this wake strength directly, a more flexible approach is adopted that allows parametric design studies to be carried out more easily: This is based on an initial guess at the magnitude of the axial velocities induced by the nozzle and hub configuration in way of the propeller plane. From this initial guess may be calculated the performance of the propeller and hence the strength of the shed slipsream vorticity.

As noted in section 2.4.4 above, the strength of the shed vorticity in the slipstream is most easily calculated via the Bernoulli equation and the known propeller thrust at each propeller radius. In order to facilitate performance calculations at bollard conditions, all velocities are non-dimensionalised with respect to the propeller tip velocity, or its equivalent, \( \pi N D \). This means that at each radius within the propeller disc the pressure increment \( \Delta p(r) \), from equation (2.31), may be rewritten in terms of the propeller thrust coefficient \( K_{TP}(r) \):

\[
\frac{\Delta p(r)}{\rho N^2 D^2} = 4 \frac{K_{TP}(r)}{\pi x^2}
\]  

(3.14)

where \( x \) is the non-dimensional radius \( (r/R) \).

The axial velocity within the vortex cylinder at radius \( r \), \( V_1(r) \), is then calculated from equation (2.30):
\[ \frac{V_i^2}{\pi^2 N^2 D^2} = \frac{2 A p(r)}{\rho \pi^2 N^2 D^2} + \frac{V_o^2}{\pi^2 N^2 D^2} \]  

(3.15)

Since the calculation of these slipstream axial velocities commences from the propeller tip radius and proceeds inwards to the propeller hub, the velocity on the outer surface of the vortex cylinder, \( V_o \), is already known, and at the tip radius itself is equal to the advance velocity \( V_a \). Finally, then, the vortex strength of the cylinder is determined:

\[ \eta(r) = \frac{V_i}{\pi N D} - \frac{V_o}{\pi N D} \]  

(3.16)

The velocities induced by the shed vorticity around the surface of the body profiles may then be calculated from the final integral of equation (3.8). This is achieved by simple trapezoidal integration, and the axial and radial velocity components, \( V_x \) and \( V_r \), are calculated directly by the methods described in reference [3]. A computer listing is made of the slipstream vorticity if required.

3.5 Propeller performance

The propeller lifting line iteration method has been described in some detail in Section 2.4.2 above, the result of which is the determination of blade section lift and drag coefficients, \( C_L(r) \) and \( C_D(r) \), at each propeller radius. The corresponding elemental thrust and torque coefficients may then be determined by the equations

\[ \Delta K_{TP}(r) = \frac{Z}{4} \frac{V_i^2}{r} \frac{c}{D} \left\{ C_L(r) \cos \beta_i - C_D(r) \sin \beta_i \right\} \]  

(3.17)

and

\[ \Delta K_{Q}(r) = \frac{Z}{8} \frac{V_i^2}{r} \frac{c}{D} \left\{ C_L(r) \sin \beta_i + C_D(r) \cos \beta_i \right\} \]  

(3.18)

where \( V_r \) is the magnitude of the resultant inflow velocity at the propeller plane and \( \beta_i \) is the pitch angle of the velocity vector, as shown in Figure 6.

For a nozzle propeller the local axial velocities induced at the propeller plane by the nozzle and hub are unknown a priori and an initial guess is made corresponding to 20% of the local wake velocity. In subsequent iterations, a check is made on the magnitude of this velocity at each propeller radius and, if the relative difference in magnitude between iterations is less than 1% at all propeller radii, then convergence is deemed to have occurred. Upon convergence a complete listing is made of the fluid flow velocity components at the propeller plane together with the blade performance characteristics at each radial station. A listing is also made, at each radial station, of the cavitation margins on both suction and pressure
surfaces of the propeller blades.

3.6 Final output

Upon convergence of the computer program both nozzle and hub thrust coefficients are calculated from the known vorticity distributions:

\[ C_{TN} = 2\pi \sum_{i=1}^{M} C_{p_i} \frac{r}{D} \frac{d(r/D)}{d\theta} \]  \hspace{1cm} (3.19)

\[ C_{TH} = 2\pi \sum_{i=1}^{M} C_{p_i} \frac{r}{D} \frac{d(r/D)}{d\theta} \]  \hspace{1cm} (3.20)

where the surface pressure coefficient is given directly by

\[ C_{p_i} = 1 - \left\{ \frac{\gamma(s)}{V_A} \right\} \]  \hspace{1cm} (3.21)

and the non-dimensionalising velocity \( V_A \) may, in the program, be either the advance velocity or the propeller tip velocity.

A second thrust coefficient, \( K_{TN} \), is also calculated from the equation

\[ K_{TN} = \frac{T}{\rho n^2 D^4} = \pi C_{TN} \frac{J^2}{8} \]  \hspace{1cm} (3.22)

This thrust coefficient is calculated for each element of a multi-element nozzle section and output with the calculated surface pressure distributions. The propeller thrust and torque coefficients, \( K_{TP} \) and \( K_Q \), are calculated by integrating equations (3.17) and (3.18) according to the second order trapezoidal rule. The total thrust is then given by

\[ K_{TT} = K_{TN} + K_{TP} \]  \hspace{1cm} (3.23)

And the Froude efficiency is calculated from the relation:

\[ \eta_F = \frac{K_{TT} J}{2K_Q} \]  \hspace{1cm} (3.24)

A second efficiency is also calculated by the computer program:

\[ \eta_G = \frac{K_{TT}}{4\pi K_Q} \left\{ \left( \frac{\pi J}{2} \right)^2 + \frac{8K_{TP}}{\pi} \right\}^{1/2} \]  \hspace{1cm} (3.25)

This efficiency represents the proportion of input power that is converted to axial momentum in the propulsor slipstream and is particularly useful in the comparison of the performance of propulsive devices operating at low advance velocities [11]. At bollard condition equation (3.25) reduces to the well-known Bendemann coefficient.
A complete listing of the computer program is given in reference [2] together with a comprehensive programmer's guide and user manual. Some of the results obtained by this analysis are described below.
SECTION 4

ASSESSMENT OF RESULTS

4.1 Open propeller performance

Figures 11, 12 and 13 illustrate open water curves for three different Wageningen B-series propellers and compare the calculated results from this analysis method to those described by the polynomials of reference [12]. It is clear that prediction of torque coefficient is excellent over the entire range of advance coefficients and pitch-diameter ratios illustrated, an observation that may also be confirmed from additional computations for three and five bladed propeller series. Unfortunately, the prediction of thrust coefficient is not quite so good: Whilst agreement may be seen to be excellent around the normal free-running design point for these propellers, the thrust coefficient is overestimated by the program at low advance ratios, particularly as the pitch-diameter ratio increases.

The major reason for this is not difficult to identify: Quite simply, the propeller theory described above is too crude. Nevertheless, it is more than adequate for the detailed study of the open propellers operating at or near their normal free-running speeds. It is this fact alone that permits the incorporation of this simple propeller theory into a performance prediction method for nozzle propellers since the interaction between the nozzle and propeller is of much greater significance than the overestimate of propeller thrust coefficient at low advance velocities. This is not to say, of course, that a more sophisticated propeller theory is of no value to a designer of nozzle propellers. But how accurately, for example, would such a theory predict the performance of the wide Kaplan-type blades commonly applied to such devices?

4.2 Nozzle and hub

Figure 14 illustrates the calculated surface pressure distribution around a solid body of revolution which was the subject of extensive wind tunnel testing by Ryan [13]. The computed results are virtually identical to those predicted by Ryan, the differences being less than the thickness of a pencil line over the whole body surface. This is not surprising since the surface singularity model employed is, except for the application of the Kutta
condition, identical.

Figures 15 & 16 illustrate the calculated surface pressure distributions around two different axisymmetric nozzles in a uniform approach flow. Both nozzles have also been extensively tested. The first, known as the A.R.L. Duct No. 8, was tested in a water tunnel by Ryal and Collins [14] and again by Ryan [13] in a wind tunnel; the second, the N.S.M.B. 19a nozzle, was tested by Gibson [9] also in a wind tunnel. As with the hub pressure distribution, the theoretical predictions made for this paper were virtually identical to those produced earlier by Ryan [13] and Gibson [9]. The interaction effects between a nozzle and a solid body of revolution placed in a uniform stream have also been calculated and, again, match the accuracy of predictions made by Ryan.

It would appear, then, that the inviscid flow calculations incorporated into this adopted flow model are as good as, if not better than, any other available method. The major drawback, of course, being the exclusion of viscous effects from the calculations. Indeed, it was this exclusion of viscous effects that was largely attributed to the discrepancies between theoretical predictions and experimental results (see reference [13]). This is itself a moot point but, as yet, no reliable theoretical model has been developed to predict the surface pressure distribution around an axisymmetric nozzle in viscous flow. In consequence, it is more usual for nozzle propeller designers to incorporate an empirical correction factor that reduces the predicted nozzle thrust coefficient, such as that proposed in references [15] and [16]. This approach has not been adopted in this paper since there are far more important questions to be answered than simply one of viscous corrections to the nozzle: one such being the limitations of the propeller theory, a second being the accuracy of the propeller-nozzle interaction. In any case, the viscous corrections are almost insignificant in magnitude.

In the case of multi-element nozzle sections, there appear not to have been any attempts at the theoretical prediction of surface pressure distributions. There have, however, been open water tests on such devices [16]. One difficulty in attempting such a theoretical model for these devices is the rather peculiar shapes of the nozzle elements, a problem which is introduced by the very real constraints imposed by construction costs. Illustrated in Figure 17, for example, are the cross sections of multi-element nozzles that have recently been tested [17], and they are clearly not what one would normally regard as a "smooth" aerodynamic shapes. Nevertheless, practical experience with the computer program over a few years had led to the conclusion that the smoothing and ill-conditioning problems encountered by
Wilkinson [7] with two-dimensional aerofoils were not of major consequence in regard to axisymmetric nozzle shapes commonly used in ships' propulsion, i.e. those with rather thick sections. The exact reasons for this are unknown, but it is suspected that the coefficient matrix for the axisymmetric case is far more diagonally dominant than its' two-dimensional equivalent and, in consequence, is less susceptible to small changes in profile geometry.

As a starting point, the profile geometry (slightly redesigned) of the main foil of Figure 17 was fed into the computer program using the input data locations indicated by Figure 18a, i.e. so as to define the sharp corners exactly. It will be recalled that these data points are interpolated to extract the pivotal point locations, and these latter are subsequently smoothed by a Lagrangian five point least squares polynomial. A computer plot of these "smoothed" pivotal points showed that the profile contained several blips, particularly in the vicinity of discontinuities of surface slope and, as might be expected, the surface pressure prediction was of little value. The most expedient way around this problem was simply to exclude the data points at the sharp corners and allow the interpolation and smoothing processes ample scope to operate effectively. The selected data points are shown in Figure 18b and the smoothed surface profile is illustrated in Figure 19 together with the calculated surface pressure distribution. Both plots are seen to be acceptable. Since, as a general principle, all new nozzle profiles should be tested for surface irregularities the above procedure is not seen as too onerous on the nozzle designer.

Figure 20 illustrates the surface pressure distributions calculated for a nozzle with a flap, the main foil being that previously discussed and illustrated in Figure 19. The effects of the flap can readily be seen: There is a lowering of the surface velocity over the entire outer surface of the main foil except for the rapid acceleration in the vicinity of the flap. On the inner surface of the main foil the surface velocity is increased and the flap itself behaves somewhat as might be expected from a study of two-dimensional multi-element aerofoils. The adopted numerical model, then, seems to be satisfactory. Furthermore, additional calculations with up to four elements have been made without any difficulty, as have the interaction effects between a hub and a multi-element nozzle. Results from these latter calculations are not presented simply because they are of no value to a nozzle designer, since the nozzle profile must always be designed with the propeller in situ and with the propulsor absorbing the power available.
4.3 Nozzle propeller performance

In order to determine whether the interaction effects between the propeller and nozzle are of the appropriate magnitude, Figure 21 compares the predicted and experimental open water curves obtained for the Wageningen B4-70 series propellers in nozzle 19a. The experimental results have been obtained from reference [18]. Firstly, it may be noted that the torque and total thrust coefficients are predicted very accurately over the whole advance coefficient range except for a pitch-diameter ratio of 1.4, an error that is equally likely to originate from either the calculations or the experimental results (see below). Secondly, may be noted that the total thrust coefficient is gradually underestimated as the advance coefficient is reduced from the free-running condition towards bollard condition, a characteristic that has already been observed with the B-series open propellers. And finally, it can be seen that thrust contribution from the nozzle is also underestimated at low advance velocities. In fact the underestimation in nozzle thrust coefficient is greater in magnitude than the underestimation in total thrust coefficient. A quick look at the open propeller characteristics of Figure 13 provides one explanation of this error: Since the propeller thrust is always overestimated at the lower advance velocities, the nozzle thrust must always be underestimated if the nozzle/propeller interaction effects are of the correct magnitude. In other words, the source of the errors is more than likely to arise from the simple propeller model rather than from the surface singularity model of the nozzle and the interaction effects. Calculations are currently being undertaken in order to test this hypothesis.

Figure 22 illustrates open water curves for the wide bladed Wageningen Ka4-70 series propellers operating within the same 19a nozzle. With these propellers, torque coefficient is predicted very accurately at low pitch-diameter ratios but is progressively underestimated as the pitch-diameter increases over the whole range of advance coefficients. Unlike the characteristics produced for the B4-70 series propellers operating in the same nozzle there is no quantum increase in error as the pitch-diameter ratio reaches 1.4, thus confirming the comment above that there may be an error in the experimental curves for the B4-70 propeller at this pitch-diameter ratio. Again from Figure 22 it may be noted that despite the underestimate of torque coefficient, the ratio of total thrust to torque is of the correct magnitude. This indicates that the empirical corrections used in the propeller theory may reasonably be applied to wide bladed propellers. In order to improve the accuracy of the predicted torque coefficient (and hence the total thrust coefficient) for wide bladed propellers, a study could be made of the full
range of Ka-series propellers that have been tested in nozzle 19a, from which a modification to the empirical corrections could be derived. Finally, from Figure 22, it may be noted that there is once more an underestimate of the nozzle thrust at low advance velocities.

The third and final open water curve featuring a single element nozzle is illustrated in Figure 23: it corresponds to the operation of the Ka4-70 series propellers operating within nozzle 37. Apart from the same observation that nozzle thrust is underestimated at low advance velocities, the prediction of both torque and total thrust coefficient are very good at all pitch-diameter ratios. The contrast to the previous figure is quite startling. If, as suggested in the previous paragraph, the empirical propeller correction factors underestimate torque for wide bladed propellers then why should the accuracy of the theoretical model be so good with nozzle 37? The answer, it is suggested, may be inferred from a study of Figure 24 which illustrates the cross-section of both nozzles 19a and 37 together with an indication of the flow direction. By applying the Kutta condition of smooth flow from the trailing edge of each nozzle, an underpressure must be experienced along the divergent inner surface of each nozzle downstream of the propeller plane. Unlike the 19a nozzle, this underpressure produces a noticable reduction in nozzle thrust on nozzle 37 which cannot be sustained in practice since the fluid separates from the inside of the nozzle. The effect of this on the theoretical model is to produce an underestimate in the magnitude of the axial velocities induced by the nozzle in the propeller plane, i.e. the propeller performance is effectively determined at a lower advance velocity than it ought to be. The result of this can be seen by a comparison of the data in Figures 22 and 23. The prognosis, then, is that the excellent correlation shown in Figure 23 for nozzle 37 is the result of two superposed errors:

(i) An underestimation of propeller torque due to the inadequacies of the empirical corrections in the propeller theory, and

(ii) an overestimation of propeller torque due to the unrealistic reduction in nozzle thrust that inevitably occurs if the fluid is considered to remain attached to the inside of the divergent outlet portion of the nozzle.

The theoretical performance curves discussed above did not include the effects of a hub as part of the surface singularity model. The reason for this is that the hub contributes very little to the axial velocity components in the propeller plane and hence has little effect on the overall performance characteristics. Secondly, the empirical propeller corrections make some allowance for the presence of the hub. Nevertheless, the hub does introduce
some contribution to the flowfield particularly around the lower propeller radii, and there have been circumstances where the facility to include the hub effects have proved valuable.

To conclude this section, a brief look is taken at one of the tentative results that have emerged from computer calculations involving multi-element nozzle propellers. The configuration featured in Figure 20 has been combined with the Ka4-70, P/D= 1.0 propeller and its' surface pressure distribution calculated at an advance coefficient of 0.4. The result is illustrated in Figure 25. The first thing to notice is that the pressure distributions around both the main foil and the flap are smooth everywhere except at the main foil leading edge. The fact that the curves are smooth indicates that the theoretical model is well-behaved and that the pressure distribution is probably representative of that obtained in practice. The underpressure peak at the main foil leading edge indicates that the forward stagnation point is on the outer surface of the nozzle and that cavitation erosion may occur due to the rapid acceleration of the fluid around the leading edge. There are two possible means to avoid this problem if the design advance coefficient is to remain at 0.4:

(i) The camber of the main foil upstream of the propeller plane must be increased until the forward stagnation point lies approximately on the leading edge, and/or

(ii) the width of the slot between main foil and flap may be reduced by either thickening the main foil trailing edge region or thickening the flap section.

Unfortunately there is as yet no experimental evidence to confirm these theoretical observations, and it is beyond the scope of this paper to describe the theoretical studies that have been undertaken to date.
SECTION 5

CONCLUSIONS

This paper has given a fairly detailed summary of a theoretical model that has been developed for the performance prediction of nozzle propellers operating in an axisymmetric flowfield. It has also presented an insight into how this theory has been modified to allow for multi-element nozzle sections and, as is usually the case when updating research work, it has turned up far more questions than it has answered.

The original theoretical model developed during the early 1970’s was severely limited by the computing power and storage capacity available at that time. To give an example: the current computer program (including the extras introduced for the multi-element nozzles) produces complete open water characteristics for a nozzle propeller using approximately two minutes CPU time on an Olivetti M280 personal computer. The active storage requirements are also derisory at just 145kb. And yet in the early 1970’s this was a considerable undertaking. It is due to the enormous advantages of modern computers over their earlier counterparts that the following suggestions for further improvements can be made:

(i) The existing propeller theory should be abandoned and replaced with an accurate lifting surface theory, and one that can accurately model not only the conventional open propellers but also the wide bladed Kaplan-type propellers commonly found in nozzle propeller installations.

(ii) A thorough search should be made for a better alternative to the surface singularity method described above. First of all, it suffers from the disadvantage of using discrete vortex rings rather than surface panels which limit its field of application to axisymmetric flows and is likely to produce many problems in the application to nozzles with flaps and slats. Secondly, the model was developed from a two-dimensional theory which contained many peculiarities which may or may not be of relevance to nozzles. Thirdly, and finally, the model is not readily adapted to incorporate the effects of viscous flow and separation.

(iii) Following from the above comment, a serious attempt should be made to incorporate the effects of viscosity particularly in regard to separation.
(iv) Whilst the slipstream-nozzle interaction effects seem to be accounted for in a reasonable manner, there have been many questions left unanswered simply due to expediency. For example, why should the nozzle induced velocities at the propeller plane be added to the inflow triangles as if they were part of the local propeller induced velocities. What is the influence of increasing the number of radial stations at which the propeller performance and slipstream strength are calculated and, more importantly, what is the consequent effect on the slipstream/nozzle interaction? Furthermore, what are the effects of slipstream contraction and deformation?

These, then, are some suggestions for improvements to the theoretical model described in this paper, and they cover every aspect of the model. Despite these sweeping statements, the method does produce very useful and informative results with very little computational effort. It is possible to produce very accurate nozzle propeller designs which, when used in conjunction with a designers "feel" for the problem, have proven to perform well in both model test and full scale applications.
SECTION 6

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LIST OF FIGURES

Figure 1. Adopted vorticity model
Figure 2. Flow model for nozzle propeller
Figure 3. Simplified flow chart of computer program
Figure 4. Ring vortex filament – induced velocity
Figure 5. Velocity components and superposition principle for NACA 16 thickness distribution and a = 0.8 meanline
Figure 6. Inflow velocity triangles and definition of cavitation numbers
Figure 7. Momentum theory of screw propeller
Figure 8. Illustration of transformation into equal steps
Figure 9. Transformation into equal steps for a two-element nozzle
Figure 10. Relative location of nozzle, hub and propeller
Figure 11. Open water curves for B4-40 series propellers
Figure 12. Open water curves for B4-55 series propellers
Figure 13. Open water curves for B4-70 series propellers
Figure 14. Surface pressure distribution on RAE (MOD 3) centrebody
Figure 15. Surface pressure distribution on A.R.L. duct no. 8
Figure 16. Surface pressure distribution on N.S.M.B. 19A
Figure 17. Cross-sections of recently tested multi-element nozzle
Figure 18. Data point locations
Figure 19. Surface pressure distribution on nozzle type 2
Figure 20. Surface pressure distribution on flapped nozzle type 2
Figure 21. Open water curves for nozzle 19A and B4-70 propeller series
Figure 22. Open water curves for nozzle 19A and Ka4-70 series propellers
Figure 23. Open water curves for nozzle 37 and Ka4-70 series propellers (ahead)
Figure 24. Comparison of nozzle cross-sections
Figure 25. Surface pressure distribution of flapped nozzle type 2 at J = 0.4
FIG. 1: ADOPTED VORTICITY MODEL.
FIG. 2: FLOW MODEL FOR NOZZLE PROPELLER.
FIG. 3: SIMPLIFIED FLOW CHART OF COMPUTER PROGRAM.
FIG. 4: RING VORTEX FILAMENT-INDUCED VELOCITY.
FIG. 5: VELOCITY COMPONENTS AND SUPERPOSITION PRINCIPLE FOR NACA 16 THICKNESS DISTRIBUTION AND $\alpha=0.8$ MEANLINE.
\[ \sigma_0 = \frac{p_a + \rho g h - \rho v}{\frac{1}{2}\rho v_A^2} \]

\[ \sigma_x = \frac{p_a + \rho g (h - r) - \rho v}{\frac{1}{2}\rho v_r^2} \]

\( V_N \) = nozzle induced velocity
\( U \) = propeller induced velocity
\( V_r \) = resultant relative velocity
\( \beta \) = advance angle
\( \beta_l \) = hydrodynamic pitch angle
\( \alpha_n \) = blade pitch angle

**FIG. 6:** INFLOW VELOCITY TRIANGLES AND DEFINITION OF CAVITATION NUMBERS.
FIG. 7: MOMENTUM THEORY OF SCREW PROPELLER.
FIG. P: TRANSFORMATION INTO EQUAL STEPS FOR A TWO-ELEMENT NOZZLE.
FIG. 15: SURFACE PRESSURE DISTRIBUTION ON A.R.L. DUCT No. 8.
FIG. 17: CROSS-SECTIONS OF RECENTLY TESTED MULTI-ELEMENT NOZZLE.
FIG. 18a: LOCATION OF DATA POINTS TO DEFINE SHARP CORNERS.

FIG. 18b: ADOPTED DATA POINT LOCATIONS.
FIG. 19: SURFACE PRESSURE DISTRIBUTION ON NOZZLE TYPE 2.
FIG. 20: SURFACE PRESSURE DISTRIBUTION ON FLAPPED NOZZLE TYPE 2.