Groundwater flow module

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Due to transient boundary conditions for example, the stability behavior of embankments varies with time. One can think of the impact of flood situations on a river dike, high tides on a sea defense or overtopping conditions in general. Nowadays the stability of an embankment is judged for steady state conditions only. A lack of knowledge about the time dependent material behavior and the unavailability of robust calculation tools make a prediction of the transient stability behavior virtually impossible.

This report presents an overview of the work that has been carried out in the frame of the Delft Cluster project ‘groundwater flow module GeoPlax’ in joint cooperation with GeoDelft, Plaxis, Alterra and Dienst Weg- en Waterbouwkunde of Rijkswaterstaat. Aim of this project was to develop a robust algorithm for modeling (un)saturated groundwater flow and to construct a Plaxis user interface that can be coupled to the already existing Plaxis deformation module. The second objective was to gain more insight in the description of material behavior under unsaturated conditions and how to describe initial and boundary conditions for a real situation. Finally the procedure was applied to a real case, the Bergambacht embankment, demonstrating the applicability of the newly developed design methodology.
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1 Summary

This report presents an overview of the work that has been carried out in the frame of the Delft Cluster project ‘groundwater flow module GeoPlax’ a joint cooperation of GeoDelft, Plaxis, Alterra and Dienst Weg- en Waterbouwkunde of Rijkswaterstaat. Aim of this project was to develop a robust algorithm for modeling (unsaturated) groundwater flow and to construct a Plaxis user interface that can be coupled to the already existing Plaxis deformation module. The second objective was to gain more insight in the description of material behavior under unsaturated conditions and to develop a procedure for describing initial and boundary conditions for a real situation. The resulting methodology is presented in two steps, the first step presents the mathematical model completed with material relations, the second step describes the numerical algorithm for solving the posed problem. The procedure is applied to a real case, the Bergambacht river embankment, demonstrating the applicability of the newly developed design methodology. Finally, two applications are presented that illustrate situations for which the use of the newly developed code may result in embankments that are constructed more economically in some cases and with a higher level of safety for more general loading, in others.

2 Introduction

Due to transient hydrological boundary conditions, the stability behavior of embankments varies with time. One can think of the impact of flood situations on a river dike, high tides on a sea defense or overtopping conditions in general. Nowadays the stability of an embankment is judged for steady state conditions only. A lack of knowledge about the time dependent material behavior and the unavailability of robust calculation tools make a prediction of the transient stability behavior virtually impossible.

Aim of this project was to develop a robust algorithm for modeling (unsaturated) groundwater flow and to construct a Plaxis user interface that can be coupled to the already existing Plaxis deformation module. Besides the work on the GeoPlax calculation code, research was carried out and presented in several reports:

- Rekenmodel onverzadigde grondwaterstroming, basisontwerp numeriek algoritme, onderzoek numeriek algoritme (01.02.01/31).
- Rekenmodel onverzadigde grondwaterstroming, heterogene drie-dimensionale toepassingen (01.02.01/66).
- Materiaalmodellen voor ontweremodel grondwaterstroming (01.02.01/57).
- Initiële condities voor ontweremodel grondwaterstroming (01.02.01/58).
- State of the art materiaalmodellen en initiële condities (01.02.01).
- In situ metingen en parameterbepalingen, validatieberekeningen ontweremodel grondwaterstroming (01.02.01/76).

This report only presents a summary of the results. Chapter 3 presents the mathematical description of unsaturated groundwater flow, including material models. In Chapter 4 the numerical model is presented for solving the posed problem in the previous chapter. Chapter 5 presents a validation of the model. Applications are given in Chapter 6 and conclusions are presented in Chapter 7.
3 Mathematical model

Bear and Verruijt [1] present the derivation of the governing equation that describes unsaturated groundwater flow. This flow equation combines a mass balance equation and Darcy’s law and is known as the Richards equation

\[
\frac{\partial}{\partial t}(nS\rho) = \frac{\partial}{\partial x_i} \left[ \frac{\rho K_{ij}}{\mu} \left( \frac{\partial S}{\partial x_j} - \rho g_{ij} \right) \right]
\] (1)

In this equation pressure \( p \) [Pa] is the primary unknown, the independent variables are \( x \) [m] specifying the position in space along the Cartesian coordinate axis and time \( t \) [s]. The fluid density \( \rho \) [kgm\(^{-3}\)] may vary with the amount of solute dissolved in the liquid phase for instance, and its dynamic viscosity \( \mu \) [kgm\(^{-1}\)s\(^{-1}\)] may vary with temperature. In this article however both are kept constant. Porosity \( n \) [-] and the saturated hydraulic conductivity tensor \( K \) [m\(^2\)] are assumed to be related to the soil only and therefore they vary in space only. The water content is described as the product of porosity and saturation \( S \) [-] which is related to fluid pressure and soil type. The relative permeability \( k \) [-] is related to saturation and soil type. Acceleration due to gravity is expressed by the gravity vector \( g \) [ms\(^{-2}\)] pointing downward in the third coordinate direction. Source terms are included as boundary conditions, leading to a more general approach.

Van Genuchten [5] presents a popular material model for saturation and relative permeability. For saturation his expression reads

\[
S = S' + \left( S' - S'' \right) \left[ 1 + \left( \frac{\rho}{C_0} \right)^{1/g_n} \right]^{1/g_n}
\] (2)

According to this description saturation varies between the residual saturation \( S'' \) [-] and saturation \( S' \) [-] at saturated conditions. The relation holds for negative pressures and is \( C_0 \) continuous only for \( p=0 \). The expression for relative permeability is given by

\[
k = (S'')^{g_n} \left[ 1 - \left( 1 - (S'')^{1/g_n} \right) \right]^{g_n}
\] (3)

Here relative permeability is related to saturation using the effective saturation \( S' \) [-] written as \( S' = (S-S')/(S''-S') \). The Van Genuchten material model contains three parameters: \( g_a \) [Pa\(^{-1}\)], \( g_l \) [-] and \( g_n \) [-] that are obtained from laboratory experiments and a fitting procedure. The missing parameter \( g_m = 1-1/g_n \) will always be positive as \( g_n > 1 \).

Based on the Van Genuchten model and a classification according to the Staring reeks and Hypres series a simplified model was constructed. Aim of this part was to define sets of materials all having more or less the same pressure-relative permeability and pressure-saturation relations. Outcome of this procedure was a classification according to figure 1.

For each zone mean Van Genuchten parameters were calculated, and a linearization procedure was carried out. This served the second objective: making the algorithm more robust. Figure 2 shows the results for an O1 material according to the Staring reeks and its equivalent by found for the linearization procedure. The linearization could be carried out directly by least squares for example, but instead of this the objective of simulating infiltration (under certain conditions) was chosen.
Now an engineer only has to specify the saturated conductivity and porosity, which are elementary parameters both, and a position in the triangle.

Figure 1. Soil classification method for Dutch soils

Figure 2. Simplified material model, original Staring O1 (green) linearized model (red)
The results for a one dimensional infiltration test carried out using the original relations and the simplified version is presented in figure 3.

Figure 3. Infiltration profiles, left O1 Staring right O1 linearized at time 2e4, 4e4, 6e4, 8e4, 1e5, 1.2e5, 2e5 and 3e5 s

Figure 4 shows an alternative material model according to Haverkamp [3] for sand and clay, which will be used for simulation. It is noted that the sandy material shows a faster decrease in saturation and relative permeability, as pressures become more negative.

Since saturation and relative permeability are related to pressure, the initial pressure field can be found from one of both and the inverse relation. More directly the initial conditions may be given as a prescribed pressure field

\[ p(x,t^0) = \bar{p}^0 \quad \text{in} \quad \Omega \]  \hspace{1cm} (4)

Alternatively, an equilibrium condition can be calculated from a set of boundary conditions. With \( \Omega \) the calculation domain, \( \Gamma \) its boundary, and an overline denoting a given value. Three types of boundary conditions are presented here. Dirichlet boundary conditions prescribe a given pressure as

\[ p = \bar{p}(x,t) \quad \text{on} \quad \Gamma_1 \]  \hspace{1cm} (5)

Neumann boundary conditions have no physical meaning for this problem, Cauchy conditions prescribe an out-flowing mass flux \( \rho q \)

\[ -\frac{\rho k K_y}{\mu} \left( \frac{\partial p}{\partial x_j} + \rho g_j \right) n_i = \rho q(x,t) \quad \text{on} \quad \Gamma_2 \]  \hspace{1cm} (6)

Only for an in-flowing fluid the right hand side mass density is known in advance, closed boundaries are formulated with a zero right hand side. Variable boundary conditions describe seepage where outflow takes place at atmospheric pressure, or precipitation in which case ponding may occur. Both conditions can be written as
Conform this notation, the volumetric precipitation in-flux has a negative sign ($q < 0$). The pressure $p$ at which ponding takes place will be zero for the seepage face without precipitation ($q = 0$). Time varying boundary conditions due to a fluctuating water level may switch from $\Gamma_1$ to $\Gamma_3$ and vice versa.

As an illustration, Figure 5 shows the infiltration profiles in two homogeneous columns; the first, Haverkamp sand and the second, Haverkamp clay. Both columns initially have a constant pressure of $-10^4$ Pa. Infiltration takes place at atmospheric pressure applied to the top of the column. The test simulations reported by Van Esch and Spierenburg [4], and compare well. Figure 6 displays the pressure distribution for a heterogeneous situation. In the first column sand is placed on top of clay, in the second clay covers sand. Initial and boundary conditions equal the conditions for the homogeneous columns. Infiltration into the sand part of the first column takes place in the same way as in the homogeneous column until the front reaches the clay material. From that moment infiltration in the clay material occurs, continuity of pressure and flux at the interface, give rise to an increase in pressure at the interface until a near hydrostatic pressure distribution has been reached in the sand.
domain. For the second column, initially not only infiltration into the clay material takes place. Here a redistribution process of fluid at the interface becomes apparent. Pressure profiles are calculated using the numerical approach presented in the next chapter on a single fixed mesh.

Figure 5. Pressure distribution for homogeneous profiles at infiltration, left figure sand $t = 4e2, 8e2, 1.2e3, 1.6e3, 2e3, 2.4e3, 2.8e3$ and $4e3$ s, right clay $t = 2e5, 4e5, 6e5, 8e5, 1e6, 1.2e6, 1.4e6$ and $2.8e6$ s.

Figure 6. Pressure distribution for heterogeneous profiles at infiltration, left sand on clay $t = 4e2, 8e2, 1.2e2, 1.6e2, 1e4, 1e5, 2e5$ and $4e5$ s, right clay on sand $t = 1e5, 2e5, 3e5, 4e5, 5e6, 6e6, 7e6$ and $1e6$ s.
4 Numerical model

This report presents a numerical algorithm with which unsaturated as well as saturated groundwater flow can be calculated. Several techniques are tested and add to a robust algorithm. An accurate solution has been found for a diagonal mass matrix. An automatic time stepping procedure based on the convergence behavior of the non-linear solver enhances the applicability of the algorithm especially for heterogeneous problems. The non-linear set of ordinary differential equations is linearized first and the solution is found by iterations. This non-linear solver called Picard iteration schema gives best results if combined with a tangential approximation of the pressure derivative term. A simple diagonal preconditioned conjugate gradient solver then solves the linear set of equations per Picard iteration step. Linear as well as quadratic (and even 15 nodded) elements can be used for discretising the spatial field. However linear shape functions are preferable, as sharp fronts can be described more accurately. For a combined deformation and fluid flow calculation linear elements are preferred as well. Only if strains and fluid pressures are interpolated in a same order of accuracy they produce a consistent stress field. This implies that for the case that linear elements are used in a fluid flow problem displacements need to be calculated using quadratic elements.

The finite element method is well explained by Verruijt [6] and will not be presented in detail here. In the following examples only bilinear quadrilateral elements and trilinear hexahedral elements are used to subdivide the modeling domain. However the implemented model is more general. The resulting set of non-linear equations when standard Galerkin finite element discretization and fully implicit finite difference time stepping is applied

\[ M_{ab}^{k+1} \frac{p_b^{k+1} - p_b^k}{\Delta t} + S_{ab}^{k+1} p_b^{k+1} = F_a^{k+1} \]  

In the set of equations superscript \( k \) denotes the current time step. The components of the mass matrix \( M_{ab} \) are given by

\[ M_{ab} = \int_{\Omega} N_a N_b (\alpha + n \beta) \rho S d\Omega + \int_{\Omega} N_a N_b n \rho \left( S_{k+1}^{k+1} - S_k^k \right) d\Omega \]

The second term includes the capacity matrix for water content change, its value is calculated numerically. Row sum lumping of the matrix improves the stability of the algorithm. The stiffness matrix \( S_{ab} \) reads

\[ S_{ab} = \int_{\Omega} \frac{\rho k K_{ij}}{\mu} \frac{\partial N_a}{\partial x_i} \frac{\partial N_b}{\partial x_j} d\Omega \]

This contribution is found using Green’s theorem for order reduction. Order reduction also contributes to the force vector \( F_a \) components

\[ F_a = \int_{\Omega} \rho g \frac{\rho k K_{ij}}{\mu} \frac{\partial N_a}{\partial x_i} d\Omega - \int_{\Gamma} \left( \rho \gamma \right)_a d\Gamma \]

Integration is done numerically by Gaussian quadrate. The non-linear equations are linearized using a Picard method

\[ \left( M_{ab}^{k+1,r} + \Delta t S_{ab}^{k+1,r} \right) p_b^{k+1,r+1} = M_{ab}^{k+1,r} p_b^k + \Delta t F_a^{k+1,r} \]

\[ 7 \]
Superscript $r$ denotes the current iterate. This process is repeated until a convergence, stated as $|p^{r+1} - p^r| - \varepsilon < \varepsilon_0$, where the vector norm has been used. Multi grid is well explained by Hackbusch [2], the method uses a grid hierarchy and is designed to solve a set of equations in $O(n)$ computational operations. For this problem, the Full Approximation Storage algorithm (FAS) is applied to resolve the non-linear set of equations generated per time step. As a criterion for refinement, the pressure gradient can be calculated. However, a better indication is found by reformulating the right hand side of Equation (1) using partial differentiation. For the resulting convection-diffusion type of equation the element Peclet number can be formulated as $Pe_i = \Delta l / k \delta x_i / \delta x_i$. Its value depends only on the element size $\Delta l$ [m] and relative permeability. Hanging grid points are handled by linear interpolation. As an example of the adaptive technique the heterogeneous column experiment with a sand layer on top of clay was extended by imbedding the column in sand. Figure 7 shows some results obtained, the two-dimensional calculation was carried out with three grid levels, only two were used for its three-dimensional counterpart.

The pictures show a reduction in the number of mesh nodes from 3321 to 870 in the 2D-case and 18081 to 4891 for the 3D-example. Adaptive refinement based on the element Peclet number is capable of capturing sharp interfaces thereby increasing the accuracy of the solution. The technique leaves the mesh unchanged in the rest of the modelling domain thereby ensuring computational efficiency. The difference between the results of the two-dimensional and three-dimensional calculation shows the influence of the missing dimension if one relies on 2D modelling.

5 Validation

The applicability of the model was investigated for a real situation. At the Bergambacht site groundwater pressures were measured at three locations, using capillary cups at 7 points in vertical direction. Figure 8 shows the hydraulic head at one such location for a certain time interval at 3.71,
3.46, 3.21, 2.96, 2.46, 1.46 and 0.86 m above N.A.P. This figure results from adding the measured pressure as depicted by figure 11 (divided by density and gravity constant) to elevation level. From Figure 8 one can immediately derive the groundwater flow direction as flow occurs from a high potential to a low potential.

The graph shows a transition from a relatively dry period to a relatively wet period. At day 225 measured hydraulic heads read 3.22 m, 3.09 m, 2.81 m, 2.63 m, 2.36 m, 2.03 m and 1.77 m. At this state a nearly stationary situation occurs and hydraulic head gradients are calculated as 0.52, 1.12, 0.72, 0.54, 0.33 and 0.43. The pressure profile -4.9 kPa, -3.7 kPa, -4.0 kPa, -3.3 kPa, -1.0 kPa, 5.7 kPa, and 9.1 kPa, shows only saturated conditions for the lowest two points. Precipitation was measured on a daily basis, Figure 9 shows the recorded signal.

The signal was corrected for evaporation, resulting in a negative precipitation value for days on which evaporation dominates precipitation. Water levels variations at the Lek river where also taken into account. The daily average levels are shown for the same time period in Figure 10.
The deepest measurement point records the change in water level quite well; their values are about one meter higher than the actual water levels due to infiltrating water. The infiltration process takes place as was shown in the previous chapters and takes place at negative pressures. All point show a strong relation to the high precipitation peaks.

A model consisting of 6944 elements and 3639 nodes was constructed and simulations were carried out with different sets of material models. Simulated pressures were compared with measured pressures. Figure 11 shows the enlarged pressure profile for the location under investigation. First laboratory results were used to construct the Van Genuchten relations. Simulation results are shown in Figure 12.
Figure 12. Predicted pressure variation, Van Genuchten material model.

The simulated water pressures also show a strong relation to precipitation. However, the amplitude of the simulated water pressure signal is smaller than measured. The second approach was to use the linearized model. Results for this simulation are presented in Figure 13.

Figure 13. Predicted pressure variation, linearized material model.

Now the amplitude is even smaller for points that remain unsaturated. Points that become saturated in time show a better reaction to precipitation. Optimizing the linear relations, does not further improve the results. Figure 14 shows the results for this simulation, that was more balanced on infiltration times of the Van Genuchten model.
Figure 14. Predicted pressure variation, optimal linearized material model

A linear relation for saturation and relative permeability to pressure based on an expert guess result in a predicted pressure signal that shows more variation in time.

Figure 15. Predicted pressure variation, expert guess material model.

The first reason for the discrepancy is thought to be the daily bases on which precipitation was described. A time scale that corresponds to the real duration of a rainfall event will lead to more variations in unsaturated pressures. The second reasons for the discrepancy is found in the homogeneous description of the embankment material, where in fact this material is very heterogeneous and highly structured. Preferential flow paths may enlarge the bulk hydraulic permeability by several orders, leading to a system that shows a faster response to changing boundary conditions. Moreover the reaction of the saturated zone will be stronger.
6 Applications

This chapter shows two situations for which time dependent groundwater flow plays an important role. Both examples were modeled using a mesh with 1600 nodes. Figure 16 shows the finite element mesh and also illustrates the initial condition. In all pictures presented here saturated groundwater pressure field are presented though actually the whole domain was then into account.

![Figure 16. Mesh and initial pressure field application.](image)

Figure 17 shows the steady state situation for which the sea water level reaches the top of the embankment. This steady state situation is used to judge the stability nowadays.

![Figure 17. Steady state solution](image)

Figure 18 illustrates the transient response to a time varying water level. The duration of the high water period is 36 ours. At 18 ours the water level reaches the top of the embankment. For this situation overtopping conditions are not taken into account.
Figure 18. Tidal wave without overtopping conditions for $t = 9h, 18h, 27h$ and $36h$.

The tidal wave and overtopping conditions were both taken into account in Figure 19. Overtopping conditions take place as long as the difference between water level and top of the embankment is less than one meter.
Figure 19. Tidal wave with overtopping conditions for $t = 9h, 18h, 27h$ and $36h$.

The results show that for the given geometry for a situation without overtopping steady state conditions lead to a safe construction overtopping conditions however lead to a smaller safety factor.
7 Conclusions

This report presents a robust algorithm for modeling unsaturated groundwater flow that has been implemented in the Plaxis environment. A simplified material model was derived and added to the model. This material model enhances calculation speed, while its parameters can be determined more easily. The code is available for the engineering practice and can be used in cooperation with the already available Plaxis deformation module. The modules linked together provide a tool to judge for instance, the time dependent stability behavior of dams.

The report also presents a methodology for constructing boundary conditions and determining the initial state. Since water content only varies one order and saturated permeability may vary multiple orders, the accuracy of the predicted water pressure field depends strongly on the saturated hydraulic conductivity. For this reason it is better to know the saturated permeability of the materials involved than the initial water content distribution. A state of the art study was focussed on determining material behavior. Field tests however showed some fundamental differences for embankment materials compared to agricultural soils.

The GeoPlax code was applied for a real situation, the Bergambacht case. For this case, discrepancies were found between measured pressures and simulated pressures at several locations. The first reason for discrepancy is thought to be the daily basis on which precipitation was described. A time scale that corresponds to the real duration of a rainfall event will lead to larger variations in unsaturated pressures. However, infiltration capacity of the soil may be limited and storage in the top soil may enlarge the infiltration period. The second reason for discrepancy is found in the homogeneous description of the embankment material, where in fact this material is very heterogeneous and highly structured. Preferential flow paths may enlarge the bulk hydraulic permeability by several orders, leading to a system that shows a faster response to changing boundary conditions. The validation test showed that better results were obtained by using the simplified material model for the wet period than by using the original Van Genuchten model.

Two applications were presented that illustrate situations for which the use of the newly developed code may result in embankments that are constructed more economically in some cases and with a higher level of safety for more general loading, in others. Future research however, should be focussed on describing the fundamental behavior of embankment materials, with the aim to develop a procedure for deriving material parameters or to construct a material database especially for these materials.

References

APPENDICES