Principles of Double Entry Bookkeeping in an Automated Control Environment

Rob Christiaanse\textsuperscript{1,2}, Joris Hulstijn\textsuperscript{1},

\textsuperscript{1} Delft University of Technology, \textsuperscript{2}EFCO Solutions, Amsterdam

r.christiaanse@efco-solutions.nl, j.hulstijn@tudelft.nl

Abstract. Developments in auditing, such as continuous auditing or compliance by design, presuppose detailed models of economic transactions, often modelled as value cycles using a money measure. Automated control environments require an automated mechanism to ensure data integrity. Data integrity is not a given, but needs to be established. From a model based perspective we show how double entry bookkeeping provides a detailed model of economic transactions and how the principles of double entry bookkeeping provide a mechanism ensuring and establishing data integrity.

Keywords: computational auditing, double entry bookkeeping, control automation, data integrity

1 Introduction

Auditors are responsible for providing reasonable assurance that (financial) information is free from material misstatements (Houston et al. 1999; Knechel et al. 2007). Due to technological developments, the traditional trade-off between the costs of control on the one hand, and the risk of missing a material misstatement on the other hand, is changing. Developments like continuous auditing or continuous control monitoring are moving in the direction of a more effective and efficient audit effort. Effectiveness of an audit relates to the nature of audit evidence, in particular its quality: relevance and reliability (IFAC 2012) and is viewed as the first order effect (Vasarhelyi et al. 2004). Efficiency can be coined as the second order effect and relates in particular to the quantity of evidence, i.e., traditionally the size of the sample. Practitioners claim that routine monitoring and auditing tasks can be done more efficiently by automation.

On the other hand there are also developments in the direction of verifying conformance of a process design (Liu et al. 2007; Lu et al. 2009; Rozinat and van der Aalst 2008). This move implies a more preventive approach. But there is a catch. Conformance checking assumes there is a reference model (‘de jure’ model) with process constraints against which the evidence of process behaviour (‘de facto’ model) can be verified. How to derive a reference model remains unspecified. If we start at a given situation, it remains unclear how the raw evidence needs to be interpreted as a ‘de facto’ model, and how to map the resulting ‘de facto’ model onto the ‘de jure’ model.

From a model based perspective we show how the value cycle provides in a detailed model of economic transactions and we state that the principles buttressing double entry bookkeeping provide a mechanism to ensure and establish data integrity of the registered data, as output of the value cycle, in an information system. Put in other words, principles underlying double entry bookkeeping can be viewed as the semantics of the actions (i.e., economic events) and the invariant of the data-structures applied in an information system, to make sure that registered transactions are well-formed. Furthermore we argue that the de facto model can be readily compared to the de jure model and vice versa. The value cycle approach modelled under these conditions allows abductive reasoning commonly practised in professional bureaucracies like auditors, financial controllers, medical practitioners et cetera.

We will develop the argument in three steps, starting from an abstract value exchange (Section 2), to the logic of double entry bookkeeping (Section 3) and finally, the use in accounting of the relationship between the flow of money and goods (Section 4).
2. Abstract Value Exchange

In general, contracts between agents constitute economic events effecting the financial position of the enterprise. They provide the necessary conditions that should be kept by both parties engaged in trade, to exchange property rights. Hence property rights are often referred to as the company’s assets.

Economic transactions can be formally modeled as a value cycle (Griffioen et al. 2000). This view is inspired by accounting models in the owner ordered accounting tradition (Starreveld et al. 1994; Blokdijk et al. 1995). Figure 1 shows a simplified version of an economic transaction for a trading company modeled as a value cycle. There are four events that affect the status of the value cycle: Buy, Sell, Pay and Collect. This basic model already provides many notions to model business processes and analyze their well-formedness. A value cycle is well-formed when all commitments are fulfilled and the difference between goods bought (cost of goods sold) and goods sold (revenue) equals the increment of the cash position.

![Figure 1: Generic overview of the value cycle. Round shapes indicate events; rectangles indicate buffers; arrows indicate an influence, either positive or negative (−/−); a dashed line indicates that an economical event should be well-formed. Note that the arrows typically make a cycle or a tour.](image)

For auditing purposes these type of models are made popular in the Netherlands by Starreveld since the 1960’s, see (Starreveld et al. 1994) and Frielink (Frielink c.s.,1997). The same model found its way in practice, for example it was Veenstra who developed the model into a practical guide for performing audits (Veenstra 1972/1976). When we use value cycle models in computer science, in most cases the purpose of these models is to analyze the representations of actions and events in a business process, and study the well-formedness of the process. Compare for example the use of the REA ontology (Geerts and McCarthy 1999), e3-value (Gordijn and Akkermans 2003), or DEMO (Dietz 2006).

Auditors are independent experts, monitoring the firm on behalf of stakeholders of the firm. For decision purposes, stakeholders need reliable financial information about the status of an organization and its performance (Knechel et al. 2007; FASB 1980). Traditionally auditors are responsible for providing reasonable assurance that (financial) information is free from material misstatements (Houston et al. 1999; Knechel et al. 2007). In this respect materiality reflects “... the magnitude of an omission or misstatement of (financial) information that, inferred in the light of surrounding circumstances, makes it probable that the judgment of a reasonable person relying on this (financial) information would have been changed or influenced by the omission or misstatement” (Houston et al. 1999, 284). In general, auditors use the audit risk model in planning the audit process.

(1) Audit Risk = Inherent Risk * Control Risk * Detection Risk

The audit risk model should be thought of as an aid in understanding how various factors affect the amount of substantive testing required by the auditor in setting up a audit program for a specific audit engagement. According to Knechel et al (2007) “[...] audit risk concerns the risk that the auditor may unknowingly fail to appropriately modify that his or her opinion on (financial) information that are materially misstated” (p. 388). Detection risks concern both failures of substantive testing and the appropriateness of audit evidence, including the strength or persuasiveness of audit evidence. Traditionally, all these factors are given,
including the quality of evidence, so the only way to reduce the audit risk to an acceptable level, is to increase the amount of substantive testing.

Figure 2. Conceptual model of responsibilities concerning audit processes

Conceptually the responsibilities concerning any audit process are embedded (Figure 2). In practice external auditors partly rely on work done by e.g. the internal audit department, risk management department, concern control et cetera, who in turn rely on work done by the business. Quality of audit evidence relates to relevance, i.e., is the data pertinent to the audit assignment, and reliability: does the data provide an accurate and complete depiction of reality? It needs no elaboration that the quality of audit evidence is largely dependable of the reliability of data. Stated otherwise: if the initial recording processes do not meet the standards which safeguard assets than the claim that routine monitoring and auditing tasks can be done more efficiently by automation is illusionary.

In practice, accounting involves (i) the measurement of economic behavior by recording primary processes, (ii) subsequent processing of the resulting data for some decision purpose, and (iii) communication of the resulting information to external stakeholders (Gray and Needles 1999, p 3-4). Bookkeeping is the process of recording and keeping financial events. Indirectly the distinction between initial recording, information processing and reporting refers to the notion of data integrity versus information integrity in modern accounting information systems. Data integrity is defined as “the state that exists when data are unchanged from its source and has not been accidently or maliciously modified, altered or destroyed” (Welke et al. 1989, p B26.1). When data are being processed and used by people for decision making, data integrity becomes a prerequisite for information integrity. By contrast, information integrity also involves the relevance and usability of information for some purpose (Boritz 2005)

3. Double entry bookkeeping

Data integrity relates to the (mathematical) concept of well-formedness of processing economical events according to the principles of double entry bookkeeping. Hereafter we show that the principles of double entry bookkeeping are closely related to the value cycle depicted in figure 1 and that the present use of the value cycle approach in money measures has some flaws from an data integrity point of view. First we look at the historical background of double entry bookkeeping practices.

What we have coined as double entry bookkeeping coincides with early developments in mathematics. Heeffer (2009) argues that “that symbolic algebra was made possible by the central idea of value as an objective quantity in mercantilism” He continues: “These two developments of the fourteenth and fifteenth century were both instrumental in the objectivation of value and they supported the reciprocal relations of exchange on which mercantilism depended.” (Heeffer 2009; p 112). Grounded in the preceding Abacus tradition, there must always be a just and true value in exchange. But Pacioli broke with the Abacus tradition by not emphasizing specific solutions to barter problems for example, but made clear that certain properties are common to a class of portioning problems. Pacioli’s Perugia manuscript (Vat. Lat. 3129) was the first to present the solution to algebraic problems as theorems with general validity (Heeffer 2009). Pacioli (1994) is also the first who describes the purpose of keeping records for a merchant, applying the symbolism as a model for designing an accounting information system, namely so the merchant can be sure that a just and true value is exchanged and that assets are safeguarded.
To understand the nature of double entry bookkeeping practices we need first to address the so called barter problems. Based on an original problem by Piero (Heefer 2009) the problem is (see Table 1):

There are two [men] who want to barter. One has cloth, the other has wool. The piece of cloth is worth 15 ducats. And he puts this to barter [at] 20 and of this he wants 1/3 in cash. And a hundred of wool is worth 7 ducats in cash. What shall they put for barter so that not one of them is being cheated?

It is easy to see that the profit (this is the difference between the booked value and the barter value) has to be the same as the difference between the two values used to determine the fair profit ratio. The profit for the merchant who sold the piece of cloth is 5 ducats. Hence if we know or if we can compute the fair profit ratio’s than we can easily compute whether that the recorded transactions is exchanged for a just and true value and the assets are safeguarded.

This inside of Pacioli (1494) buttressed the rules of double entry bookkeeping (Particularis de computis et scripturis). Chapter 36 of the Summa gives a summary of the rules for keeping a ledger (de Pacioli 1494; Geijsbeek 1914). This is the main reason why books on mathematics like the *Summa de arithmetica, geometricha, proportioni et proportionalita* (de Pacioli 1494) also contained sections on how a merchant should keep his books and why.

In case we apply double entry bookkeeping logic to the simple value cycle depicted in Figure 1, than the buy transaction and the sell transaction in the value cycle can be depicted as in Figure 2.

**Figure 2.** Value cycle for transactions applying double entry bookkeeping logic, where \( Ppr = \) purchasing price per good, \( Spr = \) sales price per good, \( Q = \) quantity of goods, \( S[t]= \) state at time \( t \). Note the dashed lines (reconciliation) between buy and sell events, and between the flow of goods and the flow of money.

Hence an important difference between Figure 1 and Figure 2 is that it adds quantities and prices. \( Q \) is a quantity measure, that holds per transaction. Also the meaning of the arrows has changed and they no longer form a circle. Here the arrows represent the increments or decrements that result from the events. For example, Buy will increment the inventory Goods with quantity \( Q \), and decrement the assets with value \( Ppr*Q \), stored under Creditors. Pay will subsequently redeem creditors, reducing Cash by \( Ppr*Q \).
Now, under this representation a single business transaction is well-formed at some specific time $t$, when all creditors have been paid, debts from all debtors have been collected, all goods or services have been sold and in the bank we find the difference between sales price and purchasing price as cash.

\[(A) \quad \text{Creditors}[t] = 0 \\
\text{Debtors}[t] = 0 \\
\text{Goods}[t] = 0 \\
\text{Cash}[t] = (Spr - Ppr)\]

Mark that this ‘ideal’ transaction -- when the equations in (A) hold -- is settled in a traditional marketplace, where buyer and seller are both present with the goods, and the goods and money are transferred simultaneously. However, when there is a distance in space or time, we may find temporary alterations to the model. For example, in case the buyer pays the seller some time before delivery or pays in a different location (e.g. over the internet) so that delivery cannot be immediately verified, the buyer will have to add the seller as a debtor to his bookkeeping system, to keep track of open or unsettled transactions. This mechanism is a well-known principle coined as the three way match.

The accounting view of the value cycle is depicted as Figure 3. This model was made popular in the Netherlands by Starreveld since the 1960’s, see (Starreveld et al. 1994), but note that it was Veenstra who developed the model into a practical guide for performing audits (Veenstra 1972/1976).

\[\text{Figure 3. Accounting view of the value cycle; note the central role of the general ledger for recording purposes, and the reconciliation relation between the flow of money and goods, and between buy and sell.}\]

The reconciliation relations shown in Figure 2, which relate the two ends of a transaction, must hold. In addition to a relationship between buy and sell events, accountants expect to find an equivalence between the goods or services delivered over a certain period of time (goods flow), and the revenues collected (money flow). These equations also hold when there is a distance in space or time between buyer and seller, because over a certain period of time, these temporary inequalities should balance out. If these equations do not hold, important information is lost and the merchant in our example will risk losing his assets due to theft, spoilage or other misbehaviors.

We use the following notation. Suppose $e$ is an event, shown as a round shape in Figure 3. The effect of events is indicated by arrows representing increments and decrements (-/-) on account variables, shown as a rectangle. For example, $\text{increase}(T, [t_0, t_1]) = \text{decrease}$ for all $e$ during $[t_0, t_1]$, and similarly for decrease
The relationship between the flow of money and goods can be expressed in two ‘laws’ (Starreveld et al. 1994). First the law of the rational relationship between sacrificed and acquired values states that, for all events $e$ that link states $S, T$ in Figure 3, we have:

(i) $\text{increase}(T, e) = f \cdot \text{decrease}(S, e)$, for some normative ratio $f$. (individual event)

(ii) $\text{increase}(T, [t_0, t_1]) = g \cdot \text{decrease}(S, [t_0, t_1])$, for some average normative ratio $g$. (aggregate)

For example, if we look at a sales event, we have: increase in accounts receivable = sales price $\cdot$ decrease in inventory. Similarly, on aggregate, we have increase in accounts receivable during $[t_0, t_1] = \text{average sales price} \cdot \text{decrease in inventory during} [t_0, t_1]$.

The second law is always formulated as an aggregate. It states that, for all states $S$, the quantity at the end of a period should equal the quantity at the beginning, with increments added, and decrements subtracted.

$$S[t_1] = S[t_0] + \text{increase}(S, [t_0, t_1]) - \text{decrease}(S, [t_0, t_1])$$

Depending on the type of business, the relationship between the flow of money and the flow of goods is stronger or weaker. For trading companies the relationship is relatively strong. The relationship is on the other hand applied in the services industry relatively weak. If we want for example to verify whether revenues are completely accounted for, than the auditor can verify for example revenues against the number of goods sold and the sales price. In the case there is a weak relationship, additional procedural controls are needed to ensure data integrity.

4. Towards a just and true model of assurance provisioning

Based on the mechanisms derived from double entry bookkeeping logic, we defended our position that the stronger the relationship between the goods and money flow, the more the auditor or monitor can rely on expected proportions. The next step is to formulate in a precise manner under which circumstances we can assume that an accounting information system will contain information about transactions that reflect just and true values of the assets of an enterprise. These predictions are formulated in the form of a number of propositions. The propositions are of two types: regular ones and default ones, compare (Reiter 1980). The regular ones are strong normative claims: they must hold by definition of the concepts used. The default claims are more like practical observations. The yare formulated using the operator ‘might’, meaning that they might hold, but only under specific ‘normal’ circumstances. An experienced auditor/monitor will know what these normal circumstances are for a specific type of business. In automating control, however, these assumptions will probably have to change. We will address this issue later on. First it is important to make the propositions explicit.

(1) If law (i) holds, then law (ii) must hold;
(2) If law (ii) holds than law (i) might hold, under some specific circumstances.
(3) If the sales price and the purchase price reflect the historically paid price, than law (i) holds iff law (ii) holds.
(4) If either the current sales price or purchase price differ from the historically paid price, than law (i) is applicable to the difference in prices.
(5) If the quantities bought or sold differ from the historical agreed upon quantities than law (i) is applicable to the difference in quantities.
(6) If the quality of the goods bought or sold differ from the historically agreed upon quality, than law (i) is applicable to the difference in quality, or proposition (4) will also hold (price difference).

These propositions allow abductive reasoning commonly practised in professional bureaucracies like auditors, financial controllers, medical practitioners et cetera. Abductive reasoning, is reasoning backwards, from the observed conclusions, to the premises that best explain them (Hobbs et al. 1993). Given evidence $E$ and candidate explanations $H_1, \ldots, H_n$ of $E$, infer the truth of that $H_i$ which best explains $E$. 


Example. Consider a tradesman in shoes. He has one large store and employs three sales assistants and a store manager. The assortment contains ladies and men shoe ware listed in varies price ranges from € 200 a pair up to € 550 a pair. On average the mark up is set by the tradesman at 40% of selling price. So on average the purchasing price ranges from € 120 up to € 330 per pair. On a daily basis he receives from his accountant the daily sales report, the stock count in shop and the purchases for shoes. Yesterday he received his daily report, stating:

<table>
<thead>
<tr>
<th>States/flows</th>
<th>Quantity</th>
<th>Account</th>
<th>Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>203 pair</td>
<td>Turnover sales</td>
<td>€ 45.675</td>
</tr>
<tr>
<td>In stock: day before yesterday</td>
<td>406 pair</td>
<td>Stock at historical cost</td>
<td>€ 53.896</td>
</tr>
<tr>
<td>In stock: today</td>
<td>203 pair</td>
<td>Stock at historical cost</td>
<td>€ 27.080</td>
</tr>
<tr>
<td>Purchases</td>
<td>nil</td>
<td></td>
<td>€ 0</td>
</tr>
<tr>
<td>Margin</td>
<td></td>
<td></td>
<td>€ 18.859</td>
</tr>
</tbody>
</table>

Table 2. Example of shoes trade

From an accounting perspective the key question is: “What information does the tradesman need so he is certain that he can be sure that a just and true value is exchanged and that assets are safeguarded?”. Theoretically the question is exactly the same form an auditing or a management monitoring point of view. In the case the tradesman accounts for all the transactions, applying the double entry bookkeeping system and principles, than the auditor or monitor simply asks himself:

If law (i) holds, than law (ii) must hold.

Remember law (i) implied:

\[
\text{increase}(T, e) = f \times \text{decrease}(S, e), \text{for some normative ratio } f \quad \text{(individual event)}
\]

\[
\text{increase} (T, [t_0, t_1]) = g \times \text{decrease}(S, [t_0, t_1]), \text{for some average normative ratio } g. \quad \text{(aggregate)}
\]

In our example the evidence is: €26.816 (€45.675 - €18.859). Since \( g \times \text{decrease}(S, [t_0, t_1]) \), for some average normative ratio \( g = \frac{45.675}{1/0.6} = 27.406 \). Surely the evidence differs from what was to be expected. The key question here is: can we infer the falsehood, since we cannot infer the truth? Remember it is a possibility that law (ii) holds:

If law (ii) holds than law (i) might hold, under some specific circumstances.

Law (ii) implies: \( S[t_1] = S[t_0] + \text{increase}(S, [t_0, t_1]) – \text{decrease}(S, [t_0, t_1]) \). In our example the evidence is: 203 = 406 + 0 – 203.

As stated, if law (ii) does hold our evidence € 26.816 might be true, under specific circumstances. These specific circumstances relate to the strong normative claim that a single business transaction must be well-formed at some specific time \( t \). This is the case when the creditor of the procured pair of shoes has been paid, that the debt from the debtor has been collected, that the pair of shoes have been delivered and that the difference between sales price and purchasing price is deposited in de bank. Hence:

\[
(A) \quad \text{Creditors}[t] = 0
\]

\[
\text{Debtors}[t] = 0
\]

\[
\text{Goods}[t] = 0
\]

\[
\text{Cash}[t] = (S_{pr} – P_{pr})
\]

The implication is that all transactions needs to be recorded on a single level and that the minimum of information needed is a timestamp, the quantity and the price. This information is ideally found in procurement systems, cash registers, bank system and accounting systems. Hence the auditor or monitor has to investigate whether the evidence produced by the cash register is integer. This can only be done by investigating whether the quantity of goods sold are correct, the selling price is authorized and the amount
is calculated correctly. It needs no elaboration that these type of security measures can be done in and automated environment.

Hence proposition (3): If the sales price and the purchase price reflect the historically paid price, than law (i) holds iff law (ii) holds. It need no further elaboration that the auditor or monitor has to investigate the claim whether the sales and purchase price are accounted for. Where do the default claims come in? As stated default claims are more like practical observations en imply cost benefit considerations. We like to argue that risk management practices are such default claims and that these type of mechanisms always handle the outcomes of assertions based on strong normative claims. We leave our example to rest.

Note that some accounts $S$ are counted in monetary currency ($€$, $\$), while others, like inventory, are counted in numbers of units (kilos, hours, boxes, ...). Practitioners and accounting theorists seem to agree upon the axiom that money measures are “the only factor that is in common to all economic events and claim that money is than the only practical unit of measurement that can produce data that are alike and can be compared” (Gray et al, 1999). Others simply ignore or do not address the problem of choice of a unit (discrete or continuous) arguing that “the problem, in the standard literature about measurement, is solved by showing that different scales introduced with respect to different units are equivalent in precise sense by using equivalent classes to get rid of units all together the problem of the choice of units is ruled out” (Balzer et al, 1991). We think that this line of reasoning is flawed for the reasons stated in our propositions.

3. Conclusions

Obviously we can’t explain the essence of accounting in a brief paper. Accountants and bookkeepers have a lot of experience and tacit knowledge. Therefore it is important to make some of these implicit assumptions explicit in a kind of ‘user manual’ for these preservation equations.

The paper has contrasted three different views of the value cycle: (i) the abstract value exchange (Figure 1) modelling the relationship between individual buy and sell economic events, (ii) the double-entry bookkeeping logic, modelling the value of assets as a result of transactions, which may involve large quantities which and which can explain the fact that profit is made from the difference between sales price and purchasing price (Figure 2), and (iii) the accounting view of the flow of money and goods, which also holds at an aggregate level and involves the recording of economic events as effects on the accounts, kept in the general ledger (Figure 3).

Apart from their usual application in accounting, these models can be applied to continuous auditing or continuous assurance (Vasarhelyi et al. 2004). They may also be used to assess and improve data integrity (Christiaanse and Hulstijn 2011). Data integrity is not a given, but needs to be established. The apparent redundancy (double entry) represents the flow of goods and money buttressing all business contracts, and has to be considered as essential to assess and improve data integrity: it creates a objective measurable norm against which integrity can be sustained and thus verified.

References


