Stellingen

behorende bij het proefschrift

A SINGLE-CHIP MULTI-CHANNEL OPTICAL TRANSMISSION SYSTEM

van Dick van den Broeke
1. Het grote voordeel van puls positie modulatie (PPM) als coderingstechniek is dat eenvoudige modulator- en demodulator schakelingen gebruikt kunnen worden voor de kwalitatief hoogwaardige overdracht van breedbandige analoge signalen.
(dit proefschrift, H.3)

2. In de aanwezigheid van additieve ruis met een vlak vermogensdichtheidspectrum is de variantie in de positie van een puls evenredig met de energie van de naar de tijd gedifferentieerde puls.
(dit proefschrift, H.3)

3. In geïntegreerde bipolaire circuits is de kleinst haalbare schakeltijd circa het tienvoudige van de transitietijd van de gebruikte transistoren.
(dit proefschrift, H.5)

4. Het dynamisch bereik van in bipolaire processen geïntegreerde tijdcontinue filters kan aanmerkelijk vergroot worden door de filters stapsgewijs in plaats van over een continu bereik af te stemmen.
(dit proefschrift, H.7)

5. Het motto "alles wordt digitaal" kan zonder gebruik te maken van analoge geluidstrillingen niet eens uitgesproken worden.

6. Het ontwerpen van analoge schakelingen is vooral de kunde van het democratisch organiseren van eigenzinnige elementen.

7. Het toepassen van overall tegenkoppeling resulteert niet altijd in het optimale circuit.
8. Het verband tussen de behaalde kwaliteit van een circuit en de benodigde ontwerpsspanning hiervoor is in benadering exponentieel.

9. De grootschalige vervanging van koperkabel door glasvezelkabel wordt minder urgent door het beschikbaar komen van efficiënte datareductietechnieken.

10. Terwijl de glasvezel door technologische innovaties steeds transparanter wordt, gebeurt met de ons omringende lucht juist het omgekeerde.


12. De stelling "een goed produkt verkoopt zichzelf" gaat in ieder geval niet op voor het produkt technologie.

13. De zorgvuldigheid waarmee de bril, welke je ongemerkt krijgt aangemeten bij het lezen van menig tijdschrift, gefocuseerd is kan zelfs door een goede opticien nauwelijks geëvenaard worden.

14. De kappersbranche heeft zijn succes tenminste evenveel te danken aan de uitvinding van de spiegel als aan de uitvinding van de schaar.
A Single-Chip Multi-Channel Optical Transmission System

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door

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Chapter 1

Introduction

Due to a considerable reduction in their cost, optical fibers have become an interesting alternative to traditional copper (coaxial) cable. Apart from the costs of their installation, both types of cable cost about US$0.5 per meter.

Compared to copper cables, optical fibers have several important advantages: (1) their bandwidth is in the order of 30THz, which is orders of magnitude higher than the bandwidth of (coaxial) copper cables, (2) the losses are less than 0.3dB/km, whereas coaxial cables exhibit losses larger than 50dB/km at frequencies of 100MHz or higher, (3) they radiate no electromagnetic field, nor are they susceptible to external electromagnetic interference and (4) they have lower weight. However, optical fibers require electro-optic and opto-electric conversion, provided by respectively light emitting diodes (LEDs) or semiconductor lasers and photodiodes, as well as driving circuitry. In addition, connecting fiber ends to the optical transmitters and receivers or to other fiber ends is relatively difficult. Consequently, despite its intrinsic advantages, optical fiber only gradually replaces copper cable.

Generally, in deciding between optical fiber and copper, the total cost per link (for point-to-point transmission) or per subscriber (for networks) is decisive. This depends on the transmission length, the nature of the signals, and, in case of applications in a network, on the hierarchical level. In many cases, we have to consider additional costs such as the depreciation costs of the existing links or networks.

As an illustration, Fig.1.1, which was provided by the Dutch PTT [1], depicts the cost of introducing optical fiber into the local telephone and cable television (CATV) networks. The fiber to the curb (FTTC) option for both "plain old telephone services" (POTS) and CATV, in which traditional copper twisted pair and coaxial cable cover the link from the curb into the home, is relatively cost effective, because the cost of the opto-electric conversion equip-
Figure 1.1: Costs per subscriber in US$ of FTTH and FTTC concepts compared with the cost of traditional copper cables.

ment are spread over about 10 subscribers. As the production volume of the equipment increases, FTTC gradually becomes available at lower cost than the traditional copper option, because one single fiber (of average length 2km) replaces several copper cables. As fiber to the home (FTTH) requires opto-electric receivers and electro-optic transmitters for each subscriber, FTTH is much less cost effective. Integrating CATV services with POTS results in additional costs because the transmitter equipment for CATV is much more expensive.

Regarding the replacement of copper by fiber, we should bear in mind that, beside the cost to the proprietor of the network, we also have to consider the cost to the subscribers. One cannot expect consumers to replace their telephone and television sets with new types which provide the required optical (and possibly digital) inputs and outputs.

In some other applications, the nature of optical signals will be a decisive factor, rather than the cost. For instance, as the absence of electromagnetic radiation prevents tapping of the transmitted signals and avoids interference to other equipment, optical fiber might be preferred in military or aircraft applications. In the video surveillance of subway stations, fibers are preferable because of their insensitivity to electromagnetic radiation caused by trains.

The outlook for optical fiber is promising and has stimulated the research and development of (high performance) semiconductor LEDs and lasers, photodiodes, various types of optical fiber, optical connectors, optical splitters,
optical amplifiers and driving circuitry. As optical fiber enters the telecommunications market, the costs of these building blocks become increasingly important.

Within the scope of this thesis, we will concentrate on building blocks for relatively simple local area networks, having applications in, for example, video conferencing, (video) surveillance and monitoring, local video distribution and the attachment of workstations to their mainframes. As the transmission lengths will be relatively short and since costs cannot be averaged over many subscribers, a low cost per unit is of primary concern. Our aim has been to develop "minimal-cost" electro-optic transmitters and opto-electric receivers, which are suitable as universal building blocks in various types of point-to-point links and networks. Such building blocks must be capable of handling various analog as well as digital signals. In addition, to bring down the cost per channel, multiplexing two or more signals into one optical fiber must be possible.

Before discussing the ideas that have been the bases for the development of the low-cost universal building blocks, we will present a brief overview of existing fiber optic transmission techniques and systems.

The best performance, in terms of information throughput, is achieved by so-called optical heterodyne systems [2], [3]. In these systems the phase or the amplitude of a coherent lightwave is modulated as if it were a radio wave. Integrated optical structures performing (de)multiplexing, (de)modulation as well as, for example, wavelength filtering, allow for the use of efficient coding techniques. A lot of effort has been put into research into coherent techniques but, unfortunately, large scale application is a long way off.

Many practical systems for a wide range of applications have been developed using commercially available light emitting diodes (LEDs) or lasers and photodiodes. In those systems, signals modulate the total intensity of the emitted light, which is not necessarily monochromatic. As modulation and multiplexing functions cannot be performed in the optical domain, the total transmission bandwidth available is determined by the modulation bandwidth of the optical transmitters and receivers and the bandwidth of the driving circuitry. Systems based on various types of (electrical) modulation that have been published or are:

- baseband modulation [4], [5], [6].

- (pulse) frequency modulation [10], [7], [8].

- pulse width modulation [10], [11], [12].

- frequency division amplitude modulation [14], [15].
• frequency division frequency modulation [13], [16], [17], [18], [19].

• digital modulation [20], [21].

The first three categories are essentially simple analog systems for the transmission of one signal. The frequency division systems allow for real-time multiplexing of several signals, but are rather complicated and require extensive hardware. Mainly because of the filters needed, the hardware needs to be implemented using discrete techniques.

The digital systems, also providing the possibility of multiplexing, are relatively complicated, but their hardware is well suited for integration in ICs, reducing the cost of the electronic circuitry (provided that the production numbers are large enough). As, the requirements to the linearity of the optical transmitters are also low, digital systems can be realized at relatively low cost. Hence, digital solutions seem to provide the "ideal" building blocks. However, digital systems require a relatively large chip area and supply power while, at the current state of art, high-performance analog-to-digital and digital-to-analog conversion is still a problem for wideband signals such as CATV signals [1].

The system presented in this thesis is based on analog coding and time-division multiplexing. In this way, analog-to-digital and digital-to-analog converters and circuits providing frequency selectivity are not required, so the complete electronic circuitry can be easily integrated in ICs. In addition, as the signals are coded in the timing of optical pulses (so called time division pulse position modulation, TDPPM), LEDs or lasers exhibiting high linearity (i.e. high-cost components) are not required. Because the costs of the ICs and the optical components are an important criterion while the highest performance is generally not required, in designing the system we preferred low cost over optimal performance. This implies, for example, simple circuitry, a minimum of external components and no special measures to reduce the noise produced by the laser diode. Provided that the production numbers are large enough to keep the cost of the ICs low, the total cost of the electro-optic and the opto-electric building blocks will be an order of magnitude lower than the cost of the modules that are currently available.

In Chap.2, we start by giving a brief introduction into the physics of optical transmitters, receivers and fibers as well as an overview of their general characteristics in terms of noise, power levels, bandwidth and distortion. Based on this knowledge, in Chap.3 we will conclude that TDPPM coding has strong preference over other coding schemes. In Chap.4, the transmitter and the receiver will be divided into functional blocks. Restricting ourselves to a bipolar technology for reasons of speed, in Chap.5, we will deal with the limitations of the electronic building blocks. In Chap.6, we will determine the general
specifications of the system such as the total available transmission bandwidth and the signal-to-noise ratio per channel. In Chap.7, the implementation of the electronic circuits will be discussed. We will end by conclusions in Chap.8.

Throughout Chapters 5..7, our discussions will be illustrated by the evaluation of a prototype system which has been realized in a 2.5GHz bipolar process. It is capable of simultaneously transmitting four semi-professional quality, full color PAL-coded TV signals.
Bibliography


Chapter 2

Optical Components

In this chapter, we discuss the physics of the optical components used: LEDs or lasers, photodiodes and fibers. Our aim is to examine their major characteristics such as modulation bandwidth, emitted power, noise performance, distortion and fiber dispersion. Later on, in Chap.3, the coding scheme will be optimally adapted to the properties of the optical components and, in Chap.6, we will determine their influence on the system specifications.

2.1 Light Emitting Diodes

As shown in Fig.2.1, light emitting diodes (LEDs) consist of a small-bandgap active layer (P or N) sandwiched between two other layers of which one is of the same type as the active layer (P or N) and the other is of the opposite type [1]. The function of the “anisotype” (upper) junction (P/N or N/P) is to

Figure 2.1: Cross section of an LED consisting of a N/P/P sandwich. Light is emitted from the active layer in all directions.

inject minorities into the active layer. To obtain high injection efficiency, the
upper side layer should exhibit a larger bandgap energy than the active layer. The isotype junction (P/P or N/N) is to confine the injected carriers in the active layer, also requiring the lower side layer to have a larger bandgap than the active layer. As the injected minorities recombine, photons are emitted with a wavelength corresponding to the bandgap energy of the active layer.

2.1.1 Power Coupled into the Fiber

The internal quantum efficiency of an LED (i.e., the ratio between emitted photons and injected electrons) can be relatively high, up to 50% [2]. However, light is emitted in all directions and only a fraction of it strikes the crystal surface at an angle less than the critical angle, which is in the order of 16°, and can thereby escape through one of the surfaces into the air. The imperfect coupling to the fiber causes a further degradation. Assuming that the core diameter of a fiber having a step-index profile is large compared to the LED surface, the coupling efficiency is approximately the square of the numerical aperture of the fiber (NA=0.2–0.5). The coupling efficiency into graded index fibers is about half the square of the NA. Consequently, the power transmitted into a fiber by an LED will be relatively low. Practical values are in the order of 50–500μW for driving currents in the range of 50–500mA.

2.1.2 Bandwidth

As the intensity of the radiated light is directly proportional to the density of the injected minorities, the modulation bandwidth depends on how fast this density changes with the injected current. When the minority lifetime is given by \( \tau \), the modulation bandwidth (-3dB bandwidth) is approximately equal to \( \frac{1}{2\pi\tau} \).

To improve the bandwidth, \( \tau \) can be shortened by increasing the doping density of the active layer: for example, in Gallium Arsenide [2], a doping density of \( 10^{17}\text{cm}^{-3} \) corresponds to \( \tau=10\text{ns} \), while increasing the density to \( 10^{19}\text{cm}^{-3} \) results in \( \tau=1\text{ns} \). Using this technique the maximal bandwidth reported is in the order of 200MHz.

Unfortunately, as high doping densities widen both the valence and the conduction band, the emitted photons have different energies, resulting in a wider emission spectrum. Typical values range from a spectral width of 35nm for low to a width of 100nm for high doping densities. Due to fiber wavelength dispersion (also called material dispersion), causing a wavelength dependent propagation delay of the fiber, a wider emission spectrum results in a decreased fiber bandwidth. To determine how this impairment depends on the wavelength, we refer to Fig.2.2, depicting the dispersion (i.e., the maximal dif-
ference in the delay times, expressed in ns per nm per km fiber length) of a standard fiber. If the maximal difference of the delay times for a given spec-

![Graph](image)

**Figure 2.2:** Wavelength dispersion of glass fiber.

tral width and fiber length is denoted by $\Delta t$, in accordance with [2], the fiber bandwidth equals approximately:

$$B_f = \frac{1}{2\Delta t}$$

(2.1)

Fig.2.3 shows the resulting overall bandwidth of the LED and the fiber as a function of the transmission length for two doping densities and for two wavelengths, 800nm and 1300nm respectively. If the transmission length is short, the bandwidth is limited by the modulation bandwidth of the LED. For a long transmission length, the bandwidth equals $B_f$ (which is limited by the fiber material dispersion in conjunction with the emission spectrum). The dispersion is minimal if the wavelength is about 1300nm, resulting in maximal fiber bandwidth.

This clearly demonstrates that the technique of increasing the doping densities only helps increase the overall bandwidth for short distances. Apart from an increased spectral width, an additional disadvantage of high doping densities is an increased amount of nonradiative recombination, causing a severely reduced power efficiency. So, even at short distances, this technique of bandwidth improvement is not recommended.
Figure 2.3: Overall bandwidth of an LED-fiber system as a function of transmission length for LEDs having doping densities of $10^{17}$ cm$^{-3}$ and $10^{19}$ cm$^{-3}$ and wavelength 800nm and 1300nm.

2.1.3 Distortion

Low-frequency distortion is caused by heating of the active region, which results in modulation of the amount of radiative recombination and photon absorption. A more subtle distortion mechanism is a current crowding effect [2]. Commercially available LEDs exhibit harmonic and intermodulation distortion products in the range of 30–40dB below the signal level.

2.1.4 Noise

In an LED two types of noise occur. First, shot noise is produced by the injection of minority carriers into the active layer and their spontaneous recombination when a photon is emitted. But, since only a small fraction of the light power is actually coupled into the fiber, this noise is negligible compared to the shot noise produced by the photo detector (Sec.2.3). Secondly, excess noise, which depends on the spectral width of the light and its power, has been observed [3]. Generally, this contribution too is below the detector shot noise.

2.2 Laser diodes

Basically, a semiconductor laser diode consists of the same sandwich structure as an LED. A fraction of the injected minorities recombines spontaneously while emitting photons; so-called spontaneous emission. Due to reflections at the layer-air facets of the active layer, and since its length equals a multiple of
2.2. LASER DIODES

the desired wavelength, the active layer serves as an optical resonator, resulting in an intensity maximum at a specific wavelength. However, as the optical waves are only partly reflected (≈ 30%), the resonator exhibits considerable losses. In lasers, compensation of these losses is obtained by increasing the minority carrier density above the "transparency density", so that photons that are accidentally absorbed within the active layer will at the same time stimulate the emission of new photons with the same wavelength (energy) and phase; so-called stimulated emission.

Since emission of photons, either spontaneously or stimulated, requires recombination of minorities, an increasing number of photons corresponds to a loss of free carriers. So, to maintain the "lasing" effect, new minorities have to be supplied by the injection current.

Fig.2.4 depicts the typical injection current-to-optical intensity characteristic of a laser diode. For low injection currents, i.e. below the threshold

![Graph showing laser output power vs injection current for various temperatures](image)

Figure 2.4: Typical characteristic of the laser output intensity versus injection current for various temperatures.

...
As a result, if the threshold current is exceeded, almost 100% of the injected minorities contribute to the stimulated emission, so the current-to-power characteristic becomes much steeper than below the threshold current.

As transparency density and non-radiative recombination are strongly temperature dependent, the characteristic of Fig. 2.4 is rather temperature dependent.

2.2.1 Power Coupled Into the Fiber

As laser light is collimated in a narrow beam along one single axis, a high percentage of the emitted light escapes from the active layer without being reflected, so high laser-to-fiber coupling efficiencies can be obtained. Overall quantum efficiencies are in the order of 10%. Practical devices transmit about 0.5–2.5mW into a single mode fiber at a driving current of 5–50mA above the threshold current.

2.2.2 Bandwidth

If the injection current is below the threshold current, the modulation bandwidth will be relatively small because, as in LEDs, the bandwidth depends on the lifetime of the injected minorities.

Above the threshold, the "feedback" mechanism stabilizes the minority density, so significantly decreasing the influence of the minority lifetimes. The improvement factor of the resulting modulation bandwidth roughly corresponds to the "loop gain" of the feedback mechanism.

To calculate the modulation bandwidth, we elaborate a laser model based on rate equations, describing the increase and decrease in the carrier and in the photon density per unit of time [1], [2] and [3]. In App.B, it is shown that the current-to-light transfer function of a laser diode is that of a second-order low-pass filter which exhibits a relatively poor damping factor. The modulation bandwidth $B_1$ is assumed to be equal to the oscillation frequency $f_1$ of the laser pulse response. So, provided that the laser operates sufficiently high above threshold, $B_1$ can be approximated as:

$$B_1 = f_1 \approx \frac{1}{2\pi} \sqrt{\frac{g n_o I - I_{th}}{\tau_n}} \frac{I_{th}}{I_{th}} \quad (2.2)$$

$$I_{th} = \frac{V n_o q}{\tau_n} \quad (2.3)$$

The parameter $g$ (m$^3$s$^{-1}$) is the average of the net increase of photons per unit of time per unit of photon density, $V$ (m$^3$) the active laser volume, $I$ the driving current, $I_{th}$ the threshold current, $q$ the electron charge, $n_o$ (m$^{-3}$) the
carrier transparency density and \( \tau_n \) the average minority lifetime. For practical lasers, operating at room temperature, some typical values are \( g = 10^{-12} \text{m}^3 \text{s}^{-1} \), \( V = 2 \cdot 10^{-16} \text{m}^3 \), \( n_o = 1.7 \cdot 10^{24} \text{m}^{-3} \) and \( \tau_n = 2.5 \text{ns} \), giving \( I_{th} \approx 20 \text{mA} \). If \( I \) is in the range of 10–100% above \( I_{th} \), \( B_1 \) is in the range 1.3–4.1GHz. We performed measurements on several commercially available multimode lasers, and found similar values of \( I_{th} \) and \( B_1 \).

As the laser cavity helps to narrow the emission spectrum (to less than 5nm), the fiber wavelength dispersion is considerably lower than with LEDs. The total bandwidth of the (multimode) laser-fiber combination (the influence of mode dispersion is ignored) is depicted in Fig.2.5. Obviously, when

![Figure 2.5: Overall bandwidth of a laser-fiber system as a function of transmission length for wavelengths 800nm and 1300nm.](image)

using lasers emitting at 1300nm, the fiber wavelength dispersion is practically negligible.

### 2.2.3 Distortion

As with LEDs, distortion results from junction heating, modulating the minority lifetime, the transparency density and the cavity length. In turn, the cavity length influences the center wavelength and the photon lifetime. The most simple semiconductor lasers exhibit harmonic and intermodulation distortion levels in the order of -35dB below the signal level. Multimode lasers, in which no special measures have been taken to inhibit secondary lasing modes, may even exhibit discontinuities in their current-to-light characteristic as the laser “jumps” from operating in one mode to another. Special types of single mode lasers have distortion levels of about -50dB.
2.2.4 Pulse Modulation Properties

Later, we will conclude that pulse position modulation (PPM) is to be used, so the laser diode will be driven by short current pulses. Due to the nonlinearities of the lasing mechanism, the laser pulse response cannot be predicted from small signal analysis. In [18] the pulse response is estimated by solving a simplified differential equation. We summarize the results and draw some conclusions.

To avoid long settling times between two laser pulses due to the relatively large carrier lifetime $\tau_n$, the bottom level of the modulating pulse current should not come below $I_{th}$. That is, the laser should never come into the "LED mode".

Fig. 2.6 shows a typical output pulse of a laser biased above threshold and driven by a rectangular current pulse. According to the approximation pre-

![Graph](image)

Figure 2.6: Typical laser output pulse response on a rectangular input pulse.

sented in [18], pp.82 and 83, starting at time $t=0$ when the current pulse is supplied, the photon population $S$, which is a direct measure for the laser output power, increases exponentially with the square of $t$ until $S$ has reached its stationary level $S_{on}$ at $t = T_{on}$:

$$S(t) = S_{bot} \exp\left(\frac{gn_o I_{on} - I_{bot} t^2}{2\tau_n I_{th}}\right)$$

(2.4)

in which $S_{bot}$ is the pulse bottom photon population, $I_{bot}$ is the pulse bottom current and $I_{on}$ is the pulse peak current.
2.2. LASER DIODES

Using Eq.(2.4), we calculate the turn on time to be:

\[ T_{on} = \frac{\sqrt{2}}{\sqrt{\frac{g_m}{\tau_n} \frac{l_{on} - l_{bot}}{l_{th}}} \sqrt{\ln S_{on}}} = \frac{\sqrt{2}}{\frac{g_m}{\tau_n} \frac{l_{on} - l_{bot}}{l_{th}}} \sqrt{\ln \frac{I_{on} - I_{th}}{I_{bot} - I_{th}}} \]  

which, provided that \( I_{bot} - I_{th} \ll I_{on} \), can be approximated by using Eq.(2.2) as:

\[ T_{on} = \frac{\sqrt{2}}{2\pi f_1} \sqrt{\ln \frac{I_{on} - I_{th}}{I_{bot} - I_{th}}} \]  

In this expression, \( f_1 \) denotes the laser oscillation frequency given by Eq.(2.2) for \( I = I_{on} \).

After turning on, the photon population stabilizes gradually to its stationary value. To understand the oscillations shown in Fig.2.6, we should bear in mind that, besides the minority lifetime \( \tau_n \), the laser incorporates a second time constant, the photon lifetime \( \tau_p \). In practice, the two time constants result in complex poles, so the laser pulse will exhibit oscillations during settling. Since the relative variations of minority and photon densities are small, the stabilization behavior can be estimated by the small signal model of App.B. Eq.(2.2) already gives the ringing frequency. The damping time constant is approximated to be:

\[ \tau_d = \frac{1}{4\pi^2 f_1^2 \tau_p} \]  

The turn off response follows from Eq.(2.4) by exchanging \( I_{bot} \) and \( I_{on} \) and substituting \( S_{on} \) for \( S_{bot} \). It is easily shown that \( T_{bot} \) equals \( T_{on} \). To estimate the settling time after \( S(t) \) has crossed the pulse bottom population \( S_{bot} \) once more, the small signal model can be elaborated.

To determine the practical values of \( T_{on} \) and \( T_{off} \), we measured \( f_1 \) of the laser used in our prototype system. Substituting \( f_1 = 3 \text{GHz} \) and \( \frac{l_{on} - l_{bot}}{l_{th}} = 5 \) in Eq.(2.6) yields \( T_{on, off} \approx 100 \text{ps} \). As a result, the minimal pulse width that can be handled by the laser is in the order of \( T_{on} + T_{off} \approx 200 \text{ps} \).

Since we wish to prevent interaction between two successive pulses, the spacing between the pulses must be large enough to settle the laser pulse tail close enough to its stationary value before the next pulse is supplied. The worst case settling time follows from assuming the initial settling error to be equal to the pulse amplitude. If we assume exponential damping with time constant \( \tau_d \), the remaining settling error is less than 1% if the pulse spacing is at least \( 5\tau_d \) and less than 0.1% with a spacing larger than \( 7\tau_d \). By substituting \( f_1 = 3 \text{GHz} \) and \( \tau_p = 2.5 \text{ps} \) into Eq.(2.7), we find \( 5\tau_d \approx 5 \text{ns} \) and \( 7\tau_d = 7 \text{ns} \).

So far, we assumed rectangular current pulses, whereas the rise and fall times (denoted by \( T_r \) and \( T_f \)) of practical pulses of course are finite. If \( T_r \) and
$T_f$ are large compared to $T_{on,off}$, the waveform of the laser pulse will track the waveform of the current. Consequently, as pulse ringing will be much less pronounced, the settling times can be reduced by choosing rise and fall times of the current pulses that are not too short. Experiments showed that a total pulse width of 2ns is feasible.

2.2.5 Noise

A suitable measure for laser noise is the so-called relative intensity noise (RIN), which is the ratio of the noise power emitted by the laser in a 1Hz bandwidth and its total emitted dc power.

The fundamental minimum of the noise produced by lasers is determined by quantum processes in the laser cavity [6] and [7]: the injection of minorities, spontaneous emission, absorption of photons, photons escaping from the cavity and, in particular, stimulated emission produce shot noise. Using the model presented in [3], App.B.3 calculates the RIN resulting from shot noise produced by stimulated emission. For frequencies below $2\pi f^2 T_p$ (when assuming the same parameters as above, this is in the order of 160MHz), the RIN is frequency independent:

$$\text{RIN} = \frac{2q}{\gamma I_{th} (g n_o \tau_p)^2} \left( \frac{I_{th}}{I - I_{th}} \right)^3$$

(2.8)

in which $\gamma \approx 3$ is a noise excess factor, denoting that the actual rates of photon absorption and photon emission related to stimulated emission are a factor $\gamma$ larger than the net rates. Substituting practical parameter values in Eq.(2.8) yields RINs ranging from -155 to -185dB/Hz for currents 10–100% above threshold. For frequencies higher than $2\pi f^2 T_p$, the RIN increases proportionally with frequency, exhibiting a maximum at $f_1$.

Unfortunately, in low-cost lasers (except at frequencies in the order of $f_1$) the contribution of shot noise is negligible compared to a second cause of laser noise, induced by waves reflecting back from the fiber into the laser. Noise resulting from reflection ratios as low as $10^{-4}$ may exceed the laser shot noise by 20–30dB. This complicated phenomenon is the subject of discussion in [9], [10], [11], [12], [13] and [18]. The reflections can be reduced by applying so-called optical isolators, but they are costly.

A third cause of noise stems from the spectral properties of the laser light in conjunction with the wavelength dependent transmission characteristics of the complete laser-fiber-photodiode system. Due to their construction, low-cost multimode lasers do not emit monochromatic light. Their emission spectrums rather consists of several peaks. As the laser stabilization mechanism only stabilizes the total photon population and not the intensity fluctuations in the individual modes, for noise frequencies up to a few tens of MHz, the RIN of each
of these emission peaks is about 30dB higher than the RIN of their sum [14]. If the coupling efficiencies from laser to fiber and from fiber to photodetector as well as the efficiency of the photodetector are not wavelength dependent, this so-called partition noise is of no concern. However, if they are, partition noise may dominate the contributions of shot noise and reflections. Single mode (monochromatic) lasers of course do not suffer from this impairment.

Finally, additional noise results if high-frequency laser phase noise is converted into amplitude noise by fiber wavelength dispersion [15], [16] and [17]. Generally, this effect can be ignored.

Note that we treated laser noise for the stationary case with no modulation. Because experiments have shown that it is a reasonable approximation, we will base further calculations on the assumption that the laser noise is additive to the output intensity.

The RIN of the 1300nm multimode lasers adopted in our prototype was measured to be in the order of -125dB/Hz.

2.3 Photodiodes

Photodiodes typically consist of an intrinsic (I) active layer sandwiched between a highly doped N and P layer (a so-called PIN diode). Within the I layer, photons are absorbed while generating free electrons and holes. Reverse biasing by an external voltage creates an electrical field across the I layer, driving electrons and holes to the N and P regions respectively. The collected carriers result in an electrical current proportional to the optical power received.

Since the refractive index of the I layer is relatively high, a coupling efficiency between fiber and diode close to 100% can be obtained by choosing the surface area of the diode larger than the fiber diameter. To absorb photons, the bandgap energy of the I layer should be lower than the photon energy so that silicon structures are only suited for wavelengths shorter than about 1000nm. Longer wavelengths require InGaAsP structures. Provided the active layer is wide enough (about 10μm), PIN devices achieve overall quantum efficiencies of approximately 100%.

In so-called "avalanche photodiodes" (APDs) [2], the photo current is amplified before it is supplied to the receiver electronic front-end. Because the amplification reduces the influence of the noise produced by the electronic circuitry, a higher sensitivity can be achieved. In between the I layer and the
highly doped P (or N) layer of an APD, a lightly doped P (or N) layer is inserted. By supplying a high voltage across this layer, the carriers that have been collected from the I layer ionize new carriers. In this way, current gains up to a factor 100 are possible.

The fundamental maximum of the bandwidth of a PIN diode depends on the width of the I layer and the velocities of the electrons and holes. If the electric field is sufficiently strong, the carriers reach their saturated drift velocities, which are $8.4 \cdot 10^6 \text{cm s}^{-1}$ for electrons and $4.4 \cdot 10^6 \text{cm s}^{-1}$ for holes. For instance, a layer width of $10\mu\text{m}$ restricts the bandwidth to about $4\text{GHz}$.

The bandwidth of APDs is restricted in the same way, since free carriers are to be collected first before multiplication. Theoretically, the bandwidth of APDs is also limited by the gain-bandwidth product of the amplification mechanism, but this product can be made large enough by proper construction and suitable doping profiles.

Noise in PIN devices consists of pure shot noise originating from the dark current of the diode and the photo current $I_{\text{ph}}$. In short distance applications, the dark current is small enough to be ignored, so the current noise power density equals

$$S_{\text{in}} = 2qI_{\text{ph}}$$  \hspace{1cm} (2.9)

APD detectors produce excess noise caused by the amplification mechanism. Depending on the doping profiles and temperature, noise excess factors, which relate the current-to-noise ratio of an APD to that of a PIN, are in the range of 4–10dB.

PIN diodes show excellent linearity over 6 decades of photo current, up to power levels of about 1mW where saturation effects decrease the carrier velocity, causing high-frequency distortion. The distortion of APDs increases for power levels higher than about 50$\mu$W, but in this region APDs generally do not provide a higher receiver sensitivity than PINs, because then the noise produced by the electronic circuitry no longer dominates.

### 2.4 Optical Fibers

Generally, as accurate positioning is costly, the costs of connecting optical fibers to LEDs, lasers, photodiodes or other fibers increase with decreasing fiber core diameter. So, in low-cost systems, fibers having a relatively large core diameter seem preferable. However, in choosing the appropriate type of fiber, we also have to consider the consequences with regard to the coupling efficiencies, fiber losses, fiber bandwidth and reflection properties. The later one is important because it influences the laser RIN as well as the crosstalk and distortion levels in TDPPM systems (to be discussed in Chap.3).
As concluded in Sec.2.1.1, the coupling efficiency of an LED depends on the numerical aperture (NA) of the fiber and its diameter $\phi$ compared to the LED surface area. Because of their large NA (typically 0.4) and $\phi$ (about 600$\mu$m), the highest coupling efficiencies are achieved when using plastic fibers. Multimode step index glass fibers (NA=0.16, $\phi$=100$\mu$m) are the second best, performing slightly better than their graded index counterparts. Theoretically, if sufficiently small LEDs were available, similar coupling efficiencies could be obtained when using single mode fibers. When applying lasers, due to their narrow collimated beams, high coupling efficiencies are obtained in conjunction with any type of fiber.

Glass fibers exhibit low attenuation, typically 3dB/km at a wavelength of 800nm and 0.3dB/km at 1300nm. Plastic fibers, on the other hand, exhibit attenuations of about 200dB/km, making them applicable only for very short distances.

The fiber bandwidth is determined by material dispersion or mode dispersion. Material dispersion, causing wavelength dependent transmission delays (Secs.2.1.2 and 2.2.2), does not depend on the index profile or the diameter of the fiber. When using LEDs, according to Fig.2.3 (at a long transmission length, where the bandwidth of the LED is not the limiting factor), bandwidth-length products are in the range 30–100MHz-km (800nm) or 300–1000MHz-km (1300nm). With multimode lasers (Fig.2.5) we find 1GHz-km (800nm) or 1THz-km (1300nm), whereas with single mode lasers still higher values are obtained.

Mode dispersion, meaning that light is conducted through the fiber along several alternative "routes", exhibiting different transmission delays, typically occurs only in multimode fibers. As the number of possible modes increases with the fiber core diameter $\phi$, the resulting bandwidth-length product decreases with increasing $\phi$. Typical values are 20MHz-km for $\phi$=200$\mu$m and 9MHz-km for $\phi$=600$\mu$m. Plastic fibers ($\phi$=1mm) achieve a bandwidth-length product of about 250KHz-km. Graded-index fibers perform considerably better: 1–4GHz-km.

In regarding laser noise, reflections from the fiber ends back into the laser cavity should be minimized. Since a laser beam is strongly collimated, a high coupling efficiency between laser cavity and fiber is obtained. Reflections from a single mode fiber are still collimated and will therefore easily enter the laser cavity again. As, on the other hand, reflections from multimode fibers are only partly capable of entering the laser cavity, the sensitivity for reflections is one order of magnitude lower.

Since reflections occur at the end of the fiber, sophisticated fiber cutting techniques may help in reducing reflections. Furthermore, optical isolators, based on polarization filters and rotation of the electromagnetic field and con-
ducting light only in one direction, can be inserted between the laser and the fiber. Unfortunately, with the current state of the art these techniques are not compatible with low cost.

When using standard connectors, exhibiting reflection coefficients of about 2% (-17dB), typical values of the "back into the cavity" reflection coefficients are 2% for single mode and about 0.2% for multimode fibers.

2.5 Summary and Conclusions

In this chapter, we discussed the most important characteristics of the low-cost optical components, which are LEDs, multimode laser diodes, PIN and APD photodiodes and different types of optical fibers. With regard to the choice of the coding scheme and the optical components to be used in a specific application, we can summarize the following conclusions:

Laser diodes were found superior to LEDs with respect to the modulation bandwidth (1–4GHz compared to 20–200MHz) and the power transmitted into the fiber (0.5–2.5mW compared to 50–500μW). In addition, due to the much higher coupling efficiencies, laser diodes require lower drive currents than LEDs. However, since special measures preventing fiber reflections from entering the laser cavity are not compatible with low cost, the relative intensity noise (RIN) of the laser light is in the order of -120 to -130dB/Hz. As a consequence, over a large range of received power levels, the laser is likely to be the dominant noise source rather than the receiver front-end.

Since the bandwidth of LEDs is limited to about 200MHz, the minimal pulse width is in the order of 10ns. We estimated that the minimal pulse width that can be handled by lasers is in the order of 2ns. The linearities of the low-cost LEDs and laser diodes are rather poor.

PIN photodiodes achieve a bandwidth comparable to that of the laser, achieve high quantum efficiencies, contribute only quantum determined noise and exhibit negligible distortion. For low received power levels, the noise contributed by the amplification mechanism of APDs is less than the noise produced by an electronic amplifier. So, for low power levels, the use of APDs may be considered. The bandwidth capabilities of APDs are comparable to those of PINs.

If a modulation bandwidth higher than 200MHz is required, lasers should be used. Generally, the received power level will be high enough to achieve the maximal receiver sensitivity by applying a simple PIN photodiode. If a relatively small modulation bandwidth is sufficient, LEDs can be used as well, combined with PINs or, if a higher receiver sensitivity is required, APDs.

To decrease the influence of wavelength dispersion, 1300nm components
or, preferably, monochromatic sources should be used. To decrease mode dispersion, we have to use (in order of increasing effectiveness) fibers having a small core diameter, graded-index fibers, or single mode fibers. However, the complexity of fiber coupling techniques, and thus cost, increases accordingly. In addition, fiber-end reflections become more harmful to the laser RIN.

As the prototype system is developed for the transmission of (“wideband”) video signals over relatively short distances, and because 800nm lasers were difficult to obtain, we decided to use a multimode 1300nm laser and a standard PIN photodiode. Since the laser and PIN are mounted in optical connectors, different types of fiber can be attached. In addition, we carried out some successful experiments with low-cost (US$2) 680nm lasers that are used in consumer CD players. As their mechanical positioning is relatively inaccurate, they can only be combined with multimode fibers.
Bibliography


Chapter 3
Coding and Multiplexing

3.1 Introduction

The first reason for using coding techniques is that coding makes possible the multiplexing of several signals into one composite signal. The second reason is that coding reduces the sensitivity to noise and other imperfections of the transmission channel such as non-linearity. Having examined the major properties of the optical components in Chap. 2, and bearing in mind that the coding and decoding circuitry is to be integrated in silicon, we will propose so-called time division multiplex pulse position modulation (TDPPM).

Next, we will optimize the shape of the transmitted TDPPM pulses and determine how the received TDPPM pulses should be handled to obtain “reshaped” pulses with minimal jitter. Since the “optimal” pulse shape and method of reshaping require impractical hardware, some modifications will be proposed. We will be using pulses of which the shape can be approximated by trapezoids and, in the receiver, a simple low-pass pulse reshaping filter instead of a matched filter. Compared to the optimal solution, the performance of this alternative will be shown to be only slightly inferior. In addition, we will examine the output signal-to-noise ratio ($\text{SNR}_o$) of the TDPPM system as a function of the transmission bandwidth and the SNR of the received pulse signal, and determine the efficiency of TDPPM coding in terms of channel capacity.

To provide the so-called frame synchronization, that is, to distinguish which of the multiplexed TDPPM pulses corresponds to a specific signal and which of the pulses should be used to synchronize the receiver clock regenerator, which is necessary for demodulation, we will propose a small modification to the shape of particular TDPPM pulses.

Next, we will discuss the distortion caused by nonuniform sampling of
the input signal and by two different types of demodulation techniques. In Chap.7, it will be shown that this type of distortion cannot be avoided because this requires high-performance sample and holds, which are very difficult to implement.

Finally, the sensitivity of TDPPM coded signals for distortion and crosstalk caused by fiber reflections will be determined.

3.2 Hardware Constraints

In designing an appropriate coding scheme, we have to take into account the typical characteristics of the (low-cost) optical components used. In addition, the coding should be such that the required coding, multiplexing, demultiplexing and decoding circuitry can be completely integrated in silicon.

In Chap.2, it was concluded that low-cost LEDs and multimode laser diodes exhibit severe non-linearities, resulting in intermodulation and harmonic distortion products at only 30–40dB below the signal level. As the non-linearities are device specific and depend on temperature, compensation techniques are difficult to implement. In addition, low-cost fiber connection techniques cannot prevent a relatively high percentage of the light from being reflected at the fiber ends, resulting in a further decrease in the accuracy of the amplitude of the received signals. Consequently, direct amplitude modulation schemes cannot be used. If we stay with amplitude modulation, only a limited number of discrete levels can be distinguished properly. Obviously, digital coding, in which generally only two levels are used, is well suited. Alternatively, various angle modulation schemes [1], such as frequency modulation (FM), phase modulation (PM), pulse width or duration modulation (PWM or PDM) or pulse position modulation (PPM) can be considered.

A further consequence of non-linear distortion is that multi-channel systems employing frequency division multiplexing (FDM) will be susceptible to crosstalk. Although crosstalk can be avoided by carefully selecting the carrier frequencies, time division multiplexing (TDM) is obviously preferable. An additional argument for TDM over FDM stems from the difficulty of integrating the required frequency selectivity in ICs.

3.3 Analog Versus Digital Coding

Since the approaches to designing systems based on digital coding are completely different from those based on analog alternatives, a first decision to be made is between analog and digital coding.
Digital systems have two important advantages over analog systems: (1) the fundamental functions such as the storage of signals, subtraction and multiplication are much easier to implement, and (2) once the signals are in digital format, there are no impairments of the signal quality, which is a particularly strong argument if repeaters are necessary. Nevertheless, in this thesis it will be shown that careful design results in an analog system that generally satisfies the requirements in the applications where low costs are a primary concern.

On the other hand, digital systems have two important disadvantages when compared with analog systems: (1) since a relatively large number of gates is required to implement a specific function and since their switching times must be relatively short, the total power consumption and the chip area will be considerably larger, and (2) for wideband signals exhibiting a high dynamic range, the conversion from analog to digital (ADC) and from digital to analog (DAC) becomes increasingly difficult. With the current state of the art (1993), 10-bit ADCs and DACs for standard video signals are available for about US$50,- each. Hence, an important advantage of an analog system is that once the IC has been designed its cost will be less than that of a digital system.

Another important criterion is the total transmission capacity that is available for the signals to be transmitted. As will be shown in Sec.3.7, digital systems achieve a total transmission capacity which is comparable to that achieved by the system based on analog TDPPM coding.

Finally, we should bear in mind that, if we choose the analog solution, this does not imply that only analog signals can be handled. Due to its transparency, the system is capable of transmitting digital signals as well.

In conclusion, the analog solution is preferred because of its relatively low cost. To achieve high performance, proper design is a primary concern.

3.4 Time Division Pulse Position Modulation

Presently, we will concentrate on analog time division multiplex (TDM) angle modulation schemes.

To transmit two or more signals in time division multiplex, the multiplexer switch sequentially selects one of the input signals, so the transmission channel is available for each of the signals only during the corresponding time slots. Basically, to transmit real time signals and to avoid parts of the signals being lost, the selections of the original signals (for example, covering one line of a video signal) have to be compressed in time to fit in the slots, see Fig.3.1. The receiver has to carry out the reverse process.

In TDM, one can exchange long time slots and a low “selection” frequency for short time slots and a high selection frequency. If the length of the slots
becomes short compared to the reciprocal bandwidth of the signals, the selections may consist of samples. So, as time compression is not necessary, the implementation of the multiplexing and demultiplexing circuitry is considerably simpler: the multiplexer consists of a switch and the demultiplexer of a switch and low-pass smoothing filters.

In further discussions, the time available for the transmission of one sample will be denoted as the slot length $T_{sl}$. One complete series of $N$ slots, containing one sample of each of the $N$ input signals, will be called a frame.

Having decided on angle modulation, on time division multiplexing and on sampling, we automatically end up with "pulses" whose timing (position) or width depends on the amplitude of the samples.

To properly distinguish between successive pulses, the pulses should preferably be separated from each other by some guard time. In addition, since non-linearities or bandwidth limitations of the transmission channel lead to inaccuracies in the measured positions or width of the pulses, the signals should preferably be carried by the positions of pulses which are equally shaped and of equal amplitude. Hence, we end up with so-called pulse position modulation (PPM). Some examples of possible shapes are shown in Fig.3.2.

The principle of time division multiplex pulse position modulation (TDPPM) is illustrated by Fig.3.3. The dots on the waveforms of the input signals (1) and (2) denote the samples that are converted into PPM pulses. As the PPM pulses (1) and (2) of the two signals remain within their assigned time slots, multiplexing into TDPPM is performed by summing both pulse trains.
3.5 Pulse Shaping and Pulse Reshaping

The shape of the TDPPM pulses transmitted by the laser diode is determined by the so-called pulse shaper. The pulses received by the photodiode will be converted into pulses with minimal jitter and a fixed amplitude by the "pulse reshaper". The reasons for using the pulse reshaper are that easy to implement demultiplexing and demodulation circuits are generally not capable of optimally detecting the positions of the TDPPM pulses (which are received in the presence of transmission noise) and exhibit unwanted amplitude sensitivity.

To achieve a maximal $\text{SNR}_o$ at the receiver output, our aim will be to determine the optimal shape of the transmitted pulses and the optimal method of pulse reshaping. On the one hand, the $\text{SNR}_o$ depends on the maximal peak-to-peak deviation of the pulse position. It is easily seen that this maximal deviation equals the slot length $T_s$ minus the width of the pulses $T_p$. On the other hand, the $\text{SNR}_o$ also depends on the jitter present in the positions of the reshaped pulses. Of particular interest is the jitter caused by (additive)
transmission noise, originating from the laser, the photodiode or the receiver front-end.

The most simple reshaper consists of a comparator, changing the polarity of its output signal when the momentary pulse amplitude exceeds the comparator threshold level. The instant at which the momentary pulse amplitude equals the threshold level will further be denoted by the decision time \( t_d \). The simple reconstructor is suited for relatively "simple" pulse waveforms such as rectangular pulses, Gaussian pulses or raised cosine pulses. Unfortunately, \( t_d \) depends only on a small portion of the pulse signal, just around the \( t_d \)'s. The rest of the pulse waveform, possibly containing relevant information as well, is disregarded.

An advanced method, based on a so-called matched filter, considers the complete pulses received. Having some knowledge about the original shape of the pulses and the spectral properties of the additive noise, correlation techniques can be used to obtain an optimal estimate of the original pulse position [3]. Basically, the optimal method of reshaping the TDPPM pulses comes down to:

- Applying a matched filter which maximizes the SNR of the filtered TDPPM pulses. The most probable pulse position is found where the filtered pulse has its maximum value.

- Detecting the pulse top position by differentiating the filtered pulses and detecting the zero crossing by a comparator.

Before examining the optimal waveform of the TDPPM pulses, we will apply the theory of matched filters to arbitrary waveforms.

### 3.5.1 Matched Filter Theory

It is assumed that after the pulses have been detected by the comparator, there are no additional signal impairments. In addition, we assume that before being supplied to the comparator the received pulse signal \( P(f) \), which is the Fourier transform of \( p(t) \), in the presence of the noise \( G_n(f) \) (noise power density) is passed by a filter having the transfer function \( H(f) \), see Fig.3.4. As the differentiation is also regarded as filtering, it is assumed to be included in the filter function \( H(f) \).

The mean square magnitude of the jitter in the decision time of the comparator, \( \sigma_{\Delta}^2 \), equals the ratio between the noise power and the squared slope magnitude of the filtered pulses at the input of the comparator. The noise power at the filter output follows from integration of the power spectrum \( G_n(f)|H(f)|^2 \). The slope magnitude of the filtered pulses at decision time
3.5. **PULSE SHAPING AND PULSE RESHAPING**

![matched filter-comparator reshaper](image)

Figure 3.4: Matched filter-comparator reshaper, receiving TDPPM pulses in the presence of additive noise.

$t_d$ is calculated from $P(f)$ by multiplication with the differentiation operator $2\pi j f$ and inverse Fourier transformation. So, we find:

$$\sigma_{t_d}^2 = \frac{\int_{-\infty}^{+\infty} G_n(f)|H(f)|^2 df}{\int_{-\infty}^{+\infty} 2\pi j f P(f)H(f)e^{j\omega t_d} df}^2$$  \hspace{1cm} (3.1)

To find its theoretical minimum, we apply the Schwartz inequality [1], p.186, yielding:

$$\sigma_{t_d}^2 \geq \frac{1}{\int_{-\infty}^{+\infty} \frac{|2\pi j f P(f)|^2}{G_n(f)} df}$$  \hspace{1cm} (3.2)

In order to achieve this optimum, the Schwarz inequality should be made an equality by modifying the receiver filter $H(f)$ to:

$$H_{\text{opt}}(f) = \frac{K(2\pi j f P(f))^* e^{-j\omega t_d}}{G_n(f)}$$  \hspace{1cm} (3.3)

in which the notation "*" refers to the complex conjugate and $K$ is an arbitrary constant. As expected, $H_{\text{opt}}(f)$ is decomposable into a filter maximizing the pulse peak power-to-noise ratio (the classical matched filter [1], p.187) and a differentiation (the operator $2\pi j f$).

The principle of matched filtering can be understood by analyzing the expression of the matched filter given by Eq.(3.3). Components which help to increase the slope magnitudes of the filtered pulses, these are the high-frequency components, are emphasized (numerator). The noisy parts of the received spectrum are suppressed (denominator). The exponential term only denotes a time delay, which is not essential.

An interesting illustration, assuming rectangular pulses in the presence of white additive noise, is shown in Fig.3.5. The pulse response of the matched filter is the inverse Fourier transform of the matched filter $(2\pi j f P(f))^* e^{-j\omega t_d}$, which yields the time-reversed differentiated input pulse. In the case of a rectangular input pulse, the pulse response has the shape of two delta pulses, one down and one up, with a spacing equal to the width of the rectangular
pulse. Convolution of the received pulse with the matched filter pulse response results in output pulses in which at time $t_d$ the leading as well as the trailing edge of the rectangular pulse contribute to a maximal slope magnitude.

![Convolution Diagram](image)

Figure 3.5: Example illustrating a matched filter for rectangular pulses in the presence of a flat noise spectrum.

If, as a second example, the noise power increases with frequency: $G_n(f) = \frac{S_0 f^2}{2}$ (where $S_0$ is a proportionality constant), the matched filter pulse response equals the time-reversed integrated rectangular pulse as shown in Fig.3.6. The matched filter now serves as an integrator (the "step" in its pulse response) and an additional attenuator for high frequencies (finite rise time of the pulse response). Now, rather than the pulse edges, the volume center of the rectangular pulses is determining $t_d$. Obviously, in the case of a triangular noise spectrum, emphasizing high frequencies would contribute more noise than useful information.

![Convolution Diagram](image)

Figure 3.6: Example illustrating a matched filter for rectangular pulses in the presence of a triangular noise spectrum.

If a matched filter is being used, the fundamental minimum of the jitter can be derived directly from the maximum given by Eq.(3.2), which depends on the shape of the TDPPM pulses and the noise power density function $G_n(f)$. Because, in practical systems, noise spectra are generally flat or triangular, only those spectra will be considered.

By defining the energy of the received pulse as:

$$E_p = \text{def} \int_{-\infty}^{+\infty} |P(f)|^2 df$$

(3.4)
and the total energy of the differentiated pulse as:

\[ E_{p'} = \text{def} \int_{-\infty}^{+\infty} |2\pi j f P(f)|^2 df \quad (3.5) \]

the minimum of the jitter given by Eq.(3.2) can be written as a function of \( E_{p'} \) or \( E_p \). If the noise exhibits a flat power density spectrum, \( G_n(f) = \frac{\mu}{2} \) (where \( \mu \) is the power density for real positive frequencies), we find:

\[ \sigma_{td}^2 = \frac{\mu}{E_{p'}} \quad (3.6) \]

In the case of the triangular noise spectrum, \( G_n(f) = \frac{S_0 f^2}{2} \), we find:

\[ \sigma_{td}^2 = \frac{S_0}{8\pi^2 E_p} \quad (3.7) \]

These results consolidate the conclusions that can be drawn from the examples assuming rectangular pulses: to minimize \( \sigma_{td} \) in the case of white noise we should maximize the steepness of the pulse edges, whereas for a triangular noise spectrum we should maximize the total pulse energy.

### 3.5.2 Optimal Pulse Shaping

We determined the fundamental minimum of the jitter in the decision time for a given \( E_p \) and \( E_{p'} \) of the received pulses. Presently, still assuming a matched filter is used, we will determine the optimum shape of the PPM pulses.

We should bear in mind that the SNR\( _{\alpha} \) at the demodulator output depends on the jitter \( \sigma_{td} \) as well as on the maximal peak-to-peak pulse deviation \( 2T_\Delta \) equaling the slot length \( T_{sl} \) minus the total pulse width \( T_p \). As the shape of the pulses influences both \( \sigma_{td} \) and \( T_\Delta \), the optimum shape is a compromise.

In the case of a flat noise spectrum, Eq.(3.6) shows that the mean square magnitude of the jitter is inversely proportional to \( E_{p'} \). Hence, if we assume that the total available pulse energy \( E_p \) is limited, the optimum pulse shape (minimizing the jitter) can be found by maximizing the quotient \( \frac{E_{p'}}{E_p} \). This is accomplished by concentrating the pulse energy into a narrow band in the highest part of the frequency spectrum available for transmission. This implies that the pulses take the shape of burst pulses, of which some examples are shown in Fig.3.7.

As concentrating pulse energy into a narrow frequency band automatically results in a large \( T_p \), minimization of \( \sigma_{td} \) conflicts with establishing a large \( T_\Delta \). Because on the one hand the ratio \( \frac{E_{p'}}{E_p} \) hardly increases, and thus the jitter
decreases, if $T_p$ becomes large compared to the period of the burst frequency $f_0$ and, on the other hand, $T_{\Delta}$ already approximates its maximal value if $T_p$ is short compared to $T_{sl}$, the optimal $T_p$ will be in the range:

$$\frac{k_1}{f_0} < T_p < \frac{T_{sl}}{k_2}$$

(3.8)

in which $k_1$ and $k_2$ are more or less arbitrary constants, having reasonable values in the range 3–5. Because the optimum is not very sensitive to the precise value of $T_p$, more accurate calculations are not presented.

For a triangular noise spectrum, Eq.(3.7) shows that pulse jitter depends only on $E_p$ and not on $E_p'$. As it results in the largest $T_{\Delta}$, the optimal pulse shape will be that of a narrow spike with maximal amplitude and, depending on the available transmission bandwidth, minimal width.

### 3.5.3 Trapezoidal Pulses and Low-Pass Filtering

In Secs.3.5.1 and 3.5.2, we determined the optimum pulse reconstruction filter and the optimum shape of the TDPPM pulses. In this section, we will deal with a type of pulse that is practically feasible. In addition, a relatively simple reshaping filter will be considered. The performance of this combination will be compared with that of the ideal solution.

The simplest laser driver just switches the laser on and off. However, the rise and fall times $T_{r,f}$ of the current pulses that drive the laser are of course finite. In Chap.5, it will be shown that $T_{r,f}$ (which are assumed to be equal) depend on the transit frequency of the components constituting the driver circuitry. If $T_{r,f}$ are large compared to the laser turn on and turn off times $T_{on,off}$ as defined in Sec.2.2.4, the shape of the optical pulses equals the shape of the current pulses. If, on the other hand, $T_{r,f}$ are short compared to $T_{on,off}$, the shape of the optical pulses is determined by the laser properties, resulting in pulses as shown in Fig.2.6. For both cases, we assume that the shape of the pulses can be approximated by a trapezoid, as depicted in Fig.3.8a. The rise and fall times will be denoted by $T_r$ and $T_f$, the pulse-high time by $T_h$ and the total pulse width by $T_p$. $A_p$ is the pulse amplitude.

Rather than a matched filter, it is simpler to use the implicit low-pass filter function of the receiver front-end instead. If the bandwidth of the receiver filter
3.5. PULSE SHAPING AND PULSE RESHAPING

Figure 3.8: (a) A trapezoid as an approximation of the waveform of optical pulses transmitted by an on-off laser driver. (b) A pulse having raised cosine edges as an approximation of the waveform of low-pass filtered TDPPM pulses.

\[ B_r \text{ is large compared to } \frac{1}{T_f}, \text{ the shape of the trapezoidal pulses is unaltered. In practical systems, this situation must be avoided because taking } B_r \text{ larger than necessary for handling the pulses only results in additional noise at the output of the filter, causing additional decision jitter. If, on the other hand, } B_r \text{ is in the order of, or small compared to } \frac{1}{T_f}, \text{ the shape of the filtered pulses will be determined by the bandwidth and the precise pole locations of the receiver filter. If the order of the filter is 2 or higher, the filtered pulses will contain little energy at frequencies beyond } B_r. \text{ Then, the shape of the filtered pulses can be approximated reasonably well by pulses having the edges of a raised cosine, see Fig.3.8b:}

\[
P(t) = \frac{A_p}{2} [1 + \cos \pi B_r (t - \frac{1}{B_r})]
\]

if \[ 0 \leq t \leq \frac{1}{B_r} \quad \text{and}\]

\[
P(t) = A_p
\]

if \[ \frac{1}{B_r} \leq t \leq \left( T_p - \frac{1}{B_r} \right) \quad \text{and}\]

\[
P(t) = \frac{A_p}{2} [1 + \cos \pi B_r (t + \frac{1}{B_r} - T_p)]
\]

if \[ \left( T_p - \frac{1}{B_r} \right) \leq t \leq T_p \]

Pulse jitter of low-pass filtered trapezoidal pulses

The decision jitter \( \sigma_{t_d} \) is calculated by determining the ratio between the noise power and the slope magnitudes of the pulses at the filter output (to be determined from Eq.(3.9)). In the case of a flat noise spectrum with single-sided power density \( \mu \), by assuming a noise bandwidth equal to \( B_r \) (which is realistic
if the order of the filter is two or higher), we find:

$$\sigma^2_{i_d} = \frac{\mu B_r}{(A_p^2 \pi B_r)^2} = \frac{4\mu}{\pi^2 A_p^2 B_r}$$  \hspace{1cm} (3.12)$$

In the case of a triangular noise spectrum with single-sided power density $S_0 f^2$, we find:

$$\sigma^2_{i_d} = \frac{S_0 B_r^3}{(A_p^2 \pi B_r)^2} = \frac{4S_0 B_r}{3\pi^2 A_p^2}$$  \hspace{1cm} (3.13)$$

Note that these results apply only if the slope magnitude of the edges of the filtered pulses is determined by the receiver bandwidth $B_r$. By requiring that the slope magnitude of the filtered pulses just equals the slope magnitude of the received trapezoidal pulses, we find the criterion:

$$B_r \leq \frac{2}{\pi T_{r,f}}$$  \hspace{1cm} (3.14)$$

As concluded above, choosing larger values of $B_r$ is not useful because it does not further increase the steepness of the filtered pulses and would only result in additional noise at the filter output.

In the case of a white noise spectrum, Eq.(3.12) demonstrates that we should take the maximal value of $B_r$, given by Eq.(3.14). If, on the other hand, the noise exhibits a triangular spectrum, according to Eq.(3.13), a small value of $B_r$ seems preferable. However, reducing $B_r$ results in a higher SNR only if $B_r$ is kept large enough to maintain the maximal amplitude of the filtered pulses. Hence, in both cases the optimal value of $B_r$ amounts to $\frac{2}{\pi T_{r,f}}$.

Because the maximal pulse deviation equals the slot length $T_{sl}$ minus the total pulse width $T_p$, we assume that $T_h$ is minimized to achieve a maximal pulse deviation. However, to maintain the full amplitude of the filtered pulses, it can be shown that the shortest possible value of $T_h$ is given by:

$$T_h = (1 - \frac{2}{\pi}) \frac{1}{B_r} = (\frac{\pi}{2} - 1)T_{r,f}$$  \hspace{1cm} (3.15)$$

Impairment due to usage of trapezoidal pulses rather than optimal pulses

Assuming that the matched filter is used, we will determine the impairment of a system based on trapezoidal pulses compared to a system based on “optimal” pulses. Subsequently, we will determine the additional impairment due to the usage of a low-pass filter rather than a matched filter.

In Sec.3.5.2, we concluded that if the spectrum of the additive noise is flat, the ratio $\frac{E_{r'}}{E_p}$ is a suitable measure for comparing the fundamental minima of
the pulse jitter obtained by different pulse shapes. For the “optimal” burst pulses, of which the burst frequency is close to the receiver band edge $B_r$, we find:

$$\frac{E_p'}{E_p} = 4\pi^2 B_r^2$$  \hfill (3.16)

For trapezoidal pulses, of which $T_{r,f}$ are assumed to be equal to $\frac{2}{\pi B_r}$, we find:

$$E_p' = 2A_p^2 \frac{T_{r,f}}{T_{r,f}} = \pi A_p^2 B_r$$  \hfill (3.17)

By putting $T_b=(\frac{2}{3} - 1)T_{r,f}$ (Eq.(3.15)), we calculate the energy of the trapezoidal pulse to be:

$$E_p = [\frac{2}{3} + (\frac{\pi}{2} - 1)]T_{r,f}A_p^2 = \frac{(3\pi - 2)A_p^2}{3\pi B_r}$$  \hfill (3.18)

giving:

$$\frac{E_p'}{E_p} = \frac{3\pi^2 B_r^2}{3\pi - 2}$$  \hfill (3.19)

Comparison of the results given by Eqs.(3.16) and (3.19), reveals an impairment of about 10dB. Thereby, we only considered the pulse jitter and not the influence of the total pulse width on the maximal pulse deviation. Hence, the actual impairment of the resulting SNR will be less.

In the case of a triangular noise spectrum, we concluded in Sec.3.5.2 that the detection jitter (when using a matched filter) depends only on $E_p$, so, for a given $E_p$, with respect to the jitter, the pulse shape is by definition optimal. Since the trapezoidal pulses exhibit approximately the minimal width (they approximate the “ideal” spike pulses), the trapezoidal pulses also achieve the maximal pulse deviation. Consequently, we conclude that in the presence of a triangular noise spectrum, the trapezoidal pulses are optimal.

**Impairment due to usage of a low-pass rather than a matched filter**

The jitter obtained when trapezoidal pulses in the presence of white noise are reshaped using a matched filter depends on $E_p'$ as given by Eq.(3.17). To find the mean square value of the jitter, we substitute $E_p'$ into Eq.(3.6), yielding $\sigma_{td}^2 = \frac{\mu}{2\pi A_p^2 B_r}$. Comparing this result to the actually achieved jitter level given by Eq.(3.12) reveals that the low-pass filter performs only 4dB below the optimum filter. This difference can be explained intuitively by considering that, when using a matched filter, the leading as well as the trailing edges of the pulses contribute to a maximal SNR at the input of the comparator, whereas in the case of the low-pass filter, only one of the edges does.
If the noise spectrum is triangular, the jitter level achieved by a matched filter follows by substituting $E_p$ given by Eq.(3.18) into Eq.(3.7), which results in:

$$
\sigma_{td}^2 = \frac{3S_oB_r}{8\pi(3\pi - 2)A_p^2} \quad (3.20)
$$

Comparing this result with the result given by Eq.(3.13) reveals that the low-pass filter performs 9dB lower than the matched filter.

### 3.5.4 Conclusions

Because of reasons emerging from implementation considerations, we have to use trapezoidal pulses and a low-pass reshaping filter. In the presence of white noise, the bandwidth of the filter should be such that the slope magnitude of the reshaped pulses is matched to the rise time of the trapezoidal pulses. Then the increase of the jitter in the positions of the reshaped pulses due to the suboptimal trapezoidal shape of the PPM pulses is about 10dB, while the filtering by a low-pass filter rather than the ideal matched filter causes a further deterioration of 4dB, giving a total impairment of 14dB. In the case of a triangular noise spectrum, only the filtering is suboptimal, resulting in an impairment of about 9dB. In return for this loss, which is affordable in many applications, we have a much simpler system.

### 3.6 Signal-to-Noise Ratio in TDPPM Systems

In the preceding section, we decided on using PPM pulses having the shape of a trapezoid and using a pulse reshaper based on the low-pass receiver filter. Presently, our objectives will be to calculate the output SNR ($SNR_o$) that can be achieved by an N-channel TDPPM system and to examine how the $SNR_o$ depends on the ratio between the bandwidth of the transmitted signals and the total transmission bandwidth available. Thereby we assume that the $SNR_o$ only depends on noise which is additive to the received PPM pulse signal. This assumption is reasonable because it applies for the most important noise sources, which are the laser diode, the photodiode (Chap.2) and the receiver front-end (Chap.5). Other possible sources, to be discussed in Chaps.5 and 7, can be kept sufficiently low by careful design.

In addition, we will determine the threshold level below which the $SNR_o$ collapses due to false pulses caused by a low value of the SNR of the received pulse signal ($SNR_r$).

In Sec.3.5, we concluded that the signal-to-noise ratio (SNR) depends on the peak-to-peak pulse deviation $2T_A$ and the jitter in the decision time $\sigma_{td}$ of the pulse reshaper. As the demodulator converts the positions of the (TD)PPM
pulses directly into the momentary amplitude of the output signal, the SNR of the demodulator output signal ($\text{SNR}_d$) follows directly from $T_\Delta$ and $\sigma_{t_d}$:

$$\text{SNR}_d = \frac{T_\Delta^2}{\sigma_{t_d}^2}$$  (3.21)

in which we substituted the “effective signal power” $\frac{T_\Delta^2}{2}$. Since the output signal of the demodulator is low-pass filtered by the smoothing filter, which removes noise outside the baseband bandwidth, $\text{SNR}_o$ at the filter output is different from $\text{SNR}_d$. So, to determine $\text{SNR}_o$, we first need to determine the influence of the smoothing filter.

### 3.6.1 Improvement of the Signal-to-Noise Ratio by the Smoothing Filter

The additive noise which is present in the receiver input signal exhibits a flat or a triangular power density spectrum (originating from the laser or photodiode and the receiver front-end respectively). As the noise is low-pass filtered by the receiver front-end and no matched filter is included, the noise spectra at the output of the front-end exhibit the same shapes, although frequencies beyond $B_r$ have been removed.

As we saw in Sec.3.5.1, the output signal of the receiver low-pass filter is supplied to the comparator of the pulse reshaper. Each time the momentary pulse amplitude exceeds the comparator threshold level (at the decision time $t_d$), the pulse position is “frozen” or “sampled”. We will assume that the sampling frequency $f_s$ is a constant, although, strictly speaking, it depends on the pulse positions. As illustrated for an arbitrary input spectrum by Fig.3.9, due to sampling the complete (double-sided) frequency spectrum of the input signal is folded around each multiple of $f_s$. Obviously, the spectrum resulting at the demodulator output is repeated, alternatively direct and frequency-reversed with respect to $f_s$, and extends to infinity. (In the figure, only a limited number of folded spectra have been drawn.)

Although the resulting noise spectrum need not be white, there are two reasons why the spectrum will be almost white: (1) the input noise power density varies only gradually with frequency, so that within one relatively narrow sideband the noise power density will vary only marginally, and (2) the contributions of upper and lower sidebands, which are reversed in frequency relative to each other, approximately average out gradual dependencies on frequency.

Since at frequencies higher than $\frac{f_s}{2}$ the frequency spectrum of the demodulator output signal only contains repetitions, the maximum bandwidth of the baseband signal at the demodulator output amounts to $\frac{f_s}{2}$. Because in low-cost
Figure 3.9: Power spectra of the received PPM pulses in the presence of additive noise and the demodulator output signal.

systems finite-order smoothing filters will be used, the actual bandwidth of the transmitted signals $B_1$ does not cover the potentially available baseband bandwidth. With regard to this so-called oversampling, we define the oversampling ratio as:

$$r = \frac{f_s}{2B_1} \quad (3.22)$$

To avoid aliasing distortion, $r$ should be one or larger. In conjunction with finite-order smoothing filters, practical values are in the range of 1.5–2.

The consequence of oversampling is that the smoothing filter will suppress the noise between $B_1$ and $\frac{f_s}{2}$. Hence, the SNR_o will be larger than the SNR_d:

$$\text{SNR}_o = \text{SNR}_d \frac{f_s}{2B_1} = \text{SNR}_d r \quad (3.23)$$

3.6.2 Signal-to-Noise Ratio at the Receiver Output

To calculate the SNR_o, we write the peak-to-peak deviation of the (TD)PPM pulses, which equals the length of the time slots $T_{sl}$ minus the pulse width $T_p$, as:

$$2T_\Delta = \frac{1}{Nf_s} - T_p = \frac{1}{2NrB_1} - T_p \quad (3.24)$$

$$f_s < \frac{1}{NT_p} \quad (3.25)$$
where \( N \) is the number of PPM signals multiplexed into one TDPPM signal (the number of slots in a frame, see Fig.3.3) and \( f_s \) is the sampling frequency of one PPM signal. The condition given by Eq.(3.25) excludes negative values of \( T_\Delta \).

Substitution of \( T_\Delta \) into Eq.(3.21) and multiplying by \( r \) yields:

\[
\text{SNR}_o = \frac{\left( \frac{1}{2N} - \frac{T_p}{B_T} \right)^2}{8\sigma_{\text{id}}^2} r \tag{3.26}
\]

Evaluation of Eq.(3.26), while bearing in mind the condition of Eq.(3.25), shows that \( \text{SNR}_o \) increases with decreasing \( r \). Obviously, the appropriate value of \( r \) is a compromise between optimal performance (\( r = 1 \)) and the cost of the smoothing filter, requiring larger values of \( r \).

Combining Eqs.(3.12), (3.13) and (3.26) and substituting \( T_p \) by the minimal width \( \frac{2}{B_T} \) of a pulse having raised cosine edges, yields:

\[
\text{SNR}_o = \left( \frac{1}{2N} - \frac{2}{B_T} \right)^2 \left( \frac{A_p}{2\pi} \right)^2 \frac{B_T}{8\mu} r \tag{3.27}
\]

for a white noise spectrum and:

\[
\text{SNR}_o = \left( \frac{1}{2N} - \frac{2}{B_T} \right)^2 \left( \frac{A_p}{2\pi} \right)^2 \frac{3}{8S_0B_T} r \tag{3.28}
\]

for a triangular noise spectrum.

Alternatively, the \( \text{SNR}_o \) can be related to the \( \text{SNR}_r \) of the received (TD)PPM pulses (measured at the input of the reshaper):

\[
\text{SNR}_o = \text{SNR}_r \left( \frac{1}{2N} - \frac{2}{B_T} \right)^2 \frac{\pi^2 B_T^2}{8} r \tag{3.29}
\]

Thereby, we define \( \text{SNR}_r \) as the SNR of the received (TD)PPM pulses at half their height because it is the decision level of the comparator:

\[
\text{SNR}_r = \text{SNR}_r \left( \frac{A_p}{\sigma_n^2} \right)^2 \tag{3.30}
\]

The denominator denotes the mean square value of the noise, which equals \( \mu B_T \) or \( S_0 \frac{B_T^3}{3} \) for a white and a triangular noise spectrum respectively.

### 3.6.3 Signal-to-Noise Ratio Versus Total Signal Bandwidth

Some interesting results are obtained if the relation between \( \text{SNR}_o \) and \( \text{SNR}_r \), given by Eq.(3.29), is considered as a function of the ratio between the so-called
total signal bandwidth $B_{\text{tot}}$ and the receiver bandwidth $B_r$. The total signal bandwidth $B_{\text{tot}}$ is defined as the product of the number of multiplexed signals $N$ and the available bandwidth per channel $B_1$:

$$B_{\text{tot}} = \text{def} \ N B_1 = N \frac{f_s}{2r} \quad (3.31)$$

Hence, Eq. (3.29) can be rewritten as:

$$\text{SNR}_o = \text{SNR}_r \left( \frac{B_r}{4rB_{\text{tot}}} - 1 \right)^2 \frac{\pi^2}{2} r \quad (3.32)$$

Fig. 3.10 shows three curves for different values of $r$. Once again it shows that

![Graph showing SNR ratio for different values of $r$]

Figure 3.10: Ratio between $\text{SNR}_o$ and $\text{SNR}_r$ versus ratio between $B_{\text{tot}}$ and $B_r$ for different values of $r$. The factor $k$ denotes the ratio between $B_{\text{tot}}$ and $B_{\text{tot, max}}$.

for the highest $\text{SNR}_o$, $r$ should preferably be close to unity.

The $\text{SNR}_o$ yields zero if the maximum pulse deviation yields zero, which is the case if $B_{\text{tot}}$ becomes so large that the related repetition frequency of the TDPPM pulses (i.e., $2N r B_1$) equals the reciprocal of the width of the TDPPM pulses $\frac{1}{T_p}$. Therefore, we define the maximum of the total signal bandwidth $B_{\text{tot}}$ as:

$$B_{\text{tot, max}} = \frac{1}{2rT_p} \quad (3.33)$$
If we assume $T_p$ to be determined by the bandwidth $B_r$ of the receiver amplifier, we may write:

$$B_{\text{tot, max}} = \frac{B_r}{4r}$$  \hspace{1cm} (3.34)

In correspondence with Eq.(3.34), for three different values of $r$, Fig.3.10 shows different values of $B_{\text{tot, max}}$.

Having defined $B_{\text{tot}}$ and $B_{\text{tot, max}}$, Eq.(3.32) can be rewritten using the definition:

$$k = \frac{B_{\text{tot}}}{B_{\text{tot, max}}} = \frac{4NrB_1}{B_r}$$ \hspace{1cm} (3.35)

as:

$$\text{SNR}_o = \text{SNR}_z \frac{(k - 1)^2 \pi^2}{k^2} r$$ \hspace{1cm} (3.36)

The factor $k$ can be regarded as a measure for the trade-off between bandwidth and SNR. The curves in Fig.3.10 are marked by some typical values of $k$. To achieve a sufficiently high SNR$_o$, practical values of $k$ are in the order of 0.8 or lower, implying that $B_{\text{tot}}$ will be 20% or more below $B_{\text{tot, max}}$.

### 3.6.4 False-Pulse Threshold Level

Up to now we assumed that the additive noise on the received pulses was small compared to the pulse amplitude, so that only the positions of the pulses are changed. However, if the momentary amplitude of the noise may (temporarily) exceed half the pulse height, false pulses will result at the reshaper output or intended pulses will be missed.

Assuming a Gaussian distribution of possible momentary noise amplitudes and considering that the bandwidth of the noise equals $B_r$, the average time between two false pulses is calculated to be (we used the approximation of the error function given in [1], p.666):

$$T_{fp} = \frac{\sqrt{2\pi}\text{SNR}_r \exp \text{SNR}_z}{2B_r}$$ \hspace{1cm} (3.37)

Since the demultiplexer is synchronized by the received TDPPM pulses (Chap.4), false or missed pulses will cause a disturbance in each of the received signals. However, the duration of the disturbance is less than one frame time because, as we will see in Chap.7, proper synchronization can be recovered within one frame.

The value of $T_{fp}$ that is just acceptable depends on the application. For example, in the case of analog video signals the disturbances will be quite acceptable if $T_{fp}$ is in the order of a second. If $B_r$ is 250MHz (as in the prototype system), this value requires an SNR$_z$ of about 15dB.
Due to the exponential term in Eq.(3.37), the required SNR\(_r\) is not very sensitive to the desired value of \(T_{fp}\). Hence we may speak of a threshold value of the SNR\(_r\) below which the SNR\(_o\) of the received signals rapidly decreases. In addition, also due to the exponential relation, the required SNR\(_r\) does not depend very much on \(B_r\). So, as a rule of thumb, we may further assume the threshold to be at an SNR\(_r\) of about 15dB.

### 3.7 Achieved Transmission Capacity

It is interesting to compare the efficiency of TDPPM coding, in terms of realized transmission capacity, with the theoretical maximum determined by the available channel bandwidth \(B_r\) and the signal-to-noise ratio SNR\(_r\). In addition, we will compare the transmission capacity of the TDPPM system with that of digital alternatives as suggested in Sec.3.3.

Shannon [1], p.587 defined the channel capacity in bits per second as:

\[
C_{ch} = B_r \log_2(1 + SNR_r) \tag{3.38}
\]

The total capacity of the TDPPM system is calculated as the sum of the capacities of the \(N\) channels having a baseband bandwidth \(B_l\) and signal-to-noise ratio SNR\(_o\):

\[
C_{TDPPM} = NB_l \log_2(1 + SNR_o) \tag{3.39}
\]

By substituting Eq.(3.36) into Eq.(3.39) and using the definitions of \(r\) and \(k\) given by Eqs.(3.22) and (3.35) to eliminate \(B_r\), \(B_l\) and \(N\), while assuming SNR\(_r\) \(\gg 1\) and SNR\(_o\) \(\gg 1\), we find:

\[
R_{TDPPM} = \text{def} \frac{C_{TDPPM}}{C_{ch}} \approx \frac{k}{4r} \left[ 1 + \frac{\ln \left( \frac{(k-1)^2 \pi^2}{k^2} \right)}{\ln \text{SNR}_r} \right] \tag{3.40}
\]

Fig.3.11 demonstrates how \(R\) depends on SNR\(_r\) for different values of \(k\) and \(r\).

As concluded before, the highest performance requires \(r=1\), whereas the limited capabilities of the smoothing filter urge for larger values of \(r\). As the practical SNR\(_r\)s are in the range from 15dB (threshold level) to 40dB (determined by the laser RIN), the value of \(R_{TDPPM}\) is in the order of 0.1. Higher values of \(R\) can be achieved by optimizing the shape of the transmitted pulses and by applying a matched filter.

Now, we compare the performance of the TDPPM system with that of an alternative system based on digital coding. We assume an \(N\) channel digital system, exhibiting the same transmission bandwidth \(B_r\), baseband bandwidth
Figure 3.11: The capacities of the TDPPM system and two digital systems compared to the theoretical capacity of the transmission channel for various \( k \) and \( r \).

\( B_1 \) and oversampling factor \( r \) as the TDPPM system. The total transmission capacity is calculated to be:

\[
C_{\text{dig}} = NB_1 \log_2(1 + \text{SNR}_{\text{dig}}) \quad (3.41)
\]

\[
\text{SNR}_{\text{dig}} = \frac{3}{8} 2^{n r} \quad (3.42)
\]

Thereby, the \( \text{SNR}_{\text{dig}} \) is assumed to be determined by the quantization noise [1], pp.434–435. The parameter \( n \) denotes the number of bits that is available for each sample. \( n \) equals the maximal transmission bitrate \( 2B_1b \) (in which \( b \) is the number of bits coded into one pulse, so the receiver distinguishes \( 2^b \) different pulse amplitudes) divided by the number of samples \( 2NB_1r \) (of all channels together) per unit of time:

\[
n = \frac{2B_1b}{2NB_1r} \quad (3.43)
\]

By combining Eqs.\((3.41), (3.42), (3.43)\) and \((3.35)\), we are able to determine \( R_{\text{dig}} \):

\[
R_{\text{dig}} = \text{def} \quad \frac{C_{\text{dig}}}{C_{\text{ch}}} = \frac{8b - 3k + k \log_2(3r)}{4r \log_2(\text{SNR}_r)} \quad (3.44)
\]

Since the non-linearity of the optical transmitter depends on the specific device and its temperature, it is unpredictable. Therefore, unless very complicated self-calibrating compensation techniques are employed, the number of
bits $b$ that can be transmitted by one pulse will be less than can be expected from the SNR$_r$.

Fig. 3.11 shows two curves of digital systems (dotted lines), assuming $b=1$ (most simple digital coding) and $b=4$ (maximum value if the non-linearity is assumed to be in the order of 5%). Obviously, a simple digital system ($b=1$) achieves about the same value of $R$ as the TDPPM system. Higher values of $R$ require $b > 1$, which, as with TDPPM, increases the system complexity.

3.8 Frame Synchronization

The received TDPPM pulses all have equal shape and amplitude. So, to distinguish which pulses belong to a particular channel an additional synchronization signal, the so-called frame synchronization (FS), needs to be provided.

Basically, as proper synchronization is maintained unless false pulses have been received, the transmission capacity required for FS is only small. This is particularly true if large acquisition times are admitted, because then averaging techniques can be used to separate the FS from noise.

Because it is necessary to avoid false pulses, the SNR$_r$ of the received TDPPM pulse signal is relatively large. So, even if the acquisition times are short, which is preferable for a proper start up behavior and a quick response to (exceptional) violent disturbances, a fraction of a few percent of the total transmission capacity turns out to be sufficient to achieve proper synchronization.

As a first possibility, FS can be added to one of the baseband input signals. For example, we may think of a small amplitude sinusoid with a frequency higher than the baseband bandwidth. A disadvantage of this method is the complexity of the circuitry required, as the pulses received are to be demodulated before the presence of the FS signal can be detected. Moreover, the acquisition time will at least be in the order of the reciprocal of the sinusoid frequency.

Shorter acquisition times, less than the duration of one frame, can be obtained by alternative approaches based on compound or orthogonal coding in addition to TDPPM coding. With compound modulation, the pulses of one of the channels, the so-called frame pulses, are modified by for example (see Fig. 3.12):

- (a) increasing the amplitude of the frame pulses
- (b) increasing the width of the frame pulses
- (c) changing the shape of the frame pulses
3.9 Distortion due to Nonuniform Sampling

As we will see in Sec. 7.3, the PPM modulator can be constructed such that it samples the input signal at a constant rate, so-called uniform sampling. The demodulator in turn converts the TDPPM pulses into uniformly spaced output pulses and passes them through a low-pass (smoothing) filter. Provided that the sample frequency $f_s$ is higher than the Nyquist frequency, distortion is in principle absent.

However, for reasons emerging from implementation considerations discussed later on in Sec. 7.3, we are interested in non-uniformly sampling modulator and demodulator circuits. Presently, we will derive a mathematical

Figure 3.12: Compound coding techniques, enabling the receiver to distinguish the frame pulses.

- (d) superimposing a small amplitude periodic signal such as a square wave with a frequency equal to the frame frequency.

The amplitude of the TDPPM pulses received cannot be assumed to be known in advance, since fiber attenuation and coupling losses cannot be predicted. Consequently, if the frame pulses are to be detected from the received TDPPM pulses themselves, the FS detection circuitry should be capable of handling a wide range of pulse amplitudes, or an automatic gain control should be provided. A simpler method is to detect the frame pulses from the output signal of the pulse reshaper because these pulses have a fixed amplitude (see Sec. 3.5.2). As, besides their positions, only the width of pulses is available, FS is preferably coded in the pulse width.

Since all TDPPM pulses should remain in their time slots, increasing the width of the frame pulses of course diminishes their maximum pulse deviation. So, the width of the frame pulses should be chosen to be just large enough to assure proper detection, independently of process inaccuracies.
description of the resulting “sample distortion”, in order to determine the maximal pulse deviation for which the distortion is still acceptable.

First, we examine a (TD)PPM transmission system using a straightforward demodulation technique, in which the PPM pulses are converted into amplitude modulated (PAM) pulses. Subsequently, we examine a system using an alternative demodulator which converts the PPM pulses into pulse duration modulated (PDM) pulses. A more detailed mathematical treatment is found in [4].

3.9.1 Demodulation Based on Conversion into PAM pulses

The straightforward demodulator converts the PPM pulses into amplitude modulated pulses (PAM) by sampling a sawtooth at the instant the PPM pulses are received (Sec.7.3). Consequently, the timing of the PAM pulses depends on the timing of the PPM pulses. Because the PAM pulses are next supplied to a low-pass smoothing filter, the shape of the PAM pulses is irrelevant. For simplicity, delta pulses are assumed.

If the transmission delay of the PPM pulses is assumed to be zero, the transfer function from the input of the modulator to the output of the demodulator can be described as a “gate” function:

\[ G(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s - T_k) \]  
\[ T_k = mT_s X(kT_s + T_k) \]

where \( \delta \) denotes the delta function, \( T_s = \frac{1}{f_c} \) is the average sampling period, \( T_k \) is the time displacement of sample \( k \) due to modulation, \( X(t) \) is the modulating input signal (assumed to have no dimension and peak-to-peak magnitude 1) and \( m \) is the modulation index (possible values in the range of 0–1). The demodulator output signal \( Y(t) \) is given by:

\[ Y(t) = X(t)G(t) \]  

As \( G(t) \) is not zero only at \( t = kT_s + T_k \), we may replace \( X(kT_s + T_k) \) by \( X(t) \), so that substitution of Eq.(3.46) in (3.45) yields:

\[ G(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s - mT_s X(t)) \]

With the help of Rowe's theorem [2], p.304, \( G(t) \) is decomposed into a series of exponential functions:

\[ G(t) = \frac{1}{T_s} (1 - mT_s X'(t)) \sum_{n=-\infty}^{\infty} \exp \frac{2\pi jn}{T_s} (t - mT_s X(t)) \]
in which $X'(t)$ denotes the time derivative of the input signal $X(t)$.

The sum term of the transfer function given by Eq.(3.49) consists of a series of constant-amplitude sinusoids having frequencies that are multiples of the sample frequency $f_s=\frac{1}{T_s}$ and a dc component ($n=0$). The magnitude of the sum term is determined by the term $1 - mT_sX'(t)$. Bearing in mind that $Y(t) = X(t)G(t)$, we conclude that there are two mechanisms contributing to the distortion of $Y(t)$: (1) The term $1 - mT_sX'(t)$ describes how the gain of the gate function depends on the time derivative of $X(t)$. Hence, $Y(t)$ contains the component $X(t)X'(t)$, which is a frequency-dependent second-order distortion. Intuitively, we may expect this behavior: The magnitude of the output signal is the product of the sampled values of $X(t)$ and the momentary repetition rate of the samples, which depends on $X'(t)$.

(2) The phase of the sinusoids is modulated by $X(t)$, producing first and higher order sidebands, similar to those produced by FM modulation. The frequency spectrum of the output signal follows by convolution of the Fourier transform of $G(t)$ with the transform of $X(t)$, so the spectrum of $X(t)$ is folded around each of the sidebands. Although the magnitudes of the sidebands diminish with increasing order $k$ (higher order Bessel functions for relatively small arguments), they may possibly overlap with the baseband.

An example of a possible spectrum of the demodulator output signal is depicted in Fig.3.13.

![Figure 3.13: Example of the output spectrum of a PPM demodulator which converts the PPM pulses into nonuniformly spaced PAM pulses.](image)

Of the two distortion mechanisms, the distortion dependent on $X'(t)$ is most likely to produce distortion products within the baseband. Compensation for this effect is possible by, for example, predistorting the signal before modulation, but is not easily performed.

An alternative demodulation technique, converting the PPM pulses into pulse duration modulated pulses (PDM), does not produce this type of distortion.
3.9.2 Demodulation Based on Conversion into PDM Pulses

One way of converting PPM pulses into PDM pulses is to start PDM pulses (leading edges) at the beginning of the time slots, and terminating them (trailing edges) at the instants the PPM pulses are actually received.

Mathematically, as illustrated by Fig.3.14, the PPM to PDM conversion can be described by subtracting the received PPM pulses from a pulse series marking the slots, and passing the resulting pulse signal through an integrator. The shape of the pulses is irrelevant, so we assume unity magnitude delta pulses for simplicity. By subtracting Eq.(3.49), which actually represents a

\[
\text{PPM pulses}
\]
\[
\text{reference pulses}
\]
\[
\text{PDM pulses}
\]

\[T_{\text{slot}}\]
\[\text{time}\]

Figure 3.14: Model describing the PPM to PDM conversion.

PPM signal (with ideal delta pulses), from a similar function describing the “slot pulses”, followed by integration, the PDM signal is written as:

\[
Y(t) = m \left\{ X(t) + \frac{\sum_{n=1}^{\infty} \left[ \sin n(\omega_s t - \phi) - \sin n\omega_s (t - mT_s X(t)) \right]}{nm\pi} \right\} \tag{3.50}
\]

in which the offset phase \(\phi\) is dependent on the average duty cycle of the PDM wave.

Eq.(3.50) clearly shows that the distortion due to modulation of the transfer gain is absent. The remaining distortion products are described by the right hand term in the summation, consisting of a series of phase-modulated sinusoids. The phase modulation produces sidebands that may overlap with the baseband. To approximate the maximum amplitude of the sidebands, we take the maximum amplitude sinusoid \(X(t) = \frac{1}{2} \sin \omega_s t\), where \(\omega_s\) is the modulating frequency, as the modulating signal. Up to now, we have assumed the modulation index \(m\) to be in the range of 0–1. However, to avoid the maximal deviation of the PPM pulses exceeding the slot length, in an \(N\)-channel system, \(m\) is limited to \(\frac{1}{N}\). For small arguments (that is, for a sufficiently small modulation index \(m\)), the Bessel functions that arise in the sideband decomposition are approximated as:

\[
J_k(z) \approx \frac{\left(\frac{z}{2}\right)^k}{k!} \tag{3.51}
\]
After some mathematical manipulation it is found that the ratio $r_k$ between the $k^{th}$ order sideband and the baseband component is limited, according to:

$$r_k \approx \frac{(nm\pi)^{k-1}}{k!} \leq \frac{(n\pi)^{k-1}}{k!}$$

(3.52)

In practical cases, we have to determine which sideband (value of $k$) just overlaps with the baseband and estimate its magnitude by applying Eq.(3.52). Thereby, we only need to consider the sidebands around $f_s$ ($n=1$) because, for higher values of $n$, only the sidebands with large $k$ will overlap with the baseband. Due to the term $k!$ in the denominator of Eq.(3.52), their magnitudes are small.

Eq.(3.52) demonstrates that in TDPPM systems, based on a non-uniformly sampling modulator and a demodulator which converts the PPM pulses into PDM pulses, sample distortion decreases with increasing number of channels $N$. This is caused by the fact that the maximal modulation index $m$ is inversely proportional to $N$. A further reduction of the distortion can be achieved by further decreasing $m$. A disadvantage of this method is that this also reduces the PPM pulse deviation and hence the SNR. If the SNR is to be maintained, there is no other solution than to use uniform sampling.

### 3.10 Influence of Optical Reflections

In the ideal case, all optical power arriving at the receiver is absorbed by the photodiode. However, in practical systems a small fraction $r_2$ is reflected back to the transmitter, see Fig.3.15. When the reflected pulses arrive at

![Figure 3.15: Reflections at the interfaces of laser/fiber and fiber/photo diode.](image)

the transmitter, a fraction $r_1$ of their power is reflected for the second time. Consequently the photodiode receives delayed and doubly reflected TDPPM pulses, the so-called triple transit echo, in addition to the intended pulses. As the echoes virtually modulate the positions of the intended pulses, small distortions and crosstalk between signals will result.

In addition, a fraction of the pulses that are reflected once will enter the cavity of the laser diode and interfere with the transmitted pulses. Due to
the nonlinearities of the lasing mechanism, the modeling of this effect is difficult. To estimate the influence of the double transit pulses, measurements are necessary.

Presently, we will concentrate on the distortion caused by the triple transit pulses and then we will deal with crosstalk. We consider the worst case situation, which is when the optical losses are small.

### 3.10.1 Distortion Caused by Triple Transit Pulses

Fig. 3.16 shows a raised cosine TDPPM pulse $P_1$ (thick line) as it appears at the output of the receiver front-end low-pass filter (Sec. 3.5.3), in the presence of a triple transit pulse $P_3$ (thin line). The amplitude of $P_1$ is denoted by $A_p$, the receiver bandwidth by $B_r$. To calculate the resulting distortion, it is assumed that the position of $P_3$ is fixed around $t=0$. The position of $P_1$ depends on the value $X$ of the modulating signal at the time $P_1$ was transmitted.

The timing difference $T_{\text{diff}}$ between $P_1$ and $P_3$ is the sum of a constant $T_c$, which depends on the transmission delay, and the deviation of $P_1$ caused by modulation:

$$T_{\text{diff}} = T_c + mT_sX$$  \hspace{1cm} (3.53)

in which $m$ is the modulation index (in the range of 0–1) and $T_s$ the sample period. The peak-to-peak amplitude of $X$ has been normalized to 1.

The shape of the raised cosine pulses is written as:

$$P_1(t) = \frac{A_p}{2} [1 + \cos\left(\frac{\omega_r}{2} (t - T_{\text{diff}})\right)]\Pi\left(\frac{4\pi}{\omega_r}, t - T_{\text{diff}}\right)$$  \hspace{1cm} (3.54)

$$P_3(t) = r_1r_2 \frac{A_p}{2} [1 + \cos\left(\frac{\omega_r}{2} t\right)]\Pi\left(\frac{4\pi}{\omega_r}, t\right)$$  \hspace{1cm} (3.55)
in which the function $\Pi(T, t_o)$ is a gate function which equals 1 between $t = t_o - T_b$ and $t = t_o + T_b$ and zero otherwise.

We determine the decision time $t_d$ when the momentary sum of $P_1$ and $P_3$ (dashed line) just equals the decision level $\frac{4\pi}{P}$. Provided that $r_1 r_2 \ll 1$, i.e., when the amplitude of $P_3$ is small compared to that of $P_1$, $t_d$ may be approximated by:

$$t_d \approx T_{\text{diff}} - \frac{\pi}{\omega_r} - \frac{P_3(T_{\text{diff}} - \frac{\pi}{\omega_r})}{P_1'(T_{\text{diff}} - \frac{\pi}{\omega_r})} \quad (3.56)$$

in which the numerator of the last term on the right hand is the momentary amplitude of $P_3$ and the denominator $P_1'$ is the momentary magnitude of the time derivative of $P_1$:

$$P_1'(t) = -\frac{A_p}{4} \sin(\frac{\omega_r}{2}(t - T_{\text{diff}}))\Pi(\frac{4\pi}{\omega_r}, t - T_{\text{diff}}) \quad (3.57)$$

Substituting Eqs.(3.55), (3.57) and (3.53) into Eq.(3.56) yields:

$$t_d = T_c + m T_s X - \frac{\pi}{\omega_r}$$
$$- \frac{2r_1 r_2}{\omega_r} \left[1 + \cos(\frac{\omega_r}{2}(T_c + m T_s X - \frac{\pi}{\omega_r}))\right]\Pi(\frac{4\pi}{\omega_r}, T_c + m T_s X - \frac{\pi}{\omega_r}) \quad (3.58)$$

Due to the nature of the distortion mechanism, intermodulation and harmonic distortion levels are unsuitable measures of the distortion. Instead, we determine the differential gain, which is defined as:

$$\text{DG} = \frac{\text{gain}_{\text{max}} - \text{gain}_{\text{min}}}{\text{gain}_{\text{average}}} \quad (3.59)$$

The DG is derived from the "effective modulation constant" of the pulses, which can be calculated by differentiation of Eq.(3.59) with respect to the modulating signal $X$:

$$\frac{d t_d}{dX(t)} = m T_s [1 + r_1 r_2 \sin(\frac{\omega_r}{2}(T_c + m T_s X - \frac{\pi}{\omega_r}))\Pi(\frac{4\pi}{\omega_r}, T_c + m T_s X - \frac{\pi}{\omega_r}) \quad (3.60)$$

Fig.3.17 depicts the normalized modulation constant as a function of $X$.

Although we disregarded the effect in our calculations, the curve will repeat itself when the pulse deviation exceeds the spacing between two successive pulses (indicated by dashes), as shown in Fig.3.17. Under normal conditions, the modulation index $m$ will be kept small enough to keep the pulses within their time slots, so we only have to consider one period of the curve.

Depending on the constant delay time $T_c$, the curve can shift along the $X$-axis. So, the level of $X$ for which the distortion occurs may vary. In addition,
in multi-channel systems, \( m \) is inversely proportional to the number of channels \( N \). So, the fraction of the range of \( X \) which exhibits distortion increases with \( N \).

From Fig.3.17, it clearly shows that the differential gain caused by the triple transit pulses equals:

\[
DG = 2r_1 r_2
\]  

(3.61)

Practical values of the reflection coefficients \( r_1 \) and \( r_2 \) are in the order of 2% (−17dB, in optical power), so the DG is practically negligible (in the order of 0.1%).

An alternative measure for the distortion is the ratio between the peak-to-peak amplitude of the total distortion signal given by the term containing the square brackets in Eq.(3.59) and the peak-to-peak pulse deviation \( 2T_\Delta \):

\[
d_{\text{tot}} = \frac{4r_1 r_2}{\omega_r 2T_\Delta} 
\]  

(3.62)

In Sec.3.6.2, we found that \( 2T_\Delta \) can be written as:

\[
2T_\Delta = \frac{1}{2N r B_1} - \frac{2}{B_r}
\]  

(3.63)

In Sec.3.6.3, we defined \( k = \frac{4N r B_1}{B_r} \), so, Eq.(3.62) can be rewritten into:

\[
d_{\text{tot}} = \frac{r_1 r_2}{\pi} \frac{k}{1 - k}
\]  

(3.64)

Obviously, the distortion depends on the ratio \( k \) between \( B_{\text{tot}} \) and \( B_{\text{tot, max}} \). For example, if \( k \) is in the range 0.5–0.8 (and \( r_1 = r_2 = 2\% \)), \( d_{\text{tot}} \) is in the range of 0.012–0.05%. 

---

**Figure 3.17**: Effective modulation constant as a result of the presence of triple transit pulses.
3.10.2 Crosstalk Caused by Triple Transit Pulses

The second consequence of pulse reflection is crosstalk. In contrast to what we did in calculating the distortion, we now assume the \( P_1 \) to be free of modulation, while \( P_3 \) is modulated with the maximal pulse deviation. The peak-to-peak modulation of \( t_d \) due to crosstalk equals the peak-to-peak deviation due to the superposition of \( P_1 \) and \( P_3 \), as described by the term between the square brackets of Eq.(3.59).

By defining the crosstalk \( C \) as the ratio between the peak-to-peak deviation of \( t_d \) caused by the triple transit pulse and the peak-to-peak deviation due to modulation, we obtain the same expression as for the total distortion:

\[
C = \frac{4r_1r_2}{\omega r^2 T_\Delta} = \frac{r_1r_2}{\pi} \frac{k}{1 - k} \tag{3.65}
\]

Assuming the same values of the parameters, the crosstalk is calculated to be in the range -78 to -66dB.

3.10.3 Experimental Results

To determine the DG of the prototype system, we used a video measurement set. We measured the DG to be in the order of 1%, which cannot be explained by our calculations. But the shape of the normalized gain curve was found to be very similar to the curve depicted in Fig.3.17, while replacing the optical fiber by another piece of fiber resulted in a different DG. In addition, separate measurements of the distortion of the modulator and demodulator revealed considerably lower values. So, we figured that the distortion in the prototype system was caused by the (once reflected) double transit pulses, which interfere with the pulses that are transmitted by the laser diode. It was concluded earlier that the modeling of this effect is difficult. Although crosstalk was not observed, since the mechanisms causing crosstalk and distortion are the same, the crosstalk will be higher than the value determined in the previous section.

To avoid the interference of double transit pulses, we have to use better fiber connection techniques or insert an optical isolator between the laser diode and the fiber. Unfortunately, these solutions are not compatible with low cost. Once more, there is a trade off between quality and costs.

3.11 Summary

Having the constraints of using low-cost optical components and low-cost (that is, in sufficiently large production numbers) integrated electronic circuitry, the aim of this chapter has been to determine the coding scheme best suited for
our transmission system. In finding compromises between quality (signal-to-noise ratio, distortion and crosstalk) and cost (requirements to the linearity and signal-to-noise ratio of the optical transmitter and the fiber reflection coefficients, complexity of the electronic circuitry, chip area and supply power), emphasis was laid on low cost.

Due to the nonlinearity of the LED or laser diode, we should preferably use digital coding, resulting in a limited number of discrete amplitude levels, or (analog or digital) angle modulation. In addition, in a multi-channel system, we should use time division multiplex rather than frequency division multiplex.

In deciding between digital and analog coding, we preferred analog coding, because it results in considerably less chip area and required supply power, while the problem of realizing high-performance analog-to-digital and digital-to-analog converters is avoided. The typical advantage of digital coding that the transmission of the coded signals does not impair the signal quality, is of minor concern because the transmission links are relatively short. In addition, the transmission capacity achieved by the system based on digital coding is comparable with the capacity achieved by a system based on the analog coding proposed in this chapter: time division multiplex pulse position modulation (TDPPM). As systems based on analog coding are much more susceptible to circuit imperfections, careful design will be a major concern.

We argued that for our system TDPPM, in which the information is carried by the positions of equally shaped PPM pulses and in which multiplexing is obtained by time multiplexing individual PPM pulse trains, is the most appropriate type of angle modulation.

If the power density spectrum of the additive transmission noise is flat (such as the spectrum of the noise produced by the laser diode), the lowest PPM pulse jitter is achieved by burst pulses, while, in the case of a triangular shaped noise spectrum, narrow spike pulses should be used. However, to keep the required electronic circuitry feasible, the practical system will be based on trapezoidal pulses. For the same reason, the receiver pulse reshaper is based on the implicit low-pass filtering of the receiver front-end rather than on a matched filter. As a consequence, the practical system will perform up to 14dB below the theoretical optimum. In the specific case of the prototype system, which was to be suited for the transmission of video signals, this was not a problem.

We calculated that the transmission capacity of the TDPPM system (in terms of bits per second) is approximately a factor 10 below the maximum defined by Shannon. The same result would apply for a simple digital system. In return for this loss, we have a low-cost system.

So-called sample distortion is caused by non-ideal modulation and demodulation techniques. It was concluded that this distortion can be kept small
by applying the PPM-to-PDM demodulator principle and by delimiting the maximum pulse deviation. The latter condition is satisfied inherently in multi-channel systems, because the length of the time slots that are available for modulation of the pulses, is inversely proportional to the number of channels.

Finally, we proved that the influence of the triple transit echoes, which result from fiber-end reflections, is sufficiently low to avoid crosstalk and distortion. Unfortunately, the influence of TDPPM pulses reflected back into the laser cavity (the double transit echo), could not be determined so, in practical systems, its influence needs to be verified by measurement.
Bibliography


Chapter 4

System Architecture

4.1 Introduction

In Chap.3, we proposed the TDPPM coding scheme, which has been optimally adapted to the characteristics of the low-cost optical components discussed in Chap.2. In this chapter, we present the global architectures of the TDPPM transmitter and receiver, which include the electronic circuits as well as the optical components. Except for some minor modifications, emerging from implementation requirements, the architectures are rather straightforward.

4.2 Transmitter Architecture

A straightforward configuration of the transmitter is depicted in Fig.4.1. We

![Diagram of Transmitter Architecture]

Figure 4.1: Straightforward configuration of the TDPPM transmitter.

distinguish the following functions:
• Multiplexer: synchronized by the clock. The multiplexer successively supplies one of the input signals to the PPM modulator.

• Modulator: samples the selected input signal and converts the samples into (TD)PPM pulses (see Fig.3.3). The clock signal is required to synchronize the modulator with the multiplexer.

• Clock: provides a (multi-phase) synchronization signal for the multiplexer and modulator. It may also serve the pulse shaper by indicating which pulses are to be the slightly widened frame pulses (dotted line).

• Pulse shaper: determines the width and the shape of the TDPPM pulses.

• Output driver: amplifies TDPPM pulses to the level required for properly driving the LED or laser and provides its biasing.

• LED or laser diode: external (that is, not integrated in the IC) lightwave transmitter, mounted in a fiber-optic connector or pig-tailed with a piece of optical fiber, converting the electrical into an optical signal.

The configuration of Fig.4.1 is straightforward, but it is not the most appropriate configuration. We will examine some of the implementation aspects of the modulator, the multiplexer and the pulse shaper, motivating the modified architecture depicted in Fig.4.2.
4.2. TRANSMITTER ARCHITECTURE

4.2.1 Implementation Aspects of Modulator and Multiplexer

In Fig.4.1 the multiplexer is located directly at the signal inputs. In this way, only one modulator is required. However, this solution has two important disadvantages: (1) any non-linearity of the multiplexer switch directly results in signal distortion and (2) in between the generation of two successive TDPPM pulses, the modulator is left only a short "recovery" time. After, for example, a PPM pulse of signal 1 has been transmitted at the end of the time slot, the next PPM pulse, belonging to signal 2, may already appear at the start of the next slot. As we will see in Chap.7, short recovery times will result in increased modulator distortion.

To avoid this type of distortion, we prefer the configuration of Fig.4.2, which uses one modulator for each input signal and in which the multiplexer is located at the modulator outputs. As a result much more recovery time is available for the modulators and, since information is coded in the timing of the PPM pulses, non-linearities of the multiplexer are of no concern. Note that we demand instead that the multiplexer exhibits a constant propagation delay. But, because this is an inevitable requirement for all of the circuits handling the (TD)PPM pulses, it will cause no additional problems.

As the multi-phase clock generator assures that the modulators transmit the PPM pulses in the appropriate slots, the multiplexing function may consist of a simple summing circuit. Consequently, the multiplexer needs no synchronization signal.

4.2.2 Pulse Shaper

In Chap.3, we concluded that the requirement of simple circuitry leads to TDPPM pulses that have the shape of a trapezoid. The output driver only needs to switch the LED or laser on and off, so pulse shaping comes down to fixing the width of the pulses and their rise and fall times.

A convenient point to fix the pulse width is where the pulses are generated, which is in the modulators. A simple modification of one of the modulators provides the slightly widened frame pulses. As a matter of course, the circuitry between the modulator and the transmitter must accurately maintain the width of the pulses.

Regarding the rise and fall times, it may be expected that they are restricted by the output driver because it handles the largest "power-bandwidth" product. This will be shown to be correct in Chap.5.

Because the pulse shaping will not be performed by an explicit circuit, the block diagram of Fig.4.2 contains no pulse shaper.
4.3 Receiver Architecture

Figure 4.3 depicts the straightforward block diagram of the receiver. It consists of the functions:

- **Photodiode**: PIN optical receiver, mounted in a fiber-optic connector or pig-tailed with a piece of optical fiber, converting the optical into an electrical signal.

- **Amplifier**: amplifies the TDPPM pulses received by the photodiode. Additionally, it performs the receiver low-pass filtering before the pulses are supplied to the pulse reshaper (see Sec.3.5.3). Although the amplitude of the received TDPPM pulses is not fixed due to different coupling and fiber losses, the gain of the amplifier need not be variable because the pulse reshaper can handle a large range of input pulse amplitudes.

- **Pulse reshaper**: reconstructs proper "square" pulses with constant amplitude and shape and minimal pulse jitter. In this way, the quality (i.e., the signal-to-noise ratio) of the received signals only depends on the performance of the pulse reshaper and not on that of the demultiplexer and the demodulator.

- **Frame pulse detector**: detects frame pulses (which are a little wider) from other TDPPM pulses, necessary for proper synchronization of the demultiplexer and clock regenerator.
4.3. RECEIVER ARCHITECTURE

- Demultiplexer: splits the TDPPM pulse train into the individual PPM pulse trains, one for each channel. The demultiplexer requires frame synchronization (FS), supplied by the frame pulse detector, and slot synchronization (SS), which can be derived directly from the TDPPM pulses or indirectly via the clock regenerator (dotted lines).

- Demodulator: converts the PPM pulses into amplitude modulated (PAM) or duration modulated (PDM) pulses (see Secs.3.9.1 and 3.9.2) so that the baseband signals can be retrieved by low-pass filtering. The time reference, indicating the zero positions of the PPM pulses, is supplied by the clock regenerator.

- Low-pass smoothing filter: removes noise and high frequency signals consisting of sidebands around the sample frequency and its harmonics (Secs.3.9.1 and 3.9.2) from the PAM or PDM signal. The baseband signal is available at the filter output.

- Clock regenerator: derives the multi-phase clock signal required for demodulation from the complete TDPPM signal (available at the pulse reshaper output) or from the frame pulses exclusively (available at one of the demultiplexer outputs). The clock signal may also serve to synchronize the demultiplexer.

The discussion of where to insert the demultiplexer, is very similar to the discussion of where to insert the multiplexer in the transmitter. Because of implementation considerations concerning the demultiplexer and the demodulator, we use one demodulator for each output and choose to demultiplex before demodulating.

Now we will discuss a better method for achieving proper slot synchronization of the demultiplexer, leading to an alternative demultiplexer configuration consisting of a counter and a decoder. In addition, we will examine which signal should be used for the clock regeneration. Fig.4.4 depicts the resulting receiver architecture, which has been slightly modified with respect to the architecture shown in Fig.4.3.

4.3.1 Modification of the Demultiplexer

Since TDPPM pulses can be located at any position in their time slots, proper operation of the demultiplexer does not allow delay errors between the TDPPM pulses and the signal determining the slot synchronization of the demultiplexer. To avoid delay errors, the demultiplexer is preferably synchronized directly by the TDPPM pulses themselves: Some supplementary circuitry alters the state of the demultiplexer after a complete TDPPM pulse has been received.
A principal disadvantage of this technique is its sensitivity to false pulses. Concerning this sensitivity, a better solution would be to have the slot synchronization signal provided by the clock signal instead; because the clock signal is regenerated by averaging over many TDPPM pulses (flywheel effect), occasional false pulses would have no influence. However, as will be shown in Chap. 7, after a disturbance it is possible to recover proper synchronization within the duration of one single frame. Consequently, when using the TDPPM pulses themselves the influence of false pulses is limited.

So far, we assumed that the demultiplexer actually passes successive PPM pulses to the appropriate demodulators, thereby maintaining the shape of the pulses. But, since information is carried only by the timing of the pulses, their shape need not be maintained. An interesting alternative demultiplexer configuration, elegantly combining demultiplexing and slot synchronization, consists of a cyclic counter and a decoder. The counter is clocked by the TDPPM pulses and restarts at the beginning of each new frame. The decoder derives the demultiplexer output signals from the counter outputs, such that for example all the zero crossings of one output signal correspond to the PPM pulses of one particular channel. Assuming a four-channel system, Fig. 4.5 illustrates this principle. Of course, since the demultiplexed signals have become square waves rather than PPM pulses, the demodulator configuration needs to be slightly modified.
4.3.2 Synchronization of the Clock Regenerator

The clock signal is regenerated by averaging the positions of the received TDPPM pulses. Practically, this function is performed by a phase lock loop (PLL).

Since the averaging time is finite, i.e. the bandwidth of the PLL is nonzero, the positions of the particular pulses that are used for clock regeneration may not be modulated by signals having frequencies in the order of the PLL bandwidth or lower. Because the system must be capable of transmitting different types of signals, possibly containing low-frequency or dc-components, particular pulses must be reserved for the purpose of clock regeneration. As frame pulses, due to their slightly larger width than other TDPPM pulses, can be easily distinguished from other pulses, we prefer to use frame pulses for clock regeneration. At the output of the demultiplexer, frame pulses are separately available. So, the demultiplexer output signal will be used to synchronize the clock regenerator.

4.4 Conclusions

The designs of the transmitter and receiver architecture are quite straightforward. Some minor modifications were necessary for reasons emerging from implementation considerations: (1) time multiplexing is carried out on the level of PPM pulses rather than on the level of the baseband signals, (2) the transmitter contains no explicit pulse shaper circuit, (3) the demultiplexer slot synchronization is provided by the received TDPPM pulses themselves rather than by the receiver clock, (4) at the demultiplexer output square waves are available rather than the received PPM pulses, and (5) only the frame pulses are used to regenerate the receiver clock signal.

The fact that the frame pulses are used for clock regeneration, means that the frame pulses may not be modulated by signals having frequencies in the order of the PLL bandwidth or lower. In addition, since the frame pulses are
slightly wider than other pulses and since the pulses should be kept within their time slots, the allowed peak-to-peak pulse deviation of the frame pulses is a little smaller than that of other TDPPM pulses. For these two reasons, the characteristics of the channel assigned to transfer the frame pulses are somewhat different to the characteristics of the other channels.
Chapter 5

Characteristics of Electronic Circuitry

5.1 Introduction

One of the aims of this thesis is to relate the performance of the system, in terms of SNR and bandwidth of the transmitted signals, to the characteristics of the low-cost optical components and the integrated electronic circuitry. Chap.2 already dealt with the optical components and concluded that the non-linearity of the optical transmitters and, in the case of a laser diode, its relatively large contribution to the transmission noise, are of primary concern.

Having presented the system architecture in Chap.4, it has become clear what types of electronic circuits are necessary in constituting the transmitter and receiver. Presently, restricting ourselves to configurations that are suitable for integration, this chapter examines the fundamental characteristics of the electronic circuits, such as maximum bandwidth, noise level and maximum signal level. Based on this knowledge, in Chap.6, we will determine the system specifications. At this stage, distortions are disregarded because they depend too much on the actual implementation.

We will start examining the bandwidth and sensitivity of the receiver front-end amplifier. Together with the bandwidth of the optical components, the bandwidth of the amplifier determines the available transmission bandwidth, while its sensitivity determines the SNR of the received TDPPM pulses when the received power level is relatively low.

Next, we will determine the switching speed of the so-called switching differential pair (SDP) and the jitter in its propagation delay. The SDP is the basic building block for limiter circuits, comparators and logic functions which are used throughout our system. It turns out that the switching time of the
SDP limits the maximal pulse rate of the TDPPM pulses that can be handled. Because the positions of the TDPPM pulses carry the sampled values of the transmitted signals and because for most of the circuits the propagation jitter does not depend on the received power level, the jitter is one of the factors that may impose an upper limit to the SNR of the demodulated signals.

Next, the phase noise produced by various types of oscillators, which are used in the clock generator and the clock regenerator, will be examined. As the oscillators provide the timing reference for modulation and demodulation of the TDPPM signals, oscillator phase noise might be another factor which possibly sets an upper limit to the SNR.

Finally, we determine the SNR (equaling the dynamic range if the signal level is optimal) of integrated continuous-time low-pass filters. Such filters will be used as smoothing filters at the demodulator outputs. The SNR of the filters too might set an upper limit to the signal SNR.

Since bipolar transistors are superior to CMOS devices at high frequencies (at the current state of art), bipolar components are required. Mixed processes, offering MOS as well as bipolar components, were not considered because their costs are higher. So, all considerations will be based on implementation in a bipolar technology.

5.2 The Receiver Amplifier

As we will see later on in Chap.7, the receiver front-end preferably consists of a negative-feedback amplifier because it combines the low input impedance required to avoid the loss of signal current into the parasitic capacitances of the PIN photodiode and the wiring between the PIN and the IC with maximum sensitivity.

In this section, we estimate the maximum bandwidth and the maximum sensitivity that can be achieved by integrated negative-feedback amplifiers. Compared to negative-feedback amplifiers consisting of discrete components, coils and transformers are not available for manipulating the locations of the poles of the amplifier transfer function and for optimizing the receiver sensitivity.

The maximal input current that can be handled by the amplifier will not be determined because in Chap.7 we will present a method for enlarging the allowed magnitude of the input current of the front-end beyond that of the amplifier.

Fig.5.1 depicts the basic configurations of negative-feedback amplifiers having a current input to provide for the low input impedance. The output current of the PIN photodiode is denoted by $I_s$. $C_s$ represents the total capacitance
of the PIN and its connections to the amplifier input. In the case of a current output, the feedback network around the active part of the amplifier (approximating the function of a nullor) [1] consists of two impedances $Z_1$ and $Z_2$. Basically, in the case of a voltage output, $Z_2$ is absent. However, since at high frequencies the loop transfer becomes sensitive to the load impedance, which affects the high frequency behavior of the negative-feedback amplifier, $Z_2$ is incorporated to represent the load impedance of the amplifier.

5.2.1 Bandwidth

In this section, we discuss the factors that limit the bandwidth of negative-feedback amplifiers. To simplify the mathematical expressions, we assume the poles of the amplifier to be located in Butterworth positions. Nevertheless, in approximation, the results apply to amplifiers having different pole locations as well.

On condition that the poles of the amplifier transfer function are in Butterworth positions, according to [1], the bandwidth $\omega_n$ of the negative-feedback amplifier depends on the so-called loop gain poles (LP) product:

\[
\omega_n = \sqrt[k]{|LP|} \tag{5.1}
\]

\[
\omega_n = \sqrt{k \left| 1 - A_o B_o \right| \prod_{j=1}^{k} |p_j|} \tag{5.2}
\]

in which $A_o$ is the low-frequency gain of the active part of the amplifier, $B_o$ the low-frequency transfer of the feedback network and $p_j$ are the $k$ dominant poles of the loop transfer function. Dominant poles are the poles of which the influence to the phase shift within the loop cannot be neglected for frequencies between zero and $\omega_n$. As a rule of thumb, a pole belongs to the dominant group if $|p_j| < 5\omega_n$.

In [1] it is concluded that properly designed stages contribute a factor $\omega_t$ to the LP product, in which $\omega_t$ is the "transit frequency" where the (current) gain
of the active components has decreased to unity. So, as long as $\omega_n$ does not exceed $\omega_c$, Eq. (5.2) suggests that any bandwidth can be achieved by cascading a sufficient number of stages. However, we still have to satisfy the condition that the poles of the amplifier transfer function are in Butterworth, or other desired positions.

Because the pole locations of the closed loop depend on the total phase shift in the loop for frequencies where the loop gain $AB$ is close to unity, we take the phase shift as a criterion. In the case of a Butterworth response, the required phase shift can be calculated by considering that, at the unity loop gain frequency, the gain of the transfer function has decreased by 3dB compared to the low frequency gain $|H_o|$. So, we have to satisfy the condition:

$$\frac{|H(AB = 1)|}{|H_o|} = \frac{1}{\sqrt{2}} \quad (5.3)$$

By rewriting the transfer $H$ as an implicit of $A$ and $B$ and by taking $|H_o| = \frac{1}{B_o}$, which is justified provided that the low-frequency loop-gain is sufficiently large, we find:

$$\frac{|A|}{|1 - AB|} |B_o| = \frac{1}{\sqrt{2}} \quad (5.4)$$

By assuming that the transfer of the feedback $B$ equals $B_o$ over the entire frequency range and by writing the loop transfer function $AB$ (at the frequency where it has unity magnitude) as $\exp j\Phi_{tot}$, in which $j = \sqrt{-1}$ and $\Phi_{tot}$ is the total phase shift within the loop, we find the condition:

$$\frac{\exp j\Phi_{tot}}{|1 + \exp j\Phi_{tot}|} = \frac{1}{\sqrt{2}} \quad (5.5)$$

Solving for negative values of $\Phi_{tot}$ yields $\Phi_{tot} = -90^\circ$.

In the following sections, we will use first order models to estimate the phase shift contributed by the active stages of the amplifier and the maximum possible compensation for this phase shift by the feedback network.

**Phase shift of non-interactive stages**

In obtaining a maximal contribution to the LP product, common emitter (CE) and common collector (CC) stages are most appropriate [1]. To start with, we consider an amplifier in which there is no interaction between successive stages. As we consider the current transfer functions, this is accomplished by driving the stages by high impedance (current) sources and termination by low impedance loads. For this purpose, common base (CB) stages can be inserted between successive stages. Rather than contributing effectively to the
5.2. THE RECEIVER AMPLIFIER

LP product (the pole of a CB stage is at a relatively high frequency, in the order of $\omega_t$), they serve as "isolators".

If the stages are non-interactive, it can be shown that each CE or CC stage contributes a single pole at the frequency $\omega_t/\beta$, where $\beta$ is the transistor low-frequency current gain. Cascading $n$ of these stages results in a total phase shift:

$$\phi_\omega = -n \arctan \frac{\omega \beta}{\omega_t}$$

$$\approx -90^\circ n \left(1 - \frac{1}{\omega \pi \beta} \right) \text{ provided that } \omega \gg \omega_t \beta$$

The bandwidth of amplifiers consisting exclusively of non-interactive stages is rather limited. For instance, if the amplifier consists of two stages, we calculate from Eq.5.6 that a total phase shift of $-90^\circ$ is obtained if $\omega = \omega_t/\beta$. Assuming the practical value $\beta=100$, the bandwidth of our amplifier is restricted to $\omega_t/100$. For amplifiers using more than two stages, still lower values result.

Reduction of the phase shift by using pole splitting techniques

The total phase shift in the active part of the amplifier can be decreased, and hence the amplifier bandwidth increased, by using "pole splitting techniques". This means that particular poles are forced to higher frequencies, while some other poles are forced to lower frequencies. Since this method reduces the loopgain for frequencies in the upper part of the passband of the amplifier, the feedback becomes less effective in reducing distortion. However, in our system this is not a problem.

The most effective way of pole splitting, which does not decrease the LP product, is based on cascading two successive CE or CC stages, without CB buffer stages. An example of this, based on two CE stages, is shown in Fig.5.2.

![Figure 5.2: Cascading of two CE stages, resulting in pole splitting.](image)

The first stage is loaded by the nonzero input impedance of the second stage. In first approximation (at this stage we disregard the influence of the
collector-substrate capacitance \( C_{cs1} \), this input impedance consists of the parallel circuit of the base-emitter and base-collector junction capacitances and the diffusion capacitance of \( Q_2 \) and the resistance \( \beta \frac{kT}{qI_c} \) where \( \beta \) is the low-frequency current gain and \( I_c \) is the emitter bias current of \( Q_2 \). If the second stage were driven by an ideal current source, the pole related to the time constant of the input impedance of \( Q_2 \) would equal \( \frac{\alpha}{\beta} \). However, at relatively high frequencies, the local feedback around \( Q_1 \) caused by its base-collector junction capacitance \( C_{bc1} \) results in a low output impedance of the first stage, forcing the pole to a higher frequency. In addition, the pole at the input of the first stage ends up at a lower frequency because the capacitive part of its input impedance is increased due to the "Miller effect".

Obviously, with the proposed two stage amplifier as depicted in Fig.5.2, it seems feasible to establish a phase shift of -90° (originating from the low-frequency pole) over a wide range of frequencies. Unfortunately, due to the presence of the collector-substrate capacitances and bulk resistances in series with the base and collector terminals of the transistors, the phase response readily deviates from the ideal -90°. Presently, we discuss the mechanisms that cause this impairment.

**Excess phase shift resulting from junction capacitances**

The capacitance \( C_{cs1} \) adds to the input capacitance of the second stage, so decreasing the bandwidth of the local feedback loop around \( Q_1 \), shifting the high frequency pole to a lower frequency.

The base-collector capacitances \( C_{bc1,2} \) conduct a fraction of the base currents directly to the collector terminals, so introducing a zero in the right half plane. These zeros contribute the same phase shift as would a pole located at the same frequency in the left half plane.

Fig.5.3 (open squares) depicts the amplitude and the phase response of the two-stage amplifier, as calculated with SPICE. We used the models of the semicustom process (\( \tau_f = 50 \)ps, corresponding to an \( f_t \) of about 3GHz) which was used for the realization of the prototype system. It shows that an additional phase shift of -25° (yielding a total phase shift of -115°) is obtained at a frequency as low as 75MHz, which is about \( \frac{f_t}{40} \).

**Excess phase shift resulting from bulk resistances**

We distinguish three mechanisms associated with the base and collector bulk resistances \( R_b \) and \( R_c \) that introduce additional phase shift. As we consider
the contribution of one resistance at a time, our models are in fact gross simplifications. Using SPICE, an estimation of the resulting total phase shift will be made.

First, as depicted in Fig.5.4a, we consider the bulk resistances $R_{c1}$ and $R_{b2}$ in series with the collector terminal of $Q_1$. Due to their presence, the local-feedback loop around $Q_1$ and $C_{bc1}$ is no longer loaded directly with the input impedance of the second stage. Consequently, the high-frequency pole, which corresponds to the bandwidth of the feedback loop, will move to a higher frequency. However, at the same time, a new pole is introduced by the transfer function from the first stage to the second. This function is an $RC$ low-pass filter, consisting of $R_{c1}$, $R_{b2}$ and $C_{bc2} + C_{bc2}$.

A second additional pole results from the collector series resistance $R_{c2}$ of $Q_2$, see Fig.5.4b. It causes current splitting at high frequencies between the collector-substrate capacitance $C_{cs2}$, $C_{bc2}$ and $R_{c2}$.

Finally, consider Fig.5.4c with the base bulk resistances $R_{b1}$ and $R_{b2}$. As
Figure 5.4: Bulk resistances at (a) the collector terminal of \( Q_1 \), (b) the collector terminal of \( Q_2 \) and (c) the base terminals of \( Q_{1,2} \) in presence of partitioned base-collector junction capacitances.

An approximation of the physical base-collector capacitances, which are distributed over the entire length of the base bulk resistances, we assume the capacitances to be partitioned into \( C_{bcA} \) and \( C_{bcB} \) at both ends of \( R_{b1} \) and \( R_{b2} \). A result of the partitioning is that a larger fraction of the supplied base signal current is passed directly to the collector terminal. Consequently, the zero in the right half plane, which results from the direct transfer, is shifted to a lower frequency.

Fig.5.3 (closed squares) illustrates the effect of bulk resistances. Compared to calculations performed with the bulk resistances set to zero (open squares), it shows that considerable additional phase shift (more than \(-25^\circ\)) results at frequencies higher than about \( \frac{f}{5} \) (in the SPICE model, \( \tau_f \) was taken to be 50ps, corresponding to an \( f_i \) of about 3GHz).
Compensation of phase shift by the feedback network

To compensate for the phase shift of the active part of the amplifier, in some cases a zero can be included in the feedback network. Preferably, the zero is a so-called phantom zero [1] because it does not introduce additional poles or zeros in the amplifier transfer function. One phantom zero compensates for a maximum of $90^\circ$ to the total phase shift at the band edge of the amplifier loop.

In the configuration of the amplifier depicted in Fig.5.1, a phantom zero is established by putting a capacitance across the resistor in place of $Z_1$. If the source impedance or $Z_2$ are capacitive, zeros can be induced also by inserting series resistances. However, if we use the source capacitance, the series resistance will contribute additional noise at the amplifier input. As noted earlier, coils are not available.

Discussion

Assuming that sufficient amplifier stages are available to obtain the required LP product, the bandwidth of negative-feedback amplifiers is restricted by the condition that the poles of the amplifier transfer function should be in specific (for instance Butterworth) positions. To locate the poles in Butterworth positions, the total phase shift within the loop should be $-90^\circ$ at the edge of the amplifier bandwidth. Other pole locations require the phase shift to be of the same order of magnitude.

We determined the phase shift of non-interactive stages and of stages elaborating pole splitting techniques. Over a relatively large range of frequencies, pole splitting ideally establishes a phase shift of $90^\circ$ per two successive stages.

Unfortunately, due to collector-substrate capacitances and bulk resistances, a larger phase shift results than one would expect at first sight. In a particular IC process, according to Fig.5.3, an excess phase of $-90^\circ$ is obtained at a frequency of about $f_{10}$. Because a phase shift of $-90^\circ$ can just be compensated for by a phantom zero, as a rule of thumb the maximum bandwidth of two-stage negative-feedback amplifiers (having for instance Butterworth pole locations) is in the order of $f_{10}$. The bandwidth of amplifiers consisting of more than two stages is restricted to still lower frequencies.

We did not consider single-stage amplifier configurations, but, as far as the phase shift is concerned, since they include fewer time constants, a higher bandwidth can be achieved. However, the resulting LP product will generally be insufficient.

Note that we only considered CE stages. The mechanisms determining the phase shift in CC stages are very similar to those in CE stages. For this reason, CC stages will not be treated here.
5.2.2 Sensitivity

We will determine the total equivalent input noise current produced by a current-input amplifier in combination with a capacitive source impedance, which, in the case of the receiver amplifier, consists of the capacitances of the PIN photodiode and the wiring between the PIN and amplifier. In accordance with [1], it is assumed that the amplifier has been designed such that only the first stage contributes to the amplifier noise. To find its fundamental minimum, the biasing and the size of this transistor will be optimized.

Noise model of the input circuit

Fig.5.5 depicts the total capacitive source impedance $C_s$, the parallel feedback resistor $R_f$ ("$Z_1$" in Fig.5.1) and the hybrid-pi model of the first transistor including its noise sources. $S_{ub}$ is the power density spectrum of the thermal noise produced by the base bulk resistance, $S_{ib}$ is the spectrum of the base current shot noise and $S_{ic}$ is the spectrum of the collector current shot noise. The spectra depend on the base bulk resistance $R_b$, the reciprocal transconductance of the transistor $r_e$ and the transistor low-frequency current gain $\beta$:

$$S_{ub} = 4kTR_b \quad S_{ib} = \frac{4kT}{2r_e\beta} \quad S_{ic} = \frac{4kT}{2r_e}$$  \hspace{1cm} (5.8)

It is assumed that $R_f$ can be chosen so large that its thermally generated current noise is small compared to transistor noise. This will be verified later on.

Equivalent input noise

We will determine the spectrum $S_{i,eq}$ of the equivalent current source in parallel with $C_s$, representing the total equivalent noise, see Fig.5.6. Its magnitude is calculated as the sum of the power spectra of the mutually uncorrelated
5.2. THE RECEIVER AMPLIFIER

Figure 5.6: Equivalent current source representing the total noise.

contributions:

\[ S_{i,eq}(\omega) = |j\omega C_s|^2 S_{ub} + |1 + j\omega C_s R_b|^2 S_{ib} \]
\[ + \left| \left( \frac{1}{\beta} + \frac{j\omega}{\omega_t} \right) + j\omega C_s \left[ \frac{r_e}{\beta} + \frac{j\omega}{\omega_t} \right] R_b \right|^2 S_{ic} \]  

(5.9)

Provided that \( \beta \gg 1 \) and \( R_b \ll 2\beta r_e \), some minor contributions can be neglected. Straightforward calculations result in:

\[ S_{i,eq}(\omega) \approx 4kT \left( \frac{1}{2\beta r_e} + \omega^2 \left[ \left( \frac{R_b + \frac{r_e}{2}}{2} \right) C_s^2 + \frac{C_s}{\omega_t} + \frac{1}{2r_e\omega_t^2} \right] + \omega^4 \frac{R_b^2 C_s^2}{2r_e\omega_t^2} \right) \]  

(5.10)

Inspection of Eq.(5.10) reveals that the \( \omega^4 \)-factor is small compared to the \( \omega^2 \)-factor for frequencies below \( \omega_t \sqrt{\frac{2r_e}{R_b}} \). So, for practical amplifiers, which have a maximal bandwidth of about \( \frac{f_t}{10} \) (Sec.5.2.1), within the passband of the amplifier the \( \omega^4 \)-proportional contribution is negligible.

The total equivalent input current noise is estimated by integrating Eq.(5.10) over the amplifier bandwidth \( B_t \):

\[ I_n^2 \approx \int_0^{B_t} S_{i,eq}(f)df \]  

(5.11)

In order to account for the biasing dependence of \( \omega_t \), we substitute:

\[ \omega_t = \frac{1}{\tau_t + \frac{C_j}{r_e}} \]  

(5.12)

where \( \tau_t \) is the transit time of the transistor and \( C_j \) is its total base-emitter and base-collector junction capacitance. Next, minimization of \( I_n^2 \) is obtained by differentiating the result of Eq.(5.11) with respect to the biasing variable \( r_e \) and setting the result to zero, giving:

\[ r_{e,opt} = \frac{\sqrt{4\pi^2 B_t^2 \beta \tau_t^2 + 3}}{2\pi B_t \sqrt{\beta (C_j + C_s)}} \]  

(5.13)
\[ \approx \sqrt{\frac{3}{\beta}} \frac{1}{2\pi B_r (C_j + C_s)} \quad \text{provided that} \quad B_r \ll \frac{1}{2\pi \sqrt{\beta}} \] (5.14)

Substitution of \( r_{e,\text{opt}} \) into Eq. (5.11) reveals:

\[ I_n^2 \approx \frac{8\pi}{3} kTB_r^2 \left[ (C_j + C_s)\sqrt{\frac{3}{\beta}} + 2\pi B_r R_b C_s^2 \right] \] (5.15)

under the same condition.

If the input transistor had no base bulk resistance and no junction capacitances, the noise power given by Eq. (5.15) would depend only on the amplifier bandwidth \( B_r \), the source capacitance \( C_s \) and the transistor low-frequency current gain \( \beta \). This is a fundamental minimum to the amplifier noise!

As the practical transistor does have base resistance and junction capacitance, the geometrical dimensions of the transistor are important. For simplicity, we assume that \( R_b \) and \( C_j \) scale linearly with the device area scale factor \( m \):

\[ R_b = \frac{R_{bo}}{m} \] (5.16)

and:

\[ C_j = mC_{jo} \] (5.17)

in which \( R_{bo} \) and \( C_{jo} \) are respectively the bulk resistance and the total junction capacitance of a transistor having standard dimensions. The optimum \( m \) is found by substitution of Eqs. (5.16) and (5.17) into Eq. (5.15), differentiation with respect to \( m \) and solving for zero, resulting in (still assuming that \( B_r \ll \frac{1}{2\pi \sqrt{\beta}} \)):

\[ m_{opt} = \sqrt{\frac{2\pi B_r C_s^2 R_{bo}}{C_{jo} \sqrt{\frac{3}{\beta}}} \sqrt{\frac{\beta}{3}}} \] (5.18)

Finally, substituting \( m_{opt} \) in Eq. (5.15) yields:

\[ I_n^2 = \frac{8\pi}{3} kTB_r^2 C_s \sqrt{\frac{3}{\beta} \left[ 1 + 2\sqrt{\frac{2\pi B_r R_{bo} C_{jo} \sqrt{\beta}}{3}} \right]} \] (5.19)

The factor preceding the parenthesis in Eq. (5.19) represents the fundamental noise minimum (no base bulk resistance and no junction capacitance). In addition, the term within the parenthesis describes the noise excess factor resulting from parasitic elements. Apart from the values of the time constant \( R_{bo} C_{jo} \) and the current gain \( \beta \), which are process dependent, the noise excess factor of the optimal amplifier depends on the amplifier bandwidth \( B_r \).
As an example, we calculated the equivalent input noise current of an amplifier realized in the process used for the prototype system. Taking $B_r = 250\text{MHz}$, $C_s = 2\text{pF}$, $\beta = 100$ and $R_{bo}C_j = 50\text{ps} \approx \tau_f$, we found $I_n = 42\text{nA}$. Thereby, the excess factor was calculated to be about 4dB. The optimum value of $r_e$ was 35Ω.

So far we considered asymmetric configurations. When doubling the values of $R_b$ and $r_e$ and halving $C_j$, Eqs.(5.9–5.19) are approximately correct for a balanced configuration as well.

At the start of our calculations, we assumed $R_f$ to be so large that the contribution of its thermally generated noise is small compared to the noise produced by the amplifier itself. To determine the minimal required value of $R_f$, we may for instance demand that the amplifier sensitivity is decreased by only 1dB. Having calculated the amplifier noise to be $I_n^2$, this criterion yields:

$$R_f > \frac{8\lambda kT B_r}{I_n^2} \quad (5.20)$$

with $\lambda = 2$ for asymmetric and $\lambda = 1$ for balanced amplifier configurations because, as is illustrated by Fig.5.7, in balanced configurations the total current noise depends on $2R_f$ rather than on $R_f$.

![Figure 5.7: Contributions of the thermal noise produced by $R_f$ in single sided and balanced amplifier configurations.](image)

5.3 The Switching Differential Pair

Many of the circuits constituting the transmitter and the receiver, such as the laser driver, the pulse reshaper, the multiplexer and the demultiplexer are based on switching differential pairs (SDPs). In this section, the minimal switching time of the SDP, its settling time and the jitter (noise) in its propagation delay will be determined.
5.3.1 Switching and Settling Times

In modeling the behavior of the SDP, we distinguish four modes in which the SDP can operate: modes (1) and (2) are the so-called saturated modes, in which the output signal of the SDP has reached its stationary value (settling). Mode (3) is the transient mode, in which the output signal changes and roughly approaches its new stationary value. The time the SDP is in mode (3), which will be determined using a large signal model, will be denoted by the so-called switching time $T_{sw}$. Subsequently, the SDP will enter mode (4), in which the output signal actually settles to its stationary value. As it would take an infinitely long time to really settle to the stationary value, we will define the settling time $T_{settle}$ by taking a certain settling error as a criterion.

Switching time of one single SDP

To start with, we concentrate on one single SDP, as shown in Fig.5.8, driven by a current source $I_s$ and terminated by the resistors $R_1$ to the positive supply voltage. The transistors $Q_{1,2}$ are characterized by their forward transit time $\tau_f$,

![Figure 5.8: Basic circuit of a switching differential pair.](image)

a feedback capacitance $C_{bc}$ and the base and collector bulk series resistances $R_b$ and $R_c$. $I_e$ denotes the tail bias current.

In the saturated modes (1) and (2), one of the transistors conducts the full current $I_e$. To determine the switching time (duration of mode (3)), we assume that the total charge that has to be supplied by $I_s$ during switching from one saturation mode to the other is given by:

$$Q_{tot} = I_e \tau_f + I_e(R_c + R_1)C_{bc} \quad (5.21)$$
in which we assumed that the charge stored in base-emitter junction capacitances $C_{be}$ is negligible compared to the base charge required for the diffusion current. This assumption is justified for collector currents larger than $\frac{C_{be}kT}{q\tau_1}$ (in the IC process used for the prototype system about 0.1mA).

The first right hand term of Eq.(5.21) denotes the charge injected into the base of the intrinsic transistors, the other term denotes the charge injected into the capacitances $C_{bc}$. If, for simplicity, we assume that the supplied source current $I_s$ changes stepwise from zero to its stationary value, the switching time required to supply $Q_{tot}$ equals:

$$T_{sw} = \frac{I_c}{I_s}(\tau_f + (R_c + R_t)C_{bc})$$  \hfill (5.22)

**Switching time of a cascade of SDPs**

Because the SDP stage in practical circuitry is driven by an identical preceding stage, we now consider a cascade of SDP stages, as depicted in Fig.5.9. To make a first approximation of the switching times, we assume that the current leaking into parasitic capacitances such as the collector-substrate capacitance $C_{cs}$ is negligible and that the resistances $R_1$ supply the constant current $\frac{I_s}{2}$, in which $I_s$ is the tail current of the driving stage. So, the effective source current $I_s$ supplied to the second stage equals plus or minus $\frac{I_s}{2}$.

To obtain an expression allowing successive stages to have different biasing currents a current-scale factor $\beta_{re}$ is defined, equal to the ratio of the tail currents of two successive stages. Still assuming a stepwise change of $I_s$, in accordance with Eq.(5.22) the switching time becomes:

$$T_{sw} = 2\beta I_e(\tau_f + (R_c + R_t)C_{bc})$$  \hfill (5.23)
Note that the influence of the loading by the next SDP is disregarded because it does not influence the total charge \( Q_{\text{tot}} \).

Unfortunately, because in fact a considerable fraction of the driving current (equaling \( \frac{I_s}{2} \)) leaks into \( R_1 \) and \( C_{\text{es}} \) and because the practical driving current will not change stepwise, our estimation of the switching time is too optimistic.

Obviously, the fraction leaking into \( R_1 \) can be minimized by choosing \( R_1 \) as large as possible. However, Eq. (5.21) shows that if \( R_1 \) increases, the required \( Q_{\text{tot}} \) increases as well. The optimum value of \( R_1 \) is determined as follows.

During switching, the voltage between the two intrinsic base terminals of \( Q_{1,2} \) gradually varies from about -0.1V to +0.1V, so the average intrinsic base-to-base voltage equals approximately zero. Consequently, the fraction \( \gamma \) (in the range 0–1) of \( \frac{I_s}{2} \) which flows into \( R_b \) equals:

\[
\gamma = \text{def.} \quad \frac{I_s}{2I_e} \approx \frac{R_1}{R_1 + R_b} \quad (5.24)
\]

By expressing \( R_1 \) in \( \gamma \) and \( R_b \), and multiplying \( T_{\text{sw}} \) given by Eq. (5.23) by the factor \( \gamma^{-1} \), we obtain:

\[
T_{\text{sw}} = \frac{2}{\gamma} \beta_I \left( \tau_f + (R_c + \frac{1}{\gamma} - 1)C_{bc} \right) \quad (5.25)
\]

The optimum value of \( \gamma \) is calculated by differentiation of Eq. (5.27) and setting the result to zero, yielding:

\[
\gamma_{\text{opt}} = \frac{\tau_f + C_{bc}R_c}{\tau_f + C_{bc}(R_c - R_b)} \left( 1 - \sqrt{ \frac{R_bC_{bc}}{R_cC_{bc} + \tau_f} } \right) \quad (5.26)
\]

If the time constants \( R_bC_{bc} \) and \( R_cC_{bc} \) are in the order of \( \tau_f \), which was true in the IC process used for our prototype system, we find \( \gamma_{\text{opt}} \approx 0.6 \). As the resulting \( T_{\text{sw}} \) is not very sensitive to the precise value of \( \gamma \), from now on we will assume \( \gamma = \frac{1}{2} \), implying that \( R_1 = R_b \) (Eq. (5.24)).

Besides leaking into \( R_1 \), another fraction of \( I_s \) leaks away into \( C_{\text{es}} \), while the driving current increases gradually rather than stepwise to its stationary value. We therefore define an excess factor \( F \):

\[
T_{\text{sw}} = 4F\beta_I \left( \tau_f + (R_c + R_b)C_{bc} \right) \quad (5.27)
\]

The practical value of \( F \) is estimated using a SPICE simulation. Fig. 5.10 (upper plot) shows the calculated waveform of the current \( I_{c1} - I_{c2} \) of one of the SDP stages. We took the parameters of the process used for our prototype system: \( \tau_f = 50 \text{ps} \), \( R_c = 150 \Omega \), \( R_b = 300 \Omega \) and \( C_{bc} = 0.17 \text{pF} \). From the figure,
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Figure 5.10: Waveforms of the collector currents in a cascade of SDP stages. Upper plot: SDP based on CE stages, lower plot: SDP based on CC and CE stages.

we see that the switching time is in the order of 0.7ns. To get the same result from Eq.(5.27), the value of $F$ must be chosen in the order of 1.4.

Provided that the time constants $R_bC_{bc}$ and $R_cC_{bc}$ are in the order of $\tau_f$, the $T_{sw}$ obtained in other IC processes can be estimated by rewriting Eq.(5.27) into:

$$T_{sw} = 4F\beta I_e 3\tau_f \approx 15\beta I_e \tau_f$$

(5.28)

**Magnitude of the biasing current $I_e$**

According to Eq.(5.28), $T_{sw}$ does not depend on the bias current $I_e$. However, this does not imply that $I_e$ can be freely chosen because the voltage swing at the input of the SDP should be large enough to drive the switch into one of its saturated modes, in which almost the full tail current flows through one of the transistors. To have the activated transistor conducting 99% or 99.9% of $I_e$, the input voltage should amount to 115mV or 170mV respectively. This voltage is the output voltage of the preceding stage, which is determined by the product $I_eR_i$. As for minimal switching times $R_i$ should be in the order of
$R_b$, we find the condition:

$$I_e(99\%) = \frac{115\text{mV}}{R_b} \quad I_e(99.9\%) = \frac{170\text{mV}}{R_b}$$ (5.29)

Because the load resistor $R_l$ exhibits a maximal tolerance of about 30%, it is recommended to choose $I_e \geq \frac{200\text{mV}}{R_b}$. Obviously, the minimum value of $I_e$ depends on $R_b$, and hence on the transistor emitter area.

The maximum value of $I_e$ can be found from the constraint that the base-collector junctions of $Q_{1,2}$ should not come in the forward mode.

**Improved switching speed by including CC buffers**

To improve the switching speed, the stages can be modified by inserting common collector (CC) buffer stages at the inputs, as depicted in Fig.5.11. At the instant the drive current $I_s$ from the preceding stage becomes available to the CC buffer stage around $Q_{1,2}$, $I_s$ flows from its base to its emitter terminals. As the inputs of the CE stage around $Q_{3,4}$ are in series with the emitter terminals of $Q_{1,2}$, almost the same drive current is available for the CE stage. In fact, the current available for the CE stage is a little less because the base-collector capacitances $C_{bc}$ of $Q_{1,2}$ require a fraction of $I_s$ as well. As the collector (signal) current which starts to flow through $Q_{1,2}$ gradually adds to the buffer output current, the magnitude of the buffer output current (driving the CE stage) readily exceeds the magnitude of $I_s$. Consequently, shorter switching times are obtained.

Fig.5.10, showing the waveforms (calculated by SPICE) of the collector currents of the CE stage in a cascade of CE stages (upper plot) compared to
the results obtained by a cascade of CC-CE stages (lower plot), illustrates that
the switching time has been decreased by a factor of \( \frac{7}{8} \). So, the switching time
given by Eq.(5.28) is decreased by the same factor, yielding:

\[
T_{sw} \approx 10\beta_{le} \tau_f
\]  
(5.30)

**Settling times**

To avoid distortion of TDPPM coded signals, relative to each other the po-
positions of the TDPPM pulses need to be accurately maintained. So, after an
SDP that handles TDPPM pulses has changed its state, the next pulse may
not arrive before the output voltage of the SDP has (approximately) reached
its stationary value.

To define a measure for the settling time, we take the criterion that the pulse
displacement \( T_e \) due to settling errors must be small compared to the pulse
deveiation caused by modulation \( (T_\Delta) \). Assuming that the slope magnitude of
the pulse edges equals the ratio between the pulse amplitude and the switching
time \( T_{sw} \), the total \( T_e \) after the pulses have passed \( n \) SDPs can be approximated
as:

\[
T_e \approx n\gamma T_{sw}
\]  
(5.31)
in which \( \gamma \) is the momentary amplitude of the pulse tail, normalized with
respect to its peak-to-peak amplitude. In Chap.3, we concluded that to achieve
reasonable values of the SNR \( T_\Delta \) should at least be in the order of the pulse
width \( T_p \) (this follows from the constraint that \( k \leq 0.8 \)). As the minimum value
of \( T_p \) is twice the switching time \( T_{sw} \), the resulting distortion-to-signal ratio is
limited according to:

\[
\frac{T_e}{T_\Delta} \leq \frac{n\gamma}{2}
\]  
(5.32)

A practical value of \( \gamma \) follows from assuming that the total number \( n \) of SDPs
handling the TDPPM pulses is about 25 and the required distortion level
should be below 0.1%, yielding \( \gamma \leq 10^{-4} \).

To determine the time required to settle the pulse tails to within a fraction
\( \gamma \) of the amplitude of the pulses, we assume an exponential damping of the
SDP output voltage. When ignoring the influence of the loading by a successive
SDP, the \( RC \) time constant at the output is found to be:

\[
\tau_{settle} = (R_c + R_t)(C_{bc} + C_{cs})
\]  
(5.33)
in which \( C_{cs} \) is the collector-substrate capacitance. Since \( e^{-9} \approx 10^{-4} \), the
10\(^{-4}\)-settling time is given by:

\[
T_{settling} = 9\tau_{settle} = 9(R_c + R_t)(C_{bc} + C_{cs})
\]  
(5.34)
In practical devices, $C_{cs}$ is of the same order of magnitude as $C_{bc}$, while the time constants $R_cC_{bc}$ and $R_iC_{bc}$ are in the order of $\tau_f$. So, we use the approximation:

$$T_{\text{settling}} \approx 35\tau_f$$  \hspace{1cm} (5.35)

Unfortunately, SPICE simulations show that our model is too simple. In the case of the unbuffered SDPs, the waveform of the output voltage of an SDP is influenced by the switching of the subsequent SDP. However, after a while the waveform exhibits the expected exponential damping. In the case of the buffered SDPs, as we will see in Sec.7.2.1, the buffers cause ringing (damped oscillations) of the output voltage. By systematically avoiding capacitive loading of the buffer stages, the oscillation damping time constant can be kept in the same order of magnitude as the $\tau_{\text{settle}}$ calculated above. Hence, our approximation still applies.

**Minimal Pulse Width**

Finally, having determined $T_{\text{sw}}$ and $T_{\text{settle}}$, we are able to estimate the total effective pulse width by taking the sum of the rise and the fall times (both in the order of $T_{\text{sw}}$) and $T_{\text{settle}}$:

$$T_{\text{pulse}} \geq 2T_{\text{sw}} + T_{\text{settling}} \approx (20\beta L + 35)\tau_f$$  \hspace{1cm} (5.36)

### 5.3.2 Propagation Jitter

In this section, we determine the propagation jitter of the SDPs. We will examine the influence of additive noise that is already present in the input signal of the SDP, as well as the influence of noise generated by the SDP itself.

To determine the noise behavior of an SDP, we need to consider what happens during switching, when the SDP exhibits the characteristics of an amplifier-limiter rather than the characteristics of a switch. As theory dealing with the behavior of limiters in the presence of noise is hardly available elsewhere, we start by presenting a general model describing the noise behavior of limiters. Subsequently, we will concentrate on the specific implementation of the limiter by an SDP.

**General limiter model describing noise behavior**

One important function of a limiter could be to prevent a signal from (temporarily) exceeding a certain amplitude. A possible application of such a limiter would be at the input of the PPM modulator, in order to ensure that the PPM pulses are kept within the appropriate time slots. Another function could be to systematically remove any variations in the amplitude of a signal, thereby
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maintaining only the positions of the zero crossing. In this case, the small signal gain of the limiter will be made large enough to obtain output pulses whose edges exhibit so-called saturated steepness: The shape of the edges remains constant, even if the amplitude of the limiter input signal is further enlarged. In fact, as in TDPPM coded signals the information is carried only in the positions of the pulses and since we wish to make the system minimally susceptible to noise and spurious signals received from other circuits, it is this function of the limiter we are interested in.

To determine the jitter in the timing of the edges of a limiter output signal, we first consider the simple model of the limiter depicted in Fig.5.12, in which an infinitely large gain is assumed. The model assumes a limited acquisition bandwidth, which is modeled by the low-pass filter \( H(f) \) at the input. In practical limiter circuits, the low-pass filter consists, for instance, of the bulk base resistances and the junction and diffusion capacitances of the transistors. The spectral power density of the noise present in the input signal is given by \( G_n(f) \). The filter is followed by an "ideal" comparator, which has infinite gain and bandwidth.

The method for determining the resulting jitter is very similar to the method proposed in Sec.3.5, in which we examined the reshaping of the received TDPPM pulses. However, at this stage, we do not assume that the input pulses have a specific shape. Instead, we assume that the waveform of the input signal around the zero crossings is described by a ramp, having a slope magnitude \( S \). It is also assumed that the slope magnitude of the filtered ramp signal equals \( S \) as well, requiring the filter bandwidth not to be too small.

At the instant the filtered input signal crosses zero, the comparator changes the sign of its output signal. The jitter in the "decision time" of the comparator is calculated by dividing the mean square value of the noise \( \sigma_n^2 \) by the square

![Figure 5.12: Model of an infinite-gain limiter having a limited acquisition bandwidth.](image-url)
of the slope magnitude $S$. The mean square magnitude of the filtered noise is calculated as:

$$\sigma_n^2 = \int_{-\infty}^{+\infty} G_n(f)|H(f)|^2 df$$  \hspace{1cm} (5.37)

and in the special but important case of white noise, with (two-sided) noise power density $G_n(f) = \frac{\mu}{2}$, we find:

$$\sigma_{td}^2 = \frac{\mu}{S^2} B_n$$  \hspace{1cm} (5.38)

in which $B_n$ represents the noise bandwidth of the low-pass filter defined by:

$$B_n = \frac{\int_0^{\infty} |H(f)|^2 df}{|H(0)|^2}$$  \hspace{1cm} (5.39)

Eq. (5.38) shows that it might be interesting to (purposely) restrict $B_n$ because this reduces jitter. However, if $B_n$ becomes so small that the slope magnitude of the comparator input signal is decreased, a net increase of $\sigma_{td}$ will result. Obviously, for minimal jitter, an optimal value of $B_n$ can be found.

The more complicated but more realistic model of Fig.5.13 assumes that the limiter consists of a cascade of limiter stages, which have a finite gain and low-pass filters at the inputs. The static input-output transfer of a single stage is depicted in Fig.5.14. The positive and negative output limiting levels are denoted by $+L$ and $-L$ respectively. In the so-called "linear region", the transfer function is characterized by the gain $g_1$ and the small signal bandwidth $B_1$.

Yet we define the aperture time $T_{ap}$ of a limiter stage as the length of the time interval during which the stage is in its linear mode. Concerning the
5.3. **THE SWITCHING DIFFERENTIAL PAIR**

![Diagram](image)

Figure 5.14: Simple model of the static input-output transfer of a single limiter stage.

The influence of the successive stages on the jitter of the limiter output signal, it will be shown that $T_{ap}$, in conjunction with the filter bandwidth $B_l$, is a crucial parameter. As each limiter stage contributes additional gain, the slope magnitude of the ramp signal is further increased after each stage. Consequently, provided that the amplitude of the input signal is large enough to drive the limiter stages into their limiting region, $T_{ap}$ decreases in successive stages (indexed by $i$):

$$T_{ap,i} = \frac{2L}{(g_i)_i S}$$  \hspace{1cm} (5.40)

in which $S$ is the slope magnitude of the limiter input signal.

However, as the low-pass filters at the inputs of the limiter stages limit the slope magnitudes of the pulses to a maximal value (denoted by $S_{\text{max}}$), $T_{ap}$ cannot become infinitely small. In addition, in practical circuitry, $S_{\text{max}}$ is also limited by slewing effects of the amplifier. Therefore it is convenient to assume that the “saturated slope magnitude” $S_{\text{max}}$ is a given parameter. Since a limiter stage is in its linear region only if its input signal is in the range $-\frac{L}{g_i}$ to $+\frac{L}{g_i}$, the minimal value of $T_{ap}$ is given by:

$$T_{ap,\text{min}} = \frac{2L}{g_i S_{\text{max}}}$$  \hspace{1cm} (5.41)

Having determined the values of $T_{ap}$ in successive stages, we distinguish three different regions in the cascade of limiter stages: stages in which (1) $T_{ap}$ is large compared to, (2) stages where $T_{ap}$ is in the order of, and (3) stages where $T_{ap}$ is small compared to $\frac{1}{B_l}$.

First, we take into consideration one of the stages operating in region 1. As the $T_{ap-1}$ of the preceding stage is large compared to $\frac{1}{B_l}$, the output signal of the filter has time to “track” its input signal, which only changes during $T_{ap-1}$. Hence the limiter stage under consideration, which regards the output signal of the filter during $T_{ap}$ (which is shorter than $T_{ap-1}$), cannot distinguish the preceding sections of the limiter cascade from “normal” non-limiting amplifiers.
Therefore, the stages operating in region 1 are said to effectively serve as linear amplifiers. The fact that they are not really in their linear mode at all times is irrelevant. As each of these stages performs additional low-pass filtering, due to the so-called bandwidth narrowing effect, the noise bandwidth depends on the number of stages operating in mode 1. Consequently the noise bandwidth decreases for decreasing slope magnitude \( S \) of the limiter input signal.

In region 3, where \( T_{\text{op}} \) is small compared to \( \frac{1}{B_1} \), the output signal of the filter is not able to track its input signal. So, in the stages operating in region 3, which are the last stages of the limiter, the waveform of the output signals of the stages is fixed. Hence, the positions of the zero crossings have been frozen.

The stage in between the stages operating in the modes 1 and 3, operating in region 2, actually determines the decision jitter. The jitter is calculated by applying Eq.(5.38), whereby the noise bandwidth \( B_n \) depends on the number of stages operating in region 1.

To derive an estimation of the maximal jitter produced by the limiter, one may take the noise bandwidth of one single stage because, due to bandwidth narrowing, the actual \( B_n \) is smaller.

**Noise Behavior of SPDs**

Having presented a general model describing the noise behavior of limiters, we now focus on the jitter produced in SPDs. The circuit diagrams of the unbuffered and the buffered version where presented in Fig.5.9 and Fig.5.11 respectively.

As the bandwidth of the CC buffer stages is large compared to that of the CE stages, for both circuits the noise bandwidth \( B_n \) depends on the CE stages. A model of the balanced CE stage, based on the hybrid-pi substitution circuit of the bipolar transistor, is shown in Fig.5.15. The low-pass \( RC \) filter at the input consists of the two base bulk resistances \( R_b \) and the total effective base-to-base input capacitance \( C_{bb} \), which approximately equals:

\[
C_{bb} = \frac{C_{bc}}{2} \left[ 1 + 2(R_l + R_c)g_m \right] + \tau_f g_m \quad (5.42)
\]

where \( C_{bc} \) is the base-collector junction capacitance, \( R_c \) the collector bulk series resistance, \( R_l \) the load resistance and \( \tau_f \) the forward transit time of the transistors constituting the CE stage. The base-emitter junction capacitance \( C_{be} \) is disregarded, which is justified if the collector currents are larger than \( \frac{C_{be} kT}{q v_T} \). The parameter \( g_m \) is the transconductance of the CE stage:

\[
g_m = \frac{q I_{c1} I_{c2}}{kT I_e} \quad (5.43)
\]
5.3. **THE SWITCHING DIFFERENTIAL PAIR**

![Diagram](image)

Figure 5.15: Model of a SDP, used to determine its noise bandwidth.

in which $I_{c1}$ and $I_{c2}$ are the collector currents of $Q_1$ and $Q_2$ respectively and $I_e$ is the tail bias current. The first term on the right hand side of Eq.(5.42) describes the influence of $C_{bc}$ due to the Miller effect. The second contribution originates from the diffusion capacitance.

Since, during switching, the currents $I_{c1}$ and $I_{c2}$ are not constant, $g_m$ varies with time. As a result, $C_{bb}$ varies starting from $\frac{C_{bc}}{2}$ to a maximum (where $I_{c1} = I_{c2} = \frac{I_e}{2}$) and then back to the starting value. So, the noise bandwidth (of one stage) as defined by Eq.(5.39), which is calculated to be:

$$B_n = \frac{1}{8R_bC_{bb}}$$

(5.44)

varies accordingly. Nevertheless, by assuming the effective $B_n$ to be constant and equal to the actual value of $B_n$ when $I_{c1} = I_{c2}$, a useful estimate of the effective noise power can be made for the following two reasons:

(1) If a particular SDP exhibits a long aperture time, the response of the subsequent SDPs depends only on the waveform of the input signal of the first SDP when the signal is close to zero, so that the condition that $I_{c1} = I_{c2}$ is satisfied automatically.

(2) If, on the other hand, the SDP exhibits an aperture time in the order of or shorter than $\frac{1}{B_n}$, the displacement of the decision time of the subsequent SDP depends on the average value of the noise signal over the aperture time. Since the small signal power gain of the first SDP, being proportional to $g_m^2$, is strongly peaked around the actual zero crossing, the average value mainly depends on the small signal transfer around the zero crossing where $I_{c1}$ is close to $I_{c2}$.

A simplified expression for the effective $C_{bb}$ is derived from Eq.(5.42) by
substituting $I_{c1} = I_{c2} = \frac{I_e}{2}$ and ignoring some small contributions:

$$C_{bb} \approx \frac{qI_e}{4kT}[C_{bc}(R_c + R_1) + \tau_f]$$  (5.45)

The related effective noise bandwidth $B_n$ is calculated using Eqs.(5.45) and (5.44):

$$B_n = \frac{kT}{2qR_bI_e[C_{bc}(R_c + R_1) + \tau_f]}$$  (5.46)

We now come to the results of our calculations, which apply for three different situations:

1. If a ramp signal in the presence of additive noise having a noise power density $\mu$ is supplied to the SDP limiter and if the noise produced by the SDPs themselves may be ignored, the maximal value of the resulting jitter (assuming $B_n$ to be determined only by the first SDP stage) is given by:

$$\sigma_{id}^2 = \frac{\mu B_n}{S^2} = \frac{\mu}{S^2} \frac{kT}{2qR_bI_e[C_{bc}(R_c + R_1) + \tau_f]}$$  (5.47)

Provided that, as is the case in the process used for the prototype system, the time constants $C_{bc}R_c$ and $C_{bc}R_1$ are in the order of $\tau_f$, we may write:

$$\sigma_{id}^2 \approx \frac{\mu}{S^2} \frac{kT}{2qI_e} \frac{1}{3R_b\tau_f}$$  (5.48)

Since $B_n$ is the same for SDPs consisting of only CE stages and SDPs using the CC buffers, this result applies to both types of SDPs.

2. If the slope magnitude $S$ of a noiseless input signal is less than $S_{\text{max}}$, the contribution of the noise produced by the first SDP stage dominates. Since the condition given by Eq.(5.29) automatically implies that:

$$I_e \gg \frac{2kT}{R_bq}$$  (5.49)

in which $R_b$ is the base bulk resistance of the transistors constituting the CC and the CE stages, the contributions of the shot noise produced by the collector currents can be ignored when compared to the contributions of the thermal noise produced by the base bulk resistances. Hence, the power density of the equivalent voltage noise at the input of the SDP is given by:

$$\mu = 4kTR_bF$$  (5.50)

in which $F = 2$ for the simple SDP and $F = 4$ for the buffered SDP. After substitution of $\mu$ into Eq.(5.48), we find:

$$\sigma_{id}^2 \approx \frac{2F(kT)^2}{S^2qI_e3\tau_f}$$  (5.51)
(3) If $S$ is equal to or larger than $S_{\text{max}}$, the SDPs (either the unbuffered or the buffered version) all contribute the same amount of jitter. The saturated slope magnitude $S_{\text{max}}$ of the intrinsic base-to-base voltage of the CE stage is estimated by dividing its peak-to-peak collector current swing, which equals $I_e$, by the transconductance $g_m$ and the switching time $T_{sw}$ (in fact, by substituting $T_{np} = T_{sw}$, this relation follows from Eq.(5.41)):

$$S_{\text{max}} \approx \frac{I_e}{g_m T_{sw}}$$  \hspace{1cm} (5.52)

Using Eq.(5.52), $S_{\text{max}}$ is calculated by substituting $I_{c1} = I_{c2} = \frac{I_e}{2}$ into Eq.(5.43), giving $g_m$, and taking $T_{sw}$ given by Eqs.(5.28) or (5.30) for the unbuffered and the buffered SDP respectively. The current-scale factor $\beta_{I_e}$ was assumed to be 1. Substitution of $S_{\text{max}}$ into Eq.(5.51) yields:

$$\sigma_{td}^2 = \frac{15^2}{12} \frac{q}{I_e} \tau_f \approx 20 \frac{q}{I_e} \tau_f$$  \hspace{1cm} (5.53)

for the unbuffered SDP and:

$$\sigma_{td}^2 = \frac{10^2}{6} \frac{q}{I_e} \tau_f \approx 15 \frac{q}{I_e} \tau_f$$  \hspace{1cm} (5.54)

for the buffered SDP. Compared to the SDP without the buffers, the modified SDP produces 3dB more equivalent input noise. But, since the switching time is a factor of 1.4 shorter, the latter exhibits a slightly lower intrinsic jitter level. As an illustration, the root mean square magnitude of the jitter produced by one SDP (buffered type) in the prototype system, where $I_e=1\text{mA}$ and $\tau_f=50\text{ps}$, amounts to 0.36ps.

### 5.3.3 Conclusions

We found that, provided that its tail bias current $I_e$ is high enough to justify neglecting the influence of the base-emitter junction capacitances, the switching time of the switching differential pair (SDP) does not depend on the precise value of $I_e$. However, to assure that the SDPs are driven into their saturated modes, in which the tail currents almost completely flow through one of the transistors constituting the actual switch, the minimal value of $I_e$ is in the order of $200\text{mV} / R_i$. The maximal value of $I_e$ follows from the condition that the base-collector junctions of the transistors should not come into forward mode. By choosing values of the load resistors $R_i$ equal to the base bulk resistances $R_b$ and by inserting CC buffer stages between the actual switches, switching times in the order of $10\tau_f$ can be achieved.
To determine the influence of external noise sources and noise sources within the SDP itself on the jitter of the output signal, we developed a general model of a limiter. We found that the noise bandwidth of an SDP, and hence the resulting jitter, irrespective of where the noise is actually produced, is inversely proportional to $I_e$.

An interesting strategy for dimensioning the SDP stages is as follows: Because $I_e$ influences only the jitter level and not the switching times, the appropriate value of $I_e$ follows from the requirements with respect to the resulting jitter. The next step is to choose $R_1$ such that for the given $I_e$ the resulting voltage swing is large enough to obtain saturated switching. Finally, to obtain minimal switching times, we scale the transistors such that $R_b$ is in the order of $R_1$.

5.4 Oscillators

Oscillators are used for clock generation in the transmitter and for clock regeneration in the receiver. The TDPPM modulator and demodulator use the clock signals as time references. Consequently, any phase noise in the clock signals directly influences the SNR of the transmitted signals.

Our discussion, which subsequentially concentrates on so-called first-order and harmonic oscillators, will be based on the available theory. To make the results comparable to each other and to make them applicable for the specific application field, we will present some modifications to models published in the literature.

We start by examining the shape of the noise power density spectrum which is common to all types of oscillators, and define a suitable measure of the oscillator phase noise. We then deal with integratable first-order oscillators and harmonic oscillators. Some examples of practical (integratable) oscillators can be found in Chap.7 of this work and in [2], [3], [4], [5] and [8].

5.4.1 Noise Power Density Spectra of Oscillators

Fig.5.16 depicts the phase noise power density $S_n(f)$ of the output signal of an oscillator as a function of the frequency distance $f$ to the carrier frequency $f_o$. Over a relatively large range of frequencies, $S_n(f)$ decreases with the square of $f$. This noise is sometimes referred to as "frequency noise" since demodulation of an oscillator signal exhibiting $\frac{1}{f}$-noise by a frequency detector results in a flat noise spectrum at the detector output.

At frequencies far from $f_o$, $S_n(f)$ is determined by other noise mechanisms causing a flat output spectrum. Also at frequencies close to $f_o$, the shape of
$S_n(f)$ deviates from its $\frac{1}{f^2}$-characteristic. A first cause for this is the contribution of physical mechanisms within the oscillator circuitry producing $\frac{1}{f}$-noise. In the output spectrum this results in $\frac{1}{f^2}$-noise. A second cause is that the "modulation index" for noise components modulating the oscillator frequency is inversely proportional to $f$, so that, in determining the resulting noise power density close to $f_o$, the contributions of higher order Bessel terms cannot be ignored. The precise shape of $S_n(f)$ close to $f_o$ is difficult to determine but, since the total oscillator power, which is finite, must by definition equal the area under the curve, we expect the curve to be flat around $f_o$. (As it would result in an infinite area, $S_n(f)$ cannot be proportional to $\frac{1}{f^3}$ down to the lowest $f$.)

We now present some frequently used measures to specify the phase noise produced by oscillators. The most common figure is the ratio between the phase noise power density at a specific frequency distance $f$ from the carrier frequency $f_o$ and the total oscillator power:

$$L(f) = \frac{S_n(f)}{P_o}$$  \hfill (5.55)

This ratio can be measured directly by a spectrum analyzer.

A second figure is the carrier-to-noise ratio (CNR), defined at a specific frequency distance $f$ from the carrier frequency. It is the inverse of $L(f)$:

$$\text{CNR} = \frac{1}{L}$$

A third figure, which can be measured at the output of a phase demodulator, is the power density spectrum of phase fluctuations (in radians$^2$ per Hz):

$$S_\Phi(f) = 2L(f)$$  \hfill (5.56)

From $S_\Phi(f)$, we can determine the mean square value of the phase jitter by
integrating over the frequency range of interest (between \( f_1 \) and \( f_2 \)):

\[
\sigma^2_\Phi = \int_{f_1}^{f_2} S_\Phi(f) \, df
\]  

(5.57)

Alternatively, we can calculate the mean square value of the time jitter as:

\[
\sigma_t^2 = \frac{1}{(2\pi f_0)^2} \int_{f_1}^{f_2} S_\Phi(f) \, df
\]  

(5.58)

In deciding which type of oscillator is to be used in our transmission system, it will be necessary to compare different types of oscillators running at different frequencies. For instance, crystal oscillators are most suited for frequencies below 50MHz, while regenerative oscillators can be used at frequencies up to 1Ghz. In a specific application, frequency multipliers and dividers can be used to obtain an oscillator signal at the desired frequency.

In comparing oscillators running at different frequencies, \( L(f) \) and \( \Phi(f) \) are not very convenient measures because they depend on the oscillator frequency. To compare different types of oscillators, a more suitable measure is the so-called mean square fractional frequency fluctuation density, which is defined by:

\[
S_f(f) = 2 \left( \frac{f}{f_o} \right)^2 L(f)
\]  

(5.59)

Because the contribution from the flat part of the noise spectrum is low enough to be ignored, we restrict our discussions to the \( \frac{1}{f^2} \) and \( \frac{1}{f^3} \) parts of the noise spectrum.

### 5.4.2 First-Order Oscillators

A typical example of a first-order oscillator [2] is depicted in Fig.5.17. First-order oscillators are popular in IC designs because they are suited for complete integration.

The capacitor \( C \) integrates the constant current \( I \) until the voltage across the capacitor equals one of the reference voltages \( V_1 \) or \( V_2 \). Then the polarity of \( I \) is reversed, so that a periodical oscillator signal results. The frequency of the oscillator signal is given by:

\[
f_o = \frac{I}{2CU_{hys}}
\]  

(5.60)

in which \( U_{hys} \) is the hysteresis voltage \( V_2 - V_1 \).

In [2] two types of noise sources are distinguished: (1) Voltage noise sources that are in series with the comparator inputs (\( V_{n1} \) and \( V_{n2} \), see Fig.5.17),
modulating $U_{\text{hys}}$, and (2) current noise sources $I_n$ that are in parallel with the integrator capacitor, modulating the integration constant $\frac{I}{C}$. As follows from Eq. (5.60), variation of $U_{\text{hys}}$ and $\frac{I}{C}$ by noise modulates the oscillator frequency $f_o$.

As the comparators will be based on switching differential pairs (SDPs), we apply the theory developed in Sec.5.3 to calculate the magnitudes of the voltage noise sources at the input of practical comparators and to estimate their noise bandwidth. Then, using the theory of [2], the resulting noise power density spectrum of the oscillator signal will be determined. Subsequently, we will focus on the contributions of current noise sources.

As other types of first-order oscillators, such as the "coupled oscillator" [5] and the "ring oscillator", exhibit similar characteristics (because the mechanism which determines the oscillator accuracy is common to all first-order oscillators), we will restrict ourselves to the type shown in Fig.5.17. In Chap.7, this configuration will be the bases of a practical implementation.

*Oscillator noise caused by voltage noise*

The simplified model of the comparator input circuit is shown in Fig.5.18 and consists of an $RC$ low-pass filter and an ideal comparator. If the first stage of

\[
\hat{u}_n = 8kTR_v
\]

![Figure 5.18: Simplified model of a comparator.](image)

the comparator circuit consists of a differential pair without additional input
buffers, \( R_{comp} \) represents twice the base bulk resistance of the transistors while, if the collector currents are large enough to ignore the influence of the shot noise, the equivalent noise resistance \( R_v \) equals approximately \( R_{comp} \). The capacitance \( C_{comp} \) equals the capacitance \( C_{bb} \) given in Eq.(5.45). The squared amplitude of the noise voltage \( \hat{u}_n^2 \) is related to the thermal noise of the resistor \( R \):

\[
\hat{u}_n^2 = 8kTR_{comp}
\]  
(5.61)

The noise bandwidth as defined by Eq.(5.39) equals:

\[
B_n = \frac{1}{4R_{comp}C_{comp}}
\]  
(5.62)

According to [2], Eq.(6.142), the relative phase noise power density of the oscillator in the \( \frac{1}{f^2} \) region resulting from voltage noise is given by:

\[
L(f) = \frac{1}{2}n_{fold}\left( \frac{f_o}{f} \right)^2 \left( \frac{\hat{u}_n}{U_{hys}} \right)^2 \left[ \cos(\pi \frac{f}{f_o}) + 1 \right]^2
\]  
(5.63)

or, equivalently:

\[
S_f(f) = n_{fold}\left( \frac{\hat{u}_n}{U_{hys}} \right)^2 \left[ \cos(\pi \frac{f}{f_o}) + 1 \right]^2
\]  
(5.64)

The factor \( n_{fold} \) accounts for the contribution of noise originating from the sidebands around the harmonics of the oscillator frequency \( f_o \). The number of sidebands \( n_{fold} \) that effectively contribute to the oscillator noise depends on the noise bandwidth of the comparator \( B_n \) and on \( f_o \) ([2], Eq.(6.135)):

\[
n_{fold} = \frac{B_n}{2f_o}
\]  
(5.65)

As we are interested in the noise close to the carrier frequency \( f_o \) (small \( f \)), the term of Eqs.(5.63) and (5.64) within the square brackets may be assumed to be 4. So, by substituting Eq.(5.62) into Eq.(5.65) and substituting Eqs.(5.61) and (5.65) into Eqs.(5.63) and (5.64), we find:

\[
L(f) = \left( \frac{f_o}{f} \right)^2 \frac{2kT}{C_{comp}f_o U_{hys}^2}
\]  
(5.66)

\[
S_f(f) = \frac{4kT}{C_{comp}f_o U_{hys}^2}
\]  
(5.67)

An important observation from Eq.(5.67) is that a higher oscillator frequency results in a lower fractional frequency fluctuation density \( S_f \). That is,
the oscillator performs better at higher frequencies. This is due to the fact that \( n_{\text{fold}} \) (for constant \( B_n \)) is inversely proportional to \( f_o \).

To minimize the oscillator noise, it follows from Eq.(5.67) that \( U_{\text{hys}} \) should be made as large as possible. A fundamental maximum is the supply voltage but, since for oscillators running at high frequencies the appropriate level shifts cannot be realized, the maximum is generally considerably lower. Practical values of \( U_{\text{hys}} \) are in the range 1–2V.

In addition, Eq.(5.67) implies that \( C_{\text{comp}} \) should be made as large as possible. However, in high frequency oscillators an impairment compared to the noise predicted by Eq.(5.67) is due to the fact that the sensitivity of the oscillator to voltage noise is actually determined by the slope magnitude \( S \) of the input signals of the intrinsic comparators (voltage across \( C_{\text{comp}} \)) rather than by the magnitude of the hysteresis \( U_{\text{hys}} \). The capacitance \( C_{\text{comp}} \), which in practical comparators has its maximal value if the comparator input signal amounts to zero (see Eq.(5.42)), is in parallel with the integrator capacitance \( C \). The total capacitance is maximal around the decision time of the comparator, so the effective \( S \) is decreased. Only if ideal comparators having zero input capacitance were used, the slope magnitudes would follow directly from the hysteresis and the oscillator frequency: \( S=2U_{\text{hys}}f_o \). So, to avoid a deterioration of \( S \), \( C_{\text{comp}} \) should be kept small compared to \( C \).

As an example, we consider a practical first-order oscillator running at a frequency of 70MHz, implemented in the 3GHz bipolar IC process, which was used for the prototype system (see Sec.7.6.5). The integrator capacitance \( C \) is about 1.5pF and \( U_{\text{hys}} \) amounts to 2V. As the biasing current of the comparators was chosen to be 1mA, by applying Eq.(5.45) the input capacitance \( C_{\text{comp}} \) was calculated to be in the order of 1pF. At a 1MHz distance from \( f_o \), Eq.(5.66) yields \( L=-128 \text{dB/Hz} \). However, with SPICE it was calculated that the resulting slope magnitude at the intrinsic transistor terminals of the SDP was a factor of 1.8 smaller than is expected from \( U_{\text{hys}} \). As the square of the comparator decision jitter is proportional to \( \frac{1}{S^2} \) (see Sec.5.3.2), the impairment amounts to 5dB, yielding \( L=-123 \text{dB/Hz} \).

Oscillator noise caused by current noise

The contributions of current noise sources (denoted by \( I_n \), see Fig.5.17) in parallel with the integrator capacitance follow from [2], Eq.(6.35):

\[
L(f) = \left( \frac{f_o}{f} \right)^2 \frac{2kT}{R_i I^2} \quad (5.68)
\]

\[
S_y(f) = \frac{4kT}{R_i I^2} \quad (5.69)
\]
in which $I$ is the integrator input current and $R_1$ is an equivalent resistor which produces the same amount of thermal noise as all the current noise sources together. The expression is valid for noise frequencies $f$ relatively small compared to the oscillator frequency $f_0$. Contributions of high frequency noise due to folding have been ignored because the integrator attenuates high frequency noise.

The oscillator developed for our prototype system uses an integrator current of 0.4mA and exhibits an equivalent noise resistance of approximately 5000Ω. At 1MHz distance from the carrier frequency, substitution in Eq.(5.68) yields $L=-123\text{dB/Hz}$.

Conclusions

From Eq.(5.67), it can be seen that, as far as the contribution of the comparator voltage noise is concerned, the oscillator noise is inversely proportional to the input capacitance $C_{\text{comp}}$ of the comparator, provided that $C_{\text{comp}}$ is kept small compared to the integrator capacitance $C$. Consequently, to reduce the contribution of the comparator noise, besides $C_{\text{comp}}$, we also have to increase the capacitance $C$. Thereby, the integrator current $I$ has to be increased accordingly.

Eq.(5.69) shows that the contributions of the current noise sources are also decreased by increasing the integrator current $I$. So, to obtain a lower oscillator noise the oscillator requires more supply current. However, as the supply current of the prototype oscillator is already in the order of 1 mA, improvements by one order of magnitude or more will require impractically large currents.

In the prototype of the system, the contributions of the voltage noise and the current noise turn out to be of the same order of magnitude, resulting in a total noise-to-carrier ratio of $L\approx-120\text{dB/Hz}$. The measurements presented in Sec.7.6.5 closely match the calculated value.

Finally, we conclude from Eqs.(5.64) and (5.69) that, if $n_{\text{fold}}$ is constant and if $f \ll f_0$, $S_f$ does not depend on the oscillator frequency.

5.4.3 Harmonic Oscillators

Harmonic oscillators, producing sinusoidal output signals, are based on two reactive elements or a resonator. The oscillator frequency depends on the values of the reactive elements or the resonator frequency. Suitable reactive elements are a capacitor and an inductor ($LC$ oscillator) or two capacitors (two-integrator oscillator, [7]). In principle, two inductors can also be used, but this option is generally found to be unattractive. The resonator can be a crystal
(see for example [6]) or a ceramic resonator. The two-integrator oscillator is the only one suited for complete integration because on-chip capacitors can be used.

Before focusing on the noise behavior, which is our main objective, we start giving a brief introduction to the principles of harmonic oscillators based on LC, crystal or ceramic resonators. The two-integrator oscillator will be discussed separately because its implementation and noise behavior are different.

**Resonator oscillators**

Fig.5.19 depicts the block diagram of a resonator oscillator, consisting of the resonator, an amplifier and (in accordance with [8]) a load. The parameters $S_{na}$, $S_{nr}$ and $S_{nl}$ will be clarified later on.

To obtain sustained oscillation, the resonator losses are compensated by sensing the current through the resonator, converting this current into a voltage (this is performed by the amplifier) and "setting" this voltage across the resonator. In this way, a low negative impedance results across the resonator terminals. This mode of oscillation is generally referred to as series resonance. Alternatively, in the so-called parallel resonance mode the amplifier senses the voltage across, and forces a current through, the resonator.

Since harmonic oscillators are basically linear networks, the amplitude of the oscillator output signal is not fixed, so additional circuitry is necessary to provide amplitude stabilization. As explained in [8], amplitude stabilization is preferably obtained by using a limiting amplifier, limiting the voltage across or the current through the resonator.

Figure 5.19: Block diagram of the resonator oscillator.
The phase noise produced in harmonic oscillators depends on the power densities of the noise produced by physical sources compared to the signal power $\frac{E^2}{2}$, but also on the Q-factor of the resonator, the small signal transfer (transimpedance or transconductance) $A$ of the limiting amplifier and the transfer (conductance or impedance) $H_o$ of the resonator at resonance frequency [8], p.47:

$$L(f) = \frac{1}{E^2} \left[ \left( \frac{f_o}{2Qf} \right)^2 (AH_oS_{na} + H_o^2S_{nr}) + S_{na} + \frac{S_{nl}}{A^2} \right]$$ (5.70)

in which $S_{na}$ is the power density spectrum of the noise sources at the amplifier input, $S_{nr}$ is the power density of the sources at the resonator input and $S_{nl}$ is the spectrum of noise sources at the input of the output buffer. All power density spectra are assumed to be white. The parameter $f_o$ denotes the oscillator frequency. Eq.(5.70) is a good approximation if the so-called excess loop gain $AH_o$ is large compared to 1, which is the loop gain required to maintain the oscillation [3]. To obtain minimal phase noise, $AH_o$ should not be chosen much larger than necessary. Practical values are in the range of 2–100.

Fig.5.20 shows an oscillator configuration based on a resonator operated in the series resonance mode. The resonator output current $I$ results in a voltage across $R_1$, which is forced to be the voltage across the resonator by means of the nullor. The maximum amplitude of this voltage is determined by the limiter. The equivalent circuit of the series resonator consists of the capacitance $C$, the inductance $L$ and the series damping resistance $R_s$. The total equivalent

![Diagram](image)

Figure 5.20: Configuration of the series resonance oscillator.

voltage noise is represented by $R_v$ and the total equivalent current noise by $R_i$. In this circuit, $AH_o$ is determined by $\frac{R_v}{R_i}$.

In the case of the series resonance oscillator, Eq.(5.70) can be rewritten as:

$$L(f) = \frac{4kT}{I^2} \left[ \left( \frac{f_o}{2Qf} \right)^2 \left( \frac{R_1}{R_1R_s} + \frac{R_v}{R_s^2} \right) + \frac{1}{R_i} \right]$$ (5.71)
in which \( I \) is the amplitude of the signal current flowing through the resonator.

To minimize the oscillator noise, it seems from inspection of Eq.(5.71) that we should select a resonator which exhibits a large value of \( R_s \). However, as \( Q \) is inversely proportional to \( R_s \), this conclusion is not correct. The best performance requires the resonator having the largest \( Q \).

To compare the performance of the resonator oscillator with the previously discussed first-order oscillator, we assume that the resonator current \( I \) and the current noise resistance \( R_i \) are equal to, respectively, the integrator current and the current noise resistance of the first-order oscillator. Additionally, we assume that the voltage amplitude across the resonator \( IR_s \) and the noise resistance \( R_v \) are equal to, respectively, the voltage amplitude \( V_{vna} \) across the integrator and voltage noise resistance \( R_{comp} \) of the first-order oscillator. Since we are only interested in the \( \frac{1}{f^2} \) part of the noise spectrum, we rewrite Eq.(5.71) as:

\[
L(f) = \frac{4kT \left( \frac{f_0}{2Qf} \right)^2 \left( \frac{R_1}{R_1R_s} + \frac{R_v}{R_s^2} \right)}{f^2} \\
S_f(f) = \frac{2kT}{f^2Q^2} \left( \frac{R_1}{R_1R_s} + \frac{R_v}{R_s^2} \right)
\]

(5.72)

Comparison of Eqs.(5.73) and (5.69) for the contributions of the current noise and Eqs.(5.73) and (5.64, substitute \( u_n^2 = 8kTR_v \)) for the contributions of the voltage noise, reveals that, apart from the influences of the excess loop gain \( \frac{R_s}{R_i} \) and the fold factor \( n_{fold} \), the CNR of harmonic oscillators is roughly a factor \( Q^2 \) better than the CNR of first-order oscillators.

Two-integrator oscillators

An example of a two-integrator oscillator is depicted in Fig.5.21. It can be

![Figure 5.21: Basic configuration of a two-integrator oscillator.](image-url)
easily shown that the oscillator frequency equals \( \frac{1}{2\pi\sqrt{R^2C^2}} \). The amplitude of the oscillator signal is stabilized by (automatically) tuning the oscillator pole positions just to the right or the left side of the imaginary axis, depending on whether the amplitude is too small or too large. For this purpose, a variable voltage amplifier (within the dashed box) having a gain close to unity can be used. An amplitude measurement circuit and an amplitude reference are needed to generate the required error signal.

To calculate \( L(f) \) in vicinity of the carrier frequency \( f_c \), we assume that the equivalent current and voltage noise at the nullor inputs are given by \( R_i \) and \( R_v \) respectively. Bearing in mind that the amplitude noise (50% of the total noise power) does not contribute to the phase noise, we find:

\[
L(f) = 2kT \left( \frac{f_c}{f} \right)^2 \left( \frac{1}{R_i I^2} + \frac{R_v}{R^2 I^2} \right) \quad (5.74)
\]

\[
S_f(f) = 4kT \left( \frac{1}{R_i I^2} + \frac{R_v}{R^2 I^2} \right) \quad (5.75)
\]

in which \( I \) is the amplitude of the signal current flowing through \( R \).

By comparing Eq.(5.75) with Eqs.(5.64) and (5.69), it appears that, for the same values of \( R_i, R_v, I \) and voltage amplitudes across the integrators: \( IR = \frac{U_{in}}{2} \), the two-integrator oscillator exhibits approximately the same \( S_f(f) \) as the first-order oscillator. For low frequencies, the two-integrator oscillator will perform slightly better, since in the two-integrator oscillator there is no mechanism folding high frequency noise to the carrier frequency.

### 5.4.4 Conclusions

The CNR obtained by first-order oscillators is comparable to the CNR obtained by two-integrator oscillators. Both types of oscillators are well suited for integration, but only the first-order oscillator facilitates the possibility of tuning the oscillator frequency by just changing the integrator current or the comparator threshold levels. Because of the inaccuracies of integrated components, tunability is considered to be an important advantage.

We determined the CNR of a 70MHz first-order oscillator implemented in the IC process used for our prototype, in which the integrator current is about 0.5mA, yielding \( L(1\text{MHz}) = -120\text{dB/Hz} \) and \( S_f = 4 \cdot 10^{-16} \). An improvement by a certain factor can only be obtained by increasing the integrator current, and hence the supply current, by the same factor.

Resonator oscillators perform better by approximately a factor \( Q^2 \). As in \( LC \) oscillators attainable \( Q \) factors are in the range 50–200, and since crystals exhibit \( Q \) factors in the order of \( 10^5 \), the improvements are considerable.
Unfortunately, resonator oscillators have the disadvantage that they require external components.

It was shown that apart from the influence of $n_{\text{fold}}$, the fractional frequency fluctuation density $S_f$ (defined by Eq.(5.59)) obtained by first-order, two-integrator and resonator oscillators is independent of the oscillator frequency.

5.5 Continuous-Time Filters

The smoothing filters at the outputs of the receiver consist of low-pass continuous-time filters. The filters can be constructed using discrete inductors and capacitors, but, for our low-cost system, it is far more attractive to integrate the filter on the chip. In [17] a general design methodology for such filters is described. Since it is the major problem in the design of such filters, the objective has been to realize a maximal (distortion free) dynamic range (DR).

The basic building blocks of integrated continuous-time filters are integrators, of which an example is shown in Fig.5.22. The resistor $R$ converts the input voltage into a current, which is integrated by the active integrator around the capacitor $C$, resulting in a voltage output. The structure of the complete filter will be discussed later on in Sec.7.11, see Fig.7.59.

At this stage, using a rough approximation, for a given supply voltage and available capacitance we will determine the DR of the low-pass filter. If we assume that the input signal of the filter can be scaled such that the filter handles the largest possible signal amplitudes, the DR of the filter determines the maximum SNR of the filter output signal.

To determine the fundamental lower limit of the noise produced by the filter, we assume that only the resistor $R$ produces (thermal) noise. In [17], the equivalent input-voltage noise $V_n^2$ of one single integrator is calculated by integrating the resulting voltage noise power density over the noise bandwidth of the filter $B_n$. As the bandwidth of the complete filter is in the order of
$\frac{1}{2\pi RC}$, we take the same value for $B_n$ yielding:

$$V_n^2 = \int_0^{\frac{1}{2\pi RC}} 4kTRdf = \frac{2kT}{\pi C}$$  \hspace{1cm} (5.76)

Given the supply voltage $V_{\text{sup}}$, the maximum peak-to-peak amplitude of integrator input and output voltages equals $V_{\text{sup}}$ and the maximum signal power is $\frac{V_{\text{sup}}^2}{8}$. By defining the DR of the integrator as the ratio of the maximum signal power and the noise power, we find:

$$\text{DR} = \frac{V_{\text{sup}}^2\pi C}{16kT}$$  \hspace{1cm} (5.77)

A rough but simple estimation of the DR of a complete low-pass filter can be made by making the following assumptions: Each integrator uses roughly the same value of the capacitor $C$, the transfer function from all integrator voltage-inputs to the filter output is that of a low-pass filter with noise bandwidth $B_n$, and the integrators only produce thermal noise related to $R$. If the filter consists of $n$ integrators, the total noise voltage power equals the sum of $n$ individual contributions. So the DR of the filter becomes:

$$\text{DR}_{\text{filter}} = \frac{V_{\text{sup}}^2\pi C}{16nkT}$$  \hspace{1cm} (5.78)

In [17] a more accurate calculation is presented which is universally applicable to all integrated continuous-time filters. But within the scope of this work our estimation is accurate enough.

As in integrated circuits capacitances of 1pF and a supply voltage of 5V are realistic while in most applications a sufficient smoothing is obtained with $n = 5$, a DR in the order of 84dB seems possible. Unfortunately, as will be shown in Sec.7.11, in practical filters implemented in bipolar processes, the amplitude of the signal is restricted to a fraction of $V_{\text{sup}}$ while, due to provisions providing the possibility of tuning the bandwidth of the filter, the noise level turns out to be higher than assumed here. So, in the practical filter a lower DR will result.

5.6 Conclusion

In this chapter, we determined the characteristics of the electronic circuits, as far as they influence the bandwidth and the sensitivity of the receiver, the width of the TDPPM pulses handled by the circuits based on SDPs, the jitter in the positions of the TDPPM pulses, the phase noise of the clock signals, and the dynamic range of the smoothing filters.
5.6. CONCLUSION

By combining the results of the characteristics of the electronic circuits with the characteristics of the optical components, which were examined in Chap. 2, we are able to determine the system specifications in a given technology. This will be the subject of Chap. 6.
Bibliography


Chapter 6

System Specifications

6.1 Introduction

In this chapter we will determine the maximum of the total signal bandwidth (denoted by $B_{\text{tot}}$, which was defined in Sec.3.6.3 as the product of the number of multiplexed signals and the bandwidth of the signals) and the signal-to-noise ratio ($\text{SNR}_0$) of the received signals. These parameters depend on the characteristics of the optical components, which were discussed in Chap.2, and on the characteristics of the electronic circuits, which were examined in Chap.5. By making these dependencies explicit, the associated trade-offs become manageable.

6.2 Total Signal Bandwidth

In Sec.3.6.3, we defined the total signal bandwidth $B_{\text{tot}}$ as the product of the number of multiplexed signals $N$ and the bandwidth of the signals $B$. The maximum of this product $B_{\text{tot,max}}$ was found to be determined by the width $T_p$ of the TDPPM pulses:

$$B_{\text{tot,max}} = \frac{1}{2rT_p}$$

(6.1)

in which $r$ is the oversampling factor $\frac{N}{2B}$. For the time being, it was assumed that the minimal value of $T_p$ is determined by the bandwidth of the receiver front-end $B_r$. Presently, we will consider all other possible limitations to $T_p$ as well.

Since the optical components and the electronic circuits handling the TDPPM pulses should not disturb the positions of the pulses, it is of primary concern to take care that the pulses do not overlap with the tails of preceding
pulses. Therefore the total pulse width $T_p$ is assumed to include the tails of the pulses.

A first possible limitation to $T_p$ originates from the optical components. Because the influence of fiber dispersion can be made small by choosing the appropriate type of fiber and because the bandwidth of standard PIN photodiodes amounts to over 1GHz, we assume that the LED or laser diode determines the minimum value of $T_p$. In Sec.2.1.2, we concluded that LEDs exhibit a maximum bandwidth $B_{LED}$ of about 200MHz (but unfortunately emit very little power). To estimate the minimum pulse width, we assume that the emitted TDPPM pulses have the shape of raised cosines (defined in Sec.3.5.3), implying:

$$T_{p,LED} \approx \frac{2}{B_{LED}} = 10\text{ns} \quad (6.2)$$

For low-cost laser diodes (because of the low-cost constraint), we argued that the minimum pulse width mainly depends on the time required to settle the pulse tails (see Sec.2.2.4). A reasonable value of the minimal pulse width is:

$$T_{p,laser} \approx 2\text{ns} \quad (6.3)$$

Two other possible limitations to $T_p$ are due to the properties of the electronic circuits. They depend on the transit time $\tau_t$ of the bipolar transistors. The first limitation is the bandwidth $B_t$ of the receiver front-end negative-feedback amplifier (Sec.5.2.1), which has a maximum value of about $\frac{1}{10} \approx \frac{1}{20\pi\tau_t}$. Assuming that the TDPPM pulses at the amplifier output have the shape of a raised cosine, we find:

$$T_{p,amp} \approx \frac{2}{B_t} = 40\pi\tau_t \quad (6.4)$$

The second limitation of the electronic circuitry originates from the switching and settling times of the switching differential pair (SDP), which is the basic building block for the modulator, the multiplexer, the laser driver, the pulse reshaper, the demultiplexer and the demodulator. In Sec.5.3.1 we approximated the $T_p$ that can be obtained by optimally designed SDPs as:

$$T_{p,SDP} = 20\beta_{\text{ld}}\tau_t + 35\tau_t \quad (6.5)$$

in which $\beta_{\text{ld}}$ is the current-scaling factor of two successive SDPs. Except for the circuits constituting the laser driver, in which the current level is to be scaled up in successive stages, $\beta_{\text{ld}}$ generally equals unity.

If the optical transmitter is an LED, limiting $T_p$ to a minimum of 10ns, $B_{tot}$ is limited to 28MHz or less (Eq.(6.1), assuming $r=1.75$). If a laser is being used, allowing $T_p$ to be in the order of 2ns, the maximum is 140MHz. By comparing Eq.(6.4) with Eq.(6.5), it can be seen that, as far as the electronic
6.3. SIGNAL-TO-NOISE RATIO

In the realm of circuitry, the parameter $T_p$ is crucial, as it is dependent on the receiver amplifier: $T_p=40\pi\tau_f$. Hence, if full profit is to be derived from the bandwidth capabilities of an LED or a laser diode, the IC process used in the fabrication of the circuits should be capable of matching this quality. For example, to match the bandwidth of the laser diode, $\tau_f$ should be in the order of 10ps, corresponding to an $f_t$ of about 10GHz. In the case of our prototype, we have $\tau_f=50$ps, so the receiver amplifier limits $T_p$ to about 6ns, yielding $B_{\text{tot,max}}=48\text{MHz}$.

As concluded in sec.3.6.3, to achieve reasonable values for the SNR$_o$, the actual value of $B_{\text{tot}}$ should be at least 20% below $B_{\text{tot,max}}$.

### 6.3 Signal-to-Noise Ratio

In Chap.3, we derived an expression giving the SNR$_o$ obtained in TDPPM systems for a given SNR$_r$ of the received pulse signal. The SNR$_r$ was defined as the ratio between the (half-amplitude) power of the received TDPPM pulses and the power of additive noise contributed by the laser diode (Sec.2.2.5) the PIN photodiode (Sec.2.3) or the receiver amplifier (Sec.5.2.2). It depends on the received signal power which of these sources dominates.

Chap.5 dealt with the various mechanisms present in the electronic circuits which result in additional jitter in the positions of the TDPPM pulses. In addition, Chap.5 discussed the phase noise produced by the oscillators, which causes jitter in the clock signals required for the modulator and demodulator. These contributions to the SNR$_o$, of course, do not depend on the received optical power.

The first aim of this section is to investigate how the SNR$_o$ at the receiver output depends on the received power level, so that for a certain application the allowable optical losses (the so-called optical budget) can be determined. The second aim is to examine how the SNR$_o$ depends on the transit frequency $f_t$ of the transistors, which can be regarded as a figure of merit of the type of IC process used. It will be shown that, depending on the $f_t$, the SNR$_o$ is determined by different noise sources.

To start with, we write the contributions of the laser, the PIN photodiode and the receiver amplifier to the SNR$_r$ as a function of the received power level and receiver bandwidth. Rather than using $f_t$, we will express our results as functions of the forward transit time of the transistors $\tau_f$, which is, for sufficiently high bias currents, approximately equal to $\frac{1}{2\pi f_t}$. The contributions from the SDPs and the oscillators will be determined in terms of the resulting pulse jitter. The next step will be to relate the calculated values of the SNR$_r$ and the jitter to the SNR$_o$.

Other possible noise sources, for example originating from the modulator
and the smoothing filter, depend too much on the circuit implementation, so they will be discussed later on in Chap.7. At this stage, we will assume those contributions to be negligible.

6.3.1 Definitions of $\text{SNR}_r$ and $P_r$

Since the decision level of the pulse reshaper is at half the height of the received TDPPM pulses and since the input quantity of the receiver front-end is a current, from now on the $\text{SNR}_r$ of the received TDPPM pulse signal will be defined as:

$$\text{SNR}_r = \text{def} \frac{(I_p)^2}{I_n^2} \tag{6.6}$$

in which $I_p$ is the peak-to-peak amplitude of the current pulses and $I_n$ is the root mean square magnitude of the noise current. In addition, the received power is defined as:

$$P_r = \text{def} \frac{(I_p^2)}{2} \tag{6.7}$$

6.3.2 Influence of Laser Noise

In Sec.2.2.5, it was argued that the RIN level obtained by low-cost lasers, which do not include optical isolators preventing impairment of the RIN due to reflections, is in the order of -125dB/Hz.

Because the TDPPM pulses are detected at half height, we use the RIN value at half the pulse height. As the noise produced by the laser is approximately white, the $\text{SNR}_r$ is calculated directly from the RIN and the receiver bandwidth $B_r$ as:

$$\text{SNR}_{r,\text{laser}} = \frac{1}{\text{RIN} B_r} \tag{6.8}$$

Since the receiver bandwidth $B_r$ equals about $\frac{1}{20\pi \tau_f}$, we can rewrite the $\text{SNR}_r$ as:

$$\text{SNR}_{r,\text{laser}} \approx \frac{20\pi \tau_f}{\text{RIN}} \tag{6.9}$$

6.3.3 Influence of PIN Shot Noise

The photodiode only produces shot noise, exhibiting a flat spectrum (Eq.(2.9)). Since the influence of the dark current can be ignored, the contribution of the photodiode is given by:

$$\text{SNR}_{r,\text{PIN}} = \frac{P_r}{2q \frac{I_p}{2} B_r} \tag{6.10}$$
By substituting Eq. (6.7) and putting \( B_r = \frac{1}{20\pi \tau_f} \) we find:

\[
\text{SNR}_{r,\text{PIN}} \approx \frac{\sqrt{P_r}}{q} 10\pi \tau_f
\]  

(6.11)

### 6.3.4 Influence of Receiver Amplifier Noise

Using Eq. (5.19), giving the mean square value of the equivalent noise current at the amplifier input (denoted by \( I_n^2 \)), and substituting \( B_r = \frac{1}{20\pi \tau_f} \), we write:

\[
\text{SNR}_{r,\text{amp}} = \frac{P_r}{\frac{2}{3\pi} k T (\frac{1}{10\tau_f})^2 C_s \sqrt{\frac{3}{\beta}} \left( 1 + 2 \sqrt{\frac{1}{10\tau_f} R_{bo} C_{jo} \sqrt{\frac{\beta}{3}}} \right)}
\]  

(6.12)

In the IC process used for our prototype system, the time constant \( R_{bo} C_{jo} \) is in the order of \( \tau_f \). Assuming \( R_{bo} C_{jo} \approx \tau_f \) to be a good approximation in general, we may conclude that the \( \text{SNR}_r \) is proportional to the square of \( \tau_f \).

### 6.3.5 Influence of Noise Produced by SDPs

The noise sources discussed so far contribute additive noise to the TDPPM pulses and hence decrease the SNR of the received TDPPM pulses. The noise produced by the SDPs does not affect the SNR of the received signals, but it does affect the SNR of the demodulated signals. Therefore we will determine the resulting pulse jitter instead.

Since the modulator, the multiplexer, the pulse reshaper, the demultiplexer and the demodulator consist of SDPs, the TDPPM pulses pass a cascade of several SDPs. The total jitter in the positions of the TDPPM pulses is the sum of the mean square magnitudes of the jitter produced by the \( n \) individual SDPs as given by Eq. (5.54), resulting in:

\[
\sigma_{\text{ln,SDP}}^2 = \sum_{j=1}^{n} \frac{15}{I_{c,j}} q \tau_f
\]  

(6.13)

Because the noise power resulting at the output of the smoothing filter depends on the shape of the power density spectrum of the jitter (Sec. 3.6.1), it is important to note here that the power spectrum of the noise produced by SDPs is white.

### 6.3.6 Influence of Noise Produced by the Oscillators

The influence of oscillator noise is determined using the model depicted in Fig. 6.1. Because it makes our calculations more legible, we represent the positions of the TDPPM pulses in the phase rather than in the time domain. The
relation between the two presentations is given by:

\[ T_k = \frac{\Phi_k}{2\pi f_s} \]  

(6.14)

in which \( f_s \) is the sample frequency of the PPM modulator, equaling the frequency of the clock oscillators, and \( k \) refers to the \( k \)-th PPM pulse. As the demodulator output signal is filtered by the smoothing filter, we ignore the sampling effects and base our model on the continuous-time function \( \Phi(t) \) and its Fourier transform \( \Phi(f) \).

Figure 6.1: Model describing the influence of phase noise produced by the transmitter and the receiver clock oscillators.

The clock in the transmitter (around osc.1) synchronizes the PPM modulator, which corresponds to a summing circuit in the phase domain. Because modulation is of no concern here, the modulator input will be set to zero. The clock signal itself is transmitted by the frame pulses. The receiver regenerates the clock signal by means of a phase lock loop (PLL) that averages the phase of the received frame pulses.

The PLL consists of a phase detector (subtractor in the phase domain) having a gain \( K_d \), a loop filter having a pole at the frequency \( \frac{1}{2\pi\tau_p} \) and a zero at the frequency \( \frac{1}{2\pi\tau_m} \), and an oscillator which is characterized by the integration constant \( K_o \). The variable \( s \) stands for the complex variable \( 2\pi j f \). The (radian) bandwidth of the PLL will be denoted by \( \omega_{PLL} \). To obtain simple expressions, we assume that the poles of the PLL are maneuvered into
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Butterworth positions, requiring:

\[ \tau_n = \frac{\sqrt{2} \omega_{PLL} \tau_p - 1}{\omega_{PLL}^2 \tau_p} \]  

(6.15)

in which \( \omega_{PLL} \) equals:

\[ \omega_{PLL}^2 = 4\pi^2 B_{PLL}^2 = \frac{K_a K_d}{\tau_p} \]  

(6.16)

The PPM demodulator consists of a subtractor. The demodulated signals are low-pass filtered by the smoothing filter having a bandwidth \( B_1 \).

The phase noise power density spectra of the transmitter and receiver oscillator signals (if the frequency-control inputs are set to a constant value) are denoted by \( S_{\Phi_1}(f) \) and \( S_{\Phi_2}(f) \) respectively. To start with, the shape of the noise spectra is assumed to obey (Sec.5.4.1):

\[ S_{\Phi}(f) = 2L(f_m) \left( \frac{f_m}{f} \right)^2 \]  

(6.17)

in which \( L(f_m) \) is the reciprocal CNR in a 1Hz bandwidth measured at a distance \( f_m \) from the carrier frequency \( f_s \). Subsequently, we will examine the influence of \( \frac{1}{f} \)-noise as well because crystal oscillators, which will be shown to be preferable in Sec.7.6.2, exhibit a relatively large range of frequencies where the \( \frac{1}{f} \)-noise dominates.

Using this model, we calculate the power transfer functions \( |H_1(f)|^2 \) and \( |H_2(f)|^2 \) describing the transfers from respectively the output of the transmitter oscillator (osc.1) and the output of the receiver oscillator (osc.2) to the demodulator output, yielding:

\[ |H_1(f)|^2 = |H_2(f)|^2 = \frac{f^2 (4\pi^2 f^2 \tau_p^2 + 1)}{4\pi^2 \tau_p^2 (f^4 + B_{PLL}^4)} \]  

(6.18)

Fig.6.2 gives a graphical presentation of \( |H(f)|^2 \). Obviously, the PLL serves as a high-pass filter for the noise produced by the oscillators. Regarding the influence of osc.1, this can be understood by considering that, at low frequencies, the PLL output signal follows its input signal, resulting in complete compensation for the noise carried by the PPM pulses. Regarding the influence of osc.2, the high-pass filtering is obvious because at low frequencies the noise present in the regenerated clock signal is suppressed due to the negative feedback within the PLL.

The jitter originating from each one of the oscillators is calculated by integrating the phase noise power density spectrum at the demodulator output.
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Figure 6.2: The oscillator phase noise power density spectrum $S_{\phi}(f)$, the transfer $|H(f)|^2$ and the resulting phase noise power density $S_{\phi}(f)|H(f)|^2$ at the demodulator output.

Over the filter bandwidth $B_1$ and by applying Eq.(6.14), yielding:

$$\sigma_{t,\text{osc1,2}}^2 = \frac{1}{4\pi^2 f_s^2} \int_0^{B_1} S_{\Phi1,2}(f)|H(f)|^2 df \quad (6.19)$$

Fig.6.2 depicts the argument of the integral as a function of the frequency. As the figure suggests, in practical PLLs the peak in the spectrum $S_{\Phi}(f)|H(f)|^2$ around the PLL band edge $B_{\text{PLL}}$ is relatively far from frequency of the zero related to $\tau_p$. Hence the numerator of Eq.(6.18) can be approximated by $4\pi^2 f^4 \tau_p^2$. In addition, since the peak in the output spectrum is at a much lower frequency than the bandwidth of the smoothing filter $B_1$ (because this is necessary to suppress the modulation of the positions of the frame pulses), the result of Eq.(6.19) can be approximated by replacing the upper bound of the integration range by infinity. Under these conditions, substitution of Eqs.(6.17) and (6.18) into Eq.(6.19) reveals:

$$\sigma_{t,\text{osc1,2}}^2 = \frac{\sqrt{2}}{8\pi} \left(\frac{f_m}{f_s}\right)^2 \frac{L(f_m)}{B_{\text{PLL}}} = \frac{\sqrt{2}}{16\pi} \frac{S_I(f_m)}{B_{\text{PLL}}} \quad (6.20)$$

The total jitter is the sum of the (identical) contributions from the transmitter and the receiver oscillators (osc.1) and (osc.2).

So far, our calculations were based on the assumption that the contribution of the oscillator noise was dominated by the $\frac{1}{f^2}$-part of the oscillator noise.
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spectrum. Now, we assume a $\frac{1}{f^3}$-spectrum instead, which is written as:

$$S_\Phi(f) = 2L(f_m)(\frac{f_m}{f})^3$$  \hspace{1cm} (6.21)

Substitution of $S_\Phi(f)$ into Eq.(6.19) yields:

$$\sigma_{i,osc,1,2}^2 = \frac{1}{8\pi} \frac{f_m^3 L(f_m)}{f_s^2 B_{PLL}^2} = \frac{f_m}{16\pi} \frac{S_\tau(f_m)}{B_{PLL}^2}$$  \hspace{1cm} (6.22)

Conclusions

In the case of a $\frac{1}{f^2}$ as well as in the case of a $\frac{1}{f^3}$-proportional oscillator phase noise power density, the resulting noise power density exhibits a maximum at the roll-off frequency of the PLL (i.e. $B_{PLL}$). As a first consequence, the influence of the flat part of the oscillator noise spectrum can be ignored if the counter frequency, where the $\frac{1}{f^2}$ or $\frac{1}{f^3}$-part of the spectrum becomes dominant, is large compared to $B_{PLL}$. This condition applies in practical cases. A second consequence of this peaking is that the smoothing filter is not effective in reducing the noise at the demodulator output.

In Sec.5.4, we concluded that the mean square frequency fluctuation density $S_\tau$ is generally independent of the oscillator frequency. Hence, the time jitter given by Eqs.(6.20) and (6.22) only depends on the type of oscillator used and on $B_{PLL}$.

The contribution of the oscillator phase noise to the jitter is minimized by choosing a large value of $B_{PLL}$. However, to remove the modulation present in the positions of the frame pulses, $B_{PLL}$ must be small compared to the lowest frequency that modulates the frame pulses. If, with this value of $B_{PLL}$, the resulting jitter is too high, we may consider using an oscillator exhibiting a lower phase noise.

Alternatively, by abandoning the possibility of using frame pulses for signal transmission, the frame pulses are available for clock-synchronization exclusively. Consequently, a wideband PLL can be used for efficiently suppressing the phase noise of the oscillators. A disadvantage of this alternative is that the jitter present in the positions of the frame pulses, caused by the noise of the laser diode, the PIN or the receiver front-end, is not suppressed. Hence, if those noise sources dominate the noise produced by the SDPs and the oscillators, the SNR$_o$ will be 3dB lower than when choosing a small PLL bandwidth.

6.3.7 Signal-to-Noise Ratio Versus Received Power

Fig.6.3 shows the SNR$_r$ of the received (TD)PPM signal as a function of the received power level $P_r$, as defined by Eq.(6.7). Since the SNR$_o$ of the receiver
output signals is proportional to the SNR_r, see Eq.(3.29), the curve giving the SNR_o exhibits the same shape. Because the actual values depend on the values of many parameters, we chose arbitrary units.

![Diagram showing the relationship between SNR_r, SNR_o, and received power level P_r, with noise sources such as laser noise, SDF noise, or oscillator noise, PIN noise, and amplifier noise.](image)

**Figure 6.3:** Signal-to-noise ratio of the received TDPPM pulse signal SNR_r and the receiver output signal SNR_o versus received power level P_r.

As the noise produced by the receiver front-end amplifier is constant, a low received power level P_r results in a low SNR_r and SNR_o. According to Eq.(6.12), SNR_r increases proportionally with P_r.

The noise produced by the PIN photodiode is proportional to the root of P_r so, above a certain level of P_r, the SNR_r is determined by the PIN photodiode. In this region, Eq.(6.11) applies, showing that the SNR_r increases proportionally to \( \sqrt{P_r} \).

The maximum value of the SNR_r, resulting for relatively high P_r, is determined by the SNR of the laser diode (given by Eq.(6.9)).

The maximum value of the SNR_o (not to be confused with the SNR_r) may be determined by the noise produced by the laser diode, but may also be determined by the noise produced by the electronic circuits such as the SDPs and the clock oscillators. If the noise produced by the SDPs limits the maximum SNR_o, the resulting SNR_o is calculated by substituting the jitter given by Eq.(6.13) into:

\[
SNR_o = \frac{(k - 1)^2}{2k^2B_r^2\sigma_i^2}r
\]

which was derived from Eq.(3.26) by substituting the factor \( k = \frac{B_{tot,max}}{B_t} = \frac{4N_rB_t}{B_t} \) and \( T_p = \frac{2}{B_r} \). In the case the oscillator phase noise dominates, we
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substitute Eqs.(6.20) or (6.22) into:

$$\text{SNR}_o = \frac{(k - 1)^2}{2k^2 B_r^2 \sigma_t^2}$$  \hspace{1cm} (6.24)

in which the improvement factor $r$ has been ignored because the noise (which exhibits a peaked power density spectrum) is hardly reduced by the smoothing filter.

In the prototype system, over the entire range of possible values of $P_r$ the SNR$_o$ that would result from the shot noise produced by the PIN photodiode was well above the SNR$_o$'s determined by the receiver amplifier and the laser diode (for optical power losses larger than and less than 15dB respectively), so the influence of the PIN noise was negligible. In addition, the laser noise just dominated the noise produced by the electronic circuits.

In Chap.3, Eq.(3.37), we approximated the value of SNR$_o$ required to avoid the reception of false TDPPM pulses to be in the order of 15dB. Now, having determined the SNR$_o$ as a function of the received power level $P_r$, we are able to calculate the value of $P_r$ that corresponds to the threshold level. As for low values of $P_r$ the noise produced by the receiver front-end dominates, we apply Eq.(6.12). In the prototype of the system, we estimated that the threshold value of $P_r$ is in the order of $10^{-13}$ which, if the sensitivity of the photodiode is assumed to be 0.8A/W, corresponds to an optical power of about -30dBm.

6.3.8 Signal-to-Noise Ratio Versus Transit Frequency

The aim of this section is to investigate how the contributions to the SNR$_o$ of the different noise sources vary as a function of $f_t$ (or $\tau_f$). For the sake of simplicity, we assume that the ratio $k$ between the total signal bandwidth $B_{tot}$ and the maximum signal bandwidth $B_{tot,max}$ is constant. In practical cases, this assumption applies if the number of channels $N$ or the bandwidth of the signals $B_1$ is proportional to $f_t$.

The constant ratio $k$ implies that the ratio $\text{SNR}^{\text{SNR}}_e$ remains constant and that the square of the TDPPM pulse deviation $T^2_\Delta$ is proportional to $\frac{1}{B_f}$ and hence to $\frac{1}{f_t}$ (Sec.3.6.3).

Fig.6.4 shows that the SNR$_o$s resulting from the laser and PIN noise, given by Eqs.(6.9) and (6.11), as well as the noise produced by the SDPs, determined by the ratio between $T^2_\Delta$ and the jitter given by Eq.(6.13), are inversely proportional to $f_t$.

The SNR$_o$ resulting from the receiver amplifier, which follows from combining Eqs.(3.36) and (6.12), decreases with the square of $f_t$. According to
Eqs. (6.20) and (6.22), if the PLL bandwidth $B_{PLL}$ is constant, the jitter produced by the oscillators is constant. As $T_\Delta^2$ is proportional to $\frac{1}{f_i^2}$, the SNR$_o$ resulting from the oscillators also decreases with the square of $f_i$.

To estimate the maximum attainable SNR$_o$, we assume that the electronic circuitry has been designed such that the laser noise dominates. So, substitution of Eqs. (6.8) and (6.9) into Eq. (3.36) yields:

$$\text{SNR}_o = \frac{\pi^2 r(k - 1)^2}{2k^2} \frac{1}{B_r\text{RIN}} = \frac{\pi^2 r(k - 1)^2}{2k^2} \frac{20\pi r_i}{\text{RIN}}$$  \hspace{1cm} (6.25)

For example, the prototype system was realized using transistors characterized by $\tau_f=50\text{ps}$, while $k=\frac{1}{2}$ and $r=1.75$. Assuming an RIN level of -125dB/Hz, we find SNR$_o=49$dB.

### 6.4 Measurements

Using a video measurement set, we measured the weighted video SNR obtained by the prototype system to be about 58dB. The corresponding SNR$_o$ is in the order of 46dB, which is 3dB lower than the calculated value. This is due to the fact that we slightly decreased the pulse deviation in order to avoid problems if the magnitudes of the input signals slightly exceed their nominal values. The variations in the measured values are due to variations in the influence of reflections on the RIN level. The laser noise was found to be dominant for received power levels higher than -15dBm.
6.5 Summary and Conclusions

The maximum of the total signal bandwidth, which is the product of the number of multiplexed signals and the bandwidth of the signals, depends on the width of the TDPPM pulses that can be handled by the optical components and the electronic circuitry. We concluded that, assuming the oversampling factor to be 1.75, low-cost laser diodes allow the product to be in the order of 140MHz, which is just in balance with the product that can be handled by the electronic circuitry (the receiver front-end) if the transmitter and receiver are integrated in an IC process exhibiting a transit frequency of 10GHz. If LEDs are being used, the product amounts to 28MHz.

If the received optical power level is relatively low, the $\text{SNR}_o$ depends on the noise produced by the receiver amplifier. If the power is relatively high, the $\text{SNR}_o$ is determined by the noise produced by either the laser diode, the SDPs or the clock oscillators. As the transit frequency of the integrated components increases, the influences of the noise produced by the amplifier and the oscillators will increase more than the influences of other noise sources. Consequently, the region where the amplifier noise dominates will be extended, while the design of the oscillators becomes increasingly critical.

We derived a simple expression giving the $\text{SNR}_o$ that can be obtained if the received optical power level is relatively high, as far as it depends on the noise produced by the (low-cost) laser diode.

The prototype system was realized in a bipolar IC process exhibiting $\tau_f=50\text{ps}$, so the attainable $B_r$ is in the order of 300MHz. The oversampling factor $r$ is 1.75, the number of channels $N$ is 4 and the signal bandwidth $B_1$ is 5MHz, yielding $k \approx \frac{1}{2}$. With these parameters and assuming the RIN to be -125dB/Hz, the $\text{SNR}_o$ is calculated to be 49dB. Due to the fact that the pulse deviation was chosen lower than its maximum value, the measurements on the prototype system revealed a 3dB lower value.
Chapter 7

Circuit Design

7.1 Introduction

In the preceding chapters, we examined the characteristics of the low-cost optical components, proposed the TDPPM coding scheme, discussed the optimal architecture of the TDPPM transmitter and receiver, determined the key parameters of the electronic circuits and related the system specifications to the available technology. Thereby, we completed the most fundamental part of this thesis.

In this chapter we will deal with the designs of circuits on the level of the components. Because the low-cost aspect is related to the complexity of the circuits, besides noise and distortion, one of the most important design objectives is minimal circuit complexity. This means that, if the performance of the circuit meets the requirements, we will prefer a more simple circuit over a qualitatively superior circuit.

To start with, a new class of logic, so-called modified emitter coupled logic (MECL), will be presented. MECL logic is based on the previously presented switching differential pair, and constitutes the basis of many of the required circuits. Next, the complete designs of the transmitter and receiver blocks will be discussed. Because most of the components are not critical, we will present the circuit diagrams without systematically determining the values of the components.

As it concerns critical circuits, the distortion produced by the modulator and the noise produced by the modulator, the receiver front-end, the clock generator and the clock regenerator will be examined in detail. For some of the circuits, we will refer to internal reports in which detailed circuit descriptions and SPICE analyses can be found.
7.2 Logic Based on MECL

An interesting characteristic of standard ECL circuitry is that it achieves short switching times, allowing pulse frequencies to be in the order of 100–500MHz. In addition, ECL has been designed to be applicable over a wide range of temperatures. Basically, these characteristics would make ECL suitable building blocks in our system. However, since in our system it is of primary concern to minimize disturbances of the positions of pulse edges, some modifications are necessary.

The first cause for disturbance of the pulse positions is gate propagation jitter. In standard ECL, bandgap references are used to fix the threshold voltage of the gates (determining whether a certain input voltage is to be regarded as a logic one or a logic zero) to halfway the nominal voltage corresponding to a logic one and to a logic zero. As bandgap reference voltages are derived using the amplified (10–20 times) differential voltage across two forwardly-biased junctions [1], the noise in the reference voltage, originating from the noise produced by the junctions, is relatively high. In MECL, balancing techniques, which contribute no significant noise, rather than bandgap references are used.

The second cause of disturbance of the pulse positions is “crosstalk” between successive pulses. If, after the trailing edge of a pulse, the gate output voltages has not been sufficiently settled, the propagation delay of the next pulse will depend on the time interval between the pulses. In ECL, the emitter outputs of the gates have to drive relatively large capacitive loads from the wiring between gates. As we will see later on, capacitive loads cause ringing resulting in increased settling times required to stabilize the output voltages to their stationary values (corresponding to the high or low voltage). In MECL, settling times have been minimized by avoiding capacitive loads at critical points.

7.2.1 Basic Configuration of MECL Gates

Fig.7.1 depicts the basic configuration of an MECL gate. Since the input and output signals are balanced, an auxiliary reference circuit providing the threshold voltage is not required. The gate consists of a switching differential pair (SDP), which was introduced previously in Sec.5.3. The load resistors $R_l$ to the positive supply voltage $V_{bb}$ provide a voltage output. The bias currents are fixed by the resistors $R_c$ and $R_{ee}$ to the negative supply voltage.

Rather than buffering the outputs by a common collector (CC) stage, which is common in ECL, in MECL the CC buffer is at the input of the SDP. Consequently, the capacitance of the (relatively long) wiring between different gates does not load the CC buffers. That this is of primary concern, is illustrated
7.2. LOGIC BASED ON MECL

by Fig.7.2, which shows the waveforms of the output voltage of the gate in the presence of small load capacitances. The waveforms, resulting when a rectangular current pulse is supplied to the output circuit, where calculated using SPICE. Compared to the case where the loading is absent (open squares), capacitive loading of the CC buffers (closed squares) causes severe ringing effects resulting in increased settling times.

The third trace in Fig.7.2 (circles) shows that capacitive loading of the tail of the CE stage should be avoided as well. Because active current sources, of which an example is shown in Fig.7.3, exhibit a capacitive output impedance due to the parasitic collector-substrate and collector-base junction capacitances, we preferred using resistors instead of current sources.

To make the logic low and high output voltages independent of process parameters, the resistors $R_l$ and $R_{ee}$ should be matched. The logic high voltage $V_h$ equals the supply voltage, whereas the single-sided voltage swing equals (in approximation):

$$V_\Delta = V_h - V_l = (V_{bb} - 2V_{be}) \frac{R_l}{R_{ee}}$$  \hspace{1cm} (7.1)

in which $V_l$ denotes the low-voltage and $V_{be}$ the base-emitter voltage of a forwardly-biased transistor.

In Sec.5.3, we explained that the appropriate magnitude of the bias currents depends on the demands on the propagation jitter. In addition, $R_l$ must be chosen such that $V_\Delta$ is large enough to obtain saturated switching, while a
Figure 7.2: Step response demonstrating ringing effects in SDP gates due to capacitive loading.

Figure 7.3: Active current source exhibiting a capacitive output impedance.

minimal switching time requires the base bulk resistance of the transistors to match $R_1$. In the prototype system, we used a biasing current of 1mA, $R_1 = 400\Omega$, and the smallest transistors available.

7.2.2 OR, NOR, AND and NAND Gates

In this section we will present MECL gates performing the basic function OR. Since the gate inputs and outputs are differential, negations are made by simply changing the input or the output terminals. By applying the appropriate
polarities to the inputs of the OR gate and by choosing the appropriate polarity of its output, NOR, AND and NAND functions can be realized with the same OR gate, see Fig.7.4.

Figure 7.4: NOR, AND and NAND functions realized with OR gates.

Fundamental logic functions such as OR and AND are based on either so-called "series" or "parallel" gating. In series gating, two or more transistors are in series, i.e. the collector of one transistor is connected to the emitter of the next transistor. Series gating is suitable for a direct implementation of AND gates, see Fig.7.5. Note that, with series gating, all inputs of the gate are symmetrical.

Figure 7.5: Configuration of an AND gate based on series and parallel gating.

In parallel gating, the emitters and collectors of two or more transistors are
in parallel. Two examples of OR gates based on parallel gating are depicted in Fig. 7.6. The left hand circuit, requiring one biasing current less than the right hand circuit, is the most simple configuration of a parallel OR gate. As the inputs have one node in common, parallel gating results in single-sided inputs. The subcircuits within the dashed boxes fix the common input nodes to the threshold voltage halfway the voltage of a logic low and that of a logic high. As the threshold reference circuit is based on a single-sided dummy gate, the threshold voltage will have the appropriate value, independently of component tolerances or temperature.

Since the gates have single-sided inputs, with parallel gating the voltage swing at the input of the SDPs is only half the voltage swing available with symmetrical inputs. To ensure that the preceding gate fully drives the switch in one of its saturated states, the input voltage swing is enlarged by modifying the preceding gate by either doubling its load resistances $R_1$ or by doubling the biasing current $I_e$ of its SDP stage. Unfortunately, since doubling $R_1$ or $I_e$ has an adverse effect on the rise and fall times of the preceding gate (Sec. 5.3.1), the rise and fall times achieved with parallel gating are larger than with series gating. Obviously, a penalty has to be paid when only one of the polarities of a signal is used.

Nevertheless, parallel gating has an important advantage over series gating. In minimizing settling times, we concluded that the tail of CE stages should not be loaded by a (parasitic) capacitance. Since in series gating one collector terminal of the lower pair serves as a capacitive load on the common emitter of the upper pair and since in parallel gating this load is absent, parallel gating achieves shorter settling times. For this reason, we preferred parallel gating.
7.2.3 EXOR Gates, Latches and Flip-Flops

Fig. 7.7 presents the design of an EXOR gate, based on parallel gating. The EXOR gate is based on two parallel, cross-coupled with respect to each other, balanced current switches, consisting of $Q_{3,4}$ and $Q_{5,6}$ respectively. Their states are determined by the A inputs. Depending on the B inputs, one of the switches is enabled while the other is disabled, resulting in the EXOR function.

Due to their symmetrical character, one of the A inputs is always high. Consequently, the two switching voltages required to enable the one and disable the other switch are supplied by the two (effectively) single-sided B inputs. To obtain saturated switching, the input signals of the B inputs should have double the voltage swing $2V_{\Delta}$. Therefore, the modifications proposed to the preceding gate required to drive an OR gate apply here as well. In addition, since the B inputs have $V_h$ of the A input as their threshold voltage, the $V_h$ and hence $V_l$ of the A input signals should be shifted down by $V_{\Delta}$. This is accomplished by supplying an additional (and constant) current to the load resistors $R_l$ in the gate driving the A inputs.

Fig. 7.8 depicts a latch. The SDP consisting of $Q_{3,4}$ passes the "data" input signal to the internal state during the track mode with the clock low. The SDP around $Q_{5,6}$ serves as a "positive feedback" conserving the state of the latch in the hold mode when the clock input is high. As the latch circuit is very similar to the EXOR gate, the data and the clock inputs are respectively of the "A
and the B type. The resistors $R_{ee}$ to the signal ground modify the internal feedback buffers to have type A outputs.

![Circuit diagram](image)

Figure 7.8: Configuration of latch based on parallel gating.

When more latches are cascaded, the data input buffers around $Q_{1,2}$ can be omitted by using the internal feedback buffers consisting of $Q_{7,8}$ of preceding latches. Additionally, when the clock inputs are common to more than one latch, only one clock buffer ($Q_{9,10}$) needs to be provided. This principle is illustrated by the configuration of the flip-flop shown in Fig.7.9 in which two latches have been cascaded.

### 7.3 Modulator

The modulator samples its input signal and converts the samples into PPM pulses whose displacements, relative to the zero positions of the PPM pulses, are proportional to the sampled values. The proportionality constant will be called the modulation constant $\xi$, expressed in seconds per volt (not to be confused with the modulation index $m$, which refers to the ratio between the peak-to-peak pulse deviation and the sample period $T_s$).

The samples are converted into PPM pulses by comparing the input voltage with a sawtooth voltage: a pulse is generated at the so-called pulse initiation
time \( t_{PI} \), when the momentary amplitude of the sawtooth voltage has reached the amplitude of the input voltage, see Fig.7.10. The slope magnitude of the sawtooth directly determines the modulation constant.

If a comparator is used to detect \( t_{PI} \), as is the case in the circuits depicted in Fig.7.11, duration modulated pulses result at the comparator output. The leading edges correspond to the sawtooth resets and the trailing edges to the
pulse initiation times. Next, a pulse shaper, which triggers on the trailing edges of the duration modulated pulses, produces the desired PPM pulses of constant pulse width.

Fig. 7.11 depicts four possible modulator configurations consisting of the sawtooth generator, the comparator and the pulse shaper. In configurations (a) and (b), the sawtooth is compared directly to the input signal with either the input signal and the sawtooth at different comparator inputs or the input signal in series with the sawtooth at the same comparator input. In both cases, the input voltage is "sampled" at time $t_{PI}$, so that the samples will not be uniformly spaced (so-called natural sampling). Typical distortions resulting from non-uniform sampling were discussed in Sec.3.9. Since the slope magnitude of the differential comparator input signal is signal dependent, the delay time of a practical comparator will become signal dependent as well. This so-called excitation modulation results in (a little) additional modulator distortion.

Configurations (c) and (d) use additional sample-and-holds which store uniformly sampled voltages during one period of the sawtooth, so there are no distortions due to non-uniform sampling and excitation modulation.

Although configurations (a) and (b) as well as (c) and (d) seem equivalent, their performances will be different if the comparator is not ideal. At time $t_{PI}$, the common-mode level in (a) and (c) equals the momentary input signal, whereas in (b) and (d) the common-mode level is constant (zero). As in non-ideal comparators the delay time will be influenced by the common-mode level, in (a) and (c) a third type of distortion results.

```
(a)
in
comparator

+ V_{saw}

pulse shaper
PPM
out

(b)
in
+ V_{saw}
 comparator
pulse shaper
PPM
out

(c)
in
sample
switch

+ V_{saw}
 comparator
pulse shaper
PPM
out

(d)
in
sample
switch

+ V_{saw}
 comparator
pulse shaper
PPM
out

C_{h}
```

Figure 7.11: Four configurations of the modulator.
As configuration (d) facilitates uniform sampling, a constant excitation and a constant common-mode level, (d) seems to be the best alternative. However, the requirements to the sample-and-hold are high. Any non-linearity directly distorts the modulator input signal and, since the time interval between the uniform sampling and the sampling by the comparator is essentially signal dependent, any ringing during the hold mode causes additional distortion. A further disadvantage of (d) is that it needs a high-performance (low distortion) circuit for summing the input and sawtooth voltages.

In Sec.7.3.2, it will be shown that the "inferior" but easy to implement configuration (a) performs well enough for most applications.

Presently, we will deal with the sawtooth generator, the comparator and the pulse shaper circuits and, because the modulator is one of the most critical circuits, determine their contributions to non-linearity and noise. Additional design considerations and SPICE simulations can be found in [3].

### 7.3.1 Sawtooth generator

As shown in Fig.7.12, the sawtooth generator basically consists of a capacitor $C$ which integrates a constant current $I$ so that a time-proportional voltage results across the capacitor. After a period has been completed the capacitor voltage is reset to its starting voltage by closing the voltage switch. The modulation constant $\xi$ can be tuned by varying the integrator input current. To keep the PPM pulses within the appropriate time slots, the reset switch is driven by an externally supplied clock signal.

Fig.7.13 depicts a simple implementation of the sawtooth generator. As the comparator requires a single-sided sawtooth input signal, we used a single-sided configuration. The current mirror consisting of $Q_{1,2}$ supplies the integrator current $I$ which can be tuned to fix the modulation constant. $Q_3$ acts as a voltage switch, resetting the integrator voltage by a direct short-circuit across the integrator capacitor. In this way peak currents through the power supply are avoided.

The starting value of the sawtooth voltage should be constant. Therefore the clock input signal is buffered by a standard SDP, removing any spurious
Figure 7.13: Simple circuit for generating the sawtooth.

signals superimposed on the clock signal. During a reset, the sawtooth voltage increases to the supply voltage minus the base-emitter voltage of $Q_3$ (and a negligibly small voltage across $R$), which is biased by the integrator current $I$. To compensate for the temperature dependence of this voltage, the most simple solution is to insert a forwardly-biased junction as a level shift in series with the signal input of the comparator as well. The level shifts at the input of the SDP prevent $Q_5$ from forward biasing its base-collector junction (voltage saturation).

To obtain a low modulator distortion, the linearity of the sawtooth is of primary concern. As the modulator is used in an $N$-channel system, high linearity is desired during the length of one slot $T_n$, which is only a fraction of the total sawtooth (sampling) period $T_s$. As a consequence, a new period of the sawtooth can be started in one of the other time slots, so avoiding settling problems.

In App.C.1 it is concluded that, if the voltage across $R_e$ is sufficiently large, the distortion caused by the Early effect of $Q_2$, which supplies the integrator current, is negligible. To keep the distortion caused by the loading of the sawtooth signal by the comparator input negligible as well, the transistors constituting the comparator input must be biased using active current sources. In the sawtooth generator designed for the prototype system, the distortion resulting from the (non-linear) junction capacitances of $Q_{2,3}$, which is in the order of 0.1% (for a 0.1V signal amplitude), dominates.

In addition, App.C.1 deals with the noise produced by the active current source around $Q_2$ and the reset switch consisting of $Q_3$. The mean square magnitude of the jitter resulting from the current source is inversely proportional to the square of the integrator current $I$. The jitter originating from the reset switch is proportional to the square of the modulation constant $\xi$. In
the prototype system, in which the integrator current was 0.1mA and \( \xi \) equals 60ns/V, we calculated 5 and 4ps respectively.

### 7.3.2 Comparator

The practical implementation of the comparator consists of switching differential pairs (SDPs), as presented earlier in Sec.5.3. To make the comparator output signal minimally susceptible to noise and interference from other circuits, the total gain of the comparator should be so large that the output pulses exhibit the maximum (saturated) slope magnitude \( S_{\text{max}} \):

\[
G_{\text{total}} \geq S_{\text{max}} \geq \frac{S_{\text{in}}}{\frac{1}{T_{\text{sw}}} \frac{V_{o,PP}}{T_{\text{sw}}}} \approx V_{o,PP} \frac{\xi}{10\tau_f} \tag{7.2}
\]

in which \( V_{o,PP} \) is the peak-to-peak output voltage of the comparator, \( T_{\text{sw}} \) is the minimal switching time of an SDP (Sec.5.3.1), \( \tau_f \) is the transit time of the transistors and \( \xi \) is the modulation constant (equal to the reciprocal of the input slope magnitude).

In the prototype system, the parameters are \( V_{o,PP} = 800\text{mV} \), \( \tau_f = 50\text{ps} \) and \( \xi = 60\text{ns/V} \), yielding \( G_{\text{total}} \geq 96 \). The small-signal gain of one SDP was calculated to be 8, requiring a minimum of three stages.

Fig.7.14 shows a four-stage comparator circuit. To keep the loading of the sawtooth generator constant (constant input current of the comparator), the first SDP stage employs active current sources. Although we previously concluded that active current sources increase settling times, this is of no concern here because the aperture time of the first SDP is so large that it effectively functions as a linear amplifier (Sec.5.3). To compensate for the offset in the sawtooth voltage, which starts one \( V_{bc} \) lower than the supply
voltage, $Q_0$ adds the same offset to the modulator input signal that is supplied to $V_{\text{in}}$. To ensure that the modulator uses the linear part of the sawtooth, the input voltage should have an offset such that the average voltage of the input signal corresponds to about half the height of the sawtooth. This additional offset voltage is established by the emitter series resistance $R_o$. Provided that all resistors are matched, this voltage will be approximately constant with temperature.

In App.C.2, we deal with the modulator distortion caused by modulation of the common-mode level and slope magnitude of the comparator input signal (so-called excitation modulation). Provided that active current sources are used to provide the biasing currents of the first comparator stage, it is shown that the distortion levels are below the distortions resulting from the previously discussed sawtooth generator. This conclusion applies if the ratio between the modulation constant $\xi$ and the transit time of the integrated components $\tau_f$ is comparable to the value of this ratio in the prototype system. It is shown that the jitter resulting from the noise produced by the comparator is proportional to the square of the modulation constant $\xi$. In the prototype system, the jitter is about 3ps, which is comparable to the jitter originating from the sawtooth generator.

### 7.3.3 Pulse Shaper

The pulse shaper generates a PPM pulse for each trailing edge of the comparator output signal. This can be done by a monostable multivibrator, but the simpler configuration of Fig.7.15 consisting of an inverter/delay and an NOR gate is also suitable. The width $T_p$ of the output pulses equals the delay time

![Figure 7.15: Pulse shaper consisting of inverter/delay and NOR gate.](image)

of the inverter.

Fig.7.16 depicts the circuit diagram of the delay/inverter and NOR gate. The SDP around $Q_{3,4}$ provides the desired time delay. The SDP is biased with a relatively low biasing current while additional junction capacitances, consisting of the junction capacitances of $Q_{5,6}$, are connected to the SDP output so that the output pulses have relatively slow edges. The delay time corresponds to
approximately half the rise time of the pulses and can be tuned by varying the 
limiter biasing current by adjusting $R_{TP}$. To provide pulses with steep edges 
to the NOR gate, additional SDP stages buffer the output pulses of the delay 
circuit.

From the instant the SDP starts "switching" from its state of current 
saturation until the SDP output signal crosses zero, the time constant, which 
results from the junction capacitances of $Q_{5,6}$ and the collector biasing resis-
tors, provides the required delay time. However, as the output voltage needs to 
be settled before it can handle the next pulse, $Q_{5,6}$ are used to shorten the time 
constant after each zero crossing of the output pulses. When the output volt-
age exceeds one base-emitter voltage, $Q_5$ or $Q_6$ behaves as a forwardly-biased 
diode, which decreases the time constant.

Unfortunately, due to junction capacitances, the NOR gate suffers from 
high frequency crosstalk from its inputs to the output. Synchronously with 
the sawtooth resets, this crosstalk causes narrow spikes at the pulse shaper 
output. To prevent the interference of the spikes with the PPM pulses from 
neighboring channels in the multiplexer, they need to be removed by additional 
SDP stages at the multiplexer input.

7.3.4 Discussion

The most simple modulator consists of a sawtooth generator, a comparator and 
a pulse shaper. It has been shown that the resulting non-uniform sampling and 
modulation of the comparator propagation delay cause only small distortions.
Far more important is the distortion originating from the non-linearity of the sawtooth, caused by junction capacitances in parallel with the integrator capacitor which generates the sawtooth slope. If necessary, the distortion can be reduced by enlarging the integrator capacitance, requiring a larger integrator current as well. An alternative solution for improving linearity could be found in balancing techniques.

Modulator noise originates from the integrator reset switch and the integrator current source, which are building blocks of the sawtooth generator, as well as from the comparator input circuit, in almost equal proportions. Since the contribution of the integrator current is inversely proportional to the square of the integrator current $I$, and since the two other contributions are proportional to the square of the modulation constant $\xi$, the SNR of the modulator can be increased by increasing $I$ and decreasing $\xi$. To maintain the same pulse deviation, we will have to enlarge the magnitude of the input signal accordingly.

In the prototype system, we used a modulation constant $\xi=60\text{ns}/V$ and an integrator current of $100\mu\text{A}$, requiring a capacitance $C$ of $6\text{pF}$. The resulting total jitter was about $7\text{ps}$, which was still below the jitter originating from the low-cost laser diode. The distortion caused by non-linear junction capacitances was about $0.1\%$.

### 7.4 Multiplexer

In Sec.4.2.1, we concluded that the multiplexer may consist of a simple summing circuit. The implementation presented in Fig.7.17 is based on an OR gate. The SDP buffers at the inputs remove the small spikes from the PPM input signals (Sec.7.3.3) as well as any other disturbing signals that may interfere with PPM pulses from neighboring channels. Moreover, the input buffers provide symmetrical inputs which make the multiplexer input signals less susceptible to crosstalk from and to other circuits or wiring. As the single-sided outputs of the buffers drive a parallel OR gate, the voltage swing of the outputs has to be twice the standard magnitude (Sec.7.2.2).

### 7.5 Laser driver

In Chap.2 we noted that the required laser bias current, which is in the range of $10-40\text{mA}$ for $1\text{mW}$ lasers, depends on temperature (Fig.2.4) and on device-specific characteristics. Therefore, commercially available lasers have a monitor photodiode mounted in the same device as the laser, so that feedback techniques can be used to stabilize the laser output power. To avoid excessively
long settling times, we concluded in Sec. 2.2.2 that the biasing current should be such that the total laser current remains above the threshold current.

To emit pulses with an amplitude of about $1$ mW, the modulating pulse current should have an amplitude of $5$–$20$ mA. The laser diode should be in forward mode and the photodiode should be in reverse mode.

Practical laser devices have three pins (Fig. 7.18): (1) anode (cathode) of the laser diode, (2) housing and cathode(s) and (or) anode(s) of the laser and photodiode and (3) anode (cathode) of the photodiode. Since the housing

![Diagram of multiplexer circuit based on an OR gate.](image)

Figure 7.17: Multiplexer circuit based on an OR gate.

![Various pinning diagrams of commercially available laser devices.](image)

Figure 7.18: Various pinning diagrams of commercially available laser devices. (a) and (b) are most common.
exhibits a severe capacitance, the laser modulation current should be supplied to pin 1 and not to pin 2.

First, we will discuss the bias circuitry and then the modulation circuitry. In [6] an alternative implementation and detailed SPICE analyses can be found.

7.5.1 Laser Bias Circuitry

To stabilize the bias current, the current through the photodiode is measured and compared to a reference current. Basically, the best method would be to measure the bottom value of the current pulses, because by stabilizing this current the total laser current will always be higher than the threshold current. If the bottom current is lower, the laser becomes slow (LED mode), if the current is higher, the laser wastes supply current while we experience an increased sensitivity to reflections.

Unfortunately, since the photodiodes are slow (due to their large area which makes them less position sensitive) only the average photo current will be available. To prevent the laser from operating below threshold, the average current should be adjusted slightly above its required level at room temperature because, apart from the threshold current, the current-to-light gain of the laser is also dependent on temperature.

Fig.7.19 depicts the complete circuit. At the collector of $Q_2$ a reference current, which can be adjusted externally by $R_{\text{ref}}$, is subtracted from the sensed photo current. The error current is amplified by the CE stage consisting of $Q_{3,4}$ and output stage around $Q_{5,6}$. $Q_{9..12}$ limit the voltage across $R_{cQ5,6}$ so that the driver cannot be damaged by too large output currents. $Q_{7,8}$ provide a level shift which is necessary to avoid voltage saturation of $Q_3$ (i.e., to avoid the base-collector junction coming into the forward mode). The external capacitor $C_{\text{ext}}$ determines the bandwidth of the feedback loop. By connecting this capacitance to the negative supply voltage, we ensure that the laser bias current starts at zero when the power supply is switched on. At the sense input, an optional external capacitance $C_{av}$ must be used if the bandwidth of the photodiode is too large to adequately average its output current.

To decrease circuit dissipation in the IC, an external series resistor $R_{\text{series}}$ is inserted between the output and the laser diode.

Provided that $R_{cQ3} \gg \beta R_{cQ5,6}$, the low frequency loop gain is:

$$H_o = \beta^2 \beta_{\text{laser}}$$

(7.3)

in which $\beta$ is the transistor current gain and $\beta_{\text{laser}}$ is the laser diode-to-
photodiode current gain (in the order of 0.05), specified at the power level $P_0$:

$$\beta_{\text{laser}} = \frac{I_{\text{photodiode},P_0}}{I_{\text{laser},P_0} - I_{\text{th}}} \quad (7.4)$$

The loop has one single dominant pole (and is thus stable) provided that:

$$\frac{C_{\text{ext}}}{C_{\text{av}}} \geq \frac{\beta^2 \beta_{\text{laser}}}{R e Q_{5,6} g_{mQ_{3,4}}} \quad (7.5)$$

in which $g_{mQ_{3,4}}$ is the transconductance of the CE stage. To assure stability under all conditions, we should assume the largest possible $\beta_{\text{laser}}$ and satisfy Eq.(7.5). The bandwidth of the single-pole loop is given by:

$$B = \frac{\beta \beta_{\text{laser}}}{2\pi C_{\text{ext}} R e Q_{5,6}} \quad (7.6)$$

The bandwidth of the loop should be sufficiently small to avoid modulation of the biasing current by the PPM pulses. As the repetition rate of the pulses is in the MHz range, practical values of the loop bandwidth are in the order of a few KHz.
In designing our circuit, we assumed a photodiode having its cathode in common with the laser diode. If the anode is in common, an additional current mirror to the positive supply voltage is required. In addition, we assumed that the laser diode has a common anode. If not, the circuit of Fig. 7.20, in which the biasing current is supplied to the common node and the modulation current to the anode, is a suitable solution. The resistor inserted between the laser diode and the positive supply voltage must be large compared to the effective series resistance of the laser diode. The decoupling capacitor must be large enough to achieve adequate decoupling but not so large that the stability of the biasing loop is endangered.

Figure 7.20: Connection of a laser diode having its cathode in common with the photodiode.

### 7.5.2 Laser Modulation Circuitry

Since PPM information is coded only in the positions of the pulses, the laser modulation circuit may consist of a cascade of SDP stages. In subsequent stages the current level of the pulses is scaled by a factor $\beta_I$, until the current is large enough to drive the laser.

Since the minimal pulse width $T_p$ that can be handled by the SDP stages depends on $\beta_I$, according to (See Sec. 5.3):

$$T_p = 2T_{sw} + T_{settle} \approx 20\beta_I \tau_f + 30\tau_f$$  \hspace{1cm} (7.7)
a certain $T_p$ allows a certain maximal value of $\beta_{I_e}$. On the other hand, because the number of stages needed is inversely proportional to $\beta_{I_e}$, $\beta_{I_e}$ should not be smaller than necessary for a certain pulse width.

Fig. 7.21 depicts the complete three-stage laser modulation circuit. $\beta_{I_e}$ has been chosen as 4 and the maximum output current amounts as 16mA. The size of the transistors has been scaled in proportion to their collector currents. Although a balanced output is not necessary to drive the laser, balancing is useful to avoid large peak currents flowing through the power supply and, if both outputs are terminated by parallel IC pins, balancing additionally reduces inductive and capacitive coupling to other circuits.

As the forward voltage across the laser diode is 1.5–2V, $Q_{17..19}$ are necessary to provide a level shift which prevents the output transistors $Q_{13..16}$ from voltage saturation. The resistors in series with the level shifts $Q_{7..10}$ damp oscillations caused by the relatively large capacitive loading of $Q_{5,6}$ by $Q_{7..12}$. The laser modulation current is determined by the total resistance between the emitter tail of $Q_{13..16}$ and the negative supply voltage. Note that this resistance is partly external ($R_{I_{mod}}$), so allowing for external adjustment and reducing dissipation in the IC. The internal part of the resistor reduces the
sensitivity of the output stage to pinning capacitance (necessary to prevent ringing). $Q_{20,21}$ protect the laser diode from reverse voltages caused by static charges when the driver is switched off.

The output impedance of the driver consists of a capacitive part originating from the collector-substrate and the collector-base capacitances of $Q_{13..16}$ and the pinning and wiring capacitance, as well as an inductive part originating from the pinning and wiring. The equivalent electrical circuit of the laser diode consists of a voltage source and a small series resistance of 1–10Ω. The capacitive and inductive components may cause severe ringing (in the GHz range) because the series resistance of the laser diode is too small to serve as a damping resistor. An effective solution providing additional damping is to insert external resistors of about 50Ω in series with the output terminals. If this is not adequate, alternative IC housings such as flat-packs should be used.

7.6 Clock Generator and Regenerator

7.6.1 Introduction

The transmitter clock generator of Fig.7.22 consists of an oscillator running at the frequency $N f_s$ (in which $N$ is the number of channels and $f_s$ is the sample frequency), an $N$-state counter and an $N$-output decoder which derives the $N$-phase output signal. One complete cycle of the counter corresponds to exactly

![Figure 7.22: Configuration of the clock generator.](image)

one frame of $N$ PPM pulses.

Fig.7.23 shows that the receiver clock regenerator uses the clock generator in a phase lock loop (PLL) that averages the positions of the frame pulses. As we saw in Sec.4.3, the frame pulses are separated from other TDPPM pulses by the demultiplexer.

In Sec.6.3.6, we concluded that since the clock generator and regenerator synchronize the modulators and demodulators, any phase noise directly results in noise at the output of the receiver. Therefore, regarding the oscillators, low phase noise is a major design objective. It will be shown that there is unfortunately no oscillator running at the desired frequency $N f_s$ available that
exhibits sufficiently low phase noise. A possible solution is the so-called two-oscillator concept, employing a low-frequency crystal oscillator and a frequency multiplier consisting of a PLL and a first-order regenerative oscillator. Next, the complete design of the crystal oscillator, the first-order oscillator, the phase detectors and the loop filters will be presented. The counter and the decoder will not be discussed here because they consist of the same circuitry as the TDPPM demultiplexer which will be discussed later on.

7.6.2 The Two-Oscillator Concept

We will discuss what type of oscillator should preferably be used in the averaging PLL and justify the two-oscillator concept.

A first consideration is the noise performance of the oscillator. In Sec.6.3.6, we related the oscillator phase noise, which is characterized by $L(f)$ or $S_T(f)$ to the resulting time jitter at the input of the demodulator:

$$\sigma^2_{t, \text{osc1,2}} = \frac{\sqrt{2}}{8\pi} \left( \frac{f_m}{f_o} \right)^2 \frac{L(f_m)}{B_{av}} = \frac{\sqrt{2}}{16\pi} \frac{S_T(f_m)}{B_{av}}$$

(7.8)

in which $f_m$ is the noise frequency where $L(f)$ or $S_T(f)$ is specified, $f_o$ is the oscillator frequency and $B_{av}$ is the bandwidth of the averaging PLL. Note that, in Sec.6.3.6 we assumed that the oscillator frequency $f_o$ is equal to the sample frequency $f_s$. Presently, since in the multi-phase clock generator $f_o$ equals $Nf_s$, Eq.(7.8) is a function of $f_o$ rather than $f_s$.

From Eq.(7.8) we see that if the oscillator exhibits a high level of phase noise, a low jitter level requires $B_{av}$ to be large. For instance, to obtain a jitter of 4ps with a regenerative oscillator running at a frequency $f_o=70$MHz and exhibiting $L(1$MHz$)=-120$dB/Hz, $B_{av}$ should be 1MHz.

A disadvantage of a large $B_{av}$ is that the transmission noise and modulation present in the positions of the received frame pulses will not be suppressed.
Obviously, as they allow a small $B_{av}$, low-noise resonator oscillators are necessary. Since resonator oscillators (Sec.5.4.3) perform better by a factor $Q^2$ ($Q$ is the quality factor of the resonator) and since $Q$ factors of 100–200 ($LC$-oscillators) or $10^4$ (crystal oscillators) are feasible, a considerably lower PLL bandwidth (below 100Hz) is possible.

Having decided on high-$Q$ resonator oscillators, a second consideration is the accuracy of the oscillator frequency. If the initial frequency can be far from the desired frequency (in the case of a completely integrated oscillator, due to tolerances of components, up to $\pm 40\%$), special measures to lock the oscillator to the desired frequency will be necessary. Due to their accuracy, crystal oscillators are clearly superior.

Finally, we have to consider the range of possible oscillator frequencies. The maximum oscillator frequency of integrated crystal oscillators (i.e. only the crystal is an external component) is restricted to about 50MHz. Higher frequencies have trouble with parasitic capacitances of the crystal as they cause parasitic oscillations.

Because in the prototype system the required oscillator frequency was in the order of 70MHz, a low-noise crystal oscillator and an additional circuit, an $M$-times frequency multiplier, transforming the oscillator signal into a signal with the appropriate frequency $Nf_s$, have been used.

The frequency multiplier consists of a PLL, of which the block diagram is shown in Fig.7.24. This additional PLL contains a second oscillator which runs at the “high” frequency but does not need to have low phase noise because the large bandwidth of the PLL ensures that the oscillator tracks the low-noise crystal oscillator.

![Diagram of the two-oscillator clock generator](image)

Figure 7.24: Configuration of the two-oscillator clock generator.

To avoid modulation of the clock output signals, the frequency multiplication factor $M$ should preferably be chosen such that $N$ is a multiple of $M$. If, for example, $N=8$, we may choose $M=2, 4,$ or $8$. As in practical systems
$f_s$ is larger than twice the bandwidth of the transmitted signals $B_1$, choosing $M=2N$ might still be acceptable, because then the resulting modulation will be removed by the smoothing filter.

### 7.6.3 Noise Analysis

In this section we will examine the influence of the phase noise produced by the clock generator and regenerator oscillators and compare the results with the results derived previously in Sec.6.3.6, where we ignored the influence of the frequency multiplier.

Fig.7.25 depicts the transmitter clock generator and the receiver clock regenerator, both consisting of a crystal oscillator and a regenerative (first-order) oscillator. The phase noise power density spectra of the oscillators are denoted by $S_{\Phi_1}(f)$ and $S_{\Phi_2}(f)$. The variable $B_{\text{av}}$ is the bandwidth, $\tau_{n1}$ and $\tau_{p1}$ are the filter time constants of the receiver averaging PLL, $B_{\text{mul}}$ is the bandwidth, and $\tau_{n2}$ and $\tau_{p2}$ are the filter time constants of the multiplier PLL. The poles of the PLL’s (closed loop) are assumed to be maneuvered into Butterworth positions, which is achieved by taking:

$$
\tau_{n1,2} = \frac{2\pi \sqrt{2} B_{1,2} \tau_{p1,2} - 1}{4\pi^2 B_{1,2}^2 \tau_{p1,2}} \quad (7.9)
$$

Provided that $\tau_{p1,2} \gg \frac{1}{2\pi \sqrt{2} B_{1,2}}$, straightforward calculations yield the noise power density spectra at the demodulator output originating from the four oscillators:

$$
S_{\Phi_1,\text{tra}}(f) = S_{\Phi_1}(f) \cdot \frac{f^2(4\pi^2 f^2 \tau_{p1}^2 + 1)}{4\pi^2 \tau_{p1}^2 (f^4 + B_1^4)} \cdot \frac{B_2^2(2f^2 + B_2^2)}{f^4 + B_2^4} \quad (7.10)
$$

$$
S_{\Phi_1,\text{rec}}(f) = S_{\Phi_1}(f) \cdot \frac{f^2(4\pi^2 f^2 \tau_{p1}^2 + 1)}{4\pi^2 \tau_{p1}^2 (f^4 + B_1^4)} \cdot \frac{B_2^2(2f^2 + B_2^2)}{f^4 + B_2^4} \quad (7.11)
$$

$$
S_{\Phi_2,\text{tra}}(f) = S_{\Phi_2}(f) \cdot \frac{f^2(4\pi^2 f^2 \tau_{p2}^2 + 1)}{4\pi^2 \tau_{p2}^2 (f^4 + B_2^4)} \cdot \frac{f^2(4\pi^2 f^2 \tau_{p1}^2 + 1)}{4\pi^2 \tau_{p1}^2 (f^4 + B_1^4)} \quad (7.12)
$$

$$
S_{\Phi_2,\text{rec}}(f) = S_{\Phi_2}(f) \cdot \frac{f^2(4\pi^2 f^2 \tau_{p2}^2 + 1)}{4\pi^2 \tau_{p2}^2 (f^4 + B_2^4)} \quad (7.13)
$$

Eqs. (7.10) and (7.11) show that the noise originating from the crystal oscillators is high-pass filtered by the averaging PLL, which is in correspondence with our results in Sec.6.3.6. In this case, however, the noise is also low-pass filtered by the multiplier PLL. But, as $S_{\Phi_1}(f)$ decreases with the frequency $f$ while $B_2 \gg B_1$, the effect of the additional high-pass filtering can be ignored.
By examining the transfer functions of the phase noise produced by the regenerative oscillators, we see that the noise is high-pass filtered by the multiplier PLLs. Compared to the calculations carried out on the influence of the crystal oscillator noise, it is as if the multiplier PLL takes over the role of the averaging PLL. Regarding Eq. (7.12), since $B_1 \ll B_2$ the influence the additional high-pass filtering by the averaging PLL can be ignored.

In conclusion, we have two contributions from the crystal oscillators. As we will see in Sec. 7.6.4, the phase noise power density spectrum of the crys-
tal oscillator signal exhibits a $\frac{1}{T^2}$-character. Hence, in correspondence with Sec.6.3.6, we find:

$$\sigma_{t_{1,tra,rec}}^2 = \frac{1}{8\pi} f_m^3 \frac{L_1(f_m)}{B_1^2} = \frac{f_m}{16\pi} \frac{S_{f1}(f_m)}{B_1^2} \quad (7.14)$$

To find the value of $B_1$ that is just acceptable, we determine the value of $B_1$ for which the noise produced by the oscillator equals the noise produced by the laser diode. Substituting the jitter (from two oscillators!) given by Eq.(7.14) into Eq.(6.24) yields the SNR, which would result from the oscillator noise. The maximum of the SNR, determined by the laser diode was calculated using Eq.(6.25). Straightforward calculations reveal that the oscillator noise can be ignored if:

$$B_1 \gg \sqrt{\frac{\pi r B_r f_m^3 L(f_m)}{2f_r^2 RIN}} = \sqrt{\frac{\pi r B_r f_m S_f(f_m)}{2RIN}} \quad (7.15)$$

In addition, there are two contributions from the regenerative oscillators, of which the power density spectrum exhibits a $\frac{1}{T^2}$-character, resulting in:

$$\sigma_{t_{2,tra,rec}}^2 = \frac{\sqrt{2}}{8\pi} \left( \frac{f_m}{f_o} \right)^2 \frac{L_2(f_m)}{B_2} = \frac{\sqrt{2}}{16\pi} \frac{S_{f2}(f_m)}{B_2} \quad (7.16)$$

The SNR, that would result from the oscillator jitter is calculated by substituting the jitter produced by the two oscillators given by Eq.(7.16) into Eq.(6.24). To keep this value above the SNR, determined by the laser diode, given by Eq.(6.25), we have to satisfy the condition:

$$B_2 \gg \frac{\pi r \sqrt{2} B_r f_m^3 L(f_m)}{4RIN f_r^2} = \sqrt{\frac{\pi \sqrt{2} B_r S_f(f_m)}{4RIN}} \quad (7.17)$$

Because the regenerative oscillator exhibits a relatively poor $L(f)$, $B_2$ needs to be relatively large. The maximum possible value of $B_2$ depends on the constraint that the PLL should not become instable.

### 7.6.4 Crystal Oscillator

The basic circuit of the crystal oscillator, as depicted in Fig.7.26, is a direct implementation of the block diagram of Fig.5.16. Transistor $Q_1$ performs the nullor function, while the limiter consists of $Q_{2,3}$. To assure proper start up, the magnitude of the loop gain $A_{H_o}$ must be larger than one, i.e.:

$$A_{H_o} = \frac{R_{cQ1}g_mQ_{2,3}R_{cQ2}}{R_s + \frac{kT}{q} + \frac{R_{cQ2}+R_{b1}}{\beta_1}} \geq 1 \quad (7.18)$$
in which $R_s$ is the effective series resistance of the crystal at the resonant frequency, $R_{b1}$ is the intrinsic base resistance, $\beta_1$ is the current gain and $I_e$ is the emitter bias current of $Q_1$. The oscillator frequency $f_o$ has been chosen as 8.8MHz because for this frequency, which is twice the color subcarrier frequency of PAL-coded video signals, crystals are widely available. The amplitude of the crystal current $I$ depends on the limiter output voltage swing, which in turn determines the signal voltage across the crystal series resistance $R_s$.

The oscillator circuit depicted in Fig.7.27 [4] has been completed with the level shifts consisting of $Q_{6,10}$ which prevent voltage saturation of the base-collector junctions of $Q_1$ and $Q_2$. A balancing technique provides the appropriate biasing voltage $V_{bias}$ to the second input of the limiter stage around $Q_{2,5}$. The capacitor $C$ has been added to stabilize the high frequency behavior of the oscillator, i.e. to prevent parasitic oscillations. To avoid impractically large values of the biasing resistors, an additional supply voltage $V_b$ is provided by the stabilization circuit around $Q_{11}$. The oscillator is equipped with output buffer stages, providing either a sinusoidal (stage around $Q_{12,13}$) or a square wave (stage around $Q_{14,15}$) output signal. An external voltage (to be connected to $V_{bb}$ or ground) selects one of these buffers. An advantage of the sinusoidal output signal is that it contains no signal power at harmonics of the oscillator frequency. As a consequence, the sinusoidal output might be useful in preventing false locks of the frequency multiplier that transforms the crystal oscillator signal into a clock signal with the desired frequency. Nevertheless, since experiments showed that false locking did not occur and since square waves are less susceptible to additive noise, a square wave output signal is preferred.

Standard available crystals exhibit tolerances of about 25ppm (parts per million) from their nominal frequencies. To allow tuning of the oscillator fre-
7.6. CLOCK GENERATOR AND REGENERATOR

Figure 7.27: Complete circuit of the crystal oscillator.

Frequency to the nominal crystal frequency, the crystals have been designed such that they should be operated in series with a nominal capacitance of 20pF. As the pullability of the crystal frequency is about 15ppm/pF, the series capacitance should be variable between 18 and 22pF.

In addition, to make the oscillator suitable for the clock regenerator, the series capacitance should be tunable by a voltage. For this purpose, we use the circuit of Fig.7.28, in which the collector-base and emitter-base junction capacitances of $Q_{\text{var}}$ can be tuned by varying their biasing voltage. To maintain the high $Q$-factor of the crystal, the biasing voltage should be supplied by a high-impedance voltage source. For this reason, the additional resistance $R_{\text{var}}$ should be in the order of 100kΩ–1MΩ, which is a relatively large value to have $R_{\text{var}}$ integrated in the IC. Because the crystal and the PLL filter capacitance are external components already requiring additional IC pins, it is no problem to use an external resistor for $R_{\text{var}}$.

The tuning constant $K_o$ (in rad.s$^{-1}$V$^{-1}$) can be determined from the voltage dependence of the capacitor $\frac{dC}{dV}$:

$$K_o = \frac{2\pi df_o}{dV_{\text{tune}}} = 2\pi f_o \frac{dC}{dV_{\text{tune}}} \text{pullability}$$  \hspace{1cm} (7.19)

Fig.7.29 shows $L(f)$ measured on the oscillator used in the prototype sys-
tem. Obviously, the oscillator suffers from a severe contribution of $\frac{1}{f}$-noise, resulting in a phase noise power density proportional to $\frac{1}{f^3}$ (Fig. 7.29) rather than $\frac{1}{f^2}$ [5]. The margins in Fig. 7.29 indicate that the total phase noise differs from crystal to crystal.

![Figure 7.28: Tapping circuit to tune the oscillator frequency.](image)

![Figure 7.29: Measured noise-to-carrier ratio of the crystal oscillator for different crystals.](image)

To ensure relatively low phase noise, the amplitude of the crystal current was chosen to be in the order of 1mA. Substituting the worst $L(f)$ measured,
\( L(f) \approx \frac{3 \times 10^{-5}}{f^3} \), into Eq.(7.15) and taking the parameters of the prototype system, \( r = 1.75 \), \( B_r = 250 \text{MHz} \), \( f_o = 8.8 \text{MHz} \) and \( \text{RIN} = -125 \text{dB/Hz} \), we find that the bandwidth of the averaging PLL \( B_1 \) should not be lower than 30Hz.

### 7.6.5 Regenerative Oscillator

Fig.7.30 depicts the basic configuration of a (balanced) regenerative oscillator. The current \( I \) charges the capacitor \( C \) until the capacitor voltage \( V_c \) equals \( V_2 \) or \( -V_1 \). Then, comparator 2 (1) resets (sets) the latch, which changes the sign of the current through \( C \), so that a triangular oscillator signal results. By putting \( V_1 = V_2 = \frac{V_{-	ext{tt}}}{2} \), the oscillator frequency is found to be:

\[
\omega_o = \frac{I}{2CV_{\text{hys}}} \quad (7.20)
\]

As the oscillator is to be used in a PLL, the current sources are controlled by an external voltage \( V_{\text{tune}} \). Assuming that the tail current \( 2I \) is supplied by a voltage-to-current amplifier with transconductance \( R^{-1} \), the oscillator constant (in rad.s\(^{-1}\)V\(^{-1}\)) is given by:

\[
K_o = \frac{\pi}{2RCV_{\text{hys}}} \quad (7.21)
\]

In Fig.7.31, showing the complete circuit, comparator 1 and 2 are implemented around \( Q_{2,4} \) and \( Q_{1,3} \) respectively. Note that \( Q_1 \) and \( Q_2 \) perform an
additional function: their base-emitter voltages are used as the voltage sources \( V_1 = V_2 \approx 0.7V \). The comparator output currents drive the latch consisting of \( Q_{5..8} \). The latch changes its state if the comparator output current exceeds the tail current \( I_{\text{latch}} \). Consequently, the comparator current \( I_{\text{comp}} \) should be chosen (for instance a factor of two) larger than \( I_{\text{latch}} \).

Since, in integrated circuits, floating capacitors are not available, the integrator capacitor consists of two series capacitances \( \frac{C}{2} \), having their common node connected to the signal ground. The current switch consists of the differential pair \( Q_{9,10} \). Its tail current \( 2I \), determining the integrator current and the oscillator frequency, depends on the tuning voltage \( V_{\text{tune}} \) supplied to the base of \( Q_{19} \), the voltage-to-current gain \( \frac{1}{R_{eQ_{19}}} \) and the offset current fixed by \( R_{eQ_{9,10}} \).

Low-noise active current sources to the positive power supply would require PNP transistors with a high transit frequency. As they are not available in simple bipolar IC processes, we use resistors instead.

The common-mode voltage of both integrator capacitors is basically undefined. In order to stabilize this voltage, the capacitor voltages are sensed by
the voltage buffers $Q_{13,14}$ and averaged by the resistors $R_{eQ_{13,14}}$. The averaged voltage is then compared to the reference voltage $V_b$. The error voltage is amplified by the series stages consisting of $Q_{15,16}$ so that feedback of the output current stabilizes the common-mode voltage to $V_b$. $R_{eQ_{15,16}}$ serve to decrease differential-mode noise originating from $Q_{15}$ and $Q_{16}$.

The oscillator output signal is buffered by the differential pair $Q_{11,12}$.

**Phase noise originating from voltage noise sources**

To estimate the magnitude of the phase noise originating from the equivalent voltage noise source at the input of the comparator, we apply the method discussed in Sec.5.6.2.

Basically, $V_{hys}$ equals twice the base-emitter voltage, but, due to the switching time delay, the capacitor voltage exceeds this value by approximately 30% (determined using SPICE), resulting in $V_{hys} = 2V$. According to Eq.5.36, a comparator tail current of $1mA$ results in an input capacitance $C$ (during switching) of about 1pF. Using SPICE it was found that, due to the comparator input capacitance in parallel with the integrator capacitor, the effective slope magnitude at the comparator input is a factor of 1.8 lower (corresponding to an impairment of about 5dB) than might be expected from $V_{hys}$ and $f_o$. Further on, the level shifts consisting of $Q_{1,2}$ increase the noise at the comparator input by a factor of 1.5 (in power), which corresponds to an additional impairment of about 1.5dB. Using Eq.5.50 with $C=1pF$, $f_o=70MHz$, $V_{hys}=2V$ and a total additional deterioration of 5+1.5=6.5dB, we obtain $L(1MHz)=-123dB/Hz$.

**Phase noise originating from current noise sources**

In Sec.5.6.2 the influence of the current noise sources on the oscillator phase noise was determined assuming an equivalent noise resistance $R_i$. Note that since the comparators sense the differential-mode integrator output voltage, only differential mode noise needs to be considered. We will determine the equivalent noise resistances of the dominant sources.

Provided that the voltages across $R_{eQ_{19}}$ and $R_{eQ_{9,10}}$ are large compared to $\frac{kT}{q}$ and if the current switch is in one of its saturated states, i.e. one of the transistors conducts the full tail current, the transconductance stage around $Q_{19}$ and $Q_{9,10}$ contributes a noise resistance:

$$R_{i,\text{trans}} \approx \frac{1}{\frac{1}{R_{eQ_{19}}} + \frac{1}{R_{eQ_{9,10}}} + \frac{R_{bQ_{19}}+R_{bQ_{9,10}}}{R_{eQ_{19}}}} \quad (7.22)$$

where the $R_{bQs}$ denote the base bulk resistances of the transistors. The
common-mode stabilization circuit contributes:

\[ R_{i,cm} \approx \frac{(R_{eQ15} + R_{eQ16})^2}{R_{eQ15} + R_{eQ16} + 2R_{BQ15,16}} \] (7.23)

Evaluation of Eqs.(7.22 and 7.23) for the prototype of the oscillator yields \( R_{i,\text{trans}} = 930 \Omega \) and \( R_{i,\text{cm}} = 1300 \Omega \). Applying Eq.5.51 and substituting the oscillator frequency \( f_o = 70 \text{MHz} \) we find \( L(1 \text{MHz}) = -123 \text{dB/Hz} \). Since reducing the contributions of the dominant sources implies a lower modulation constant (larger \( R_{eQ19} \)) and a lower loopgain of the common-mode circuit (larger \( R_{eQ15,16} \)), further reduction is practically impossible.

We may conclude that the total noise contribution from the current sources equals that of the voltage noise sources. As a result, the total phase noise will be in the order of \( L(1 \text{MHz}) = -120 \text{dB/Hz} \). Fig.7.32 depicts the measured phase noise power density spectrum, which corresponds well with our calculations.

![Figure 7.32: Measured noise-to-carrier ratio of the 70MHz regenerative oscillator.](image)

Substituting the parameters of the prototype system, \( r = 1.75 \), \( B_r = 250 \text{MHz} \), \( \text{RIN} = -125 \text{dB/Hz} \) and \( f_o = 70 \text{MHz} \) into Eq.(7.17), we find the bandwidth of the multiplier PLL \( B_2 \) should be larger than 320KHz.

### 7.6.6 Phase Detectors and Loop Filters

Fig.7.33 depicts the complete phase detector circuit, based on an EXOR gate. The difference with the EXOR gate presented in Sec.7.2.3 is that this configuration is based on series gating. Because the phase detector output signal is
filtered by the PLL loop filter, the relatively long settling times resulting from series gating are no problem.

In discussing the crystal oscillator, it was noted that it might be useful to use the sinusoidal output signal of the oscillator, because this reduces the chance of false locks of the multiplier PLL to frequencies related to the harmonics of the oscillator frequency. In this case, to have the phase detector properly handle sinusoidal signals (which are to be supplied to the B inputs), the phase detector needs to be modified by placing a resistor between the emitter terminals of $Q_{11}$ and $Q_{12}$. However, as false locks did not occur in the prototype system and since square wave signals are less susceptible to additive noise, we decided to use square waves.

The output of the phase detector supplies a current to the filter of the PLL, of which an example is shown in Fig. 7.34. It has a pole in the origin (if the output impedance of the phase detector is assumed to be infinitely large) and a zero at the frequency $\frac{1}{R_\text{e}C_p}$. The output signal of the filter, which controls the oscillator frequency, is a voltage. Depending on the frequency inaccuracy of the oscillator used, the tuning voltage in the lock mode can vary over a relatively large range of several volts, which may cause voltage saturation of the phase detector output circuit. Reducing this voltage range by using a voltage amplifier at the filter output is not recommended. Due to the amplification
of the noise contributed by the amplifier, this severely decreases the signal-to-
oise ratio of the oscillator input signal, resulting in increased oscillator phase
noise. Instead, we maximized the allowed voltage swing at the phase detector
output by using current mirrors to the negative and positive voltage supplies.

A disadvantage of the current mirrors is that they restrict the bandwidth of
the phase detector to the (relatively low) transit frequency of the (lateral) PNP
transistors. But, as the PLL bandwidth is generally an order of magnitude
lower, this causes no problem.

The transfer function of the phase detector \( I_f \) has the shape of a triangle
with amplitude \( 2I_{cQ11,12} \), period \( 2\pi \), and offset \( \frac{\pi}{2} \), i.e. \( I_0 \) is zero for \( \phi = \pm \frac{\pi}{2} \).
Then the gain factor \( K_d \) equals \( \frac{4I_{cQ11,12}}{\pi} \).

7.6.7 Discussion

The two-oscillator solution provides a clock signal, exhibiting the stability of
a crystal oscillator, at the desired frequency. To maintain the stability of the
crystal oscillator, the phase noise produced by the second oscillator in the
PLL frequency multiplier should be suppressed by choosing a sufficiently large
bandwidth for the multiplier PLL. However, to prevent the PLL from undesired
oscillations, the PLL bandwidth is practically restricted to about one fifth of
the frequency of the crystal oscillator.

We have discussed the designs of the crystal oscillator, the regenerative
oscillator, the phase detectors and the loop filter. Having determined the
oscillator constants \( K_o \) and the detector constants \( K_d \), the averaging and the
multiplying PLL can be designed to have the desired bandwidth by selecting
the appropriate filter components \( C_p \) and \( R_n \).

7.7 Receiver Front-End

In Sec.5.2, we already assumed that the receiver front-end consists of a negative-
feedback amplifier. Since our objective was to determine the limitations of the
7.7. RECEIVER FRONT-END

electronic circuitry, we concentrated on the bandwidth and sensitivity of the negative-feedback amplifier.

In this section, we present the complete configuration of the front-end, which does not only consist of the amplifier, but includes additional circuitry to handle large input signals as well. After examining the influence of this additional circuitry on the sensitivity of the front-end, we will deal with the circuit diagram. Details can be found in [8] and [9].

Basic Configuration

As the capacitance of the wiring between the PIN photodiode and the front-end adds to the capacitance of the PIN itself, the total capacitance at the input of the front-end (further denoted as $C_{\text{PIN}}$) is relatively large (1–2pF). The input circuit of the front-end is preferably based on a negative-feedback amplifier because it combines low input impedance, required to avoid loss of signal current into $C_{\text{PIN}}$, with low equivalent input noise [10]. The output signal of the amplifier is either a voltage or a current, obtained with the configurations depicted in Fig.7.35(a) and Fig.7.35(b) respectively.

![Figure 7.35: Basic configurations of a negative-feedback amplifier having a current-to-voltage (a) and a current-to-current transfer (b).](image)

As concluded in Sec.5.2.2, the receiver sensitivity of a properly designed amplifier is determined only by the noise originating from the first stage of the amplifier [10]. This noise is minimized by using a CE stage at the amplifier input and optimizing its bias current and geometry. By choosing a sufficiently large feedback resistor $R_f$ and amplifier gain, the contributions of respectively noise originating from $R_f$ and successive circuitry can be made negligibly small.

As the input current causes a voltage across $R_f$, the maximum input current is restricted by $R_f$ and by the maximum signal voltage that can be handled by the output stage of the amplifier. In current amplifiers, the maximum input current is also restricted by the gain of the amplifier and its maximum output current. Therefore, a high saturation current demands a small value of $R_f$ and, in the case of a current amplifier, a low current gain. Consequently, the requirements for a high saturation level conflict with the requirements for a low noise level.
A successful method of enlarging the maximum allowed input signal level, while maintaining optimal sensitivity for small signals, is the insertion of a gain control at the input of the amplifier. Unfortunately, continuously variable gain control circuits show poor noise behavior [11]. An alternative solution, suitable for current amplifiers, uses a current switch at the input which bypasses large input signals directly to the output of the amplifier.

The current switch consists of two common-base (CB) stages (Fig. 7.36). Depending on the voltage $V_{sw}$ across the two bases, the emitter input current is conducted to either the input or the output of the amplifier.

![Figure 7.36: The current-to-current amplifier with a bypass switch at its input.](image)

An important advantage of this specific type of switch is that it effectively isolates the source capacitance $C_{PIN}$ from the feedback loop of the amplifier. Consequently, unlike the situation in conventional receivers, the high-frequency response of the amplifier does not depend on the actual value of $C_{PIN}$.

The bandwidth of the CB stage amounts to the transit frequency $f_t$ of the transistors. As the maximum bandwidth of negative-feedback amplifiers exhibiting a proper high-frequency behavior is practically restricted to about $f_t/10$, the bandwidth of the switch will be sufficiently large in all cases and needs no further consideration. The noise behavior of the switch is more complicated and requires more detailed treatment.

**Noise Behavior of Current Switch**

First, we will compare the equivalent input noise of CB stages (adopted in the switch), to that of common-emitter (CE) stages (adopted in the amplifier), when they are connected to capacitive source impedances. Next, we will show that the total noise produced by the switch and amplifier together is only slightly larger than the noise of an optimally matched amplifier which is connected directly to the PIN photodiode.

The noise produced by a bipolar transistor can be represented by an equivalent noise current source $i_n$ between its base and the emitter terminal and an equivalent noise voltage source $u_n$ in series with its base terminal. Fig. 7.37
shows the transformation of $u_n$ and $i_n$ to the input and output terminals of the CB stage. $C_s$ is the total source capacitance. The contributions of the

![Diagram](image)

Figure 7.37: Determination of the equivalent input noise from $u_n$ and $i_n$ of the CB stage.

resulting voltage sources at the collector terminal can be ignored because of the large voltage gain and transimpedance of the CB stage. Transformation of $u_n$ and $i_n$ in the case of a CE stage, as depicted in Fig.7.38, reveals exactly the same equivalent sources at the input. Obviously, CB stages produce the same equivalent input noise as CE stages.

In Sec.5.2.2, we calculated the total equivalent input noise current by integrating the total noise current in parallel with $C_s$ over the amplifier bandwidth $B_r$, demonstrating how the total noise power depends on $C_s$. In the case of the switch, where the same result applies, $C_s$ consists of the total capacitance $C_{PIN}$ of the PIN photodiode and its wiring, whereas in the case of the successive amplifier, $C_s$ consists of the output capacitance of the switch $C_s$.

As the switch has unity current gain, the amplifier noise adds to the noise

![Diagram](image)

Figure 7.38: Determination of the equivalent input noise from $u_n$ and $i_n$ of a CE stage.
produced by the switch. But, since $C_o$, consisting of the relatively small collector-to-substrate and collector-to-base capacitances of the CB stage, is small compared to $C_{PIN}$, the noise contributed by the amplifier may be ignored. Hence, the noise behavior of the front-end is dominated by the switch. Since its equivalent input noise equals that of an amplifier using a CE stage at its input, we may conclude that the noise performance is hardly affected by inserting the switch.

To avoid clipping, the dc collector current of the CB stage should be larger than the maximum signal current, so optimal biasing is not always possible. However, since for practical implementations the total noise power $I_n^2$ is not very sensitive to variation of $I_c$, this is of minor concern.

**Complete Circuit of the Receiver Front-End**

Fig.7.39 shows a balanced configuration of the receiver employing the current switches at the input.

![Circuit Diagram](image)

Figure 7.39: Circuit diagram of the low-noise front-end with current switches at the input.

The switches consist of $Q_{8,9}$ (activated when using the amplifier) and $Q_{7,10}$ (activated in the bypass mode). The output current of the first stage, which
consists of $Q_1$ and $Q_2$, is buffered by $Q_3$ and $Q_4$ before being supplied to the output stage around $Q_5$ and $Q_6$. The buffer stage helps to maneuver the poles of the negative-feedback amplifier into their proper positions. The feedback network is constituted around $R_{1,3}$. Frequency compensation is provided by the base-emitter and base-collector junction capacitances of $Q_{11,12}$ and $Q_{13,14}$. The high-frequency response of the amplifier depends on the reverse biasing voltages across these junctions.

The reverse biasing voltage across the junctions is achieved via $R_{10,11}$, having a resistance of about $20k\Omega$ to avoid a short circuit to signal ground. The biasing voltage is fixed by the voltage divider $R_{12,13}$ but it can also be supplied externally, allowing trimming of the high-frequency response.

The dc biasing of the current switch and the amplifier is performed by $Q_{15,16}$ and $R_{4,9}$. The circuit within the dashed box drives the current switches and blocks the output stage of the amplifier in the bypass mode.

An additional circuit is required to detect the amplitude of the output signal, deciding the state of the switches. Hysteresis is necessary to avoid repetitive switching between amplifier and bypass for certain amplitudes of the input signal. A simple circuit is appropriate to implement this function.

By substituting $C_j,_{\text{balanced}}=0.1pF$, $C_s=2pF$, $\beta=100$ and $R_{b,\text{balanced}}=300\Omega$ and $B_r=250MHz$ into Eq.(5.15), the total equivalent input noise current was calculated to be in the order of $60nA$. Fig.7.40 depicts the measured noise spectrum at the amplifier output. We have calibrated the $Y$ axis in terms of equivalent input noise by applying a reference level input signal. Integrating

![Graph showing input noise current vs. frequency](image)

Figure 7.40: Measured noise power density spectrum of the receiver.

the measured noise power density in a bandwidth of $220MHz$ results in a total equivalent input noise current of about $75nA$, which is only $2dB$ higher than
predicted. For the lowest and the highest frequencies the measured spectrum does not fully obey Eq.5.6, due to the noise produced by the measurement equipment and the influence of the low-pass filtering by the amplifier respectively.

The frequency response of the receiver, depicted in Fig.7.41, was measured for different values of the compensation capacitance (tuned with an externally supplied voltage). To obtain minimal pulse widening and to avoid ringing, the poles should be in Bessel-Thomson positions.

![Graph showing frequency response of the receiver](image)

**Figure 7.41:** Frequency response of the receiver for different values of the compensation capacitance.

### 7.8 Pulse Reshaper

At its input, the pulse reshaper receives TDPPM pulses from the front-end. The shape of these pulses depends on the laser dynamic properties, the fiber dispersion and the filter characteristic of the front-end, while the amplitude depends on the power of the transmitted pulses, the losses of the fiber and the connectors and the gain of the front-end. As the maximal pulse peak power of the laser is about 1mW while the threshold level is in the order of 1μW (Sec.3.6.4), the amplitude of the received TDPPM pulses may vary as much as 60dB. The task of the reshaper is to derive proper square pulses with constant amplitude and minimal pulse jitter that are suitable for further processing.

In Sec.3.5 it was concluded that a practical method for reshaping the TDPPM pulses is to pass them through a low-pass filter and a comparator.
As the front-end already performs the low-pass function, at this stage, we concentrate on the comparator function.

The comparator threshold level should be set to the level at which the pulse edges exhibit maximal steepness. If, in approximation, the maximum steepness is at half the pulse height, we may use the simple reshaper shown in Fig. 7.42. It uses two peak detectors and a sum circuit to set the threshold level.

![Diagram of pulse reshaper](image)

Figure 7.42: Pulse reshaper in which the threshold level is set to half the pulse height.

If the pulses do not exhibit their maximum steepness at half the pulse height, more advanced circuits as shown in Fig. 7.43, using, for instance, differentiators to find the steepest part of the pulse edges, are required. The actual reshaping function is performed by comparator 2, while the comparators 1 and 3 have a threshold level which is respectively a fixed voltage $V$ lower and higher than the threshold level of comparator 2. The comparator outputs determine the states of two set-reset flip-flops in such a way that the width of their output pulses is inversely proportional to the slope magnitudes of the TDPPM pulses just below and just above the threshold level of comparator 2. After subtraction and low-pass filtering, a feedback signal is available which
tunes the threshold voltage of comparator 2 to the level at which the slope magnitude is maximal.

Because of the large uncertainty in pulse amplitude, the presented reshaper circuits require peak detectors and comparators exhibiting very low offset voltages. As these requirements cannot be met in practical circuitry, we will use the alternative reshaper circuit of Fig. 7.44. The comparator output pulses are low-pass filtered and the resulting dc voltage is compared to a reference voltage \( V_{\text{dut}} \), which is determined from the duty cycle \( D \) of the received TDPPM pulses according to:

\[
V_{\text{dut}} = V_I + D(V_h - V_I)
\]  

(7.24)

in which \( V_I \) and \( V_h \) are the low and high comparator output voltage. Feedback of the error signal will adapt the threshold level such that the average duty cycle of the comparator output pulses equals the (indirectly) adjusted value of \( D \). As the pulses have finite rise and fall times, an improperly adjusted \( V_{\text{dut}} \) results in a slightly too low or too high threshold level.

In the prototype system, \( V_{\text{dut}} \) is adjusted manually. A method to automatically derive the (approximately) correct value of \( V_{\text{dut}} \) would be to generate dummy PPM pulses in the receiver, using the same circuitry as in the transmitter, and to pass them through a low-pass filter.

The balanced implementation shown in Fig.7.45 consists of four limiter stages around \( Q_{1..20} \). The number of stages has been chosen large enough to derive output pulses with saturated steepness, even if the input pulses have minimal amplitude. As the input signal is a current, the reshaper can be connected directly to the front-end output.

Rather than a reference voltage, the circuit uses a (balanced) reference current. This current is supplied to the emitters of \( Q_{23,24} \), while low-pass filtering is performed on the error current rather than on the individual currents. As a consequence, if the reference current is derived from dummy pulses, one single low-pass filter is sufficient. The reference current can be adjusted manually by
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Figure 7.45: Complete pulse reshaper circuit with manual adjustment of the duty cycle.

\( R_{\text{dut}} \). The filter output voltage, which is available across the external capacitors \( C_{\text{ext}} \), is added to the pulse voltage across \( R_{1,2} \) and then supplied to the comparator input by the transistors \( Q_{21,22} \). The common-mode voltage at the comparator input is stabilized by \( R_{3,4} \) and \( Q_{23,25} \). The collectors of \( Q_{21,22} \) can be used as outputs to monitor the output signal of the receiver front-end.

7.9 Demultiplexer

In Sec.4.3.1, we concluded that demultiplexing can best be performed by a counter and a decoder, because this method automatically ensures proper slot synchronization. Using this technique, the actual PPM pulses are not available at the demultiplexer outputs. Instead, each output supplies a rectangular wave of which the leading, the trailing or both the leading and the trailing edges correspond exclusively to the PPM pulses of one of the channels.

An additional circuit, the frame pulse identifier (FID), is necessary to ensure that the PPM pulses are demultiplexed to the appropriate outputs (the so-called frame synchronization). A handy method would be to reset the counter to its zero state each time a frame pulse has been detected. However, counter circuits having reset inputs are more complicated than counters without. So, we prefer an alternative solution based on a blocking circuit which blocks input TDPPM pulses until proper synchronization is obtained. Fig.7.46 schematically depicts how frame synchronization is obtained. Alternative (a) checks after each detected frame pulse whether the counter is in its zero state. If not, the successive TDPPM pulse is blocked. After a maximum of \( N-1 \) frame pulses, taking \( N-1 \) pulses, proper synchronization is obtained. Alternative (b) checks during the zero state of the counter whether a frame pulse has been
Figure 7.46: Synchronization of the multiplexer to the frame pulses based on the blocking method. Depending on the configuration, a maximum of $N-1$ frames (a) or $N-1$ pulses (b) is required to obtain proper frame synchronization.

detected. If not, the successive TDPPM pulse is blocked. Because proper synchronization is obtained within $N-1$ pulses, i.e. within one frame, the principle of (b) is preferred over that of (a).

As the demultiplexer counter and decoder are based on standard MECL logic such as OR gates, latches, flip-flops and buffers, we will only discuss the design at the level of a block diagram. The FID does not consist of standard cells, so the design of the FID will be discussed in more detail.

7.9.1 Counter and Decoder

Basically, two types of demultiplexer output signals are possible: (1) the leading edges correspond to the PPM pulses of one channel, while the trailing edges correspond to pulses of one of the other channels, and (2) both the leading and the trailing edges correspond exclusively to the PPM pulses of one of the channels. As illustrated by Fig.7.47, the first option requires an $N$-state counter (the figure shows only one of the possible waveforms), whereas the second alternative requires an $2N$-state counter.

The counter is constructed with cells that have finite propagation delays, so, as demonstrated in Fig.7.48, when using conventional counters, improper design may easily result in intermediate states. As a first consequence, intermediate states may cause false pulses at the decoder output, making the demultiplexer worthless. Secondly, if the intermediate states remain so short that false pulses do not appear, intermediate states do result in uncertainty in delay times, which makes the circuit more susceptible to noise, distortion and crosstalk. Fig.7.48 also demonstrates that in so-called Gray counters, in which
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![Diagram of demultiplexers based on N-state and 2N-state counter]

Figure 7.47: Demultiplexers based on an N-state and a 2N-state counter.

![Diagram of binary and Gray counter transitions]

Figure 7.48: State diagrams of a conventional binary counter and a Gray counter. The arrows indicate the state transitions. The binary counter has intermediate states, causing false pulses or timing errors.

For each clock pulse only one of the output bits changes its value, intermediate states cannot occur.

Gray counters having an even number of states can be implemented using a ring of D-flip-flops, which includes one inversion and some additional circuitry preventing start-up in improper modes. Fig. 7.49 shows examples of a 4-state and a 6-state Gray counter. The 6-state counter has one undesired mode with sequence 101-010, which is prohibited by the AND gate. It detects the 010 state and forces the counter to the state 000.

An interesting option is to convert the TDPPM signal into a square wave by a single flip-flop Gray counter and to use this square wave as the clock signal for a second Gray counter of which, beside the flip-flop outputs, the outputs of the internal latches are also made available. After each even TDPPM pulse, the
Figure 7.49: Examples of (a) a 4-state and (b) a 6-state Gray counter.

flip-flop outputs, and after each odd pulse the outputs of the internal latches change their states. In this way, the \( N \)-state counter has become an \( 2N \)-state counter.

A further advantage of Gray counters is that, in contrast to conventional binary counters, at each output a square wave with a period of \( N \) (or \( 2N \)) cycles is available. Consequently, the counter output signals need no further decoding, so omitting another possible cause of false pulses. Still, small disturbances of the output signals may result from crosstalk between the clock inputs and the counter outputs. This effect can be reduced by additionally buffering the output signals by standard limiters.

### 7.9.2 Frame Identification and Blocking Circuit

Besides their function of frame synchronization, frame pulses can be used to transmit signals, just as other TDPPM pulses. To keep the maximum fraction of the time slots available for modulation of the frame pulses, the FID circuit should be designed such that for proper identification the frame pulses need to be only slightly wider than other pulses.

We will discuss three methods for detecting frame pulses, of which the
method of converting the width of the TDPPM pulses into PAM pulses will be worked out to a realization.

**Identification by counting the cycles of a reference frequency**

To identify frame pulses, a first possibility is to measure the pulse duration by counting the cycles of a reference high-frequency oscillator signal. Since the width of the TDPPM pulses as well as the minimum period of the oscillator signal is determined by the $\tau_f$ of the integrated components, the resolution of the counter identification circuit (i.e., the difference in pulse width that can just be detected) will be in the order of one pulse width, which is relatively poor. In addition, a disadvantage of having the required oscillator on the same IC is that it may, due to parasitic coupling, induce spurious on other signals.

**Identification by comparing to reference pulses**

A second possibility is to generate reference pulses whose length is automatically tuned to the average length of the TDPPM pulses, and to compare the length of the reference pulses with that of the TDPPM pulses.

In the circuit shown in Fig. 7.50, the reference pulses are started by the TDPPM pulses. The AND gate ("=2") produces a logic high if the TDPPM

![Diagram](image)

**Figure 7.50:** Frame pulse identifier based on subtraction of the received TDPPM pulses from automatically generated reference pulses.

signal is still high after the reference pulse has ended, that is, when a frame pulse is received. The output pulses of the AND gate, which trigger the pulse blocking circuit, will be called identification (ID) pulses. By subtracting the reference pulses from the TDPPM pulses, low-pass filtering and feedback, the width of the reference pulses is tuned automatically to its appropriate value. To compensate for the time required to start up a reference pulse, the TDPPM pulses are delayed by a buffer before they are supplied to the AND gate.

The resolution of the identifier is limited by the minimum pulse width that can be handled by the AND gate. As this pulse width is determined by $\tau_f$, the resolution will be in the same order of magnitude as that of the counter FID.
Improved resolution is obtained by subtracting the TDPPM pulses from the reference pulses and subsequent integration, as demonstrated in Fig.7.51. The integrator converts the difference of pulse duration into pulse height and keeps the measured signal available for some time $T_h$ (hold time). Then, by closing the switch, the integrator output voltage is reset to zero before the next pulse is received. By means of a comparator and some circuitry determining a suitable threshold level, the ID signal can be derived from the integrator output signal.

Using this technique, the resolution is no longer determined by the propagation delay of the gates. In addition, as the resulting integrator output voltage depends only on the difference in pulse width, delay compensation (which is the task of the buffer) has become less critical.

**Identification by conversion to PAM pulses**

Rather than subtracting reference pulses from the TDPPM pulses and converting the pulse length difference into a voltage, it is simpler to convert the TDPPM pulses directly into amplitude modulated (PAM) pulses [12]. The converter depicted in Fig.7.52 integrates a constant current when the TDPPM signal is high, stores the integrator voltage for some time $T_h$ and then resets the integrator voltage to its initial value before the next TDPPM pulse arrives. The pulse amplitude is directly proportional to the pulse width. In contrast to the pulses generated in the converter presented above, the average amplitude of the PAM pulses does not equal zero. An appropriate value of $T_h$ is in the order of the TDPPM pulse width, because shorter pulses cannot be handled by the comparator, whereas longer pulses will result in longer identification times so that the next TDPPM pulse might arrive before the reset.

Fig.7.53 depicts the circuit diagram of the pulse width-to-amplitude converter. The TDPPM pulses drive the current switch $Q_{9,10}$ so that the integrator output voltage across the capacitor $C$ decreases while a pulse is present. The
buffer, exhibiting a delay time $T_h$ (hold time), and the AND gate generate the integrator reset pulses which start $T_h$ seconds after the trailing edges and stop after the leading edges of the TDPPM pulses. The reset pulses drive the voltage switch $Q_{12}$, resetting the integrator voltage to its starting value $V_{bb} - 2V_{be}$. ($V_{be}$ denotes the voltage across the forwardly-biased base-emitter junctions of the transistors $Q_{11,12}$.) The related waveforms of the TDPPM input signal and the PAM output signal are shown in the figure.

To optimize sensitivity, the amplitude of the output pulses should be maximal. The level shifts consisting of $Q_{1-8}$ avoid voltage saturation of $Q_9$ for large pulse amplitudes.

To separate the PAM pulses with the largest amplitude (those pulses correspond to the TDPPM frame pulses) from the other pulses, we use the comparator circuit shown in Fig.7.54. The actual comparator, separating the ID pulses, consists of $Q_{1,4}$. The rest of the circuit serves to fix the threshold voltage at the second input of the comparator.

During the intervals when no frame pulses are received, the comparator output voltage is low. Then, a small current $I_{low}$ slowly charges the capacitor
Figure 7.54: Circuit diagram of the comparator with asymmetrical output and feedback.

C. Each time the input voltage drops below the capacitor voltage, which is after each frame pulse, the comparator, and hence the current switch around \( Q_{13,14} \), change their state, so the peak current \( I_{\text{peak}} \) rapidly charges the capacitor voltage until it has reached the momentary input voltage. The remaining part of the cycle, the comparator voltage will be low. The ratio between \( I_{\text{low}} \) and \( I_{\text{peak}} \) depends on the ratio between the emitter resistances of \( Q_{15} \) and \( Q_{17} \).

Although the circuit is relatively sensitive to variations in the amplitude of the integrator pulses (width of the actual frame pulses), reliable identification is obtained since the pulse width is approximately constant or varies only slowly (temperature effects). Experiments with the prototype system showed that good results are obtained with frame pulses that are 20% wider than other pulses.

**Complete configuration of FID and demultiplexer**

In adding the demultiplexer, the FID and the blocking circuit together, an important concern is to ensure adequate timing of the various signals. In [12], a detailed analysis of a possible configuration, in which synchronization is obtained within \( N-1 \) frames, is described. Earlier, in Fig.7.46b, we illustrated an alternative solution in which synchronization is obtained within one frame time.

The block diagram of the required circuitry and the waveforms of the various signals are depicted in Fig.7.55. The blocking circuit consists of an AND gate. The vertical arrows indicate the order of subsequent events.
Figure 7.55: (a) ID circuit, blocking circuit and demultiplexer counter added together. (b) Timing diagram of relevant signals.

If the counter comes into its zero state, the flip-flop is set, so the blocking circuit will be ready to block the next pulse. If the last TDPPM pulse received turns out to be a frame pulse, proper synchronization has already been obtained so that blocking is cancelled by resetting the flip-flop. Otherwise, if no frame pulse was detected, all the following TDPPM pulses are blocked until a frame pulse has been detected.

7.10 Demodulator

We will discuss two demodulator principles. The first one is based on taking samples of a sawtooth reference signal at the instant the PPM pulses are received. In this way, the PPM pulses are converted into amplitude modulated (PAM) pulses. The second method, which has been adopted in our prototype system, is based on conversion of the PPM pulses into duration modulated pulses (PDM). Subsequently, to retrieve the baseband signals, the PAM or PDM pulses are low-pass filtered by the smoothing filter.

7.10.1 Demodulation Based on Conversion into PAM Pulses

Fig. 7.56a shows the block diagram of the demodulator. It consists of a saw-
Figure 7.56: Two configurations of the demodulator using a sawtooth generator and sample-and-hold switches.

tooth generator, which is synchronized by the receiver clock regenerator, and which is similar to the sawtooth generator used in the modulator. Note that, rather than PPM pulses, the demodulator has to handle square waves. The pulse shaper serves to derive short pulses for driving the sample switch from the square wave output signals of the demultiplexer.

As illustrated in the waveform diagram, sampling of the sawtooth directly results in the momentary amplitude of the baseband signal $V_1$. However, since the timing of the sampling is signal dependent, uniform spacing of the output pulses requires an additional sample-and-hold. As illustrated in configuration (b), this second sample-and-hold samples the output of the first at a constant rate ($V_2$). The system is free of sample distortion (see Sec.3.9) if the “double sampling demodulator” is used in combination with the double sampling modulator in the transmitter (configurations (c) and (d) in Fig.7.11).

In [13] the design of the double sampling demodulator has been worked out. It was found that, with some special measures, the non-linearity of a uniform sampling sample-and-hold can be below 1%. However, as the time interval between a sample of the first sample-and-hold and that of the second one is signal dependent, the demodulator will be quite susceptible to ringing and any spurious signals on the output signal of the first sample-and-hold $V_1$. Therefore, additional distortion is difficult to avoid. In addition, sample-and-hold circuits have the disadvantage that they induce relatively large peak currents into the substrate and into the power supply, possibly influencing other circuits on the same IC.
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7.10.2 Demodulation Based on Conversion into PDM Pulses

The second method, based on conversion into PDM pulses is more simple and less critical. The configuration of Fig.7.57a is used if only the leading or only the trailing edges carry information of the same baseband signal. The leading (trailing) edge sets the flip-flop, whereas the clock signal resets the flip-flop. Configuration (b) uses an exclusive OR (EXOR) gate and is suitable if both the leading and the trailing edges of the demultiplexed signal belong to the same channel. Provided that the time between two pulse edges is large enough to avoid settling problems, due to their simplicity, these types of demodulator exhibit very low distortion.

Fig.7.58 depicts a practical implementation of the EXOR based demodulator. Except for the level shifts, which allow for a large voltage swing at the output so that the demodulator can be connected directly to various type of low-pass filters, the demodulator is a standard EXOR gate.

Besides the baseband signal, the spectrum of the PDM output signal contains components at multiples of the sampling frequency. Eq.(3.26) shows that, depending on the average duty cycle of the PDM output signal and due to the relatively small modulation index, the magnitude of those components can be
large compared to the magnitude of the baseband signal. In the prototype system, for instance, their amplitude was up to 20dB higher than the amplitude of the baseband signal. To avoid distortion caused by these high frequency components, the low-pass filter must be capable of handling the large input signals. In addition the filter should have a large attenuation for high frequencies.

The requirements to the filter can be relaxed by minimizing the magnitude of the high-frequency components. This can be accomplished by subtracting a second square wave signal from the PDM output signal. The second square wave has to have the same average amplitude and duty cycle as the PDM wave.

7.11 Smoothing Filter

In the prototype of the system, we used a low-pass smoothing filter consisting of discrete capacitors and inductors. If one does not have the practical restrictions of semicustom ICs, the filter can be integrated as well. We will describe the design of such a filter and report some experimental results.

Integrated continuous-time filters consist of a series of integrators, all using capacitors as reactive elements, see for example [14], [15] and [16]. By applying the optimization theory of [17], it can be shown that, for a given amount of capacitance and supply voltage and if the spectral power density of the input signal is assumed to be flat (within the passband of the filter), an almost optimal dynamic range is obtained by a so-called Leapfrog structure [18], [20]. Fig.7.59 shows how the integrators, having two or three (balanced) inputs and one output, are mutually interconnected. By choosing the appropriate integrator constant (gain) for each of the integrator inputs, the filter bandwidth and the locations of the poles can be fixed.

![Figure 7.59: Low-pass filter consisting of integrators in a Leapfrog structure.](image)

In the prototype system the SNR is in the order of 50dB (determined by the laser diode). So, the dynamic range of the filter should preferably be in the order of 60dB or larger. In addition, since the magnitudes of the high frequency components present in the output signal of the demodulator are relatively large and since the magnitudes are decreased by successive integrator sections, the first two or three integrators need to have a larger dynamic range than the other integrators.
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For a sufficiently large suppression of the high frequency components, 5 was chosen as the order of the experimental filter. By applying Eq.(5.78) we find that, provided that the peak-to-peak amplitude of the integrator output voltage is 5V and for a dynamic range of 60dB, a capacitance of only 5fF for each integrator would be sufficient.

The integrator constant, and hence the filter bandwidth, depend on the product of the integrator capacitor $C$ and a resistance $R$. Since in integrated circuits both components exhibit tolerances of plus or minus 10%, the filter bandwidth requires trimming afterwards over a range of plus or minus 20%. Basically, in a bipolar technology, only junction capacitances or trans-linear current multipliers (for example, so-called Gilbert quads) can be used for tuning. Because the first alternative restricts the maximum signal voltage in avoiding non linear distortion and because the second alternative introduces considerable shot noise, the dynamic range of bipolar tunable filters is much lower than might be expected considering the available capacitance and supply voltage. Nevertheless, we will present a filter based on trans-linear current multipliers. A second design presented here, achieving a higher dynamic range, is based on using switches which add capacitance in steps to fixed capacitors until the filter bandwidth has approximated its desired value. As the prototype system was used for the transmission of video signals, the filter bandwidth was about 5MHz.

The principles of automatically adjusting the appropriate tuning voltage (or switch setting) can be found in [21].

7.11.1 Tunable Transconductance-Capacitor Integrators

Fig.7.60 depicts a complete (multiple-input) integrator. To distinguish the signal circuit from the biasing circuit, the signal circuit has been drawn using thick lines. The input stages consisting of $Q_{1,2}$ and $R_e$ convert the input voltages into currents and adds the currents together. The current is scaled by the Gilbert multiplier around $Q_{3,6}$ by a factor which equals the ratio between the biasing currents of $Q_{5,6}$ and $Q_{3,4}$. Subsequently, the current is buffered by the CB stage consisting of $Q_{7,8}$. The output current is integrated by the capacitors $C$, so the integrator has a voltage output.

The dc current through the resistors $R_e$ provides a level shift, allowing coupling of the integrator outputs directly to the inputs of other integrators. With a 10V supply voltage, the maximum peak-to-peak signal amplitude is 3V. The common-mode voltage of the integrator output is sensed and stabilized by the circuit around $Q_{9,10,11}$. The externally-supplied voltages $V_b1$ and $V_b2$ serve to adjust the common-mode level of the capacitor voltages and current scaling factor.
Figure 7.60: Integrator cell which is tunable by varying the biasing currents of the Gilbert multiplier.

For proper filtering, the operating range of the integrator, i.e. the range of frequencies within which the phase shift for sinusoidal signals is approximately 90°, should extend over a relatively large range around the roll-off frequency. The function of the CB stage is to increase the output impedance of the Gilbert multiplier so that the frequency range of the integrator is extended to lower frequencies, while small resistances are inserted in series with the capacitors C to improve the high frequency response of the integrator. Between 1MHz and 50MHz, the phase response is in the range 90°±10°.

In the experimental filter, we used metal-to-metal interconnect capacitances of about 1pF each. The (distortion free) dynamic range measured was 60dB. The measured filter characteristic is shown in Fig. 7.61 for various biasing currents resulting in roll-off frequencies ranging from 4.5–7.5MHz. More details can be found in [18] and [19].

7.11.2 Resistor-Stepwise Tunable Active Impedance Integrators

Basically, the active integrator shown in Fig.7.62 consists of one single stage \( Q_{1,2} \) and the capacitors \( C \). We found that one single stage causes no stability
Figure 7.61: Measured filter response of the experimental smoothing filter using transconductance-current multiplier-capacitor integrators for various biasing currents.

Figure 7.62: Integrator cell which is tunable by the stepwise addition of additional capacitance.

problems at high frequencies, while the loop gain was large enough for proper low-frequency behavior. The switches consisting of $Q_{9..12}$, adding additional capacitance to the integrator capacitors $C$, are activated by making $V_1$ and/or $V_2$ low. The output buffer around $Q_{3..8}$ provides a voltage level shift and a low output impedance.

The capacitors $C$ were assumed to be about 1pF. Using SPICE, we calcu-
lated that the phase response is $90^\circ \pm 10^\circ$ for frequencies between 700 KHz and 100 MHz. With a 3V peak-to-peak amplitude of the output signal, the (distortion free) dynamic range of a 5th order filter based on this type of integrator was determined to be in the order of 80 dB.

7.12 Conclusions

Many of the circuits, such as the pulse shaper, multiplexer, laser driver, pulse reshaper, demultiplexer and demodulator, are based on a modified type of emitter coupled logic (ECL), so-called MECL. Compared to standard ECL, MECL exhibits lower gate jitter and shorter settling times, so MECL is optimally suited to handle (TD)PPM pulses. Since MECL gates consist of switching differential pairs (SDPs), the switching times and propagation jitter can be determined by applying the theory developed in Chap. 5.

The PPM modulator consists of a sawtooth generator, a comparator and a pulse shaper. The distortion mainly originates from parasitic junction capacitances in parallel with the capacitor that constitutes the sawtooth generator. Further reduction of this distortion requires a larger capacitance, and hence more supply current accordingly. In the prototype system, we found that three different mechanisms contributed about the same amount of jitter to the transmitted PPM pulses. The total jitter is still below the jitter produced by the laser diode, but can, if necessary, be reduced by decreasing the modulation constant and increasing the integrator current of the sawtooth generator. To keep the PPM pulse deviation constant, enlarging the modulation constant requires the modulator input signal to be enlarged accordingly.

To obtain a small width of the transmitted (TD)PPM pulses, the laser driver is implemented as a cascade of SDPs, in which the current level is gradually increased in successive stages. To decrease the dissipation in the IC and to avoid ringing of the driver output current, two external resistors were provided.

To suppress modulation and jitter in the positions of the frame pulses, the clock regenerator PLL needs to have a small bandwidth. As the sensitivity to oscillator phase noise is inversely proportional to the bandwidth of the PLL, the small PLL bandwidth necessitates the use of a low-noise crystal oscillator. To transform the crystal oscillator signal into the appropriate frequency, a frequency multiplier, consisting of a second PLL and a regenerative oscillator, is employed. The influence of the phase noise produced by the regenerative oscillator is kept small by choosing a large bandwidth for the multiplier PLL.

Optimal sensitivity of the receiver front-end is obtained by a low-noise current amplifier. The maximum input current of the front-end was increased
significantly by bypassing large input currents directly to the output of the amplifier by means of a low-noise current switch at the amplifier input.

As the receiver front-end already provides the low-pass filtering, the pulse reshaper consists only of a comparator. Due to the large variation in the amplitude of the TDPPM pulses (up to 60dB), the threshold level of the comparator cannot be determined from the received TDPPM pulses themselves. Instead, the threshold level is adjusted using a small bandwidth feedback loop around the comparator.

In Chap.4, we explained that, as it automatically obtains proper slot synchronization, the demultiplexer is to be implemented as a counter and a decoder. Because false output pulses should be avoided, intermediate counter states should be avoided, which is accomplished by using a so-called Gray counter. An additional advantage of a Gray counter is that it implicitly performs the decoder function. To synchronize the counter such that its output pulses correspond with the appropriate TDPPM pulses (frame synchronization), an additional circuit is used to identify the frame pulses. Because it achieves a high resolution and requires simple circuitry, the identifier based on a pulse duration-to-amplitude converter and a comparator is the best alternative.

Low demodulator distortion is obtained by demodulating the PPM signals by converting the PPM pulses into pulse duration modulated (PDM) pulses. In this way high performance sample-and-hold gates and a linear sawtooth are not required.

In a full custom IC process, complete integration of the low-pass smoothing filter is feasible. Due to the inaccuracy of the integrated components, integrated filters require tuning of the roll-off frequency. Unfortunately, since only bipolar transistors are available, the tuning circuits are far from ideal and hence significantly decrease the dynamic range of the filter. Therefore, a better solution is to use stepwise tunable filters, in which switches add capacitance (or conductance) until the filter exhibits approximately the desired roll-off frequency.
Bibliography


Chapter 8

Conclusions

In the lower levels of communication networks and in relatively short point-to-point links, the decision to use optical fiber, which has important advantages over the traditional copper cable, mainly depends on the cost of the electro-optic and opto-electric transmitters and receivers required. An effective strategy for minimizing these costs, is to make use of low-cost optical components such as LEDs or multimode semiconductor lasers (for example the lasers developed for CD players) and PIN photodiodes, and to integrate the complete electronic circuitry necessary for coding and multiplexing into an IC. As ICs are cost effective only in sufficiently large production volumes, the ICs should preferably be applicable in different application fields. Therefore they should be capable of handling different types of signal, for example computer data and digitized or analog audio and video. In addition, in order to reduce the cost per channel, they should facilitate real time multiplexing of two or more signals into one optical fiber.

Because low-cost LEDs or lasers exhibit considerable nonlinearity for which is difficult to compensate, the channel coding should be based on either multi-level amplitude modulation, in which the digitized data is represented by a limited number of discrete amplitude levels, or on phase modulation. In addition, as non-linearity introduces intermodulation products which may easily result in crosstalk between the transmitted signals, time division multiplexing is preferred to frequency division multiplexing. Regarding the constraint of low-cost, analog coding is more appropriate than digital coding because it does not require analog-to-digital and digital-to-analog converters, which are cost enhancing in the case of wideband high-SNR signals. Additionally, by using analog coding, the required silicon area and power consumption will be significantly reduced. The proposed time division multiplex pulse position modulation (TDPPM) is the most suitable coding scheme because it combines

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the required features of analog coding, phase modulation and time division multiplexing with the possibility of integrating the complete electronic circuitry in an IC.

Because of reasons emerging from implementation considerations (under the constraint of low-cost), we decided that the transmitted TDPPM pulses should have the shape of trapezoidals and the pulse receiver should be based on a simple low-pass filter rather than a matched filter. Compared to a system elaborating optimally shaped TDPPM pulses and matched filters, depending on the received optical power, we calculated the impairment of the SNR to be in the range $3-13$dB.

In the TDPPM system, the maximum of $B_{tot}$, being the product of the number of multiplexed signals and the bandwidth of the signals to be transmitted, depends on the maximal repetition frequency of the TDPPM pulses. When using special types of LEDs, which achieve a modulation bandwidth of about $200$MHz, $B_{tot}$ is limited to about $30$MHz, while the low-cost laser diodes allow $B_{tot}$ to be in the order of $150$MHz. To exploit the potentially available $B_{tot}$, we have to use an IC process exhibiting a sufficiently high transit frequency. For example, if $B_{tot}$ is to be $150$MHz, the transit frequency should be $10$GHz or higher. We should bear in mind that to achieve useful values of the SNR, the actual value of $B_{tot}$ should be chosen to be at least 20% lower than its maximum.

Since optical isolators, avoiding the generation of excess noise by the laser diode, are not compatible with low cost, the noise contributed by the laser diode is one of the factors that may determine the maximum SNR if the received optical power is relatively high. Another factor, which becomes increasingly important if the system is designed for higher values of $B_{tot}$, is the noise produced by the oscillators that provide the clock signals for the PPM modulators and demodulators. In the prototype of the system, thanks to a technique based on a crystal oscillator and a PLL frequency multiplier, we succeeded in keeping the contributions of the oscillator noise below that of the laser diode. If the received optical power is relatively low (in the prototype of the system when the optical losses exceed 15dB), the SNR depends on the noise produced by the receiver front-end.

As the major design objective has been low cost, we preferred simple circuits over theoretically optimal circuits. Two previously mentioned consequences of this decision are the trapezoidal shape of the TDPPM pulses and the absence of a matched filter. Another consequence is that the timing of the sampling by the PPM modulator has become signal dependent, resulting in so-called sample distortion. However, in multi-channel systems, the maximal time displacement of the samples is small enough to keep the distortion products below the noise level. In addition, to relax the requirements to the
selectivity of the smoothing filter, rather than unity, the oversampling factor has been chosen to be in the order of 1.75. This results in a decreased pulse deviation, and hence a lower SNR. Finally, the appropriate threshold level of the receiver pulse reshapera is not, as would be optimal, determined from the received TDPPM pulses themselves but from the pulse reshape output pulses. As a result, to achieve the maximal possible SNR, it might be necessary to make a manual adjustment, but the adjustment is not very critical.

To make the system minimally susceptible to undesired coupling between different circuits, we preferred balanced circuits and signals. Many of the circuits handling the TDPPM pulses (such as the modulator, the multiplexer, the laser driver, the pulse reshapera, the demultiplexer and the demodulator) are based on the so-called switching differential pair (SDP). The switching speed of the SDP was maximized by proper dimensioning of its load resistors and by inserting common collector buffers at its inputs. The settling times of the SDP stages were minimized by systematically avoiding the presence of capacitive loads to the emitter terminals of the constituent transistors. To assure proper handling of a large range of possible received power levels, the receiver front-end was equipped with a low-noise bypass switch.

We determined the noise production of the critical circuits as a function of the supply current, so, for a given specification of the SNR, we are able to minimize the total supply current.

We realized a four-channel prototype of the system using an external 1300nm multimode laser and PIN photodiode, both mounted in optical connectors, and a 2.5GHz semicustom bipolar IC process. In designing the prototype, the aim has been that it should be capable of handling standard PAL-coded video signals. The bandwidth is 5MHz per channel. The SNR (for optical losses up to 15dB) and the differential gain of the complete system, measured by a video measurement system, amount to 58dB (weighted) and 1% respectively. Crosstalk was not observed. The threshold power was estimated to be in the order of -30dBm.

The distortion produced by the electronic circuits was measured to be considerably less than the distortion produced by the complete system, including the optical components. In addition, the nature of the distortion corresponds well to distortions caused by reflections. We calculated that the influence of the (twice reflected) triple transit pulses is negligible, leading to the conclusion that the distortion is caused by the double transit pulses, which interfere with the lasing process within the laser cavity. Obviously, if (low-cost) optical isolators were available, the maximal SNR as well as the linearity can be improved.

Finally, this thesis provides some generally applicable secondary results: (1) we presented a general model describing the noise behavior of a cascade of
limiters, (2) we discussed how the bandwidth of wideband negative-feedback amplifiers depends on the phase shift caused by bipolar transistors, (3) we determined the fundamental minimum of the equivalent input noise of current-input amplifiers having capacitive source impedances and (4) we proposed stepwise tuning of the bandwidth of integrated continuous time filters implemented in bipolar technologies, as a technique to achieve a maximal dynamic range.
## Appendix A

### List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_f$</td>
<td>Fiber bandwidth</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$q$</td>
<td>Elementary charge</td>
<td>$C$</td>
</tr>
<tr>
<td>$B_l$</td>
<td>Laser bandwidth</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$I_{th}$</td>
<td>Threshold current of laser diode</td>
<td>$A$</td>
</tr>
<tr>
<td>$T_{on,off}$</td>
<td>Turn on,off time of laser diode</td>
<td>$s$</td>
</tr>
<tr>
<td>$\tau_d$</td>
<td>Laser ringing damping time constant</td>
<td>$s$</td>
</tr>
<tr>
<td>RIN</td>
<td>Laser relative intensity noise</td>
<td>$s$</td>
</tr>
<tr>
<td>$T_{sl}$</td>
<td>Length of time slot available for 1 TDPPM pulse</td>
<td>$s$</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Sampling frequency</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Time between two samples of one signal ($\frac{1}{f_s}$)</td>
<td>$s$</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Total width of TDPPM pulse</td>
<td>$s$</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Energy of TDPPM pulse</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$E_{p'}$</td>
<td>Energy of differentiated TDPPM pulse</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$T_{r,f}$</td>
<td>Rise, fall time of TDPPM pulses</td>
<td>$s$</td>
</tr>
<tr>
<td>$B_p$</td>
<td>&quot;Bandwidth&quot; of TDPPM pulse</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$t_d$</td>
<td>Decision time of TDPPM pulse reshaper</td>
<td>$s$</td>
</tr>
<tr>
<td>$\sigma_{td}$</td>
<td>Root mean square value of the jitter in $t_d$</td>
<td>$s$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Power density of &quot;white noise&quot;</td>
<td>$s$</td>
</tr>
<tr>
<td>$S_o$</td>
<td>Proportionality factor of frequency-proportional noise</td>
<td>$s^2$</td>
</tr>
<tr>
<td>$T_A$</td>
<td>Amplitude of TDPPM pulse deviation</td>
<td>$s$</td>
</tr>
<tr>
<td>$m$</td>
<td>Modulation index:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ratio between the peak-to-peak pulse deviation and $T_s$</td>
<td></td>
</tr>
<tr>
<td>$\xi$</td>
<td>Modulation constant</td>
<td>$sV^{-1}$</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of multiplexed signals</td>
<td></td>
</tr>
<tr>
<td>$B_r$</td>
<td>Bandwidth of receiver front-end</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$A_p$</td>
<td>Peak-to-peak amplitude of TDPPM pulses</td>
<td></td>
</tr>
</tbody>
</table>

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APPENDIX A. LIST OF SYMBOLS

- $\text{SNR}_o$: Signal-to-noise ratio at output of receiver smoothing filter
- $\text{SNR}_d$: Signal-to-noise ratio at demodulator output
- $\text{SNR}_r$: Signal-to-noise ratio at receiver input
- $T_{fp}$: Average time between two false TDPPM pulses
- $B_i$: Bandwidth of transmitted signals (equals bandwidth of smoothing filter)
- $B_{tot}$: Total signal bandwidth of TDPPM system $NB_i$
- $B_{tot,max}$: Maximal available $B_{tot}$
- $k$: $\frac{B_{tot}}{B_{tot,max}}$
- $r$: Oversampling factor $\frac{f_t}{2B_i}$
- DG: Differential gain
- $d_{tot}$: Relative amplitude of distortion
- $T_{sw}$: Switching time
- $\beta_{fe}$: Current scale factor of successive SDP stages
- $\tau_f$: Forward transit time of bipolar transistors
- $f_t$: Transit frequency of bipolar transistors ($\approx \frac{1}{2\pi\tau_f}$)
- $g_m$: Transconductance
- $k$: Boltzmann constant $\approx 1.4 \cdot 10^{-23}$
- $T$: Absolute temperature (assumed 300$K$)
- $T_{settling}$: Settling time of pulse circuits
- $B_n$: Noise bandwidth
- $T_{ap}$: Aperture time of limiter stage
- $L(f)$: Relative oscillator phase noise power density per Hz bandwidth at distance $f$ from the carrier frequency
- $S_\Phi(f)$: Power density of oscillator phase fluctuations
- $S_t(f)$: Mean square fractional frequency fluctuation density
- $n_{fold}$: Effective number of sidebands contributing to the phase noise of oscillators
- $R_v$: Resistance representing the total equivalent voltage noise $\Omega$
- $R_i$: Resistance representing the total equivalent current noise $\Omega$
- $Q$: Quality factor of a resonator
- $B_{PLL}$: Bandwidth of PLL
Appendix B

Laser Model

B.1 Rate Equations

The dynamic behavior of laser diodes can be calculated using a model based on rate equations [1] and [3]:

\[
\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_n} - F(N)S \quad (B.1)
\]

\[
\frac{dS}{dt} = \frac{N}{\tau_n} - \frac{S}{\tau_p} + F(N)S \quad (B.2)
\]

\[
F(N) = \frac{1}{\tau_p} + G(N - N_0) \quad (B.3)
\]

Eq.(B.1) describes the increment of free electrons (population N) in the laser cavity per unit of time. Electrons, having the elementary charge q, are supplied by the injection current I, and disappear due to (1) spontaneous recombination (average lifetime \(\tau_n\)) and (2) stimulated emission (last term).

Eq.(B.2) describes the increment of photons (population S) in the cavity per unit of time. New photons are supplied by (1) spontaneous emission, whereby a fraction \(\alpha\) of the recombining free electrons contributes effectively to the lasing mode, and (2) stimulated emission (last term). Practical values of \(\alpha\) are in the range 1 \(\cdot\) 10\(^{-5}\) to 1 \(\cdot\) 10\(^{-2}\), so in the lasing mode the contribution of the spontaneous emission will be small compared to that of the stimulated emission. Photons disappear mainly because of the imperfect cavity-air mirrors, their average lifetime being \(\tau_p\).

Eq.(B.3) gives the proportionality constant determining the net rate of stimulated emission, which equals \(F(N)S\). This rate is directly proportional to the photon population \(S\) and becomes effective if the electron population \(N\) exceeds the transparency population \(N_0\), i.e., if the stimulated emission is just capable of compensating for the cavity losses.
B.2 Small Signal Behavior

To derive the small signal behavior, Eqs.(B.1 and B.2) are linearized by assuming small deviations $\Delta N$, $\Delta S$ and $\Delta I$ around their bias values $N$, $S$ and $I$. The linearized differential equations are:

\[
\frac{d\Delta N}{dt} = \frac{\Delta I}{q} - \left(\frac{1}{\tau_n} + GS\right)\Delta N - \left(\frac{1}{\tau_p} + G(N - N_0)\right)\Delta S \quad (B.4)
\]

\[
\frac{d\Delta S}{dt} = \left(\frac{\alpha}{\tau_n} + GS\right)\Delta N + G(N - N_0)\Delta S \quad (B.5)
\]

Eqs.(B.4 and B.5) describe a second order system. Its oscillation frequency $\omega_1$ can be determined by ignoring the damping terms and substituting the differential operator by the Laplace operator $s = j\omega$:

\[
s\Delta N = \frac{\Delta I}{q} - \left(\frac{1}{\tau_p} + G(N - N_0)\right)\Delta S \quad (B.6)
\]

\[
s\Delta S = \left(\frac{\alpha}{\tau_n} + GS\right)\Delta N \quad (B.7)
\]

To determine the magnitude of the term within the square brackets of Eq.(B.6), we assume $S$ constant, so the left hand term of Eq.(B.2) yields zero. As the spontaneous emission, equaling $\frac{N}{\tau_s}$, is small compared to the stimulated emission given by $F(N)S$, we see from Eq.(B.2) that $F(N)S$ is approximately equal to $\frac{1}{\tau_p}$. Consequently, the term within the square brackets of Eq.(B.6) which, according to Eq.(B.3) equals $F(N)$, is approximated by $\frac{1}{\tau_p}$. Using Eq.(B.13), which is defined later on, the term $GS$ of Eq.(B.7) can be replaced by $-\frac{\alpha N_0}{\tau_s(N - N_0)}$. Because, in the lasing mode, $N$ is close to $N_0$, the term $\frac{\alpha}{\tau_n}$ can be ignored. Calculation of the oscillation frequency from Eqs.(B.6) and (B.7) yields:

\[
\omega_1 = \sqrt{\frac{SG}{\tau_p}} \quad (B.8)
\]

The damping time constant $\tau_d$ is calculated by only considering the damping terms of Eqs.(B.4 and B.5):

\[
s\Delta N = -\left(\frac{1}{\tau_n} + GS\right)\Delta N \quad (B.9)
\]

\[
s\Delta S = G(N - N_0)\Delta S \quad (B.10)
\]

For sufficiently large $S$, i.e., if the laser operates well above threshold, the term $GS$ of Eq.(B.9) is large compared to other terms, so the damping time constant is given by:

\[
\tau_d = \frac{1}{GS} \quad (B.11)
\]
To express the bias values \( N \) and \( S \) as functions of the laser drive current \( I \), its threshold current \( I_{th} \) and the time constants \( \tau_n \) and \( \tau_p \), we require \( N \) and \( S \) to be constant, by setting Eqs.(B.1 and B.2) to zero:

\[
\frac{I}{q} - \frac{N}{\tau_n} - \frac{S}{\tau_p} - G(N - N_0)S = 0 \quad (B.12)
\]

\[
\frac{\alpha N}{\tau_n} + G(N - N_0)S = 0 \quad (B.13)
\]

Elimination of the term \( G(N - N_0)S \) by summing Eq.(B.13) and Eq.(B.12) yields:

\[
\frac{I}{q} = \frac{N}{\tau_n} (1 - \alpha) + \frac{S}{\tau_p} \quad (B.14)
\]

If the laser operates sufficiently high above threshold, \( S \) will be relatively large, so Eq.(B.13) demonstrates that \( N \) must be close to \( N_0 \). Now, Eq.(B.14) can be rewritten as:

\[
\frac{S}{\tau_p} = \frac{I}{q} - \frac{N_0}{\tau_n} (1 - \alpha) \quad (B.15)
\]

Obviously, in the stationary case, i.e. for low modulation frequencies, the photon population \( S \) has become directly proportional to the injection current \( I \). The offset term \( \frac{N_0}{\tau_n} (1 - \alpha) \) denotes the threshold current, normalized with respect to \( q \). Hence, we may write:

\[
S = \frac{I - I_{th}}{q} \tau_p \quad (B.16)
\]

in which, provided that \( \alpha \ll 1 \), the offset current can be approximated as:

\[
I_{th} \approx q \frac{N_0}{\tau_n} \quad (B.17)
\]

In addition, we argued that \( N \) is close to \( N_0 \), so we find:

\[
N = \frac{I_{th} \tau_n}{q} \quad (B.18)
\]

Having determined \( S \) as an explicit function of \( I \) and \( I_{th} \) we rewrite Eq.(B.8) as:

\[
\omega_1 = \sqrt{\frac{G(I - I_{th})}{q}} \quad (B.19)
\]

or, by eliminating \( q \) (Eq.(B.17)), substituting \( N_0 \) by \( V n_0 \), in which \( n_0 \) is the transparency density and \( V \) is the cavity volume, and substituting \( G \) by \( \xi \), in
which \( g \) is the average of the net increase of photons per unit of time per unit of photon density:

\[
\omega_1 = \sqrt{\frac{gn_0 (I - I_{th})}{\tau_n I_{th}}}
\]  \hspace{1cm} (B.20)

We concluded earlier that, for low frequencies, \( S \) is directly proportional to \( I - I_{th} \). Obviously, the laser behaves as a low-pass filter whose roll-off frequency is determined by \( \omega_1 \) and the damping time constant is \( \tau_d \). For simplicity, we will assume that the modulation bandwidth \( B \) equals \( \frac{\omega_1}{2\pi} \). The time constant \( \tau_d \) can be written as a function of \( \omega_1 \) by combining Eqs. (B.11) and (B.8):

\[
\tau_d = \frac{1}{\omega_1^2 \tau_p}
\]  \hspace{1cm} (B.21)

For a practical laser at room temperature, typical values are \( g = 1 \cdot 10^{-12} \text{m}^3 \text{s}^{-1} \), \( n_0 = 1.7 \cdot 10^{24} \text{m}^{-3} \), \( \tau_n = 2.5 \text{ns} \) and \( \tau_p = 1 \text{ps} \). Assuming a laser volume \( V = 2 \cdot 10^{-16} \text{m}^3 \) and applying Eq. (B.17), while putting \( N_0 = n_0 V \) and \( q = 1.6 \cdot 10^{-19} \), it follows that \( I_{th} = 22 \text{mA} \).

For bias currents \( I \) close to \( I_{th} \), our approximations are no longer valid. Numerical calculations have shown that the bandwidth gradually decreases to the "LED bandwidth" \( \frac{1}{\tau_n} \).

### B.3 Laser Noise

Based on rate equations, we will derive a simple estimation of the laser relative intensity noise (RIN). The major objective is to calculate the order of magnitude of the fundamental limit to the RIN, and to clarify how the RIN depends on the driving current of the laser.

To determine the shot noise originating from stimulated emission, we assume that the injection current in excess of \( I_{th} \) completely contributes to stimulated emission, resulting in the net rate of stimulated emission:

\[
R_{st} \approx \frac{I - I_{th}}{q} = \frac{I - I_{th}}{I_{th}} \frac{n_0 V}{\tau_n}
\]  \hspace{1cm} (B.22)

As the shot noise depends on the actual rate of released and absorbed photons or free carriers, which is in practice a factor of \( \gamma \approx 3 \) larger than the net rate, the power density of shot noise is given by:

\[
S_{st} = 2\gamma R_{st}
\]  \hspace{1cm} (B.23)

Since the release of one photon requires recombination of one free carrier, the influence of the shot noise on the photon population is determined by
adding the noise source $X_n(t)$ to Eq.(B.1) and a source $-X_n(t)$ to Eq.(B.2). Solving Eqs.(B.1) and (B.2) for low frequencies, i.e. setting $\frac{dN}{dt} = 0$ and $\frac{dS}{dt} = 0$ yields:

$$S \approx \tau_p \left( \frac{I}{q} - \frac{N_0}{\tau_n} \right) + \tau_p \frac{N_0 - N}{\alpha N} X_n(t)$$  \hspace{1cm} (B.24)

in which the first right hand term is the total average photon population and the second is the noise. By expressing $N$ and $N_0$ in $I$, $I_{th}$, $n_0$ and $g$ (use Eqs.(B.13), (B.16) and (B.17)), we obtain:

$$S \approx \tau_p \frac{I - I_{th}}{q} + \frac{I_{th}}{(I - I_{th}) gn_0} X_n(t)$$  \hspace{1cm} (B.25)

Finally, as the RIN is defined as:

$$\text{RIN} = \frac{|\frac{dS}{dX_n}|^2 S_{st}}{S_{tot}}$$  \hspace{1cm} (B.26)

we obtain:

$$\text{RIN} = \frac{2q}{\gamma I_{th} (gn_0 \tau_p)^2} \left( \frac{I_{th}}{I - I_{th}} \right)^3$$  \hspace{1cm} (B.27)

Obviously, the RIN can be improved by increasing the driving current.
Bibliography


Appendix C

Distortion and Noise of the Modulator

In this appendix, we calculate the distortion and noise (resulting in pulse jitter) produced by the PPM modulator whose block diagram was presented in Fig.7.11a. Subsequently, we will concentrate on the contributions of the sawtooth generator and the comparator.

C.1 Sawtooth Generator

A first cause for sawtooth non-linearity, and hence modulator distortion, is the Early effect of the transistor that supplies the integrator current to the sawtooth generator. Second and third causes are the presence of junction capacitances in parallel with the actual integrator capacitor and the loading of the integrator output by the comparator input circuit.

Subsequently, we will determine the contributions of the noise originating from the integrator current source and the integrator reset switch.

C.1.1 Distortion Caused by the Early Effect

The influence of the Early voltage of the bipolar transistors on the non-linearity of the sawtooth, and hence on the signal distortion, is determined by using the model depicted in Fig.C.1. In correspondence with the circuit of the sawtooth generator presented in Fig.7.13, \( Q_2 \) is the transistor which supplies the integrator current to the integrator capacitance \( C \). At the start of a new sawtooth period (then, we assume \( t=0 \)), the switch has just opened so the voltage across the capacitor \( V_c \) equals the initial voltage \( V_o \). After \( t=0 \), the integrator voltage decreases until the start of a new sawtooth period (at \( t = T_s \), the sample pe-
The comparator detects that the momentary sawtooth voltage is equal to the input voltage at the pulse initiation time $t_{PI}$.

In approximation, the collector current of $Q_2$ (i.e., the integrator current $I$) depends on the base-collector voltage $V_{bc}$ and the so-called forward Early voltage $V_{eaf}$ according to:

$$I \approx I_o(1 - \frac{V_{bc}}{V_{eaf}}) = I_o(1 - \frac{V_c - V_{bco}}{V_{eaf}} \frac{1}{H_1})$$

(C.1)

in which $I_o$ is collector current if $V_{bc}=0$, $V_c$ is the capacitor voltage, $V_{bco}$ is the base-collector voltage if $V_c=0$ and $H_1$ is the low frequency loopgain of the series stage around $Q_2$:

$$H_1 = \frac{1}{\frac{1}{\beta} + \frac{V_t}{V_{Re}}}$$

(C.2)

In this formula, $\beta$ denotes the low-frequency current gain of the transistor, $V_t$ is the thermal voltage $kT/q$ and $V_{Re}$ is the dc voltage across $R_e$.

The capacitor voltage $V_c$ is calculated from the differential equation:

$$\frac{dV_c}{dt} = \frac{-I}{C}$$

(C.3)

$V_c(t = 0) = V_o$ (C.4)

Substitution of Eq.(C.1) into Eq.(C.3) and solving yields (for $0 \leq t < T_s$):

$$V_c(t) = (V_o - V_{bco} + H_1 V_{eaf}) \exp\left(-\frac{t I_o}{C V_{eaf} H_1}\right) + V_{bco} - H_1 V_{eaf}$$

(C.5)

At the instant $V_c$ equals the input voltage $V_i$, at time $t_{PI}$, a PPM pulse is transmitted. Solving $t_{PI}$ and expanding the equation into a Taylor polynomial while putting $\xi_o = \frac{I_o}{V_c}$ results in:

$$t_{PI} \approx \xi_o H_1 V_{eaf} \left[ \ln\left(\frac{H_1 V_{eaf} - V_{bco} + V_o}{H_1 V_{eaf} - V_{bco}}\right) - \frac{1}{H_1 V_{eaf}} V_i \right.$$

$$+ \frac{1}{2 H_1^2 V_{eaf}^2} V_i^2 - \frac{1}{3 H_1^3 V_{eaf}^3} V_i^3 + ... \right]$$

(C.6)
in which it has been assumed that \( H_1 V_{\text{ef}} \gg |V_{\text{bco}}| \) and \( H_1 V_{\text{ef}} \gg |V_o| \). The negative signs in the odd terms stem from the fact that a positive input voltage results in a negative pulse deviation.

So far, we assumed that the sawtooth period starts at \( t=0 \), but in fact a new sawtooth starts at the beginning of each frame. Therefore, the calculated \( t_{\text{PI}} \) can be regarded as the deviation of the PPM pulses relative to their zero positions.

The first-order term (in \( V_i \)) of the polynomial given by Eq.(C.6) is the intended pulse deviation, the higher-order terms describe the distortion due to sawtooth non-linearity.

We denote the coefficients of the polynomial by \( k_1, k_2, \ldots \). The magnitudes of \( V_i \) for which the second and third order harmonic distortion products would have the same magnitude as \( V_i \) (the so-called second and third order intercept voltages) are:

\[
V_{\text{int},2} = \left| \frac{k_1}{k_2} \right| = 2H_1 V_{\text{ef}} = \frac{2V_{\text{ef}}}{\frac{1}{\beta} + \frac{V_i}{V_{R_e}}} \tag{C.7}
\]

\[
V_{\text{int},3} = \sqrt{\left| \frac{k_1}{k_3} \right|} = \sqrt{3}H_1 V_{\text{ef}} = \frac{\sqrt{3}V_{\text{ef}}}{\frac{1}{\beta} + \frac{V_i}{V_{R_e}}} \tag{C.8}
\]

So, the distortion caused by the Early effect can be made small by choosing a sufficiently large ratio \( \frac{V_{R_e}}{V_i} \).

To estimate practical values, we take the parameters of the IC process used for our prototype system: \( V_{\text{ef}}=20V \), \( \beta=100 \) and \( \frac{V_{R_e}}{V_i} \approx 100 \), yielding values of \( V_{\text{int},2,3} \) larger than 1000V. As the amplitude of the input signal \( V_i \) in the prototype system is in the order of 0.1V, the influence of the Early effect can generally be ignored.

### C.1.2 Distortion Caused by Junction Capacitances

A second cause for sawtooth non-linearity is the non-linearity of the integrating capacitor. If we use, for example, the capacitance from the first interconnect to a shallow N or P diffusion or to the second interconnect, the capacitance itself will be approximately linear. However, in parallel with \( C \) are the non-linear junction capacitances of Q2,3, see Fig.7.13. Since the reverse voltages across these junction capacitances vary in the same direction, they do not compensate for each other. Therefore, we assume the total capacitance \( C \) to be the sum of the linear capacitor \( C_{\text{lin}} \) and one single voltage dependent junction capacitance:

\[
C = C_{\text{lin}} + \frac{C_{J_n}}{(1 + \frac{V_{V_r} - V_{V_n}}{V_{J_n}})^p} \tag{C.9}
\]
\( V_c \) is the \textit{signal} voltage across the capacitor \( C \), \( V_{co} \) is the junction reverse \textit{bias} voltage, \( V_{jo} \) is the build-in voltage of the junction capacitor and \( C_{j0} \) is its zero voltage capacitance. The grading coefficient \( p \) is in the range 0.33–0.5.

In correspondence with the assumptions made in the preceding section, it is assumed that the initial voltage of the capacitor \( C \) is \( V_o \) and that \( C \) is charged with a constant current \(-I\). Unfortunately, this time the capacitor voltage cannot be written as an explicit function of time. However, we do know the modulation constant \( \xi \), which is the inverse of the slope magnitude of the capacitor voltage at time \( t_{PI} \) (then \( V_c \) equals \( V_i \)):

\[
\xi = \frac{d t_{PI}}{d V_i} = \frac{C(V_i)}{I} \quad \text{(C.10)}
\]

To determine a closed expression for \( t_{PI} \), we make use of the equality:

\[
t_{PI} = \int_0^{V_i} \frac{d t_{PI}}{d V_i} d V_i + \text{constant} = \int_0^{V_i} \xi(V_i) d V_i + \text{constant} \quad \text{(C.11)}
\]

Substitution of Eq.(C.9) into Eq.(C.10) and expanding the equation into a Taylor polynomial, while putting \( \xi_o = \frac{C_{j0}}{C_{lin}} \), yields:

\[
\xi(V_i) = \xi_o \left[ 1 + \frac{C_{j0}}{C_{lin}} \left( \frac{V_{jo} - V_{co}}{V_{jo}} \right)^{-p} \left( 1 + \frac{p}{V_{co} - V_{jo}} V_i + \frac{p(p+1)}{2(V_{co} - V_{jo})^2} V_i^2 + \ldots \right) \right] \quad \text{(C.12)}
\]

After substitution of \( \xi(V_i) \) into Eq.(C.11) we obtain:

\[
t_{PI} = \text{constant} + \xi_o \left[ 1 + \left( \frac{V_{jo} - V_{co}}{V_{jo}} \right)^{-p} \right] + \xi_o \left( \frac{V_{jo} - V_{co}}{V_{jo}} \right)^{-p} \left[ \frac{p}{2(V_{co} - V_{jo})} V_i^2 + \frac{p(p+1)}{6(V_{co} - V_{jo})^2} V_i^3 + \ldots \right] \quad \text{(C.13)}
\]

The second and third order intercept voltages are:

\[
V_{int,2} = \left| k_{12} \right| = 2 \frac{V_{jo} - V_{co}}{p} \left[ 1 + \frac{C_{j0}}{C_{lin}} \left( \frac{V_{jo} - V_{co}}{V_{jo}} \right)^p \right] \quad \text{(C.14)}
\]

\[
V_{int,3} = \sqrt[3]{|k_{13}|} = \sqrt[3]{\frac{6}{p(p+1)}} \left[ 1 + \frac{C_{j0}}{C_{lin}} \left( \frac{V_{jo} - V_{co}}{V_{jo}} \right)^p \right] (V_{jo} - V_{co}) \quad \text{(C.15)}
\]

From Eqs.(C.14) and (C.15) it is clear that, within the voltage range available, we should make the reverse biasing voltage \( V_{co} \) across the junction capacitors as large as possible. Additionally, \( C_{lin} \) should be large compared to \( C_{j0} \).

Assuming \( \frac{C_{j0}}{C_{lin}} = 10 \), \( V_{jo} = 0.8\, \text{V} \), \( V_{co} = -1\, \text{V} \) and \( p = 0.5 \), we find \( V_{int,2} = 115\, \text{V} \) and \( V_{int,3} = 20\, \text{V} \). Obviously, the distortions caused by junction capacitances are an order of magnitude larger than the distortions caused by the Early effect. If the amplitude of \( V_i \) is in the order of 0.1\, \text{V}, the distortion level is about 0.1\%.
C.1.3 Distortion Caused by Loading

The sawtooth output is loaded by the input of the comparator. To avoid large inaccuracies in the slope magnitude, the comparator input current should be an order of magnitude lower than the charge current of the integrator capacitor. If necessary, the comparator input current can be decreased by using a CC buffer stage, biased by a sufficiently low emitter current. A disadvantage of this buffer is that it produces additional noise at the comparator input (the comparator noise will be discussed in App.C.2).

To avoid distortion, the comparator input current should remain constant. Consequently, the first stage of the comparator as well as any buffer stages must be biased by active current sources rather than by simple resistors.

In principle, a small voltage dependence of the comparator input current is caused by the Early effect of the buffer or comparator input transistors. If active current sources are being used for biasing these stages, it can be shown that the influences are comparable with influences of the Early effect caused by the current source driving the integrator (Sec.C.1.1).

C.1.4 Noise Induced by the Current Source

We assume the current noise power density present in the integrator current to be:

\[ S_n = \frac{4kT}{R_i} \]  \hspace{1cm} (C.16)

in which \( R_i \) is the equivalent noise resistance. Between the start of a new period of the sawtooth at \( t_0 \) and the instant the PPM pulse is transmitted at \( t_{PI} \), the current noise is integrated by the capacitor \( C \), resulting in a mean square noise voltage (at time \( t_{PI} \)):

\[ V_n^2 = \frac{4kT}{R_i} \frac{1}{C^2} (t_{PI} - t_0) \]  \hspace{1cm} (C.17)

By putting \( \xi = \frac{C}{I} \) we find:

\[ \sigma_{ID}^2 = \xi^2 V_n^2 = \frac{4kT}{I^2 R_i} (t_{PI} - t_0) \]  \hspace{1cm} (C.18)

Obviously, this contribution can be kept small by making the signal-to-noise ratio of integrator current large (requiring a large integrator capacitor \( C \) as well) and minimizing the integration time. Substituting the parameters of the prototype system: \( I = 0.1 \text{mA}, \ R_i = 1 \text{k}\Omega \) and \( t_{PI} - t_0 = 20 \text{ns} \), we find \( \sigma_{ID} = 5 \text{ps} \).
C.1.5 Noise Induced by the Reset Switch

After the initial integrator voltage has been restored during a reset, $Q_3$ (see Fig. 7.13) and the resistor $R$ in series with its base to the supply voltage will induce noise on the integrator capacitor. As soon as $Q_3$ is released from its reset state, this noise voltage is "frozen" (i.e. sampled) on the integrator capacitor.

To estimate its magnitude, we use the model of Fig. C.2, where $V_n$ is the equivalent noise voltage at the input of the switch, $R'_b$ is the total resistance in series with the base terminal of $Q_3$ consisting of its intrinsic base resistance and the external resistance $R$, and $C$ is the integrator capacitor. $\beta$ denotes the low-frequency current gain, $g_m = \frac{qI}{kT}$ is the transconductance and $\tau_f$ is the transit time of $Q_3$. As the influence of the junction capacitances is relatively small and since we are interested in a rough approximation, junction capacitances are ignored.

By assuming $\beta \gg 1$ and $\beta \gg R'_b g_m$, the small signal transfer from $V_n$ to $V_c$ equals ($s$ denotes the Laplacian operator $2\pi j f$):

$$\frac{V_C}{V_n} = \frac{\frac{\tau_f s + 1}{CR'_b \tau_f}}{s^2 + s \frac{C g_m \tau_f}{CR'_b g_m \tau_f} + \frac{1}{CR'_b \tau_f}} \quad \text{(C.19)}$$

which describes a second order low-pass filtering. Because of its high frequency, the zero can be ignored. The small-signal bandwidth is determined by two complex poles if:

$$R'_b > \frac{(C + g_m \tau_f)^2}{4C g_m^2 \tau_f} \quad \text{(C.20)}$$
which, in practical circuits, is generally the case. By assuming the noise bandwidth equal to the roll-off frequency we find:

$$B_n \approx \frac{1}{2\pi} \sqrt{\frac{1}{C R_b' \tau_f}}$$

(C.21)

The voltage noise spectral power density equals:

$$S_n = 4kT(R_b' + \frac{kT}{2qI})$$

(C.22)

So the sampled noise voltage becomes:

$$V_n^2 = B_n S_n = \frac{2kT}{\pi}(R_b' + \frac{kT}{2qI}) \sqrt{\frac{1}{C R_b' \tau_f}}$$

(C.23)

and the resulting jitter:

$$\sigma_{id}^2 = \xi^2 V_n^2 = \xi^2 \frac{2kT}{\pi}(R_b' + \frac{kT}{2qI}) \sqrt{\frac{1}{C R_b' \tau_f}}$$

(C.24)

Substituting $$\xi=60\text{ns}/V$$, $$I=0.1\text{mA}$$, $$R_b'=700\Omega$$ and $$\tau_f=50\text{ps}$$ we find $$\sigma_{id}=4\text{ps}$$. So, the contributions of the integrator current source and the reset switch to noise in the sawtooth output signal are of the same order of magnitude, 5 and 4ps respectively.

### C.2 Comparator

Besides the sawtooth generator, modulator distortion and modulator noise may result from the comparator as well. We will successively deal with the distortion caused by modulation of the common-mode level and the slope magnitude of the comparator input signal and the jitter produced by the comparator itself.

#### C.2.1 Distortion Caused by Modulation of the Common-Mode Level

As we preferred the modulator configuration of Fig.7.11a, when the comparator switches, the common-mode level of the comparator input signal just equals the input voltage $$V_i$$. Consequently, the first possible cause for comparator induced distortion is the dependence of the propagation delay of the comparator on this common-mode level.

As only the first comparator stage is influenced by the input common-mode level, we need only consider the delay of the first stage, consisting of
the transistors $Q_{0..4}$, see Fig.7.14. Since the slope magnitude of the sawtooth is relatively small, the first stage exhibits a large aperture time and can thus be regarded as a linear amplifier. As the bandwidth of the CC buffer around $Q_{0..2}$ is relatively large, the delay time mainly depends on the time constant of the $RC$ network at the input of the CE stage (Sec.5.3, Fig.5.15). This time constant is given by (see also Eq.(5.42)):

$$\tau = 2R_b C_{bb} = 2R_b \left[ \tau_f g_m + \frac{C_{je}}{2} \left( 1 + 2R'_c g_m \right) + \frac{C_{jc}}{2} \right] \quad (C.25)$$

$C_{bb}$ denotes the effective input capacitance of the CE stage, $R_b$ is the base bulk resistance, $R'_c$ is the total collector resistance (intrinsic collector resistance and external load resistor), $C_{je}$ is the base-emitter junction capacitance, $C_{jc}$ is the base-collector junction capacitance, $\tau_f$ is the transit time of the transistors and $g_m$ is the transconductance of the CE stage.

Three mechanisms are responsible for variation of this time constant:

1. **Modulation of the transconductance**

   If the CE stage consisting of $Q_{3,4}$ were biased by a resistor, its tail current $I_t$ and hence the transconductance $g_m$ depend on the common-mode level. Provided that the voltage across the biasing resistor is large compared to the thermal voltage $kT/q$ and if $\tau_f$ were constant, $\tau$ would vary linearly with the common-mode voltage. This linear modulation would effectively increase the modulation constant $\xi$ and cause no distortion. However, the precise value of $\tau_f$ depends on the biasing current [2], pp.102–105. An effective and simple provision is to keep $I_t$ constant by using an active current source instead of a resistor.

   As we concluded in discussing the linearity of the sawtooth generator, an additional reason for using a constant bias current is that it keeps the comparator input current constant.

2. **Modulation of the base-collector junction capacitances**

   The common-mode level modulates the base-collector voltages of $Q_{3,4}$ (see Fig.7.14) and thus the collector-to-base junction capacitances $C_{jc}$ (see model in Fig.5.15). To determine its influence, $C_{jc}$ is written as a function of the voltage across the base-collector junctions:

   $$C_{jc} = \frac{C_{jo}}{\left( 1 + \frac{V_t - V_{bce}}{V_{jo}} \right)^p} \quad (C.26)$$

   $C_{jo}$ is the zero-voltage capacitance, $V_{bce}$ is the (reverse) bias voltage of the junction, $V_{jo}$ is the build-in voltage and $V_t$ is the input voltage of the modulator.
In practical components, the constant \( p \) is in the range 0.33–0.5. Expansion of Eq.(C.26) into a Taylor polynomial around the biasing voltage \( V_{bco} \) and subsequent substitution into Eq.(C.25) yields:

\[
\tau = \text{constant} + \frac{p \tau_c}{V_{bco} - V_{jo}} V_i \\
+ \frac{p(p+1) \tau_c}{2(V_{bco} - V_{jo})^2} V_i^2 + \frac{p(p+1)(p+2) \tau_c}{6(V_{bco} - V_{jo})^3} V_i^3 + \ldots \tag{C.27}
\]

\[
\tau_c = R_b(2R_cg_m + 1) \frac{C_{jo}}{(V_{jo} - V_{bco})^p} \tag{C.28}
\]

Since the intended modulation constant \( \xi \) is large compared to the \( V_i \)-proportional term of Eq.(C.27), the second and third order intercept voltages can be approximated as:

\[
V_{int,2} = \left| \frac{k_1}{k_2} \right| \approx \frac{2\xi(V_{bco} - V_{jo})^2}{p(p+1)\tau_c} \tag{C.29}
\]

\[
V_{int,3} = \sqrt{\left| \frac{k_1}{k_3} \right|} \approx \sqrt{\frac{6\xi(V_{jo} - V_{bco})^3}{p(p+1)(p+2)\tau_c}} \tag{C.30}
\]

To estimate the orders of magnitude we assume \( p=0.5, C_{jo}=0.1 \text{pF}, R_b=300 \Omega, V_{jo}=0.8 \text{V}, R'_c=400 \Omega, g_m=0.01 \text{A/V}, V_{bco}=-2 \text{V} \) and \( \xi=60 \text{ns/V} \), yielding \( V_{int,2}=8700 \text{V} \) and \( V_{int,3}=170 \text{V} \). If the amplitude of the input voltage is in the order of 0.1V, the resulting distortion will be negligible.

Note that the intercept voltages determined in the preceding sections do not depend on \( \xi \) and process dependent time constants. This time however, as it is reasonable to assume that \( \tau_c \) scales proportionally with the transit time \( \tau_f \), the intercept voltages depend on the ratio between \( \xi \) and \( \tau_f \).

\( \text{(3) Modulation of transit time} \)

The base-collector junctions of \( Q_{3,4} \) are inversely biased. The depletion depth into their base regions varies with the common-mode voltage, so the transit time \( \tau_f \) of \( Q_{3,4} \) is not a constant. According to [2], pp.104, \( \tau_f \) can be written as a function of the forward and reverse Early voltages \( V_{ear} \) and \( V_{eaf} \):

\[
\tau_f = \tau_{fo}(1 + \frac{V_{be}}{V_{ear}} + \frac{V_{bc}}{V_{eaf}})^2 \tag{C.31}
\]

where \( \tau_{fo} \) is the transit time which would result if the base-emitter and base-collector voltages were extrapolated to zero. Taylor expansion around \( V_{bco} \)
with variable $V_i = V_{bc} - V_{bco}$ results in:

$$
\tau_f = \tau_{fo}[(1 + \frac{V_{be}}{V_{ear}} + \frac{V_{bco}}{V_{eaf}})^2 \\
+ 2V_i(\frac{V_{bco}}{V_{eaf}^2} + \frac{V_{be}}{V_{eaf} V_{ear}} + \frac{1}{V_{eaf}}) + V_i^2 \frac{1}{V_{eaf}^2}] \quad (C.32)
$$

Substitution of $\tau_f$ into Eq. (C.25) yields:

$$
\tau = \text{constant} + 4R_b g_m \tau_{fo} \left( \frac{V_{bco}}{V_{eaf}^2} + \frac{V_{be}}{V_{eaf} V_{ear}} + \frac{1}{V_{eaf}} \right)^2 V_i \\
+ 2R_b g_m \tau_{fo} \frac{1}{V_{eaf}^2} V_i^2 \quad (C.33)
$$

Once again, the $V_i$-proportional term is small compared to $\xi$, so we estimate the second order intercept voltage by:

$$
V_{int,2} = \frac{|k_1|}{|k_2|} \approx \frac{\xi V_{eaf}^2}{2R_b g_m \tau_{fo}} \quad (C.34)
$$

Using this model, the third order intercept voltage is implicitly infinite. With $V_{eaf}=25V$ we find $V_{int,2}=12 \cdot 10^3 V$, so this mechanism can be practically ignored. Obviously, this type of distortion also depends on the ratio between $\xi$ and $\tau_f$.

### C.2.2 Distortion Caused by Modulation of the Excitation

As no sample-and-hold is employed at the input of the comparator (Fig.7.11, configuration(a)), the slope magnitude of the differential input voltage varies with the time derivative of the input signal $V_i$:

$$
S_i = \frac{1}{\xi} + (\frac{dV_i}{dt}) \quad (C.35)
$$

In the first stages of the comparator, where the aperture times are long compared to the $RC$ time constants of the SDP stages, the propagation delays are approximately equal to the time constants and are thus constant. In the last stages, which handle saturated pulse edges, all amplitude modulation has been removed, so they exhibit constant delay times as well. However, in the intermediate stages the switching times decrease with increasing slope magnitude, resulting in modulation of the total delay time of the comparator. The variation of the delay time due to variation of the slope magnitude will subsequently be referred to as the excitation modulation.
In the presence of excitation modulation of the comparator, the total pulse deviation from the zero positions can be written in the general form:

\[ t_{PI} = k_0 + \xi V_i + k_1 \left( \frac{dV_i}{dt} \right) + k_2 \left( \frac{dV_i}{dt} \right)^2 + k_3 \left( \frac{dV_i}{dt} \right)^3 + \ldots \]  

(C.36)

in which \( k_0 \) is an offset, \( \xi \) is the modulation constant and the operator \( \frac{d}{dt} \) denotes the time derivative. Since the term determined by \( k_1 \) does not cause components at new frequencies but only emphasizes high frequencies, a non-zero value of \( k_1 \) effectively results in a zero in the modulator transfer function at the frequency \( \frac{\xi}{2\pi k_1} \). In the case of the prototype, using SPICE, we estimated \( k_1 \) to be in the order of \( 10^{-18} \) s/V. Non-linear distortion results from the higher order terms and, because of the differentiation, its magnitude increases with the frequency. We estimated \( k_2 \) to be about \( 10^{-27} \) s/V, \( k_3 \) could not be determined because it is too small.

Because the distortion is frequency dependent, we calculate the harmonic distortion at one specific frequency (determining the magnitude \( \frac{dV_i}{dt} \)) rather than the intercept voltage:

\[ d_{harm,2} = \frac{|k_2| |\frac{dV_i}{dt}|^2}{|\xi V_i|} \]  

and:

\[ d_{harm,3} = \frac{|k_3| |\frac{dV_i}{dt}|^3}{|\xi V_i|} \]  

(C.37)  

(C.38)

To estimate the maximal the value of \( d_{harm,2} \), we assume the input signal \( V_i \) to be a sinusoidal \( A \cos(2\pi f_m t) \), in which \( A \) is the amplitude and \( f_m \) the maximal modulation frequency, resulting in:

\[ d_{harm,2} = \frac{k_2 A}{\xi \omega_m^2} \]  

(C.39)

Substituting \( \xi = 60 \text{ns/V}, A = 0.05 \text{V} \) (corresponding to a peak-to-peak amplitude of 0.1V) and the maximum frequency of the baseband signal \( f_m = 5 \text{MHz} \) yields \( d_{harm,2} \approx 0.01\% \). The third-order distortion is still lower.

As it is reasonable to assume that \( k_2 \) and \( k_3 \) are proportional to \( \tau_f \), this type of distortion also depends on the ratio between \( \xi \) and \( \tau_f \).

C.2.3 Noise Induced by the Comparator

In Sec.5.3, we determined the equivalent input voltage noise produced by an SDP stage and the noise bandwidth of a cascade of SDP stages. By substituting
\[ S = \frac{1}{\xi} \] and \( F = 4 \) into Eq.(5.51) we find:

\[ \sigma_{td}^2 \approx \frac{8(\xi kT)^2}{qI_e 3\tau_f} \] (C.40)

Taking \( \xi = 60 \text{ns/V} \), \( I_e = 1 \text{mA} \) and \( \tau_f = 50 \text{ps} \), yields \( \sigma_{td} \approx 3 \text{ps} \). Compared to noise generated in the sawtooth generator, the contribution of the comparator is of the same order of magnitude.
Appendix D

Summary

Optical fiber exhibits important advantages over the traditional copper cable, such as a larger bandwidth, minimal losses, insensitivity to electromagnetic interferences and lower weight. Nevertheless, due to the high costs of the required electro-optic transmitters and opto-electric receivers, copper cable is still preferable in many applications.

In this thesis, within the constraint of minimal costs, a system consisting of a transmitter and receiver, capable of handling various types of analog as well as digital signals, has been developed. The strategy of minimizing cost has been to make use of low-cost multimode semiconductor laser diodes and PIN photodiodes, to facilitate the possibility of simultaneously transmitting two or more signals, and to integrate the complete electronic circuitry necessary for coding and multiplexing in an IC. To minimize the required silicon area and supply current of the IC and to minimize susceptibility to nonlinearities of the laser diodes, the system is based on (analog) time division multiplex pulse position modulation (TDPPM). An examination is made of how the total transmission capacity and the quality of the received signals depend on the characteristics of the optical and electronic components.

A prototype of the system, capable of simultaneously transmitting up to four PAL-coded video signals, has been realized and tested successfully, proving the feasibility of the system.
Appendix E

Samenvatting

Vergeleken met de traditionele koperkabel, biedt glasvezel belangrijke voordeelen, zoals een grotere bandbreedte, minder verliezen, ongevoeligheid voor elektromagnetische stoorvelden en een geringere fysieke omvang. Toch wordt in veel toepassingen nog steeds de voorkeur gegeven aan de koperkabel, vanwege de hoge kosten van de benodigde elektro-optische zenders en optisch-elektrische ontvangers.

Met als uitgangspunt een minimalisatie van de kosten, wordt in dit proefschrift een systeem ontwikkeld, bestaande uit een zender en een ontvanger, dat geschikt is voor de transmissie van verschillende typen analoog en digitaal signalen. Om de kosten te minimaliseren, wordt gebruik gemaakt van goedkope typen halfgeleiderlasers en fotodioden, worden meerdere signalen gelijktijdig door dezelfde glasvezel verstuurd, en is de voor het coderen en samenvoegen van de signalen benodigde elektronica kompleet geïntegreerd in een IC. Om de gevoeligheid voor de niet-lineariteiten van de goedkope lasers, het benodigde siliciumoppervlak van het IC en de totaal benodigde voedingsstroom te minimaliseren, is het systeem gebaseerd op (analoog) tijdverdeelmultiplex puls positie modulatie (TDPPM). Er is onderzocht hoe de totaal beschikbare transmissiecapaciteit en de kwaliteit van de overgedragen signalen afhangen van de eigenschappen van de optische en elektronische componenten.

De haalbaarheid van een dergelijk systeem is aangetoond door de realisatie van een prototype van het systeem, dat geschikt is voor gelijktijdige overdracht van 4 PAL-gecodeerde videosignalen.
Appendix F

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Appendix G

Biography

Dick van den Broeke was born on the 19\textsuperscript{th} of January, 1966. He started his study in the Faculty of Electrical Engineering at the Delft University in 1984, and graduated in 1989. Subsequently, he started his Ph.D. work in the Electronics Laboratory of the same faculty.