RISK ANALYSIS FOR MARINE SYSTEMS: AN INTRODUCTION
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Abstract - The safety analysis of marine systems is studied. One assumes that the required results of the hazard analysis are available. The identification of the appropriate performance function and the relevant reliability calculation are discussed in detail. The procedure is exemplified for some basic categories of marine systems.

Introduction

Hydrology was a pioneer within civil engineering in suggesting probabilistic design criteria for hydraulic structures involving hydrologic input. Nevertheless traditional design of marine structures is based on the concepts of design load (defined on a probabilistic base) and safety margin. The safety margin is the difference between the design action and the design value of the system carrying capacity. Since the carrying capacity itself is a random variable partial safety factors can also be introduced. In modern system reliability [1][2][3][4][5] this approach is regarded as a level I of investigation. An action fractile with low (5%) probability of being exceeded in a reference period is multiplied by its own safety factor to define the design load. Then it is compared with the design resistance. It is a resistance fractile with high probability (95%) of being exceeded, divided by a resistance partial safety factor. This is a design criterion rather than a risk evaluation process since it does not permit the evaluation of the probability of failure of
the system.

By contrast, within upper levels of sophistication, one either approximates (level II) or computes (level III) the actual probability of failure of the system and assumes the results as a performance index. Furthermore, by the introduction of consequence functions specific for the case under investigation, one can also proceed to assess the generalized risk (level IV).

This paper gives some informative details on the last three levels and provides some applications to different marine system typologies.

**Reliability-Based Design Procedures**

Classical level-I exceeding-probability schemes are based on the concept of return period $T_r$; i.e. the number of years required to exceed at least once a considered value of a time variant random variable [2][4]. Let $W_1$ be the maximum value in one year of the variable of interest $W$ (say the external action) and $P_{W_1}(W)$ its probability distribution; then [2]:

$$T_r(W) = \frac{1}{1 - P_{W_1}(W)} \quad (01)$$

Given a hydrological event, one selects a return period $T_r$ (f.i. 100 years) for the variable which describes the phenomenon. The inversion of Eq. (01), then gives the design value of $W$, say $\bar{W}$, provided that the probability distribution $P_W(W)$ was assessed on hydrological based. A first partial safety factor $\gamma_W$ (which can also be 1) leads one to the design value of the action:

$$W_D = \gamma_W \bar{W} = \gamma_W P_W (1 - \frac{1}{T_r}) \quad (02)$$
It must be lower than or equal to the resistance design value

\[ R_D = \frac{R_K}{\gamma_R} \quad (03) \]

which, on turn, is the ratio between a low fractile of the resistance distribution and the corresponding partial safety factor. (Note that the partial safety factors are always conceived to be larger than 1).

Assume that:

1) the design is made for

\[ W_D = R_D \quad (04) \]

2) the resistance is a deterministic variable

\[ R = \gamma_R R_D \quad (05) \]

It follows that

\[ R = \gamma_W \gamma_R \bar{W} \quad (06) \]

where \((\gamma_W \gamma_R - 1)\bar{W}\) is often denoted as "the empirical freeboard" [6].

Then, the probability of failure in one year of the system is the probability that the action value exceeds \(R\):

\[ P_{f_1} = 1 - P_{W_1}(\gamma_W \gamma_R \bar{W}) \leq 1 - P_{W_1}(\bar{W}) \quad (07) \]

Its value strongly depends on the mathematical model which describes the probability distribution function \(P_W(W)\) and hence this approach does not provide coherency between different designs.

Moreover, Eq.(07) does not account for:

1) the actual life expectancy (lifetime \(T\)) of the hydraulic structures to be designed;

2) the randomness of the system carrying capacity \(R\).

When \(R\) is a random variable (independent of \(W\)), Eq.(07) becomes:
where \( p_R(R) \) is the probability density function of \( R \). The variability of the relationship between \( P_{f1} \) and the product \( \gamma = \gamma_W \gamma_R \) is illustrated in Ref. [2] for different combinations of mathematical models for \( P_W(W) \) and \( p_R(R) \). Let the resistance be normally distributed with coefficient of variation (the ratio between the standard deviation and the mean) varying between 0.05 and 0.2. For an action of extreme type distribution, for instance, the probability of failure for \( \gamma = 1.4 \) ranges between \( 10^{-3} \) and \( 5 \times 10^{-5} \) when the coefficients of variation of the action is 0.1 and between \( 5 \times 10^{-3} \) and \( 10^{-3} \) when the coefficient of variation is 0.3! (Figure 1). Figure 1 shows that for an extreme type distribution of action, with significant coefficient of variation, Eq.(07) can be used as an accurate approximation of Eq.(08): the uncertainty of the action, in fact, is predominant. In general, however, Eq.(08), i.e. the effect of resistance randomness, can result of basic importance in the assessment of the structural safety.

Three further elements are developed in the next Sections: incorporated:

a) the generalization of the performance function \( R - W \geq 0 \) till now adopted for describing the good behaviour of the system, so that the uncertainty of the resistance parameters be included:

b) the procedures for assessing the probability of failure;

c) the variability in time in view of the study of systems which can undergo damage.
Figure 1 - Relation between $P_f$ in Eq. (08) and $\gamma$ for different probabilistic models of $P_u$ and $P_B$ [2].
The performance function and level III methods

Assume that the state of a system be of a Boolean nature: either safe or unsafe. Let this state be depending on the vector \( \{X\} \) of \( N \) time invariant random basic variables (action, strength of materials, geometrical and mechanical parameters) of joint probability density function (JPDF) \( p_{\{X\}}(\{X\}) \).

Points of the space of the variables \( \{X\} \) correspond to one of the two states. The surface between the two domains (safe \( S \) and unsafe \( U \)) is the failure surface \( \Sigma \). A function \( g(\{X\}) \) is a performance function if:

\[
\begin{align*}
g(\{X\}) > 0 & \text{ denotes safe states, i.e. } \{X\} \in S \\
g(\{X\}) = 0 & \text{ denotes limit states, i.e. } \{X\} \in \Sigma \\
g(\{X\}) < 0 & \text{ denotes failures states, i.e. } \{X\} \in U
\end{align*}
\]

The probability that the random vector \( \{X\} \) falls into \( U \) is the probability of failure:

\[
P_f = \text{Prob} [\{X\} \in U] = \int_U p_{\{X\}}(\{X\}) \, d\{X\} \tag{09}
\]

The reliability of the system is the complement to 1 of \( P_f \).

Methods which use the complete distribution for assessing the probability of failure are called level III methods.

Nevertheless, Eq.(09) can be adopted in a rigorous way just for a limited number \( N \) of random variables \( \{X\} \), say \( N \leq 5 \). A tremendous simplification is obtained when \( \{X\} \) is formed by two variables (the action and the resistance) and they can be assumed to be stochastically independent (see Eq.(08)).

In the general case, simulation methods must be adopted for assessing the value of the probability of failure. One simulates realizations \( \{X\}_j \) of the random vector \( \{X\} \); a sequence
of pseudo-random number in \((0,1)\) is carried out by any of the available algorithms; then for any random variable \(X_i\), the associated probability distribution is considered by the procedure illustrate in Figure 2, which leads to the simulated value \(X_{ij}\) of the vector \(\{X\}_j\). Then \(g(\{X\}_j) = g_j\) is evaluated and the member \(n_f\) of negative values is counted. The probability of failure is estimated by

\[
P_f = \frac{n_f}{n} \tag{10}
\]

The sample size in the Monte Carlo simulation approach must be very large in order to obtain a sufficiently reliable estimate of \(p_f\). Variance reduction techniques can be adopted in order to improve the sampling process [7]. Special aspects of simulation in system reliability assessment are called 'importance sampling' and 'directoril simulation' for which the reader is referred to the appropriate literature [3][5].

Level II methods

Let \(\{X\}\) be a normal random vector. It is mapped into a standardized and uncorrelated vector \(\{Z\}\) with expected value \(E[\{Z\}] = 0\) and covariance matrix \(\text{Cov} [\{Z\}\{Z\}^T] = [\Sigma_Z] = [I]\), \([I]\) being the identity matrix:

\[
\{Z\} = [U]^T (\{X\} - E[\{Z\}]) \tag{11}
\]

with \([U]\) the normalization eigenvector matrix of \([\Sigma_A]\).

In the space of the variable \(\{Z\}\) the performance function is mapped into the function \(g_{\{Z\}}(\{Z\})\). Since the JPDF \(P_{\{Z\}}(\{Z\})\) is radially symmetric, the point \(Z^*\) of the surface \(g_{\{Z\}}(\{Z\}) = 0\) which is closest to the origin of the \(\{Z\}\) space is the most likely failure point [2]. Therefore the distance
\[
B(\{Z\}) = \min (\{Z\}^T \{Z\})^{1/2}, \quad \{Z\}|g_{\{Z\}}(\{Z\}) = 0
\]

(12)
is the safety index proposed by Hasofer and Lind as a measure of the probability of failure. Figure 3 provides the geometrical meeting of this safety index.

In the simple case of independent action and resistance
\[
g(W, R) = R - W = z
\]
(13)
z being the safety margin. The distance \(B\) coincides in this case with the ratio between the mean value and the standard deviation of the safety margin
\[
B = \mu_z/\sigma_z
\]
(14)

In general, the basic variables \{X\} are not normally distributed. A suitable transformation of the vector \{X\} into a set of random variables \{Z\}, uncorrelated, standardized and normally distributed can be obtained by means of the Rosenblatt transformation [3].

**Time-variant behaviour**

Assume first that one variable of the vector \{X\} is varying in time, for instance the action. Eq. (08), where \(R\) is a function of the remaining \(N-1\) variables \{X\}, provides the probability of failure in one year. Then if the events of each year are independent of the ones of the previous year, the probability of failure in \(t\) years in the complement to 1 of the probability of no failure during \(T\) years:
\[
P_f(t) = 1 - (1 - P_{f1})^t
\]
(15)

Alternatively one can write:
\[
P_f(t) = \int_0^{\infty} (1 - P_{Wt}(R)) P_R(R) \, dR
\]
(16)
Figure 3 - Graphical explanation of the meaning of safety index $B$
when $W_t$ is the maximum value of $W$ in $t$ year and \[ P_{W_t}(W) = (P_{W_1}(W))^t \] (17)

Replacing $W_1$ with $W_t$ into $\{X\}$, then, the level II method can be used to assess the relevant safety index.

When several actions varying in time must be considered one must idealize the action combination. Several engineer-oriented approximations are available in the literature [3], but the rigorous solution is still lacking. It must be pursued within stochastic process theory and can also incorporate the aging of the system [3][5]. The basic idea is to compute the mean outcrossing rate $\nu^+(\tau)$ of $\mathcal{D}$ at time $\tau$, so that

\[
P_f(t) = 1 - \exp \left[ - \int_0^t \nu^+(\tau) \, d\tau \right] \tag{18}
\]

Eq. (18) is based on the assumption that the upcrossing are non-homogeneous Poisson events with mean rate $\nu^+(\tau)$.

Note that for a stationary behaviour, the probability of failure in one year is

\[
P_{f_1} = 1 - \exp (-\nu^+(1)) \tag{19}
\]

and for high reliability levels

\[
\nu^+(1) = P_{f_1} \tag{20}
\]

Therefore

\[
P_f(t) = 1 - \exp (-P_{f_1} \, t) \tag{21}
\]

which is an approximation of Eq. (15) holding for large $t$.

In the following $t$ will be omitted and all the calculations will be performed over the lifetime period $(0,T)$. 

1-10
Reliability-based design

Let reliability be the basis of the design criterion. The system has to be designed so that the actual reliability \((1 - P_f(T))\) over the design life \(T\) exceeds a reliability target \((1 - \hat{P}_f)\) which must be given as part of the design information.

\[
(1 - P_f(T)) \geq (1 - \hat{P}_f) \quad (22)
\]

For each choice of the design variable \(\{X\}\) one obtains a particular value of \(P_f(T)\) and Eq. (22) can be met for many sets of parameters \(\{X\}\).

An optimization problem with an objective function to be minimized (for instance the system cost) is conveniently introduced. It is constrained by Eq. (22) as well as by technological constraints:

\[
\{X\} \in \mathcal{J} \quad (23)
\]

\(\mathcal{J}\) being the domain of the feasible solution.

The design stages are outlined in the top part of the scheme of Figure 4. One starts from hydrological data and, by means of the appropriate model, one derives the basic variable \(W(t)\) and its variability. The hydrologic variable \(W(t)\) is not generally the variable of interest and must be transferred by hydraulic relations into the effects \(\{S\}\) of the action. Similarly, the basic structural parameters must be transferred by structural relations into the set of parameters \(\{R\}\) which together with \(\{S\}\) form the vector \(\{X\}\) to be transferred into the standardized variables \(\{Z\}\). In this space, the reliability methods work by either Eq. (09) or Eq. (12).

Note that the method for assessing the probability of failure, repeated for different values of \(t\), provides the
Figure 4 - Schematic representation of the reliability concept
dependence on time of the system reliability.

Often the previous scheme is just applied to components of
the systems and systems reliability is then evaluated by means of
tools as event trees and fault trees [8][9] in connection with
algebra operations holding for parallel and series systems [3].
When the assemblage of the basic components cannot be reduced to
simple schemes, stochastic finite element algorithms [3][10]
should be introduced for propagating to the response the
uncertainty on the input variables.

**Fragility curves**

When a single external action is predominant over the
reliability assessment process one can maintain separated hazard
and fragility analyses. Fragility is the probability of failure
of the system conditional on a specific hazard [3]. The
unconditional probability can then be obtained by integrating
over the entire range of hazard intensities.

Fragility is calculated in selected situations and plotted
against the excitation intensity (fragility curves). Most
fragility curves for local response are rather steep, i.e. small
changes in the input result in large changes in the probability
of failure. For complex geometries and combination of components
the fragility curves are expected to be flatter. The flatter the
curve, the better is the design.

In evaluating the fragility, uncertainties can arise from an
insufficient understanding of structural material properties and
failure modes. It follows that there is a possibility of
representing the component fragility as a family of fragility
curves. A probability of being true is then associated with each curve.

Examples of performance functions

a) Flood levee case [6]

The problem is to accommodate a discharge $Q$ related to the water level at gage by a known hydrological model. The variable $Q$ can be transferred in a deterministic way to a channel dimension $h$ by the transformation

$$h = h(Q)$$  \hspace{1cm} (24)

which usually involves Manning's equation. A freeboard $\Delta h$ is then added to a design value of $h$, say $h_S = h(Q_S)$ in order to define the levee height $h_R$

$$h_R = h(Q_S) + \Delta h$$  \hspace{1cm} (25)

The performance function is then simply

$$g(Q, \Delta h) = h_R - h(Q) \geq 0$$  \hspace{1cm} (26)

Note that there are a number of causes for the dike to be lower than specified and make $h_R$ a random variable. For example this could be due to settlement or the height might be limited due to soil mechanical problems such as piping or slope failure.

Examples of the dimension of canal to be determined which were considered in the literature (see Ref.[6]) include: the stage in an open channel; the stage at an upstream point in a tidal river under the influence of a tidal surge at the mouth of a river; the stage of a downstream point in an open channel when the stage at the channel inlet is given in terms of a flood wave.

The hydrologic model for obtaining $Q$ from hydrological data can be as simple as a measured stage-discharge relation or as
complicated as a simulation model of run-off in which the catchment model is subjected to time functions which have been generated artificially by means of the hydrological records. The hydraulic transformation $h(Q)$ can be determined from mathematical models of the flow in open channels or, experimentally, by means of a physical model of a river in a hydraulic laboratory.

Note that the hydraulic reliability is not equal to hydrologic reliability because there usually is no equivalent between the design discharge $Q_S$, selected on the basis of its exceedance probability, and the design stage $h_R$, since a probability distribution of values $h_R$ correspond to $Q_S$, rather then a single value. Conversely failure can also occur for values of $h$ lower than the expected one, i.e. also the resistance is characterized by a probability distribution ($\Delta h$ is a random variable). The resistance is indeed a true random variable because it is not possible to predict the exact properties of the level at any one time. Its effective height can vary unpredictably due to natural influences such as settlement, animal actions, tree roots or due to human factors such as variation in methods of construction or maintenance problems.

b) Determining the upstream flooding probability of a culvert design [9]

The following study is presented in Ref.[9] "as an illustration of the procedure, not for it detailed completeness and precision in the analysis".

The highway culvert should drain the run off from a surface A watershed of farm land. The culvert is 1 in length, consisting
of \( n_p \) 1\(_1\)-long circular concrete pipes on a \( \mu \% \) slope. The entrance to the culvert is a vertical head wall set flush with the pipe and symmetric 45\( ^\circ \) wingwalls. The pipe invert at the entrance is at elevation \( H \). The outlet has a flushed end wall with a short apron to protect the downstream from erosion. Under the design condition, the maximum allowable headwater elevation upstream from the culvert is \( H_p \) and upstream storage is negligible. The expected tail water elevation range between \( H_1 \) and \( H_2 \).

One assumes to know the formula relating the rainfall intensity \( i \) to the return period \( T_p \) and the duration \( t_d \):

\[
i = \frac{a T_p^m}{t_d^k + b}
\]

(27)

in which \( a, b, m \) and \( k \) are known parameters. The duration is given by the time of concentration; by Kirpich's formula it is a function of the basic length \( L \) and the average slope \( s \) \((t_d = c_1(L/\sqrt{s})^c)\). The design culvert service period is 50 years. Failure is defined as the flow from upstream \( Q_L \) exceeding the culvert capacity \( Q_c \):

\[
g(Q_L, Q_c) = Q_c - Q_L
\]

(28)

Figure 5 shows the flume tree of this system from which

\[
P_f = \text{Prob} \left[ (L_L \cup L_t \cup L_f) \cap (C_s \cup C_r) \right]
\]

(29)

In a deterministic context the design discharge can be written

\[
Q_o = c_o i A
\]

(30)

where \( c_o \) is the run-off coefficient. The culvert design critical condition is difficult to identify since up to 27 cases were classified. On possible result, using Manning formula and entrance, exit and other losses (\( K_{\text{ent}}, K_{\text{exit}} \)), is
\[ Q = \pi \sqrt{(y_u - y_d) \left[ \frac{c n^2}{d \xi_1} + (K_{ent} + K_{exit}) \frac{8}{9d \xi_2} \right]^{-\frac{1}{2}}} \]  

(31)

where the upstream and downstream water surface elevation of flow are \( y_u \) and \( y_d \) respectively, \( c, \xi_1 \) and \( \xi_2 \) are coefficients and \( n \) ranges from 0.011 and 0.15(?). Eq.(31) was used to find the design value of \( D \).

Next step is to associate a coefficient of variation to each variable and to apply a level II method for estimating the result safety index, provided that the previous values are the distribution modes.

Further developments

a) Design of strategic systems

Two designs which ensure the same reliability are consistent each with the other. Nevertheless this consistency can be unjustified for two piers, for instance, of quite different economical revenue. In these cases one can introduce a consequence function which quantifies the consequence that are to be expected from selecting a set of variable \( \{X\} \) (level IV).

The consequence of a failure can be generally expressed as costs \( C(\{X\}) \); for instance the sum of the costs of the structures plus the cost that results from its operation and maintenance plus the expected costs should the structure fail. Then the generalized risk can be expressed as

\[ R_k = \int_{\Theta} C(\{X\}) P_{\{X\}}(\{X\}) \, d\{X\} \]  

(32)

where \( \Theta \) is the whole space of the \( \{X\} \) variables. Eq.(32)
expressed in terms of benefit rather than cost is denoted as expected utility note that a binary definition of $C\{X\}$ (0 for $\{X\} \in \mathcal{S}$ ; 1 for $\{X\} \in \mathcal{U}$) made $R_k = P_f$ (see Eq.09).

The design can be based on a minimization of $R_k$, which. It can be regarded as a mix of safety with economics aspects or as a mix of responsibilities. In real terms Eq.(32) cannot be sometimes used since a value cannot be generally given to the so-called intangibles [2] (as loss of lives and so on) A multiobjective optimization scheme, where a simultaneous maximization of the expected utility and of the reliability is pursued (see Figure 7) was proposed in [11] and the development of operative procedures are presently in progress.

An interactive multiobjective programming for the evaluation of alternative ocean disposal sites is discussed in [12].

b) Monitoring and maintenance

Due to aging, the reliability of the system is generally a decreasing function of time. Monitoring of the system gives rise to a re-evaluation of reliability (see Figure 8). The new piece of information are generally included in a Bayesian upgrading scheme which can be easily implemented in an expert system prototype [3]. The basic problem of maintenance is then the optimal stopping-repairing of the system. For this purpose it is necessary to have [13]: the history of the action; a theory for predicting the probability of failure; an optimal stopping rule; a method of optimization and an instantaneous identification of system failure.
Figure 7 - Multiobjective optimization and Pareto optima

Figure 8 - Reliability degradation in time and is updating by bayesian techniques
Let $c$ be the cost of a preventive stopping of the system performance and $(c+d)$ be the cost of failure. Moreover assume that the only possible action is to stop the performance at discrete times $t_n, n=1,2,...$ For a monotonic degradation process, the optimum stopping rule turn out to be the following [12]:

- the performance should be stopped as soon as the conditional probability of failure during the $(n+1)$-th step will exceed or be equal to the value $c/dn$, provided that failure did not happen before.

A method which takes into account the new information gathered during the operation phases of the system in a probabilistic context was pursued in [14]. A software capable of optimizing decisions concerning the inspection and maintenance of the system was implemented during that research project. A prototype expert system capable of guiding the decision making process was its final objective.

Conclusions

When dealing with marine systems, two different tasks can be envisaged: the design and the maintenance.

Reliability based design criteria require a target value be defined. For this aim the decision maker, or alternatively the writers of rules and standards, gives the state of the art of that particular type of systems and ensures legal protection to the design.

Design by risk may be more objective but cannot be a mere loss-function minimization [2][11]. A general definition of risk,
in fact, should include the consequences of the failure event in technical and socio-economical terms [3].

Monitoring for off-line upgrading of the system reliability is the field of on-going research. Its development seems to be of specific interest in hydraulic engineering, where the maintenance of existing systems is becoming the all day problem.
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