Identification and Feedforward Control of a Drop-on-demand Inkjet Printhead

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Identification and Feedforward Control of a Drop-on-demand Inkjet Printhead

MASTER OF SCIENCE THESIS

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Inkjet technology is a very important key-technology from an industrial point of view. The variety of its applications is very wide, with the document printing the most common one. Usually, applications of inkjet technology are accompanied with some performance criteria concerned with several drop properties such as the drop-velocity and the drop-volume, as well as the consistency of these drop properties. In addition, requirements with respect to the productivity and stability of the jetting process itself are also frequently imposed. These performance criteria are expected to become more tighter for future applications.

For a typical piezoelectric inkjet printhead, a standard actuation pulse is applied so as to meet the drop-on-demand requirement. However, two performance limitations are generally encountered for such an approach: residual pressure oscillations and cross-talk. The former one relates to the fact that the ink inside the channel is not at rest immediately after a drop ejection, while the later one refers to the fact that the drop properties of the jetting channel are affected if its neighboring channels are simultaneously actuated. Both phenomena would limit the drop consistency and the productivity considerably.

In this thesis, a systems and control method is proposed to improve the printing performance. First, the modeling of an inkjet printhead is taken to provide good insights for the system dynamics. Given the complicated jetting conditions, in this thesis, system identification method is adopted to estimate the models. Based on these models, an off-line optimization-based method is used in a SISO case to design an optimal input actuation pulse for the piezo actuator. Then the damping of residual pressure oscillations inside one ink channel can be improved. In the MIMO case for reducing the cross-talk, the method used is just a more complex vision, except that an additional input delay between neighboring channels is also optimized for.

Simulation results found with the identified models show good applicability of the proposed method. In addition, based on a real printhead setup, the experimental results also demonstrate a significant improvement of the printing performance with respect to the drop-velocity consistency, e.g., the maximal velocity variation in DoD curve can be reduced from 6 m/s to
$2 \text{ m/s}$ in direct-channel, the velocity caused by cross-talk can also be eliminated in multiple channels.
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Chapter 1

Introduction

The ability of inkjet technology to deposit materials, with diverse chemical and physical properties, onto a substrate has made it an important subject for both industry and academic research. In last fifties, a rapid development of inkjet technology started off and since then, the inkjet technology has been a very versatile technique with a wide range of applications due to the low operational costs. Apart from the conventional document printing, the inkjet technology has also been applied in the production of solar panels [1][2], LED and LCD fabrication [3][4], organic tissue and organ printing [5], as well as 3D rapid prototyping [6], etc.

In this thesis, a piezoelectric inkjet printhead with two arrays of ink channels is addressed. Each channel is equipped with one piezo actuator. When one droplet is needed, a specified actuation pulse is applied to the piezo actuator and then a single ink drop can be ejected from the nozzle [7]. After the ejection, the ink channel is not immediately at rest and one should wait until the residual pressure oscillations are damped out before another actuation pulse can be applied. Otherwise, the properties of the subsequent droplets cannot be guaranteed. For this reason, the attainable jetting frequency is limited and that would then degrade the printhead performance. Another limiting factor is often encountered during the jetting of several channels. Given the so-called cross-talk, the drop properties through an ink channel are affected when its neighboring channels are simultaneously actuated. Other undesired phenomena include the formation of satellite droplet, variations in ink properties and disturbances in nozzle [8].

However, in nowadays inkjet printheads, a fixed actuation pulse which does not take the aforementioned problems into account is mostly used. Generally speaking, the actuation pulse only consists of one positive trapezoidal pulse. This particular pulse is also called the standard pulse. The main drawback of this standard pulse is that it would generate great residual oscillations. In the literature, there are various methods for the design of an actuation pulse which help damp the residual oscillations [8][9][10][11]. While, in this thesis, the method to be used is a further study of that used in reference [12]. That paper proposed an optimization-based method to design the actuation pulse for the piezo actuator in order to
improve the damping of the residual oscillations in the meniscus velocity. Here, the meniscus is an interface between the ink and the air in a nozzle. However, it is thought to be very difficult to experimentally measure the meniscus position and velocity during the jetting process. Therefore, in this thesis, the integration of a piezo-unit sensored signal is used to indicate the pressure signal inside an ink channel because of the self-sensing characteristic of a piezo-unit [13]. Briefly speaking, a piezo-unit can be simultaneously used as an actuator and a sensor. When used as a sensor, its sensored signal is proportional to the derivative of the ink channel pressure.

Thus, a similar systematic model-based approach as the reference [12] is applied in this research. First, we require a model of the system that we want to control. Several models such as the ‘two-port’ model [11] and the ‘narrow-gap’ model [14] have been used in other research. However, the accuracies of these models are to some extent limited due to the complicated physical relationships in the printhead system. Therefore, in order to improve the model accuracy, a system identification method is chosen to describe the printhead dynamics. A model can be estimated by relating the actuation pulse voltage (i.e, the input) to the piezo-unit sensored signal (i.e, the output). Then, using this model, we are able to obtain the optimal actuation pulse with an optimization-based feedforward control method. A time-delay between neighboring channels is also optimized to reduce the cross-talk in multi-channel system [15]. In conclusion,

Estimate an accurate experimental model to describe the relevant dynamics of the inkjet printhead and then use this model for the implementation of an optimization-based feedforward control. Finally investigate the effects of this approach to reduce the two performance limitations: residual pressure oscillations and cross-talk.

The problem statement above can be clarified with the following research objectives:

- Identify the model by making use of the piezo-unit sensored signal to describe the system dynamics.
- Implement an optimization-based feedforward control method, which is based on the integrated sensor signal, in a single channel so as to damp the residual pressure oscillations in practice. This is referred to as the SISO case.
- Apply an input time-delay between neighboring channels so as to reduce the effects of the cross-talk in multiple channels system. This is referred to as the MIMO case.

This thesis report is organized as follows. In chapter 2, structure of a typical printhead and its working principle are introduced at first and then, two performance limitations are described in details. Moreover, the experimental setup and its sensor functionalities are discussed. In chapter 3, the modeling of the printhead is treated with system identification based on piezo-sensor functionality. Both the models for direct-channel and cross-talk channels are estimated. In next chapter, an optimization-based feedforward control method is introduced in direct-channel system, as well as an additional time-delay implementation in multi-channel system. At the ends of these two cases, the experiment results are shown to verify the improvements. Finally, chapter 5 presents the conclusions and gives some recommendations for future research.
Chapter 2

Printhead description

The basis of a drop-on-demand (DoD) inkjet printhead is the use of a piezoelectric crystal to convert an electric pulse into mechanical pressure. This pressure then is able to overcome the surface tension forces holding the ink in a nozzle [14]. The name drop-on-demand comes from the fact that one droplet is only jetted when an actuation pulse is provided. The varieties encountered in piezoelectric inkjet printhead designs are enormous, however, there are three common fundamentals.

- **Ink channel design**
  Four basic components like the channel, nozzle, ink supply and piezo-unit are required for an ink drop jetting.

- **Working principle**
  The operation of such a printhead is very complicated while the acoustics inside the ink channel play the most important role.

- **Operational limitations**
  The printing performances are affected mainly by two factors that are associated with the design and operation of a printhead: residual pressure oscillations and cross-talk.

Although the research presented in this thesis is elaborated on one particular printhead setup at Océ Technologies, Netherlands, the results throughout are still generally applicable in other piezoelectric inkjet printheads. In this chapter, the above mentioned three aspects are discussed explicitly.
2-1 Ink channel description

Several processes take place before an ink drop can be jetted [14]. In Fig.2-1, the prototype drawing for a typical DoD inkjet printhead is depicted. In the jetting case, it starts with melting the ink in the melting unit (a). A good heat transfer and draining of the melted ink at a given temperature is necessary. Then the ink is filtered with unit (b). The next unit is the reservoir (c) with enough volume to keep the total printhead dimensions within certain limits. For the hose (d), it is used for closing static pressure in the reservoir. The lower part of the printhead is the central part (e) where drop formation happens. More details are given later. The required electric voltage for jetting is supplied by the electronic flex (f).

![Figure 2-1: 3D CAD drawing of a printhead prototype showing (a) the melting unit, (b) the filtering unit, (c) the reservoir, (d) the static pressure hose, (e) the central part and (f) the electronic driving supply.](image)

The printhead setup used in our experiments consists of two parallel arrays of 128 ink channels each. All these ink channels are assumed to be identical. A cross-section view of such an ink channel is shown in Fig.2-2.
As we can see, an ink channel is carved in channel plate. On the top of plate, a filter is placed between the reservoir and channel to remove impurities from the melted ink. A metallic plate with drilled holes is attached at the bottom of the channel plate. Every hole acts as a nozzle for jetting. One wall of the ink channel is formed by a flexible foil with a piezo-unit attached. On the application of a voltage, the piezo-unit, as an actuator here, would deform the flexible foil and then this formation would generate pressure waves in the channel. When specific conditions are met, an ink drop is jetted [12].

In Fig.2-3, a cross-section view of a typical printhead is schematically depicted. It is clear to see that all these piezo-units are connected to the same substrate and actuate their corresponding ink channels.
2-2 Working principle

In this section, a description of the working principle of the used piezoelectric inkjet print-head is given. A schematic side view of an ink channel and its working principle is illustrated in Fig.2-4.

In order to jet a droplet, a positive trapezoidal pulse is provided to the piezo actuator and then, the following five steps occur inside the ink channel [11].

To start with, a negative pressure wave is generated by enlarging the volume in the channel with the actuation voltage increasing.

Then, this pressure wave splits up and propagates in both directions.

Since the reservoir and the nozzle are different boundaries, these pressure waves are reflected in different ways when they reach the ends.

Next, by decreasing the actuation voltage, a positive pressure wave is superimposed on the reflected waves exactly when they get to the middle of the channel.

Consequently, the wave traveling towards the reservoir is canceled whereas the pressure wave traveling towards the nozzle is amplified to be large enough to jet a droplet.

Figure 2-4: A schematic side view of an ink channel and its working principle.
2-3 Operational limitations

For a piezoelectric inkjet printhead, some important drop properties, such as drop velocity, drop volume and the consistency for them, are related to certain printing performances, for example, to avoid irregularities on printed images. In practical applications, these performance requirements are severely affected by the following two operational limitations [14].

2-3-1 Residual pressure oscillations

Once an ink drop is jetted, due to the presence of the traveling pressure waves, the fluid-mechanics within an ink channel are not at rest immediately. Generally, a fixed actuation pulse is designed under the assumption that an ink channel is in steady state. If the subsequent ink drop is jetted before the settling of these residual vibrations, the resulting drop properties will be different from the previous one. So one has to wait for these residual pressure oscillations to be sufficiently damped out to guarantee consistent drop properties. Since it takes several microseconds for the pressure oscillations inside the channel to decay, the maximally attainable jetting frequency will be limited. If the residual pressure oscillations are ignored and the jetting frequency increases nonetheless, the resulted drop properties would start varying.

Therefore, an important characteristic named DoD curve, which represents the ink drop velocity as a function of the jetting frequency, is proposed to show the effect of residual vibrations on drop velocity consistency. Ideally, as we can imagine, such a DoD curve must be flat. However, due to the aforementioned reason, the curve is far from flat in practice.

Here we take Fig.2-5 below as an example, in plot (a), when a standard actuation pulse is applied, obviously, the pressure waves inside the channel take about $30 - 40$ microseconds to damp out after the drop ejection. Therefore, the attainable jetting frequency would be limited. If we generally increase the jetting frequency, the residual pressure oscillations cannot be damped out sufficiently. Furthermore, the resulted drop properties cannot be guaranteed. As shown in plot (b), considerable fluctuation results in DoD curve. The maximum speed variation, as we can see, is as large as $6 \text{ m/s}$. 
Figure 2-5: Residual vibrations (a) and its effect on DoD curve (b).

2-3-2 Cross-talk

The phenomenon that one ink channel cannot be actuated without affecting the fluid-mechanics in its neighboring channels is called cross-talk. It mainly occurs in two ways. First, the acoustic cross-talk, which occurs via the ink reservoir, i.e., the pressure waves within one channel
influence other channels. Generally, its overall influence can be considered small and here we mainly introduce the second one, the *structural cross-talk*, which can again occur in two ways.

For example, since all piezo-units are connected to a substrate, as shown in Fig.2-3, when a piezo-unit is actuated, the reaction force of the substrate will be guided to its neighboring non-actuated piezo-units. And then the channels which are not actuated will deform, too. As shown in Fig.2-6 (a), the resulting deformation of the neighboring channels is opposite to the deformation which is necessary for jetting a drop. Therefore, when a neighboring channel is actuated together with the direct-channel, the drop velocity of the direct-channel will be lower than that of only direct-channel is actuated.

![Figure 2-6: Front view of the channel structures. (a) The reaction force of the substrate is guided to the neighboring channels and this results in an opposite deformation of the neighboring channels (b) The deformation of one channel results in an enlargement of its neighboring channels.](image)

Another path is via the deformation of a channel itself since a positive pressure inside the actuated channel would result a deformation of its neighboring channels. As shown in Fig.2-6 (b), the deformation of one channel results an enlargement of its neighboring channels and diminution of the channel pressure, thus the drop velocity will also get lower [14].

In Fig.2-7, with actuation in the neighboring two channels, the effect of cross-talk on the direct-channel pressure and the DoD curves are depicted. As we can see in plot (a), although the pressure waves of two neighboring channels (red and green) are much smaller than that of the direct channel (blue), this part can not be easily ignored. In plot (b), the resulting drop speed of direct-channel (blue) is depicted when in turn the first neighboring channel is actuated (black), as well as the first and second neighboring channels are simultaneously actuated.
actuated (green), then we can see obvious drop-velocity decreases among these conditions. This in turn shows that the effect of cross-talk on the drop-speed is particularly substantial when multiple channels are simultaneously actuated.

Figure 2-7: Cross-talk (left) and its effect on the drop-speed (right).
2-4 Experimental setup description

In references [12] and [15], a discrete-time model relating the piezo input voltage to the meniscus velocity was used. Namely, the input is the piezo actuation pulse and the output is the meniscus velocity. The meniscus is the interface between ink and air in a nozzle. However, it is very difficult to experimentally measure the meniscus position and velocity while jetting an ink drop [16]. Thus in this thesis, we adopt to use the piezo-unit as both an actuator and a sensor simultaneously because of its so-called ‘self-sensing’ capability. A schematic overview of the experimental setup is depicted in Fig.2-8.

With the computer, the actuation signals can be programmed and processed, then they are sent to a waveform generator. Through an amplifier, the actuation signal is fed to switch board, which is used to determine which channel is provided with the appropriate actuation pulse. An oscilloscope is used for the tracing of both the actuation and sensor signals. The CCD camera equipped with a microscope is used to observe the generated droplets. Since both the time duration and the distance a droplet has traveled are known, an estimate of the drop velocity can easily be obtained. Unfortunately, all these measurements only give details on the ink flow outside the printhead, while for the phenomena inside the ink channels need to be investigated with the piezo-unit sensed signal. Some fundamental explanations for piezo-unit’s self-sensing characteristic are introduced as follows.

![Figure 2-8: A schematic overview of the experimental setup.](image-url)
As generally known, a piezo-unit can be used as an actuator or a sensor [17]. For the actuator effect, if an electrical potential $V$ is applied to the piezo-unit, a deformation $u$ of the piezo-unit is generated. While for the sensor effect, if a force $F$ is applied to the piezo’s surface, an electric charge $q$ generates. Together, this behavior can be described as in equation (2-1).

$$\begin{bmatrix} u \\ q \end{bmatrix} = \begin{bmatrix} d & 1/k \\ C & d \end{bmatrix} \begin{bmatrix} V \\ F \end{bmatrix}$$ (2-1)

with $C$ the piezo’s capacity, $d$ the piezoelectric charge constant, $k$ the piezo’s stiffness coefficient.

When a single piezo-unit is used both as an actuator and a sensor simultaneously, it possesses several advantages over the use of two separate units acting as individual actuator and sensor, e.g. much lighter and costlier. Furthermore, the collocation of actuating and sensing allows the control signal to be applied at the point of measured response, thereby eliminating the capacitive coupling between individual actuator and sensor [13]. In practical applications, specially designed electric circuit as introduced in reference [14], the so-called bridge circuit, is used to realize the self-sensing concept.

For a piezo-unit sensor, its measured signal is an electric charge $q$. In our experiments, the measured signal $q$ is made up of two contributions, as shown in Fig.2-9. One is generated from the applied actuation voltage $V$ via the piezo’s capacity $C$, which is also referred to as the direct-path. The other one originates from the force $F$, exerted by the ink inside the channel, via the piezoelectric charge constant $d$ and is also called as the indirect-path.

![Division into a piezo- and ink-block diagram.](image)

As the indirect-path contribution is related to the jetting process inside the ink channel, it is the required sensor signal and needs to be extracted from the measured output signal $q$. However, since the contribution of the direct-path is considerably larger than that of the indirect-path, it is quite difficult to acquire this part while using the piezo-unit as an actuator. In this thesis, we use the off-line compensation approach proposed in [18] to reconstruct the sensor signal.
As schematically depicted in Fig.2-10, two experiments are carried out with applying the same input. The first one is carried out with the ink inside the channel and the measured piezo output signal $q_{\text{fill}}$ is stored. During the second experiment, the channel is kept empty by removing the ink from the printhead, then the measured output signal $q_{\text{empty}}$ is stored. By a subtraction of the signal $q_{\text{empty}}$ from the signal $q_{\text{fill}}$, the required sensor signal $q$ is obtained. Here during the experiments in two situations, piezo capacity differences caused by temperature difference and deformation of substrate are neglected.

![Figure 2-10](image)

**Figure 2-10:** The basic principle to obtain the actuation and sensor signal simultaneously as used in the piezo-sensing device.

Having discussed the technical implementation of using piezo-unit as an actuator and a sensor, as well as the practical approach to obtain the required sensor signal, the next question is to know the representation of this sensor signal. Physically speaking, a piezo-unit senses the force which results from the pressure distribution in the channel. This force creates a charge on the piezo-unit, as shown in equation (2-1). Therefore, we have a voltage as our final sensor signal. This voltage signal is thought to be proportional to the derivative of the pressure inside the ink channel because the force on the piezo-unit is induced from the channel pressure [8]. As a result, the obtained sensor signal can be regarded as a representation for the jetting process and plays an important role in investigating a systems and control approach to improve printing performance. This part will be introduced in next chapter.
Chapter  3

Identification of ink channels

Generally, the piezo actuation pulse only consists of one positive trapezoidal pulse, namely the standard pulse as shown in Fig.2-5 (a). It is determined mainly by manual tuning on a printhead setup. The main drawback of this standard pulse is that it generates residual pressure oscillations, as discussed in section 2-3-1. Thus the printing quality and the jetting frequency would be limited. Therefore, in this thesis, a systems and control approach is used to design the actuation pulse for the DoD inkjet printhead so as to improve the printing performance.

For this purpose, we require a model of the system that we want to control, at first. Basically, there are two different ways for modeling [19]

- **physical modeling**
  A model is derived by formulating the physical laws that the system obeys, such as mass-balance, energy-balance and so on.

For ink channel systems, two such models can be found in the literature. One is the ‘two-port’ model, in the reference [8], the ink channel is divided into several functional subsystems. Each of them then is modeled as a two-port system using first principle modeling only. At last, a modeling approach based on the notion of bilaterally coupled systems is proposed. The other model is called the ‘narrow-gap’ model [14], which is an analytical model. It describes the system dynamics mainly based on viscous and thermal wave propagation principles.

- **system identification**
  A model is constructed by fitting a parametric model to the measured input and output (I/O) data, without concerning the physical interpretation of the model parameters.

As in many engineering problems, system identification has been successfully applied and sometimes, it is the only method that can be applied to do the modeling because
of high system complexity or insufficient knowledge about the physical behaviors. However, due to the desired simplicity, a model estimated from system identification cannot perfectly describe a complex system.

Although some analytic and numerical models are available, they are more appropriate for printhead system design and not sufficiently accurate for the purpose of control design. For high performance control problem, it requires accurate models. So in this thesis, system identification method is applied as the best modeling method to get an accurate model for further control design. At the very beginning, a brief introduction about system identification procedures is given.

### 3-1 Identification procedure

As shown in Fig.3-1, the system identification procedure has a natural logical flow: first is collecting data, then choosing a model set and picking the ‘best’ model from this set. It is quite likely that the model first obtained will not pass the model validation tests. Various steps of the procedure thus need revision [20].

![Figure 3-1: A flow diagram of system identification.](image)

Generally, the I/O data $Z^N = \{y(t), u(t) | t = 1, ..., N\}$ is generated according to a true system $S$.

$$ S : y(t) = G_0(q)u(t) + H_0(q)e(t) $$

The choice of input signals is of very substantial influence since the input signals determine the operating point of the system and which modes of the system are excited during the experiments [21]. For the identification of a linear system, there are some basic facts governing
the choices of the input signal. First, from practical considerations, the input signal must have limited amplitudes. Second, it should be persistently exciting of a certain order, which at least should be equal to the number of parameters to be estimated in the plant $G(\theta)$. For example, if the numerator and denominator of a transfer-function model have a same number of parameters $n$, then the input signal should be persistently exciting of order $2n$. This means that the spectrum of the input $\Phi_u(\omega)$ should be nonzero at $2n$ points.

Next to the choice of the input signal, the selection of the sampling interval is also of importance. Generally, the information loss caused by sampling is best described in frequency domain. To avoid aliasing, the signal being sampled should not contain frequencies beyond the Nyquist-frequency $f_N$, which is defined as half of the sampling frequency $f_S$ [22].

The choice of model structure is quite subjective. Generally, we denote the model structures by $\mathcal{M}$, while a particular model corresponding to the parameter value $\theta$ is described as $\mathcal{M}(\theta)$. Such a parametrization is instrumental in searching the ‘best’ models. For example, given a model structure below

$$\mathcal{M} : \{G(q, \theta) , H(q, \theta) \mid \theta \in D_M\}$$

Then one aspect of the choice of a good model structure is to select $\mathcal{M}$ so that $S \in \mathcal{M}$ holds for the given description of $S$, namely, $S = \mathcal{M}(\theta_0)$ for some values $\theta_0$ [20]. Normally, $S$ is unknown and this will typically involve trials of several different structures $\mathcal{M}$.

The choice of an appropriate model structure is based on the insights about the system to be identified. It is conceivable that various nonparametric techniques could be helpful for finding suitable model structures and estimating the order of a linear system. Usually, the empirical transfer function estimate (ETFE) is applied to give valuable information about the resonance peaks and the high frequency roll-off. All these would give a hint as to what model orders would be required to give an adequate description of the interested dynamics. Based on the data set $Z^N$ over the interval $1 \leq t \leq N$, the estimate of the frequency response of the true plant transfer function $G_0(e^{i\omega})$ can be described as

$$\hat{G}_N(e^{i\omega}) = \frac{Y_N(\omega)}{U_N(\omega)}$$  \hspace{1cm} (3-1)

with two Fourier transforms

$$Y_N(\omega) = \frac{1}{\sqrt{N}} \sum_{t=1}^{N} y(t)e^{-i\omega t} , \quad U_N(\omega) = \frac{1}{\sqrt{N}} \sum_{t=1}^{N} u(t)e^{-i\omega t}$$

In practice, several model sets are commonly used, such as ARX, OE, BJ, etc. Here, as an example, we consider the model structure $\mathcal{M}$ corresponding to a generalized transfer-function structure

$$y(t) = \frac{B(q)}{F(q)} u(t) + \frac{C(q)}{D(q)}e(t)$$  \hspace{1cm} (3-2)
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with the parameter vector

$$\theta = [b_1, \cdots, b_{n_b}, f_1, \cdots, f_{n_f}, c_1, \cdots, c_{n_c}, d_1, \cdots, d_{n_d}]$$

Here $\theta$ includes all the coefficients of the polynomials involved. The orders of the polynomials are $n_b$, $n_f$ and so on. By tuning the orders, a corresponding true system $S$ can be uniquely represented such that $S = M(\theta_0)$ holds.

As we know, the selection of a parameterized model structure is vital for identification problem. This is the link between system identification and parameter estimation techniques. Basically, there are two methods, prediction-error identification method (PEM) and subspace model identification (SMI).

In the PEM framework, the prediction error $\{\epsilon(t, \theta)\}$ is used to create a cost-function which is then optimized with respect to the parameters of the model. The prediction error is

$$\epsilon(t, \theta) = y(t) - \hat{y}(t, \theta)$$

with the predicted output given by

$$\hat{y}(t, \theta) = H(q, \theta)G(q, \theta)u(t) + (1 - H(q, \theta)^{-1})y(t)$$

The cost-function is usually defined as the average of two-norm of the prediction error

$$V_N(\theta) = \frac{1}{N} \sum_{t=1}^{N} \|y(t) - \hat{y}(t, \theta)\|_2^2$$  \hspace{1cm} (3-3)

The resulting estimation of $\hat{\theta}_N$ can be obtained by the minimization of equation (3-3)

$$\hat{\theta}_N = \arg \min_{\theta} V_N(\theta)$$

Usually, the PEM is considered to be the basic approach for system identification because of three important advantages. First, it is applicable to general model structures. Second, it has an optimal asymptotic accuracy when the true system can be represented within the model structure ($S \in M$). Third, it has reasonable properties when the true system cannot be represented within the model structure ($S \notin M$).

For a linear system with several outputs, which requires a model structure with many parameters, the SMI methods form a valuable alternative. They have the advantage of allowing an estimate by using efficient and numerically robust calculations without iterative search. For detailed information about SMI, we can refer to the reference [23].
With the parameter estimation procedure, a ‘best’ model within the chosen model structure can be obtained yet. Then, *model validation* is the next crucial step to check if the model performs sufficiently well. There exist several methods with different characters. At first, a particular useful technique, the so-called residuals analysis is introduced.

The residuals here mean the differences between the model predicted output and the measured output. Thus, the residuals represent the portion of the data which are not explained by the model.

\[
\epsilon(t) = \epsilon(t, \hat{\theta}_N) = y(t) - \hat{y}(t, \hat{\theta}_N)
\]

Generally, residuals analysis consists of two tests: the whiteness test \( \hat{R}_\epsilon \) and the independence test \( \hat{R}_{\epsilon u} \), given as following two equations.

\[
\hat{R}_\epsilon^N(\tau) = \frac{1}{N} \sum_{t=1}^{N} \epsilon(t)\epsilon(t-\tau)
\]

\[
\hat{R}_{\epsilon u}^N(\tau) = \frac{1}{N} \sum_{t=1}^{N} \epsilon(t)u(t-\tau)
\]

The numbers \( \hat{R}_\epsilon^N(\tau) \) carry the information about whether the residuals can be regarded as white. If they are not small for \( \tau \neq 0 \), then part of \( \epsilon(t) \) could have been predicted from past data. This means that \( y(t) \) could have been predicted from past data, which is a sign of deficiency of the model. The number \( \hat{R}_{\epsilon u}^N(\tau) \) express the covariance between residuals and past inputs. If they are not small, then there are traces of past inputs in the residuals. This means there is a part of \( y(t) \) that originates from the past inputs has not been properly picked up by the model. Hence, the model can be improved.

Another commonly used validation method is to compare the fit between the model simulated output and the measured output:

\[
\text{Fit} = (1 - \frac{|y(t) - \hat{y}(t, \hat{\theta}_N)|}{|y(t)|}) \times 100\% \tag{3-5}
\]

If a model can pass the validation tests, it is regarded to be sufficient for intended application. Otherwise, the identification procedure should go back to previous steps, such as choosing new input signals or model structures, and find a better model.
### 3-2 Multi-channel system

As we previously discussed in Chapter 2, this thesis mainly focuses on improving the printing performance of a printhead by restraining two operational factors, residual pressure oscillations in a single ink channel and cross-talk among several ink channels, respectively. The following figure shows a diagram for a multi-channel system. Here, only five channels are considered.

![Diagram of the multi-channel system](image)

**Figure 3-2:** A block diagram of the multi-channel system.

As depicted in Fig.3-2, the models have two sub-index. Each of them has an input and output in the channel denoted by the second and first index, respectively. For example, an input for
channel $n + 1$, denoted as $u_{n+1}$, goes through model $H_{n(n+1)}$ to have an effect on the output of channel $n$, which is represented by $y_n$. Moreover, $y_n$ is the sum of the outputs from five models, $H_{n(n-2)}, H_{n(n-2)}, H_{nn}, H_{n(n+1)}$ and $H_{n(n+2)}$:

$$y_n = y_{n(n-2)} + y_{n(n-2)} + y_{nn} + y_{n(n+1)} + y_{n(n+2)}$$

In our case, two assumptions are made: all channels in the printhead are identical and bilaterally symmetrically distributed. Thus, with the same input for every channel, the output effect on the middle channel would be identical, no matter from the left neighboring channel or the right one. As a result, we can simplify such a multi-channel system into three models, denoted as follows

$$
\begin{align*}
    H_{nn} &= H_d \\
    H_{n(n-1)} &= H_{n(n+1)} = H_{c1} \\
    H_{n(n-2)} &= H_{n(n+2)} = H_{c2}
\end{align*}
$$

Here, $H_d$ means the model for the direct-channel dynamics. $H_{c1}$ and $H_{c2}$ represent the models for the cross coupled dynamics from the first and second neighboring channel, respectively.

Now, the system identification problem for a printhead can be solved by estimating three SISO models. In subsequent sections, these models are discussed individually.
3-3 Direct-channel $H_d$ identification

In this section, we will introduce the practical system identification procedures for direct-channel. A model for $H_d$, as shown in Fig.3-2 is given in the end.

To collect data for further analysis, a few decisions have to be made. During the experiments, we selected a sampling interval of 0.1 $\mu$s. The input was chosen to be a periodic signal since the drop jetting is an iterative process. Due to the practical limitations on the waveform shape of the actuation pulse, the input signal we used should be in trapezoidal form. As shown in Fig.3-3, a standard pulse was tried at first. However, this pulse would cause residual pressure oscillations, as discussed in section 2-3-1, the identification results were not good especially when we considered different jetting frequency conditions.

![Actuation input signals](image)

Figure 3-3: Test pulse (blue) used as input signal and a standard pulse (red).

In reference [12], an additional pulse is applied after the standard pulse in order to damp the residual oscillations in the meniscus velocity, namely the test pulse as shown in Fig.3-3. And we decided to choose it as our input signal for the identification problem. The output signal is the piezo-unit sensed signal as discussed in section 2-4, which is a subtraction of the piezo outputs from the ink filled-channel and empty-channel.
A record of 60,000 samples was collected and a portion of the data is shown in Fig. 3-4 below. The data set was then split into two halves, to be used for estimation and validation, respectively.

For the input signal, we also checked its spectrum. For example, as shown in Fig. 3-5, there were a large number (16) of nonzero values in different frequencies up to 330 kHz. Thus we thought this input signal was persistently exciting enough for our model estimation.
With a high sampling frequency of 10 MHz, if we applied the ETFE to have a first insight about the system, we can find the data contains high-frequency noise outside the frequency range of the system dynamics, as shown in Fig.3-6.

Therefore, a resampling of the data signals without information loss is necessary and helpful. In our experiments, a resampling factor 15 was selected. As shown in Fig.3-7, from the ETFE of the resampled data, we can clearly see two resonance peaks in a frequency interval from 20 kHz to 200 kHz. Thus, at least a fourth-order model is required.

Since our modeling focuses on the dynamics rather than the disturbance properties, a output-error (OE) model would be sufficient. Its structure can be represented by

\[ y(t) = \frac{B(q)}{F(q)} u(t) + e(t) \]
After several trials, an eighth-order OE model was chosen, with its frequency response shown in Fig. 3-7. Obviously, it has a good fit within the frequency range from 20 kHz to 200 kHz.
For model validation, here we use two approaches. Fig.3-8 shows that this chosen OE model has a Best Fit of 94.76 to the validation data. That means the model can reproduce the data set quite well.

![Image of Figure 3-8](image_url)

**Figure 3-8:** Best Fit between measured and simulated outputs.

Fig.3-9 shows the residual analysis results.

![Image of Figure 3-9](image_url)

**Figure 3-9:** Residual tests for direct-channel.
As we just mentioned, for a output error model, the modeling focus is on the dynamics $G(q, \theta)$. So the model could just show independence test of the residual $\epsilon(t)$ and the input $u(t)$, while pay less attention to the whiteness test of $\epsilon(t)$. Generally, for a good model, the required correlation function should not go significantly outside a confidence region, as we introduced in section 3-1. The confidence region corresponds to the range of residual values with a specific probability of being statistically insignificant for the system. The system identification toolbox in Matlab uses the estimated uncertainty in the model structures to calculate this confidence intervals. Here in Fig.3-9, we use a 95% confidence region, which means that the region around zero represents the range of residual values that have a 95% probability of being statistically insignificant.

As shown in Fig.3-9, the cross-correlation $\hat{R}_{u\epsilon}$ passes the residuals test. This allows to determine a suitable model set $\mathcal{M}$. Besides the residuals analysis, we can also compare the frequency response between the model $G(e^{j\omega}, \hat{\theta}_N)$ and the standard deviation of its variance $\sqrt{\text{cov}(G(e^{j\omega}, \hat{\theta}_N))}$. For example, if we want to use the model for control, the modeling error (measured by $\sqrt{\text{cov}(G(e^{j\omega}, \hat{\theta}_N))}$) has to be small up to the bandwidth. A rule of thumb is

$$\sqrt{\text{cov}(G(e^{j\omega}, \hat{\theta}_N))} < 0.1 \|G(e^{j\omega}, \hat{\theta}_N)\| \quad (3-7)$$

![Figure 3-10: Comparison between $G(e^{j\omega}, \hat{\theta}_N)$ (red) and $\sqrt{\text{cov}(G(e^{j\omega}, \hat{\theta}_N))}$ (blue).](image-url)
As shown in Fig.3-10, the equation (3-7) holds, which means the uncertainty is small and the model identification is good enough. Thus it seems reasonable to pick this eighth-order OE model as the final choice for our direct-channel model $H_d$.

Its numerical value is

$$H_d : \quad y(t) = \left[ B(q)/F(q) \right] + e(t)$$

$$B(q) = 0.0001165 - 0.001259q^{-1} + 0.001601q^{-2} + 0.002929q^{-3}$$
$$\quad \quad \quad - 0.009304q^{-4} + 0.01049q^{-5} - 0.006042q^{-6} + 0.001469q^{-7}$$

$$F(q) = 1 - 3.477q^{-1} + 5.893q^{-2} - 6.732q^{-3}$$
$$\quad \quad \quad + 5.592q^{-4} - 3.296q^{-5} + 1.236q^{-6} - 0.2782q^{-7} + 0.06331q^{-8}$$

### 3-4 Cross-talk channel identification

A quite similar procedure with previous section is used for the cross-talk channel estimation. The models for $H_{c1}$ and $H_{c2}$, mentioned in section 3-2, are given in the end of the following two subsections, respectively.

#### 3-4-1 $H_{c1}$ identification

To estimate a ‘best’ model for the first cross-talk channel, we proceeded as follows. A record of 60,000 samples was collected during the experiments. The input signal is the test pulse applied in the first neighboring channel ($n + 1$), while the output signal is the piezo-unit sensed signal of channel ($n$). Due to the setup limitations, some high-frequency disturbances intervened. As shown in Fig.3-11, with a fast Fourier transform of the output data, we can clearly see some disturbances around 1 MHz caused by piezo actuator dynamics, which are beyond the frequency range of interest for the system dynamics.
Since the input-output relation for a linear system will not be changed by filtering the input and output data through a same filter [20], a digital low-pass filter of 500 kHz was applied to preprocessing the data. A portion of the filtered data is shown in Fig.3-12. Then the filtered data was split into an estimation set, consisting of the first 30,000 samples, and a validation set, consisting of the remaining 30,000 samples.
Then a factor of 15 was selected to resample the estimation data, the ETFE of this data set is depicted in Fig.3-13. As we can see, there are three resonance peaks, thus, a sixth-order model is needed, at least. Due to the reasons we discussed in direct-channel identification problem, a OE model structure was also chosen for the estimation problem here. After several trials, a ninth-order OE model was obtained, with its frequency response depicted in Fig.3-13.

Figure 3-13: ETFE and FR of an estimated model for first cross-talk channel.
Next task is the model validation.

According to Fig.3-14, we see a best fit of 70.09 between the measured and predicted data. The residuals analysis is given in Fig.3-15. The values of cross-correlation $R_{e\epsilon}$ are inside a 95% confidence region, which means the model passes the residuals test.
Furthermore, we take a look at the frequency response comparison between the model $G(e^{i\omega}, \hat{\theta}_N)$ and the standard deviation of its variance $\sqrt{\operatorname{cov}(G(e^{i\omega}, \hat{\theta}_N))}$.

As shown in Fig. 3.16, the equation (3-7) holds, which means the uncertainty is small and the model identification is good enough. So it seems quite reasonable that we did a good modeling job with a fairly simple model OE990 for our first cross-talk channel model $H_{c1}$.

Its numerical value is

$$H_{c1}: \quad y(t) = \frac{B(q)}{F(q)} + e(t)$$

$$B(q) = 0.0002768 - 0.001107q^{-1} + 0.001826q^{-2} - 0.001446q^{-3} - 0.0001079q^{-4} + 0.001686q^{-5} - 0.002164q^{-6} + 0.001432q^{-7} - 0.0004426q^{-8}$$

$$F(q) = 1 - 1.894q^{-1} + 1.215q^{-2} + 0.4509q^{-3} - 1.56q^{-4} + 1.525q^{-5} - 0.7671q^{-6} - 0.07883q^{-7} + 0.2018q^{-8} - 0.02906q^{-9}$$
3-4-2 $H_{c2}$ identification

The identification procedure for the second cross-talk channel is quite the same with that for the first one. So here only a brief introduction is given. After the I/O data was collected, a low-pass filter was also used to remove the disturbances of high frequencies that we do not want to be included in the modeling process. Then the filtered data was split into two parts for estimation and validation, respectively. A portion of that data is given in Fig.3-17 below.

![Time plot for the first cross-talk channel.](image)

In Fig.3-18, we can see the ETFE of the resampled data with a resampling factor of 15. There are three resonance peaks in the frequency range from 10 kHz to 200 kHz. So the order of a sufficient model should be at least six. For the model structure, we still chose the output-error form. After several trials, a ninth-order OE model was adopted. Its frequency response is also displayed in Fig.3-18.
Figure 3-18: ETFE and FR of an estimated model for second cross-talk channel.
With the following two figures, model validation is discussed.

**Figure 3-19:** Best Fit between measured and simulated outputs.

**Figure 3-20:** Residual tests for second cross-talk channel.

A best fit of 79.02 between the measured and predicted data, as shown in Fig.3-19, together with the residuals analysis in Fig.3-20 illustrate the model identification is good enough.
Furthermore, let us have a look at the frequency response comparison between the model $G(e^{i\omega}, \hat{\theta}_N)$ and the standard deviation of its variance $\sqrt{\text{cov}(G(e^{i\omega}, \hat{\theta}_N))}$.

As shown in Fig.3-21, the uncertainty of the model is small and such a OE990 model is a reasonable choice for the second cross-talk channel model $H_{c2}$. Its numerical value is

\[
H_{c2} : \quad y(t) = \left[ B(q)/F(q) \right] + e(t)
\]

\[
B(q) = -0.0002019 + 0.0003613q^{-1} - 0.0001565q^{-2} - 0.000714q^{-3} + 0.0009255q^{-6} + 0.0005931q^{-7} - 0.002703q^{-8}
\]

\[
F(q) = 1 - 1.376q^{-1} + 0.5288q^{-2} + 1.317q^{-3} - 2.199q^{-4} + 1.515q^{-5} - 0.0376q^{-6} - q^{-7} + 0.8364q^{-8} - 0.4067q^{-9}
\]
3-5 Concluding remark

Till now, we have got three models for the direct-channel and two cross-talk channels, namely \( H_d, H_{c1}, H_{c2} \) shown in Fig.3-2. The frequency responses with their corresponding model uncertainties of these three models are depicted in following Fig.3-22.

As we can see, the amplitude of direct-channel \( H_d \) is much larger than the other two models. Now we briefly discuss about the model uncertainty. When we estimate the model parameters from the data, we can obtain their nominal values which are accurate within a confidence region. For the size of this region, it is determined by the values of the parameter uncertainties during estimation process. The magnitudes of the uncertainty provide us a measure of the model reliability. For \( H_d \), the plot shows that the uncertainty is very low within \( 20 - 200 \) kHz frequency range. That indicates the estimated model \( H_d \) is quite reliable in this frequency interval. While for \( H_{c1} \) and \( H_{c2} \), large uncertainties happen at some frequency resonances.

In next chapter, these three models for direct-channel and two cross-talk channels are used for the control application.
Chapter 4

Feedforward control

In previous section 2-3, the performance requirements of a printhead as well as the corresponding limitations are discussed in detail. For our thesis, the control objective is focused on improving the printing performance with respect to the following two requirements, drop-consistency and productivity. Furthermore, the residual pressure oscillations and cross-talk are the major performance limiting phenomena when considering the above two requirements. Therefore, the control objective can be shifted to minimize the residual pressure oscillations and cross-talk. In this chapter, the corresponding research will be discussed.

4-1 Feedforward control design

For the printhead setup under investigation, feedback control is not applicable due to the following limitations:

- No sensor is available for real-time measurement of the ink channel pressure.
- The driving electronics limit the waveform of the actuation pulse. Only trapezoidal shape can be used in practice.
- Sampling time would be very short for the control computation due to a high jetting frequency.

Thus, a feedforward strategy, as discussed in reference [12], is applied in this thesis. The ultimate goal is to generate a trapezoidal actuation pulse for the piezo actuator to meet the control objective. In general, a single positive trapezoidal pulse, namely the standard pulse, is applied to jet a single droplet with specified properties. The parameters describing such a pulse can be found by extensive examination of an experimental setup. However, this pulse cannot damp the residual pressure oscillations inside the ink channel after jetting an ink drop. With the difference from the reference [12], in which the meniscus velocity inside the nozzle is used as an image of the channel pressure, the integrated piezo-unit sensed signal...
is adopted in this thesis. As discussed in section 2-4, the piezo-unit can be also used as a sensor. Its sensored signal represents the derivative of the pressure inside the ink channel. Numerically, damping this derivative to zero does not imply the channel is at rest. Therefore, the sensored signal needs to be first integrated so as to focus on the channel pressure itself. Here, in Fig.4-1, a plot of the integrated sensor signal resulting from a standard pulse with the direct-channel model $H_d(q)$ is given to show the pressure oscillations, as well as the time instant of drop-ejection is indicated, around 9 $\mu$s.

![Figure 4-1: Pressure oscillations of direct channel model due to a standard actuation pulse input.](image)

As can be seen in Fig.4-1, it takes about 30 to 40 $\mu$s for the oscillations to be sufficiently damped out. Therefore, a negative trapezoidal pulse is usually added to the standard pulse so as to damp the residual pressure oscillations, as depicted in Fig.4-2.
As we can see, this actuation signal then consists of two parts. A positive trapezoidal pulse, also called resonating pulse, is responsible for jetting an ink drop. The following negative trapezoidal pulse, which is named the quenching pulse, is responsible for damping the oscillations. This actuation pulse can be characterized by the rise time ($t_r$), dwell time ($t_w$), fall time ($t_f$) and the amplitude ($V_R$ and $V_Q$) for both the resonating pulse and the quenching pulse, besides a time interval $t_{dQ}$ between them. Thus the input signal $u(k, \theta)$ can be defined by using the parameter vector $\theta$, with the time units given in microseconds ($\mu$s).

$$\theta = [t_{rR}, t_{wR}, t_{fR}, V_R, t_{dQ}, t_{rQ}, t_{wQ}, t_{fQ}, V_Q]^T$$

As opposed to the physical approaches [9], in this thesis, an optimal parameter vector of the actuation pulse is determined by using a systematic optimization-based approach. In order to define an optimization problem and then find the optimal parameter vector $\theta_{opt}$. At first, a reference trajectory $y_{ref}$ as the desired ink channel pressure needs to be defined.

As shown in Fig.4-3, the integrated sensor signal, which is proportional to the channel pressure, presents a response from a standard pulse in two parts.
Part A is the response to the resonating pulse which allows the ink drop to be jetted and part B contains the unwanted residual pressure oscillations which would influence the correct jetting of subsequent droplet. Therefore, a desired reference signal $y_{\text{ref}}$ is to keep part A unchanged and brings part B to zero gradually.

![Graph](image.png)

**Figure 4-3:** Reference trajectory for the integrated sensor signal.

Generally, there are two important constraints for the reference trajectory construction. First, the pressure oscillations are not brought to a rest immediately after the ejection so as to ensure the refill of the nozzle. Second, a gradually damping can avoid a high actuation voltage.

If an optimal actuation pulse can found in such a way that the actual channel pressure $y(k)$ follows the reference trajectory $y_{\text{ref}}(k)$, the channel will come to a rest state very quickly after the drop ejection. As a result, the residual oscillations can be reduced and then the attainable jetting frequency can be increased. The procedure to find such an optimal actuation pulse is introduced in the next section.

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4-2 SISO control

In this section, we examine a single ink channel and apply the optimization-based approach to improve its performance with respect to reducing the residual pressure oscillations.

At first, we can formulate the optimization problem. The objective function can be defined as the following sum of square errors between the reference signal $y_{ref}(k)$ and the integrated sensor signal $y(k)$.

$$J(\theta) = \frac{1}{N} \sum_{k=0}^{N} \left( y_{ref}(k) - y(k, u(k, \theta)) \right)^2$$

$$= \frac{1}{N} \sum_{k=0}^{N} \left( y_{ref}(k) - \sum_{p=0}^{k} S(p) \right)^2$$

(4-1)

$$S(p) = H_d(q) u(p, \theta)$$

where $N = \frac{T}{T_s}$ with $T_s$ the sampling time, $T$ the chosen response time. $H_d(q)$ is the direct-channel model we estimated in section 3-3. $q$ is the forward shift operator and $u(p, \theta)$ is the proposed actuation pulse parameterized by $\theta$. A summation of $H_d(q) u(p, \theta)$ expresses the integration calculation.

Thus, the optimal actuation pulse $\theta_{opt}$ is the parameter vector $\theta$ by solving the optimization problem

$$\min_{\theta} J(\theta),$$

subject to

$$\theta_{LB} \leq \theta \leq \theta_{UB}$$

with $\theta_{LB}$ and $\theta_{UB}$ the vectors containing the lower and upper bounds on each element of the parameter vector $\theta$. Generally, the parameter bounds are determined by physical insights of the printhead.

This problem is a constrained nonlinear optimization problem and can be solved off-line with standard algorithms. Recall that part A of the reference signal is kept the same as a standard pulse response, so we can also adopt such standard pulse as our resonating pulse. The Optimization toolbox in Matlab is used to solve this optimization problem with an initial value chosen as

$$\theta_{init} = [1.5 \ 3 \ 1.5 \ 25 \ 0 \ 0 \ 0 \ 0]^T$$

An optimal parameter vector $\theta_{opt}$ for the optimal actuation pulse can be obtained

$$\theta_{opt} = [1.5 \ 3 \ 1.5 \ 25 \ 6.5 \ 3.5 \ 1.0 \ 1.1 \ -7.11]^T$$
4-2-1 Simulation results

Now, we compare the standard pulse and optimal actuation pulse which is parameterized by $\theta_{opt}$ in Fig.4-4(b).

![Graph showing comparison between standard and optimal actuation pulses]

Figure 4-4: Channel pressure response to standard and optimal actuation pulses.

As expected, the optimal input pulse contains two components, the resonating pulse and the quenching pulse. The quenching pulse plays an important part in damping the residual oscillation. This also enables the channel pressure signal to track the reference trajectory very closely, as shown in Fig.4-4(a). The sum of square errors with optimal pulse response is 78.11% smaller than that of the standard pulse response (from 5.799 to 1.269). Since the ink channel is brought to rest very quickly, the ink drops can thus be jetted with higher frequencies using this optimal actuation pulse.
4-2-2 Experimental results

The simulation results show that significant improvements can be achieved by using the optimal input \( u(k, \theta_{opt}) \). In this subsection, we present the experimental results to validate this claim.

First, a standard actuation pulse was applied for the direct-channel while the jetting frequency was set as 20 kHz. As introduced in section 2-4, the experiment consists of two conditions: the ink-filled channel and the empty channel. In these two conditions, the corresponding output signals were stored. Then, the sensed signal for standard pulse can be obtained by a subtraction of these two outputs. The same experiment was also implemented for the optimal actuation pulse and another sensed signal for optimal pulse can be obtained. The sensed signals were then numerically integrated to express the pressure situation inside the direct-channel during jetting process. In Fig.4-5, the integrated sensor signals from a standard pulse and an optimal pulse are compared.

![Experiment result comparison](image)

Figure 4-5: Experiment results comparison between standard and optimal actuation pulses.

As can be seen from Fig.4-5, with the standard actuation pulse, it takes about 30 \( \mu \)s to gradually damp out the residual pressure oscillations. While with the optimal pulse, the integrated
sensor signal is brought to zero much better.

As previously discussed in section 2-3-1, the DoD curve is also an important performance indicator for the minimization of the residual pressure oscillations. In the experiments, with the jetting frequency creasing from 10 kHz to 52 kHz, a CCD camera was used to measure the drop velocity. Here a DoD curve comparison with the standard pulse and the optimal pulse is depicted in Fig.4-6.

![DoD curves for direct channel](image)

**Figure 4-6:** Comparison between standard and optimal actuation pulses with DoD curves.

From Fig.4-6, we can find that the speed fluctuation with standard pulse is quite large, especially within high frequencies. The maximum of the speed variation is approximate 5.8 m/s (2.2 m/s-8.0 m/s). However, with the optimal actuation pulse, it is well substituted by a rather flat curve, with a smaller velocity variation of 2 m/s (5.1 m/s-7.1 m/s). This also demonstrates a reasonable good result in pressure oscillations reduction with the proposed optimization-based method.
In a real printhead, besides improving the performance of a single channel, the performances when multiple ink channels are simultaneously actuated, should also be taken into account. As shown in Fig.3-2, we know that an input for one channel will influence the output of its neighboring channels, i.e. the cross-talk. In this section, possible method to compensate the cross-talk is discussed.

Fig.4-7 shows a diagram in which cross-talk phenomena are displayed. In this thesis, the situation that at most five channels are simultaneously actuated is considered, since the effects from the third or further neighboring channels are very small and can be negligible during this research. All these channels are applied with the same input signal as we got from the SISO case in previous section. Moreover, the output $y_n$ is the sum of the outputs from the direct-channel model $H_d$, two cross-talk channel models $H_{c1}$ and $H_{c2}$. The effects from the left or right side are identical as we assumed.

![Diagram of cross-talk phenomena](image)

**Figure 4-7:** Inputs of neighboring channels affect output through cross-talk.
With a consideration that an input pulse can either be applied or not, there are 16 permutations in which the four cross-talk channels \((n - 2, n - 1, n + 1, n + 2)\) are jetting an ink drop or not. And due to the fact that there exist different permutations, the objective function to be finally optimized should be in the worst case.

Based on the reference [15], we know that in a three-channel system, the worst case occurs when the bilateral symmetry channels are actuated simultaneously. So in this thesis, for a five channels system, we only need to consider three possible worst permutations: the first neighboring channels \((n - 1)\&(n + 1)\), the second neighboring channels \((n - 2)\&(n + 2)\), as well as all these channels \((n - 2), (n - 1), (n + 1)\&(n + 2)\) are simultaneously actuated with direct-channel \((n)\). The experiments were also separate into two conditions, ink-filled channel and empty channel and then the sensed signals were obtained by a subtraction of those two output signals. The corresponding integrated sensor signals for the direct-channel \(y_n\) are shown in Fig.4-8.

The largest difference, from the response of direct-channel actuated only, happens when we apply the same input pulse to all five channels. This could also shown with numerical values. The objective function in these three permutations, which is defined in equation (4-1) as the sum of square errors between the reference signal \(y_{ref}(k)\) and the integrated sensor signal \(y(k)\), can be calculated and compared. The values are 6.404, 15.043 and 23.417 for above mentioned three permutations. The largest one comes from the condition that five channels are simultaneously actuated. So in the sequel, this situation is considered as the worst case to be investigated.
One manner with which the cross-talk can be reduced is to introduce a time-delay between the neighboring channels. This is investigated by making a distinction between odd and even channels. That is, all the odd channels \((n - 1), (n + 1)\) will be actuated simultaneously, as will the even channels \((n - 2), (n), (n + 2)\), but the odd and even channels will be delayed relative to each other. Fig.4-9 shows the block diagrams when the channel \((n)\) is an odd channel and Fig.4-10 the channel \((n)\) is an even channel, with the time-delay unit denoted by \(q^{-d}\) block.
Figure 4-9: Odd channel system: inputs of odd channels are delayed with respect to even channels.

Figure 4-10: Even channel system: inputs of even channels are delayed with respect to odd channels.
Then an optimization problem wherein the time-delay \( d \) will be optimized is proposed. The corresponding objective function \( J(\theta_{\text{opt}}, d) \) only has a variable \( d \) since we set the input pulse as the optimal input parameter vector \( \theta_{\text{opt}} \) which we got from the SISO case. Moreover, because a differentiation is made between odd and even channel systems, the final choice for the objective function should be the worst case. Such an optimization problem can be described as follows

\[
d_{\text{opt}} = \arg\min_d J(\theta_{\text{opt}}, d)
\]

subject to

\[
d_{\text{LB}} \leq d \leq d_{\text{UB}}
\]

where

\[
J(\theta_{\text{opt}}, d) = \max \left[ J_{\text{odd}}(\theta_{\text{opt}}, d), J_{\text{even}}(\theta_{\text{opt}}, d) \right]
\]

\[
J_{\text{odd}}(\theta_{\text{opt}}, d) = \frac{1}{N} \sum_{k=0}^{N} \left( y_{\text{ref}}(k) - \sum_{p=0}^{k} S_1(p) \right)^2
\]

\[
S_1(p) = H_d(q)u(p, \theta_{\text{opt}}) + 2H_{c1}(q)u(p - d, \theta_{\text{opt}}) + 2H_{c2}(q)u(p, \theta_{\text{opt}})
\]

\[
J_{\text{even}}(\theta_{\text{opt}}, d) = \frac{1}{N} \sum_{k=0}^{N} \left( y_{\text{ref}}(k - d) - \sum_{p=0}^{k} S_2(p) \right)^2
\]

\[
S_2(p) = H_d(q)u(p - d, \theta_{\text{opt}}) + 2H_{c1}(q)u(p, \theta_{\text{opt}}) + 2H_{c2}(q)u(p - d, \theta_{\text{opt}})
\]

with

- \( d_{\text{LB}} \) Lower bound for the input delay
- \( d_{\text{UB}} \) Upper bound for the input delay
- \( N \) Number of time samples
- \( q \) Discrete-time forward shift operator
- \( H_d \) Model for direct-channel
- \( H_{c1} \) Model for first cross-talk channel
- \( H_{c2} \) Model for second cross-talk channel
- \( d \) Delay of the samples
- \( \theta_{\text{opt}} \) Optimal parameter vector for the input pulse and here equal to

\[
[1.5 \ 3 \ 1.5 \ 25 \ 6.5 \ 3.5 \ 1.0 \ 1.1 \ -7.11]^T
\]

The values for the objective function \( J(\theta_{\text{opt}}, d) \) can be calculated and the result is depicted in Fig.4-11. An optimal value for the time-delay is equal to 4.4 \( \mu s \).
4-3-1 Simulation results

In this section, simulation results are given. First, the worst-case as depicted in Fig.4-8 (c) is used here to show the integrated sensor signal when all five channels are simultaneously actuated without an input time-delay considered. Then, the aforementioned odd and even systems are adopted together with the input time-delay between neighboring two channels. The integrated sensor signals are also shown to make a comparison. Fig.4-12 shows the simulation results of the integrated sensor signal for even and odd channel system.

As we can see, with an optimal time-delay applied, the pressure signal inside the direct-channel is much more close to the reference trajectory than that without such a time-delay, both in even channel system (a) and odd channel system (b). This can also be proved by the sum of square errors between the reference signal and the optimal response. For the odd system, the value reduced from 23.417 to 19.374, an improvement of 17.3% can be obtained. For the even system, the value with an input delay is 16.8% smaller than that of without the optimal time-delay, reduced from 23.417 to 19.491.

Figure 4-11: Values of objective function $J(\theta_{opt}, d)$. 
Figure 4-12: Cross-talk compensation with input delay 4.4 \( \mu s \).

Basically, we can combine the optimization problem concerning with the input parameter \( \theta \) and time-delay \( d \) together. And then we can get a new optimal actuation input pulse, as shown in Fig.4-13, and a new optimal time-delay. Furthermore, we compare the integrated sensor signals with this pulse and time-delay to previous ones, here, we take the even channel system for an example.
As we can see, there is very slight change between these two results, which can also be verified by the sum of square errors between the reference signal $y_{\text{ref}}(k)$ and the integrated sensor signal $y(k)$ in these two conditions, 19.491 and 19.879 for old optimal pulse and new optimal pulse, respectively. So we just keep the optimal pulse which we got from SISO case the optimal time-delay from MIMO case to test the experimental setup.

Figure 4-13: Cross-talk compensation with new optimal input and input delay.
4-3-2 Experimental results

Here, we also use the DoD curve to show the improvement in cross-talk compensation. Fig.4-14 and Fig.4-15 shows the DoD curve for even and odd channel system, respectively. With the optimal actuation pulse applied, as we can see, if all five channels are simultaneously actuated, the DoD curve is lower than that of only the direct-channel is actuated. The drop velocity has a decrease of 0.5 m/s along the jetting frequencies. The reason for that is the cross-talk, as we discussed in section 2-3-2. After the time-delay is used, as we supposed, the performance can be improved. The DoD curves of five-channel systems are quite close (less than 0.1 m/s) with that of only direct-channel system, both for even and odd channel systems.

![DoD curves for even channel system with time-delay](image)

**Figure 4-14:** DoD curves for even channel system with time-delay.
Figure 4-15: DoD curves for odd channel system with time-delay.

This also refers to the fact that the cross-talk in multi-channel system can be reduced by applying such an optimal time-delay between the neighboring channels, with a respect of drop-consistency in velocity.
The ability to deposit various types of material onto a substrate in certain patterns has made inkjet technology a very important technology. For document printing, a typical design of a piezoelectric inkjet printhead comprises a large array of piezo-actuated channels. The corresponding actuation pulse can be tuned to get the required drop-on-demand results. Generally, there are two operational issues in practical that would deteriorate the printing performance, that is, residual pressure oscillations and cross-talk. The former one relates to the fact that the ink in a pressure is not at rest state immediately after jetting a droplet, while cross-talk results from the fact that one channel would be influenced when its neighboring channels are actuated simultaneously. These two phenomena would limit the productivity as well as the drop consistency considerably, and then the printing performance of such a piezoelectric inkjet printhead. In this report, a systems and control approach is adopted to solve these limitations.

First is the modeling of such a printhead so as to provide good insight. Unfortunately, the internal physical relationships are quite difficult to get a clear recognition. So we use the system identification method to estimate models. With some assumptions, the task could be simplified to identify three models including one for direct channel and two for the first two neighboring channels. With the experimental setup, we choose the piezo-unit as both the actuator and the sensor because of its so-called ‘self-sensing’ capability. Then we have the piezo-sensor signal, which is proportional to the deviation of the channel pressure, as the output signal. Together with the actuation input signals, we can get good models with the system identification toolbox in Matlab.

Given the restrictions of feedback control, an optimization-based feedforward control method is used to tune the shape of the actuation pulse. With the identified direct-channel model, the result of a standard positive trapezoidal pulse adding a negative pulse could follow a reference trajectory quite well. This optimal actuation pulse would damp the ink channel pressure oscillations quickly. However, even with this optimal pulse, the resulted cross-talk is still very significant. Furthermore, a time-delay between the neighboring channels are considered to compensate that. By the optimization toolbox in Matlab, this off-line nonlinear optimization problem can be solved.
Finally, the optimal actuation pulse and input delay are implemented on printhead setup. The experimental results demonstrate that the research objectives are achieved by a considerable improvement of the printing performance with respect to the productivity and drop-consistency, e.g., the maximum velocity variation of the DoD curve in direct-channel can be reduced by 66.7%.

Several recommendations are provided for further increase of the performance quality.

First, more accurate models can be obtained by the adjustments of more precise equilibrium of the bridge circuit for ‘self sensing’ character. Second, the piezo-unit sensor signal is now collected by subtracting the output signals of filled-channel and empty-channel. Some problems, like the temperature difference or empty-channel with some ink left, would influence the quality of the sensor signal. So a structure design of the printhead can be investigated. That is, in a printhead, one ink channel can be settled in empty state all the time. This would also save a lot of efforts since we can collect the sensor signal with only one experiment. Third, the uncertainty in the parameters of the estimated models needs to be considered since there exists a rather large model variance when the identification procedure is based on piezo-unit sensed signal. A robust feedforward control can be used to extend the optimization-based technique.
Bibliography


List of Acronyms

DoD        drop-on-demand
PEM        prediction-error identification method
SMI        subspace model identification
SISO       single input single output
MIMO       multi input multi output
I/O        input and output
ETFE       empirical transfer function estimate
OE         output error
LCD        liquid crystal display
LED        light emitting diode
3D         three dimensional