CHARACTERISTICS OF TURBULENCE STRUCTURE AND UNDERTOW IN THE SURF ZONE

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CHARACTERISTICS OF TURBULENCE STRUCTURE AND
UNDERTOW IN THE SURF ZONE

砕波帯内における乱れ構造と戻り流れに関する研究

by

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ABSTRACT

The characteristic motion of water under breaking waves and the turbulence structure in the surf zone were investigated through detailed two-dimensional velocity measurements in a wave flume. Significant difference was found between the breaking processes in the outer and inner regions of the surf zone. The velocity field in each region consists of steady current, periodic wave motion, organized vortex motion and turbulence. It was found that the organized vortex motion caused by wave breaking was an important fluid motion connecting the wave motion and the turbulence.

The vertical profiles of the undertow, which is the steady current below the wave trough level, were investigated from velocity histories measured by hot-film and laser-Doppler-velocimeters. The turbulence generated in the upper layer by wave breaking prevents the development of the bottom boundary layer in the inner region. The vertical distribution of the mean Reynolds stress and the mean eddy viscosity coefficient in the inner region can be approximated by linear functions of the vertical elevation. The offshore-directed mean shear stress on the bottom is so large that it can not be neglected in the modeling of the undertow.

The transition point which was the boundary between the outer and inner regions of the surf zone was defined as the offshore limit of the quasi-steady breaking region. The distance from the breaking point to the transition point was expressed in terms of the breaking water depth and the bottom slope.

In order to describe the mechanism of the energy transfer during wave breaking accurately, a model was presented in which the organized large vortexes were taken into account as a transmitter of energy in the energy transfer process from wave motion to turbulence. The distribution of the turbulence energy calculated by this model agreed with the experimental results qualitatively.
The mass and momentum fluxes by the organized large vortexes were also discussed. The mass transport by breaking waves was found to be induced by the wave motion and the organized large vortexes.

By using the models of the energy distribution and the mass transport, a model was presented for the two-dimensional distribution of the undertow. The Reynolds stress and the eddy viscosity coefficient were quantitatively evaluated from the energy dissipation rate on the basis of the dimensional analysis. The variation of the mean water level in the surf zone was also predicted with a good accuracy by considering the momentum flux by the organized vortexes. The model can evaluate the distribution of the undertow on an arbitrary beach topography from the incident wave condition.
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<thead>
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<th>Description</th>
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<tr>
<td>$A$</td>
<td>total area of organized large vortex</td>
</tr>
<tr>
<td>$A_1$</td>
<td>section of an organized large vortex</td>
</tr>
<tr>
<td>$a$</td>
<td>amplitude of wave</td>
</tr>
<tr>
<td>$a_l$</td>
<td>amplitude of incident wave</td>
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<tr>
<td>$a_r$</td>
<td>factor of function of sheer stress distribution</td>
</tr>
<tr>
<td>$a'_r$</td>
<td>modified factor of function of sheer stress distribution</td>
</tr>
<tr>
<td>$a_\nu$</td>
<td>factor of function of eddy viscosity distribution</td>
</tr>
<tr>
<td>$B$</td>
<td>width of wave flume</td>
</tr>
<tr>
<td>$b_r$</td>
<td>factor of function of sheer stress distribution</td>
</tr>
<tr>
<td>$b'_r$</td>
<td>modified factor of function of sheer stress distribution</td>
</tr>
<tr>
<td>$C_r$</td>
<td>coefficient for evaluating $\tau_m$ from $D_B$</td>
</tr>
<tr>
<td>$C_\nu$</td>
<td>coefficient for evaluating $\nu_m$ from $D_B$</td>
</tr>
<tr>
<td>$c$</td>
<td>wave celerity</td>
</tr>
<tr>
<td>$c_{bore}$</td>
<td>celerity of bore</td>
</tr>
<tr>
<td>$c_g$</td>
<td>group velocity of wave</td>
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<tr>
<td>$c_{sol}$</td>
<td>celerity of solitary wave</td>
</tr>
<tr>
<td>$c_0$</td>
<td>wave celerity in offshore region</td>
</tr>
<tr>
<td>$D_B$</td>
<td>rate of energy dissipation by wave breaking</td>
</tr>
<tr>
<td>$D_{b+w}$</td>
<td>rate of energy dissipation by bottom and wall friction</td>
</tr>
<tr>
<td>$d_c$</td>
<td>water depth at wave crest</td>
</tr>
<tr>
<td>$d_t$</td>
<td>water depth at wave trough</td>
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<tr>
<td>$E_k$</td>
<td>kinetic energy of wave motion</td>
</tr>
<tr>
<td>$E_p$</td>
<td>potential energy of wave motion</td>
</tr>
<tr>
<td>$E_t$</td>
<td>total energy of wave</td>
</tr>
<tr>
<td>$E_v$</td>
<td>energy of organized large vortex</td>
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$E_w$  energy of wave motion
$E_1$  energy of rotational component of an organized large vortex
$E_2$  energy of parallel component of an organized large vortex
$E_3$  total energy of an organized large vortex
$e_v$  energy of organized large vortex per unit mass
$f_A$  total transfer factor of energy
$f_T$  energy transfer factor by breaking
$f_{b+w}$  dissipation factor by bottom and wall friction
$g$  acceleration of gravity
$H$  wave height
$H_b$  wave height at breaking point
$H_i$  incident wave height
$H_0$  deep water wave height
$h$  mean water depth
$h^*$  representative water depth
$h_b$  mean water depth at breaking point
$h_i$  water depth in offshore constant depth region
$h_0$  still water depth
$h_{0b}$  still water depth at breaking point
$h_{0p}$  still water depth at plunging point
$h_{0t}$  still water depth at transition point
$k$  wave number ($=2\pi/L$)
$L$  wave length
$L_b$  wave length at breaking point
$L_0$  deep water wave length
$l$  representative length of turbulence
$l_b$  distance from crest breaking point to breaking point
$l_d$ length of dissipation of energy of organized large vortex

$l_p$ distance from breaking point to plunging point

$l_t$ distance from breaking point to transition point

$M_{LLW}$ mass flux calculated by linear long wave theory

$M_{SFM}$ mass flux calculated by stream function method

$M_t$ mass flux by wave

$M_v$ mass flux by organized large vortex

$M_w$ mass flux by wave motion

$n$ ratio between group velocity and wave celerity

$p$ pressure

$p_s$ static pressure

$Q$ flow rate

$Q_r$ flow rate for wave recovering

$\hat{Q}$ amplitude of flow rate

$\hat{Q}_x$ $x$-component of amplitude of flow rate

$\hat{Q}_y$ $y$-component of amplitude of flow rate

$q$ representative velocity of turbulence

$R_p$ ratio of potential energy to kinetic energy of wave

$R_r$ ratio between mean shear stress at trough and bottom

$r$ radial coordinate in organized large vortex

$S$ section of an organized large vortex

$S_v$ excess momentum flux by organized large vortex

$S_w$ excess momentum flux by wave motion

$S_{xx}$ radiation stress

$T$ wave period

$T_B$ transfer rate of energy from $E_w$ to $E_v$

$t$ time
$t_d$ contribution to local dissipation rate by $T_B$ at certain value of $x$

$U$ $x$-component of steady current

$U_m$ $x$-component of mean steady current below trough level

$U_v$ contribution to $U_m$ from mass flux by organized large vortexes

$U_w$ contribution to $U_m$ from mass flux by wave motion

$U_{off}$ steady current in offshore region

$u$ $x$-component of water particle velocity

$u'$ $x$-component of turbulent fluctuation

$u_m'$ square root of vertical average of mean turbulence energy

$u_1$ rotational component of velocity of organized large vortex

$u_2$ parallel component of velocity of organized large vortex

$\hat{u}$ amplitude of horizontal water particle velocity at still water level

$\bar{u}$ $x$-component of moving averaged velocity

$\langle u \rangle$ $x$-component of ensemble mean velocity

$V$ $y$-component of steady current

$v$ $y$-component of water particle velocity

$v'$ $y$-component of turbulent fluctuation

$W$ $z$-component of steady current

$w$ $z$-component of water particle velocity

$w'$ $z$-component of turbulent fluctuation

$w_m'$ square root of vertical average of mean turbulence energy

$\bar{w}$ $z$-component of moving averaged velocity

$\langle w \rangle$ $z$-component of ensemble mean velocity

$x$ horizontal coordinate in cross-shore direction

$x_b$ breaking point

$x'_b$ crest breaking point

$x_p$ plunging point
transition point

spacing between mesh points

horizontal coordinate in longshore direction

vertical coordinate

vertical distance from bottom

coefficient multiplied to energy transfer factor

inclination of bottom

ratio of water particle velocity to wave celerity

ratio of water particle velocity to wave celerity at wave breaking

ratio of wave height to water depth

ratio of water particle velocity to wave celerity for wave recovering

ratio of water particle velocity to wave celerity on constant slope

energy of turbulence per unit mass

energy dissipation rate per unit mass

water surface elevation

elevation of boundary between organized large vortex and wave motion

elevation of wave crest

water surface elevation of reflected wave

elevation of wave trough

amplitude of water surface elevation

wave setup and setdown

wave length of laser light

kinematic viscosity

eddy viscosity

mean eddy viscosity averaged vertically below trough level

total viscosity

eddy viscosity in horizontal plane
\[ \begin{align*}
\xi & \quad \text{complex amplitude of wave} \\
\xi_0 & \quad \text{surf similarity parameter} \\
\rho & \quad \text{density of water} \\
\sigma & \quad \text{angular frequency (}=2\pi/T) \\
\tau & \quad \text{mean shear stress acting on horizontal plane in } x \text{-direction} \\
\tau_m & \quad \text{mean shear stress averaged vertically below trough level} \\
\phi_0 & \quad \text{amplitude of velocity potential at mean water level}
\end{align*} \]
CHAPTER 1
General Introduction

The velocity field and the energy process in the surf zone are of great importance since they exert wide influences over the phenomena such as the wave attenuation, sediment transport and material diffusion which actively take place due to wave breaking. As the velocity field in the surf zone is rather complicated because of its variation in time and space as well as its turbulent characteristics, only qualitative descriptions of breaking waves were presented early in the last decade through observation and visualization of the flow pattern in laboratories. With the recent development of the velocity measurement techniques, many researches have been carried out thereafter to clarify the structure of the turbulent velocity field in the surf zone by measuring the velocity fluctuation. However, the understanding of the characteristics of the velocity field in the surf zone and the process of the energy transfer due to wave breaking is not yet satisfactory.

The mass transport due to breaking waves, which is larger than that due to non-breaking waves, causes the undertow which is offshore-directed steady current below the wave trough level. Since such mass movement induces a macro-scale circulation in the surf zone, it plays significant roles in the sand movement and the material diffusion in combination with micro-scale eddies, i.e. turbulence, generated by the wave breaking. The relatively large velocity of the undertow causes large shear stress near the trough level and on the bottom. Although some models to estimate the distribution of the undertow were presented by adopting rather crude assumptions, the application was limited to the inner region in which the wave breaking can be regarded as a quasi-steady process. Since most of these models are constructed by using the local properties of waves and turbulence, the accurate prediction of the wave field and the energy dissipation in the surf zone is required.
Numerical models for predicting the wave height or wave energy variation in the surf zone from the incident wave condition have been proposed based on simple modeling of the breaking process. In these models, the wave energy is considered to be directly transferred to the turbulence energy. However, the rotational organized motion of water formed at the crests of the breaking waves cannot be neglected because the existence of the vortex motion shows an important characteristic of the energy transfer and also of the mass transport and momentum exchange in the surf zone. Most of the energy of irrotational wave motion is converted firstly to the energy of the organized large vortexes, although it finally dissipates to heat through turbulent dissipation. The previous models therefore are not able to explain the experimental results on the characteristics of the turbulent velocity field such as the spatial distribution of turbulence energy. In order to establish a practical model which is capable of dealing with various problems due to wave breaking, the comprehensive understanding of the surf zone dynamics is necessary.

The mechanisms of wave breaking in the outer and inner regions of the surf zone are much different. In the outer region, a rapid transition of wave profiles takes place with energy transfer from wave motion to the organized vortexes, especially in case of plunging breakers. On the other hand, a quasi-steady breaking occurs in the inner region and the energy transfer from the wave motion to the turbulence through the vortex is in an equilibrium state. Hence the definition and determination of the transition point which is the boundary between these two regions are important for the general description of the surf zone dynamics.

Motivated by the background mentioned above, the characteristics of the turbulent velocity field in the surf zone were investigated in the present study through detailed measurement of the two-dimensional velocity field under breaking waves. Then a model which describes the energy transfer during wave breaking is presented to evaluate the energy variation and the energy dissipation in the surf zone. In the
model, the wave energy is assumed to be transmitted to the turbulence energy through the organized large vortexes. A model for evaluating the distribution of the undertow is also proposed on the basis of the accurate description of the wave field and energy dissipation in the surf zone.

The first objective of the present study is to investigate the velocity field and the turbulence structure in the surf zone. The second objective is to describe the energy transfer in the surf zone on the basis of the understanding of the physical process and to formulate a model for estimating the vertical distribution of the steady current below the trough level which is valid throughout the surf zone.

The present dissertation is divided into two parts according to the above subjects. The former part, Chapter 2, deals with the characteristics of the velocity field through detailed and precise laboratory experiments. The organized motion of the water particle under breaking waves is investigated in terms of the velocity, vorticity and turbulent intensity. Next, the distributions of the turbulence properties such as the turbulence energy, Reynolds stress and eddy viscosity are investigated. In the last section of this chapter, the position of the transition point is discussed.

In the latter part, Chapter 3, a model which estimates the two-dimensional distribution of the undertow is presented. The transfer of energy in the surf zone is firstly described based on the results obtained in Chapter 2. Then the energy variation and energy dissipation rate in the surf zone are estimated by applying the time-dependent mild slope equation. The organized large vortexes formed at the wave crests are taken into account as transmitters of energy from the wave motion to the turbulence in order to accurately describe the energy distribution in the surf zone.
CHAPTER 2

Turbulence Structure in the Surf Zone

2.1 Introduction

The velocity field in the surf zone is much concerned with the phenomena there. Hence, it is a very important technical problem to make it clear. Since the velocity field under breaking waves is rather complicated because of its variation in time and space as well as its turbulent characteristics, only qualitative descriptions of breaking waves were presented early in the last decade through visual observation in laboratories.

Svendsen et al. (1978) separated the surf zone into two regions which were outer and inner regions (Fig. 2.1.1). In the outer region, rapid transitions of wave shape occurs, particularly in case of plunging breakers. Sawaragi and Iwata (1974) suggested the existence of a large scale vortex formed just below the wave plunging. It is frequently called “horizontal roller” or “plunger vortex”. Miller (1976) showed the process of formation of the plunger vortex and the recirculating motion near the water surface caused by the splash of wave plunging by using photography. He also observed the same processes with a relatively smaller scale in spilling breakers. In the inner region, quasi-steady breaking waves which are similar to bores propagate shoreward with attenuation. Both such bore-like waves and plunger vortexes seem to give strong effect on the generation of turbulence in the surf zone.

With the recent development of the velocity measurement techniques, many works have been carried out to clarify the structure of the turbulent velocity field in the surf zone by measuring the velocity. Hattori and Aono (1985) measured the velocity field under breaking by using a hot-film-velocimeter. They separated the
Fig. 2.1.1 Wave characteristics in the surf zone.
turbulence from measured velocity considering the coherence between the velocity and the water surface elevation and discussed the turbulence structure in the inner region, which they pointed out to be strongly three-dimensional. Nadaoka et al. (1989) found from flow visualization and measurement by a laser-Doppler-velocimeter that two-dimensional horizontal vortexes developed near the water surface in front of the wave crests and obliquely downward stretched eddies with strong three-dimensionality were formed behind the wave crests. Sakai et al. (1984) discussed the variation of the Reynolds stress in the surf zone. Okayasu et al. (1986) verified the initial breaking process by detailed measurements of the velocity field in the outer region and Mizuguchi (1986) also discussed the rapid transition of waves near the breaking points.

The existence of "undertow" caused by the mass transport of breaking waves was first observed by Bagnold (1940) and has been recognized for many years [see e.g. Hansen and Svendsen (1984)]. The undertow plays an important role for the circulation of material or the sediment transport in the surf zone. However our knowledge on the interaction between the turbulence and the undertow or the mass flux by the vortexes as well as on the breaking process in the surf zone is not yet satisfactory. In order to clarify the turbulence structure, the flow field and the energy dissipation process in the surf zone, it is important to investigate the motion and roles of the large vortexes which are considered to generate turbulence and characterize the velocity field in the surf zone.

In this chapter, these characteristics of the velocity field in the surf zone will be clarified through detailed and precise laboratory experiments. The distributions of the turbulent properties such as the turbulence energy, the Reynolds stress and the eddy viscosity will be investigated. The energy dissipation in the surf zone will also be discussed in relation to the variation of the wave energy.
The mechanisms of wave breaking in the outer and inner regions are much different as already mentioned. Hence, the position of the transition point which is the boundary between the two regions is also important for comprehensive descriptions of the surf zone. However, no systematic investigation to determine the position of the transition point was carried out. In this chapter, the position of the transition point and the plunging point will be discussed later.
2.2 Experiments on Two-Dimensional Velocity Field in the Surf Zone

2.2.1 Experimental Facilities and Arrangement

Laboratory experiments were carried out in a two-dimensional wave flume which was 23 m long and 0.8 m wide with a wave generator at one end. The generator was controlled by electric signals from a function oscillator which generated regular sinusoidal signals in the present experiments. The beach models which had smooth beds were set at the other side of the flume. A partition was set at the center of the beach models for the sake of keeping the phenomena to be two-dimensional, i.e., uniform in the longshore direction. Figure 2.2.1 shows a sketch of the flume.

The experiments were divided into series A and B. The detailed measurements of the velocity fields were performed in series A to get a basic and essential understanding of the velocity field throughout the surf zone. For case A–1, the step-type beach profile was used to elucidate the behavior of the plunger vortex. The beach model consisted of a 1/10 slope and a horizontal bed. The still water depth on the horizontal bed was 10 cm. The reasons why this type of beach topography was chosen were to fix the breaking point of each wave and to make a region deep enough for the measurement. A large amount of thin plastic strings were put on the bed at the shoreward end instead of a slope to prevent the wave reflection. The measuring area was from 0.5 cm to 14.5 cm above the bottom and 80 cm long in an cross-shore directed vertical plane. The measuring points were arranged to make 2 cm grids. The arrangement is shown in Fig. 2.2.2.

For cases A–2 and A–3, a constant slope of 1/20 was used to measure the whole area of the surf zone. The measuring areas were about 250 cm long in both cases, but the arrangements of measuring points were different. The 2 cm grids arrangement was adopted for case A–2 to obtain the vorticity distribution. The
Fig. 2.2.1 Plan and side view of the wave flume (case A-3).
Fig. 2.2.2 Arrangement of measuring points (case A-1).
lowest points were 7mm above the bottom. The detailed arrangement is shown in Fig. 2.2.3. On the other hand, for case A–3, a grid which was sparse in the horizontal direction and dense in the vertical direction was used for the purpose of detailed measurements of the mass flux variation and the vertical profiles of the undertow. The measuring points were arranged with distance of 10 cm in the cross-shore direction and distance from 2.5 to 10 mm in the vertical direction. On every measuring line, at least seven measuring points were set below the trough level for accurate evaluation of the mass flux. The lowest points were 2 mm above the bottom. The arrangement is shown in Fig. 2.2.4 which is exaggerated in the vertical direction.

The measuring points were arranged only close to the bottom for case A–4 to investigate the details of the bottom boundary layer in the surf zone. Nine measuring lines were allocated every 40 cm from the offshore side beyond the breaking point to the still water shoreline. The measuring points were arranged 1, 2, 3, 5, 10 and 20 mm above the bottom along each line.

For series B, velocity fields were measured for various beach topographies and incident wave conditions. Three kinds of beach topographies were used. They were a 1/20 uniform slope (from case B–1 to B–4), a 1/30 uniform slope with 1 m of a 1/20 slope at its offshore end (from case B–5 to B–9) and a step-type topography which consisted of 4.11 m of a 1/10 slope, 2.85 m of a horizontal bed and a 1/10 slope at its onshore end (cases B–10 and B–11). The still water depth on the horizontal bed were 6 cm for cases B–10 and B–11. The arrangements of the measuring points were sparse in the horizontal direction and dense in the vertical direction for the purpose of detailed measurements of the undertow profiles. The measuring points were arranged along 6 or 7 vertical measuring lines in every case but case B–11. The first measuring lines were set on the wave breaking points. The second lines were located on the plunging points in case of plunging breakers, or the intermediate
Fig. 2.2.3  Arrangement of measuring points (case A-2).
Fig. 2.2.4 Arrangement of measuring points (case A-3).
point between lines 1 and 3 for spilling breakers. From the third, the measuring lines were arranged in the inner regions where the bore-like waves developed well. In case B–11, the measuring lines were arranged to cover the wave recovery point which was at \( x = -115 \) cm by visual observation. The lowest measuring points in each measuring line are 1mm above the bottom in cases B–1 to B–4, and for cases B–5 to B–11, they are 2 mm above the bottom. The vertical distance of those points were from 1 to 20 mm and the highest points are near the mean water levels. As examples, the arrangement of cases B–1, B–5 and B–11 are shown in Fig. 2.2.5.

2.2.2 Experimental Conditions

The \( x \)-axis and \( z \)-axis were set to be shoreward and vertically upward, respectively. The origin of the coordinates in case A–1 was at the offshore end of the measuring area as shown in Fig. 2.2.2. In the other cases, the shoreline at the still water level was set to be the origin. The conditions of the experiments of series A and series B are listed in Table 2.2.1 and 2.2.2, respectively. In the table, \( T \) is the wave period, \( h_i \) the water depth in the offshore constant depth region, \( H_i \) the wave height there, \( H_0/L_0 \) the deep-water wave steepness, \( x_b \) the breaking point, \( x_p \) the plunging point.

<table>
<thead>
<tr>
<th>case</th>
<th>slope</th>
<th>( T (s) )</th>
<th>( h_i ) (cm)</th>
<th>( H_i ) (cm)</th>
<th>( H_0/L_0 )</th>
<th>( x_b ) (cm)</th>
<th>( x_p ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A–1</td>
<td>1/10+flat</td>
<td>1.55</td>
<td>40.0</td>
<td>6.95</td>
<td>0.0201</td>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>A–2</td>
<td>1/20</td>
<td>1.50</td>
<td>40.0</td>
<td>7.36</td>
<td>0.0226</td>
<td>-225</td>
<td>-162</td>
</tr>
<tr>
<td>A–3</td>
<td>1/20</td>
<td>1.50</td>
<td>36.4</td>
<td>8.15</td>
<td>0.0249</td>
<td>-250</td>
<td>-215</td>
</tr>
<tr>
<td>A–4</td>
<td>1/20</td>
<td>1.50</td>
<td>39.5</td>
<td>7.48</td>
<td>0.0230</td>
<td>-250</td>
<td>-200</td>
</tr>
</tbody>
</table>
Fig. 2.2.5 Arrangement of measuring points (cases B-1, B-5 and B-11).
Table 2.2.2  Conditions of experiments of series B.

<table>
<thead>
<tr>
<th>case</th>
<th>slope</th>
<th>$T$ (s)</th>
<th>$h_i$ (cm)</th>
<th>$H_i$ (cm)</th>
<th>$H_0/L_0$</th>
<th>$x_b$ (cm)</th>
<th>$x_p$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>1/20</td>
<td>2.00</td>
<td>40.0</td>
<td>8.50</td>
<td>0.0139</td>
<td>-270</td>
<td>-220</td>
</tr>
<tr>
<td>B-2</td>
<td>1/20</td>
<td>2.00</td>
<td>40.0</td>
<td>5.63</td>
<td>0.0092</td>
<td>-200</td>
<td>-165</td>
</tr>
<tr>
<td>B-3</td>
<td>1/20</td>
<td>1.17</td>
<td>40.0</td>
<td>9.87</td>
<td>0.0502</td>
<td>-275</td>
<td>-10-1)</td>
</tr>
<tr>
<td>B-4</td>
<td>1/20</td>
<td>0.91</td>
<td>40.0</td>
<td>6.69</td>
<td>0.0542</td>
<td>-200</td>
<td>-10-1)</td>
</tr>
<tr>
<td>B-5</td>
<td>1/30</td>
<td>1.61</td>
<td>40.0</td>
<td>8.80</td>
<td>0.0232</td>
<td>-410</td>
<td>-10-1)</td>
</tr>
<tr>
<td>B-6</td>
<td>1/30</td>
<td>1.97</td>
<td>40.0</td>
<td>6.17</td>
<td>0.0104</td>
<td>-290</td>
<td>-230</td>
</tr>
<tr>
<td>B-7</td>
<td>1/30</td>
<td>1.96</td>
<td>40.0</td>
<td>8.22</td>
<td>0.0140</td>
<td>-410</td>
<td>-10-1)</td>
</tr>
<tr>
<td>B-8</td>
<td>1/30</td>
<td>1.12</td>
<td>40.0</td>
<td>8.26</td>
<td>0.0457</td>
<td>-350</td>
<td>-10-1)</td>
</tr>
<tr>
<td>B-9</td>
<td>1/30</td>
<td>1.23</td>
<td>40.0</td>
<td>6.05</td>
<td>0.0279</td>
<td>-290</td>
<td>-10-1)</td>
</tr>
<tr>
<td>B-10</td>
<td>1/10+flat</td>
<td>2.01</td>
<td>47.1</td>
<td>10.82</td>
<td>0.0179</td>
<td>-440</td>
<td>-385</td>
</tr>
<tr>
<td>B-11</td>
<td>1/10+flat</td>
<td>1.20</td>
<td>47.1</td>
<td>9.24</td>
<td>0.0447</td>
<td>-415</td>
<td>-365</td>
</tr>
</tbody>
</table>

1) spilling breaker

In addition to the table, the wave height to the water depth ratios, $H_b/h_b$, at the breaking points were 1.02, 0.901 and 0.954 in cases A-1, A-2 and A-3, respectively. The breaking points (abbreviated to “B.P.”) and the plunging points (“P.P.”) were indicated in the figures. After breaking, the wave propagates shoreward forming successive vortexes on the water surface. In case A-3, the point where these vortexes formed a rather stable bore-like waves and the point where the vortexes formed just in front of the wave crests began to attenuate were at $x = -170$ cm and $x = -80$ cm, respectively. The mean shoreline was at $x = 50$ cm in case A-3.

2.2.3  Data Processing and Analysis

A hot-film-velocimeter with a split film probe was used to measure the histories of two-dimensional velocity vectors lying on the $xz$ plane for cases A-1 and A-2. The split-hot-film sensor consists of two platinum films on a single quartz fiber whose length is 1.2 mm and the diameter is 0.15 mm. The output from the two films provides a set of data of the magnitude of the velocity component normal to
the axis of the fiber, and of the component normal to the split plane. The magnitude of the velocity component parallel to the split plane is calculated from these two values though its sign cannot be detected. In case A-1, the sign of the cross-shore component of the velocity could not be determined. An assumption was therefore made that the surface profile and the cross-shore velocity are in phase. Hence, the sign of the velocity was reversed once in every wave period according to the surface profile. In case A-2, velocities were measured twice, once for the cross-shore and the other time for the vertical direction to avoid such an uncertainty.

Velocities for cases A-3 and A-4 and all cases of Series B were measured by a two-component laser-Doppler-velocimeter equipped with a 4 W Ar-ion laser tube. The laser beam projected from the laser tube is separated to a green light (the wave length $\lambda = 514.5$ nm) and a blue light ($\lambda = 488$ nm) by color filters. Since the two lights do not interfere each other, the two-component laser-Doppler-velocimeter works as if two completely individual velocimeters. Consequently, used with frequency shifters, full information of velocities for two component can be obtained. Moreover, the laser-Doppler-velocimeter has a great advantage for the measurements of turbulent velocity fields that it works without physical contact to the fluid. If the laser beams have transverse intensity distributions which obey the Gaussian distribution, the form of the probe of the velocimeter is considered to be ellipsoid. The longitudinal diameter of the probe was about 3.7 mm (green beam) or 3.5 mm (blue beam) and transverse diameter was 0.17 mm (green beam) or 0.15 mm (blue beam) in the present experiments.

The module was used as three-beam backscatter mode with frequency shifting. The laser-Doppler-velocimeter was capable of evaluating two components of velocities in the normal to the optic axis of the module by detecting the Doppler-shift frequency of the green and the blue lights scattered by particles in the focus of the laser beams. In order to obtain the scattered light with sufficient intensity, a
small amount of white paint which contained acrylic emulsion was seeded in the flume. Detailed measurements of velocity components \((u, w)\) in a vertical plane were conducted by injecting the laser beams perpendicularly through the side wall of the flume. Though the laser beams projected with a little angle of depression for the measuring points close to the bottom, the deviation of velocities is negligible. Generally, it is difficult to measure the velocity at a point very close to the solid boundaries because of increasing noise by the diffused reflection. It became a serious problem especially for case A-4 in which the velocities near the bottom were primarily measured for investigation of the bottom boundary. Plastic mirrors were therefore placed on the bottom in case A-4 to prevent reflected laser lights from disturbing the signal. It was proved to be successful in getting velocity data with high S/N (signal to noise) ratio.

The velocity data were sampled every 10 ms for all cases except for case A-2 in which the sample timing was 12 ms and were converted into digital data over 30 (for cases A-1 and A-2) or 100 (for the other cases) wave periods. The data of the water surface elevation over the measuring point were also taken simultaneously by using a capacitance-type wave gage. The ensemble mean (equi-phase-mean) value of velocity which is expressed by \(<u>\) in \(x\)-direction or \(<w>\) in \(z\)-direction was calculated as the average of the velocity at the same phase of every wave. The steady current components \((U, W)\) were calculated from those ensemble mean values. The turbulence components denoted by \(u'\) and \(w'\) was determined as the deviation from the ensemble mean value.

The Reynolds stress was calculated from the turbulence as \(-\rho uu'w'\) in case that the laser-Doppler-velocimeter was used, where \(\rho\) is the water density. The mean Reynolds stress \(-\rho uu'w'\) was obtained by averaging the Reynolds stress over one wave period. From the steady current and the mean Reynolds stress, the mean eddy viscosity coefficient \(\nu_e\) was evaluated by using the eddy viscosity model. It
means that the mean eddy viscosity coefficient was obtained not by averaging the instantaneous values of the eddy viscosity coefficient but from the vertical gradient of the steady current and the mean Reynolds stress.
2.3 Velocity Field and Turbulence Structure in the Surf Zone

The results of the experiments will be shown in this section. The spatial distributions and time histories of ensemble mean velocities, vorticities and turbulent intensities were obtained. The time averaged values of each quantity were also obtained. The mean Reynolds stress and the mean eddy viscosity were calculated for cases in which the laser-Doppler-velocimeter was used. From these results, the characteristics of the velocity field in the surf zone will be discussed.

2.3.1 Difference of Turbulence Component due to the Definition

Figure 2.3.1 shows a comparison between the ensemble mean values of velocity \( \langle u \rangle \) and the moving averages \( \bar{u} \) of 0.11 s (time averages over 0.11 s of the instantaneous values of the velocity) at the point \( x = -100 \) cm and 2 cm above the bottom in case A-3. It is found in the surf zone that the variation of the velocity in one wave by the organized motion, which is caused by the fluctuation of the breaking point etc., is far larger than the turbulence in higher frequency which is beyond 10 Hz. Considering the previous concept of turbulence in which the turbulence does not include the organized variation, the definition of turbulence by the ensemble-mean method may not be proper. Sakai et al. (1984) defined the turbulence by the moving-average method to obtain the Reynolds stress in the surf zone. However, the mean Reynolds stress calculated by using the ensemble-mean method as 
\[
-\rho(\langle u \rangle)(w - \langle w \rangle)
\]

is far larger than that by the moving-average method calculated as 
\[
-\rho(u - \bar{u})(w - \bar{w})
\]

as shown in Fig. 2.3.2 for case A-3. The values by the moving-average method is so small that it can hardly be distinguished in the figure. It means that the fluctuation in low frequency by the organized motion is predominant for the momentum exchange in the surf zone. It can therefore be considered that the ensemble-mean method is better for defining the turbulent component to
Fig. 2.3.1  Comparison between the ensemble mean values and the moving averages of velocity.
Fig. 2.3.2 Mean Reynolds stress obtained by the ensemble mean method and the moving average method (case A-3).
investigate the steady current distribution by using the eddy viscosity model which will be discussed later in the present study.

2.3.2 Organized Vortex Motion Caused by Wave Breaking

Figure 2.3.3 gives a result for case A-1. The field of the ensemble mean velocities, the vorticity field and the turbulent intensity field at the time immediately after the wave plunging are shown in the figures. A large scale vortex can be observed obviously by the velocity vectors around $x = 22$ cm and $z = 3$ cm in the figure. The high positive (clockwise in the figure) vorticity and turbulent intensity regions are in good agreement with the velocity vectors which suggest the existence of the plunger vortex.

Large magnitude of the velocity vectors can be seen at the wave crest. Another large value of the vector is seen at $x = 26$ cm. It can be considered that this small wave crest is caused by the large mass of water which penetrates into the water surface by the wave plunging pushing up the neighboring water body as Peregrine (1983) described. These crests have large magnitudes of positive vorticities, although turbulent intensities are not so large. There should be large vortexes, which rotate clockwise in this figure, also at the crests. On the other hand, negative (counterclockwise) vorticities can be seen between these vortexes. These negative vorticities should be made to cancel the positive vorticities caused by the large vortexes, although the spatially averaged vorticity is positive. The most intense turbulence exists between the regions which have positive values of vorticity and negative values of vorticity.

Figure 2.3.4 is the successive figures of the ensemble mean velocities after plunging. The process that the first plunging pushes up the neighboring water body and the second plunging occurs in a smaller scale is shown. The plunger vortex formed by the first plunging does not move according with the crest and goes downward.
Fig. 2.3.3 Velocity, vorticity and turbulent intensity immediately after wave plunging (case A-1).
Fig. 2.3.4  Ensemble mean values of velocity vectors (case A–1).
The contour maps of the turbulent intensity at the same phases are shown in Fig. 2.3.5. The high turbulent regions caused by the first and second wave plunging go ahead slowly behind the wave crests, spread, and then dissipate. These figures show that the most part of the turbulence was produced near the surface and the turbulence generated near the bottom is little.

Figure 2.3.6 shows the distribution of the dropout rate for the measurements by the laser-Doppler-velocimeter for case A–3. The dropout rate is defined as the ratio of the time in which the intensity of the scattered light was not sufficient to resolve velocity component to the total time of the measurements. Although it is natural that the figure indicates so large values of the dropout rate above the trough level where the measuring points were beyond the water surface for a while, the values are also large in the region from \( x = -120 \) cm to \( x = -180 \) cm below the trough level. It can be considered that a large amount of air bubbles injected from the water surface obstructed the laser beams, which also confirms the existence of the plunger vortexes.

Figure 2.3.7 gives the successive contour maps of the turbulent intensity for case A–2. The figure shows a high turbulent region in front of the wave crest. It should be noticed that this turbulence is caused by the vortex motion as described by Nadaoka (1986) which is formed there. In the figure, it is also confirmed that the turbulence spreads downward, so the vortex motion in front of the wave crest should influence the turbulent flow field there and the bottom boundary layer. The figure indicates another turbulent region from \( x = -40 \) cm to \( x = -80 \) cm at the phases \( t/T =0.0–0.6 \). Since this high turbulent region stays there after the wave crest passes, it can be said that this turbulence is due to the interaction between the bore-like wave and the return flow from the swash zone. The high turbulent region caused by the wave plunging is also ascertained in this figure.
Fig. 2.3.5  Distribution of turbulent intensity (case A-1).
Fig. 2.3.6    Dropout rate of signals of the laser-Doppler-velocimeter (case A–3).
Fig. 2.3.7  Distribution of turbulent intensity (case A-2).
Based on these facts, it can be concluded that the generation of the plunger vortex has much influences on the characteristics of the velocity field and is much concerned with the energy dissipation process under plunging waves. The vortex motion formed in front of the wave crest has also important roles in the energy dissipation process. As the vortexes must have some rotational kinetic energy, it also relates to the energy transfer of breaking waves in the surf zone. Both of these vortex motions should not be included in the wave component, but they are not turbulence, either. In the present study, they are called "organized large vortexes". Since the vortex which is formed by the wave plunging is usually called "plunger vortex", the term "organized large vortexes" generally means the vortex motion formed in front of the crests of the bore-like waves hereafter.

Up to now, the velocity field in the surf zone were often described as the sum of a steady current, periodic wave motion and turbulence. This time, however, the measured results showed that the periodic component consists of the irrotational wave motion and organized rotational motion due to the wave breaking. It can be said that the organized vortexes are an important fluid motion connecting the wave motion and turbulence. Hence, it is proposed that the velocity field in the surf zone should be divided into four components as \([\text{steady current}] + [\text{irrotational wave motion}] + [\text{organized vortex motion}] + [\text{turbulence}]\).

In the present study, the turbulence is defined as the deviation from the ensemble mean values of velocity. This definition is based on the idea that the waves are completely periodic so that the vortexes are formed at the same position and phase at every time. However, the turbulence defined here includes a part of the organized motion, which is caused by the fluctuation of the breaking point \(etc\). as already shown in §2.3.1. The situation becomes much more complicated in case of irregular waves for which the ensemble mean does not make any sense. Hence, it may be
better to define the "organized vortex motion" as "a coherent structure with large-scale eddies which depends on the flow geometry concerned" as defined by Nadaoka et al. (1989) when the physical process of the wave breaking is considered.

2.3.3 Vertical Distribution and Average of Undertow

Figure 2.3.8 shows the steady current distribution for case A-3. It can be seen that the shape of the undertow profiles in the inner region is significantly different from that around the breaking point. The gradient of the profiles $\frac{\partial U}{\partial z}$ below the trough level is negative in the outer region, but it is positive or zero in the inner region except near the bottom. The large magnitude of the gradient shows that large shear stress works near the mean water level. The steady current distribution for case A-4 is shown in Fig. 2.3.9. In the inner region the velocity at the elevation of 1 mm above the bottom indicates large value in the offshore direction, while it still directs onshore at the plunging point. This is because the bottom boundary layer in the inner region does not develop well due to agitation of the turbulence from the upper layer. The influence by the bottom to the steady current can be seen up to 3 mm above the bottom there.

The cross-shore variation of the mean steady current $U_m$ averaged vertically from the bottom to the trough level was shown in Fig. 2.3.10 for case A-3. The vertically averaged undertow in the bore region is larger than that at the plunging point. It keeps almost the same value from the breaking point to the plunging point, then rapidly increases from the plunging point despite the monotonous decrease of the wave height. It continues to take a large value in the region where the bore-like waves were fully developed, and then gradually decreases to the shore line. The increment of the value should be caused by the organized large vortexes formed in front of the crests of the bore-like waves.
Fig. 2.3.8  Distribution of steady currents (case A-3).
Fig. 2.3.9 Distribution of steady currents (case A-4).

Fig. 2.3.10 Cross-shore variation of vertically averaged steady current (case A-3).
The distribution of the steady current for case B–11 in which the breaking waves recovered on the flat bed is shown in Fig. 2.3.11. The bore-like waves almost completely recovered at $x = -115$ cm. The profiles of the undertow in the recovery zone are much more similar to those at the outer region rather than those in the inner region. The wave height to the water depth ratio at the breaking point was 0.904. On the other hand, it was 0.294 in the wave recovery zone ($x = -105$ cm). The small magnitude of the steady current in $x \geq -205$ cm means that the mass transport by the bore-like waves is little.

Figure 2.3.12 gives the distribution of the turbulent intensity for case B–11. Since the turbulence is defined as the deviation from the ensemble means, the turbulence of the horizontal component near the bottom is large by fluctuation for each wave. The turbulent intensity at the trough level is still large in $x \geq -205$ cm where the steady current is small. It is consistent with the visual observation that the profile and magnitude of the distribution at $x = -105$ cm are about the same as those at the breaking point ($x = -415$ cm). It can be considered that the mass transport by the organized large vortexes are almost zero in $x \geq -205$ cm.

2.3.4 Distributions of the Reynolds Stress and the Eddy Viscosity Coefficient

Figure 2.3.13 shows the distributions of the mean Reynolds stress of cases B–2 and B–7. The distributions of the mean eddy viscosity coefficient of those cases are given in Fig. 2.3.14. In the figures, it can be seen that both of them decrease linearly from the trough level to the bottom in the inner region. And on the bottom, the value of the mean eddy viscosity coefficient is very small compared with that at the trough level. This should correspond to the fact that the turbulence produced by the large vortex on the front face of the wave crests is far larger than that generated near the bottom. But with respect to the mean Reynolds stress, it is observed that the offshoreward shear stress is too large to be neglected. The result of this study for
Fig. 2.3.11 Distribution of steady currents (case B-11).

Fig. 2.3.12 Distribution of turbulent intensity (case B-11).
Fig. 2.3.13  Distributions of mean Reynolds stress (cases B–2 and B–7).
Fig. 2.3.14  Distributions of mean eddy viscosity coefficient (cases B–2 and B–7).
the vertical variation of the mean eddy viscosity coefficient is in contradiction to the analysis by Svendsen (1987) that the mean eddy viscosity coefficient is independent of the vertical coordinate.

2.3.5 Wave Height Attenuation and Cross- Shore Distributions of the Turbulence Energy

Figure 2.3.15 shows the cross-shore distribution of the vertically averaged mean turbulence energy $u_m^2 = \frac{1}{dt} \int_0^{d_t} \overline{u'^2} \, dz'$ and $w_m^2 = \frac{1}{dt} \int_0^{d_t} \overline{w'^2} \, dz'$ for cases B-2 and B-6. The wave height variations are also given in the figures. The wave height rapidly decreases after wave breaking in both cases, however the turbulence does not increase until near the plunging points. The tendency is similar in case of the spilling breakers, although the change becomes gentle as shown in Fig. 2.3.16 for cases B-3 and B-7.

Since the energy of the turbulence which is rapidly transferred to heat corresponds to the energy dissipation and the wave height attenuation implies rapid decrease of the wave energy, the figures indicate that the wave energy loss is not equal to the local energy dissipation. It is inconsistent with the previous models of the energy decay or the wave attenuation in the surf zone in which the wave energy directly dissipates to heat. One reason may be that the wave height attenuation does not mean the wave energy decay. However, the primary reason should be the existence of the organized large vortexes including both of the plunger vortexes and those formed as bore-like waves. These vortexes must have some energy which was not considered in the previous models. The little turbulence energy in the outer region means that the total energy of breaking waves which includes the vortex energy does not decrease so much. Hence, the transformation of waves in the outer region should be the process in which the energy of wave motion is transferred to the energy of the organized large vortexes. The concept in the previous models
of the energy dissipation that the wave energy decay equals the generation of the turbulence energy is not adequate. The further investigation about the magnitude of the energy of the organized large vortexes will be discussed later in Chapter 3.
2.4 Experiments on Length of Outer Region

2.4.1 Definition of Transition Point

The surf zone is divided into the “outer region (transition region)” where a rapid transformation of the wave profiles take place and the “inner region” where rather stable bore-like waves propagate according to the dynamics of wave breaking. This concept was first introduced by Svendsen et al. (1978) from the visual observations of wave behavior after breaking. The experimental results shown in §2.3 revealed the difference between the two regions in the distributions of the turbulence and the steady current. Since it can be considered that the mechanism of wave breaking in the outer region is essentially different from that in the inner region, it is necessary to distinguish them for modeling the energy transfer or dissipation process in the surf zone.

Svendsen (1984a) found that the mean water level is almost horizontal over some distance (5–8 times the mean water depth at the breaking point) from the breaking point, although a rather sharp increase in its slope occurs in the onshore side. He termed the point where the slope of the mean water level changes as the “transition point”, and defined it as the limit between the outer and the inner region. Basco (1985) divided the surf zone into three regions which were termed Zones I, II and III according to the slope of the mean water level and the wave height variation. He mentioned that Zone I covers roughly 25–30% of the wavelength at the breaking point and it corresponds to the outer region defined by Svendsen (1984a) from the slope of the mean water level. However, he also mentioned that “the transition point will probably occur during the latter stages of the breaking process”, which suggested that the boundary between the outer and the inner region may not be the onshore side of Zone I according to his understanding.
Yamashita et al. (1988) described the motion of the organized large vortexes formed by spilling breakers precisely by using a video recording system. They also separated the surf zone into three regions according to the behavior of the vortexes, which are the "rapid transformation region", "large vortex region" and "quasi-steady vortex region". They found that the size of the vortexes and the time intervals of their generations rapidly decrease in the former two regions, on the other hand, they become almost stable in the "quasi-steady vortex region".

As mentioned above, the boundary between the outer region and the inner region has not been investigated systematically yet, and its definition is not obvious, despite the importance in the energy process in the surf zone. The plunging point which is also important for the dynamics of wave breaking as the point where the turbulence increases abruptly has not been investigated systematically, either. In the present study, the transition point which is the boundary between the outer and the inner region is defined as the "point where fully developed bore-like waves are formed", and is interpreted as the offshore end of the quasi-steady energy decaying region. Since the criterion is based on the visual observation of the large vortexes formed in front of the wave crests, it is considered that the transition point defined here corresponds to the boundary between the "large vortex region" and the "quasi-steady vortex region" described by Yamashita et al. (1988).

The transition points and the plunging points were measured for various incident waves on different bottom slopes. The relation between the distances from the breaking point to the transition point and the bottom slope or the wave steepness will be investigated. The distance to the plunging point will also be investigated.

2.4.2 Experimental Facilities, Procedures and Conditions

Laboratory experiments were performed in a two-dimensional wave flume which was 12 m long and 20 cm wide with a glass side wall. The beach models were made
of 1/10, 1/20, 1/30 and 1/50 constant slopes which had 1/20 slope of 1 m at their 
feet in cases of the 1/30 and 1/50. The breaking point, plunging point (in case of 
the plunging breaker) and bore developing point were measured for various regular 
wave conditions.

The breaking point was defined as the point where the wave height took its 
maximum value, and the "bore developing point" which is the limit between the 
outer region and the inner region was determined as the point where bore-like 
waves which looked stable enough were formed. It may not be proper for a strict 
argument to decide the boundary by the visual observation because it is inevitable 
to imply the subjectivity. However, the determination by the visual observation is 
far more convenient than the way adopted by Yamashita et al. (1988) and it should 
be considered that the change in the wave attenuation, the slope of the mean water 
level or the distribution of the turbulent intensity is the secondary phenomenon 
caused by the difference of the energy transfer process of the waves.

The breaking points and others were averaged over about 10 waves because 
they fluctuated a little by the wave reflection or the oscillation by long period 
waves in the flume. The periods and the heights of the incident waves were 0.6–1.8 
s and 5–8 cm for every beach slope. The still water depth in the offshore constant 
depth region was 18–21 cm. The detailed experimental conditions are listed in 
Table 2.4.1–4 together with the experimental results. In the tables, \( h_{ob} \) is the still 
water depth at the breaking point, \( l_p \) the distance from the breaking point to the 
plunging point and \( l_t \) the distance from the breaking point to the transition point.
Table 2.4.1  Conditions and results of experiments of series C-1 (\(\tan \beta = 1/10\)).

<table>
<thead>
<tr>
<th>case</th>
<th>(T) (s)</th>
<th>(h_i) (cm)</th>
<th>(H_i) (cm)</th>
<th>(H_0/L_0)</th>
<th>(h_{ob}) (cm)</th>
<th>(l_p/h_{ob})</th>
<th>(l_l/h_{ob})</th>
</tr>
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<tbody>
<tr>
<td>C-1-1</td>
<td>0.617</td>
<td>19.67</td>
<td>5.92</td>
<td>0.1040</td>
<td>7.34</td>
<td>-1)</td>
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<td>0.0882</td>
<td>7.35</td>
<td>2.04</td>
<td>4.76</td>
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<td>C-1-3</td>
<td>0.800</td>
<td>19.69</td>
<td>7.78</td>
<td>0.0940</td>
<td>8.25</td>
<td>2.31</td>
<td>4.85</td>
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<tr>
<td>C-1-4</td>
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<td>0.0454</td>
<td>7.37</td>
<td>2.71</td>
<td>6.51</td>
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<tr>
<td>C-1-6</td>
<td>1.105</td>
<td>19.72</td>
<td>6.79</td>
<td>0.0383</td>
<td>7.39</td>
<td>3.38</td>
<td>7.44</td>
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<td>19.80</td>
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<td>0.0223</td>
<td>6.98</td>
<td>3.72</td>
<td>7.16</td>
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1) spilling breaker

Table 2.4.2  Conditions and results of experiments of series C-2 (\(\tan \beta = 1/20\)).

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<th>(h_{ob}) (cm)</th>
<th>(l_p/h_{ob})</th>
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<td>0.1100</td>
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<td>0.700</td>
<td>18.13</td>
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<td>0.794</td>
<td>18.14</td>
<td>6.80</td>
<td>0.0755</td>
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<td>0.910</td>
<td>18.15</td>
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<td>0.0247</td>
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<td>1.276</td>
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<td>8.14</td>
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</table>

1) spilling breaker
Table 2.4.3 Conditions and results of experiments of series C-3 (\(\tan \beta = 1/30\)).

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<th>(T) (s)</th>
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<th>(H_i) (cm)</th>
<th>(H_0/L_0)</th>
<th>(h_{ob}) (cm)</th>
<th>(l_p/h_{ob})</th>
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<td>0.796</td>
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<td>19.89</td>
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<td>0.0511</td>
<td>10.39</td>
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<td>9.63</td>
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1) spilling breaker

Table 2.4.4 Conditions and results of experiments of series C-4 (\(\tan \beta = 1/50\)).

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<th>(H_i) (cm)</th>
<th>(H_0/L_0)</th>
<th>(h_{ob}) (cm)</th>
<th>(l_p/h_{ob})</th>
<th>(l_i/h_{ob})</th>
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<tr>
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<td>C-4-7</td>
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<td>7.71</td>
<td>3.24</td>
<td>11.03</td>
</tr>
</tbody>
</table>

1) spilling breaker
2.4.3 Distance from Breaking Point to Plunging and Transition Points

Figure 2.4.1 shows the dimensionless property, \( l_t/h_{ob} \), which is obtained from the ratio of the distance between the breaking point and the transition point to the still water depth at the breaking point as a function of the deep water wave steepness \( H_0/L_0 \). As for the plunging points, the distance, \( l_p \), from the breaking point divided by \( h_{ob} \) is shown by the dotted lines in the figure. The quantity \( l_t/h_{ob} \) is nearly independent of the wave steepness and takes an almost constant value for each slope, although it fluctuates a little. Since the incident wave height does not vary so much, it shows that the distance to the transition point does not depend on the wavelength. It can also be considered that the distance to the plunging point is slightly influenced by the wave steepness though the influence due to the bottom slope is not obvious because plunging-type breaking happened only in few cases on a mild slope such as 1/30 or 1/50.

Figure 2.4.2 shows the relation between the dimensionless value \( (h_{ob} - h_{ot})/h_{ob} \) \( (h_{ob} \) and \( h_{ot} \): the still water depths at the breaking and transition points) and the surf similarity parameter \( \xi_0 \) \( (\equiv \tan\beta/\sqrt{H_0/L_0}) \) in order to compare with the criterion presented by Basco and Yamashita (1986). It is obvious from the figure that it is difficult to express a general relation in terms of such parameters.

The relation between the averaged value of \( l_t/h_{ob} \) for each slope and the bottom slope is shown in Fig. 2.4.3. The marks in the figure express the averages and the vertical lines express the fluctuations. The solid line expresses

\[
l_t = \left( \frac{1}{5 \tan\beta} + 4 \right) h_{ob}
\]

(2.4.1)

by which the transition point can roughly be estimated.

As for the plunging points, the averages are nearly constant and do not depend on the bottom slope, although the data for plunging breakers on milder slopes
Fig. 2.4.1  Relationships between $H_0/L_0$ and $l_t/h_{0b}$.

Fig. 2.4.2  Relationships between $\xi_0$ and $\frac{h_{0b} - h_{0t}}{h_{0b}}$. 
Fig. 2.4.3  Relationships between $\frac{1}{\tan \beta}$ and $l_t/h_{ob}$. 
are few as mentioned before. Svendsen and Hansen (1986) adopted the relation expressed by $h_{op}/h_{ob} = 0.8$, where $h_{op}$ is the still water depth at the plunging point, to determine the plunging point. The experimental result of the present study is contrary to it.

If the wave plunging is subject to the free fall and the water particle velocity at the top of the wave and the vertical distance for the water particle falling are assumed to be the wave celerity by the small amplitude wave theory $\sqrt{gh_{ob}}$ and the wave height $H_b \simeq h_{ob}$, respectively, the distance from the breaking point to the plunging point $l_p$ must be $\sqrt{2}h_{ob}$ which depends only on the water depth. On the other hand, the distance is expressed roughly as

$$l_p = 2.5h_{ob} \quad (2.4.2)$$

in Fig. 2.4.3. It can be considered that the reasons why the factor in Eq. (2.4.2) is larger are the plunging points were determined by the onshore end of the plunging jets and the water particle velocity at the top of the wave crests should be larger than $\sqrt{gh_{ob}}$.

On the contrary, Eq. (2.4.1) can be interpreted as the sum of the distance given by the water depth and the distance depending on the bottom slope (or the surf zone width).
CHAPTER 3

Modeling of Undertow

3.1 Introduction

In order to predict the sediment transport rate, material diffusion or wave attenuation rate in the surf zone, it is necessary to estimate the velocity distribution with high accuracy. Especially for the sediment transport rate and the bottom friction, the prediction of the velocity near the bottom is necessary.

Longuet-Higgins (1953) derived a laminar solution of mass transport velocity in the fluid with assumption that non-linear acceleration term in the conduction equation is negligible compared with the viscous term. The solution obtained by Longuet-Higgins cannot be applied to the turbulent flow in the surf zone. Svendsen (1984c) presented a model of vertical distribution of cross-shore steady current in the surf zone applying the eddy viscosity model. He used a boundary condition which was the mass transport velocity on the bottom given by the Stokes wave theory. Okayasu et al. (1986) gave one of the boundary conditions by the slip condition on the bottom. Stive and Wind (1986) and Tsuchiya et al. (1988) investigated a model with a boundary condition at the trough level. Nadaoka and Hirose (1986) evaluated the diffusion coefficient in the surf zone. They also obtained the vertical distribution of the steady current from that of the mean vorticity.

However, the above mentioned studies are not adequate to evaluate the velocity distribution close to the bottom or the shear stress on the bottom. From this point of view, Okayasu et al. (1988) presented a model in which the distribution of the mean eddy viscosity coefficient was assumed to be a linear function of the vertical coordinate and to be zero at the bottom, while the above mentioned models were formulated with the depth-independent eddy viscosity. Svendsen et al. (1987) also
evolved Svendsen's model considering the bottom boundary layer and the bottom shear stress theoretically, but the application of the models is limited to the inner region.

The variation of the wave height or the cross-shore distribution of the energies in the surf zone is necessary to evaluate the undertow and to consider other various practical problems as well. Mizuguchi (1980) presented a model to estimate the wave height variation in the surf zone with formulation of the energy dissipation rate by wave breaking. Izumiya and Horikawa (1984) presented a model for evaluating the change of the wave energy in the surf zone without converting to the variation of the wave height so that the model can estimate the radiation stress accurately. These models were constructed on the concept that the loss of the wave energy is locally balanced with the energy dissipation. However the organized large vortexes whose importance was already verified in Chapter 2 should be taken into account as the transmitter of mass, momentum and energy. A comprehensive model which describes the wave and velocity field throughout the surf zone on the basis of the accurate understanding of the energy process is therefore required for practical uses.

In this chapter, based on the results obtained in Chapter 2, a model which estimates the two-dimensional distribution of the undertow will be presented. First the transfer of energy in the surf zone will be described with consideration of the physical process of the energy transfer in the surf zone. Then applying the time-dependent mild slope equation, the energy variation and energy dissipation rate in the surf zone will be estimated. The existence of the large vortexes gives a specific feature to the energy dissipation process and to the mass flux in the surf zone. Hence, the organized large vortexes formed in the wave crests will be considered for an accurate description of the wave field and velocity field in the surf zone.

An attempt to combine the previous models for the undertow and the longshore
current was made by Svendsen and Lorenz (1989) to cope with much more general conditions. The present model can easily be expanded to the comprehensive three-dimensional model by virtue of the applicability of the time-dependent mild slope equation.
3.2 Energy Transfer in the Surf Zone

The evaluation of the steady current distribution is very important for estimating the convective diffusion of materials, the sediment transport in the surf zone and others. However, the wave field in the surf zone is so different from that in offshore region that it is necessary to investigate the velocity field in all its aspects. The most different point compared with the offshore region is the wave breaking in the surf zone. The energy conveyed from the offshore region dissipates by the wave breaking. The dissipation rate of the energy is far larger than that in the offshore region. Strong turbulence is generated in the dissipation process. The turbulence gives a specific feature to the velocity field in the surf zone as shown in Chapter 2.

The bore-like waves give another specific feature to the velocity field in the surf zone. Organized large vortexes are formed in front of the crests of the bore-like waves. The large vortexes transport a large amount of mass onshoreward; therefore it is natural that the large vortexes also have considerable energy flux. Hence, it can be considered that the organized large vortexes play an important role in the generation process of the turbulence. The high turbulent intensity observed near the surface confirms the concept. The organized large vortexes are the transporter of the large mass, the transmitter of the energy and the generator of the turbulence.

First of all, an outline of the energy transfer in the surf zone will be described qualitatively in this section. It will be mentioned later, in §3.5, how each part of the energy will be estimated in the actual calculation in this model. For the proper description of the energy transfer in the surf zone, it is necessary to take the energy of the large vortexes into account as mentioned above. The total energy in the surf zone should therefore be described as

$$E_t = E_w + E_v$$ (3.2.1)
where $E_t$ is the total energy, $E_w$ the energy of wave motion and $E_v$ the energy of organized large vortexes. Generally, the wave energy can be divided into two parts: the potential energy $E_p$ and the kinetic energy $E_k$ as

$$E_w = E_p + E_k$$  \hspace{1cm} (3.2.2)

Energy exchange between the potential and kinetic energies takes place during propagation of waves. The energy $E_k$ in Eq. (3.2.2) is therefore defined as the transferable kinetic energy. Though the energy of the organized large vortexes should be classified as the kinetic energy in the general sense, it is denoted as another variable here. The reason is that it can be considered that the energy of the large vortexes is conveyed to smaller size eddies, never transferred back to the wave motion.

In the early stage of the breaking process, the turbulent intensity is still small, although the wave height rapidly decreases as shown in §2.3.5. It can be considered that the energy transferred to the kinetic energy cannot be regarded as the wave kinetic energy $E_k$ because the potential and kinetic energies of the wave motion is in balance in some way as mentioned just before. A little part of the lost energy should be directly transferred to the turbulence, but it is natural to consider that the most part of the energy is transferred to the energy of the organized large vortexes. Hence, in this model, it is assumed that the wave energy is first converted to that of the organized large vortexes formed in front of the wave crests. After the conversion, the energy is transferred to smaller size eddies, then dissipates.

Since the energy flux by wave motion is denoted by $E_wc_y$, the transfer rate from the wave energy to the energy of the vortexes per unit length and unit width is expressed as

$$T_B = \frac{d{(E_wc_y)}}{dx}$$  \hspace{1cm} (3.2.3)
If the energy which is directly transferred from the wave energy to the turbulence is little, the energy flux by the large vortexes can be denoted as

$$\frac{d(E_t c_g)}{dx} = T_B - D_B$$

(3.2.4)

where $D_B$ is the dissipation rate per unit area by wave breaking. If it is assumed that the organized large vortexes propagate with the wave crests, the energy flux should be described by $E_t c$. But the group velocity $c_g$ is nearly equals to the phase velocity $c$ in the surf zone and it is not clear how fast the vortexes are. Hence the phase velocity is replaced by the group velocity for the simplicity of the model.

From Eqs. (3.2.1), (3.2.3) and (3.2.4), $D_B$ is also expressed as

$$D_B = -\frac{d(E_t c_g)}{dx}$$

(3.2.5)
3.3 Mass Balance in the Surf Zone

The steady current distribution in the surf zone is much different from that of any wave theories solved in the inviscid condition. The organized large vortexes which are formed just in front of the wave crest of the bore-like wave propagating in the surf zone affects the mass flux balance and the turbulence generation as shown in Chapter 2. In this section, the general aspects of the mass balance in the surf zone will be discussed. Then it will be considered that how it can be expressed in terms of the energy.

3.3.1 Mass Flux in the Surf Zone

Since the wave field which is completely two-dimensional is considered in this study, the velocity component in the longshore direction is assumed to be zero. The continuity equation of time averaged values of the velocity components becomes

$$\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0$$  \hspace{1cm} (3.3.1)

where $U$ and $W$ are the steady current components in the cross-shore and the vertical direction, respectively. Since the boundary condition is given as $U = 0$ at the shoreline, the integral property in the vertical direction of Eq. (3.3.1) is obtained as

$$\int_{-h_0}^{\zeta_c} U \, dz = 0$$  \hspace{1cm} (3.3.2)

where $h_0$ is the still water depth and $\zeta_c$ is the wave crest level. The above equation can also be expressed as

$$M_t = \int_{-\zeta_t}^{\zeta_c} U \, dz$$

$$= - \int_{-h_0}^{\zeta_t} U \, dz$$  \hspace{1cm} (3.3.3)

where $\zeta_t$ is the wave trough level. The value $M_t$ is defined as the mass flux by waves which is the mass transport above the trough level per unit time and unit width.
The second line in Eq. (3.3.3) can be considered as the compensatory flow of the mass flux by waves and is termed "undertow" in the surf zone.

Outside the surf zone, the mass flux $M_t$ is caused only by the wave motion. But in the surf zone, the mass flux by organized large vortexes is considerably large as found in §2.3.3. In other words, the total mass flux by breaking waves should be expressed as

$$M_t = M_w + M_v$$

in which $M_w$ is the mass flux by wave motion and $M_v$ is the mass flux by organized large vortexes. Actually, it is impossible to divide the mass flux in the two parts exactly, because it is difficult to define the boundary of the organized vortexes [Peregrine and Svendsen (1978)].

### 3.3.2 Mass Flux due to Wave Motion

The wave component of the water particle velocity calculated by the stream function method which was presented by Dean (1965) is reliable as reported by Isobe et al. (1979), but the computation takes much time. Moreover, the stream function method needs full information of the time history of water surface elevation which is difficult to obtain in the surf zone. If the mass flux by wave motion can be estimated from the wave energy distribution, the problem becomes much easier. Hence in this section, application of the linear long wave theory to the estimation of the mass flux by wave motion will be considered by comparing with the stream function method.

By using the linear long wave theory, the mass flux by wave motion is described as

$$M_{LLW} = \int_{\zeta_i}^{\zeta} u \, dz$$
where $M_{LLW}$ is the mass flux calculated by the linear wave theory, $\zeta$ the water surface elevation, $c$ the wave celerity and $h$ the mean water depth. Since $\bar{\zeta}$ expresses the time average, $\bar{\zeta}$ is the wave setup.

Figure 3.3.1 shows the comparisons between the mass flux variation calculated by the stream function method and the linear long wave theory from measured water surface profiles in the surf zone. Examples are given for cases B-2, B-3 and B-5 presented in Chapter 2. The incident wave conditions have been already shown in Table 2.2.2. In Fig. 3.3.1, the solid lines show the values calculated by the stream function method and the dashed lines show those by the linear long wave theory. Though difference exists between the solid lines and the dashed lines, they agree qualitatively. From the experiments of Series B, it is found that the values calculated by the stream function method can be approximated by the linear long wave theory as

$$M_w \simeq M_{SPM} \simeq 0.8M_{LLW}$$

(3.3.6)

where $M_{SPM}$ is the mass flux calculated by the stream function method. The coefficient in Eq. (3.3.6) is obtained by averaging the ratio of the two values for all the cases of Series B. The values modified by multiplying 0.8 to $M_{LLW}$ are also shown in Fig. 3.3.1 with $M_{SPM}$ and $M_{LLW}$. They agree well, so that the mass flux by wave motion can be evaluated by using the linear long wave theory.

Since Eq. (3.3.5) also needs the time history of the surface elevation, it is of little advantage. The potential energy of waves is defined as

$$E_p = \frac{1}{T} \int_0^T \rho g (\zeta - \bar{\zeta}) \, dt = \frac{1}{2} \rho g (\zeta - \bar{\zeta})^2$$

(3.3.7)
Fig. 3.3.1 Comparison of the cross-shore variations of $M_{SFM}$, $M_{LLW}$ and $0.8M_{LLW}$ (cases B-2, B-3 and B-5).
where $T$ is the wave period. Then from Eq. (3.3.7), Eq. (3.3.5) can be replaced by

$$M_{LLW} = \frac{2c}{\rho g h} E_p \quad (3.3.8)$$

This means the mass flux by the wave motion can be estimated from the potential energy. Consequently, the mass flux by the wave motion can be evaluated by the following equation:

$$M_w = \frac{1.6c}{\rho g h} E_p \quad (3.3.9)$$

Equation (3.3.9) is obtained by regarding the area occupied by the organized vortexes as the area of the wave motion. However, the area of vortexes should be much smaller than that of wave motion. Svendsen (1984b) approximated the area of the vortex by $0.9H^2$ based on the experimental results on a breaker behind a towed hydrofoil obtained by Duncan (1981). Okayasu et al. (1986) found that it can be expressed by $0.06HL$ which was confirmed by Hansen (1989) later. In both cases, the area is small compared to the cross section of a wave.

### 3.3.3 Mass Flux due to Organized Large Vortexes

In this section, the velocity distribution inside the organized large vortexes formed in front of wave crests will be assumed. From the assumption, the area of the section and the mass transport by the vortexes will be estimated.

The velocity distribution inside one vortex is assumed as shown in Fig. 3.3.2 (a). Although the vortexes generated in front of the wave crests decelerate and go downward, the mean horizontal velocity can be assumed the same as the wave celerity at the first stage. The diameter of the vortex is given by the wave height $H$.

The velocity can be divided into the rotational component $u_1$ and the parallel component $u_2$. Figure 3.3.2 (b) shows the former part of the velocity and Fig. 3.3.2 (c) shows the latter. The total velocity $u$ is expressed as

$$u = u_1 + u_2 \quad (3.3.10)$$
Fig. 3.3.2 Assumption of velocity distribution of an organized large vortex.
\( u_1 = \frac{2c}{H} r, \quad u_2 = c \) \hspace{1cm} (3.3.11)

where \( r \) is the radial coordinate. The energy, \( E_1 \), of the rotational component per unit width is given as

\[
E_1 = \int_0^\frac{4}{h} \frac{1}{2} \rho u_1^2 2\pi r \, dr \\
= \frac{1}{16} \pi \rho c^2 H^2
\] \hspace{1cm} (3.3.12)

The parallel component \( E_2 \) is also given as

\[
E_2 = \frac{1}{8} \pi \rho c^2 H^2
\] \hspace{1cm} (3.3.13)

The total energy of one vortex per unit width \( E_3 \) is obtained as

\[
E_3 = E_1 + E_2 \\
= \frac{3}{16} \pi \rho c^2 H^2
\] \hspace{1cm} (3.3.14)

Even if the vortex is skewed or the real velocity distribution is slightly different from the assumption, it influences only the constant value in Eq. (3.3.14). The total energy should be proportional to \( c^2 H^2 \). Svendsen (1984b) presented the model that there is one skewed vortex, so-called “surface roller” in front of the waves. On the other hand, Nadaoka (1986) described that there should be plural vortexes (Fig. 3.3.3). The following discussion applies to both models.

The section of one vortex \( A_1 \) is given as

\[
A_1 = \frac{\pi}{4} H^2
\] \hspace{1cm} (3.3.15)

From Eqs. (3.3.14) and (3.3.15), the energy, \( e_v \), of the vortex per unit mass is obtained as

\[
e_v \equiv \frac{1}{\rho A_1} E_3 = \frac{3}{4} c^2
\] \hspace{1cm} (3.3.16)
Fig. 3.3.3 Conceptual illustration of plural vortexes in one wave.

[Nadaoka (1986)]
If there are some vortexes in one wave crest, for example as shown in Fig. 3.3.5, the energy, $E_v$, of the vortexes per unit width and unit length can be expressed as

$$E_v = \rho \frac{Ae_v}{L} \quad (3.3.17)$$

where $A$ is the total area of the vortexes per wave and $L$ is the local wave length. Substituting Eq. (3.3.16) into Eq. (3.3.17), $A$ is expressed as

$$A = \frac{4}{3} \frac{L}{\rho c^2} E_v \quad (3.3.18)$$

The mass flux, $M_v$, due to the vortexes is expressed by the following equation:

$$M_v = \frac{\rho c A}{L} = \frac{4}{3c} E_v \quad (3.3.19)$$

The above formula implies the mass flux due to organized large vortexes can be estimated from the energy of the vortexes. In actual calculation, the factor $\frac{4}{3}$ in Eq. (3.3.19) is modified to 2 so as to fit the measured values. Hence, Eq. (3.3.19) is replaced by

$$M_v = \frac{2}{c} E_v \quad (3.3.20)$$

Finally, from Eqs. (3.3.4), (3.3.9) and (3.3.20), the total mass flux due to breaking waves can be described in terms of the potential energy and the energy of organized large vortexes as

$$M_t = \frac{1.6c}{\rho gh} E_p + \frac{2}{c} E_v \quad (3.3.21)$$
3.4 Momentum Balance in the Surf Zone

In case of cross-shore vertically two-dimensional (2-VD) models such as discussed in this study, the longshore variation of the momentum flux need not be considered, but the momentum flux variation in the cross-shore direction causes the wave setup which can not be neglected to describe the wave field accurately. The radiation stress which is defined as the excess momentum flux due to waves is expressed as

\[ S_{xx} = \int_{-h_0}^{\zeta} (\rho u^2 + p) \, dz - \frac{1}{2} \rho gh^2 \]  (3.4.1)

where \( p \) is the pressure. The gradient of \( S_{xx} \) in the \( x \)-direction should balance the gradient of the mean water elevation, that is

\[ \frac{dS_{xx}}{dx} = -\rho gh \frac{d\zeta}{dx} \]  (3.4.2)

By the small amplitude wave theory, the radiation stress \( S_{xx} \) is expressed in terms of the wave energy \( E_w \) as

\[ S_{xx} = \left( \frac{1}{2} + \frac{2kh}{\sinh 2kh} \right) E_w \]  (3.4.3)

for monochromatic waves. For the shallow water condition, the value inside the parentheses in Eq. (3.4.3) can be replaced by 3/2.

Most of the previous studies adopted a simple equation (3.4.3) for calculating the wave setup in the surf zone because of the inordinate complication of wave field. But in making an effort to describe the momentum balance correctly, at least one should consider the momentum flux by the organized large vortexes in the surf zone. If the velocity field of wave motion and the velocity field of large vortexes do not affect each other as already introduced in §3.2, the radiation stress can also be divided into two parts as

\[ S_{xx} = S_w + S_v \]  (3.4.4)
where $S_w$ is the excess momentum flux by wave motion and $S_v$ is that by organized large vortexes.

Generally the small amplitude theory cannot express the wave motion adequately in shallow water condition because of the strong effect of the non-linearity of waves. The finite amplitude theories can express it much more accurately offshoreward of the breaking point; however, they cannot either be applied to the description of the wave field in the surf zone as mentioned before. The small amplitude wave theory is therefore adopted for convenience to calculate the radiation stress by wave motion in this study. The radiation stress, $S_w$, due to wave motion is calculated in this model according to Watanabe and Dibajnia (1988) for a complex wave field.

To calculate the total radiation stress $S_{xx}$ by Eq. (3.4.1), it is necessary to estimate the radiation stress, $S_v$, due to organized large vortexes. $S_v$ is defined as

$$S_v = \int_{\zeta_b}^{\xi} \left( \rho u^2 + (p - p_s) \right) dz$$

(3.4.5)

where $\zeta_b$ is the boundary between the velocity field of wave motion and that of vortexes and $p_s$ the static pressure. If a vortex is flat, the quantity $\frac{\partial w}{\partial t}$ is so small that the term $(p - p_s)$ is negligible except at the both ends of the vortex. At the front and rear ends, $\frac{\partial w}{\partial t}$ is large so that the pressure deviates a lot from the static value. It can be considered that it makes the fluctuating upper surface boundary. Since the deviation of the pressure is negligible in the most part of a vortex, the following equation can be obtained:

$$S_v = \int_{\zeta_b}^{\xi} \rho u^2 dz$$

(3.4.6)

Here, the velocity distribution is assumed again as shown in Fig. 3.3.2 (a). The radiation stress by one vortex $S_1$ is denoted as

$$S_1 = \frac{1}{L} \int_{A_1} \rho u^2 dx dz$$
\[ \frac{1}{L} \int_{A_1} \rho (u_1 + u_2)^2 \, dx \, dz \]

\[ = \frac{5}{L} E_3 \]  

\[ (3.4.7) \]

where \( A_1 \) is the area occupied by the vortex. From Eqs. (3.3.16) and (3.3.17), the energy of the vortexes is expressed as

\[ E_v = \frac{A}{A_1} \frac{E_3}{L} \]  

\[ (3.4.8) \]

Then the total radiation stress by the organized large vortexes is obtained as

\[ S_v = \frac{A}{A_1} S_1 = \frac{5}{3} \frac{A}{A_1} \frac{E_3}{L} \]

\[ = \frac{5}{3} E_v \]  

\[ (3.4.9) \]

As well as for the wave induced momentum flux, the momentum flux due to organized large vortexes is expressed as a function of the energy of the vortexes.
3.5 Estimation of Energy Distribution in the Surf Zone

In this section, the wave energy distribution will be estimated by using the time-dependent mild slope equation. The time-dependent mild slope equation can deal with wave transformation under combined reflection, refraction, diffraction and breaking in horizontal two-dimensional (2-HD) models. The present model which is developed by considering the phenomena in the surf zone more precisely has also an advantage that it can easily be combined with the 2-HD nearshore wave field models such as Watanabe and Maruyama (1986) and extended to three-dimensional (3-D) nearshore current models.

Since the mild slope equation was originally derived by using the linear wave theory, the energy calculated by the time-dependent mild slope equation should be considered as the energy of wave motion which is denoted as $E_w$ in Eq. (3.2.1). Consequently, the energy of the organized large vortexes is not included in the calculation because it can be considered that the energy of the large vortexes is never transferred to the wave motion as already mentioned in §3.2. The estimation of the energy of the organized large vortexes needs another equation based on Eq. (3.2.4). The method will also be discussed in this section.

3.5.1 Governing Equations of Time-Dependent Mild Slope Equation

The mild slope equation was first obtained by Berkhoff (1972) for a stationary wave field as

$$\nabla \cdot (c G \nabla \hat{\phi}_0) + \sigma^2 \frac{c_g}{c} \hat{\phi}_0 = 0$$

(3.5.1)

where $\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right)$, $y$ is the horizontal coordinate in the longshore direction, $c$ the phase velocity, $c_g$ the group velocity, $\sigma$ the angular frequency and $\hat{\phi}_0$ the amplitude of the velocity potential at the mean water level given by the small-amplitude wave theory. Watanabe and Maruyama (1986) derived a set of equations...
equivalent to Eq. (3.5.1) in terms of the water surface elevation $\zeta$ and the flow rate $Q$ as follows:

\[
\frac{\partial Q}{\partial t} + \frac{c^2}{n} \nabla (n\zeta) = 0 \tag{3.5.2}
\]

\[
\frac{\partial \zeta}{\partial t} + \nabla \cdot Q = 0 \tag{3.5.3}
\]

where

\[
n = \frac{1}{2} \left( 1 + \frac{2kh}{\sinh 2kh} \right) \tag{3.5.4}
\]

\[
Q = (Q_x, Q_y) \tag{3.5.5}
\]

\[
Q_x = \int_{-h}^{0} u \, dz, \quad Q_y = \int_{-h}^{0} v \, dz \tag{3.5.6}
\]

and $u$ and $v$ are horizontal velocity components in $x$ and $y$ direction, respectively. A set of equations completely equivalent to Eq. (3.5.1) was also presented by Nishimura et al. (1983) as

\[
\frac{\partial Q'}{\partial t} + c^2 \nabla \zeta = 0 \tag{3.5.7}
\]

\[
\frac{\partial \zeta}{\partial t} + \frac{1}{n} \nabla \cdot (nQ) = 0 \tag{3.5.8}
\]

The variable $Q'$ has a difference of order of $\nabla n$ from the flow rate $Q$ in Eq. (3.5.5).

Watanabe and Maruyama (1986) added a term to Eq. (3.5.2) so that the equations could express the energy attenuation due to the breaking. The equation is as follows:

\[
\frac{\partial Q}{\partial t} + \frac{c^2}{n} \nabla (n\zeta) + f_T Q = 0 \tag{3.5.9}
\]

The third term of Eq. (3.5.9) is generally called “the energy dissipation term”, but it is termed “the energy transfer term” in this study because the lost energy does not dissipate directly but is once transferred to the energy of the organized large vortexes in the present model. The value $f_T$ which is also termed “the transfer factor” by the breaking is determined as

\[
f_T = \alpha_T \tan \beta \sqrt{\frac{g}{h} \left( \frac{\hat{Q}}{Q_r} - 1 \right)} \tag{3.5.10}
\]
\[
\dot{Q} = \sqrt{\dot{Q}_x^2 + \dot{Q}_y^2}, \quad Q_r = \gamma' \sqrt{gh^3}
\]  
(3.5.11)

where \(\tan\beta\) is the averaged bottom slope, \(\dot{Q}_x\) and \(\dot{Q}_y\) are the amplitude of the flow rate components. \(\alpha_T\) and \(\gamma'\) were set to 2.5 and 0.25, respectively. By setting \(f_T = 0\) after breaking (if \(\dot{Q} < Q_r\)), wave recovery can be described as well as wave decay in the surf zone. In case of progressive waves, Eqs. (3.5.3) and (3.5.9) reduce to the wave energy equation, that is expressed as

\[
\frac{\partial(E_w c_y)}{\partial x} = -f_T n E_w
\]  
(3.5.12)

where \(E_w\) is the wave energy per unit area. Considering the physical meaning and the experimental results, Isobe (1987) presented the following equation

\[
f'_r = n f_T = -\frac{5}{2} \tan \beta \sqrt{\frac{g}{h}} \left(\frac{\gamma - \gamma_r}{\gamma_s - \gamma_r}\right) f_d(kh)
\]  
(3.5.13)

where \(\gamma = k \sqrt{2E_w / \tanh kh}\) is the ratio of water particle velocity to wave celerity. The quantity \(f_d\) comes from the formulation in terms of the ratio of water particle velocity to wave celerity instead of wave height to water depth ratio. The value \(f_d\) is unity for small \(kh\). The values of \(\gamma_s\) and \(\gamma_r\) were given as

\[
\gamma_s = 0.4 (0.57 + 5.3 \tan \beta)
\]  
(3.5.14)

\[
\gamma_r = 0.135
\]  
(3.5.15)

Watanabe and Dibajnia (1989) added the energy transfer term to Eq. (3.5.7) in the same manner as Watanabe and Maruyama and obtained an one-dimensional model as follows:

\[
\frac{\partial Q}{\partial t} + c^2 \frac{\partial \zeta}{\partial x} + f_T Q = 0
\]  
(3.5.16)

\[
\frac{\partial \zeta}{\partial t} + \frac{1}{n} \frac{\partial (n Q)}{\partial x} = 0
\]  
(3.5.17)

The energy transfer factor \(f_T\) was given as

\[
f_T = \alpha_T \tan \beta \sqrt{\frac{g}{h}} \left(\frac{\gamma - \gamma_r}{\gamma_s - \gamma_r}\right)
\]  
(3.5.18)
with the shallow water approximation of Eq. (3.5.13). The value $\alpha_T$ is a coefficient which varies from 0 to 2.5 around the braking point for preventing numerical reflections. In Eq. (3.5.18), $\gamma_s$ was given by Eq. (3.5.14). According to Maruyama and Shimizu (1986), $\gamma_r$ was replaced by

$$\gamma_r = 0.4 \left( \frac{a}{h_b} \right)$$

(3.5.19)

where $\left( \frac{a}{h_b} \right)$ is the ratio of the wave amplitude to the mean water depth at the breaking point.

The time-dependent mild slope equation was obtained by using the small amplitude wave theory. However, waves in the surf zone are no more small amplitude waves. In that sense, the time-dependent mild slope equation may not be appropriate for waves in the surf zone, but it is also a fact that there is no wave theory which can express the waves in the surf zone adequately. It was reported by the above mentioned researchers that the wave energy calculated by the time-dependent mild slope equation fits well with the measured wave energy. Hence, in the present study, Eqs. (3.5.16) and (3.5.17) are essentially adopted as the governing equations to estimate the wave energy variation in the surf zone. The wave height distribution is not discussed in this study.

The wave energy dissipation due to bottom friction should be taken into account for accurate estimation of the wave energy variation. The dissipation by wall friction should also be considered in case of laboratory experiments. Since the computational results will be compared with laboratory experiments in the present study, the dissipation factor by the bottom and wall friction $f_{b+w}$ is added to the attenuation factor by breaking.

The bottom and wall boundary layers in the surf zone may not be laminar flow, but the dissipation by wave breaking is so large that the damping due to the
bottom and wall friction is negligible. In the present study, the energy dissipation by the friction is calculated from the theory for laminar flow.

Iwagaki and Tsuchiya (1966) dealt with laminar damping due to bottom and wall friction for finite amplitude waves. When deducing in the linearized theory, the dissipation factor by friction $f_{b+w}$ is expressed as

$$f_{b+w} = \frac{1}{n} \sqrt{2 \nu \sigma} \left( \frac{1}{B} + \frac{k}{\sinh 2kh} \right)$$

(3.5.20)

where $\nu$ is the kinematic viscosity and $B$ is the width of the wave flume. The dissipation rate by the bottom and wall friction is obtained as

$$D_{b+w} = f_{b+w} n E_w$$

(3.5.21)

if no reflected wave exists in the field.

The lost energy by the wave breaking is transferred from the wave energy $E_w$ to the energy, $E_v$, of large vortexes, but on the contrary, it can be considered that the lost energy by the bottom and wall friction directly dissipates to heat. A series of equations from Eq. (3.2.3) to Eq. (3.2.5) is therefore replaced by following equations:

$$\frac{d(E_w c_g)}{dx} = -T_B - D_{b+w}$$

(3.5.22)

$$\frac{d(E_v c_g)}{dx} = T_B - D_B$$

(3.5.23)

$$\frac{d(E_{\ell} c_g)}{dx} = -D_B - D_{b+w}$$

(3.5.24)

On the slope milder than $\tan \beta = 1/10$, the reflection by wave breaking is so small that $D_{b+w}$ is evaluated by Eq. (3.5.21). The variation of $E_w$ is calculated by the scheme expressed as

$$\frac{\partial Q}{\partial t} + c^2 \frac{\partial \zeta}{\partial x} + f_A Q = 0$$

(3.5.25)

$$\frac{\partial \zeta}{\partial t} + \frac{1}{n} \frac{\partial (nQ)}{\partial x} = 0$$

(3.5.26)
which are obtained by replacing the energy transfer factor $f_T$ in Eq. (3.5.16) by the total attenuation factor $f_A$ determined as

$$f_A = f_T + f_{b+w} \quad (3.5.27)$$

The energy transfer factor, $f_T$, of breaking waves expressed by Eq. (3.5.18) is used after Watanabe and Dibajnia. The detailed description of the coefficients $\gamma_s$ and $\gamma_r$ in Eq. (3.5.18) will be mentioned in §3.5.4. Since the wave energy is calculated by Eqs. (3.5.25) and (3.5.26), the energy transfer rate can be obtained by using Eq. (3.5.22).

The shoreline and offshore boundary conditions are given according to Watanabe and Dibajnia (1989). They imposed the shoreline boundary condition as $Q = 0$. As for the offshore boundary condition, a non-reflective boundary and sinusoidal incident waves are considered. The incident wave condition is given in terms of the water surface elevation $\zeta$ as

$$\zeta(x_0) = \zeta(x_0 + \Delta x) + a_I[\sin(kx_0 - \sigma t) - \sin\{k(x_0 + \Delta x) - \sigma(t - \tau_0)\}] \quad (3.5.28)$$

where $\tau_0 = \frac{\Delta x}{c_0}$, $\Delta x$ is the spacing between mesh points, and $\sigma$ and $a_I$ are the frequency and the amplitude of the incident waves, respectively. The subscript 0 denotes quantities at the offshore boundary.

The method of the numerical computation and the adopted mesh scheme are the same as those employed by Watanabe and Dibajnia. But the convergence of solution is assumed when the absolute errors between the values obtained from two successive cycles of the calculation at every point are less than a tolerance error throughout the field. The required absolute error is equal to $0.02a_I$ for both the wave amplitudes and the setup calculation in the present study.

The functional form of $\alpha_T$ which is different from that proposed by Watanabe and Dibajnia will be mentioned later in §3.5.3. The wave celerity is modified to
prevent the variation of $f_T$ from increasing the numerical reflection. It will be discussed in §3.5.5.

3.5.2 Estimation of Energy Dissipation Rate

Since the time-dependent mild slope equation was obtained by the small amplitude wave theory, the energy calculated by the equation is the energy due to the wave motion. From the discussion in §3.2 it is assumed that the wave energy is first transferred to that of the organized large vortexes formed in front of the wave crests. The transferred energy which is obtained from $T_B$ should dissipate anyhow, because the total amount of the transferred energy throughout the surf zone should be equal to the total amount of the dissipation.

Here it is considered that after the transfer, the energy is converted to smaller size eddies, and then dissipates. This idea is the same as so called "the energy cascade model". Hence, there should be retardation of dissipation. It also means that the energy transferred to the large vortexes takes some distance during the dissipation because the vortexes propagate onshore with the waves.

The problem is how long the transferred energy takes to dissipate. Yamashita et al. (1988) performed detailed measurements of behavior of the organized large vortexes in the surf zone and obtained a result that the time interval of the generations of vortexes in front of one propagating wave are almost constant in the inner region and are much larger around the breaking point. The interval should have a relation to the life of a vortex.

In the present model, the energy transferred at one point constantly dissipates within the distance given as

$$l_d = \begin{cases} 
4 \frac{x_t - x}{x_t - x_b} + 4) h & (x \leq x_t) \\
4h & (x > x_t)
\end{cases}$$

(3.5.29)
where \( x'_b \) is the crest breaking point which will be discussed in the next section, and \( x_t \) the transition point which is given by Eq. (2.4.1). A schematic illustration of Eq. (3.5.29) is shown in Fig. 3.5.1. The values of the constants in Eq. (3.5.29) need further verifications.

A contribution from the energy transferred at \( x = x' \) (i.e. \( T_B(x') \)) to the local dissipation rate is expressed as

\[
t_d(x, x') = \begin{cases} 
0 & (x \leq x') \\
\frac{T_B(x')}{l_d(x')} & (x' \leq x < x' + l_d(x')) \\
0 & (x' + l_d(x') \leq x)
\end{cases} \quad (3.5.30)
\]

Finally, the total dissipation rate, \( D_B \), by wave breaking is obtained as

\[
D_B(x) = \int_{-\infty}^{x} t_d(x, x') \, dx' \quad (3.5.31)
\]

### 3.5.3 Energy Transfer Factor around the Breaking Point

In relation to the breaking of composite waves, Watanabe et al. (1984) proposed a breaker index in terms of the ratio of water particle velocity to the wave celerity so as to agree with the breaking criterion proposed by Goda (1970). This criterion was determined to give an accurate prediction of the breaking point when the water particle velocity and the wave celerity by the linear wave theory were used. For convenience for the numerical calculations, Isobe (1987) approximated it by the following formula:

\[
\gamma_b = \left( \frac{\hat{u}}{c} \right)_b = 0.53 - 0.3 \exp\{-3\sqrt{h_b/L_0}\} + 5 \tan^{3/2} \beta \exp\{-45(\sqrt{h_b/L_0} - 0.1)^2\} \quad (3.5.32)
\]

where \( \hat{u} \) is the amplitude of horizontal water particle velocity at the still water level, \( L_0 \) the deep-water wavelength, and subscript \( b \) denotes the quantity at the breaking point. The above equation is adopted in the present study to predict the breaking point.
Fig. 3.5.1 Setting of dissipation distance $l_d$.

Fig. 3.5.2 Setting of coefficient $\alpha_T$. 

key: 
- crest breaking point
- breaking point (maximum wave height)
- transition point
- $x_b$, $x_b$, $x_t$
Watanabe and Maruyama (1986) linearly changed the coefficient $\alpha_T$ in Eq. (3.5.18) around the breaking point to avoid numerical reflection caused by the sudden increase of the transfer factor $f_T$. They set $\alpha_T = 0$ at the point $L_b/4$ before the breaking point and linearly increased to 2.5 at the point $L_b/4$ after the breaking point. The distance $L_b$ is the local wave length at the breaking point. However, the distance $L_b/4$ has no physical reason. The dissipation and the transfer of the energy should occur from the breaking point.

The breaking points slightly differ in accordance with its definition. Seyama and Kimura (1988) measured cross-shore variations of the wave height on constant slopes in the surf zone for irregular wave condition. They defined the breaking point as the point where the crests of propagating waves begin to break and gave a formula presenting the wave height variation of irregular waves for various bottom slopes. From their formula, the distance $l_b$ from the breaking point according to their definition to the point where the wave height takes its maximum value was given as

$$
l_b = x_b - x'_b
= \frac{h_b}{\tan \beta} \left\{ \frac{1.65 \exp(-2.3 \tan \beta) + 0.01 \tan \beta + 0.37}{1.54 \exp(-11.5 \tan \beta) - 0.0458 \tan(8.41 \tan \beta) - 0.67 \exp(-3.1 \tan \beta) + 0.01 \tan \beta + 1.37} - 1 \right\}
$$

(3.5.33)

in which $x'_b$ is the point where the crest of propagating waves begin to break and $x_b$ is the point where the wave height takes its maximum value.

The energy transfer from the wave motion to the organized large vortexes and also the energy dissipation should occur from the point where the wave crests begin to break. On the other hand, though the breaker index proposed by Watanabe et al. deals with the ratio of the water particle velocity to the wave celerity, it can be considered that their index presents the point where the wave height takes its
maximum value for an analogy from the criterion by Goda. The formula proposed by Isobe is also for the same point.

Also in this model, the breaking point is defined as the point where the wave height takes its maximum value, but the energy transfer and dissipation are made to take place from offshoreward of the breaking point determined by using Isobe's formula for the exact description of the energy transfer around the breaking point. The distance between the two points can be calculated by Eq. (3.5.33), and found to be

\[ l_b = \begin{cases} 
2.55h_b & (\tan \beta = 1/20) \\
1.96h_b & (\tan \beta = 1/30) 
\end{cases} \]  

(3.5.34)

The distance is given by \( 2h_b \) for the simplicity of the calculation in the present study. Then the crest breaking point \( x'_b \) is given as

\[ x'_b = x_b - 2h_b \]  

(3.5.35)

The coefficient \( a_T \) in Eq. (3.5.18) is set to be 0 at the crest breaking point \( x'_b \) and to be the maximum value 2.5 at the transition point \( x_t \). In other words, \( a_T \) is increased in the transition region and fixed in the inner region in the model.

\[ a_T = \begin{cases} 
0 & (x \leq x'_b) \\
2.5 \frac{x - x'_b}{x_t - x'_b} & (x'_b < x \leq x_t) \\
2.5 & (x_t < x) 
\end{cases} \]  

(3.5.36)

The variation of \( a_T \) is illustrated in Fig. 3.5.2.

### 3.5.4 Ratio of Potential Energy to Kinetic Energy of Waves

The value of the potential energy is equal to that of the kinetic energy according to the small amplitude wave theory, but the actual ratio of the kinetic energy to the potential energy is not unity in general. Dibajnia et al. (1988) calculated the ratio by using finite amplitude wave theories. They obtained the result that the
maximum ratio of the kinetic energy to the potential energy calculated by the finite amplitude wave theory is 1.2 as far as the calculation converged.

Watanabe and Maruyama (1986) imposed the energy dissipation factor from $L_b/4$ before the breaking point to cope with the rapid decreasing of the potential energy after breaking. In this model, although the energy transfer from wave energy takes place from the crest breaking point $x_b^c$, the ratio $R_p = \frac{E_p}{E_b}$ is made to change linearly from 1 to its minimum at the wave plunging point. Because it can be considered that the breaking waves keep the minimum value as for the ratio $R_p$. The relationship is expressed as

$$R_p = \begin{cases} 1 & (x \leq x_b^c) \\ 1 - \left(1 - \frac{1}{1.2}\right) \frac{x - x_b^c}{x_p - x_b^c} & (x_b^c < x \leq x_p) \\ \frac{1}{1.2} & (x_p < x) \end{cases}$$

(3.5.37)

where $x_p$ is the wave plunging point. Though the distance from the breaking point to the plunging point is not clear yet, the plunging point $x_p (= x_b + l_p)$ is estimated by Eq. (2.4.2) for convenience.

Adopting Eq. (3.5.37), the calculated potential energy is reduced to about 90% of $\frac{1}{2}E_w$. The constant coefficient 0.4 in Eq. (3.5.14) for the value $\gamma_s$ in Eq. (3.5.18) which can be considered to express the strength of the wave decay was determined so that the potential energy which was given by $\frac{1}{2}E_w$ agreed well with the measured value. It should be reduced in proportion to the decrease of the potential energy calculated by means of Eq. (3.5.37), because the variation of $R_p$ accelerates the decrease of the potential energy apparently. Hence, Eq. (3.5.14) is replaced by

$$\gamma_s = 0.35 (0.57 + 5.3 \tan \beta)$$

(3.5.38)

For the same reason, the constant coefficient 0.4 in Eq. (3.5.19) should be increased to keep the potential energy in the recovery zone. $\gamma_r$ in this model is determined
3.5.5 Modification of the Wave Celerity

The energy dissipation by wave breaking occurs with little wave reflection, but the calculation by using the time-dependent mild slope equation generates numerical reflection. It is necessary to keep it small for the accurate description of the wave field. Since a rapid change of the energy transfer factor generates considerable wave reflection, one of the reasons why the coefficient \( \alpha_T \) in Eq. (3.5.18) is made to change linearly around the breaking point is to diminish the numerical reflection. In this section, another way to diminish the numerical reflection will be discussed.

The governing equations of the time-dependent mild slope equation applied to this model are given by Eqs. (3.5.25) and (3.5.26). If considering a wave field bounded into two regions in which only the transfer factors \( f_T \) are different, namely \( f_T \) is zero in the incident wave side region and takes non-zero value \( f_2 \) in the other region, the reflected wave \( \zeta_r \) generated by the sudden change of \( f_T \) is expressed as

\[
\zeta_r = \left\{ \left( \frac{1}{2} - a_k \right) p_2 + b_k q_2 \right\} + i \left\{ -b_k p_2 + \left( \frac{1}{2} - a_k \right) q_2 \right\} e^{-i(kx + \sigma t)}
\]  

(3.5.40)

where \( p_2 \) and \( q_2 \) are the real part and the imaginary part of the complex amplitude of the transmitted wave, respectively, and \( k \) is the wave number calculated by the small amplitude wave theory. \( a_k \) and \( b_k \) in Eq. (3.5.40) are given as follows:

\[
a_k = \sqrt{1 + \sqrt{1 + \frac{f_2^2}{\sigma^2}}} , \quad b_k = \frac{f_2}{2\sqrt{2} \sigma \sqrt{1 + \sqrt{1 + \frac{f_2^2}{\sigma^2}}}}
\]  

(3.5.41)

It is possible to decrease the energy of the reflected wave by changing the wave number in the onshore region. Replacing \( k \) in Eq. (3.5.40) by \( k' \), the minimum ratio
$R_{\min}$ of the reflected wave energy to the transmitted wave energy is obtained as

$$R_{\min} = \frac{1}{4} \frac{b_k^2}{a_k^2 + b_k^2}$$  \hspace{1cm} (3.5.42)

when

$$k' = \frac{a_k}{2(a_k^2 + b_k^2)}$$  \hspace{1cm} (3.5.43)

The details are shown in Appendix A. The wave celerity $c'$ calculated by using $k'$ in Eq. (3.5.43) is larger than that by the small amplitude wave theory.

It is known that the actual wave celerity in the surf zone is also different from the value calculated by the small amplitude wave theory. Svendsen et al. (1978) found that the measured wave celerity in the surf zone is larger than the value $\sqrt{gh}$ obtained as the shallow water approximation of the small amplitude wave theory, and evaluated it by the celerity of a periodic bore which is given by

$$c_{\text{bore}} = \sqrt{gh \frac{1}{2} d_c d_t (d_c + d_t)}$$  \hspace{1cm} (3.5.44)

where $d_c$ and $d_t$ are the water depth at the wave crest and the wave trough, respectively. In general, $c_{\text{bore}} \geq \sqrt{gh}$. Horikawa and Isobe (1980) mentioned that the wave celerity in the surf zone can be predicted fairly well by the solitary wave theory. The wave celerity by the solitary wave theory is described as

$$c_{\text{sol}} = \sqrt{g(d_t + H)}$$  \hspace{1cm} (3.5.45)

From the discussion above, it can be concluded that using $k'$ in Eq. (3.5.43) instead of $k$ is adequate in the sense that the wave celerity is getting closer to the actual wave celerity in the surf zone. In the present model, the transfer factor $f_T$ takes discrete values and gradually change in the surf zone. In this case, the modified wave number $k'$ should be calculated from the transfer factors at the neighboring mesh points. However, little difference was recognized on the effect
between the methods, Eq. (3.5.43) is therefore adopted to give the wave celerity in this model with replacement of \( f_2 \) in Eq. (3.5.41) by \( f_T \) for the simplicity. Since \( f_T = 0 \) offshoreward of the breaking point, \( k' \) equals \( k \) in the offshore region. Adopting the wave height to water depth ratio \( \gamma_H = 0.78 \) in the surf zone, \( c_{sol} \) can be approximated as

\[
c_{sol} \approx \sqrt{g (1.39h)} \\
\approx 1.2 \sqrt{gh}
\]

(3.5.46)

The upper limit of the wave celerity is therefore made as

\[
\frac{c'}{c} = \frac{k}{k'} \leq 1.2
\]  

(3.5.47)
3.6 Dimensional Analysis of Turbulence Properties

In this section, representative values of length and velocity of turbulence, mean energy dissipation by turbulence and mean eddy viscosity will be discussed by using the dimensional analysis after Battjes (1975).

A dimensional analysis of turbulence in the surf zone concerning to horizontal dispersion in the longshore direction was done by Battjes (1975). In the first half of this part, the outline of his work will be introduced. Battjes adopted the energy cascade model to the energy dissipation process in the surf zone. It means that the turbulence consists of small-size eddies, the energy is transferred from larger size eddies to the smaller size, and then dissipates to heat.

If the wave energy dissipates through turbulence, the energy dissipation rate $D_B$ by wave breaking per unit width and unit length could be expressed in terms of energy dissipation rate of turbulence as

$$D_B \approx \rho h^* \varepsilon$$  \hspace{1cm} (3.6.1)

where $h^*$ is the representative water depth and $\varepsilon$ is vertically averaged energy dissipation rate of turbulence per unit mass. By using the representative length $l$ and the representative velocity $q$ for the turbulence, i.e. eddies, the energy of the eddies per unit mass and the representative frequency were given as $q^2$ and $q/l$, respectively. Then the energy transfer to heat through the higher frequency, that is energy dissipation rate, $\varepsilon$, per unit mass was determined as

$$\varepsilon \approx q^2 \cdot \frac{q}{l} = \frac{q^3}{l}$$  \hspace{1cm} (3.6.2)

Battjes considered that the scale of the eddies was restricted by the vertical length rather than the horizontal length. Replacing $l$ by $h^*$, and putting $h^* = h$, the representative velocity was obtained as

$$q \approx \left(\frac{D_B}{\rho}\right)^{1/3}$$  \hspace{1cm} (3.6.3)
By the eddy viscosity model, the Reynolds stress in the longshore direction was expressed as

$$-ho u'v' = \nu_h \frac{dV}{dx}$$

(3.6.4)

in which $\nu_h$ is the eddy viscosity coefficient in the horizontal plane, $u'$ and $V$ are the turbulent component and the steady component of velocity in the longshore direction, respectively. It was considered that $\nu_h$ had a dimension of $ql$. Hence, $\nu_h$ could be given from Eq. (3.6.3) as

$$\nu_h \approx qh \approx h \left( \frac{D_B}{\rho} \right)^{1/3}$$

(3.6.5)

By using the linear wave theory on a constant slope, the energy dissipation rate was expressed as follows:

$$D_B \approx -\frac{d(E_{wnc})}{dx} \approx -\frac{1}{8} \rho g H^2 c \left( \frac{1}{\rho g} \right)$$

(3.6.6)

Substituting $H = \gamma_H h$ and $c = \sqrt{gh}$, Battjes obtained

$$D_B \approx -\frac{d}{dx} \left( \frac{1}{8} \rho g^{3/2} \gamma_H^{2} h^{5/2} \right)$$

$$= -\frac{5}{16} \rho g^{3/2} \gamma_H^{2} h^{3/2} \frac{dh}{dx}$$

$$= \frac{5}{16} \rho g^{3/2} \tan \beta \gamma_H^{2} h^{3/2}$$

(3.6.7)

where $\gamma_H$ is the wave height to water depth ratio. Battjes dealt with the horizontal dispersion, but the analysis can be applied in the vertical direction. The energy of turbulence and eddy viscosity coefficient will be discussed hereafter.

The vertically averaged energy, $\epsilon$, of turbulence per unit mass is described from Eq. (3.6.3) as

$$\epsilon \approx q^2 \approx \left( \frac{D_B}{\rho} \right)^{2/3}$$

(3.6.8)

As for the eddy viscosity in the vertical plane, it can be considered that the discussion about $\nu_h$ can be applied. Hence, $\nu_t$ which is the eddy viscosity coefficient
in the vertical plane is expressed by using the energy dissipation rate $D_B$ and the mean water depth $h$ as

$$\nu_e \approx q^l \approx h \left( \frac{D_B}{\rho} \right)^{1/3} \quad (3.6.9)$$

The Reynolds stress should be essentially determined by the flow field. However, if $u'w'$ is proportional to $\epsilon$, it can be described as

$$\overline{u'w'} \approx \left( \frac{D_B}{\rho} \right)^{2/3} \quad (3.6.10)$$

If the small amplitude theory holds, the energy of turbulence is expressed in terms of water depth, celerity and bottom slopes by substituting Eq. (3.6.7) into Eq. (3.6.8) as

$$\epsilon \approx \left( \frac{5}{16} \left[ g^{3/2} \tan \beta \gamma_H^2 h^{3/2} \right] \right)^{2/3}$$

$$= \left( \frac{5}{16} \gamma_H^2 \right)^{2/3} c^2 \tan^{2/3} \beta \quad (3.6.11)$$

The above equation means that $\epsilon$ is in proportion to the square of the celerity, and the power of two third of the bottom slope. However, one should take into account the variation of the wave height to water depth ratio $\gamma_H$.

In the same manner, $\nu_e$ is expressed from Eq. (3.6.9) as

$$\nu_e \approx \left( \frac{5}{16} \gamma_H^2 \right)^{1/3} c h \tan^{1/3} \beta \quad (3.6.12)$$

The quantity is in proportion to the water depth, the celerity and the third power of the slope.
3.7 Modeling of Vertical Distribution of Undertow

The mass flux by waves which is equal to the vertically integrated value of the undertow is determined by the wave field model shown in the previous sections. The discussion in §3.6 pointed out the possibility that the vertically averaged values of the mean Reynolds stress and the mean eddy viscosity coefficient can also be obtained from \( D_B \) which is estimated by the model of the energy dissipation process.

The vertical distribution of the mean Reynolds stress and the mean eddy viscosity coefficient will be assumed as linear functions of the vertical elevation \( z \) based on the experimental results. The undertow profile will be determined from these distributions by using the eddy viscosity model. The undertow can be evaluated throughout the surf zone.

3.7.1 Modeling of Vertical Distribution of the Mean Reynolds Stress and the Mean Eddy Viscosity

Based on the experimental results shown in §2.3, the vertical distributions of the mean Reynolds stress and the mean eddy viscosity coefficient are assumed as linear functions of the vertical elevation \( z \) in each measuring line. This assumption is different from that adopted by Svendsen and Hansen (1988) or Tsuchiya et al. (1988). Figure 3.7.1 shows this simple model for the estimation.

First the coefficients of the linear functions for the measuring line 4 and 5 in every case of series B of the experiments described in Chapter 2 were obtained by using the regression analysis. To investigate consistently among the measuring lines and for various incident wave conditions, it should be proper to non-dimensionalize the coefficients by the water density \( \rho \), the celerity \( c \) and the trough level \( d_t \), which is probably proportional to the mean water depth \( h \) in the inner region, according to Eqs. (3.6.11) and (3.6.12). Equations (3.6.11) and (3.6.12) were derived with the assumption that the energy dissipation is expressed by Eq.(3.6.6), which should hold
Fig. 3.7.1 Assumed distributions of mean Reynolds stress and mean eddy viscosity coefficient.
in the inner region where waves can be regarded as quasi-steady, although the energy
dissipation rate will be actually evaluated by the model described in §3.5 because it
can apply throughout the surf zone. Then the average of the dimensionless values
of the measuring line 4 and 5 was taken for each case.

Using these parameters, the linear functions for the mean Reynolds stress
$-\rho u'w'$ and the mean eddy viscosity coefficient $\nu_e$ are expressed as

\begin{align*}
-\rho u'w' &= \alpha_1 \rho \frac{c^2}{d_t} z' + \beta_1 \rho c^2 \\
\nu_e &= \alpha_2 cz' + \beta_2 cd_t
\end{align*}

(3.7.1) \hspace{1cm} (3.7.2)

where $\alpha_1, \beta_1, \alpha_2$ and $\beta_2$ are dimensionless parameters and $z'$ is the vertical elevation
from the bottom. The wave celerity is expressed as

$$c = \sqrt{g(d_t + H)}$$

(3.7.3)

based on the solitary wave theory, where $H$ is the local wave height. Table 3.7.1 is
the list of the dimensionless parameters.

Table 3.7.1  Dimensionless parameters for the distribution of $-\rho u'w'$ and $\nu_e$.

<table>
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<th>case</th>
<th>slope</th>
<th>$\alpha_1$</th>
<th>$\beta_1$</th>
<th>$\gamma_1$</th>
<th>$\alpha_2$</th>
<th>$\beta_2$</th>
<th>$\gamma_2$</th>
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<td>0.0017</td>
<td>0.015</td>
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</tbody>
</table>
In the table, the parameter \( \beta_2 \) is the dimensionless value of \( \nu_e \) on the bottom \((z' = 0)\) and \( \gamma_2 \) is the value at the trough level \((z' = d_t)\). Though the magnitude of the parameters are much smaller than the values obtained from Eqs. (3.6.11) and (3.6.12), the fluctuation of the parameters are so small that they can be regarded as constants for each bottom slope. The fact that \( \beta_2 \) is far smaller than \( \gamma_2 \) is consistent with the experimental result mentioned in §2.3.4. It should therefore be possible to take \( \nu_e = 0 \) on the bottom. Equation (3.7.2) can be approximated as

\[
\nu_e = \alpha_2 c z' \quad (3.7.4)
\]

To deal with the mean Reynolds stress and the mean eddy viscosity coefficient in the surf zone which include the outer region or the wave recovery zone, it is necessary to estimate the magnitude of them by means of \( D_B \) calculated by the model described in §3.5. For this sake, it is convenient that the values are expressed in terms of dimensional parameters. Since the mean Reynolds stress can be transformed to the mean shear stress, the mean shear stress \( \tau \) acting on the horizontal plane and the the mean eddy viscosity coefficient \( \nu_e \) can be denoted as

\[
\tau = a_\tau z' + b_\tau \quad (3.7.5)
\]

\[
\nu_e = a_\nu z' \quad (3.7.6)
\]

where \( z' (= z + h_0) \) is the vertical elevation measured from the bottom and \( a_\tau, b_\tau \) and \( a_\nu \) are functions of \( x \) which can be treated as constants at a certain value of \( x \).

By using the vertically averaged mean shear stress \( \tau_m \) which is defined as

\[
\tau_m = \frac{1}{d_t} \int_0^{d_t} \tau \, dz'
\]

(3.7.7)

and the ratio of the value at the trough level to the value at the bottom

\[
R_\tau \equiv \frac{\tau_{z' = d_t}}{\tau_{z' = 0}}
\]

(3.7.8)
the values \( a_r \) and \( b_r \) are denoted as

\[
a_r = -\frac{2(1 - R_r)}{d_t(1 + R_r)} \tau_m, \quad b_r = \frac{2}{1 + R_r} \tau_m
\]  

(3.7.9)

The mean shear stress is expressed as

\[
\tau = \frac{2}{1 + R_r} \tau_m \left\{ - (1 - R_r) \frac{z'}{d_t} + 1 \right\}
\]

(3.7.10)

From Eq. (3.6.10), it can be considered that the vertically averaged mean shear stress is expressed as

\[
\tau_m = C_r \rho^{1/3} D_B^{2/3}
\]

(3.7.11)

where \( C_r \) is a constant. In the present study, \( C_r \) is determined from the experimental results as

\[
C_r = 0.02
\]

(3.7.12)

The value is far smaller than unity. One reason is that the representative length is given by \( h \) in §3.6 though the actual diameter of turbulent eddies should be much smaller. The ratio \( R_r \) is determined from the values \( \beta_1 \) and \( \gamma_1 \) as

\[
R_r = -4.0
\]

(3.7.13)

It should be noticed that \( \tau \) becomes always zero at \( z' = \frac{1}{5} d_t \) in this case. Substituting Eqs. (3.7.11), (3.7.12) and (3.7.13) into (3.7.10), \( \tau \) is obtained as

\[
\tau = \frac{0.04}{3} \rho^{1/3} D_B^{2/3} \left( \frac{5}{d_t} z' - 1 \right)
\]

(3.7.14)

The value \( d_t \) is calculated in the model as

\[
d_t = h - a
\]

(3.7.15)

where \( a \) is the wave amplitude calculated by the small amplitude wave theory. Although the small amplitude wave theory cannot express the wave height distribution in the surf zone accurately, it is adopted for convenience of the calculation.
The vertical distribution of the mean eddy viscosity $\nu_e$ can be obtained in the same manner. Since the value at the bottom was assumed to be zero in §2.3.4, $a_\nu$ in Eq. (3.7.6) is expressed as

$$a_\nu = 2 \frac{\nu_m}{d_t} \quad (3.7.16)$$

where

$$\nu_m = \frac{1}{d_t} \int_0^{d_t} \nu_e \, dz' \quad (3.7.17)$$

From Eq. (3.6.9), $\nu_m$ can be denoted as

$$\nu_m = C_\nu h \rho^{-1/3} D_B^{1/3} \quad (3.7.18)$$

where $C_\nu$ is determined as

$$C_\nu = 0.03 \quad (3.7.19)$$

from the experimental results. Then Eq. (3.7.6) is replaced by

$$\nu_e = 2\nu_m \frac{z'}{d_t}$$

$$= 0.06\rho^{-1/3} D_B^{1/3} h \frac{h}{d_t} z' \quad (3.7.20)$$

### 3.7.2 Expression of Undertow Profile by Using the Eddy Viscosity Model

It was assumed that the eddy viscosity coefficient takes the value zero at the bottom in the previous section, but there exists the kinematic viscosity $\nu$. Though the value $\nu$ is far smaller than $\nu_e$ in the surf zone in general, it cannot be neglected near the bottom or in the offshore region where the field is regarded as a laminar flow. Therefore, the total viscosity is defined as follows:

$$\nu_t = \nu_e + \nu \quad (3.7.21)$$

By using the eddy viscosity model, the relation between the mean shear stress $\tau$ acting on the horizontal plane and the steady current $U$ in $x$-direction is expressed as

$$\tau = \rho \nu_t \frac{\partial U}{\partial z} \quad (3.7.22)$$
Substituting Eqs. (3.7.5), (3.7.6) and (3.7.21) into Eq. (3.7.22) and replacing $z$ by $z'$, the steady current $U$ can be expressed in a first-order linear differential equation as

$$\frac{\partial U}{\partial z'} = \frac{a_\tau z' + b_\tau}{a_\nu z' + \nu}$$  \hspace{1cm} (3.7.23)

The general solution of Eq. (3.7.23) is obtained as

$$U = \int \frac{a_\tau z' + b_\tau}{a_\nu z' + \nu} dz' = \frac{a_\tau}{a_\nu} z' + a_\nu b_\tau - a_\tau \nu a_\nu^2 \log (a_\nu z' + \nu) + C_1$$  \hspace{1cm} (3.7.24)

where $C_1$ is an integral constant. The vertically averaged value of the undertow is defined as

$$U_m = \frac{1}{d_t} \int_{-h_0}^{d_t} U dz' = \frac{1}{d_t} \int_{-h_0}^{d_t} U dz$$  \hspace{1cm} (3.7.25)

From Eq. (3.3.3), it is obvious that the integrated undertow should compensate the mass flux above the trough level. Hence, $U_m$ is calculated by using Eq. (3.3.3) as

$$U_m = -\frac{1}{d_t} M_t$$  \hspace{1cm} (3.7.26)

Substituting Eq. (3.7.24) into Eq. (3.7.25), $C_1$ is determined as

$$C_1 = U_m - \frac{1}{2} \frac{a_\tau}{a_\nu} d_t - \frac{a_\nu b_\tau - a_\tau \nu}{a_\nu^3 d_t} (a_\nu d_t + \nu) \log (a_\nu d_t + \nu) - \nu \log \nu - a_\nu d_t$$  \hspace{1cm} (3.7.27)

The value $\tau$ calculated by Eq. (3.7.14) becomes zero in the offshore region or in the wave recovery zone because $D_B$ is zero in these regions. Then the steady current calculated by Eq. (3.7.24) becomes a uniform flow. Longuet-Higgins (1953) derived the Eulerian mass transport velocity on the assumption that the non-linear inertia terms can be neglected compared with the viscous terms as

$$U_E = \frac{a_\nu^2 \sigma k}{4 \sinh^2 kh} \left\{ 3 + kh \left( 3 \frac{z^2}{h^2} + 4 \frac{z}{h} + 1 \right) \sinh 2kh \right\}$$
Equation (3.7.28) produces no net mass transport when using the small amplitude wave theory. In the present model, the steady current in the offshore region and the wave recovery zone is determined as

$$U_{\text{off}} = U_E + C_2$$

(3.7.29)

where $C_2$ is the compensatory value to satisfy

$$\int_{-h_0}^{\zeta_t} U_{\text{off}} \, dz = -M_t$$

(3.7.30)

From Eqs. (3.7.26), (3.7.28), (3.7.29) and (3.7.30), $C_2$ is obtained as

$$C_2 = U_m - \frac{a^2\sigma k}{4(\zeta_t + h) \sinh^2 kh} \left\{ \left( \frac{kh \sinh 2kh + \frac{3 \sinh 2kh}{2kh}}{2} \right) \frac{\zeta_t^3}{h^2} + \left( 2kh \sinh 2kh \right) \frac{\zeta_t^2}{h} + \left( kh \sinh 2kh - \frac{3 \sinh 2kh}{2kh} - \frac{3}{2} \right) \zeta_t \right\}$$

(3.7.31)

Then $U_{\text{off}}$ is expressed as

$$U_{\text{off}} = \frac{a^2\sigma k}{4 \sinh^2 kh} \left[ \left( 3kh \sinh 2kh + \frac{3 \sinh 2kh}{2kh} + \frac{9}{2} \right) \frac{z'^2}{h^2} + \left( -2kh \sinh 2kh - \frac{6 \sinh 2kh}{2kh} - 9 \right) \frac{z'}{h} + \frac{d_t}{h} \left\{ \left( 1 - \frac{d_t}{h} \right) kh \sinh 2kh + \left( 3 - \frac{d_t}{h} \right) \frac{\sinh 2kh}{2kh} + \frac{3}{2} \left( 3 - \frac{d_t}{h} \right) \right\} \right] + U_m$$

(3.7.32)

For the sake of smoothing the solution near the breaking point or the wave recovery point, Eq. (3.7.24) is replaced as

$$U = \frac{a_r'}{a_r} \left( z' - \frac{d_t}{2} \right) + \frac{a_r b_r' - a_r' v}{a_r^2} \left( 1 + \log \frac{a_r z' + v}{a_r d_t + v} - \frac{v}{a_r d_t} \log \frac{a_r d_t + v}{\nu} \right) + U_m$$

(3.7.33)
where
\[ a'_r = a_r + \frac{\nu a^2 \sigma k}{2h^2 \sinh^2 kh} \left( 3kh \sinh 2kh + \frac{3 \sinh 2kh}{2kh} + \frac{9}{2} \right) \]
\[ b'_r = b_r - \frac{\nu a^2 \sigma k}{4h \sinh^2 kh} \left( 2kh \sinh 2kh + \frac{6 \sinh 2kh}{2kh} + 9 \right) \]
\[ (3.7.34) \]

The values of the second terms in Eq. (3.7.34) are far smaller than those of the first terms in the inner region of the surf zone. In the offshore region where \( D_B = 0 \), Eq. (3.7.33) reduces to
\[ \frac{U}{U_m} = \frac{a'_r}{2\nu} z'^2 + \frac{b'_r}{\nu} z' - \frac{a'_r}{6\nu} d'_t - \frac{b'_r}{2\nu} d_t + U_m \]
\[ (3.7.35) \]
which is exactly same as Eq. (3.7.32) [see Appendix B]. In case of \( D_B \ll 1 \), the undertow profile is calculated by Eq. (3.7.35) to prevent the calculation from diverging.
3.8 Computational Results

By using the present model, at first the potential and kinetic energies of the wave motion and the energy of the organized large vortexes are obtained. The wave setup is also calculated. From the energy variations, the vertically averaged value of undertow is estimated. Finally, the profiles of the undertow are obtained with estimation of the mean Reynolds stress and the mean eddy viscosity coefficient from the energy dissipation rate.

In this section, the calculated values will be compared with the measured values which were obtained by the laboratory experiments described in §2.2. The energy variations will also be discussed with the experimental results obtained by Nagayama (1983).

3.8.1 Variations of Energies

The variation of the calculated values of the potential energy of wave motion $E_p$, the energy of the organized large vortexes $E_v$ and the total energy of wave $E_t$ for case B-1 which was described in §2.2 are shown in Fig. 3.8.1. The measured potential energy of waves is shown together in the figure. The measured and calculated breaking points, plunging points and transition points are also indicated.

For the calculation by the present model, only beach topography, period and height (or amplitude) of the incident waves are necessary as inputs. The spacing between the mesh points and the time step of the calculation were respectively 2 cm and 0.005 s for all the cases. The calculation was started from the points 100 cm offshoreward of the foot of the slopes, which was $x = -900$ cm in case B-1. The convergence of solution was assumed when the absolute errors between the values obtained from two successive cycles of the calculation at every point were less than 2% of the amplitude of the incident waves throughout the field for both the wave
Fig. 3.8.1 Calculated and measured energy variations (case B–1).
amplitudes and the setup calculation. Other needed factors and coefficients were already given in the previous sections.

In the figure, the calculated and measured potential energy $E_p$ are in good agreement in the surf zone; however the calculated $E_p$ is slightly smaller than the measured value at $x = -500$ cm. The numerical reflection can be seen in the offshore region. The breaking point is estimated slightly offshoreward in spite of the smaller value of the evaluated potential energy in the offshore region. The estimated plunging point is much offshore, but the transition point is well estimated if the difference of the breaking points is taken into account. The calculated $E_p$ begins to attenuate from the estimated breaking point in the figure, although Watanabe and Maruyama (1986) mentioned that the calculated energy still increased at the breaking point when setting $\alpha_T = 0$ at the breaking point. The reasons should be that $\alpha_T$ increases from the crest breaking point in this case and that the ratio between the potential energy and the kinetic energy is modified after breaking.

On the other hand, the total energy $E_t$ does not change so much around the breaking point and rapidly decreases from the measured plunging point to the transition point. It is caused by the existence of the organized large vortexes. The magnitude of the energy of the organized large vortexes $E_v$ is comparable to that of the potential energy $E_p$. The energy of the vortexes increases from the crest breaking point and takes its maximum value which is almost the same as the value of $E_p$ near the calculated transition point.

The calculated energy transfer rate $T_B$ from the wave energy to the energy of the vortexes and the energy dissipation rate $D_B$ are shown in Fig. 3.8.2. The hatched area corresponds to the rate of change of the vortex energy. The qualitative agreement with the vertically averaged energy of turbulence $u'_m^2$ and $w'_m^2$ shown in Figs. 2.3.15 and 2.3.16 is good except near the plunging point where the
Fig. 3.8.2  Rates of energy transfer and dissipation (case B-1).

Fig. 3.8.3  Potential energies calculated by the present model and measured by Nagayama (1983) on a uniform slope (case 1).
calculated value of the energy dissipation is much larger than the measured value of the turbulence energy. But in general, it can be said that the difference of the spatial distributions between the attenuation of the wave energy and the energy dissipation is well expressed by the present model.

Figure 3.8.3 gives the comparison with the variation of the potential energy measured by Nagayama (1983) on a 1/20 constant slope (case 1). The incident wave condition is listed in Table 3.8.1. The potential energy is under-estimated in the surf zone. Since the potential energy rapidly decreases after breaking, it can be considered that the prediction of the breaking point influences much to the variations of the calculated energies in the surf zone. The calculated potential energy increases gradually up to the wave breaking point, although the measured value keeps almost same value. It should be because the small amplitude theory by which the wave energy is over-estimated in shallow water is adopted in this model [Dibajnia et al. (1988)].

Table 3.8.1  Conditions of experiments performed by Nagayama (1983).

<table>
<thead>
<tr>
<th>case</th>
<th>beach type</th>
<th>T (s)</th>
<th>$h_i$ (cm)</th>
<th>$H_i$ (cm)</th>
<th>$H_0/L_0$</th>
<th>$x_b$ (cm)</th>
<th>$x_p$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>uniform</td>
<td>1.19</td>
<td>27.1</td>
<td>5.44</td>
<td>0.027</td>
<td>-182</td>
<td>-132</td>
</tr>
<tr>
<td>3</td>
<td>step</td>
<td>1.19</td>
<td>28.4</td>
<td>5.07</td>
<td>0.025</td>
<td>-358</td>
<td>-306</td>
</tr>
<tr>
<td>5</td>
<td>step</td>
<td>0.96</td>
<td>28.4</td>
<td>7.19</td>
<td>0.055</td>
<td>-387</td>
<td>-308</td>
</tr>
<tr>
<td>6</td>
<td>bar</td>
<td>0.94</td>
<td>28.4</td>
<td>6.48</td>
<td>0.051</td>
<td>-357</td>
<td>-331</td>
</tr>
<tr>
<td>7</td>
<td>bar</td>
<td>0.95</td>
<td>28.4</td>
<td>5.85</td>
<td>0.045</td>
<td>-357</td>
<td>-314</td>
</tr>
</tbody>
</table>

The comparisons between the calculated and measured potential energies are shown in Figs. 3.8.4 and 3.8.5 for cases 3 and 5 of the experiments by Nagayama on a step-type beach with the variations of the transfer factor $f_T$. The transfer factors increase from the crest breaking points according to the coefficient $\alpha_T$, then
Potential energies calculated by the present model and measured by Nagayama (1983) on a step-type beach (case 3).

Potential energies calculated by the present model and measured by Nagayama (1983) on a step-type beach (case 5).
decrease from the transition points on the flat bed. As the transfer factor becomes zero on the flat bed in case 5, waves recover at that point in calculation. On the other hand, they do not recover by calculation for case 3, although the variation of the measured potential energy suggests the wave recovery around $x = -150$ cm. However, in general, it can be said that the calculated values predict the actual energy variations well in both cases. The energy variations for case B–11 are shown in Fig. 3.8.6. The relative magnitude of the vortex energy is larger than other cases because of the steeper bottom slope offshoreward of the flat bed. The agreement is good.

Figures 3.8.7 and 3.8.8 give the comparisons of the calculated and measured potential energies for cases 6 and 7 of Nagayama’s experiments on a bar-type beach. The agreement is good. However, the estimation of the second breaking points is not accurate in both cases. It is probably due to the fact that waves which have once recovered tend to break easily for the second time. The reflection at the first breaking points is too large to be neglected.

3.8.2 Vertically Averaged Undertow

Figure 3.8.9 shows the comparison of the variations of the calculated and measured values $U_m$ of the steady current averaged vertically below the trough level for case B–1. The contributions to the calculated value of $U_m$ from the mass flux by wave motion $U_w \left( \equiv -\frac{M_w}{d_t} \right)$ and the organized large vortexes $U_v \left( \equiv -\frac{M_v}{d_t} \right)$ are also given in the figure. The measured value is smaller than the calculated value in the outer region but is larger in the inner region. Since the measured value is almost the same as the value $U_w$ offshoreward of the plunging point, it is considered that the actual magnitude of the contribution by the organized large vortexes is much smaller than the calculated value.
Fig. 3.8.6  Calculated and measured energy variations (case B-11).
Fig. 3.8.7  Potential energies calculated by the present model and measured by Nagayama (1983) on a bar-type beach (case 6).

Fig. 3.8.8  Potential energies calculated by the present model and measured by Nagayama (1983) on a bar-type beach (case 7).
The calculated value of the energy dissipation around the plunging points is much larger than the measured value of the turbulence energy as already mentioned. It means that the actual energy dissipation may be smaller than the value evaluated by this model. Since the calculated potential energy agrees with the measured value as shown in Fig. 3.8.1, little energy dissipation means that the energy of vortexes must be much larger. But in the present model, the large value of the vortex energy at the plunging point makes the mass flux by the vortexes large there, which is contrary to the results shown in Fig. 3.8.9. It is probable that the structure of the organized large vortexes in the outer region is different from that in the inner region. The magnitude of the vortex energy depends on the dissipation length of vortex energy $l_d$ which is given by Eq. (3.5.29). The influence by the difference of the structure of the organized large vortex is partly considered in Eq. (3.5.29) in this model.

The comparisons between the calculated value and measured value of $U_m$ for cases B–4 and B–8 are shown in Figs. 3.8.10 and 3.8.11. The vertically averaged undertow is well predicted in case B–4 in the outer region, however it is rather over-estimated in case B–8. The reason for such a difference is not clear, but at least, it can be said that the prediction of the breaking point affects the evaluation of $U_m$ especially in the outer region.

In the inner region, the calculated value is larger than the measured value in case B–4. In case B–8, the measured value fluctuates in the inner region. It cannot be concluded whether the fluctuation of the measured value is caused by the three-dimensionality of the steady current in the inner region or the error of the measurement.

3.8.3 Profiles of Undertow
Fig. 3.8.9  Variations of calculated and measured $U_m$ (case B-1).

Fig. 3.8.10  Variations of calculated and measured $U_m$ (case B-4).

Fig. 3.8.11  Variations of calculated and measured $U_m$ (case B-8).
The comparison of the calculated and measured distribution of the steady current in the cross-shore direction $U$ for case B–1 is shown in Fig. 3.8.12 with the bottom profile. The variation of the mean water level which will be discussed in the next part of this section is given together. From the figure, it can be said that the profiles in the inner region are well evaluated. The calculated profiles in the outer region which decrease with the vertical elevation $z'$ are different from the measured profiles which increase with $z'$. In the present model, the mean shear stress $\tau$ is almost zero at $z' = \frac{1}{5}d$, in the surf zone as mentioned in §3.7.1, $\frac{\partial U}{\partial z'}$ becomes also zero at that point. However, the profiles are greatly depend on the energy dissipation rate $D_B$. If $D_B$ is very small as it is at the breaking point, the profile becomes almost vertical as shown around the predicted breaking point in Fig. 3.8.12. In that case, the agreement of the calculated and measured profiles is fairy good. On the other hand, as the calculated energy dissipation is larger than the measured turbulent energy near the plunging points as already mentioned in the previous part, the calculated profiles become much different from the measured value. If the evaluated energy dissipation becomes in better agreement, the profiles will also approach to the measured value near the plunging point. Since $D_B$ changes rapidly in the outer region as shown in Fig. 3.8.2, the prediction of the breaking points has a great importance for the accurate estimation of $D_B$, i.e., the profiles of the steady current in the outer region. The comparisons for cases B–4 and B–8 are shown in Figs. 3.8.13 and 3.8.14. The tendency is the same, but the agreement in the outer region is rather good for case B–4.

Figure 3.8.15 gives the undertow profiles for case B–11. The calculated profile does not fit to the measured profile at the plunging point, either in this case. On the offshore part of the flat bed, the form of the calculated profiles is good, although the magnitude is smaller than the measured values. The onshore side on the flat bed is a wave recovery zone. The energy transfer factor $f_T$ becomes zero at $x = -240$
Fig. 3.8.12  Steady current distribution and the variation of mean water level (case B-1).

Fig. 3.8.13  Steady current distribution and the variation of mean water level (case B-4).
Fig. 3.8.14  Steady current distribution and the variation of mean water level (case B–8).

Fig. 3.8.15  Steady current distribution and the variation of mean water level (case B–11).
cm, but $D_B$ has some value there and it becomes zero around $x = -200$ cm where the waves are regarded to recover by the calculation. The observed recovery point was $x = -115$ cm. The measured profiles are almost vertical and the magnitude is small at the measuring lines at $x = -145$ and $x = -105$ cm where the calculated profiles are the same as those by Longuet-Higgins (1953) because $D_B$ is zero there. The figure shows that the computational result expresses the distribution of the steady current under recovering waves on the onshore side of the flat bed well. Hence, it can be concluded that the present model can deal with the steady current distributions not only for uniform but also for composite beach topographies.

### 3.8.4 Wave Setup and Setdown

The variations of the mean water level, that is the wave setup or setdown, are shown in Figs. 3.8.12 - 3.8.15. The agreement between the calculated and measured setup is good in all cases. The wave setdown is slightly over-estimated at the breaking points, because the radiation stress is over-estimated there by the small amplitude theory. Since the total wave energy $E_t$ by the calculation does not change so much till the plunging point, the wave setup in the outer region is under-estimated. In other words, the wave setup lag behind the deformation of the waves. It is evident in case B-11, but the setup on the flat bed is well predicted by the present model.
CHAPTER 4

Conclusions

The present study has investigated the characteristic motion of water under breaking waves and the turbulence structure in the surf zone through detailed and precise laboratory experiments. A model for estimating the cross-shore vertical distribution of the undertow has been presented with a model of the energy transfer process in the surf zone. The main conclusions obtained in the this study are summarized as follows:

1. The organized vortex motion caused by wave breaking is an important fluid motion which transmits the wave energy to the turbulence energy. The velocity field in the surf zone can therefore be divided into four components which are the steady current, wave motion, organized vortex motion and turbulence.

2. The bottom boundary layer in the inner region of the surf zone does not develop well due to agitation of the turbulence from the upper layer. In such a quasi-steady breaking region, the magnitude of the steady current velocity near the bottom is large in the offshoreward direction.

3. The mean Reynolds stress and the mean eddy viscosity coefficient in the inner region can be regarded as linear functions of the vertical elevation. The offshore directed mean shear stress on the bottom is so large that it cannot be neglected. The vertically averaged values of these two quantities can be expressed in terms of the energy dissipation rate.

4. The transition point which is the boundary between the outer region and the inner region should be defined as the offshore end of the quasi-steady breaking region. The distance from the breaking point to the transition point can be expressed in terms of the water depth at the breaking point and the bottom
slope. The distance to the plunging point does not depend on the bottom slope.

(5) From the distribution of the energy of wave motion which can be calculated by using the time-dependent mild slope equation, the distribution of the vortex energy and the variation of the energy dissipation rate in the surf zone can be evaluated by a model. In the model, the organized large vortexes are taken into account as transmitters of energy from wave motion to turbulence in the energy transfer process. The model is able to explain the spatial difference between the wave attenuation and the production of the turbulence energy.

(6) The mass transport by breaking waves can be evaluated as the sum of two components. One component is the contribution from the wave motion which can be estimated by modifying the linear long wave theory. The other component is that by the organized large vortexes which can be estimated from the vortex energy with an assumption of the internal velocity distribution of the vortex.

(7) The momentum flux by the organized vortexes should also be taken into account. The little change of the mean water level around the wave plunging points can be rationally explained by considering the momentum flux by the vortexes.

(8) The cross-shore vertically two-dimensional distribution of the undertow which is calculated by the present model only from the incident wave conditions agree well with the measured values in the surf zone including the wave recovery zone.

The present study has introduced a new concept of energy transfer in the surf zone considering the organized vortex motion due to wave breaking. Although this idea is based on the experimental results obtained in the former half of this study, it is based on some assumptions as well. Investigations of the energy transfer process
from micro-scopic views are therefore necessary for a much more accurate estimation of the energy distribution and the impregnable description of the turbulent velocity field in the surf zone. For that sake, detailed measurements of the velocity field in the upper layer and the bottom layer are required. Though the present study dealt only with the on-offshore cross-sectional distribution of the undertow, it can be applied to the prediction of the three-dimensional distribution of the nearshore current.
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APPENDIX A

The governing equations of the time-dependent mild slope equation applied to this model are given by

\[ \frac{\partial Q}{\partial t} + c^2 \frac{\partial \zeta}{\partial x} + f_A Q = 0 \] (3.5.25)

\[ \frac{\partial \zeta}{\partial t} + \frac{1}{n} \frac{\partial (nQ)}{\partial x} = 0 \] (3.5.26)

In the derivation of the time-dependent mild slope equation, the following equations have been assumed:

\[ \begin{align*}
\zeta &= \zeta e^{-i\sigma t} \\
Q &= \hat{Q} e^{-i\sigma t}
\end{align*} \] (A.1)

where \( \hat{\zeta} \) and \( \hat{Q} \) are the complex amplitudes of the surface elevation and the flow rate, respectively. Substituting Eq. (A.1), Eqs. (3.5.25) and (3.5.26) can be replaced by

\[ (f_A - i\sigma) \hat{Q} + c^2 \frac{\partial \hat{\zeta}}{\partial x} = 0 \] (A.2)

\[ -i\sigma \hat{\zeta} + \frac{1}{n} \frac{\partial (n\hat{Q})}{\partial x} = 0 \] (A.3)

Eliminating \( \hat{Q} \) from Eqs. (A.2) and (A.3), a single equation which includes the second derivative of \( \hat{\zeta} \) is obtained as

\[ i\sigma \hat{\zeta} + \frac{c^2}{(f_A - i\sigma)} \frac{\partial^2 \hat{\zeta}}{\partial x^2} + \frac{1}{n} \frac{\partial}{\partial x} \left( \frac{n c^2}{(f_A - i\sigma)} \right) \frac{\partial \hat{\zeta}}{\partial x} = 0 \] (A.4)

If the water depth \( h \) and the energy transfer factor \( f_A \) are constant whole the region, \( n \) and \( c \) become also constants. Then the third term of Eq. (A.4) vanishes.

\[ i\sigma \hat{\zeta} + \frac{c^2}{(f_A - i\sigma)} \frac{\partial^2 \hat{\zeta}}{\partial x^2} = 0 \] (A.5)

Replacing \( c \) by \( \sigma/k \), Eq. (A.5) is expressed as

\[ k^2 \left( 1 + \frac{if_A}{\sigma} \right) \hat{\zeta} + \frac{\partial^2 \hat{\zeta}}{\partial x^2} = 0 \] (A.6)
Assuming the form of the solution as

$$\zeta = \xi e^{\kappa x}$$  \hspace{1cm} (A.8)

where $\xi$ is a complex amplitude and $\kappa$ is a complex quantity which expresses the energy decay and the phase, Eq. (A.6) can be described as

$$k^2 (1 + \frac{f_A}{\sigma}) \xi e^{\kappa x} + \xi \kappa^2 e^{\kappa x} = 0$$  \hspace{1cm} (A.7)

The solution is obtained as

$$\zeta = \xi e^{\frac{k f_A}{\sqrt{2 \sigma}} \sqrt{1 + \sqrt{1 + \frac{f_A^2}{\sigma^2}}} x} e^{i k \sqrt{1 + \sqrt{1 + \frac{f_A^2}{\sigma^2}}}} x$$  \hspace{1cm} (A.8)

Now a wave field which has two regions bounded at $x = 0$ as shown in Fig. A.1 is supposed. Here it can be treated as $f_A = f_T$, because $f_{h+w}$ given by Eq. (3.5.20) is far smaller than $f_T$ in the surf zone and it changes little whole the field. The transfer factor $f_T$ is zero in Region I at the left hand side of the field and takes some value $f_2$ in Region II at the right hand side. Though the water depth is constant in both regions, the wave number $k_1$ in Region I and $k_2$ in Region II are different according to the value of $f_T$ as inferred from Eq. (A.8). If the wave comes from the left hand side, the complex amplitude function of the surface elevation in Region I $\tilde{\zeta}_1$ consists of the incident wave component $\tilde{\zeta}_i$ and the reflected wave component $\tilde{\zeta}_r$, which is expressed as

$$\tilde{\zeta}_1 = \tilde{\zeta}_i + \tilde{\zeta}_r$$

$$\tilde{\zeta}_i = \xi e^{i k_1 x}, \quad \tilde{\zeta}_r = \xi e^{-i k_1 x}$$  \hspace{1cm} (A.9)

which has no decaying factor because of $f_T = 0$. On the other hand, $\tilde{\zeta}_2$ in Region II contains the decaying factor $r_2$ and is denoted as

$$\tilde{\zeta}_2 = \xi_2 e^{(r_2 + i k_2) x}$$  \hspace{1cm} (A.10)
\( \xi_i, \xi_r \) and \( \xi_2 \) are complex amplitudes which are determined as

\[
\xi_i = p_i + iq_i, \quad \xi_r = p_r + iq_r, \quad \xi_2 = p_2 + iq_2
\]  
(A.11)

where \( p_i, q_i, p_r, q_r, p_2 \) and \( q_2 \) are real numbers. The boundary conditions of the surface elevation and the velocity at \( x = 0 \) are expressed as

\[
\hat{\xi}_1 = \hat{\xi}_2
\]  
(A.12)

\[
\frac{\partial \hat{\xi}_1}{\partial x} = \frac{\partial \hat{\xi}_2}{\partial x}
\]  
(A.13)

Substituting Eqs. (A.10) and (A.11) into Eqs. (A.12) and (A.13)

\[
\begin{align*}
 p_i + p_r &= p_2 \\
 q_i + q_r &= q_2
\end{align*}
\]  
(A.14)

and

\[
\begin{align*}
 k_1 (p_2 - 2p_r) &= k_2 p_2 + r_2 q_2 \\
 -k_1 (q_2 - 2q_r) &= r_2 p_2 - k_2 q_2
\end{align*}
\]  
(A.15)

can be obtained. From Eqs. (A.14) and (A.15), \( p_r \) and \( q_r \) is expressed as follows:

\[
\begin{align*}
 p_r &= \frac{1}{2} \left( p_2 - \frac{r_2 p_2 - k_2 q_2}{k_1} \right) \\
 q_r &= \frac{1}{2} \left( q_2 + \frac{k_2 p_2 - r_2 q_2}{k_1} \right)
\end{align*}
\]  
(A.16)

Considering Eq. (A.8), the wave number and the decaying factor in Region II are denoted as

\[
\begin{align*}
 k_2 &= 2a_k k_1 \\
 r_2 &= -2b_k k_1
\end{align*}
\]  
(A.17)

where

\[
a_k = \frac{\sqrt{1 + \sqrt{1 + \frac{f_2}{\sigma^2}}}}{2\sqrt{2}}, \quad b_k = \frac{f_2}{2\sqrt{2}\sigma\sqrt{1 + \sqrt{1 + \frac{f_2}{\sigma^2}}}}
\]  
(A.18)

Then the values \( p_r \) and \( q_r \) are obtained as

\[
\begin{align*}
 p_r &= \left( \frac{1}{2} - a_k \right) p_2 + b_k q_2 \\
 q_r &= -b_k p_2 + \left( \frac{1}{2} - a_k \right) q_2
\end{align*}
\]  
(A.19)
Substituting Eq. (A.19) with Eq. (A.11), the complex amplitude function of water surface elevation of the reflected wave can be denoted as

\[ \hat{\zeta}_r = (p_r + iq_r) e^{-ik_1 z} = \left[ \left\{ \left( \frac{1}{2} - a_k \right) p_2 + b_k q_2 \right\} + i \left\{ -b_k p_2 + \left( \frac{1}{2} - a_k \right) q_2 \right\} \right] e^{-ik_1 z} \] (A.20)

It can be concluded that the reflected wave component expressed by \( \hat{\zeta}_r \) in Eq. (A.20) is generated by the sudden change of the energy transfer factor. It is possible to restrain the energy of the reflected wave by apparently changing the wave number in Region II.

If using \( k' \) which does not satisfy the dispersion relation in stead of \( k_1 \) in Region II, Eq. (A.17) is replaced by

\[ k_2 = \frac{2a_k k'}{r_2} = -2b_k k' \] (A.21)

\( p_r \) and \( q_r \) are also obtained as

\[ p_r = \left( \frac{1}{2} - \frac{k'}{k_1} a_k \right) p_2 + \frac{k'}{k_1} b_k q_2 \]
\[ q_r = -\frac{k'}{k_1} b_k p_2 + \left( \frac{1}{2} - \frac{k'}{k_1} a_k \right) q_2 \] (A.22)

Then the energy of the reflected wave \( E_R \) can be expressed in terms of the energy of the transmitted wave \( E_T \) as

\[ E_R = \left\{ \left( \frac{1}{2} - \frac{k'}{k_1} a_k \right)^2 + \left( \frac{k'}{k_1} b_k \right)^2 \right\} E_T \] (A.23)

where

\[ E_R = \frac{1}{2} \rho g (p_r^2 + q_r^2), \quad E_T = \frac{1}{2} \rho g (p_2^2 + q_2^2) \] (A.24)

The ratio of the reflected wave energy to the transmitted wave energy is calculated by

\[ R_E \equiv \frac{E_R}{E_T} \]
Finally, the minimum value of $R_E$ is given by

$$R_E = \frac{1}{4} \frac{b_k^2}{a_k^2 + b_k^2}$$  \hspace{1cm} (A.26)$$

when

$$k' = \frac{a_k}{2(a_k^2 + b_k^2)} k_1$$  \hspace{1cm} (A.27)$$
APPENDIX B

The expansion of \( \log(1 + x) \) to a power series around \( x = 0 \) is denoted as

\[
\log(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad (B.1)
\]

Hence \( \log(a_v z' + \nu) \) and \( \log(a_v d_t + \nu) \) can be expressed as

\[
\log(a_v z' + \nu) = \log \left( 1 + \frac{z'}{\nu} a_v \right)
\]

\[
= \log \nu + \log \left( 1 + \frac{z'}{\nu} a_v \right)
= \log \nu + \frac{z'}{\nu} a_v - \frac{1}{2} \left( \frac{z'}{\nu} a_v \right)^2 + \frac{1}{3} \left( \frac{z'}{\nu} a_v \right)^3 - \frac{1}{4} \left( \frac{z'}{\nu} a_v \right)^4 + \cdots \quad (B.2)
\]

\[
\log(a_v d_t + \nu) = \log \left( 1 + \frac{d_t}{\nu} a_v \right)
\]

\[
= \log \nu + \log \left( 1 + \frac{d_t}{\nu} a_v \right)
= \log \nu + \frac{d_t}{\nu} a_v - \frac{1}{2} \left( \frac{d_t}{\nu} a_v \right)^2 + \frac{1}{3} \left( \frac{d_t}{\nu} a_v \right)^3 - \frac{1}{4} \left( \frac{d_t}{\nu} a_v \right)^4 + \cdots \quad (B.3)
\]

Substituting Eqs. (B.2) and (B.3), \( U \) in Eq. (3.7.33) is transformed as

\[
U = \frac{a_v}{a_{\nu}} \left( z' - \frac{d_t}{2} \right) + \frac{a_v b_{\nu} - a_{\nu} \nu}{a_{\nu}^2} \left( 1 + \log \frac{a_v z' + \nu}{a_v d_t + \nu} - \frac{\nu}{a_v d_t} \log \frac{a_v d_t + \nu}{\nu} \right) + U_m
\]

\[
= \frac{a_v}{a_{\nu}} \left( z' - \frac{d_t}{2} \right) + \frac{a_v b_{\nu} - a_{\nu} \nu}{a_{\nu}^2} \left[ 1 + \left\{ \log \nu + \frac{a_v}{\nu} z' - \frac{1}{2} \left( \frac{a_v}{\nu} z' \right)^2 + \cdots \right\} \right.
\]

\[
- \left\{ \log \nu + \frac{a_v}{\nu} d_t - \frac{1}{2} \left( \frac{a_v}{\nu} d_t \right)^2 + \cdots \right\}
\]

\[
- \frac{\nu}{a_v d_t} \left( \left\{ \log \nu + \frac{a_v}{\nu} d_t - \frac{1}{2} \left( \frac{a_v}{\nu} d_t \right)^2 + \frac{1}{3} \left( \frac{a_v}{\nu} d_t \right)^3 + \cdots \right\} - \log \nu \right) \right] + U_m
\]

\[
= \frac{a_v}{a_{\nu}} \left( z' - \frac{d_t}{2} \right) + \frac{a_v b_{\nu} - a_{\nu} \nu}{a_{\nu}^2} \left[ \frac{a_v}{\nu} z' - \frac{1}{2} \left( \frac{a_v}{\nu} z' \right)^2 + \cdots \right.
\]

\[
- \frac{a_v}{\nu} d_t + \frac{1}{2} \left( \frac{a_v}{\nu} d_t \right)^2 + \cdots + \frac{a_v}{\nu} d_t - \frac{1}{3} \left( \frac{a_v}{\nu} d_t \right)^2 + \cdots \right] + U_m \quad (B.4)
\]
The steady current $U$ in the limit when $a_\nu$ approaches to zero (namely $D_B$ approaches to zero) is given as

\[
\lim_{a_\nu \to 0} U = \lim_{a_\nu \to 0} \left[ \frac{a_r'}{a_\nu} \left( z' - \frac{d_t}{2} \right) + \frac{a_\nu b_r'}{a_\nu^2} \left\{ \frac{a_\nu}{\nu} z' - \frac{1}{2} \left( \frac{a_\nu}{\nu} z' \right)^2 - \frac{1}{2} \frac{a_\nu}{\nu} d_t + \frac{1}{6} \left( \frac{a_\nu}{\nu} d_t \right)^2 + \cdots \right\} + U_m \right]
\]

\[
= \lim_{a_\nu \to 0} \left[ \frac{a_r'}{\nu} \left( \frac{z'}{2} - \frac{d_t^2}{6} \right) + \frac{b_r'}{\nu} \left( z' - \frac{d_t}{2} \right) + a_\nu \frac{b_r'}{\nu^2} \left( -\frac{z'^2}{2} + \frac{d_t^2}{6} \right) + U_m \right]
\]

\[
= \frac{a_r'}{2\nu} z'^2 + \frac{b_r'}{\nu} z' - \frac{a_r'}{6\nu} d_t^2 - \frac{b_r'}{2\nu} d_t + U_m
\]

\[
= \frac{1}{2
nu} \frac{\nu a^2 \sigma k}{2 h^2 \sinh^2 kh} \left( 3 k h \sinh 2 k h + \frac{3 \sinh 2 k h}{2 k h} + \frac{9}{2} \right) \frac{z'^2}{h^2}
\]

\[
- \frac{1}{\nu} \frac{\nu a^2 \sigma k}{4 h \sinh^2 kh} \left( 2 k h \sinh 2 k h + \frac{6 \sinh 2 k h}{2 k h} + 9 \right) \frac{z'}{h}
\]

\[
- \frac{1}{6\nu} \frac{\nu a^2 \sigma k}{2 h^2 \sinh^2 kh} \left( 3 k h \sinh 2 k h + \frac{3 \sinh 2 k h}{2 k h} + \frac{9}{2} \right) \frac{d_t^2}{h^2}
\]

\[
+ \frac{1}{2\nu} \frac{\nu a^2 \sigma k}{4 h \sinh^2 kh} \left( 2 k h \sinh 2 k h + \frac{6 \sinh 2 k h}{2 k h} + 9 \right) \frac{d_t}{h}
\]

\[
+ U_m
\]

\[
= \frac{a^2 \sigma k}{4 \sinh^2 kh} \left\{ \left( 3 k h \sinh 2 k h + \frac{3 \sinh 2 k h}{2 k h} + \frac{9}{2} \right) \frac{z'^2}{h^2}
\]

\[
- \left( 2 k h \sinh 2 k h + \frac{6 \sinh 2 k h}{2 k h} + 9 \right) \frac{z'}{h}
\]

\[
- \left( k h \sinh 2 k h + \frac{\sinh 2 k h}{2 k h} + \frac{3}{2} \right) \frac{d_t^2}{h^2}
\]

\[
+ \left( k h \sinh 2 k h + \frac{3 \sinh 2 k h}{2 k h} + \frac{9}{2} \right) \frac{d_t}{h}
\}

+ U_m
\]

\[= U_{\text{off}} \] (B.5)