CHAPTER 6

PRACTICAL USE OF ELECTRIC NETWORKS TO SIMULATE OR PREDICT SEICHE CONDITIONS IN HARBORS

By

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I. INTRODUCTION

The successful design of a marina with respect to seiche conditions presupposes two categories of knowledge. Of these the more difficult to obtain is an adequate description of the local long period wave environment. Somewhat easier, but equally important, is a detailed knowledge of the responses various basin configurations present as a result of irradiation under some standardized wave environment. One such environment would be that provided by sinusoidal waves issuing from a distant line source maintained at unit amplitude, constant frequency, and fixed direction. Independently varying both the source orientation and frequency and measuring resulting responses at fixed locations is a procedure common to many wave scattering experiments. It is generally supposed that these scattered wave fields later may be superimposed. This will be the case if linear equations adequately describe the wave motion, as will be assumed here. However, in a medium with spacially inhomogeneous propagation properties, there is no unambiguous distinction between the incident and scattered waves; only in the case of uniform propagation are the real and imaginary parts of \( e^{i(k \cdot r - \omega t)} \) identical, apart from a translation in the direction of \( k \). Interfering reflections and other scattering effects due to variable depth cannot be eliminated or treated separately from peripheral reflections. The complexity inherent in most practical situations requires that some sort of model be used.

Two computational techniques based on potential flow formulations and suitable for programming on a digital computer have been discussed by Stoker (1957) and Raichlen (1965). General purpose analogue computers are also known to have been used to solve seiche problems (Ref. 19). A mathematical formulation of the boundary-value problem for a harbor of arbitrary shape and constant depth has been given by Miles and Munk (1961a, Ref. 12) with subsequent application to a rectangular harbor. Adaptations of numerical methods used previously for predicting tides or storm surges (summarized in References 1, 3, 6, 9, and 21) also afford promising approaches. A classical technique is construction of a suitably scaled-down hydraulic model. Another type of physical model is due to Ishiguro and is based on a physical analogy with electric networks. Such models have been tested and found to yield results quite close to those observed in nature (References 2, 7, 8, and 10). This paper reports on the methods and use of such an analogue model.

II. DISCUSSION

Suppose a fixed volume of fluid having a free surface \( \xi(x, y; t) \) occupies a specified region, \( R \), above a bottom boundary surface of slowly varying
Consider first that the only external forces acting are gravity and a forcing function whose space derivatives furnish time-varying components of force or pressure. For irrotational flow with no local energy dissipation, the resulting motion is derivable from a velocity potential function. In the case where $R$ is bounded horizontally by an impenetrable closed perimeter $C$, the normal modes for free oscillations, or seiches, of the fluid within the variable depth region could be obtained from an equation involving the potential function by setting the forcing function equal to zero and requiring that on $C$ normal components of velocity vanish. In general there exists a discrete, but infinite, set of natural frequencies $\omega_j$ and time independent modal amplitude distributions $A_j(x, y)$ such that $A_j(x, y)e^{\pm i\omega_j t}$ satisfies the reduced equation and boundary condition.

If no mechanisms of internal energy dissipation or coupling are included, motion initiated in a particular mode would continue oscillating with the same amplitudes indefinitely; thus the "$Q$"$^*$ for that mode would be infinite. Suppose now that the basin perimeter described by the curve $C$ were not completely closed, and that this were simulated by a suitably absorbing segment inserted to represent the entrance and also to effect the closure. The initial energy of the oscillations then could decay by radiative losses through this entrance segment and the $Q$ of the mode would no longer be infinite. Other mechanisms which limit $Q$ values are local bottom friction, fluid viscosity, turbulence, and imperfectly reflecting boundaries. A periodic forcing function may be able to weakly excite certain modes even though these differ slightly in frequency.

Often the effects of local energy dissipation are important. In such cases equations of motion and continuity may be written in terms of momentum and mass fluxes where the horizontal velocity components are assumed to take on their vertically averaged values. By ignoring horizontal and vertical velocity differences between vertically separated points such "long-wave" equations only approximately describe the real motions. Equations (1) are of this type, but remain complicated by Coriolis effects and various non-linearities.

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*The quantity $2\pi/Q$ is a linear measure of wave damping. In the time domain it is the average relative decrement of energy per cycle in the decay of free oscillations; the value $Q = 1/2$ corresponds to the case of critical damping. In the frequency domain, the sharpness of resonance in any particular mode of oscillation is also given by the $Q$ for that mode. For large $Q$ this may be taken as the frequency at peak resonance divided by the frequency bandwidth between the half-power points on a response curve. Resonant amplification is not confined only to discrete points along the frequency axis. (References 12, 13, and 15 provide discussions of $Q$ as applied to seiches.)
Approximate solutions are commonly obtained from a linearized set of equations (with corresponding boundary conditions) for cases where: (1) $\eta$ is everywhere very small when compared to $h$; (2) only oscillations with periods very small compared to a pendulum day are considered; (3) the field acceleration terms are small compared to $u_t$ and $v_t$; (4) the quadratic term for tangential stress exerted by the bottom is replaced over some range by one depending on the first power of the velocity. If some prior estimate of the mean current speeds can be made, a form $[\rho y^2 (|u|) u]$ can be used.* The seiches which occur in most harbors, marinas and many natural embayments fairly well meet these conditions.

The superposition property of linear equations is of great practical importance for this problem. Certainly if non-linear interactions between progressing waves or standing wave modes are to be realistically taken into account, much more has to be known regarding the specific interactions which take place. For instance, Groves (1964) points out that if there is a non-linear bottom stress, then any mode will be influenced by all others as well as by any steady flows present. A normally non-dispersive wave phenomenon might also exhibit dispersion due to frictional effects. Long period waves in nature are seldom confined to a single frequency or incident direction. Moreover, any particular realization of an exciting force time history whose case might be treated exactly, has very low probability of recurrence. Thus a statistical description seems to be indicated. For engineering purposes, the techniques of spectral representation applied to a linear process are widely used while those appropriate for other processes are presently in much more limited use.

III. THE ELECTRIC NETWORK ANALOGY

The equations which may be solved through physical analogy with a two-dimensional passive electrical network made up of linear** lumped

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* Harleman and Ippen (1961) state that for waves of initially small amplitude-to-depth ratio, the friction should be essentially independent of wave amplitude.

**i.e., length-distributed circuit elements which are independent of the applied voltage or current and localized as single points at which they assume their average values taken over a small interval $\Delta t$; for instance, inductance $L_j = \text{henrys/meter} \cdot \Delta t_j$.  

---
inductive, capacitive and resistive elements are:

\[
(uh)_t = -gh\eta_x - Fuh
\]

\[
(vh)_t = -gh\eta_y - Fvh
\]

\[
[(\eta+h)dx
dy]_t = \left\{ \left[(uh)dx\right]_x + \left[(vh)dx\right]_y \right\}
\]

where F is a locally constant friction coefficient. The wave motion described by these equations exhibits neither frequency or amplitude dispersion.

Neumann (1944), Ishiguro (1950), Kajiura (1961), and Miles and Munk (1961) in varying degrees have applied the impedance function concept to long-wave phenomena. The real part of the impedance is identified with the combined effects of radiative resistance of a basin entrance and linear internal dissipation. The imaginary part also consists of two effects: an inductive impedance associated with the local inertia of an accelerating flow and another reactive impedance associated with the required volume flux (electric current) to raise the local water level (charge a condenser).

Unit impedances, each a function of the local depth h(x, y), could be considered jointly in combinations, thus providing a system with many degrees of freedom. The power amplification function or characteristic power spectrum obtained at one location \((x_1, y_1)\) due to sinusoidal excitations introduced at a different location \((x_2, y_2)\) would be \(1/|H(x_1, y_1, x_2, y_2)|^2\) where \(H\) represents the overall (amplitude or voltage) transfer function between the two points in the indicated sense. Local resonances at \((x_1, y_1)\) would be determined from the zeros (or minima) of this transfer function.

Ishiguro (1959) published the electrical analogue technique for solving Equations (2) subject to boundary conditions of variable depth, lateral perimeter and connection with the open sea. Besides providing some additional examples, this paper will serve for engineering use, to augment Ishiguro's earlier work.

Consider Figure 1. If electric wave propagation in a two-dimensional network is to simulate the propagation of long water waves over an irregular bottom, certain constant scaling factors must be established. These relate:

1. the elevation of the surface from the mean level, \(\eta\), to voltage, \(e\), across a capacitor.

\[
\eta = K e \text{ (units of } K = \text{ cm/volt)}
\]

2. the volumetric flow components \(w_1\) and \(w_2\) to the electric current flowing in mesh branches in the positive x and y directions.

*Unit density is assumed.
Figure 1. Illustrations showing the physical analogy.
4.) \[ w_x = K_I x \text{ (units of } K_i: \text{ cm}^3 \text{ sec}^{-1} / \text{ampere}) \]

\[ w_y = K_I y \]

and (3) dimensionless scale factors \(K_i\) and \(K_e\) relating model time and space to real time and space, respectively,

5.) \[ t = K_t e \]

\[ t_e = K_e t \]

Henceforth a subscript "e" will be used to distinguish quantities in the electrical system from corresponding quantities in the hydraulic system.

Supposing that \(h = 0\), letting \(dx = dy \propto \Delta t\), and then multiplying the first two of Equations (2) by \(\Delta t\), and the third by \(1/(\Delta t)^2\) yields equations in terms of flow components

\[ \frac{\partial w_x}{\partial t} = -gh\Delta t \eta_x - Fw_x \]

6.) \[ \frac{\partial w_y}{\partial t} = -gh\Delta t \eta_y - Fw_y \]

\[ \frac{\partial \eta}{\partial t} = -\left( \frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} \right) \]

The currents and voltages in the mesh unit shown in Figure 1.b, obey*

\[ \frac{\partial I_x}{\partial t_e} = -\frac{1}{L} (\Delta E_x) - \frac{R}{L} (I_x) \]

7.) \[ \frac{\partial I_y}{\partial t_e} = -\frac{1}{L} (\Delta E_y) - \frac{R}{L} (I_y) \]

\[ \frac{\partial e}{\partial t_e} = -\frac{1}{C} (\Delta I_x + \Delta I_y) \]

So substituting (3), (4) and (5) in place of the hydrodynamic variables in (6) gives

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*In Figure 1.b, the positive direction for "e" is upward and the positive direction for the currents, indicated by small arrows.
SEICHE CONDITION SIMULATION

\[
\frac{\partial I}{\partial t} = -gh \left( \frac{K e}{K_l} \right) \frac{\partial e}{\partial x} - F(K e I)
\]

8.)

\[
\frac{\partial I}{\partial t} = -gh \left( \frac{K e}{K_l} \right) \frac{\partial e}{\partial y} - F(K e I)
\]

\[
\frac{\partial e}{\partial t} = - \left( \frac{1}{K e} \right) \frac{1}{K_l^2} \frac{1}{dt} \left[ \frac{\partial I}{\partial x} + \frac{\partial I}{\partial y} \right]
\]

Let

\[
\begin{align*}
\frac{dt (\frac{\partial e}{\partial x})}{\partial x} &= \Delta E_x \\
\frac{dt (\frac{\partial e}{\partial y})}{\partial y} &= \Delta E_y \\
\frac{\partial I}{\partial x} &= \Delta I_x \\
\frac{\partial I}{\partial y} &= \Delta I_y
\end{align*}
\]

Comparing (7) and (8), one finds (when c.g.s. units are used),

\[
L = \left( \frac{K e}{K_l} \right) \left( \frac{1}{K e} \right) \frac{1}{gh} \quad \text{(volts Ampere}^{-1} \text{sec, or Henrys)}
\]

9.)

\[
C = \left( \frac{K e}{K_l} \right) \left( \frac{1}{K e} \right) \frac{\mu^2}{gh} \quad \text{(volts}^{-1} \text{Ampere sec, or Farads)}
\]

\[
R = \left( \frac{K e}{K_l} \right) \frac{F}{gh} \quad \text{(volts Ampere}^{-1}, \text{or Ohms)}
\]

After \( K_e, K_l, \) and \( K \) have been fixed, these formulas can be used to relate the values of the circuit elements \( (L, C, \) and \( R) \) in each electrical "mesh" (Figure 1.b) to the mean depths, \( h, \) of the fluid cells, each of which is taken to have finite surface area \( (\Delta l)^2 \). The water depth is assumed constant over each cell; however, it changes from cell to cell as indicated in Figure 1.c. The circuit elements in the electrical meshes shown in Figure 1.d, set in accordance with Equations (9) above, will then be an analogous medium, in principle, for linear long-wave propagation.

Three types of interrelated "bandwidths" must be considered in such a model. These concern: (1) the frequency range of interest, (2) the depth range to be spanned, and (3) the requirement that the area of interest should
not be large compared to the total area modeled. Also, the sizes of three increments $\Delta t$, $\Delta h$, and $\Delta L$ must be established.

The mesh sizes chosen must be small in comparison to the major features of the bottom topography. Munk, Snodgrass and Gilbert (1964), represent smooth depth profiles as a series of discontinuous steps. Their equations are solved for each constant depth "layer", and the solution "patched" by requiring continuity in the flux of mass and momentum across the steps. A result of Volterra (1887) is cited, which demonstrates for this case that the exact solution can be obtained to any required precision by making the depth steps sufficiently small. The same result, in principle, holds for the analogous electric network, where one notes that Kirchhoff's Laws automatically take care of the continuity requirements.

As in all finite-difference computations, there is an error due to the coarseness of the spacing of computational points in the horizontal plane since $\Delta t$ is finite. It is convenient to define a quantity $k$ as being the number of computational points, or electrical meshes, per wavelength, $k = \lambda/(\Delta t)$. The wavelength here refers to its in situ value; thus $k$ may vary from place to place. The errors in estimates of the phase and amplitude for a particular wave frequency are functions of $k$ and decrease as $k$ increases. Clearly, for a given mesh size, the lowest accuracy is obtained for the case of highest frequency, and shallowest depth, that the modeling must accommodate.

The finite-mesh-size parameter $k$ expressed in the two systems is

\[
10. \quad k = \frac{\lambda}{(\Delta t)} = \frac{2\pi/gh}{w(\Delta t)} = \frac{2\pi}{\omega \sqrt{LC}}
\]

A dimensionless dissipation factor, $s$, also useful in describing both systems is

\[
11. \quad s = \frac{2\pi/gh}{F(\Delta t)} = \frac{2\pi}{R} \sqrt{\frac{L}{C}}
\]

In terms of these two parameters, Ishiguro (1959) examined the error for the case of one-dimensional propagation with friction, and his paper should be consulted for this discussion. Briefly, for the case with $s = 10$ and $k \approx 2\pi$, the error in the estimate of amplitude is approximately 10%.

A related error occurs if a complicated time-varying excitation function is represented by a set of $n$ values spaced $\Delta t$ apart in time. In order to maintain low harmonic distortion of the excitation waveform, the time $\Delta t$ must be small compared to the period of the highest-frequency component, in the decomposition of the forcing function, which possesses any significant amplitude and which the model is to accommodate. For analogue measurements, the times between sweep repetitions when the sets of $(n\Delta t)$ voltage values are inserted must also be long compared to the lowest frequency contained in the transient pulse so that the oscillations of the response are given adequate time.
to die out before another excitation cycle begins. If the excitation is a continuous function, say from a sine wave voltage generator, this discussion is of little interest since only steady state responses are normally desired.

After the parameters above are fixed, economic design of the electric network for simulating long-wave phenomena rests crucially on the selection of the constants $K_1$, $K_2$, and $K_3$.

**IV. OUTLINE OF PROCEDURE FOR ELECTRIC NETWORK DESIGN**

The process of selection of suitable and practical values for $K_i$ and the ratio $K_i/K_o$ continues somewhat along the following lines. First set down the initial constraints, which are: (1) frequency bandwidth required by the problem, (2) number of meshes required, (3) bathymetric accuracy and characteristics ($h_{\text{max}}/h_{\text{min}}$) of the modeled region, (4) accuracy sought, (5) types of circuit components available, (6) accuracy required in electric component values, (7) types of generators and test equipment available, or on hand, and (8) funds available or required.

The selection of electrical components is especially important. Inductors are usually the most expensive items, and here the electrical "Q", required of the component makes a large difference. This shifts attention back to the hydrodynamic system in order to ascertain what "Q's" are required in order to analogously reproduce, in the electrical system, similar conditions. For mesh design, only local dissipation--and not radiative losses--are considered. Assuming a sinusoidal velocity $u_0 \cos \omega t$ and a quadratic friction law, the mean relative dissipation of energy $E$ per cycle per unit area is*

\[
12. \ldots \quad Q = \frac{w\langle E \rangle}{d\langle E \rangle/\text{dt}} = \frac{3\pi}{16\gamma^2} \frac{wh^2}{A_o \sqrt{gh}} = \frac{3\pi h}{16\gamma^2 u_o^2}
\]

Accordingly, we define a new linearized friction coefficient $r = 16\gamma^2 u_o/3\pi \leq u_o h$ that $F = r/h$ (Equations 9). Then,

\[
13. \ldots \quad Q_e = \frac{w(L(h))}{R(h)}
\]

*Using: $\langle E \rangle = 1/4 \rho g A_o^2$ for standing waves; $u_o h = A_o \sqrt{gh}$ for long waves; $\langle u \rangle_t = \sqrt{2} \int [u_o \cos \omega t]^{3/2} \text{d}\langle E \rangle/\text{dt}$.
Minimum Inductor $Q_{eq}$ Required at Various Depths and Wave Periods

Figure 2. Reciprocal damping factor, or hydrodynamic $Q$, for waves having various periods propagating in water of fixed depth for a constant value of frictional coefficient, $r$. 
<table>
<thead>
<tr>
<th>Inductance</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.13mH</td>
<td>160 turns (7/44 Litz Wire)</td>
</tr>
<tr>
<td>1.62mH</td>
<td>100</td>
</tr>
<tr>
<td>645μH</td>
<td>63</td>
</tr>
<tr>
<td>258μH</td>
<td>40</td>
</tr>
</tbody>
</table>

3H1 ferrite material for cores having $A_L = 160\text{mH}/10^3$ turns

Figure 3. "$Q_e$" vs. frequency characteristics of 14mm inductors for various windings and inductances.

(Taken from Ferroxcube Corporation of America, bulletin no. 213.)
Figure 2 illustrates the effect of depth on internal $Q$ for various wave periods and for an assumed average value of $r$. The corresponding relationship for inductors is not so simple, but can be obtained by reploting data similar to that shown in Figure 3. Inductance is inversely proportional to the depth and directly proportional to the square of the number of coil turns, but each winding has a different curve $Q_e(f_e)$. If the type of inductor core whose characteristics are shown in Figure 3 were to prove adequate, one should also arrange to have the range in required depths spanned by coils containing 40 to 160 turns, the $Q_e$ characteristics being somewhat degraded outside of this range.

Inductor cores are distinguished by two major parameters: (1) core material type, the molecular and magnetic properties of which determine about frequency range, $L$ and $Q_e$ values obtainable, and (2) a quantity $A_{L'}$ per thousand coil turns. Curves like those of Figure 3 may be obtained from manufacturers.

Starting with a likely inductor core candidate, one constructs a graph having two abscissa scales, wave period $T$ and electrical frequency $f_e$, related to each other by the as-yet-unselected factor $K_1$. In similar fashion, the graph shown in Figure 4 has three independent ordinate scales, which are allowed to slide relative to each other. These ordinates consist of scales of depth, inductance and coil turns. A trial choice for $K_1$ fixes the abscissa scales relative to each other; similarly, choice of the ratio $K_1/K_e$ fixes the depth axis relative to the inductance axis, the inductance and turns scales being already linked through $A_{L'}$. Now, for each point in the plane, one is able to assign values for $Q_e$ using Figure 2 (the average value of $r$ being that obtained through calculation or current measurements), and values for $Q_e$ using curves similar to those of Figure 3. A valid design over the depth and period ranges defined by the intervals $[h_{\min} \leq h \leq h_{\max}]$ and $[T_{\min} \leq T \leq T_{\max}]$ requires that $Q_e(L, f_e) \geq Q(h, T)$. After values of $K_1/K_e$ and $K_e$ obtaining this condition with have been determined, it is advisable to adjust these values in connection with the mesh size coefficient ($M$) to so as to obtain a capacitor size which is commonly available, rechecking then to verify that the above validity remains.

The value $K_1$ must be chosen so that the current flowing through the arms (Figures 1.b and 1.d) is well below the core saturation level of the inductors, yet high enough to provide a good signal-to-noise ratio over the complete velocity range of interest when using available current measuring instruments. With trial values of $K_1/K_e$ and $K_e$ determined, $K_e$ is set automatically. The range of values that $L$, $C$, and $R$ may take on can be calculated using Equations (9). The unit mesh impedances can then also be calculated using Equations (9).

*Note: The $Q_e(f_e)$ values used in Figure 4 were for a different inductor than that described in Figure 3.
## Seiche Condition Simulation

<table>
<thead>
<tr>
<th>Depth (meters)</th>
<th>Inclination (1°)</th>
<th>Call Number</th>
<th>Tuna</th>
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<td>70</td>
<td>8</td>
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</table>

### Figure 4
**Range of Model Validity**

- **m/s**
- **w/s**

### Electrical Parameters (in %)

<table>
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<th>Frequency (Hz)</th>
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<th>500</th>
<th>1000</th>
<th>2000</th>
<th>4000</th>
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<td>20</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

**Note:**
- Frequency values are in Hz.
- **m/s** and **w/s** indicate meters per second and wavelengths, respectively.
calculated. If the overall driving-boundary impedance using these choices turns out to be too low, too great power will be required to excite the network, and the above process must then be repeated, or modified, so as to achieve more reasonable values. Transmission line theory provides an approximate means of calculating these boundary impedances.

For the model measurements to be meaningful, it is imperative that the electrical characteristics of the network be unaltered by the presence of the wave generator or the measuring equipment. It is therefore necessary to have the impedance across these input and output equipment terminals much higher than the impedances across the terminals of the network to which they are connected. This is not a difficult condition to achieve for the case of the measuring instruments, as their input impedances are very high.

In the case of the wave generator, however, the additional conflicting requirement for efficient power transfer complicates the situation, and a compromise condition must be tolerated. Here the lowest-possible isolating impedance, corresponding to the maximum error, is estimated. Next, considering the sensitivity of the measuring equipment and the maximum and minimum voltages required in the network, determine what power will be required from the wave generators. For transient excitations, the peak source power required must be estimated. For a single mesh where a wave of height $\zeta$ is required, this can be no less than

$$P_s = \frac{1}{2Z_o} \left( \frac{2\zeta}{K_e} \right) = \frac{2\zeta^2 (bl)/gh}{K_e K_1}$$

(Watts peak to peak);

where n meshes are so driven, $P_s = 2\zeta^2 / K_e \sum_{j=1}^{n} \left( 1/Z_{oj} \right)$.

The landward perimeter of the water area is simulated by an open circuit in that direction. In this case, $Z = \infty$ and perfect wave reflections occur although, if knowledge permits, other high values ($Z > Z_o$) could be specified, allowing for imperfect reflections.

In the electric model, those boundaries for which a connection to the open sea is to be simulated require special attention. It would be unrealistic to have any outgoing waves reflected from this boundary, or diminish in amplitude on approaching it. Consequently, this boundary should be terminated in its characteristic impedance. Reasonable approximation to this condition is achieved by connecting (to ground) resistors having values calculated using the real part of

$$Z_i = \frac{K_i}{K_e} \frac{1}{\sqrt{gh}} \frac{1}{(\Delta t)} \sqrt{1 + \frac{Tr}{2\pi K_t h}}$$
where the local value of $h$ is used. These values may later be adjusted experimentally. If the network is properly designed, this equivalent impedance will be purely resistive to an excellent approximation.

The capacitors selected should have dissipation factors $(1/Q)$ certainly no greater than that of the inductors. This presents no problem, but indicates the need for capacitors having mica, polystyrene, glass, or other low-dissipation, high-stability, dielectric material.

Little need be mentioned concerning the resistance components, save that they must be stable and have sufficient resolution to be consistent with the overall accuracy sought.

V. EXAMPLES OF EXPERIMENTAL RESULTS

Values of scale and other parameters that have been used for two long wave analogues are compared below.

<table>
<thead>
<tr>
<th>Chesapeake Bay</th>
<th>Small Boat Harbor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_t = 2.0 \times 10^7$</td>
<td>$K_t = 1.0 \times 10^6$</td>
</tr>
<tr>
<td>$K_i = 2.0 \times 10^{12}$ cm$^3$sec$^{-1}$/Amp.</td>
<td>$K_i = 8.56 \times 10^9$ cm$^3$sec$^{-1}$/Amp.</td>
</tr>
<tr>
<td>$K_e = 40$ cm/volt</td>
<td>$K_e = 1.0$ cm/volt</td>
</tr>
<tr>
<td>$T_{min} = 10$ minutes</td>
<td>$T_{min} = 25$ seconds</td>
</tr>
<tr>
<td>$\Delta t = 3.0$ Km; 1.0 Km; 0.5 Km</td>
<td>$\Delta t = 920$ ft; 240 ft; 80 ft</td>
</tr>
<tr>
<td>$h = 3.0 - 33$ meters</td>
<td>$h = 2.0 - 16$ meters</td>
</tr>
</tbody>
</table>

Changes in distance scale in the electric analogue can be easily achieved by changing capacitor values.

Before using an electrical analogue model to attack general seiche problems, it can be very reassuring to have successfully set it to the task of solving a long-wave problem where the prototype solution is known in advance. The observed tides around Chesapeake Bay nearly provided such a problem and also allowed the opportunity to adjust regional frictional values, so as to more correctly simulate the tidal effects. This afforded an approximate model calibration.

Figure 5 shows the positions of seventeen standard tide stations located around Chesapeake Bay. Figure 6 shows the results of the harmonic analysis of the tides observed at these stations. These consist of sinusoids of fixed frequencies whose amplitudes and phases have been adjusted so as to realize some stretch of tidal elevation time history. Sinusoidal voltages whose
Figure 5. Positions of standard tide stations (listed in Figure 6) for which harmonic components were recently computed and used for comparison with results of analog model.
Figure 6.

AMPLITUDES AND PHASES OF THE MAJOR CONSTITUENTS OF THE OBSERVED TIDES AROUND CHESAPEAKE BAY

<table>
<thead>
<tr>
<th>U.S.C. B. G. S. TIDE STATION</th>
<th>LENGTH OF TIDE RECORD ANALYZED (Yrs)</th>
<th>M_2</th>
<th>S_2</th>
<th>N_2</th>
<th>K_1</th>
<th>O_1</th>
<th>P_1</th>
<th>Q_1</th>
<th>S_1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>40 days 20 years</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2180 Old Point Comfort (76°06'N, 76°20'W)</td>
<td>y 1</td>
<td>246°</td>
<td>1°</td>
<td>36.61</td>
<td>277°</td>
<td>0°</td>
<td>6.92</td>
<td>229°</td>
<td>0°</td>
</tr>
<tr>
<td>2181 Port Royal, Frederick (76°45'N, 76°45'W)</td>
<td>y 1</td>
<td>254°</td>
<td>1°</td>
<td>40.64</td>
<td>255°</td>
<td>4°</td>
<td>156°</td>
<td>2°</td>
<td>1.63</td>
</tr>
<tr>
<td>2125 Cape Charles (37°41'N, 76°07'W)</td>
<td>y 1</td>
<td>252°</td>
<td>0°</td>
<td>34.00</td>
<td>279°</td>
<td>7°</td>
<td>6.89</td>
<td>231°</td>
<td>5°</td>
</tr>
<tr>
<td>402 Hampton Roads (36°55'N, 76°20'W)</td>
<td>y 2</td>
<td>257°</td>
<td>1°</td>
<td>36.56</td>
<td>285°</td>
<td>1°</td>
<td>7.10</td>
<td>239°</td>
<td>1°</td>
</tr>
<tr>
<td>493 Sherris Island Light (36°36'N, 76°25'W)</td>
<td>y 2</td>
<td>400°</td>
<td>14°</td>
<td>17.7</td>
<td>214°</td>
<td>1°</td>
<td>5.1</td>
<td>163°</td>
<td>3.1</td>
</tr>
<tr>
<td>491 Holland Island Bar Light (36°00'N, 76°06'W)</td>
<td>y 2</td>
<td>412°</td>
<td>21°</td>
<td>255°</td>
<td>33°</td>
<td>122°</td>
<td>3.3</td>
<td>349°</td>
<td>4.2</td>
</tr>
<tr>
<td>2180 Annapolis (38°55'N, 76°26'W)</td>
<td>y 1</td>
<td>143°</td>
<td>21°</td>
<td>114°</td>
<td>170°</td>
<td>9°</td>
<td>125°</td>
<td>247°</td>
<td>2.36</td>
</tr>
<tr>
<td>487 St. Mary Point Light (37°54'N, 76°16'W)</td>
<td>y 2</td>
<td>296°</td>
<td>50°</td>
<td>16.1</td>
<td>329°</td>
<td>5°</td>
<td>116°</td>
<td>45°</td>
<td>0.0</td>
</tr>
<tr>
<td>488 Great Wicomico Light (37°45'N, 76°16'W)</td>
<td>y 2</td>
<td>333°</td>
<td>9°</td>
<td>15.5</td>
<td>337°</td>
<td>2°</td>
<td>157°</td>
<td>27°</td>
<td>2.4</td>
</tr>
<tr>
<td>692 Solomon's Island (39°00'N, 76°25'W)</td>
<td>y 2</td>
<td>470°</td>
<td>14°</td>
<td>166°</td>
<td>17°</td>
<td>159°</td>
<td>2.6</td>
<td>18°</td>
<td>156°</td>
</tr>
<tr>
<td>2181 Baltimore (39°40'N, 76°04'W)</td>
<td>y 4</td>
<td>190°</td>
<td>30°</td>
<td>47°</td>
<td>212°</td>
<td>3°</td>
<td>33°</td>
<td>267°</td>
<td>3.3</td>
</tr>
<tr>
<td>499 Reedy Island Light (39°17'N, 76°16'W)</td>
<td>y 4</td>
<td>212°</td>
<td>32°</td>
<td>17.4</td>
<td>259°</td>
<td>4°</td>
<td>22°</td>
<td>163°</td>
<td>4.8</td>
</tr>
<tr>
<td>2130 Washington, D.C. (38°55'N, 77°00'W)</td>
<td>y 4</td>
<td>228°</td>
<td>42°</td>
<td>286°</td>
<td>259°</td>
<td>6°</td>
<td>206°</td>
<td>338°</td>
<td>8.1</td>
</tr>
<tr>
<td>2180 Cambridge, Del. (38°40'N, 76°05'W)</td>
<td>y 1</td>
<td>107°</td>
<td>22°</td>
<td>21.5°</td>
<td>230°</td>
<td>5°</td>
<td>37°</td>
<td>240°</td>
<td>5.07</td>
</tr>
<tr>
<td>495 Love Point Light (39°03'N, 74°17'W)</td>
<td>y 2</td>
<td>168°</td>
<td>282°</td>
<td>3.8</td>
<td>219°</td>
<td>2°</td>
<td>23°</td>
<td>163°</td>
<td>5.4</td>
</tr>
<tr>
<td>500 Elk River Entrance-Lovett Point (39°02'N, 75°55'W)</td>
<td>y 2</td>
<td>246°</td>
<td>50°</td>
<td>30°</td>
<td>286°</td>
<td>37°</td>
<td>5.2</td>
<td>219°</td>
<td>351°</td>
</tr>
<tr>
<td>501 Chesapeake City, Md. (39°32'N, 75°49'W)</td>
<td>y 2</td>
<td>263°</td>
<td>37°</td>
<td>31°</td>
<td>35°</td>
<td>39°</td>
<td>4.1</td>
<td>238°</td>
<td>37°</td>
</tr>
<tr>
<td>653 Court House Point, Md. (39°31'N, 75°53'W)</td>
<td>y 4</td>
<td>269°</td>
<td>38°</td>
<td>31°</td>
<td>40°</td>
<td>4.1</td>
<td>239°</td>
<td>37°</td>
<td>6.10</td>
</tr>
<tr>
<td>496 Seven Foot Knoll Light, Md. (39°05'N, 76°25'W)</td>
<td>y 2</td>
<td>185°</td>
<td>29°</td>
<td>13.4</td>
<td>22°</td>
<td>30°</td>
<td>2.7</td>
<td>215°</td>
<td>2.8</td>
</tr>
</tbody>
</table>

# - Represents the phase log of a tidal constituent behind that of the corresponding equilibrium tide at the same position.

# - Represents phase lags corrected to a common meridion, Old Point Comfort (76°18'W). Note: A maximum error of 2° may result here because of some approximations.

A - Amplitude in centimeters.

NOTE: Interpolated values are shown by means of parentheses to distinguish from direct observations.

Figure 6. Tidal Conditions Simulation SEICHE CONDITION SIMULATION
frequencies corresponded in model time to the tidal components listed were introduced well offshore from the bay mouth. Voltage amplitudes and phases were measured at positions in the model corresponding to those of the seventeen tide stations. A normalizing station was chosen near the bay entrance. The amplitude ratios and phase shifts of the remaining set of sixteen stations could be compared in both the model and the prototype in this way.

When the $M_2$ tidal frequency was used and the electric current flowing in various mesh arms measured to obtain local tidal-current vectors, it was found that these exceeded measured $M_2$ speed by about 50%. Initially, a uniform value for bottom friction was used throughout the bay. Following these measurements, new values of $r$, the linearized friction coefficient, were computed using the simulated $M_2$ tidal currents measured in the model previously; friction was then nonuniformly distributed. After this, the observed model currents were lower than those observed in nature, but with much smaller local deviations than before. The tide station comparisons were more or less improved also, but not so markedly. Only one such iteration for the bottom frictional distribution has been tried to date.

Since tides are not small amplitude oscillations and Coriolis effects are present, it would be surprising if substantial errors were not observed using this linear model for tides. A certain regularity, however, was noticed in the accuracy with which tidal simulation was possible at some stations. At Stations 5, 6, 11 and 14 in Figure 5 prediction was consistently good, while at Stations 10 and 17 it was consistently poor. For the set of seven tidal components and seventeen tide stations, the phase difference error averaged about 10° and the amplitude ratios deviated about 25% when compared to those from the prototype. Such prediction errors are presumably smaller for shorter period excitations.

Figure 7 shows an example of amplitude response spectra obtained for three locations in Chesapeake Bay. In each case two curves are superimposed. These show the effect of varying the direction of initial wave approach. Apparently certain modes of oscillation are excited preferentially by waves from different directions. Especially at low frequencies, the constraint imposed by the location of the wave source becomes more important. One notices in Figure 7, however, that the directional effect diminishes at low frequencies as it might be expected to do.

Figure 8 shows examples of impulse responses also obtained at various modeled locations in Chesapeake Bay. Here the excitation pulse time history appears as the top trace in each frame, while the lower oscilloscope beam, after some delay, traces out the response time histories.

Examination of response spectra from a number of locations may indicate consistent large activity or resonance amplification at certain frequencies. In such cases it is desirable to examine a particular mode of oscillation in detail. Figure 9 shows an example of such a case for one basin configuration that had
Figure 7. Model response spectra for indicated positions showing the effect of variation in the initial direction of wave approach. Boundaries B (•) and D (+)
0.5-Kilometer Meshes

Figure 8. Impulse responses.
Figure 9.
A configuration for a small-boat harbor illustrating one severe case.
RMS wave amplitude contoured in volts (or cm).
Frequency = 13.4 kc, Period (T) = 74.6 sec.

BOUNDARY DRIVER, 35 volts RMS
REFERENCE, 1 volt RMS
AVERAGE DEPTH, 17.82 feet
AVERAGE WAVE LENGTH, 1790 feet
1790/80 = 22 meshes

Figure 10.
Another configuration for a small-boat harbor showing a severe case.
RMS wave amplitude contoured in volts (or cm).
Seiche Condition Simulation

Insertion of simulated solid mole at node of standing wave, illustrating the mode of oscillation (shown in previous figure) effectively eliminated.
been proposed for a small boat harbor. The grid shown is on 80-foot centers and the quantity contoured is RMS voltage as measured in the model. This particular mode exhibited resonant amplification factors \( e_{ij}/e_{ref} \) of as large as sixteen, over a variety of initial wave headings. Figure 10 shows a similar case for another simple basin configuration, for which approximate hand computations can be made. The basin depth in this case was relatively constant except near the entrance and along the borders. High amplification factors are again noted. Figure 11 shows the same basin with a simulated solid mole or sheet-pile obstruction inserted at a critical point. Conditions are otherwise the same as for the previous figure; however, that mode of oscillation has now been effectively eliminated. The numbers toward the top of each square are the mesh voltages, \( e \), and the bottom entries are the phase differences existing between each of these points and the driver, located many wavelengths away. The examples shown here are fairly simple ones; quite complex patterns also may be found.

Frequencies which produce large resonant amplifications at points internal to the basins can be identified and their modes of oscillation investigated. Steps can then be taken to suppress those which may be particularly undesirable. Each change in lateral or bottom basin configuration may introduce new modes, or change the \( Q \)'s of pre-existing ones, and these must be re-examined. Nevertheless, to some extent, it is possible to attack the inverse problem of arriving at three dimensional basin shapes which yield desired responses. It appears that, given sufficient description of the long-wave environment, the described technique will aid in the design of shelters for vessels so that these will fulfill their major function well in the environment found.

VI. ACKNOWLEDGMENTS

The author gratefully acknowledges the useful discussions and criticisms of Mr. Robert R. Putz during the preparation of this paper and to Dr. Shizuwo Ishiguro for his previous invaluable advice.
SEICHE CONDITION SIMULATION

LIST OF SYMBOLS

Hydrodynamic Quantities

\( g \)  gravitational acceleration
\( t \)  time
\( T \)  period for one wave cycle
\( f \)  frequency of wave component
\( \omega \)  angular frequency = \( 2\pi f \)
\( c \)  phase velocity
\( x, y \)  horizontal coordinates in a right-handed, rectangular Cartesian system
\( z \)  vertical coordinate, positive upward
\( \xi \)  horizontal distance
\( \Delta \)  increment in horizontal distance (mesh size)
\( h \)  water depth
\( \vec{k} \)  propagation vector
\( \rho \)  fluid density
\( \eta \)  surface elevation function
\( u, v \)  horizontal velocity components averaged vertically
\( \overline{w}_x, \overline{w}_y \)  fluid flow averaged vertically, its horizontal components (\( \overline{w}_x = uhdx \))
\( \tau \)  stress due to friction with the bottom
\( F \)  friction coefficient
\( \gamma^2 \)  a coefficient related by \( \gamma^2 - g/C_f \) to De Chézy's coefficient, \( C_f \)
\( \rho \)  Ishiguro's linearized friction coefficient (cm/sec)
\( (E) \)  mean energy per unit surface area
\( Q \)  reciprocal damping factor; \( Q = \omega (E) \sqrt{\frac{d(E)}{dt}} = \frac{2\pi}{T} \frac{h}{r^2} \) (h in cm)
\( \zeta \)  trough to crest wave height; \( \eta_{\text{max}} - \eta_{\text{min}} \) in general, \( 2A \) for sinusoidal waves
Electrical Quantities

t_e\quad \text{electrical time}

T_e\quad \text{period to complete one cycle, electrical time}

f_e\quad \text{electrical frequency}

\omega_e\quad \text{angular frequency of voltage} = \frac{T}{2\pi t_e}

c_e\quad \text{phase speed in electrical network}

\ell_e\quad \text{horizontal distance in electrical network corresponding to physical distance } \ell

x_e, y_e\quad \text{horizontal coordinates corresponding to } x, y

\Delta E_x, \Delta E_y\quad \text{voltages across one arm of mesh representing } x \text{ and } y \text{ component}

\text{e}\quad \text{voltage across capacitor, at an arbitrary point}

I_x, I_y\quad \text{electric currents flowing in } x \text{ and } y \text{ mesh arms}

L\quad \text{inductance of mesh}

C\quad \text{capacitance of mesh}

R\quad \text{resistance of mesh}

Z_0\quad \text{mesh impedance}

Z_1\quad \text{iterative impedance of one dimensional transmission line}

Q_e\quad \text{electrical "quality" factor } \frac{\text{E}}{\text{R}} (\text{for inductors})

Scale and Error Parameters

K_t\quad \text{time scale coefficient defined as } K_t = \frac{t}{t_e}

K_\ell\quad \text{space scale coefficient defined as } K_\ell = \frac{\ell}{\ell e}

K_e\quad \text{transformation from water level to voltage} K_e = \eta / e

K_1\quad \text{transformation from fluid flow in some direction to electric current flowing in that direction} K_1 = \frac{w}{I_x}

k\quad \text{ratio: mesh junctions per wavelength}

s\quad \text{dimensionless dissipation factor}


