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Floating node method with domain-based interaction integral for generic 2D crack growths

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ABSTRACT

The Floating Node Method (FNM), first developed for modeling the fracture behavior of laminate composites, is here combined with a domain-based interaction integral approach for the generic fracture modeling of quasi-brittle materials from crack nucleation, propagation to final failure. In this framework, FNM is used to represent the kinematics of cracks, crack tips and material interfaces in the mesh. The values of stress intensity factor are obtained from the FNM solution using domain-based interaction integral approach. To demonstrate the accuracy and effectiveness of the proposed method, four benchmark examples of fracture mechanics are considered. Predictions obtained with the current numerical framework compare well against literature/theoretical results.

1. Introduction

Fracture is one of the most important failure phenomena of structural components. A reliable and efficient modeling of the fracture process plays a vital role in the safety assessment of the structural components. In the last few decades, several methodologies have been proposed for the robust and accurate modeling of evolving discontinuities in the structural domains. The Finite Element Method (FEM) has been widely used as a numerical tool for damage and fracture mechanics problems. However, the modeling of crack evolution process can be difficult with standard FEM, as such problems require conformal meshes with respect to the locations of discontinuities. Thus, in FEM, remeshing is required for each step of crack propagation, which needs to transfer the history variables from the old mesh to the newly generated one. A more robust framework for the prediction of progressive failure is thus desirable. In the last two decades, many methods have been proposed in the literature, such as boundary elements method [44], meshfree method [3,53], extended finite element method [4,13], phantom node method [56,47], phase field method [34], floating node method [10], virtual node XFEM [26], coupled FE_EFG approach [39,39,42], and extended isogeometric analysis [5]. A thorough account of all these methods is outside the scope of this paper, hence only the ones closely related to this work are reviewed in the following paragraphs.

The Extended or Generalized Finite Element Method (XFEM/GEFM) [52,43], based on a partition of unity concept [33], is a popular choice for

the simulation of fracture mechanics problems. In XFEM/GFEM, a level set technique with enrichment functions is used to represent the crack in the domain, hence avoiding the requirement of remeshing during the crack propagation. This method has since been extended for interfacial crack [57,36,27,23], fatigue crack growth [12,55,40,21,35], elasto-plastic crack growth [15,25,28,30], three dimensional crack growth [1,48,38], dynamic crack growth [64,49,29] fatigue crack growth in functionally graded materials [54,7,7] and interaction of multiple cracks [22]. Despite its success in many types of problems, there exist some limitations: (1) it introduces an error during the mapping of discontinuities from the physical space to the natural space [17]; (2) the implementation in FEM can be complicated as blending elements are generally required for connecting the enriched elements to standard elements; (3) the numerical solution is sensitive to the numerical integration scheme used for the enriched elements [46]; and (4) different enrichment functions are usually required to tackle different material problems.

An alternative approach proposed by Hansbo and Hansbo [20] models the discontinuities within an element via additional nodal degrees of freedom. This concept was further developed into the Phantom Node Method (PNM) by Song and co-workers [56]. In PNM, when an element is cut by a discontinuity, extra nodes are superposed with the original nodes and two superposing sub-elements are formed. For the modeling of a strong discontinuity, the solution by PNM is equivalent to the XFEM/GFEM with only Heaviside enrichment function. Accordingly, the same error as XFEM/GFEM occurs in PNM when mapping the physical discontinuities to the natural space.

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Recently, Chen et al. [10] proposed the Floating Node Method (FNM), which has a similar computational architecture to the PNM. In FNM, discontinuities are modeled locally within an element by partitioning the original finite element into sub-elements with the use of additional nodes. These additional nodes, known as floating nodes, are initially dormant and do not coincide with the real nodes. Instead, the floating nodes are only activated at the location of discontinuity within the said element during the crack initiation and propagation process. The FNM thus does not introduce any error during the mapping of crack geometry (discontinuity) from physical to natural space. Moreover, the integration of sub-elements is straight forward, without requiring any sub-domain integration. Conceptually, it is also very intuitive and can be readily implemented in finite element codes.

So far, the work on FNM is limited to the problems of composite laminates [10,11]. Thus, in the current paper, the FNM is combined with a domain-based interaction integral approach, for describing the arbitrary crack trajectories during the fracture of quasi-brittle materials. In our framework, the FNM captures the kinematics of discontinuities i.e. crack, material-interface, and the domain based interaction integral approach adopted for determining the values of Stress Intensity Factors (SIFs). An equivalent SIF value is next compared with the material fracture toughness to determine the point of failure.

To demonstrate the capability of the proposed approach, four benchmark problems of fracture mechanics are considered, and the predictions obtained by FNM compared against available literature/theoretical results. For the first problem, an edge crack plate under mode-I cyclic loading is considered. In the second problem, a dog-bone specimen is subjected to mixed mode cyclic loading conditions. The Rankine's failure criterion is adopted for the nucleation of a crack, and its subsequent propagation and failure is controlled by the fracture mechanics criteria. In the third problem, a bi-material interface edge crack is subjected to mode-I cyclic loading. The toughness of the interface is assumed to be higher than those of the base materials, such that the crack propagates into one of the base materials. In the fourth problem, a bi-material plate with an edge crack perpendicular to material interface is subjected to mode-I loading. The effect of the material interface is observed on the J -integral.

The paper is organized as follows: Section 2 describes the governing equations and the mathematical formulation of the FNM. The computation of stress intensity factors for arbitrary brittle crack propagation based on the domain interaction integral is provided in Section 3. Section 4 elaborates on the crack nucleation and crack trajectory criteria. Four benchmark problems of fracture mechanics are presented in Section 5 to demonstrate the capability of the FNM framework adopted. Finally, the conclusions of the present work are presented in Section 6.

2. Numerical framework

In this section, the governing equations for the static analysis of an elastic medium containing traction-free cracks are briefly summarized for completeness. A brief review of FNM is also presented. In the framework adopted here, we incorporate an additional floating node to maintain the aspect ratio of sub-elements during the integration process.

2.1. Governing equations

Consider a domain (Ω) as shown in Fig. 1, which is partitioned into displacement (Γ_u), traction (Γ_t) and crack surface (Γ_c) boundaries. The strong form of the static equilibrium equations and boundary conditions are given as:

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \quad \text{in } \Omega \quad (1a)$$

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{n}} = \bar{\mathbf{t}} \quad \text{on } \Gamma_t \quad (1b)$$

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{n}} = \mathbf{0} \quad \text{on } \Gamma_c \quad (1c)$$

Fig. 1. A cracked domain with boundary conditions.

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \Gamma_u \quad (1d)$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{u}$ is the displacement field vector, $\mathbf{b}$ is the body force vector, $\bar{\mathbf{t}}$ is the traction vector and $\hat{\mathbf{n}}$ is the unit outward normal. For small strains and displacements, the strain-displacement relation can be written as:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\mathbf{u}) = \nabla_s \mathbf{u} \quad (2)$$

where ∇_s is a symmetric gradient operator and $\boldsymbol{\varepsilon}$ is the linear strain tensor. The constitutive relation for linear elastic material is given by Hooke's law,

$$\boldsymbol{\sigma} = \mathbf{D} \boldsymbol{\varepsilon} \quad (3)$$

where $\mathbf{D}$ is the Hooke's tensor.

The weak form of the equilibrium equation can be expressed as,

$$\int_{\Omega} \boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{v}) d\Omega = \int_{\Omega} \mathbf{b} \cdot \mathbf{v} d\Omega + \int_{\Gamma_t} \bar{\mathbf{t}} \cdot \mathbf{v} d\Gamma \quad (4)$$

In FEM, the above weak form can be transformed into the following discrete set of equations,

$$\mathbf{K} \mathbf{d} = \mathbf{f} \quad (5)$$

where $\mathbf{K}$ is the global stiffness matrix, $\mathbf{d}$ is the vector of nodal unknowns and is the external force vector.

2.2. FNM formulation

The FNM is used to simulate the arbitrary crack growth in quasi-brittle materials. The discretized mesh thus comprises of either intact elements, or elements that each encompasses a crack. The floating nodes in an intact element are dormant, and the resulting element is thus identical to a standard finite element. Once a crack appears in the element, these floating nodes are activated to model the crack in the element (see Fig. 2). The intersections between the crack and the element edges are defined (points with coordinates $\mathbf{x}_r$ and $\mathbf{x}_s$), and the element is split into two sub-elements Ω_A and Ω_B as shown in Fig. 2. The vectors of nodal coordinates of sub-elements are defined as:

$$\mathbf{x}_{\Omega_A}^T = [\mathbf{x}_r^T, \mathbf{x}_s^T, \mathbf{x}_3^T, \mathbf{x}_4^T] \quad \text{and} \quad \mathbf{x}_{\Omega_B}^T = [\mathbf{x}_1^T, \mathbf{x}_2^T, \mathbf{x}_s^T, \mathbf{x}_r^T] \quad (6)$$

Each sub-element (Ω_A and Ω_B) has a separate Jacobian given by:

$$\mathbf{J}_A = \frac{d\mathbf{x}}{d\xi} = \frac{d\mathbf{N}}{d\xi} \mathbf{x}_{\Omega_A} \quad \text{and} \quad \mathbf{J}_B = \frac{d\mathbf{x}}{d\xi} = \frac{d\mathbf{N}}{d\xi} \mathbf{x}_{\Omega_B} \quad (7)$$

The displacement u_A and u_B in the sub-elements are interpolated separately from the respective degrees of freedom d_A and d_B of the sub-element Ω_A and Ω_B respectively:

$$\mathbf{u}_A = \mathbf{N} \mathbf{d}_A \quad \text{and} \quad \mathbf{u}_B = \mathbf{N} \mathbf{d}_B \quad (8)$$

where $\mathbf{d}_A^T = [d_8^T, d_7^T, d_3^T, d_4^T]$ and $\mathbf{d}_B^T = [d_1^T, d_2^T, d_6^T, d_5^T]$.

Fig. 2. Splitting of an element with strong discontinuity into two sub-quadrilateral elements using floating node method [10].

The stiffness matrices and force vectors of the sub-elements are thus defined as:

$$\mathbf{K}_A = \int_{\Omega_A} \mathbf{B}_A^T \mathbf{D} \mathbf{B}_A \det(\mathbf{J}_A) d\Omega \quad \text{and} \quad \mathbf{K}_B = \int_{\Omega_B} \mathbf{B}_B^T \mathbf{D} \mathbf{B}_B \det(\mathbf{J}_B) d\Omega \quad (9)$$

$$\mathbf{f}_A = \int_{\Omega_A} \mathbf{N}^T \mathbf{b} \det(\mathbf{J}_A) d\Omega + \int_{\Gamma \cap \Omega_A} \mathbf{N}^T \bar{\mathbf{t}} \det(\mathbf{J}_A) d\Gamma \quad (10a)$$

$$\mathbf{f}_B = \int_{\Omega_B} \mathbf{N}^T \mathbf{b} \det(\mathbf{J}_B) d\Omega + \int_{\Gamma \cap \Omega_B} \mathbf{N}^T \bar{\mathbf{t}} \det(\mathbf{J}_B) d\Gamma \quad (10b)$$

The equilibrium equations for both sub-elements are written as:

$$\mathbf{K}_A \mathbf{d}_A = \mathbf{f}_A \quad \text{and} \quad \mathbf{K}_B \mathbf{d}_B = \mathbf{f}_B \quad (11)$$

Finally, the equilibrium equation of the floating node element is the assembly of the two sub-elements:

$$\mathbf{K} \mathbf{d} = \mathbf{f} \quad (12)$$

$$\text{where } \mathbf{K} = \begin{bmatrix} \mathbf{K}_A & \\ & \mathbf{K}_B \end{bmatrix}, \mathbf{d}^T = [\mathbf{d}_A^T, \mathbf{d}_B^T] \text{ and } \mathbf{f}^T = [\mathbf{f}_A^T, \mathbf{f}_B^T]$$

2.3. Different cases of discontinuities and integration procedure

In a generic fracture analysis, the scenario illustrated in Fig. 2 is only one possibility. An overview of the different possible cases of a cracked element is available in literature [10]. For curved crack problems, depending on the intersections of element edges with the crack, accurate integration is not possible in cases where the aspect ratios of triangular sub-elements are poor. An illustration is shown in Fig. 3a, where one of the elements (see Fig. 3b) is divided into two sub-domains, a triangle and a pentagon. While the integration over the triangle is straightforward, integration over the pentagon is not directly possible. A convenient approach is to divide the pentagon into three triangular elements [11], though with poor aspect ratios as illustrated in Fig. 3c, which could result in poor evaluations of stresses in that element.

Our strategy to overcome this issue is to introduce one (additional)

internal floating node to divide the pentagon domain into five sub-triangular elements, as shown in Fig. 3d. The introduction of this internal node, the position of which is determined iteratively, helps to maintain an acceptable aspect ratio for all the sub-triangular elements. The detailed procedure to maintain the aspect ratios of triangular elements is provided in the algorithm presented in Fig. 4. The procedure to update the position of internal floating node is as follows:

- Find acute angle with internal floating node of sub-triangular element with poor aspect ratio
- If acute angle $< 20^\circ$
move internal floating node towards the edge opposite to the corner for which angle is measured
Else if acute angle $> 160^\circ$
move internal floating node away from the edge opposite to the corner for which angle is measured
End

2.4. Crack tip modeling using FNM

An accurate crack tip modeling is vital for the fracture analysis of a structure. In FNM, a crack is modeled by splitting the intact element into two sub-elements by activating the floating nodes. The failed element has two floating nodes at the edge E2 of the intact element, where the crack terminates (see Fig. 5a). Thus, the crack tip remains open at the edge E2 as shown in Fig. 5b. The crack tip closing can be effected by constraining the two sets of degrees of freedom (dofs) from both floating nodes at the crack tip to be identical. However, this procedure may lead to an artifact at the crack tip as illustrated in Fig. 5c, i.e. the cracked element at the crack tip does not have adequate topology for connecting to the adjacent intact element, resulting in a lack of displacement compatibility at the element edge E2. To mitigate this issue, the dofs at the crack tip are interpolated from the neighboring dofs as shown in Fig. 5d. Thereafter, the intact element ahead of the crack tip is

Fig. 3. A schematic diagram for the modeling of curved crack using five floating nodes.

modeled as a transition element (see Fig. 5e) to maintain a displacement compatibility between failed and intact elements. The detailed procedure can be found in the literature [10].

3. Computation of stress intensity factors

In fracture mechanics, SIF is a critical parameter that has frequently been used in the failure criterion for brittle materials. In Chen et al. [10], a virtual crack closure technique (VCCT) [51] is used with FNM to

obtain the energy release rates for mode-I and mode-II. Thereafter, the values of SIFs are obtained from the energy release rates using linear elastic relations. In VCCT, the crack propagation is assumed to be always self-similar and a uniform mesh is generally required to obtain accurate predictions of the energy release rates.

To overcome the above limitations, in this paper, a domain based interaction integral approach is used in conjunction with FNM to compute the values of SIFs. In this approach, the SIF values are computed using a domain near the crack tip, which makes it more accurate

Fig. 4. An algorithm to maintain the aspect ratio of sub-elements obtained by dividing the five node element during integration.

Fig. 5. A schematic procedure for representing the crack tip in FNM using interpolation technique and transition element.

and very effective for calculating SIF values for both mode-I and mixed mode loading conditions. Since, the interaction integral is derived from the J -integral [50] for two admissible states (actual and auxiliary fields), hence the auxiliary field is discussed first in detail.

3.1. The auxiliary fields

There are several choices of auxiliary fields to extract the mixed mode SIFs. Here, we utilized the expressions provided in Matos et al. [32]. These expressions of auxiliary displacement fields in local x_1 – x_2 co-ordinate system (Fig. 6) can be written as [57],

$$u_i = \begin{cases} \frac{1}{4\mu_1 \cosh(\pi\epsilon)} \sqrt{\frac{r}{2\pi}} f_i(r, \theta, \epsilon, \kappa_1) & \text{for upper half plane} \\ \frac{1}{4\mu_2 \cosh(\pi\epsilon)} \sqrt{\frac{r}{2\pi}} f_i(r, \theta, \epsilon, \kappa_2) & \text{for lower half plane} \end{cases} \quad (13)$$

Fig. 6. A schematic of bi-material interface crack.

where μ is the shear modulus, ϵ and κ are bi-material constants and defined as,

$$\epsilon = \frac{1}{2\pi} \log \left(\frac{1-\bar{\beta}}{1+\bar{\beta}} \right) \quad \text{and} \quad \kappa_i = \begin{cases} \frac{3-\nu_i}{1+\nu_i} & (\text{plane stress}) \\ 3-4\nu_i & (\text{plane strain}) \end{cases}$$

where $\bar{\beta}$ is a Dundurs parameter and defined as,

$$\bar{\beta} = \frac{\mu_1(\kappa_2-1)-\mu_2(\kappa_1-1)}{\mu_1(\kappa_2+1)+\mu_2(\kappa_1+1)}$$

To extract the mode-I SIF, the functions f_1 and f_2 are given as,

$$f_1 = A + 2\delta \sin\theta \sin\varphi \quad \text{and} \quad f_2 = -B - 2\delta \sin\theta \cos\varphi$$

whereas to compute the mode-II SIF, the functions f_1 and f_2 are written as,

$$f_1 = -B + 2\delta \sin\theta \cos\varphi \quad \text{and} \quad f_2 = -A + 2\delta \sin\theta \sin\varphi$$

The parameters (δ , φ , A , B) used in the above expressions are defined as,

$$\delta = \begin{cases} e^{-(\pi-\theta)\epsilon} & \text{for upper half plane} \\ e^{(\pi+\theta)\epsilon} & \text{for lower half plane} \end{cases}, \quad \varphi = \epsilon \log r + \frac{\theta}{2}$$

$$A = \beta \gamma \cos \frac{\theta}{2} + \beta' \gamma' \sin \frac{\theta}{2}, \quad B = \beta' \gamma \cos \frac{\theta}{2} - \beta \gamma' \sin \frac{\theta}{2}$$

$$\beta = \frac{0.5 \cos(\epsilon \log r) + \epsilon \sin(\epsilon \log r)}{0.25 + \epsilon^2}, \quad \beta' = \frac{0.5 \sin(\epsilon \log r) - \epsilon \cos(\epsilon \log r)}{0.25 + \epsilon^2}$$

$$\gamma = \kappa \delta - \frac{1}{\delta}, \quad \gamma' = \kappa \delta' - \frac{1}{\delta'}$$

The auxiliary strain components are the symmetric gradient of the auxiliary displacement components, and defined as [61],

$$\epsilon_{ij}^{aux} = \frac{1}{2} (u_{i,j}^{aux} + u_{j,i}^{aux}), \quad (i, j = 1, 2) \quad (14)$$

where u_i^{aux} and ϵ_{ij}^{aux} are the auxiliary displacement and strain fields

Fig. 7. Detailed procedure in floating node method for the modeling of crack initiation to final failure.

respectively. The detailed procedure to obtain the displacement gradients of the auxiliary fields are given in Sukumar et al. [57], and can be written as,

$$u_{i,j}^{\text{aux}} = C \left(Df_{i,j} + \frac{r_{,j} f_i}{4\pi B} \right), \quad (i, j = 1, 2) \quad (15)$$

where a comma denotes a partial derivative with respect to the co-ordinates.

3.2. Domain form of interaction integral

The standard J -integral [50] can be written as,

$$J = \int_{\Gamma} (W \delta_{1,j} - \sigma_{ij} u_{i,1}) n_j d\Gamma \quad (16)$$

where $W = \frac{1}{2} \sigma_{ij} \varepsilon_{ij}$ is the strain energy density, σ_{ij} is the strain tensor, u_i is displacement field vector, n_j is the unit vector and δ_{ij} is the Kronecker's delta.

The J -integral remains path independent for bi-material interface crack problems, if the material homogeneity exists in the direction parallel to the crack surface [24]. In this case, the mixed mode SIF values can be readily computed using domain interaction integral. The contour interaction energy integral can be written as [14,62,37],

$$I = \int_{\Gamma} (\sigma_{ik} \varepsilon_{ik}^{\text{aux}} \delta_{1j} - \sigma_{ij} u_{i,1}^{\text{aux}} - \sigma_{ij}^{\text{aux}} u_{i,1}) n_j d\Gamma \quad (17)$$

The domain form of the interaction integral is a well-defined technique to evaluate the mixed mode SIF values of interface crack problems. Now, using the divergence theorem, the domain form of the contour interaction integral is written as [57,19,63],

$$I = - \int_A (\sigma_{ik} \varepsilon_{ik}^{\text{aux}} \delta_{1j} - \sigma_{ij} u_{i,1}^{\text{aux}} - \sigma_{ij}^{\text{aux}} u_{i,1}) q_j dA \quad (18)$$

In linear elastic fracture mechanics (LEFM), the interaction integral is related to the SIFs through the relation,

$$I = \frac{2}{E^* \cosh^2(\pi \varepsilon)} (K_I K_I^{\text{aux}} + K_{II} K_{II}^{\text{aux}}) \quad (19)$$

where K_I^{aux} and K_{II}^{aux} are local auxiliary stress intensity factors for the auxiliary fields, and E^* is defined as,

$$\frac{2}{E^*} = \frac{1}{E_1} + \frac{1}{E_2}, \quad E_i = \begin{cases} E_i & (\text{plane stress}) \\ \frac{E_i}{1 - \nu_i^2} & (\text{plane strain}) \end{cases} \quad (i = 1, 2)$$

The mixed mode SIF values can be obtained from Eq. (19) using $K_I^{\text{aux}} = 1$ and $K_{II}^{\text{aux}} = 0$ and vice versa,

$$K_I = \frac{E^* \cosh^2(\pi \varepsilon)}{2} I_1, \quad K_{II} = \frac{E^* \cosh^2(\pi \varepsilon)}{2} I_2 \quad (20)$$

4. Criteria for crack nucleation and propagation

In the following, we furthermore carried out fatigue crack growth simulations under constant amplitude cyclic loading with the proposed framework. The discrete set of equations is solved to compute the primary variables (displacements), before computing for the values of SIFs from Eq. (20). The range of SIF for both mode-I and mixed mode under constant amplitude cyclic loading is defined as,

$$\Delta K = K_{\text{max}} - K_{\text{min}} \quad (21)$$

where K_{max} and K_{min} are the SIF values corresponding to maximum and minimum applied loads respectively.

In this paper, a generic curved crack trajectory is captured through many small line segments. Rankine's failure criterion is used for the crack nucleation in the domain by comparing the maximum principal stress value with the yield stress of the material. As the maximum principal stress in a particular element exceeds the yield stress of the material, a crack is assumed to nucleate in that element. The direction

of the crack nucleation is also obtained as the maximum principal stress plane direction. Once a crack has nucleated, its propagation, as well as the subsequent failure of the material, is determined using fracture mechanics criteria. For this purpose, an equivalent mode-I SIF and the direction of crack growth θ_c at each crack increment is determined as follows [16],

$$\Delta K_{\text{Ieq}} = \Delta K_I \cos^3 \left(\frac{\theta_c}{2} \right) - 3 \Delta K_{II} \cos^2 \left(\frac{\theta_c}{2} \right) \sin \left(\frac{\theta_c}{2} \right) \quad (22)$$

$$\theta_c = 2 \tan^{-1} \left(\frac{K_I - \sqrt{K_I^2 + K_{II}^2}}{4 K_{II}} \right) \quad (23)$$

Failure takes place whenever $(\Delta K_{\text{Ieq}})_{\text{max}} > K_{IC}$, where $(\Delta K_{\text{Ieq}})_{\text{max}}$ is the equivalent stress intensity factor corresponding to maximum load and K_{IC} is the fracture toughness of the material. The detailed procedure of modeling from crack nucleation, propagation to final failure is illustrated in Fig. 7.

5. Numerical results and discussion

Advanced materials such as composites, metal-ceramic interfaces, adhesive joints etc. have many important industrial applications. Their behavior is strongly dependent on the existence of the material interfaces. The failure of interface is very common in these materials, hence a deep understanding of failure mechanism is required for the designing purpose. The fracture characteristics in the vicinity of and at interface are strongly influenced by the properties of interface and of the material either side of the interface [60,59]. To solve such problems through enrichment methods such as XFEM and meshfree, separate set of enrichment functions is required when the crack tip is at the interface [9]. Thereafter, when the crack deflects into the material either side of interface, these enrichment functions are to be replaced with the enrichment functions of homogeneous materials, which makes these approaches quite cumbersome.

Therefore, to illustrate the effectiveness and accuracy of FNM, four fracture mechanics examples are solved and the results obtained by FNM are compared with the available literature/theoretical results. In the first problem, an edge crack plate is simulated under cyclic loading. In the second problem, a dog-bone specimen is considered for the simulation under mixed mode cyclic loading. In the third problem, a plate with a bi-material interfacial edge crack is simulated under cyclic tensile loading. In the fourth problem, a bi-material plate with edge crack perpendicular to interface is simulated under tensile loading. All simulations are performed under plane stress condition. LEFM theory is considered for the simulations, where the elastic fields are singular at the crack tip. Thus, special elements are required to capture the singularity at the crack tip. In the following, however, quadrilateral elements of the same size are used in the entire domain. To model the crack tip, special triangular transition element within the FNM framework is used (see Fig. 5) just ahead of the crack tip, which improves the accuracy of the results. In addition to this, the stress intensity factor is obtained based on global quantities far from the crack tip using domain based interaction integral approach, which reduces the effect of singularity on the numerical results. Several investigators [45,56,18] have used the same kind of elements in the entire domain to solve crack problems, and found that the usage of global quantities reduces the effect of singularity on the numerical results.

5.1. An edge crack plate under tensile loading

A rectangular plate of size 50 mm × 100 mm with an initial edge crack of length $a = 10$ mm as shown in Fig. 8 is considered. The plate is subjected to a tensile load of intensity $\sigma = 50$ MPa at the top edge of the plate, while the bottom edge of the plate is constrained as shown in Fig. 8. The thickness of the plate is 1.0 mm and the material properties

Fig. 8. A homogenous plate with an edge crack.

Fig. 9. Convergence study for FNM; stress intensity factor variation with crack length.

are taken as Young's modulus $E = 200$ GPa, Poisson's ratio $\nu = 0.30$ and fracture toughness $K_{IC} = 60$ MPa·m^{0.5}. It is well known that the numerical results are strongly dependent on the mesh size, hence a mesh convergence study is performed. The variation of SIF with crack length is provided in Fig. 9 for different mesh sizes. From the results, it is observed that there is negligible variation in the results for different mesh sizes, hence a mesh of 9730 elements (70 by 139 elements in x- and in y-directions respectively) is adopted for the simulation.

To compare the FNM results, the problem is also solved by FEM and XFEM. In FEM, a non-uniform mesh of 13,425 elements (very fine mesh at the crack tip) is considered, while a uniform mesh of 4851 elements (49 elements in x-direction and 99 elements in y-direction) is adopted for XFEM. In addition to this, the SIF values are also computed theoretically. Further, the SIF values obtained by FNM through interaction

Fig. 10. Comparison of FNM with FEM, XFEM and Analytical results: stress intensity factor variation with crack length.

Fig. 11. Final deformed shape of an edge crack plate for FNM.

integral approach are compared with the analytical (theoretical), FEM and XFEM approaches, with good agreements as illustrated in Fig. 10. The final deformed shape (at scale $\times 10^6$) obtained by FNM is depicted in Fig. 11. A small portion of the crack growth path is magnified and shown in the same figure to illustrate the split and transition elements. Finally, the normal stress contour plot (σ_{yy}) obtained in floating node method is illustrated in Fig. 12.

The simulation is done on a computer model with specifications: Intel (R) Core (TM) i7-6700 CPU @ 3.40 GHz, 4 Core, 8 GB RAM. The computational time required to simulate the first load step is 26 s and 33 s for FNM and extrinsic enriched XFEM respectively. While the difference in computational time is small, but observable, for this simple problem, it is expected to be more significant for non-linear problems.

5.2. Dog-bone specimen under mixed mode loading

A dog-bone specimen is subjected to mixed mode loading as shown in Fig. 13. The dimensions of the specimen are taken as; $L = 50$ mm, $H = 100$ mm and $D = 50$ mm. A mixed mode traction of intensity $\sigma_x = 10$ MPa and $\sigma_y = 25$ MPa is applied at the top edge of the plate, with the bottom edge of the plate constrained. The thickness of the specimen is taken as 1.0 mm. The material properties are taken as Young's modulus $E = 200$ GPa, Poisson's ratio $\nu = 0.30$, yield strength $\sigma_{yt} = 250$ MPa and fracture toughness $K_{IC} = 60$ MPa·m^{0.5}. An initial mesh of 9730 elements is used for the simulation.

Fig. 12. Normal stress distribution (σ_{yy}) in MPa for an edge crack plate.

A dog-bone specimen without crack is considered for the simulation under mixed mode cyclic loading. Rankine's failure criterion is utilized for the crack nucleation in the domain. As the maximum principal stress in a particular element exceeds the yield stress of the material, a crack is assumed to nucleate in that element. The direction of the crack nucleation is also obtained as the maximum principal stress plane direction. Once, the crack is nucleated in the domain, the rest of the analysis is performed based on fracture mechanics criteria as described in Sections 3 and 4.

The variation of von-Mises stress in the dog-bone specimen is shown in Fig. 14a just before the crack nucleation. After crack nucleation, the SIF values are obtained using the domain based interaction integral (as described in Section 3). The value of equivalent SIF is then calculated for each crack growth step and compared with the fracture toughness of the material to check the failure status of the specimen. The variation of von-Mises stress in the specimen after fifteen crack growth steps and for the final crack length are shown in Fig. 14b and c respectively. From the above simulations, it is observed that the proposed approach is able to model the phenomenon of crack nucleation, propagation to final failure of the structure.

5.3. Interfacial edge crack plate under tensile loading

A bi-material plate of size 50 mm × 100 mm with an interfacial edge crack of initial length $a = 10$ mm is taken for the simulation as shown in Fig. 15, following the geometry in Kumar et al. [26]. A tensile load of intensity $\sigma = 50$ MPa is applied at the top edge of the plate, while the bottom edge of the plate is constrained as shown in Fig. 15. The thickness of the plate is assumed as 1.0 mm. An initial uniform mesh of 9730 elements is used for simulation.

The fracture toughness of the interface is set higher than both base materials, hence a crack may propagate into either of the base materials depending on their material properties (see Table 1). The computed ΔK_{Ieq} is compared with the local fracture toughness of the two materials to determine the crack trajectory. For this, two ratios R_1 and R_2 are calculated as [6],

$$R_1 = \frac{(\Delta K_{Ieq})_{m1}}{(K_{IC})_{m1}} \quad \text{and} \quad R_2 = \frac{(\Delta K_{Ieq})_{m2}}{(K_{IC})_{m2}} \quad (24)$$

where $m1$ and $m2$ represent the material-1 and material-2 respectively. If the $R_1 > R_2$, the crack will propagate in the first material along $\theta = \theta_c$.

Fig. 13. Geometrical dimensions of dog-bone specimen under mixed mode loading.

Otherwise, it will propagate into the second material.

The SIF values obtained by FNM using interaction integral approach are compared with available results in literature, and a good agreement is obtained as shown in Fig. 16. Finally, the normal stress contour plot (σ_{yy}) obtained in floating node method is illustrated in Fig. 17.

As a further illustration of the predictive capability and accuracy of the FNM, we consider another bi-material plate problem that was reported in Liu et al. [31], shown again schematically in Fig. 15. The plate dimensions are scaled with $L = 3$ units, $H = 9$ units, and a tensile loading of intensity $\sigma = 1$ unit is applied at the top and bottom edges of the plate. The Poisson's ratios of both materials are 0.3. In order to demonstrate the capability of the FNM for interfacial crack problems, solutions are obtained for two ratios of Young's moduli, i.e. $E_1/E_2 = 2$ and 10 (keeping E_2 fixed as 100 units), with different crack lengths $a/L = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7$. The bi-material plate is discretized with 7301 quadrilaterals elements with the material interface conforming to the element edges. The values of normalized stress intensity factors obtained for different E_1/E_2 and a/L ratios are listed in Table 2. It is easily observed that the FNM predictions compare well against the results in literature.

5.4. An edge crack perpendicular to interface under tensile loading

A bi-material plate of size 50 mm × 100 mm with an edge crack, perpendicular to interface, of initial length $a = 10$ mm is next considered, as shown in Fig. 18. The bi-material plate consists of two materials: metal and ceramic (Al_2O_3). The mechanical properties of both materials are given in Table 3 [2]. The behavior of ceramic is assumed as linear elastic, while the elasto-plastic Ramberg's Osgood constitutive is adopted for the metal. A uniformly distributed tensile load of intensity $\sigma = 70$ MPa is applied at the top and bottom edges of the plate as shown in Fig. 18. The bi-material plate is discretized with an initial uniform mesh of 9730 quadrilateral elements, with the material interface conforming to the element edges. The material model parameters are derived from the stress-strain behavior of the metal [2]. Details on the elasto-plastic model and J -integral computation can be found in literature [30].

Fig. 14. Von-Mises stress (in MPa) contour plots in dumb-bell specimen for different crack length.

Fig. 15. Bi-material plate with an interfacial edge crack.

Table 1 Properties of the constituents of bi-materials.			
	Young's Modulus, E (GPa)	Poisson's ratio, ν	Fracture toughness, K_{IC} (MPa·m ^{0.5})
Material-1	74	0.30	40
Material-2	200	0.30	60

Fig. 16. Comparison of literature and FNM results; stress intensity factor variation with crack length.

In the simulations, the effect of the crack position, in terms of normalized crack length a/h_m ($h_m = h_c$), is observed via the J -integral. The variation of J -integral with the normalized crack length is shown in Fig. 19. In the first case, the initial crack occurs in the metal and propagates towards the ceramic, for which Dundurs parameter (α) is equal to -0.65 . In the second case, the initial crack occurs in the ceramic and propagates towards the metal, for which the value of $\alpha = 0.65$. From the simulations, it is observed that the value of J -integral increases with the increase of a/h_m ratio for both cases. A critical value of ratio a/h_m is determined as ~ 0.5 , after which the value of J -integral increases significantly. This increase in J -integral is more significant when the crack propagates from the ceramic (brittle material) to metal (ductile material), and reaches its maximum value at the vicinity of the interface ($a/h_m \rightarrow 1$). Conversely, the value of the J -integral decrease near to the

Fig. 17. Stress distribution of σ_{yy} (MPa) for a bi-material interfacial edge crack plate.

Table 2
Comparison of normalized mode-I and mode-II stress intensity factor values of an interface edge crack rectangular plate.

E_1/E_2	a/L	Floating Node Method		Liu et al. [31]	
		$K_I/\sigma\sqrt{\pi a}$	$K_{II}/\sigma\sqrt{\pi a}$	$K_I/\sigma\sqrt{\pi a}$	$K_{II}/\sigma\sqrt{\pi a}$
2	0.2	1.357	−0.136	1.374	−0.137
	0.3	1.652	−0.157	1.669	−0.159
	0.4	2.103	−0.193	2.125	−0.198
	0.5	2.804	−0.264	2.844	−0.267
	0.6	3.981	−0.395	4.062	−0.394
	0.7	6.266	−0.667	6.402	−0.664
10	0.2	1.351	−0.352	1.379	−0.354
	0.3	1.617	−0.396	1.661	−0.403
	0.4	2.074	−0.491	2.109	−0.500
	0.5	2.785	−0.657	2.819	−0.668
	0.6	3.973	−0.974	4.018	−0.986
	0.7	6.251	−1.641	6.319	−1.661

interface when the crack propagates from metal to ceramic. A similar observation is made by Suresh et al. [58] during the fatigue tests of bimetallic composites. The decrease in J -integral value is due to the decrease in stress level at the crack tip with the increase in rigidity of the materials ($E_{\text{ceramic}}/E_{\text{metal}}$), which can be observed in Fig. 20a, where the stress level is lower at the crack tip than the stress level at the material interface in the ceramic. The variation of normal stress distribution for different crack tip position is shown in Figs. 20 and 21 when the crack is propagating from metal to ceramic and ceramic to metal respectively. From Fig. 20, it can be observed that the stress level increases after the interface, because the crack tip is developing in the ceramic, which is stiffer than the metal. Conversely, when the crack propagates from ceramic to metal in Fig. 21, the stress level decreases as the crack tip develops in the softer material.

Note that problems 3 & 4 considered two types of discontinuities: the discontinuous displacement field along the crack face and the discontinuous derivative along the material interface. The FNM is more advantageous than XFEM/GFEM to solve such problems. In XFEM/GFEM, the material interface requires special enrichment functions, which adds additional degrees of freedom into the solution. XFEM/GFEM also requires special treatment of interface elements during the

Fig. 18. Geometrical model of a plate with an edge crack perpendicular to interface.

Table 3
Properties of the constituents of bi-materials.

	Young's Modulus, E (GPa)	Poisson's ratio, ν	Yield Stress, σ_y (MPa)
Ceramic (Al_2O_3)	345	0.30	–
Metal	72	0.33	300

Fig. 19. Comparison of FNM with the literature results; variation of J -integral versus a/h_m .

integration procedure, as the material interface does not lie along the element edges. However, In FNM, crack growth is modeled through element partitioning with the help of floating nodes and the material interface lies along the sub-element edges. The integration of elements along the material interface is thus performed directly as per standard

Fig. 20. Stress distribution of σ_{yy} (MPa) for a bi-material plate, where material-1 (metal) is softer than material-2 (Al_2O_3): (a) $a/h_m = 0.9$, (b) $a/h_m = 1.0$, and (c) $a/h_m = 1.2$.

Fig. 21. Stress distribution of σ_{yy} (MPa) for a bi-material plate, where material-1 (Al_2O_3) is stiffer than material-2 (metal): (a) $a/h_m = 0.9$, (b) $a/h_m = 1.0$, and (c) $a/h_m = 1.2$.

FEM. From the numerical results, it is easily observed that the FNM is able to efficiently model the two issues concerning crack growth and material interface. It is highlighted that the predicted results are very close to literature results, despite a simpler numerical framework.

6. Conclusions

In this contribution, FNM is combined with a domain-based

interaction integral approach to model generic 2D crack growths. An internal floating node is used to maintain the aspect ratios of the sub-elements after the partition of the original element with respect to oblique crack propagation. The combination of FNM with the domain-based interaction integral offers accurate and path independent evaluations of SIFs ahead of the crack tip. From the simulations, it is observed that the combination of FNM and domain-based interaction integral is a viable alternative to solve fracture mechanics problems.

When dealing with material interface and continuous to discontinuous material modeling, that FNM offers several advantages over XFEM/GFEM:

- It does not require crack tip enrichment functions, which are problem specific;
- It does not require additional enrichment functions to model the material interface;
- Integration is simpler as it does not require integration over a part of domain after mapping into natural space;
- It does not require additional enrichment functions to model crack perpendicular to material interface.

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