The influence of inelastic rock behaviour on hydraulic fracture geometry

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Introduction

Hydraulic fracturing is the fracturing of solid material, under the influence of fluid pressure inside the fracture. It comprises deformation and fracturing of the solid, and fluid flow processes (see figure 1.1). In this thesis we study aspects of the hydraulic fracturing processes in scaled experiments on rock that is confined under in-situ stress states. We concentrate on the fracturing and deformation processes near the fracture tip.

Figure 1.1. Schematic view of the hydraulic fracturing process. Fluid flows inside the fracture, while the fracture opens and propagates. Fluid is pumped into the fracture mouth to maintain fracture propagation.
1. Introduction

Application of hydraulic fracturing
Hydraulic fracturing of oil- and gas wells is a technique that has been applied on a routine basis since the 1970s. Its main aim is to create a permeable flow path in order to facilitate the flow of oil and gas towards the wellbore. Fluid is pumped into a sealed part of the wellbore. When the pressure has become high enough, the rock breaks and fluid flows into the created fracture. If pumping continues, fluid will continue to flow into the fracture, which will propagate further. The fluid that creates the fracture usually contains solid particles, which keep the fracture open after the fluid has leaked away into the rock. Increasing the permeability near the wellbore is often very effective, as most of the pressure drop in the reservoir usually takes place in the vicinity of the wellbore (because of the radial flow pattern or permeability reduction by near-wellbore damage).

Originally only employed in reservoirs of low permeability, hydraulic fracturing of soft and permeable formations was introduced in the 1980s (e.g. Smith et al., 1987, and Ayoub et al., 1992). This technique has been combined with gravelpackin ("frac-and-pack", e.g. Wong et al., 1993). In addition to the stimulation of oil- and gas wells, hydraulic fracturing is used to measure in-situ stresses (also referred to as "micro hydraulic fracturing", e.g. Desroches, 1995). Waterflooding for secondary recovery can take place under fracturing conditions (e.g. Hagoort, 1981). In this technique, a fracture propagates relatively slowly, while fluid leaking away from the fracture faces floods the reservoir. Furthermore, hydraulic fracturing is often of interest for stability of wellbores and tunnels.

Besides the field of petroleum engineering, the hydraulic fracturing technique is used in other fields as well. In geothermal energy extraction in "hot dry rock", fluid is pumped through a hydraulically fractured granite rock in order to increase the temperature of the fluid. The aim is to create a fracture pattern in which much heat flow takes place from the rock towards the fluid that flows through the fracture system, which connects two wellbores (Nemat-Nasser, 1982). Furthermore, hydraulic fractures can be created during waste injection (Rothenburger et al., 1994). In civil engineering, hydraulic fracturing has several applications as well. It can be used to facilitate waste remediation techniques (Du et al., 1993), or unwanted hydraulic fracturing can occur during field permeability testing (Bjerrum et al., 1972).

Besides these technical applications, hydraulic fracturing also occurs in geological processes (e.g. Shoji and Takenouchi, 1982, and Pollard et al., 1975). Examples are dikes formed by magma, or extension of fractures by local pore pressure increase (Lacazette and Engelder, 1992).

Problem statement and motivation of research
An optimum fracture geometry can be estimated before fracturing a petroleum reservoir, based on expected productivity increase and treatment costs. Hence, a correct prediction of fracture geometry is important, as is the prediction of net pressure which controls growth
of the hydraulic fracture into regions with a higher least principal stress (usually referred to as "height growth" into "stress barriers"). Numerical or analytical models are used for the design of a hydraulic fracture in the field.

The net pressure is usually the only variable that is used to provide information about the geometry of the created fracture. Various methods have been used to obtain more information about the geometry of hydraulic fractures in the field and validate the simulations. Tiltmeters placed at the surface (e.g. Castillo et al., 1997) or in a nearby borehole (e.g. Warpinsky et al., 1997), and active or passive acoustic monitoring (e.g. Vinegar et al., 1992, and Block et al., 1994), give information about the fracture geometry. More unorthodox methods are coring of a hydraulic fracture (Warpinsky et al., 1993), or placement of width and pressure transducers along an already created fracture, followed by mining the fracture (Warpinsky, 1983). Wellbore logs (e.g. temperature, borehole televIEWERS) give information about the fracture close to the wellbore. Furthermore, radioactive tracers can be added to the proppant to give information about the fracture width (Reis et al., 1996).

These methods can yield valuable information about the hydraulic fracturing process on a field scale. However, a number of them are not suitable yet to use routinely, and others give only limited information. The pressure needed to propagate the fracture is still the main source of information for predicting the fracture geometry. This again emphasises the importance of correct modelling of the hydraulic fracturing process, because a model couples the pressure to the fracture geometry. Measurements of the pressure are ideally made downhole, close to the entrance of the fracture.

Models of hydraulic fracturing usually predict the net pressure, which is defined as the difference between the fluid pressure in the fracture and the in-situ stress working perpendicularly to the fracture surface. The net pressure is usually small in comparison with the in-situ stresses. Even when downhole pressure measurements are available, the accuracy of the determination of the net pressure relies heavily on the accuracy of the determination of the in-situ stress working perpendicularly to the fracture surface.

In various cases reported in the literature, elastic models are not able to explain the net pressures observed in field hydraulic fracturing treatments (Medlin and Fitch, 1983, and Palmer and Veatch, 1987). Such net pressures being higher than expected, are usually referred to as "high net pressures". High net pressures are not only seen during fracturing of intact rock, but also when reopening a previously created fracture (Shlyapobersky and Chudnovsky, 1994). More problems with elastic simulators are reported. Wright et al. (1993) observed a dependence of net pressure on flow rate and fracturing fluid viscosity that was weaker than predicted by current models. Another indication that current modelling of the hydraulic fracturing process needs improvement, is that various hydraulic fracture simulators predict significantly different propagation pressures and fracture geometries for the same input (Warpinsky et al., 1994 and Cleary, 1994).
1. Introduction

The assumption of infinite tensile stress at the fracture tip in elastic models is certainly unrealistic. Furthermore, the in-situ stresses cause a large deviatoric stress near the tip, where the stress is approximately released in one direction. Because of this, inelastic rock deformation is expected to occur in the tip region. Inelastic rock behaviour at the fracture tip plays a central role in a number of possible explanations for observed high net pressures. Hydraulic fracturing models predict a large pressure gradient close to the fracture tip, because of the small fracture width near the tip (see Warpinsky, 1983, and Desroches et al., 1994). The possible squeezing effect of inelastic rock deformation on the fracture width is proposed as a mechanism that can cause high net pressures (Johnson and Cleary, 1991). Shlyapobersky and Chudnovsky (1994) propose a rate- and scale-dependent fracture mechanism at the tip, which increases the energy needed for fracture propagation by several orders of magnitude. Modelling of the processes that take place at the tip of the fracture is still subject for research in hydraulic fracturing modelling.

Objective and method

The aim of the present study was to investigate the influence of inelastic rock behaviour at the fracture tip on the pressure and geometry of a propagating hydraulic fracture. Inelastic rock behaviour is a wide notion for deviations from elasticity. The rocks used in this study will be described by plasticity theory, so we will frequently use the term "inelastic" instead of "inelastic". We started our research by performing scaled hydraulic fracturing model experiments on various rocks. We used rock material in which we expected that plasticity was negligible, and rocks in which we expected plasticity to be significant. We compared the results with a fully coupled linear elastic model (Barr, 1991). Besides varying the rock properties, we also varied the in-situ stress states, which are thought to influence the relative importance of plasticity as well.

We characterised our rocks in triaxial tests, tensile tests, and other rock mechanical tests. We coupled the behaviour of the rocks in these tests with the results of the hydraulic fracturing model experiments. We also developed a numerical simulation program for hydraulic fracturing in an elastic-plastic solid. In order to compare this with the experiments, we used the results of the material characterisation experiments as input for our numerical model. The hydraulic fracturing model experiments, the elastic-plastic simulations, and the material characterisation form the three main ingredients of this study.

The emphasis of this study is the investigation of the hydraulic fracture propagation phase. However, the experiments performed contain more information, which is partly coupled to the propagation phase. This information is in the closure phase of the hydraulic fractures, and in the roughness of the fracture surfaces. We analysed these measurements, and present possible explanations.
1. Introduction

Organisation of the thesis
Chapter 2 of this thesis starts with the background theory of the processes of interest: fluid flow, rock fracturing, and rock deformation. It also discusses the coupling of the equations describing these processes, and shows how scaling laws for the experiments can be derived. The experimental part starts in Chapter 3, which describes the experimental set-up and method. Chapter 4 contains the experimental characterisation of the rocks and fracturing fluid. Chapter 5 presents the results of the hydraulic fracturing model experiments. The numerical simulation method and results are described in Chapter 6, and a comparison with the experimental results is made. Chapter 7 contains a general discussion of the results, and integrates the results of the other chapters. It also discusses the implications of the results for field applications. Chapter 8 presents the main conclusions of this study.
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Analysis of hydraulic fracturing processes

The hydraulic fracturing process is a combination of rock fracturing, fracture opening, and fluid flow. This chapter contains the basic theory used for modelling these processes. We list the assumptions that are made in applying the models to hydraulic fracturing, and discuss their validity. Section 2.1 describes elastic and plastic rock deformation. Fracture mechanics and the fracturing process are discussed in sections 2.2 to 2.4. Section 2.5 gives a description of fluid flow inside the fracture and leak-off into the rock. Section 2.6 gives a short review of various ways to obtain approximate analytical solutions of the governing equations for elastic rock behaviour, and includes an overview of fully coupled numerical models.

In the various processes taking place during hydraulic fracturing, energy is stored or dissipated. The energy rates associated with each process are presented in section 2.7. Using the basic equations describing hydraulic fracturing, we can derive dimensionless groups to scale our model experiments. Section 2.8 shows the derivation of these dimensionless groups. In section 2.9, we present a new model that can explain the occurrence of fracture surface roughness. This model is based on the minimisation of the total energy dissipated during fracture propagation.
2. Hydraulic fracturing processes

2.1 Rock mechanics

The mechanical behaviour of rock can often be described using continuum mechanics. This section contains some basic concepts on which the numerical simulations presented in Chapter 6 are based, and which can be found in for example Fetter and Walecka (1980) and Mase (1970).

**Deformation**

In the following, we define the variables with respect to a Cartesian coordinate system, and use an Eulerian description of deformation. Consider two points A and B in space that are close to each other at initial positions \(x^A\) and \(x^B\). Let the deformation move these points to positions \(y^A = x^A + u^A\) and \(y^B = x^B + u^B\). The distance between the points after the deformation is:

\[
\delta y = y^A - y^B = x^A - x^B + u^A - u^B = \delta x + \delta u
\]  
(2.1)

in which \(\delta x = x^A - x^B\) and \(\delta u = u^A - u^B\). \(\delta u\) equals the change in vector separation between points A and B, caused by the deformation. When we take the \(\delta\)'s small and use a first-order Taylor expansion, we can write the components of \(\delta u\) as:

\[
\delta u_i = \frac{\partial u_i}{\partial x_j} \sum_{j=1}^{3} \left( x^A_j - x^B_j \right)
\]  
(2.2)

where the partial derivative is evaluated in \(x\), while we assume that \(u\) is differentiable.

Equation (2.2) shows that the relative change in vector separation between two points as a result of deformation is determined by the coefficients \(\frac{\partial u_i}{\partial x_j}\). We can write these coefficients as:

\[
\frac{\partial u_i}{\partial x_j} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) = \epsilon_{ij} + \omega_{ij}
\]  
(2.3)

The first term in brackets in this equation comprises the coefficients \(\epsilon_{ij}\) of a symmetric tensor, which describes internal deformation and is named the infinitesimal strain tensor. This strain described by this tensor is invariant under rigid body rotations (which are of no interest to our problem and are assumed to be zero). The second term in brackets comprises the coefficients \(\omega_{ij}\) of an anti-symmetric tensor, which describes a local rigid rotation and does not correspond to an internal deformation (e.g. Fetter and Walecka,
1980). We made the assumption of small strain. In elastic-plastic problems, large strains can develop in shear bands, but we will not consider this quantitatively in this study.

The strain field can be calculated from the displacement field. If the components $\epsilon_{ij}$ of the strain tensor were independent, we would have six degrees of freedom, while the displacement field has only three degrees of freedom. The compatibility equations provide a restriction to the strain components and ensure a kinematically admissible strain field (e.g. Mase, 1970). They are obtained by differentiating the definition of $\epsilon_{ij}$ two times with respect to $x_1$, $x_2$, or $x_3$, interchange the order of differentiation, and eliminate the displacements. Using the symmetry of the strain tensor, six distinct equations are found:

$$\frac{\partial^2 \epsilon_{ii}}{\partial x_i^2} + \frac{\partial^2 \epsilon_{jj}}{\partial x_j^2} - 2 \frac{\partial^2 \epsilon_{ij}}{\partial x_i \partial x_j} = 0$$

$$i \neq j, i \neq l, \text{ and } j \neq l \quad (2.4)$$

Within this framework, fracture surfaces are regarded as boundaries.

**Stress**

The forces that work on a small volume element of a continuum can be divided into body and surface forces. In geomechanical problems, usually the only body force is the gravitational force. Although this force causes large stresses in petroleum reservoirs, it is negligible in this study. This is caused by the relatively small depth variation within the rock volumes considered, compared to the total depth. The overburden stress is taken as a boundary condition.

The surface forces are given by the stress tensor. We can define the stress tensor with components $\sigma_{ij}$ which equals the $i^{th}$ component of the surface stress in the $j$-direction. Then $\sum_{j=1}^{3} \sigma_{ij}dA_j$ is the $i^{th}$ component of the force acting on a surface $dA = dA \hat{n}$, with outward normal $\hat{n}$ (Fetter and Walecka, 1980). From the rate of change of the angular momentum in a small volume element, and using conservation of momentum, it can be shown that the stress tensor is symmetric (Fetter and Walecka, 1980), so that

$$\sigma_{ij} = \sigma_{ji} \quad (2.5)$$

Because of the quasi-static assumption in our problem, force balance of a volume element is required. Neglecting body forces, the force balance can be stated as (Mase, 1970):
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\[ \sum_{j=1}^{3} \frac{\partial \sigma_{ji}}{\partial x_j} = 0 \]  

(2.6)

Throughout this thesis, we follow the usual convention in earth sciences and take compressive stress positive.

Constitutive behaviour

In general, the constitutive behaviour couples the deformation of a material element with the forces that work on it as a function of time. Here we will consider linear elastic and elastic-plastic constitutive models, and assume time-independent material behaviour. We also neglect influences of temperature.

The linear elastic constitutive model is based on Hooke’s law, which states that the elastic deformation is linearly proportional to the applied load. For an isotropic material, the components of the symmetric stress and strain tensors are related by (Jaeger and Cook, 1969):

\[
\begin{pmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{12} \\
\varepsilon_{13} \\
\varepsilon_{23}
\end{pmatrix} = \frac{1}{E} \begin{pmatrix}
1 & -\nu & -\nu & 0 & 0 & 0 \\
-\nu & 1 & -\nu & 0 & 0 & 0 \\
-\nu & -\nu & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1+\nu & 0 & 0 \\
0 & 0 & 0 & 0 & 1+\nu & 0 \\
0 & 0 & 0 & 0 & 0 & 1+\nu
\end{pmatrix} \begin{pmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{13} \\
\sigma_{23}
\end{pmatrix}
\]  

(2.7)

\( E \) is the Young’s modulus, and \( \nu \) is Poisson’s ratio.

Because the amount of plastic deformation depends on the stress path, the constitutive behaviour of elastic-plastic materials is given in incremental form. The total strain \( d\epsilon_{ij} \) is assumed to be the sum of the linear elastic strain \( d\epsilon_{ij}^e \) (given by equation (2.7)) and the plastic strain \( d\epsilon_{ij}^p \):

\[ d\epsilon_{ij} = d\epsilon_{ij}^e + d\epsilon_{ij}^p \]  

(2.8)

Plastic deformation takes place when the yield criterion is fulfilled (e.g. Chen and Mizuno, 1990):

\[ f(\sigma_{ij}) = k_y \]  

(2.9)

and

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\[ df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} \geq 0 \]  \hspace{1cm} (2.10)

The equality sign is for perfect plasticity. \( k \) is a yield stress; \( f \) is the yield function, and is assumed to depend on the stress situation. It can be viewed as a surface in stress space.

When the yield criterion is satisfied, the plastic strain is calculated from a potential function \( g(\sigma_{ij}) \) by the flow rule:

\[ d\varepsilon_{ij}^p = d\lambda \frac{dg}{d\sigma_{ij}} \]  \hspace{1cm} (2.11)

\( d\lambda \) is the plastic multiplier, which is a scalar factor of proportionality. This flow rule expresses that a plastic strain component is proportional to the change in the potential function as a result of the corresponding stress change. This implies that the direction of the plastic strains is normal to the equipotential surface, which can be made visible by superimposing the strain coordinates on the stress coordinates. This predicts dilatant behaviour of the plastic strains when the potential \( g \) increases with the mean pressure (in this thesis referred to as "dilatancy", "dilation", or "dilatant behaviour"). When \( g \) coincides with the yield function \( f \), the flow rule is called associated (otherwise, it is called non-associated). The plastic volume increase predicted by the associated flow rule is too large for many rock types (Chen and Mizuno, 1990).

Often, an increase in yield stress is observed when plastic deformation occurs, which is called hardening. It can be represented by a shifting of the yield surface in stress space, and makes the yield- and potential function dependent on the stress history. The shifting of the yield surface is assumed to be governed by a hardening parameter. There is a certain degree of arbitrariness in the choice of the hardening parameter. Two choices are generally in use (Chen, 1982): a hardening parameter based on the plastic work done, or based on an integration of some combination of the plastic strain increments.

Drucker's stability postulate, or the requirement that the work done is positive when plastic deformations occur (irreversibility condition), leads to the conclusion that the yield surface must be convex, and that the plastic strain increment is normal to the yield surface (Chen, 1982). These conditions of convexity and normality assure that the solution to a boundary value problem is unique. For a non-associated flow rule, the normality condition is violated and uniqueness is in general not assured, and Drucker's postulate can be violated.

The plastic multiplier \( d\lambda \) in equation (2.11) is in principle undetermined, so that the plastic strain increment is undetermined when the stress increment is prescribed. Instead, in numerical simulations usually the strain is prescribed and from that the factor \( d\lambda \) and hence the plastic strains are calculated (Chen, 1982).
2.2 Linear elastic fracture mechanics

Linear elastic fracture mechanics (LEFM) describes fracture opening and propagation in a linear elastic continuum. It is one of the basic elements of hydraulic fracturing models. This section contains the main results concerning the fracture propagation criterion, fracture opening, and the stress field near the fracture tip.

Fracture propagation criterion

The problem of a fracture in a linear elastic material under far-field stresses can be handled using the superposition principle (e.g., Mase, 1970). Because of the linear relationship between elastic deformation and applied load, the sum of two loading situations of the same body is again a solution of the elasticity equations. Applied to our problem, the superposition principle states that the stress loading can be subdivided into loading of the far-field compressive stress and loading of the net pressure inside the fracture (see figure 2.1). The final solution is obtained by adding the stresses and displacements of both solutions. The superposition shows that fracture opening under confined and unconfined conditions is equal when the net pressure loading on the fracture surfaces is equal. The net pressure \( p_{\text{net}} \) is defined as the fluid pressure \( p_f \) inside the fracture minus the confining stress \( \sigma_c \) working perpendicular to the fracture surface. The displacements and stress intensity can now be expressed in terms of the net pressure loading at the fracture surfaces.

For a radial fracture geometry, the stress intensity \( K_I \) as a function of the net pressure is given by (Sneddon and Lowengrub, 1969):

\[
K_I = \frac{2}{\sqrt{\pi R}} \int_0^R \frac{r p_{\text{net}}(r) dr}{\sqrt{R^2 - r^2}}
\]  

(2.12)

where \( R \) is the fracture radius, and \( r \) is the radial coordinate (see figure 2.2). The stress intensity defined by this formula is a factor \( \sqrt{\pi} \) larger than in Sneddon and Lowengrub (1969), and is a more commonly used definition. The stress intensity is a measure for the amount of tensile strain energy that is stored in the stress field around the tip as a result of the loading of the net pressure. It appears in expressions for the stress field around the tip and energy release rate associated with fracture propagation. When the stress intensity reaches a critical value, the fracture propagates. For a uniform net pressure in the fracture, the stress intensity at the tip is given by:

\[
K_{Ic} = \frac{2}{\sqrt{\pi}} p_{\text{net}} \sqrt{R}
\]

(2.13)
for a fracture with a radial geometry, and

$$K_{ic} = p_{net} \sqrt{\pi L} \quad (2.14)$$

for a plane-strain fracture, in which $L$ is the fracture half length (see figure 2.2).

Close to the tip, the stress field is proportional to $\frac{1}{\sqrt{s}}$, where $|s|$ is the distance to the fracture tip (see equation (2.24)). This corresponds with a finite amount of tensile strain energy release per unit fracture area during fracture propagation. According to the Griffith energy balance, the strain energy release per unit fracture area equals twice the fracture surface energy $\Gamma$, which is the energy needed for creating a unit fracture area. For plane strain, the critical stress intensity is related to $\Gamma$ by (e.g. Sneddon and Lowengrub, 1969):

$$K_{ic} = \sqrt{2\Gamma E} \quad (2.15)$$

where $E$ is the plane strain modulus, defined as $E/(1-\nu^2)$.

**Fracture opening**

Using the superposition principle, the fracture width $b$ is given by the loading of the net pressure (Sneddon and Lowengrub, 1969):

$$b(r) = \frac{2}{\pi E} \int_{r}^{R} \frac{du}{r \sqrt{u^2 - r^2}} \int_{0}^{u} \frac{p_{net}(s)ds}{\sqrt{u^2 - s^2}} \quad (2.16)$$

$E$ is the crack opening modulus, defined as $E/(4(1-\nu^2))$. $u$ and $s$ are here spatial variables along the fracture radius.

**Stress field near the fracture tip**

In two-dimensional elasticity problems it is possible to calculate the stress field from complex potential functions $\varphi$ and $\psi$ in the following way (Muskhelishvili, 1953):

$$\sigma_{xx} + \sigma_{yy} = -2\left(\varphi'(\zeta) + \overline{\varphi'(\zeta)}\right) \quad (2.17)$$

$$\sigma_{yy} - \sigma_{xx} + 2i\sigma_{xy} = -2\left(\overline{\zeta}\varphi''(\zeta) + \psi'(\zeta)\right) \quad (2.18)$$

where $\zeta$ is the complex variable $x+iy$, $x$ and $y$ are coordinates along orthogonal axes, and $'$
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Figure 2.1. The total loading can be subdivided into two separate situations (only the loading perpendicular to the fracture surface is shown). The sum of the stresses and displacements in situation II and III gives the solution to the problem of a fracture under confining stress $\sigma_c$ with internal fluid pressure $p_f$ (situation I). In situation II, the displacements on the fracture face and the stress intensity are zero. So, the displacements and stress intensity are given by situation III.

plane strain:                      radial geometry:

Figure 2.2. Coordinate system for plane strain and radial fracture geometry (also called "penny-shaped"). For the plane strain geometry, the origin is chosen in the centre of the crack, or at the crack tip. In the plane strain geometry, the strain in the $z$-direction stays unaltered compared to the initial situation.
indicates differentiation with respect to $\zeta$ here. A minus sign is included on the right-hand side of these equations because we take compressive stress positive. For plane strain, the displacements are given by:

$$u_x + iu_y = \frac{(1 + \nu)}{E}\left((3 - 4\nu)\varphi(\zeta) - \zeta\varphi'(\zeta) - \overline{\varphi(\zeta)}\right)$$

(2.19)

We now choose the origin at the centre of the fracture and the $x$- and $y$-axis according to figure 2.2. After Westergaard (1939) a potential function can be defined for the stress field around a plane strain fracture with half length $L$ and uniform internal fluid pressure $P_0$:

$$\varphi'(\zeta) = \frac{P_0}{2} \left( \frac{\zeta}{\sqrt{\zeta^2 - L^2}} - 1 \right)$$

(2.20)

and

$$\psi'(\zeta) = -\zeta \varphi''(\zeta)$$

(2.21)

so that

$$\psi(\zeta) = -\zeta \varphi'(\zeta) + \varphi(\zeta)$$

(2.22)

The integration constant in equation (2.22) is taken zero, so that the displacements at the crack tip are zero. Equation (2.21) ensures that $\sigma_{xy} = 0$ at $y = 0$. This choice for the potential functions satisfies the following boundary conditions, which correspond to a plane strain fracture filled with fluid under uniform pressure:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Boundary Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 0$</td>
<td>$\sigma_{xx} = \sigma_{yy}, \sigma_{xy} = 0$</td>
</tr>
<tr>
<td>$-L &lt; x &lt; L, y = 0$</td>
<td>$\sigma_{xx} = \sigma_{yy} = p_0$</td>
</tr>
<tr>
<td>$</td>
<td>x</td>
</tr>
<tr>
<td>$</td>
<td>x</td>
</tr>
</tbody>
</table>

We can shift the origin of the $x,y$-coordinate system to the crack tip by taking $\zeta' = \zeta - L$. Making this substitution, we have:

$$\varphi'(\zeta) = \frac{P_0}{2} \left( \frac{(\zeta' + L)}{\sqrt{(\zeta' + L)^2 - L^2}} - 1 \right)$$

(2.23)
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When we assume that $L \gg \zeta$, we can approximate equation (2.23) by:

$$
\phi'(\zeta) = \frac{p_0}{2} \left( \frac{\sqrt{L} \zeta}{\sqrt{2}} \left( \frac{1}{2} \right)^2 - 1 + O \left( \frac{\zeta}{L} \right)^{\frac{1}{2}} \right)
$$

(2.24)

The lowest-order approximation in $\left( \frac{\zeta}{L} \right)^{\frac{1}{2}}$ is valid in the region where the real part of $\left( \frac{\zeta}{L} \right)^{\frac{1}{2}} << 1$, which is in general more restrictive than the requirement imposed by the imaginary part of $\left( \frac{\zeta}{L} \right)^{\frac{1}{2}}$, which is that $\frac{\zeta}{L} << 1$. When we calculate the stress field, the first restriction coincides with the condition that $-\sigma_{yy} >> p_0$.

The Westergaard potential function has a constant pressure on the fracture surface. However, a strong pressure gradient is usually present near the tip of a hydraulic fracture, caused by fluid flow. Desroches et al. (1994) developed a solution in which the equations for fluid flow and fracture opening are both satisfied. To obtain this solution, a generalised form of the potential function was used:

$$
\phi(\zeta) = \frac{A}{2\alpha} \zeta^a
$$

(2.25)

Together with equation (2.21), this potential function satisfies the boundary conditions:

$$
y = 0: \quad \sigma_{xx} = \sigma_{yy}, \sigma_{xy} = 0$$

$$|x| \geq L, \ y = 0: \quad u_y = 0$$

Taking $\sigma_{xy}=0$ at $y=0$ implies that the shear stress exerted by the fluid on the fracture wall is negligible. This follows from the equations for fluid flow presented later: equation (2.35) shows that the ratio of shear stress to fluid pressure scales as the ratio of fracture width and a characteristic length in the radial direction. This ratio is usually much smaller than one.

The asymptotic solution for semi-infinite fracture length that couples fluid flow in the fracture (modelling locally as Poiseuille flow between parallel plates) and elastic opening of the fracture yields that $\alpha=2/3$ and $A = \frac{4}{3} \left( \frac{2E^2 \mu L}{15} \right)^{\frac{1}{3}}$ for Newtonian fluid behaviour (Desroches et al., 1994). The fluid pressure at the tip goes to minus infinity for this
potential function. Note that for $\alpha=1/2$ and $A = \frac{p_0\sqrt{L}}{\sqrt{2}}$ (while $\left|\frac{\xi}{L}\right|^2 << 1$), the potential functions of equations (2.25) and (2.24) coincide. $A$ is related to the stress intensity by

$$A = \frac{K_L}{\sqrt{2\pi}}.$$

When we add compressive far-field stresses, the potential functions (2.20) or (2.25), and (2.21) change by adding an extra term containing these stresses. As an example, we will take the potential functions of equations (2.25) and (2.21), which become:

$$\varphi(\zeta) = \frac{A}{2\alpha} \frac{\zeta^\alpha}{\zeta^*} - \frac{\zeta}{4} \left( \sigma_c + \sigma_h \right)$$

(2.26)

and

$$\psi'(\zeta) = -\zeta \varphi'(\zeta) + \frac{\left( \sigma_h - \sigma_c \right)}{2}$$

(2.27)

where $\sigma_h$ and $\sigma_c$ are the compressive far-field stresses in the $x$- and $y$-direction respectively. Because of the neglected terms in equation (2.24), adding the far-field stresses to the approximated potential functions is only useful when $\sigma_c >> p_0$.

When we substitute equations (2.26) and (2.27) in equations (2.17), (2.18), and (2.19), and use $\zeta = \frac{\zeta}{L}$, we get:

$$\sigma_{xx} + \sigma_{yy} \equiv -\left( A \zeta^\alpha + \overline{A} \zeta^\alpha \right) + \left( \sigma_h + \sigma_c \right)$$

(2.28)

$$\sigma_{yy} - \sigma_{xx} + 2i\sigma_{xy} = -\left( \overline{A} \zeta^\alpha - \zeta \right) (\alpha - 1) A^\alpha - 2 \left( \sigma_h - \sigma_c \right)$$

(2.29)

$$u_x + iu_y = \frac{(1 + \nu)}{E} \left( \begin{array}{c}
\left( 3 - 4\nu \right) \left( A \zeta^\alpha - \overline{A} \zeta^\alpha \right) - \zeta \left( \sigma_c + \sigma_h \right) \\
- \left( \zeta - \overline{\zeta} \right) \left( 2 \zeta^\alpha - \frac{\sigma_c + \sigma_h}{4} \right) \\
- \left( \frac{A \zeta^\alpha}{2} - \zeta \frac{\sigma_c + \sigma_h}{4} + \frac{\sigma_h - \sigma_c}{2} \right)
\end{array} \right)$$

(2.30)

For a plane strain fracture we have for the out-of-plane principal stress with far-field value $\sigma_h$:
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\[ \sigma_{zz} = \sigma_h - v(\sigma_h - \sigma_{xx} + \sigma_c - \sigma_{yy}) \]  

(2.31)

From equations (2.28), (2.29), and (2.31) it is possible to calculate the stress components near the tip.

2.3 Review of the fracturing process in rock

The preceding section described linear elastic fracture mechanics (LEFM), which is the basic framework for lots of studies on fracture mechanics. However, the stress singularity at the fracture tip in LEFM is certainly an unrealistic prediction. In this section we discuss the fracturing process in rock. We start with an overview of various failure mechanism and associated plastic yielding. We continue with a literature review of tensile rock fracturing, and discuss what factors can influence it. Various methods to model deviations from LEFM are discussed. For scaling field-scale hydraulic fractures to laboratory experiments, the dependence of fracture surface energy on fracture length, propagation rate, pore pressure, and grain size is important.

**Failure mechanisms in rock**

In fracture mechanics, three fracturing modes are usually identified (e.g. Ewalds and Wanhill, 1984). First, we have tensile fracturing, usually referred to as mode I fracturing. In this mode, the fracture surfaces open with respect to each other after fracturing has taken place. The energy needed for creation of the surface is provided by tensile strain energy. Second, there is shear fracturing (mode II), in which the fracture energy is provided by shear strain relative to the fracture plane at the tip. After fracturing has taken place, the surfaces move in opposite directions, in the direction of and away from the direction of fracture growth. This movement has no components in the directions perpendicular to the fracture growth direction. Third, there is the tearing mode (mode III), in which a torsional component is applied to the fracture front. Combinations of these fracturing modes can occur.

In rocks, tensile failure is associated with separation of the grains, while shear failure goes together with sliding of the grains with respect to each other. However, sliding of the grains also causes separation of grains. Furthermore, under global confining stresses, tensile stress can exist between the grains caused by the granular structure of the rock (see e.g. Hettema, 1996). This makes it likely that shear failure is also associated with tensile failure on a grain scale. Experiments show that macroscopic shear failure in fact can consist of small tensile cracks that link up (e.g. Gramberg, 1989, van Mier and Vonk,
1991, Horii and Nemat-Nasser, 1985), although this behaviour probably depends on rock type and stress level (Einstein and Dershowitz, 1990). The distinction between tensile failure and shear failure is especially useful when we accept the continuum hypothesis for rock. In discussions about fracture mechanisms at the tip of a hydraulic fracture, we will neglect the possibility that shear fracturing may in fact consist of tensile fracturing on a smaller scale.

We already mentioned the three fracturing modes that are usually distinguished. In addition to this, in practical rock mechanical tests without torsional loading usually four failure mechanisms are distinguished: tensile failure, axial splitting, shear failure, and pore collapse. Tensile failure and shear failure were already discussed. Axial splitting is a failure mode that is observed in uniaxial compressive tests and triaxial compression tests at low confining stress, especially on brittle rocks. In this failure mode the cylindrical rock sample fails parallel to the cylinder axis. The resemblance of this failure mechanism to tensile fracturing was argued in Gramberg (1989), based on the agreement in fracture surface morphology. The resemblance of axial splitting to tensile fracturing can be understood from the fact that the tensile strain energy builds up in the radial direction of an axially compressed cylindrical sample. The main difference between the two modes lies in the externally applied stress, which is tensile in tensile fracturing, but compressive in axial splitting. This difference in stress situation might cause some differences in the failure process.

Pore collapse is the fourth failure mechanism that occurs in practical tests without torsional loading. This failure mode refers to failure of the porous structure of the rock as a result of high mean stress, which results in densification of the rock. Pore collapse is especially of interest for high-porosity rocks. Several micro-mechanisms can occur during pore collapse, one of them being sliding of grains with respect to each other. An example of the change in microstructure of chalk after pore collapse is given in Johnson et al. (1988).

The occurrence of these four failure mechanisms depends on material and stress. The mechanisms may not be clearly distinguishable, when for example the loading is a combination of tensile and compressive stress, or when shear fracturing takes place at high mean stress. When we take a general failure envelope, the four mechanisms discussed occur for different mean pressure according to figure 4.7 in Chapter 4. In general it can be said that for rocks the occurrence of opening of fractures becomes less likely with increasing mean pressure.

Plastic deformation of rock is defined as permanent deformation on a continuum scale, which remains after removal of the applied stress. Similar to failure, several grain-scale micromechanisms can be identified, which cause plastic deformations. As plastic deformation precedes failure, the differentiation of failure mechanisms made above can also be applied to plastic yielding. For a wide variety of rock types under stresses typical
for petroleum reservoirs, plastic deformation that precedes shear failure and goes together with shearing of grains which respect to each other is the mechanism of interest.

This discussion of course neglects many aspects of rock failure, for example intragranular fracturing and plastic deformation within grains and crystals. Still, it gives an overview of the main mechanisms of failure that are of interest in this study.

**Description of tensile fracturing process in rock**

Schlangen and van Mier (1992) refer to Mindess (1991) for the following description of the fracture process zone: the fracture process zone is the zone of discontinuous microcracking ahead of a continuous macrocrack. In Horii and Ichinomiya (1991) the name "process zone" is used for the bridging zone and the microcracking zone in front of the bridging zone. In the bridging zone or cohesive zone, there is a cohesive force between the two crack faces, caused by ligaments of yielded material (also called "bridges"). Schlangen and van Mier (1995) observed during tensile tests on sandstone that a fracture propagates by linking up of overlapping cracks, which form in front of the continuous main crack. The "offsets" between the overlapping fractures form the bridges between the two crack faces. The same mechanism was observed in concrete fracture (van Mier, 1990, and Schlangen and van Mier, 1992). In this work, we will consider the bridging zone or cohesive zone to be part of the main crack. The continuous transition from cohesive zone to microcracking zone will make the choice of the crack tip position somewhat arbitrary for real fractures. Figure 2.3 shows the definition of the tip region that we use in this thesis.

A process zone is observed in hydraulic fracturing laboratory experiments on Colton sandstone (De Pater et al., 1994b). In several other experimental studies, in which the loading mechanism was not hydraulic, a process zone at the tip of a tensile crack in rock is observed as well (e.g. Hoagland et al., 1973, Wang Chengyong et al., 1990, and Nolen-Hoeksema and Gordon, 1987) and concrete (e.g. Krstulovic-Opara, 1993, Schlangen and van Mier, 1992, and Horii and Ichinomiya, 1991). The reported size of the microcracked zone shows much variation. According to Krstulovic-Opara (1993), this is probably caused by differences in the detection technique that is used, and by differences in sample size and geometry. Schlangen and van Mier (1992) explain the large variation in observed process zone sizes by the argument that this size depends on the exact structural conditions at the fracture tip, which encompasses all influences that originate from the testing technique (such as boundary conditions and shape of the sample) and not from material properties.

Hoagland et al. (1973) describe the initiation and propagation of a tensile fracture from a saw-cut notch in three stages. During stage 1, the initial loading stages, few microcracks form and the load-displacement curve is still linear. During stage 2, when microcracking becomes more intense, the load-displacement curve becomes non-linear. The damaged
zone is still able to carry an increasing load and the conditions for crack extension are still unsatisfied. At the onset of stage 3, the main crack starts to advance and the load is about at its maximum. The damaged zone may still grow faster than the main crack during this initial stage of crack growth, as suggested by the growing resistance to crack propagation. After some distance a steady state is achieved during which the damage zone and main crack grow at the same rate, and the resistance to crack growth approximates its asymptotic limit, which is determined by the size of the process zone and the rate of microcrack production within it.

Figure 2.3. Schematic representation of the fracture tip region with definition of process zone, cohesive zone, microcracking zone, and fracture tip, as used in this thesis. In the cohesive zone, a tensile force acts between the fracture surfaces, caused by ligaments of unfractured material. In reality, the transition between the cohesive zone and microcracking zone will in general be gradual.

Models of the microcracking zone

The deviation from LEFM in rocks lies in the occurrence of plastic deformation or the process zone around the tip of the fracture. There are several ways to account for this effect, each based on a different model for the process zone. We first want to make the following quantitative reasoning. The fracturing process can be considered as a balance between the energy that is needed for the creation of new fracture area and the strain energy that is released by the propagation of the fracture, which is the original idea of Griffith (Griffith, 1921). Microcracking and other inelastic processes lead to loss of part of the strain energy, so that it cannot contribute to the strain energy release needed for the propagation of the main fracture. On the other hand, microcracking leads to weakening of
the material. The balance between the screening of the remote loading and the weakening of the material determines the influence of microcracking on the resistance to fracture propagation.

A way to quantify the effect of a process zone is to consider its influence on the stress intensity at the tip of the main crack, which is assumed to be sharp and have stress-free surfaces. There are basically three ways to describe the effect that microcracks have on the stress intensity of the main crack (Wu and Chudnovsky, 1993).

First, effective elastic properties (dependent on the intensity of microcracking) of the material around the tip can be introduced to describe the shielding effect. This approach was followed for example by Ortiz (1988). He described the softening effect of a linear row of microcracks ahead of the fracture tip by a decreased toughness $K_{tc}$ of the material. He found that shielding and toughness degradation seem to balance each other almost exactly, and that these effects are solely dependent on the microcrack density and not on the (undamaged) elastic properties of the solid. One possible reason for the cancelling out of shielding and weakening is that all microcracks lie in the plane of the fracture, so that no energy will be lost. This would imply that microcracks that form outside the plane of the fracture do provide effective shielding.

Second, the crack-microcrack interaction for every microcrack can be described precisely. This approach was followed for instance by Rubinstein (1986). He calculated the effect that one microcrack, aligned with the main crack, has on the stress intensity factor of the main crack. For a certain fixed distance to the main crack tip, an increase in the stress intensity was found for a microcrack positioned at an angle smaller that 62 degrees with respect to the plane of the main fracture. For larger angles, the microcrack reduced the stress intensity of the main crack, so that it had a shielding effect. Furthermore, the maximum increases in stress intensity was caused by a microcrack with a position angle of approximately 20 degrees from the main crack plane. As a result of the influence of the microcrack on the local stress field near the tip, the main crack is directed away from its original direction, causing a wavy crack growth path.

Third, in Wu and Chudnovsky (1993) a statistical approach was used in which the effect of a microcracked zone was formulated in terms of statistical distributions of microcrack sizes, densities, and orientations. It was found that the stress intensity factor of the main fracture could be halved. Weakening of the material as a result of the microcracks was not considered.

The methods discussed are valuable for obtaining insight in the interaction between main crack and process zone. However, they are not easily employed in practical modelling of fractures in rock, primarily because little is known about actual microcrack distributions. We can ask whether plasticity theory can describe microfracturing in the process zone. Although fracturing conflicts with the assumption of a continuous displacement field, required for defining the strain tensor, this might not be a problem as long as the scale of the microfractures is small enough. Qualitatively, plasticity theory will
lead to similar bulk rock behaviour as microfracturing during tensile loading: larger deformations and possible dilatancy. In both cases, strain energy is lost into respectively irreversible deformations and fracture surface energy of the microcracks. However, during reloading differences may occur, as microcracking will not necessarily lead to irreversible strains. Also, the localisation of the deformations may be different. A constitutive model based on elastic-plastic theory is used to describe the behaviour of jointed rock in e.g. Cai and Horii (1992).

Microcracking caused by tensile failure is often the dominant failure mode in the process zone under unconfined conditions, because rocks have a low tensile strength compared to their compressive strength. However, under the high confining stresses in petroleum reservoirs the rock in the process zone will in general behave less brittle than in tensile tests, so plasticity theory may be more appropriate then. Furthermore, the brittleness will vary between the various rock types, so that the applicability of plasticity theory may also depend on rock type.

**Cohesive-zone model**

In the preceding section, models of the process zone were discussed. Here, we discuss yet another way to model the process zone, which are the "strip-yield" models. In these models, the main fracture is extended in the plastic or process zone while cohesive forces work between the fracture faces in the process zone. While in the previous section the microcracks were assumed to surround a sharp fracture with stress-free surfaces, here the fracturing process is assumed to be localised in a thin strip in the plane of the fracture, where possibly existing microcracks link up. All other possible influences between main fracture and microcracks are neglected.

The cohesive stress can be uniform over the crack length in the plastic zone and equal to the yield stress (Dugdale model, Dugdale, 1960), or can be allowed to vary with position and opening displacement (Barenblatt, 1962). In these models the cohesive stress distribution is such that the stress singularity at the crack tip is removed. The maximum value of the cohesive stress is the yield stress or the tensile strength of the material.

A strip-yield model proposed for rocks is the cohesive-zone model (Hillerborg et al., 1976). The fracturing process is assumed to localise in a thin zone in the plane of the fracture. An explanation for the cohesive stress working between the surfaces is the formation of bridges, that form as a result of overlapping cracks in the process zone (Schlangen and van Mier, 1992 and 1995). The ability of the material to bear the tensile loading will depend on the distance between the surfaces of the opening main fracture. The cohesive-zone model assumes a unique relationship between the cohesive stress working between the fracture faces and fracture opening. This relationship can be measured in uniaxial tension tests (see figure 6.1). Figure 2.4 shows a comparison between the stress
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distribution near the tip in linear elastic fracture mechanics, and in the cohesive-zone model.

The fracture surface energy $\Gamma$ is related to the work per unit area done in propagating the fracture with a cohesive zone by (Rice, 1968):

$$2\Gamma = \int_{0}^{b^c} \sigma(b) db$$  \hspace{1cm} (2.32)

The cohesive-zone model is used in modelling fracture propagation (e.g. Papanastasiou and Thiercelin, 1993), and we will use it in our numerical simulations as well (see section 6.1). We will combine it with plasticity theory in order to model plastic shear strain caused by the in-situ compressive stresses.

**Figure 2.4. Stress distribution in linear elastic fracture mechanics and cohesive-zone model. The contribution of the tip-stresses to the total stress field is sketched.**

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Influence of confining stress on tensile fracturing

First, we will discuss the influence of confining stress on the cohesive-zone model. The influence of the confining stress on the tensile fracture mechanism can be investigated in triaxial extension tests on notched specimens, in which the axial stress is unloaded and subsequently becomes tensile (no loading is present inside the notch, which opens in the axial direction). These tests were performed by Visser (1998) on Felser sandstone, and showed a decrease of the maximum axial tensile stress with increasing radial confining pressure. For high enough radial pressures, the material already failed before the axial stress became tensile. Note that this decrease in minimum principal stress can easily be inferred from the general shape of the failure envelope.

The above observations suggest that the magnitude of the maximum stress transmitted in the cohesive zone is influenced by the radial confining stress, and that failure can be reached at the closed tip of the fracture when the least principal stress is still compressive. This can be explained by the contribution of the radial confining stress to the tensile strain field. Mode I tensile failure will occur when the tensile strain energy stored around the tip reaches a critical value. The radial compressive stress field parallel to the fracture plane contributes to the tensile strain field and releases energy during fracturing, as a result of which axial splitting occurs. This also implies that shear failure will occur when the radial stress is high enough.

When shear fracturing near the tip takes place, fracture opening is zero at positions where the rock is fractured. When we define the fracture tip as the point where the fracture starts opening, the fracture will propagate through pre-fractured rock. This would lead to the conclusion that modelling of hydraulic fracturing under a high enough deviatoric stress is best done assuming zero toughness or a reduced tensile strength in the cohesive-zone model. Then, the compressive energy as a result of the lateral stresses provides energy for fracturing.

Second, we consider the influence of confining stress on the measured fracture surface energy. The fracture surface energy determined in experimental studies usually consists of energy needed to separate the grains that form the fracture surface, and inelastic processes that take place at some distance from the fracture plane. In the present study, we make a distinction between these two energies, as the latter also includes plastic deformations around the tip. We will indicate the energy that is purely needed for creating a fracture surface by the fracture surface energy $\Gamma$, equivalent to its definition in LEFM. The energy that is needed to create both fracture surfaces and includes inelastic processes that do not contribute directly to creating the fracture surface, will be indicated by the separation energy $\Gamma_S$. When $\Gamma$ is the energy per unit of fracture area dissipated in inelastic processes that do not directly contribute to the creation of new fracture area, we have:

$$\Gamma_S = 2(\Gamma + \Gamma_f)$$  \hspace{1cm} (2.33)
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It is the separation energy that is usually measured in experimental studies. When alternatively the fracture toughness is measured, this usually should be regarded as an "effective fracture toughness", which includes inelastic processes at some distance from the fracture. When inelastic processes take place, residual stresses may remain, with associated elastic deformations. Although these elastic deformations are in principle recoverable, they should be added to the right-hand side of equation (2.33), as long as this elastic energy has not been released after fracturing of the sample. As a result, the energy dissipated in inelastic processes underestimates the energy consumed by inelastic processes. However, we can disregard the elastically stored energy as a result of residual stress when we speak about inelastic energy dissipation in a qualitative sense.

In experimental studies a significant effect of hydrostatic pressure on fracture toughness is measured (Biret et al., 1989, Schmidt and Huddle, 1977, Abou-Sayed, 1978, Müller, 1986). They studied the influence of hydrostatic stress by performing experiments in a pressure vessel, while the pressurising fluid was not allowed to enter the crack. The toughness was observed to increase several times, and increased approximately linearly with hydrostatic pressure. From these results, we make two main observations. First, linear elastic fracture mechanics cannot explain these observations, which is an indication for the fracture process to deviate from LEFM. Second, the observed change in toughness is moderate. For a hydrostatic stress of 50 MPa, an increase in the toughness of 2 to 3 three times is measured in most of these studies.

In Abou-Sayed (1978), the externally applied confining stress had a deviatoric part, instead of being hydrostatic. The simultaneous increase of the stresses working in the plane of the fracture in the direction of fracture growth, and perpendicular to the fracture plane, was responsible for the toughness increase. However, in these tests the samples were hydraulically fractured, which complicates the test interpretation. The toughness was inferred from the fracture length and the maximum fluid pressure in the wellbore (breakdown point). From the hydraulic fracturing experiments on plaster and cement paste in the present study, no significant effect of stress on initiation pressure could be determined (see Appendix A, table A.2).

We are left with the uncertainty whether the fracture surface energy depends on confining stress in the same way as the separation energy. It is not possible to answer this question from the studies referred to. However, the results of Medlin and Massé (1986) may give information about this question. In their study, the separation energy is thought to consist of the fracture surface energy and a term that accounts for plastic deformation, equivalent to equation (2.33). They analysed hydraulic fracturing model experiments on Carthage limestone, in which propagation was dominated by fracture toughness (according to the criterion given in section 2.7). The separation energy was calculated from the work done by the net pressure in opening the fracture, using a balance between separation energy and stored elastic energy per fracture area (Biot et al., 1986). In these experiments, the largest externally applied stress on the sample worked in the main propagation
direction of the fracture, while the stresses in the other directions were approximately equal. The largest externally applied stress was typically 1.5 times the smallest externally applied stress.

The results of the experiments show an increase of the total work per fracture area, done by the net pressure in opening the fracture, with increasing fracture length. This was attributed to a plastic energy contribution to the separation energy being linearly proportional to the fracture length. However, extrapolating this value to field scale yielded values of the effective fracture toughness that were too high to match observed net pressures in reservoirs of the Carthage limestone type (Medlin and Massé, 1986). This shows the incorrectness of the concept of the plastic contribution to the separation energy being simply proportional to the fracture length. The observed increase in the experiments may correspond with the commonly observed increase of effective toughness with fracture length for small fractures (see below in this section).

Adopting the hypothesis that the increase of separation energy with fracture length for small fractures is due to the developing process zone, the fracture surface energy can be inferred from the separation energy by extrapolating the results to zero fracture length. This was done by Medlin and Massé (1986) for several values of the confining stress. It yielded that the fracture surface energy - corresponding with the separation energy for infinitely small fracture length - was independent of the confining stress. The increase of separation energy with fracture length was stronger for higher confining stress, corresponding with the increase in separation energy with confining stress that is often found in laboratory experiments. This shows that an increase in separation energy with confining stress does not necessarily mean that the fracture surface energy increases too.

**Influence of scale**

For small fractures in laboratory experiments, an increase of separation energy with fracture length is generally found. When the fracture reaches a certain length, the fracture toughness becomes approximately constant (e.g. Hoagland *et al.*, 1973). The fracture length at which the separation energy becomes approximately constant, can be estimated by \(2.5 \left( \frac{K_{lc}}{T_0} \right)^2\). The numerical factor 2.5 originates from standard testing of metals, and may be somewhat conservative for some rock types (Whittaker *et al.*, 1992). This behaviour can be explained by assuming that for fractures smaller than this length scale, the process zone is unable to develop completely before the crack propagates.

In Perkins and Bartlett (1963), the measured separation energy was constant for crack lengths larger than the mentioned length scale and up to 45 cm, for various rock types. Van Vliet and van Mier (1998) performed a series of tensile tests on concrete samples with a length ranging from 3 cm to 96 cm, and with a constant thickness of 10 cm. For this
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A large range of sample sizes, the separation energy was fairly constant. Only at the smallest sample size, the separation energy appeared to be somewhat lower. A similar series of experiments on Felser sandstone showed a moderate increase of measured separation energy with sample size, which approached a constant value (van Vliet, 2000). From these laboratory experiments, there is no evidence of a fundamental increase of fracture surface energy with fracture size up to 1 metre. Assuming that the fracture surface energy depends in the same way on scale as the measured separation energy, the extrapolation of these results to hydraulic fracture field size (10^1 - 10^2 m) would imply an unaltered fracture surface energy.

Van Vliet and van Mier (1998), and van Vliet (2000), measured a decrease in tensile strength with increasing fracture size. This implies that the parameters in the cohesive-zone model also depend on the fracture size. However, the effect of size on tensile strength is moderate, and predicts a decrease in tensile strength of approximately 2 times for a field scale compared with the lab.

**Influence of pore pressure**

Laboratory experiments on a wide variety of rocks show that shear failure is described by effective stress (defined according to Terzaghi's principle) (see e.g. Paterson, 1978). Bruno and Nakagawa (1991) found experimental indications for tensile failure to be determined by the effective stress too. Their experiments showed that an effective tensile stress caused by pore pressure increase initiated failure at effective stresses that agreed with uniaxial tensile tests. In addition, they found from experiments on Colton sandstone that a local increase in pore pressure around the fracture tip facilitates fracture extension. From an energy balance, it was inferred that the strain energy release by the pore fluid contributes to the "crack driving force" (compare Griffith's energy balance approach). In a comment Detournay and Boone (1993) reply that the pore pressure field is not singular and that a singularity is needed to obtain a finite contribution to the strain energy release rate. In their reply Bruno and Nakagawa (1994) argue that the singularity is a mathematical nonphysical concept and that they therefore prefer the energy balance approach.

Experiments in which the effect of pore pressure on toughness is measured also indicate that tensile fracturing is governed by effective stress. From experiments on Indiana limestone performed by Abou-Sayed (1978), it followed that the measured fracture toughness stayed equal if the pore pressure was increased with the same amount as the confining stress. Müller (1986) also found that for Ruhr sandstone the measured separation energy stayed unaltered when the pore pressure was increased with the same value as the hydrostatic confining stress. These experiments were done at zero effective stress.

The previously mentioned experiments suggest that the separation energy is independent of the pore pressure. In hydraulic fracturing, the situation is more complicated because of the possibility of flow of pore fluid near the tip. For certain conditions, a fluid
lag develops at the fracture tip (see section 2.6). The presence of the fluid lag and the lower pore pressure near the tip as a result of unloading of the stress, will cause flow of pore fluid respectively out of and towards the rock near the tip. When the pore pressure is maintained by flow towards the near-tip region, the effective stress will become negative and tensile fracturing can be enhanced. The reverse can also occur, when fluid only flows towards the fluid lag. Whether this occurs depends on the local permeabilities, and the time scale needed to increase the pore pressure in the unloaded zone, in comparison with the time needed for the fracture to propagate over the unloaded zone. The problem of flow of pore fluid towards the fluid lag was addressed in Cleary (1979, 1990). Note that in our experiments, we do not apply pore pressure. Any possible effect of flow of pore fluid near the tip on the effective stress situation and fracturing mechanism will be absent in the experimental results.

**Influence of propagation velocity**

Besides the influence of fracture length, pore pressure, and confining stress, the dependence of fracture surface energy on propagation rate and grain size are topics of importance. The correctness of the scaling of our hydraulic fracturing experiments depends on the correctness of the assumption that the fracture surface energy is approximately independent of these factors. An assumed scale dependence is used as a physical justification for the use of a high toughness in hydraulic fracturing field treatments (Shlyapobersky, 1985).

Müller (1986) measured an increase of fracture toughness with increasing crack mouth opening displacement rate in three-point bending tests on various sandstones. His measurements would suggest an increase in toughness smaller than approximately two times for rates in the laboratory compared with rates in the field. Whittaker et al. (1992) mentions laboratory investigations which showed that the loading rate in laboratory experiment did not significantly influence the measured toughness. For high loading rates in general lower values of the toughness are measured, which is explained by the process zone having insufficient time to develop completely. In Zhang et al. (1999), an approximately constant fracture toughness was measured for the loading rates of interest for hydraulic fracturing.

**Influence of grain size**

Wang Chengyong et al. (1990) found that an increasing grain size increases the size of the fracture process zone. This is also found in other experimental studies (Schlangen and van Mier, 1992, see also Whittaker et al., 1992 for an overview). One explanation for this is that when the grain size is small, more weak spots are present close to the plane of the fracture, so that microcracks can grow easily close to the plane of the fracture.
increase of the process zone size does not mean an increase in separation energy. According to a literature review in Whittaker et al. (1992), the separation energy in general decreases with grain size for grain sizes larger than $10^{-4}$ m. This is, however, a relatively small effect.

Under confined conditions the tensile zone size is smaller than under unconfined conditions. Furthermore, in laboratory experiments the fracture width and process zone length are smaller than in the field. Due to these reasons, differences in the ratio of grain size to one of these length scales may occur. To prevent this, scaling of hydraulic fracturing experiments would require scaling down of the grain size with respect to the crack width and tensile zone size. All rocks in our experiments had a relatively small grainsize ($\sim 10^{-3}$ m, see figure 4.1).

2.4 Development of plastic zone near the tip

The high tensile stress of the linear elastic stress field cannot be sustained at the fracture tip. In the preceding section it was argued that the material will release these stresses by redistribution of the excess stress in a zone of material failure. Under unconfined conditions, tensile yielding will be dominant, because the tensile strength is much smaller than the shear strength for most rocks. The tensile stress is redistributed in a cohesive zone or process zone. When confining stresses are present around the rock sample, a zone of shear fracturing can also develop. In that case, the tensile unloading causes a large deviatoric stress and subsequent shear yielding. Tensile failure and shear failure will interact.

In this section we will consider the influence of the externally applied stresses on the zone of material failure around the fracture tip. In doing this, we will start from a fracture according to linear elastic fracture mechanics. Besides this, we calculate the stress path followed by a material element near the tip. These observations give some insight in the role of the in-situ stresses on the fracturing mechanism. Analysis of the stress path is also of interest for the design of triaxial tests, done for the determination of material behaviour.

Calculation of tensile and plastic zone size

The plastic zone around the tip is defined as the zone where plastic deformation is taking place, as a result of shear yielding. In addition, a tensile zone is defined in which tensile failure is taking place. We made a first estimate of the size of the plastic and tensile zone. This was done by applying a failure criterion to the plane strain linear elastic stress field around the tip (equation (2.20), with in-situ stresses added according to equations (2.26)
and (2.27)), and calculating the position of failure. The real plastic zone or tensile zone will be different because of stress redistribution and mutual influence of shear and tensile failure. Furthermore, the determination of the real plastic zone size boundary is somewhat arbitrary, as small plasticity will already develop at a low stress difference.

We used a Mohr-Coulomb failure criterion to calculate a measure of the plastic zone size. This reads for compressive stress positive:

\[ \tau = s_0 + \mu f \sigma_n \] (2.34)

\( \tau \) is the shear stress across the sliding plane, and \( \sigma_n \) is the normal stress. The tensile zone size was calculated by applying a maximum tensile stress criterion to the stress field. As an example of the influence of stress on the plastic and tensile zone, figure 2.5 shows the shape of the plastic and tensile zone around the fracture tip for three stress conditions. We took as base case the stress perpendicular to the fracture plane \( \sigma_c=8 \) MPa, and the stress parallel to the fracture plane \( \sigma_0=20 \) MPa. We made two variations by taking \( \sigma_c=26 \) MPa, and \( \sigma_c=6 \) MPa, keeping the other stress constant. Besides the externally applied stresses, the size of the plastic zone depends on the critical stress intensity or fracture toughness, the cohesion, and the friction angle. The tensile zone size depends on the fracture toughness and tensile strength. For these parameters, we used the following values: tensile strength \( T_0=3.5 \) MPa, cohesion \( S_0=6.2 \) MPa, friction coefficient \( \mu_f=0.47 \), and fracture toughness 0.3 MPa\( \sqrt{m} \). These values correspond with material properties of strong plaster (see Chapter 5, table 5.2).

Figure 2.5 shows that the plastic zone is influenced strongly by the stress parallel to the fracture plane, unlike the tensile zone. Both are influenced by the stress perpendicular to the fracture plane. Furthermore, both the tensile zone and the plastic zone have a maximum x-value out of the plane of the fracture, which may have consequences for the fracturing mechanism.

**Stress path followed by a material element near the tip**

Figure 2.6 shows the elastic stress path for an infinitesimally small material element near a plane strain fracture in two projections of the principal stress space. The path in space is a line parallel to the fracture plane, at a height \( y \) above the fracture plane (see figure 2.7). Displacement of the material element as a result of fracture opening is ignored. The curves stop once the tensile strength is reached, or when the x-coordinate is smaller than zero. As externally applied stresses, we took \( \sigma_h=12 \) MPa and \( \sigma_c=8 \) MPa. We took Poisson's ratio \( \nu=0.20 \), while the other material properties of interest were the same as for figure 2.5. We studied the case in which the stress path is given by LEFM (equation (2.20), with in-situ stresses added according to equations (2.26) and (2.27)).
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\[ \sigma_c = 8 \text{ MPa and } \sigma_n = 20 \text{ MPa:} \]

\[ \sigma_c = 8 \text{ MPa and } \sigma_n = 26 \text{ MPa:} \]

\[ \sigma_c = 6 \text{ MPa and } \sigma_n = 20 \text{ MPa:} \]

Figure 2.5. Approximated plastic and tensile zone for strong plaster, with various values of the applied stresses of \( \sigma_c \) and \( \sigma_n \). The crack tip is at (0,0).
Figure 2.6 shows that the deviatoric stress in the x-y plane is constant in the plane of the fracture. The stress in the z-direction is the largest principal stress. For positions outside the plane of the fracture, the stress path is approximately equal to the stress path in triaxial extension tests.

**Implications for fracture mechanism**

When we assume that material failure is determined by the stress situation, we can determine the failure mode by looking at the stress path near the tip. The failure mode is then given by the intersection of the failure envelope by the stress path experienced by a material element near the tip (see also figure 4.7 in Chapter 4). Figure 2.8 shows an example of this. In the following we will discuss the consequences of this for the fracture mechanism.

We first consider the situation in the plane of the fracture. Shear failure as a result of the deviator of the stresses in the x-y plane ($\sigma_2$ and $\sigma_3$) will in most cases be preceded by tensile failure (except in figure 2.5). But when we take the stress in the z-direction into account, a shear failure criterion can be satisfied more easily before the tensile failure criterion is satisfied. Limited plastic strain in the z-direction can develop because of elastic unloading, keeping the total strain zero. It is however doubtful whether this deformation is enough to create failure. The limited plastic deformation in the z-direction may damage the material and thus influence the fracturing process.

Material failure obviously is not limited to the plane of the fracture. The largest x-value for which the shear failure criterion is satisfied lies outside the plane of the fracture. The same thing can be expected from the fact that the tensile failure criterion is also first satisfied outside the plane of the fracture. This implies that the least principal stress as a function of the angle with the x-axis for constant distance to the fracture tip, has a minimum outside the plane of the fracture. This can be readily checked by the principal stresses in the near-tip stress field, as for example given in Ewalds and Wanhill (1984). For comparison, in dynamic fracture propagation, the occurrence of a minimum outside the plane of the fracture in the angular stress component is used as an explanation for crack tip branching (e.g. Lawn, 1993). Although the tensile strain energy density has a maximum in the plane of the fracture, the minimum of the least principal stress outside the plane of the fracture can promote crack tip deflection, for example by interaction with microcracks.

Regarding the situation of a tensile zone in combination with a shear zone, it is likely that the tensile fracture localises in the plastic zone, where the material is weakened. Only in cases where the tensile zone is much larger than the plastic zone, pure tensile fracturing is expected to happen. When the size of plastic and tensile zone is comparable, we expect mixed mode fracturing. When the plastic zone is much larger than the tensile zone, the material may be weakened so much that pure shear failure takes place at the tip.
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![Graphs showing stress paths in principal stress space and in the π-plane. σ₁ is the stress perpendicular to the plane of the strains. The curves stop when the tensile strength is reached, or the x-coordinate becomes negative.](image)

Figure 2.6 Stress path in principal stress space and in the π-plane. σ₁ is the stress perpendicular to the plane of the strains. The curves stop when the tensile strength is reached, or the x-coordinate becomes negative.
Figure 2.7. Paths in space along which the stresses experienced by a material element are calculated according to the linear elastic stress field, at various heights \( y \) from the fracture plane.

Figure 2.8. Stress path for strong plaster with failure envelope for strong plaster according to the Mohr-Coulomb criterion applied to the stresses in the \( x-y \) plane. The cohesion and friction angle are from table 5.2. The stress in the \( z \)-direction is calculated from the plane strain condition and initial stresses \( \sigma_3 = 6 \text{ MPa} \) and \( \sigma_2 = \sigma_1 = 20 \text{ MPa} \).
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2.5 Fluid flow and leak-off

In this section we discuss the modelling of the fluid flow processes. We mention or analyse the assumptions that are usually made.

Flow regime

The Reynolds number of the flow in the bulk of the fracture is given by \( N_{Re} = \frac{V_T W \rho}{\mu} \), in which \( W \) is a characteristic fracture width, \( \rho \) is the fluid density, \( \mu \) is the dynamic viscosity, and \( V_T \) is a characteristic radial fluid flow velocity. The Reynolds number is used for dynamic scaling of the flow field in the fracture between field and experiments. Our experiments do not scale the Reynolds number. In the experiments the Reynolds number is typically \( 10^{-7} \), while in practice it can be as large as \( 10^3 \), which implies the possibility of turbulent flow and entrance effects. Furthermore, fluid leak-off and fracture surface roughness may influence the flow regime. For flow between parallel plates, deviations from Poiseuille flow due to turbulence occurred at a Reynolds number of about 1500 in a study of Beavers and Joseph (1967). The amplitude of the roughness of the plates was about six times smaller than the channel width, which was 1.6 mm.

Going towards the tip, the product of width and fluid velocity becomes smaller because of leak-off, increasing radius, and increasing fracture opening rate, so that the Reynolds number is significantly smaller for the tip region. It is important to realise this, because the tip region usually has the largest influence on the wellbore pressure. Because of this, deviations from laminar flow in the bulk of the fracture usually have limited influence on the wellbore pressure and fracture geometry. Even in this case, modelling of the flow in the bulk of the fracture using laminar solutions appears to be appropriate, as long as the flow in the tip-region is laminar.

Approximations for flow in the fracture

The width of a hydraulic fracture varies in time and along the radial coordinate. Furthermore, fluid flows through the permeable fracture walls. In the following, we will analyse the conditions under which we can model the flow in the fracture by locally using the solution for Poiseuille flow between parallel plates with constant width. The effect of temporal width variations and leak-off on the local flow rate is included in the continuity equation (see next part of this section). Apart from this, the influence of leak-off and spatial and temporal width variations must be negligible when locally using the Poiseuille solution. Furthermore, we assume here that the fluid is incompressible.
Based on symmetry considerations, the angular velocity components and all angular derivatives are zero for a radial fracture geometry. Then, the Navier-Stokes equations (see e.g. Batchelor, 1990) in cylindrical coordinates reduce to:

\[
\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_y \frac{\partial v_r}{\partial y} \right) = \mu \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) + \frac{\partial}{\partial y} \frac{\partial v_r}{\partial y} \right) - \frac{\partial p}{\partial r} + \rho g_r \quad (2.35)
\]

\[
\rho \left( \frac{\partial v_y}{\partial t} + v_r \frac{\partial v_y}{\partial r} + v_y \frac{\partial v_y}{\partial y} \right) = \mu \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_y}{\partial r} \right) + \frac{\partial}{\partial y} \frac{\partial v_y}{\partial y} \right) - \frac{\partial p}{\partial y} + \rho g_y \quad (2.36)
\]

where \( t \) is the time, \( v \) is the fluid velocity, and \( g \) is the gravity acceleration. The directions of the \( r \)- and \( y \)-components of these vectors are given in figure 2.2. For the assumption of Poiseuille flow between parallel plates to be correct, the left hand side of the Navier-Stokes equations must be negligible. For a radial hydraulic fracture, this is most critical near the fracture tip and near the wellbore. Near the tip, the time derivatives and the gradient in the width profile are large, while near the wellbore the radius of curvature and flow rate are relatively large. A further assumption in the Poiseuille solution is that the pressure gradient and flow in the fracture are one-dimensional. Leak-off and temporal width variations can possibly invalidate this assumption.

We will first analyse the situation near the tip. We make the Navier-Stokes equations (2.35) and (2.36) dimensionless using a characteristic length scale \( W \) in the \( y \)-direction, a characteristic length scale \( R_T \) in the radial direction (representing the length of the tip region), a characteristic velocity \( V_T \) in the \( r \)-direction (representing the fracture propagation velocity and the radial fluid flow velocity), a characteristic velocity \( V_y \) in the \( y \)-direction, characteristic time scales \( R_T/V_T \) and \( W/V_T \) for flow in the \( r \)- and \( y \)-direction respectively, and characteristic pressure differences \( P_R \) and \( P_y \) in the \( r \)- and \( y \)-direction. A characteristic length scale for the total fracture radius disappears from the equations. Furthermore, the gravity terms are negligible in our problem. We then find:

\[
\frac{V_T^2}{R_T^2} \frac{\partial v_r^*}{\partial t^*} + \frac{V_T^2}{R_T^2} \frac{v_r^*}{r} \frac{\partial v_r^*}{\partial r^*} + \frac{V_T V_y}{W} \frac{\partial v_r^*}{\partial y^*} = \mu \left( \frac{V_T}{R_T} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r^*}{\partial r^*} \right) + \frac{V_T}{W^2} \frac{\partial}{\partial y} \frac{\partial v_r^*}{\partial y^*} \right) - \frac{P_R}{R_T} \frac{\partial p^*}{\partial r^*} \quad (2.37)
\]
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\[ \rho \left( \frac{V_y^2}{W} \frac{\partial v_y^*}{\partial t^*} + \frac{V_T}{R_T} v_r^* \frac{\partial v_r^*}{\partial r^*} + \frac{V_T^2}{W} v_y^* \frac{\partial v_y^*}{\partial y^*} \right) = \mu \left( \frac{1}{R_T^2} r^* \frac{\partial}{\partial r^*} \left( r^* \frac{\partial v_y^*}{\partial r^*} \right) + \frac{V_T}{W^2} \frac{\partial}{\partial y^*} \left( \frac{\partial v_y^*}{\partial y^*} \right) \right) - \frac{P_T}{V_T} \frac{\partial p^*}{\partial y^*} \]  

(2.38)

Using \( W \ll R_T \) for a fracture, so that terms of order \[ \left( \frac{W}{R_T} \right)^2 \] can be neglected with respect to terms of order 1, these equations reduce to:

\[ \rho \left( \frac{\partial v_r^*}{\partial t^*} + v_y^* \frac{\partial}{\partial r^*} \frac{V_T}{W} v_y^* \frac{\partial v_r^*}{\partial y^*} \right) = \mu \left( \frac{R_T}{V_T W^2} \frac{\partial}{\partial y^*} \left( \frac{\partial v_y^*}{\partial y^*} \right) \right) - \frac{P_T}{V_T^2} \frac{\partial p^*}{\partial r^*} \]  

(2.39)

\[ \rho \left( \frac{\partial v_y^*}{\partial t^*} + \frac{W V_T}{R_T V_T} v_y^* \frac{\partial}{\partial r^*} \frac{V_T}{W} v_y^* \frac{\partial v_y^*}{\partial y^*} \right) = \mu \left( \frac{1}{W V_T} \frac{\partial}{\partial y^*} \left( \frac{\partial v_y^*}{\partial y^*} \right) \right) - \frac{P_T}{V_T^2} \frac{\partial p^*}{\partial y^*} \]  

(2.40)

We consider two possibilities to express the characteristic fluid velocity in the \( y \)-direction \( V_y \). For impermeable fracture walls, \( V_y = W/(R_T/V_T) \). When leak-off takes place, the velocity in the \( y \)-direction scales with \( C_l \sqrt{V_T/R_T} \) (see equation (2.53) for the definition of the leak-off coefficient \( C_l \)). In the last case we assumed that the leak-off velocity is much larger than the velocity of the fracture surface in the \( y \)-direction, so that

\[ C_l \gg \frac{W V_T}{R_T}. \]

The two assumptions in the Poiseuille solution are that the left-hand side of equation (2.39) is negligible, and that the flow is one-dimensional. For impermeable fracture walls, we see from equation (2.39) that the first assumption is fulfilled when \[ \frac{\rho}{\mu} \frac{W^2 V_T}{R_T} \ll 1, \] in which case the left-hand side of equation (2.40) is also negligible with respect to the other terms in that equation. We will assume that the flow in the fracture is one-dimensional if \( V_y \ll V_T \). This leads to the requirement that \[ \frac{W}{R_T} \ll 1, \] which is usually the case. Equations (2.39) and (2.40) show that this is equivalent to requiring that the pressure gradient is much smaller in the \( y \)-direction than in the \( r \)-direction.

For permeable fracture walls, the assumption that the left-hand side of equation (2.39) is small in comparison with the terms on the right-hand side introduces the requirement
that \( \frac{\rho W C_l \sqrt{V_T}}{\mu \sqrt{R_T}} \ll 1 \), in which case the same holds for equation (2.40). Furthermore, the fluid velocity in the y-direction is much smaller than the velocity in the r-direction when \( \frac{C_l}{\sqrt{V_T R_T}} \ll 1 \).

To analyse these three requirements, we need a value for \( R_T \). As a minimum for \( R_T \), we can use the size of the fluid lag. For all materials in our experiments, these requirements are fulfilled. For a typical field case (using \( R_T = 0.5 \) m, \( V_T = 10^{-2} \) m/s, \( W = 0.01 \) m, \( C_l = 10^{-3} \) m/\( \sqrt{\text{s}} \), and \( \mu = 0.1 \) Pa-s), these requirements are fulfilled too. However, in field practice in particular inertia effects can become important for the flow in the fracture near the tip, when the parameters of interest have values different from this example.

A similar analysis can be made for the situation near the wellbore. We use the wellbore radius \( R_w \) as the characteristic length scale in the radial direction, and \( \frac{Q}{WR_w} \) as characteristic fluid flow velocity in the radial direction (\( Q \) is the flow rate flowing from the wellbore into the fracture). We neglect the y-components of the velocity because the radial flow velocity is much larger than the fracture propagation velocity. We then find the requirement that \( \frac{\rho W Q}{\mu R_w^2} \ll 1 \) for the Poiseuille solution to be used locally. Again, in our experiments this is the case, so that the local use of the Poiseuille solution for flow between parallel plates is correct for modelling the experiments. However, for a typical field case this requirement is not fulfilled (using \( Q = 0.1 \) m\(^3\)/s and \( R_w = 0.1 \) m). This implies that for field hydraulic fractures, fluid inertia is important for the flow near the wellbore. However, this conclusion is restricted to the radial fracture geometry transversely to the wellbore. For a plane strain geometry for example, the requirements for the bulk of the fracture apply to the wellbore region too. Besides the possibility that the influence of fluid inertia on the wellbore pressure may be small, this makes that the experiments are still representative for situations occurring in field practice.

Barr (1991) analysed the pressure distribution inside the fracture tip region using a two-dimensional wedge-shaped fracture geometry. He found that the deviation from the local Poiseuille flow assumption is negligible for typical conditions, and for impermeable fracture walls.

Integration of equation (2.39) over the y-coordinate while neglecting the left-hand side gives the equation that relates the local pressure gradient to the local flow rate per perimeter \( q_f \) for Poiseuille flow between parallel plates:

\[
\frac{\partial P}{\partial r} = -\frac{12\mu}{b^3(r)} q_f'(r) \quad (2.41)
\]
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Taking into account the previous considerations, this equation will give a satisfactory approximation of the pressure gradient in the fracture in many cases. Furthermore, the effect of surface tension at the fluid front is negligible in our experiments, as well as in most practical cases.

**Continuity equation**

Conservation of mass for an incompressible fluid can be stated as a local condition:

\[ \nabla \cdot \mathbf{v} = 0 \]  

(2.42)

For an axisymmetric geometry, all angular components are zero. Equation (2.42) can be integrated over the fracture width:

\[
\frac{1}{b} \int_{-b/2}^{b/2} \left( \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_y}{\partial y} \right) dy = 0
\]  

(2.43)

Bui and Parnes (1982) show that after integration, this equation can be written as:

\[
\frac{\partial b}{\partial t} + 2v_t + \frac{1}{r} \frac{\partial (r q'_f)}{\partial r} - \frac{\partial b}{\partial r} v_{r,s} = 0
\]  

(2.44)

in which we included the term \( v_t \), which is the leak-off velocity through one fracture surface; \( v_{r,s} \) is the fluid velocity at the fracture surface in the radial direction. Scaling this equation, using the same characteristic values as in the preceding section, yields:

\[
\frac{\partial b^*}{\partial t^*} + \frac{C_l}{W} \frac{R_T}{V_T} 2v_{t^*} + \frac{1}{r^*} \frac{\partial (r^* q^*_{f')}}{\partial r^*} - \frac{V_S}{V_T} \frac{\partial b^*}{\partial r^*} = 0
\]  

(2.45)

where \( V_S \) is a characteristic velocity for the radial component of the flow at the fracture surface. Neglecting \( V_S \) with respect to \( V_T \) is correct for impermeable rock, and permeable rock where the fracture permeability at the position of the fluid front is much larger than the rock permeability (see below in this section). Equation (2.45) also shows the relative magnitude of the other terms. For \( V_S \ll V_T \), we have for the continuity equation:

\[
\frac{\partial b}{\partial t} + 2v_t + \frac{1}{r} \frac{\partial (r q'_f)}{\partial r} = 0
\]  

(2.46)
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Wall roughness
Rough fracture walls influence the pressure gradient inside the fracture. This effect can become significant for narrow fractures. Zimmerman et al. (1991) analysed the influence of a sinusoidal surface of a two-dimensional channel with parallel walls on lubrication flow through it. From this analysis, and from the presented simulation results from others, it appears that the problem can be described by two parameters: the hydraulic (or effective) aperture divided by the mean aperture, and the mean aperture divided by the standard deviation of the roughness. For zero standard deviation, the hydraulic aperture and the mean aperture are equal. For a standard deviation being one half of the mean aperture, the hydraulic aperture can be 0.7 times the mean aperture. For Reynolds number significantly larger than one, fluid inertia as a result of relatively large roughness may become important too (comparable to Forchheimer's correction to Darcy's law). However, for a Reynolds number of 20 and the roughness amplitude being equal to the distance between the plates, Poiseuille flow still describes the flow reasonably well (Beavers and Joseph, 1967). Their results also show that, in case the roughness is small in comparison with the fracture width, Poiseuille flow describes the flow in the fracture for high Reynolds number until turbulence sets in.

A small increase in hydraulic fracture width is sufficient to cancel out an increase in pressure, because of the non-linearity of the width in equation (2.41). This can occur, because the fracture opens as a result of the pressure inside the fracture. The width increase will also diminish the influence of the roughness. Also, the 3-dimensionality of the real problem reduces the effect of roughness. Furthermore, the surface roughness profiles of the hydraulic fractures in our experiments consist of grooves in the direction of fracture growth, which also diminishes the effect of roughness. Altogether, we do not expect a significant influence of the roughness on our experimental results.

Parallel flow
In flow through a channel with parallel walls made of a permeable porous medium, the no-slip condition at the permeable walls is not met. This is caused by the fluid flow in the permeable porous medium parallel to the channel, which affects the flow in the channel. We refer to the flow in the porous medium as "parallel flow". This effect is experimentally observed and analysed by for instance Beavers and Joseph (1967), Richardson and Parr (1991), Neale and Nader (1974), and Berkowitz (1989). The effect of flow in the porous medium can be represented by a slip-velocity at the wall of the channel, which increases the flow rate through the channel. Equivalently, this effect can be expressed by an increased "effective width" of the channel. An analytical expression for this effective width $b_e$ can be derived from the increase in flow rate as a result of parallel flow according to Berkowitz (1989), and can be expressed as:
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\[ b_e = b \left( 1 + \frac{6\sqrt{k}}{\kappa b} + \frac{12k}{b^2} \right) \]

(2.47)

where \( b \) is the geometrical channel width, \( k \) is the permeability of the porous medium, and \( \kappa \) is a parameter which expresses the square root of the ratio of the velocity gradient in the channel and the porous medium (which is described by Brinkman's extension of Darcy's law). A uniform pressure gradient in the channel and porous medium is assumed.

\( \kappa \) must be determined experimentally; Richardson and Parr (1991) found the empirical relationship for uniform glass beads:

\[ \kappa = 55 \left( \frac{k}{k_0} \right)^{\frac{1}{4}} \]

(2.48)

with \( k_0=1 \text{ m}^2 \). This relationship was based on measurements in which the permeability of the porous medium was larger than 1 D (≈0.986 \( \text{10}^{-12} \text{ m}^2 \)). Beavers and Joseph (1967) suggested the linear dependence of \( \kappa \) on a measure of the average pore diameter, based on the consideration that \( \kappa \) is determined by the structural constants of the material. Their data on two materials with different pore structure show a value of \( \kappa \) that is an order of magnitude larger than predicted by equation (2.48). Furthermore, we mention that the effect of parallel flow as described by equations (2.47) and (2.48) is not scaled in our experiments (see section 2.8 for scaling), and predicts a potentially larger effect in the experiments.

We now analyse whether the influence of parallel flow is significant in hydraulic fracturing experiments on plaster. Of the rocks we used, plaster would be the most susceptible to effects of parallel flow because it has the largest permeability, while the fracture width is of the same order of magnitude in all rocks. Provided that the pressure gradient inside the porous medium is already established, equation (2.48) gives \( \kappa=0.017 \) which leads to an increase in the effective width of 1.4 times for a permeability of 40 mD and a channel width of 30 \( \mu \text{m} \) (which are representative values at the fluid-front position in experiments on plaster). This increase in effective width depends strongly on the value of \( \kappa \), which depends on the materials microstructure. Using the measurements of Beavers and Joseph (1967) would already diminish the increase in effective width to 1.2 times. Furthermore, the width increases fast in the tip region, so that parallel flow becomes insignificant quickly at positions away from the fluid front. In addition, the effect on the final wellbore pressure might be small, because width changes tend to partly counteract changes in the pressure gradient in a hydraulic fracture. Besides the uncertainty of equation (2.48) for permeabilities smaller than 1 D, these arguments make us expect that the effect of parallel flow in plaster is not significant.
Description of leak-off

The pressure gradient of one-phase flow of fluid through a porous medium is given by Darcy's law (Batchelor, 1990):

$$\frac{\partial p_f}{\partial x_i} - g_i = \frac{v_i \mu}{k} \quad (2.49)$$

where $p_f$ is the fluid pressure, $x_i$ is the spatial coordinate, $v_i$ is the flow rate in the $i$-direction per unit rock area, $\mu$ is the dynamic viscosity, and $k$ is the permeability, which is assumed to be isotropic and homogeneous. The gravity acceleration $g_i$ is negligible under our conditions. Equation (2.49) is valid when the Reynolds number based on the grain size is much smaller than one.

Usually, the assumption of one-dimensional leak-off is made. This is valid when the fluid velocity in the fracture is much larger than the radial component of the fluid velocity in the rock adjacent to the fracture. This is the case when the fracture permeability at the fluid front is larger than the rock permeability, which is usually the case. Furthermore, we assume one-phase flow, so that the fluid that leaks away from the fracture into the reservoir completely saturates the reservoir. We neglect the effect of capillary pressure at the fluid front. The leak-off rate of fracturing fluid into the formation is then governed by a one-dimensional diffusion equation (e.g. Hagoort, 1981):

$$\frac{\partial p_f}{\partial t} = \frac{k}{\varphi_p \mu c_t} \frac{\partial^2 p_f}{\partial y^2} \quad (2.50)$$

where $c_t$ is the compressibility of the fluid and $\varphi_p$ is the porosity of the rock. Here, the assumptions are made that the grain compressibility is much larger than the fluid compressibility, and that pore volume changes as a result of pressure variations make only an insignificant contribution to the local mass balance.

We now consider a situation in which fluid leaks into rock, while the pressure at the fluid front stays zero. This situation is representative for our experiments. When we assume zero fluid compressibility, so that $\frac{\partial^2 p_f}{\partial y^2} = 0$, and assume that the pressure applied at the fracture surface is constant in time, we find from equation (2.49) and conservation of mass for the leak-off velocity $v_i$ (being the flow rate per unit area that flows into the reservoir):

$$v_i = \frac{k \varphi_p \Delta p_f}{\sqrt{2 \mu (t - t_0)}} \quad (2.51)$$
where \( t_0 \) is the time at the moment the rock surface was first exposed to the fluid pressure. A constant pressure difference \( \Delta p_f \) between fracture and reservoir is assumed.

When the fluid is compressible and pressure diffusion controls the leak-off rate, the leak-off velocity is given by:

\[
\nu_l = \frac{k \rho_p c_i}{\sqrt{\pi \mu(t-t_0)}} \Delta p_f
\]  \hspace{1cm} (2.52)

In both cases, the leak-off velocity is proportional to \( 1/\sqrt{t} \), but the dependence on the pressure difference \( \Delta p_f \) is different. In both cases the Carter equation can be used, which states that:

\[
\nu_l = \frac{C_l}{\sqrt{t-t_0}}
\]  \hspace{1cm} (2.53)

where \( C_l \) is the Carter leak-off coefficient.

### 2.6 Coupling of radial hydraulic fracturing equations

Characteristic for hydraulic fracturing is the coupling of the fluid flow, rock fracturing, and fracture opening processes. These processes are represented by equations (2.12), (2.16), (2.41), and (2.46), and govern propagation of a penny-shaped hydraulic fracture in a linear elastic impermeable solid. Even in the linear elastic case, there is no analytical solution for this problem. However, when certain assumptions are made, useful analytical solutions can be derived. In this section we give a review of these analytical solutions and mention the approximations made. In addition, we shortly discuss numerical models that solve the governing equations. We give an overview of numerical simulations presented in the literature, together with the most important results. We start with linear elastic simulations. After that, we discuss simulations which in various ways incorporated the process zone in the model.

**Propagation regimes**

The first simplification of the governing equations that can be made, is to distinguish two propagation regimes for linear elastic hydraulic fracture propagation. These are usually named the toughness-dominated and viscosity-dominated regime. In the viscosity-dominated regime, the resistance to fluid flow in the radial direction is caused by the
viscosity of the fluid. The fluid does not reach the tip, and the fluid pressure at the fluid front position equals the pore pressure in the fluid lag (neglecting the interfacial tension). The toughness can be neglected in this case. Because the fluid does not reach the fracture tip, a fluid lag (also called non-penetrated zone or dry tip) is present adjacent to the fracture tip. This fluid lag forms because the rock cannot bear the fluid pressure loading needed to establish the viscous fluid flow up to the tip. In this way the fluid lag stabilises the fracturing process, which is effectuated by the confining stress working over the fluid lag (De Pater et al., 1994a).

When the toughness becomes so large that the fluid lag disappears, transition to the toughness-dominated regime takes place. The fluid is able to reach the tip, and the pressure gradient inside the fracture disappears. The resistance to fluid flow inside the fracture in the radial direction is now caused by the toughness of the rock. The fluid viscosity can be neglected in this case.

In the toughness-dominated regime the net pressure \( p_{\text{net}} \) and width \( b_w \) at the wellbore as a function of fracture radius \( R \) can be deduced from equations (2.13) and (2.16):

\[
p_{\text{net}} = \frac{\sqrt{\pi}}{2} \frac{K_{lc}}{\sqrt{R}}
\]

\[
b_w = \frac{1}{\sqrt{\pi}} \frac{K_{lc} \sqrt{R}}{E}
\]

(2.54)

(2.55)

(using that \( p_{\text{net}} \) is constant in the fracture and taking the wellbore width \( b_w \) at \( r=0 \). \( K_{lc} \) is the fracture toughness, and \( E \) is the crack opening modulus (defined as \( E/(4(1-\nu^2)) \), \( E \) is Young’s modulus and \( \nu \) is Poisson’s ratio).

**Analytical approximations in the viscosity-dominated regime**

Various approximations can be made in order to obtain useful analytical solutions in the viscosity-dominated regime. In Perkins and Kern (1961), a constant fracture width was used in order to determine the pressure profile in the fracture. This leads to a logarithmic pressure profile:

\[
p(r) = p_w - \frac{Q \mu}{2\pi b_{av}^3} \ln \left( \frac{r}{R_w} \right)
\]

(2.56)

where \( \mu \) is the channel flow viscosity (being 12 times the viscosity \( \mu \)), \( Q \) is the flow rate flowing into the fracture, \( R_w \) is the radius of the wellbore, \( b_{av} \) is an average width along the radial coordinate, and \( p_w \) is the fluid pressure at the wellbore. The width profile is assumed
to be elliptical, and is assumed to be determined by a constant pressure in the fracture, equal to the average pressure of the pressure profile of equation (2.56). Also, the effect of fracture opening rate on the flow rate was neglected. In Geertsma and de Klerk (1969), the same assumptions for the pressure profile were used. However, the width profile was calculated from the logarithmic pressure profile loading. In Abé et al. (1976), the influence of the time derivative of the width on the pressure singularity at the tip was recognised. In Spence and Sharp (1985), a self-similar formulation of the fully coupled problem is given. From this, the time exponents of the solution were derived, for a flow rate proportional to \( t^\beta \) (\( t \) is time).

In Crockett et al. (1986), the governing equations were simplified by introducing integration factors, which represent the shape of the width- and pressure profile. These factors are regarded as being constant during propagation and are of \( O(1) \), which also implies self-similar propagation. Then, the only variables are fracture length, net pressure and fracture width at the wellbore, and time. When leak-off is neglected and after elimination of time, these equations lead to expressions for the net pressure and width at the wellbore that have a similar time dependency as in Spence and Sharp (1985) for constant flow rate. We can write this solution as a function of fracture radius \( R \):

\[
P_{\text{net},w} \propto \left[ \frac{\mu Q E}{R^3} \right]^{\frac{1}{4}} \tag{2.57}
\]

\[
b_w \propto \left[ \frac{\mu Q R}{E} \right]^{\frac{1}{4}} \tag{2.58}
\]

Because of the change in fracture opening as a result of changes in net pressure, the wellbore pressure responds less strongly to variations in flow rate and viscosity than when the width stays fixed. This leads to the power 1/4 dependency of the net pressure on flow rate \( Q \) and viscosity \( \mu \) in equation (2.57), instead of the linear relationship in case the width would have been fixed (which is, for example, visible in equation (2.56) for fixed average width).

Desroches et al. (1994) derived an asymptotic solution for the tip region of an impermeable fracture. They assumed self-similar propagation in order to remove the time derivative of the width from the continuity equation. The potential function given in equation (2.25) was used to represent the stress-strain behaviour in the tip region, which made it possible to satisfy both the elasticity and fluid flow equations. The fluid reached the fracture tip in this solution, but was allowed to go to minus infinity. Despite these assumptions, a comparison with a fully coupled numerical simulation program gave a
good match of the width profile. A similar result was obtained by Lenoach (1995) for the permeable case (including equation (2.53) in the continuity equation). From these results, Desroches et al. (1993) conclude that the fluid lag is a local mechanism that does not affect the global response directly. The major influence of the fluid lag is that it negates the effect of the fracture toughness.

The asymptotic tip-model was implied in a penny-shaped geometry in Savitski and Detournay (1999). They found a solution of the pressure profile that satisfied the logarithmic behaviour at the wellbore, and the plane strain asymptotic solution near the tip.

While in the previous solutions for the viscosity-dominated regime fracture toughness was neglected, Garagash and Detournay (1998) extended the asymptotic tip model to include toughness. A universal relationship between dimensionless toughness \( K_{ic}^* \) and dimensionless fluid lag size \( \omega^* \) was obtained. This model directly predicts the size of the fluid lag as a function of treatment parameters and material properties (which may be important for determining the propagation mode). \( K_{ic}^* \) and \( \omega^* \) are defined as:

\[
K_{ic}^* = \frac{K_{ic}}{E} \sqrt{\frac{2\sigma_c}{\pi \mu \dot{L}}} \tag{2.59}
\]

\[
\omega^* = \frac{\omega}{\mu \dot{L}} \frac{\sigma_c^3}{16E^2} \tag{2.60}
\]

where \( \dot{L} \) is the propagation velocity of the fracture, and \( \omega \) is the length of the fluid lag. Near the tip, the coefficient \( \alpha \) in the potential function given by equation (2.25) was found to be 1/2, while further away from the tip \( \alpha \) was 2/3. This model is valid when the fracture length is much larger than the length scale of the tip region \( L_{tip} \), which is defined by (Garagash and Detournay, 1998):

\[
L_{tip} = \frac{1}{\mu \dot{L}} \frac{\sigma_c^3}{16E^2} \tag{2.61}
\]

\( L_{tip} \) is related to the size of the fluid lag corresponding with zero toughness. This model is compared with our experiments in section 5.1.

**Numerical simulation of linear elastic hydraulic fracture propagation**

To obtain complete solutions of the hydraulic fracturing equations, numerical simulation programs were developed. The first code that satisfies all the equations (called "fully
coupled") is developed by Barr (1991), and will be used in the present study for comparison with the experimental results. Other simulation programs for the radial geometry were developed by Gardner (1992) and Desroches and Thiercelin (1993).

Gardner (1992) combined the governing equations to eliminate the pressure terms in the equation that describes fracture opening, resulting in an equation that only contains the fracture width. Barr (1991) iteratively solved all equations, while the double integration in equation (2.16) was reduced to a single integration. Convergence of the solution depended strongly on the initial estimate for the crack opening rate. The model of Desroches and Thiercelin (1993) used a variational formulation of the equation describing elastic fracture opening.

The numerical simulation results show that for viscosity-dominated propagation, the proportionality factors in equations (2.57) and (2.58) are reasonably constant during propagation, for the fracture radius of interest. Gardner (1992) showed that the pressure in the fluid lag influences the borehole pressure and width only slightly. When the pore pressure in the fluid lag is decreased, the fracturing fluid must penetrate closer to the fracture tip to establish the stress intensity that is needed for propagation. Apparently, the higher pressure that is needed for this is cancelled out by the smaller pressure that exists at the fluid front.

**Modelling of process zone in numerical simulations**

The process zone, being a deviation from LEFM, can be incorporated in various ways in numerical models. The simplest way is to increase the toughness in a linear elastic model (Shlyapobersky, 1985, Gardner, 1992, Desroches et al., 1993, and Medlin and Massé, 1986). In this case, toughness-dominated propagation can be achieved. The physical justification of this is that the process zone leads to an increase in energy dissipated during the creation of new fracture area.

A second way is to model the process zone as a cohesive zone, assuming that the fracturing process localises in a narrow tensile zone (see section 2.3). Here, the influence of shear yielding on the fracturing process is neglected. This was done in Desroches et al. (1993) using a fully coupled finite element program. Variation of the cohesive zone parameters had very little influence on the global fracture geometry, as long as the fluid did not reach the cohesive zone (which was assumed to be impermeable). The influence of the cohesive zone parameters on the tip region is discussed in section 6.3, where we present similar numerical simulations.

Probably the most realistic model is to couple the cohesive zone with elastic-plastic rock behaviour. This was done in Papanastasiou and Thiercelin (1993) and Desroches et al. (1993), who also investigated the role of dilatancy. Plasticity near the tip increased the width in the bulk of the fracture that was needed for propagation, because the plastic deformation screened the fluid pressure loading: part of the strain energy was lost in
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plastic deformation, so that not enough strain energy was available to act as the crack driving force and create new fracture area. To create this extra width, the fluid front has to be closer to the tip.

For one case, plasticity was seen to increase the width of the fracture approximately 1.5 times, but influenced the wellbore pressure only slightly. Apparently, the effect of extra penetration and changes in width profile on the wellbore pressure cancel out. However, if the fluid reaches the tip of the fracture, the pressure at the borehole is increased significantly (Papanastasiou and Thiercelin, 1993, and Papanastasiou, 1999a). This could be described as "plasticity-dominated propagation". In this case the plastic energy dissipation is much higher than the strain energy release rate that is needed for fracture propagation (see also Desroches et al., 1993). The plastic deformation could increase the value of the effective toughness typically 20 - 30 times.

When rock dilation is introduced in elastic-plastic simulations, it is seen that the width of the fracture further increases in comparison with the elastic-plastic case with zero dilation (Papanastasiou and Thiercelin, 1993). It appears that dilation just increases the screening effect of the plastic zone. The wellbore pressure is nearly unaffected and the fluid lag becomes smaller.

Simulations that did not couple fluid flow and rock deformation were done by van den Hoek et al. (1993) and Yew and Liu (1993). They both found that plastic deformation and dilatancy can screen the tip from the fluid pressure loading. However, the problem was solved uncoupled, assuming a constant pressure in the fracture. This makes the value of other conclusions they made questionable, because the coupled elastic-plastic simulations showed the importance of the interaction between fluid flow and fracture opening.

A fourth way to model deviations from linear elastic fracture mechanics is to introduce an artificial constraining of the fracture width in the region of the fluid lag (Barr, 1991 and Gardner, 1992). In this way, higher net pressures can be reached. As fracture opening is still elastic, the width increases proportional to the net pressure. This method yields similar results as the elastic models in which the toughness is increased. The physical motivation for the constraining of the tip region is the dilation of rock, as a result of the plastic deformation (dilation hypothesis as suggested by Johnson and Cleary, 1991). What effectively is done, is that a larger modulus near the tip is assumed. As a result of the higher net pressure, a larger fluid lag develops.

2.7 Energy rates of hydraulic fracturing processes

In this section we present expressions for energy rates associated with hydraulic fracture propagation. Energy considerations give useful understanding of the hydraulic fracturing
process. The relative magnitude of the energy rates occurring during propagation can be coupled with the propagation regime of a hydraulic fracture. We use this to derive a parameter whose magnitude predicts the propagation regime in the linear elastic case.

**Energy dissipated or stored in various processes**

In linear elastic fracture mechanics, the energy dissipated in the fracturing process is given by the product of fracture area and fracture surface energy $\Gamma$:

$$U_F = \pi R^2 2\Gamma$$  \hfill (2.62)

The energy dissipation rate is obtained by taking the time derivative:

$$\dot{U}_F = 4\pi R R \Gamma$$  \hfill (2.63)

The energy dissipated in fluid flow equals the work done in propagating the fluid:

$$\dot{U}_F = - \int_0^{V_f} \nabla p dV_f$$  \hfill (2.64)

where $V_f$ is the fluid volume in the fracture. When we use the channel-flow approximation (2.41) for the pressure gradient, and integrate over the fracture width, this becomes:

$$\dot{U}_F = 2\mu \int_{A_f} \frac{-\nabla r^2}{b} dA_f = \frac{-\mu}{\pi} \int_{r=0}^{R_f} b^3(r) r dr$$  \hfill (2.65)

$A_f$ is the fracture area covered by fracturing fluid, $R_f$ is the radius of the fluid front, $\nabla r$ is the radial velocity averaged over the fracture width, and $q_f$ is the flow rate in the fracture.

During opening of a fracture without propagation, the work $W_e$ done by the fluid on the fracture surface is given by:

$$W_e = \int_0^R \left( \sigma_c + \frac{1}{2} p_{net}(r) \right) b(r) 2\pi r dr$$  \hfill (2.66)

The confining stress $\sigma_c$ is the far-field compressive stress in the $y$-direction, working perpendicular to the fracture plane. This work is transferred into elastic strain energy $U_e$ stored in the rock, and work done against the far-field stresses. For our situation with a cubic block with ribs with length $L_{ib}$ and faces with area $A_{ib}$, we have:
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\[ W_e = U_e + \sigma_c A_{bl} \Delta L_{bl,y} \]  
(2.67)

\[ \Delta L_{bl,y} \] is the surface average of the increase in block length in the \( y \)-direction (perpendicular to the fracture plane). We neglected the work done in the directions parallel to the fracture plane, as the displacements in these directions are usually much smaller than the displacements perpendicular to the fracture plane, while the stresses are of the same order of magnitude.

The superposition principle for linear elasticity (see figure 2.1) shows that the relationship between the width distribution of the fracture and the block elongation as a result of fracture opening is independent of the confining stress. We now apply the elastic reciprocity theorem (see e.g. Jaeger and Cook, 1969) on the equilibrium states II and III in figure 2.1. This yields that:

\[ \int_0^R \sigma_c b(r) 2\pi r dr - \sigma_c A_{bl} \Delta L_{bl,y} = 0 \]  
(2.68)

From equations (2.66), (2.67), and (2.68), we find for the elastically stored energy \( U_e \) in the block:

\[ U_e = \int_0^R \left( \frac{1}{2} P_{net}(r) b(r) \right) 2\pi r dr \]  
(2.69)

For the volume of the fracture \( V_{frac} \) we find from equation (2.68) that:

\[ V_{frac} = A_{bl} \Delta L_{bl,y} \]  
(2.70)

The extra work done by the fluid as a result of the presence of the confining stress \( \sigma_c \) is completely consumed by the work done against the confining stress. This indicates that in energy considerations of hydraulic fracturing, we can leave out the confining stresses (as was done by several authors, for example in Biot et al., 1986 and De Pater et al., 1994a). Using \( b(r) \) equals zero at \( r=R \), the elastic energy storage rate then is:

\[ \dot{U}_e = \int_0^R \left( \frac{1}{2} P_{net}(r) b(r) + \left( \frac{1}{2} P_{net}(r) \right) \dot{b}(r) \right) 2\pi r dr \]  
(2.71)

The energy rate \( \dot{U}_f \) dissipated in leak-off is obtained by integrating the power dissipated in leak-off over the fracture surface:
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\[ \dot{U}_t = \int_0^R \left[ p_{net}(r) + \alpha_c \right] 2\nu_l(r) 2\pi r dr \]  

(2.72)

where \( v_l \) is the leak-off velocity through one fracture surface.

We can write the strain energy rate in the plastic zone around the tip as the sum of an elastic part and a plastic part (Mase, 1970):

\[ \int_{V_p} \left( \sigma_{ij} \varepsilon_{ji}^p \right) dV_p = \int_{V_p} \left( \sigma_{ij} \varepsilon_{ji}^e \right) dV_p + \int_{V_p} \left( \sigma_{ij} \varepsilon_{ji}^p \right) dV_p \]  

(2.73)

\( V_p \) is the volume of the active plastic zone, being the region where plastic deformation is currently taking place. Using force equilibrium over each material element so that \( \sigma_{ji} = \sigma_{ij} \), we find using equation (2.73) that the energy dissipated in plastic deformation in the active plastic zone is given by:

\[ \dot{U}_p(t) = \int_{V_p} \sigma_{ij} d \varepsilon_{ij}^p dV_p \]  

(2.74)

In order to find the order of magnitude of the plastic energy dissipation rate, we non-dimensionalise equation (2.74). We take a radial fracture geometry, and assume that the size of the plastic zone stays constant during propagation over a distance equal to the active plastic zone size \( l_p \). We take as characteristic time the time needed for the fracture to propagate over a length \( l_p \), and as characteristic volume for the active plastic zone \( V_p \approx 2\pi^2 l_p^2 R \). Furthermore, we use a characteristic plastic strain \( \varepsilon^p \) to scale \( \varepsilon_{ij}^p \), and a characteristic shear stress \( \sigma_{ij} \) to scale \( \sigma_{ij} \). We then find that:

\[ \dot{U}_p(t) = 2\pi^2 S_c \varepsilon_{ij}^p l_p R \int_{V_p} \sigma_{ij} d \varepsilon_{ij}^p dV_p \]  

(2.75)

**Energy rates and propagation regime**

We will consider the ratio of the energy rates associated with fluid flow and rock fracturing in the linear elastic case. This ratio is much larger than one for toughness-dominated propagation, and much smaller than one for viscosity-dominated propagation, and indicates which propagation mode is present. We use the expressions for the energy rates given in equations (2.63) and (2.65). In these equations, we use characteristic
quantities for the flow rate \( q(r) \), width \( b(r) \), and \( 2\pi R \dot{R} \). These are respectively \( Q \), \( b_w \), and \( Q/b_w \).

When we use equation (2.58) for the width at the wellbore, we find for the viscosity-dominated regime:

\[
\frac{\dot{U}_r}{\dot{U}_f} \propto \Gamma \left( \frac{R}{Q \mu E} \right)^{\frac{1}{2}}
\]

(2.76)

In the toughness-dominated regime, we find using equation (2.55) for \( b_w \):

\[
\frac{\dot{U}_r}{\dot{U}_f} \propto \Gamma^2 \left( \frac{R}{Q \mu E} \right)
\]

(2.77)

We see that for both regimes, the factor \( \Gamma^2 \left( \frac{R}{Q \mu E} \right) \) determines the ratio, and the propagation mode. As equations (2.63) and (2.65) are derived for impermeable fracture walls, this criterion holds as long as leak-off does not invalidate these equations. For most practical cases, however, leak-off has only a small influence on the wellbore width (see e.g. the simulations presented in Lenoach, 1995). Without any motivation, a criterion based on this same factor is given by Cleary (1990), who claims that \( \Gamma^2 \left( \frac{R}{Q \mu E} \right) \) must at least be of order \( 10^2 \) for toughness to become important.

In all our experiments, this factor is much smaller than one, so that we expect viscosity-dominated propagation. We used the fracture surface energy \( \Gamma \) - and accompanying toughness - for the energy dissipated in the fracturing process, excluding energy dissipated in plastic deformation. Plasticity may cause a transition to a plasticity-dominated propagation mode. For a typical field case in weak rock, the factor \( \Gamma^2 \left( \frac{R}{Q \mu E} \right) \) equals \( 10^{-3} \), indicating viscosity-dominated propagation (using \( \Gamma = 100 \) N/m, \( R = 10 \) m, \( \mu = 0.1 \) Pa·s, \( Q = 0.1 \) m³/s, \( E = 1 \) GPa).

In each propagation mode, the maximum of the energies dissipated in fluid flow and fracturing typically balances the energy stored in rock deformation. By using the expressions for net pressure and width at the wellbore (equation (2.54) and (2.55), or (2.57) and (2.58)) in equations (2.63), (2.65), and (2.71), we find that the dominant energy rate (associated with fluid flow or fracturing) is of the same order of magnitude as the rate of energy stored in rock deformation. This agrees with intuition, because we expect that
the pressurised fluid in the wellbore prefers to transfer its energy to the process that costs the minimum energy. This typically leads to a balance.

2.8 Scaling of elastic-plastic hydraulic fracture propagation

In this section we present scaling of elastic radial hydraulic fracture propagation. Using the scaling, we can translate our experimental results to a field scale.

**Elastic scaling**

We can make the governing equations for linear elastic hydraulic fracture propagation (2.16), (2.12), (2.41), and (2.46) dimensionless according to de Pater et al. (1994b). In these equations, we use the crack opening modulus $E$ to make the net pressure dimensionless, a characteristic radius $R_c$ to make the width and radius dimensionless, a characteristic time $T$ to make the time dimensionless, a flow rate $Q/R_c$ to make the flow rate per unit perimeter $q'$ dimensionless, and a characteristic leak-off velocity $C/\sqrt{T}$ to make the leak-off velocity $v_l$ dimensionless.

Using $E$ to make the net pressure dimensionless, and using the characteristic radius $R_c$ to make the width dimensionless, can be motivated by the equation describing fracture opening (2.16) (see also equation (5.7) for a simplified version). However, equations (2.54), (2.55), (2.57), and (2.58) show that in general the net pressure and width are not linearly proportional to respectively $E$ and $R_c$. Using $E$ and $R_c$ to make the net pressure and width dimensionless, assumes a similar ratio between width and radius in the lab as in the field, or equivalently the same ratio between net pressure and Young's modulus in the lab as in the field. This somewhat limits the applicability of this scaling to order of magnitude analysis.

The non-dimensionalised equations then are:

$$1 = \frac{E\sqrt{R_c}}{K_c} \frac{2}{\sqrt{\pi R^*}} \int_0^R \frac{r^* p_{\text{net}}(r^*)dr^*}{\sqrt{R^*}}$$

$$\frac{\partial p^*}{\partial r^*} = -\frac{\mu Q}{ER_c^3} \frac{q_f^*(r^*)}{b^3(r^*)}$$

(2.78) 

(2.79)
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\[
b^* (r) = \frac{2}{\pi} \int_{s^*}^{w^*} \frac{dw^*}{\sqrt{w^*^2 - r^*^2}} \int_0^{w^*} s^* \left( p^* (s^*) - \frac{\sigma^*_c}{E} \right) ds^* \tag{2.80}
\]

\[
\frac{R_c}{T} \frac{\partial b^*}{\partial t^*} + 2 \frac{C_l}{\sqrt{T}} v^*_l + q^*_f \frac{1}{R_c^2} \frac{1}{r^*} \frac{\partial (r^* q^*_f)}{\partial r^*} = 0 \tag{2.81}
\]

which can be written as:

\[
\frac{R_c^2}{\sqrt{T}} \frac{\partial b^*}{\partial t^*} + 2C_l \sqrt{\frac{R_c}{Q}} v^*_l + \frac{\sqrt{QT}}{R_c^3} \frac{1}{r^*} \frac{\partial (r^* q^*_f)}{\partial r^*} = 0 \tag{2.82}
\]

The dimensionless groups in equations (2.78), (2.79), (2.80), (2.81), and (2.82) are:

\[
N_T = \frac{E\sqrt{R_c}}{K_{lc}} \tag{2.83}
\]

\[
N_E = \frac{\mu Q}{ER_c^3} \tag{2.84}
\]

\[
N_{\sigma_c} = \frac{\sigma^*_c}{E} \tag{2.85}
\]

\[
N_{C_l} = C_l \sqrt{\frac{R_c}{Q}} \tag{2.86}
\]

\[
N_T = \frac{TQ}{R_c^3} \tag{2.87}
\]

These dimensionless groups form a complete set according to the \( \Pi \)-theorem. We use these dimensionless groups to scale our experiments. Instead of \( N_{C_l} \), we can also use the fracture efficiency to scale leak-off (the fracture efficiency is defined as the fracture volume divided by the injected volume). This assumes that the influence of leak-off on the
local mass balance does not influence the pressure profile significantly. Numerical simulations presented in Lenoach (1995) show that this assumption is reasonable for a wide range of values of the leak-off coefficient.

Under the assumptions stated ($p_{net}$ proportional to $\bar{E}$, $b_w$ proportional to fracture radius, and insignificant effect of leak-off), $N_I$ can be interpreted as the proportion between the energy rates associated with fracture opening and rock fracturing, and $N_E$ can be interpreted as the proportion between the energy rates associated with fluid flow in the fracture and fracture opening (De Pater et al., 1994a). When we consider the solutions for viscosity- and toughness-dominated propagation regimes, the factor $\Gamma^2 \left( \frac{R}{Q\mu\bar{E}} \right)$ determines the proportion between the energy rates dissipated in rock fracturing and fluid flow. This factor can be expressed by $N_I$ and $N_E$:

$$\Gamma^2 \left( \frac{R}{Q\mu\bar{E}} \right) = \frac{1}{16N_I^4N_E}$$  \hspace{1cm} (2.88)

This implies that the scaling of these dimensionless groups ensures the same elastic propagation regime in the lab as in the field.

The scaling procedure using the above-mentioned dimensionless groups can be as follows. In our laboratory experiments, we choose to use the same $\bar{E}$ and $\sigma_c$ as in the field, so that $N_{\sigma_c}$ is the same in lab and field. Because $R_c$ is at least an order of magnitude smaller in the lab as in the field, a smaller value of $K_{lc}$ has to be used in the lab in order to have the same $N_I$ in lab and field. The scaling factor of $R_c$ determines the required scaling factor of $K_{lc}$. We choose a time scale $T$ in the lab of $10^3$ s, in order to have sufficient time for measurements. Comparison with the time scale in the field determines $Q$ via $N_T$. Because $T$ is larger and $R_c$ is smaller in the lab, $Q$ must be much smaller in the lab. $\bar{\mu}$ is determined by $N_E$, and is increased in the lab. Finally, the required leak-off coefficient in the lab is smaller than in the field, and is determined by $N_{Cl}$.

**Scaling of elastic-plastic fracture propagation**

In this section we present scaling of elastic-plastic fracture propagation. Starting point is the elastic scaling of hydraulic fracture propagation of the previous section. This yielded that the confining stress and Young's modulus are the same in field and lab. We also assume that the far-field stresses working in the plane of the fracture are the same in lab and field.

The rock behaviour near the tip is governed by three processes. These are fracture propagation, elastic deformation, and plastic deformation. We distinguish two regions near the tip, which form as a result of the presence of the fracture tip. First, surrounding the tip,
a plastic region with length scale $l_p$. Second, this plastic region is surrounded by an elastic region with length scale $l_e$. The boundaries of these regions are not well defined, but can be chosen to contain a certain large part of the elastic or plastic deformation energy, induced by the fracture tip.

As a result of the loading on the fracture surface, elastic shear strain with respect to the $x$-$y$ coordinates is built up around the tip. The intensity of the stress field near the tip is critical, so that the fracture is in a state of propagation. When the fracture propagates over a small distance $dx$, strain energy close to the fracture tip is released. We assume that this strain energy is proportional to the shear strain energy in the region adjacent to the tip with horizontal extent $dx$ and vertical extent $l_e$. Then, a fracture propagation criterion states that:

$$\frac{1}{G} \left( \int_{y=0}^{l_p} \sigma_{xy}^2 dy + \int_{y=l_p}^{l_e} \sigma_{xy}^2 dy \right)_{x=x_{tip}} \propto \Gamma \quad (2.89)$$

where $G$ is the shear modulus. In the plastic region, the stress scales with a characteristic shear stress $S_c$, which is determined by the material strength and externally applied stresses. In the elastic region we assume that the linear elastic stress field is present, so that we have $K_{lc} / \sqrt{l_e}$ as characteristic stress. Making equation (2.89) dimensionless yields:

$$\frac{S_c^2 l_p}{G \Gamma} \left( \int_{y^*=0}^{l_p} \sigma_{xy}^{*2} dy^* + \int_{y^*=l_e}^{l_e} \sigma_{xy}^{*2} dy^* \right) \propto 1 \quad (2.90)$$

We have two dimensionless groups: $\frac{S_c^2 l_p}{G \Gamma}$ and $\frac{l_e}{l_p}$. We choose the externally applied stresses and stress-strain behaviour to be the same in lab and field, so that $S_c$ and $G$ are the same. Then the scaling requires that $\frac{l_p}{\Gamma}$ and $\frac{l_e}{l_p}$ are the same in lab and field. In the following we will show that this is the case as a result of the elastic scaling.

**Scaling of the plastic zone size**

There are various length scales associated with the fracture tip region that can be used to scale the plastic zone size. First, we can infer a length scale from the linear elastic stress
field around the tip, in combination with a yield criterion. Equations (2.28) and (2.29) give
the stress situation around the tip, which determines the stresses in a yield criterion. We
take the argument of the complex variable \( \zeta \) in these equations constant in order to
calculate the plastic zone size \( l_p \) under that angle. In that case, the stresses are determined
by the variable \( A x^{\alpha-1} \) and the in-situ stresses \( \sigma_c \) and \( \sigma_p \). We assumed that the in-situ
stresses and strength are similar in the experiments and the field. Under that assumption,
we have for the scaling of the plastic zone size that:

\[
l_p \propto A \left( \frac{1}{1-\alpha} \right) \tag{2.91}\]

In the case of the linear elastic fracture mechanics stress field, \( \alpha=1/2 \) and \( A = \frac{K_{lc}}{\sqrt{2\pi}} \). Then,
we have from equation (2.91) for the plastic zone size:

\[
l_p \propto K_{lc}^2 \tag{2.92}\]

When the stress field is determined by the pressure profile and not by the fracture
toughness, \( \alpha=2/3 \) and \( A = \frac{4}{3} \left( \frac{2E^2\mu L}{K_{lc}} \right)^\frac{1}{3} \) (Desroches et al., 1994). Then, we find that

\[
l_p \propto \frac{E^2\mu L}{K_{lc}} \tag{2.93}\]

Equations (2.28) and (2.29) - on which equation (2.91) is based - can be used at
positions where the change in \( \sigma_{yy} \) as a result of the crack tip stress field is much larger than
\( p_0 \) in equation (2.24). Using \( K_{lc}=0.3 \text{ MPa}\sqrt{\text{m}} \) and \( L=0.1 \text{ m} \) being representative values for
strong plaster, this requirement results in a required change in \( \sigma_{yy} \) that is much larger than
0.5 MPa. This is approximately true for most of our experiments, which implies that it is
justified to use equation (2.91) for scaling purposes.

A second way to scale the plastic zone size is to take the size of the zone of stress
release in the plane of the fracture. For this, we can take the tensile zone in the plane of the
fracture, which is also a measure of the cohesive zone. The size of the tensile zone scales
in a similar way as the plastic zone size. Using equation (2.28) and (2.29), we find in the
plane of the fracture for the tensile zone size:

\[
l_T \propto \left( \frac{A}{T_0 + \sigma_c} \right)^\left( \frac{1}{1-\alpha} \right) \tag{2.94}\]

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The assumptions for this equations to be valid (as discussed after equation (2.91)) are approximately satisfied.

A third length scale of the tip region is the size of the fluid lag zone. In Garagash and Detournay (1998), a value for the size of the fluid lag $\omega$ was given, based on a similar asymptotic solution as in Desroches et al., (1994). The maximum value of this fluid lag - corresponding with zero toughness - is given to be proportional to:

$$\omega \propto \frac{E^2 \mu L}{\sigma_c^3}$$  \hspace{1cm} (2.95)

When the confining stress $\sigma_c$ in the laboratory and the field is similar, the scaling factor for the fluid lag is equal to that of the plastic zone (2.93).

Summarising, we have two possibilities to scale the plastic zone size: equations (2.92) and (2.93). In both cases, the tensile zone size scales in the same way, so that $\frac{l_e}{l_p}$ is the same in the lab as in the field. In the case of LEFM, equation (2.92) assures that $\frac{l_p}{l_p}$ is the same in lab and field, assuming the same Young's modulus. When equation (2.93) is used for $l_p$, $\frac{l_p}{\Gamma}$ is scaled based on the scaling of $N_E$ and $N_\Gamma$ in the elastic scaling, using that the Young's modulus is the same in lab and field.

### 2.9 Influence of plasticity on the fracturing process

In this section we present a model that considers the influence of plasticity on the fracturing process and fracture surface roughness of a quasi-statically propagating fracture. The model is based on the hypothesis that the total energy dissipated in the creation of new area and the plastic deformation processes associated with fracturing, is minimised. The \textit{in-situ} stresses are driving forces for the plastic deformation. Because of this, reorientation of the fracture plane with respect to the \textit{in-situ} principal stress directions can decrease the plastic energy dissipation.

**Model**

We can write the energy dissipated in the fracturing process as the sum of the fracture surface energy $\Gamma$ per unit of newly created global fracture area dissipated in creating
2. Hydraulic fracturing processes

fracture surface (equal to the previously defined fracture surface energy), and the energy per new global fracture area $\Gamma_p$ dissipated in plastic deformation. The term global fracture area means that we consider the area on a scale much larger than the surface roughness. We neglect fracture roughness, which makes the real area larger than the global area. We now consider these energies when the fracture front is locally twisted at the tip (see figure 2.11). We assume that the plastic zone size is still much smaller than the length of the fracture front during twisting.

![Twist and Tilt Diagram](image)

**Figure 2.9. Possible deflections of the fracture plane, and definition of twist angle $\alpha_d$. The naming is according to Lawn (1993).**

For a twist angle $\alpha_d$ with respect to the global fracture plane (perpendicular to the least principal stress), the total fracture area will increase by a factor $\frac{1}{\cos(\alpha_d)}$, so that $\Gamma$ is:

$$\Gamma = \frac{1}{\cos(\alpha_d)} \Gamma_0$$

(2.96)

$\Gamma_0$ is the fracture surface energy for $\alpha_d = 0$. When the fracture plane twists, the longer fracture front will similarly increase $\Gamma_p$ with a factor $\frac{1}{\cos(\alpha_d)}$. However, $\Gamma_p$ is also
influenced by the size of the plastic zone. The size of the plastic zone decreases when the fracture twists, because the confining stress working perpendicular to the fracture plane increases (see figure 2.5). The influence of this stress on the plastic zone size depends on the stress situation and material properties. When we assume that the plastic zone is influenced in the same way by the confining stress as the tensile zone, a relationship between plastic zone and confining stress is given by equation (2.94). Using this, we find an increase in $l_p$ with a factor $\cos^{1-\alpha}(\alpha_d)$. The stress in the $x$-direction is independent from the tilt angle. Furthermore, we neglect plastic deformation as a result of mode III loading of the fracture front, which is present after twisting. This is allowed when the mode III displacement is much smaller than the mode I displacement.

To express $\Gamma_p$ as a function of $\alpha_d$, we need a relationship between $\Gamma_p$ and the plastic zone size. When we assume that the average stress and average plastic strain stay equal during twisting, equation (2.75) predicts a linear relationship between $\Gamma_p$ and $l_p$. We then have for the energy dissipated in plastic deformation per unit of global fracture area:

$$\Gamma_p = \cos^{1-\alpha}(\alpha_d)\Gamma_{p,0}$$

(2.97)

The total energy $\Gamma_{tot}$ per new fracture area consumed in the fracturing process is then:

$$\Gamma_{tot} = \cos^{1-\alpha}(\alpha_d)\Gamma_{p,0} + \frac{\Gamma_0}{\cos(\alpha_d)}$$

(2.98)

We now take $\alpha=1/2$, which coincides with the LEFM stress field around the tip. The conclusions will qualitatively be the same when we take $\alpha=2/3$, which corresponds with the asymptotic solution. We try to find a minimum of $\Gamma_{tot}$ as a function of $\alpha_d$:

$$\frac{d\Gamma_{tot}}{d\alpha_d} = \Gamma_0 \sin(\alpha_d) \left( \frac{1}{\cos(\alpha_d)} - \frac{\Gamma_{p,0}}{\Gamma_0} \right)$$

(2.99)

When $\Gamma_0 > \Gamma_{p,0}$, $\Gamma_{tot}$ has a minimum for $\alpha_d=0$. When $\Gamma_0 < \Gamma_{p,0}$, the minimum at $\alpha_d=0$ becomes a maximum and $\Gamma_{tot}$ has a minimum for

$$\cos(\alpha_d) = \left( \frac{\Gamma_0}{\Gamma_{p,0}} \right)$$

(2.100)

Figure 2.10 show $\Gamma_{tot}/\Gamma_0$ as a function of $\alpha_d$ for various values of $\Gamma_{p,0}/\Gamma_0$. A similar theory can be made for tilting of the fracture plane (corresponding with a rotation around the $z$-axis). This rotation reduces plastic energy dissipation even more, as also the
2. Hydraulic fracturing processes

horizontal stress that induces the plastic strains becomes lower. A combination of both rotations could lead to the geometry sketched in figure 2.11.

![Graph](image)

**Figure 2.10.** Dimensionless total energy per unit of global fracture area dissipated in fracturing and plastic deformation, as a function of the twist angle $\alpha_d$.

![Diagram](image)

**Figure 2.11.** Schematic view of possible groove development as a result of twisting and tilting of the fracture plane.

*Model predictions*

The presented model is a simplification of reality, but incorporates the basic elements of stable fracture propagation. Although the presented model cannot predict the final fracture
surface roughness, it can make qualitative predictions about the roughness. The model predicts that tilting and twisting of the fracture surface is energetically favoured when $\Gamma_{p,0} > \Gamma_0$, while a plane fracture is favoured when plastic energy dissipation is relatively small. This will lead to a tendency for the fracture to twist and tilt when the plastic energy dissipation is large enough. This tendency is contrary to the behaviour of a fracture governed by linear elastic fracture mechanics, which predicts reorientation of the fracture so that mode III and mode II loading is minimised (Lawn, 1993). When the plastic energy dissipation is small enough, the predictions of the model agree with LEFM.

When the energy rates associated with fluid flow and fracture opening are much larger than the energy rate associated with plastic deformation around the tip, the tilting will only occur as a local phenomenon. The fracture plane will stay perpendicular to the least principal stress (which is energetically favourable for opening, and as a result also for fluid flow), and the tilting will only lead to an increase in fracture roughness. When the energy dissipated in plastic deformation is significant with respect to the other energy rates, twisting and tilting of the whole fracture surface is predicted. As a result of twist starting at a uniform fracture front, splitting up of the fracture can occur. Besides explaining roughness of fractures, equation (2.100) predicts an increase of $\alpha_d$ with increasing plastic zone size.

2.10 Conclusions

In this chapter we presented and discussed basic processes of interest for hydraulic fracturing. We can make the following conclusions.

- Although linear elastic fracture mechanics is used extensively, there are clear experimental observations that deviations occur.
- Large-scale laboratory experiments show that the energy needed for the creation of the fracture surfaces of rock on a field scale is approximately equal to the fracture surface energy measured in laboratory experiments. Propagation velocity also shows to have an insignificant effect on stable fracture propagation, for propagation velocities ranging from those in the present study to those in the field.
- Laboratory experiments show that tensile rock fracturing appears to be controlled by effective stress. As a result, pore pressure effects can influence the fracturing process.
- Externally applied stresses perpendicular to and parallel to the fracture plane, have a significant influence on the size of the plastic zone around the fracture tip.
- Analysis of the stress path and plastic zone size indicates that a shear failure criterion can be satisfied before a tensile failure criterion, which implies that the fracturing mechanism at the tip might deviate from pure tensile fracturing.
2. Hydraulic fracturing processes

- For the description of fluid flow in our experiments, we can use locally the solution for Poiseuille flow between parallel plates.
- Scaling of linear elastic hydraulic fracture propagation equations yields dimensionless groups, which can be used for the scaling of our experiments to a field scale. Additional parameters for scaling plastic deformation near the tip are presented.
- There are various ways to model the process zone in hydraulic fracturing simulations. Only in fully coupled elastic-plastic simulations, plastic rock deformation can increase the width of a fracture without changing the wellbore pressure.
- We determined a parameter which indicates whether we can expect viscosity- or toughness-dominated propagation.
3

Experimental method and set-up

In this chapter we present the experimental method and set-up used for hydraulic fracturing model experiments, material characterisation, and measurement of fracture surface roughness. In addition, we describe the experimental procedure and show an example of the measurements done during an hydraulic fracturing experiment. We describe the sample preparation, and discuss the accuracy of the measurements and the stress situation in the block.

3.1 Experimental method

Measurements and variation of parameters
In order to determine the influence of plastic rock deformation on hydraulic fracturing, we took the following approach in the research. We started with experiments on cement in order to validate the elastic model. We compared the results with a fully coupled linear elastic simulator (Barr, 1991). The strength of cement is high, and tensile failure is brittle (see section 4.4), so plastic deformation is not expected to occur for the applied stresses.

Then, plaster and diatomite as weak rocks were used for which plastic deformation near the tip is expected to be significant. Besides this variation of strength by variation of the material, we also varied the externally applied stresses, which are expected to influence the amount of plastic deformation near the tip too. We determined elastic properties and plastic yield- and failure envelopes in triaxial tests and tension tests.
3. Experimental set-up

The measurements done during propagation and closure focused on determining fracture geometry and pressure. We measured fracture width and pressure at the borehole. A measure of the fracture volume was obtained from the block deformation during propagation. With acoustic measurements we measured the radius of the fluid front, the tip radius, and a measure of the size of the fluid lag in cement and plaster. After splitting the block, we could check on the acoustically determined fracture radius and dry tip size when no fracture growth after shut-in took place. In cement, we could measure the width profile of the fracture with a method developed in Groenenboom (1998). We also measured the fracture surface roughness.

Scaling
We choose the mechanical behaviour of our (artificial) rock materials to be representative for reservoir rocks. This requires values for stresses and pressure that are comparable with in-situ values. The length scale of the laboratory fracture is determined by the maximum sample size for our triaxial machine, which is 0.30 m cubic. The data acquisition speed imposes a lower boundary on the time scale for fracture propagation of $10^2$ s, while the maximum practical propagation time is $10^4$ s.

In section 2.8 scaling laws for hydraulic fracture propagation were derived. From the practical limitations just mentioned, the scaling yields that fracturing fluids with a relatively high viscosity are needed, and that the fracture toughness and permeability of the rock must be lower than in the field. Tables 4.2, 4.5, 4.6, and 5.1 give the (elastic) rock- and fluid properties used.

The scaling is based on a representative field case (see section 2.7), in which fracture propagation is expected to be viscosity-dominated. Based on the scaling for linear elastic hydraulic fracture propagation, we also expect viscosity-dominated propagation in the lab. We apply the elastic scaling to the weak rocks as well. In that case, the elastic-plastic scaling in section 2.8 showed that the relative importance of plasticity in the field and in the lab is similar for similar stress-strain behaviour.

3.2 Experimental set-up for hydraulic fracturing experiments

The experimental set-up used for the hydraulic fracturing experiments consists of three parts: the true triaxial compression machine, the hydraulic fracturing set-up, and the acoustic monitoring part. In this section, we describe these three elements.
3. Experimental set-up

Figure 3.1. Photo and schematic view of the true triaxial compression machine. The photo also shows the high pressure pump and acoustic tower.
3. Experimental set-up

True triaxial machine
We simulate the in-situ stresses by loading the blocks in a true triaxial machine, in which the force in each direction can be applied independently from the force in the other directions. As this is an open system, we do not apply pore pressure. The maximum force is 3500 kN, corresponding with a maximum stress of 39 MPa on cubes of 0.30 m size. The forces can be set with an accuracy of 3 kN, corresponding with a stress of 0.03 MPa for cubes of 0.30 m size. The friction in the cylinders is smaller than 2.5 kN. This low value is achieved by hydrostatic bearing of the cylinders.

Figure 3.1 shows a photo and a simplified schematic view of this machine. The pump that supplies the oil pressure can give a main pressure between 0 and 350 bar, which is always somewhat higher than the maximum pressure needed in the cylinders. By adjusting the valves, the control PC controls the reduction of the pump pressure to the main pressure and the reduction of the main pressure to the pressures in the cylinders. The pressure in each cylinder can be set independently from the other cylinders. A second pump gives a constant pressure of 200 bar, which is used for the hydrostatic bearing of the cylinders, and enables us to let the cylinders return to their original position after the experiment has ended. The reference synthesizer PC runs the data acquisition program and sends the pressure set-points to the control PC.

A cooling machine is connected to a closed system filled with water, and cools the hydraulic oil via a heat exchanger. During an experiment of several hours, the temperature of the oil increases from 18 °C to almost 30 °C, especially when high stress differences are applied.

Figure 3.2. Schematic view of the compression set-up in one direction of the true triaxial machine. The LVDTs are connected directly to the end-platens.
The load frame of the true triaxial machine consists basically of three uniaxial frames, which can move independently from each other. Figure 3.2 shows a schematic view of the compression part in one direction. The end-platen are mounted on spherical seats, lubricated with thick grease. The piston and the opposing end-platen are connected with four tension bars. In each direction two LVDTs (Linear Variable Differential Transformer) at opposing corners of the block measure the deformation of the block. These LVDTs are connected to the aluminium end-platen (see figure 3.2). When calculating the block deformation, we corrected for the deformation of the aluminium.

The steel parts on which the end-platen are mounted contain holes for the transducers and cable ducts for the transducer cables (see figure 3.3). The end-platen are made out of aluminium. We used two different designs of end-platen. The old one was 0.28 x 0.28 m in size, while the new one was 0.294 x 0.294 m and had more holes in which the acoustic transducers could be placed. In the new design, we had the choice between end-platen with holes and without holes (with thicknesses of 6 and 20 mm respectively). The end-platen with holes enabled us to put the transducers directly on the block (see figure 3.4), giving a better acoustic signal. However, during experiments on weak blocks we used the end-platen without holes, similar to the old design, to prevent the sample from flowing into the holes. By hindering the lateral movement of the block over the end-platen, such flow can influence the stress situation inside the block significantly.

Between the aluminium end-platen and the block we put a sheet of teflon of 0.1 mm thickness. We greased the teflon at both sides with vaseline to improve acoustical coupling and to diminish shear stresses on the block faces. When we used the aluminium end-platen with holes, we made holes in the teflon sheets at the transducer positions.

**Hydraulic fracturing measurement set-up**

Figure 3.5 shows a schematic overview of the hydraulic fracturing measuring system. It includes the interaction with the acoustic scanning system and the true triaxial machine set-up. At the beginning of an experiment, the time between the three systems is synchronised. During an experiment, the hydraulic fracturing measurement program controls the moment that the true triaxial machine takes a measurement, so that the times in the separate files are equal. The acoustic scanning system acquires its data independently.

Figure 3.6 schematically shows the block set-up, together with the pump, triaxial machine measurements, and acoustic transducers. A high pressure pump injects fluid into the wellbore. Inside the wellbore we mounted an LVDT with clamps (see also De Pater et al., 1994b), which measures the width of the fracture at the wellbore. Figure 3.7 shows this in more detail. The LVDT is mounted in a steel part, which is attached with clamps to the borehole wall. The clamps are pushed against the borehole wall by tightening the nut.
3. Experimental set-up

Figure 3.3. Photo's of the end-platen with holes for the transducers, and the steel part with holes for the acoustic transducers (new design).

Figure 3.4. Schematic view of transducer set-up in the case of massive end-platens, and in the case of end-platens with holes.
3. Experimental set-up

Figure 3.5. Overview of the hydraulic fracturing set-up, and the interaction with the true triaxial machine and the acoustic scanning system.

Figure 3.6. Schematic view of the block set-up.
3. Experimental set-up

The slender screw on which the steel core of the LVDT is mounted, is pushed against a table with a spring. The table is attached with clamps to the borehole wall in a similar way, at the opposing side of the fracture mouth. When the fracture opens, the core will move inside the LVDT, which yields the fracture width. The measuring interval of the LVDT, which is determined by the attachment points of the clamps, is approximately 3 cm effectively. A 3 mm deep notch in the wellbore wall controls the fracture location. Pressure transducers measure the pressure at a dead string and at the position where the tube enters the triaxial machine, almost at the edge of the block. The pressure at the dead string equals the pressure in the wellbore at the fracture mouth. The only flow that takes place in the dead string is caused by the decompression flow rate, which leads to a negligible pressure drop. The data acquisition program that runs on the PC measures the two pressures, the displacement of the LVDT, and the pressure, volume and flow rate at the pump.

At the ends of the borehole aluminium or steel borehole cylinders were glued to form a smooth surface on which O-rings could seal. The dead string and the supply tube, which were placed at the outer ends of the borehole, sealed with rubber O-rings on these smooth surfaces (see figure 3.7). The fluid pressure inside the borehole pushed these parts against the end-platens. A hole of 4 cm diameter was made in the teflon to prevent these parts from pushing against the teflon, which could yield undesirable time-dependent movement during an experiment. During an experiment the end-platens move and the block deforms, which subsequently causes movement of the end-platens. The variation of the borehole volume as a result of movement of the dead string and the supply tube, which are pushed against the end-platens, can be neglected, because it is much smaller than the fracture volume (the proportion of these respective volumes is of the order $R_w^2/4a_{hi}$ in which $R_w$ is the wellbore radius and $A_{hi}$ is the area of one block face). At the end of the dead string is a high pressure transit for the electric wires, connected to the LVDT in the borehole. The dead string has a plug at the other end, which makes contact with the electric wires of the LVDT. The supply tube and the dead string each seal with two rubber O-rings at one of the borehole cylinders, which are glued at the ends of the borehole.

As fracturing fluid we used polymeric silicone oil with viscosities between 12.5 and 500 Pa·s, which is approximately Newtonian at the shear rates of interest (see section 4.7). The volume of the pump and tubes was minimised in order to minimise the decompression flow rate (see section 5.1), which can complicate the pressure response and promote excessive fracture growth after shut-in. The radius of the borehole inside the epoxy layer was 23 mm. Taking the radius too small yields a strong pressure gradient in the fracture near the borehole, which is unfavourable for interpretation of the experiments. To further minimise the decompression volume, we designed a borehole set-up with a non-return valve (see figure 3.7). This valve closes at shut-in and disconnects the pump from the decompression volume. In this way, the system stiffness could be doubled. The non-return
Figure 3.7. Schematic view of the borehole set-up. The borehole set-up aims to fill much of the borehole volume, in order to minimise the decompression volume. Space is left for the fluid so that the dead string is hydraulically connected with the fracture mouth and the whole borehole wall is pressurised by the fluid.
3. Experimental set-up

valve was made out of brass, which sealed directly on the steel part in the borehole. This non-return valve was used in most experiments on cement, but not in plaster and diatomite.

Acoustic scanning system
During an experiment we did active acoustic measurements using compressional (P) and shear (S) wave ultrasonic transducers. Every transducer can act as a source or as a receiver. The transducers have a circular contact area of 14.4 mm diameter and a nominal element size of 12.7 mm (type Panametrics V103-RM and V153-RM). The transducers generate waveforms with a centre frequency of 1 MHz and a signal length of about 5 microseconds. A maximum of 48 transducers can be used. It took approximately 45 s to measure and store the results of all combinations between these transducers, and this time could be reduced by reducing the amount of measured combinations. The data acquisition system used for the acoustic measurements is described extensively in Savic (1995), Romijn and Groenenboom (1997), and Groenenboom (1998).

Figure 3.8 shows a schematic view of the set-up used for the ultrasonic measurements. The switchbox connects the pulser to the transmitting transducer, and the receiving transducers to the filter and gain unit. The connection between the transducers established by the switchbox enables each transducer to operate in one of three modes: the active (transmitting) mode, the passive (receiving) mode, and the pulse-echo (transmitting and receiving) mode. Prior to A/D conversion in the transient recorder, the signal is high- and low-pass filtered and amplified in the filter-and gain unit. A PC controls the acoustic measurements and stores the data on a hard disk. During an experiment, the acoustic data can be monitored almost real-time, which allows us to shut-in the pump at the moment the fracture radius has reached the desired length.

![Figure 3.8. Schematic view of the acoustic data acquisition system.](image-url)
3.3 Experimental procedure for hydraulic fracturing experiments

We did the experiments according to a fairly fixed procedure. After the block preparation, we mount the borehole set-up inside the borehole. We apply vaseline and teflon to the end-platens and place the block in the true triaxial machine. One day before the test, we filled the pump with the viscous silicone oil in order to give the air bubbles enough time to escape from the fluid. When using cement blocks, we then apply the stresses in the true triaxial machine, after which we fill the block and tubes with fracturing fluid, for which we use part of the pump volume. In the case of diatomite and plaster, we fill the borehole and tubes before loading the block, giving the fluid at least one day to let air bubbles escape. In plaster this is done because most of the pump volume is needed to create the fracture, while in diatomite the pressure needed to fill the system in a reasonable amount of time would be too high in comparison with the applied stresses and material strength. Filling the borehole before loading the block also enables us to increase the fluid pressure inside the borehole during loading. This is done to assure wellbore stability. Then, we pressurise the borehole and create the fracture. After fracture closure, we depressurise the borehole and subsequently unload the block. At the end, we saw the block in pieces and split the parts over the fracture surface, or saw slices for microscopic visual inspection.

3.4 Measurements during hydraulic fracturing

During loading of the block, we determine the elastic properties of the rock by measuring the applied force and the block deformation. During hydraulic fracture propagation, we measure the wellbore pressure at the dead string, the pressure at the pump, and the pressure at the inlet of the block. The fracture width is measured by the LVDT in the borehole. The injected volume is measured at the pump. The block deformation during propagation is a measure for the fracture volume. Figure 3.9 shows an example of these measurements.

Acoustic scanning can yield the radius of the fluid front and fracture tip, and the fracture width profile. Figure 3.10 shows the classification of the source-receiver combinations that were of interest for this study. We found that the tip of the fracture and the fluid front both cause a diffraction of the ultrasonic signal. This diffraction is clearly visible after subtraction of the original wavefield (without a fracture in the block), which is shown in figure 3.11.

From the diffraction record, we know the travel time of the diffraction as a function of experiment time. We also know the transducer positions relative to the block, with an
3. Experimental set-up

accuracy of 3 mm. We assume that the diffraction originates from the position on the fracture radius where the travel time from source to receiver has a minimum. Only diffractions from this region are expected to show constructive interference (Groenenboom, 1998). Furthermore, we know the velocities in the block and in the end-platen, and we take into account the small delays in the teflon sheets, vaseline, and in the scanning system. We assume that the fracture grows from the centre of the block where the notch is located, perpendicular to the least compressive stress. Using Snell's law, the travel path can be constructed in which the fracture radius is the only unknown. In this way, we can determine the fracture radius, the radius of the fluid front and subsequently the fluid lag size. The source-receiver combinations used for radius determination from the diffractions always consist of one transducer located at the side of the block, and one transducer at the top or at the bottom of the block (see figure 3.10 for an example of such a combination).

3.5 Sample preparation

We used Portland B cement paste and plaster as artificial model materials, and diatomite as natural rock. The artificially made model blocks were poured in a mould of 0.30 x 0.30 m, and 0.38 m high. A steel rod was centered in the middle, in order to make the borehole and space for the borehole cylinders. Then we poured the cement or plaster paste inside the mould and vibrated it on a vibrating table for several minutes, in order to remove air bubbles. After curing, the steel rod was removed and approximately 5 cm was sawn from the top of the block at a height of about 0.305 m. The remaining material was ground away. The final block had dimensions of 300 ± 0.1 mm, and the faces were parallel and at right angles to each other with an accuracy of ± 0.1 mm. The faces were globally flat with an accuracy of ± 0.02 mm for the ground face, while the other faces were somewhat more rough. The diatomite blocks were treated in the same way, with the difference that six sides were ground. Two sides were approximately parallel to the bedding plane. Because of the large deformations when loading diatomite, we made cubes of 0.31 m size. The borehole and room for the borehole cylinders were bored.

The water/cement weight ratio of the cement paste is 0.40. After pouring, the cement was placed on a vibrating table for 30 to 120 s to remove air bubbles. This cement sets relatively slowly, which gave us enough time to pour and vibrate the block. The cement blocks cured for at least 28 days before testing and were kept in a 100 % humidity environment.
Figure 3.9. Example of raw data measured during hydraulic fracturing (experiment sp04). The first figure shows the measured pressure $p$ at the dead string and the inlet of the block, and the fracture width $b_w$ measured at the wellbore. The second figure shows the flow rate $Q_{pump}$ at the pump, the effective flow rate $Q_{eff}$ flowing into the fracture, and the fracture volume $V_{frac}$ calculated from the block deformation according to equation (2.70). Characteristic points in the pressure development during hydraulic fracturing are indicated in the upper figure.
3. Experimental set-up

Figure 3.10. Classification of acoustic transducer combinations as used in this study.

Figure 3.11. Example of P-wave diffraction record of a propagating fracture in weak plaster (upper left, experiment wp01) and cement (lower left, experiment cc04). In both experiments, the diffraction from the tip is visible as the first relatively weak event, followed by the stronger diffraction from the fluid front. The figure at the right shows an S-wave transmission record in strong plaster (experiment sp05). After fracture closure, the transmission energy is restored almost completely.
3. Experimental set-up

To make the plaster blocks, we used β-hemihydrate (see section 4.1 for the material description). The water/hemihydrate weight ratio was 0.50. We added the hemihydrate to the water while stirring the mixture using two boring-machines. It took 3 to 5 minutes to have a homogeneous mixture. After pouring, the mixture was placed on a vibrating table for 2 minutes to remove air bubbles. We had added a retarder which increased the setting time with approximately 5 minutes, so that we had 10 minutes from the start of mixing to the end of vibrating. One day after pouring, we placed the blocks in a 100 % humidity room for several days to complete the conversion into dihydrate. Afterwards, the blocks were dried in a ventilated oven at 40 °C to remove water from the pores.

The water content affects the strength and Young's modulus of the plaster (see section 4.2). When the water content becomes lower than approximately 0.03, the strength and modulus increase sharply. In our experiments, we used plaster with two different water contents. First, we used plaster blocks that were kept in the oven until the weight stayed constant (typically 15 days), indicating that no water leaves the pores anymore. The constant weight also indicates that at our drying conditions very little water was removed from the crystal structure. Then, the blocks were kept at atmospheric conditions, and a weight increase of approximately 20 g was measured, most probably due to the humidity of the air. This air-dried plaster will be referred to as "strong plaster". Second, we used plaster blocks that we dried until the water content was between 0.04 and 0.06 (the water content is defined as the weight of the absorbed water divided by the weight of the air-dried rock matrix). The material properties are approximately constant in this range of water contents (see section 4.2). We will refer to these blocks as "weak plaster".

Our diatomite blocks had a very clear layering, and inherent anisotropic properties. The water content of the blocks was between 0.42 and 1.03. We found that the mechanical properties are approximately constant for these saturations (see section 4.2). When the blocks dried out too far, they fractured along the bedding planes.

The final preparation of the blocks started with glueing the borehole cylinders in the block, on which the O-rings can seal (see figure 3.7). Care was taken that the borehole cylinders were about 1 mm below the block surface, so that they could not touch the end-platens during compression in the triaxial machine. At the same time, a 0.5 mm thick glue layer was made at the interior of the borehole. This was done by putting a teflonised steel rod - or a steel rod wrapped with teflon foil in the case of diatomite - inside the borehole. The rod fitted exactly in the borehole cylinders, while leaving 0.5 mm space to the borehole wall. This space was then filled with epoxy. After removing the steel rod, a notch was sawn transversely in the borehole wall. We did this by placing the block on a rotating table, and placing a horizontal saw in a boring-machine. The eccentricity of the axis of the boring-machine with respect to the borehole was increased until the notch depth was 3 mm. To enable the tubes to enter the borehole, which connect the pump and the pressure transducer at the dead string to the borehole, two grooves were ground from the borehole
3. Experimental set-up

to an edge of the block (visible on figure 3.7). The depth and width of these grooves is approximately 0.5 cm.

3.6 Stress situation in the block

There are several causes that can make the stress situation in the block deviate from the expected stresses, based on the applied forces and block size. We quantified the effect of friction between end-platen and block, the steel borehole cylinders, and non-total coverage of the block by the end-platens. Full details can be found in van Dam and De Pater, 1997a.

Shear forces between block and end-platen

Shear forces that build up at the contact planes between the sample and the end-platens influence the stress state inside the block. These shear forces are caused by a difference in displacement - in a direction parallel to the block surface - between block and end-platen in combination with friction between block and end-platen. This effect can influence the uniaxial compressive strength of concrete specimens considerably (e.g. Kotsovos, 1982) and the effect of frictional restraint on the stress in the block is formidable in true triaxial testing, according to Jaeger and Cook (1969), § 6.7. This is caused by the cubic sample shape in true triaxial tests, in combination with the fact that the deformations parallel to the end-platen surface tend to be relative large (caused by the loading in these directions). van Mier and Vonk (1991) found a significant influence of boundary conditions on softening behaviour in uniaxial and true triaxial tests. They studied softening using dry end-platens (without the application of any friction-reducing medium between end-platen and block), end-platens with brushes, and slightly greased end-platens covered with thin teflon sheets. A strong effect of boundary conditions on the lateral deformation in biaxial tests was found in Gerstle et al. (1980). This lateral deformation was much lower when no friction-reducing method was applied.

Two-dimensional finite element calculations (Veeken, 1988) showed that the high friction between end-platen and block can reduce the stress in the middle of the block to 60% of its expected value, even falling to zero near the boundaries. We found a 50 % reduction in the centre of the block using the two-dimensional finite element program FLAC (van Dam and De Pater, 1997a). The importance of using a lubricant between block and end-platen for the stress situation in the block was also recognised in Stuart (1992). Two hydraulic fracturing experiments on cement, in which the steel parts containing the transducers (see figure 3.3) were placed directly on the block without the application of any friction-reducing method, indeed showed a strong effect of the boundary conditions on
the stress inside the block (see van Dam and De Pater, 1997b). Fracture initiation and propagation pressures indicated that the stress perpendicular to the fracture plane was decreased with approximately 35% compared to experiments where a friction-reducing method was applied (which is described below).

To establish a uniform and known stress state in the block, the shear stress at the surface must be diminished as much as possible. To measure the maximum value of this shear stress, we did a "direct double shear test" (see Labuz and Bridell, 1993). A block was loaded uniaxially in the x-direction of the true triaxial machine and a force in the z-direction was applied. We increased this force until the static friction was overcome and the block started to move. If the friction was very low, we applied no force in the z-direction but instead diminished the compressive force in the x-direction with discrete steps until the block started to move under its own weight. Because the dynamic friction is lower than the static friction, we could infer a maximum for the possible shear stress at the surface of the end-platen. Additionally, we compared different friction-reducing methods with each other.

To reduce the friction, we mainly looked at two options for the friction reducing material (called "solid lubricant"). The configurations which we studied are (going from end-platen through the solid lubricant into the block): end-platen (made of aluminium) - vaseline - 1. teflon sheet of 0.1 mm thickness or 2. lead of 1 mm thickness - block. Lead greased with vaseline was used as solid lubricant in Weijers (1995). Because the end-platens are smoother than the cement, we expect that during hydraulic fracturing experiments the movement will take place between end-platen and solid lubricant. To ensure that the movement during these friction experiments occurred between the solid lubricant and the end-platen, we omitted putting vaseline between the cement and the solid lubricant. Because the compression of a cement block between lead plates gave problems with tensile splitting of the block, we also used a steel block instead of cement. Actually, this steel block was made out of 4 separate blocks with dimensions 75 x 300 x 300 mm. We expect that there will be little difference in the measured friction when a cement or steel block is used in combination with lead, while there could be a difference when using 0.1 mm thin teflon because of the difference in surface roughness of cement and steel.

The friction coefficient is defined as the ratio of the force parallel to the surface needed to obtain movement, to the force working perpendicular to the surface. Figure 3.12 shows that the friction coefficient that we measured in the case of aluminium-vaseline-teflon-steel can be as low as 0.001, while the friction coefficient of teflon on polished steel without the application of vaseline is approximately 0.05 (Mokha et al., 1993). The vaseline probably forms a lubricating film (whose thickness decreases with time and increasing confining stress). In the case of aluminium-vaseline-teflon the measured friction coefficient is between 0.001 and 0.008. In the case of aluminium-vaseline-lead, the measured friction coefficients were between 0.008 and 0.02. Figure 3.12 shows the measured friction coefficients as a function of experiment time (experiment time t=0
corresponds with the moment that loading in the $x$-direction started). An increasing friction coefficient with time was measured for both lead and teflon. The effect of stress on the friction coefficient was not noticed: we did too few measurements and the time dependence of the friction coefficient for the times of interest (several hours) appeared to be stronger. The confining stress varied between 8.25 and 33 MPa for lead and between 10 and 20 MPa for teflon.

These experiments were done with end-platens without holes. The holes probably accelerate the decrease of the vaseline layer thickness, giving a higher friction coefficient. The difference between experiments 1 and 2 with lead can be attributed to the fact that we placed the parts of the steel blocks in experiment 1 with the joints against the end-platen: the joints promote the formation of channels between lead and vaseline through which the vaseline can flow away. The variation in the amount of vaseline that we had put on the end-platen before the experiment started is another reason than can explain the difference between the two experiments.

![Graph showing friction coefficient $\mu_f$ as a function of time for different loading configurations.](image)

Figure 3.12. Friction coefficient $\mu_f$ as a function of time for different loading configurations.

From these experiments we can conclude that teflon in combination with vaseline yields lower friction coefficients than lead in combination with vaseline using cement blocks or blocks with a similar or smaller surface roughness. For this reason, we used teflon in almost all hydraulic fracturing experiments. Although the confining stress using teflon (10 - 20 MPa) was lower than the maximum during hydraulic fracturing experiments (23 - 33 MPa), we expect - based on the experiments with lead - that the

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measured friction coefficients are also representative for the friction under these higher confining stresses. With a value of the friction coefficient of 0.01 we have a maximum relative distortion of 0.02 times the ratio of horizontal to confining stress, which is acceptable.

**Influence of the borehole cylinders**
Except for the shear forces on the block faces, the stress inside the block could be disturbed by the steel or aluminium borehole cylinders that are glued at the ends of the borehole. Calculations using FLAC with an axisymmetric configuration show that the borehole cylinders lower the confining stress near the borehole in the centre of the block by approximately 5% (see van Dam and De Pater, 1997a).

**Influence of non-total coverage of the block surface**
The end-platens leave stress free strips on the block surface, necessary to avoid touching of the sides of adjacent end-platens. This causes a slight concentration of stress in the centre of the block. To estimate how large this effect is, we did simulations using FLAC. As boundary conditions we took uniform stress, while no restrictions on displacement were applied. This is a reasonable boundary condition because of the presence of the relatively soft solid lubricant and the vaseline. We found that in a two-dimensional situation, the stress in the centre is increased with 8%. However, due to the stress free strips in the lateral directions the stress in the centre perpendicular to the fracture surface is decreased. The effects approximately cancel each other for typical experimental conditions. This effect was confirmed by measurement of strains inside the block in the centre and 3.2 cm from the surface (see van Dam and De Pater, 1997a). Halfway through the research, the size of the stress free strips was decreased from 10 to 3 mm in the new end-platens design in order to diminish the effect and increase the accuracy of the experiments.

### 3.7 Accuracy of the measurements during hydraulic fracturing

Variability of experimental results in rock mechanical tests is usually largely determined by sample variability. Besides this variability, systematic errors are present in our measurements. In the fracture width measurement at the wellbore during hydraulic fracturing, a systematic error was made by measuring the width over a certain interval. The net fluid pressure that opens the fracture, also compresses the rock inside this interval,
3. Experimental set-up

which causes the measured width to be systematically too low. The length of this interval is difficult to determine, because the clamps are attached to the wellbore over a certain length. However, during the loading phase the effective length of this interval can be determined. Especially for small radius, when the net pressure is large, the error in the width measurement is significant. We correct for this effect by calculating the displacement of the rock material in the measuring interval (see section 5.1).

Due to fluid compressibility and deformation of wellbore and tubes, the volume of the fluid in the system (consisting of pump, wellbore, and tubes) varies with the fluid pressure. This volume variation causes fluid storage in the system, so that the flow rate applied at the pump differs from the flow rate that effectively flows into the fracture. In section 5.1 we show how we determine the effective flow rate.

The third systematic error is the difference between the real and expected stress in the sample. The stress situation in the block was analysed in the previous section and in van Dam and de Pater (1997a). The deviations of the stress in the region of interest from the expected value, caused by various sources, partly cancel out. It was found that the stress in the region of interest differed less than 10% of the expected stress based on the applied force and the block size.

3.8 Set-up for triaxial tests and leak-off tests

We did triaxial tests to determine the strength and deformation properties of samples made of our materials. In these tests, the sample was loaded with compressive stresses. We used two triaxial cells for these experiments. The first one, with which we performed only few experiments, was a Hoek cell. It contains a sleeve within a steel cylinder with radial oil pressure (see Hoek and Franklin, 1968). The sample - placed in the cylinder - is placed under a uniaxial compression machine, which has spherical seats. The samples in this cell have a length and diameter of approximately 110 and 50 mm respectively. This cell offers the possibility of measurement of axial stress, radial stress, and axial deformation using LVDTs placed outside the cell. The second triaxial cell offers the possibility of measuring axial and radial stress, and deformations in both the axial and radial direction. In this second triaxial cell, we performed triaxial tests and leak-off tests. This cell will be described in the following. In both cells, the axial force is measured accurately with a load cell.

Figure 3.13 shows a photo and schematic view of the triaxial cell. The samples in this cell had a length and diameter of approximately 80 and 40 mm respectively. The devices used for strain measurement, the load cell, and the pressure vessel that contains the oil giving the radial pressure, were made by TerraTek Systems, Salt Lake City, U.S.A. Axial
3. Experimental set-up

Figure 3.13. Photo of displacement measurement set-up and schematic view of the triaxial compression cell.
3. Experimental set-up

dehoration is measured by strain gauges on four cantilevers, pushed aside by a conical ring that is connected to the upper end-piece. The radial deformation is measured by four cantilevers with strain gauges. A load cell measured the axial stress. The displacement and force measurements are compensated for changes in temperature with additional strain gauges. As a result, the strain measurements do not respond to isotropic loading as a result of pressure changes of the oil in the cell. The measured force was corrected for this effect using the measured radial pressure. The friction of the piston at the rubber O-rings depended on the radial pressure. The friction was measured and incorporated in the control and acquisition program. The measurement of axial strain was corrected for deformation of the steel end-pieces between sample and attachment points of the axial gauges, up and below the sample (see figure 3.13). We determined the stiffness of the steel over this measuring interval using an aluminium sample with known properties. A similar procedure to correct the displacements was used in the Hoek cell. Uniaxial compression tests were performed using a similar uniaxial compression machine as was used for the Hoek cell. The samples in the uniaxial compression tests had a length and diameter of approximately 80 and 40 mm respectively.

The surface of the end-piece in the triaxial cell contained a pattern of grooves, which made it possible to apply fluid on the axial face of the sample. In our leak-off experiments, we applied fluid pressure - being lower than the radial and axial stress on the sample - on one axial face of the sample and kept the other axial face at atmospheric pressure. We used a high pressure pump to apply this pressure, and measured the flow rate at the pump. The pressure was measured using a pressure transducer positioned as closely as possible to the sample, in order to make the pressure drop to the sample as small as possible.

In the triaxial tests, 1.5 mm thick steel end-platen's with a surface roughness smaller than 4 μm were placed on the end-pieces to prevent the sample from flowing into the grooves. On the steel end-pieces a layer of 0.1 mm thick teflon was placed, while we applied 0.025 g vaseline between the teflon and end-platen. This was done to prevent shear forces between sample and end-platen and to establish a uniform stress situation in the sample. The same configuration of teflon and vaseline was used in the triaxial tests in the Hoek cell, and in the uniaxial compression tests. We measured the deformation of teflon and vaseline using an aluminium sample and found that it was negligible.

We used a teflon shrink fit sleeve of 0.5 mm thickness with a tensile strength of approximately 20 MPa, and a Young's modulus of 0.4 GPa. Even in diatomite, using these sleeves has an insignificant influence on the measurements in the axial direction. Using an aluminium sample, the effect of the sleeve on the radial deformation was found to be negligible. The sleeve had to be heated to a temperature above 90 °C to make it shrink.
3.9 Set-up for tensile tests

We did three kinds of experiments in which tensile stresses are induced in the sample. These are: uniaxial tensile tests, three-point bending tests, and indirect tensile tests (Brazilian tests). The parameters that we determined from these tests are separation energy or fracture surface energy, Young’s modulus in tension, and strength under tensile and combined compressive and tensile loading.

**Uniaxial tensile tests**

Uniaxial tensile tests were performed in order to determine the stress-strain behaviour before fracturing has taken place, the softening curve of the fracture, and the separation energy of the created fracture. Uniaxial tensile tests that yield this information were performed at the faculty of Civil Engineering of Delft University of Technology, by Van Vliet and Van Mier. In uniaxial tensile tests, the sample is subjected to a tensile stress in one direction. The tensile force is measured, together with the displacement in the direction in which the tensile force works. The experiments were deformation controlled, in order to try to measure the softening branch.

![Sample shape for uniaxial tensile tests](image)

*Figure 3.14. Sample shape for uniaxial tensile tests (after van Vliet and van Mier, 1997). Experiments were done using samples with \( w_t = 50, 100, \) and \( 200 \) mm.*

The experiments were done on "dog-bone" shaped samples with a thickness of 100 mm and a proportion between smallest and largest width of 0.6. Experiments with three
different sample sizes were done, keeping all proportions constant (see figure 3.14). The three sample sizes had smallest widths of 30 mm, 60 mm, and 120 mm (referred to as sample A, B, and C respectively). The stiff end-platens are glued to the ends of the sample with epoxy. This hinders lateral deformation, and could lead to a measured Young’s modulus that is somewhat too large. The deformations were measured using four LVDTs located at the corners of the specimen. The tensile force was applied eccentric to the centre of the specimen, with a distance from the centre of 2 % of the largest width. This was done in order to promote stable propagation from one side to the other. The end-platens could rotate freely, which promotes cracking from one side to the other, without the occurrence of secondary cracks from the other side. Further information about this type of test can be found in van Vliet and van Mier (1997) and van Vliet (2000).

Three-point bending tests
To determine the influence of the water content on the fracture surface energy in plaster, we performed 3-point bending tests on strong and weak plaster and compared the results. This was done because the uniaxial tensile tests on strong plaster were unstable after reaching the peak load, so no separation energy could be determined from these tests. In a 3-point bending test, the sample is supported at two position, while in between these positions a force is applied. We calculated the fracture surface energy from the loading force and the displacement at the loading point (see section 4.5).

The three-point bending tests were done using a test set-up that is in accordance with the ISRM standards (Ouclerlony, 1988). The sample diameter was approximately 49 mm, in which a Chevron notch was sawn with a thickness of 1.6 mm. The remaining fracture surface was 11.6 cm². The tests were done on a displacement controlled uniaxial compression machine. The crack mouth opening displacement was measured, as well as the displacement and the force at the loading point, which is located 180 degrees from the crack mouth.

Brazilian tests
We performed Brazilian tests on weak and strong plaster samples with diameter between 40 and 55 mm, and a thickness of about 25 mm. In a Brazilian test, a cylindrical sample is diametrically compressed, in which a lateral tensile stress is induced in the sample. These tests yield information about the strength under combined tensile-compressive loading. The Brazilian tests were done using a uniaxial compression machine with flat steel end-platens, which were placed directly onto the sample.
3.10 Laser profilometer

We measured the surface roughness profile of a created fracture using a laser profilometer, which is based on the triangulation method (Blum et al., 1990). The profilometer measures the roughness profile along a straight line. In this technique a laser illuminates a spot on the fracture surface. Part of the diffusely reflected light is projected on a light sensitive detector via a lens (see figure 3.15). The position of the projection on the detector depends on the distance between the spot position and the detector. The configuration of lenses project the spot of the laser sharply on the detector when the sample is within the vertical measuring range. This vertical measuring range is approximately 1 cm. In the profilometer that we used, two measuring devices are present. By averaging both measurements proportional to their intensity, the influence of shadow caused by the irregular fracture surface is diminished. Also, the sensitivity to lateral deviations is increased significantly. The vertical accuracy of the measurements is 10 μm, the spot size of the laser is smaller than 35 μm, and the sampling interval is 25 μm.

![Diagram of laser profilometer setup](image)

*Figure 3.15. Schematic view of the laser profilometer. The diffusely reflected laser light is projected on the detectors. The laser profilometer moves with a constant speed over a rail, and scans the sample along a straight line.*

3.11 Conclusions

In this chapter, we described the experimental set-up used in hydraulic fracturing experiments and the set-up used for material characterisation experiments. We conclude by listing the measurements we can make.
3. Experimental set-up

- We can measure the net pressure and fracture width at the wellbore, the fracture radius, the fracture volume, the fluid lag size, and in cement the entire fracture width profile.
- By preventing friction between the end-platens of the true triaxial machine and the surfaces of the block, we can accurately apply uniform stresses inside the block.
- For the entire range of mean stress, we can determine the failure envelope, using various rock mechanical tests. We can measure plastic shear and volumetric strain in triaxial compression and extension tests.
- We can measure the permeability or leak-off coefficient of the rocks using fracturing fluid, under confined conditions in the triaxial cell.
- We have two independent tests, which can be used for the determination of the separation energy.
- We can measure the roughness profile of the fracture surfaces along a straight line.
4

Material characterisation

This chapter contains the results of tests done to characterise the rock materials we used in hydraulic fracturing model experiments. Its main purpose is to characterise the material by its deformation and failure behaviour under applied stress. The material properties obtained are used for the interpretation of hydraulic fracturing model experiments, and form input for numerical simulations. We also give the most important petrophysical properties of the materials. In addition, we describe the properties of the fluid used to create the hydraulic fractures.

4.1 Microstructure of rocks

Cement
The basic materials for the fabrication of Portland cement are chalk (CaCO₃), aluminium oxide (Al₂O₃), quartz (SiO₂), iron oxide (Fe₂O₃), and small amounts of other substances such as magnesium oxide, sodium- and potassium oxide, and some metal oxides (Reinhardt, 1985). The most important components of the cement are di- and tricalciumaluminate (2CaO·SiO₂ and 3CaO·SiO₂), tricalciumaluminate (3CaO·Al₂O₃), and calciumaluminateferrite (4CaO·Al₂O₃·Fe₂O₃) (van Breugel, 1995). After addition of water the cement reacts in a complicated series of reactions to cement stone, which consists of several composites and has an irregular structure. The hardened cement paste consists of individual particles of hydration products. After 28 days most of the hydration has taken place and the properties of the cement stone are approximately constant (Reinhardt, 1985).
4. Material characterisation

Figure 4.1. Microstructure of cement, plaster, and diatomite (from top to bottom).

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The cement has its strength mainly from calciumsilicahydrates, which form crystals sheets. These sheets roll up to form small needles or fibres (Reinhardt, 1985). The pore size diameter of most of the pore space is smaller than 1 μm (Reinhardt, 1985), while the size of the individual cement particles can be between 1 and 100 μm (van Breugel, 1995). Figure 4.1 shows an example of the microstructure of cement.

**Plaster**

We used β-hemihydrate (CaSO₄·1/2H₂O) to make our plaster blocks. The gypsum (CaSO₄·2H₂O) from which the hemihydrate was fabricated was obtained from a Zechstein gypsum formation near Walckenried, Germany. The hemihydrate is available under the name Almod from the company Boergardts GmbH, Walckenried, Germany.

Plaster consists of crystals of calciumsulphate (CaSO₄·2H₂O), which form a structure of needles (see figure 4.1 and e.g. Coquard and Boistelle, 1994). Under applied stress, several deformation mechanisms are possible in plaster. These were studied by de Meer (1995) in creep experiments, but similar mechanisms may occur when plaster deforms plastically in our experiments. At room temperature, it was concluded that pressure solution at grain contacts, presumably in combination with sliding of grains, was the dominant deformation mechanism in uniaxial creep experiments on wet gypsum. Crystal plasticity or microcracking had an insignificant contribution to the creep.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \varphi_p ) (-)</th>
<th>( \rho_{bulk} ) (10³ kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cement *</td>
<td>0.021 ± 0.006 **</td>
<td>2.04 ± 0.01</td>
</tr>
<tr>
<td>plaster ***</td>
<td>0.426 ± 0.012</td>
<td>1.331 ± 0.017 (air dried)</td>
</tr>
<tr>
<td>diatomite ****</td>
<td>0.704 ± 0.010</td>
<td>0.64 ± 0.02 (air dried)</td>
</tr>
</tbody>
</table>

* Water saturation is 0.7 ± 0.2 (Weijers, 1995)
** After Weijers, 1995
*** Cores taken from three different blocks.
**** Cores taken from a single block.
4. Material characterisation

Diatomite
Diatomite is a natural rock and consists mainly of partly broken silica shells of diatoms, which are single-cell organisms that live in seas and lakes. Figure 4.1 shows the microstructure of the diatomite. Diatomite is a laminated rock, with a clear bedding plane orientation (see figure 4.30). The diatomite was obtained from a quarry in Lompoc, CA, U.S.A., which is in the Miocene Monterey formation. Table 4.1 shows measured values of porosity and density. These measurements agree approximately with properties of diatomite given in De Rouffignac and Bondor (1995).

4.2 Influence of water content on mechanical properties and strength

An effect of water content on modulus and strength in conventional rock mechanical tests is measured for a wide variety of rock types, for example sandstones (Hawkins and McConnell, 1992), shale (Van Eckhout, 1976), and chalk (Schroeder et al., 1998). In plaster, a similar effect is measured (Russel, 1960, de Meer, 1995, Kato et al., 1980, and Coquard and Boistelle, 1994). The last two studies show that when other fluids than water were used to saturate plaster, the magnitude of the reduction in strength changed.

Van Eckhout (1976) mentions various possible explanations for the effect of water content on the strength and mechanical properties of rock. First, the capillary force of the fluid menisci between the grains contributes to the strength. A slight increase in the water content can make these menisci disappear, followed by a decrease in strength. Second, the measured surface energy of the solid depends on the surrounding medium. Kato et al. measured a decrease of the mechanical strength of saturated gypsum with increasing surface energy of the saturating liquid. Third, the liquid can act as a lubricant between sliding grains.

Furthermore, water can lead to solution processes at grain boundaries. De Meer (1995) shows that this mechanism can explain measurements of creep in wet gypsum. Coquard and Boistelle (1994) found that liquids that have a high gypsum solubility as well as a high dielectric constant reduced the strength and modulus most, while solubility alone was not the controlling factor. The increase and decrease in strength after wetting and drying was repeatable many times (also after flushing the sample), which violated the hypothesis that small amounts of some kind of cement are responsible for the strength of dry plaster. They suggest that electrostatic interactions between the gypsum crystals in contact are important for the strength and modulus of plaster (see also Israelachvili, 1991), and that the fluids can shield the electrostatic forces between the gypsum crystals in contact.
4. Material characterisation

Figure 4.2. Uniaxial compressive strength $U_0$ and Young's modulus $E$ determined in unloading loops of plaster as a function of the water content $w$.

Figure 4.3. Uniaxial compressive strength $U_0$ and Young's modulus $E$ determined in unloading loops of diatomite as a function of the water content $w$.

To determine the sensitivity of the mechanical properties and strength on the water content, we carried out a series of uniaxial tests on plaster and diatomite for various water contents. The water content $w$ is defined as the weight of the water in the pores divided by the weight of the dry sample, while the water saturation is the volume fraction of the pore space that is occupied by water. Figures 4.2 and 4.3 show that below a certain value of the
water content the material becomes considerably stiffer and stronger. Above this value - and well below 100 % water saturation, where pore pressures can build up - the unloading Young's modulus and uniaxial compressive strength are approximately independent of the water content. Figure 4.2 shows that for plaster this value is about 0.04, and in diatomite it is about 0.30. Russel (1960) found an increase in the uniaxial compressive strength at water contents below 0.05 in plaster with water/gypsum ratio's of 0.55, 0.70, and 0.85. At a water content of 0.012, the strength had increased about 10 % for a water/gypsum ratio of 0.55. A further decrease in water content resulted in a very sharp increase of about 2.5 times for oven dried samples. A similar limit of 0.05 in the water content was found by Coquard and Boistelle (1994) by determining the hardness of a plaster with a porosity of 0.525. This hardness was determined by measuring the resistance to the indentation of a needle.

In the hydraulic fracturing experiments we wanted to have conditions that were well-defined and easy to establish. Therefore, we did experiments with plaster blocks that are air-dried (referred to as strong plaster) and plaster that has a water content between 0.04 and 0.10 (referred to as weak plaster). Making the water content too high could give complications due to build-up of pore pressure and changes in permeability. The diatomite blocks we used in the hydraulic fracturing experiments had a water content between 0.29 and 1.03. In the triaxial tests, the water content was between 0.03 and 0.08 for plaster and between 0.23 and 0.53 for diatomite. From the triaxial test results, no distinction could be drawn between the water contents within this range. The water content in diatomite and weak plaster is above the level below which the rock becomes significantly stronger, and low enough to prevent pore pressures building up. For the strong plaster, possible variation of the moisture content in the air could influence the mechanical properties and strength of air-dried plaster somewhat.

We performed triaxial tests under high confining stress, in which we measured the leak-off rate of silicone oil into plaster (see section 3.8). The same type of silicone oil was used as fracturing fluid in the hydraulic fracturing experiments. Because a significant part of the sample was saturated with silicone oil in these experiments and the applied stress was relatively high, we can also infer a possible effect of the silicone oil on the strength of plaster. From these experiments, we can conclude that the silicone oil does not influence the Young's modulus and strength of the plaster significantly. No significant increase in deformation of the sample was measured as a result of the saturation with silicone oil.

In cement, drying causes shrinkage cracks (see e.g. Reinhardt, 1985). To prevent this, we placed our blocks in a 100 % humidity room, or kept our cement blocks under water. According to Weijers (1995), this leads to a water saturation of 0.7 ± 0.2. Before an experiment, the cement was exposed to air for a few hours maximum.
Table 4.2. Static elastic properties of all materials determined in various tests. The Young’s modulus $E$ and Poisson’s ratio $\nu$ were determined in unloading loops.

<table>
<thead>
<tr>
<th></th>
<th>uniaxial tension</th>
<th>uniaxial compression</th>
<th>triaxial test</th>
<th>loading hydraulic fracturing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Young’s modulus $E$</strong> (GPa)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cement</td>
<td>17 *</td>
<td>20 ± 4</td>
<td>19 ± 3 **</td>
<td>21 ± 2</td>
</tr>
<tr>
<td>strong plaster</td>
<td>10 ± 3</td>
<td>8.9 ± 1.5</td>
<td>9.0 ± 0.7</td>
<td>8.9 ± 1.4</td>
</tr>
<tr>
<td>weak plaster</td>
<td>6.2 ± 1.3</td>
<td>5.2 ± 0.9</td>
<td>5.4 ± 0.8</td>
<td>5.4 ± 0.5</td>
</tr>
<tr>
<td>diatomite ⊥ bedding</td>
<td>-</td>
<td>0.17 ± 0.02</td>
<td>0.18 ± 0.03</td>
<td>0.20 ± 0.06</td>
</tr>
<tr>
<td>diatomite // bedding</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.40 ± 0.18</td>
</tr>
<tr>
<td><strong>Poisson’s ratio $\nu$ (-)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cement</td>
<td>-</td>
<td>0.18 ± 0.05</td>
<td>0.24 ± 0.10</td>
<td>0.25 ± 0.02</td>
</tr>
<tr>
<td>strong plaster</td>
<td>-</td>
<td>0.16 ± 0.02</td>
<td>-</td>
<td>0.21 ± 0.02</td>
</tr>
<tr>
<td>weak plaster</td>
<td>-</td>
<td>0.20 ± 0.02</td>
<td>0.20 ± 0.02</td>
<td>0.19 ± 0.07</td>
</tr>
<tr>
<td>diatomite ⊥ bedding</td>
<td>-</td>
<td>0.07 ± 0.01</td>
<td>-</td>
<td>0.11 ± 0.09</td>
</tr>
<tr>
<td>diatomite // bedding***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* Based on one test.
** Without friction-reducing material.
*** Ratio between strain parallel to and perpendicular to the bedding plane, as a result of unloading stress loops perpendicular to the bedding plane.

4.3 Elastic rock properties

We determined the Young’s modulus and Poisson’s ratio of the materials in unloading loops in uniaxial tension and compression tests, triaxial tests, and during loading before a hydraulic fracturing experiment. The experimental set-up used in these experiments is described in Chapter 3. In general, mechanical properties depend on the stress situation.
4. Material characterisation

We compared the elastic properties determined from unloading loops at different stress levels for all materials. However, no significant effect of stress on elastic properties was found for our materials. Densification, for example as a result of material failure, can also influence the mechanical properties. However, in a hydrostatic compression test on weak plaster with significant plastic volume reduction, we did not see significant changes in the unloading bulk modulus. This test is plotted in figure 4.22, in which the slope of the unloading loops can be seen to be independent of the total amount of plastic strain, after correction for creep.

Table 4.2 shows the Young’s modulus and Poisson’s ratio for all materials, determined from the different tests. The results from the various tests show a reasonable agreement. We also calculated the dynamic Young’s modulus and Poisson’s ratio from the compressional and shear wave velocities, and the bulk density (see table 4.3). In dry plaster, the static unloading modulus and the dynamic modulus agree. This indicates that there is little effect of strain amplitude and strain rate on elastic properties. In weak plaster there is a difference between the dynamic and static unloading modulus. These observations agree with the common observation in rocks that the dynamic modulus is equal to or larger than the static modulus (Jaeger and Cook, 1969).

<table>
<thead>
<tr>
<th>Material:</th>
<th>$C_P$  (km/s)</th>
<th>$C_S$  (km/s)</th>
<th>$E_{dyn}$ (GPa)</th>
<th>$v_{dyn}$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cement</td>
<td>3.85 ± 0.04</td>
<td>2.14 ± 0.02</td>
<td>23.9 ± 0.5</td>
<td>0.276 ± 0.004</td>
</tr>
<tr>
<td>strong plaster</td>
<td>2.80 ± 0.14</td>
<td>1.66 ± 0.15</td>
<td>8.9 ± 1.2</td>
<td>0.21 ± 0.07</td>
</tr>
<tr>
<td>weak plaster</td>
<td>2.45 ± 0.08</td>
<td>1.47 ± 0.05</td>
<td>7.3 ± 0.4</td>
<td>0.21 ± 0.02</td>
</tr>
</tbody>
</table>

4.4 Plastic rock deformation and failure envelopes

This section presents an analysis of the measured plastic deformations, failure envelopes, and constitutive behaviour. To characterise the rock materials, we used results from triaxial tests, uniaxial tension and compression tests, and Brazilian tensile tests.
4. Material characterisation

Representation of data

Characteristic for the failure of geomaterials in general is the dependence of the maximum shear stress in the material on the mean stress. This behaviour can be expressed by a failure envelope in which this maximum shear stress is plotted as a function of the mean stress. In triaxial extension and compression tests, principal stresses and strains are measured. To describe our tests, we will use as stress and strain variables:

\[
p = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3)
\]

(4.1)

\[
q = \sigma_1 - \sigma_3
\]

(4.2)

\[
\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3
\]

(4.3)

\[
\gamma = \varepsilon_1 - \varepsilon_3
\]

(4.4)

\(\sigma_i\) and \(\varepsilon_i\) are the principal stresses and strains (\(\sigma_1\) is the largest compressive stress). \(p\) and \(q\) respectively represent the mean normal stress and twice the maximum shear stress in the sample, here expressed by the principal stresses. For triaxial extension and compression tests \(\sqrt{3}p\) and \(\sqrt{\frac{2}{3}}q\) equal the length of the vectors along the hydrostatic axis and in the deviatoric plane respectively in the principal stress space (Chen and Mizuno, 1990). \(\varepsilon_v\) is the volumetric compressive strain, and \(\gamma\) is the maximum shear strain in the sample.

The description of the 3-dimensional failure envelope in a 2-dimensional picture may give a wrong view when the failure envelope is not symmetric around the hydrostatic axis, and the path in the deviatoric plane is different for the various tests. We take care of this by identifying the type of test when plotting the results of various tests together.

The characterisation of a material ideally results in the determination of a constitutive model. To determine such a model, the basic characteristics of the constitutive behaviour must be determined first, to which we will restrict ourselves in this chapter. The characteristics that we determine here are 1) the shape of the failure envelope and the failure modes, 2) the occurrence of plastic deformation, 3) the possible non-associativeness of the flow rule for the plastic strains, and 4) the influence of the intermediate principal stress. The occurrence of time-dependent plastic deformations (creep) will be discussed as well.
4. Material characterisation

Description of tests
The conventional way to determine the amount of plastic strain as a function of stress is the triaxial test, in which a cylindrical core is deformed by axial and radial stresses. Section 3.8 describes the triaxial cell we used for these experiments. The largest and smallest principal stress can be varied independently, while the intermediate principal stress $\sigma_2$ is equal to the minimum principal stress $\sigma_3$ (compression test) or maximum principal stress $\sigma_1$ (extension test). We determined the amount of plastic strain mainly in triaxial extension tests in which, after hydrostatic compression, the axial stress $\sigma_{ax}$ was released and the radial stress $\sigma_{rad}$ stayed constant. If the sample had not failed after the axial stress approached zero, we reloaded the material in the axial direction until the hydrostatic stress state was restored. The test was ended after subsequent hydrostatic unloading.

We used the triaxial extension test because it best resembles the stress path followed by a material element near the fracture tip (see section 2.4). Besides these extension tests, we also determined plastic deformation and failure in hydrostatic compression tests, triaxial compression tests (increase of $\sigma_{ax}$ with constant $\sigma_{rad}$), and uniaxial tension tests. In addition, we determined the failure stress in diametral compression tests (Brazilian tests). Figure 4.4 shows the stress path followed in all tests. Sections 3.8 and 3.9 describe the setup used for these tests.

![Diagram showing stress path for various tests.](image)

**Figure 4.4. Stress path for various tests.**

Figure 4.5 shows an example of the raw data of a triaxial extension test on weak plaster. The test consists of hydrostatic compression, followed by unloading in the axial direction.
In this test, stress loops in the axial direction are made. When the loading direction of the axial stress changes sign, the stress stays constant for some time (typically 400 seconds in plaster), because of the friction in the sealing O-rings of the axial plunger (see section 3.8). This gives an indication of the creep rate. The slope of the unloading and reloading curves during stress loops also give an indication of the amount of creep. The data show that during the extension period, creep in the axial direction is fairly limited (except close to failure), while creep in the radial direction can be significant.

Figure 4.5. Example of raw data of a triaxial extension test on weak plaster.
4. Material characterisation

![Graph](image)

*Figure 4.6. Example of raw data of a triaxial extension test on diatomite.*

The maximum extension of the samples in our cell was reached when $\sigma_{ax}$ was almost zero. If failure had not occurred then, we reloaded the material again in the axial direction. Figure 4.6 shows an example of this reloading in diatomite, which also clearly shows the permanent plastic deformation. The loading rate in most tests was $7 \times 10^{-3}$ MPa/s, while in some tests on strong plaster it was twice as high. This is comparable with the loading rate during a hydraulic fracturing experiment. The control in the tests on diatomite was done by hand, so the loading rate showed more variation than in plaster.
**Determination of plastic deformation and failure**

To determine the amount of plasticity in triaxial extension tests, we took the following approach. As zero point, we took the stress and deformation at the moment unloading in the axial direction started, just after completion of the hydrostatic compression phase. Then, we used the Young's modulus and Poisson's ratio from table 4.2 to calculate the elastic strains, using the measured stress changes relative to the zero point. We calculated the plastic strains from the total strains minus the calculated elastic strains. When failure of the sample had not taken place, we reloaded the material in the axial direction until the stress state was again hydrostatic, in order to check whether the strain interpreted as being plastic indeed was permanent.

Conventional plasticity theory (see section 2.1) assumes time-independent constitutive behaviour. However, significant creep occurred in weak plaster (see figure 4.5). The plastic strain as a result of creep during the unloading loops was omitted from the calculated plastic strain. A similar approach was used in the triaxial compression tests and uniaxial tensile tests. In these tests, the moment the stress deviator started to increase was taken as zero point for the calculation of the plastic strain. The loading rate in the triaxial tests was comparable with the loading rate in the hydraulic fracturing experiments. Because of this, hydrostatic and deviatoric creep during extension are expected to be the same in the triaxial tests as in the hydraulic fracturing experiments. Hence we did not correct for the creep during the extension, and considered it to be part of time-independent stress-strain behaviour when modelling the experiments.

When material properties are derived from triaxial test results and are used as input for numerical simulations, care must be taken in assuming that the measured strains are homogeneous inside the entire sample. Localisation of plastic strain into shear bands can make the measured plastic deformations non-representative for the sample volume. The low strength and very large strains in the shear band can dominate the measured stress-strain behaviour completely. In an overview given by Santarelli and Brown (1989), the ratio between the axial stress at the onset of strain localisation and the peak stress in triaxial compression tests, varied between 0.6 to 0.99 for various rock types. In extreme cases, inspection of the samples after the test gives information about the homogeneity of the strain within the sample. Another source of information about strain localisation is the comparison of the radial strains that are measured in two orthogonal directions. These will in general start to deviate when strain localises in the central part of the sample (Santarelli and Brown, 1989).

Figures 4.11, 4.19, and 4.30 show pictures of samples after the test, for all types of tests. Some of these samples demonstrate the formation of shear bands. There is some uncertainty about the point at which the formation of shear bands started to influence sample deformation significantly. However, the deformations in the triaxial tests at low and high radial stress show a coherent view, even close to the failure point. Furthermore, no significant deviations between the two radial stresses were measured until failure.
occurred. The two radial deformations are coherent in all the strain measurements given later in this chapter. From this and the visual inspection of the cores, it appears that the strains in the tests with low radial stress are homogeneous. From this and the quite good reproducibility of the tests, we can infer that the strains in the test at high radial stress are most probably homogeneous for the largest part of the stress path too.

The failure mechanism of rocks generally differs for various values of the mean stress (see also the discussion in section 2.3). Figure 4.7 shows a general overview of possible failure mechanisms along the failure envelope. Visual inspection of the samples can help in determining the failure mechanism. The sample damage could have been increased by the way we unloaded the sample after failure. Due to safety reasons, we had to release the axial stress just before releasing the radial stress. This can cause progressive failure of the sample.

![Schematic overview of failure envelope with possible failure mechanisms.](image)

Figure 4.7. Schematic overview of failure envelope with possible failure mechanisms.

In the extension tests, we defined the failure point as the point where the maximum shear stress was reached, and excessive strains were measured. In the hydrostatic and uniaxial compression tests (with significant volumetric yielding), we defined the failure point as the maximum in the volumetric stress-strain curve. In the Brazilian tests, we assumed plane stress so that $\sigma_2 = 0$, and we took the assumed stress in the centre of the core as the failure stress. Here, the compressive stress is three times the induced tensile stress according to the linear elastic solution (Jaeger and Cook, 1969). However, this interpretation is a bit ambiguous, because a zone with high compressive stresses is present near the loading platens where failure can start. This is especially relevant for plaster,
where the ratio of tensile and compressive stress is relatively high (see Jaeger and Cook, 1969).

**Results for cement**
The uniaxial compressive strength of cement is approximately $89 \pm 7$ MPa (see table 4.4). Weijers (1995) found a uniaxial compressive strength of cement paste of $54 \pm 9$ MPa. The difference with our results is probably caused by difference in the age of the block from which the cores were taken, which was more than one year in our case (see also Reinhardt (1985) for the dependence of the uniaxial compressive strength on time). Note that the Young's modulus in uniaxial compression given by Weijers (1995) was $19 \pm 1$ MPa, which agrees with our value of $20 \pm 4$ MPa (see table 4.2).

The highest applied stress in the hydraulic fracturing experiments is 33 MPa, which is much lower than the uniaxial compressive strength. Figure 4.8 shows a uniaxial compression experiment on cement paste. The difference between the loading and unloading modulus is interpreted as to be caused by plastic deformation, although part of it may be anelastic. For stress states close to failure, plasticity is present and failure is not completely brittle (as was also measured in Spooner et al., 1976). However, for the stresses of interest the amount of plastic strain is small compared to the elastic strain. Furthermore, part of the plastic deformations describe volume decrease. This is unimportant for the stress path followed by a material element near the tip, for which the mean pressure is decreasing.

![Graph showing plastic and total axial and radial strain in a uniaxial compression test on cement.](graph)

*Figure 4.8. Plastic and total axial and radial strain in a uniaxial compression test on cement.*
4. Material characterisation

Figure 4.9 shows that failure in uniaxial tension experiments is brittle, while the loading modulus in these tests is close to the unloading modulus up to the point of failure. The tensile strength in these tests was $3.0 \pm 0.8$ MPa (see table 4.4). As both in compression and in tension the plastic shear strains showed to be relatively small, we can expect that in hydraulic fracturing experiments on cement plastic deformation will be relatively small too.

![Graph showing plastic axial strain $\varepsilon_{ax}^p$ and total axial strain $\varepsilon_{ax}$ in a uniaxial tensile test on cement.](image)

*Figure 4.9. Plastic axial strain $\varepsilon_{ax}^p$ and total axial strain $\varepsilon_{ax}$ in a uniaxial tensile test on cement.*

**Results for strong plaster**

Figure 4.10 shows the failure envelope of strong plaster, which is determined using triaxial compression and extension tests, uniaxial tension and compression tests, and Brazilian tests. Table 4.4 gives the strength in the uniaxial tests. In the hydrostatic compression test, a possible maximum in the volumetric stress-strain curve was not reached yet because of failure of the sleeve. The given failure stress is probably a slight underestimate. Figure 4.11 shows pictures of the cores, and describes the failure modes. Uniaxial compressive tests show a combination of axial splitting and shear fracturing. Triaxial extension tests on cores under radial stresses of 12, 16, and 20 MPa show no visible damage, because the confining stress was too low to reach the failure envelope. Extension tests with higher confining stresses show shear fracturing and shear bands.
4. Material characterisation

Table 4.4. Strength of the materials in three standard tests: the uniaxial tensile strength $T_0$, the tensile strength $T_{0,B}$ inferred from diametral compression tests (Brazilian tests), and the uniaxial compressive strength $U_0$. In the uniaxial tensile tests on strong plaster, samples of the smallest and largest size were used (see figure 3.14), which showed no significant variation in tensile strength. For cement, the largest sample size was used, and for soft plaster the intermediate one.

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_0$ (MPa)</th>
<th>$T_{0,B}$ (MPa)</th>
<th>$U_0$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cement</td>
<td>3.0 ± 0.8</td>
<td>-</td>
<td>89 ± 7</td>
</tr>
<tr>
<td>strong plaster</td>
<td>3.8 ± 0.4</td>
<td>3.5 ± 0.9</td>
<td>20 ± 6</td>
</tr>
<tr>
<td>weak plaster</td>
<td>1.0 ± 0.2</td>
<td>1.5 ± 0.5</td>
<td>8.2 ± 1.0</td>
</tr>
<tr>
<td>diatomite</td>
<td>-</td>
<td>-</td>
<td>1.42 ± 0.12</td>
</tr>
</tbody>
</table>

Figure 4.10. Failure envelope for strong plaster. The filled symbols indicate the uniaxial and triaxial compression tests. In the hydrostatic compression test, failure as a maximum in the stress-strain curve was not completely reached.
4. Material characterisation

Axial splitting and shear fracturing in a uniaxial compression test (sample size is 8 x 4 cm).

Shear bands in a triaxial extension test (sample size is 5.5 x 11 cm, $\sigma_{\text{rad}}$=20 MPa).

Shear fracturing in a triaxial extension test (sample size is 8 x 4 cm, $\sigma_{\text{rad}}$=28 MPa).

Shear fracturing in a triaxial extension test (sample size is 8 x 4 cm, $\sigma_{\text{rad}}$=24 MPa).

Shear fracturing in a triaxial extension test (sample size is 5.5 x 11 cm, $\sigma_{\text{rad}}$=23 MPa).

Shear fracturing in a triaxial compression test (sample size is 5.5 x 11 cm, $\sigma_{\text{rad}}$=6 MPa).

Figure 4.11. Description of cores of tests on strong plaster.
Fracture pattern in a Brazilian indirect tensile test (sample diameter is 5.5 cm, thickness is 2.7 cm).

Figure 4.11 (continued).

Figure 4.12 shows the measured plastic shear strain and shear stress for strong plaster. This material clearly shows hardening as a result of plastic shear strain (the plastic volumetric strain is negligible during most of the stress path, see figure 4.16). Figure 4.13 gives the evolution of the yield surface when we take the plastic axial strain as hardening parameter. Figure 4.14 shows the stress-strain diagram in hydrostatic compression. We have two experiments, which show a clear difference. This is most probably due to small variations in water content. Figure 4.14 clearly shows hardening of strong plaster as a result of plastic volumetric strain. Figure 4.15 shows that no plastic strain occurs in strong plaster in uniaxial tension.

Figure 4.13 shows an increase of the failure stress with mean pressure (except for the highest radial stress case), while figure 4.16 shows that the plastic volume change is approximately zero for the largest part of the stress path towards failure in these tests. This indicates that the flow rule is non-associated for at least part of the stress space (see also section 2.2). When coming close to the failure envelope, figure 4.16 shows that significant dilatant behaviour occurs, except for the highest radial stress case. The measured dilatancy could be caused by the occurrence of shear bands. No significant creep was measured in strong plaster.
4. Material characterisation

Figure 4.12. Plastic shear strain and deviator stress in extension tests on strong plaster for various radial stresses.

Figure 4.13. Yield envelope for strong plaster with $\varepsilon_{ax}^P$ as hardening parameter.

Figure 4.10 shows that the failure envelope in extension differs from that in compression when $q$ is plotted against $p$, for the region at low mean stress where pore collapse is expected to be unimportant. This indicates that a failure criterion that is symmetric around the hydrostatic axis in the principal stress space does not describe these data. Figure 4.17 shows these data plotted as $q/2$ against $(\sigma_1 + \sigma_3)/2$, after which the data of extension and compression tests fall on the same line for low mean pressure. This shows
that the intermediate principal stress $\sigma_2$ is unimportant for failure in strong plaster at low mean pressure, and that the failure envelope is best described by a failure criterion that does not depend on $\sigma_2$. For high mean pressure, where pore collapse is expected to be important, figures 4.10 and 4.17 show that the failure envelopes of the compression and extension tests coincide when $q$ is plotted against $p$, and do not coincide when $q$ is plotted against $(\sigma_1+\sigma_3)/2$. This indicates that the intermediate principal stress is important for
4. Material characterisation

Figure 4.16. Dilatant behaviour of strong plaster in triaxial extension, for various radial pressures.

Figure 4.17. Failure envelope for strong plaster. The uniaxial and triaxial compression test are indicated by black dots.

describing failure at high mean stress, which agrees with the expectation that pore collapse is important in this region.
Results for weak plaster

Figure 4.18 shows the failure envelope, which is determined using triaxial compression and extension tests, uniaxial tension and compression tests, Brazilian tests, and a hydrostatic compression test. In the hydrostatic compression test, a maximum in the apstress was reached. Figure 4.19 shows pictures of the cores, and describes the failure modes. Triaxial extension tests on cores under radial stresses of 4.5 and 6 MPa show no visible damage, because the confining stress was too low to reach the failure envelope. Extension tests with higher confining stresses show shear fracturing (figure 4.19). It also shows that shear bands can develop in this material.

![Failure envelope for weak plaster. The filled symbols indicate the uniaxial and triaxial compression tests.](image)

Figure 4.20 shows the measured plastic shear strain and shear stress for weak plaster in extension tests. This material clearly shows hardening as a result of plastic shear strain (the volumetric strain during most of the stress path is negligible in most tests, see figure 4.24). Figure 4.21 gives the evolution of the yield surface when we take the plastic axial strain as hardening parameter. Figure 4.22 shows the stress-strain diagram in hydrostatic compression. Hardening as a result of plastic volumetric strain occurs, although part of the apparent hardening is due to creep. Figure 4.23 shows that significant plastic strain and hardening occurs in weak plaster in uniaxial tension. Creep is insignificant in uniaxial tension, as is shown by the unloading loops.
Axial splitting and shear fracturing in a uniaxial compression test (sample size is 8 x 4 cm).

Shear failure and shear bands in an extension test (sample size is 8 x 4 cm, $\sigma_{rad}=10$ MPa).

No visible damage in a hydrostatic compression test (sample size is 4 x 8 cm).

Visually undamaged sample in an extension test (sample size is 4 x 8 cm, $\sigma_{rad}=8$ MPa).

Shear failure and shear bands in an extension test (sample size is 8 x 4 cm, $\sigma_{rad}=12$ MPa).

No visible damage in a triaxial compression test (sample size is 4 x 8 cm, $\sigma_{rad}=2$ MPa).

Figure 4.19. Description of cores of tests on weak plaster.
Fracture pattern in a Brazilian indirect tensile test (sample diameter is 4 cm, thickness is 2.5 cm).

Figure 4.19 (continued).

Figure 4.20. Plastic shear strain and deviator stress in extension tests on weak plaster for various radial stresses.
4. Material characterisation

Figures 4.21 and 4.24 show an increase of the failure stress with mean pressure, while the plastic volume change is approximately zero for most of the stress path towards failure. This indicates that the flow rule is non-associated for at least part of the stress space. When coming close to the failure envelope, figure 4.24 shows that some dilatant behaviour occurs, except for the cases with high radial stress.

![Figure 4.21. Yield envelopes for weak plaster with $\varepsilon^p_{ax}$ as hardening parameter.](image)

![Figure 4.22. Plastic and elastic strains in a hydrostatic compression test on weak plaster.](image)
4. Material characterisation

![Graph showing plastic and elastic strain in two uniaxial tension tests on weak plaster.](image)

*Figure 4.23. Plastic and elastic strain in two uniaxial tension tests on weak plaster.*

![Graph showing dilatant behaviour of weak plaster in triaxial extension, for various values of the radial stress.](image)

*Figure 4.24. Dilatant behaviour of weak plaster in triaxial extension, for various values of the radial stress.*

Figures 4.25 and 4.26 show the plastic shear strain and plastic volumetric strain of weak plaster in triaxial compression tests. As in the extension tests, hardening is visible. The failure envelope in figure 4.18 shows that the failure point in these tests is already close to the hydrostatic cap, which explains the weak dependence of maximum shear stress on the radial stress. Possible volume increase caused by shear failure is masked by the plastic volumetric strain as a result of the increasing mean pressure.
4. Material characterisation

Figure 4.25. Plastic shear strain and deviator stress in triaxial compression tests on weak plaster for various radial stresses.

Figure 4.26. Plastic volumetric strain in weak plaster in triaxial compression.

Figure 4.27 shows the development of the yield envelope when we take the plastic deviatoric strain as hardening parameter, which is not influenced by the plastic volumetric strain. The data of compression and extension tests show a reasonable agreement, comparable to figure 4.18. The agreement of the tests becomes somewhat worse when we take \((\sigma_1 + \sigma_3)/2\) instead of \(\sigma\) on the horizontal axis in these figures. This indicates that for the mean pressures in these tests, the intermediate principal stress is important for
4. Material characterisation

describing failure and plastic yielding. This agrees with the expectation that pore collapse is already important in this range of mean pressure. From these data, no information can be inferred about the influence of the intermediate principal stress at low mean pressure.

When comparing plastic shear strain and failure envelopes between extension and compression tests, we assume that plastic volumetric strain as a result of high mean stress does not influence the results. A sample that is extended in a triaxial extension test, has already been subjected to a higher mean stress. Contrary to this, the mean stress is only increasing in a compression test (see figure 4.4). If the change in porosity as a result of plastic volumetric strain is only little - as is the case in our experiments - this appears to be a reasonable assumption.

![Figure 4.27. Evolution of yield envelope with $\gamma$ as hardening parameter. The dashed lines are the triaxial extension tests, while the solid lines are the triaxial compression tests.](image)

Conventional plasticity theory assumes time-independent constitutive behaviour. However, significant creep can occur in weak plaster. Figure 4.28 shows the volumetric creep rate as a function of the hydrostatic stress for weak plaster. The creep rates were calculated at the onset and turning point of stress loops, where the stress stayed constant for some time because of the friction in the O-rings of the axial plunger (see section 3.8). Influence of possible strain hardening in the experiments on the creep rate was neglected.

Creep is unimportant in hydraulic fracturing experiments when the strain as a result of creep is much smaller than the elastic strain. The elastic strain is of the order of $p_{ne}/E$, which is 1 mstrain for plaster. When we consider a propagation time of $10^3$ s, the creep
rate must be much smaller than $10^{-3}$ mstrain per second. For volumetric creep, figure 4.28 shows that this is the case for a hydrostatic pressure smaller than 6 MPa.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4.28}
\caption{Volumetric creep rate of weak plaster as a function of the hydrostatic stress $p$.}
\end{figure}

**Results for diatomite**

Figure 4.29 shows the failure envelope, which is determined using triaxial extension tests and uniaxial compression tests. In these tests, failure of the sample was never reached, but excessive plastic strains developed while the deviatoric stress stayed almost equal. This stress was then taken as the failure stress. In all tests on diatomite, the bedding planes were oriented perpendicular to the axis of the cylindrical sample. Figure 4.30 shows pictures of the cores, and describes the failure modes. Cores that show large deformations only show irregularities in the diameter as visible damage.

We did not measure the tensile strength of diatomite in the direction perpendicular to its bedding plane (which is the strength of interest, because we created the hydraulic fractures parallel to the bedding plane). However, diatomite breaks easily along its bedding planes, so that we expect that the strength in a tensile test would be negligible. We expect that the tensile strength of the material in between the bedding planes is also relatively small, because of the loose grain structure of diatomite.
Figure 4.29. Failure envelope for diatomite. The filled symbols indicate the uniaxial compression tests.

Development of irregular diameter in uniaxial compression (sample size is 4 x 7 cm).

Development of irregular diameter in triaxial extension (sample size is 4 x 8 cm, $\sigma_{rad}=2.3$ MPa).

Development of irregular diameter and visible reduction of diameter in the centre in triaxial extension (sample size is 4 x 8 cm, $\sigma_{rad}=3.1$ MPa).

Figure 4.30. Description of cores of tests on diatomite.
4. Material characterisation

Figure 4.31 shows the measured plastic shear strain and shear stress for diatomite. This material clearly shows hardening as a result of plastic strain. Volumetric yielding takes place almost from the beginning of hydrostatic loading (see figure 4.6). Figure 4.32 gives the evolution of the yield surface during extension when we take the plastic axial strain as hardening parameter.

**Figure 4.31. Plastic shear strain and deviator stress in extension tests on diatomite for various radial stresses.**

**Figure 4.32. Yield envelopes for diatomite with εₚ as hardening parameter.**
Combination of figures 4.31 and 4.33 shows that the flow rule is non-associated, in order to explain the initial zero plastic volume changes. When reaching the failure envelope, figure 4.33 shows that significant dilatant behaviour occurs in the high radial stress cases.

Creep can be significant in diatomite, but we do not have systematic measurements of it.

![Diagram showing dilatant behaviour of diatomite in triaxial extension, for various values of the radial stress.](image)

Figure 4.33. Dilatant behaviour of diatomite in triaxial extension, for various values of the radial stress.

**Comparison of rock materials**

Figure 4.34 shows that the failure envelope normalised using the uniaxial strength agree for strong and weak plaster. For diatomite, the failure envelope agrees with plaster for low mean pressure. For high mean pressure, plaster shows pore collapse, while the hydrostatic pressure for which diatomite fails lies at a relatively higher mean pressure.

Strong plaster, weak plaster, and diatomite show strain hardening. We compare the importance of plasticity in these materials by calculating the plastic strain normalised by the elastic strain in tension, shear, and hydrostatic compression. We again normalise the stresses by the uniaxial compressive strength according to table 4.4.

Figure 4.35 shows the plastic strain divided by the elastic strain for weak and strong plaster in hydrostatic compression. The mean normal pressure is normalised by the uniaxial compressive strength. This figure shows that plastic volumetric strain in weak and strong plaster will be approximately of the same relative magnitude. Figure 4.36 shows the plastic strain divided by the elastic strain for weak and strong plaster in uniaxial tension. This figure shows that plastic strain in weak plaster is very significant, while in strong
4. Material characterisation

plaster it is negligible. Figure 4.37 shows that the plastic strain normalised by the elastic strain in triaxial extension tests is about equal in weak plaster and diatomite, while it is significantly less in strong plaster.

Figure 4.34. Failure envelope for strong plaster, weak plaster, and diatomite. The stresses are scaled with the uniaxial compressive strength $U_0$ according to table 4.4.

Figure 4.35. Plastic volumetric strain divided by elastic volumetric strain in isotropic compression of weak and strong plaster. The mean normal pressure $p$ is normalised by the uniaxial compressive strength according to table 4.4.
4. Material characterisation

Figure 4.36. Plastic strain divided by elastic strain in uniaxial tensile tests on weak and strong plaster. The tensile stress $\sigma_{ax}$ is normalised by the uniaxial compressive strength according to table 4.4.

Figure 4.37. Comparison of proportion between plastic and elastic shear strain in triaxial extension tests on strong plaster, weak plaster, and diatomite. The stresses are normalised by the uniaxial compressive strength according to table 4.4. For all tests, the radial stress divided by the uniaxial compressive strength $\sigma_{rad}/U_0=1.2$.

We can conclude that the main difference between weak and strong plaster lies in the plastic strain that develops in shear and tension. For stress situations sufficiently far away
from hydrostatic yielding, weak plaster and diatomite show similar behaviour under shear loading. We did not do tensile tests on diatomite. However, we do not expect that large plastic strain develops in uniaxial tension, because of the relatively low tensile strength as a result of the weak bedding planes and loose grain structure. If this is the case, the effect of plastic strain, that develops during tensile loading, on hydraulic fracturing can be experimentally investigated by comparing results in weak plaster with diatomite. The effect of plastic strain that develops under shear loading, can then be investigated by comparing results in strong plaster and diatomite.

4.5 Separation energy

The fracture surface energy is defined as the energy needed to create a unit area of fracture surface. Confusion can exist about the area (global fracture area or real area taking into account surface roughness), and whether losses in plastic deformation around the fracture plane are included in the definition of the fracture surface energy. In the present study we use the term "fracture surface energy" for the energy needed to create a certain amount of global fracture area (with global we mean at the scale of the fracture size), while we exclude energy losses from plastic deformation of the material around the fracture. So, in our definition we use the same area for a fracture with a high and low roughness (assuming that the length scale of the roughness is much smaller than the length scale of the fracture area). The separation energy also contains energy dissipated in inelastic processes that do not directly contribute to the creation of fracture area, and is defined in equation (2.33).

We measured the separation energy in two ways: in a direct tensile test, and in a three-point bending test (according to the ISRM standard, Ouchterlony, 1988). The tensile tests were carried out in collaboration with the faculty of Civil Engineering of Delft University of Technology (van Vliet and van Mier, 1997). They were done on dog-bone shaped samples without a notch. The minimal cross-sectional area was 100 x 60 mm (see section 3.9 and van Vliet and van Mier, 1997). Rotating boundary conditions were applied. A small eccentricity in the force was applied to ensure that the fracture started growing at one side of the sample. The three-point bending tests were done on cylindrical samples with a diameter of 4.9 cm and a chevron notch.

Table 4.5 shows the fracture surface energies calculated from these tests for strong plaster and weak plaster. In the uniaxial tests, we calculated the separation energy from the area under the load-displacement curve. The work that was done in plastic deformation within the measuring interval of the LVDTs before the peak load was reached, was excluded in order to calculate the separation energy. This plastic deformation was
determined using the initial elastic modulus at low stress, or the unloading modulus determined in stress loops. Despite this correction, the presented value still includes energy dissipated in inelastic processes that occur during fracture propagation and stored elastic energy as a result of residual stress, which do not contribute to the creation of the fracture surface.

Table 4.5. Measured separation energy $\Gamma_S$ and corresponding critical stress intensity $K_{IC}$ of strong and weak plaster in three-point bending tests and uniaxial tensile tests. The values corrected for plasticity in the three-point bending tests are also given.

<table>
<thead>
<tr>
<th></th>
<th>strong plaster</th>
<th>weak plaster</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Gamma_S$ (J/m$^2$)</td>
<td>$K_{IC}$ (MPa$\sqrt{m}$)</td>
</tr>
<tr>
<td>tensile test</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>three-point</td>
<td>7.3 ± 1.7</td>
<td>0.26 ± 0.04</td>
</tr>
<tr>
<td>bending test</td>
<td>5.1 $^*$</td>
<td>0.21 ± 0.02 $^*$</td>
</tr>
</tbody>
</table>

$^*$ Corrected for plasticity during fracture propagation.

The energy in the three-point bending tests was calculated from the load and the displacement at the loading point. In this test, we determined the plastic part of the deformation measured at the loading point by making frequent unloading loops. For elastic behaviour, the extrapolation of these loops crosses the point where deformation and load are both zero. Deformations larger than zero are attributed to plasticity, although incomplete closure due to surface roughness could also have contributed to these deformations. This plastic deformation is interpreted as being caused by compression of material directly under the loading point, and by plastic deformation of material surrounding the fracture tip. A curve was fitted through the points, which yielded a relationship between total deformation and plastic deformation. Subtracting the plastic deformation from the total deformation yielded the deformation that is associated with work done in creating the fracture surfaces, defined as the fracture surface energy.

We see that the values differ for the different tests on weak plaster. Besides the above-mentioned contribution of inelastic processes in the tensile tests, this may be attributed to the differences in sample size and fracture surface roughness (the roughness in the tensile tests was larger than in the three-point bending tests). However, the differences found
between the two tests are relatively small. In Mechtcherine and Müller (1997), these tests were also compared with each other for equal sample size, comparable with the size in our direct tensile test. They found that the separation energy of three types of concrete in the direct tensile tests was 3 to 6 times the energy in the bending tests, while the surfaces also had a larger roughness in the tensile tests. These tensile tests were performed with non-rotating loading platens. Table 4.5 also shows the calculated value of the critical stress intensity, using equation (2.15) and the Young’s modulus. As critical stress intensity of cement we used 0.5 MPa√m (Reinhardt, 1995).

4.6 Permeability

We measured the permeability of the various rocks in three ways. First, a cylindrical sample of 2.5 cm length and 2.7 cm diameter was evacuated and then saturated with silicone oil with a viscosity of 5·10⁻³ Pa·s, which was assumed to be incompressible. A pressure difference of approximately 0.2 bar was applied (this set-up is described in Kelder, 1997). This yielded a permeability for dry plaster of 35 ± 7 mD (see table 4.6).

Table 4.6. Measured permeabilities of the materials used in hydraulic fracturing experiments. The permeability calculated from the leak-off coefficient depends on the applied fluid pressure.

<table>
<thead>
<tr>
<th>Material</th>
<th>low pressure $k$ (mD)</th>
<th>from leak-off coefficient $k$ (mD)</th>
<th>from infiltration zone $k$ (mD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cement</td>
<td>-</td>
<td>0.003 ± 0.003</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(\Delta p_f \approx 30 \text{ MPa})$</td>
<td></td>
</tr>
<tr>
<td>plaster</td>
<td>35 ± 7</td>
<td>43 - 59</td>
<td>59 ± 18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(2 \text{ MPa} &lt; \Delta p_f &lt; 20 \text{ MPa})$</td>
<td></td>
</tr>
<tr>
<td>diatomite</td>
<td>-</td>
<td>0.19 - 0.23</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.6 \text{ MPa} &lt; \Delta p_f &lt; 2.0 \text{ MPa})$</td>
<td></td>
</tr>
</tbody>
</table>
We also determined the permeability of plaster and diatomite in leak-off tests in a triaxial cell (see section 3.8). The permeability in diatomite was measured perpendicular to the bedding planes. We applied fluid pressure $\Delta p_f$ at one axial face of the cylindrical core, while the other face was connected to air at atmospheric pressure. We kept the fluid pressure at a constant level and measured the flow into the sample. In these experiments we used the same type of silicone oil as was used in the hydraulic fracturing experiments. From the leak-off volume we could determine the Carter leak-off coefficient (see equation (2.53)), and the permeability when we assume incompressible fluid behaviour using equation (2.51).

Figure 4.38 shows the product of the square root of viscosity and the measured Carter leak-off coefficient in plaster and diatomite, as a function of the square root of pressure $\Delta p_f$. An approximately linear relationship was found, as could be expected from equation (2.51). No significant influence of the water content of the plaster (being lower than 0.08) in comparison with air-dried samples was found. In plaster, silicone oil of 500 and 12.5 Pa·s was used, while in diatomite we used viscosities of 12.5 and 100 Pa·s. The water content in diatomite was approximately between 0.3 and 1.0.

The permeability can also be calculated using the pressure difference and the length of the infiltration zone (see figure 4.39), again assuming incompressible fluid behaviour, complete saturation, and neglecting capillary pressure. By measuring the weight of the sample, we could determine the saturation in the infiltration zone. This was $0.92 \pm 0.13$ for plaster. In diatomite it was difficult to measure the saturation, because of the small leak-off volume and the changing water saturation due to evaporation.

Table 4.6 shows the permeabilities calculated in these three ways. The permeability calculated from the leak-off coefficient depends on the applied fluid pressure, because of the fact that the straight line that can be drawn through the points in figure 4.38 crosses the $y$-axis somewhat below zero. This could be caused by the rheology of the fracturing fluid or by a systematic error in the experimental set-up. Capillary forces are probably not the cause for this, because the effect of them would be expected to be much larger in diatomite compared to plaster, where the permeability and applied pressures are much higher. The results in table 4.6 agree with each other within the experimental error.

Because of the very low leak-off rates in cement, it was difficult to measure the leak-off coefficient in the way described above. Small temperature changes influenced the volume in the pump and hence the apparent leak-off rate significantly. Instead, we used a pressure vessel in which we placed the sample, and which we filled with the silicone oil. Then, we increased the pressure linearly with time. The volume needed for the pressure increase goes into leak-off into the sample and compressibility of the system. The latter quantity was inferred from a similar experiment with a steel sample. In this way we could calculate the leak-off coefficient. We assumed that the leak-off infiltration depth was only small in comparison with the sample dimension, and that the pore pressure inside the sample stayed zero.
4. Material characterisation

Figure 4.38. Carter leak-off coefficient $C_i$ times the square root of the viscosity $\mu$ in a) plaster and b) diatomite, using mainly silicone oil with viscosities of 500 and 100 Pa·s respectively. Both figures also include some measurements using 12.5 Pa·s silicone oil.

In the hydraulic fracturing experiments described in the next chapter, we found an unexpected pressure decline after shut-in. A strongly decreasing pressure decline rate was observed, while a constant decline rate could be expected (see Appendix B). This motivated us to perform yet another type of leak-off experiment on plaster, in which we simulated the pressurisation history of the hydraulic fracture surface. We applied fluid at a constant pressure for a time approximately equal to the propagation time of a hydraulic fracture. Then, we stopped the pump, and the pressure fell because of the continuing leak-off rate into the sample. Figure 4.40 shows an example of the measured pressure decline.
From the pressure decline and the system compressibility we can calculate the leak-off rate. By making the system compressibility small enough, we could make the initial pressure decline rate as large as in the hydraulic fracturing experiments.

Figure 4.39. Dark-coloured infiltration zone of about 7 mm in a leak-off experiment on plaster. The cylindrical sample was sawn through the centre parallel to the axis. The sample size is 4 x 8 cm.

Figure 4.40. Direct simulation of hydraulic fracturing pressurisation history and shut-in in a leak-off test on strong plaster.
4.7 Fracturing fluid

As fracturing fluid we used silicone oil, fabricated by Dow Corning Chemicals. According to the data supplied by the manufacturer, this fluid has a density of $0.97 \times 10^3$ kg/m$^3$, a surface tension of $22 \times 10^3$ N/m, and a dielectric constant of 2.8 (at 25 °C and atmospheric pressure). The fluids we use have viscosities between 12.5 and 500 Pa·s. The manufacturer gives a doubling of the viscosity for a pressure increase from 0.1 MPa to 44 MPa for silicone oil with 1 Pa·s viscosity. These data also show that the dependence of viscosity on pressure becomes less for fluids with a higher viscosity.

We measured the steady state shear viscosity and visco-elastic behaviour of the 500 Pa·s silicone oil as a function of shear rate in a cone and plate rheometer. In the dynamic measurements a sinusoidal strain rate was applied. These measurements yield the complex viscosity, which is defined by combining the viscous and elastic behaviour of a linear viscoelastic fluid (see e.g. Macosko, 1994). For certain fluids (e.g. polymer melts), the magnitude of complex viscosity $\mu'$ shows approximately the same behaviour as the shear viscosity $\mu$ measured in steady state tests (Cox-Merz rule, see e.g. Macosko, 1994). We also calculated the effective viscosity $\mu_{eff}$ for flow of the silicone oil through a pipe, assuming Newtonian fluid behaviour. The diameters of the pipes used were 2.1 and 3.9 mm, while the lengths varied from 5 to 69 cm.

Figure 4.41 shows the rheological behaviour in these tests, which all show shear-thinning behaviour at high shear rates. In the pipe flow experiment, we plot on the horizontal axis the apparent shear rate, i.e. the shear rate for Newtonian fluid behaviour averaged over the pipe cross section. For low shear rate, the steady state and dynamic measurements in the cone and plate rheometer yield a viscosity of 466 ± 14 Pa·s. For the whole range of shear rates, the dynamic measurements in the cone and plate rheometer show a reasonable agreement with the pipe flow measurements. The scatter in the pipe flow measurements is probably caused by variations in the pressure level in the pipe, which varied between 1 and 16 MPa in the various tests. This might influence the measured apparent viscosity, because the dependence of viscosity on pressure. At the same time we can conclude that the pressure has no dramatic effect on the viscosity (assuming that the Cox-Merz rule holds). For high shear rates, the silicone oil leaked between the plates of the cone and plate rheometer during the steady state measurements, making the measured viscosity too small. This could explain the deviation from the other measurements. The elastic behaviour contributes significantly to the magnitude of the complex viscosity in the dynamic measurements for frequencies larger than 1 rad/s.

The dynamic and steady state measurements were done at a temperature of 25 °C. By doing the same experiments at a temperature of 35 °C, a temperature dependence of about 8 Pa·s/°C was determined, while the shear-thinning behaviour was influenced relatively little.
4. Material characterisation

Figure 4.41. Shear viscosity as a function of deformation rate in three different tests.

For a radial hydraulic fracture, the shear rate is highest near the tip (small width) and near the wellbore (large fluid velocity). For typical conditions in the experiments using 500 Pa·s oil (fluid velocity of $10^{-4}$ m/s and width of $10^{-5}$ m for the tip region) the maximum shear rate is of order $10^3$ s$^{-1}$. From figure 4.41 we can conclude that the assumption of Newtonian fluid behaviour is reasonable. For the experiments using 12.5 Pa·s oil, the shear rate can be an order of magnitude larger. However, according to the data supplied by the manufacturer, significant shear-thinning for the 12.5 Pa·s oil begins only at a shear rate of $10^3$.

4.8 Conclusions

In this chapter, we characterised the rock- and fluid properties of interest for the hydraulic fracturing experiments. We can draw the following conclusions about the material behaviour.

- We determined static elastic parameters for the materials, which were independent of the stress level and of damage due to failure. Consistent dynamic elasticity parameters were determined.
- We determined failure envelopes for plaster and diatomite. Failure as a result of pore collapse can become of interest in plaster for a high mean stress. At a low mean stress, the intermediate principal stress is unimportant for failure in strong plaster, and the failure envelope of compression and extension tests coincide.
4. Material characterisation

- Cement behaved essentially linear elastically for the stresses of interest. We determined yield envelopes for plaster and diatomite in triaxial extension, and for weak plaster also in triaxial compression. Weak plaster and diatomite showed significant plastic shear strain, contrary to strong plaster. Weak plaster showed significant plastic strains in uniaxial tension, contrary to the other materials. Plastic volumetric strain is of interest in plaster for high mean stress, while in diatomite plastic volumetric strain is already important at low stress levels. Dilatancy is not very strong in weak plaster. Volumetric creep can be significant in weak plaster, depending on the stress level.
- We determined the separation energy of weak plaster in three-point bending tests and uniaxial tensile tests. The results of these tests show a reasonable agreement.
- We determined the permeability of the rock materials. Experiments on plaster showed that leak-off can be described using incompressible fluid behaviour.
- We measured the shear viscosity of the 500 Pa·s silicone oil as a function of shear rate. For the shear rates of interest, the behaviour is approximately Newtonian. The pressure dependence appears to be relatively small.
5

Results of hydraulic fracturing model experiments

This chapter contains the results of hydraulic fracturing model experiments. The experimental results are discussed and compared with results from literature. The first section concentrates on the measurements of the geometry during propagation. It contains measurements of the net pressure, the width and width profile, and the size of the fluid lag. We compare these measurements with linear elastic numerical simulations.

The fracture surface roughness reflects the fracturing process, and can also be of practical interest. The second section of this chapter contains the measurements of the roughness profiles of the fracture surfaces. We experimentally determined the parameters that influence this roughness.

The closure phase of hydraulic fractures is used in field practice to infer information about the in-situ stress state and rock properties. In the third section, we present the measurements during hydraulic fracture closure. These measurements show special properties of the closure process itself, but are also influenced by the fracture geometry caused during propagation.

5.1 Pressure and geometry of a propagating hydraulic fracture

This section gives the measurements of the net pressure, fracture width, and radius of a propagating radial hydraulic fracture. The raw measured data are given, together with a dimensionless representation of wellbore width and net pressure. Deviations from
5. Experimental results

elasticity are correlated with the size of the plastic zone around the fracture tip. Furthermore, the measured size of the fluid lag at the fracture tip is compared with linear elastic predictions. We also present measurements of observed fracture patterns in the tip region.

Representation of data

The conventional representation of the data is to plot the measurements as a function of experiment time. Besides this, we will also plot the measured net pressure and width at the wellbore as a function of the measured fracture radius. This is done to avoid the influence of leak-off, which strongly influences the measured net pressure and width as a function of time. However, moderate leak-off is expected to have a negligible influence on the net pressure and width as a function of radius (as long as poro-elastic effects are negligible). This can be seen from numerical simulations, for example those presented in Lenoach (1995).

In order to compare experiments under different experimental conditions on rocks with different properties, we present the measurements of net pressure and fracture width at the wellbore in dimensionless form. Elastic scaling provides a way to scale curves obtained for various experimental conditions to an elastic curve, that is approximately constant under these conditions. By doing this, influence of plasticity can be detected by deviations from this general elastic curve. The net pressure $p_{net,w}^*$ and width $b_w^*$ at the wellbore are made dimensionless according to the expressions for the viscosity-dominated regime (see Crockett et al., 1986 and section 2.6):

$$p_{net,w}^* = p_{net,w} \left[ \frac{\mu Q_{eff} E}{R_w^3} \right]^{\frac{1}{4}}$$  \hspace{1cm} (5.1)

$$b_w^* = b_w \left[ \frac{\mu Q_{eff} R_w}{E} \right]^{\frac{1}{4}}$$  \hspace{1cm} (5.2)

$p_{net,w}^*$ and $b_w^*$ are respectively the dimensionless net pressure and fracture width at the wellbore, $R_w$ is the wellbore radius, $\mu$ equals 12 times the fluid viscosity $\mu$, $E$ is the crack opening modulus (defined as $E/(4(1-v^2))$, $E$ is Young's modulus and $v$ is Poisson's ratio), and $Q_{eff}$ is the flow rate flowing effectively into the fracture (defined in equation (5.4)). We used the Young's modulus determined in unloading loops to make the measurements dimensionless. $Q_{eff}$ is used for the non-dimensionalisation because it can strongly differ in
time, while the flow rate applied at the pump is constant (see figure 5.1). The influence of deviations from self-similar propagation is neglected here.

The fracture radius is regarded as the independent variable, and is made dimensionless using the wellbore radius:

$$R^* = \frac{R}{R_w}$$  \hspace{1cm} (5.3)

The wellbore radius is the same in all experiments and simulations, and has a value of 0.0115 m. In the viscosity-dominated regime for radial fractures transverse to the wellbore, the ratio of wellbore radius to fracture radius has physical meaning. For a relatively small wellbore radius and constant wellbore width, a logarithmic singularity in the wellbore pressure will develop (e.g. Perkins and Kern, 1961, see also section 2.6). The contribution of this singularity to the wellbore pressure in comparison with the effect of the tip region, is influenced by the ratio of wellbore radius and fracture radius (Savitski and Detournay, 1999). Note, however, that numerical simulations show that this logarithmic singularity is unimportant in our experiments (see Chapter 6, figure 6.7).

The experimental conditions are given in table 5.1. In addition to the measured curves, we calculated parameters that give information about the global fracture geometry. These parameters are defined later in this chapter (equations (5.6) and (5.7)) , and are presented in Appendix A, table A.1. In combination with the measured curves, they give a comprehensive view of the measurements done during propagation. Table A.2 gives the values of the net pressure and fracture geometry at characteristic points during hydraulic fracture propagation (see figure 3.9 in Chapter 3 for the definition of these characteristic points).

Possible influence of plasticity on the geometry of hydraulic fractures is expected to show up when we try to correlate the measurements of geometry with the plastic zone size. We calculated a measure of the plastic zone size by applying the Mohr-Coulomb criterion on the toughness-dominated linear elastic stress field around the fracture tip, in the same way as was done in section 2.4. Table 5.2 gives the material properties that were used here. These values were determined from the material characterisation experiments in Chapter 4. We choose to calculate the plastic zone size at a value of the hardening parameter that coincided with material failure, which is a somewhat arbitrary choice. The inaccuracy in the friction coefficient is relatively large, because of the lack of data in the failure envelope at low mean pressure. Furthermore, the friction coefficient changes as hardening takes place (see Chapter 4, figures 4.13, 4.21, and 4.32). Figure 4.34 shows that the shape of the failure envelope is approximately equal for all materials, so we took equal values for the friction coefficient in strong plaster, weak plaster, and diatomite. Using the friction coefficient, the cohesion was determined from the uniaxial compressive strength.
5. Experimental results

Table 5.1. Overview of the experimental conditions in all experiments. $\sigma_{h,1}$ and $\sigma_{h,2}$ are the largest and smallest applied stress in the plane of the fracture, $\sigma_c$ is the stress perpendicular to the fracture plane, $Q_{pump}$ is the flow rate applied at the pump, and $\mu$ is the fracturing fluid viscosity.

<table>
<thead>
<tr>
<th>Material</th>
<th>name</th>
<th>$\sigma_{h,1}$ (MPa)</th>
<th>$\sigma_{h,2}$ (MPa)</th>
<th>$\sigma_c$ (MPa)</th>
<th>$Q_{pump}$ (cc/min)</th>
<th>$\mu$ (Pa·s)</th>
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</thead>
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<td>33</td>
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<td>1.5</td>
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</tbody>
</table>

* intermediate water content.
** after previously creating a fracture with a small width using water.
Table 5.2. Parameters used to calculate the measure of the plastic zone size: cohesion $S_0$, friction coefficient $\mu_f$ and toughness $K_{fc}$. An estimate of the toughness of diatomite is made, based on the average ratio of toughness and uniaxial compressive strength of strong and weak plaster. The other parameters are determined in independent tests (see Chapter 4).

<table>
<thead>
<tr>
<th></th>
<th>$S_0$ (MPa)</th>
<th>$\mu_f$ (-)</th>
<th>$K_{fc}$ (MPa$\cdot$m)</th>
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<td>0.3</td>
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</table>

As measure of the plastic zone size we took the component perpendicular to the fracture plane of the maximum radius from the fracture tip to the boundary of the plastic zone, while we only considered positions with a larger x-coordinate than the fracture tip (see figure 2.2 for coordinate system).

**Corrections and accuracy of experiments**

Due to fluid compressibility and deformation of wellbore and tubes, the volume of the fluid in the system (consisting of pump, wellbore, and tubes) varies with the fluid pressure $p_f$. This volume variation causes fluid storage in the system, which influences the flow rate that effectively flows into the fracture. This effective flow rate $Q_{eff}$ is calculated using

$$Q_{eff} = Q_{pump} - \frac{dV_{system}}{dp_f} \frac{dp_f}{dt}$$  \hspace{1cm} (5.4)

where $t$ is the time, $Q_{pump}$ is the flow rate applied at the pump, and $V_{system}$ is the volume of the system, which equals the volume of the fluid. $p_f$ is assumed to be the same everywhere in the system.

Figure 5.1 shows a typical example of the effective flow rate in cement. In the cement experiments, the effective flow rate is almost two times larger than the flow rate applied at the pump during most of the propagation time. In the plaster experiments, the high flow rate and relatively low fracture efficiency make the effective flow rate about equal to the flow rate applied at the pump. In diatomite, the effective flow rate and flow rate at the pump are about equal because of the relatively small change in net pressure during propagation.
Figure 5.1. Characteristic example of the difference between the effective flow rate $Q_{\text{eff}}$ and the flow rate at the pump $Q_{\text{pump}}$ in cement experiments (experiment ce02).

The inaccuracy in the measurement of the pressure is usually negligible (relative error smaller than 2%). Only in experiments on diatomite, the net pressure is so low that the noise in the measuring signal becomes visible (see figure A.10). The error in the net pressure is largely determined by the error in the confining stress. Systematic deviations of the stress situation in the block from the expected stresses were found to be within 10\% (see sections 3.6 and 3.7).

In the measurement of the width, deformation of the rock between notch and clamps is included. In order to correct for this systematic error, we determined the effective length of the measuring interval during loading of the block. The deformation of the rock inside this interval is influenced by changes in the net pressure in the fracture and pore pressure changes as a result of leak-off. When we assume that the depth of the leak-off zone $l_{\text{lo}}$ varies with the square root of time, we can calculate the value of $l_{\text{lo}}$ during propagation from the value at shut-in. The depth of the leak-off zone $l_{\text{lo,si}}$ at the moment of shut-in is approximately equal to the depth that is measured after opening of the block in plaster, because this depth changes only little after shut-in (plaster is the only material for which the leak-off correction is of interest). We assumed zero lateral deformation in the direction parallel to the fracture plane, which assumption is checked using the numerical program FLAC and gives a satisfactory approximation. The correction in $b_w$ is then given by:

$$\Delta b_w = \left( l_{\text{ti}} p_{\text{net,w}} - 2l_{\text{lo,si}} \frac{t - t_{\text{ti}}}{t_{\text{si}} - t_{\text{ti}}} \frac{1}{p_p} \left( \frac{1 - 2v}{E} \right) \right)$$  \hspace{1cm} (5.5)
where $l_i$ is the length of the measuring interval of the LVDT used for the width measurement, $r$ is the experiment time, $t_i$ is the moment of fracture initiation, and $t_{si}$ is the time at the moment of shut-in. $p_p$ is the average pore pressure in the leak-off zone. We assumed that effective stress (according to Terzaghi's principle) controlled the deformation, which coincides with a value of 1 for Biot's coefficient of effective stress. This value is probably somewhat too high, but the approximation is sufficient for our purpose as the first term on the right-hand side of equation (5.5) usually makes the largest contribution to the correction. For a fracture of 10 cm radius and a measuring interval of 4 cm between the clamps, the correction is about 10% of the measured width. This relative error becomes larger for smaller radius, because the net pressure becomes larger and the wellbore width smaller. Another cause for an error in the width measurement is asymmetry of the fracture, although this is usually of smaller importance.

In all experiments on plaster, only the acoustic signal caused by the diffraction at the fluid front position was strong enough during the complete propagation time to be used for calculation of the fracture radius. There is however a difference between the position of the fluid front and the fracture tip, because of the presence of the fluid lag. In almost all experiments on plaster this difference is smaller than 1 cm (except for experiment sp01, where it is approximately 2 cm at the moment of shut-in), and introduces a systematic error in the radius measurement smaller than approximately 10%. We did not correct for this error. When an acoustically measurable fluid lag size occurred in cement, the diffraction coming from the fracture tip was strong enough during the whole propagation time to use it for the radius determination. As a result, we have a negligible systematic error in the radius measurement in cement, even when a significant fluid lag is present.

Moderate fracture asymmetry will cancel out in the radius measurements, as we always average the acoustic measurements over the four sides of the block. We also average the acoustic radius determinations from source-receiver combinations having one transducer at the top and combinations that have one transducer at the bottom of the block, in order to cancel out small tilting of the fracture plane with respect to the expected fracture plane.

In order to determine the accuracy of the acoustic radius determination, we compared the acoustically measured fluid front position at shut-in with the fluid front position measured after splitting of the block in plaster (see Appendix A, table A.2 for the post-mortem fracture radius and the acoustic radius, and table A.1 for the size of the dry tip). The acoustic diffraction shows that the fracture stops growing immediately after shut-in, because the diffraction disappears or stays at the same travel time for a short time. The average difference between the acoustic and post-mortem radius is 3 mm, which shows that the acoustic measurements are accurate and have a relatively small error.

Besides the measuring errors discussed so far, we are confronted with uncertainty in the material properties of interest when we non-dimensionalise the measurements. We estimate relative errors of 10% in the Young's modulus, and 20% in the viscosity.
5. Experimental results

Furthermore, we estimate a random error of 15 % in the flow rate and we take into account an error in the radius of 10 %. Then, using equations (5.1) and (5.2), the relative errors in the dimensionless net pressure and dimensionless width are 12 % and 7 % respectively.

We duplicated a few experiments in order to determine the repeatability of the experiments. In cement, the experimental conditions (material, externally applied stresses, flow rate, and viscosity) were the same in experiments ce01 - ce02 - ce03 - ce06, and in experiments ce04 - ce07. In strong plaster, also two experimental conditions were repeated. The conditions in experiments sp01 - sp05 were the same, and in sp04 - sp06. These tests show that the variation in the dimensionless net pressure is typically 10 %, while in the wellbore width it is typically 15 % for a radius of approximately 7 cm or larger (see figures 5.3, 5.4, 5.6, and 5.7). The variation in the net pressure compares well to the variation expected from the uncertainty in material properties, flow rate, and the error in the radius. The variation in the wellbore width is somewhat larger than expected. Fracture asymmetry could play a role here.

The accuracy of the stress situation inside the block was discussed in section 3.6. Based on measurements and simulations, it was concluded that the stress in the block differed less than 10 % from the expected value. We analysed the results of hydraulic fracturing experiments in cement in order to try to find any systematic influence of the applied stresses on the net pressure, which would reflect deviations in the stress situation inside the block. The only difference between the groups of experiments ce01 - ce02 - ce03 - ce06 and ce04 - ce07, is the externally applied stresses. If the confining stress inside the block would differ systematically from the expected value, we would expect this deviation to be larger in experiments ce01 - ce02 - ce03 - ce06, where the applied stresses are larger. Comparison of the dimensionless net pressure in these two groups of experiments shows no systematic difference. This indicates that the influence of a possible systematic deviation in the confining stress in the block from the expected value is insignificant in these experiments, and most probably also in the other experiments. Note that the ratio of the stress parallel to and perpendicular to the fracture plane is almost equal in the two groups of experiments. As a result, no conclusions can be made from the observations mentioned above about the stress situation in experiments in which this ratio was strongly different.

Concluding, we have a random variation of 10 % in the net pressure. The systematic error in the net pressure as a result of inaccuracy in the applied stress, is relatively small. We have a random variation of 15 % in the wellbore width. Furthermore, the systematic error in the radius as a result of the presence of a fluid lag is smaller than 10 % in plaster, while it is negligible in cement.
5. Experimental results

Raw data
We performed hydraulic fracturing experiments on four different materials: cement paste, strong plaster, weak plaster, and diatomite. We also varied the externally applied stresses, and in some cases the fracturing fluid viscosity and flow rate. Table 5.1 contains these experimental conditions. The measured net pressure at the wellbore, width at the wellbore, and fracture radius, showed a clear influence of flow rate, fluid viscosity, and material properties. To have an idea of the influence of some of these parameters, figure 5.2 shows the measured wellbore pressure and width at the wellbore (in this figure not corrected for the deformation over the width interval). The fluid viscosity is the same in all experiments, while the confining stress $\sigma_c$ is the same in the experiments on cement (ce04), strong plaster (sp05), and weak plaster (wp02). The flow rate in strong plaster and weak plaster is equal, so that the difference in material properties causes the lower net pressure and larger width in weak plaster. Comparing strong plaster with cement shows that the change in material properties and difference in flow rate (lower in cement) approximately cancel out for the net pressure, but yield a much larger width in strong plaster. In diatomite, the rock properties (Young's modulus and strength) differ strongly from the other experiments, which has a strong effect on the net pressure.

In Appendix A, figures A.1 through A.11 show the measured wellbore pressure $p_{net,w}$, the width $b_w$ at the wellbore and the fracture radius $R$ as a function of the experiment time $t$ minus the initiation time $t_i$ for all propagation experiments. The time scale of propagation varies two orders of magnitude in these figures, which is caused by the flow rate, viscosity, and rock permeability. The fracture efficiency (fracture volume divided by injected volume) of the fractures created in cement is almost 1, in plaster it is approximately 0.1, and in most diatomite experiments it is approximately 0.8. The measurement of wellbore radius shows some irregularities at the end. This is caused by the diminishing number of diffraction records which builds up a radius measurement curve.

The moment of fracture initiation is defined as the maximum in the pressure derivative during pressurisation of the borehole. After initiation, the pressure continues to increase, until the break-down point is reached at the point the pressure reaches its maximum (see figure 3.9 in Chapter 3). The initiation and breakdown pressures in all experiments are listed in Appendix A, table A.2, together with the values of the wellbore width, wellbore pressure, and fracture radius at the moment of shut-in.

Dimensionless net pressure and width in cement
Figure 5.3 shows the dimensionless net pressure $p^*_{net,w}$ as a function of fracture radius $R$ in cement, while figure 5.4 shows the dimensionless wellbore width $b^*_w$ as a function of dimensionless fracture radius $R^*$. The non-dimensionalisation shows that the influence of the flow rate and fluid viscosity is correctly described by equations (5.1) and (5.2). Similar
Figure 5.2. Net pressure and fracture width at the wellbore $p_{\text{net,w}}$ and $b_w$ for one experiment in all materials. The fluid viscosity in these experiments is the same, as also is the confining stress $\sigma_c$ in experiments ce04, sp05, and wp02 (diatomite experiment di01 is shown). The experimental conditions are listed in table 5.1. The results show the large influence of the Young's modulus, and also the influence of flow rate.
5. Experimental results

Figure 5.3. Dimensionless net pressure at the wellbore $p_{w*}$ in cement. The sudden decrease of the dimensionless net pressure in experiment ce07 is caused by a sudden increase in the flow rate.

Figure 5.4. Dimensionless fracture width at the wellbore $b_{w*}$ in cement.
scaling laws - based on the expressions for net pressure, width, and radius as a function of time - were also confirmed in experiments on Colton sandstone and cement, and interface separation of rubber (de Pater et al., 1994b, and Johnson and Cleary, 1991). Furthermore, the externally applied stresses have no significant influence on the net pressure and fracture geometry, which is also shown in Appendix A, figures A.1 through A.3. This was expected from the fully coupled numerical simulations of hydraulic fracture propagation in linear elastic material according to Barr (1991).

Furthermore, figures 5.3 and 5.4 show that the measurements agree with the fully coupled numerical simulations based on linear elastic fracture mechanics (Barr, 1991). Only for experiment ce05, there is a significant difference between the net pressure in experiment and simulation. This may be caused by inaccuracy in the applied stress state, which could have been caused by the large ratio of the externally applied stress parallel to and perpendicular to the fracture plane, or by insufficient functioning of the lubrication by teflon and vaseline in this specific experiment (see section 3.6). In Barr (1991), Desroches and Thiercelin (1993), De Pater et al. (1994a, 1994b, and 1996), also agreement between measurements and coupled simulations during propagation in cement was found.

The pressure curves in the raw data show a maximum (the break-down pressure), at a radius between 3 and 4 cm (see Appendix A, figures A.1 and A.3). This coincides with a dimensionless radius $R^*$ between 2.6 and 3.5. Figure 5.3 shows that after non-dimensionalisation, this characteristic break-down point in the pressure curve has disappeared. Instead, the pressure curves are continuously decreasing, similar to the fully coupled linear elastic simulation. The disappearance of the break-down point indicates that the viscosity-dominated scaling already applies before the break-down pressure is reached in figure A.1, and that this peak in the pressure is not caused by fracture toughness. This is confirmed by the value of the parameter that indicates whether the viscosity-dominated regime is present (see section 2.6), for a fracture radius of 2 cm.

In order to investigate directly the importance of toughness in the cement experiments, we did an experiment (numbered ce08 in table 5.1) in which we prefractured the block with water. The radius of the water fracture was 8.8 cm, while the maximum attained width was 18 μm. The fracture surface was smooth, and the fracture closed almost completely after shut-in, which was indicated by the acoustic transmission measurements. Figure 5.5 shows a picture of the fracture surface, obtained after splitting of the block over the fracture surface after the experiment had ended. We reopened this fracture with silicone oil (see table 5.1). Figures 5.3 and 5.4 show that the net pressure and fracture width at the wellbore during reopening agreed with the fractures created in intact cement. Also, the break-down pressure is unaffected by the absence of toughness (see figure A.1 and table A.2). This again indicates that the influence of fracture toughness is negligible in cement under our conditions of stress, flow rate, and viscosity.

We also reopened two fractures that were first created with silicone oil, and had reached a maximum width of the order of 100 μm (experiments ce06 and ce07). The
maximum net pressure in these experiments was lower than during creation of the first fracture (see van Dam et al., 1999). One could erroneously conclude that this is caused by the absence of fracture toughness. That this is not the case, was shown by the above-mentioned results. The lower break-down pressure is most probably caused by the remaining width due to a thin layer of silicone oil that stays in the fracture because of the extremely low leak-off rate. Acoustic transmission measurements indeed show that these fractures never closed completely. Furthermore, indications were found for fluid flow inside the fracture before the fluid pressure reached the confining stress level, which was inferred from the time derivative of the wellbore pressure and acoustic shear wave diffractions originating from the propagating fluid front. The maximum in the net pressure approximately coincided with the moment the fluid front had reached the previously created fracture radius. Further details of the experiments on fracture reopening can be found in van Dam et al. (1999).

Figure 5.5. One quarter of the fracture surface of experiment ce08 after splitting of the block. The borehole is in the upper left corner. The final radius of the water fracture is visible as an irregularity on the smooth fracture surface. The dark-coloured area is caused by residue of the fracturing fluid.
5. Experimental results

Figure 5.6. Dimensionless net pressure at the wellbore $p_{net,w}^*$ in strong plaster.

Figure 5.7. Dimensionless fracture width at the wellbore $b_w^*$ in strong plaster.
Figure 5.8. Dimensionless net pressure at the wellbore $p_{\text{net, w}}^*$ in weak plaster.

Figure 5.9. Dimensionless fracture width at the wellbore $b_w^*$ in weak plaster.
5. Experimental results

**Dimensionless net pressure and width in strong and weak plaster**

Figures 5.6 and 5.7 show the dimensionless net pressure and wellbore width in strong plaster as a function of the dimensionless fracture radius $R^*$. In most experiments, the measurements agree within the experimental error with the fully coupled linear elastic simulations. The best agreement is obtained for experiment sp02, with a relatively small externally applied stress deviator. The largest difference between experiment and simulation is for experiments sp09 and sp10, where a very large stress difference was applied. The net pressure is generally somewhat lower than in the simulation, which is the same kind of deviation as in cement. This suggests a systematic deviation between the measurements and the simulation, which however is within the experimental error.

Figure 5.8 shows the dimensionless net pressure $p_{net,w}^*$ as a function of fracture radius $R$ in weak plaster, while figure 5.9 shows the dimensionless wellbore width $b_w^*$ as a function of fracture radius. The net pressure at the wellbore is significantly lower than the linear elastic model, while the wellbore width is larger. This is the same deviation from the elastic model as appeared in strong plaster when the deviator stress was very large (experiments sp09 and sp10). The wellbore pressure already deviates significantly from the linear elastic simulation for small radius, from which we can conclude that this deviation is not caused by boundary effects due to the finite dimensions of the block. As indicated in table 5.1, two experiments on weak plaster (wp01 and wp03) had a water content that probably was too low, so the properties of the blocks in these experiments were intermediate between what we defined as strong plaster and weak plaster (see section 3.5). The Young's moduli of these blocks were 7.0 and 7.6 GPa respectively (compare table 4.2 in Chapter 4), which values we used to make the measurements dimensionless.

Triaxial tests show that creep may be important in weak plaster (see section 4.4). We did three experiments (wp04, wp05, and wp06) to determine the significance of creep in this material. In these experiments, the fracture efficiency and product of viscosity and flow rate were equal, which would lead to the same net pressure in the elastic case. The time scales of propagation were $10^3$ s (wp04), $10^3$ s (wp05), and $10^2$ s (wp06). From these experiments, it appears that creep leads to a smaller wellbore pressure and larger wellbore width, although the effect is within the normally observed variation (see figures 5.8 and 5.9).

**Dimensionless net pressure and width in diatomite**

Because the attenuation of the ultrasonic waves in diatomite was too strong, we could not measure the fracture radius during fracture propagation. We measured the fracture radius after splitting the block. Combination of the net pressure and width at the moment of shut-in with the radius after splitting of the block, gives one point that can be compared with the simulations (we then neglect possible radius growth after shut-in). Figures 5.10 and
5. Experimental results

Figure 5.10. Dimensionless net pressure at the wellbore $p_{net,w}^*$ in diatomite.

Figure 5.11. Dimensionless fracture width at the wellbore $b_w^*$ in diatomite.
5. Experimental results

5.11 show these points of dimensionless net pressure at the wellbore $p_{net,w}^*$ and fracture width $b_0^*$, as a function of the dimensionless radius $R^*$.

Deviations from the linear elastic simulation were seen in experiments di02 and di05. In experiment di02, a very asymmetric fracture developed. At the short side of the fracture, two non-overlapping fracture planes were present that were not connected. Directly after closure an almost instant decrease of the wellbore pressure of about 50% of the net pressure takes place, while the wellbore width stays approximately constant. This indicates that an obstacle to fluid flow is present between the wellbore and the fracture. After splitting of the block, indeed a somewhat irregular fracture geometry near the wellbore is visible. In experiment di05, both the fracture width and net pressure are higher than expected. However, the interpretation of this experiment is unclear, because a large irregularity from geologic origin was present in the block at the fracture plane. Note that experiments di02 and di05 were the two experiments where the product of flow rate and viscosity was relatively low. The low net pressure that is a result of this, increases the relative influence of systematic errors, for instance deviations in the stress in the sample from expected values.

Diatomite is a laminated rock, with a clear bedding plane orientation and inherent anisotropic elastic properties (see Chapter 4, table 4.2). Diatomite tends to split along the bedding planes. However, the hydraulic fractures in our experiments appear not to be influenced by the lower cohesion on these planes. This is clearly visible in experiment di01, where the hydraulic fracture plane makes a small angle with the bedding planes and crosses several laminae (see figure 5.12).

Net pressure and width in other materials from literature

In van den Hock et al. (1993), experiments performed on Colton sandstone were presented. The net pressure and width at the wellbore at the moment of shut-in were compared with a linear elastic model. For one experiment with a relatively low flow rate and high confining stress $\sigma_c$, the measured net pressure was significantly higher than the model, while the width agreed with the model. No explanation for this was given. This agrees with the experiments on diatomite, where also the low flow rate experiments showed deviations from the simulation. However, in our experiments possible causes have been identified, as discussed above. Other experiments on Colton sandstone show a reasonable agreement with fully coupled numerical simulations during propagation (de Pater et al., 1994b and 1996).

Barr (1991) presents a comparison between fully coupled hydraulic fracturing simulations and the results of interface separation experiments of rubber and PMMA, which is relatively stiff. In these experiments the fluid pressure is measured at the wellbore and at several locations along the fracture radius. The experimental results are corrected
for the smaller width because of the stiff PMMA, and for the friction between rubber and PMMA. The wellbore pressure and fracture radius, both as a function of time, and the pressure profile agree with the numerical simulations. The pressure profile also agreed with fully coupled simulations in Desroches and Thiercelin (1993), who used a different simulator.

![Image](image.png)

**Figure 5.12. Path of a hydraulic fracture in diatomite (experiment di01).** The wellbore is at the left, while the hydraulic fracture is dark-coloured and has a radius of approximately 9 cm. The path of the hydraulic fracture is apparently not influenced by the bedding planes with low cohesion. Note that the deviation between the hydraulic fracture plane and the horizontal plane usually was smaller in the tests.

**Width profile**

The width profile in our experiments was determined in two ways. First, we used a technique developed in Groenenboom (1998), which could be used in cement. This method is based on calculating the influence of the multiple reflections at the stiff fracture walls on the acoustic signal. The measured signal is matched to the calculated signal by adjusting the width in the calculations. Figure 5.13 shows an example of a width profile determined using this method compared with the fully coupled numerical simulations, together with the borehole width measurement and the acoustic radius measurement. The example is for experiment ce09 at time \( t-t_0 = 2505 \) s, when the flow rate at the pump is 0.022 cc/min. According to equation (5.4), the effective flow rate then is 0.042 cc/min, which is used in the numerical simulations. In this example, the width profile based on the
acoustic measurements, acoustically determined fracture radius, and width measurement at the wellbore, shows a very good agreement with the fully coupled simulation.

At a certain moment the flow rate is increased strongly in experiment ce09 (see table 5.1). After the pressure has reached its maximum and the flow rate is approximately constant, the acoustic measurements show a deviation from the simulation (see Groenenboom (1998), where the same experiment is discussed). This may be caused by the discontinuous flow rate. However, measurement of the width profile in another experiment with a relatively high flow rate (experiment ce10) - which stayed constant during the whole experiment - showed a similar deviation of the width profile. Interpretation of these observations is difficult, because the measuring error in the acoustically determined width profile near the tip is not completely clear (especially when a significant fluid lag is present). Shear-thinning behaviour of the fracturing fluid might also play a role at these high flow rates.

![Diagram showing acoustic measurements and width profile](image)

Figure 5.13. Acoustically determined width $b$ as a function of radius $r$ in cement experiment ce09, together with the borehole measurement and fully coupled linear elastic numerical simulation. The dark grey area at the left represents the wellbore, while the light grey area represents the cement. The black dots - connected with dotted lines - are the acoustic measurements and the width measurement at the wellbore. The acoustically determined radius is indicated by the centre of the circle at $r=6.2$ cm. The width profile of the fully coupled numerical simulation (Barr, 1991) is represented by black lines. The apparent fracture opening in front of the fracture tip is probably caused by the block deformation.
Another measurement that yields information about the width profile, is the relationship between fracture volume, fracture radius, and fracture width at the wellbore. The fracture volume can be inferred from the block deformation in the direction perpendicular to the fracture plane (see section 2.7). However, this measurement can be influenced by creep, and by plastic shear deformation and pore collapse inside the block. In addition, friction between block and end platens of the true triaxial machine can influence these measurements. In all experiments that showed unrealistic block deformations, there were reasons to assume that one of these causes was present. Figure 5.14 shows an example of the comparison between the measured fracture volume and the effectively injected volume in cement experiment ce04. Numerical simulations show that the efficiency of this fracture is almost one. From this, we expect agreement between the injected volume and fracture volume, which figure 5.14 shows to be the case.

![Graph showing comparison between injected volume and fracture volume](image)

**Figure 5.14. Comparison between injected volume $V_{\text{eff}}$ and fracture volume $V_{\text{frac}}$ which is calculated from the block deformation (experiment ce04).**

The relationship between fracture volume and width profile can be expressed by:

$$V_{\text{frac}} = 2\gamma_v b_w R^2$$  \hspace{1cm} (5.6)

$\gamma_v$ is an integral of the width profile, and is a geometric factor. For a constant pressure in the fracture, the width profile is elliptical and $\gamma_v$ equals $\pi/3$ ($\approx 1.05$). The fully coupled numerical simulations yield $\gamma_v = 0.91$. The maximum possible value for $\gamma_v$ is $\pi/2$. 

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![Graph showing $\gamma_v$ as a function of $\sigma_c$ for various materials.](image)

**Figure 5.15.** $\gamma_v$ as a function of the confining stress $\sigma_c$ for various materials.

![Graph showing $\gamma_v$ versus $l_p$ in plaster.](image)

**Figure 5.16.** Measured value of $\gamma_v$ versus the calculated measure of the plastic zone size $l_p$ in plaster.

corresponding with straight parallel fracture walls up to the rectangular blunt tip. Values of $\gamma_v$ being larger than $\pi/2$ indicate significant plastic deformation in the plastic zone before the tip. These values are included in table A.1 in Appendix A, but were omitted in the figures. The measurements that were significantly influenced by friction or creep, leading to too small values of $\gamma_v$, were omitted. Table A.1 and figure 5.15 show the measured values of $\gamma_v$. For cement, there is a good agreement with the numerical simulations. For
plaster, the values can be significantly larger. When interpreting this, there is always the uncertainty to what extent plastic deformation in the plastic zone inside the block has influenced the measured block deformation.

Figure 5.16 shows $\gamma_e$ as a function of the calculated measure of the plastic zone size, according to section 2.4. From this figure, it appears that - at least for strong plaster - the value of $\gamma_e$ increases with the measure of the plastic zone size, although the number of experiments is not enough to make definitive conclusions. The acoustic width determination of Groenenboom (1998) cannot be used here, because of the strong scattering from the tip and the presence of the infiltration zone due to leak-off, while also the fracture surface roughness may play a disturbing role.

**Relation between net pressure and width**

Linear elastic fracture mechanics predicts a relation between fracture opening and net pressure inside the fracture. This relation can be written as (Crockett *et al.*, 1986):

$$b_w = \gamma_1 \frac{P_{net,w}R}{E}$$  \hspace{1cm} (5.7)

$\gamma_1$ is an integral of the pressure profile (see equation (2.16)). For a constant pressure in the fracture, $\gamma_1$ is $2/\pi$. Fully coupled simulations show that for viscosity-dominated hydraulic fracture propagation, the value of $\gamma_1$ is approximately equal to 0.50.

![Figure 5.17. Value of $\gamma_1$ for all materials as a function of the confining stress $\sigma_c$. As indicated, one point for weak plaster falls outside the vertical scale.](image-url)
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![Graph showing measured values of y1 versus lp in plaster and diatomite.]

Figure 5.18. Measured value of y1 versus the calculated measure of the plastic zone size lp in plaster and diatomite.

Table A.1 in Appendix A and figure 5.17 show the measured values of y1 in all experiments. When possible, the values were taken at a radius of 7.5 cm. When no radius measurement was obtained, the value at shut-in was taken. It is clear that some experiments show a significantly larger value of y1 than predicted by elastic theory. One experiment on diatomite (di02) shows a significantly lower value. In this experiment, the fracture geometry deviated strongly from the assumed radial geometry. For strong plaster, figure 5.18 appears to indicate that the value of y1 starts to increase with the plastic zone size when the plastic zone size exceeds a certain threshold value. For weak plaster and diatomite, the measurements show more variation, although a similar behaviour as for strong plaster is not contradicted.

Fluid lag size and importance of toughness

The fluid lag is defined as the distance from the fluid front to the tip of the fracture. When using the cohesive-zone model (see section 2.3), we define the fluid lag length as the distance from the fluid front to the position where rock fracturing starts (the position of maximum tensile stress). We measured the size of the fluid lag during propagation and closure in two ways. Firstly, we determined the difference in radius between the acoustic diffractions which are assumed to originate from the fracture tip and fluid front positions (see section 3.4). This yields an acoustically determined measure for the size of the fluid lag \( \omega_f \). The accuracy of these measurements is mainly determined by the interference between both diffractions, and by the uncertainty about the exact cause of the diffractions.
Secondly, we identified a dry zone at the tip in experiments on plaster after splitting of the block after the experiment (see figure 5.19). This dry zone has the same surface roughness profile as the wetted part of the fracture. The acoustic measurements indicate that the fluid front stays at the same position after shut-in, so we can compare the acoustically determined measure of the fluid lag $\omega_f$ with the size of the visual dry zone $\omega_d$ at the tip, also indicated with "dry tip".

![Figure 5.19. Roughness pattern and dry tip after splitting of the block in experiment sp06 on strong plaster. The wellbore is located in the upper right corner. The contrast in this picture is increased, compared to the original photograph.]

Figures 5.20 and 5.21 show the acoustically determined measure of the fluid lag size in two cement experiments, together with the acoustic measurements of fracture radius and position of the fluid front. The figures show that radius growth continues while the pressure falls after shut-in. At the same time, the size of the fluid lag decreases. This observation confirms the concept of the fluid lag that creates a balance between toughness and fluid pressure loading. The stress intensity at the tip stays critical - which can be inferred from the fact that the fracture is propagating - while the pressure is decreasing, which is possible because the size of the fluid lag decreases. We already mentioned the experiment ce08 in which we prefractured the block with water, which indicates that the
influence of fracture toughness is negligible in cement under the conditions of stress and net pressure. Instead of toughness, the stress intensity as a result of net pressure must be counterbalanced by the confining stress working over the fluid lag. This observation is consistent with the observed interaction mechanism.

Figure 5.20. Fracture radius \( R \), fluid front radius \( R_f \), and the acoustically determined measure of the fluid lag size \( \omega_a \) in cement experiment ce04.

Figure 5.21. Fracture radius \( R \), fluid front radius \( R_f \), and the acoustically determined measure of the fluid lag size \( \omega_a \) in cement experiment ce05.
The acoustically determined fluid lag size in cement during propagation is compared with the fully coupled numerical simulations in figure 5.22. It shows that the fluid lag size in experiment ce04 is somewhat larger than the simulation. The difference between the experiment and the simulation for small radius is caused by differences in net pressure between experiment and simulation. These are a result of the varying flow rate in the experiments (see figure 5.1), while the simulations have a flow rate that is constant in time. In experiment ce05, a significantly larger fluid lag size is measured. Figure 5.3 shows that this experiment also has a lower net pressure than all other experiments. Compared to experiment ce04, experiment ce05 has a much larger value of $\sigma_h$. Analysis of the stress situation in the block shows that this can influence the stress in the direction perpendicular to the fracture plane (see section 3.6). Another possibility is that the lubrication by teflon and vaseline functioned insufficiently in this specific experiment. Both the large value of the fluid lag size (figure 5.21) and the value of the wellbore pressure (figure 5.3) can be explained by assuming a confining stress of 4 instead of 8 MPa. The use of presenting the results of experiment ce05 is the qualitative agreement with experiment ce04.

![Figure 5.22. Comparison between the acoustically determined fluid lag size $\omega_a$ in cement experiments ce04 and ce05, and fully coupled linear elastic numerical simulations with a confining stress $\sigma_c=8$ MPa.](image)

Another way to compare the experimentally observed fluid lag size with an elastic model is to use the semi-analytical model of Garagash and Detournay (1998) (see also section 2.6). They calculated a universal curve for the dimensionless toughness $K'^*_c$ versus
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dimensionless fluid lag size $\omega^*$, for a plane strain fracture of semi-infinite length. We apply this model to a radial fracture. When we take $\dot{R}$ for the fracture propagation velocity, the variables are defined by:

$$K_{lc}^* = \frac{K_{lc}}{E} \sqrt{\frac{2\sigma_c}{\pi \mu \dot{R}}} \quad (5.8)$$

$$\omega^* = \frac{\omega}{\frac{\sigma_c^3}{\mu \dot{R} 16E^2}} \quad (5.9)$$

where $\omega$ is the size of the fluid lag. Figures 5.23 and 5.25 show the results for experiments ce04 and ce05. Figures 5.24 and 5.26 show the condition for which the semi-infinite approximation is valid, which is that the length scale of the tip region $L_{tip}$ is much smaller than the fracture radius. This length scale of the tip region is related to the size of the fluid lag corresponding with zero toughness, and is defined by equation (2.61).

The conclusion that we can draw from figures 5.23 through 5.26, is that the measured size of the fluid lag agrees approximately with the model when the approximation of semi-infinite fracture length is valid (taking into account the uncertainty in the confining stress in experiment ce05, which was discussed above). The measurements confirm the interaction between fluid lag and fracture toughness, as predicted by the model. However, the fluid lag size is rather small in comparison with the measuring error for the points where the semi-infinite approximation is valid. When the approximation of semi-infinite fracture length is not valid, the fluid lag size in the experiments is smaller than in the model. This can be expected, because the smaller loading area of a finite fracture leads to a smaller fluid lag size.

Figure 5.27 shows the acoustically measured size of the fluid lag $\omega_a$ in three plaster experiments (two strong plaster blocks, and one with an intermediate water content). This figure also shows the size of the dry tip $\omega_d$ measured after splitting of the block. We see a reasonable agreement between the acoustically measured fluid lag size $\omega_a$ and the size of the dry tip $\omega_d$ measured after splitting of the block. All measurements are averaged over the four sides of the block. The numerical simulations give a value of the fluid lag size $\omega$ that is smaller than $\omega_d$ for both values of the confining stress $\sigma_c$, while the net pressure in the simulations is even somewhat higher than in the experiments.
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Figure 5.23. Dimensionless toughness $K_{IC}^*$ versus the acoustically determined dimensionless fluid lag size $\omega_a^*$ for cement experiment ce04, in comparison with the self-similar elastic model of Garagash and Detournay (1998) for a semi-infinite plane strain fracture.

Figure 5.24. Ratio of the near-tip length scale $L_{tip}$ and the fracture radius $R$, versus the dimensionless fluid lag size $\omega_a^*$ for cement experiment ce04.
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![Graph showing dimensionless toughness $K_{lc}^*$ versus the acoustically determined dimensionless fluid lag size $\omega_a^*$ for cement experiment ce05, in comparison with the self-similar elastic model of Garagash and Detournay (1998) for a semi-infinite plane strain fracture.](image)

Figure 5.25. Dimensionless toughness $K_{lc}^*$ versus the acoustically determined dimensionless fluid lag size $\omega_a^*$ for cement experiment ce05, in comparison with the self-similar elastic model of Garagash and Detournay (1998) for a semi-infinite plane strain fracture.

![Graph showing ratio of near-tip length scale $L_{tip}$ and the fracture radius $R$ versus the dimensionless fluid lag size $\omega_a^*$ for cement experiment ce05.](image)

Figure 5.26. Ratio of the near-tip length scale $L_{tip}$ and the fracture radius $R$, versus the dimensionless fluid lag size $\omega_a^*$ for cement experiment ce05.
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Figure 5.27. Comparison between the fluid lag size $\omega_d$ in plaster measured with acoustic diffractions, the dry tip size $\omega_d$ measured after splitting of the block, and the fluid lag size $\omega$ in the fully coupled linear elastic numerical simulations (Barr, 1991). The measurements of $\omega_d$ and $\omega_d$ with corresponding symbols are done in the same experiment. The simulations were done with $K_{lc} = 0.3 \text{ MPa} \cdot \text{m}$.

Figure 5.28. Value of the dry tip size $\omega_d$ in plaster as a function of the maximum value of the fluid lag size $\omega_{\text{max}}$ according to equation (5.10), using $K_{lc} = 0.3 \text{ MPa} \cdot \text{m}$. The simulated values of the fluid lag $\omega$ versus $\omega_{\text{max}}$ were obtained using the fully coupled linear elastic numerical program of Barr (1991). In these simulations, representative input parameters were used for experiments on weak and strong plaster: $K_{lc} = 0.3 \text{ MPa} \cdot \text{m}$, $\sigma_c$ is between 4 and 8 MPa, $R$ is between 0.10 and 0.11 m, and $p_{net}$ varies between 6 and 13 MPa.
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We plotted the size of the dry tip $\omega_d$ measured after splitting the block as a function of the maximum size of the fluid lag predicted by linear elastic fracture mechanics. This maximum value corresponds to a constant pressure in the fracture up to the fluid lag. When we insert this pressure distribution in equation (2.12), an expression for the maximum value of the fluid lag size $\omega_{max}$ is obtained (assuming that $\omega_{max} \ll R$):

$$
\omega_{max} = \frac{R}{2} \left[ \frac{K_{lc} \pi}{2} \frac{\pi}{R} \right]^{2} \left[ \frac{P_{net,w} - \frac{K_{lc}}{2} \sqrt{\frac{\pi}{R}}}{\sigma_c + P_{net,w}} \right]^{2}
$$

(5.10)

Figure 5.28 shows that the size of the dry tip $\omega_d$ in strong and weak plaster is larger than the fluid lag size predicted by the fully coupled linear elastic simulations. In section 6.3, we compare these results with the results from our elastic-plastic simulation program.

In a few experiments, no dry tip was visible after splitting the block (see table A.1, figure 5.29, and table 5.1 for the experimental conditions). In experiments wp02 and wp08 on weak plaster, this could be caused by the rather small values of $\omega_{max}$ of 3 and 2 mm respectively. In two experiments on strong plaster however (sp09, and sp10), $\omega_{max}$ is 7 and 11 mm. In these experiments, the visible dry tip is absent. The absence of the dry tip goes together with a large fracture surface roughness, and a larger wellbore width and smaller net pressure in comparison to the linear elastic case.

In Quinn (1992), fluid lag sizes in experiments on cement were also measured. The fluid lag sizes were larger than coupled linear elastic simulations, but were smaller than the maximum value given by equation (5.10). However, the fluid lag sizes were measured at the moment the fracture tip was close to the sample boundary, so that boundary effects could have influenced the measurements significantly. Still, these observations are not contradictory to the values of the fluid lag measured in cement in the present study, because of the uncertainty about the exact locations from which the diffractions from the tip region in cement originate. Although we have a good agreement between the acoustically determined fluid lag size and the visually observed dry tip size in plaster, such a comparison is not available in cement, because fracture growth continues after shut-in.

The shape of the fluid front is more irregular for rough fractures. Figure 5.29 shows the shape of the fluid front under various conditions for strong and weak plaster. This could be caused by the grooves on the fracture surface, but could also be indicative for a smaller width at the position of the fluid front.

Visual inspection of fractures in tip region

For moderate values of the externally applied stresses, a clear distinction between the hydraulic fracture surface and the surface caused by splitting of the block is visible by
differences in surface texture. When the block is loaded almost to failure, this clear distinction disappears (see for example figure 5.29), which was the case in experiments sp09, sp10, and wp08. Instead, the surface roughness structure disappears gradually going away from the fluid front. This indicates that the process zone is larger in the latter case.

Figure 5.29. Shape of the fluid front and disappearance of the visible dry tip under various stress conditions: at the upper left experiment wp08 (σ_c=2.5 MPa, σ_h=9.5 MPa), at the upper right experiment wp04 (σ_c=2.5 MPa, σ_h=4 MPa), at the lower left experiment sp09 (σ_c=8 MPa, σ_h=26 MPa), and below experiment sp05 (σ_c=8 MPa, σ_h=12 MPa). The fracture face wetted by silicone oil is dark-coloured. In the left figures, the fluid front at the moment the block was split, is indicated with a pencil line.

Figure 5.30 shows a hydraulic fracture from the side (experiment sp10). First, the leak-off zone is apparent. Second, shear fractures are visible that end on the fracture surface or in the plane of the fracture. These fractures may be pre-existing as a result of the external loading of the block. However, the fact that they all end on the fracture surface shows that they are at least destabilised (in case they pre-existed) by the passing hydraulic fracture.
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We also see that the fluid tends to follow one such shear fracture. This was observed more clearly in other tests, and shows that these fractures are not formed during splitting of the block after the test. We conclude that under influence of the hydraulic fractures shear fractures start growing, which might interact with or originate from existing shear fractures. Whether the fluid follows the induced shear fracture or forms a new fracture within one experiment, may depend on the fracture propagation velocity. The occurrence of shear fractures and the fact that the fluid can follow such a shear fracture, is also seen in other experiments with large deviatoric stresses (sp07, sp09).

Figure 5.31 show a cross-sectional view of the fracture plane, perpendicular to the radial direction (experiment sp09). The fracture surface appears to consist of planes that partly overlap each other. A few fractures are visible that indicate that the fracture planes indeed overlap. This is not apparent in plaster experiments with a lower roughness. This might be due to the scale, or because the apparently overlapping planes do not develop in experiments with a small roughness. In Murdoch (1993) a similar roughness pattern was seen, in which some of the linear grooves were offsets between overlapping fracture planes. Overlapping fracture planes were common for high values of the externally applied stress deviator \((2(\sigma_H-\sigma_T)/(\sigma_H+\sigma_T)\) between 0.7 and 1.1), which is similar to our observations.

Figure 5.30. Side-view of the hydraulic fracture (white dotted line) and infiltration zone. Shear fractures are visible (black dotted lines) near the fracture tip (experiment sp10). The horizontal scale of the picture is approximately 10 cm.
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$r=9.5$ cm at the centre, the radial coordinate is directed out of the paper:

$r=5.2$ cm at the centre, the radial coordinate is directed out of the paper:

$r=5.2$ cm at the centre, the radial coordinate is directed into the paper:

Figure 5.31. Cross-section perpendicular to the radial direction, together with fracture surface, for experiment sp09. The fracture surface appears to consist of planes that partly overlap each other. Furthermore, a few fractures are visible (indicated by dotted lines), that indicate that the fracture planes indeed overlap.
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5.2 Fracture surface roughness

The hydraulic fracture surfaces show a characteristic surface roughness pattern. Figure 5.32 shows an example of the surface roughness for all materials we tested. In this section we will describe this roughness pattern. We characterised the pattern and made correlations with the applied stresses and plastic zone size around the fracture tip.

Figure 5.32. Example of surface roughness pattern for all materials used: cement, plaster, and diatomite. At the upper left experiment ce01 ($\sigma_c=23$ MPa, $\sigma_h=33$ MPa), at the upper right experiment sp04 ($\sigma_c=8$ MPa, $\sigma_h=20$ MPa), at the lower left experiment di03 ($\sigma_c=0.85$ MPa, $\sigma_h=2.125$ MPa). The fracture face wetted by silicone oil is dark-coloured.
Roughness measurement and characterisation
The surface roughness in the experiments on plaster and diatomite showed a pattern of radial grooves (see figures 5.19, 5.29, 5.32, and 5.34). The amplitude of this roughness pattern is much larger than the grain size, which is approximately $10^{-5}$ m (see section 4.1). We measured the surface roughness profile using a laser profilometer (see section 3.10). The roughness was measured along a straight line approximately tangential to the fracture front. The measuring interval length was 8 cm and the radius at the measuring points was between 7 and 8 cm. In its centre, the measuring interval was perpendicular to the radial direction.

There are various ways to quantify the roughness of a surface, which vary from comparing the profile with standardised profiles to calculating the Fourier spectrum. We characterised the profile by two characteristic quantities, which are representative for the amplitude and average slope of the profile. We choose these quantities because they are suitable for physical interpretation.

As characteristic quantity for the amplitude of the roughness profile, we used the root mean square roughness, which is a measure for the deviation of the roughness profile from a centre line through the profile. This centre line is used to filter out the wavelengths that are of a much larger scale than the scale of the fracturing process, and which would influence the root mean square roughness. The definition of a centre line through the profile has a certain degree of arbitrariness. We constructed the centre line in the following way. The slope of the centre line is equal to the average slope of the roughness profile over a 1 cm interval. Furthermore, the average amplitude of the centre line equals the average amplitude of the roughness profile over the entire measuring interval. Figure 5.33 shows an example of a roughness profile and centre line.

The root mean square roughness is defined as:

$$RMS = \left( \frac{1}{n} \sum_{i=1}^{n} \left( y_{\text{profile},i} - y_{\text{centreline},i} \right)^2 \right)^{\frac{1}{2}}$$  \hspace{1cm} (5.11)

where $n$ is the number of points measured and $y_{\text{profile},i}$ is the height of the measured profile at measuring point $i$.

Besides characterising the amplitude of the roughness profiles, we also determined an average slope for the measured profiles. This average slope $\varphi_s$ is defined as:

$$\varphi_s = \frac{1}{n-1} \sum_{i=2}^{n} \arctan \left( \frac{y_{\text{profile},i} - y_{\text{profile},i-1}}{x_{\text{profile},i} - x_{\text{profile},i-1}} \right)$$  \hspace{1cm} (5.12)
Before calculating \( \phi_n \), the measured profile was tilted until a least-square linear fit through the profile had zero slope.

**Influence of stress and rock type on roughness**

The visually observed roughness increases with \( \sigma_h \) and decreases with \( \sigma_c \). Figure 5.34 shows an example of this. A similar roughness pattern and qualitatively similar dependence of roughness on confining stress \( \sigma_c \) was seen in experiments performed on Colton sandstone (Bohmeier, 1995). Medlin and Massé (1984) also reported a consistent increase of fracture surface roughness with decreasing confining stress \( \sigma_c \). In Murdoch (1993) a similar roughness pattern was seen, in which some of the linear grooves were offsets between overlapping or neighbouring fracture planes. Overlapping fracture planes were common for high values of the externally applied stress deviator.

![Graph showing roughness profile and center line](image)

*Figure 5.33. Roughness profile and centre line as a function of the distance along the measuring interval for an experiment on strong plaster with \( \sigma_c = 8 \text{ MPa} \) and \( \sigma_h = 20 \text{ MPa} \)*

The observed dependence of roughness on stress motivated us to plot the RMS roughness as a function of the applied stress. Figure 5.35 shows the RMS roughness as a function of the ratio of \( \sigma_h - \sigma_c \) and \( (\sigma_h + \sigma_c)/2 \). Besides the hydraulic fracturing experiments on plaster and diatomite, also the roughness of uniaxial tensile tests and of a uniaxial compressive test were included in this figure. Figure 5.36 shows the average slope \( \phi_3 \) as a function of \( 2(\sigma_h - \sigma_c)/(\sigma_h + \sigma_c) \).
Figure 5.34. Example of roughness pattern for various stress states. At the upper left experiment sp02 (σ_c=16 MPa, σ_h=20 MPa), at the upper right experiment sp05 (σ_c=8 MPa, σ_h=12 MPa), and below experiment sp06 (σ_c=8 MPa, σ_h=20 MPa). The fracture face wetted by silicone oil is dark-coloured. The contrast in these pictures is increased, compared to the original photographs.

Medlin and Massé (1984) also mention that the surface roughness depended on the rock type (they used Carthage and Lueders limestone, and Mesa Verde sandstone). We also found a dependence on rock properties (see figure 5.32). For cement experiment ce04, the stresses were equal to strong plaster experiments sp01 and sp05. The amplitude of the
5. Experimental results

roughness profile in cement is smaller than $10^{-5}$ m (being the measuring accuracy of the laser profilometer), while the surface of plaster is much more rough.

The RMS roughness in figure 5.35 shows a sharp increase when $2(\sigma_0-\sigma_v)/(\sigma_0+\sigma_v)$ reaches approximately the value 1. A measure of the size of the plastic zone calculated for typical material parameters as a function of stress, shows a similar behaviour (see figure 5.35). With this in mind, a first approach to analyse the cause for the dependence of roughness on applied stress is to correlate the RMS roughness with the size of the plastic zone. Figure 5.37 shows that there is a reasonable correlation between the RMS roughness and the calculated size of the plastic zone. The points for strong plaster and diatomite fall on the same line, while the points for weak plaster fall on a separate line. From the two lines, it would appear that the relationship between calculated plastic zone size and roughness depends on material behaviour. This may be true in general, but cannot be concluded from this picture. This is because the vertical difference of the two lines is not significant with respect to the measuring error of the roughness, which is used to calculate the plastic zone size. Figure 5.36 shows that the average slope $\varphi_s$ as a function of the applied stresses increases with the relative stress difference. When we plot $\varphi_s$ as a function of the plastic zone size, we see that also $\varphi_s$ increase with the plastic zone size (figure 5.38).

In most experiments, the horizontal stresses (parallel to the fracture plane) in both directions were equal. For determining the mechanism that causes the roughness, it is important to know which horizontal stress has the largest influence on the roughness. Therefore we did two experiments in which the horizontal stresses within one experiment were different. In strong plaster, the horizontal stresses were 17 and 24 MPa, while the confining stress $\sigma_c$ was 10 MPa (experiment sp08). Figure 5.39 shows the measured RMS roughness as a function of fracture radius. In a similar experiment on diatomite (experiment di05), the stresses were 2.0, 1.5, and 1.0 MPa (see figure 5.40). Figures 5.39 and 5.40 show that, for relatively large radius, the roughness is larger when the horizontal stress in the direction of fracture growth is larger. This agrees with visual inspection of the block surfaces. However, near the wellbore the reverse appears to be the case, which might be caused by the pressurised wellbore influencing the stress situation. The stress in the radial direction near the wellbore is increased by the wellbore pressure, which makes the relative difference between the two directions smaller. The stress perpendicular to the direction of fracture growth however, shows a relatively large difference. Near the wellbore, a higher compressive stress in the direction perpendicular to fracture growth correlates with a larger roughness. Although these measurements are not conclusive, it appears that the horizontal stress in the direction of fracture growth has a larger influence on the roughness than the horizontal stress perpendicular to the direction of fracture growth.
5. Experimental results

Figure 5.35. Measured RMS roughness as a function of the externally applied stress $\sigma_c$ and $\sigma_h$ ($\sigma_h$ is taken in the direction of fracture growth). The dotted lines indicate the measure of the plastic zone size $l_p$ for strong plaster, calculated with varying $\sigma_c$ and a constant value $\sigma_h$ and for constant $\sigma_c$ and varying $\sigma_h$.

Figure 5.36. Angle $\varphi_s$ as a function of the externally applied stresses.
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Figure 5.37. Correlation of plastic zone size with RMS roughness.

Figure 5.38. Correlation between angle $\phi_s$ and plastic zone size $l_p$. 
Figure 5.39. RMS roughness in strong plaster as a function of radius at four sides of the block, with different horizontal stresses $\sigma_{h,x}$ in the direction of fracture growth and $\sigma_{h,z}$ in the direction perpendicular to fracture growth (experiment sp08).

Figure 5.40. RMS roughness in diatomite as a function of radius at four sides of the block, with different horizontal stresses $\sigma_{h,x}$ in the direction of fracture growth and $\sigma_{h,z}$ in the direction perpendicular to fracture growth (experiment di05).
5. Experimental results

**Dependence of roughness on radius and propagation velocity**

As shown above, we can successfully correlate the roughness with a measure of the plastic zone size. We can also consider other influences on roughness: we might expect that the roughness increases with net pressure, fracture radius, and propagation velocity.

Figure 5.41 shows the measured roughness as a function of radius. The length of the measuring interval changes proportional with the fracture radius. This ensures that for all measurements, the variation of radius along the measuring line is relatively the same (the same was done in obtaining figures 5.39 and 5.40). The figures show that the influence of radius is small, if present at all. In experiments in which the propagation velocity varied two orders of magnitude (between $10^{-3}$ and $10^{-5}$ m/s) and which had identical net pressure and radius (experiments wp04, wp05, and wp06), no significant variation in roughness was found. Within one experiment, the propagation velocity and net pressure are typically respectively 2 and 5 times larger for small radius (5 cm) in comparison with larger radius (10 cm). As is apparent from the dependence of roughness on radius, the data do not support the possible hypothesis that the roughness is influenced significantly by propagation velocity, net pressure, or radius.

In the experiments on Colton sandstone in Bohlmeier (1995), the propagation velocity varied between $10^{-2}$ and $10^{-4}$ m/s. There was no clear visible effect on the fracture surface roughness. Uniaxial tensile tests on weak plaster showed that the roughness of samples that fractured unstably and stably (propagation speed of $10^{-5}$ m/s) was comparable.

![Figure 5.41. RMS roughness as a function of fracture radius for various experiments.](image-url)
5.3 Hydraulic fracture closure

The closure period of hydraulic fractures contains information about the fracture geometry and rock properties. In field practice, analysis of closure of minifracs is used to infer values of the leak-off coefficient, fracture efficiency, and least in-situ stress. These are key parameters in the design of the main hydraulic fracture. The fracture geometry created during propagation will also influence the closure period.

In this section, we will present measurements done during closure of hydraulic fractures in our laboratory experiments. We will show the significance of plastic deformation for the pressure decline during closure. We will first give typical examples of the closure period in the rock materials we used. Then, we present results of fracture closure in strong plaster, which can be understood based on linear elastic fracture mechanics. This will be used to analyse closure in weak plaster, where plasticity has a significant influence. More information about the analysis of the closure period can be found in section 6.3, 7.4, and in van Dam et al. (1998).

Typical examples of fracture geometry and pressure during closure

We used the Nolte model (Nolte, 1986) to present our measurements during closure. This model is based on the assumption of constant radius after shut-in, constant leak-off coefficient according to Carter's law (equation 2.53), and a linear relationship between net pressure and fracture opening. The model takes into account the difference in pressurisation history of the fracture surface for different positions on the surface, which leads to a smaller leak-off velocity for positions closer to the wellbore. The predicted pressure decline rate is determined by the total leak-off rate through the fracture surfaces, and the compliance of the fracture. The fracture compliance is determined by the crack opening modulus and the fracture radius, which are assumed to be constant during closure. A uniform pressure inside the fracture and an elliptical width profile are assumed.

Figures 5.42 through 5.45 show the decline of the wellbore pressure $p_w$ and the fracture width at the wellbore $b_w$ as a function of the dimensionless $G$-function in the limit of low leak-off (see Appendix B). Assuming constant fracture compliance, the Nolte model predicts a straight line when plotting the pressure against the dimensionless $G$-function (see equation (B.13)). However, the pressure decline curve in all experiments has a strongly curved shape. In weak and strong plaster, the initial pressure decline rate agrees with the Nolte model, using the independently determined leak-off coefficient from figure 4.38, the unloading modulus, the fracture radius, and the system compliance (see table 5.3). The system compliance is needed in order to correct for the fluid flowing from the wellbore into the fracture as a result of system decompression (see equation (5.4)). The total time needed to reach closure at the wellbore in plaster, is larger than predicted by the
5. Experimental results

Nolte model. In cement and diatomite, the initial pressure decline rate shortly after shut-in is much larger than predicted by the Nolte model (see table 5.3).

Table 5.3. Comparison of the leak-off coefficient determine in the triaxial cell, and from the pressure decline directly after shut-in in hydraulic fracturing experiments.

<table>
<thead>
<tr>
<th>Material</th>
<th>triaxial cell</th>
<th>hydraulic fracturing decline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_l$</td>
<td>$C_l$</td>
</tr>
<tr>
<td></td>
<td>($10^{-5}$ ms$^{1/2}$)</td>
<td>($10^{-5}$ ms$^{1/2}$)</td>
</tr>
<tr>
<td>cement</td>
<td>0.005 ± 0.005</td>
<td>0.37 ± 0.11</td>
</tr>
<tr>
<td>plaster</td>
<td>2.1 ± 0.2</td>
<td>2.5 ± 0.9</td>
</tr>
<tr>
<td>diatomite</td>
<td>0.12 ± 0.06</td>
<td>1.3 ± 0.6</td>
</tr>
</tbody>
</table>

Another characteristic in the experimental results is that the wellbore width at the moment of closure is typically larger than zero in cement and plaster (see also Appendix A, figures A.2, A.5, and A.8). This non-zero width remains approximately constant, while the wellbore pressure continues to decrease. We call the fracture to be closed then, despite of the non-zero final width. In diatomite however, the wellbore width tends to return to zero (see figures 5.45 and 5.51). The signal of acoustic shear transmission measurements is almost completely recovered in closed fractures in plaster, which indicates mechanical contact of the fracture surfaces. In cement, this signal was never even partly recovered during closure of fractures filled with silicone oil, while it was recovered almost completely after closure of the fracture in cement that was created using water. In combination with the fact that fractures in cement are very smooth (see figures 5.5 and 5.32) and the extremely low leak-off velocity of the silicone oil into the cement, we conclude that the remaining width in cement is caused by a remaining thin film of silicone oil. In plaster we will refer to the width that remains after closure as the residual width, which is defined as the width caused by the non-matching surfaces in mechanical contact.

One difference between closure of fractures in strong plaster and weak plaster, is the fact that all fractures in weak plaster closed at the wellbore at a fluid pressure at the wellbore that is significantly lower than the confining stress working perpendicular to the fracture surface (see figure 5.44). In the following we will concentrate on this phenomenon. Therefore we will first present additional measurements in strong plaster, so that we can understand the pressure decline in this material.
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Figure 5.42. Typical example of wellbore pressure decline and fracture width at the wellbore as a function of dimensionless G-function for cement (experiment ce04).

Figure 5.43. Typical example of wellbore pressure decline and fracture width at the wellbore as a function of dimensionless G-function for strong plaster (experiment sp06).
5. Experimental results

Figure 5.44. Typical example of wellbore pressure decline and fracture width at the wellbore as a function of dimensionless $G$-function for weak plaster (experiment wp03).

Figure 5.45. Typical example of wellbore pressure decline and fracture width at the wellbore as a function of dimensionless $G$-function for diatomite (experiment di04).

Elastic analysis of pressure decline in strong plaster

In the preceding section we saw deviations from the Nolte model in all materials. In order to explain the deviations in strong plaster, we validated the assumptions of the Nolte model and analysed their influence (see also van Dam et al., 1998). First, we considered the assumption of constant leak-off coefficient. Figure 4.38 in Chapter 4 shows the
measured leak-off coefficient in plaster as a function of pressure, which indeed is fairly constant for the pressure range of interest. The expected leak-off behaviour is further validated experimentally in another type of leak-off test. In this test, fluid leaked away into a constant area of rock from a system of constant compressibility, without pumping fluid into the system (see section 4.6). The system compressibility was such that the pressure decline rate was about equal to the initial pressure decline in the hydraulic fracturing experiments. This leak-off was preceded by about 1000 s pressurisation with constant pressure, thus simulating injection and subsequent shut-in in a hydraulic fracturing experiment. Figure 4.40 in Chapter 4 shows a typical example of the pressure decline in this type of test. The leak-off experiment shows a pressure decline rate that is fairly constant in time for the pressure range of interest, as was expected from the approximately constant leak-off coefficient. Concluding, the assumption of the leak-off coefficient being constant is reasonable.

Second, the fracture surfaces are not smooth, which is expected to cause a residual width after closure. This is caused by a small displacement of the fracture surfaces with respect to each other, as a result of which the opposing surfaces do not match. The influence of roughness on the closure process is discussed in section 7.4. Figures A.5 and A.8 show that all experiments on plaster show a residual width after closure at the wellbore has taken place. However, we did not find a correlation between the roughness and the value of the residual width after closure at the wellbore in strong plaster. The fact that the pressure at which the fracture closes mechanically is slightly larger than the confining stress (see figure 5.42) in most experiments on strong plaster may also be associated with this effect. Poro-elastic effects are not expected to have caused this, because the residual width stays approximately constant while the pressure declines after closure at the wellbore has taken place.

Third, the radius does not stay constant during closure. Acoustic diffraction measurements show fracture growth after shut-in in cement (see figures 5.20 and 5.21). Acoustic transmission measurements after shut-in show radius decline during closure in plaster (see figure 5.46). This figure shows that the recovery of the transmitted signal is a gradual process, which complicates the picking of the closure time. Therefore, we calibrated the picking of the closure time from the transmission measurements by comparing the measurements during propagation with the diffractions. The ultimate result shows approximately a linear decay of radius with time after shut-in (see figure 5.47). This figure also shows that the acoustic measurements are in line with the closure time detected from the width measurement at the wellbore.

Radius recession is a possible cause for the observed decreasing pressure decline rate in strong plaster. We adjusted the conventional Nolte model in order to take into account the radius recession, and investigate whether it can explain the observed pressure decline. We approximate the measured radius decline in figure 5.47 by a linear decrease of radius with time. When we assume that no leak-off takes place over the closed part of the fracture, we
find a reasonable prediction for the measured volume after shut-in in the experiment on strong plaster (see figure 5.48). This model is described and discussed in Appendix B.

Besides the leak-off rate, the pressure response is determined by the mechanical behaviour of the fracture. Figure 5.49 shows the value of \( \gamma \) during closure for the assumption of constant radius after shut-in, and for radius receding linearly with time according to the measurements in figure 5.47. When we assume constant fracture radius, the value of \( \gamma \) becomes smaller than during propagation. Assuming a receding radius, the value of \( \gamma \) is approximately constant, although it becomes somewhat higher than expected. This can be explained from the difference between the measured receding radius and the linear fit, plastic rock deformation, and incomplete closure due to fracture roughness. The calculated values of \( \gamma \) in figure 5.49 show that the receding radius must be taken into account in order to understand the relation between net pressure and width in the experiments.

Based on the above elastic analysis, we understand both the observed leak-off behaviour and the mechanical fracture behaviour during closure in strong plaster, accepting that radius recession takes place. As these two processes determine pressure decline, we now understand the observed pressure decline in strong plaster. Comparing figures 5.43 and 5.44, we see very different behaviour in closure for strong and weak plaster. Based on the measurements during propagation and the material property determination, we expect that these differences are caused by plastic deformation, which is significant in weak plaster.

![Figure 5.46. Transmitted shear-energy during closure in experiment sp06 on strong plaster, for various transmission combinations (see figure 3.10). The radial coordinate of the transmission ray path at the fracture plane is indicated in the figure.](image-url)

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Figure 5.47. Acoustically determined receding radius $R_{rec}$ after shut-in in three experiments on strong plaster. The measurement after splitting of the block ($t=t_{si}$) and at the moment of closure at the wellbore ($R_{rec}=0.0115$ m) are also in this figure.

Figure 5.48. Comparison of measured leak-off volume and modelled leak-off volume in strong plaster (experiment sp06), using the Nolte model which assumes constant radius and the model which incorporates the receding radius (see Appendix B). The fracture volume is calculated from the wellbore width and receding radius, assuming an elliptical width profile.
5. Experimental results

![Graph of \( \gamma_1 \) values for experiments sp06 on strong plaster, showing both constant and receding radius over time.]

Figure 5.49. Value of \( \gamma_1 \) in experiment sp06 on strong plaster, for both constant and receding radius after shut-in (using the approximation of radius receding linearly with time).

**Pressure decline in weak plaster**

The pressure decline curve after shut-in in weak plaster shows the same curved shape as in strong plaster. There is one striking difference, which is that in all experiments on weak plaster the fracture closed at a pressure that was well below the confining stress level (see figure 5.44). Acoustic transmission measurements show that at zero net pressure the fracture was open over a considerable part of the fracture radius, which shows that this effect is not limited to the wellbore region. The fracture width at the wellbore at zero net pressure varied between the experiments. Experiments on strong plaster and diatomite with a relatively large plastic zone showed the same phenomenon.

Table A.1 in Appendix A gives an overview of the width \( b_0 \) at the moment the net pressure is zero, in the experiments which showed fracture closure at a wellbore pressure below the confining stress. We plotted the measured value of \( b_0 \) versus the calculated size of the plastic zone around the tip \( L_p \) in figure 5.50. This figure shows that \( b_0 \) appears to increase with \( L_p \) for the experiments in weak and strong plaster, except for one experiment in which the volumetric creep rate was relatively large. However, the amount of measurements is not large enough to make definitive conclusions.
Figure 5.50. Wellbore with $b_0$ at zero net pressure during closure, versus the measure of the size of the plastic zone $l_p$. In one experiment on weak plaster (wp02), the effect of volumetric creep was probably large, because of the high applied stresses and relatively large propagation time.

In the experiments on weak and strong plaster which show fracture closure at a level below the confining stress, the closure times were increased, while the leak-off coefficient was comparable to the experiments on strong plaster. The difference in closure time is visible for example in figure 5.47 for experiments sp05, sp06, and sp09, and also in the raw data in Appendix A. The larger closure time of experiment sp09 in comparison with experiments sp05 and sp06 is partly caused by the larger fracture width.

Furthermore, in strong plaster closure is visible as a break in the pressure decline, and also as a clear break in the width decline (see figure 5.43). All experiments on plaster that show closure below the confining stress do not show a break in the G-plot of the pressure decline (see figure 5.44). Some experiments on diatomite do show a break in the pressure decline, even when the fracture closes at a pressure below the confining stress (see figure 5.51). When plotting the pressure decline on a log-log scale, we can identify closure as a local minimum in the pressure derivative for strong plaster. For the experiments in which the pressure decline does not show a clear break (experiments on weak plaster and cement), a similar minimum in the derivative can be identified (see van Dam et al., 1998).
5. Experimental results

![Graph showing closure stress and wellbore width (b) against G(t_D)](image)

*Figure 5.51. G-plot in diatomite (experiment di03).*

5.4 Conclusions

In this chapter we showed the main experimental results. From this, we can draw the following conclusions.

- The experimental results in cement confirm the scaling of the net pressure and fracture width at the wellbore in the viscosity-dominated regime, according to equations (5.1) and (5.2). In the experiments, the flow rate, confining stress, and the fracture radius varied.

- The net pressure and wellbore width agree with the fully coupled linear elastic simulation in cement. This comparison comprises the net pressure and fracture width at the wellbore, the width profile, the fracture volume, and a measure of the fluid lag size.

- In strong plaster, the fracture width and net pressure at the wellbore in experiment and linear elastic simulation agree for limited values of the externally applied stress deviator. For relatively large values of the externally applied stress deviator in strong plaster, and for all experiments in weak plaster, the wellbore measurements show significant differences with the linear elastic simulation. The wellbore width is larger, while the net pressure is smaller than in the simulation. Creep in weak plaster contributes to the wellbore width being larger and the net pressure being smaller than the linear elastic simulation predicts.

- In strong plaster, the fracture volume measurements indicate a development of bluntness of the tip region with increasing plastic zone size.
• The measured dry tip size in plaster for small and moderate values of the externally applied stress deviator is in general larger than predicted by fully coupled numerical simulations based on linear elastic fracture mechanics. When the block is loaded almost to failure, the dry tip in plaster vanishes completely.

• The width and net pressure at the wellbore in experiments on diatomite agreed with the linear elastic simulations.

• The fracture surface roughness for stable hydraulic fracture propagation is determined by the externally applied stress and rock properties. The roughness shows a correlation with a measure of the size of the plastic zone around the fracture tip. We did not see any effect of the propagation speed on the roughness. As the surface roughness reflects the fracturing process, this observation indicates that the fracturing process is influenced by the plastic zone. Possibly, the failure process localises in the plastic (shear) zone, so that the actual fracturing process is a combination of tensile and shear failure. This view is supported by observations of fracture patterns around the tip under extreme loading conditions.

• Radius recession takes place in plaster. When radius recession is taken into account, the fracture closure process in strong plaster can be understood quantitatively using a linear relationship between fracture opening and net pressure. In weak plaster, a significant deviation from elastic behaviour occurs: the fracture closes at a wellbore pressure that is significantly lower than the confining stress working at the fracture surfaces. The degree of this effect appears to increase with the estimated size of the plastic zone.
where $\dot{R}$ is the fracture propagation velocity. This equation was used in many studies (for example in Desroches et al., 1994), and was found to give satisfactory results. We use it in our numerical simulations for a finite fracture length. Below we show which approximations are made in obtaining equation (6.3) from the assumption of self-similar propagation, when the fracture length is finite.

Self-similar propagation implies that we can express $b/b_w$ (fracture width $b$ divided by the width at the wellbore $b_w$) as a function of $r/R$ (radial coordinate $r$ normalised by the fracture radius $R$), which is constant in time:

$$b(r,t) = b_w(t) f\left(\frac{r}{R(t)}\right)$$

(6.4)

Taking the derivative to $r$ and $t$ yields:

$$\frac{\partial b}{\partial t} = \frac{\partial b_w}{\partial t} - b_w \frac{\partial f\left(\frac{r}{R}\right)}{\partial \left(\frac{r}{R}\right)} \frac{r}{R}$$

(6.5)

$$\frac{\partial b}{\partial r} = b_w \frac{\partial f\left(\frac{r}{R}\right)}{\partial \left(\frac{r}{R}\right)} \frac{1}{R}$$

(6.6)

Combination of these equations leads to:

$$\frac{\partial b}{\partial t} = \frac{\partial b_w}{\partial t} - \frac{r}{R} \frac{\partial b}{\partial r}$$

(6.7)

The influence of the time derivative of the width to the local flow rate is relatively important near the tip, while it is relatively unimportant near the wellbore. This is illustrated in the fully coupled linear elastic simulation used for validation (see figure 6.7), in which the ratio between the time derivative of the width and the average fluid velocity in the radial direction is $3 \cdot 10^{-2}$ near the tip, and $3 \cdot 10^{-4}$ near the wellbore. Furthermore, the second term on the right-hand side of equation (6.7) is much larger than the first term on the right-hand side near the tip, where $r = R$. Based on these considerations, we derive equation (6.3) by writing the time derivative of the width as:

$$\frac{\partial b}{\partial t} = \frac{\partial b_w}{\partial t} + \frac{R-r}{R} \frac{\partial b}{\partial r} - R \frac{\partial b}{\partial r} = -R \frac{\partial b}{\partial r}$$

(6.8)
This expression is used to calculate the local flow rate. It will give the correct behaviour near the tip. Near the wellbore the neglected terms in brackets may be significant for calculating \( \frac{\partial b}{\partial t} \), but here the influence of \( \frac{\partial b}{\partial t} \) on the local flow rate is relatively small. Furthermore, the neglected terms will partly cancel out. Using equation (6.3) turns out to be a reasonable approximation, at least for the situations considered here (see figure 6.8).

Combination of the self similar propagation assumption expressed by equation (6.8) and the continuity equation (2.46) yields for impermeable fracture walls:

\[
\frac{\partial q_f'}{\partial r} = R \frac{\partial b}{\partial r} - \frac{q_f'}{r}
\]  
(6.9)

Integration of this equation yields (using that the width and flow rate are zero at \( r=R \)):

\[
q_f'(r) = \int_r^R \frac{q_f'(r)}{r} \, dr + R b(r)
\]  
(6.10)

where \( q_f' \) is the flow rate divided by \( 2\pi r \). We use this equation to calculate the local flow rate in the numerical simulations, which is used in the Poiseuille equation (2.41).

**Tip model**

We assumed a cohesive-zone model (see section 2.3) for the fracture tip region, according to Hillerborg et al. (1976). In this model the already fractured rock still can bear tensile stress after the maximum tensile stress has been reached, which can be observed in displacement controlled tensile tests. In the cohesive-zone model, a tensile stress \( \sigma_{coh} \) is applied on the fracture faces after the tensile strength has been reached (indicating that the material has started to fracture). The applied stress \( \sigma_{coh} \) is a function of the fracture width \( b \), and \( \sigma_{coh} \) is zero as \( b \) has reached a critical value \( b_c \) (indicating that complete separation of the opposing fracture surfaces has occurred). The relationship between tensile stress and width is obtained from uniaxial tensile tests (see figure 6.1). In the simulations presented, we used a linear relationship between opening \( b \) and stress \( \sigma_{coh} \) after the maximum stress \( T_0 \) was reached:

\[
\sigma_{coh} = T_0 \left( 1 - \frac{b}{b_c} \right), \quad b \leq b_c
\]  
(6.11)

\[
\sigma_{coh} = 0, \quad b > b_c
\]
Numerical simulation of hydraulic fracturing in an elastic-plastic solid

In this chapter we present a numerical model and simulation results of hydraulic fracture propagation and closure in an elastic-plastic solid. We simulate a radial fracture geometry oriented transversely to the wellbore, similar to the experiments. The aim of the numerical simulations is to study the basic mechanisms that are responsible for the deviations from linear elastic fracture mechanics predictions that we observed in our experiments (see Chapter 5).

6.1 Model description

 Mechanical behaviour of the solid
We use the program FLAC to solve the mechanical equations within the solid. The constitutive behaviour of the solid is based on elasticity and plasticity theory, as described in section 2.1. We use a Mohr-Coulomb model for the plastic yield function. The use of the Mohr-Coulomb model is motivated by the unimportance of the intermediate principal stress in the failure of strong plaster at low mean pressure (see section 4.4). The cohesion and friction angle vary as a function of the accumulated plastic strain $\gamma^p$, whose increment is defined as:
6. Numerical simulations

\[ d\gamma^p = \left( \frac{1}{2} (d\gamma_1^p - d\gamma_2^p)^2 + \frac{1}{2} (d\gamma_2^p)^2 + \frac{1}{2} (d\gamma_3^p - d\gamma_m^p)^2 \right)^{\frac{1}{2}} \]

\[ d\gamma_m^p = \frac{1}{3} (d\gamma_1^p + d\gamma_3^p) \]  

\( d\gamma_1^p \) and \( d\gamma_3^p \) are principal plastic shear strain increments, which are given by the principal strain increments:

\[ d\gamma_1^p = |de_2^p - de_3^p| \]

\[ d\gamma_3^p = |de_1^p - de_2^p| \]

\( de_i^p \) is the principal plastic shear strain in the \( i \)-direction.

**Fluid flow**

We consider a radial fracture, in which fluid is injected from a wellbore. We use the Poiseuille equation (2.41) to calculate the local pressure gradient, as a function of the local width and flow rate. The flow rate is calculated using the continuity equation (2.46). We assume impermeable fracture walls, because for the efficiencies in our experiments leak-off has only a small effect on the pressure gradient and width profile for fractures compared at equal fracture length (see simulations presented in Lenoach, 1995).

The time derivative of the fracture width appears in the continuity equation (2.46). Convergence of the numerical integration scheme is facilitated by expressing the time derivative of the width by the width gradient. This relationship between the time derivative of the width and the width gradient was provided by using the assumption of self-similar propagation. The assumption of self-similar propagation is not fully correct, indicated by the fact that the width profile does change in shape somewhat during the simulation. However, the effect of this deviation will show to be only small, at least for our conditions. It is important to emphasize that by using the assumption of self-similarity we do not impose a restriction on the shape of the width profile during the simulation. Instead, we only use it in order to calculate the local flow rate via the continuity equation (2.46).

For a semi-infinite fracture radius, the assumption of self-similar growth relates the time derivative of the width to the width gradient:

\[ \frac{\partial b}{\partial t} = -R \frac{\partial b}{\partial r} \]  

(6.3)
Figure 6.1. Schematic view of a stable uniaxial tensile test. The relationship between $\sigma_{\text{coh}}$ and fracture opening $b$ is given by the descending line. Using this line neglects the deformation over the measuring interval.

We assume that the cohesive zone is permeable for fluid flow. The cohesive zone is thought to consist of ligaments that connect the opposite fracture faces, which are expected not to hinder fluid flow significantly. The assumption of a permeable cohesive zone is further supported by the value of $b_c$ for weak plaster being almost as large as the fracture width measured at the wellbore during the model experiments of hydraulic fracture propagation. The fact that the fluid actually is flowing into the fracture, supports the assumption of a permeable cohesive zone. In the numerical simulations, the tensile cohesive stress in the cohesive zone is added to the fluid pressure loading in the cohesive zone.

**Required input parameters**

As required input parameters for the described model, we have:

- Constitutive behaviour: elastic bulk modulus $K$ and shear modulus $G$, and hardening tables for plastic yield function as a function of accumulated plastic strain.
- Flow rate $Q$ at the wellbore and fracturing fluid viscosity $\mu$.
- The uniaxial tensile strength $T_0$ and the critical opening $b_c$ for the cohesive zone.
6.2 Numerical method

Program FLAC
We used the two-dimensional finite difference program FLAC, which uses an explicit time-marching scheme. FLAC calculates static solutions by solving the dynamical equations of motion for each grid point, and uses mechanical damping of the grid-point velocities. From the grid-point displacements, new strains and stresses are calculated using the constitutive behaviour of the solid. We used an axisymmetric geometry. Further information about FLAC can be found in Appendix C.

We used the programming language embedded within FLAC for the adjustment of boundary conditions. These were altered as a function of the grid-point displacements and time, according to the iteration scheme representing hydraulic fracture propagation. We ran the program on a PC, with a 233 MHz Intel Pentium processor. Propagation of the hydraulic fracture in the presented results took typically 24 hours.

Grid and boundary conditions
Figure 6.2 shows the boundary conditions and axisymmetric grid. We modelled only the top of the block, because of symmetry considerations. Around the tip a fine grid is needed. As there is no practical remeshing possibility, a fine grid was required around the whole fracture. The used grid size is fine enough to describe strong pressure gradients near the fracture tip (see for example figure 6.10). We expect that our grid is fine enough is this respect, based on the validation of the model using the model of Barr (1991) (see next section). The grid size near the tip was approximately 1.5 mm, which is approximately equal to the size of the plastic zone calculated for example in figure 5.37 in Chapter 5. However, these plastic zone sizes were based on strength parameters that describe failure. Plastic deformation already takes place before failure is reached, so that we expect that the grid size is much smaller than the plastic zone. The results confirm this expectation. The grid size is small enough to identify the mechanisms near the tip, without aiming at a full quantitative comparison with the experimental results. Because of this and because of computing time, we did not refine the grid in order to check whether significant differences would occur. At the end of the calculation steps, the maximum unbalanced force per element in the grid was typically 1 % of the force per element applied at the boundary.

We applied stress boundary conditions on the external surfaces of the block, inside the wellbore, and on the pressurised part of the fracture. The grid-point coordinates in the plane of the fracture were initially fixed in the direction perpendicular to the fracture plane. Fracturing of these grid points took place when the tensile stress exceeded the tensile strength. Fracturing of a grid point was modelled by removing the displacement
constraint. After fracturing, the grid-point positions were fixed again when the initial position was reached (which happens after unloading of the pressure working on the fracture surface).

Figure 6.2. Boundary conditions and grid.

**Iteration scheme and coupling of equations**

Each simulation started with application of the externally applied stresses and the wellbore pressure, for elastic material behaviour. After convergence, we opened an elastic fracture with small radius. After this, plasticity parameters were introduced in case of elastic-plastic constitutive behaviour. This leads to a local exaggeration of the plastic strain, which is visible in the final width profile.

When simulating hydraulic fracture propagation, we want to calculate the pressure- and width profile for a certain fracture radius. After obtaining these profiles, the fracture radius is increased, and new pressure- and width profiles are calculated. In the calculation procedure for a certain radius, the usual approach in numerical simulations is to take the
fracture tip fixed and let the fluid front position vary in the calculation (e.g. Papanastasiou and Thiercelin, 1993). We used a different approach, in which we fixed the position of the fluid front and let the fracture tip position vary. Fracture propagation is represented by increment of the fluid front position.

By using equation (6.10), the local flow rate is calculated from the fracture propagation velocity \( \dot{R} \) and the width profile, without using the flow rate at the wellbore \( Q \). We calculate \( Q \) from the change in fracture volume \( \Delta V_{\text{frac}} \) for two subsequent fluid front positions, separated by a distance \( \Delta R \):

\[
Q = \Delta V_{\text{frac}} \frac{\dot{R}}{\Delta R}
\]  

(6.12)

When calculating the pressure and width profiles for a certain radius, the fracture propagation velocity was varied. Directly after each increment of the fluid front, the pressure on the surfaces was diminished strongly by approximately halving the propagation velocity. When equilibrium of the fluid flow equations and mechanical behaviour of the solid was reached for this low propagation speed, the calculated flow rate was too low. Then, the propagation speed was increased, and again equilibrium was reached using this new propagation speed. Increasing the flow rate leads to an increase in fracture volume, so that equation (6.12) shows that the flow rate \( Q \) increases when the fracture propagation velocity \( \dot{R} \) increases. Increasing the flow rate and reaching equilibrium was repeated until the calculated flow rate equalled the desired flow rate, which was constant in time.

This approach had the following advantages. First, the stress path on the fracture surface was one of monotonically increasing loading. This is important for elastic-plastic calculations. When the loading is too large, unrealistic plastic strains will develop, which are irrecoverable. Furthermore, we allowed the tip of the cohesive zone to propagate during loading of the fracture surfaces. In this way, the fracture is always in a state of propagation when the tensile stress is reached at the beginning of the cohesive zone. In that case, the fracture propagation criterion is always satisfied, and does not need to be considered further in the simulations. Using this method, fractures with zero toughness can be propagated too (which is of interest for fracture reopening, for example).

As explained, reaching the required flow rate was used as propagation criterion instead of reaching a critical stress intensity. This approach is possible because of the specific values of the parameters in our problem, for which a fluid lag is present. Then, the tensile failure stress at the tip is always reached. The limitation of this approach becomes visible when the fluid front reaches the crack tip. In that case the propagation criterion is not fulfilled automatically. We have two criteria that have to be fulfilled: both the flow rate and the tensile stress must be critical. We must reach this situation by increasing both the
propagation speed and the tip pressure. Although it appears in principle possible to have useful results also in this case, we did not pursue the simulations for this situation.

For a given radius of the fluid front, the fluid flow equations and mechanical behaviour of the solid must be solved in order to reach equilibrium. These equations are solved simultaneously. Every ten calculation steps of FLAC, the new pressure distribution is calculated from the fluid flow equations, which use the width profile as input. Reaching equilibrium took in general less than 2000 steps.

The previously described iteration scheme consists of three iteration levels. At the most basic level, the fluid flow equations, fracture propagation in the cohesive zone, and mechanical equations of the solid are solved, for a fixed fluid front position and propagation velocity. One level higher, the propagation velocity is increased, for a fixed fluid front position. At the highest level, the position of the fluid front is increased. Figure 6.3 gives a schematic view of the previously described iteration scheme.

<table>
<thead>
<tr>
<th>main loop:</th>
<th>basic routine:</th>
</tr>
</thead>
<tbody>
<tr>
<td>do while fluid front position &lt; final position</td>
<td>do 200 times:</td>
</tr>
<tr>
<td>if fluid front is in initial position</td>
<td>do 10 steps:</td>
</tr>
<tr>
<td>do initial routine</td>
<td>check displacements and stress in fracture plane, and apply displacement boundary conditions</td>
</tr>
<tr>
<td>end if</td>
<td>calculate and apply new pressure distribution</td>
</tr>
<tr>
<td>do while current flow rate &lt; desired flow rate</td>
<td>calculate and apply new stresses in cohesive zone</td>
</tr>
<tr>
<td>do basic routine (start integration of pressure profile at the fluid front position)</td>
<td>end loop</td>
</tr>
<tr>
<td>increase propagation speed</td>
<td>initial routine:</td>
</tr>
<tr>
<td>end do</td>
<td>do while fluid pressure at fluid front is zero</td>
</tr>
<tr>
<td>increase fluid front position</td>
<td>do basic routine (start integration of pressure profile at the wellbore)</td>
</tr>
<tr>
<td>decrease propagation speed</td>
<td>increase wellbore pressure</td>
</tr>
<tr>
<td>end do</td>
<td>end do</td>
</tr>
<tr>
<td></td>
<td>introduce plastic parameters (optional)</td>
</tr>
</tbody>
</table>

Figure 6.3. Basic iteration scheme of hydraulic fracturing simulation program, as used after the application of the externally applied stresses $\sigma_c$ and $\sigma_w$, and the wellbore pressure.
Figure 6.4. Comparison between numerical model and triaxial extension test results for weak plaster. For larger confining stress, the experimental results are influenced significantly by creep in the radial direction during the turning of the axial plunger after hydrostatic compression. The stress loops in the experiments are removed from the experimental data.
6.3 Numerical results

**Input parameters**
We determined the friction angle, cohesion, and cap pressure for volumetric yielding as a function of the hardening parameters used in FLAC. Using these data, we simulated our triaxial extension tests on weak plaster in a one-element test. Figure 6.4 shows the results of axial strain versus axial stress. We assumed zero dilatancy angle in these simulations, which can be justified by the results in figure 4.24. There is a reasonable match between simulations and experiments, except for the case in which the radial pressure was 12 MPa. This was probably due to creep. Figure 6.5 shows a comparison between two uniaxial tensile tests on plaster, and a FLAC simulation. It was not possible to model the plastic deformations in the extension test and the tensile test with the same parameters. Compared to the extension test, we reduced the cohesion in order to match the tensile test.

![Graph showing the comparison between numerical model and uniaxial tensile test results for weak plaster.](image)

**Figure 6.5. Comparison between numerical model and uniaxial tensile test results for weak plaster.**

In all simulations, we used $Q=2.5$ cc/s, $\mu=100$ Pa-s, $K=3.11$ GPa, and $G=2.33$ GPa. The cohesive zone parameters were varied to determine their influence in the elastic situation. In the elastic-plastic simulations, we used $T_0=1.0$ MPa, and $b_r=62$ $\mu$m. These properties correspond with weak plaster. The externally applied stresses varied between the simulations, as also was the constitutive behaviour. For the constitutive behaviour, we had three choices: linear elastic, elastic-plastic according the extension tests, and elastic-plastic according to the extension tests and tensile tests. In the last case, the change in the Mohr-
Coulomb parameters was made when the stress component perpendicular to the fracture plane became tensile. This leads to an irregular yield envelope (see figure 6.6). However, it will at least give the correct behaviour qualitatively, which is our basic aim. Unless stated otherwise, we used a dilatancy angle of zero in the simulations. In order to determine the influence of dilatancy, we also used associated plasticity.

We did not include the volumetric yield mechanism under influence of mean pressure (also called pore collapse) in the hydraulic fracturing simulations presented. It masks the mechanisms associated with shear and tensile yielding, which is our primary aim of study. However, after application of the confining stresses, volumetric yielding can possibly be of interest when the net pressure in the fracture becomes larger than zero. We have to consider the possibility of volumetric yielding when comparing the simulations with the experiments. Therefore, we will discuss its influence later in this section.

![Figure 6.6. Schematic view of irregular yield envelope when the material behaviour in tensile tests and extension tests is combined. The volumetric yield envelope as used in FLAC is also drawn here.](image)

**Validation**

To validate our program, we made a comparison with a fully coupled simulation using the program of Barr (1991). Figure 6.7 shows that the width and pressure profiles of both programs agree well. This supports the plausibility of the self-similar assumption, and indicates that the grid is fine enough and that the solution has indeed converged far enough. The confining stress \(\sigma_c\) in the simulation was 8 MPa, because the code of Barr did not converge when the confining stress became too low.
Figure 6.7. Comparison of FLAC simulation with fully coupled elastic model (Barr, 1991): fluid pressure in the fracture and width as a function of radius along the fracture. The fracture tip is at 0.07 m. In the case of the FLAC simulations, the stress profile ahead of the fracture tip is also shown. The confining stress $\sigma_c$ is 8 MPa.

In the FLAC simulations, we calculated the crack opening rate from the width gradient and crack propagation velocity using equation (6.8). Because we made approximations to obtain equation (6.8), we compared the calculated crack opening rate with the fully coupled linear elastic simulations of Barr (1991). Figure 6.8 shows that there is a reasonable agreement between the two programs. The agreement near the wellbore shows that apparently the terms between brackets in equation (6.7) cancel out for the largest part. Note that the influence of the time derivative on the local flow rate is rather small in this
example. This is shown by the ratio of $\partial b/\partial t$ and the average radial fluid velocity in the fully coupled simulations, which has a maximum value of $3 \times 10^{-2}$.

![Graph](image)

**Figure 6.8. Comparison of partial time derivative of the fracture width $\partial b/\partial t$ between the FLAC model (calculated from $\partial b/\partial r$ and the propagation speed, according to equation (6.8)) and the fully coupled linear elastic simulation according to Barr (1991).**

**Stress path and importance of pore collapse**

The stress path experienced by a material element close to the fracture tip, is shown in figure 6.9. The unloading part of the stress path approximately coincides with the stress path in figure 2.7 in Chapter 2. For the position considered in this example, quite large hydrostatic stresses developed after the net pressure had become larger than zero, which would lead to a pore collapse mechanism of failure for weak plaster. When we compare the simulation results - which do not include a pore collapse type of yielding - to the experiments in weak plaster, we have to know what influence pore collapse has.

We modelled the volumetric yielding according to the hydrostatic compression part of the extension tests, using a cap pressure in the constitutive behaviour (see figure 6.4). The volumetric yielding in these tests is partly caused by creep. From a simulation which included the hydrostatic cap in the yield function, we found an insignificant influence of pore collapse for $\sigma_b = 2.5$ MPa and $\sigma_b = 4$ MPa. As part of the modelled volumetric yielding is time dependent, this conclusion may not be true for all time scales. In section 4.4 we saw that volumetric creep is unimportant for a hydrostatic stress of 6 MPa or smaller, when the experiment time is $10^3$ s. The stress path in figure 6.9 shows that volumetric creep might even be important for the lowest externally applied stresses ($\sigma_b = 2.5$ MPa and
$\sigma_i=4$ MPa), because the mean pressure $p$ becomes larger than 6 MPa (except for experiment wp06, where the propagation time was $10^2$ s).

![Graph](image)

$\sigma_2-\sigma_3$ (MPa) vs. $p$ (MPa)

**Figure 6.9.** Stress path experienced by a material element adjacent to the fracture plane, with its centre located at 0.9 mm from the fracture plane and at a radial coordinate $r=5$ cm (38.5 mm from the wellbore). $\sigma_2$ and $\sigma_3$ are the principal stresses in the $r$-$y$ plane (see figure 2.2). The constitutive behaviour is elastic, $T_0=1$ MPa, $b_c=62$ $\mu$m, $\sigma_c=2.5$ MPa, and $\sigma_b=4$ MPa.

**Influence of tip model**

We used a cohesive-zone model at the fracture tip. In order to see its influence in the elastic case, we varied the critical width $b_c$ and tensile strength $T_0$, keeping the fracture surface energy constant. Figure 6.10 shows the pressure and width profile at the tip. The wellbore width and pressure are hardly influenced. However, we see that the size of the fluid lag or dry tip (defined as the distance from the fluid front to the fracture tip, where $\sigma_{coh}=T_0$) increases when the tensile strength decreases. Table 6.1 gives an overview of the dependence of the size of the dry tip on the cohesive zone parameters. In these simulations, we used $\sigma_c=2.5$ MPa and $\sigma_b=4$ MPa as externally applied stresses working respectively perpendicular and parallel to the fracture plane.

In Desroches *et al.* (1993) a fully coupled finite element program was used in order to study the influence of the cohesive zone parameters. They also found that the length of the dry tip increased by decreasing the tensile strength of the material, while the energy needed to propagate the crack was kept constant. Also, variation of the cohesive zone parameters had very little influence on the global fracture geometry and pressure profile,
as long as the fluid did not reach the cohesive zone (which was assumed to be impermeable in their study). This is expected for viscosity-dominated propagation, and equivalent to the insensitivity to the fracture toughness (as also observed in simulations presented in de Pater et al., 1994b).

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_c$ (MPa)</th>
<th>$\sigma_b$ (MPa)</th>
<th>$T_0$ (MPa)</th>
<th>$b_c$ (µm)</th>
<th>$\omega$ (cm)</th>
<th>$\omega/\omega_{max}$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>elastic</td>
<td>2.5</td>
<td>4</td>
<td>0</td>
<td>-</td>
<td>2.00</td>
<td>1.08</td>
</tr>
<tr>
<td>elastic</td>
<td>2.5</td>
<td>4</td>
<td>0.5</td>
<td>124</td>
<td>1.57</td>
<td>0.89</td>
</tr>
<tr>
<td>elastic</td>
<td>2.5</td>
<td>4</td>
<td>1</td>
<td>62</td>
<td>1.43</td>
<td>0.82</td>
</tr>
<tr>
<td>elastic</td>
<td>2.5</td>
<td>4</td>
<td>10</td>
<td>62</td>
<td>0.28</td>
<td>0.19</td>
</tr>
<tr>
<td>elastic-plastic case 1</td>
<td>2.5</td>
<td>4</td>
<td>1</td>
<td>62</td>
<td>1.14</td>
<td>0.73</td>
</tr>
<tr>
<td>elastic-plastic case 2</td>
<td>2.5</td>
<td>4</td>
<td>1</td>
<td>62</td>
<td>0.57</td>
<td>0.41</td>
</tr>
<tr>
<td>elastic-plastic case 3</td>
<td>2.5</td>
<td>7.5</td>
<td>1</td>
<td>62</td>
<td>0.86</td>
<td>0.67</td>
</tr>
</tbody>
</table>

In Chapter 5 (figure 5.28) we compared the measured size of the dry tip in plaster with the value of the fluid lag obtained by fully coupled linear elastic numerical simulations (using the code of Barr, 1991). In figure 6.11, the values obtained by the FLAC simulations are added to this. First, the FLAC simulation with a tensile strength of 10 MPa approximately agrees with the linear elastic simulation according to Barr (1991). By using a high tensile stress, we approximate the LEFM stress field (the maximum stress we can use is limited by the grid size near the tip). The fluid lag consisted only of a few grid points in this case, which can explain the difference with the program of Barr (1991). Second, for small and moderate values of the externally applied stress deviator, we see a fair agreement between the size of the cohesive zone in the FLAC simulations and the experiments. When plasticity is introduced (see next section for details), the size of the dry tip still agrees with the experiments, although it becomes somewhat smaller. For extreme values of the externally applied stress deviator (block loaded close to failure), the dry tip was not seen after splitting of the block. We did not model this situation in FLAC. From these results we conclude that the cohesive-zone model predicts a realistic size of the dry
tip in plaster for small and moderate values of the externally applied stress deviator, contrary to the linear elastic fracture mechanics tip model.

![Graphs showing the effect of variation of the tensile strength on the situation near the tip.](image)

*Figure 6.10. Effect of variation of the tensile strength on the situation near the tip.*
Figure 6.11. Measured size of the dry tip $\omega_d$ and simulated fluid lag size $\omega$ as a function of the maximum size $\omega_{\text{max}}$ (corresponding to a constant pressure in the fracture, see equation (5.10)); $K_e=0.3$ MPa$\cdot$m was used to calculate $\omega_{\text{max}}$. The dashed line connects the points that were obtained from the fully coupled linear elastic simulations, using representative input parameters for the experiments on weak and strong plaster. These simulation results and the experimental data of strong and weak plaster are equal to figure 5.28.

**Tip mechanism in elastic-plastic fractures**

In the elastic-plastic simulations, we studied three different situations (see table 6.1). In the first situation, we used material parameters according to the extension tests. The externally applied stresses were $\sigma_r=2.5$ MPa and $\sigma_s=4$ MPa. We will refer to this situation as case 1. In the second situation we also included material parameters according to the tensile tests (referred to as case 2). In case 3, we increased the horizontal stress from 4 to 7.5 MPa, while the constitutive behaviour was the same as in case 1 (Mohr-Coulomb parameters according to the extension tests). In both case 1, case 2, and case 3, the dilatancy angle was zero.

Figures 6.12 (case 1), 6.13 (case 2), and 6.14 (case 3) show the pressure and width profile in the elastic-plastic calculations for the final calculation step. The profiles are compared with the elastic case, for equal fracture length. The initial fracture radius (fluid front position at $r=3.15$ cm) is still visible in the final width profile of the elastic-plastic calculations, and causes a small irregularity in the pressure profile. These figures show that in the elastic-plastic case, the fluid comes closer to the fracture tip, so that the fluid lag length is smaller. Also when we take the change in wellbore pressure into account, the size of the fluid lag becomes relatively smaller (see value $\omega_d\omega_{\text{max}}$ in table 6.1 and figure 6.11). This shows that plasticity "screens" the fluid pressure loading, so that stronger loading...
(closer to the tip) is present during propagation. Identical results were obtained in Papanastasiou and Thiercelin (1993). They calculated the J-integral around the plastic zone at the fracture tip, and found that the energy dissipated in the plastic zone was more than two orders of magnitude larger than the energy needed for creating the fracture surfaces. An even larger increase in plastic energy dissipation was obtained in Papanastasiou (1999a).

![Graphs](image)

Figure 6.12. Stress in the plane of the fracture and width profile for elastic and elastic-plastic simulation. The fracture tip in both cases is at the same position. Tensile test results are not included in the material behaviour (case 1).
Figure 6.13. Stress in the plane of the fracture and width profile for elastic and elastic-plastic simulation. The fracture tip in both cases is at the same position. Tensile test results are included in the material behaviour (case 2).
Figure 6.14. Stress in the plane of the fracture and width profile for elastic and elastic-plastic simulation. The fracture tip in both cases is at the same position. Tensile test results are not included in the material behaviour. The externally applied stresses are $\sigma_y = 2.5 \text{ MPa}$ and $\sigma_r = 7.5 \text{ MPa}$ (case 3).

Figure 6.15 shows an example of contours of accumulated plastic strain that develop around the fracture tip. This figure also shows that large plastic shear strains develop around the borehole, while permanent plastic strain around the initial fracture radius is visible too. Figure 6.15 shows that the fracture has grown away far enough from the wellbore, so that the tip region is not disturbed significantly by these effects in the wellbore region. Hence, we can make conclusions about the effect of plasticity on the tip region from our simulations.
When the plastic strain in the rock is so large that shear bands develop, quantitative comparison with numerical simulations is questionable. Figure 6.16 shows the total strain in the direction perpendicular to the fracture plane in the elastic-plastic simulation (case 1). The strain around the tip is larger than -0.6 mstrain. This is comparable with the strain level that occurred in a triaxial extension test for a radial stress of 4.5 MPa (figure 6.4). This experiment did not show indications for the development of shear bands inside the sample (see section 4.4). This indicates that the simulations are representative for the experiments (assuming that the material behaviour under the stress states in extension tests represents the material behaviour in the tip region, and neglecting the influence of sample geometry on strain localisation). However, the surface roughness shows that a failure zone around the tip is present in the experiments, which is of the order of the grid size. As discussed before, this may complicate a quantitative comparison between experiment and simulation. However, it is not expected to influence the qualitative behaviour, which is our basic object of study.

Figure 6.15. Example of contours of accumulated plastic shear strain $\gamma^p$ (millistrain) (see equation (6.1)) around the fracture, for case 1. The interval between the contours is 0.05 mstrain. Plastic deformation around the initial fracture radius is visible, and large plastic strains develop around the borehole.
Figure 6.16. Contours of the total strain (millistrain) in the y-direction $\varepsilon_{yy}$ for an elastic-plastic simulation (case 1).

When larger radial stresses are applied, or the behaviour of tensile tests is included in the material behaviour, the tensile strain around the tip becomes larger. Based on the strain level, failure in a larger zone around the tip can be expected, possibly related to the formation of shear bands. This supports explanations for fracture surface roughness, as given in Chapter 7. As the modelling of this failure process is lacking in the simulations, quantitative comparison with the experiments is difficult. However, a qualitative comparison is still valid, which is our basic aim.

**Influence of plasticity on global width profile and net pressure**

From figures 6.12 (case 1), 6.13 (case 2), and 6.14 (case 3), we see that plasticity leads to a larger wellbore width and smaller net pressure at the wellbore. This qualitatively agrees with the experimental observations. This effect becomes stronger when the size of the plastic zone increases (cases 2 and 3 with respect to case 1). When we compare the elastic-plastic simulations with the elastic case, we can identify two processes that influence the wellbore pressure. First, in the main part of the fracture the pressure gradient is smaller, because of the larger width. Second, the width at the position of the fluid front is smaller for the elastic-plastic fractures, which increases the pressure gradient in this region. Both effects influence the wellbore pressure. In our experiments, the influence of the smaller gradient in the main part of the fracture is dominant. In a fully coupled simulation presented in Papanastasiou and Thiercelin (1993), both effects cancel out. We cannot exclude that there are conditions in which the influence of the tip gradient dominates the
effect of the gradient in the main part of the fracture, so that a high net pressure results while the fluid does not reach the fracture tip.

In Papanastasiou (1997), a 30% increase in the net pressure was obtained in a case where the fluid front reached the cohesive zone (which was assumed to be impermeable). After increasing the flow rate and fracturing fluid viscosity, a 10% increase of the net pressure in comparison to elasticity was obtained. This can be interpreted as a transition from a plasticity-dominated towards a viscosity-dominated propagation regime. It was not clear whether in the last case the fluid actually reached the cohesive zone, or that it just came very close to it. In case the fluid did not reach the cohesive zone, this is an example in which the effect of the pressure gradient at the tip on the wellbore pressure is larger than the effect of the pressure gradient in the main part of the fracture.

**Influence of dilatancy**

The simulations presented above were for zero dilatancy angle. To determine the influence of dilatancy, we repeated the calculations of case 1, 2, and 3 with an associated flow rule (see equation (2.11) and below). In this way, we can determine to what extent dilatancy will change the observed mechanisms.

Figure 6.17 shows the pressure and width profiles for case 3, in combination with zero dilatancy angle and in combination with associated plasticity. The friction angle in the simulations varied depending on hardening, but was about 20 degrees on average. It shows that dilatancy increases the previously mentioned effects of plasticity; a shorter fluid lag, a smaller width at the position of the fluid front, and a larger width in the main part of the fracture. The "extra" effect of dilatancy on plasticity is relatively small in case 3. For case 2, dilatancy made the fluid lag disappear, so we could not determine its influence in that case.

The results can be understood by recognising that the plastic volume increase leads to an extra elongation in the direction perpendicular to the fracture plane. This leads to a stronger screening at the tip of the fluid pressure loading. Apparently, the volume increase of the material surrounding the fracture as a result of dilatancy is of smaller importance, as the fracture width is larger along the entire profile.

**Influence of plasticity on closure mechanism**

Figures 6.18 through 6.20 show the stress and opening during closure of the fracture. The closure process is modelled by a constant pressure in the open part of the fracture, which is equal to the confining stress. The figures show that the elastic-plastic fractures stay open over a certain radius, which is smaller than the original radius. In the closed part of the fracture, a high stress perpendicular to the fracture plane develops.

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Figure 6.17. Stress in the plane of the fracture and width profile of the elastic-plastic simulation in case 3 for dilatant behaviour corresponding with an associated flow rule, and for a non-associated flow rule with zero dilatancy angle.

Figure 6.21 shows the directions of the plastic deformation that occurs in the plastic zone. Elongation in the direction perpendicular to the fracture plane occurs, and contraction in the direction parallel to the fracture plane. It is this elongation that acts like a wedge during closure, which causes a high compressive stresses working perpendicular to the fracture plane that tends to keep the fracture open. Similar results were obtained in Papanastasiou (1999b). The plastic contraction in the direction parallel to the fracture plane also hinders the fracture surface from returning to its original position during
closure. This also tends to keep the fracture open, but causes lower stresses at the closed fracture surfaces.

![Diagram](image)

Figure 6.18. Stress in the plane of the fracture and width profile for elastic and elastic-plastic simulation, after fracture closure. In the open part of the fracture, the applied fluid pressure equals the confining stress. Tensile test results are not included in the material behaviour (case 1).
Figure 6.19. Stress in the plane of the fracture and width profile for elastic and elastic-plastic simulation, after fracture closure. In the open part of the fracture, the applied fluid pressure equals the confining stress. Tensile test results are included in the material behaviour (case 2).
Figure 6.20. Stress in the plane of the fracture and width profile for elastic and elastic-plastic simulation, after fracture closure. In the open part of the fracture, the applied fluid pressure equals the confining stress. Tensile test results are not included in the material behaviour. The externally applied stresses are $\sigma_c=2.5$ MPa and $\sigma_b=7.5$ MPa (case 3).
Figure 6.21. Schematic qualitative view of direction of plastic deformation around the fracture tip. As a result of plasticity, a material element deforms into the dashed shape. This results in plastic elongation in the direction perpendicular to the fracture plane, and plastic contraction in the direction parallel to the fracture plane.

Scale dependence of closure mechanism
The influence of the plastic strain around the fracture tip that tends to keep the fracture open at the wellbore during fracture closure, is determined by three length scales: the size of the plastic zone, the width of the fracture, and the length of the fracture. For very long and wide fractures with a small plastic zone, the influence of plastic strain around the tip will be relatively small. When the tip is far away from the wellbore, the plastic elongation in the direction perpendicular to the fracture plane instead will cause the fracture to close at a fluid pressure inside the fracture that is somewhat higher than the confining stress. Furthermore, we note that imperfectly matching fracture surfaces due to surface roughness, can cause an effect similar to plastic elongation in the direction perpendicular to the fracture plane, according to a similar mechanism.

6.4 Conclusions

From the results presented in this chapter, we can make the following conclusions.

- Compared to the elastic case, plasticity causes a smaller fluid lag and a larger width in the main part of the fracture, while the width at the position of the fluid front is smaller. The first effect increases the wellbore pressure, while the second effect
6. Numerical simulations

reduces it. Dilatancy augments this behaviour. The net effect in our simulations and experiments was a decrease in wellbore pressure.

- Numerical simulations with a cohesive-zone model quantitatively predict the size of the dry tip in plaster reasonably well for small and moderate values of the applied stress deviator.
- Plastic deformation around the fracture tip influences the relationship between wellbore pressure and fracture width during closure. For the conditions in our experiments on weak plaster, plastic deformation around the tip tends to keep the fracture open at the wellbore at a lower pressure than in the elastic case.
Discussion of results

In this chapter we combine and discuss the results of the previous chapters. We start with listing the main results concerning the geometry and pressure of a propagating hydraulic fracture, which is based on Chapters 4, 5, and 6. After that, we concentrate on two main topics that determine the geometry and pressure during propagation: first, the interaction between fluid flow and (plastic) rock deformation near the tip, and second, the fracture mechanism at the tip. This is followed by a discussion about fracture closure. We end with discussing possible implications for field practice.

7.1 Fracture geometry and net pressure during propagation

The fully coupled linear elastic model was confirmed by the experiments in cement. The net pressure, width at the wellbore, width profile, and fracture volume in the experiments all agreed quantitatively with the simulations. The influence of variation of the externally applied stresses and the injection flow rate was as predicted by the linear elastic model. The acoustically determined measure of the fluid lag size also agreed with the linear elastic model, for varying propagation speed. Reopening of a completely closed fracture confirmed the unimportance of fracture toughness for viscosity-dominated propagation.

Fracture propagation in weak rocks (plaster and diatomite) was also compared to these linear elastic simulations. In strong plaster, agreement was found between this model and measured fracture width and net pressure at the wellbore, for moderate values of the externally applied stress deviator. Reliable determination of the acoustic width profile in
plaster is not possible with the current interpretation of the ultrasonic measurements. Changes in the size of the dry tip, observed after an experiment on plaster, as a result of variations in the externally applied stresses, agreed qualitatively with predictions based on linear elastic fracture mechanics and the cohesive-zone model. However, the size of this dry tip agreed quantitatively with the cohesive-zone model, and disagreed with the fluid lag predicted by linear elastic fracture mechanics. Furthermore, for large values of the externally applied stress deviator in strong plaster, the net pressure became lower and the fracture width larger than predicted by the linear elastic simulations. At the same time, the dry tip that was normally seen after splitting a plaster block after the experiment, had disappeared completely. Similar deviations were seen in weak plaster. In this material, the dry tip also disappeared for a large externally applied stress deviator. The low net pressure and large fracture width were seen in all experiments on weak plaster.

The results in diatomite essentially agree with the linear elastic model. In the two experiments with a low flow rate, higher net pressures than predicted by elasticity were measured. However, these deviations could be explained from irregularities in the specific samples. No information could be inferred from the ultrasonic measurements in diatomite, because the ultrasonic waves were attenuated too strongly.

Plastic shear deformation near the tip is most prominent in weak plaster. Coupled elastic-plastic simulations of hydraulic fracture propagation in weak plaster showed similar phenomena as were seen in the experiments. In the elastic-plastic simulations, a lower net pressure was obtained when the elastic and elastic-plastic fractures were compared at equal fracture radius, and the fracture width was larger. The relative size of the fluid lag in the simulations became smaller when the material constitutive behaviour was changed so that larger plastic strains developed around the tip, or when the externally applied stress in the plane of the fracture became larger. A similar dependence of the relative size of the fluid lag on the externally applied stress in the plane of the fracture could not be detected in the experiments on weak plaster (probably due to the measuring accuracy and the relatively small fluid lag size), but was detected in experiments on strong plaster.

The agreement of the elastic-plastic simulation results with the experimental observations gives us confidence in the interaction mechanism at the tip between fluid and (plastic) rock deformation that was observed in the simulations (referred to as "tip mechanism"). Although identified for one specific situation, this tip mechanism is believed to apply to general cases of hydraulic fracturing. It is discussed in section 7.2.

The material characterisation presented in Chapter 4 showed that the behaviour of weak plaster in tensile tests differed strongly from strong plaster and diatomite. Weak plaster showed significant plastic deformation in uniaxial tension, unlike strong plaster. Diatomite has a relatively small tensile strength, so we do not expect that significant plasticity develops in uniaxial tension in diatomite before tensile failure occurs. The behaviour under compressive shear loading was similar in weak plaster and diatomite. The hydraulic
fracturing experiments show agreement with the linear elastic model for strong plaster (for moderate values of the externally applied stress deviator) and diatomite, while in weak plaster deviations occur. This indicates that the development of plasticity in tensile tests is most indicative for the occurrence of deviations from the elastic model, under moderate values of the externally applied stress deviator.

7.2 Tip mechanism

Before discussing the role of plasticity, we first discuss the elastic tip mechanism for viscosity-dominated propagation. In this propagation regime, a fluid lag forms at the tip of the fracture. This fluid lag forms because the rock is not able to bear the fluid pressure loading that is needed to overcome the viscous resistance to fluid flow. This mechanism was confirmed during fracture closure in cement (see section 5.1). The fracture propagation velocity decreased, together with the fluid pressure at the wellbore. This went together with a diminishing size of the fluid lag, which confirms the before-mentioned idea of the dependence of the fluid lag on the pressure in the fracture. The predicted relationship between fluid lag and net pressure by the linear elastic model of Garagash and Detournay (1998) for a semi-infinite fracture, was confirmed by the measured net pressure and the acoustically determined measure of the fluid lag.

Besides the above-mentioned relationship between fluid lag size and net pressure, the fluid lag also reacts to the resistance of the rock to fracturing. Numerical simulations show that the fluid lag size decreases for increasing fracture toughness (e.g. Desroches et al., 1993 and de Pater et al., 1994b). A relationship between the dimensionless fluid lag size and fracture toughness was predicted by the model of Garagash and Detournay (1998) (see sections 2.6 and 5.1). This prediction was confirmed by the measurements of fluid lag in cement, together with the other variables of interest. The results of the numerical simulations in section 6.3 also show that the fluid lag becomes smaller with increasing tensile strength of the rock. The net pressure and width at the wellbore are hardly affected, which was also found by others (e.g. Desroches et al., 1993).

Linear elastic fracture mechanics predicts that an increase in confining stress $\sigma_c$ decreases the size of the fluid lag. This can be understood by the contributions to the stress intensity of the net pressure in the bulk of the fracture and the fluid lag (Jeffrey, 1989). The fluid lag has a negative contribution to the stress intensity. A larger confining stress increases this negative contribution, so that the fluid lag can become shorter in case the confining stress increases and the net pressure in the bulk of the fracture stays equal. This was qualitatively confirmed by the dependence of the dry tip size in strong plaster on confining stress. In cement, a fluid lag was measured at low confining stress (8 MPa),
while at high confining stress (23 MPa) it could not be measured, probably due to its small size. This is again a qualitative confirmation of the expected dependence of the fluid lag size on the confining stress.

The fluid lag was seen to react to changes in rock strength and confining stress. Despite the change in fluid lag size, the net pressure and width at the wellbore are hardly affected. According to fully coupled linear elastic simulations by Gardner (1992), the same holds for changes in pore pressure in the fluid lag. Furthermore, the fluid lag reacts to changes in the net pressure, and forms because of the high pressure needed to overcome the viscous resistance to fluid flow. It is important to recognize this causality, in order to prevent possibly wrong theories that start from the assumption that the fluid lag size directly determines the net pressure. When a significant fluid lag is present, any theory that considers the effect of the fluid lag as a direct cause for high net pressures appears to be wrong, because in that case the net pressure is fully determined by flow and fracture opening. We can approve the conclusion of Desroches et al. (1993), who state that the fluid lag is a local mechanism, which does not significantly influence the pressure response. Its major effect is to negate the effect of fracture toughness. The fluid lag is a passive device, and only becomes important for the net pressure when it disappears.

In Chapter 6 we identified the following mechanism at the fracture tip. As a result of plasticity the size of the fluid lag diminishes, and the width at the position of the fluid front decreases. However, the average width gradient in the fluid-filled part of the tip region is larger, so that the width in the main part of the fracture is larger. The influence on the wellbore pressure is determined by the magnitude of both effects. In the following we compare this mechanism with existing theories and hypotheses.

The "dilation hypothesis" (Johnson and Cleary, 1991, see also section 2.6) predicts a smaller width near the tip and a larger fluid lag size as a result of inelastic rock dilation. A way to model this dilation is to use plasticity theory, which was done in Chapter 6. Using an associated flow rule gives a maximum to the plastic volume increase, while using zero dilation angle leads to zero plastic volume changes. Section 6.3 shows that dilation has a similar effect as plasticity with zero dilation, and that dilation augments the effect of plasticity with zero dilation. The plastic volume increase leads to stronger screening at the tip of the fluid pressure loading, and a smaller fluid lag size.

In Barr (1991) and Gardner (1992), the dilation hypothesis was modelled by artificially constraining the width near the fracture tip. This is equivalent to assuming a larger Young's modulus near the tip. This leads to a larger net pressure, and as a result to a larger fluid lag. The simulations from Chapter 6 show that plasticity and dilation indeed lead to a smaller width at the position of the fluid front, similar to what was assumed in the dilation hypothesis. However, the plasticity that causes this has two other effects. First, the loading at the tip is screened. This makes the fluid lag size smaller instead of larger. Second, the width going from the fluid front towards the wellbore increases more quickly as a result of
plasticity. The dilation hypothesis neglects these two additional effects of plastic rock deformation, which also can have a significant effect on the wellbore pressure.

Another hypothesis that explains high net pressures in the field, is that the separation energy during fracture propagation is in some way increased by typically two orders of magnitude (Shlyapobersky, 1985). This hypothesis is based on the following reasoning. "When net pressures are much higher than predicted by an elastic model, the quasi-static fracture propagation must be dominated by resistance to rock fracturing. The correct way to model fracture propagation, is to assume that in some way the separation energy is increased significantly".

This reasoning seems to be logical, but there are some practical problems with it. First, the physical cause for the increase in separation energy seems to be lacking. This increase is attributed to the size of the process zone, which is assumed to increase with fracture length and propagation speed (Shlyapobersky and Chudnovski, 1994). However, no indication from laboratory experiments can be found for such an increase of separation energy with scale or speed (see section 2.3). Second, high net pressures are also seen during reopening of a previously created fracture (Shlyapobersky and Chudnovski, 1994), which would not be expected when rock strength is important for the net pressure.

Plastic deformation near the tip is another possible cause for explaining this increase in toughness. The simulation results indicate that plasticity decreases the size of the fluid lag, not only absolutely but also relative to the maximum elastic value based on constant fluid pressure up to the fluid front (see table 6.1). In two experiments with a large plastic zone, the dry tip after splitting the block was absent, while a measurable fluid lag size was expected based on the maximum elastic value (see figure 5.28). From these observations, we conclude that plasticity is able to diminish the size of the fluid lag, which would make a transition into a "plasticity-dominated" propagation regime possible. This is a possible mechanism for the occurrence of high net pressures.

The simulations of Papanastasiou (1999a) yield values of the energy dissipated in plastic deformation that indeed were more than two orders of magnitude larger than the energy dissipated in creating the fracture surfaces. In these simulations the cohesive zone was assumed to be impermeable, so that fluid pressure could increase at the beginning of the cohesive zone. The development of the large plastic energy dissipation depends partly on this pressure build-up. However, it is doubtful whether this cohesive zone is still impermeable when such a large pressure develops at its entrance. The cohesive zone was assumed to be fully permeable in our simulations in Chapter 6. The permeability of the cohesive zone, including effects like parallel flow (see section 2.5), is a subject suitable for further research.

The second problem in elastic-plastic simulations, is to what extent plasticity weakens the material. The fact that fracture roughness is proportional to the plastic zone size, indicates that the influence of plasticity on the fracturing process is of interest. It appears that, when the material is weakened by plastic shear deformation, large plastic strains
calculated in elastic-plastic simulations are not very realistic. Instead, it is likely that fracturing in the plastic zone will occur before the tensile fracture propagation criterion in the idealised fracture plane is satisfied. The same thing can be concluded from other aspects of the fracturing mechanism at the tip, as discussed in the next section.

In numerical simulations, the shielding effect of plasticity should not be decoupled from the weakening effect. This becomes especially important when the fluid reaches the tip of the fracture. The simplest way to model the weakening, is to change the cohesive-zone parameters. A better way would be to simulate shear bands, and to allow for deviations of the fracture from the idealised straight fracture plane. Note that the absence of these mechanisms in the numerical simulations in Chapter 6 complicates the quantitative interpretation, but does not affect the value of the observed (qualitative) mechanism.

7.3 Fracture mechanism

In this section, we discuss the fracturing mechanism based on the experimental results from section 5.2, and the theoretical predictions from sections 2.4 and 2.9. The fracturing mechanism is reflected in the roughness of the fracture surfaces. Determining the fracturing mechanism is equivalent to finding an explanation for the observed fracture surface roughness, on which question we concentrate this discussion. We start with a review of surface roughness patterns in fracturing experiments and geologic formations. Subsequently, we discuss explanations for the observed roughness pattern.

Observations of fracture surface roughness

Hydraulic fractures show a characteristic surface roughness pattern. It consists of radial grooves in the direction of fracture growth, which have a much larger amplitude than the grain size. This pattern develops under pure mode I loading conditions. When a mode II or III loading component is present, irregularities are expected to develop because of crack deflection (see e.g. Lawn, 1993). However, we will here only discuss and assume mode I loading. This characteristic pattern is observed in many materials in hydraulic fracturing laboratory studies, for example in plaster (Daneshy, 1973, van Dam et al., 1997c, and this study), Colton sandstone (Bohlmeier, 1995), Carthage Limestone (Daneshy, 1973), silty clay (Murdoch, 1993), diatomite (this study), and plexiglass (Rummel, 1991).

Roughness patterns similar to the hydraulic fracture surfaces in our experiments are seen on numerous fracture surfaces from geological origin. Examples of this are given in Kulander et al. (1990), where these patterns were seen in shale. Another example is
Lacazette and Engelder (1992), where natural hydraulic fracturing under the influence of natural gas pressure is believed to have caused fractures in siltstone. Tensile stresses as a result of cooling have caused fracturing of basaltic lava into columnar joints (e.g. DeGraff and Aydin, 1987). The surfaces of these columnar joints show stepwise fracture propagation. At the end of each step, the surface is more rough. Ryan and Sammis (1978) explain this by the increased ductility of the cooling basaltic lava as the crack grows into a region with higher temperature. Note that this is an indication of the relationship between plasticity and surface roughness.

Similar roughness patterns develop in axial splitting and tensile tests on rocks (see e.g. Gramberg, 1989). Here, fracture propagation is dynamic, while in most previously mentioned hydraulic fracturing experiments fracture propagation is quasi-static (only in Rummel (1991) fracture growth consisted of unstable increments). Dynamic propagation in uniaxial tensile tests on glass shows a similar characteristic surface pattern as in similar tests on rock (e.g. Smekal, 1936, and Field, 1971). At the origin of crack growth, a "mirror" zone exists, where the surface is very smooth. As the fracture propagates, a "mist" zone develops, where the fracture surface is more rough. Further from the origin, a rough zone is present. The transition into the rough zone goes together with crack tip branching, and a constant propagation velocity, which was increasing in the mirror zone (Field, 1971).

Explanations for roughness of dynamically propagating fractures

Lawn (1993) discusses the occurrence of roughness that develops in dynamically propagating cracks in brittle material without planes of weakness, such as glass. He mentions that this phenomenon has no explanation in quasi-static fracture mechanics, and gives three possible explanations for it.

First, the crack tip stress field for a propagating fracture is considered. Above a certain velocity, the kinetic energy of the material around the tip becomes important. Yoffe (1951) showed how this affects the angular component of the tensile linear elastic stress field around the fracture, which becomes maximum under an angle with the plane of the fracture tip. The idea is that this leads to branching and associated fracture roughness. Lawn (1993) mentions two discrepancies between theory and observation. First, the speed at which branching occurs is lower than expected. Furthermore, the angle under which the tensile stress field has its maximum is much larger than the actually observed bifurcation angle.

As a second explanation, Lawn (1993) mentions the presence of microcracks ahead of the crack tip. They link up in front of the crack tip, and link to the crack later. This causes deviations from the original fracture surface, which form the roughness. The idea is that only for high speed is the stress field intense enough to activate the microcracks ahead of the tip. Smekal (1936) gives a similar explanation, but attributes the higher intensity of the
stress field to the reduced cross section of the tensile test specimen. One of the problems
with this model is that microfractures in glass are not observed.

The third explanation is that acoustic waves influence the stress field, and cause surface
roughness. This is an explanation for the occurrence of Wallner lines, as observed on the
mirror-surface of glass (e.g. Smekal, 1950). The effect is even used as a measurement
technique for the fracture propagation speed. An ultrasonic pulse of fixed frequency
propagates through the sample in which the crack is propagating, which leaves a ripple
mark on the fracture surface (see e.g. Field, 1971).

Application of dynamic theories to stable fracture propagation

In the hydraulic fracturing experiments, fracture propagation was stable. Furthermore,
fracturing took place under externally applied stresses. Despite these differences in
experimental conditions, the roughness of hydraulic fracture surfaces (see section 5.2)
shows a strong resemblance with surfaces of dynamically propagating fractures, for
example the rough zone in tensile tests on glass and rocks. We could think of the
possibility that the slow propagation of the fracture consists in fact of small unstable
increments. However, it is hard to believe that the length scale of these increments is much
larger than the grain size. This implies stable fracture propagation on a continuum scale.
The continuum hypothesis is adopted in two of the three above-mentioned theories that
consider the influence of velocity on surface roughness.

To our knowledge, no explanation is given in the literature for the observed fracture
surface roughness in stable propagation. Heterogeneity and grain size of the material may
play a role, but the experimental observations suggest another fundamental mechanism. To
reveal the cause for the development of the roughness, we analysed the influence of
variation of experimental parameters. Section 5.2 showed that the externally applied
stresses and rock material had a significant influence on the roughness. Propagation
velocity, fracture size, and net pressure had an insignificant influence. Even plaster
samples that fractured unstable in uniaxial compressive and tensile tests, showed the same
quantitative roughness as the stable propagation.

We will first analyse whether the existing theories for dynamically propagating cracks
can somehow be applied to our experiments. The first of the above-mentioned theories
does not apply to plaster, because of its dynamic nature. The second of the above-
mentioned theories is applicable to our situation. We replace the term microcrack with
weak spots, which are expected to be present and can be enhanced by the shear
deformation. Then, it can explain the correlation with plastic zone size. It also predicts that
the roughness is independent of the other parameters, which agrees with the experimental
results. Although it can explain the variation of roughness with experimental parameters, it
cannot explain the radial pattern of grooves.
The third of the above-mentioned theories needs to be considered, as in addition to the acoustic pulses caused by the fracturing process, the active ultrasonic measurements also influence the fracturing process. Because of the low fracture propagation velocity, no Wallner lines will be generated and any influence of acoustic waves is expected to be random. This is in disagreement with the observed radial grooves, which imply some structure. Furthermore, the fracture surfaces do not show any ripple marks, which should have a length in the order of one centimetre, based on the acoustic measuring interval and the propagation velocity. We neither saw any influence of the measuring interval (which varied between the tests) or other variation in the acoustic monitoring technique (for example transducers placed directly on the block, or with loading platens in between). Furthermore, in most of the other studies that showed similar roughness profiles, no active acoustic monitoring was done. The apparent unimportance of the acoustic waves for the fracture surface roughness is probably caused by the fact that the stress levels that cause fracturing are much larger than the stress amplitude of the acoustic waves. This is the case because the loading of the fracture is a balance between two large loading forces (pressure inside the fracture and confining stress working over the fluid lag).

**Explanations for roughness in stable fracture propagation**

The exact mechanism that causes the roughness in our tests is therefore not completely resolved. In the following some new ideas and observations are presented, which can enlighten the phenomenon.

First, we can think of the possibility that we have a small mode III component that influences the deviation from the mode I loading situation. However, the phenomenon is observed under many different loading conditions. In our experiments, the roughness was in general uniform over the fracture surface, which would not be expected if a small mode III component would have a large influence. Furthermore, the roughness was also strongly influenced by the stress perpendicular to the fracture plane, which is not expected to induce a mode III component at the tip. The fact that fractures in cement showed smooth surfaces also indicates that mode III loading is not significant. From these observations we conclude that we are indeed studying a phenomenon that occurs as a result of mode I loading.

A first explanation is based on the idea that the fracture mechanism differs from pure tensile fracturing. Analysis of the stress path for a material element shows that the failure envelope is first reached outside the plane of the fracture (see section 2.4). The position at which the stress path reaches the failure envelope determines the fracturing mechanism (tensile failure, axial splitting, or shear fracturing).

Due to plasticity and displacement restrictions, reaching the failure envelope in an elastic analysis will not necessarily lead to complete fracturing of the rock. However, weakening of the rock inside the plastic zone results in preferential breaking of the rock in
that zone. This is thought to cause the roughness, which is typically of the order of magnitude of the plastic zone. This view is confirmed by the amplitude of the roughness pattern, which agrees with the plastic zone size (see section 5.2). The occurrence of induced fractures and overlapping fracture planes, as observed in section 5.2 for extreme loading conditions, also confirm this view. This model has similarities with the previously described "microcrack model". Regions of large plastic strain inside the plastic zone act as initiation planes for the rock to separate, while in the microcrack model microcracks act as initiation points in the process zone.

The orientation of failure planes with respect to the largest principal stress coincides with the observed groove pattern. Furthermore, it can explain the dependence of the average slope of the roughness profile on the externally applied stresses (see figure 5.36). When the confining stress perpendicular to the fracture plane becomes smaller, or the externally applied stress parallel to the fracture plane becomes larger, failure will occur at a higher mean normal stress. This can be seen by constructing the stress path similar to figure 2.6, and realising that the starting point of the stress paths in this figure shifts when the externally applied stresses change. Because of the convex shape of the failure envelope (see section 4.4), the smallest angle between the largest principal stress and the failure surface will become larger for higher values of the mean normal pressure.

A second approach is to state that the total energy in the fracturing process is minimised. This total energy is the sum of the fracture surface energy and the energy dissipated in plastic deformation. The sum of the energies can be minimised by twisting and tilting of the fracture surface, in case the plastic energy dissipation is large enough compared with the fracture surface energy. The twisting and tilting increases the surface area and thus the fracture surface energy, but the size of the plastic zone becomes smaller when the fracture plane is rotated with respect to the externally applied stresses. A model that describes this behaviour is presented in section 2.9. The idea is that this tendency to twist and tilt causes the roughness. Because the energy dissipated in fluid flow and stored in elastic fracture opening dominate in our experiments, the global fracture area will be perpendicular to the least compressive stress.

This model can qualitatively explain the correlation of the roughness with the plastic zone size, and also the correlation between the slope and the plastic zone size (see section 5.2). Although it cannot predict the final situation, the model qualitatively predicts that the deviations causing the roughness become larger when the plastic zone size increases. Furthermore, section 5.2 shows that when the in-situ stresses are close to failure, the fracture surface shows somewhat twisted and overlapping fracture planes. It is not clear whether the process that causes this is fundamentally different from the process causing the radial groove pattern. The observation that for a large plastic zone the fracture surface shows these twisted overlapping planes agrees with the model of section 2.9.

When the energy dissipated in plastic deformation is of the same order of magnitude as the dominant energy rates, twisting and tilting of the complete fracture surface is possible
according to the model in section 2.9. This can easily lead to fracture branching and parallel fractures. For a hydraulic fracture, the twist and tilt is limited by the condition that the net pressure must be larger than zero. Besides our experiments, fracture geometries that support this mechanism are observed in geological formations (Pollard et al., 1975, see also Germanovich et al., 1997), although it is not clear whether a mode III component was present here. An example of a sudden twisting and splitting up of a fracture was given in Helgeson and Aydin (1991). This happened at the moment a fracture in siltstone started to propagate into shale, which is more ductile than siltstone. The driving mechanism of these fractures is not clear, although in the same Appalachian Plateau (New York, U.S.A.) indications for natural hydraulic fracturing in siltstone were found (Lacazette and Engelder, 1992). The fracturing fluid here was believed to be gas, which is expected to cause fracture propagation that is dominated by the fracturing process at the tip. The model of section 2.9 can explain this twisting and splitting up for plasticity-dominated fracture propagation. Again, other explanations like changes in stress directions can also explain these observations.

The geologic observations suggest the presence of a length scale of the groove spacing. However, in our experiments the power spectrum of the measured profiles does not show a local maximum. Instead, it is a continuously decreasing function, as observed for numerous other fracture surfaces (e.g. Power and Tullis, 1991).

The two above-mentioned approaches to explain the roughness combine well. Both mechanisms cause the fracture to deviate from its original path. The planes of weakness caused by plastic shear deformation might form initiation planes for the twisting and tilting fracture planes. Also, the shear failure might cause small-scale roughness on the twisting and tilting that leads to roughness on a larger scale. Which mechanism will be dominant is expected to depend on material properties and experimental conditions.

7.4 Hydraulic fracture closure

The experimental results show three significant phenomena during fracture closure. These are radius growth, radius recession, and closure at a wellbore pressure that is lower than the confining stress. In this section, we will discuss the mechanisms causing this.

Our experimental results showed that radius growth after shut-in was accompanied by a diminishing size of the fluid lag. This confirms the concept of a fluid lag that establishes a large apparent toughness which ensures balance between fluid pressure and stress intensity. During closure, when the fluid pressure decreases, the critical stress intensity - and thus fracture growth - is maintained by a diminishing size of the fluid lag. This implies
that fracture growth after shut-in is indicative for the presence of a fluid lag during propagation.

During radius recession the stress intensity at the receding tip is zero, because the fractured rock cannot bear any stress intensity. The tip recession can also be described as “an increasing part of the fracture where the fracture width is zero”. Equation (2.16) gives the fracture width as a function of the fluid pressure loading. This equation shows that radius recession in a linear elastic material requires a negative net pressure in a part of the fracture.

There are several possible mechanisms that can cause radius recession in our tests. First, a continuous redistribution of fluid in the fracture takes place during closure. This is caused by the higher leak-off velocity at the tip and the smaller width near the tip, so that the leak-off has a relatively large effect on the local mass balance. The pressure gradient caused by this flow towards the tip inside the fracture causes smaller opening near the tip, which subsequently makes the pressure gradient larger. In this way, the net pressure can become negative, and the surfaces touch. After touching, the net pressure remains negative or ultimately becomes zero. The combination of fluid flow, decreasing width and decreasing wellbore pressure was already given in Nolte (1988) as an explanation for radius recession.

Fully coupled numerical simulations based on local Poiseuille flow between parallel plates, Carter leak-off, and elastic rock deformation indeed show a receding radius (de Pater et al., 1996, and Desroches and Thiercelin, 1993). The previous reasoning would imply that radius recession is impossible for zero confining stress. Also, modelling of fluid flow during closure would be essential to simulate recession. The surface roughness can enhance this process by increasing the pressure gradient. Variation between roughness profiles in different directions might even cause variations in recession velocity.

Whether fracture growth or recession takes place after shut-in, appears to depend on the rock permeability. This is the only material parameter that very strongly varies between strong plaster and cement. When the permeability is too large - like in plaster - the fluid is simply not able to reach the tip and make the fracture propagate.

As a second mechanism for recession, both the surface roughness and possible plastic elongation of the material surrounding the fracture in a direction perpendicular to the fracture surfaces can make the surfaces touch at pressures where otherwise no contact would be present. Also, poro-elastic expansion of the rock surrounding the fracture can cause contact of the surfaces.

We observed that the fluid pressure at which the fracture closes in weak plaster is significantly lower than the closure stress working perpendicular to the fracture surface. When we look at the material behaviour of weak plaster in comparison with relatively strong plaster, we expect larger plastic deformations, and pore collapse at a lower mean stress (see section 4.4). Two processes can possibly cause the measured effect: either pore collapse of the plaster around the fracture, or plastic deformation in a direction
perpendicular to the fracture plane that forms a wedge at the tip. Numerical simulations show that the "wedge-mechanism" can qualitatively explain the experimental results, without the introduction of a hydrostatic cap in the yield envelope. In most practical situations, pore collapse will not be significant.

Plastic elongation of the material close to the fracture surface in the direction perpendicular to the fracture surface, and imperfectly matching fracture surfaces due to roughness, have a similar effect during closure. Both make that the fracture surfaces touch earlier during closure. As mentioned in section 6.3, it depends on the length scales of interest (fracture length, fracture width, and plastic zone size or roughness) whether this effect increases or decreases the pressure at which the fracture closes at the wellbore. On the one hand, the surfaces will touch earlier near the tip, thus forming a wedge. This wedge tends to keep the fracture open, causing a closure pressure lower than the confining stress. On the other hand however, the roughness or plastic elongation will also cause the fracture to close earlier at the wellbore. In most experiments on strong plaster, it appears that the residual width near the tip caused by the fracture roughness is so small, that the increases in closure pressure at the wellbore dominates when we consider the influence of roughness. Furthermore, the residual width may be smaller near the tip, where the maximum separation of the surfaces is smaller. In weak plaster, the lowering of the closure pressure appears to be dominant, which indicates that the length scale of the plastic elongation is much larger than the residual width near the tip as a result of fracture roughness.

7.5 Implications for field practice

Our experiments are scaled in order to represent experiments on a field scale (see section 2.8). The most important assumption here is that fracture toughness does not change significantly with fracture length and speed, which is supported by large-scale laboratory experiments (see section 2.3). This scaling ensures that phenomena and mechanisms observed in the laboratory experiments are representative for hydraulic fracture propagation on a field scale. This enables us to list several implications for hydraulic fracturing field practice from the experimental results. These will be discussed in the following.

*In-situ stress state*

The significance of our results concerning fracture surface roughness for field practice first depends on the *in-situ* stress states. These stresses influence the plastic zone size, and as a
result the effect of plasticity. In the absence of geological activity, the maximum and minimum in-situ stresses can be given by $\sigma_1 = \sigma_3 (1-\nu) / \nu$, so that $\sigma_1 = 4\sigma_3$ for $\nu = 0.2$ (Jaeger and Cook, 1969). For typical rock properties, this can be close to yielding. Tectonically induced stresses can bring the stress state closer to yielding. In addition, pore pressure reduces the mean effective stress, and increases the deviatoric stress. For rock with a low cohesion, this can promote rock failure, which is governed by effective stress. We conclude that the in-situ stresses can be such that the experimentally observed phenomena are significant for field practice.

**Fracture geometry and net pressure**

The previously described results can have serious implications for hydraulic fracturing in field applications. First, we observed a larger width in the bulk of the fracture for elastic-plastic fractures, which leads to a larger volume compared with the elastic case for the same radius. When the net pressure compared at equal radius would be the same in the elastic and elastic-plastic case, the net pressure as a function of time would be larger in the elastic-plastic case. This is caused by the larger volume for the same radius, in combination with the decrease in net pressure with radius.

The observed tip mechanism can increase or decrease the net pressure, while the width in the bulk of the fracture will always be larger in the elastic case. These differences can easily be 20%, but can also be larger depending on the specific situation. When the fluid reaches the tip of the fracture, numerical models show that the net pressure can be increased significantly. However, the application of the cohesive-zone model under these circumstances is questionable. The weakening effect of the plastic strain could cause localisation of the fracturing process in the plastic zone, which effectively decreases the strength of the material. This should be incorporated in numerical models.

When the two far-field stresses parallel to the fracture plane differ in magnitude, the plastic zone in both directions will differ. When the plasticity leads to a change in the net pressure, this can easily cause a preferred growth direction (we do not take stress- or permeability barriers into account here). Assuming that the vertical in-situ stress is the maximum principal stress, plasticity will cause height growth if it decreases the net pressure. If it increases the net pressure, height growth will be suppressed.

When plasticity becomes the dominant dissipation mechanism, turning of the fracture surfaces and splitting up of the fracture was predicted by the model in section 2.9. Another effect that can occur is that fluid starts to follow the induced shear fracture at the tip, as was observed in our experiments (see for example the first part of the fluid front in figure 5.30). These effects can obviously have large effects on the fracture geometry, and need to be investigated further.

When we assume that the direction of fracture growth is unaffected by plasticity, we can ask whether we can use an elastic model. The simplest objection against an elastic
model is that the relation between net pressure and fracture opening is different for elastic-plastic models. When the enlarged width in the main part of the fracture controls the wellbore pressure, an elastic model with a reduced modulus may give acceptable results. When the tip gradient is dominant, an increased modulus may predict the pressure correctly, but underestimates the width severely. To match the pressure, the toughness can be enhanced too. Only when this is done with a reduced modulus, an approximately correct prediction of the geometry might be obtained. The correction factors for modulus and toughness in an elastic model could be used in practice if these correction factors that account for the previously described effects could be identified to be constant for an acceptable wide range of practical cases.

Current hydraulic fracturing models usually only consider viscous forces in modelling fluid flow in the fracture. However, scaling of the Navier-Stokes equations indicates that in some practical cases, fluid flow near the wellbore is typically governed by inertia terms (see section 2.5).

Fracture reopening

One of the problems with high net pressures is that they are also observed during reopening of a fracture. Although we only presented one reopening experiment in this thesis, we performed some more of them (see van Dam et al., 1999). We found that when the first fracture has enough residual conductivity (due to roughness or remaining fluid inside the fracture), the maximum in the net pressure is reached when the fluid reaches the tip of the previously created fracture. When the residual conductivity is high enough, this could imply that the apparent initiation in the field is in fact the moment that the previously created fracture is already filled, and fracturing of intact rock begins. Then, we can expect a similar elevation of the net pressure during "reopening", as during the first fracturing phase. When the high net pressure is caused by turning of the fracture plane with respect to the in-situ stresses, a high net pressure during reopening may be just caused by the higher stress working perpendicular to the fracture plane.

Hydraulic fracture closure

Radius growth and recession took place in the experiments. Radius growth after shut-in is indicative for the presence of a fluid lag during propagation. Whether growth or recession takes place appears to be influenced by rock permeability, all other parameters being equal. Radius recession slowed down the pressure decline rate and increased the closure time in the experiments. In that case, determination of the leak-off coefficient from the pressure decline must be done immediately after shut-in. However, when radius growth takes place, this leads to a value of the leak-off coefficient that is too large.
The occurrence of radius changes after shut-in was recognised in Nolte (1990). A systematic way to take this into account, is the so-called "3/4 rule", that proposes to calculate the leak-off coefficient from the decline curve at 3/4 of the net pressure at shut-in (Nolte et al., 1993). This is based on an assumed transition from fracture growth to fracture recession at 3/4 of the net pressure at shut-in. When there is no further theoretical motivation for this method, its results should be used very cautiously. For our laboratory tests, the 3/4 rule was invalid, as the radius in plaster receded immediately after shut-in, while in cement the fracture kept propagating constantly.

A mechanism was identified by which plastic deformation can decrease the measured closure stress significantly (see section 7.4). This has important implications for stress measurements and determination of net pressure. Errors in the determination of confining stress can also be a cause for deviations in the (apparent) net pressure from the linear elastic model. This closure mechanism has other consequences too. When the fluid pressure has decreased significantly, the stress working on the fracture faces in the bulk of the fracture will be reduced significantly. This will influence the stability of proppant packs and the material around the fracture (Papanastasiou, 1999c). When the fracture tip is relatively far away, plasticity can also lead to a somewhat larger closure stress (see section 7.4).

In strong plaster, a clear break was present in the G-plot at the moment the fracture closed at the wellbore. In weak plaster, where plastic deformation is significant, no break in the G-plot was present. However, the closure point was visible as a local minimum in the log-log plot of the pressure derivative against time (van Dam et al., 1998).

Rough fracture surfaces usually do not fit perfectly when they are closed. We observed this in our experiments (see Chapter 5 and Appendix A), while it is for example also observed in field tests (Warpinsky et al., 1997). This imperfect closing is caused by the roughness due to the fracturing process, in combination with slight shifting of the surfaces with respect to each other. The fracture surface roughness has qualitatively the same effect on fracture width during the closure process as plasticity. However, the non-matching rough surfaces form a channel that is permeable for fluid flow. The permeability caused by this residual width can be so large, that proppant volumes can be reduced or even omitted (Mayrhofer and Meehan, 1998).

Our results indicate that roughness of hydraulic fracture surfaces can be predicted from rock properties and in-situ stresses. The material properties might be "calibrated" by performing a uniaxial tensile test. The prediction of roughness is certainly important for the prediction of the residual conductivity. However, the residual width is also influenced by the shifting of the fracture surfaces with respect to each other, an effect which is still hard to predict.
7.6 Conclusions

In this chapter, we discussed three topics: the interaction between fluid flow and (plastic) deformation at the tip, the fracture mechanism at the tip, and the fracture closure mechanism. Furthermore, we discussed the implications for field practice. This discussion was primarily based on results from previous chapters and literature, and contained the main results of the study. The conclusions that can be made from the combination of results from the previous chapters, are listed here:

- An extensive experimental verification of most aspects of the linear elastic model, showed that this model was confirmed in cement. Deviations were seen in weak plaster, and in strong plaster under a large values of the externally applied stress deviator.
- Combination of material behaviour in standard tests with the results of hydraulic fracturing experiments, showed that the development of plasticity in tensile tests is most indicative for the occurrence of deviations from the elastic model, under moderate values of the externally applied stress deviator.
- Combination of several points of view lead to the conclusion that the fracturing process at the tip deviates from pure tensile fracturing, but rather is a combination of tensile and shear fracturing.
- Plasticity near the tip can decrease the net pressure needed for propagation. Based on the identified interaction mechanism between fluid flow and plastic rock deformation, we expect that an increase in net pressure is also possible.
- Based on the experimental and numerical results, plasticity-dominated propagation and incorrect determination of the minimum confining stress were identified as possible causes for the occurrence of high net pressures.
Conclusions and recommendations

In this chapter, we list the main conclusions of the present study. The purpose of the chapter is to provide the reader with a concise summary of the most important results. The conclusions overlap the conclusions made at the end of each chapter. For a further foundation of these conclusions, we refer to the chapters of interest. In addition, we make recommendations about possible extension of this study.

Conclusions

- The elastic model that couples fluid flow, rock deformation, and rock fracturing, is confirmed in the experiments on cement. An extensive experimental verification was performed, in which the net pressure and fracture width at the wellbore, the width profile along the fracture radius, the fracture volume, and a measure of the fluid lag size were measured. The injection flow rate and in-situ stresses were varied.

- The concept of the fluid lag that forms to create a balance between the fluid pressure loading and the resistance to fracturing of the rock, is confirmed in experiments on cement, and in coupled numerical simulations. Coupled numerical simulations show that a cohesive-zone model for the tip predicts the size of the fluid lag correctly in plaster, contrary to the tip model of linear elastic fracture mechanics.

- Plasticity near the fracture tip leads to a shorter fluid lag, a larger width in the main part of the fracture, and a smaller width at the position of the fluid front. This was shown in numerical simulations, while the influence of plasticity on the size of the
fluid lag and width at the wellbore was confirmed in experiments on plaster. Furthermore, the effect of plasticity on the wellbore pressure was similar in simulations of and experiments on weak plaster.

- The development of plasticity in tensile tests is most indicative for the occurrence of deviations from the elastic model, for situations in which the externally applied stress deviator has small or moderate values. This is concluded from the hydraulic fracturing results in plaster and diatomite, in combination with the material characterisation.

- The fracture surface roughness of hydraulic fracturing experiments on cement, plaster, and diatomite is determined by rock properties and externally applied stresses. A correlation with a measure of the size of the plastic zone was found.

- The fracturing process at the fracture tip is a combination of tensile and shear fracturing. This is concluded from the following observations. The fracture surface roughness of hydraulic fracturing experiments on plaster and diatomite shows a correlation with a calculated measure of the size of the plastic zone around the fracture tip. The elastic stress path shows that shear fracturing can precede tensile fracturing. Experiments show that under extreme conditions shear fractures near the tip are induced by the passage of the hydraulic fracture.

- We observed radius growth and recession after shut-in in respectively cement paste and plaster. This different behaviour is most probably caused by differences in permeability. These radius changes influence the pressure decline rate and closure time significantly.

- As a result of plasticity near the tip, hydraulic fractures close at the wellbore at a negative net pressure. This was observed in both experiments and numerical simulations. From the simulations, a mechanism was observed that causes this behaviour, in which the plastic deformations acted like a wedge.

**Recommendations**

Recommendations for further research follow from the open questions that arise from the present work. One of these questions is to what extent the "bridges" that transmit the stress in the cohesive zone, hinder the fluid flow in this zone. Furthermore, the interaction between plastic shear deformation and the fracturing process must be investigated further. This is important with respect to the question whether plasticity can cause an effective toughness that is one or two orders of magnitude larger than measured in the laboratory.
Appendix A

Raw data of hydraulic fracturing model experiments

This appendix contains the data of the hydraulic fracturing experiments, as they were measured. The following figures show the net pressure at the wellbore $p_{net,w}$, the width at the wellbore $b_w$, and the acoustically determined fracture radius $R$. The wellbore width in these figures is not corrected for deformation in the measurement interval of the width.

The figure are followed by two tables. Table A.1 shows the measurements as they were used to make the figures in Chapter 5. Table A.2 shows the values of the net pressure, the wellbore width, and/or the radius at the characteristic points during hydraulic fracturing (see figure 3.9).
Figure A.1. Net pressure $p_{\text{net, w}}$ at the wellbore in cement as a function of experiment time $t$ minus initiation time $t_i$.

Figure A.2. Width $b_w$ at the wellbore in cement as a function of experiment time $t$ minus initiation time $t_i$.  

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Figure A.7. Net pressure $p_{net,w}$ at the wellbore in weak plaster as a function of experiment time $t$ minus initiation time $t_i$.

Figure A.8. Width $b_w$ at the wellbore in weak plaster as a function of experiment time $t$ minus initiation time $t_i$. 

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Figure A.3. Fracture radius $R$ in cement as a function of experiment time $t$ minus initiation time $t_i$. In experiments ce09 and ce10, the fracture grew out of the block. The measured radius shows some irregularities at the end of the curves, because some diffraction combinations disappear. Note that radius growth takes place after shut-in.

Figure A.4. Net pressure $p_{net,w}$ at the wellbore in strong plaster as a function of experiment time $t$ minus initiation time $t_i$. 
Figure A.9. Fracture radius $R$ in weak plaster as a function of experiment time $t$ minus initiation time $t_i$.

Figure A.10. Net pressure $p_{net,w}$ at the wellbore in diatomite as a function of experiment time $t$ minus initiation time $t_i$. The time of the curves indicated with * is shrunk ten times, in order make it fit on the time scale.
Figure A.5. Width $b_W$ at the wellbore in strong plaster as a function of experiment time $t$ minus initiation time $t_r$.

Figure A.6. Fracture radius $R$ in strong plaster as a function of experiment time $t$ minus initiation time $t_r$.
Figure A.11. Width $b_w$ at the wellbore in diatomite as a function of experiment time $t$ minus initiation time $t_i$. The time of the curves indicated with * is shrunk ten times, in order to make it fit on the time scale.
Table A.1. Results of all experiments. $\gamma_c$ and $\gamma_t$ are coefficients describing fracture opening and shape, $\omega_d$ is the size of the dry tip measured after opening of the block, RMS is a measure for the fracture surface roughness, $\phi_s$ is the average slope of the roughness pattern, and $b_0$ is the width of the open fracture when the net pressure is zero.

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* intermediate water content.
** after previously creating a fracture with a small width using water.
Table A.2. Net pressure and width at the wellbore, and fracture radius, at characteristic points during hydraulic fracturing. Besides the measuring error, the difference between acoustic radius and post-mortem radius (radius measured after splitting of the block) is caused by radius growth after shut-in in cement, and by the size of the dry tip in plaster.

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* intermediate water content.
** after previously creating a fracture with a small width using water.
Figure A.7. Net pressure $p_{net, w}$ at the wellbore in weak plaster as a function of experiment time $t$ minus initiation time $t_i$.

Figure A.8. Width $b_w$ at the wellbore in weak plaster as a function of experiment time $t$ minus initiation time $t_i$. 

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Figure A.9. Fracture radius $R$ in weak plaster as a function of experiment time $t$ minus initiation time $t_i$.

Figure A.10. Net pressure $p_{net,w}$ at the wellbore in diatomite as a function of experiment time $t$ minus initiation time $t_i$. The time of the curves indicated with * is shrunk ten times, in order make it fit on the time scale.
Appendix B

Closure model with receding radius

This appendix contains a derivation of the leak-off volume during fracture closure for two different assumptions about radius behaviour: constant radius and radius receding linearly with time. The derivation for constant radius is analogous to the one presented in Nolte (1986). We compare the closure times for both cases, which gives us an idea about the impact of radius recession on closure times.

We assume a growth function for the radius $R_f$ of the fluid front given by:

$$ R_f = K_1 t^\beta $$

so that

$$ A_f = K_2 t^{2\beta} \quad (B.1) $$

$A_f = \pi R_f^2$ is the area over which fluid leak-off takes place, $K_1$ and $K_2$ (which equals $\pi K_1^2$) are constants, and $\beta$ is the fracture growth coefficient. The leak-off velocity $v_i$ through both fracture faces at a certain position is described by the Carter equation:

$$ v_i = \frac{2C_i}{\sqrt{t-t_0}} \quad (B.2) $$

$C_i$ is the leak-off coefficient, which is assumed to be independent of time, and $t_0$ is the time the position at the fracture faces was first exposed to fluid pressure. Using equations (B.1) and (B.2), we can write the leak-off velocity $v_i$ at a position $r = \sqrt{\frac{a}{\pi}}$ as:

$$ v_i = \frac{2C_i}{\sqrt{t-t_0} \sqrt{1-\frac{1}{\alpha_i^{2\beta}}}} = \frac{2C_i}{\sqrt{t \sqrt{1-\alpha_i^{2\beta}}}} \quad (B.3) $$
in which we put \( \alpha_r = \frac{a}{A_f} \). The total leak-off rate \( q_t \) through the area \( a \) is (excluding the borehole area \( A_w \), while the fracture area is perpendicular to the wellbore):

\[
q_t = \int \gamma_t da = \frac{2C_i A_f}{\sqrt{t}} \int_{\alpha_w}^{\alpha_r} \frac{1}{\sqrt{1-\alpha_r^{2\beta}}} d\alpha_r \tag{B.4}
\]

in which \( \alpha_w = \frac{A_w}{A_f} \). We now consider the analytical solution for \( \beta = \frac{1}{2} \), which approximately corresponds to the low limit of leak-off (Nolte, 1986). For a constant flow rate and impermeable fracture walls, \( \beta = \frac{4}{9} \) for viscosity-dominated propagation (Crockett et al., 1986) and \( \beta = \frac{2}{5} \) for toughness-dominated propagation (which can be inferred from equation (2.55)). Taking \( \beta = \frac{1}{2} \) approximately agrees with the measured value of \( \beta \) in plaster, which is 0.4. Note that the efficiency of the hydraulic fractures in the other materials we used is higher, giving a value of \( \beta \) closer to 0.5. Using \( \beta = \frac{1}{2} \), integration of equation (B.4) yields:

\[
q_t = \frac{4C_i A_f}{\sqrt{t}} \left[ \sqrt{1-\alpha_w} - \sqrt{1-\alpha_r} \right] \tag{B.5}
\]

Because of the assumed fracture growth function in equation (B.1), we can express the area \( A_f \) by time \( t \) and a reference value and time, for which we take the moment of shut-in, so that:

\[
A_f = A_{f,\text{ref}} \frac{t}{t_i} \tag{B.6}
\]

Then equation (B.5) yields:

\[
q_t = \frac{4C_i A_{f,\text{ref}}}{\sqrt{t_i}} \sqrt{t_i + 1} \left[ \sqrt{1-\alpha_w} - \sqrt{1-\alpha_r} \right] \tag{B.7}
\]

in which \( t_D = \frac{t}{t_i} - 1 \); \( t_i \) is the injection time and \( A_{f,\text{ref}} \) is the fluid area at shut-in.
Until now we formally considered a propagating fracture. The closure is incorporated by adjusting the value of \( \alpha_r \). For a constant fracture area after shut-in we have:

\[
\alpha_r = \frac{a}{A_f} = \frac{A_{f,si}}{A_f} = \frac{1}{t_D + 1}
\]

(B.8)

and

\[
\alpha_w = \frac{A_w}{A_{f,si}} \frac{1}{t_D + 1}
\]

(B.9)

For the case that the fluid front radius recedes linearly with time, and reaches the borehole at the moment \( t = t_i + t_c \) when \( b_w = 0 \), we have:

\[
\alpha_r = \frac{a}{A_f} = \frac{A_{f,si}}{A_f} \left( \frac{1}{t_c} \right)^2 \left( \frac{1}{t_i} \right)^2 \frac{1}{t_D + 1}
\]

(B.10)

We assumed that in the closed part of the fracture the leak-off velocity is zero. Now we can calculate the total leak-off volume \( V_l \) during closure:

\[
V_l = \int_{t_i}^{t} q_i \, dt
\]

(B.11)

For constant radius after shut-in, this yields

\[
V_l = 2C_i A_{f,si} \sqrt{t_i} \left( \frac{4}{3} \left( t_i + 1 - \frac{A_w}{A_{f,si}} \right)^2 - \left( 1 - \frac{A_w}{A_{f,si}} \right)^2 - t_D^2 \right)
\]

(B.12)

For \( A_w = 0 \) we can write this as (as was obtained in Nolte, 1986):

\[
V_l = 2C_i A_{f,si} \sqrt{t_i} \left( \frac{4}{3} \left( t_i + 1 \right)^2 - 1 - t_D^2 \right) = 2C_i A_{f,si} \sqrt{t_i} G(t_D)
\]

(B.13)

This defines the dimensionless G-function, and shows that the leak-off volume as a function of \( G(t_D) \) is a straight line for constant leak-off coefficient and leak-off area.
For the radius receding linearly with time, we find:

\[
V_t = 2C_l A_f \sqrt{\frac{t_t}{t_l}} \sqrt{t_t - t_l} \\
V_t = 2C_l A_f \sqrt{\frac{t_t}{t_l}} \sqrt{t_t - t_l} \left[ \frac{4}{3} \left( t_D + 1 - \frac{A_w}{A_{f,si}} \right)^2 - \left( 1 - \frac{A_w}{A_{f,si}} \right)^2 \right] \right]
\]

(B.14)

in which \( t_H = \left( \frac{t_t}{t_l} \right)^2 + 2 \frac{t_t}{t_l} \). From equations (B.12) and (B.14) we find, as an example, that the closure time is 4 times larger when the radius is receding in comparison with the constant radius case (for \( t_l = 1000, t_c = 665 \) s, and \( A_w=0 \)).
Appendix C

Description of FLAC

In this appendix we describe the program FLAC with which we did two-dimensional simulations of hydraulic fracture propagation. FLAC is a Lagrangian method and uses an explicit time-marching numerical scheme. This appendix shortly describes how the mechanical equations are solved in FLAC. More information can be found in Cundall and Board (1988).

**Governing equations**

In order to obtain quasi-static solutions, the equation of motion for a grid point is solved using a pseudo-time. The equation of motion for a grid point is:

\[
\rho_{\text{bulk}} \frac{\partial u_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho_{\text{bulk}} g_i
\]  

(C.1)

\(\rho_{\text{bulk}}\) is the density of the solid, \(g_i\) is a component of the gravitational acceleration, and \(u_i\) is a component of the position vector relative to a fixed frame of reference. From this equation, the velocity \(\dot{u}_{ij}\) at a grid point is calculated. The grid-point velocities yield the velocities of strain \(\varepsilon_{ij}\) and rotation \(\omega_{ij}\) for a grid element:

\[
\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i} \right)
\]  

(C.2)
\[
\omega_{ij} = \frac{1}{2} \left( \frac{\partial \dot{u}_i}{\partial x_j} - \frac{\partial \dot{u}_j}{\partial x_i} \right)
\]

(C.3)

The new stresses are calculated with the aid of the constitutive equation, which is in the case of isotropic elasticity:

\[
\sigma_{ij}(t + \Delta t) = \sigma_{ij}(t) + \left\{ \delta_{ij} \left[ K - \frac{2}{3} G \right] \dot{\varepsilon}_{kk} + 2G \dot{\varepsilon}_{ij} + \omega_{ij}(t) \sigma_{kk}(t) - \sigma_{kk}(t) \omega_{ij} \right\} \Delta t
\]

(C.4)

where \( K \) is the bulk modulus, \( G \) is the shear modulus of the solid, and \( \delta_{ij} \) is the Kronecker delta.

**Numerical formulation**

The grid consists of quadrilateral elements. Lines between the grid points at opposite corners subdivide an element into two triangles in two different ways. In this way, two sets of two generally different triangles are defined (each set forming the original quadrilateral). These triangular elements have constant stress and strain. In FLAC, a "mixed discretization" scheme is used. The isotropic parts of stress and strain are averaged over the four triangles, while the deviatoric parts remain unchanged for each triangle. Constant stress/strain triangular elements are used. This procedure has advantages when plastic flow occurs, in which case requirements on plastic volume change in combination with the plane strain condition impose constraints on the grid-point deformations (Marti and Cundall, 1982). Also, the use of the triangular elements prevents problems with "hourglass" deformation of the triangular element, which consist of non-zero grid-point displacements which yield zero strain of the quadrilateral element.

In order to calculate the average strain rate in an element from the grid-point displacements, Gauss' divergence theorem is used:

\[
\int_{A'} n_j \dot{u}_i \, dA' = \int_{V'} \frac{\partial \dot{u}_i}{\partial x_j} \, dV'
\]

(C.5)

where \( A' \) is the closed boundary of the volume \( V' \), and \( n_i \) is the outward normal to \( A' \). Because the problem is two-dimensional, contributions in the out of plane direction cancel out in the integral in the left-hand side of (C.5), and are zero in the integral on the right-hand side. In 2D, \( A' \) can be regarded as the closed contour forming the boundary of the area \( V' \).
After discretisation, the value of $\dot{u}_i$ is uniform over the part of the closed contour $s$ that connects two grid points. For a triangular element, we then can write equation (C.5) as:

$$\frac{\partial u_i}{\partial x_j} = \sum_s n_j u_i \Delta s$$  \hspace{1cm} (C.6)

$\frac{\partial u_i}{\partial x_j}$ is the volume average of the strain rate in the triangular element, and $\Delta s$ is the side of such an element; the summation is over the three sides of the element. In order to calculate the average strain rate in a triangular element from the grid-point velocities, FLAC uses:

$$\varepsilon_{ij} = \frac{\partial u_i}{\partial x_j} = \frac{1}{2A} \sum_{s} (u_i + u_j) n_j \Delta s$$  \hspace{1cm} (C.7)

where $u_i^A$ and $u_i^B$ are the grid-point velocities at the consecutive grid points A and B of the triangular element.

With the use of the constitutive equation new stresses are calculated. The stress at one side of the triangle is taken to be equivalent to two forces working at the ends of that side at the grid points, so the force at a grid point has at least two contributions. On grid points where triangular elements from the same set meet the forces are summed. In these grid points there are four contributions to the total force from one set of triangles. Then, the forces are averaged over the two sets of triangles.

The new velocities and grid-point positions are calculated using

$$\dot{u}_i(t + \frac{\Delta t}{2}) = \dot{u}_i(t - \frac{\Delta t}{2}) + \left( \sum F_i(t) - 0.8 \sum F_i(t) \text{sign} \left( \dot{u}_i(t - \frac{\Delta t}{2}) \right) \right) \frac{\Delta t}{m}$$  \hspace{1cm} (C.8)

$$u_i(t + \Delta t) = u_i(t) + \dot{u}_i(t + \frac{\Delta t}{2}) \Delta t$$  \hspace{1cm} (C.9)

$m$ is the mass associated with a grid point. Using this discretisation the first order error disappears. The time step is so small that only nearest neighbour interaction is taken into account, using the maximum speed at which strains can propagate through the solid. A damping term is included in equation (C.8).
The (pseudo-)time step in FLAC is set to unity and the grid-point masses are adjusted for optimum convergence. This can be done because the goal of the calculation is to obtain a static solution: the grid-point masses only control the relaxation. The stability condition for an elastic solid with element size $\Delta x$ is:

$$\Delta t < \frac{\Delta x}{C}$$  \hspace{1cm} (C.10)

$\Delta t$ is the time step and $C$ is the maximum speed at which information can propagate in the solid. Expressing $C$ - which typically equals the P-wave speed - in terms of the elastic bulk- and shear modulus leads to an expression for the grid-point mass.

Damping of the equation of motion is necessary to obtain static solutions. A term that gives this damping is included in equation (C.8). The damping in FLAC is such that body forces from the damping vanish for steady-state conditions, and the amount of damping can vary from point to point. Furthermore, the damping constant is dimensionless and independent of material properties and boundary conditions.
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References


# List of symbols

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<td>stress intensity</td>
<td>$\text{Pa} \cdot \text{m}^{-1/2}$</td>
</tr>
<tr>
<td>$K_{IC}$</td>
<td>critical stress intensity or fracture toughness</td>
<td>$\text{Pa} \cdot \text{m}^{-1/2}$</td>
</tr>
<tr>
<td>$k$</td>
<td>rock permeability</td>
<td>$\text{m}^2$</td>
</tr>
<tr>
<td>$k_y$</td>
<td>yield stress</td>
<td>$\text{Pa}$</td>
</tr>
<tr>
<td>$L$</td>
<td>fracture half length for plane strain fracture geometry</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$L$</td>
<td>fracture propagation velocity for plane strain fracture geometry</td>
<td>$\text{m} \cdot \text{s}^{-1}$</td>
</tr>
<tr>
<td>$l_p$</td>
<td>measure of size of the plastic zone</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$l_T$</td>
<td>measure of size of the tensile zone</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$l_w$</td>
<td>measuring length interval for wellbore width</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$l_{lo}$</td>
<td>depth of leak-off zone at the moment of shut-in</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$m$</td>
<td>mass</td>
<td>kg</td>
</tr>
<tr>
<td>$N_{Cl}$</td>
<td>dimensionless number for leak-off</td>
<td>-</td>
</tr>
<tr>
<td>$N_E$</td>
<td>dimensionless number for fracture opening</td>
<td>-</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>$N_{Re}$</td>
<td>Reynolds number</td>
<td>-</td>
</tr>
<tr>
<td>$N_T$</td>
<td>dimensionless number for time</td>
<td>-</td>
</tr>
<tr>
<td>$N_f$</td>
<td>dimensionless number for fracture surface energy</td>
<td>-</td>
</tr>
<tr>
<td>$N_{\sigma_c}$</td>
<td>dimensionless number for fluid lag</td>
<td>-</td>
</tr>
<tr>
<td>$P_R$</td>
<td>characteristic pressure difference in $r$-direction</td>
<td>Pa</td>
</tr>
<tr>
<td>$P_Y$</td>
<td>characteristic pressure difference in $y$-direction</td>
<td>Pa</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure or mean stress in the solid</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_f$</td>
<td>fluid pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_{net}$</td>
<td>net pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_{net,w}$</td>
<td>net pressure at the wellbore</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_p$</td>
<td>pore pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_0$</td>
<td>average pore pressure in the leak-off zone</td>
<td>Pa</td>
</tr>
<tr>
<td>$\Delta p_f$</td>
<td>pressure difference driving leak-off</td>
<td>Pa</td>
</tr>
<tr>
<td>$Q$</td>
<td>flow rate of fracturing fluid at the wellbore</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$Q_{eff}$</td>
<td>flow rate flowing effectively into the fracture</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$Q_{pump}$</td>
<td>flow rate applied at the pump</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$q$</td>
<td>difference between largest and smallest principal stress</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$q_f$</td>
<td>flow rate inside the fracture</td>
<td>$m^2/s$</td>
</tr>
<tr>
<td>$q_f'$</td>
<td>flow rate inside the fracture per unit perimeter ($q_f$ divided by $2\pi r$)</td>
<td>$m^2/s$</td>
</tr>
<tr>
<td>$R$</td>
<td>fracture radius</td>
<td>m</td>
</tr>
<tr>
<td>$R_c$</td>
<td>characteristic radius</td>
<td>m</td>
</tr>
<tr>
<td>$R_f$</td>
<td>radius of fluid front</td>
<td>m</td>
</tr>
<tr>
<td>$R_{rec}$</td>
<td>receding radius</td>
<td>m</td>
</tr>
<tr>
<td>$R_T$</td>
<td>characteristic length of the tip region</td>
<td>m</td>
</tr>
<tr>
<td>$R_w$</td>
<td>wellbore radius</td>
<td>m</td>
</tr>
<tr>
<td>$RMS$</td>
<td>fracture surface roughness</td>
<td>m</td>
</tr>
<tr>
<td>$r$</td>
<td>radial coordinate</td>
<td>m</td>
</tr>
<tr>
<td>$S_w$</td>
<td>water saturation</td>
<td>-</td>
</tr>
<tr>
<td>$S_0$</td>
<td>cohesion</td>
<td>Pa</td>
</tr>
<tr>
<td>$T$</td>
<td>characteristic time for hydraulic fracture propagation</td>
<td>s</td>
</tr>
<tr>
<td>$T_0$</td>
<td>tensile strength</td>
<td>Pa</td>
</tr>
<tr>
<td>$T_{0,B}$</td>
<td>Brazilian tensile strength</td>
<td>s</td>
</tr>
<tr>
<td>$t$</td>
<td>experiment time</td>
<td>s</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time step in simulations</td>
<td>s</td>
</tr>
<tr>
<td>$t_i$</td>
<td>initiation time of a hydraulic fracture</td>
<td>s</td>
</tr>
<tr>
<td>$t_{si}$</td>
<td>moment of shut-in</td>
<td>s</td>
</tr>
<tr>
<td>$t_0$</td>
<td>moment the rock was first exposed to fluid pressure</td>
<td>s</td>
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<tr>
<td>$u$</td>
<td>displacement vector</td>
<td>m</td>
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<tr>
<td>$U_e$</td>
<td>elastic energy stored in fracture opening</td>
<td>N-m$^3$</td>
</tr>
<tr>
<td>$U_f$</td>
<td>energy dissipated in fluid flow</td>
<td>N-m$^3$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>$U_l$</td>
<td>energy dissipated in leak-off</td>
<td>N·m$^3$</td>
</tr>
<tr>
<td>$U_p$</td>
<td>energy dissipated in plastic rock deformation</td>
<td>N·m$^3$</td>
</tr>
<tr>
<td>$U_T$</td>
<td>energy dissipated in creating fracture surface</td>
<td>N·m$^3$</td>
</tr>
<tr>
<td>$V_l$</td>
<td>leak-off volume after shut-in</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_T$</td>
<td>characteristic velocity in the radial direction</td>
<td>m·s$^{-1}$</td>
</tr>
<tr>
<td>$V_{\text{eff}}$</td>
<td>effectively injected volume</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_f$</td>
<td>fluid volume in the fracture</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_{\text{frac}}$</td>
<td>volume of fracture</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_p$</td>
<td>volume of active plastic zone</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_{\text{system}}$</td>
<td>volume of total fluid-filled system</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_S$</td>
<td>characteristic fluid velocity at the fracture surface</td>
<td>m·s$^{-1}$</td>
</tr>
<tr>
<td>$v_l$</td>
<td>leak-off velocity through one fracture surface</td>
<td>m·s$^{-1}$</td>
</tr>
<tr>
<td>$v$</td>
<td>fluid velocity in the fracture or in the rock</td>
<td>m·s$^{-1}$</td>
</tr>
<tr>
<td>$v_{r,s}$</td>
<td>radial component of fluid velocity at the fracture surface</td>
<td>m·s$^{-1}$</td>
</tr>
<tr>
<td>$W$</td>
<td>characteristic fracture width</td>
<td>m</td>
</tr>
<tr>
<td>$w_e$</td>
<td>work done by the fluid in opening of the fracture</td>
<td>N·m$^3$</td>
</tr>
<tr>
<td>$w$</td>
<td>water content (water mass divided by dry rock mass)</td>
<td>-</td>
</tr>
<tr>
<td>$w_t$</td>
<td>width of sample used in tensile test</td>
<td>m</td>
</tr>
<tr>
<td>$x$</td>
<td>cartesian coordinate in direction of fracture growth</td>
<td>m</td>
</tr>
<tr>
<td>$x$</td>
<td>position vector</td>
<td>m</td>
</tr>
<tr>
<td>$y$</td>
<td>cartesian coordinate perpendicular to the fracture plane</td>
<td>m</td>
</tr>
<tr>
<td>$y_{\text{centreline,}i}$</td>
<td>y-coordinate of centre line at measuring point $i$</td>
<td>m</td>
</tr>
<tr>
<td>$z$</td>
<td>cartesian coordinate perpendicular to fracture growth and fracture planem</td>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tr>
<td>$\alpha$</td>
<td>coefficient in stress field potential</td>
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<tr>
<td>$\alpha_d$</td>
<td>deviation angle of fracture plane</td>
<td>deg</td>
</tr>
<tr>
<td>$\beta$</td>
<td>general time exponent</td>
<td>-</td>
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<tr>
<td>$\Gamma$</td>
<td>fracture surface energy per unit of global fracture area</td>
<td>N·m$^{-1}$</td>
</tr>
<tr>
<td>$\Gamma_s$</td>
<td>separation energy per unit of global fracture area</td>
<td>N·m$^{-1}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>maximum shear strain</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma_{pa}$</td>
<td>accumulated plastic shear strain, hardening parameter</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>principal plastic shear strain</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>coefficient describing fracture opening</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma_e$</td>
<td>coefficient describing width profile</td>
<td>-</td>
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<tr>
<td>$\delta_{ij}$</td>
<td>component of the strain tensor</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>average plastic strain in active plastic zone</td>
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</tr>
<tr>
<td>$\varphi$</td>
<td>potential function for two-dimensional stress field</td>
<td>Pa·m</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>porosity</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>average slope of roughness profile</td>
<td>deg</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>parameter expressing proportion between velocity gradients</td>
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### Symbols

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<th>Symbol</th>
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<tr>
<td>$\mu$</td>
<td>dynamic viscosity</td>
<td>Pa·s</td>
</tr>
<tr>
<td>$\bar{\mu}$</td>
<td>channel flow viscosity, being 12 $\mu$</td>
<td>Pa·s</td>
</tr>
<tr>
<td>$\mu^c$</td>
<td>magnitude of the complex viscosity in a dynamic test</td>
<td>Pa·s</td>
</tr>
<tr>
<td>$\mu^{\text{eff}}$</td>
<td>effective viscosity in pipe flow test</td>
<td>Pa·s</td>
</tr>
<tr>
<td>$\mu_f$</td>
<td>friction coefficient</td>
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</tr>
<tr>
<td>$\nu$</td>
<td>Poisson's ratio</td>
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<tr>
<td>$\rho$</td>
<td>fluid density</td>
<td>kg·m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_{\text{bulk}}$</td>
<td>bulk density of rock</td>
<td>kg·m$^{-3}$</td>
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<td>$\sigma_{ij}$</td>
<td>component of stress tensor</td>
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</tr>
<tr>
<td>$\sigma_c$</td>
<td>confining stress perpendicular to the fracture plane</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>externally applied stress parallel to the fracture plane</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_{h,1}$, $\sigma_{h,2}$</td>
<td>externally applied stress parallel to the fracture plane</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>stress normal to the failure plane in the Mohr-Coulomb criterion</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>largest principal compressive stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>intermediate principal compressive stress</td>
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</tr>
<tr>
<td>$\sigma_3$</td>
<td>smallest principal compressive stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$\tau$</td>
<td>shear stress along failure plane in Mohr-Coulomb criterion</td>
<td>Pa</td>
</tr>
<tr>
<td>$\omega_{ij}$</td>
<td>element of rotation tensor</td>
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<td>$\omega$</td>
<td>size of the fluid lag</td>
<td>m</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>acoustically determined measure of the fluid lag size</td>
<td>m</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td>size of the dry tip observed after splitting the block</td>
<td>m</td>
</tr>
<tr>
<td>$\omega_{\text{max}}$</td>
<td>maximum fluid lag size</td>
<td>m</td>
</tr>
<tr>
<td>$\psi$</td>
<td>potential function for two-dimensional stress field</td>
<td>Pa·m</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>complex variable with crack centre as origin</td>
<td>m</td>
</tr>
<tr>
<td>$\zeta^*$</td>
<td>complex variable with fracture tip as origin</td>
<td>m</td>
</tr>
</tbody>
</table>

### Subscripts

- $^e$ elastic
- $^p$ plastic
- $^*$ non-dimensionalised quantity
- $ax$ axial
- $bd$ break-down
- $\text{dyn}$ dynamic value
- $ij$ tensor indices according to cartesian coordinate system
- $\text{ini}$ initiation
- $\text{si}$ shut-in
- $r, \text{rad}$ radial component
- $x$ component in $x$-direction

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component in y-direction
component in z-direction
Summary

Hydraulic fracturing of reservoir rock is a much used technique for stimulation of oil- and gas wells. The success of a hydraulic fracturing treatment depends heavily on the created fracture geometry. To predict the fracture geometry, correct modelling of the hydraulic fracturing process is essential. This can only be done if all processes of interest are modelled correctly. Till today, a lot of dispute exists about created fracture geometries, how to model hydraulic fracturing, and especially about the fracturing process at the tip.

The objective of this research is to understand how plastic rock deformation around the fracture tip influences the geometry of the hydraulic fracture. Plastic rock deformation around the fracture tip is one of the mechanisms that are lacking in current practical fracture models, and may explain field observations of "high net pressures". We investigated this by performing scaled hydraulic fracturing model experiments. In these experiments, we varied the rock material and the applied stress in order to vary the size of the plastic zone around the fracture tip. Furthermore, we developed a numerical model that was used to reveal the mechanisms that can explain the experimental results.

The size of our rock samples was 0.30 m cubic. The samples were placed in a true triaxial machine to simulate in-situ stress states. During fracture propagation, pressure and width at the wellbore were measured. Also, ultrasonic monitoring was applied, which yielded the fracture radius and a measure of the fluid lag size. To characterise our materials, triaxial tests and other rock mechanical tests were performed. The roughness of the fracture surfaces was measured with a laser profilometer.

In linear elastic rock material (cement), our experimental results agree with linear elastic numerical simulations that couple fluid flow, rock deformation, and the rock fracturing process. This agreement comprises the net pressure at the wellbore, the width profile along the radial coordinate, and the size of the fluid lag. In weak rocks (moist plaster) we measured significant deviations from linear elastic fracture mechanics. Hydraulic fractures tend to become wider than predicted by linear elastic models. At the same time, the net pressure was lower.

Elastic-plastic simulations show that the observations in weak plaster are related to the width increase in the bulk of the fracture, which decreases the pressure gradient. The simulations also show that the fluid has to come closer to the fracture tip, because plasticity screens the fluid pressure loading. The extra wellbore pressure needed for this is smaller than the pressure decrease as a result of the larger width in the bulk of the fracture, for our experimental conditions. However, this mechanism might lead to a higher wellbore pressure under conditions different from our experiments. The disappearance of the fluid...
Summary

lag for large plastic zone sizes was confirmed in the experiments. This forms an indication for a transition into a "plasticity-dominated" propagation mode.

Hydraulic fracture width and net pressure at the wellbore in diatomite and strong plaster approximately agreed with the fully coupled linear elastic simulations. Especially for diatomite this is remarkable, because this material shows large plastic shear deformations in triaxial tests. Among the three weak materials we used (strong plaster, weak plaster, and diatomite), weak plaster has the unique property of showing large plastic strain under applied tensile stress. We conclude that this behaviour is most indicative for the occurrence of deviations of linear elastic simulations of hydraulic fracturing.

The surface roughness of the fractures created during the experiments reflects the fracturing process at the tip. The roughness shows a clear correlation with the size of the plastic zone around the tip. This indicates that the actual fracturing process at the tip is not purely tensile - as assumed in many models - but mixed mode between shear and tension. Analysis of the stress path followed by the material near the tip supports this hypothesis.

Analysis of hydraulic fracture closure is important for inferring information about the least in-situ stress and the leak-off coefficient in field practice. The experimental results show various deviations from conventional model assumptions. Radius growth or recession after shut-in occurs, rough fracture surfaces influence fracture closure, and plasticity near the tip influences the pressure decline significantly.
Acknowledgements/Dankwoord

Bij het totstand komen van dit proefschrift is de bijdrage van veel mensen onmisbaar geweest. Allereerst wil ik Gerard Mathu, Jan Etienne, Karel Heller, en André Hoving bedanken voor hun bijdrage aan het experimentele werk. Gerard, mijn dank voor je inzet en ondersteuning bij de hydraulic fracturing experimenten, en voor de vakkundige preparatie van de gesteenteblokken. Mijn andere steunpilaar bij de experimenten was Jan Etienne, met wiens hulp de triaxialproeven op zeer efficiënte wijze zijn gedaan. Ook wil ik Jan Webbink bedanken voor het nauwkeurige maken van de samples. Met veel plezier heb ik ook de proeven op de uniaxiale drukbank bij Wim Verwaal en Arno Mulder gedaan. Verder dank ik Rob Witting voor zijn inleiding tot de geheimen van de triaxiale drukbank. Het commentaar van Marc Hettema op dit proefschrift heb ik zeer gewaardeerd, evenals de open discussies over gesteentemechanica. Verder dank ik allen voor hun hulp en bijdrage aan de goede atmosfeer die ik ondervonden heb in de Petrophysica-sectie, het Dietz-lab en de rest van de faculteit.

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I would like to thank the members of the Technical Steering Committee of the project for the discussion and comments to the results. This research was supported by the Dutch Technology Foundation (S.T.W.).
Samenvatting

De titel van dit proefschrift is "De invloed van inelastisch gesteentegedrag op de geometrie van hydraulische scheuren". Hydraulisch scheuren van reservoirgesteente is een veelgebruikte techniek voor het vergroten van de opbrengst van olie- en gasputten. Het succes van de hydraulische-scheurtechniek hangt voor een groot deel af van de gecreëerde scheurgeometrie. Om deze goed te voorspellen, is een correcte beschrijving van het hydraulische scheurproces essentieel. Dit kan alleen gedaan worden, als alle deelprocessen goed gemodeleerd zijn. Tot op de dag van vandaag bestaan er grote meningsverschillen over gecreëerde scheurgeometriën, het modelleren van hydraulisch scheuren, en in het bijzonder over het modelleren van het scheurproces bij de tip.

Het doel van dit onderzoek is het begrijpen van de invloed van plastische gesteendeformatie, die plaatsvindt rondom de scheurtip, op de hydraulische-scheurgeometrie. Dit is één van de mechanismen die ontbreken in de simulatiemodellen die in de huidige praktijk gebruikt worden, en zou een mogelijke verklaring kunnen vormen voor het optreden van "hoge netto drukken". We hebben dit onderzocht door middel van geschaalde modelexperimenten. In deze experimenten varieerden we het gesteentemateriaal en de uitwendig aangebrachte spanningen, met als doel de grootte van de plastische zone rondom de tip te variëren. Verder ontwikkelden we een numeriek model, wat gebruikt is om de mechanismen te ontrafelen die de experimentele resultaten konden verklaren.

Onze kubusvormige gesteentemonsters hadden een grootte van 0,30 m. We plaatsten deze monsters in een polyaxiale drukbank, om de spanningen in de aarde na te bootsen. Tijdens scheurpropagatie hebben we de scheurwijdte en vloeistofdruk bij het boorgat gemeten. We hebben de materialen gekarakteriseerd in triaxiaaltesten, en andere gesteentemechanische proeven. De ruwheid van de scheuroppervlakken is gemeten met behulp van een laser profilometer.

Onze resultaten in lineair elastisch materiaal (cement) komen overeen met lineair elastische simulaties die vloeistofstroming, gesteendeformatie, en het scheurproces koppelden. Deze overeenkomst omvat de netto druk bij het boorgat, het wijdteprofiel langs de radiale coördinaat, en de grootte van de vloeistofvrije zone. In zacht gesteente (zacht gips) hebben we belangrijke afwijkingen van lineair elastische breukmechanica gemeten. Hydraulische scheuren werden wijder dan voorspeld door lineair elastische simulaties, terwijl de netto druk lager werd.

Elastisch-plastische simulaties laten zien dat de resultaten in zacht gips gerelateerd zijn aan de toegenomen wijdte in de bulk van de scheur, hetgeen de drukgradient vermindert.
De simulaties laten ook zien dat de vloeistof dichter bij de tip is gekomen, omdat plasticiteit de belasting door de vloeistofdruk afschermt. De extra vloeistofdruk die hiervoor nodig is ter plaatse van het boorgat, is kleiner dan de afname van de vloeistofdruk als gevolg van de toename in wijdte in de bulk van de scheur, onder onze experimentele omstandigheden. Dit mechanisme zou echter tot een hogere boorgatdruk kunnen leiden voor omstandigheden die verschillen van die in onze experimenten. Het verdwijnen van de vloeistof-vrije zone is bevestigd in de experimenten. Dit vormt een aanwijzing voor een overgang naar een "plasticiteitsgedomineerd" propagatie regime.

De netto druk en scheurwijde bij het boorgat in diatomiet en sterk gips komen ongeveer overeen met de volledig gekoppelde lineair elastische simulaties. Dit is in het bijzonder opmerkelijk voor diatomiet, omdat dit materiaal significante plastische schuifdeformatie laat zien is triaxiaaltesten. Van de drie zachte materialen die we gebruikten (sterk gips, zacht gips en diatomiet), laat alleen zacht gips grote plastische deformatie zien onder uniaxiale trekbelasting. Hieruit concluderen we dat dit gedrag de beste aanwijzing is voor het optreden van afwijkingen ten opzichte van lineair elastische simulaties van hydraulisch scheuren.

De ruwheid van de scheuroppervlakken reflecteert het scheurproces bij de tip. De ruwheid laat een duidelijke correlatie zien met de grootte van de plastische zone rondom de scheurtip. Dit laat zien dat het breukproces niet puur uit trekbreuk bestaat, maar ook uit schuifbreuk. Analyse van het spanningspad dat gevolgd wordt door een materiaalelementje bij de tip, ondersteunt deze hypothese.

Analyse van het sluiten van hydraulische scheuren is belangrijk voor het afleiden van de kleinste hoofdspanning in de aarde, en de weglek-coëfficiënt. De experimentele resultaten laten verschillende afwijkingen zien van aannames die normaalgesproken in modellen worden gedaan. Zowel groei als recessie van de straal van de scheur kwamen voor na het insluiten, evenals beïnvloeding van het sluiten door scheurruwheid en plasticiteit bij de tip.
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