Damage Tolerance Aspects of a Full Composite Airplane Fuselage

Requirements, Modelling, Predictions, Experiments

July 1993

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- LR report 728 -

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Summary.

With the introduction of composite materials new difficulties may arise with respect to the damage tolerance behaviour of the structure, especially when Carbon fibres are used which are brittle by nature. This problem occurred with Carbon/PEI laminates which were used in the facings of a honeycomb sandwich in a preliminary design of a full composite fuselage. Therefore, in order to use this material, suitable measures must be taken to overcome this.

Since cyclic loading generally is not critical to composite structures, only impact damage is considered. Impact damage can greatly reduce the strength of composite materials, even when no damage is visible. In order to model a worst case impact damage a crack was taken with the size similar to that of the impact damaged area.

Linear elastic fracture mechanics are applied to the distinct material in a fixed 0-90/±45/0-90 laminate lay-up. With the use of a Feddersen approach the plane stress fracture toughness is obtained for cracks in the direction of the principal material axes. The critical mode I stress intensity factor for the Carbon/PEI material found under these circumstances is 33.81 MPa/m.

With this fracture toughness the behaviour of a panel provided with damage stoppers can be calculated. For this calculation the computer program CRACKS is used. For tuning of the program a series of three similar test panels provided with crack stoppers are produced and tested. After proper tuning of the program the residual strength behaviour of a new panel with other measurements is predicted. From a test on this new panel it is concluded that the computer program works fairly well and can be used for giving tendencies in the performance of different stiffened panel configurations.

From the various tests on the panels follows that the application of damage stoppers results in a satisfactory damage tolerance behaviour.

Further research deals with a calculation method for geometry factors, which play an important role in the linear elastic fracture mechanics. Although application of this method is limited here to a case with three stress raisers (i.e. holes or cracks), also other geometries can be evaluated in a similar manner. From an evaluation with solutions found in literature, it can be concluded that the calculation method is very good. Additional tests on this subject confirm the applicability of the method for several geometries although in some cases deviations are found.

From observation of the fracture mechanism at the crack tip it can be concluded that the material shows no noteworthy delaminations during a tensile test on a cracked specimen. Furthermore, examining the crack surface of the applied fabric shows that practically always intralaminar fracture occurs.

Strain gauge measurements show that strains can be predicted fairly accurate. Application of a K-gage at the crack tip of several cracked specimens learns that this gauge can not be used for determination of the crack tip stress intensity factor since the radius of the gage is to large with respect to the considered crack lengths.
(Preface)
(Summary)

Contents:

List of Symbols and Abbreviations ............................................. ii
1 Introduction and Problem Statement .................................... 1
2 Subjects Related to Composite Fuselage Design .................... 4
   2.1 Different Design Concepts ............................................. 4
   2.2 Aviation Requirements ............................................... 6
   2.3 Loading Conditions .................................................. 18
   2.4 Impact Characteristics .............................................. 24
3 Linear Elastic Fracture Mechanics ........................................ 32
   3.1 General Description .................................................. 32
   3.2 Calculations of Some Geometry Factors ............................ 40
4 Residual Strength Tests on Unstiffened Panels ...................... 43
   4.1 Description of Tests .................................................. 43
   4.2 Test Results ........................................................... 46
5 Theory on Stiffened Panels .................................................. 56
   5.1 General Description .................................................. 56
   5.2 Calculations on a Stiffened Panel .................................. 61
6 Residual Strength Tests on Stiffened Panels .......................... 63
   6.1 Description of Tests .................................................. 63
   6.2 Test Results ........................................................... 66
Conclusions ................................................................. 71
Recommendations ............................................................. 74
References ................................................................. 75
Tables ................................................................. 79
Figures ................................................................. 87

Appendix A: Fuselage Skin Stresses During Flight.
Appendix B: Stress Calculation Formulae.
List of Symbols.

\( a_c \)  Critical (half) crack length
\( a_0 \)  Initial (half) crack length
\( B \)  Gauge factor for K-gage
\( b \)  Ellipse semi y-axis
\( C_R \)  Tip stress reduction factor
\( d \)  Ellipse semi x-axis
\( E \)  Elastic Modulus (Young's Modulus)
\( F \)  Work performed by external forces
\( G \)  Energy release rate
\( G \)  Shear Modulus
\( K_{Ic} \)  Critical stress intensity factor (fracture toughness)
\( K_{Ie} \)  Apparent stress intensity factor (relates initial crack length with critical stress level)
\( K_{II} \)  Stress intensity factor at initiation of crack growth
\( L_s \)  Stringer load concentration factor
\( M \)  Bending and twisting moments
\( N \)  In-plane forces
\( R \)  Radius of hole
\( R \)  Stress ratio of the applied cyclic load (\( R = \sigma_{min}/\sigma_{max} \))
\( S \)  Shear strength
\( S' \)  Negative shear strength (as a positive number)
\( t \)  Thickness
\( U \)  Energy
\( u,v,w \)  Displacements
\( W \)  Width of damage stopper
\( x,y,z \)  Coordinate directions
\( X,Y,Z \)  Tensile strengths
\( X',Y',Z' \)  Compressive strengths (as a positive number)
\( \gamma \)  Shear strain
\( \gamma_c \)  Elastic surface energy
\( \epsilon \)  Engineering strains
\( \theta \)  Angle
\( \nu \)  Poisson's ratio
\( \sigma \)  Normal stress
\( \sigma_c \)  Critical stress (stress at which unstable crack growth occurs)
\( \sigma_i \)  Stress at initiation of crack growth
\( \sigma_x, \sigma_y, \sigma_z \)  Stresses in directions of principal coordinate axes
\( \sigma_0 \)  Unnotched tensile strength
\( \sigma_1, \sigma_2, \sigma_3 \)  Stresses in directions of material symmetry (lamina of orthotropic material)
\( \tau \)  Shear stress
Indices.

c  critical
I  mode I crack loading (opening mode)
II  mode II crack loading (sliding mode)
III mode III crack loading (tearing mode)
u  ultimate
y  yield

Abbreviations.

BVID  Barely Visible Impact Damage
CCT  Centre Cracked Plate Tension Specimen
COD  Crack Opening Displacement
DOC  Direct Operating Costs
FAR  Federal Airworthiness Regulations
JAR  Joint Aviation Requirements
PEEK  Poly Ether Ether Ketone
PEI  Poly Ether Imide
SEM  Scanning Electron Microscope
TOD  Threshold of Detectability
1 Introduction.

In 1915 Hugo Junkers designed the first metal airplane. The fuselage of this airplane was built up from a thin skin stiffened by stringers, longerons and frames. Although this stiffened shell principle leads to relatively light structures, it has the disadvantage of a great number of parts. All these parts, stiffeners, longerons and frames, need to be assembled which leads to high production costs.

With the development of new materials, new constructions become possible. The application of these new materials in aircraft structures however demands new concepts in aircraft design through which the expected advantages (e.g. weight reduction, reduction in the number of parts) can be attained without reducing the safety level of the aircraft.

A new type of materials was developed during the last decades. These materials are built up as a combination of two or more materials, differing in form or composition on a macroscale. The constituents retain their identities; that is, they do not dissolve or merge completely into one another although they act in concert. Normally, the components can be physically identified and exhibit an interface between one another. The general name given to this kind of materials is composites.

Before such newly developed materials become interesting to introduce into new aircraft design and structure, a lot of research has to be done to become familiar with the material behaviour. The application of composite materials in aircraft structures started off with their introduction into secondary structures, where damage or failure does not have catastrophic results. With increasing knowledge of the material behaviour, application in primary structures (e.g. the vertical fin of the Airbus A320) became a fact.

A composite airplane design may reduce the weight of the structure. This weight reduction can be used for an increase of the payload. In this sense the application of composites improves the flight performance of an airplane. In Fig. 1.a (Johnson et al. (1985)) the increase in payload is given as a function of the weight reduction along the span of a large commercial airplane. Note especially that the one on one gain obtained at the fuselage structure offers great possibilities. Structural weight savings can also be translated into a reduction of the overall weight of an airplane which in turn means a decrease in fuel consumption. Although this effect can be rather impressive, due to the so-called 'snowball effect', it only yields a marginal decrease of the direct operating costs (DOC). Reduced fuel consumption itself is very important in relation to the current environmental concerns.

Fig. 1.b (Johnson et al. (1985)) indicates another benefit of the introduction of composites into a fuselage structure. A conventional fuselage contains one third of the total number of parts of an airplane. With the application of composites in the fuselage structure this number can be reduced by approximately 20 per cent (Johnson et al. (1985)). Such a reduction will directly lead to a reduction in production costs.
One of the greatest disadvantages of the application of composite materials is their poor residual strength. Especially in the case of a reinforcement of Carbon fibres, which are brittle by nature, presence of cracks may lead to catastrophic failures.

In the preliminary design process of a full composite fuselage of a major airliner, performed by Lagendijk (1991), such problems became apparent.

In this preliminary design the fuselage of an Airbus A320 was chosen since much data is available on this particular airplane. The chosen design was a sandwich structure built up from a Nomex honeycomb and Carbon fibre reinforced Polyetherimide (PEI) faces.

A simple test performed on a single face demonstrated the poor fracture toughness qualities of the proposed facing material.

The questions raised from this study are:

* Is it possible to predict fracture of the distinct material with the application of the K-concept? And if so, what is the influence of the size of the tested panels on the value of the stress intensity factor for the material?

* Is there a possibility within the given preliminary design to overcome the poor fracture toughness properties of the material used? A potential solution should meet the given aviation requirements valid for the given airplane.

In trying to find answers to these questions the present report starts with a general description of composite fuselage structures. In section 2.1 several design concepts for composite fuselages are mentioned. To get a better insight into problems of damage tolerance the appropriate rules for fuselage structures, drawn up by the Airworthiness Authorities, are discussed in section 2.2. Section 2.3 deals with the loads on a fuselage structure while the impact characteristics of composite materials are handled in section 2.4.

In section 3.1 a theory is presented which brings prediction of strength under the presence of cracks within reach. This prediction is based on the basic concepts of fracture mechanics and especially the relatively simple stress intensity factor concept is discussed. Section 3.2 discusses a method to calculate some geometry factors that play an important role in this concept. The theory of chapter 3 is supported with some experiments, the results of which are presented in chapter 4. With the results of these tests it is possible to make some estimations on the strength of stiffened panels.

In chapter 5 these stiffened panels are discussed. The theoretical background is given in section 5.1 while in section 5.2 some details are given on a calculation method suitable for calculation of the relevant properties of a stiffened panel.

In chapter 6 a stiffened panel which may improve the damage tolerance capabilities of the given composite sandwich fuselage structure is designed. A stiffened panel is produced and tested to check the predicted residual strength behaviour of the panel. Finally the report will be completed with the provision of conclusions from the research and some recommendations for future research.
From the earlier mentioned preliminary design study the following restrictions, which hold for the experimental part of this work, can be given in advance:

The material to be used in the experiments is Carbon Fibre Reinforced Polyetherimide CD282 in a 5 Harness weave (see Fig. 1.c), produced by Ten Cate composites. The material is provided as a one metre wide roll of prepreg with the warp direction of the prepreg lengthwise.

The laminate construction is taken as a 0-90/±45/0-90 laminate, similar to the one proposed in the preliminary design study.

Furthermore, fatigue damage will be neglected in the test series and only a single face is considered to cut down the amount of work in the present research. A situation of plane stress is considered to gain a better insight into the different associated problems. Research sequel to this work can be directed to the complete sandwich construction with curvature and loaded in multiple directions under repeated loads or otherwise.
2 Subjects Related to Composite Fuselage Design.


2.1 Different Design Concepts.

In the introduction of this report the major benefits of a composite fuselage design are mentioned. These benefits can be summarized as lighter structure, less parts and lower production costs. Despite the high initial material expenses for the composite materials, the lower production costs and the longer service life of the structure lead to a high cost efficiency. To meet with these advantages different design concepts can be adapted. From Johnson et al. (1985) the following five concepts are stated (Fig. 2.1.a):

1) Full-depth honeycomb core
2) Laminated skin with cocured I-stringers
3) Laminated skin with cocured foam-filled hat section stringers
4) Honeycomb core with cocured I-stringers
5) Honeycomb core with cocured foam-filled hat section stringers

1) Full-depth honeycomb core:

This concept is called full-depth honeycomb since a single honeycomb replaces the total conventional structure built up from skin and stringers. In this concept, which is actually a special form of sandwich construction, the faces are necessary to carry the in-plane tensile, compressive, and shear stresses, induced by bending, in-plane shear, and normal loads. The honeycomb core is required to take up the out of plane shear stresses. Furthermore, the core keeps the facings at a certain distance, thus achieving a large specific bending stiffness for the sandwich structure. This concept is by far the simplest and least labour intensive in the production cycle. The number of parts which have to be assembled is kept to a minimum and the production of the panels is suitable for excessive automation. Production tolerances must be kept very small with this concept due to the stiffness of the panels after curing. In the future the application of in-situ foamed core in sandwich construction may become important since with such a concept the production of the structure can be simplified even more. With the use of ultrasonics this type of construction can be thoroughly inspected.

2) Laminated skin with cocured I-stringers:

This concept resembles the traditional fuselage structure. The stringers carry the axial loadings on the fuselage and provide for sufficient buckling stability. A reduction of parts may be obtained by cocuring the stiffeners directly to the skin. Hard tooling can be used to define the stringer shape and therefore autoclave
pressures can be relatively high which stimulates good consolidation of the laminate. The inspectability of a construction of this type is very good.

3) Laminated skin with cocured foam-filled hat section stringers:

This concept is in principle equal to the former concept. The foam in the hat stringers provides extra lateral stability to the stringer webs and flanges. The foam can also be used to define the shape of the stringer in the curing process which directly reduces the amount of required tooling. Unfortunately, using the foam to define the stringer shape limits the pressure which can be applied in the curing cycle. The major concern with regard to these foam-filled stringers is that in-service inspection of possible cracks and delaminations can not be carried out easily with current inspection techniques because of the locked in foam core.

4) Honeycomb core with cocured I-stringers:

In this concept as well as in the following one the honeycomb core is not as thick as in the case of the full-depth honeycomb. Therefore stringers are required for the same reasons as in the second and third concept. However, these stringers can be lighter than in the previous cases. This concept obviously tries to combine the advantages of the first and second configurations. The disadvantage of this concept as well as the next one is the high costs in comparison with the first three concepts. Furthermore autoclave pressures are limited due to the honeycomb core. This increases the potential for laminate porosity, a dangerous phenomenon in relation to sandwiches.

5) Honeycomb core with cocured foam-filled hat section stringers:

This concept is in principle equal to the former concept except for the higher lateral stability of the stringers. Here also the inspectability of the structure is very poor.

In Johnson et al. (1985) it is stated that in all five mentioned concepts frames will be necessary to provide lateral stability of the fuselage shell structure. Furthermore, these frames contribute to the damage tolerance of the complete structure. In the preliminary design of a sandwich fuselage by Lagendijk (1991) however, it was found that no such frames are necessary, at least not for the provision of lateral stability. In the case of the Beech Starship also no visible frames are applied although the frames may be cocured here within the sandwich skin.

In this work only the sandwich fuselage will be discussed. If the choice of material is already fixed the only design parameters of such a fuselage are the core thickness and the face thickness. The values of these parameters will be different for the different sections of the fuselage (crown, side panels and keel; see Fig. 2.1.b) and will also vary within these sections along the longitudinal axis of the fuselage.
2.2 Aviation Requirements.

Each airplane has to be certified by the Aviation Authorities before it is allowed to fly. The rules according to which the different Authorities judge an airplane can be found in the Joint Aviation Requirements (JAR) for Europe and in the Federal Airworthiness Regulations (FAR) for the United States. Furthermore, each specific nation can have its own additional rules which hold for that nation only. Normally, once a certification according to either JAR or FAR is obtained from one of the participating Authorities, permission to fly the airplane in most other countries can be expected since the various rules are nearly the same in all associated countries and there are only some slight differences between the JAR and the FAR.

Both the JAR and the FAR are divided into several parts. The probably best-known and most used parts for an airplane designer are part 23, which holds for small airplanes, and part 25, which holds for large airplanes. Furthermore there is a part which contains all definitions used by FAR/JAR, also there are several parts which concentrate on aviation legislation, etcetera. This report will only deal with part 25, valid for large airplanes, since it treats the type of commercial aircraft which are of great importance for the airplane industry and since this part is the most strict part to comply with for all airplane structures.

Once the requirements for certification according to part 25 are met, the requirements according to part 23 are satisfied automatically, and what's more, in some special cases additional requirements from part 25 are needed to acquire a licence corresponding to part 23. One of those special cases was the certification of the Starship, an all-composite business aircraft of the Beech Aircraft Corporation, which had to meet the damage tolerance requirements from part 25 since the composite honeycomb structure was completely unknown to the Aviation Authorities and the strength of it was uncertain up to then.

Part 25 of the JAR is divided into several Subparts, each containing several paragraphs related to a certain subject. Besides these paragraphs the JAR also contain Acceptable Means of Compliance and Interpretative Material in which possible methods to comply with a specific paragraph or interpretations of a paragraph are given.

For a composite fuselage not all paragraphs are of interest. Therefore, only some particular paragraphs of the Subparts C and D which deal with Structure, and Design and Construction, respectively, will be discussed in more detail here.

**JAR 25.301: Loads.**

The definitions of the loads which have to be used in strength calculations follow from this paragraph of the JAR. These loads are either limit loads (the maximum loads to be expected in service) or ultimate loads (limit loads multiplied by prescribed factors of safety). Unless otherwise specified, prescribed loads are limit loads.

Ultimate load is very unlikely to occur during the operational life of the aircraft but when ultimate load appears complete fracture of the structure can well be expected.
JAR 25.303: Factor of safety.

From JAR 25.301 it can be concluded that the way to obtain ultimate loads for aircraft primary structures (i.e. structures whose failure would cause a complete loss of the aircraft) is to establish the limit load and multiply this limit load with a prescribed factor that is called the factor of safety. Such a factor of safety is used to deal with uncertainties in 1) material properties, workmanship, quality control and maintenance, 2) load and service control, and 3) accuracy in assessing strength.

For aircraft primary structures normally the value 1.5 is used as the factor of safety. This value originates from a composition of two factors, which can be formulated as: SF = n1n2. In this simple equation the separate values of n1 and n2 are given by:

\[ n1 = 1.2 \]
\[ n2 = 1.25 \]

where n1 deals with scatter in material properties and size, inaccuracy in calculations and built in stresses (quality), and

n2 deals with the occurrence of very high loads with a very small probability.

It seems strange that the scatter in material properties is included in n1, because in the definition of A- and B-allowables the scatter in material properties is also included (see JAR 25.615). It would be advisable to lower the value of n1 and if necessary increase the required certainty levels in the definition of A- and B-allowables.

JAR 25.305: Strength and deformation.

In this paragraph the deformation which is allowed in each of the earlier defined loading cases is established. It also contains an extra safety requirement to account for the rare situation of the occurrence of ultimate load. This load has to be sustained for at least 3 seconds before final failure, unless the strength is shown by a dynamic test.

JAR 25.307: Proof of structure.

This paragraph prescribes the methods by which it can be shown that the structure can sustain the given (flight) loading conditions.


This paragraph gives a general description of the flight loads. The flight load factor is defined as the ratio of the aerodynamic force component to the weight of the airplane.

JAR 25.365: Pressure cabin loads.

In this paragraph some regulations for a pressurised cabin are given. The main concern of this paragraph is that the pressurized cabin has to be strong enough to withstand flight loads combined with pressure differential loads, which seems quite obvious. A factor of 1.33 is to be applied on the maximum load which can occur due to pressurization of the fuselage cylinder. This factor is necessary to create extra safety for the pressurized cabin since this maximum load occurs once per flight. Additional statements refer to the probability of a sudden pressure release due to openings in the pressure cabin which can occur during flight. Partitions, bulkheads, and floors must be able to withstand the effects of such a sudden pressure release.
JAR 25.571: Damage-tolerance and fatigue evaluation of structures.

This paragraph of the JAR is the most important one in relation to the present work. The main statement of this paragraph is that catastrophic failure due to fatigue, corrosion, or accidental damage must be avoided throughout the operational life of the airplane. To fulfill this objective different failure evaluations are discussed together with their respective requirements. In the ACJ belonging to this JAR 25.571 some suggestions which can be used to comply with the damage tolerance requirements are given. In subparagraph (b) of JAR 25.571 it is explained how to obtain damage tolerance by the use of fail-safety. Locations and modes of damage have to be established, growth of damage and corresponding change in residual strength have to be evaluated and inspection intervals have to be deduced from this analysis. Of course these considerations are all based on experience with stiffened shell structure made of metal. This means that fatigue cracking is seen as the main form of damage and these cracks are supposed to initiate and grow in a multiple load path (fail-safe) structure.

Later on it will be shown that such an approach is not very useful in judging the damage tolerance behaviour of composite sandwich structures but it was felt necessary to discuss the approach for metal structures first to get a better insight in the requirements as stated in this paragraph of the JAR. More details on the following discussion on fail-safe structures can be found in Broek et al. (1974).

As a result of uncertainties in design loads and stress analysis and the possible occurrence of minor damage to the structure it has to be expected that cracks or partial failures will occur long before the aircraft life is expended. Safety then requires a structural design, that can withstand an appreciable load under the presence of cracks or failed parts. It also requires that the damage can be detected before it has extended to a dangerous size. The structure that meets these requirements is considered fail-safe. For metal structures this fail-safety can be achieved in various ways. The two most applied methods for fail-safe design are known as provision of a multiple load path in the structure and the damage tolerance method.

The method of a multiple load path can be described as follows: (see fig 2.2.a)

The structure consists of different parallel elements, each carrying a part of the load. When one of these elements has failed, other elements can take over its task by accepting a higher load. This method is often the most preferable one. A disadvantage of this method is that all other elements of the group have experienced more or less the same load history and therefore more of them may be close to failure. Moreover, their load is increased due to failure of the first element. This implies that more elements may soon fail eventually resulting in an unacceptable safety level. The time between failure of the first element and failure of the element which results in an unsafe structure must be sufficient to detect failure. Only if the failure of the first element was a premature failure induced by minor damage (i.e. intrinsic or external damage), the inspection period can be fairly long. However, if the first failure was not caused by special circumstances the period may fully depend upon the scatter in fatigue lives.
The method of damage tolerance can be described as follows: (see fig 2.2.b)

The structure is built up from materials with a high fracture toughness (see section 3.1 of this report), thus leading to a low crack growth rate\(^1\) and high residual strength, and a design with inherent crack stopping capabilities is adopted. After some period a crack may be initiated but this crack is still so small that it will not be detected by any of the existing inspection techniques. During further operation the crack will grow to a detectable size. While the crack further increases in length the remaining strength of the structure will gradually deteriorate until it will drop below the acceptable fail-safe strength. This acceptable strength level may be below ultimate load if this situation occurs only for a short period of time but it may never reach limit load (see Fig. 2.2.b). For safe operation of the aircraft it is necessary that in the available period of time to detect the crack at least two or three inspections take place, since a crack of the minimum detectable length may just escape attention during an inspection. Furthermore it is important that relatively small cracks are detected early even if the maximum tolerable crack length is relatively large. The reason for this is that the crack will stay relatively small during the greater part of time during crack growth.

Crack detection is of vital importance for the two previous described cases of fail-safety. Therefore various methods of crack detection have been developed. A structure cannot be made fail-safe with the described methods if the critical crack is so small that it will not be detected. However, the ability to detect small cracks is still increasing with the development of new inspection techniques. With this ability inspection periods may become longer and thus economically more favourable.

If the fail-safe philosophy is applied to composite materials various problems will follow. The first problem is formed by the definition of terms used to describe the fail-safe capabilities according to JAR 25.571 (b). A sandwich fuselage without stringers and frames can be regarded either as a multiple or as a single load path structure. If it is regarded as a multiple load path structure which is also fail-safe, the damage tolerance requirements are fulfilled by definition. If the approach of a single load path structure is used, a totally different strategy to the damage tolerance evaluation will be necessary. Therefore it seems appropriate to abandon the term fail-safety for integrated composite structures and henceforth only speak of damage tolerance, since fail-safe considerations require load path definitions besides load value considerations. Especially for composite parts with cocured stiffening or strengthening parts or layers these load paths are hard to define, thus making the residual strength analysis of damaged parts difficult. This remark should of course not be interpreted as a negative side of composite structures. It only shows that the damage tolerance behaviour of composite structures is not closely linked to fail-safe design, which is an intrinsic property of stiffened shell structures

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\(^1\)A structure can also be made fail safe if cracks propagate fast as long as the inspection interval is made short enough. However, these short inspection periods are not very economical.
because of their large number of discrete elements. For composite materials other approaches can be applied as will be shown later on (see discussion on ACJ 25.603).

Another problem is that damage in composite materials is not likely to grow under repeated loads or at least not in the same amount as in metal structures. If non-critical, visible damage does not propagate under repeated loading, the no-growth concept is usually adopted. In this concept it is assumed that after crack stabilization (i.e. setting of final damage size after a restricted number of load cycles) no further crack growth as a result of cyclic loads will occur. If this concept is considered acceptable, visibly undetectable damage may never be critical. It is also clear that, in case of no-growth damage, damage in the structure will either be catastrophic or not catastrophic and that in the latter case damage is not prone to become catastrophic. In the no-growth philosophy impact damage is a major source of damage. Subparagraph (e) of JAR 25.571 refers to this problem. Because of the importance of impact damage a further discussion on this subject will be given in section 2.4 of this report. If it is found impossible to establish a no-growth concept for the material under study, other approaches have to be used to obtain a certification of the airplane. A more detailed discussion on this topic can be found in ACJ 25.603 where composite materials are addressed in further detail. In subparagraph (c) of JAR 25.571 it is explained that compliance with paragraph 25.571 for a structure can also be obtained if it is shown by analysis, supported by test evidence, that the structure in question can withstand all loads during the aircraft life without showing any detectable cracks. This approach is also known under the name safe-life. Since safe-life deals with fatigue damage of undamaged structures it essentially differs from the earlier mentioned no-growth concept which concerns damage growth under fatigue loading within damaged structures. Therefore validation of a safe-life normally demands another approach. Rouchon (1992) states that for safe-life demonstration normally more flights are required in the testing program. Subparagraph (d) of JAR 25.571 deals with sonic fatigue and therefore will be neglected in this present work. In subparagraph (e) of JAR 25.571 it is explained how to obtain damage tolerance in the case of discrete source damage (see fig 2.2.c). In this special case the damage is so obvious that the rest of the flight in which it occurred will be short and the pilot will be carefully manoeuvring and terminate the flight at the nearest airport. It is unlikely that a high load will occur in this period and therefore the acceptable residual strength level may be lower than in other cases. Since the damage is obvious it will be repaired prior to the next flight.

JAR 25.581: Lightning protection.

For a composite fuselage the electrical current resulting from a lightning strike must be diverted so as not to endanger the airplane. A successful means to obtain this objective is the use of thin metal threads which can be woven in between the fibres in the outer skin of the fuselage. A similar approach is also used in the Beech Starship (Abbot (1989)), where a hybrid woven graphite/aluminium fabric with 0.1 mm aluminium wires is used in the surface ply in all exterior surfaces. Damage from a 200,000 ampère strike is said to be limited to the outer ply and repair is relatively simple.
In this paragraph it is stated that the airplane may not have design features that
can be hazardous or unreliable.

JAR 25.603: Materials.
This paragraph states that material data should be established on the basis of experience
or tests, conform to approved specification, and considering the effects of environmental
conditions.
In ACJ 25.603 an acceptable means of showing compliance with the provisions of JAR 25
regarding airworthiness type certification for composite structures is given. This ACJ is
equal to Advisory Circular 20-107A issued by the Federal Aviation Administration which
also deals with composite aircraft structures. Since this present work deals with a full
composite fuselage structure it may be obvious that this ACJ is of great importance. It
seems strange that this ACJ is coupled with paragraph 603 of the JAR since this paragraph
only deals with materials, while the ACJ concerns complete composite aircraft structures.
ACJ 25.603 starts off with a list of definitions which is also incorporated in the glossary of
terms that can be found at the beginning of this report. In the ACJ itself it is already
stated that this ACJ is expected to be modified periodically according to advances in
composite technology.
To provide an adequate design data base suitable for composite materials, extensive
testing under various conditions will be necessary. Especially repeated load data for
composite materials, which shows an extensive amount of scatter, yields serious problems
in certification procedures. Laméris (1990) mentions four different approaches which can
be used to obtain reliable structural data on a statistical basis (see also JAR 25.613 &
25.615) under repeated loading conditions. These approaches are:
1) Life scatter approach
2) Load enhancement factor approach
3) Ultimate strength approach
4) Change in spectrum approach

1) Life scatter approach.
The basic principle of this method is to apply enough repeated load damage to
a single test specimen to make it representative for the condition of the weakest
member of the population after a specified service life. The life scatter factor is
defined as the ratio of the average repeated load life and the B-basis repeated
load life. Because of the large scatter in load life, obtaining a B-basis reliability
at one lifetime requires a proved load life of at least 13.3 lifetimes (Laméris
(1990)) in a single article. Thus to guarantee one lifetime demands a test duration
of 13.3 times the desired lifetime. This means that the time in which reliable
repeated load data can be obtained becomes very long and this is not very
economical.

2) Load enhancement factor approach.
The basic principle of this method is to increase the applied loads in the repeated load test to obtain a certain reliability level with a shorter test duration. An additional requirement related to this approach is that the mode of failure of the test article may not change significantly. The load enhancement factor gives the value by which the applied loads must be multiplied to obtain the predefined certainty level by one lifetime of testing. This method of testing can be combined with the life scatter approach, yielding a combination of test conditions as given in the table. Rouchon (1992) reports that a load enhancement factor of 1.17 (associated with one lifetime on a B-basis probability to cover material scatter) or a combination of a load enhancement factor with a life scatter factor (e.g. 1.15 associated with 1.5 lifetimes) is demanded by the airworthiness Authorities. Although it seems strange that these factors are identical for any design made out of composite materials, a study carried out in the United States on composite fatigue variability proved the applicability of these rather severe factors. If, however, experiments on representative materials and design prove that a lower factor can be justified the Aviation Authorities can be convinced that in that particular case lower factors may be applied. An example of this is the certification of the ATR 72 composite outer wing where a load enhancement factor of 1.10 associated with one lifetime was used. Unfortunately, use of the load enhancement factor is limited since the resulting maximum peak stress may never exceed ultimate load. Especially if the load enhancement factor is to be combined with other load increasing factors (e.g. an environmental knock down factor) the applicability of it may become impossible. Also when hybrid materials are used, a raise of the stress level may introduce severe problems for one of the consisting components. For ARALL® or GLARE® similar problems occur. If the stress level in the aluminium constituents of these materials becomes to high, plastic deformation will occur, changing the failure mechanism drastically. This will probably not be accepted by the aviation Authorities.

3) Ultimate strength approach.

The basic principle of this method is to design the structure in such a way that design ultimate load is always below the repeated load threshold (i.e. fatigue loading stress level below which no fatigue damage occurs) to demonstrate adequate repeated load life. This approach is very conservative but when it is satisfied, no full scale repeated load test is necessary. The applied strategy is justified by the fact that the path of the material strength as a function of the number of load cycles for composite materials is relatively flat as compared to
aluminium. Therefore the load threshold found at the end of the total lifetime of the airplane lies at a relatively high portion of the initial static strength. Consequently, only a small increase in structural weight will reassure that repeated load damage will not occur. The present approach, as distinct from the earlier mentioned no-growth concept (see discussion on JAR 25.571), can only be adopted for sound structure. If a damaged structure is considered, repeated load data will still be necessary to obtain the necessary damage tolerance certification.

4) Change in spectrum approach.

The basic principle of this method is to change the load spectrum to be used in repeated load testing on aircraft structures in order to make it more relevant for composite materials. In the past it was tried to come up with one load spectrum which would give satisfactory results for both metallic as well as composite structures. In theory this can be done by changing the spectrum in such a manner that B-basis stress-life plots overlap for both material systems. However, because of the different paths of the curves showing the residual strength as a function of load cycles (i.e. composite materials have a significantly flatter slope of the curve as compared to metallic materials) no such generally applicable spectrum can be found. Since the availability of reliable experience on composite materials under repeated load is limited up to now, this concept of changing the load spectrum has not yet been fully developed.

In present design practice normally a combination of the life scatter approach and the load enhancement factor approach is used. The ultimate strength approach is frequently used in military aircraft. Future developments may be directed more to a change in the load spectrum related with a growing knowledge of the behaviour of composite materials under cyclic loading. More details on cyclic loading of composite materials can be found in section 2.3 which deals with the loads on an aircraft fuselage.

As stated before, also environmental effects on the design properties of the material system should be established. Environmental design criteria should be developed that identify the most critical environmental exposures to which the material in the application under evaluation may be exposed. In the case of the earlier mentioned Beech Starship extensive research on the environmental effects on different material properties was carried out (Abbott (1989), Wong et al. (1990)).

To acquire reliable standards on the environmental conditions to which the airplane could be exposed a USAF sponsored survey of 158 bases worldwide was carried out. The research identified the Andersen Air Base at Guam as the worst-case base for moisture exposure with March as the most humid month (average humidity 85.3 %). In the test chamber a relative humidity of 87 % was used to generate maximum moisture content for static testing and a slightly lower percentage was used in fatigue and flaw growth testing based on a mission/moisture analysis. Death Valley, California was pointed out as the worst-case base for temperature exposure. The highest ever recorded temperature was 57 °C which, when combined with solar radiation, can lead to a temperature of 65 °C
on the upper wing surface. Fortunately this temperature needs not be considered in combination with flight loads since the airplane systems are not qualified for take-off if the ambient temperature exceeds 50 °C.

It can be assumed that the aircraft becomes conditioned while stationed on the runway and cools during take-off and flight. With this in mind it can be noted that it is not necessarily the most critical loading of the aircraft that has to be combined with the worst environmental conditions. For each part of the structure the right combination of load and environment has to be established. If for instance the fuselage is considered, the maximum internal pressure occurs relatively long after take-off and only at high altitudes, therefore this loading condition will never occur in a hot/wet material condition.

In the research of the composite materials to be used for the Beech Starship it was found that moisture content only had negligible effect on flaw growth and residual strength under spectrum loading. Nevertheless, a precautionary factor was included to account for possible environmental effects.

Another important issue stated in ACJ 25.603 concerns impact damage. Impact damage is generally accommodated by limiting the design strain level. In section 2.4 of this report this matter will be further discussed.

The principles used to assure proof of structure (both static and dynamic) made of composite materials are not essentially different from the methods used for conventional materials. The main difference between both classes of material lies in the behaviour of composite materials under repeated loads but it can be covered with different approaches as discussed before. A damage tolerance analysis can, just as with metallic structures, be divided into three areas, covering fatigue, corrosion, and accidental damage respectively. However, for composite materials the relative importance of each of these areas is changed compared to these metallic materials. Also some new approaches may be used to comply with the damage tolerance requirements of the structure. One of those approaches is the validation of the no-growth concept to composite materials, which can be established by extensive repeated load testing or appropriate in-service experience.

Rouchon (1992) states that, as long as no analytical tools for the prediction of damage growth for composite structures are available, only three possibilities remain to demonstrate adequate airworthiness regarding fatigue. These possibilities are 1) safe-life approach, 2) crack arrest or no-growth concept, and 3) fail-safe approach by redundancy. Since these methods are mentioned before no further discussion will be given here, although it is felt that, under the present state of knowledge on the behaviour of composites, particularly the second method is very suitable.

Further regulations from ACJ 25.603, valid for composite materials, concern matters as quality control, production specifications, inspection and maintenance, and substantiation of repair.

**JAR 25.605: Fabrication methods.**

This paragraph states that production methods should be such that it is unlikely that a major defect will remain in any primary structural component after manufacture.
Furthermore, each new fabrication method must be substantiated by a test programme. With the use of composite materials new fabrication methods will be introduced. Particularly in the case of thermoplastics, which are very rare in the aircraft industry up to this very moment, much work has to be done to comply with this paragraph of the JAR.

**JAR 25.609: Protection of structure.**

The protection of structure meant by this paragraph of the JAR refers to weathering, corrosion, and abrasion. One of the major advantages of the use of composite materials is that these affections are significantly restricted. However, if the composite material makes contact with a metallic part, the possibility of galvanic corrosion should well be considered. Especially with the use of carbon fibre reinforcements in contact with metallic parts, extensive affection of the metal surface can take place.

**JAR 25.613: Material strength properties and design values.**

Material strength properties must be based on a sufficient number of tests to establish design values on a statistical basis. To be able to apply the generally accepted Weibull distribution on the strength properties at least 200 test results are required. Such a distribution is fairly accurate and therefore very useful to obtain the necessary material data. Nevertheless, this also means that each new material has to be tested extensively before it can be applied to an aircraft construction. Especially when composite materials are used, where each new laminate construction will yield other strength values, an introduction into the airplane demands an extensive test program to allow for certification. With the use of ARALL® or GLARE® similar problems occurred. Each new grade of one of these materials had to be certified before application was allowed in any aircraft structure.

An additional problem related to the use of composites is that the fibre directions may contain several uncertainties, strongly depending on the applied production method. For winding structures fibre directions are known within reasonable bounds. If, however, rubber forming is used prediction of fibre orientation creates many difficulties, especially for parts with double curvature. In such cases strength values may differ from one point to another and it is hard to define one single value for the material strength. To overcome this problem probably extensive models have to be created which are able to predict fibre directions from which strength values can be defined. With the use of fabrics the fibre orientations are limited by the mutual connection between fibre bundles and therefore a model which is able to calculate the shear angles of the fabric may yield reasonable results. With the use of unidirectional layers however, forming of the final product may lead to large deviations from intended fibre orientations which can lead to considerable variations in strength and stiffness. In the description of JAR 25.619 further discussion on this subject will follow.

In JAR 25.613 it is further reported that temperature effects must be considered when significant for allowable stress values. The probability of disastrous fatigue failure must be minimised. Material specifications must be according to certain standards set by the Authorities or persons which are allowed to act in their name.
JAR 25.615: Design properties.

In this paragraph the design properties that have to be used in the airplane design are defined. Normally the guaranteed minimum mechanical properties should be applied although sometimes a less stringent standard is required. A-basis values must be used for structural members, the failure of which would result in the loss of the structural integrity of a component. B-basis values may be used for redundant structures, in which failure of the individual elements only results in a redistribution of loads over the other elements. Deviation from these rules is only allowed if a premium selection of the material is made and, moreover, strength properties used in the design process are shown to be below their actual values.

JAR 25.619: Special factors.

In addition to the earlier mentioned factor of safety of 1.5 (see JAR 25.303) a supplementary factor has to be used in some special cases. This extra factor is therefore called a special factor in the JAR and it has to be applied to each part of the structure the strength of which is -

(a) uncertain;
(b) likely to deteriorate in service before normal replacement; or
(c) subject to appreciable variability because of uncertainties in manufacturing processes or inspection methods.

In paragraphs 25.621 through 25.625 of the JAR different situations are presented for which a special factor has to be applied. The highest pertinent special factor of safety found from those paragraphs has to be chosen. This special factor has to be multiplied with the ordinary factor of safety before calculating the ultimate load of the structure.

(a) Uncertain strength.

Strength can only be predicted with reasonable accuracy if the material and structural behaviour of the finished part can be described within reasonable bounds. This will not always be the case with composites. If, however, extensive research, combined with service experience, is used to establish the material strength on a statistical basis as prescribed in JAR 25.613, the uncertainty in material strength will be virtually overcome. Normally a special factor to cover uncertain strength will only be used where extensive testing is not very economical (e.g. in a special (sub)component which is very expensive and only limited in number). Another problem concerning the strength of composite materials is the great variability in strength in different material directions. To make a reasonably accurate strength prediction fibre directions must be known. These fibre directions however can differ from one point to another due to the material processing. More remarks on this subject will be given later.

(b) Strength likely to deteriorate.

Deterioration of the material means a loss of structural properties, but unlike damage, this loss can not be detected by nondestructive testing. Such a
deterioration can be caused by environmental effects or fatigue loading on the structure during service. Normal practice is to cover this effect with some special factor. A well known knock down factor in the line of this paragraph is the so called environmental factor (see discussion of ACJ 25.603), in which all environmental effects are included. A separate factor to describe degradation of the composite material due to ultraviolet radiation may be considered if other environmental effects can be neglected.

(c) Strength subject to appreciable variability.

This subparagraph of JAR 25.619 can be related to the variation in material properties from one part to the other but it can also be utilized to describe the variation of material properties within one distinct part. If the first approach is used the discussion is basically similar to the one stated at subparagraph (a). If the second approach is used several problems arise with the application of composite materials. Further discussion of this subject will be focused on the variation of strength within one particular (composite) material.

As mentioned before, the fibre orientations in the product will contain some uncertainties when a structural part is made out of fabrics by rubber pressing. A composite material can be defined as a mixture of fibres and resin. This definition implies that the material properties vary on a very small scale from point to point within the material. The concept of establishing A- or B-basis design values for a component may then become impracticable, since this concept can only be applied directly to the resin and the fibres separately. A model which describes the relation between fibre orientation, constituent properties and composite mechanical properties is required to calculate local material properties. This model used for the prediction of material properties should comply with the concept of A- or B-basis values. Uncertainties in the prediction of fibre orientation could be coped with by introduction of a special factor in the same way as done for castings, bearings, or fittings (see JAR 25.621 through 25.625).

Tape laying or hand lay-up will give better predictable fibre placement and therefore can do with smaller special factors.

If short fibre reinforced plastics are applied, the orientation of the fibres will also be uncertain. Prediction models are even more complicated for this production process, probably even impossible. Therefore a special factor, an injection moulding factor, will have to be used with a larger value than the factor proposed for fabric processing.

In addition to the uncertainties in fibre direction other uncertainties are possible. If, for instance, thermoplastic composites, processed with thermoforming, are considered, the degree of consolidation can vary within the product. Furthermore, after processing thermoplastic matrix materials some parts in the material will be crystalline while other parts will be amorphous. This will lead to a variation in mechanical properties. This variation can also be covered with a special factor.

JAR 25.621: Casting factors.
In this paragraph a special factor as described in JAR 25.619 is defined for castings. Normally a casting factor of at least 1.25 is to be used which leads to a total factor of safety of 1.875, but also casting factors exceeding 2.0 are possible, depending on the function of the casting. Only if approved quality control supported with extensive testing on coupon level is used to guarantee the mechanical properties of the casted material, a smaller value can be applied. Of course the casting factor is never less than 1.0.

JAR 25.623: Bearing factors.

In this paragraph a special factor as described in JAR 25.619 is defined to provide for the bearing strength of each part that is able to move relatively to its connecting part(s). No specific values are given in the requirements so the only certainty here is the lower boundary for the bearing factor of 1.0. The value of this factor depends on the amount of relative motion and the loads and vibrations expected for the specific joint. For hinges on control surfaces, which combine large loadings with frequent motion, the bearing factor can get as high as 6.67 to provide for adequate safety.

JAR 25.625: Fitting factors.

In this paragraph a special factor as described in JAR 25.619 is defined for fittings. The lower boundary for this value is set on 1.15, unless testing and experience allow for a lower value. The minimum value is also 1.0.

From the present section of this report it can be concluded that the design margins for an airplane structure are small within the different regulations. Each new construction type has to be excessively tested and it must be proven that the chances of fatal accidents are extremely remote. Furthermore, where the Authority is not satisfied with the approaches used by the manufacturer to obtain the necessary certificates, other appropriate measures must be taken.

The introduction of composite materials into airplane structures gives rise to new problems. At present the main difficulties related to the airworthiness requirements are formed by the uncertainty of strength properties, not only varying from material to material but also strongly varying within each material, and the very limited knowledge on the behaviour under fatigue loadings. As a result, future research might be concentrating on the behaviour of complete structures rather than on single material properties. Only if suitable models, able to predict material properties in every point of the material and fatigue behaviour of the structure, are realised certification of composite materials may become an easier task.

2.3 Loading Conditions.

During its operational life the loading spectrum of a pressurized fuselage is built up by three repeating schemes. The first part of the loading spectrum consists of flight loads (gust and manoeuvre), the second part consists of ground loads (taxing, landing impact, turning, engine runup, braking and towing), and the third part is the pressurisation load. The mechanical equivalents of these loading conditions are three normal forces in the
directions of the airplane major axes and three bending moments around these axes, as pictured in Fig. 2.3.a.

Combination of these normal forces and bending moments result in stresses in the structure. It is normal practice to distinguish three different stress components, each having its own particular effect. These three stress components are called 1) axial stress or longitudinal stress, 2) hoop stress or circumferential stress, and 3) shear stress. Such a division is justified by the possibility to study each component separately and afterwards look at their mutual influence. Later on it will become clear that this division is very useful.

The axial stress component during flight originates from a combination of several loads acting on the fuselage. The differential pressure yields a contribution which is primarily determined by the fuselage radius, skin thickness, and the magnitude of the differential pressure. It is of dynamic nature with a frequency of 1 cycle/flight. The R-value of the load spectrum, which is defined as \( \sigma_{\text{min}}/\sigma_{\text{max}} \) equals zero. Another contribution originates from the distributed weight of the airframe plus payload and the vertical accelerations due to gust and manoeuvring. The spectrum of the dynamic component of this load is rather complex, consisting of a sequence of loads with different magnitudes and frequencies. Due to the different loading conditions on different parts of the fuselage the axial stress component will strongly depend upon the position on the fuselage. Especially the variation in the bending moment around the y-axis of the fuselage is of importance for the axial stress component. To get an impression of this variation in bending moment around the y-axis of the airplane the ultimate envelope of this bending moment for an Airbus A320 in flight is given in Fig. 2.3.b.

The hoop stress is much easier to describe. For the main part it originates from the differential pressure with a small contribution of the other loadings. The hoop stress is practically equal for all zones of the fuselage. The magnitude of this stress component is only influenced by the fuselage radius and skin thickness, and the differential pressure. The frequency of the load is 1 cycle/flight, for this component \( R=0 \) also holds.

The shear stress originates from the shear loads and torsional moments which act on the fuselage. These shear loads are initiated by the inertia loads, resulting from airplane manoeuvres and gust, in combination with tail-loads and aerodynamic loads. The latter usually can be neglected. The inertia forces which act on the fuselage have the largest significance in the direction of the z-axis and a smaller effect in the direction of the y-axis. The tail-loads can generate a rather large torsional moment around the airplane length axis but the induced stresses are normally negligible. The shear stress will be largest in the side panels. Both at the top side of the fuselage and at the bottom, the shear stress is almost zero.

Each of the different stress components will have its own particular influence on the structure. In the case of a composite structure, the mutual influence of the components may cause severe problems. In an aluminium fuselage the critical situation occurs due to fatigue cracks in axial direction. This makes the residual strength in hoop direction critical. In composite materials, damage different from fatigue cracking will occur giving
different damage growth mechanisms and therefore probably leading to different critical situations. A situation in which the damage growth can be driven by axial loading while residual strength becomes critical in hoop direction is imaginable. A similar situation was reported by Bristow (1985) for a wing box where flap loads produced local delaminations which not reduced the tensile strength in the loading direction but significantly reduced the compressive strength in the direction of the wing bending loads.

In appendix A the fuselage skin stresses during one flight cycle are presented. The given stress distributions follow from calculations performed by Deutsche Airbus on the fuselage of the A330-300. The three stress components (longitudinal skin stress, circumferential skin stress, and shear skin stress) are given in four points along the circumference for two different positions along the fuselage. From this information a good impression of the evolution of the stresses during flight in the different parts of the fuselage is obtained.

The discussion on the stress distribution provided in appendix A yields some interesting aspects. First it is to be noticed that the circumferential stress during flight is the same for each part of the fuselage. As stated before, this stress component primarily depends on the pressurization of the fuselage. Further it can be observed that the ratio between the circumferential stress and the longitudinal stress, which is 2:1 for a pressurized cylinder, can shift from roughly 1:1 in the crown of the fuselage to roughly 1:0 in the keel of the fuselage, depending on the axial position. It is also reported in appendix A that the qualitative stress distribution is practically similar for every airplane fuselage although other fuselage measurements will obviously yield other absolute stress levels. From the information provided in appendix A the loads on the different parts of the fuselage can be deduced. This will be done here for the parts of the fuselage structure as shown in Fig. 2.1.b.

The upper part of the fuselage, the crown, will primarily undergo tensile loadings, both in the axial and in the hoop direction. The stress in axial direction varies with a rather large amplitude and at high frequency around a positive value. For the two frame positions given in appendix A, the axial stress has always a positive value, even during ground handling. This is due to the position of the frames considered. For frames near the middle of the nose-wheel and the main landing gear also negative axial stresses can occur although these compressive stresses will never be very large because of the balancing of the total aircraft around the main landing gear. Of course, these compressive stresses must be considered in the stability analysis of the crown section although not much problems are to be expected. The stress in hoop direction varies only with a small amplitude and at lower frequencies since this stress is almost exclusively caused by pressurization of the fuselage, as stated already. Even the loading case airbrakes fully extended, as applied to one of the frames in appendix A, hardly influences the hoop stress. For this part of the fuselage the circumferential component of the skin stress is also greater than zero in all thinkable cases. The shear stresses present in the crown can be neglected because of its small values.

The side panels of the fuselage will also undergo tensile loading in both the axial and the hoop direction but these stresses will be accompanied by a shear loading in this
section. The axial stress in the side panels varies around approximately one third of the axial stress in the crown section, depending on the axial position considered. Although the amplitude of the axial stress component is reduced the frequency of it stays high. The change in both the highest value as well as the amplitude of this longitudinal loading can be explained by the fact that it is a summation of the pressurization loads and the bending moments on the fuselage, which are largest around the y-axis. The neutral axis of the fuselage for this bending moment will be just below the middle of the fuselage cylinder according to the distribution of the masses of the airplane and its payload. If for a position on the side panel a point on the neutral axis is chosen one should find a stress distribution similar to the one for the hoop stress since the axial stress will now also solely be determined by the pressurization load. The value of this axial stress should then be half the value of the hoop stress, since this is the ratio between these two stresses for a cylinder loaded by an internal pressure only. The unchanged frequency of the axial stress component is obvious since this frequency mainly depends on the variation in bending moment which stays unchanged. During ground handling and just after takeoff or before landing, the axial stress in this part of the fuselage may become negative due to the bending moment around the y-axis and the absence of a pressurization load. The hoop stress for this part of the fuselage is nearly similar to the one for the crown section although small negative values can occur in the ground cycle. The shear stress is a consequence of the fuselage bending around the y-axis, as was mentioned before. Although the absolute value of this stress component is not nearly so high as for the other two components, this shear stress may fully determine the applied construction type of these panels. This fact can be explained by the much lower value of the shear strength as compared to the tensile strengths. For aluminium the shear strength is roughly 55 % of the tensile strength, for the 0-90/±45/0-90 carbon-PEI laminate roughly 30 %, and for a unidirectional carbon-epoxy roughly 5 %. Although this last laminate construction shall never be used in practice for this kind of structure, because of the poor tensile properties perpendicular to the fibres, the value given shows a good reason for concern.

The lower part of the fuselage, the keel, will primarily undergo a cyclic load around approximately zero in axial direction and a cyclic tensile load in hoop direction. The stress in axial direction varies with a rather large amplitude and at high frequency. The mean value of this stress component, which is roughly zero, depending on the position on the fuselage, comes forth from the large compressive loads induced by the bending moment around the y-axis, which almost fully eliminate the tensile stress as a result of pressurisation. For one of the frames given in appendix A, the mean value of the axial stress is definitely higher than zero. This might be a consequence of the position of this frame, which is near the attachment of the wing to the fuselage, for which a heavier structure is applied. during the ground cycle of the airplane, the axial stresses in this part of the fuselage are definitely negative. Again, the hoop stress for this part of the fuselage is nearly similar to the one for the crown section although here also small negative values can occur in the ground cycle. The shear stresses present in the keel can be neglected because of its small values.
Now that the loading conditions for each part of the fuselage are known, design parameters can be proposed for each of these parts. The main observation of the loading conditions leads to the conclusions that the crown is to be designed for static strength and repeated tensile loads, the side panels are to be designed to withstand static shear loads (stability) and repeated shear loads, and the keel is to be designed mainly for a combination of tensile stresses in the hoop direction and a cyclic load with a mean value of zero in the axial direction.

In contrast to metals, where fracture under static or fatigue loads result from the nucleation and growth of a single dominant flaw, the fracture of fibre-reinforced plastic is characterized by the initiation and progression of multiple failures of different modes such as matrix cracks, interfacial debonding, fibre breaks and delaminations between adjacent plies of laminae. The kinds of occurring failures, their distribution, time sequence and possible interactions depend on many parameters such as the properties of the fibre/matrix system, the stacking order and curing process, the influence of the environment, etc. The problem is further complicated by different failure modes under static and dynamic load applications, and by the possibility of fatigue failure in the compressive as well as in the tensile load regime.

Toward the end of the specimen life, adjacent failure modes tend to interconnect and form failure paths which, because of their stochastic nature, differ from specimen to specimen and lead to discrepant life spans of the test specimens. With the gradually increasing damage state is a reduction of the overall laminate stiffness. After development of a critical combination of failure modes rapid deterioration of the stiffness occurs in only a few cycles and the specimen will fail relatively fast. Because of this multiplicity of failure modes and the sensitivity of composites to out-of-plane stresses, the development of a damage tolerance approach to composites is greatly complicated in comparison to metals. Different loading spectra can lead to different failure modes, while static loads frequently show a complete different failure mode compared to that of cyclic loadings.

Compared to static loading more delamination and transverse matrix cracks develop under cyclic fatigue loading. Static strength seems to be controlled by the fibres and the fatigue limit to be dominated by the matrix. Growing damage causes deformation increase (stiffness reduction) at cyclic loading and loss of static strength. In the case of notched laminates a loss of strength may be preceded by an increase of residual strength.

In section 2.2 of this report the resulting problems were already briefly addressed in relation to the airworthiness requirements and therefore a further discussion on this will be left out. For the rest of this section it will be enough to shortly refer to the behaviour of composite materials under different loading types.

Tensile loading (Baker et al. (1985)).

Significant defects for composites under tensile loading are holes, scratches and deep cuts. It is found that, depending on ply configuration, fatigue cycling under tensile loading tends to remove stress concentrations by causing delaminations and splitting at the edges of the hole or notch, increasing the residual strength.
in some cases to net-section values. Thus interest centres more on static strength than on residual strength following fatigue cycling. With this knowledge a development of a damage tolerance approach for composite materials under tensile loadings seems well possible.

Challenger (1986) remarks concerning this matter that the stress concentration created by through-the-thickness notches and broken fibres is reduced by formation of a damage zone in the vicinity of these defects when a tensile load is applied. This damage zone consists of splitting within and delamination between the plies. The tougher matrix materials (e.g. PEEK, PEI) resist this type of damage better than lower toughness materials (e.g. Epoxy). Thus the reduction of the stress concentrations is smaller for the tough matrix materials, resulting in an increased notch sensitivity for these tougher materials. An advantage of the tougher matrix materials is that when impact occurs the damage will be less severe than for the more brittle matrix materials. But this effect will thus be (partly) annulled by the earlier stated increase in notch sensitivity for the tougher matrix materials.

Compressive loading (Baker et al. (1985)).

The situation concerning damage tolerance under compressive loading is much more complex than for tensile loading. Under cyclic compressive loading, although the degree of stress concentration is initially reduced by fatigue, the loss in stiffness due to formation and growth of delaminations and general cracking can lead to propagation of damage by local deformation of surrounding material. Thus the residual strength may fall as a function of the number of cycles. Further, since the behaviour depends on the support offered to the fibres and plies against buckling, the behaviour is matrix-sensitive.

Cyclic loading.

Fatigue loading in compression-compression or tension-compression is more detrimental than fatigue loading in tension-tension. The fatigue strength of composite materials decreases when compression becomes a larger part of the load cycle. It has been observed many times that damage in polymer matrix, starting at hole edges, grows parallel to the load axis in the form of ply cracks in the 0°-plies and in the form of delaminations following these ply-cracks. Associated with the damage growing longitudinally is an effective notch blunting with a pertinent increase in residual strength. Delamination is always observed at a notch and under cyclic loading it is here also more extensive than during static loading. Delamination growth increases near the end of life, thereby enlarging the deformation or strain of the laminate at the peak loads of the cycle.

In contrast to metals, fibre composite materials show a large damaging influence of a notch on static strength and low cycle fatigue strength, and only a small influence on the high cycle fatigue strength. This notch effect is most pronounced if the fibre composite material is loaded parallel to the fibres and it is observed to be universal for fibre reinforced polymer matrix composites (Gerharz (1989)).

23
When cyclic loading is applied to a laminate fibre fracture and delamination patterns are influenced by the load level used. Lower load levels produce a greater extent of delamination in the material, whereas higher load levels produce more extensive matrix cracking (Razvan et al. (1990)). This observation is in agreement with the argument that delamination is a growth phenomenon and that matrix cracking is initiation dominated. Furthermore it was noted that the two different failure patterns (i.e. delamination growth and matrix cracking) interacted in some way. Since in some cases the fibre fracture seems to follow delamination and in other cases delamination seems to follow fibre fracture the assumed relationship is not yet clear. In operational life different load levels will be encountered on a rather arbitrary basis and large scatter in fatigue lives can result from this.

Effects of modification of load spectra on the fatigue life (Gerharz (1989)).

Regarding simplifications, the largest pay-off will result from omission of low loads with high frequency of occurrence, whereas modifications at the high load end of the spectrum will be of more importance to demands with respect to flight load recording and simulation. Fatigue life for composite materials is very sensitive to omission of high loads. Fortunately omission of the lowest load levels of the spectrum shows promise for achieving large reduction in test time without significantly changing test results.

Degradation of strength occurs at constant amplitude loading. The designer is confronted with the requirement that the structure, despite the deterioration accompanying exposure to environment and repeated loading in service, is able to withstand ultimate design loads at all times during one lifetime. Therefore information on how deterioration develops during exposure to loading and environment occurring in one lifetime is needed.

2.4 Impact Characteristics.

When considering impact of any material, the first distinction that has to be made is between the terms damage resistance and damage tolerance. The damage resistance indicates the ability of a material/structure to undergo an event without the formation of damage, while the damage tolerance refers to the ability of a material/structure to perform with a preexisting amount of damage. In the aviation requirements only impact damage is mentioned and therefore only the damage tolerance capabilities are considered. In everyday handling of an aircraft structure the damage resistance may be of greater importance since avoiding the creation of damage also means avoiding all problems related to damage (e.g. damage tolerance, repair). In this report most interest will be focused on impact damage although some remarks on damage resistance will be given also.

In general practice, where impact data is given in the form of impactor kinetic energy versus residual strength, the distinction between damage resistance and damage tolerance
is no longer pronounced and a comparison between different materials in the design process can become rather complicated. Other parameters such as damage dept and size should be known to choose the most suitable materials. If the threshold of detectability (TOD) damage level, which is defined as the energy level at which damage is just visible to the naked eye on the surface of the structure, is used the above mentioned problem will be overcome.

In the discussion on ACJ 25.603 of the JAR, provided in section 2.2 of this report, it was already stated that impact damage in composite materials is generally accommodated by limiting the design strain level. With the discussion in this section it will be shown that such an approach is very limited and by far always satisfactory. Other approaches can yield results which are much more in accordance with reality and therefore these other approaches might be preferred.

Additional statements in the JAR concerning impact damage can be found in paragraph 25.571 and in ACJ 25.603. Another important statement is, for instance, that it should be shown that impact damage below the established threshold of detectability for the selected inspection procedure, will not reduce the structural strength below ultimate load capability.

Three important questions involved with any damage on any material are:
1) how is the damage formed?
2) how will the damage influence the structure?
3) how can the damage be handled?

1) How is the damage formed?

For composite materials to be used in the airplane, several sources of damage can be given. These sources are a) intrinsic or discrete damage (e.g. voids, varying fibre directions), b) large manufacturing flaws (e.g. delaminations), c) repeated load damage, and d) accidental damage. The first two of these damage types are quite obvious while the third is already excessively handled in the previous section. Therefore, only the occurrence of accidental damage will be further discussed below.

Accidental damage, or impact damage, will be dominated by hail, runway debris, dropped tools, damage caused by servicing equipment, especially near airplane doors and tank openings, and impact of engine debris after serious engine failure. The results of an impact can be classified as no damage, non-visible damage, barely visible damage and readily visible damage. For no damage evidently no problems are to be expected. The structure will maintain exactly the same safe operational level as before the occurrence of impact. As mentioned before, the structure is called damage resistant to this kind of impacts.

If at higher loads damage does occur, the nature of this damage will depend on both the structural parameters (ply configuration, shape and size of the component) and the material parameters (stiffness of the layers and the critical strain energy release rates for inter- and intra-ply failure). Also the velocity of the impacting object is of interest when the failure mode of the structure is observed. At high velocities the failure mode is dominated by fibre cracking while at lower velocities mainly interlaminar and intralaminar debonding takes place. Higher velocities lead to a larger damage zone up
to a velocity where the damage is characterised by a punch-out mechanism. Besides the material parameters the velocity dependence is influenced by the support boundary conditions. For a narrow support the impact resistance is different from that for a wide support of the specimen. At low impactor speeds the plate behaves like statically loaded. The energy is stored mainly near the load introduction at the impact locus and the supports, that means in a relatively large material volume. At very high speeds the punching affects an area only slightly larger than the cross section of the projectile. At moderate velocities a mixture of these two mechanisms is evident. While considering the third question a discussion on the present day handling of impact damage will be provided.

2) how will the damage influence the structure?

Damage in a structure can influence both the behaviour of the structure under static loading as well as under dynamic loading. Concerning the behaviour of the structure under static loads, residual strength properties will be of major importance while under dynamic loads also damage growth might be of significant relevance. First some remarks on static and dynamic loading behaviour will be given, followed by a general overview on the effects of damage in the different parts of the fuselage, regardless of the severity of the damage.

Behaviour of the damaged structure under static loading.

Earlier research on the residual strength under the presence of impact damage of Graphite/Epoxy and Kevlar/Epoxy composites from Cairns et al. (1990) led to the conclusion that the measured residual strength is only a function of the damage present. The manner in which the damage is introduced has no influence. With this in mind all kinds of damage can be simulated with a suitable impact event which yields the same amount of damage as in realistic operation of the structure. The amount of damage created should be the driver in such impact research, not a fixed energy level to create the damage. In order to come to a good definition of such an amount of damage in service experience on present day airplane structures can play an important role.

If such an approach of the amount of damage can be validated, this means that also the damage present after cyclic loading might possibly be simulated with a single impact event, as long as the damage in both cases is similar. Extensive research on the influence of fatigue loading on the damage size and mode of damage, combined with the behaviour of the structure under impact loading might lead to a large reduction in time necessary to provide structural characteristics of a damaged structure under cyclic loading. In fact, a similar approach is used in the research of fatigue of aluminium where an artificial crack (i.e. a saw cut) can be used to simulate the first part of the fatigue life.

Behaviour of the damaged structure under cyclic loading.

In section 2.3 of this report some loose remarks on the results of repeated loading of composite materials were given. But, up to now only little is known about the general material behaviour under these conditions and consequently no suitable models to predict damage growth in composite materials are available. Because of this, mostly a combination of limiting the strain level (e.g. $\epsilon_x, \epsilon_y < 0.4\%$) and a no-growth concept is
adopted in the treatment of damage in composite materials. The approach of a limiting strain level is based on the typical shape of the ultimate static strain versus impact energy curves, showing a nearly asymptotic branch. Despite the uncertainty of the influence of dynamic loading on the material it seems clear that different loading conditions yield different material behaviour. Therefore a discussion on the effect of impact damage on the different parts of the fuselage for which the loading conditions were discussed in section 2.3 will be provided herein.

Influence of damage on crown.

As stated before, the loading spectrum on the crown is characterized by tension-tension loadings. It is generally assumed that for these kind of loadings composite materials show no repeated load damage during normal operational life (90,000 flights). In order to convince the Airworthiness Authorities of the applicability of such an assumption considerable repeated load testing or service experience will evidently always be necessary. Of course, in handling the fatigue life of the structure, a difference should be made here between damages which occur within the material and damages in the interface between the materials (e.g. joints), which can lead to a decrease of structural stiffness due to deterioration of the interface. As mentioned in the introduction of this report, fortunately the number of parts can be greatly reduced with the application of composite materials, thus automatically leading to a great reduction in the number of joints.

Accidental damage on the crown section will mainly be dominated by hail impact. For this type of impact prescribed energy levels are available that should be withstood without the formation of fatal damage. The amount of damage however will also be dictated by the form of the impactor and its properties (e.g. elastic behaviour). Therefore, this fact has to be accounted for in the establishment of a proper test set-up too.

Influence of damage on side panels.

The loading spectrum on the side panels also mainly consists of tension-tension loadings although in this part it is combined with considerable shear loading. In section 2.3 it was reported that this shear loading often dominates the applied structure and therefore in this part of the fuselage cyclic shear becomes really important. Since this kind of loading is very difficult to simulate in laboratory circumstances little is known about the influence of this loading type. Especially when composite materials are used, with its varying properties in varying directions, it will be difficult to make a good strength prediction because of the lack of thorough knowledge of the off-axis properties of these materials.

Accidental damage on the side panels will mainly be dominated by impact of service equipment or in case of serious emergencies by engine debris. For both damage types the locations of the impact event are limited to a distinct area and therefore special measures can be taken to deal with it.

Influence of damage on keel.
The loading spectrum on the keel is primarily built up from tension-tension loadings in the hoop direction and tension-compression loadings in the axial direction. Especially for this second kind of loading composite materials show a strongly unfavourable behaviour under repeated load. Compressive loading is usually accompanied with delaminations and their growth. Large delaminations will allow for fibre buckling in compressive loading thus leading to failure of (part of) the material. Combination of the different loading directions can also lead to the nasty situation of damage growth due to the axial compressive stresses, leading to understrength of the structure in the hoop direction. Of course, as in case of the crown section, a difference should be made here between damages which occur within the material and damages in the joints, which once again can lead to decrease of structural stiffness due to degeneration of the joints.

Accidental damage on the lower panels will mainly be dominated by impact of runway debris (e.g. shrapnel of tires, spattering rocks, etcetera). With this kind of impact rather large amounts of energy can be involved.

3) how can the damage be handled?

If the four different sources mentioned in the treatment of the first question are handled separately, this results in the following statements. With the discussion on the airworthiness requirements it was stated already how to cope with the intrinsic or discrete damage. Part of it is covered by the factor of safety and the definition of A- and B-basis values and the rest can be assured by a special factor as discussed with JAR 25.619.

The second damage type (i.e. damage resulting from large manufacturing flaws) can be greatly reduced by good quality control of the manufactured parts. If however this kind of damage is present or if its absence can not be guaranteed with the applied inspection method, a no-growth concept should be established for the largest damage size possible unless it can be shown that during growth of this damage it will be detected before it reaches a dangerous size. The damage present in such case of manufacturing flaws with a no-growth verification should never reduce the strength of the structure below design ultimate load.

For the third type of damage, the repeated load damage, a scatter factor can be applied as discussed with the treatment of ACJ 25.603. With fatigue testing of the material the necessary design parameters for this kind of damage will be defined. Or if the concept of the ultimate strength approach is adopted (see discussion on ACJ 25.603) fatigue testing can be avoided since this damage will by definition not occur in sound structure.

The effect of accidental damage is much harder to describe since it is liable to appreciable variability and contains several uncertainties. A short discussion on this subject will be given in the sequence of this section. More details on this particular subject can be found in Rouchon (1992).

Damage in composite structures can possibly reduce the static strength below design ultimate load. For non-visible and barely visible damage it is clear that such a reduction is never allowed. For readily visible damage this cannot be stated in the same simple way.
If the structure maintains ultimate load carrying capabilities and the no-growth concept is established, the damage need never be detected. Suppose that the ultimate strength level is chosen such that the residual strength of the structure will not decrease with a further increase of the impact energy. In this case the structure should never be repaired during operational life since the ultimate strength requirement is always met. Repair of visible damage in this case will only give an apparent increase in the safety level. Non-visible damage could give the same reduction of the residual strength while no repair is prescribed.

 Normally repair is used to bring back the strength of the structure to the initial value and thus a key issue regarding this matter is the development of a need-to-repair criterion (Jones et al. (1987)). Some bonded repairs made in the past were found to be unnecessary. In some other cases repair even led to a reduction of the service life and had such flaws been unrepaired, the structure would have been just as strong and lasted much longer.

If the strength of the structure is reduced below ultimate load another approach will be necessary. In such a situation the damage may according to the airworthiness authorities never reduce the strength level below limit load. If the no-growth concept can be demonstrated the damage will never reach the critical size corresponding to limit load carrying capability. So, whatever the inspection intervals are, the damage will implicitly be detected before its critical size and the basic requirement of damage tolerance will automatically be met. In Fig. 2.4.a this philosophy is demonstrated for both metallic and composite structures. Note that especially for composite structures a large period of time below ultimate load carrying capabilities is possible.

From ACJ 25.603 it follows that in selecting the inspection intervals, the residual strength level associated with the assumed damage should be considered. This can be interpreted as the larger the strength reduction below ultimate load, the sooner the damage should be detected. However, since the occurrence of accidental damage is an unpredictable event, also a probability approach has to be applied. Combining these two features leads to the statement that for a given static strength reduction below ultimate load, the more likely damage may occur, the sooner it should be detected. The general outline of such an approach is given in Fig. 2.4.b. In this figure only the area where both the probability of damage is above 10E-9 and the residual strength is below ultimate load is of interest. The line defining the acceptability level is chosen such that the probability of having the combination of a strength reduction with a catastrophic gust loading for that situation is extremely remote. For the application of this probability approach an extensive data base considering impact damage and loading conditions is absolutely necessary. With the gradual introduction of composite materials into airplane structures and careful examination of its behaviour such a data base may become a fact.

If it can be assumed that the result of different impact damages may be superimposed the operational life of an aircraft will depend on probability studies for the different kind of impact damages. Ultimate failure of the aircraft will then be dictated by a large amount of impact damages on or close to earlier positions of impact. Due to the total of these damages the residual strength decreases below an acceptable level of safety.
With this philosophy a factor of safety (via the probability of occurrence of impact) can be agreed which in fact determines the aircraft operational life.

Fortunately, the severity of impact damage to be considered for each aircraft structure is limited by two parameters. These parameters are the threshold of detectability (TOD) of the damage introduced and the realistic level of energy which together with the impact characteristics determines the maximum damage size to be taken into account. For the selection of the TOD the choice of the barely visible impact damage (BVID), corresponding to the detectable indentation size at the most favourable conditions for a visual inspection, is generally accepted by the Authorities.

Davis et al. (1981) states on the BVID that it most likely will not be detected until the next inspection period therefore either a safe-life concept should apply (which implies that the damage need never be detected) or the structure should be shown to be capable of withstanding operational loads with the damage present for the number of flight hours between inspections. Readily visible damage undoubtedly would be repaired prior to the next flight which means that the structure should be of adequate strength so that the flight can be safely completed after sustaining such damage.

In the present work, where the influence of damage on the behaviour of the fuselage is considered, especially the case of accidental damage is of importance. As announced in the introduction of this report a reduction of the amount of work involved in the present research is made by neglecting fatigue damage. For tension loaded areas (i.e. crown of the fuselage) such a simplification can be justified by the fact that earlier research performed by Lagendijk (1991) showed that damage caused by impact will not grow under cyclic tensile loading. Therefore a simple residual strength approach can be used for the damage tolerance evaluation of the structure under these loading conditions. Certainly accidental damage in the other parts of the fuselage will be influenced by fatigue loading which yields sufficient possibilities for future research.

Further assumptions concerning the modelling of the impact damage follow from Cairns et al. (1990) and Vlot (1991).

In Cairns et al. (1990) it is stated that a damaged area in any material can be conceived as an inclusion in the material with reduced stiffness properties. In other words the damage present can be modelled as an elastic inclusion. The extreme limit of such an elastic inclusion is an open hole for in that case the elastic properties are all reduced to zero. Since composite materials are known as notch sensitive, such a model will result in some lower boundary value which can be applied to the material under study. Combining this open hole approach with the findings of Vlot (1991) that a saw cut yields lower residual strength values than an open hole (see Fig. 2.4.c), it is found that with a saw cut the worst case of impact damage can be simulated. Therefore, in the present work a test specimen provided with a saw cut is used to yield the lowest boundary for the residual strength of a material with impact damage.

The only choice that is left now are the measurements of the saw cut. Of course the width of it must be kept as small as possible, which can be achieved by using a fine fretsaw. The length of the saw cut can be chosen freely, depending on the amount of
impact damage that has to be simulated. Since in first instance the damaged area was modelled as an open hole which thereupon was represented with a saw cut with the length similar to the diameter of the hole, the length of the saw cut has to be similar to the size of the impact damaged area.

In this work it is found necessary to model a critical situation which is as close to normal day practice as possible. Therefore, the threshold of detectability has to be established for the material under study and an impression of the impact energies encountered during aircraft operation is essential. From an investigation performed at the Delft University of Technology by Haars (1992) on the impact behaviour of sandwich shells this information can be obtained. It was found by Haars that the BVID was in the order of magnitude of 25 mm. Therefore in the experimental part of the present work a crack length of 25 mm was chosen as the most critical situation.
3 Linear Elastic Fracture Mechanics.


Fracture mechanics can be very useful in the solution of damage tolerance problems since with the use of it predictions of residual strength and crack propagation can be made. With this residual strength and crack propagation one can prescribe a safe inspection interval or ensure safety throughout a given interval of time. Of course data from experiments will be necessary to check on the predicted values. In the present work, where only tensile loading is considered and thus impact damage can be simulated with a through crack, fracture mechanics is used to predict the residual strength of the material under study. As argued in section 2.4, this residual strength value can be used as a worst case simulation of impact damage.

Since this report deals only with sheet structures, a state of plane stress can be assumed. Apart from that, a plane stress situation is of main importance for almost all present aircraft structures since the greatest part of these structures consists of relatively flat and thin sheets and sheet structures. Even for a fuselage structure, which is definitely a curved structure, only for a small fuselage radius or a relatively thick construction, stresses which act perpendicular to the skin may become important. Even when the structure bulges out, which is indeed very unlikely for the well supported skin of a sandwich fuselage, the stress state remains plane.

A further restriction made is that only uniaxial stress states will be considered. As stated in section 2.3, the stress distribution in a pressurized fuselage is far from a uniaxial one. By neglecting the axial stress and thus accepting a uniaxial stress state, a worst case is created since a biaxial stress field with a tensile stress added to the crack direction tends to reduce the stress peak at the crack tip. Besides this, experimental data under a biaxial stress state is very difficult to obtain. This restriction means that only the upper part of the fuselage is considered here, since only in this part a tension-tension load is found.

Attention will only be focused on the relatively simple linear elastic fracture mechanics to predict residual strength properties for the panels. Of course, also other fracture mechanics concepts can be used to describe and or predict fracture. One of these other concepts which becomes more and more frequently applied is the J-integral concept. This concept will not be treated here since application of the linear elastic fracture mechanics yields sufficient possibilities concerning the problems dealt with in this report.

3.1 General Description.

In this section a general description of the linear elastic fracture mechanics is presented. At first attention will be focused on the historical background of this concept as deduced for isotropic materials. Although the underlying thoughts of this concept may not be directly applicable to composite materials this background may result in a better insight
on the general idea. Following this historical background, a presentation is given on how to apply the concept in a rational and straightforward manner to centre cracked tension panels. This presentation will be important for the evaluation of the experimental results as demonstrated in the next chapter.

Linear elastic fracture mechanics, also known as the stress intensity factor concept, is related to a concept which was first formulated by Griffith in 1920 when he was considering the propagation of brittle cracks in glass. He stated that an existing crack will propagate if thereby the total energy of the system is lowered. With this in mind Griffith assumed that there is a simple energy balance, consisting of a decrease in elastic strain energy within the stressed body as the crack extends, counteracted by the energy needed to create the new crack surfaces. This can be formulated as:

\[ U = U_0 + U_a + U_\gamma - F \]  

[3.1]

In which \( U_0 \) is the elastic energy of the loaded uncracked plate (constant under crack extension), \( U_a \) is the change in elastic energy caused by introducing the crack in the plate, \( U_\gamma \) is the change in elastic surface energy caused by the formation of the crack surfaces, and \( F \) is the work performed by external forces. The equilibrium condition for crack extension is obtained by setting \( dU/da \) equal to zero. In Fig. 3.1a it is illustrated that when, with the absence of work by external forces, the release of elastic energy due to a potential increment in crack growth, \( da \), outweighs the demand for surface energy for the same crack growth, the introduction of a crack will lead to its unstable propagation.

Griffith also deduced the values for \( U_a \) and \( U_\gamma \) as:

\[ |U_a| = \frac{\pi \sigma^2 a^2}{E}, \quad U_\gamma = 2(2a \gamma_e) \]  

[3.2]

Where \( \sigma \) stands for the uniform tensile stress applied at infinity, \( a \) stands for the half crack length, and \( \gamma_e \) is the elastic surface energy of the material.

If no work is done by external forces, the change of elastic energy caused by introducing the crack in the plate is negative because of the loss in stiffness which makes the load applied at infinity drop. Setting \( dU/da \) equal to zero yields the following requirement:

\[ \frac{d}{da} \left( U_0 - \frac{\pi \sigma^2 a^2}{E} + 4a \gamma_e \right) = 0 \Rightarrow \frac{2\pi \sigma^2 a}{E} = 4\gamma_e \]  

[3.3]

In this equation \( E \) and \( \gamma_e \) are material properties and therefore crack extension occurs when \( \sigma/a \) attains a certain (constant) critical value.

Irwin extended this energy balance as a balance between (1) the stored strain energy and (2) the surface energy plus the work done in plastic deformation. This can be done by replacing \( \gamma_e \) in the equations by \( (\gamma_e + \gamma_p) \) where \( \gamma_p \) denotes the plastic energy of the material.
Irwin also defined a material property $G$ as the total energy absorbed during cracking per unit increase in crack length and per unit thickness. $G$ is called the 'energy release rate' or 'crack driving force'. $G$ is defined as:

$$ G = \frac{\pi \sigma^2 a}{E} \quad [3.4] $$

If $G$ reaches a critical value, $G_c$, fracture will occur.

In the middle 1950s Irwin showed that the energy approach is equivalent to a stress intensity ($K$) approach, according to which fracture occurs when a critical stress distribution ahead of the crack tip is reached. The material property governing fracture may therefore be stated as a critical stress intensity, $K_c$, or in terms of energy as a critical value $G_c$. The relation between these two approaches is given by:

$$ G_c = \frac{(1-\nu)}{2G} K_c^2 = \frac{(1-\nu^2)}{E} K_c^2 \quad \text{for plane strain} \quad [3.5] $$

$$ G_c = \frac{1}{2G(1+\nu)} K_c^2 = \frac{K_c^2}{E} \quad \text{for plane stress} $$

In which the $G$ without an index denotes the shear modulus of the given material which is not to be mistaken with the energy release rate.

As stated already in the introduction of this section, this formulation might not be directly applicable to composite materials due to the differing failure modes. It might well be possible to deduce similar formulations for composite materials depending on the different material behaviour but this is not of much interest to this present work.

The attraction of the stress intensity factor approach for isotropic materials lies in the fact that the stress distribution around and close to a crack tip is always the same, independent off the material in use. With this knowledge tests on suitably shaped and loaded specimens to determine $K_c$ performed on isotropic materials proved that it is possible to determine what flaws are tolerable in an actual structure under given conditions. In this work it is assumed that a similar approach is valid for composite materials which enables a prediction of tolerable flaws and residual strength for structures made out of these materials. Later on experimental results on composite materials will be used to verify the validity of this assumption.

The stress distribution at the crack tip ($r \rightarrow 0$) for isotropic materials can be given with the following formula (see Fig. 3.1.b):

$$ \sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \quad [3.6] $$
where $K$ denotes the stress intensity factor. If anisotropic materials are used the stress distribution is also influenced by the material parameters which can be implemented in the function $f_{ij}(\theta)$.

Cracks under load can be opened in different ways. This is shown in Fig. 3.1.c. In mode I the crack is opened perpendicular to the fracture surface. Modes II and III, where the displacements occur in the plane of the crack, are known as shear modes. The stress intensity factor determines the $1/r$ singularity which appears at the crack tip for each of these three opening modes. If the same coordinate system is used as in Fig. 3.1.c, the stress intensity factor for each crack opening mode can be given as:

\[
K_I = \lim_{r \to 0} \left[ \sqrt{2\pi r} \cdot \sigma y_y(r, \theta = 0) \right] \\
K_{II} = \lim_{r \to 0} \left[ \sqrt{2\pi r} \cdot \tau y_y(r, \theta = 0) \right] \\
K_{III} = \lim_{r \to 0} \left[ \sqrt{2\pi r} \cdot \tau y_z(r, \theta = 0) \right]
\]  

[3.7]

It can be noted from these equations that the stress intensity factor has the dimension of $[\text{stress} \cdot \sqrt{\text{length}}]$.

In this report only cracks of opening mode I will be further discussed because only a uniaxial stress state is considered acting perpendicular to the crack direction. In the side panel of the fuselage also cracks under opening mode II are likely to appear as a result of the loading conditions but this will not be treated here. Opening mode III, the out-of-plane sliding of crack faces parallel to the crack front, is not very likely to occur in the case of a sandwich construction because of the mutual support of the two facings. Only when bulging of the crack fronts occurs this opening mode might become important.

As seen before, to distinguish between the different opening modes an index referring to the regarded mode is added thus yielding $K_{IC}$ for the critical stress intensity factor for a mode I loading case. This parameter governs tensile fracture of a particular material and therefore $K_{IC}$ is called the fracture toughness of the material. If a structure is built up from material with a higher fracture toughness the residual strength of this structure will be increased.

Since in this work composite materials are considered it is important to consider the directional dependence of the fracture toughness for it might be possible that cracks under different angles with the principal material axes show different results. To establish a relation between the fracture toughness and the angle of the crack with the principal material axes it is necessary to perform a series of off-axis tests on the material under study. However, at first instance this dependence is not of much interest to this work so it was chosen to only consider cracks in the direction of one of the in-plane material principal axes. Of course the choice which of these two axes is considered makes no difference here since the material properties are similar in both directions. If the linear
elastic fracture mechanics as discussed in this chapter have proven its worth, establishing such a relationship might become attractive.

Of course a plate with infinite dimensions is not a very realistic object although, when the total crack length, 2a, is sufficiently small as compared to the plate width, W, the finite plate dimensions will not have any influence on the stress distribution around the crack and the situation can well be approximated by assuming infinite plate dimensions. The ratio between the crack length and the plate width to justify such an infinite plate approximation differs for each material. A ratio of 2a/W = 1/3 yields satisfactory results for isotropic materials, however, finite element calculations on the stress distribution around a hole in anisotropic materials showed a decrease of this ratio down to 1/10 in some particular cases of anisotropic materials (Fujita). If experiments on any material are to be used for residual strength analysis, strain gauges can be applied to determine whether or not an infinite plate solution can be used.

To provide for finite plate dimensions and other factors which influence the stress intensity at the crack tip (e.g. cut-outs, frames, stringers, doubler plates, door openings, etcetera), the general formulation of the stress intensity factor is given by:

$$K = C \sigma \sqrt{\pi a}$$  \[3.8\]

In which $\sigma$ is the stress applied at considerable distance away from the crack, a is the half crack length, and C is a non-dimensional factor to correct for all deviations from the infinite plate solution for the stress intensity factor. Formula [3.8] still holds for each of the three different loading cases, although the correction factor may have different values for the different loading cases. Since this factor is a function of the geometry of the structure around the crack it is often called the geometry factor. The great advantage of such a geometry factor is its independence of the material in use.

The geometry factor plays a very important role in the stress intensity factor concept since it virtually determines the complete solution of the situation under study. In section 3.2 more details on stress intensity factor solutions in the form of a geometry factor are given as well as a method to calculate these important geometry factors. Some of the results of these calculations will be given later on.

In normal practice the fracture toughness of a material is obtained under plane strain conditions by a standard method published by the American Society for Testing and Materials (ASTM). In this standard method the specimen measurements and loading conditions are accurately prescribed in order to define one incontrovertible $K_{IC}$ value. In the present work however, where a plane stress situation is considered, it is not of interest to obtain this standard $K_{IC}$ value.

In order to distinguish between the plane strain and plane stress fracture toughness, in the literature the symbol $K_C$ is also used to refer to the situation of plane stress while $K_{IC}$ is reserved for plane strain. However, since in this work only plane stress situations are considered and therefore never causes any confusion, the symbol used throughout this work is $K_{IC}$.

No standard test method is available for plane stress fracture toughness testing and therefore the engineering approach as proposed by Feddersen (1971) will be applied
here. This method is the only one suitable for practical use besides R-curve testing. By using the Feddersen approach it will become clear how the stress intensity factor concept for mode I loading situations can be presented in a general way in order to distinguish between different materials in a rational manner. By using this general formulation both the utility and generality of $K_I$ and the simplicity of an elementary stress-flaw relationship is maintained.

At first the inverse relationship between the stress $\sigma$ and the crack length $2a$ is illustrated in Fig. 3.1.d. This relationship is the same as the formulation of Eq. [3.8] with the assumption of an infinite plate (i.e. $C=1$). In theory this curve covers the whole quadrant pictured in the figure. In reality however, the applied material is limited in its strength and in its measurements, thus yielding a maximum in the stress direction by the tensile strength and a maximum in the crack length dimension by the finite panel width. It should be noted here that, in contrary to isotropic materials, no plasticity is included in this analysis because of the totally elastic behaviour observed in earlier experiments on the material under study. The two boundaries define a net section fracture limit which is pictured by a diagonal straight line in Fig. 3.1.e.

It can be observed that each line drawn from the panel width limit (i.e. $2a=W$) that crosses the positive stress axis is a line of constant average stress on the remaining cross section of the panel. The value of this averaged net section stress can be found on the intersection point of the stress axis.

Combination of the two curves of Figs. 3.1.d and 3.1.e produces a general limitation of the fracture data. Earlier experimental results however showed remarkable deviation from these fracture lines in the areas of the extremes of stress and flaw size. In order to reduce the experimental results to a simple, yet general form, for engineering applications Feddersen (1971) suggested to take the tangents to the idealised $K$ curve (see Fig. 3.1.f) to obtain a smooth and continuous curve of fracture behaviour over the full range of crack lengths. Fracture data on several materials demonstrated the very good agreement of such an approach with experimental results. Therefore such an approach is nowadays widely applied in the presentation of plane stress fracture toughness data. What seemed to be a rough simplification actually turned out to be a very useful possibility to present plane stress fracture toughness data on an engineering level which for long could not be improved. Only recently the R-curve concept was proposed and this concept becomes more and more frequently applied.

If the slope of the $K$-curve at any point is determined, the tangency conditions of the now obtained fracture curve can be calculated. This slope can be given as:
\[
\frac{d\sigma}{d(2a)} = \frac{d}{d(2a)} \left[ \frac{K}{\sqrt{\pi a}} \right] = \frac{K}{2a\sqrt{\pi a}} \cdot \frac{1}{2} - \frac{\sigma}{4a} \tag{3.9}
\]

And with this the tangency condition at the left hand side of the curve becomes:

\[
\frac{d\sigma}{d(2a)} = -\frac{\sigma}{4a} - \frac{\sigma_{\text{max}} - \sigma}{2a} \Rightarrow \sigma = \frac{2}{3} \sigma_{\text{max}} \tag{3.10}
\]

Which implies that the left hand side tangency point always occurs at a stress level of two thirds of \( \sigma_{\text{max}} \). For the right hand side a similar argumentation can be given, thus yielding:

\[
\frac{d\sigma}{d(2a)} = -\frac{\sigma}{4a} - \frac{\sigma}{W-2a} \Rightarrow 2a = \frac{1}{3} W \tag{3.11}
\]

Which implies that the right hand side tangency point always occurs at one third of the panel width. Feddersen (1971) refers to this right hand side of the curve as the region with finite width or boundary effects. As stated before, it should be kept in mind that this curve is deduced for isotropic materials and the applicability of it for composite materials can only be shown by experiments. Once again it is mentioned here that earlier performed finite element calculations on an open hole (Fujita) showed that a ratio of \( 2R/W = 1/10 \) is to be used in some particular cases of anisotropic materials in order to justify an infinite plate approximation.

If a general applicability of this theory is assumed, the fracture curve can be divided into three separate parts for each of which another data model is to be used. The first part (i.e. \( \sigma > 2/3 \sigma_{\text{max}} \)) consists of the left hand tangent to the K curve, the second part (i.e. \( \sigma_{\text{max}} > 3 \sigma_{\text{max}} \wedge 2a > W/3 \)) consists of the K curve itself, and the third part (i.e. \( 2a > W/3 \)) consists of the right hand tangent to the K curve. In order to obtain the stress intensity factor for a particular material it is therefore important to reassure that the panel size is chosen properly and to check if the fracture stress is not to high since this might result in a wrong \( K_I \) value.

Feddersen (1971) obtained further simplification of the presentation of test results by plotting fracture curves for different panel widths in one and the same figure. This simplification is justified by the fact that for each different panel width practically the same K value is found. The basics of such a presentation can be found in Fig. 3.1.g. In this figure it is also shown that there is a minimum panel width that can be used to obtain the \( K_I \) value.

In the next chapter a threefold fracture curve will be used on the Carbon/PEI material in a given laminate lay-up to check its validity for this specific material.

Normally, in a uniaxially loaded panel with a central transverse crack of length \( 2a_0 \), the stress can be raised to \( \sigma_1 \) before the crack will start to grow slowly. This crack growth is stable, i.e. crack propagation can be maintained only if the load is raised further. Ultimately, at a stress \( \sigma_c \) the crack will reach a length \( 2a_c \) where it will propagate.
unstably while the stress level will drop, resulting in final failure of the panel. The longer
the initial crack, the lower the values of the initial crack growth stress and the stress at
final fracture but there is more slow crack growth, as illustrated by the curves in Fig.
3.1.h. The middle (dashed) curve relates the fracture stress directly to the initial crack
length $2a_0$ and is therefore often referred to as apparent instability curve.

An important feature of the stress intensity factor solutions under a mode I loading
situation is the superposition principle. In this principle it is stated that the total stress
field in the vicinity of a crack tip can be obtained by an algebraic summation of the
respective stress intensity factors. With this knowledge various difficult fracture situations
can be solved by dividing them into separate known solutions via which the total solution
can be found. In Fig. 3.1.i an example is given for a crack under internal pressure. With
this principle it can be deduced that $K^D_I - K^A_I - K^C_I = 0$, $K^C_I = -a/\pi a$. In chapter 5, in which
stiffened panels are treated, this principle will also be used.

Further use of the mode I stress intensity factor is directed to the prediction of crack
growth rates in isotropic materials. Since this subject is not of interest to this present
work, in which the attention is mainly focused on residual strength, it will not be treated
any further here.

The stress intensity factor approach can well be applied for purely elastic materials and
materials with low ductility. The linear elastic approach may need some adjustment for
more ductile materials (e.g. the frequently used 2024 aluminium alloy) in order to
characterise the fracture mechanics of the material. A regularly used method for this
purpose is considering an effective crack length in case of ductile materials rather than
the physical crack length. This effective crack length is larger than the physical crack
length in order to account for the redistribution of stresses due to yielding of the
material at the crack tip.

Another linear elastic fracture mechanics concept, which does account for the crack tip
plasticity, is the R-curve concept which becomes more and more frequently applied. In
this concept $R$ is the total energy consumed in the plastic zone ahead of the advancing
crack. This total energy contains elastic energy and plastic energy, the latter of which is
by far the largest. During slow stable crack growth there is a continuous balance between
released and consumed energy. The energy release rate $G$ can be measured during crack
growth and an increasing $G$ appears to be required to maintain slow growth. Apparently,
the energy consumption $R$ increases as the crack proceeds. In a graphical presentation
of the R-curve the fracture condition is given by the point of tangency of the curve (see
Fig. 3.1.k).

Because of the absence of a plastic zone at the crack tip in the material used in the
present work (see discussion in the experimental part presented in chapter 4), these
methods for taking account of the plastic zone will not be treated here in further detail.
3.2 Calculations of Some Geometry Factors.

As stated in the former section, the non-dimensional geometry factor, $C$, is used to deal with all deviations of the stress intensity factor from the infinite plate solution (see Eq. [3.8]). This factor owes its name to the fact that it is a function of the geometry of the structure around the crack. In the past many research is done in order to find geometry factors for different cases of cracks under loading. Murakami (1987) gives a rather complete enumeration of directly available geometry factors. With the knowledge of the geometry factor of the examined crack in the stress field present, the fracture stress of the cracked material can be predicted, provided that the stress intensity factor for the material in use for the particular loading direction is known.

In this section an analytical method is used to calculate some geometry factors for some specific situations. The calculations are based on a program that calculates the stress distribution around a row of three elliptical holes in an infinite plate. This program was earlier developed at the Delft University of Technology by Tooren et al. (1992). The basic formulas upon which this program is built can be found in appendix B of this report. A crack is simulated by letting one of the axes of an ellipse approach to zero (see appendix B.4). Some modifications on the program were necessary in order to make it suitable for the direct calculation of geometry factors.

The procedure which is followed for the calculation of the $K$ values can be described as follows: first the stress distribution around the crack tip in the disturbed stress field is calculated and then this solution is divided by the exact stress distribution which is available for a crack with the same length in a further undisturbed infinite plate. This value, which gives the deviation of the crack tip stress field with respect to a crack in an infinite plate, is by definition equal to the desired geometry factor. This procedure can be formulated as follows:

$$
C = \frac{\sigma_{\text{total disturbance}}}{\sigma_{\text{single crack}}} \text{crack tip}
$$

[3.12]

In which the addition 'crack tip' should be noted since the stress field around the crack tip is only locally defined by the stress intensity factor and therefore a good solution can only be found for (the area very close to) the crack tip. If the area around the crack tip is again given with Fig. 3.1.b, this means that only for small values of $r/a$ the solution found by this calculation method is valid. Calculations on several materials showed convergence of the geometry factor to a specific value for values of around $r/a < 1/4$, depending on the material under study. Only small changes ($< 0.01\%$) in geometry factor could be observed for values below $r/a = 1/1000$.

At this time a major restriction is formed by the fact that with the use of the earlier mentioned calculation program only stress distributions around three holes (i.e. also cracks, see appendix B) in an infinite plate can be found. If calculation programs for cracks in other situation become within reach the described method can easily be applied.
to this new situation, thus directly yielding a good possibility to calculate the geometry factor.

First implementation of the calculation of geometry factors in the program was based on the use of a K-gage (further details will be given in chapter four), leading to a summation of strains in y-direction along a circle around the crack tip. This total strain was divided by the total strain around the crack tip in an infinite plate. As stated before, only for circles very close to the crack tip \((r \rightarrow 0)\) the geometry factor is unequivocally determined. Later on it was found that by calculation of a single strain value, approaching the crack tip along the x-axis, led to the same geometry factor if the point in which the strain was calculated was close enough to the crack tip. With this latter calculation method a large reduction in calculation time could be obtained since instead of calculating a series of strains along a circle (92 in case of a K-gage), only calculation of one strain value was necessary. A further simplification was made by avoiding the calculation of strains and directly dividing the obtained stress values. This simplification also led to a minor reduction in calculation time since instead of multiplying the two stress values by a constant factor (i.e. Hooke's law) before dividing them, they are now directly divided. This method of determining the ratio of stresses in a single point very close to the crack tip is applied in all following calculations since this is by far the easiest calculation procedure.

A further modification of the calculation program concerns the number of terms that are used in the Taylor series expansion in the calculation program (see appendix B, Eq. [B.44]). By writing the necessary Taylor expansion as a series expansion, instead of programming each Taylor expansion as was done in the original program, results in principle in the possibility of an infinite number of terms in the Taylor expansion thus giving the exact value for \(\zeta_k\). After this relatively easy modification an even greater improvement was reached by writing the necessary derivatives of the stress functions as a series expansion rather than writing out each derivative. This alteration led to the possibility of extending the number of terms used in the stress functions (see Eq. [B.43]) to each desired value. Extending these series to infinity leads to an exact solution for the stress functions and thus to the exact desired stresses and displacements.

The first remark concerning this matter is however that the accuracy of the solution is restricted by the maximum length of the numbers used in the calculations, thus limiting the accuracy of the total solution. A second boundary is formed by the system limits of the available computer system. In the present case, where a hp 9000 series 300 was used, this second restriction resulted in a series expansion with a maximum of 12 terms. For a larger number of terms the available memory was insufficient. This problem may however be partly overcome by a more structured programming of the same calculation program, resulting in a more sophisticated use of the memory available. Another solution is of course an expansion of this memory or switching to an even larger computer system.

Evaluation of this new calculation method of geometry factors was realised by calculating different geometry factors with this method and comparing the results with the solution given in Murakami (1987). Again the limitation of the calculation program formed by a constant number of three holes means a restriction in the possibilities. Nevertheless, two
interesting cases given by Murakami (1987) yield sufficient opportunity for evaluation. These two cases concern a row of three equally sized collinear cracks and a crack between two circular holes.

The comparison between the given solution and the newly calculated one can be found in tables 3.2.a and 3.2.b for a row of three collinear cracks with equal length. In the first table the solution of Murakami (1987) is presented, while in the second table the solution of the calculation is presented together with for each value of the geometry factor its percentage of deviation with respect to the first table. It can be seen that the maximum deviation between the two solutions is slightly less than 1 per cent which is a very satisfactory result. With truncation of the Taylor series after 8 terms and only observing 8 terms in the stress function (i.e. the original calculation program) a maximum deviation of 2.95 % is found, which is about three times higher. The difference between the given solution and the calculated one is largest if the cracks are closest to each other.

A comparison of the results for a crack in the middle of two equally sized circular holes is given by Figs. 3.2.a and 3.2.b. From these two figures also the good agreement between the given and the calculated solution can be noticed. The solution given in Murakami (1987) is based on a Laurent series expansion and is said to have an accuracy of less than 1 %. The figure based on the calculation method is realized by calculating the geometry factor in different points and subsequently drawing a graph through these results. A total of 143 calculated points is used for this single figure.

Both cases demonstrate the very good applicability of the present calculation method and it seems well possible to calculate some geometry factors which are not yet tabulated.

Since this report is dealing with a fuselage structure it seems logical to obtain a solution for the geometry factor for a crack in the window section of the fuselage. This area is the most heavily loaded part of the fuselage because of the disturbance of the stress distribution by the large cutouts to attach the windows. In the experimental part concerning the calculation of geometry factors some examples can be found (see chapter 4).
4 Residual Strength Tests on Unstiffened Panels.

References: none.

In this chapter the experiments performed on unstiffened panels will be presented. The experiments started off with a series of centre cracked specimens in order to obtain the fracture toughness of the laminate under study. Further some tests were carried out to gain knowledge on the fracture mechanism present at the crack tip. Finally some experiments were done in order to check upon the calculation of geometry factors as presented in section 3.2.

In the first section of this chapter a description of the test will be given while in the second section the results will be discussed.

4.1 Description of Tests.

As stated in the opening of this chapter, the first series of tests was performed in order to obtain the fracture toughness of the material. In the introduction of this report some restrictions that came forth from an earlier performed preliminary design on a composite airplane fuselage were mentioned. The restrictions which are important for the fracture toughness tests at issue consist of the choice of material and the fixed laminate lay-up.

In the preliminary design study it was found that the material to be used for the facings of the sandwich fuselage construction is a Carbon fibre reinforced 0-90/±45/0-90 laminate. Since Carbon/PEI CD282 in a 5 Harness weave, produced by Ten Cate composites, is readily available in the structures and materials laboratory this material was chosen in the preliminary design.

In order to obtain the desired fracture toughness data centre cracked plate tension specimens (CCT) were used (see Fig. 4.1.a for the basic configuration of these specimens). All centre cracked tension specimens were cut from a 500 mm x 500 mm large panel made out of the specific laminate. The laminae used to produce these panels were cut from a one metre wide roll of prepreg to the 500 mm squares, for each two squares cut in the direction of the roll one square had to be cut under an angle of 45 degrees. After cutting of the laminae the solvent remaining from the production of prepreg was damped out by placing the laminae in an air heated oven at 270 °C for about twelve hours. Next to this drying cycle the laminae were stacked to a 0-90/±45/0-90 lay-up and cured in a hot platen hydraulic press. For this purpose the press plates were heated to a temperature of 310° C, the stacked laminae were inserted and right after that a pressure of 15 to 20 bar was applied. After half an hour the plates were cooled down with the laminate in between them and the pressure still applied in order to consolidate the laminate. When temperature dropped below 100° C the press was opened and the laminate taken out. After this production cycle the laminates were ready to be tested.

First each laminate was provided with four aluminum tabs in order to enable proper clamping in the tensile testing device. Since each tab was 50 mm wide, the free length of the test specimens (i.e. the length between the two clappings) was 400 mm. After the tabs were attached, the 500 mm x 500 mm plates were cut to smaller specimens. Next
Damage Tolerance Aspects of a Full Composite Airplane Fuselage.

to this the tabs were provided with $10$ mm bolt holes and in the middle of each specimen a $1$ mm hole was drilled from which a crack was introduced using a fret saw. In this way a total of 25 specimens was produced with varying specimen widths and crack sizes. The exact measurements of each of these specimens will be given in section 4.2. In Fig. 4.1.b one of the finished test specimens is shown with the clamping plates attached to it by means of $10$ mm bolts.

After production of the test specimens the testing was performed in a stroke controlled tensile testing device, using a $0.01$ mm/sec crosshead speed. From the test $F_{\text{max}}$ was measured.

The second test series was utilised in order to gain some information about the failure mechanism at the crack tip. Because of the sudden rupture of the test specimens from the first series of tests practically no stable crack growth was observed. Therefore it was reasoned to provide a single test specimen with two similar cracks. If it is assumed that each crack experiences the same history during the tensile test and this assumption is combined with the knowledge that only one of the two cracks will show complete rupture after the test, studying the crack that is still intact will yield information about the phase just before failure.

At first this test method was applied to test specimens similar to those of the first test series with a free length of $400$ mm provided with two parallel cracks of $2a=25$ mm at a mutual distance of $10$ mm. The results of these test however showed a significant increase in the fracture toughness values obtained from the tests as will be discussed in the next section. In order to overcome this problem of an increase in fracture toughness, that might also indicate a change in failure mechanism, larger test specimens were necessary in order to enable a larger mutual distance between the two cracks.

Because the prepreg is provided on a one metre wide roll and the laminate construction dictates a $\pm 45$ degrees layer in the middle of the laminate, the total length of the test specimen is limited. A further restriction is formed by the maximum free length of the tensile testing device, which is about $1350$ mm. This free length will not only be occupied by the free length of the specimens but also partly by the clamping devices in which the specimen is to be tightened. As a consequence, it was chosen to produce the specimens out of a laminated plate with a length of $850$ mm which implies a maximum specimen width equal to $\frac{1}{2} \times (1000 - 1/2 \times \sqrt{2} \times 850) = 564$ mm, dictated by the $\pm 45^\circ$ layer.

A total of two of these large plates were produced in a similar fashion as discussed before (cut, dry, cure) although curing in the hot platen hydraulic press had to be replaced by curing in the autoclave because of the large panel size. In this autoclave curing cycle at first a vacuum was applied to the stacked laminae after which the laminate was inserted in the autoclave. After closure of the door a differential pressure of $10$ bar was applied and then the interior temperature of the autoclave was raised to $320^\circ$ C. This temperature was held for about $20$ minutes after which cooling down was allowed. When the laminate was at room temperature the pressure was released and the laminate was taken out.

Sequential to this laminate production the panels were provided with tabs and out of each panel three $160$ mm wide specimens were cut, leading to a total of six specimens.
Each of these specimens was provided with the necessary bolting holes and two similar parallel cracks perpendicular to the loading direction at a mutual distance of 200 mm. To enable placement of a clip gauge, suitable for crack opening displacement (COD) measurements, the central holes from which the crack was introduced were enlarged to φ3.2 mm and supplied with knife edges by means of a countersink drill. Before testing of the specimens each hole was checked on delaminations by means of C-scan measurements.

These larger double cracked test specimens were also tested in a stroke controlled tensile testing device, again using a 0.01 mm/sec crosshead speed. From the test the crack opening displacements of both cracks as well as $F_{\text{max}}$ were measured. After complete rupture of one of the cracks the cracked surface in the vicinity of the crack tip was studied using a scanning electron microscope (SEM) and the intact crack was once again investigated with a C-scan recording.

The third and last test series of this chapter consisted of six different specimens to check upon the calculation of geometry factors as discussed in section 3.2. In order find strength values which are significantly below those of a single crack, it was found that the test specimens had to be designed such that a geometry factor of at least 1.15 was found. Again, in order to be able to calculate the geometry factor, the situations that can be verified are restricted to a row of three holes or cracks or a combination of these geometries. Furthermore, it would be interesting to see if the calculation examples given in section 3.2 (a row of three equally sized collinear cracks and a crack between two holes) can be verified by experiment. These qualifications led to the following six specimens: 1) a single crack between two circular holes, 2) three equally sized collinear cracks, 3) two equal collinear cracks with a smaller crack in between, 4) a single crack centred between two elliptical holes, 5) a single excentric crack of the same size as in four between two elliptical holes, and 6) a single excentric crack of a larger size than in five between two elliptical holes.

Since the geometry factor is by definition only geometry dependent, applicability of the previously prescribed calculation method can in principle be tested with any material. However, when yielding of the material occurs deviation from the linear elastic material behaviour will be found and therefore a deviation from the stress intensity factor approach. To avoid this problem for the six test specimens that will be used here, the carbon/PEI laminate was chosen since earlier experiments on this material showed good agreement with the K-concept. The greatest handicap of this particular material is formed by the physical dimensions of the fibre bundles. To simulate an infinite plate within the given test equipment limits the total measurements of disturbing holes and cracks. Therefore, holes as well as cracks are approaching the micro scale of the total material while a check on the level of the macro scale is performed.

The first three specimens were produced out of 500 mm* 500 mm panels cured in the hot platen hydraulic press while the latter three were made out of the 850 mm* 560 mm panels cured in the autoclave. The elliptical holes in the last three specimens have a ratio between the short axis and the long axis similar to that of an airplane window while
the spacing between the holes is the scaled down window spacing. In this way an impression of the effects of damage between two windows can be obtained.

The intended measurements of each of the specimens are given in table 4.1.a together with the calculated geometry factors. In table 4.1.b the actual measurements of the specimens, measured after production, are given, again accompanied by their respective geometry factor. Although the geometry factors between the two situations are practically the same, the values of the second table will be used in further calculations since these values are in accordance with the correct measurements of the specimens. In Fig. 4.1.c test specimen 2 is shown after production.

Each specimen was provided with a set of five linear strain gauges to see if the infinite plate approach can be justified and in order to check a proper clamping of the specimen in the testing device. In some cases additional linear strain gauges were applied in order to be able to check on the strain calculation which can be performed with the calculation program. Furthermore, around some of the crack tips a K-gage was applied. The intention of the manufacturer of this K-gage is that it is possible to directly measure the geometry factor at the crack tip. In this work these K-gages are applied in order to check their applicability for this purpose.

All six specimens were tested up to failure in a stroke controlled tensile test bench with a crosshead speed of 0.002 mm/sec. In Fig. 4.1.d a picture of test specimen 6, fixed in the testing device, is shown while in Fig. 4.1.e the equipment for strain measurement is displayed.

After this description of the different test series the results together with a discussion on these results will be given in the next section.

4.2 Test Results.

The results obtained from the first and second series of experiments, from which the fracture toughness has to be deduced, can be found in table 4.2.a. The experiments from the second test series, the double cracked specimens with the lines of the cracks parallel to each other, perpendicular to the loading direction, are indicated in this table with a superscript 1 for cracks close to each other (a total of 3 specimens) and with a superscript 2 for cracks further apart (a total of 6 specimens). The width, the crack length, and the maximum force measured from the experiment are shown for each specimen, as well as the geometry factor and the calculated value of $K_{IC}$.

The given values of the geometry factors are calculated according to a relation proposed by Feddersen to correct for the finite panel width. This relation is based on experimental results and can be given as:

$$C = \sqrt{\frac{\sec \left( \frac{\pi a}{W} \right)}{\cos \left( \frac{\pi a}{W} \right)}}$$  \hspace{1cm} [4.1]
The values obtained by this method show very good agreement with another calculation method that is presented in Murakami (1987).

The values of $K_{IC}$ are calculated in accordance with Eq. [3.8]. If all specimens are taken into account this results in a mean mode I stress intensity factor of 34.23 MPa/m. A typical value of this factor for aluminium 7075-T6 alclad, under the condition of plane stress, is 66.8 MPa/m, which is approximately twice as high. It should be noted that the stress intensity factor for the Carbon/PEI laminate was obtained in the same direction as the principal material axes and that its value alters if these directions are changed. If the values are presented in a Feddersen approach in the same manner as discussed in the previous chapter, Fig. 4.2.a is found. In this figure also the standard deviation for the stress intensity factors is given from which a reasonably good mutual agreement can be concluded.

From better observation of the results in table 4.2.a it can be noticed that all three results of the double cracked specimens with the cracks close to each other lead to a rather high K-value. A good explanation for this fact was found in Murakami (1987) in which geometry factors for two parallel cracks aligned to tensile direction are presented. From this reference it can be found that in case of the three specimens, with $2a/d=2.5$, a geometry factor of approximately 0.75 is to be applied. If this value is used for the cases at issue their fracture toughnesses drop from the maximum measured value of approximately 38 down to 28.5 MPa/m which would mean a minimum value for the fracture toughness. In reality the fracture toughness for the three specimens in question will probably be somewhere between these two extreme values.

No large deviation from the mean value can be seen for the other six double cracked specimens. This can be dedicated to the fact that now the cracks are at respectable distance from each other. If once again the Feddersen approach is used for presentation of the mode I stress intensity factor, yet with leaving out the three specimens with cracks close to each other for the reason discussed above, Fig. 4.2.b is found. Note that now an even somewhat lower mean fracture toughness of 33.81 MPa/m is found and the standard deviation has greatly improved. In the continuation of this work this latter value of the fracture toughness will be used as the proper value for the Carbon/PEI material in a 0-90/±45/0-90 laminate construction. Especially in chapter six, where the properties of a stiffened panel are determined, this value of the fracture toughness plays an important role.

As stated in the previous section, during the tensile test of the double cracked specimen also crack opening displacement (COD) measurements were performed. Since in this method the opening at the crack tip plays the most important role, this method is also referred to as crack Tip opening displacement although the same abbreviation is used. The COD approach was first used by Wells in 1961. This approach is based on the fact that stresses and strains in the vicinity of the crack are responsible for failure. At the crack tips the stresses will exceed the yield strength of the material and plastic deformation will occur. Wells assumed that the stress at the crack tip always reaches the critical value, leading to a plastic zone in the crack tip region that controls fracture. A measure of the amount of crack tip plasticity is the displacement of the crack flanks,
especially at or very close to the tip. Therefore it might be expected that at the onset of fracture this COD has a characteristic critical value for a particular material and hence can be used as a fracture criterion.

The crack flank displacement $v$ at any position away from the crack tip is given with the following formula:

$$v = \frac{2\sigma}{E} \sqrt{a^2 - x^2} \quad \text{(plane stress)} \quad [4.2]$$

in which $a$ is half the crack length and $x$ is the coordinate along the crack line, measured from the centre of the crack. The total crack opening at a position on the crack can be given with $\delta = 2v$. If crack tip plasticity is not accounted for, the displacement at the crack tip, $v_t$, and the crack opening at the crack tip, $\delta_t$, are both equal to zero. If the existence of a plastic zone at the crack tip cannot be neglected, the following expression can be deduced for the displacement at the crack tip:

$$2v_t = \delta_t = \frac{8\sigma_{ys} a}{\pi E} \ln \sec \left( \frac{\pi \sigma}{2\sigma_{ys}} \right) \quad [4.3]$$

in which $\sigma_{ys}$ denotes the yield strength of the material. Details on the derivation of this formula will not be given here because it is far beyond the scope of this work. Eq. [4.3] is the starting point for most COD considerations in the literature. The major disadvantage of Eq. [4.3] is that it is valid only for infinite plates and it is not possible to obtain similar formulations for more practical geometries.

If $\sigma/\sigma_{ys}$ is much less than unity (linear elastic materials) the $\ln \sec$ term can be written as a series expansion. If now only the first term of this series expansion is considered an approximation for the crack tip opening displacement can be found. The expression for such an approximation is given by:

$$\delta_t = \frac{\pi \sigma^2 a}{E \sigma_{ys}} = \frac{K_1^2}{E \sigma_{ys}} \quad [4.4]$$

from which directly a relation between the COD and the linear elastic fracture mechanics can be noticed.

In the literature further extension of the theory on COD measurements can be found. However, since the experimental part of this work is directed only to composite material, for which no suitable expressions for the crack opening displacement could be found, a further extension of the theory is not considered very useful to this work.

As stated in section 4.1, a total of six specimens were provided with a clip gauge for COD measurements. Each of these specimens contained two cracks of the same length with the respective clip gauges attached at the centres of the cracks (i.e. $x=0$ in Eq. [4.2]). In the experiments three different $2a/W$ ratios were used. From this test series it can be concluded that the COD measurements provide good reproducibility of the results. At
present it is not clear however how these results can be used in the fracture mechanics research of the material under study. Deviation from a linear curve in the region of higher forces can probably be explained by the very irregular fracture surface which is formed when the crack extends. If this explanation holds it implies that in the linear part of the curve no crack extension occurred but only a crack opening by increasing force and that in this material only a small amount of crack extension is found at a relatively high load as compared to the load at failure.

Other additional tests on the double cracked tension specimens concerned the failure mechanism at the crack tip. Information on this subject was accomplished by studying both cracks before and after failure of the specimens. As stated in the previous section, for this purpose c-scan recordings were used for a check upon delaminations and the fracture surface was studied with the use of a scanning electron microscope. The area around all cracks was scanned for delaminations before the tensile test was performed. This led to a total of 12 c-scan plots, two for each specimen. From these plots it can be concluded that practically no delaminations were present before the test was performed. Only in some cases very small delaminations can be noticed at the crack edges which were probably initiated together with the introduction of the crack. In all cases the area away from the crack was very good, leading to the conclusion that the production cycle of the test specimens was well performed. After tensile test to failure of each specimen the crack that was still intact was studied again. Visual inspection of those cracks showed no change in the form of the crack, no crack extension or whatsoever. After visual inspection the crack was scanned again and the resulting plot was compared with the earlier one for the same crack. If any difference between the two plots could be found it was certainly negligible and this led to the conclusion that the tensile stress applied to the specimens did not introduce any delaminations.

The fracture surfaces of the cracks that failed were investigated with the use of a scanning electron microscope in the area close to the crack tip (see Figs. 4.2.c to 4.2.e). From the figures the very bad interfacial bonding between the fibres and the matrix material should be noted.

In Fig. 4.2.c a top view of the fracture surface is shown. At the left hand side of this figure the crack tip can be noticed and from the remaining part it can be seen that the fracture surface is very irregular.

It should be noted that, although the laminate is built up from only three layers, it seems as if the total laminate consists of six layers. At the top of the figure a layer with fibres more or less in the direction of the line of the crack can be noticed, followed by a layer with the fibre direction perpendicular to the plane of the figure. In the middle of the figure two layers of fibres can be noticed with an angle of +45 and -45 degrees with respect to the plane of the figure. Then again a fibre layer perpendicular to the figure can be seen, accompanied with a layer in the direction of the crack which is not so clear due to a speck of dust at the lower surface of the specimen.

A good explanation of the fact that it seems that the laminate consists of six layers can of course be found in the recognition that each of the three layers involve a woven fabric which contains as many fibres in one single direction as fibres perpendicular to this
direction. In the outer layer of the crack surface at the lower part of Fig. 4.2.c a cross-over of the fibre bundles can be noticed by shifting of the fibres perpendicular to the plane of the figure from the inner part of the layer (at the left hand side of the figure) to the outer part of the layer (at the right hand side).

In Fig. 4.2.d another view on the crack surface is presented. Here the camera position is just above the crack. In this figure a rather large part of the crack can be seen, extending from the left hand side of the figure to approximately two thirds of the picture. In this figure especially the protruding part at the outer surface of the laminate is of interest since from the remaining imprint of matrix material at this part it can be concluded that intralaminar fracture has occurred.

From investigation of more fracture surfaces it can be concluded that this form of intralaminar failure took place very frequently. Only in limited cases interlaminar fracture was observed although this mechanism also occurred (see Fig. 4.2.e). A clarification of this follows from the fact that between the two consisting layers of each lamina the fibre directions differ maximum (i.e. 90°).

Also the crossing of fibres over each other, necessary to obtain a fabric, contributes to the loss of strength in fibre direction. A possible solution to this problem is to build up the skins of the fuselage from unidirectional prepregs. This would eliminate the crossing of the fibres and provides for a possibility to alter the stacking sequence of the different layers, which can reduce the giant leap in fibre directions from 90 degrees to 45 degrees as a minimum if the same lamina angles are to be used.

At the Fokker airplane company an engineering guideline for the constructor of composite parts says that the change in fibre direction of two neighbouring layers never may exceed the 45 degrees limit. From this point of view it can also be concluded that a woven fabric in which the fibres are perpendicular to each other never can be used in a vulnerable airplane structure.

Now only the results from the test series to check upon the calculated geometry factors remain to be handled. Once again it is mentioned here that the results of the calculations of the geometry factor of all six experiments can be found in table 4.1.b. In the evaluation of the results the mode I stress intensity factor of 33.81 MPa/m, found at the beginning of this section, will be used. At first the calculation of the geometry factors will be checked and after that the results from the strain gauge and K-gage measurements will be evaluated.

The results obtained for the verification of the geometry factor calculations can be found in table 4.2.b. In this table the calculated geometry factor is copied from table 4.1.b. The width of the specimens follows from the size of the basic panel from which the specimens were produced.

It can be noted that for all specimens the thickness is the same. In reality this is not true since after production of the laminates the thickness can vary slightly over the surface. However, because each panel is made out of three layers of woven fabric, it may be assumed that at each position of the panel the same amount of fibres is found. These fibres ultimately determine the maximum allowable stress and strain and therefore the
thickness can be chosen as a constant. Furthermore, in the determination of the critical stress intensity factor of the laminate, the same thickness was used and therefore the effect of this parameter always cancels out in the calculations.

Table 4.2.b continues with the crack length of the most critical crack that can be found in the specimen. Calculation of the geometry factor at each crack tip showed that this critical crack was always the central crack. This observation could be expected from the results of earlier calculations shown in tables 3.2.a and 3.2.b. $F_{\text{max}}$ is measured from the experiments and $C_{\text{test}}$ is calculated with the use of Eq. [3.8] and the critical mode I stress intensity factor obtained from earlier experiments. For specimen 6 no $F_{\text{max}}$ was measured since the test was stopped before final failure of the specimen in order to preserve it for possible further research.

The last column of table 4.2.b contains the ratio between the measured and the calculated value of the geometry factor. From this ratios it can be concluded that especially in the case of the specimens with three cracks deviation of experiment from the theory is large. From the earlier calculations performed to compare the outcome of the calculation with values of the geometry factors found in literature, it may be concluded that the calculated values are almost perfect and therefore another explanation for the difference between calculated and experimental geometry factors must be given. This explanation can probably be found in the earlier mentioned microscale of the composite material. Especially in the case of a crack, where one of the dimensions is assumed to be zero, the microscale of the material can certainly not be neglected. The results of specimens 4 and 5 agree reasonably well with the calculated geometry factors and specimen 1 can be considered as quite successful.

Next the result of the strain gauge measurements will be evaluated. In Figs. 4.2.f to 4.2.k the strains are plotted as a function of the applied force. In these figures a reduction of data points was applied for ease of survey, however, in order not to alter the path of the curve, this reduction was only applied in the linear part of the curves.

From the figures it can be noted that above a certain value of the applied force, the relation between the measured strains and the applied force is purely linear. The reason for an onset in the strain values can be found in the fact that the specimens were slightly curved. Therefore, in the region of low applied forces, it is possible that the laminate under certain strain gauges is not yet (fully) stressed, thus producing a somewhat strange behaviour at lower forces. During the experiments it was noticed that especially at the two free edges of each test specimen this effect plays an important role. Sometimes it took an applied force of at least 7.5 kN to stress these free edges.

The curvature of the specimens comes forth from the curvature of the prepreg, used to produce the laminate. Since this prepreg is provided on a large roll, a piece that is cut from the roll is curved in the rolling direction. If a symmetric laminate with an even number of layers is produced, it is possible to stack the layers so as to avoid curvature of the total laminate by letting the curvature of each single layer cancel out to each other. If however an unsymmetric laminate or a laminate with an odd number of layers is produced, as in the present work, it is not possible to escape some amount of curvature. After the curing cycle of the laminate it is not perfectly flat.
From the Figs. 4.2.f to 4.2.k it can be noted that in all cases the lines through the data points of the strain gauges 1 and 3 are more or less parallel to each other. A similar story goes for the gauges 2, 4 and 5. However, the first two gauges show a stronger increase in local strain with an increase in the force applied at the specimen edge than the latter three gauges. If linear elastic material behaviour is assumed, and it is recognised that zero applied force automatically results in zero strain, it can be concluded from these observations of linear progression that the strains at the positions 1 and 3 are slightly higher than those at the positions 2, 4 and 5. Since the stiffness of the material is invariable with the position on the specimen, also the local stress in positions 1 and 3 will be higher than in the other three positions.

As stated in section 3.2, if experiments on any material are to be used for residual strength analysis, strain gauges can be applied to determine whether or not an infinite plate solution can be used. From the strain measurements of gauges 1 to 5 it may be concluded that an infinite plate yields at least a good approximation for the present test specimens. The remaining strain gauges that were attached at the boundary of some of the holes were only used for an evaluation of the strain calculations.

In order to evaluate the applicability of the stress calculation program the strain for each strain gauge position was calculated and compared with the values found during the experiments. The strain that is determined in the calculation program is taken as the average of ten positions on a line perpendicular to the strain direction for reasons of accuracy. The distance between those positions is exactly similar to the distance between the threads of a single strain gauge, as measured with a microscope. The middle of the ten positions is placed at the centre of the actual strain gauge.

The moduli of elasticity that were used in the calculations follow from Hoek (1992). In this reference the moduli of a single layer of the fabric were obtained from a four point bending test. A difference was made between the moduli of elasticity in warp and in weft direction. Since the differences between these values were only small, here only the average value will be used. The shear modulus and the Poisson’s ratio were obtained from an official ten Cate data sheet. The thickness used in the calculations was considered similar for all specimens for earlier mentioned reasons. The following data (with respect to tensile behaviour) were applied for each layer:

<table>
<thead>
<tr>
<th>$E_x$ [GPa]</th>
<th>$E_y$ [GPa]</th>
<th>$G_{xy}$ [GPa]</th>
<th>$\nu_{xy}$</th>
<th>$t$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>56.0</td>
<td>56.0</td>
<td>5.2</td>
<td>0.27</td>
<td>0.305</td>
</tr>
</tbody>
</table>

Since the experiments as well as the measurements show a linear behaviour, it is sufficient to compare the experimental and calculated strain values for only one value of the applied force. The only condition for this value is that it should be sufficiently high to reach the area of linear strains in the experiments. In table 4.2.c both the measured strains as well as the calculated ones can be found. It should be noted here that this table also contains the K-gage measurements that will be discussed further on.
From the strain gauge results it can be concluded that generally a good agreement between the measured and the calculated strains is obtained. Only for case 2 the deviations are very large (measured strain is approximately twice the calculated value) and for this no good explanation could be found. In other cases where large deviations between the measured values and the calculated ones are observed (e.g. gauge 3 of case 3, gauge 3 of case 5, and gauge 5 of case 6) an explanation is found in the earlier mentioned onset necessary to stress the gauges. It can be noted from the figures of the strain gauge measurements that once the strain gauge recordings become linear with the applied force, the slope is approximately equal for all strain gauge recordings. If it is again realised that zero force results in zero strain, this slope plays more important role than the absolute value of the measured strain. If a correction for the start values is made, measured and calculated strains agree quite well.

The difference between theoretical and recorded strain is somewhat larger for the gauges applied at the opening edge of the holes and in the row of gauges on specimen 5. This difference is due to a large stress gradient near the holes. As a result of this, slight misplacement of the gauge, which can be effected in gauge position as well as in gauge angle, will yield large deviations in the measured strains. Furthermore, in this area close to the holes the modelling of the strain gauge as 10 discrete positions might be too rough.

The calculated strain values are within 10 per cent accuracy for other strain gauges. In some cases deviations from less than one per cent were found. From consideration of all strain values presented in table 4.2.c it can be concluded that in general the calculated strains agree very well with the measured values although over all the strains predicted by the calculations are slightly higher than the experimental values.

Now only the K-gage recordings remain to be discussed. These K-gages were developed in order to directly measure the geometry factor for a crack tip from an experiment. The great advantage of this is that even for difficult geometries, for which no geometry factor solution is readily available, or, for instance, during the service life of an aircraft, the stress intensity factor can be directly obtained from a single measurement.

As stated in section 3.2, the K-gage consists of 92 positions in a circle around the crack tip at which the strain is measured. Since these positions are connected to each other through one single thread, the K-gage can be looked upon as a simple strain gauge. The modelling of a K-gage is then similar to that of a normal strain gauge only now the average strain value over a circle of 92 positions is to be obtained instead of over a straight line consisting of 10 positions.

One single K-gage contains 4 bridges for strain measurements. The inner two of these bridges, which are closest to the crack tip, are oriented in the direction of the crack and can therefore be used to obtain the mode II stress intensity factor. The outer two bridges are oriented in a direction perpendicular to the crack and can be used for mode I stress intensity factors.

As in the case of the normal strain gauges, the measured values are compared with calculations. The results of these calculations can again be found in table 4.2.c. Once again, test specimen 2 shows a great difference in the measured and the calculated
values. In general, it can be noted that the strain calculation of the mode I bridge is in better agreement with the recorded values.

In normal practice, when a K-gage is applied to obtain the stress intensity factor for a particular situation, first a test series to obtain K-gage reference values is conducted. These tests consist of K-gage readings on a large plates made out of the same material as the real structure and provided with a crack of a certain length. From these measurements a so called gauge factor can be obtained defined as \( B = K/\varepsilon \). In which \( K \) stands for the known stress intensity factor for a crack with a certain length (see Eq. [3.8] with \( C = 1.0 \)) in an infinite plate and \( \varepsilon \) is the (strain) reading of the K-gage. If this gauge factor is tabulated or given in a graphical presentation the in-the-field K-gage measurements can directly be transformed to the stress intensity factor at the crack tip. In order to check the applicability of the K-gage for the material under study a table of the gauge factor, or at least K-gage readings on large reference plates with crack lengths similar to the lengths in the test series, should therefore be known. Since this information is not readily available for the material under study, evaluation of the K-gage in this sense is not possible.

It is possible however, to obtain a gauge factor from the stress calculation program with which the results can be checked. This calculated gauge factor can be obtained in a way similar to the one prescribed for the K-gage. In table 4.2.d the calculated strain readings for the mode I bridges of the K-gage are given, together with the calculated geometry factors and the resulting gauge factors. In this table also the cross-sectional area of the specimens is given, necessary to calculate the stress. This cross-section is obtained by multiplying the panel width with the value of the thickness, which again is chosen as a constant value of 0.915 mm.

Since for the calculation the infinite plate solution for a crack was used, as described in part B.4 from appendix B, these calculated gauge factors are at least very close to reality. If now the crack in the disturbed stress field is considered, the gauge recordings can be transformed into the K-factor by multiplying them with this gauge factor. The results of this are shown in table 4.2.e.

The columns with the strain values are taken from table 4.2.c and from these values the stress intensity factor is obtained by multiplying them with the gauge factor of table 4.2.d. The average value of the two (at either side of the crack tip) mode I bridges was taken for the gauge readings from the experiments. For case 6, where two K-gages were applied, the values of the right hand side of the crack were selected. The two resulting K-values can be found in the table. The ratio between the K values obtained by this method is similar to the ratio observed in the strain values since the values were only multiplied with a constant factor.

The last two columns of table 4.2.e contain the calculated geometry factor as calculated in table 4.1.b and the stress intensity factor calculated by multiplying the geometry factor with the stress intensity factor for an infinite plate.

From the table it can be concluded that the K-gage does not yield proper results. Even if both the K-gage reading and the reference stress intensity factor are calculated with the calculation program, the deviation between the values is large. This observation is
in accordance with earlier observations that only for very small values of r/a the stress intensity factor approaches to one distinct value. The bridges of the strain gauge are too far from the crack tip to come to a reliable stress intensity factor reading. Only for very large cracks it might be expected that the K-gage as used in the experiments can be applied.

From this present chapter it can be concluded that a linear elastic fracture mechanics approach can be applied to composite materials. However, as stated in the previous chapter, the fracture toughness may vary with the angle between the crack and the principal material axes of the material. From the experiments, in which the crack was always along one of the principal material axes, a fracture toughness of 33.81 MPa/m was found.

The COD measurements were not found useful for the investigation in this work. A good interpretation for the obtained values could not be found for the composite material under study. However, from the COD measurements it might be concluded that some amount of crack growth occurs in the very last stage of the loading. The tests on the failure mechanism showed that mostly intralaminar fracture occurred and that final fracture occurred very sudden, not showing any amount of damage just before fracture. From the test series to check the calculated value of the geometry factor it can be concluded that if the crack was between two holes the experiments agree rather well with the predicted values, however, for the situations of three collinear cracks large deviations were found.

Observation of the measured strains in the specimens to check upon the geometry factor showed a good similarity with calculated strain values. Both for a normal strain gauge as for a K-gage the results were satisfactory. From the results of the calculations it was concluded that the application of the K-gage is not possible for the situations under consideration since the circle along which the strains are measured is too far from the crack tip.
5 Theory on Stiffened Panels.


As stated in the introduction of this report and shown by the results of the previous chapter, the fracture toughness of the Carbon/PEI skin material is very poor. Therefore suitable measures must be taken to overcome this problem or at least to make it survivable.

From the Airworthiness requirements the following statement can be quoted: "Practical design measures to minimize the risk of catastrophic damage may include for example, possible redundant design of crack stoppers to limit the dynamic propagation of tears which have been caused by debris impact". This requirement comes forth from JAR 25.903 in which the hazards of an uncontained engine failure are under consideration but, if this requirement can be extended to all kinds of impact damage, it forms a good potential to deal with the problems discussed in this work. It should be stated here however that, due to the extent of damage caused by impact of engine debris, the problem of uncontained engine failure may not be (fully) accounted for with the solution of damage handling as presented in this report. Probably, the area which is under danger in case of uncontained engine failure must be protected in a better manner.

Since in this work cracks are used to model the worst case of impact damage, which can occur during the aircraft service life, the term damage stopper rather than crack stopper is adopted because it better describes the function of the additional elements. Nevertheless, the statement from the requirements as mentioned above yields a relatively easy possibility to meet the requirements without a complete change of the present fuselage design. With sufficiently small damage stoppers, the complete sandwich construction can be preserved. The damage stopper should be designed to prevent the existing damage to grow to any catastrophic size.

The function of the damage stopper is to take over part of the load of the damaged skin, thus reducing the stress around the damage or, in case of a single crack, to reduce the crack tip stress intensity factor. Since the damage stoppers cause the same in-plane effects on the skin as the application of stringers and frames in a conventional fuselage structure, it is very useful to briefly discuss the theoretical background on stiffened panels. With this knowledge a better understanding on the use of different damage stoppers is obtained which can be useful in the design process of the ultimate damage tolerant panel. This general description of the theory on stiffened panels will be given in section 5.1 while in section 5.2 a calculation performed on a stiffened panel will be briefly discussed.

5.1 General Description.

Similar to the previous chapters, a state of plane stress will be considered. Once the residual strength properties of an unstiffened panel are known, the residual strength properties of a stiffened panel can be predicted fairly accurate.
For application to stiffened panels it is not strictly necessary that the events described obey a constant K concept, i.e. $K_{tp}$, $K_{te}$, and $K_{te}$ need not be constants, but it is useful to describe the events in terms of the stress intensity factor. This problem will be discussed later in this chapter in more detail.

When the panel is equipped with stringers the stress distribution in the cracked region is altered. The stringers provide extra stiffness that tends to decrease the stress at the crack tip by load transfer from sheet to stringer. The higher the load in the stringer near the crack tip, the more the stress intensity factor of the sheet material is reduced. From this, it is obvious that calculation of the sheet-stringer interaction forces is an essential part in the prediction of the residual strength properties of a stiffened panel.

With respect to the sheet-stringer interaction two significant dimensionless parameters can be introduced, namely the tip stress reduction factor $C_R$ and the stringer load concentration factor $L_S$.

The tip stress reduction factor $C_R$ is defined as the ratio of stress intensity factors for sheets with and without stringers:

$$C_R = \frac{K_{\text{stiffened}}}{K_{\text{unstiffened}}} < 1 \quad [5.1]$$

Observation of this formula shows that the stress tip reduction factor can also be seen as a geometry factor. Instead of the influence of other holes or cracks on the crack tip stress distribution, as in the previous chapters, this factor gives the influence of the presence of stringers on the crack tip stress distribution.

Since the value of $C_R$ is generally smaller than one, the stress intensity factor will be reduced by the presence of the stringer, thus increasing the residual strength. Only in case of a broken stiffener and a relatively small crack in the vicinity of this broken stiffener the value of $C_R$ can be higher than one. According to formula [5.1] this will lead to an increased stress intensity factor as compared to an unstiffened sheet. This effect is due to the fact that the load, which is normally carried by the stringer, is now transferred to the skin, thus locally enlarging the prevailing stress field. When the crack starts to grow the stringers that are still intact will take over part of the load hence again reducing the crack tip stress intensity factor to a level below that of an unstiffened sheet.

The stringer load concentration factor is defined as the ratio of the maximum stringer load and the load in the stringer remote from the cracked section:

$$L_S = \left(\frac{F_{\text{max}}}{F_{\infty}}\right)_{\text{stiffener}} > 1 \quad [5.2]$$
Since the value of $L_S$ is greater than one, the load in the stringer will locally be increased. This increase is largest at the position on the stringer which lies on the line of the crack. With this local increase of the stringer load, the structure might become stringer critical. Problems of this kind will be addressed at a later stage.

The values of $C_R$ and $L_S$ depend upon the stiffening ratio, the stiffness of the attachment, and the ratio of crack length to stringer spacing. In section 5.2 a calculation program will be discussed which is able to calculate the values of $C_R$ and $L_S$ for various configurations of cracked panels. With this program also one configuration of a stiffened C/PEI panel with the skin material made out of the for this work so important 0-90/±45/0-90 laminate will be evaluated. However, for a qualitative discussion it is sufficient to know how the values of $C_R$ and $L_S$ vary with varying stiffening ratio and/or crack length. Such a qualitative representation is given in Figs. 5.1.a and 5.1.b where $C_R$ and $L_S$ as a function of the crack size are presented respectively.

In Fig. 5.1.c a residual strength diagram of a panel with two stiffeners and a central crack is given. In this figure the lines a,b and c represent the residual strength curves for an unstiffened panel (compare with Fig. 3.1.h). As said before, the stringers cause a reduction in the stress intensity factor by a factor $C_R < 1$. If it is assumed that crack propagation in the stiffened panel occurs at the same stress intensity factor as in the unstiffened panel, the stress to propagate a crack will be increased by a factor $1/C_R$. This means that the lines a and c will be raised to e and f, respectively. These curves e and f show a maximum for a crack slightly larger than the stringer spacing because the maximum tip stress reduction occurs when the crack extends slightly beyond the stringer centre line (see Fig. 5.1.a).

As stated before, the possibility of stringer failure should also be considered in case of a stiffened panel. Stringer failure is presented by line g in Fig. 5.1.c. When there is no crack the stringer will fail at the ultimate tensile strength, which in case of Fig. 5.1.c is equal for both stringer and skin material. When the crack approaches the stringer, the load concentration factor $L_S$ will increase, so that the stringer will fail at a lower nominal stress. The line g is therefore determined by the equation:

$$\sigma_{\text{failure}} = \frac{\sigma_{\text{ultimate tensile strength stiffener}}}{L_S} \quad [5.3]$$

In this equation it is assumed that no yielding occurs in the stringer before final failure. If yielding does occur $L_S$ has to be corrected for stress values which exceed the yielding stress.

In principle, the residual strength diagram is fully determined by the lines e,f and g. Therefore, some examples will be given from which the reaction of the panel on an increased loading for different initial crack sizes becomes clear.

If the crack is still small at the onset of instability ($2a < \approx 2s$, in which $2s$ is the stringer spacing), the stress condition at the crack tip will hardly be influenced by the stringer and the stress at unstable crack initiation will be the same as that for an unstiffened panel of the same size. When the unstably growing crack reaches the stiffener, the load concentration of the stiffener will be so high that the stiffener fails without stopping the unstable crack growth (see line ABCD in Fig. 5.1.c).
In case of a crack that extends almost from one stiffener to the other (2a=2s), the stringer will be very effective in reducing the peak stress at the crack tips (this means that $C_R$ is small), resulting in a higher value of the stress at crack growth initiation at point F (Fig. 5.1.c). With increasing load, the crack will grow stably to the stiffener (line EFGH) and due to the increase in stiffener effectiveness, the crack growth will remain stable. (For cracks larger than 2a$_2$, no unstable crack growth will occur). Fracture of the panel will in this case occur at a stress level $\bar{\sigma}$ due to the fact that the stiffener has reached its failure stress and the stress reduction in the skin is no longer effective after stringer failure.

For cracks of intermediate size (2a=2a$_1$, see Fig. 5.1.c), there will be unstable crack growth at a stress level slightly above the fracture strength of the unstiffened sheet (point M) but this will be stopped under the stiffeners at N. After this crack arrest the panel load can be further increased at the cost of some additional stable crack growth until H, where the ultimate stringer load is reached, again at the stress level $\bar{\sigma}$.

The actual residual strength curve for the simple panel presented in Fig. 5.1.c is indicated by the solid line. This curve contains a horizontal part which is determined by the intersection of the lines e and g. This flat part of the curve constitutes a lower bound of the residual strength.

Since the tip stress reduction factor ($C_R$) and the stringer load concentration factor ($L_S$) depend on the stiffening ratio (see Figs. 5.1.a and 5.1.b) this implies that the residual strength diagram presented in Fig. 5.1.c is not unique. In this figure the stringer failure is the critical event. Skin failure may become the critical event for other stiffening ratios (see Fig. 5.1.d). In this case, due to a low stringer load concentration factor, the lines e and g do not intersect. A crack with size 2a$_1$ will show stable crack growth at point B and becomes unstable at point C. At point D crack arrest occurs from where again further slow crack growth can occur if the load is raised. Finally at point E the crack will become unstable again, resulting in final panel failure.

Another situation in which no intersection of the lines of skin failure and stringer failure can be found is introduced when the line for stiffener failure lies totally below the one for skin failure. In this situation failure of the stiffener always occurs before the skin fracture toughness will be fully exhausted. The subsequent failure mode of the panel will depend on the relative location of other stiffeners.

Apparently, a criterion for crack arrest has to include the two alternatives of stringer failure and skin failure, depending on the relative stiffness of sheet and stringer.

As mentioned before, the stiffness of the attachment also influences the values of $C_R$ and $L_S$. The effect of fastener failure is depicted in Fig. 5.1.e. In this figure the line h presents the line of fastener failure. Since at zero crack length the fasteners do not carry any load, this line tends to infinity for 2a$_a$=0. For the situation given in Fig. 5.1.e the residual strength is no longer solely determined by stringer failure (dashed horizontal line through H) but possibly by fastener failure (point K). A crack of length 2a$_1$ will show slow crack growth from E to F and unstable growth from F to G. After crack arrest at G further slow crack growth occurs until at K the fasteners fail. This fastener failure will lead to a new residual strength diagram in which the stringers become less effective in
taking over part of the load. As a result of this line f will be lowered to f' and line g will
be raised to g'. As long as the intersection of these new lines (H') is above the point of
fastener failure (K) the residual strength will still be determined by stringer failure at H'.
In case of riveted stringers, fastener failure will be very easy to imagine but also for
bonded stringers a kind of fastener failure can occur, as a result of local debonding of
the stringer in the region of the crack tip. Tests performed by Vlieger (1983) showed that
even a small amount of debonding (10 mm at either side of the line of the crack) already
had a large effect on the behaviour of the panels during the residual strength test. Not
only faster stable crack growth was observed but also a change in the behaviour at
fracture instability. In case of panels with intact bonding layers, skin and stringers
apparently failed at the same stress level. For (partly) debonded panels instability of the
skin crack made it extend until the panel edges, leaving all the stiffeners intact.

Of course, when composite materials are used, offering the possibility of an integrally
manufactured structure, fastener failure can be avoided or at least be brought back to
a non-critical level. When the same matrix material is used skin and stringer can be
coupled in a single production cycle. To increase the bond strength, the stringers can also
be stitched to the skin before curing. Another possibility, which can only be used if the
stringers are sufficiently flat, is addition of the stiffening material in between the
different layers of the skin.

Up till now, only a simple cracked panel with only two stringers has been discussed. The
behaviour of panels with more stringers is not essentially different from this simple panel.
Therefore a graphical representation of the influence of two stringers on each side of the
crack is considered sufficient. This is shown in Fig. 5.1.f for different stringer stiffening
ratios. It should be noted that in this figure it is assumed that, even as the crack extends
to twice the stringer spacing, all stringers remain intact. Especially the stringer closest
to the centre of the crack may become heavily loaded and therefore stringer failure (or
fastener failure) might occur, leading to a new residual strength diagram.

In the present work, where instead of stiffeners damage stops are considered which
are used to obtain damage tolerance of the structure, the stress level which confines the
lower bound of the residual strength should be sufficiently high. Furthermore, if crack
arrest undoubtedly takes place, it may be desirable that the difference between the stress
level at which onset of crack growth occurs and the stress level at which ultimate fracture
takes place is large in order to simplify the detection of cracks or damage. Under these
circumstances crack extension may occur in the early crack life after which crack arrest
follows and a stable situation arises with sufficient load carrying capabilities.

From the damage tolerance point of view the most important requirement that can be
made for the stiffened panel is that the chances of reaching the stress level at which final
fracture of the panel occurs are extremely remote. After the crack has reached the
stiffeners it can well be expected that it will be detected during a walk around inspection.
However, it is still important that when such a situation occurs the airplane can safely
continue its flight.
5.2 Calculations on a Stiffened Panel.

As stated in the previous section, the stress intensity of a flat stiffened panel with a crack is affected by the presence of the stringers. The influence of these stiffeners can be determined by means of a displacement compatibility analysis. The essence of this method is that the displacements in the cracked sheet are made compatible with the displacements in the stiffeners at the locations of the fasteners.

The general procedure which is applied in the displacement compatibility analysis of a reinforced panel is illustrated in upper part of Fig. 5.2.a. In this figure the fastener forces are denoted by an F, and the displacements of the sheet and stiffener are denoted by \( v_{sh} \) and \( v_{st} \) respectively. The fastener shear displacement, which is not given in the figure, is denoted by \( v_f \) here.

The displacements in the sheet, \( v_{sh} \), are obtained by a superposition of the three situations shown in the lower part of Fig. 5.2.a. These three situations can be described as follows: 1) \( v_a \), the displacement in the cracked sheet caused by the remote stress \( \sigma \), 2) \( v_b \), the displacements in the uncracked sheet caused by the stiffener fastener loads \( F \), and 3) \( v_c \), the displacements in the cracked sheet caused by stresses applied to the crack faces equal and opposite to the stresses caused by the fastener loads. These stresses must be exerted on the crack edges to provide stress-free crack edges.

The displacements in the stringers, \( v_{st} \), depend on the axial loads and bending introduced by the fasteners in combination with the axial loads already present in the stringer.

The displacements of the fasteners, \( v_f \), are obtained by choosing a linear approximation of the elastic-plastic behaviour in case of a rivetted joint and an approximation by two linear parts of the displacement curve in the case of a bonded joint.

The still unknown interaction forces \( F \) can now be determined by the sheet-stringer compatibility in the corresponding attachment points. If each of these points is denoted by a unique number, say \( k \), the compatibility condition can be given with:

\[
v_{sh}^{(k)} - v_{st}^{(k)} + v_f^{(k)} = 0, \quad k = 1, \ldots, \text{(total number of fasteners)}
\]  \hspace{1cm} [5.4]

and if this equation is used in combination with appropriate expressions for the earlier mentioned displacements, the interaction forces can be found. More details on the calculation procedure can be found in Bos (1988) and Broek et al. (1974). In these publications also a comprehensive enumeration of the different formulations for the displacements can be found.

The positions of the fasteners are fully determined for riveted structures. In case of a bonded structure however, no discrete fastener locations can be indicated. Therefore, when a bond layer is applied this layer has to be divided into a number of separate segments and it has to be assumed that the shear stress is constant over each individual segment. Particularly in the vicinity of the crack the segments should be sufficiently small in order to justify this assumption. In Fig. 5.2.b this split of the bond layer is shown. In this figure the bond layer is only split along its length axis and expressions for the displacements at the edge of each segment are used in the calculation procedure. Since
the variation in fastener forces is largest in the vicinity of the crack, the length of the bond elements are smallest closest to the line of the crack and increase quadratically with the distance to this line. It is obvious that a division of the bond layer in discrete points yields a rough simplification of reality. To meet this handicap to some degree, the bond layer can be further divided in the direction of the crack line (i.e. perpendicular to the length axis of the bond layer) thus yielding more points of attachment.

Once the interaction forces are calculated by means of the displacement compatibility method, the crack tip stress intensity factor can be found by considering the cracked sheet as a superposition of the three situations given in the lower part of Fig. 5.2.a. The first part is equal to an unstiffened panel with a crack of length 2a loaded by a remote stress. For this situation Eq. [3.8] holds with C = 1. The second part does not contribute to the stress intensity factor since under this condition no crack is present. The third part produces again a part of the total stress intensity, the formulation of which is not of much interest to the present work.

The calculation method as illustrated above can be implemented in a calculation program which, after providing basic material properties and measurements of a panel with a crack, can calculate the stress tip reduction factor $C_R$ and the stringer load concentration factor $L_S$. Such a program was written under the name of 'CRACKS' at the Delft University of Technology by Bos (1988) in the program language fortran. Comparison of the required parameters calculated with this program and those calculate with both a similar program written at Fokker (FRACTA) as well as a program written at the NLR (BOND) showed the validity of the program CRACKS.

In the next chapter, in which the experiments on a stiffened panel are presented, the program CRACKS will be used to perform a calculation on the Carbon/PEI panel provided with damage stoppers. The result of this calculation are presented in Figs. 6.1.b and 6.1.c. A further discussion on this will also be given in the next chapter.
6 Residual Strength Tests on Stiffened Panels.

References: None.

6.1 Development of Test Specimen.

The starting-point of this investigation is a sandwich fuselage structure, without stringers or frames. To give the structure sufficient damage tolerance some kind of damage stoppers have to be applied. Since these damage stoppers have to be incorporated within the sandwich shells, only stiffeners which do not greatly alter the thickness of the facings can be applied. This immediately results in sections consisting of unidirectional tape added to the facings. Any other appearance of stiffener elements will completely change the design of the fuselage.

As prescribed in chapter 5, the computer program CRACKS (Bos (1988)) can give a first impression of the application of stiffening elements to the skin. Especially the influence of the material properties (e.g. variation of stiffnesses) of the applied stiffening elements is of interest as well as the effect of the different dimensions of the stiffeners and the skin.

The damage stoppers that were applied to the Carbon/PEI skin material were made out of strips of Unidirectional Carbon/PEI with a fibre direction parallel to the length axis of these stiffening elements. The basic material for these stringers came from a large roll of prepreg with a width of 240 mm and the fibres in the rolling direction. To produce these damage stoppers a special mould in which the material was completely locked in during the production cycle, had to be constructed. This enclosure of the material is necessary since, if the temperature of the matrix material is raised to a level above its glass transition temperature, the material becomes very fluid. If the pressure, necessary for the consolidation of the material, is applied on this fluid it will flow out, using every possible chink to leave the scene.

The mould was made out of aluminium plates with thicknesses varying from 5 mm up to 20 mm in order to avoid large deformations of the mould in the production cycle. The complete mould, together with a piece of the produced material is shown in Fig. 6.1.a. With this mould panels with a width of 180 mm and a length of 500 mm can be produced. Later on the stiffeners had to be cut from these panels to the desired dimensions.

In the tests the damage stoppers were bonded to the panel material with AF-163-2K. This is an epoxy based thermoset adhesive film with a knotted carrier of a thickness of 0.250 mm. Since the thermoplastic matrix material in skin and damage stopper is equal to each other, joining the different parts could also be achieved by locally heating the different parts to a level above the glass transition temperature of the material and applying some pressure during cooling down. In the production of the stiffened panels for the test series however this method was not applied because in this case another mould is necessary to prevent the stringers from floating apart. For a stiffened panel
which is to be used in a real airplane structure an integrally produced panel generates good possibilities.

In order to calculate the tip stress reduction factor and the stringer load concentration factor, the Young's modulus in the length direction of the stiffening elements is required. Furthermore the ultimate tensile strength of the stiffeners is necessary to obtain the residual strength diagram for the stiffeners.

The modulus of elasticity was obtained by a flutter test on one of the stiffeners. For this purpose the stiffener was fixed at one end by clamping it firmly between two plates. The free length of the specimen and the cross sectional properties were accurately measured. The specimen was brought into vibration by giving it an arbitrary initial displacement perpendicular to its plane and subsequently releasing it. With the use of a single strain gauge connected with an oscilloscope, the natural frequency of the stiffener could be reliably assessed. In the next section it will be shown how from these data the modulus of elasticity can be obtained.

The tensile strength was obtained by a series of four specimens made out of the Unidirectional Carbon material. The specimens were provided with aluminium tabs and tested in a tensile test up to failure. From the maximum force obtained by the experiment the ultimate tensile stress can be calculated by dividing this value by its cross-sectional area.

As stated in the previous chapter, the residual strength properties of the panel with damage stoppers were calculated with the program CRACKS (Bos (1988)). The problem with this program is the large number of variables that have to be tuned before a good strength prediction becomes within reach. Therefore it was found necessary to perform tests on a particular panel configuration which could be used for tuning the program parameters, followed by another test on a different panel configuration to validate the strength prediction obtained by the program. If this validation comes out to be correct the program can be used for other configurations as well and then it becomes a useful designing tool.

A panel of width \( W = 480 \text{ mm} \) with four damage stoppers of width \( d = 40 \text{ mm} \) attached to one side was used for tuning of the program. A total of three of these panels was tested, one of which provided with a number of strain gauges.

A single panel of the same width \( W = 480 \text{ mm} \) with four damage stoppers of width \( d = 20 \text{ mm} \) attached to it was used for the evaluation of the program CRACKS.

Tuning of the program was done by calculating the values of \( C_R \) and \( L_S \) for a large range of crack lengths in the panel under consideration, thus obtaining a residual strength diagram for the particular panel. Variation in program parameters such as bonding flexibility, total length of the stiffening element, number of attachment points, etcetera obviously yield different diagrams. The set of parameters which yield the diagram which is closest to the results obtained by the experiment are chosen as the initial parameters for the program used for the residual strength prediction. For the first test series, the stringer pitch can be calculated as \( (480-40)/3 = 146.67 \text{ mm} \). The stringers were flat straps made out of the Unidirectional Carbon/PEI with a thickness of 1.80 mm and a width of 40 mm. These stiffeners were bonded to the panel over its complete length. The choice
of these stringer dimensions is rather arbitrary although preliminary calculations showed that values in this order of magnitude exhibited reasonable results.

In order to properly introduce the tensile forces into the test specimens, again aluminium tabs were used. At the side of the panel at which the damage stoppers were attached, the positions of the damage stoppers were chemically milled out of the tabs in order to apply the force to the complete panel right from the clamping edges. After bonding the tabs to the panels, \( \phi 10 \) mm bolt holes were drilled into the panel and an initial crack of \( 25 \) mm was introduced with a small saw in the middle of the specimen.

Again the testing was performed in a stroke controlled tensile testing device, using a \( 0.01 \) mm/sec crosshead speed. During each test the force-displacement diagram was directly recorded on a plotter and the maximum force was registered.

The result of the calculation with the computer program CRACKS, with the different program parameters based on the experiments which will be discussed in the next section, can be found in Fig. 6.1.b. From this figure it can be noted that the curve for failure of the damage stoppers is always considerably above that of skin failure (note the difference in y-axis scaling in this figure) and therefore it can be concluded that the structure is skin critical. In the figure the line of skin failure for an unstiffened skin, which is similar to the theoretical K-curve shown in Fig. 3.1.d, is also given. An increase in residual strength of the skin is found for crack lengths approaching the damage stopper, induced by taking over part of the load of the skin by the damage stopper.

In order to check the validity of the program another test had to be performed with different measurements of the panel and/or stringers. For this purpose a panel was chosen with a width \( W = 480 \) mm, similar to that of the earlier described panel, and four stiffeners with a width of \( d = 20 \) mm. With these values the stringer pitch now equals \((480-20)/3 = 153.33\) mm. All other dimensions remained similar, as well as the materials used. Only one panel of this configuration was produced and tested in a way corresponding to that of the previous described panel.

The results from the calculation of the residual strength diagram for this single panel, again with the program parameters based on the first experiment, can be found in Fig. 6.1.c. It can be observed from this figure that the stringer load concentration factor has increased for larger crack lengths (i.e. the ultimate failure stress for the stiffener is lowered in the vicinity of the stiffener centreline). However, the complete structure is still skin critical since the curve for skin failure is still far below the one for stiffener failure.

Another observation from this figure is that the line for ultimate skin stress shows less increase for crack lengths approaching the damage stoppers. Without such a strength increase the panel will never obtain any degree of damage tolerance since, once the crack starts to grow in such a situation, it will not be stopped by the damage stoppers and thus crack growth will directly lead to complete panel failure.

From a safety point of view it is difficult to say just how much strength increase is desired for cracks approaching the damage stoppers. If for the peak in the diagram ultimate load is chosen and for the minimum limit load (i.e. the ratio of the two values equals the factor of safety) safe operation is always guaranteed. Structures with some amount of damage can safely fly under all loading conditions below limit load, the
damage will not be extended under these conditions. If during the life of this structure a load higher than limit load is encountered, the damage will extent in some amount until the size is in the order of magnitude of the stringer pitch. However, the structure will be damage tolerant since the load level can still be raised up to ultimate load before final failure occurs. Damages as large as the stringer pitch will be directly detected and repaired and a new situation will be created in which no damage is present. This strategy, however, will result in heavy structures which are therefore not very economical. It can be argued that crack lengths in the order of magnitude of the damage stopper pitch will easily be detected, even during flight. Therefore the factor of safety might be lowered from 1.5 to 1.25 for instance in such cases, leading to a smaller peak in the residual strength diagram. This strategy has no influence on the undamaged structure since in this case the residual strength is equal to the actual material strength which is sufficiently high. However, with a reduction of the ultimate strength level problems might generate from the undetected damages. It should be borne in mind that, as long as the damage is undetected, a factor of safety of 1.5 is prescribed. In other words, for small damages the peak in the diagram must be sufficiently high to stop the cracks from growing beyond the damage stopper.

From a manufacturing point of view, a large stringer pitch would be pleasant. Especially for the outer panels of the sandwich fuselage, where a smooth surface is desired for the aerodynamics of the airplane, sinking of the stringers into the honeycomb is necessary. This can be accomplished by milling out the position of the stringers from the core, which means an extra production cycle. The milling has to be carried out very accurately in order to obtain a perfect panel. In this respect, the earlier mentioned in-situ foaming process would mean a great advantage in the production process.

6.2 Test Results.

First the different tests on the Unidirectional Carbon/PEI are discussed. From the vibration test on a stringer the modulus of elasticity can be obtained using the following equation:

\[ f_n = \frac{K_n}{2\pi} \sqrt{\frac{EIg}{wll^4}} \]  

[6.1]

In which \( f_n \) stands for the natural frequency where \( n \) refers to the mode of vibration, \( E \) is the desired modulus of elasticity, \( I \) is the area moment of inertia, \( g \) is the gravitational acceleration, \( K_n \) is a constant depending on the mode of vibration, \( w \) is the uniform load per unit length, including the beam weight, and \( l \) is the free length of the beam. After substituting the test results, a modulus of elasticity of 111.8 GPa was found. From the literature it can be concluded that the modulus of elasticity for unidirectional carbon fibre reinforced composites varies somewhere between 100 GPa and 140 GPa and therefore the result obtained from this vibration test is found reliable.
From all four tensile tests on the unidirectional specimens the maximum load was obtained. From these maximum loads a tensile strength of 1720.1 MPa was calculated. Again this value agrees well with values reported in literature and therefore this maximum load can be used later in the calculation of the residual strength diagram of the stringers.

Now only the results from the tensile tests on the stiffened panels remain to be discussed. As prescribed before, a total of four of these panels, each possessing four damage stoppers, were tested. Three of them had damage stoppers of a width of 40 mm while the fourth panel had damage stoppers of a width of 20 mm. During each test the behaviour of a central crack with an initial length of 25 mm, placed in the middle of the inner two damage stoppers was examined.

From Fig. 6.1.b it can be noticed that a total crack length of 25 mm should in all cases result in a sudden instability of the crack growth, followed by crack arrest leading to a crack length practically equal to the spacing of the damage stoppers. In contradiction to this expectation, a little amount of stable crack growth was observed just before crack growth instability occurred. This observation follows from the stroke control used in the experiment, leading to an immediate drop in the load level during crack extension, thus avoiding sudden force corrections of the test bench and preserving the test panel. This enables visual insight in the event of crack extension. With a further increase in the stroke of the bench also the load in the panel will be raised again, resulting in further crack extension and finally complete rupture of the panel.

As mentioned before, from the first set of tests the different program parameters for the computer program CRACKS were deduced while the last test was used for program validation purposes. From each test only the total force at the panel end is known. Therefore, before comparison of calculation with the experimental part concerning panels with damage stopper is possible, the panel end force has to be transformed into stresses within the panel. For this purpose the following formula can be used:

$$F_{\text{panel}} = \sigma_{\text{skin}} \cdot A_{\text{skin}} + \sigma_{\text{damage stopper}} \cdot A_{\text{damage stopper}}$$  \[6.2\]

which holds for the undisturbed part of the panel (i.e. no influence from the grips or from the crack). With the well-known formula $\sigma = E \cdot \varepsilon$ and the compatibility equation for the strains (i.e. $\varepsilon_{\text{skin}} = \varepsilon_{\text{damage stopper}}$) the next equation can be deduced:

$$\frac{\sigma_{\text{skin}}}{E_{\text{skin}}} = \frac{\sigma_{\text{damage stopper}}}{E_{\text{damage stopper}}} = \sigma_{\text{damage stopper}} = \frac{E_{\text{damage stopper}}}{E_{\text{skin}}} \cdot \sigma_{\text{skin}}$$  \[6.3\]

and with this Eq. [6.2] transforms to:

$$F_{\text{panel}} = \sigma_{\text{skin}} \left\{ A_{\text{skin}} + \frac{E_{\text{damage stopper}}}{E_{\text{skin}}} \cdot A_{\text{damage stopper}} \right\}$$  \[6.4\]
From which the skin stress can be readily calculated from the panel end force if the
areas and moduli of skin and stringers are known.
In Fig. 6.2.a one of the panels with the damage stoppers is shown, fixed in the testing
device. On the surface of the panel the strain gauges can be noticed. While increasing
the load, instable crack growth was noticed at approximately 165 kN (146 MPa, average
value of three panels) and the crack suddenly extended to a position under the stringers.
This situation is shown in Fig. 6.2.b. After crack arrest the load could be raised further
to 230 kN (200.4 MPa, average value of three panels). At this load final failure of the
panel occurred. From the experiment it could not be seen if the failure was skin critical
or fastener critical, leading to instability of the skin crack growth. In Fig. 6.2.c the panel
is shown after failure.
From the measured maximum skin stress for crack lengths equal to the stringer pitch, the
residual strength diagram could be calculated with CRACKS. In this calculation the different
program parameters were set to obtain a similar maximum. The result of this calculation
can be found in Fig. 6.1.b.
In Fig. 6.1.c the residual strength diagram is shown for the panel with the smaller
damage stoppers. For this calculation the same parameters as deduced from the first set
of experiments was used. The program predicts a maximum skin stress of 151 MPa. In
the experiment a maximum force of 129.8 kN (159.5 MPa skin stress) was found. This
means that a deviation of only 5.3 % was found between calculated and measured values.
This result is very satisfactory, especially if it is realised that only one panel was tested
for this evaluation instead of taking an average value from a series of test.

As stated before, one of the panels was provided with strain gauges. In Fig. 6.2.a this
panel is shown. In Figs. 6.2.d and 6.2.e these results are visualised.
From the result it can be noticed that the slope of the strain force diagram for all strain
gauges is practically similar in the region of lower forces (up to approximately 125 kN).
With the earlier discussed correction for the initial value, it can be concluded that at a
fixed load level the strain in each position over the panel is similar. However, since in
these panels different materials are used, the stress will still vary because the local
stiffnesses are different.
Above the level of 125 kN a diversion from the straight line can be noticed for some
strain gauges. This load level might therefore indicate the beginning of the crack growth.
At 155 kN a discontinuity can be observed for several strain gauge recordings. Especially
for the gauges 3, 3', and 8 this discontinuity is rather large. A sudden increase in the
measured strains is observed for gauges 3 and 3' while the strain at position 8 decreases.
This observation can easily be justified by the instable extension of the crack up to the
position of the stringers. After this discontinuity, especially strain gauge 3', now situated
at the crack tip, shows a rapid increase.

As stated before, if the method is to be applied to real structures the proper dimensions
for the panels have to be established. For this actual design process a finite element
method might be preferable. However, with the parameters for the program CRACKS as
found from the experiments, it is possible to monitor in what manner the residual
strength properties change with a change in stiffener properties and dimensions. The
results of these calculations can be found in Figs. 6.2.f till 6.2.i. A Carbon/PEI skin of
1.0 mm thickness, provided with Unidirectional Carbon/PEI damage stoppers of width
W = 40 mm, thickness t = 1.8 mm, and a modulus of 126900 MPa at a pitch of 600 mm
was chosen as the basic panel configuration in these figures. All dimensions were rather
arbitrary, however, for the desired purpose it does not matter.
In each of the figures the minimum and maximum skin stress from the residual strength
diagram and the ratio of these two can be found. This ratio might become important for
the choice of the safety operation level of the structure. If for instance a ratio of 0.8 is
found the maximum skin stress is 1.25 times the minimum value. With the discussion at
the end of section 6.1 in mind this might result in a safe operation of the structure as
long as small cracks are quickly detected.
It can be noted that each figure contains two different lines for calculations with or
without eccentricity. This eccentricity will cause bending stresses in the panel which lower
the residual strength values. From the figures it can be noted that for all cases a
practically parallel shift of the results is found. Furthermore, especially the maximum
residual stress of the skin is altered. This can be explained with the higher bending
stresses which obviously occur at higher loadings resulting in a larger deviation from the
situation without eccentricity.
In the actually tested panels the eccentricity was not restrained and therefore the
calculated results can be considered reliable. In case of a sandwich structure however,
both the symmetric construction as well as the support by the core material will reduce
the bending stresses, resulting in better residual strength properties. In reality the
eccentricity will probably not be reduced to zero and therefore the actual solution will
be somewhere between the two given lines.
It should be understood that especially the maximum peak stress found is very important
since this peak determines the ultimate strength of the structure under the presence of
cracks. As long as the peak is high enough, safe operation of the airplane can be
guaranteed. From Figs 6.2.f till 6.2.i it can be seen that with increasing modulus, width
or thickness of the damage stoppers, the peak in the residual strength diagram is raised
while an increase in pitch lowers the peak. This behaviour is completely according
expectations.
In order to find out if the chosen stiffener is effective enough in the residual strength
behaviour of the chosen design another approach is needed. For this purpose the damage
stopper area was altered with a change in the damage stopper pitch such that the
smeared area of the damage stopper (i.e. total damage stopper area divided by its pitch)
remained constant. The smeared area was chosen such that a pitch of 600 mm yielded
a damage stopper area of 40 mm x 1.8 mm. The result of the calculation can be found
in Fig. 6.2.j. In this figure only the solution without eccentricity is provided since
changing the dimensions of the damage stopper area will change both its moment of
inertia as well as the distance of the centre of gravity, thus influencing the eccentricity
in two different ways. From this figure it can be noted that the ratio of the minimum and
the maximum skin stress has a fist maximum value at a pitch of approximately 250 mm
and a minimum at approximately 450 mm. After this minimum the ratio rapidly increases
due to a strong drop in the maximum skin stress. From the figure it is concluded that a stringer pitch from 600 mm is satisfactory although the maximum in the residual strength diagram is rapidly decreasing for larger pitches.
Conclusions.

In this part of the report a short description of each chapter is given, followed by the most important conclusions that were obtained.

Chapter 1 started of with an introduction and the definition of the problem which had to be solved in this work. This problem concerned the poor fracture toughness of Carbon/PET laminates, which were used in the facings of a honeycomb sandwich in a preliminary design of a full composite fuselage. Poor fracture toughness of any material leads to a low residual strength of a damaged structure made out of this material. In order to use the distinct material after all, suitable measures must be taken to overcome this.

In Chapter 2 a general description of some subjects in fuselage design is presented. The main attention of this chapter lies on composite materials. First some composite fuselage designs are mentioned, followed by a description of the part of the aviation requirements that concerns fuselages. Further on in this chapter the loadings on a fuselage are considered and the chapter is finished with a discussion on impact characteristics of composite materials.

From this se

71 chapter it can be concluded that nowadays it is an enormous task to certify any airplane structure. Especially in the case of composite materials some problems might pop up that never occurred before. These new problems partly follow from the fact that relatively little is known about the behaviour of composite materials and partly from the fact that requirements drawn up in the past were made for metals.

As long as no satisfactory methods for the prediction of damage growth in composite materials are available, only three methods can be used to demonstrate adequate airworthiness regarding cyclic loading. These methods include 1) safe-life approach, 2) crack arrest or no-growth concept, and 3) fail-safe approach by redundancy.

Other requirements related problems concern the variation (both in loading direction as in position in the laminate) of strength for composite materials for which no closely-reasoned definition could be found. Unlike metals, strength and stiffness values may vary in every direction which yearns for a totally new approach with respect to allowables.

From the discussion of the loading conditions it can be concluded that a division of the loads on the fuselage skin in three distinct parts is well possible. Each of these parts can be treated separately. Different conditions are found for each part of the fuselage, possibly leading to different designs. Cyclic compressive loadings may cause reason for concern, especially for composite materials and hybrid laminates. Under these loadings delaminations can easily develop, in the end leading to understrength of the structure.

For the description of the effect of impact on a structure it is important to not only look at the amount of energy of the object of impact but also consider its shape. Although this may lead to a large number of different situations of impact during certification of the
airplane it is the only possible way to achieve reliable service behaviour predictions. Furthermore, once the different situations are defined, the impact testing is independent of the material in use.

The impact characteristics of composite materials differ greatly from that of metals. Even single impact can cause a reduction in strength of the structure of around 40 %, also when nothing can be noticed from the surface of the impacted structure.

Impact can be treated with a probability approach. This approach basically boils down to the following: for a given strength reduction below ultimate load, the more likely damage may occur, the sooner it should be detected.

It was found here that a single crack, the size according to that of the impact damaged area may be considered as a worst case simulation model. Therefore for the rest of the work panels with cracks were considered. The critical crack size in the experiments was chosen as $a = 25 \text{ mm}$ because this was the boundary of the barely visible impact damage as obtained from Haars (1992).

In Chapter 3 linear elastic fracture mechanics are discussed. Although the background on which this theory is built differs from the behaviour of composite materials, it was found that this approach yields a relatively easy, yet sufficient, way to describe the fracture behaviour of the material under study. It should be noted that in this work only cracks in the principal material axes were investigated and no attempt was made to establish a stress intensity factor in of-axis situations.

In the stress intensity factor concept the geometry factor plays a very important role. In order to be able to calculate some of these geometry factors in some restricted cases, a calculation program for the calculation of stresses and strains around a row of three holes in an infinite plate was modified.

For an evaluation of these calculations, two known solutions from the literature were recalculated and compared. From this evaluation it can be concluded that the calculation method is very good.

In Chapter 4 experiments on unstiffened panels were performed. Tests included determination of the fracture toughness of the material, observation of the fracture mechanism at the crack tip, and some experiments to verify geometry factor calculations.

From the results it can be concluded that the critical mode I stress intensity factor for the Carbon/PEI material is $33.81 \text{ MPa} \cdot \text{m}$, measured in the direction of the material principal axes along which the highest stiffnesses are reached. From the C-scan tests it can be concluded that the material shows no noteworthy delaminations during a tensile test on a cracked specimen.

Observation of the fracture surfaces at the crack tip learned that almost in all layers intralaminar fracture occurred while interlaminar fracture was very rare. This is caused by the giant leap in fibre orientation which is unavoidable if fabrics are used in which the fibre bundles are placed perpendicular to each other.
The COD measurements were not found useful for the investigation in this work. A good interpretation for the obtained values could not be found for the composite material under study. However, from the COD measurements it might be concluded that some amount of crack growth occurs in the very last stage of the loading. The tests on the failure mechanism showed that mostly intralaminar fracture occurred and that final fracture occurred very sudden, not showing any amount of damage just before fracture.

From the test series to check the calculated value of the geometry factor it can be concluded that if the crack was between two holes the experiments agree rather well with the predicted values, however, for the situations of three collinear cracks large deviations were found. Since calculated geometry factors of similar situations agree well with literature (see section 3.2) it can be assumed that the calculated values are reliable and therefore another reason has to be found for these deviations. A possible explanation can be found in the fact that instead of a real crack ($r_{\text{crack tip}} = 0$) the crack was introduced with a fret saw.

Observation of the measured strains in the specimens to check upon the geometry factor showed a good similarity with calculated strain values. Both for a normal strain gauge as for a K-gage the results were satisfactory. From the results of the calculations it was concluded that the application of the K-gage is not possible for the situations under consideration since the circle along which the strains are measured is too far from the crack tip.

In Chapter 5 a background is given on the theory of stiffened panels. This knowledge is very useful for the understanding of panels provided with damage stoppers. In the first section a general description is presented and in the second section a method to compute the influence of the stiffening elements on the stress distribution in the panel. With this method a computer program was developed at the Delft University of Technology in 1988. With this program calculation of a stiffened panel is performed as will be discussed below.

In the Chapter 6 three test were performed on panels provided with crack stoppers. First the computer program CRACKS was used to select a panel configuration for the experiments. It was realised that dimensions of the panel might not be very realistic but for establishing the applicability of the damage stoppers the panels were sufficient.

The computer program CRACKS seems suitable for the calculation of the residual strength properties of a stiffened panel. After the different parameters of the program are tuned with a first test series of stiffened panels, prediction of the behaviour of other panels becomes possible.

Although the residual strength behaviour from the test panels differed from the behaviour predicted by the calculations, a level of damage tolerance was reached. From the results it was concluded that application of damage stoppers yields a good potential to meet the aviation requirements without a complete change of the sandwich fuselage structure.
Recommendations.

Recommendations for future research include:

Investigate the relation of $K_{Ic}$ as a function of $\alpha$, in which $\alpha$ is the angle between the direction of the crack and the principal material axes.

Perform test on panels with a more realistic stringer pitch. For this tests a pitch above 500 mm seems reasonable.

Perform tests on panels with curvature. By doing so, a better approximation for the fuselage model is obtained.

Perform tests on a complete sandwich structure with either one or both facings cracked. Try to correlate the results from this present investigation with the newly found results.
References.


Bristow, John W. (1985), Structural Composites Airworthiness in Civil Aircraft. Civil Aviation Authority, United Kingdom.


Fujita, Kazuhisa, titel. To be published - Delft University of Technology.


Tooren, M.J.L. van (1992), private communications.


Vlot, Ad. (1991), Low-velocity Impact Loading - on Fibre Reinforced Aluminium Laminates (ARALL) and Other Aircraft Sheet Materials. Delft University of Technology.


Table 3.2.a: Geometry factors according to Murakami (1986).

<table>
<thead>
<tr>
<th>2a/d</th>
<th>C_A</th>
<th>C_B</th>
<th>C_C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>1.00083</td>
<td>1.00040</td>
<td>1.00063</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00150</td>
<td>1.00164</td>
<td>1.00252</td>
</tr>
<tr>
<td>0.2</td>
<td>1.00585</td>
<td>1.00702</td>
<td>1.01030</td>
</tr>
<tr>
<td>0.3</td>
<td>1.01296</td>
<td>1.01710</td>
<td>1.02407</td>
</tr>
<tr>
<td>0.4</td>
<td>1.02297</td>
<td>1.03353</td>
<td>1.04529</td>
</tr>
<tr>
<td>0.5</td>
<td>1.03631</td>
<td>1.05913</td>
<td>1.07663</td>
</tr>
<tr>
<td>0.6</td>
<td>1.05383</td>
<td>1.09915</td>
<td>1.12316</td>
</tr>
<tr>
<td>0.7</td>
<td>1.07724</td>
<td>1.16456</td>
<td>1.19558</td>
</tr>
<tr>
<td>0.8</td>
<td>1.11032</td>
<td>1.28348</td>
<td>1.32136</td>
</tr>
<tr>
<td>0.9</td>
<td>1.16439</td>
<td>1.56454</td>
<td>1.60685</td>
</tr>
</tbody>
</table>
Table 3.2.b: Calculated geometry factors.
(percentage between parentheses gives the deviation from table 3.2.a)

<table>
<thead>
<tr>
<th>2a/d</th>
<th>C_A</th>
<th>C_B</th>
<th>C_C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>1.00198 (0.11 %)</td>
<td>1.00200 (0.16 %)</td>
<td>1.00063 (&lt; 0.01 %)</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00128 (0.02 %)</td>
<td>1.00142 (0.02 %)</td>
<td>1.00252 (&lt; 0.01 %)</td>
</tr>
<tr>
<td>0.2</td>
<td>1.00562 (0.02 %)</td>
<td>1.00679 (0.02 %)</td>
<td>1.01030 (&lt; 0.01 %)</td>
</tr>
<tr>
<td>0.3</td>
<td>1.01275 (0.02 %)</td>
<td>1.01689 (0.02 %)</td>
<td>1.02407 (&lt; 0.01 %)</td>
</tr>
<tr>
<td>0.4</td>
<td>1.02274 (0.02 %)</td>
<td>1.03353 (&lt; 0.01 %)</td>
<td>1.04529 (&lt; 0.01 %)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.03646 (0.01 %)</td>
<td>1.05913 (&lt; 0.01 %)</td>
<td>1.07663 (&lt; 0.01 %)</td>
</tr>
<tr>
<td>0.6</td>
<td>1.05394 (0.01 %)</td>
<td>1.09915 (&lt; 0.01 %)</td>
<td>1.12316 (&lt; 0.01 %)</td>
</tr>
<tr>
<td>0.7</td>
<td>1.07735 (0.01 %)</td>
<td>1.16466 (0.01 %)</td>
<td>1.19555 (&lt; 0.01 %)</td>
</tr>
<tr>
<td>0.8</td>
<td>1.11018 (0.01 %)</td>
<td>1.28294 (0.04 %)</td>
<td>1.32071 (0.05 %)</td>
</tr>
<tr>
<td>0.9</td>
<td>1.16218 (0.19 %)</td>
<td>1.55007 (0.92 %)</td>
<td>1.59112 (0.98 %)</td>
</tr>
</tbody>
</table>
### Table 4.1.a: Intended measurements of test specimens to check upon geometry factor calculation, together with calculated geometry factor.

<table>
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<tr>
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<th>Centre</th>
<th>Right side</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2d [mm]</td>
<td>2b [mm]</td>
<td>s [mm]</td>
<td>2d [mm]</td>
</tr>
<tr>
<td>1</td>
<td>21.0</td>
<td>21.0</td>
<td>30.0</td>
<td>21.0</td>
</tr>
<tr>
<td>2</td>
<td>21.0</td>
<td>0.0</td>
<td>30.0</td>
<td>21.0</td>
</tr>
<tr>
<td>3</td>
<td>21.0</td>
<td>0.0</td>
<td>24.5</td>
<td>10.0</td>
</tr>
<tr>
<td>4</td>
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<td>60.0</td>
<td>45.0</td>
<td>10.0</td>
</tr>
<tr>
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<td>45.0</td>
<td>60.0</td>
<td>35.0</td>
<td>10.0</td>
</tr>
<tr>
<td>6</td>
<td>45.0</td>
<td>60.0</td>
<td>37.5</td>
<td>15.0</td>
</tr>
</tbody>
</table>

### Table 4.1.b: Actual measurements of test specimens to check upon geometry factor calculation, together with calculated geometry factor.

<table>
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<th>Centre</th>
<th>Right side</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2d [mm]</td>
<td>2b [mm]</td>
<td>s [mm]</td>
<td>2d [mm]</td>
</tr>
<tr>
<td>1</td>
<td>20.5</td>
<td>20.5</td>
<td>30.45</td>
<td>22.5</td>
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<tr>
<td>2</td>
<td>21.7</td>
<td>0.0</td>
<td>30.67</td>
<td>21.95</td>
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<tr>
<td>3</td>
<td>21.95</td>
<td>0.0</td>
<td>24.6</td>
<td>9.55</td>
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<tr>
<td>4</td>
<td>45.0</td>
<td>60.0</td>
<td>44.75</td>
<td>9.8</td>
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<tr>
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<td>45.0</td>
<td>60.0</td>
<td>34.8</td>
<td>8.6</td>
</tr>
<tr>
<td>6</td>
<td>45.0</td>
<td>60.0</td>
<td>37.8</td>
<td>13.8</td>
</tr>
</tbody>
</table>
Table 4.2.a: Results from the CCT specimens.

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>W [mm]</th>
<th>2a [mm]</th>
<th>F_{max} [kN]</th>
<th>C</th>
<th>K_{lc} [MPa/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>99.5</td>
<td>16.8</td>
<td>17.972</td>
<td>1.0180</td>
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<tr>
<td>2</td>
<td>99.6</td>
<td>25.3</td>
<td>14.648</td>
<td>1.0418</td>
<td>33.3797</td>
</tr>
<tr>
<td>3</td>
<td>99.7</td>
<td>25.6</td>
<td>15.886</td>
<td>1.0427</td>
<td>36.4117</td>
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<tr>
<td>4</td>
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<td>24.3</td>
<td>15.873</td>
<td>1.0381</td>
<td>35.2487</td>
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<td>99.9</td>
<td>26.3</td>
<td>14.906</td>
<td>1.0450</td>
<td>34.6365</td>
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<td>100</td>
<td>50.2</td>
<td>8.80</td>
<td>1.1911</td>
<td>32.1673</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>50.2</td>
<td>9.282</td>
<td>1.1911</td>
<td>33.9292</td>
</tr>
<tr>
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<td>100.42</td>
<td>15.3</td>
<td>19.16</td>
<td>1.0146</td>
<td>32.7974</td>
</tr>
<tr>
<td>9</td>
<td>100.5</td>
<td>15.5</td>
<td>19.322</td>
<td>1.01493</td>
<td>33.2756</td>
</tr>
<tr>
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<td>16.45</td>
<td>17.75</td>
<td>1.0168</td>
<td>31.5315</td>
</tr>
<tr>
<td>11</td>
<td>150.1</td>
<td>38.3</td>
<td>17.406</td>
<td>1.0421</td>
<td>32.3957</td>
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<tr>
<td>12</td>
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<td>38.7</td>
<td>17.14</td>
<td>1.0430</td>
<td>32.0489</td>
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<tr>
<td>13</td>
<td>150.4</td>
<td>22.55</td>
<td>24.41</td>
<td>1.0141</td>
<td>33.8541</td>
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<tr>
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<td>150.4</td>
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<td>25.465</td>
<td>1.0159</td>
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<td>24.4</td>
<td>21.785</td>
<td>1.0165</td>
<td>31.5045</td>
</tr>
<tr>
<td>16</td>
<td>150.4</td>
<td>24.65</td>
<td>22.795</td>
<td>1.0169</td>
<td>33.1449</td>
</tr>
<tr>
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<td>24.7</td>
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<td>1.0170</td>
<td>31.5505</td>
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<tr>
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<td>25.1</td>
<td>24.57</td>
<td>1.0175</td>
<td>36.0729</td>
</tr>
<tr>
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<td>25.2</td>
<td>21.455</td>
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<td>24.275</td>
<td>1.0186</td>
<td>36.2070</td>
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</table>

(to be continued at the next page).
Table 4.2. a continued:

<table>
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<tr>
<th>Specimen Number</th>
<th>W [mm]</th>
<th>2a [mm]</th>
<th>$F_{\text{max}}$ [kN]</th>
<th>C</th>
<th>$K_{\text{lc}}$ [MPa/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>150.5</td>
<td>25.2</td>
<td>23.715</td>
<td>1.0177</td>
<td>34.8918</td>
</tr>
<tr>
<td>22$^1$</td>
<td>150.5</td>
<td>25.9</td>
<td>25.705</td>
<td>1.0187</td>
<td>38.3533</td>
</tr>
<tr>
<td>23</td>
<td>150.8</td>
<td>37.9</td>
<td>18.868</td>
<td>1.0408</td>
<td>34.7267</td>
</tr>
<tr>
<td>24$^1$</td>
<td>151.4</td>
<td>24.82</td>
<td>26.725</td>
<td>1.0169</td>
<td>38.7358</td>
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<tr>
<td>25$^1$</td>
<td>151.6</td>
<td>25.5</td>
<td>26.265</td>
<td>1.0178</td>
<td>38.5706</td>
</tr>
<tr>
<td>26$^2$</td>
<td>160</td>
<td>26.67</td>
<td>23.87</td>
<td>1.0175</td>
<td>33.9557</td>
</tr>
<tr>
<td>27$^2$</td>
<td>160</td>
<td>26.67</td>
<td>24.23</td>
<td>1.0175</td>
<td>34.4678</td>
</tr>
<tr>
<td>28$^2$</td>
<td>160</td>
<td>40</td>
<td>19.93</td>
<td>1.0404</td>
<td>35.5016</td>
</tr>
<tr>
<td>29$^2$</td>
<td>160</td>
<td>40</td>
<td>20.25</td>
<td>1.0404</td>
<td>36.0717</td>
</tr>
<tr>
<td>30$^2$</td>
<td>160</td>
<td>53.3</td>
<td>15.35</td>
<td>1.0745</td>
<td>32.5975</td>
</tr>
<tr>
<td>31$^2$</td>
<td>160</td>
<td>53.3</td>
<td>15.49</td>
<td>1.0745</td>
<td>32.8948</td>
</tr>
<tr>
<td>32</td>
<td>170.5</td>
<td>24.3</td>
<td>27.45</td>
<td>1.0127</td>
<td>34.9276</td>
</tr>
<tr>
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<td>199.4</td>
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<td>26.745</td>
<td>1.0174</td>
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<td>199.5</td>
<td>33.2</td>
<td>27.295</td>
<td>1.0174</td>
<td>34.7419</td>
</tr>
</tbody>
</table>

1: Specimen with two parallel cracks close to each other (d=10 mm).
2: Specimen with two parallel cracks at a considerable distance (d=200 mm).
Table 4.2.b: Results from the test specimens to check upon geometry factor calculations.

<table>
<thead>
<tr>
<th>Case</th>
<th>( C_{\text{calc}} ) [mm]</th>
<th>( W ) [mm]</th>
<th>( t ) [mm]</th>
<th>( 2a ) [mm]</th>
<th>( F_{\text{max}} ) [kN]</th>
<th>( C_{\text{test}} )</th>
<th>( \frac{C_{\text{test}}}{C_{\text{calc}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2614</td>
<td>480</td>
<td>0.915</td>
<td>22.5</td>
<td>60.15</td>
<td>1.3132</td>
<td>1.0411</td>
</tr>
<tr>
<td>2</td>
<td>1.2168</td>
<td>480</td>
<td>0.915</td>
<td>21.95</td>
<td>95.63</td>
<td>0.8363</td>
<td>0.6873</td>
</tr>
<tr>
<td>3</td>
<td>1.2515</td>
<td>480</td>
<td>0.915</td>
<td>9.55</td>
<td>69.86</td>
<td>1.7355</td>
<td>1.3867</td>
</tr>
<tr>
<td>4</td>
<td>1.5614</td>
<td>520</td>
<td>0.915</td>
<td>9.8</td>
<td>69.01</td>
<td>1.8788</td>
<td>1.2033</td>
</tr>
<tr>
<td>5</td>
<td>2.1330</td>
<td>520</td>
<td>0.915</td>
<td>8.6</td>
<td>60.74</td>
<td>2.2787</td>
<td>1.1400</td>
</tr>
<tr>
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<td>1.9988</td>
<td>520</td>
<td>0.915</td>
<td>13.8</td>
<td>?</td>
<td>?</td>
<td>-</td>
</tr>
</tbody>
</table>
### Table 4.2.c: Measured strains (first column) and calculated strains (second column) for a single value of F and all six test specimens.

<table>
<thead>
<tr>
<th>F [kN]</th>
<th>case 1</th>
<th>case 2</th>
<th>case 3</th>
<th>case 4</th>
<th>case 5</th>
<th>case 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.37</td>
<td>.257</td>
<td>.245</td>
<td>.184</td>
<td>.297</td>
<td>.249</td>
<td>.248</td>
</tr>
<tr>
<td>50.01</td>
<td>.219</td>
<td>.240</td>
<td>.160</td>
<td>.293</td>
<td>.222</td>
<td>.245</td>
</tr>
<tr>
<td>49.02</td>
<td>.221</td>
<td>.244</td>
<td>.162</td>
<td>.298</td>
<td>.237</td>
<td>.248</td>
</tr>
<tr>
<td>41.80</td>
<td>.215</td>
<td>.244</td>
<td>.156</td>
<td>.298</td>
<td>.252</td>
<td>.248</td>
</tr>
<tr>
<td>1 [-]</td>
<td>-.103</td>
<td>-.135</td>
<td>.257</td>
<td>.469</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2 [-]</td>
<td>.336</td>
<td>.405</td>
<td>-.026</td>
<td>-.079</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3 [-]</td>
<td>-.021</td>
<td>-.071</td>
<td>.255</td>
<td>.469</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>4 [-]</td>
<td>-.105</td>
<td>-.071</td>
<td>-.007</td>
<td>-.079</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>5 [-]</td>
<td>.337</td>
<td>.405</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>6 [-]</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>7 [-]</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>8 [-]</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>9 [-]</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>10 [-]</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
Table 4.2.d: Calculation of the gauge factor B with the calculation program.

<table>
<thead>
<tr>
<th>Case</th>
<th>load [kN]</th>
<th>2a [mm]</th>
<th>A [mm²]</th>
<th>K_{ref} [MPa/m]</th>
<th>ε_{ref} [%]</th>
<th>B = K/ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49.37</td>
<td>22.5</td>
<td>439.2</td>
<td>21.13</td>
<td>0.2691</td>
<td>78.530</td>
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<tr>
<td>2</td>
<td>60.01</td>
<td>21.95</td>
<td>439.2</td>
<td>14.31</td>
<td>0.3245</td>
<td>44.111</td>
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<tr>
<td>3</td>
<td>50.07</td>
<td>9.55</td>
<td>439.2</td>
<td>13.96</td>
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<td>13.05</td>
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Table 4.2.e: K-values from experiments and calculations.

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<th>Case</th>
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<th>ε_{calculated}</th>
<th>K_{measured} [MPa/m]</th>
<th>K_{calculated} [MPa/m]</th>
<th>C</th>
<th>C*K_{ref} [MPa/m]</th>
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<td>1.2614</td>
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<td>0.469</td>
<td>11.29</td>
<td>20.69</td>
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<tr>
<td>3</td>
<td>-----</td>
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<td>-----</td>
<td>-----</td>
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<tr>
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<td>19.30</td>
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Figure 1.a: Effectiveness of structural weight saving as function of semispan

Figure 1.b: Structural parts division (conventional fuselage structure)
Figure 1c: Satin weave
<table>
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<th>Configuration</th>
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<td><strong>COCURED I-STRINGERS</strong></td>
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<td><strong>LAMINATE SKIN</strong></td>
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<td><strong>COCURED FOAM-FILLED HAT SECTION STRINGERS</strong></td>
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<td><strong>HONEYCOMB CORE</strong></td>
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<tr>
<td><strong>COCURED FOAM-FILLED HAT SECTION STRINGERS</strong></td>
<td></td>
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</tbody>
</table>

Figure 2.1.a: Possible composite fuselage shell configurations
Figure 2.1.b: Circumferential segmentation of the fuselage

Figure 2.2.a: Damage tolerance by multiple load path
Figure 2.2.b: Residual strength as a function of time of a damage tolerant metal fuselage structure with developing fatigue damage.

Figure 2.2.c: Residual strength as a function of time of a composite fuselage structure subjected to accidental damage.
Figure 2.3.a: Loads on the cross-section of the fuselage

Figure 2.3.b: Variation of bending moment around the y-axis of an Airbus A320 during flight
Figure 2.4.a: Required damage tolerance behaviour of metallic and composite structures

Figure 2.4.b: Acceptability of accidental damage in composite structures defined by probability and strength reduction
Figure 2.4.c: Effect of cut-outs, saw cuts and impact damage on the residual strength of ARALL 2H32
Figure 3.1.a: Griffith's energy balance approach of fracture

Figure 3.1.b: Crack tip stress field
Damage Tolerance Aspects of a Full Composite Airplane Fuselage.

Figure 3.1.c: Opening modes of cracks

Figure 3.1.d: K-curve
Figure 3.1.e: Net section strength of cracked specimens

Figure 3.1.f: The Feddersen (1971) approach for plane stress fracture toughness
Figure 3.1.g: Residual strength for various panel sizes

Figure 3.1.h: Crack growth curve
Figure 3.1.i: Summation of stress intensity factors to obtain problem solution

Figure 3.1.k: An example of an R-curve
Figure 3.2.a: Stress intensity factor solution for a crack between two circular holes as obtained from Murakami (1987)
Geometry Factors
Crack between two circular holes

Figure 3.2.b: Stress intensity factor solution for a crack between two circular holes as obtained from the stress calculation program
Figure 4.1.a: Outline of a centre cracked tension specimen

Figure 4.1.b: One of the specimens provided with clamping plates
Figure 4.1.c: Second specimen of the test series to check the geometry factor calculations after production

Figure 4.1.d: Specimen six as mounted in the test bench
Figure 4.1e: Equipment for strain gauge measurements with on the left hand side specimen six in the test bench and the autoclave at the background
Fracture curve C/PEI 0-90/±45/0-90
LEFM approach

K value statistics:
mean: 34.23 MPa√m
max: 38.74 MPa√m
min: 31.50 MPa√m
standard deviation: 2.017

\[ K = C\sigma \sqrt{\pi a} = 34.23 \text{ MPa}\sqrt{\text{m}} \]

Figure 4.2.a: Fracture toughness curve according to Feddersen (1971) for the total series of 34 data points
Fracture curve C/PEI 0-90/±45/0-90
LEFM approach

K value statistics:
mean: 33.81 MPa√m
max : 36.42 MPa√m
min : 31.50 MPa√m
standard deviation: 1.558

\[ K = C\sigma\sqrt{\pi a} = 33.81 \text{ MPa}\sqrt{\text{m}} \]

Figure 4.2.b: Fracture toughness curve according to Feddersen (1971) after neglecting the three double cracked specimens with cracks close to each other
Figure 4.2.c: Top view of the fracture surface close to the crack tip

Figure 4.2.d: Fracture surface with intralaminar failure
Figure 4.2.e: Interlaminar failure somewhere near the crack tip
Strain gauge measurements
test specimen 1

Figure 4.2.1: Strain gauge measurements of specimen 1
Figure 4.2.g: Strain gauge measurements of specimen 2
Strain gauge measurements

test specimen 3

Figure 4.2.h: Strain gauge measurements of specimen 3
Strain gauge measurements

Figure 4.2.i: Strain gauge measurements of specimen 4
Strain gauge measurements

test specimen 5

Figure 4.2.j: Strain gauge measurements of specimen 5
Strain gauge measurements

Test specimen 6

Figure 4.2.k: Strain gauge measurements of specimen 6
Figure 5.1.a: Tip stress reduction factor as function of the crack length

Figure 5.1.b: Stringer load concentration factor
Figure 5.1.c: Residual strength diagram for a stiffened panel in case of a stringer critical panel

Figure 5.1.d: Residual strength diagram for a stiffened panel in case of a skin critical panel
**Figure 5.1.e:** Effect of fastener failure on the residual strength diagram for a stiffened panel

\[
C_R = \frac{K}{\sigma \sqrt{\mu a}}
\]

\[\mu = \text{STIFFENING RATIO}\]
\[\mu = 0.21 \text{ (LIGHT STRINGER)}\]
\[0.41\]
\[0.58 \text{ (HEAVY STRINGER)}\]

**Figure 5.1.f:** The effect of more stringers on the residual strength diagram
Figure 5.2.a: Outline of the displacement compatibility analysis for stiffened panels
Figure 5.2.b: Modelling of the bond layer in segments
Figure 6.1.a: Mould for the production of unidirectional laminates with a piece of produced material
Figure 6.1.b: Residual strength diagram as obtained from CRACKS
Figure 6.1.c: Residual strength diagram as obtained from CRACKS
Figure 6.2.a: Panel with damage stoppers as mounted in the test bench

Figure 6.2.b: Crack extension till stringers, after crack arrest
Figure 6.2.c: Panel with damage stoppers after failure
Figure 6.2.d: Strain gauge recordings from one of the panels with damage stoppers.
Figure 6.2.e: Strain gauge recordings from one of the panels with damage stoppers
Figure 6.2.f: The effect of pitch variation on the residual strength diagram
Width variation of damage stopper

$E = 126900 \text{ MPa}, \ t = 1.8 \text{ mm}, \ \text{pitch} = 600 \text{ mm}$

Figure 62.6: The effect of width variation on the residual strength diagram

Damage Tolerance Aspects of a Full Composite Airplane Fuselage
 Thickness variation of damage stopper
E = 126900 MPa, W = 40 mm, pitch = 600 mm

Figure 6.2.4: The effect of thickness variation on the residual strength diagram

- No eccentricity
- With eccentricity

Skin stress [MPa] vs. damage stopper thickness [mm]

- $\sigma_{\text{max}}$
- $\sigma_{\text{min}}$
- Ratio $\sigma_{\text{min}}/\sigma_{\text{max}}$
Figure 62.6: The effect of modulus variation on the residual strength diagram

Modulus variation of damage stopper

$W=40\ \text{mm},\ t=1.8\ \text{mm},\ \text{pitch}=600\ \text{mm}$

- $\sigma_{\text{max}}$
- $\sigma_{\text{min}}$
- Ratio

Skin stress [MPa]

$\text{ratio (}\sigma_{\text{min}}/\sigma_{\text{max}}\text{)}$

Damage stopper modulus [MPa]

No eccentricity

With eccentricity
Figure 6.2.j: The effect of pitch variation on the residual strength diagram (constant smeared thickness)
Appendix A: Fuselage Skin Stresses During Flight.

This appendix contains the fuselage skin stresses during flight as obtained from calculations performed by Deutsche Airbus on the fuselage of the Airbus A330-300.

The Airbus A330 is of a large-capacity wide-bodied medium/long-range commercial transport aircraft type. The development programme for this aircraft was launched as a combined programme with the Airbus A340 in June 1987. Combining the development of both aircraft will reduce development costs. The wing, cockpit, tail unit, and basic fuselage will be the same in all versions of these aircraft. Furthermore, the aircraft will have much in common with existing Airbus wide-bodies (e.g. A310 and A320).

The main difference between the Airbus A330 and A340 lies in the number of engines and in engine-related systems. The Airbus A330 is a twin-engined medium/long-range version while the Airbus A340 is a four-engined long-range version of basically the same aircraft. The aircraft basic configuration has a seat capacity of 375 (standard) or 440 (optional).

The Airbus A330-300 is the longer-range version of A330, able to carry typical load of 335 passengers over non-stop distance of 9820 km with a maximum take-off weight of 223,000 kg. This aircraft is under study and scheduled for delivery in 1996. The outside configuration of this aeroplane is given in Fig. A.1.

Figure A.1: Configuration of the Airbus A330.
At the fuselage positions of frame 38 and 58, situated just before the wing and halfway the aft-body respectively (see Fig. A.2), the typical stress time histories during a short range mission are given in four stringer positions along these frames. The location of these four positions is given in Fig. A.3. The stress time histories are based on finite element method calculations. The simulated flight is of type AZ which means the most severe flight type. All other flight types have the same evolution (sequence) of stresses during flight although the number of cycles as well as the amplitude of the cycles will be reduced if the flight is less severe.

At each position on the frame the circumferential skin stress, the longitudinal skin stress, and the shear skin stress is given. In the Figs. A.4 to A.15 the skin stresses at frame 38 are given while in Figs. A.16 to A.27 the skin stresses at frame 58 are given.

It can be noted that for the evaluation of stresses at frame 38 the number of points used in the calculation is approximately twice as low as in the case of frame 58. This difference can be explained by the fact that there are no lateral gust and manoeuvre loads at frame 38 in contrast to frame 58. Another difference is the significant drop of the longitudinal and shear stress for frame 58. This feature is a result from the loading case "Airbrakes fully extended" which is to be applied in 25% of all flights.

For both frames the sequence of stresses is roughly the same. The only exception on this is the shear stress at the stringer positions 2 and 3, for which the signs are reversed if one shifts from frame 38 to frame 58 or the other way around (see Figs. A.9, A.12, A.21 and A.24). The orders of magnitude of the stresses also correspond to each other although the amplitude of the stresses is highest at frame 58.

Another important feature of the evolution of stresses at the different positions is the similarity in circumferential stress at all four stringer positions and the same magnitude and sequence of this stress component at both frames (see Figs. A.5, A.8, A.11, A.14, A.17, A.20, A.23 and A.26). This distinct characteristic is due to the fact that the circumferential stress is merely dictated by the pressurization cycle of the fuselage which occurs once per flight. Other loading conditions have practically no influence on the circumferential stress. Although the influence of the pressurization cycle can also be noted in the progress of the longitudinal stresses these stress components are unequal at different stringer positions due to fuselage bending loads.

When a closed cylinder is loaded with internal pressure only, this will lead to a circumferential stress which is twice as high as the axial stress. In the case of a fuselage construction, where this pressurization is combined with bending loads, this ratio between the stress components will be reduced from 2:1 to roughly 1:1 in the top of the cylinder (crown of the fuselage, see Figs. A.4, A.5, A.16 and A.17) and to roughly 1:0 in the bottom of the cylinder (keel of the fuselage, see Figs. A.13, A.14, A.25 and A.26). It is once more emphasized that in this change of the ratio between the circumferential stress and the axial stress the circumferential stress itself does not alter.

With the load time histories presented in this appendix a good impression on the qualitative stress distribution of an aeroplane fuselage is obtained. For every aeroplane fuselage this distribution will be practically the same although other fuselage measurements will obviously yield other stress levels.

figure A.3: Different Stringer Positions along the Frames.
Load Time History of: 3AZ

maximum: 10.76/load case: 3205110/point: 526,
minimum: 0.53/load case: 3106322/point: 1017,
number of points: 1025,
scale: 50 points/cm.
figure A.5: Circumferential Skin Stress at Stringer Position 1 of Frame 38.

A.5
figure A.6: Shear Skin Stress at Stringer Position 1 of Frame 38.
figure A.7: Longitudinal Skin Stress at Stringer Position 2 of Frame 38.
A330-300 SR
CQUAD4 380175 T=2.0
CIRCUMFERENTIAL STRESS

Load Time History of: 3AZ

maximum: 6.42/load case: 3205120/point: 527,
minimum: -0.36/load case: 3105120/point: 1011,
number of points: 1025,
scale: 50 points/cm.

STRESS AT CQUAD4 380175Y

load case sequence

0.00 100.00 200.00 300.00 400.00 500.00 600.00 700.00 800.00 900.00 1000.00 1100.00
Figure A.9: Shear Skin Stress at Stringer Position 2 of Frame 38.
figure A.10: Longitudinal Skin Stress at Stringer Position 3 of Frame 38.
A330-300 SR
CQUAD4 380325 T=2.0
CIRCUMFERENTIAL STRESS

Load Time History of: 3AZ

maximum: 6.24/load case: 3205120/point: 527,
minimum: -0.15/load case: 3105110/point: 1010,
number of points: 1025,
scale: 50 points/cm.
figure A.12: Shear Skin Stress at Stringer Position 3 of Frame 38.
figure A.13: Longitudinal Skin Stress at Stringer Position 4 of Frame 38.
figure A.14: Circumferential Skin Stress at Stringer Position 4 of Frame 38.
**figure A.15**: Shear Skin Stress at Stringer Position 4 of Frame 38.
figure A.16: Longitudinal Skin Stress at Stringer Position 1 of Frame 58.
Figure A.17: Circumferential Skin Stress at Stringer Position 1 of Frame 58.
figure A.18: Shear Skin Stress at Stringer Position 1 of Frame 58.
Figure A.19: Longitudinal Skin Stress at Stringer Position 2 of Frame 58.
figure A.20: Circumferential Skin Stress at Stringer Position 2 of Frame 58.
**figure A.21:** Shear Skin Stress at Stringer Position 2 of Frame 58.
A.22

figure A.22: Longitudinal Skin Stress at Stringer Position 3 Frame 58.
Figure A.23: Circumferential Skin Stress at Stringer Position 3 of Frame 58.
figure A.24: Shear Skin Stress at Stringer Position 3 of Frame 58.
**Figure A.25:** Longitudinal Skin Stress at Stringer Position 4 of Frame 58.
**figure A.26**: Circumferential Skin Stress at Stringer Position 4 of Frame 58.
Figure A.27: Shear Skin Stress at Stringer Position 4 of Frame 58.
Appendix B: Stress Calculation Formulae.

This appendix contains the basic stress calculation formulae upon which the calculation program is based. More details can be found in Jong (1984), Lekhnitskii (1968), Scholte Albers (1991) and Tooren et al. (1992).

B.1 General Equations for Stresses and Strains

For the calculation of stresses and deformations in a plate it is assumed that the plate is a continuous medium. The state of stress in a point of such a medium, which is either in rest or in motion as a result of external forces, can be given with the stress components on three mutually perpendicular planes through the point (see figure). If Cartesian coordinates are used the following equations apply for the stress components on an inclined plane with normal $n$:

$$
X_n = \sigma_x \cos(n,x) + \tau_{xy} \cos(n,y) + \tau_{xz} \cos(n,z)
$$

$$
Y_n = \tau_{xy} \cos(n,x) + \sigma_y \cos(n,y) + \tau_{yz} \cos(n,z)
$$

$$
Z_n = \tau_{xz} \cos(n,x) + \tau_{yz} \cos(n,y) + \sigma_z \cos(n,z)
$$

[B.1]

In which $X_n$, $Y_n$ and $Z_n$ are the stresses along the coordinate axes acting on the inclined plane.

If a situation of equilibrium on a stationary body is considered the following equilibrium equations must be satisfied:

$$
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X = 0
$$

$$
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + Y = 0
$$

$$
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + Z = 0
$$

[B.2]

In which $X$, $Y$ and $Z$ designate the forces acting on the body referred to a unit volume in the directions $x$, $y$ and $z$ respectively.

Six components of deformation characterize the state of deformation in the neighbourhood of a given point. These components are given as three relative elongations $\varepsilon_x$, $\varepsilon_y$ and $\varepsilon_z$ and three relative shear deformations $\gamma_{yx}$, $\gamma_{xz}$ and $\gamma_{xy}$. In the case of small deformations, when the derivatives of the displacements are small compared with unity, the relation between the deformations and the displacements $u, v$ and $w$, in the directions $x, y$ and $z$ respectively, are:

$$
\varepsilon_x = \frac{\partial u}{\partial x}, \quad \gamma_{yx} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}
$$

$$
\varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}
$$

[B.3]
\[ \varepsilon_z = \frac{\partial w}{\partial z}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \]

With the equations [B.1] through [B.3] it is impossible to solve the problems of equilibrium of an elastic body. Additional relations between the components of stress and deformation are necessary. In most calculations on elastic bodies the generalized Hooke’s law is assumed, that is the components of deformation are linear functions of the components of stress. In this work, as well as in the calculation program, Hooke’s law is considered to be valid for all situations.

An elastic body is called isotropic when its elastic properties are identical in all directions, and anisotropic when its elastic properties are different for different directions. A body is called homogeneous when its elastic properties are identical in all parallel directions passing through any of its points.

If a homogeneous elastic body with anisotropy of a general type (no elastic symmetry) is considered a number of 6 X 6 elastic constants will be necessary to describe the relation between the (six) stresses and the (six) deformations. From the mechanics of continuous media it follows that these constants have a certain symmetry and so the number of independent constants will be 21.

If the body contains one plane of symmetry different elastic constants cancel out and the number of independent elastic properties reduces to 13. Since this case is important for the description of anisotropic plates the formulation of the generalized Hooke’s law in this situation is given here:

\[ \varepsilon_x = a_{11} \sigma_x + a_{12} \sigma_y + a_{13} \sigma_z + a_{16} \tau_{xy}, \quad \gamma_{yz} = a_{44} \tau_{yz} + a_{45} \tau_{xz} \]

\[ \varepsilon_y = a_{12} \sigma_x + a_{22} \sigma_y + a_{23} \sigma_z + a_{26} \tau_{xy}, \quad \gamma_{xz} = a_{45} \tau_{yz} + a_{55} \tau_{xz} \] \[ \text{[B.4]} \]

\[ \varepsilon_z = a_{13} \sigma_x + a_{23} \sigma_y + a_{33} \sigma_z + a_{36} \tau_{xy}, \quad \gamma_{xy} = a_{16} \sigma_x + a_{26} \sigma_y + a_{36} \sigma_z + a_{66} \tau_{xy} \]

If the body contains three mutual perpendicular planes of symmetry, the body is then called orthogonally-anisotropic or in short orthotropic, even more elastic constants cancel out. In this case a total of 9 independent elastic properties describes the relation between deformation and stress.

For this latter situation the following formulae apply:

\[ \varepsilon_x = a_{11} \sigma_x + a_{12} \sigma_y + a_{13} \sigma_z, \quad \gamma_{yz} = a_{44} \tau_{yz} \]

\[ \varepsilon_y = a_{12} \sigma_x + a_{22} \sigma_y + a_{23} \sigma_z, \quad \gamma_{xz} = a_{55} \tau_{xz} \] \[ \text{[B.5]} \]

\[ \varepsilon_z = a_{13} \sigma_x + a_{23} \sigma_y + a_{33} \sigma_z, \quad \gamma_{xy} = a_{66} \tau_{xy} \]

In which \( x, y \) and \( z \) are the principal axes of elasticity.

Instead of the coefficients of deformation \( a_i \) also the engineering constants can be used to describe the relation between deformation and stress. This results in:
\[
\varepsilon_x = \frac{1}{E_1} \sigma_x - \frac{\nu_{21}}{E_2} \sigma_y - \frac{\nu_{31}}{E_3} \sigma_z , \quad \gamma_{yz} = \frac{1}{G_{23}} \tau_{yz} \\
\varepsilon_y = -\frac{\nu_{12}}{E_1} \sigma_x + \frac{1}{E_2} \sigma_y - \frac{\nu_{32}}{E_3} \sigma_z , \quad \gamma_{xz} = \frac{1}{G_{13}} \tau_{xz} \\
\varepsilon_z = -\frac{\nu_{13}}{E_1} \sigma_x - \frac{\nu_{23}}{E_2} \sigma_y + \frac{1}{E_3} \sigma_z , \quad \gamma_{xy} = \frac{1}{G_{12}} \tau_{xy} \tag{B.6}
\]

Which contains 12 elastic constants, three Young's moduli \( E_j \) for tension (compression) along the principal directions of elasticity, six Poisson's ratios \( \nu_{ij} \) which characterize the lateral contraction of the body for tension (compression) along the principal directions of elasticity, and three shear moduli \( G_{ij} \) which characterize changes of angles between principal directions.

With the following relations between the Young's moduli and the Poisson's ratios the number of independent elastic constants is again reduced to 9:

\[
E_1 \nu_{21} = E_2 \nu_{12} , \quad E_2 \nu_{32} = E_3 \nu_{23} , \quad E_3 \nu_{13} = E_1 \nu_{31} \tag{B.7}
\]

When through every point of a given body passes a plane in which all directions are equivalent with respect to the elastic properties, the body is then called transversely-isotropic, the generalized Hooke's law reduces further to

\[
\varepsilon_x = a_{11} \sigma_x + a_{12} \sigma_y + a_{13} \sigma_z , \quad \gamma_{yz} = a_{44} \tau_{yz} \\
\varepsilon_y = a_{12} \sigma_x + a_{11} \sigma_y + a_{13} \sigma_z , \quad \gamma_{xz} = a_{44} \tau_{xz} \tag{B.8}
\]

\[
\varepsilon_z = a_{13} (\sigma_x + \sigma_y) + a_{33} \sigma_z , \quad \gamma_{xy} = 2 (a_{11} - a_{12}) \tau_{xy}
\]

Which can be stated again in engineering constants as follows:

\[
\varepsilon_x = \frac{1}{E_1} (\sigma_x - \nu \sigma_y) - \frac{\nu'}{E'} \sigma_z , \quad \gamma_{yz} = \frac{1}{G'} \tau_{yz} \\
\varepsilon_y = \frac{1}{E_2} (\sigma_y - \nu \sigma_x) - \frac{\nu'}{E'} \sigma_z , \quad \gamma_{xz} = \frac{1}{G'} \tau_{xz} \tag{B.9}
\]

\[
\varepsilon_z = -\frac{\nu'}{E'} (\sigma_x + \sigma_y) + \frac{1}{E'} \sigma_z , \quad \gamma_{xy} = \frac{1}{G} \tau_{xy}
\]

In which \( E \) stands for the Young's modulus for directions in the plane of isotropy, \( E' \) the Young's modulus for directions perpendicular to this plane, \( \nu \) the Poisson's ratio which characterizes the contraction in the plane of isotropy when tension (compression) is applied in the same plane, \( \nu' \) the Poisson's ratio which characterizes the contraction in the plane of isotropy when tension (compression) is applied in the direction perpendicular to this plane, \( G = (E/(2(1+\nu))) \) the shear modulus for the plane of isotropy and \( G' \) the shear modulus which characterizes the distortion of angles between the isotropy plane and the normal.

With the given relation for \( G \) equations [B.8] and [B.9] each contain five independent elastic constants. The last simplification of the generalized Hooke's law is obtained by regarding a complete symmetric body which reduces the problem to that of an isotropic body. This yields (in engineering constants):

B.3
\[ \varepsilon_x = \frac{1}{E} \left( \sigma_x - \nu (\sigma_y + \sigma_z) \right), \quad \gamma_{yz} = \frac{1}{G} \tau_{yz} \]

\[ \varepsilon_y = \frac{1}{E} \left( \sigma_y - \nu (\sigma_x + \sigma_z) \right), \quad \gamma_{xz} = \frac{1}{G} \tau_{xz} \]

\[ \varepsilon_z = \frac{1}{E} \left( \sigma_z - \nu (\sigma_x + \sigma_y) \right), \quad \gamma_{xy} = \frac{1}{G} \tau_{xy} \]  

[B.10]

With the well-known relation \( G = E/(2(1+\nu)) \) the number of independent elastic constants in this case is two.

With nine functions (six stress components and three displacements) it is possible to fully describe the state of stress and deformation of a given elastic body. In order to determine these nine functions nine independent equations are required. In the case of a flat plate those nine required basic equations can be obtained by combining equation [B.2], which gives the stresses that follow from the equilibrium of the body, [B.3], which gives the relation between the deformations and the displacements, and equation [B.4], which gives the relation between the stresses and the deformations.

Since the system of equations which is now obtained contains several derivatives the equations have to be integrated and boundary conditions have to be given to completely solve the problem.

### B.2 Equations for Plane Problems of an Anisotropic Body

In order to describe the condition of an anisotropic plate, three assumptions will be made in advance. Those assumptions are 1) at each point of the plate there is a plane of elastic symmetry which is parallel to the middle plane, 2) the forces applied to the edge and the body forces are acting within planes which are parallel to the middle plane and they are distributed symmetrically with respect to this plane and they change slightly with the plate thickness, and 3) deformations of the plate are small. A state of stress in a plate which satisfies these three conditions is called a state of generalized plane stress.

The middle plane of the plate is chosen as the coordinate plane xy with o as the origin and the axes x and y in an arbitrary direction, the plate thickness is called \( h \), and \( a_{11}, a_{12}, \ldots, a_{66} \) are the elastic constants of the material in the coordinate system \( xyz \).

In studying the state of plane stress the stress components and displacements are given by its average values with respect to the thickness of the body. This can be formulated as:

\[ \bar{\sigma}_x = \frac{1}{h} \int_{-h/2}^{h/2} \sigma_x \, dz \quad \bar{\sigma}_y = \frac{1}{h} \int_{-h/2}^{h/2} \sigma_y \, dz \quad \bar{\tau}_{xy} = \frac{1}{h} \int_{-h/2}^{h/2} \tau_{xy} \, dz \]

[B.11]

\[ \bar{u} = \frac{1}{h} \int_{-h/2}^{h/2} u \, dz \quad \bar{v} = \frac{1}{h} \int_{-h/2}^{h/2} v \, dz \]

Note that \( \sigma_z, \tau_{xz}, \tau_{yz} \) are not mentioned in this equation since these stresses are all zero by the definition of plane stress. Furthermore \( w \) is not of interest in the case of plane stress. The five average (with respect to the thickness) values (three stresses and two displacements) of equation [B.11] can be calculated from the earlier mentioned equilibrium equations integrated with respect to \( z \) from \(-h/2\) to \( h/2\).

For the sake of simplicity the bars over the symbols for stresses and displacements, which indicate average values, will be omitted and from now on the symbols \( \sigma_x, \sigma_y, \tau_{xy} \) and \( u \) and \( v \) stand for the average values with respect to the thickness.

With the earlier mentioned assumptions the next system of equations is obtained:

B.4
\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0, \quad \text{with} \quad X = X = \frac{1}{h} \int_{-h/2}^{h/2} X \, dz
\]

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0, \quad \text{with} \quad Y = Y = \frac{1}{h} \int_{-h/2}^{h/2} Y \, dz
\]

\[
\varepsilon_x = a_{11} \sigma_x + a_{12} \sigma_y + a_{16} \tau_{xy}, \quad \varepsilon_x = \frac{\partial u}{\partial x}
\]

\[
\varepsilon_y = a_{12} \sigma_x + a_{22} \sigma_y + a_{26} \tau_{xy}, \quad \varepsilon_y = \frac{\partial v}{\partial y}
\]

\[
\gamma_{xy} = a_{16} \sigma_x + a_{26} \sigma_y + a_{66} \tau_{xy}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\]

If the body forces \(X, Y\) are assumed to be derivable from a potential \(U(x,y)\) then

\[
X = -\frac{\partial U}{\partial x}, \quad Y = -\frac{\partial U}{\partial y}
\]

In which \(U\) stands for the potential of the body forces as averaged with respect to the thickness.

Now the equilibrium equations of [B.12] will be satisfied by the introduction of the Airy stress function \(F(x,y)\) and by assuming that:

\[
\sigma_x = \frac{\partial^2 F}{\partial y^2} + U, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} + U, \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}
\]

The compatibility condition is obtained by differentiation of the deformations:

\[
\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = 0
\]

With [B.13] through [B.15] the system of equations from [B.12] can now be formulated as the following differential equation:

\[
a_{22} \frac{\partial^4 F}{\partial x^4} - 2a_{26} \frac{\partial^4 F}{\partial x^3 \partial y} + (2a_{12} + a_{66}) \frac{\partial^4 F}{\partial x^2 \partial y^2} - 2a_{16} \frac{\partial^4 F}{\partial x \partial y^3} + a_{11} \frac{\partial^4 F}{\partial y^4} - \\
-(a_{12} + a_{22}) \frac{\partial^2 U}{\partial x^2} + (a_{16} + a_{26}) \frac{\partial^2 U}{\partial x \partial y} - (a_{11} + a_{12}) \frac{\partial^2 U}{\partial y^2}
\]

In the case of the absence of body forces the right hand side of this equation reduces to zero and a homogeneous equation is to be solved.

The boundary conditions for the differential equation [B.16] follow from either 1) the external forces and body forces on the surface, 2) the displacements and body forces on the surface, or 3) a combination of 1) and 2). These boundary conditions are transformed into the conditions at the contour edge of a plane \(S\), situated on the plane of symmetry of the body.
The plane problem of the theory of elasticity is now reduced to the determination of a stress function $F(x,y)$ in region $S$ on the plane of symmetry of the body, which satisfies equation [B.16] and the boundary conditions at the contour. If such a function is found the equilibrium equations will be automatically satisfied by the stresses which follow from [B.14] and the displacements will satisfy the compatibility equation [B.15]. If the case of generalized plane stress with the absence of body forces is considered, the right hand side of equation [B.16] vanishes and the function $F$ must satisfy the equation:

$$a_{22} \frac{\partial^4 F}{\partial x^4} - 2a_{26} \frac{\partial^4 F}{\partial x^3 \partial y} + (2a_{12} + a_{66}) \frac{\partial^4 F}{\partial x^2 \partial y^2} - 2a_{16} \frac{\partial^4 F}{\partial x \partial y^3} + a_{11} \frac{\partial^4 F}{\partial y^4} = 0$$  \[B.17\]

With the use of four linear differential operators of the first order this equation can be written symbolically as:

$$D_1 D_2 D_3 D_4 F = 0 , \quad \text{with} \quad D_k = \frac{\partial}{\partial y} - \mu_k \frac{\partial}{\partial x} , \quad k = 1.4 \tag{B.18}$$

with $\mu_k$ out of: $a_{11} \mu^4 - 2a_{16} \mu^2 + (2a_{12} + a_{66}) \mu^2 - 2a_{26} \mu + a_{22} = 0$

The characteristic equation in $\mu$ from equation [B.18] can have either complex or purely imaginary roots but never has real roots in case of any ideal elastic body with elastic constants $a_{11}, a_{12}, a_{22}$ and $a_{66}$ finite and not equal to zero. The four roots are designated as $\mu_1, \mu_2$ and their two conjugates. Two possible situations exist, depending on the relations between the elastic constants.

In the first situation all four roots of the equation are different:

$$\mu_1 = \alpha + \beta i , \quad \mu_2 = \gamma + \delta i , \quad \mu_{\bar{1}} = \alpha - \beta i , \quad \mu_{\bar{2}} = \gamma - \delta i \quad (\beta > 0 , \delta > 0) \tag{B.19}$$

In which $\alpha, \beta, \gamma$ and $\delta$ are real numbers.

In the second situation the four roots are pairwise equal:

$$\mu_1 - \mu_2 = \alpha + \beta i , \quad \mu_{\bar{1}} - \mu_{\bar{2}} = \alpha - \beta i \quad (\beta > 0) \tag{B.20}$$

Where again $\alpha$ and $\beta$ are real numbers.

The quantities $\mu_1$ and $\mu_2$ are called the complex parameters of the first order of plane stress (or plane strain), or simply the complex parameters. These parameters can be considered as numbers which characterize the degree of anisotropy in the case of plane problems. Their values can be used to indicate the difference between the given body and an isotropic body, for which always $\mu_1 = \mu_2 = i$ and $|\mu_1| = |\mu_2| = 1$.

With the use of equation [B.18] a new expression for $F$ can be found, depending on different values of the complex parameters. In the case of four different complex parameters:

$$F = F_1 (x + \mu_1 y) + F_2 (x + \mu_2 y) + F_3 (x + \mu_{\bar{1}} y) + F_4 (x + \mu_{\bar{2}} y) \tag{B.21}$$

And in the case of pairwise equal complex parameters:

$$F = F_1 (x + \mu_1 y) + (x + \mu_{\bar{1}} y) F_2 (x + \mu_1 y) + F_3 (x + \mu_{\bar{1}} y) + (x + \mu_1 y) F_4 (x + \mu_{\bar{1}} y) \tag{B.22}$$

In which $F_1..F_4$ are arbitrary functions of complex and generalized variables. If the following designations are introduced:

$$z_1 = x + \mu_1 y , \quad z_2 = x + \mu_2 y , \quad \bar{z}_1 = x + \mu_{\bar{1}} y , \quad \bar{z}_2 = x + \mu_{\bar{2}} y \tag{B.23}$$

B.6
and it is realized that the stress functions should be a real function of variables $x$ and $y$ to obtain real stresses, the real parts of $F_1$ and $F_2$ will be equal to each other while their imaginary parts will cancel out. The same story goes for $F_2$ and $F_4$. Now expression [B.21] can also be given as:

$$F = 2 \text{ Re } [F_1(z_1) + F_2(z_2)]$$  \hspace{1cm} [B.24]$$

and expression [B.22] can also be given as:

$$F = 2 \text{ Re } [F_1(z_1) + \overline{z_1} F_2(z_1)]$$  \hspace{1cm} [B.25]$$

Where Re designates the real part of any complex expression.

As shown in equation [B.24] the stress function in the case of unequal complex parameters is expressed by two arbitrary analytical functions of complex variables $z_1 = x + \mu_1 y$ and $z_2 = x + \mu_2 y$. In the case of equal parameters only one complex variable $z_1$ is obtained as shown in equation [B.25].

From now on only the case with different complex parameters will be considered. If the following expressions are introduced:

$$\Phi_1(z_1) = \frac{dF_1}{dz_1}, \quad \Phi_2(z_2) = \frac{dF_2}{dz_2}, \quad \Phi'_1(z_1) = \frac{d\Phi_1}{dz_1}, \quad \Phi'_2(z_2) = \frac{d\Phi_2}{dz_2}$$  \hspace{1cm} [B.26]$$

The stresses which follow from equation [B.14] are given by:

$$\sigma_x = 2\text{Re} \left[ \mu_1^2 \Phi'_1(z_1) + \mu_2^2 \Phi'_2(z_2) \right]$$

$$\sigma_y = 2\text{Re} \left[ \Phi'_1(z_1) + \Phi'_2(z_2) \right]$$  \hspace{1cm} [B.27]$$

$$\tau_{xy} = -\text{Re} \left[ \mu_1 \Phi'_1(z_1) + \mu_2 \Phi'_2(z_2) \right]$$

Which can be calculated if the expression for $\Phi_1'$ and $\Phi_2'$ are known.

### B.3 Equations for the Stress Distribution in a Plate with an Elliptical Opening.

In this part of appendix B determination of stresses in a plate weakened by an elliptical opening and deformed by forces acting on the middle plane is considered. It is assumed that the opening is small in comparison with the plate size and that it is not located at the edge of the plate. Now the plate can be assumed as being infinite and the effect of the external edge of the plate can be disregarded.

Let $x$ and $y$ be the coordinate axes along the axes of the elliptical hole, $a_{ij}$ the (known) elastic constants of the generalized Hooke's law (equation [B.4]), $d$ and $b$ the semi-axes of the ellipse, $h$ the plate thickness, $X_n$ and $Y_n$ the forces acting at the opening edge given in coordinate directions, and $P_x$ and $P_y$ the resultants of these forces in coordinate directions, (see figure).
To obtain the functions $\Phi_1$ and $\Phi_2$, necessary to calculate the stresses according to [B.27], the elliptical contour of the hole is written in a parametric form as follows:

$$x = d \cos \theta, \quad y = b \sin \theta$$  \[B.28\]

In this case the external forces at the contour of the ellipse embody the boundary conditions necessary to solve the problem. If it is assumed that the given forces $X_n$ and $Y_n$ are in equilibrium over the boundary, these forces can be expanded in Fourier series as follows:

$$-\int_0^s Y_n ds + c_1 = -\alpha_0 + \sum_{m=1}^\infty (\alpha_m \sigma^m + \overline{\alpha}_m \overline{\sigma}^m)$$  \[B.29\]

$$\int_0^s X_n ds + c_2 = -\beta_0 + \sum_{m=1}^\infty (\beta_m \sigma^m + \overline{\beta}_m \overline{\sigma}^m)$$

in which $\sigma = e^{i\theta}$ represents the circle of unity in a complex representation, $\alpha_m$ and $\beta_m$ are known (usually complex) coefficients which depend on the load distribution at the opening edge, and $\alpha_0$ and $\beta_0$ are arbitrary constants.

With this equation the boundary conditions of this particular problem can be formulated as:

$$2\text{Re}[\Phi_1(z_1) + \Phi_2(z_2)] = -\frac{P_y}{2\pi h} \phi + \alpha_0 + \sum_{m=1}^\infty (\alpha_m \sigma^m + \overline{\alpha}_m \overline{\sigma}^m)$$  \[B.30\]

$$2\text{Re}[\mu_1\Phi_1(z_1) + \mu_2\Phi_2(z_2)] = -\frac{P_x}{2\pi h} \phi + \beta_0 + \sum_{m=1}^\infty (\beta_m \sigma^m + \overline{\beta}_m \overline{\sigma}^m)$$

Where $P_x$ and $P_y$ are the earlier mentioned resultants of the applied force in coordinate directions thus symbolising the deviation from equilibrium of the forces at the boundary. If the displacements at the contour of the hole are known, equations similar to equation [B.29] can be given. Since this is not of much interest in this present work those equations will not be stated here.

With equations [B.29] the basic solution, resulting from the presence of the loaded hole can now be given as follows:

$$\Phi_1(z_1) = A_0 + A \ln \zeta_1 + \sum_{m=1}^\infty \frac{\beta_m - \mu_2 \overline{\alpha}_m}{\mu_1 - \mu_2} \zeta_1^{-m}, \quad \text{with} \quad \zeta_1 = \frac{z_1 + \sqrt{z_1^2 - d^2 - \mu_1^2 b^2}}{d - i\mu_1 b}$$  \[B.31\]

$$\Phi_2(z_2) = B_0 + B \ln \zeta_2 + \sum_{m=1}^\infty \frac{\beta_m - \mu_1 \overline{\alpha}_m}{\mu_2 - \mu_1} \zeta_2^{-m}, \quad \text{with} \quad \zeta_2 = \frac{z_2 + \sqrt{z_2^2 - d^2 - \mu_2^2 b^2}}{d - i\mu_2 b}$$

These functions have the same $\sigma = e^{i\theta}$ at the opening edge. $A_0$ and $B_0$ are arbitrary constants, and $A$ and $B$ are constants which follow from:
\[
A + B = \bar{A} - \bar{B} = \frac{P_y}{2\pi h_i}
\]

\[
\mu_1A + \mu_2 - \bar{\mu}_1\bar{A} - \bar{\mu}_2\bar{B} = \frac{P_x}{2\pi h_i}
\]

\[
\mu_1^2A + \mu_2^2B - \bar{\mu}_1^2\bar{A} - \bar{\mu}_2^2\bar{B} = -\frac{a_{16}}{a_{11}} \cdot \frac{P_x}{2\pi h_i} - \frac{a_{12}}{a_{11}} \cdot \frac{P_y}{2\pi h_i}
\]

\[
\frac{1}{\mu_1}A + \frac{1}{\mu_2}B - \frac{1}{\bar{\mu}_1}\bar{A} - \frac{1}{\bar{\mu}_2}\bar{B} = -\frac{a_{12}}{a_{22}} \cdot \frac{P_x}{2\pi h_i} - \frac{a_{26}}{a_{22}} \cdot \frac{P_y}{2\pi h_i}
\]

Equation [B.32]

As can be seen from these equations, the terms with these constants A and B represent the influence of the resultant of the forces at the opening edge on the total stress field. This part of the stress field is fully determined by this resultant of the forces (via \(P_x\) and \(P_y\)) and the material parameters (\(a_{11}, a_{66}, \mu_1\) and \(\mu_2\)). In the case of equilibrium of forces, these terms vanish from equation [B.31].

For both \(\Phi_1\) and \(\Phi_2\) the part of equation [B.31] which contains the summation represents the disturbance of the stress field due to the distributed forces at the opening of the elliptical hole. This part of the stress field is determined by the axes of the ellipse (d and b), the load distribution at the opening edge (via \(\alpha_m\) and \(\beta_m\)), and the material parameters (via \(\mu_1\) and \(\mu_2\)). This disturbance only consists of negative powers of \(\zeta\) in order to limit its influence to the area around the hole.

The derivatives of \(\Phi_1\) and \(\Phi_2\) (equation [B.30]) are given by:

\[
\Phi_1'(z_1) = \frac{1}{\sqrt{z_1^2 - d^2 - \mu_1b^2}} \left\{ A - \sum_{m=1}^\infty \frac{\beta_m}{\mu_1} - \frac{\mu_2\beta_m}{\mu_2 - \mu_1} \zeta^{-m} \right\}
\]

\[
\Phi_2'(z_2) = \frac{1}{\sqrt{z_2^2 - d^2 - \mu_2b^2}} \left\{ B - \sum_{m=1}^\infty \frac{\beta_m}{\mu_2} - \frac{\mu_1\beta_m}{\mu_1 - \mu_2} \zeta^{-m} \right\}
\]

Equation [B.33]

If these derivatives are substituted into equation [B.27] the equations for the stresses are known as a function of \(z_1\) and \(z_2\). With equation [B.23] in mind the stresses in every point of the plate are now defined.

If it is assumed that an infinite plate loaded at infinity is disturbed by an elliptical hole, similar formulas can be deduced. Let again x and y be the coordinate axes along the axes of the ellipse, \(a_i\) the (known) elastic constants of the generalized Hooke’s law (equation [B.4]), d and b the semi-axes of the ellipse, h the plate thickness, P the force applied at the infinity, and \(\varphi\) the angle between the x-axis and the applied force (see figure).
If there are no forces at the opening edge, the logarithmic term of [B.31] will be zero and only the part of [B.31] with the negative power of $\varphi$ will determine the influence of the hole on the total stress field. The total stress field can now be found by summation of the stresses in a solid plate subjected to uniform tension given by:

$$\sigma_x = P \cos^2 \varphi, \quad \sigma_y = P \sin^2 \varphi, \quad \tau_{xy} = P \sin \varphi \cos \varphi$$  \[B.34\]

together with the stresses which give the disturbance of the stress field due to the presence of the elliptical hole. This disturbing stress field follows from $\Phi_1$ and $\Phi_2$ which are given by:

$$\Phi_1(z_1) = \frac{\bar{\beta}_1 - \mu_2 \bar{\alpha}_1}{\mu_1 - \mu_2} \cdot \frac{1}{\xi_1}$$

$$\Phi_2(z_2) = \frac{\bar{\beta}_1 - \mu_1 \bar{\alpha}_1}{\mu_2 - \mu_1} \cdot \frac{1}{\xi_2}$$  \[B.35\]

with $\bar{\alpha}_1 = \frac{P \sin \varphi}{2} (d \sin \varphi - ib \cos \varphi)$

$$\bar{\beta}_1 = \frac{P \cos \varphi}{2} (d \sin \varphi - ib \cos \varphi)$$

Note that these functions only contain one term of $\varphi$. With the expressions for $\xi_1$ and $\xi_2$ obtained from equation [B.31] the derivatives of these functions, which are to be used in the calculation of the stresses, can be given with:

$$\Phi_1'(z_1) = -\frac{1}{\sqrt{z_1^2 - d^2 - \mu_2^2 b^2}} \frac{\bar{\beta}_1 - \mu_2 \bar{\alpha}_1}{\mu_1 - \mu_2} \frac{d - i \mu_1 b}{z_1 + \sqrt{z_1^2 - d^2 - \mu_1^2 b^2}}$$  \[B.36\]

$$\Phi_2'(z_2) = -\frac{1}{\sqrt{z_2^2 - d^2 - \mu_1^2 b^2}} \frac{\bar{\beta}_1 - \mu_1 \bar{\alpha}_1}{\mu_2 - \mu_1} \frac{d - i \mu_2 b}{z_2 + \sqrt{z_2^2 - d^2 - \mu_2^2 b^2}}$$

In which $\alpha$ and $\beta$ are equivalent to those mentioned in equation [B.35].

**B.4 Equations for the stress distribution around a crack.**

With the knowledge of paragraph B.3, the stress distribution as a result of a crack in an infinite plate can be found by letting one of the axes of the ellipse approach to zero.

If a crack along the x-axis is considered (i.e. $b = 0$) loaded perpendicular to the crack in the y-direction (i.e. $\varphi = \pi/2$), equation [B.36] reduces to the equation:
\[ \Phi'_1(z_1) = -\frac{1}{\sqrt{z_1^2 - d^2}} \frac{Pd\mu_2}{2(\mu_1 - \mu_2)} \frac{d}{z_1 + \sqrt{z_1^2 - d^2}} \]  
\[ \Phi'_2(z_2) = -\frac{1}{\sqrt{z_2^2 - d^2}} \frac{Pd\mu_1}{2(\mu_2 - \mu_1)} \frac{d}{z_2 + \sqrt{z_2^2 - d^2}} \]  

And with the knowledge of equation [B.27] the stresses can be found.
The total stress field due to a crack in an infinite plate loaded in the y-direction at infinity can now be given with:

\[ \sigma_x = \text{Re} \left[ \frac{Pd^2\mu_1\mu_2}{\mu_2 - \mu_1} \left\{ \frac{\mu_1}{\sqrt{z_1^2 - d^2} + z_1\sqrt{z_1^2 - d^2}} - \frac{\mu_2}{\sqrt{z_2^2 - d^2} + z_2\sqrt{z_2^2 - d^2}} \right\} \right] \]  
\[ \sigma_y = P + \text{Re} \left[ \frac{Pd^2}{\mu_2 - \mu_1} \left\{ \frac{\mu_2}{\sqrt{z_1^2 - d^2} + z_1\sqrt{z_1^2 - d^2}} - \frac{\mu_1}{\sqrt{z_2^2 - d^2} + z_2\sqrt{z_2^2 - d^2}} \right\} \right] \]  
\[ \tau_{xy} = -\text{Re} \left[ \frac{Pd^2\mu_1\mu_2}{\mu_2 - \mu_1} \left\{ \frac{1}{\sqrt{z_1^2 - d^2} + z_1\sqrt{z_1^2 - d^2}} - \frac{1}{\sqrt{z_2^2 - d^2} + z_2\sqrt{z_2^2 - d^2}} \right\} \right] \]  

In which the single P in the formulation for \( \sigma_y \) comes forth from equation [B.34] and denotes the influence of the undisturbed stress field.

These equations [B.38] for the stress distribution around a single crack in an infinite plate are used in the calculation program in order to calculate geometry factors as described in paragraph 3.2 of this report.

**B.5 The Stress Calculation Program.**

How can the equations from the former paragraphs be used in a calculation program intended to calculate the stress distribution around a row of three holes?

In the case of a row of three holes the stress functions are summations of three double (both for \( \Phi_1 \) and \( \Phi_2 \)) sums. Each sum represents the influence of a particular elliptical hole on the total stress field. By satisfying the load boundary conditions of each of the three holes, an infinite set of linear equations of the coefficients of the stress functions is obtained. If a truncated set of these equations converges, an approximate solution for the coefficients is obtained which can be implemented in a calculation program.

The stress functions \( \Phi_1 \) and \( \Phi_2 \) for the row of three holes can be given as:

\[ \Phi_1(z_1) = \Phi_1^{\text{hom}}(z_1) + \Phi_1^{\text{dist}}(z_1) \]  
\[ \Phi_2(z_2) = \Phi_2^{\text{hom}}(z_2) + \Phi_2^{\text{dist}}(z_2) \]  

B.11
Damage Tolerance Aspects of a Full Composite Airplane Fuselage.

In which \( \Phi^\text{hom} \) denotes the undisturbed homogeneous stress field resulting in a plate without holes loaded at infinity (see equation [B.34]) and \( \Phi^\text{dist} \) denotes the disturbance of the stress field by the combined influence of the three elliptical holes. This disturbance will be defined by the measurements of the holes, the hole spacings, the material parameters, and the load distribution at the edge of each hole. The two stress functions are found by simultaneously solving the boundary conditions at the edges of the holes. For this purpose first the boundary conditions for each hole are drawn up separately and thereupon all equations are combined to come to solution of the stress functions. The approach is thus basically similar to the one treated for a single hole, however, now also the influence of the two other holes on the opening edge of this single hole has to be considered. This can be formulated as:

\[
\Phi^\text{dist}_k(z_k) = \left[ \Phi^\text{dist}_k(z_k) \right]_{p=c-1} + \left[ \Phi^\text{dist}_k(z_k) \right]_{p=c+0} + \left[ \Phi^\text{dist}_k(z_k) \right]_{p=c+1}, \quad k=1,2, \quad c=-1,0,+1 \tag{B.40}
\]

In which \( p \) is a hole index, used to distinguish between the three holes, and \( c \) is an index used to distinguish between the three independent boundary condition evaluations. For each value of \( c \) the hole index of the hole whereof the boundary conditions are considered becomes equal to zero (\( c=-1 \to \text{right hole} \), \( c=0 \to \text{middle hole} \), and \( c=1 \to \text{left hole} \).

If \( s \) is used to indicate the distance between the holes, which need not be similar for the left side and the right side, then the stress functions that define the disturbing stress field can be given as:

\[
\Phi^\text{dist}_k(z_k) = \sum_{n=1}^{\infty} F_n(z_k-ps) \mu_n^{-1} + \sum_{n=1}^{\infty} G_n(z_k-ps) \mu_n^{-1} + \sum_{n=1}^{\infty} H_n(z_k-ps) \mu_n^{-1}, \quad k=1,2, \quad c=-1,0,+1 \tag{B.41}
\]

In which \( F_n, G_n \) and \( H_n \) are coefficients that are determined by the boundary conditions. From each boundary condition evaluation a total of \( 3n \) coefficients is found (\( nF+nG+nH \)), together with \( n \) equations. Consequently, all three boundary condition evaluations have to be regarded in order to find enough equations to determine all \( 3n \) coefficients.

To calculate the different coefficients, the origin of the axis system is moved to the centre of the hole under consideration. With a transformation similar to that of equation [B.31] the contour of this elliptical hole is now transformed into a unit circle. If \( d \) and \( b \) are again the semi axes of the elliptical holes, coupled with one of the three holes through the hole index number \( p \), this transformation can be expressed as:

\[
\zeta_k^{(p)} = \frac{(z_k - ps) \pm \sqrt{(z_k - ps)^2 - \mu_k b^2 - d^2}}{d - i \mu_k b}, \quad k=1,2 \tag{B.42}
\]

And with this transformation equation [B.41] alters to:

\[
\Phi^\text{dist}_k(z_k) = \sum_{n=1}^{\infty} f_n(z_k^{(p-1)}) \mu_n^{-1} + \sum_{n=1}^{\infty} g_n(z_k^{(p+0)}) \mu_n^{-1} + \sum_{n=1}^{\infty} h_n(z_k^{(p+1)}) \mu_n^{-1}, \quad k=1,2, \quad c=-1,0,+1 \tag{B.43}
\]

In which the hole whereof \( p=0 \) is transformed into the unit circle \( \sigma = e^{i\theta} \). For the remaining two holes a Taylor series expansion around the origin of the axis system (i.e. \( z_k = 0 \)) is applied in order to find suitable expressions for their coefficients so as to solve the boundary conditions of the single hole. This Taylor series expansion is formulated as follows:
\[
\left( s_k^{(p)} \right)^{-n} = \left( s_k^{(p)} \right)_{z_k=0}^{-n} + \frac{z_k}{1!} \left[ \frac{d}{dz_k} \left( s_k^{(p)} \right)^{-n} \right]_{z_k=0} + \frac{z_k^2}{2!} \left[ \frac{d^2}{dz_k^2} \left( s_k^{(p)} \right)^{-n} \right]_{z_k=0} + \ldots
\]  

[B.44]

Such a Taylor series expansion yields an approximation of the value of \( (s_k^{(p)})^{-n} \) which becomes better as the number of terms is increased.

After completing this operation for one of boundary conditions of the stress functions, part of the equations is expressed in terms of the unit circle while the remainder is expressed in powers of \( z_k \) following from the Taylor series expansion. To be able to use these Taylor series expansions for determination of the coefficients \( f_n, g_n \), and \( h_n \), the powers of \( z_k \) have to be related to powers of \( \sigma \). This can be accomplished with a relation given by the following expression:

\[
z_k^m = \left( \frac{d - i\mu_k b}{2} \right)^m \sum_{r=0}^{m} \left( \frac{d + i\mu_k b}{d - i\mu_k b} \right)^r \left( \frac{m}{r} \right) \sigma^{m-2r}
\]  

[B.45]

After completing this transformation for all powers of \( z_k \), the total influence of the three holes on the stress field is found in terms of the unit circle \( \sigma \). Together with the boundary conditions at the edge of the holes, the desired coefficients can now be calculated.

With the boundary conditions for a single hole, \( n \) expressions for the coefficients \( f_n, g_n \), and \( h_n \) can be found. By following this strategy for all three holes, a set of \( 3^n \) expressions containing coefficients \( f_n, g_n \), and \( h_n \) is obtained. If these expressions are solved, both \( (k=1,2) \) stress functions for the disturbing stress field are known and with the use of equation [B.27] the stresses can be found. As before, a summation of this disturbing stress field with the undisturbed stress field returns the total stress field.

Of course, in order to make the equations suitable for application in a calculation program, a truncation of terms has to be applied. In the original calculation program (see Scholte Albers (1991) and Tooren et al. (1992)) a truncation of the series of equation [B.43] was made after a total of eight terms. This led to a number of 24 equations with 24 unknowns for each of the two stress functions. Since in the calculations the complex coefficients must be solved for both their real part as well as their imaginary part, this led to a total of 96 equations with 96 unknowns. In the original program, also the Taylor series expansion (equation [B.44]) was truncated after the eight term.

After an extension of the calculation program the number of terms can in principle be expanded to infinity, thus resulting in an exact solution. However, since the capacity of the computer used is only limited, a number of 12 terms was found as a maximum, thus resulting in a total of 144 equations with 144 unknowns. The number of Taylor terms which is used in the approximation of the two remaining holes is always kept equal to the number of terms considered in equation [B.43].

B.13