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Fast Energy-Minimization Method for Surface-Related-Multiple Removal

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SUMMARY

A fast and efficient optimization procedure is presented to meet the minimum-energy criterion for the determination of the source wavelet in the process of elimination of surface-related wavefield phenomena. The energy criterion is formulated in the frequency domain and a minimization over all frequencies and traces is taken as point of departure. A modified conjugate-gradient iterative method for the non-linear optimization problem is presented, where the conjugate-gradient directions are chosen in such a way that, at each iteration, the causality of the unknown source wavelet is enforced. The method is applied to marine seismic field data and leads to satisfactory results.

INTRODUCTION

The presence of surface related wave phenomena in geophysical data as water-surface multiples in the marine case leads to problems in further analysis of the data in migration or inversion. The removal of these multiples has to be effected without changing any relevant subsurface information present in the recorded data. Verschuur *et al.* (1988) have presented a multiple-elimination method that does not require any knowledge about the subsurface structure, but only about the source wavelet. It can be shown that the fundamental theory is based on Rayleigh's reciprocity theorem, see e.g., Fokkema and Van den Berg (1990, 1993), and Van Borselen *et al.* (1991). Assuming that the source wavelet $\hat{q}(\omega)$ is given in the frequency domain, the multiple-free pressure wavefield $\hat{p}^r$, reflected by the earth, is the solution of the operator equation in the frequency domain:

$$\hat{p}^r(x, \omega) + [\hat{q}(\omega)]^{-1} \mathcal{K} \hat{p}^r(x, \omega) = \hat{p}^{dgh}(x, \omega), \quad (1)$$

where $\hat{p}^{dgh}$ is the known source- and receiver-deghosted wavefield and $\mathcal{K}$ is the operator. Here, x stands for the spatial dependence (trace). This equation may be solved by a Neumann-series solution

$$\hat{p}^r(x, \omega) = \sum_{j=0}^{\infty} [-\hat{q}(\omega)]^{-j} \mathcal{K}^j \hat{p}^{dgh}(x, \omega), \quad (2)$$

where $\mathcal{K}^j \hat{p}^{dgh}(x, \omega) = \mathcal{K} \mathcal{K}^{j-1} \hat{p}^{dgh}(x, \omega)$, $j = 1, 2, \dots$, is the repeated operation with $\mathcal{K}$, while $\mathcal{K}^0 \hat{p}^{dgh}(x, \omega) = \hat{p}^{dgh}(x, \omega)$. Verschuur *et al.* (1992) estimated the source wavelet by minimizing the total "energy" in space-time domain, i.e.,

$$E = \sum_{\text{all } x} \int_{\text{all } t} |\hat{p}^r(x, t)|^2 dt, \quad (3)$$

in which the time-domain counterpart of Eq. (2) is substituted. This criterion is minimized by a Monte-Carlo type of procedure, with a computationally time-consuming procedure of switching between the frequency domain and the time domain.

The objective of the present paper is not to discuss the validity of the energy criterion for the determination of the source wavelet, but rather to develop an efficient optimization scheme to solve the pertaining non-linear problem. As point of departure, the frequency-domain counterpart of the energy criterion is taken.

FORMULATION OF THE PROBLEM

The objective is to solve the source wavelet $\hat{q}(\omega)$ by minimisation of the energy norm

$$E = \frac{1}{\pi} \sum_{\text{all } x} \int_0^{\infty} |\hat{p}^r(x, \omega)|^2 d\omega, \quad (4)$$

where

$$\hat{p}^r(x, \omega) = \sum_{j=0}^{\infty} [-\hat{q}(\omega)]^{-j} \mathcal{K}^j \hat{p}^{dgh}(x, \omega), \quad (5)$$

with the a priori information that the source wavelet is causal, i.e.,

$$q(t) = \mathcal{F}^{-1}\{\hat{q}(\omega)\} = 0 \quad \text{when } t < 0. \quad (6)$$

Note that $\mathcal{F}$ denotes the temporal Fourier transform and $\mathcal{F}^{-1}$ its inverse, viz.,

$$\hat{u}(\omega) = \mathcal{F}\{u(t)\} = \int_{t \in \mathbb{R}} \exp(-j\omega t) u(t) dt, \quad (7)$$

$$u(t) = \mathcal{F}^{-1}\{\hat{u}(\omega)\} = \frac{1}{2\pi} \int_{\omega \in \mathbb{R}} \exp(j\omega t) \hat{u}(\omega) d\omega. \quad (8)$$

The causality of the real source wavelet $q(t)$ is enforced by the Hilbert transforms between the real part and imaginary part of $\hat{q}(\omega)$ in the frequency domain. This means that the source wavelet $\hat{q}(\omega)$ is related through its real part $\hat{r}(\omega) = \text{Re}[\hat{q}(\omega)]$ via a linear operator $\mathcal{L}$, viz. (see Fokkema and Van den Berg, 1993, p.33),

$$\hat{q}(\omega) = \mathcal{L}\hat{r}(\omega) = 4\mathcal{F}\left\{\chi(t)\text{Re}\left[\mathcal{F}^{-1}\{\chi(\omega)\hat{r}(\omega)\}\right]\right\}, \quad (9)$$

where $\chi(y)$ is the characteristic function such that $\chi(y) = 0$ when $y < 0$. We observe that $\hat{r}(\omega)$ is the fundamental unknown function of the problem at hand. In practice, the characteristic function $\chi(y)$ is replaced by a function that filters large values of y as well.

ITERATIVE SOLUTION

Let the minimization of the energy norm be carried out via the iterative updating of the real part of the complex source wavelet

$$\begin{aligned}\hat{r}_0(\omega) &= \text{arbitrary}, \\ \hat{r}_n(\omega) &= \hat{r}_{n-1}(\omega) + \alpha_n \hat{c}_n(\omega), \quad n = 1, 2, \dots, \quad (10)\end{aligned}$$

then the updating of the source wavelet is given by

$$\hat{q}_n(\omega) = \hat{q}_{n-1}(\omega) + \alpha_n \hat{d}_n(\omega), \quad n = 1, 2, \dots, \quad (11)$$

where (see Eq. (9))

$$\hat{q}_n(\omega) = \mathcal{L}\hat{r}_n(\omega) \quad \text{and} \quad \hat{d}_n(\omega) = \mathcal{L}\hat{c}_n(\omega). \quad (12)$$

The function $\hat{c}_n(\omega)$ is a suitably chosen real update direction. The real parameter α_n has to be determined in such a way that the cost functional of Eq. (4) is minimized. The value of this cost functional at the n th step is

$$E_n = \int_0^\infty \sum_{j=0}^J \sum_{k=0}^J [-\hat{q}_n(\omega)]^{-j} [-\hat{q}_n^*(\omega)]^{-k} K_{j,k}(\omega) d\omega, \quad (13)$$

where $\hat{q}_n(\omega)$ is given by Eq. (11), while

$$K_{j,k}(\omega) = \frac{1}{\pi} \sum_{\text{all } x} \mathcal{K}^j \hat{p}^{dgh}(x, \omega) [\mathcal{K}^k \hat{p}^{dgh}(x, \omega)]^*. \quad (14)$$

The asterisk denotes complex conjugation. Note that all quantities occurring in the cost functional are functions of ω only. The values of $K_{j,k}$ are obtained as summations over all traces that we would take into account and these values are computed once at the start of the iterative scheme. For known $\hat{d}_n(\omega)$, the value α_n that minimizes the cost functional E_n can be determined with a one-dimensional search method. We employ a routine of a one-dimensional search with first derivatives (routines MNBRK and DBRENT, Press *et al.*, 1986). In the numerical procedure it is advantageous to use a normalized quantity of the cost functional. We define it as

$$\bar{E}_n = \frac{E_n}{\int_0^\infty K_{0,0}(\omega) d\omega}, \quad (15)$$

where the denominator is the total energy of the deghosted wavefield.

CONJUGATE GRADIENT DIRECTION

For the directions in the updating of $\hat{r}_n(\omega)$ we choose the Polak-Ribière conjugate gradient direction (Brodley, 1977)

$$\begin{aligned}\hat{c}_1(\omega) &= \hat{g}_1(\omega), \\ \hat{c}_n(\omega) &= \hat{g}_n(\omega) + \gamma_n \hat{c}_{n-1}(\omega), \quad n = 2, 3, \dots, \quad (16)\end{aligned}$$

where $\hat{g}_n$ is a real gradient direction and γ_n is a real parameter given by

$$\gamma_n = \frac{\int_0^\infty \hat{g}_n(\omega) [\hat{g}_n(\omega) - \hat{g}_{n-1}(\omega)] d\omega}{\int_0^\infty [\hat{g}_{n-1}(\omega)]^2 d\omega}. \quad (17)$$

Instead of computing the direction $\hat{c}_n$, we rather want to compute $\hat{d}_n$. The updating of $\hat{d}_n$ is obtained as

$$\begin{aligned}\hat{d}_1(\omega) &= \mathcal{L}\hat{g}_1(\omega), \\ \hat{d}_n(\omega) &= \mathcal{L}\hat{g}_n(\omega) + \gamma_n \hat{d}_{n-1}(\omega), \quad n = 2, 3, \dots, \quad (18)\end{aligned}$$

and shows that the determination of $\hat{c}_n$ is superfluous.

GRADIENT DIRECTION

In order to determine the gradient direction $\hat{g}_n(\omega)$, we proceed as follows. Substituting the update of Eq. (11) into the expression for $\hat{p}^r$ of Eq. (5), we obtain an update for $\hat{p}^r$ as a non-linear expression in α_n . The linearized form of this expression is obtained as

$$\begin{aligned}\hat{p}^r(x, \omega) &= \sum_{j=0}^J \{ [-\hat{q}_{n-1}(\omega)]^{-j} + j [-\hat{q}_{n-1}(\omega)]^{-j-1} \alpha_n \hat{d}_n(\omega) \} \\ &\quad \times \mathcal{K}^j \hat{p}^{dgh}(x, \omega), \quad (19)\end{aligned}$$

when $\alpha_n \rightarrow 0$. In the linearized form, the derivative with respect to α_n is obtained as

$$\frac{\partial \hat{p}^r(x, \omega)}{\partial \alpha_n} = \sum_{j=1}^J j [-\hat{q}_{n-1}(\omega)]^{-j-1} \hat{d}_n(\omega) \mathcal{K}^j \hat{p}^{dgh}(x, \omega), \quad (20)$$

when $\alpha_n \rightarrow 0$. The cost functional has now a unique minimum as a function of α_n . Differentiating the cost functional E_n with respect to α_n we note that this cost functional is minimized when

$$\text{Re} \left[\sum_{\text{all } x} \int_0^\infty \hat{p}^r(x, \omega) \left(\frac{\partial \hat{p}^r(x, \omega)}{\partial \alpha_n} \right)^* d\omega \right] = 0. \quad (21)$$

After substituting Eqs. (19) and (20) in Eq. (21) we obtain

$$\alpha_n = -\text{Re} \left[\frac{\int_0^\infty \hat{d}_n^*(\omega) \hat{f}_n(\omega) d\omega}{\int_0^\infty |\hat{d}_n(\omega)|^2 \hat{h}_n(\omega) d\omega} \right], \quad (22)$$

where

$$\hat{f}_n(\omega) = \sum_{j=0}^J \sum_{k=1}^J [-\hat{q}_{n-1}(\omega)]^{-j} k [-\hat{q}_{n-1}^*(\omega)]^{-k-1} K_{j,k}(\omega) \quad (23)$$

and

$$\hat{h}_n(\omega) = \sum_{j=1}^J \sum_{k=1}^J j [-\hat{q}_{n-1}(\omega)]^{-j-1} k [-\hat{q}_{n-1}^*(\omega)]^{-k-1} K_{j,k}(\omega), \quad (24)$$

while $K_{j,k}$ is defined in Eq. (14). This value of α_n is used as a starting guess of the one-dimensional search routine MNBRK (Press *et al.*, 1986). Since, the denominator of Eq. (22) is real, improvement ($\alpha_n \neq 0$) is obtained if we require that the real part of the numerator of Eq. (22) does not vanish. We subsequently substitute $d_n = \mathcal{L}\hat{g}_n$ in this numerator. Then, the improvement condition becomes

$$\text{Re} \left[\int_0^\infty [\mathcal{L}\hat{g}_n(\omega)]^* \hat{f}_n(\omega) d\omega \right] \neq 0. \quad (25)$$

With the definition of the operator $\mathcal{L}$ and repeated application of Parseval's theorem, the improvement condition may be written as

$$\int_0^\infty \hat{g}_n(\omega) \text{Re} [\mathcal{L}\hat{f}_n(\omega)] d\omega \neq 0, \quad (26)$$

and we observe that the left-hand side does not vanish when we choose the gradient direction to be

$$\hat{g}_n(\omega) = \text{Re} [\mathcal{L}\hat{f}_n(\omega)]. \quad (27)$$

This gradient is used in Eq. (18) to arrive at the conjugate direction, while the conjugate direction is used in the minimization of the cost functional of Eq. (13).

INITIAL GUESS

We have observed that the convergence of the present scheme depends not heavily on the initial guess, $\hat{q}_0(\omega)$, for the source wavelet. The only a priori condition is that the initial guess must also represent a causal function in time. One initial guess is to start with a constant complex value. Taking into account only the first two terms of the right-hand side of Eq. (5), we obtain

$$\hat{p}^*(x, \omega) \simeq \hat{p}^{dgh}(x, \omega) - \hat{A} \mathcal{K} \hat{p}^{dgh}(x, \omega), \quad (28)$$

where $\hat{A}$ is the inverse of the complex constant that serves as our initial guess for the source wavelet. This constant is determined by minimizing the energy norm of Eq. (4),

$$E \simeq \frac{1}{\pi} \sum_{\text{all } x} \int_0^\infty |\hat{p}^{dgh}(x, \omega) - \hat{A} \mathcal{K} \hat{p}^{dgh}(x, \omega)|^2 d\omega. \quad (29)$$

Differentiating the cost functional with respect to $\hat{A}^*$, we note that this cost functional is minimized when

$$\sum_{\text{all } x} \int_0^\infty [\hat{p}^{dgh}(x, \omega) - \hat{A} \mathcal{K} \hat{p}^{dgh}(x, \omega)] [\mathcal{K} \hat{p}^{dgh}(x, \omega)]^* d\omega = 0, \quad (30)$$

or

$$\hat{A} = \frac{\int_0^\infty K_{0,1}(\omega) d\omega}{\int_0^\infty K_{1,1}(\omega) d\omega}. \quad (31)$$

The causal initial choice is now directly obtained by operating with the causality operator $\mathcal{L}$, viz.,

$$\hat{q}^{init}(\omega) = \frac{1}{2} \mathcal{L} \{ \hat{A}^{-1} \}. \quad (32)$$

However, this causality operation has destroyed the initial minimum of the energy. To correct this, we take

$$\hat{q}_0(\omega) = B^{-1} \hat{q}^{init}(\omega) \quad (33)$$

as starting value of the source wavelet, in which B is a real constant that follows from the minimization of

$$E \simeq \frac{1}{\pi} \sum_{\text{all } x} \int_0^\infty |\hat{p}^{dgh}(x, \omega) - B [\hat{q}^{init}(\omega)]^{-1} \mathcal{K} \hat{p}^{dgh}(x, \omega)|^2 d\omega. \quad (34)$$

We now arrive at

$$B = \text{Re} \left[\frac{\int_0^\infty [\hat{q}^{init}(\omega)]^{-1} K_{1,0}(\omega) d\omega}{\int_0^\infty [\hat{q}^{init}(\omega)]^{-2} K_{1,1}(\omega) d\omega} \right]. \quad (35)$$

NUMERICAL EXAMPLE

In the discretized version of the scheme, the Fourier transforms are replaced by FFT routines and the pertaining integrals over the frequencies are replaced by summations over the frequency samples. In the implementation we have used the actual field, rather than its deghosted counterpart. In order to test the present scheme, a high-resolution marine field data example from the North Sea was processed. The data was shot by a single seismic source. Source and receiver were both located at a depth of 2.5 m. The nearest offset was 18 m. Shot and receiver spacing were both 12.5 m. The time sampling rate was 0.5 ms. From this seismic line 99 input shot records with 48 traces each were used. A raw input shot record is shown in Fig. 1. Note the good signal to noise ratio: multiples M1, M2, M3, primaries P1 and P2 and peg-leg L1 are clearly distinguishable. The following pre-processing steps were applied to the data: high-cut frequency filtering above 100 Hz and resampling to a regular receiver sampling from 12.5 m to 6.0 m to avoid spatial aliasing. Furthermore, the missing near offsets were estimated from the first available offsets. The direct wave has been removed. In Fig. 2 the pre-processed shot record is shown. Next, the $K_{j,k}$ coefficients from Eq. (14) were calculated. Using these coefficients as input, the iteration scheme minimized the energy. In Fig. 3 the normalized energy in the seismogram versus the number of iterations is presented. Note the small number of iterations that was required to reach the final "minimum energy". The result of the multiple removal scheme (Fokkema and Van den Berg, 1993) is presented in Fig. 4. From this result we can conclude that indeed the multiples M1, M2, M3, and the peg-leg L1 are suppressed in

the data, whereas the primary P2 remains unaffected. Computation of the $K_{j,k}$ coefficients took about 5 CPU minutes on a SUN-Sparc 10-40, while the estimation of the wavelet took 10 seconds. The removal of the surface-related wave phenomena amounted to 1 CPU minute.

Figure 1. Raw input shot record.

Figure 2. Input shot record after pre-processing.

Figure 3. Convergence of the iteration scheme.

Figure 4. Shot record after multiple removal.

CONCLUSIONS

In the procedure of elimination surface-related wave phenomena, using the minimum-energy criterion, we have developed an iterative scheme to compute an estimate of the causal source wavelet. After some initial computations, the actual minimization procedure is carried out in the frequency domain only. In the improvement directions, causality is enforced. The method is computationally very fast and its application to marine seismic field data leads to promising results.

REFERENCES

- Brodie, K.W., 1977, Unconstrained minimization, Chapter III in: *The State of the Art in Numerical Analysis*, ed. by D.A.H. Jacobs, Academic Press, London, pp. 255-257.
- J.T. Fokkema and P.M. van den Berg, 1990, Removal of surface related wave phenomena: the marine case, *Expanded Abstracts*, 60th SEG Meeting, San Francisco, U.S.A., pp. 1689-1692.
- J.T. Fokkema and P.M. van den Berg, 1993, *Seismic Applications of Acoustic Reciprocity*, Elsevier, Amsterdam, 350 pp.
- Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Vetterling, 1986, *Numerical Recipes, The Art of Scientific Computing*, Cambridge Univ. Press, New York.
- D.J. Verschuur, A.J. Berkhout, and C.P.A. Wapenaar, 1992, Adaptive surface-related multiple elimination, *Geophysics*, 57, pp. 1166-1178.
- D.J. Verschuur, P. Herrmann, N.A. Kinneging, C.P.A. Wapenaar and A.J. Berkhout, 1988, Elimination of surface-related multiply reflected and converted waves, *Expanded Abstracts*, 58th SEG Meeting, Anaheim, U.S.A., pp. 1017-1020.
- R.G. van Borselen, J. Thorbecke, J.T. Fokkema and P.M. van den Berg, 1991, Surface related multiple elimination based on reciprocity, *Expanded Abstracts*, 61th SEG Meeting, Houston, U.S.A., pp. 1339-1342.