Modelling and parameterizing the hydro- and morphodynamics of curved open channels

Willem Ottevanger
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Proefschrift

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This research has been supported by the Dutch Technology Foundation (STW, applied science division of NWO) under grant DCB.7780 and by Deltares.

Cover photo: The River Sempt in Germany © Klaus Leidorf (http://www.leidorf.de/).

Printed by Ipskamp Drukkers B.V., the Netherlands.
ISBN: 978-94-6191-925-0
doi:10.4233/uuid:9f19d0ea-5d89-4c15-b99d-3fb02fc28eb7

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Acknowledgements

During the years that I worked on this thesis I have received considerable support from many people. In this short text I would like to thank everybody who helped me on my journey to the completion of my Ph.D, hoping to forget no-one.

First of all, I would like to thank my promotors Wim Uijttewaal and Huib de Vriend. Wim, thank you for your guidance and help with organising the research. You were always available for short questions and discussions on the research. Huib, thank you for giving me the opportunity to work on this research project. Furthermore, your to-the-point analyses helped me complete this work. Furthermore, I would especially like to thank my daily supervisor Koen Blanckaert. I greatly appreciate your always swift responses and forcing me to simplify my message to the reader. Besides that, I would like to thank you for allowing me to work with your two very useful measurement sets. Kees Sloff, thank you for presenting the research opportunity to me and for useful discussions on the research and other aspects of morphodynamic modelling.

Further thanks goes to:

- Technology foundation STW and Deltares, for making this study financially possible. In particular I would like to thank STW (and Cor de Boer) for the organisation of user-committee meetings in which my project could be discussed with experienced professionals from the field.

- User-committee members Gert Jan Akkerman, Bram Bliek, Bert Jagers, Tom de Mulder, Arjan Sieben, Harmen Talstra, Gert Jan Weltje and Anne Wijbenga for your time and valuable input to the project;

- Ingo Schnauder of the IGB Berlin, thank you for the measurements of the Tollense River and the many interesting discussions we had on the topic. It was great to cooperate with you on the modelling of the Tollense River.

- Yun Zhang and Dong Chen, it was great working together with you. I hope we find the time in the future to further collaborate.
• Arco van Sabben, it was a true pleasure to work together with you on the simulation of strongly curved bend flow.

• Wim van Balen for the discussions and the LES simulation results of strongly curved bend flow forming the previous phase of the research project.

• Frank Everdij, thank you for keeping the cluster up and running so my LES simulations could also run.

• Fedor Baart and Tycho Scholtens, thanks the for help with the \LaTeX settings.

• To all of the colleagues at the Civil Engineering Faculty at the TU Delft and Deltares (ZWS-RIV), thank you very much for making the working environment a very nice place to be.

• Nicolette Volp, Steven te Slaa, Pascal Boderie and Evelyn Aparicio for being such pleasant office mates and for the many interesting discussions. Shahid Ali, Patrick Andeweg, Jaap van Duin, Otti Kievits, Sanjay Giri, Miguel de Lucas Pardo, Mohamed Nabi and Andres Vargas Lluna for the all the interesting discussions we had.

• Scientific colleagues who I had discussions with, both nationally and internationally. Although the research field is large and vastly spread over the world, the community feels small and is very accessible.

• Family and friends who were always very interested in my progress and my results.

• My paranymphs, Wout, for all the comments and last minute reading and Sigrid, for flying from Paramaribo to join the ceremony.

And finally, Hanneke, you have made my home situation such that I had the opportunity to do this project. Your love, your patience, your support and words of wisdom really pulled me through this period. Thank you for everything.

Willem, September 2013, Utrecht
Summary

Meandering rivers are interesting features of the landscape due to their aesthetically pleasing forms. There is an abundance of scientific studies on meandering rivers, however, their behaviour is still not fully understood. The complexity of the flow, bed morphodynamics and bank stability, the related uncertainties in water and sediment motion, sediment properties and geotechnical properties of the banks, as well as the large time and length scales suggest that it is impossible to exactly predict the evolution of a meandering river.

In order to improve our understanding of these processes and the modelling thereof, a research framework was setup which consisted of four interconnected projects on the hydro-morphological processes occurring in (strongly) curved open channel bends:

1. field measurements providing detailed data on the hydrodynamics, morphodynamics and ecology in natural meander bends;

2. laboratory experiments in schematized bends under controlled circumstances which allow variation of e.g. bed roughness, bank roughness, or water depth;

3. development and validation of a detailed three-dimensional (3D) numerical model solving the complex turbulent hydrodynamics, in order to extend the results from the laboratory measurements, which are inaccurate near the fixed boundaries and the free surface; and

4. development and validation of reduced-order models which are typically an order of magnitude faster than their 3D counterparts, by depth- (and width-) averaging while including parameterizations of the relevant 3D processes obtained from theory and from the other components of the present research. By reducing the order of the model, this knowledge can finally be applied to the length and time scales occurring in natural meandering rivers.

The present study reports the findings of the fourth phase of this research framework. In Chapter 2 a width- and depth-averaged reduced-order model
is applied to naturally occurring strongly curved channels. The results show that models which are only valid for mildly curved bends tend to overestimate the secondary flow and the cross-stream redistribution of streamwise momentum by secondary flow it brings about. The inclusion of a non-linear feedback mechanism for the secondary flow is crucial in strongly curved bends. Furthermore, it is shown that streamwise changes in channel curvature, an aspect which is also neglected in many mild-curvature reduced models, is a dominant mechanism for the redistribution of streamwise momentum in strongly curved channels.

A reduced-order model for bed morphology valid for mildly and strongly curved channels and allowing large-amplitude bed level and streamwise velocity perturbations is derived and validated in Chapter 3. The inclusion of the non-linear feedback mechanism in the secondary flow strength is used to refine the model quantifying the gravitational pull on the sediment particles. Neglecting the feedback mechanism, again typical of mild-curvature and mild-amplitude models, is shown to lead to a strong overestimation of the transverse slope in strongly curved channels.

In Chapter 4 a quasi-3D reduced-order model is presented which includes the non-linear feedback mechanism for secondary flow. The secondary flow influences the redistribution of momentum through the so-called dispersion stresses. These dispersion stresses are shown to be especially important for the redistribution of streamwise momentum over slowly varying bathymetries. Over strongly varying bathymetries the role of their role is less important than the effect of momentum redistribution by topographic steering. The magnitude and the direction of the bed shear stresses are also investigated further in this chapter. The results indicate that the direction and magnitude of the bed shear stress are modelled reasonably well for mildly varying bathymetries, whereas for strongly varying bathymetries these quantities are not accurately modelled, by lack of sufficient freedom in the profiles imposed through the secondary flow parameterization model.

A parameter study using large eddy simulations of fully developed flow in curved open channels with a rectangular cross-section is described in Chapter 5. The results show that the outer bank cell locally has a negative influence on the bank stability as high-momentum fluid is forced towards the toe of the bank. The overall effect of the outer bank cell on the bank is however not distinguishable, since it would require comparison with a simulation lacking the outer bank cell. The study reveals that the outer bank shear stress can be parameterized by a quadratic relationship with the near-bank velocity multiplied by a correction factor depending on the aspect ratio and the ratio between depth and curvature radius.

To demonstrate the practical applicability of the currently developed models the expected morphological evolution of a meandering side channel of the
river Vecht near Junne is predicted by the reduced-order 1DH+ model (Appendix A). The results show the 1DH+ model can be used as a tool for rapid assessment of meandering rivers. In the future the 1DH+ model will be extended to include improved bank erosion formulations, which is expected to yield still better predictions.

The final chapter presents the main findings of the present research, focussing on typical phenomena occurring in (strongly) curved open channel flow, and discussing how each of the considered models succeeds in the modelling thereof. Furthermore, recommendations for further research are presented.
Meanderende rivieren zijn interessante kenmerken van het landschap vanwege hun esthetisch mooie vormen. Er bestaat een grote hoeveelheid wetenschappelijke literatuur over meanderende rivieren. Desondanks is hun gedrag nog steeds niet volledig begrepen. De complexiteit van de stroming, bodemmorfordynamica en oeverstabiliteit, de gerelateerde onzekerheden in de water- en sedimentbeweging, sediment eigenschappen en geotechnische eigenschappen van de oevers, zowel als de grote tijd- en lengteschalen, maken dat het onmogelijk is om de evolutie van een meanderende rivier exact te voorspellen.

Om ons begrip van deze processen en hun modellering te verbeteren, is een onderzoeksstudie opgezet bestaande uit vier gekoppelde projecten over de hydromorfologische processen in (sterk) gekromde bochtstroming:

1. veldmetingen die gedetailleerde data opleveren over de hydrodynamica, morfordynamica en ecologie in natuurlijk meander bochten;

2. laboratorium metingen in geschematiseerde bochten onder gecontroleerde omstandigheden die variatie van e.g. bodemruwheid, oeverruwheid of waterdiepte toe laten;

3. ontwikkeling en validatie van een gedetailleerd drie-dimensionaal (3D) numeriek model die de complexe turbulente hydrodynamica oplost, om de resultaten van de lab metingen aan te vullen, die onnauwkeurig zijn in de buurt van vaste grensoppervlakken en het vrije oppervlak; en

4. ontwikkeling en validatie van gereduceerde-orde modellen, die veelal een orde sneller zijn dan de 3D modellen, door middel van diepte- (en breedte-) middeling terwijl parametrisaties van de relevante 3D processen afgeleid uit theorie en uit andere componenten van het onderzoeksstuk. Na het reduceren van de orde van het model, kan deze kennis worden toegepast op de aanwezige lengte en tijd schalen in natuurlijk meanderende rivieren.

Het huidige onderzoek bericht over de bevindingen van de vierde fase van dit onderzoeksstuk. In Hoofdstuk 2 wordt een diepte en breedtegemiddeld
gereduceerde-orde model toegepast op scherp gekromde geulen die voorkomen in de natuur. De resultaten laten zien dat modellen die slechts geldig zijn voor mild gekromde bochten de secundaire stroming en de daarbij behorende herverdeling van langsimpuls overschatten. Het toevoegen van een niet-lineair terugkoppelmechanisme is cruciaal in sterk gekromde bochten. Bovendien blijkt dat de verandering van de kromtestraal langs de rivier, een aspect dat veelal wordt verwaarloosd in mild gekromde gereduceerde-orde modellen, een dominant mechanisme voor de herverdeling van langsimpuls is in sterk gekromde kanalen.


In Hoofdstuk 4 wordt een quasi-3d gereduceerde-orde model gepresenteerd die het niet-lineair terugkoppelmechanisme voor secundaire stroming bevat. De secundaire stroming beïnvloedt de herverdeling van langsimpuls door de zogenaamde dispersiespanningen. Deze dispersiespanningen zijn vooral belangrijk voor de herverdeling van langsimpuls over langzaam variërende bodemliggingen. Over sterk variërende bodemliggingen is hun rol minder belangrijk dan het effect van impulsherverdeling door topografische sturing. De grootte en richting van de bodemschuifspanning worden ook verder onderzocht in dit hoofdstuk. De resultaten laten zien dat de richting en grootte van de bodemschuifspanning goed gemodelleerd kunnen worden over mild variërende bodemliggingen, terwijl over sterk variërende bodemligging deze grootheden niet accuraat kunnen worden gemedeleerd, vanwege onvoldoende vrijheidsgraden in de profielen die worden opgelegd door het parametrisatiemodel voor de secundaire stroming.

Een parameter studie gebruikmakend van grote wervel simulaties (large eddy simulations) van volledig ontwikkelde stroming in gekromde geul met een rechthoekige doorsnede wordt beschreven in Hoofdstuk 5. De resultaten laten zien dat de secundaire cel bij de buitenoever met tegengestelde draairichting aan de secundaire stroming op de geul, lokaal een negatief effect hebben op de oeverstabiliteit aangezien hoge impulsstroom richting de teen van de oever wordt geforceerd. De gehele invloed van de buitenoevercell op de oeverstabiliteit is op basis van de simulaties niet vast te stellen aangezien daarvoor een simulatie zonder buitenoevercell nodig zou zijn. De studie laat zien dat de buitenoeververschuifspanning geparameatriseerd kan worden met een kwadratische afhankelijkheid van de snelheid nabij de oever vermengdigvuldigd met een correc-
tiefactor die afhangt van de breedte-diepte verhouding en de diepte-bochtstraal ratio.

Om de praktische toepasbaarheid van de huidig ontwikkelde modellen te tonen is de verwachte morfologische evolutie van een nevengeul van de Vecht bij Junne voorspeld met behulp van het gereduceerde-orde 1DH+ model (Appendix A). De resultaten laten zien dat het 1DH+ model gebruikt kan worden als een middel voor snelle beoordeling van meanderende rivieren. In de toekomst zal het 1DH+ model worden uitgebreid met een verbeterde oevererosieformulering, waarmee nog betere voorspellingen verwacht worden.

Het laatste hoofdstuk presenteert de hoofdbevindingen van het huidige onderzoek, gericht op typische fenomenen die voorkomen in (sterk) gekromde stroming, en een discussie over hoe goed de verschillende modellen die fenomenen kunnen modeleren. Bovendien, worden ook aanbevelingen voor verder onderzoek gepresenteerd.
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Chapter 1

Introduction

1.1 Background

The river is an ever changing environment (Heraclitus, 535BC - 475BC, “You cannot step in the same river twice”). The river is permanently in motion and is the conveyor of water, sediments and nutrients from the catchment area (ultimately to a sea or lake). Typically in low slope alluvial planes, rivers follow an aesthetically attractive winding serpentine pattern (see Figure 1.1), which is referred to as meandering. During floods, fine and nutrient-rich sediment settles on the banks and fertilizes the lands. This fertile land, fresh water and the efficient shipping opportunities, drove people to settle close to the river. Since these times, understanding and predicting the river behaviour and even influencing it have been ways in which mankind and nature coexist with the river.

Accurately predicting water levels allows the timely evacuation of people and livestock and timely removal of crops, and it produces design criteria for dike heights which provide sufficient safety from flooding. The accurate prediction of flow patterns enables predicting the dispersion of suspended materials (e.g. nutrients, pollutants, or cooling water).

Moreover, the flowing water interacts with the sediments on the river bed. For example, on the inner bank of a river bend a point bar forms, which form an obstacle to navigation on the river. The prediction of bed levels and understanding the underlying processes can be the basis of efficient and cost-reducing dredging strategies.

The flowing water also interacts with the river banks. Predicting the forces of the water on the bank and vice versa are of interest to the prediction of bank stability and failure (i.e. land loss, safety from flooding). In addition understanding the interaction between bank vegetation and the flow leads to
CHAPTER 1. INTRODUCTION

Figure 1.1: The meandering Innoko River in Alaska

improved maintenance strategies of the river bank vegetation. In meandering streams, correctly predicting the migration rate of the eroding and the accreting bank provide information on how much room should be given to that stream, in order not to lose valuable farmland and infrastructure, and to keep the ecologically rich.

1.2 Open channel bend dynamics

Open channel bends have been studied in numerous investigations, ranging from field measurements, laboratory experiments to computational modelling.

The hydrodynamics can be described by a streamwise flow in downstream direction and spanwise flow perpendicular to it. The spanwise velocity can be subdivided into a depth-averaged component (cross flow) and the vertical deviations from it (secondary flow). The secondary flow can be subdivided into the centre-region cell covering most of the channel cross-section and the outer-bank cell which often occurs near the outer bank.

The centre region cell is a well studied phenomenon in curved open channel flows with Thomson (1876) as its pioneer. It forms because of two counteracting mechanisms, the first is the centrifugal acceleration which is proportional
1.2. OPEN CHANNEL BEND DYNAMICS

Depth averaged streamwise velocity: $U_s$
Spanwise velocity: $v_n$
Streamwise velocity: $v_s$
Secondary flow: $(v_n, v_z)$
Outer-bank cell
Center-region cell

Figure 1.2: Schematized bend flow

To the velocity squared, and the second is a centripetal pressure gradient delivered by the cross-stream water surface slope. This overall centripetal pressure gradient is uniformly distributed over the water column and proportional to the depth-averaged value of the squared velocity. Hence it is not sufficient to make the fastest flowing water near the surface follow the bend curvature, whereas it gives the slower flow further down in the water column a stronger curvature. Hence the water in the upper part of the water column moves towards the outer bank and the water lower in the column towards the inner bank. The resulting flow pattern is a helical flow path through the river bend.

The outer bank cell is a more subtle feature of bend flow first observed by Bathurst et al. (1977). It is a combination of a Taylor-Görtler type instability (de Vriend, 1981a) and turbulence anisotropy, slightly amplified by the motion of the centre-region cell (Blanckaert and de Vriend, 2004; van Balen, 2010). Accurate modelling of the flow in open channel bends requires a computational solver for three-dimensional flow with advanced turbulence modelling capabilities. However, when modelling meandering river reaches, this approach will soon become computationally too demanding and less demanding models will become attractive. Such models can be obtained by reducing the dimension of the problem space by e.g. depth- (and width-) averaging and will therefore be classified as reduced-order models. When performing such
averaging operations, three-dimensional effects needed to be captured via pa-
parameterisation. An example of such a parameterization is the secondary flow
closure model. Coupling a depth-averaged model to a parametric secondary
flow closure model yields a quasi-3D model. Similarly, coupling a depth- and
width-averaged model to a secondary flow closure model yields in a 1DH+
model.

The parameterization of secondary flow is obtained by assuming a stream-
wise flow profile (e.g. logarithmic, \(1/7^{th}\) power, or von Karman) in streamwise
direction and subsequently solving a simplified transverse momentum balance
at the heart of the channel (Rozovskii, 1957; de Vriend, 1977; Engelund, 1974;
Jansen et al., 1979) to obtain a secondary flow profile. These parametric mod-
els are derived for mildly curved flow (width \(B\) is much smaller than the radius
of curvature \(R\)) which limits their applicability to mildly curved bends. In
strongly curved bends the secondary flow and streamwise flow components in-
teract. Blanckaert and de Vriend (2003) present a non-linear parametric model
for the secondary flow which takes this interaction into account and therefore
can also be applied to strongly curved bends. The secondary flow redistributes
the streamwise momentum over the channel (see e.g. Kalkwijk and de Vriend
(1980)), thus changing the cross-stream distribution of the main flow.

The near-bed streamwise and spanwise vertical velocity gradients coincide
with the direction of the bed shear stress which is important for determining
the bed morphology. Typically in a river bend the secondary flow in the lower
part of the water column is directed towards the inner bank. Since the sediment
content will be highest near the bed, there is a net sediment transport towards
the inner bank leading to the development of a point bar. Reduced-models
therefore account for the deviation of the bed shear stress from the depth-
averaged flow direction. Neglecting the influence of the secondary flow leads
to a pool rather than a point bar at the inner bank.

Point bars are responsible for an extra redistribution of streamwise mo-
momentum towards the outer bank through the so-called topographic steering
mechanism (Nelson, 1988; Dietrich and Whiting, 1989), which drives the wa-
ter around instead of over the point bar.

Planform changes are due to two mechanisms, viz. bank erosion and bank
accretion (typically on opposite banks). Over long length and time scales
the width of a meandering river is generally assumed to be a uniform (i.e. the
outer bank erosion and inner bank accretion occur at the same pace). The rate
of bank erosion is generally modelled as a function of the near-bank velocity
(excess) or water depth.
1.3 Research framework

To improve our understanding of the flow in river bends a research framework was setup in which different aspects were addressed namely: measurements in the field, laboratory experiments, detailed numerical modelling and reduced numerical modelling.

1.3.1 Field measurements

The Institute of Freshwater Ecology and Inland Fisheries (IGB) in Berlin, Germany carried out various measurement campaigns in amongst others the Spree, the Ledra, and the Tollense River (Blanckaert et al., 2010a; Schnauder and Sukhodolov, 2012; Sukhodolov, 2012). The measurements provide detailed data on the hydrodynamics, morphodynamics and ecology, which aim to improve the understanding of relevant fluvial processes occurring in natural river meanders and their interactions. Moreover, these measurements serve as validation data for computational models.

1.3.2 Laboratory experiments

The second phase of the research framework consisted of laboratory experiments which took place in the Environmental Fluid Mechanics Laboratory at the Delft University of Technology (Netherlands) and at the Ecole Polytechnique Fédérale in Lausanne (EPFL) in Switzerland. (Booij, 2003; Duarte, 2008; Blanckaert, 2009, 2010; Blanckaert et al., 2010b) These experiments were performed under controlled circumstances and therefore parameters such as water depth, bed roughness and bank roughness can be varied more easily than in the field. These variations are introduced to isolate different mechanisms in mildly and strongly curved bends. Comparison with data obtained from naturally occurring river bends allow analysis of Reynolds number dependency and show the relevance of laboratory experiments to the understanding of natural meandering. Furthermore the measured data can be used for model validation.

1.3.3 Detailed modelling of curved open channel flow

The next phase of the research (Booij, 2003; van Balen, 2010) focussed on validating and understanding the flows in moderate Reynolds number flows (up to 90,000) with a computational flow model which resolved the turbulent flow field by means of Large Eddy Simulation (LES). Large eddy simulation resolves the larger eddies, whereas the influence of the smaller eddies is parametrized. The detailed flow field obtained from LES augments experimental data which are often inaccurate close to solid boundaries and near the free surface. The
CHAPTER 1. INTRODUCTION

Figure 1.3: Graphical overview of the research framework: From top to bottom
a) Field measurements (Strongly curved Tollense bend field measurement campaign by the IGB, courtesy Ingo Schnauder); b) Lab measurements (Strongly curved flume in Lausanne, Switzerland, courtesy Koen Blanckaert); c) Detailed modelling of streamwise velocity in a strongly curved river bend by Large Eddy simulation (courtesy Wim van Balen) and d) Reduced modelling of the streamwise depth averaged flow field in a strongly curved bend.
LES-results are translated into parameterizations which can be included in commonly used engineering models.

1.3.4 Reduced modelling of river bends

The present phase of the research framework consists in implementing the knowledge obtained from previous phases into reduced-order models which can be used in engineering practice. These models are subsequently validated by data obtained from the previous phases. This thesis is part of this component of the research framework.

1.4 Objective of the thesis

The flow in river bends is three-dimensional and to model it accurately requires the use of a three-dimensional model. The time and memory constraints associated with the large spatial and temporal scales often involved in the modelling of real-life meandering rivers drive a search for reduced-order models that include these three-dimensional processes in simplified form while still yielding a reasonable representation of the flow. Such solutions already exist for mild-curvature meandering rivers, but have are still in development for strongly curved flow.

The thesis objectives are defined as follows:

1. To analyze the mechanisms underlying flow redistribution in naturally occurring, strongly curved river bends using the non-linear 1DH+ model developed by Blanckaert and de Vriend (2010) (Chapter 2);

2. To develop and validate a non-linear 1DH+ model for the evolution of the bed and analysis of the driving mechanisms for the formation of the bar pool pattern in mildly and strongly curved bends (Chapter 3);

3. Develop and validate a non-linear quasi-3D hydrodynamic model and analyze the role of the dispersive stresses on the redistribution of streamwise momentum and to understand the influence of secondary circulation on the bed shear stress modelling in quasi-3d models in strongly curved bends (Chapter 4);

4. to understand how curvature, roughness, aspect ratio and secondary flow affect the bank shear stresses in curved open channel bends and to develop a parameterization which improves the modelling of bank shear stresses in reduced models in schematized bends (Chapter 5).
Chapter 2

1DH+ hydrodynamics of sharp open channel bends

Abstract

Insight is provided in hydrodynamic processes governing the velocity redistribution in sharp river bends based on simulations of three recent experiments by means of Blanckaert and de Vriend’s (2003, 2010) reduced-order nonlinear model without curvature restrictions. This model successfully simulated the flow redistribution and the secondary flow in all three experiments. The results indicate that the flow redistribution is primarily governed by topographic steering, curvature variations and secondary flow, in a broad range of different configurations, including mildly to sharply curved bends, narrow to shallow bends, smooth to rough bends, bends with additional complexities such as horizontal recirculation zones or patches of riverbed vegetation. The relative importance of these three dominant processes is case dependent, and controlled by the parameters $C_f^{-1}H/B$, $R/B$ and streamwise curvature variations. The first parameter characterizes a river reach, whereas the second and third parameters are characteristics of individual bends. Major differences exist between the hydrodynamic processes in mildly and sharply curved bends. First, velocity redistribution induced by curvature variations is negligible in mildly curved bends, but the dominant process in sharp bends. This result is relevant, because most meander models are based on the assumption of weak-curvature variations. Second, nonlinear hydrodynamic interactions play a dominant role in sharp bends, where mild-curvature models overpredict the secondary flow.

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and in some cases even falsely identify it as the dominant process governing the velocity redistribution, which leads to unsatisfactory flow predictions. The reduction in secondary flow strength provoked by the nonlinear hydrodynamic interactions is accompanied by a reduction in the transverse bed slope, which reduces the effect of topographic steering.

2.1 Introduction

The winding of single-thread rivers in their alluvial plane, known as meandering, has interested many scientists since the Renaissance (e.g. da Vinci, 1503–1508; Fargue, 1868; Boussinesq, 1868; Thomson, 1876) from various disciplines: meanders are studied by fluid dynamicists, morphologists, ecologists, geomorphologists, and petroleum engineers (see Camporeale et al., 2007, and the references therein). The recent attention for renaturalization projects has lead decision makers to consider the partial remeandering of previously trained rivers. Factors such as navigation and man-made infrastructure along the river set limits for the maximum migration of such rivers. Therefore, models that can predict the evolution of meandering rivers are required.

Mathematical models that are used to study meandering rivers generally consist of three interconnected components: i) a hydrodynamic component, ii) a channel bed morphology component and iii) a channel bank migration component. The hydrodynamic component describes the flow field and provides the shear stresses near the bed, which are the driving force behind the morphology component, which describes the adaptation of the riverbed, ultimately resulting in a sequence of alternating stable and migrating bars. Similarly the shear stresses near the bank are important for predicting the migration of the banks, which generally happens over longer time scales than the bed adaptation. The adaptation of the bed and the banks in turn influence the flow field and in this manner the system is interconnected. The focus of this paper will be on the hydrodynamic component.

Recently, Rüther and Olsen (2007) showed the feasibility of a three dimensional (3D) meander model by simulation of the 72-hour-lasting experiment of Friedkin (1945). The flow in their meander model was solved using Reynolds averaged Navier-Stokes equations with a k-ε turbulence closure (Rodi, 1980), producing a detailed description of the flow. Their detailed hydrodynamic model provided all the shear stresses, which could be fed into the bed and bank adaptation components. Typical lengths of meandering rivers are much larger than the 40 m long tilting flume in Friedkin (1945)’s experiment. Moreover, the time that is necessary for meandering rivers to migrate over the distance of a channel width is of the order of years instead of 72 hours as in Friedkin’s experiment. Therefore, it is expected that performing a simulation of a real river is not feasible using a detailed 3D hydrodynamic model as it would be
computationally too expensive. Reduced-order hydrodynamic models, which are typically one-dimensional (1D) or two-dimensional (2D) models are computationally faster and have the advantage of being more insightful by clearly revealing the processes governing the flow redistribution. But they provide a less detailed description of the flow field and require a parameterization of 3D flow effects, which are known to play an important role in natural river bends.

An example of such a parameterization is the secondary flow (schematically indicated in Figure 2.1), which occurs in the plane perpendicular to the streamwise flow. This secondary flow is related to the river planform and can primarily be expressed as a convolution function of the channel curvature. This secondary flow has two important effects on the flow redistribution. First, it induces a transverse component of the bed shear stress, which conditions the development of a transverse bed slope with increasing flow depth in outward direction (Olesen, 1987; Camporeale et al., 2007). This transverse bed slope scales with the inverse of the radius of curvature (Ikeda et al., 1981; Odgaard, 1981). According to Chézy’s law, the depth-averaged velocity scales with the square of the flow depth, implying that higher/lower velocities will be attracted to the deeper/shallower parts of the cross section. This process is often called topographic steering (Dietrich and Smith, 1983; Blanckaert, 2010). Second, the secondary flow redistributes momentum, causing velocities to increase in outwards direction. The accompanying higher/lower sediment transport over the deeper/shallower parts strengthen the development of the transverse bed slope and lead to a positive feedback between the flow field and the transverse bed slope.

The secondary flow was first parameterized by van Bendegom (1947) and Rozovskii (1957), followed by many others (e.g. Engelund, 1974; Ikeda, 1975; de Vriend, 1977; Johannesson and Parker, 1989a). These models are invariably based on mild-curvature assumptions, implying that $R/B$ and $R/H$ are sufficiently large and that the curvature radius varies slowly in streamwise direction. Here $R$ is the radius of curvature at the centerline, $B$ is the width and $H$ is the width-averaged flow depth (Figure 2.1). In that case the interaction between the streamwise flow and the secondary flow is negligible, resulting in a secondary flow strength that is linearly proportional to the ratio $H/R$ and only a function of the roughness. It has been shown that mild-curvature secondary flow parameterizations considerably overestimate the secondary flow in moderately and sharply curved bends, because they neglect the nonlinear interactions between the streamwise flow and the secondary flow (de Vriend, 1981b; Yeh and Kennedy, 1993a; Blanckaert and de Vriend, 2003). Blanckaert (2009) has shown that the secondary flow does not increase when the curvature is increased in very sharp bends, and he called this process the saturation of the secondary flow.

Blanckaert and de Vriend (2003, 2010) developed and validated a nonlinear
reduced-order hydrodynamic model that accounts for these nonlinear interactions and successfully simulates the saturation of the secondary flow. In its mild-curvature formulation, their model reduces to Johannesson and Parker (1989b)’s linear model. Therefore, Blanckaert and de Vriend’s model extends the parameterization of the secondary flow to sharply curved bends. Moreover, their model is neither restricted to mild curvatures nor to slow variations of the curvature in streamwise direction. Blanckaert and de Vriend (2010) predicted by means of a scaling analysis that these streamwise variations in curvature are a dominant driving force of the velocity redistribution in sharply curved bends. Blanckaert (2011) predicted by means of an analytical analysis for the case of axi-symmetric curved flow (infinite length bend, also referred to as fully-developed flow) that nonlinear hydrodynamic interactions are important in sharp open-channel bends.

Knowledge on the processes in sharply-curved bends is of practical relevance. Outer-banks in sharply-curved bends are particularly vulnerable to bank erosion. Moreover, cut-off events, which are an essential process in the long-term and large-scale meander dynamics, typically occur in sharply-curved bends. The aim of this paper is to extend foregoing investigations by enhancing the insight into the processes governing the velocity redistribution in sharply-curved open-channel bends, by addressing the following questions:

1. What are the dominant processes with respect to the velocity redistribution in sharply-curved open channels, and do they differ from the dominant processes in mildly-curved bends? Special attention is paid to the role of the streamwise variations in curvature. Sharp bends are typically relatively short and characterized by pronounced streamwise curvature variations. Linear models, which are based on the assumption of weak variations in curvature, are inherently unable to estimate the relevance of these variations.

2. How important are nonlinear hydrodynamic interactions, and especially the saturation of the secondary flow, with respect to the velocity redistribution in sharp open-channel bends?

3. Is the velocity redistribution in sharply-curved laboratory flumes governed by the same processes as in sharp natural river bends characterized by additional processes such as horizontal recirculation zones or patches of vegetation?

These questions will be addressed by means of simulations performed with Blanckaert and de Vriend (2003, 2010)’s model. This model will be briefly reported in Section 2.2; reference is made to Blanckaert and de Vriend (2003, 2010) for a more detailed and complete description. The questions will be addressed by investigating three configurations of sharply-curved open-channel bends with different characteristics. The first case concerns a sharply-curved narrow laboratory flume with rectangular cross-section and smooth boundaries. The second case concerns an even narrower sharp natural bend characterized by
2.2 Reduced-order nonlinear hydrodynamic model

Figure 2.1 schematically defines the variables: $\Delta H$ and $\Delta U$ are the height and velocity excess at the outer bank w.r.t. their width-averaged values $H$ and $U$, respectively. The streamwise, transverse and vertical coordinates are given by $s$, $n$ and $z$ respectively. The three dimensional velocity is denoted by $v_j$, where the subscript $j$ refers to the component in the direction of the respective coordinate $s$, $n$, or $z$. In a similar way, $U_j$ is the depth averaged flow in $s$ or $n$ direction. The water and bed levels are given by $z_s$ and $z_b$, respectively. The centerline radius of curvature $R$ is positive (negative) for right (left) turning bends. Blanckaert and de Vriend (2003)’s nonlinear hydrodynamic model is based on a reduction of the three-dimensional flow equations by means of the near absence of secondary flow and a nearly flat bed topography. The third case concerns a sharp natural bend with gradually varying width, horizontal recirculation zones and patches of vegetation. These three cases are presented in Section 2.3. Section 2.4 focuses on the research questions.
profile functions with one degree of freedom for the transverse distributions of
the depth-averaged streamwise velocity $U_s$ and the local depth $h$:

$$U_s \approx U \left(1 + \frac{n}{R} \right)^{\alpha_s} \approx U \left(1 + \frac{\alpha_s}{R} n \right). \quad (2.1)$$

$$h = z_s - z_b \approx H \left(1 + \frac{n}{R} \right)^{F_r^2 + A} \approx H \left(1 + \frac{F_r^2 + A}{R} n \right). \quad (2.2)$$

The degree of freedom in Equation (2.1) is represented by the dimensionless number $\alpha_s$. Values of $\alpha_s = -1$ and $\alpha_s = 1$ correspond to potential and forced vortex distributions, respectively (cf. Vardy, 1990; Blanckaert and de Vriend, 2003, Figure 7). The square of the dimensionless Froude number, $F_r^2$, parameterizes the transverse inclination of the water surface (often called superelvation). The so-called scour factor $A$ Engelund (1974); Zimmermann and Kennedy (1978); Odgaard (1981) parameterizes the transverse bed level gradient and is typically between 2.5 and 6 in natural open-channel bends (Ikeda et al., 1981; Odgaard, 1981). Obviously the adoption of transverse profile functions will only allow describing processes that occur on a length scale that is larger than the channel width. But the adoption of such simplified transverse distributions does not preclude the model to account for nonlinear processes.

Using the above parameterizations, Blanckaert and de Vriend (2003, 2010)’s reduced-order nonlinear model describes the velocity redistribution in open-channel bends by means of the following nonlinear relaxation equation in $\alpha_s/R$:

$$\lambda_{\alpha_s/R} \frac{\partial}{\partial s} \left( \frac{\alpha_s}{R} \right) + \frac{\alpha_s}{R} = F_{\alpha_s/R} \quad (2.3)$$

where the factor $\lambda_{\alpha_s/R}$ is the flow adaptation length, and the system is subject to the forcing term $F_{\alpha_s/R}$. The adaptation length $\lambda_{\alpha_s/R}$ is defined as:

$$\lambda_{\alpha_s/R} = \frac{1}{2} \frac{H}{\psi C_f} \left( 1 - \frac{m \alpha_s + 1}{12} \frac{B^2}{R} \right) \quad (2.4)$$

where $C_f$ parameterizes the straight channel roughness, and the parameter $\psi$ (provided by Blanckaert and de Vriend (2003, 2010) and Blanckaert (2009)), parameterizes additional curvature-induced friction losses. The variable $m$ is a binary integer, which is set to 1 in the nonlinear model and to 0 in the
2.2. REDUCED-ORDER NONLINEAR HYDRODYNAMIC MODEL

The mild-curvature formulation of the model. The forcing $F_{\alpha s/R}$ is described as:

$$F_{\alpha s/R} = \frac{1}{2} F_{r^2 + A - 1}^{\alpha s/R} \left( 1 - \frac{m}{6} \frac{B^2}{R^2} \right) \left( 1 + \frac{m}{12} \frac{(F_{r^2 + A + 3}^2) B^2}{R^2} \right) \left( \frac{1}{R} \right) \left( \frac{1-R}{R} \right) \left( \frac{1}{R} \right)$$

It clearly displays the processes governing the velocity redistribution. Its first line (I) in Equation (2.5) relates to the transverse gradient of the water depth (cf. equation (2.2)) i.e. the effect of topographic steering (Nelson, 1990; Blanckaert, 2010). The second line (II) is related to streamwise changes in channel curvature. This term will be shown to be of predominant importance in sharply-curved bends and merits therefore some further explanation. The transverse tilting of the water surface (also called superelevation) is in first approximation given by $\partial z_s/\partial n \approx F_{r^2} H/R$. Streamwise changes in $\partial R^{-1}/\partial s$ will thus lead to streamwise variations in the transverse tilting of the water surface that are accompanied by variations in the streamwise water surface gradient. An increase in curvature, for example, $\partial R^{-1}/\partial s > 0$, will lead to an increase in the transverse tilting of the water surface. As a result, the streamwise water surface gradient will decrease/increase in the outer/inner part of the cross-section, leading to flow deceleration/acceleration and flow redistribution. The third line (III) relates to the redistribution of the streamwise velocity by the secondary flow and the final line (IV) relates to streamwise changes in the transverse bed and water surface gradients. The brackets $\langle \rangle$ represent depth-averaged values.

The velocity redistribution by the secondary flow, represented by $\langle v_s v_n \rangle/U^2$ is obtained from the following equation:

$$\lambda \frac{\partial}{\partial s} \left\{ \frac{\langle v_s v_n \rangle}{U^2} \right\} + \frac{\langle v_s v_n \rangle}{U^2} = \frac{\langle v_s v_n \rangle_{\infty}}{U^2}$$

where $\lambda$ is the adaptation length defined in Johannesson and Parker (1989a) (cf. Blanckaert and de Vriend (2010)). The expression on the right hand side of equation (2.6) denotes the value obtained for axi-symmetric flow and is computed as follows:

$$\frac{\langle v_s v_n \rangle_{\infty}}{U^2} = \frac{\langle v_s v_n \rangle_{\infty}}{\langle v_s v_n \rangle_0} \frac{\langle v_s v_n \rangle_0}{U^2} \approx \frac{fct(\beta)}{U^2} \frac{\langle v_s v_n \rangle_0}{U^2}.$$  (2.7)

The index 0 indicates linear-model solutions, which grow linearly with the ratio $H/R$ and uniquely depend on the friction factor $C_f$ (e.g. van Bendegom, 1947; Rozovskii, 1957; Engelund, 1974; Ikeda, 1975; de Vriend, 1977; Johannesson and Parker, 1989a). The nonlinear hydrodynamic interactions between the
secondary flow and the streamwise flow are parameterized by means of the correction factor $\langle v_s v_n \rangle_\infty / \langle v_s v_n \rangle_0 = fct(\beta)$, which uniquely depends on the so-called bend parameter (Blanckaert and de Vriend, 2003):

$$
\beta = C_f^{-0.275} \left( \frac{H}{R} \right)^{0.5} (\alpha_s + 1)^{0.25}
$$

(2.8)

Figure 2.2 provides the graphical solution of the correction factor $fct(\beta)$, which is obtained by the model of Blanckaert and de Vriend (2003).

Inclusion of the correction factor $fct(\beta)$ makes the variables $\alpha_s / R$ and $\langle v_s v_n \rangle / U^2$ mutually dependent according to Equations (2.3), (2.5), (2.7) and (2.8), indicating the nonlinearity of the hydrodynamics. A mild-curvature formulation is obtained by setting $m = 0$ (Equations (2.4) and (2.5)), setting $fct(\beta) = 1$ (Equations (2.6) and (2.7)), and neglecting additional curvature-induced friction losses ($\psi = 1$). This mild-curvature formulation is identical to the linear model of Johannesson and Parker (1989b). Comparison of Blanckaert and de Vriend (2010)’s model without curvature restrictions to the model of Johannesson and Parker (1989b) therefore reveals the influence and relevance of nonlinear hydrodynamic effects. Extensions and improvements of
2.3. Investigated sharp open channel bends

2.3.1 Kinoshita flume

The first investigated case is the laboratory experiment carried out and reported by Abad and García (2009a) (Figure 2.3 & Table 2.1). Their flume consists of seven consecutive bends of alternating direction, which centerline...
Table 2.1: Hydraulic and geometric properties of the measured bends. \( H \) is the bend averaged water depth, \( Q \) is the flow discharge, \( C_f \) is the dimensionless friction factor, \( B \) is the bend averaged channel width, \( |R|_{ap} \) is the radius of curvature at the bend apex, \( |R|_{ap}/B_{ap} \) is the radius to width ratio at the bend apex, \( |R|_{ap}/H_{ap} \) is the radius to depth ratio at the bend apex, \( |R|/B \), \( |R|/H \), \( C_f^{-1} H/B \) and \( C_f^{-1} H/|R| \) bend averaged scaling parameters and \( \beta \) is the bend averaged bend parameter (Equation (2.8)).

| Case                  | \( H \) [m] | \( Q \) [m\(^3\)s\(^{-1}\)] | \( C_f \) | \( B \) [m] | \( |R|_{ap} \) [m] | \( |R|_{ap}/B_{ap} \) | \( |R|_{ap}/H_{ap} \) | \( |R|/B \) | \( |R|/H \) | \( C_f^{-1} H/B \) | \( C_f^{-1} H/|R| \) | \( \beta \) |
|-----------------------|-------------|-----------------|---------|---------|----------------|----------------|--------------------|-------|-------|-------------|-------------|------|
| Kinoshita flume       | 0.143       | 0.025           | 0.0048  | 0.6     | 0.72          | 1.2            | 1.0                | 5     | 2.4   | 10          | 50          | 21   |
| Polblue Creek         | 0.65        | 0.3             | 0.005   | 1.5     | 1.6           | 1.05           | 1.56               | 2.4   | 3     | 7           | 87          | 29   |
| Tollense River        | 1.5         | 1.48            | 0.03    | 19.1    | 16.4          | 0.7            |                    |       |       |             |             |      |

\[ \frac{1}{R} = -\frac{2\pi}{\lambda} \theta_0 \left\{ \cos \left( \frac{2\pi}{\lambda} s \right) - 3\theta_0^2 \left[ J_s \sin \left( \frac{3\pi}{\lambda} s \right) + J_f \cos \left( \frac{3\pi}{\lambda} s \right) \right] \right\} \] (2.9)

The terms between square brackets cause an asymmetrical form of the curve, whereas the first term in the right-hand-side represents the family of symmetric sine-generated curves defined by Leopold and Wolman (1960). These curves are a kind of averaged idealized representation of the planform of natural meandering rivers. Abad and Garcia (2009a) adopted a maximum angular amplitude of \( \theta_0 = 110^\circ \), which is characteristic of sharp bends close to cut-off, an arc wavelength of \( \lambda = 10 \) m, as well as values of the skewness and flatness of \( J_s = 1/132 \) and \( J_f = 1/192 \), respectively. The flume had a rectangular cross section with a constant width of \( B = 0.6 \) m and smooth boundaries parameterized by a friction factor of \( C_f = 0.0048 \). The average flow depth in the experiment was 0.143 m. The smooth boundaries, flat bed, narrow cross-section (\( B/H \approx 4 \)), low values of the ratios \( R_{ap}/B_{ap} = 1.2 \), and \( R_{ap}/H_{ap} = 5 \) (\( R_{ap} \) is the centerline radius of curvature in the bend apex), indicate that this is a very sharply curved bend. Although such bend characteristics will rarely be encountered in nature (Table 2 in Blanckaert, 2011, and its discussion), this case is an appropriate test case for the objectives of the present paper.

Figure 2.3 shows the velocity distribution measured in the flume. Due to the change in direction between successive bends, the velocity distribution at the bend entrance is inwards skewed (velocities decrease from the inner bank towards the outer bank). This inwards skewed velocity distribution gets more pronounced in the region of strong curvature increase between the bend en-
2.3. INVESTIGATED SHARP OPEN CHANNEL BENDS

...trance and the bend apex. Downstream of the apex, outwards velocity redistribution is discernable, which results in an outwards skewed velocity distribution at the bend exit. Figure 2.3 also shows the transverse profile functions with one degree of freedom that have been fitted to the experimental data according to Equation (2.1). These profiles satisfactorily represent the global features of the velocity redistribution through the bend.

This experiment was simulated by means of the nonlinear hydrodynamic model as well as its mild-curvature formulation (Section 2.2). At the inflow of the flume, a uniform velocity distribution ($\alpha_s = 0$) was imposed without secondary flow. Furthermore a constant discharge was imposed upstream (see Table 2.1). The measured flow depth at the end of the flume was imposed as boundary condition at the flume exit. The streamwise water level is part of the model solution (see Figure 2.3(b)). The difference between the linear and non-linear model solutions can be explained by the additional curvature induced friction losses parameterized by $\psi$ (see Equations (2.4) and (2.5)).

2.3.2 Polblue Creek Bend 3

The second investigated case is the sharply curved bend number 3 in the Polblue Creek, Barrington Tops National Park, New South Wales, Australia ($31^\circ 57' 20.33''$S, $151^\circ 24' 42.90''$E), which was investigated by Nanson (2010). A schematic overview of the bend is given in Figure 2.4(a). Nanson (2010) performed flow measurements in the seven cross-sections crs1 to crs7 shown in Figure 2.4. The radius of curvature and the width of the bend have been estimated by digitizing the coordinates of the centerline and the banklines. The curvature radius, for example, is obtained from the mathematical definition of curvature (e.g. Legleiter and Kyriakidis, 2006) as:

$$\frac{1}{R(s)} = \frac{x(s)'y(s)'' - x(s)''y(s)'}{[x(s)'^2 + y(s)'^2]^{3/2}} \quad (2.10)$$

where $s$ is the intrinsic coordinate along the centerline, and $(x, y)$ are the coordinates of the centerline in an arbitrary Cartesian reference system. The centerline radius of curvature at the apex was found to be about $R_{ap} = 1.6$ m, which is only slightly more than the average width of about $B = 1.5$ m. The width varied slightly through the bend (Figure 2.4(e)). The average flow depth in the bend was about 0.65 m. The measured flow depth in cross-section crs6 was substantially lower (Figure 2.4(d)), which is mainly due to the presence of a riffle in that cross-section (Nanson, 2010) and a slightly increased width (Figure 2.4(c)). This riffle could be due to the presence of a stable surface layer (e.g. a non-erodible layer). Figure 2.4(b) illustrates the morphology of this narrow and deep peatland channel. The banks are quasi-vertical due to root reinforcement. From literature we find the following friction factor for...
Figure 2.4: Experiment performed in Polblue Creek bend 3 by Nanson (2010). (a) Planform, depth-averaged streamwise velocity distribution measured in the cross-sections crs1 to crs7 and corresponding fitted profile function according to Equation (2.1); (b) Bed topography measured in the cross-sections crs1 to crs7 and corresponding fitted profile function according to Equation (2.2); measurement-based model input for the streamwise evolution of (c) the parameter \( A/R \) (derived from figure 4(b)), (d) the cross-sectional averaged flow depth \( H \); (e) and the width (at the water surface).
2.3. INVESTIGATED SHARP OPEN CHANNEL BENDS

bend 3 in the Polblue Creek, namely $C_f = 0.02$ (Nanson, 2010) and $C_f = 0.005$ (Nanson et al., 2010, Table 1: measurement P6 just downstream of bend 3). (Nanson, 2010) only assumes this value while in Nanson et al. (2010) the friction factor is analyzed in more detail along the Polblue Creek. Therefore, we believe that $C_f = 0.005$ given in Nanson et al. (2010) characterizes the friction factor in the Polblue Creek. Most notable are the absence of a point bar and the quasi-horizontal bed, which cannot solely be explained by the limited sediment supply, but must be related to the hydrodynamic forcing. Figure 2.4(b) also shows the transverse profile functions with one degree of freedom that have been fitted to the measured bed topography according to Equation (2.2), whereas Figure 4(c) shows the corresponding evolution through the bend of the scour factor $A/R$. As mentioned before, these profiles are not intended to represent the local morphological features, but only the features occurring on a spatial scale larger than the channel width. The ratios $R_{ap}/B_{ap} = 1.05$ and $R_{ap}/H_{ap} = 2.4$ and the very narrow cross-sections indicate that this is a very sharply curved bend. In spite of this very sharp curvature, Nanson (2010) measurements revealed that there was hardly any secondary flow throughout the bend, which complements the observation of the quasi-horizontal bed topography. Figure 2.4(a) shows the velocity distribution measured by Nanson (2010). The velocity is about uniform over the width at the bend entrance. Similar to the velocity redistribution in the Kinoshita flume, the velocity skews inwards between the bend entrance and the bend apex, and subsequently skews outwards. In general, however, the skewing of the velocity profiles is weak and the distributions are remarkably uniform. Figure 2.4(a) also shows the transverse profile functions with one degree of freedom that have been fitted to the experimental data according to Equation (2.1). These profiles satisfactorily represent the global features of the velocity redistribution through the bend.

For this case the simulation was done imposing the measured velocity distribution based on Equation (2.1) and velocity redistribution caused by secondary flow was imposed at the upstream boundary as $\langle vs vn \rangle_\infty /U^2 = fct(\beta(\alpha_s,crs)) \cdot \langle vs vn \rangle_0 /U^2$. The present simulations took into account the gradual variations of the width and depth by means of the smoothed interpolation curves shown in Figure 2.4(d) and 2.4(e). The cross-sectionally averaged streamwise velocity subsequently follows as the discharge $Q$ along the reach is considered to be constant.

2.3.3 Tollense River

The third investigated case considers a meander bend in the Tollense River, Germany (53° 37' 50.00" N, 13° 15' 12.28" E), which has been investigated by a team of the Leibniz-Institute of Freshwater Ecology and Inland Fisheries (IGB, Berlin, Germany) (Schnauder and Sukhodolov, 2012). The Tollense bend is
Figure 2.5: Experiment performed in Tollense River bend (courtesy Schnuder and Sukhodolov). (a) Planform, depth-averaged streamwise velocity distribution measured in the cross-sections crs1 to crs7 and corresponding fitted profile function according to Equation (2.1); (b) Bed topography measured in the cross-sections crs1 to crs7 and corresponding fitted profile function according to Equation (2.2); measurement-based model input for the streamwise evolution of (c) the parameter $A/R$ (derived from figure 5(b)), (d) the cross-sectional averaged flow depth $H$; (e) the width (at the water surface); and (f) the cross-sectional averaged friction factor.
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schematically shown in Figure 2.5. This bend is characterized by additional complexities in the form of horizontal recirculation zones, which are found at crs3 (outer bank) and at crs4 (inner bank). Schnauder and Sukhodolov (2012), computed the friction factor from the water slope as \( C_f \approx 0.086 \). Local values of friction factor were also derived from the vertical profiles of point measurements of velocity and Reynolds shear stresses. At locations without riverbed vegetation, the stress profiles were linearly extrapolated to the bed level to yield the bed shear stress. Where vegetation was abundant, the measured peak stress in the water column associated with the top of the vegetation canopy was taken as reference to determine the friction factor. The patchiness of the riverbed vegetation can be recognized in the spatial variations of the friction factor through the bend (Figure 2.5(f) and Schnauder and Sukhodolov (2012)). The average friction factor found using the point measurements of velocity and Reynolds shear stresses is \( C_f = 0.03 \) which is lower than the value obtained from the water slope. This difference can be explained because of the losses from the horizontal mixing layers and the topographic steering effects which cannot be captured using a model with a single degree of freedom for the bed-elevation (A). Furthermore, the varying water depth along the bend and the low streamwise velocity (\( U = 0.05 \) m/s) introduce an added uncertainty for the correct determination of the friction factor from the water slope. We believe that the friction factor determined from point measurements of velocity and the Reynolds stresses is a more accurate representation of the bed-friction and therefore this value will be used in the simulation. The centerline radius of curvature and the width were estimated according to the same method applied for the Polblue bend. The centerline radius of curvature at the apex was found to be about \( R_{ap} = 16.4 \) m, which is smaller than the average width of about \( B = 19.1 \) m. The width varies through the bend (Figure 2.5) and attains maximum values in the cross-sections where outer-bank flow separation occurs (crs3) and at the bend apex (crs4). The average flow depth in the bend was about 1.5 m, but varies considerably through the bend (Figure 2.5). The ratios \( R_{ap}/B_{ap} = 0.7 \) and \( R_{ap}/H_{ap} = 9.2 \) indicate that this is a sharply curved bend.

Figure 2.5(b) shows that the bed morphology varies throughout the bend: crs2 and crs3 have a compound channel appearance that is related to the zones of horizontal flow recirculation, whereas the other cross-sections have morphologies that are more typical of single-thread rivers. Steep outer banks occur downstream of the bend apex. Figure 2.5(b) also shows the transverse profile functions with one degree of freedom that have been fitted to the measured bed topography according to Equation (2.2), whereas Figure 2.5(b) show the corresponding evolution through the bend of the scour factor \( A/R \). Similarly, Figure 2.5(a) shows the measured velocity distribution and the transverse profile functions fitted to them. The influence of the right-turning upstream bend is clearly visible in the transverse profiles of the bed topography and
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the velocity. The measured velocities at crs3 (see Figure 2.5), reveal flow in the upstream direction which is a signature of the horizontal flow recirculation zone.

For the simulation of this case the measured velocity distribution at crs1 was imposed at the upstream boundary according to equation (2.1). The secondary flow value at the upstream boundary was computed as \( \langle v_s v_n \rangle_\infty / U^2 = fct(\beta(\alpha_{s,crs1})) \cdot \langle v_s v_n \rangle_0 / U^2 \). Moreover, the simulations took into account the gradual variations of the width (including the horizontal recirculation zones) and the depth, by means of the smoothed interpolation curves shown in Figure 2.5(d) and 2.5(e) similarly to the method used for the Polblue Creek bend 3 simulation.

2.4 Analysis of processes governing the velocity redistribution

As indicated in the model description (Section 2.2), the secondary flow is of particular importance with respect to the velocity redistribution and with respect to the effect and relevance of nonlinear hydrodynamic interactions. Figure 2.6 (left column) compares the measured evolution of the width averaged secondary flow strength around the bend to simulations with the nonlinear hydrodynamic model and to simulations with the linear mild-curvature formulation of this model.

The width averaged secondary flow strength is defined as

\[
\tilde{I} = \frac{1}{BH} \int_{-B/2}^{B/2} \left\langle \frac{v_n - U_n}{U} \right\rangle^2 \cdot \text{sign}(v_{n,\text{surface}}) \, dn
\]

(2.11)

The brackets \( \langle \rangle \) indicate depth-averaged values. Notice that this parameter remains valid in straight reaches where \( R^{-1} = 0 \). The addition of \( \text{sign}(v_{n,\text{surface}}) \) allows accounting for the sense of rotation of the secondary flow cell. Averaging the secondary flow over the width of the river, allows us to get a qualitative idea about the secondary flow strength through the bend as local features which appear on less than the width scale are filtered out.

For all three cases the linear model predicts higher absolute values of the width averaged secondary flow than the nonlinear model. Nonlinear hydrodynamic effects reduce the secondary flow \( \tilde{I} \) by as much as 79%, 95% and 27% in the three respective cases. For all cases, the nonlinear model predictions agree satisfactorily with the experimental data.

Figure 2.6 (right column) compares the streamwise velocity distribution inferred from the measurement to simulations with the nonlinear and linear models, respectively. In line with the observations on the secondary flow, the
2.4. ANALYSIS OF PROCESSES GOVERNING THE VELOCITY REDISTRIBUTION

Figure 2.6: Width-averaged secondary flow strength $\bar{I}$ (1/m; Equation (2.11)) (left column) and transverse flow structure parameterized by $\alpha_s/R$ (1/m; Equation (2.1)) (right column). Comparison of experimental data to predictions by Blanckaert and de Vriend (2003, 2010)’s nonlinear model without curvature restrictions and to their model in its linear mild-curvature formulation.
nonlinear model simulates satisfactorily the global velocity redistribution for all three cases. The linear model fails dramatically for the Polblue bend with values that deviate by an order of magnitude from the measurements, and shows errors of about 100% for the Kinoshita flume. Only for the Tollense bend, the linear model provides satisfactory results.

These results indicate that the nonlinear model is a reliable tool for predicting the flow distribution in open-channel bends, irrespective of their curvature. The success of the model for the Tollense bend indicates that, although the flow patterns can become increasingly complex in real meander bends (horizontal recirculation zones, riverbed vegetation, pool riffle sequences), the non-linear hydrodynamic flow model is of practical relevance for the modelling of large scale and long term meander development as it accurately captures the large scale velocity redistribution through the bend. Locally, differences may occur, such as the difference between the measured velocity distribution and the nonlinear model prediction in cross-sections crs4 and crs5 of the Tollense bend. This difference could well be attributed to the patch of vegetation on the point bar near the inner bank, which causes an outwards skewing of the velocities. Obviously, the reduced-order nonlinear hydrodynamic model cannot account for such features occurring on a spatial scale that is smaller than the channel width.

The successful simulation of the secondary flow and the velocity redistribution in the three investigated sharp open-channel bends further validates Blanckaert and de Vriend (2003, 2010) nonlinear hydrodynamic model as a tool to investigate the processes that govern the flow redistribution in sharp river bends. The relative importance of the different processes will be assessed by evaluating the different contributions to the forcing term (Equation 2.5) in the model equation of the velocity distribution (Equation 2.3). The nonlinear and linear models will again be compared to provide further insight in the role of the nonlinear hydrodynamic interactions.

For the Kinoshita flume and the Polblue Creek Figure 2.7 clearly reveals that the erroneous linear model predictions of the flow redistribution are essentially due to the pronounced overestimation of the effect of the secondary flow (term III in Equation 2.5), which is erroneously identified as the dominant process with respect to the velocity redistribution. As mentioned before, non-

Figure 2.7 (facing page): Total forcing term of the velocity redistribution and different contributions according to Equation (2.5) estimated from Blanckaert and de Vriend (2003, 2010)’s nonlinear model without curvature restrictions (left column) and their model in its linear mild-curvature formulation (right column). The bend averaged values of the absolute forcing terms for each case are included in the shaded rectangle.
2.4. ANALYSIS OF PROCESSES GOVERNING THE VELOCITY REDISTRIBUTION

**Kinoshita flume**

- Linear
- Non-linear

**Polblue Creek bend 3**

- Linear
- Non-linear

**Tollense River bend**

- Linear
- Non-linear

---

**Legend:**
- I: 0.3
- II: 8.1
- III: 15.5
- IV: 0.0

---

**Figures:**
- Kinoshita flume
- Polblue Creek bend 3
- Tollense River bend
linear hydrodynamic effects reduce the secondary flow strength in the Tollense bend by 27% (Figure 2.6). Nevertheless, the linear and nonlinear model predictions of the velocity distribution (Figure 2.6) are hardly different. This can be explained by the spatial distribution of the forcing terms around the bend: secondary flow effects are an order of magnitude smaller than the effects of curvature variations in the upstream half of the Tollense bend, and they only become relevant in the downstream half of the bend. Nonlinear hydrodynamic effects also affect the change in curvature mechanism (term II in Equation 2.5, see bend-averaged values included in Figure 2.7). Again the effect is more pronounced in the Kinoshita flume and the Polblue Creek bend, than in the Tollense River bend.

The streamwise change in curvature (term II in Equation 2.5) is the dominant process in all three investigated cases. This confirms the scaling analysis of Blanckaert and de Vriend (2010), which suggested that this process is negligible in mildly curved bends but of leading order of magnitude if $C_{f}^{-1}H/R = 0(1)$. This scaling parameter attains values of about 21, 29 and 2 in the Kinoshita, Polblue and Tollense bends, respectively.

Velocity redistribution by the secondary flow is a process of dominant order of magnitude in all three cases. The scaling analysis of Blanckaert and de Vriend (2010) suggested that this process is of dominant order of magnitude in narrow rivers $B/H < 10$, but negligible in shallow ones $B/H > 50$. The aspect ratio for the Kinoshita flume, Polblue Creek and the Tollense River are 4.2, 2.3 and 12.6 respectively. This means that although nonlinear hydrodynamic interactions considerably reduce secondary flow effects in sharply curved river bends, they remain of dominant order of magnitude in bends that are not very shallow.

Topographic steering of the flow (term I in equation 2.5) is obviously negligible in the Kinoshita flume with horizontal bed. It is a process of dominant order of magnitude in the Tollense bend, but also negligible in the Polblue bend. This seemingly contradicts the scaling analysis of Blanckaert and de Vriend (2010), which suggested that this process is never negligible. Their scaling analysis was based on values of the scour parameter $A$ in the range of 2.5 to 6, however, which is typical for alluvial rivers Ikeda et al. (1981); Odgaard (1981). The quasi-horizontal bed topography in the Polblue bend leads to a scour factor that is an order of magnitude smaller, and explains why topographic steering is negligible.

The influence of streamwise variations in the transverse bed and water surface slopes (term IV in Equation 2.5) is negligible in all three cases, as predicted by Blanckaert & de Vriend’s scaling analysis.

These results extend the results of Blanckaert (2011), who investigated the asymptotic case of axi-symmetric curved flow (an infinite bend of constant curvature). Blanckaert showed the relative importance of secondary flow and
topographic steering as a function of the two control parameters $C_f^{-1}H/B$ and $R/B$ and moreover he analyzed the influence of non-linear hydrodynamic effects as a function of these same two parameters. The first parameter, accounting for the roughness and the shallowness, characterizes a river reach, whereas the second parameter quantifies the curvature of individual bends. Although this case of axi-symmetric curved flow is by definition unable to account for curvature changes, it solution can be seen as a first approximation for the importance of secondary flow effects and topographic steering in natural river bends with varying curvature.

As mentioned in the introduction, the secondary flow has two important effects on the flow redistribution. Besides the advective redistribution of momentum analyzed above, the secondary flow also induces a transverse component of the bed shear stress, which conditions the development of a transverse bed slope (Olesen, 1987; Camporeale et al., 2007) that in its turn leads to topographic steering. Averaged over the bend, the scour factors, $A$, in the Polblue and Tollense bends are about 0.1 and 0.35, respectively, which is considerably lower than the typical values for alluvial rivers of $A = 2.5$ to 6 reported by Ikeda et al. (1981) and Odgaard (1981). These low values may be ascribed to two main causes. The first is the reduction of the secondary flow strength due to non-linear hydrodynamic interactions. The second reason is the high roughness in the vegetated Tollense River ($C_f \approx 0.03$), which is higher than the typical roughness found in naturally meandering channels ($C_f \approx 0.008$; Crosato (2008), Table 7.1). An increase in roughness leads to weaker secondary flow (cf. Blanckaert and de Vriend (2003), Figure 4).

### 2.5 Conclusions

The processes governing the velocity redistribution in sharp meander bends are still poorly understood, which can largely be attributed to the validity range of most meander models (i.e. limited to mildly-curved bends and slow streamwise curvature variations), and also to the scarcity of experimental data in sharp bends. This paper provided new insight in hydrodynamic processes occurring in sharp river bends based on numerical simulations of three recent experiments by means of Blanckaert and de Vriend (2003, 2010)’s reduced-order nonlinear model without curvature restrictions. These three experiments concerned very sharply curved open-channel flows in configurations with different characteristics. The Kinoshita laboratory flume had a narrow rectangular cross-section and smooth boundaries, the even narrower Polblue bend was characterized by the quasi-absence of secondary flow, and the Tollense had marked horizontal recirculation zones and patches of riverbed vegetation.

Blanckaert & de Vriend’s nonlinear model includes the following processes that contribute to the velocity redistribution in bends: topographic steering,
effects of streamwise variations in river curvature and effects of secondary flow. Moreover it accounts for nonlinear hydrodynamic interactions that reduce the secondary flow strength. The model successfully simulated the global mean flow redistribution and the characteristics of the secondary flow in all three experiments, which further validated the model as a tool to investigate the processes governing the global flow redistribution. The results suggest that the global flow redistribution is primarily governed by topographic steering, curvature changes and secondary flow, in a broad range of different configurations, including mildly to sharply curved bends, narrow to shallow bends, smooth to rough bends, bends with additional complexities such as horizontal recirculation zones or patches of riverbed vegetation. The relative importance of these three dominant processes is case dependent, and essentially controlled by the parameters $C_f^{-1}H/B$, $R/B$ and streamwise variations in curvature, $\partial R^{-1}/\partial s$ which are determined by the river planform. The first parameter characterizes a river reach, whereas the second and third parameters are characteristics of individual bends.

Major differences exist between the processes governing the velocity redistribution in mildly and sharply curved bends. First, velocity redistribution induced by curvature variations is negligible in mildly curved bends, but was found to be the dominant process in the three investigated sharp bends. These results confirm the scaling analysis of Blanckaert and de Vriend (2010), which suggested that flow redistribution induced by streamwise curvature variations scales with the control parameter $C_f^{-1}H/R$. This result is relevant, because most meander models are based on the assumption of weak-curvature variations and therefore intrinsically unable to represent accurately the velocity redistribution in sharp river bends. Second, nonlinear hydrodynamic processes play a dominant role in sharp bends, as revealed by comparison of predictions with Blanckaert & de Vriend’s nonlinear hydrodynamic model to predictions with their model in its linear mild-curvature formulation. Linear models were not as accurate in predicting the secondary flow strength in the three investigated sharp bends. For the Tollense the linear model provided the correct order of magnitude, however for the Kinoshita flume and the Polblue Creek it overpredicted the secondary flow strength by an order of magnitude, and falsely identified the secondary flow as the dominant process governing the velocity redistribution, ultimately leading to unsatisfactory predictions of the velocity redistribution. The inclusion of nonlinear hydrodynamic interactions in the non-linear model reduces the growth of the secondary flow with increasing curvature. This mechanism, which Blanckaert (2009) called the saturation of the secondary flow, was also demonstrated by van Balen (2010) using a Large Eddy Simulation model applied to a wide range of axi-symmetric flow cases. This saturation of the secondary flow is accompanied by a reduction in the velocity redistribution induced by the secondary flow, as well as a reduction
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in the transverse bed slope induced by the near-bed secondary flow velocities. This reduced transverse bed slope reduces the effect of topographic steering.

It should be recalled that this paper focused on the hydrodynamic processes occurring in sharp open-channel bends. The nonlinear hydrodynamic model has to be coupled to models for the bed morphology and for the bank migration that are valid in the sharp-curvature range, in order to obtain a model for meander dynamics without curvature restrictions.

Acknowledgements

The Dutch Technology Foundation (STW, applied science division of NWO) is acknowledged for funding the PhD research of the first author under grant DCB.7780. The support of Deltares is also gratefully acknowledged. The development of the nonlinear hydrodynamic model by the second author was funded by the Swiss National Science Foundation (SNF) under grants 2100052257, 2000 059392, 2100066992, 20020103932, and 200020119835. The second author further acknowledges funding by Chinese Academy of Sciences fellowship for young international scientists under grant 2009YA1-2 and by the Sino-Swiss Science And Technology Cooperation for the joint research project GJH20908. The authors thank Jorge Abad for the measurement data from the Kinoshita Flume, Rachel Nanson for the measurement data from the Polblue Creek and Ingo Schnauder and Alex Sukhodolov for the Tollense River field data. The acquisition of the field data of the Tollense River was funded by the Deutsche Forschungsgemeinschaft (DFG) and the Netherlands Organization for Scientific Research (NWO) under grants SU 405/3-1 and DN66-149 in the framework of their bilateral cooperation program. Finally, Mohamed Nabi is thanked for reviewing the chapter and Hanneke Nijhof is thanked for help with the production of the figures.
Chapter 3

Meander dynamics: A reduced order non-linear model without curvature restrictions for flow and bed morphology

Abstract

Reduced-order models remain essential tools for meander modelling, especially for processes at large length scales and long time scales, probabilistic simulations, rapid assessments or when input data are scarce or uncertain. Present reduced order meander models either consider their dependent variables as small amplitude variations compared to a basic state (linearity) or as varying gradually in a spatial sense (gradual variation). In a prequel, Blanckaert and de Vriend (2010) derived a non-linear reduced-order hydrodynamic model without curvature restrictions and showed that linearity or gradual curvature variations assumptions do not hold in strongly curved channels. Moreover, in strongly curved channels, a non-linear feedback mechanism causes the secondary flow strength to be smaller than its linear mild curvature equivalent. In the limit of mild amplitude variations and mild-curvature, their non-linear meander flow model simplifies to a well-known linear formulation. The present paper extends this nonlinear modelling to the bed morphology in strongly curved bends, making use of Exner’s sediment conservation principle.

CHAPTER 3. REDUCED MEANDER FLOW AND BED MORPHOLOGY

Furthermore, the model quantifying the relative influence of the downslope gravitational force is refined by considering non-linear effects. The coupled non-linear flow and bed morphology model yields satisfactory results for the bed topography, whereas the corresponding linear model strongly overpredicts the magnitude of the transverse bed slope. Analyses of the forcing mechanisms indicate that this erroneous behaviour is caused by an overestimation of the upslope drag force due to the secondary flow.

3.1 Introduction

Meandering rivers are landscape features which have interested scientists, civil engineers, geomorphologists and decision makers for centuries. Although great progress has been made in the study of meandering rivers over the last few decades, their behaviour remains to be fully understood.

Meander models generally consider three interconnected processes: the hydrodynamics, bed morphodynamics and bank morphodynamics (cf. Figure 3.1, Camporeale et al. (2007, Figure 2) for a graphic overview of these interconnected processes). The flowing water induces shear stresses on the river bed which move the bed sediments, ultimately leading to a pattern of stable and migrating morphological features. The flow of water also induces shear stresses on the banks which cause them to erode, whereas banks in zones of low shear stress may accrete, thus changing the river’s course. As the banks are generally made of more cohesive material or bound by riparian vegetation, the time scales of the bank adaptation are generally longer than those of the bed adaptation. This implies that the bed and bank morphodynamics can be considered separately. The morphological changes of the bed and the banks in their turn influence the hydrodynamics, thus giving rise to further morphological changes, and so on. An alluvial river is thus a complex adaptive dynamic system. As a step towards unravelling this system, this article focuses on the bed morphodynamics.

At present it is possible to model meandering channels in great detail. Rüther and Olsen (2007) simulated the three day meander experiment by Friedkin (1945) with a three-dimensional meander model, solving the hydrodynamics by means of a Reynolds-averaged Navier Stokes model with a linear $k-\epsilon$ turbulence closure. Typical length and time scales for meander evolution are much larger than the 72 hours and the 40 meters of the Friedkin (1945) experiment. The simulation of longer time and length scales motivates the use of reduced order models (obtained by depth-averaging (and width-averaging) operations), which are generally an order of magnitude faster than their three-dimensional counterparts. Moreover, reduced order models require less input, which is useful when data are scarce or uncertain.

Present reduced order meander models are either based on the linearity or
MEANDER MODEL

Large scale 1D morphodynamic model
\[ U, H, Z^n = \text{fct} \left( C_f, Q, B, S, a, b \right) \]
\[ C_f = \psi C_{\beta} = \left( \psi_{\text{secondary flow}} w, \psi_{\text{bottom}} \right) C_{\beta} \]

Meander morphology submodel
\[ G = \text{fct} \left( \tau^n, g, \Delta, D \right) \]
\[ s^n = \text{fct} \left( \tau^n, \alpha, b, g, \Delta, D, G \right) \]
\[ \frac{A}{R} = \text{fct} \left( s^n \right) \]

Meander hydrodynamics submodel
\[ \frac{\alpha^n}{R} = \text{fct} \left( C_f, \frac{1}{R}, H, B, \frac{A}{R}, \text{Fr}, \frac{f_s}{R} \right) \]
\[ \frac{\alpha^s}{R} = \text{fct} \left( C_f, \frac{1}{R}, H, \frac{\alpha^s}{R} \right) \]
\[ \psi = \text{fct} \left( C_f, \frac{1}{R}, H, \frac{\alpha^s}{R}, B \right) \]
\[ \tau^n = \text{fct} \left( \psi, C_f, U, H, \frac{\alpha^n}{R}, \frac{\alpha^s}{R} \right) \]

Meander migration submodel
\[ M = \text{fct} \left( \frac{\alpha^n}{R}, \frac{A}{R} \right) \]

Figure 3.1: Meander model with dependent variables illustrating the connections between the different components. Adapted from Blanckaert and de Vriend (2010).
gradual variation assumption. The linearity assumption implies that the variables are assumed to be small with reference to a basic state, a condition which is often justified in mild curvature bends. The gradual variation assumption allows large variations of the variables as long as they vary gradually in space, a condition which is generally justified in mild curvature bends (cf. discussion in Bolla Pittaluga et al. (2009)).

Models which are based on the linearity assumption include Struiksma (1983a), de Vriend and Struiksma (1984), Odgaard (1989), Lancaster and Bras (2002), Abad and Garcia (2006) and Crosato (2008), whereas models based on the gradual variation assumption include Zolezzi and Seminara (2001) and Bolla Pittaluga et al. (2009).

In strongly curved channels, the linearity and gradual variation assumptions no longer hold. Furthermore in strongly curved bends the frequently used linear parameterization of secondary flow (Rozovskii, 1957; Engelund, 1974; de Vriend, 1977), which is proportional to the depth to radius of curvature ratio, no longer holds. This is due to a non-linear feedback mechanism which causes the reduction of the secondary flow strength compared to its linear equivalent (Blanckaert and de Vriend (2003, 2010), Chapter 2). These issues motivate the development of non-linear reduced order meander models which also allow sudden variations in curvature and include a non-linear treatment of the secondary flow (in short: non-linear, without curvature restrictions). Blanckaert and de Vriend (2003, 2010) extended the hydrodynamic modelling to strongly curved bends. In this paper the morphodynamics will be extended in a similar manner.

Morphological development is related to the sediment transport field. In the cases considered herein, this transport field can be described by a sediment transport formula relating the transport rate and direction to local flow properties. Most sediment transport formulae are derived from straight flume measurements and are related to the drag force imposed by the fluid on the sediment particles. In curved channels a transverse bed slope $\partial z_b/\partial n$ (the gradient of the bed level $z_b$ in transverse direction $n$) exists with a bed that usually deepens in outward direction (cf. Figure 3.1). Gravity exerts a downhill force on sediment particles positioned on the transversally inclined bed. The combination of these forces allows us to determine the sediment transport direction on sloping beds as found in curved channels.

Fargue (1868) (cf. Hager (2003)) was the first to report that the magnitude of the transverse bed slope in a river bend correlates well with the inverse radius of curvature $1/R$ of the bend. Later research showed the correlation of the transverse bed slope in a bend to the water depth $H$ as well (van Bendegom, 1947). By inserting a constant of proportionality $A$ (also known as the scour factor) the transverse bed level gradient can be expressed in the following
manner:

\[
\frac{\partial z_b}{\partial n} = -H \frac{A}{R_i}
\]  

Allen (1978) reported that van Bendegom (1947) derived a first estimate of the scour factor \( A = 10 \) from the force balance on a stationary sediment particle. Later Rozovskii (1957) independently found \( A = 11 \), which he also justified by comparison to field and laboratory data. Engelund (1974, 1975) obtained \( A = 21 \) for an annular flume with a moving lid and \( A = 7 \) for open channel field conditions. Ikeda et al. (1981) and Odgaard (1981) reported values of the scour factor \( A \) between 2.5 and 6 for alluvial rivers. Zimmermann and Kennedy (1978) found an expression for \( A \) related to the friction factor (based on an empirical relation found by Nunner (1956)), a ratio relating the projected surface area of a non-spherical particle to its volume and the particle densimetric Froude number \((U/\sqrt{g\Delta D})\), where \( U \) is the bulk velocity, \( g \) is the gravitational constant, \( \Delta (\approx 1.65) \) is the relative density of sediment and \( D \) is a characteristic sediment diameter. The above mentioned findings are all based on the assumption of mildly sloping streamwise bathymetries. Recently, Seminara et al. (2002) and Parker et al. (2003) developed an implicit theoretical model for bed load transport at low shield stresses, based on the force balance on a particle in motion along arbitrarily sloping beds. Their results can however not easily be expressed in terms of equation (3.1).

The range of the scour factor in strongly curved bends remains to be investigated, but measurements in the field by Nanson (2010), Schnauder and Sukhodolov (2012) and measurements in laboratory flumes by Blanckaert (2010) and Abad and Garcia (2009b) reveal scour factors of \( A = 0.1, 0.35, 2 \) and 2, respectively, which suggest that in sharp bends significantly lower scour factors apply than those typically found in more mildly curved channels.

The objectives of the present paper are (i) to investigate the mechanisms responsible for the generation of the bed morphology in sharp river bends; (ii) to develop a non-linear model for the bed morphology without curvature restrictions, encompassing existing linear models; (iii) to improve the model of the gravitational pull on the sediment particles and to analyze the sensitivity of results to this model and (iv) to analyze the importance of non-linear effects in the prediction of the bed morphology.

### 3.2 Model description

As stated in the introduction, meander models consist of three interacting components that account for the flow, the bed morphology, and the planform, respectively. Blanckaert and de Vriend (2010) presented a meander model
framework of which the different components are summarized in Figure 3.1 for a schematised river bend.

### 3.2.1 Meander flow submodel

Because morphological processes in rivers are mainly driven by the flow, the dominant flow variables playing a role in the hydrodynamic submodel will first be described briefly. A detailed description is reported in Blanckaert and de Vriend (2003), Blanckaert (2009) and Blanckaert and de Vriend (2010).

The hydrodynamic submodel computes the cross-stream distribution of the depth-averaged downstream velocity $U_s$ via the parameter

$$\alpha_s = \frac{1}{R} \frac{\partial U_s}{\partial n},$$  \hspace{1cm} (3.2)

When $\alpha_s = -1$ the downstream velocity distribution corresponds with a potential vortex distribution and when $\alpha_s = 1$ it corresponds to a forced vortex distribution (Vardy, 1990).

In curved open channels, $\alpha_s$, is the result of different processes described in detail by Johannesson and Parker (1989b) and Blanckaert and de Vriend (2010). A dominant factor is the curvature-induced secondary flow which requires adequate representation in reduced order hydrodynamic models (Johannesson and Parker, 1989b; Finnie et al., 1999; Blanckaert and Graf, 2004).

The fluid motion normal to the streamwise flow $v_s$, is referred to as spanwise flow $v_n$, which is the sum of the depth averaged cross-flow $U_n$ and the secondary flow $v_n^*$. In the paper we consider the linear secondary flow model by de Vriend (1977) and the non-linear secondary flow model by Blanckaert and de Vriend (2003).

The nonlinear flow model of Blanckaert and de Vriend (2003, 2010) also computes the direction of the bed shear stress, which may be written as the ratio of the transverse and streamwise components of the bed shear stress ($\tau_{bn}$, $\tau_{bs}$ respectively), which is known to be dominant in the formation of the transversally inclined bed (e.g. van Bendegom, 1947; Engelund, 1974; Camporeale et al., 2007; Blanckaert and de Vriend, 2010)). The transverse component of the bed shear stress is composed of two components $\tau_{bn}^\gamma$ and $\tau_{bn}^*$. The former is related to the crossflow $U_n$ and the latter is the bed shear stress induced by the secondary flow (cf. Figure 3.1).

$$\frac{\tau_{bn}}{\tau_{bs}} = \frac{\tau_{bn}^\gamma}{\tau_{bs}} + \frac{\tau_{bn}^*}{\tau_{bs}} = \frac{U_n}{U_s} + \frac{\tau_{bn}^*}{\tau_{bs}}$$  \hspace{1cm} (3.3)
3.2. MODEL DESCRIPTION

The direction of the bed shear stress induced by secondary flow can be approximated as follows:

\[
\frac{\tau_{bn}}{\tau_{bs}} = \frac{\tau_{bn}}{\tau_{bs}}|_{n=0} = H g_\tau \frac{\alpha_{\tau\infty}}{R} \tag{3.4}
\]

\[
= H g_\tau \left[ \frac{\alpha_{\tau\infty}}{\alpha_{\tau0}} \right] \left[ \frac{\alpha_{\tau0}}{R} \right] \tag{3.4}
\]

\[
= H g_\tau \left[ \text{ftc}(\beta) \right] \left[ \frac{2}{\kappa^2} \left( 1 - \frac{\sqrt{C_f^*}}{\kappa} \right) \right] \frac{1}{R} \tag{3.4}
\]

where \( g_\tau(n) \) is the distribution function of the bed shear stress direction over the channel width and \( \kappa(\approx 0.4) \) is the von Kármán constant. The contributions \( \alpha_{\tau\infty}/R \) and \( \alpha_{\tau0}/R \) represent the solution of the bed shear stress direction as computed by the non-linear and linear model, respectively. The present paper adopts the linear-model solution of de Vriend (1977) for \( \alpha_{\tau0}/R \), which is included in Equation (3.4). The equation shows that the reduction of the secondary flow strength due to the non-linear feedback mechanism also reduces the angle between the transverse and streamwise components of the bed shear stress vector. Blanckaert and de Vriend (2003)’s nonlinear meander flow model quantifies this reduction by means of the correction factor \( \alpha_{\tau\infty}/\alpha_{\tau0} \), which is also included in Equation (3.4). Their non-linear meander flow model provides this correction factor as a function of the parameter \( \beta = C_f^{-0.275} (H/R)^{0.5} (\alpha_s + 1)^{0.25} \). Figure 3.2 graphically shows how this factor varies with \( \beta \).

The hydrodynamic submodel also computes the magnitude of the bed shear stress vector as:

\[
\|\tau_b\| = \rho C_f U_{tot}^2 = \rho \psi C_{f0} U_{tot}^2, \tag{3.5}
\]

where \( C_{f0} \) is the straight channel friction factor and \( U_{tot} \) is the magnitude of the depth-averaged velocity vector. The factor \( \psi \) accounts for additional energy losses in a curved open-channel flow as compared to a straight open-channel flow. These curvature-induced energy losses are related to increased near-bed gradients of the deformed velocity profiles caused by the non-linear secondary flow effects, the additional transverse component of the bed shear stress and additional curvature-induced turbulence production (for details see Blanckaert and de Vriend (2003) and Blanckaert (2009)). These additional curvature-induced energy losses are neglected in mildly-curved bends, implying that \( \psi = 1 \). They are important, however, in strongly-curved bends. Figure 3.2b graphically shows the value of \( \psi \) as a function of \( \beta C_f^{0.15} \) according to the non-linear meander secondary flow model of Blanckaert and de Vriend (2003). Other sources of the increased energy loss in the bend are also accounted
Figure 3.2: Solutions of Blanckaert and de Vriend (2003)'s nonlinear meander flow model without curvature restrictions. (a) The direction of the bed shear stress parameterized by $\alpha_{\tau}$. The solution is given as a correction factor to the linear model solution $\alpha_{\tau 0}$ (Eq. 3.4). When plotted against the bend parameter $\beta$, all solution points nearly collapse on a single curve. (b) The curvature-induced energy loss factor $\psi$ (Eq. (3.5)). Minimum scatter around a single curve is obtained when expressing $\psi$ as a function of $\beta C_f^{0.15}$.

for in the hydrodynamic submodel (for details see Blanckaert (2009)). The magnitude and direction of the bed shear stress, as computed from (3.4) and (3.5), are important ingredients to the morphology submodel described below.

3.2.2 Morphology submodel

3.2.2.1 Basic governing processes and equations

The bed morphology in a bend is determined by the conservation of sediment mass, as described by Exner’s balance equation:

$$\frac{1}{1 - \epsilon_p} \frac{\partial z_b}{\partial t} + \frac{1}{1 + n/R} \frac{\partial s_{bs}}{\partial s} + \frac{\partial s_{bn}}{\partial n} + \frac{1}{1 + n/R} \frac{s_{bn}}{R} = 0,$$

(3.6)

where $z_b$ denotes the bed level, $s_b = [s_{bs}, s_{bn}]$ is the sediment transport capacity and $\epsilon_p$ denotes the porosity of the bed. Typical values of $\epsilon_p$ for clean uniform sand range between 0.29 and 0.50 (Lambe and Whitman, 1979, p. 31).

The transport is assumed to be at capacity always and everywhere. Moreover, it is assumed that the transport rate and direction can be expressed in terms of local flow properties, such as the flow velocity. Mosselman (2005) gave
an overview of many such sediment transport formulae, one class of which is
the so-called Engelund and Hansen (1967) type formula given by

\[ s_b = \alpha \sqrt{g \Delta D^3} (\theta)^{b/2} = \alpha \sqrt{g \Delta D^3} \left( \frac{\tau_b}{\rho g \Delta D} \right)^{b/2} \]

\[ = \alpha \sqrt{g \Delta D^3} \left( \frac{\psi C_f U_{tot}^2}{g \Delta D} \right)^{b/2} = a(\psi C_f U_{tot}^2)^{b/2}, \quad (3.7) \]

where \( \theta \) denotes the dimensionless bed shear stress, \( \alpha \) is a non-dimensional O(1) calibration coefficient and \( b \) is an exponent indicating the non-linearity of the sediment transport formula (e.g. for Engelund and Hansen (1967) \( b = 5 \)).

Although Engelund-Hansen’s sediment transport formula is a formula for the total sediment load, i.e. bed load and suspended load, it should only be applied in the present model to configurations where bedload transport is dominant. The dimensionless Rouse number \( P = w_s/(\kappa \sqrt{C_f U}) \) represents the ratio between the settling velocity \( w_s \) and the shear velocity multiplied by the von Kármán constant. A Rouse number larger than 2.5 is often used as an approximate criterion for the predominance of bedload transport (Fryirs and Brierly, 2013). Both Zeng et al. (2005) and Wu et al. (2000) have shown that ignoring the suspended sediment transport does not change significantly the equilibrium bed morphology predictions in meander bends if the bed load accounts for more than 75% of the total sediment load. Moreover, for cases in which the suspended load is negligible, the model of Engelund and Hansen (1967) was shown to give comparable or slightly better predictions than the van Rijn (1984a) model in moderately curved meander bends (e.g., see Zeng (2006)). It was also shown to satisfactorily predict the bed morphology in Blanckaert (2010)’s bedload dominant strongly curved M89 experiment (Zeng et al., 2008a), which will also be simulated in the present paper.

The direction of the bed load transport is given by (cf. van Bendegom (1947); Engelund (1974)).

\[ \frac{s_{bn}}{s_{bs}} = \frac{\tau_{bn}}{\parallel \tau_b \parallel} - G \frac{\partial z_b}{\partial n} \]

\[ \frac{\tau_{bn}}{\parallel \tau_b \parallel} - G \frac{\partial z_b}{1+n/R} \frac{\partial z_b}{\partial s} \quad (3.8) \]

For mildly sloping streamwise slopes a simplified expression given by Olesen (1987) may be used which makes an error of at most 10% for \( \tau_{bn}/\tau_{bs} < 0.5 \):

\[ \frac{s_{bn}}{s_{bs}} = \frac{\tau_{bn}}{\tau_{bs}} - G \frac{\partial z_b}{\partial n} \quad \text{Eq.(3.3), Eq.(3.4)} \]

\[ \Leftrightarrow \quad \frac{U_n}{U_s} - H \frac{\alpha \tau}{R} g - G \frac{\partial z_b}{\partial n}. \quad (3.9) \]

This expression shows that the direction of the bed load transport vector deviates from the direction of the depth-averaged velocity, \( U_n/U_s \), due to two
effects. First, the transverse component of the bed shear stress induced by the secondary flow, \( H \alpha - g / R \) (Equation (3.4)) exerts a drag force on the sediment particles which is typically uphill. Second, gravity exerts a downhill force on the sediment grains, parameterized by the last term in Equation (3.9). This gravitational force is modelled as being proportional to the transverse bed slope, with a factor of proportionality that is commonly called the gravitational pull \( G \).

In the past, different investigations have expressed the dependency of the gravitational pull model on the dimensionless shear stress, either based on theoretical derivations, empirical evidence or numerical particle simulations (cf. Mosselman, 2005). Van Bendegom [1947] was the first to do so and proposed \( G = (1.5 \theta)^{-1} \). Engelund (1974) proposed \( G \propto \theta^0 \). Ikeda (1982), Hasegawa (1984), Parker and Andrews (1985), Olesen (1987), Struiksma (1988) Talmon et al. (1995) proposed relations \( G \propto \theta^{-0.5} \) and finally Sekine and Parker (1992) proposed a similarity solution \( G \propto \theta^{-0.25} \) based on numerical simulations of the motion of saltating particles. To this end three different gravitational pull models relating \( G \) to \( \theta \) to the power -1, -0.5 and -0.25 are introduced, which will be referred to as van Bendegom, Struiksma and Sekine and Parker type, respectively.

Experimental gravitational pull studies are either based on results in curved flumes (e.g. Engelund (1974), Zimmermann and Kennedy (1978) or in straight flumes (Talmon et al., 1995; Talmon and Wiesemann, 2006; Francalanci and Solari, 2007). The experiments of Zimmermann and Kennedy (1978) were performed in a circular flume which was long enough for the transverse slope to reach an equilibrium. The transverse slope angle in these experiments varied between 3 and 15 degrees which encompasses both mild and strong transverse slopes. (Talmon et al., 1995; Talmon and Wiesemann, 2006) performed bed levelling experiments of mildly sloping transverse slopes from which they derived their gravitational pull model. Francalanci and Solari (2007) performed experiments in a straight flume where the streamwise slope angle and transverse slope angle were varied. As the instantaneous behaviour of the sediment particles was observed, the experimental setup was very suitable to study gravitational slope effects in strongly sloping beds. Francalanci and Solari (2007) used steel discs with a major diameter of 3 mm and a minor diameter of 0.6 mm instead of almost spherical particles. It is questionable, whether the theoretical model by Seminara et al. (2002); Parker et al. (2003), derived for spherical particles, can be applied directly to such ellipsoidal particles without any adaptation. For example, the drag force upon such a particle could vary up to a factor five depending on the orientation of the particle in the flow. For that reason the experimental data of Francalanci and Solari (2007) have not been considered in the present study.

Olesen (1987) and Struiksma (1988) analyzed the experiments of Zimmer-


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\textit{mann and Kennedy} (1978) to compute the gravitational pull $G$. In these experiments, the flow and bed morphology reached a so-called fully-developed state, which means that all longitudinal gradients vanish, $\partial/\partial s = 0$, and no depth-averaged transverse velocity exists, $U_n = 0$. Hence, equation (3.9) reduces to:

$$
\frac{\partial z_{b,e}}{\partial n} = -\frac{H}{G} \frac{\alpha_{\tau,e}}{R} g_T = -\frac{H A_e}{R} g_T
$$

(3.10)

where the subscript $e$ indicates fully-developed conditions. The value of the gravitational pull $G$ can be obtained by measuring the transverse bed slope in the experiments, and by estimating the direction of the bed shear stress according to a meander flow model.

\textit{Olesen} (1987) and \textit{Struiksma} (1988) estimated $\tau_{bn}^*/\tau_{bs}$ according to the linear secondary flow model of \textit{de Vriend} (1977), (cf. Equation (3.4)). In the present paper, the gravitational pull $G$ is re-evaluated by estimating $\tau_{bn}^*/\tau_{bs}$ according to the nonlinear secondary flow model of \textit{Blanckaert and de Vriend} (2003). The three types (Van Bendegom, Struiksma and Sekine and Parker) are evaluated in Figure 3.3 using the above equation (3.10). Figure 3.3 also shows the newly computed values as well as the old fit by Struiksma (1988) and the model by \textit{van Bendegom} (1947). The newly computed values obtained for the gravitational pull parameter $G$ are smaller than previously found by \textit{Olesen} (1987). This means that the effect of the gravitational pull in determining the direction of the bed load transport is weaker than previously derived, and the equilibrium transverse bed slope steeper (see (3.10)). The performance of the three newly derived models will be evaluated in Section 3.3.2.

3.2.2.2 Reduced-order bed morphology model without curvature restrictions

The steady state Exner equation ((3.6) with $\partial/\partial t=0$) with Eq. (3.9) substituted in it, elaborated term by term and subsequently divided by $s_{bs}/(1+n/R)$
Figure 3.3: Gravitational pull $G$ determined from experiments (Zimmermann and Kennedy, 1978) with varying radii of curvature $R = 1.80$ mm, 2.55 m and 3.30 m and two different sediment diameters $D_{50} = 0.21$ mm and 0.55 mm. The filled lines indicate gravitational pull models based on linear secondary flow model (van Bendegom, 1947; Struiksma, 1988), whereas the dashed lines indicate gravitational pull models based on a non-linear secondary flow model.

yields:

$$
\begin{align*}
&\frac{b}{2} \frac{\partial \psi}{\partial s} + \frac{b}{U_s} \frac{\partial U_s}{\partial s} + \frac{b - 1}{2} \frac{1}{1 + \frac{U_n^2}{U_s^2}} \frac{\partial}{\partial s} \left( \frac{U_n^2}{U_s^2} \right) \\
&+ \left[ (1 + n/R) \frac{b}{U_s} \frac{\partial U_s}{\partial n} + \frac{1}{R} \right] \left[ \frac{U_n}{U_s} - \frac{H \alpha_t g_r - G \partial z_b}{\partial n} \right] \\
&+ \left[ \frac{b - 1}{2} (1 + n/R) \frac{1}{1 + \frac{U_n^2}{U_s^2}} \frac{\partial}{\partial n} \left( \frac{U_n^2}{U_s^2} \right) \right] \left[ \frac{U_n}{U_s} - \frac{H \alpha_t g_r - G \partial z_b}{\partial n} \right] \\
&+ (1 + n/R) \left[ -\frac{H \alpha_t g_r}{R} \frac{\partial g_r}{\partial n} - G \frac{\partial^2 z_b}{\partial n^2} \right] + (1 + n/R) \frac{\partial}{\partial n} \left( \frac{U_n}{U_s} \right) = 0,
\end{align*}
$$

(3.11)
in which the first line is related to downstream sediment transport gradient and the second to fourth lines are related to the transverse transport.

Using the equation of continuity for the flow

\[
\frac{1}{1 + n/R} \frac{\partial h U_s}{\partial s} + \frac{\partial h U_n}{\partial n} + \frac{1}{1 + n/R} \frac{h U_n}{R} = 0, \tag{3.12}
\]

the term \( \partial / \partial n(U_n/U_s) \) in (3.11) can be rewritten, to yield:

\[
\frac{b}{2} \frac{\partial \psi}{\partial s} + \frac{b - 1}{U_s} \frac{\partial U_s}{\partial n} + \frac{b - 1}{2} \frac{1}{1 + U_s^2} \frac{\partial}{\partial s} \left( \frac{U_n^2}{U_s^2} \right)
\]

\[+ \left[ (1 + n/R) \frac{b}{U_s} \frac{\partial U_s}{\partial n} + \frac{1}{R} \right] \left[ \frac{U_n}{U_s} - \frac{H}{R} \alpha g - G \frac{\partial z_b}{\partial n} \right]
\]

\[+ \left[ \frac{b - 1}{2} \frac{1}{1 + U_s^2} \frac{\partial}{\partial n} \left( \frac{U_n^2}{U_s^2} \right) \right] \left[ \frac{U_n}{U_s} - \frac{H}{R} \alpha g - G \frac{\partial z_b}{\partial n} \right]
\]

\[+ (1 + n/R) \left[ - \frac{H}{R} \alpha g - G \frac{\partial^2 z_b}{\partial n^2} \right] - \frac{1}{h} \frac{\partial h}{\partial s}
\]

\[- \frac{U_n}{U_s} \left[ (1 + n/R) \frac{1}{U_s} \frac{\partial U_s}{\partial n} + \frac{1}{R} + (1 + n/R) \frac{1}{h} \frac{\partial h}{\partial n} \right] = 0. \tag{3.13}
\]

### 3.2.2.2.1 Expressing the bed shear stress angle as function of the dependent variables

Equation (3.13) is further elaborated by expressing the bed shear stress angle and the bed level as functions of the water depths. The difference between the fully developed and the developing transverse bed slope is approximately equal to the difference in fully developed and the developing transverse water depth gradients:

\[
\frac{\partial z_{b,e}}{\partial n} - \frac{\partial z_b}{\partial n} \approx \frac{\partial h_e}{\partial n} + \frac{\partial h}{\partial n}. \tag{3.14}
\]

The above equation in combination with Equation (3.10) leads to:

\[
- \frac{H}{G} \frac{\alpha g}{R} \frac{\partial z_b}{\partial n} \approx \frac{\partial h_e}{\partial n} + \frac{\partial h}{\partial n}, \tag{3.15}
\]

If \( U_n/U_s \) is expressed as its centreline value multiplied by a transverse...
distribution function $g_c(n)$, the substitution of (3.15) into (3.13) yields:

\[
\frac{b}{2} \frac{1}{\psi} \frac{\partial \psi}{\partial s} + \frac{b-1}{U_s} \frac{\partial U_s}{\partial n} + \frac{b-1}{2} \frac{1}{1 + \frac{U_n^2}{U^2} g_c} \frac{\partial}{\partial s} \left( \frac{U_n^2}{U^2} g_c^2 \right) + \left[ \frac{(1+n/R)}{U_s} \frac{\partial U_s}{\partial n} + \frac{1}{R} \right] \left[ \frac{U_n}{U} g_c - G \frac{\partial h_c}{\partial n} + G \frac{\partial h}{\partial n} \right] \\
\left[ b - 1 \right] \frac{(1+n/R)}{2} \frac{1}{1 + \frac{U_n^2}{U^2} g_c^2} \frac{\partial}{\partial n} \left( \frac{U_n^2}{U^2} g_c^2 \right) \left[ \frac{U_n}{U} g_c - G \frac{\partial h_c}{\partial n} + G \frac{\partial h}{\partial n} \right] \\
+ (1 + n/R) \left[ -G \frac{\partial^2 h_c}{\partial n^2} + G \frac{\partial^2 h}{\partial n^2} \right] - \frac{1}{h} \frac{\partial h}{\partial s} - \frac{U_n}{U} g_c \left[ (1+n/R) \frac{\partial U_s}{\partial n} + \frac{1}{R} + (1+n/R) \frac{1}{h} \frac{\partial h}{\partial n} \right] = 0. \tag{3.16}
\]

*Blanckaert and de Vriend* (2010) derived the transverse velocity component at the centreline:

\[
U_n = U_n \bigg|_{n=0} = U \frac{B^2}{8} \frac{\partial}{\partial s} \left( \frac{\alpha_s}{R} + \frac{A}{R} + \frac{Fr^2}{R} \right). \tag{3.17}
\]

### 3.2.2.2 Choice of profile distribution functions

To simplify the depth-averaged model, transverse profile distributions with one degree of freedom are assumed for the depth averaged streamwise velocity $U_s$, water depth $h$ and equilibrium water depth $h_e$. In the derivation of meander models different assumptions have been made regarding the transverse profile functions for the bed level and the downstream velocity. *Odgaard* (1989) and *Johannesson and Parker* (1989b) considered a linear profile approach, *Struiksma* (1983a), *Struiksma et al.* (1985), *Struiksma and Crosato* (1989) and *Crosato* (2008) considered sinusoidal profiles and *Blanckaert and de Vriend* (2010) considered an exponential approach for their non-linear hydrodynamic submodel. These profiles, shown in Figure 3.4, all represent first-order approximations of the real bed morphology, which is sufficient because the smallest relevant length scales in a reduced model are of the order of the channel width. The three types of profiles are quite similar in representing the main features of the bed topography and have comparable accuracy (*Blanckaert and de Vriend*, 2010, Figure 4).
Figure 3.4: Exponential, linear and sinusoidal bed level distribution functions for a scour factor $A = 2$, a radius of curvature $R = 5$ m, a channel width $B = 2$ m, and a mean water depth $H = 0.2$ m.

From now on sinusoidal profile functions will be adopted:

$$h = H \left(1 + \frac{B}{\pi R} \sin \left(\frac{\pi}{B} n\right)\right) = H \left(1 + \frac{B}{\pi R} f_s\right), \quad (3.18)$$

$$h_e = H \left(1 + \frac{B_e}{\pi R} \sin \left(\frac{\pi}{B} n\right)\right) = H \left(1 + \frac{B}{\pi R} f_e\right), \quad (3.19)$$

$$U_s = U \left(1 + \frac{B}{\pi R} \alpha_s \sin \left(\frac{\pi}{B} n\right)\right) = U \left(1 + \frac{B}{\pi R} f_s\right), \quad (3.20)$$

$$g_c = \cos \left(\frac{\pi}{B} n\right) = \frac{B}{\pi} \frac{\partial f_s}{\partial n} = f_c, \quad (3.21)$$

where $f_s$ and $f_c$ are shorthand notations for the sines and cosines. Substituting
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these profiles (3.18)-(3.21) into (3.16) leads to:

\[
\begin{align*}
\left(3.221\right) & \quad \frac{b}{2} \frac{1}{\psi} \frac{\partial \psi}{\partial s} + \frac{b - 1}{\left\{1 + \frac{b}{\pi} \frac{\alpha_s}{n/R} f_s\right\}} \frac{\partial}{\partial s} \left\{\frac{\alpha_s}{n/R} B \frac{1}{\pi} f_s\right\} + \frac{b - 1}{2} \frac{1}{\left\{1 + \frac{U_0^2}{U^2} f_s^2\right\}} \frac{\partial}{\partial s} \left(\frac{U_n^2}{U^2}\right) (f_c)^2 \\
\left(3.221I\right) & \quad + \left(1 + n/R\right) \frac{b}{\left\{1 + \frac{b}{\pi} \frac{\alpha_s}{n/R} f_s\right\}} \frac{\alpha_s}{n/R} f_c + \frac{1}{R} \left[U_{n0} f_c - GHA f_c + GH \frac{A}{R} f_c\right] \\
\left(3.221II\right) & \quad + \left(1 + n/R\right) \frac{1}{2} \frac{U_{n0}^2}{U^2} \left(-2 \pi f_C f_s\right) \left[U_{n0} f_c - GHA f_c + GH \frac{A}{R} f_c\right] \\
\left(3.221III\right) & \quad + \left(1 + n/R\right) \left[GH \frac{\pi A}{R} f_s - GHA \frac{A}{R} f_s\right] - \frac{1}{\left\{1 + \frac{b}{\pi} \frac{\alpha_s}{n/R} f_s\right\}} \frac{B f_s}{\pi} \frac{\partial}{\partial s} \left(\frac{A}{R}\right) \\
\left(3.221IV\right) & \quad - \frac{U_{n0}}{U} f_c \left[1 + n/R\right] \frac{1}{\left\{1 + \frac{b}{\pi} \frac{\alpha_s}{n/R} f_s\right\}} \frac{\alpha_s}{n/R} f_c + \frac{1}{R} \left[1 + n/R\right] \frac{A}{R} f_c = 0.
\end{align*}
\]

(3.22)

Previous linear meander bed morphology models rely on the assumptions of mild curvature and mild amplitude perturbations, which means all denominators in curly brackets in equation (3.22) are approximately equal to one. For the strongly curved mobile bed experiments in the strongly curved flumes of Abad and Garcia (2009b) and Blanckaert (2010) the value of $B/\pi \cdot A/R$ is about 0.9, so these contributions to the denominators can no longer be neglected. The denominators in curly brackets in equation (3.22) are therefore retained.

3.2.2.2.3 A nonlinear meander bed morphology equation without curvature restrictions

Equation (3.22) still applies to all points $(s, n)$ of the river. In order to simplify it to a one-dimensional equation in $s$, the transverse dimension requires elimination. Since the bed level is approximated by a sine profile, the reduced version of (3.22) considers what each of the terms contributes to the sine profile. These contributions are obtained by projecting the Equation (3.22) onto a sine profile using an operation similar to a Fourier analysis:

\[
\text{Eq. (3.24)} \approx \frac{1}{B} \int_{-B/2}^{B/2} \left\{\text{Eq. (3.22)}\right\} f_s \, dn \cdot f_s.
\]

(3.23)
Projecting Equation (3.22) onto sine profiles then yields:

\[
\begin{align*}
&\left[GH \frac{\pi}{B} \frac{A_e}{R} - GH \frac{\pi}{B} \frac{A}{R}\right] \\
&\approx \text{Eq. 3.22I} \\
&+ (b - 1) \frac{\partial}{\partial s} \left(\frac{\alpha_s}{R}\right) \frac{B}{\pi} f_{P1} \left(\frac{B}{\pi} \frac{\alpha_s}{R}\right) \\
&\approx \text{Eq. 3.22II} \\
&- \frac{B}{\pi} \frac{\partial}{\partial s} \left(\frac{A}{R}\right) f_{P1} \left(\frac{B}{\pi} \frac{A}{R}\right) \\
&\approx \text{Eq. 3.22III} \\
&+ \left[\frac{b \alpha_s}{R}\right] \left[\frac{U_{n0}}{U} - GH \frac{A_e}{R} + GH \frac{A}{R}\right] f_{P2} \left(\frac{B}{\pi} \frac{\alpha_s}{R}, \frac{B}{R}\right) \\
&\approx \text{Eq. 3.22IV} \\
&- \frac{U_{n0}}{U} \left[\frac{\alpha_s}{R}\right] f_{P2} \left(\frac{B}{\pi} \frac{\alpha_s}{R}, \frac{B}{R}\right) f_s - \frac{U_{n0}}{U} \left[\frac{A}{R}\right] f_{P2} \left(\frac{B}{\pi} \frac{A}{R}\right) \\
&\approx \text{Eq. 3.22VIII} \\
&+ \frac{b - 1}{2} \frac{U_{n0}^2}{U^2} \left(-\frac{2 \pi}{B}\right) \left[\frac{U_{n0}}{U} - GH \frac{A_e}{R} + GH \frac{A}{R}\right] f_{P3} \left(\frac{U_{n0}^2}{U^2}\right) = 0, \quad (3.24)
\end{align*}
\]

where \(f_s\) is eliminated as it occurs in every term.

The functions \(f_{P1}, f_{P2}\) and \(f_{P3}\) are infinite series of which the first few terms are given below:

\[
\begin{align*}
f_{P1} \left(\frac{B}{\pi} \frac{A}{R}\right) &= 1 + 3 \left(\frac{B}{\pi} \frac{A}{R}\right)^2 + 5 \left(\frac{B}{\pi} \frac{A}{R}\right)^4 + 35 \left(\frac{B}{\pi} \frac{A}{R}\right)^6 + \cdots, \quad (3.25) \\
f_{P2} \left(\frac{B}{\pi} \frac{A}{R}\right) &= \frac{8}{9 \pi^2} \frac{B}{R} - \frac{1}{4} \left(\frac{B}{\pi} \frac{A}{R}\right)^2 + \frac{105}{225 \pi^2} \frac{B}{R} \left(\frac{B}{\pi} \frac{A}{R}\right)^2 - \frac{1}{8} \frac{B}{R} \left(\frac{B}{\pi} \frac{A}{R}\right)^3 + \cdots, \quad (3.26) \\
f_{P3} \left(\frac{U_{n0}^2}{U^2}\right) &= \frac{1}{4} - \frac{1}{8} \frac{U_{n0}^2}{U^2} + \frac{5}{64} \frac{U_{n0}^4}{U^4} - \frac{7}{128} \frac{U_{n0}^6}{U^6} + \cdots. \quad (3.27)
\end{align*}
\]

Figure 3.5 shows the distribution functions graphically. The coupled non-linear meander flow and bed morphology model, which truncates the approximation functions \(f_{P1}, f_{P2}\) and \(f_{P3}\) at 25 terms, predicts the transverse bed slope \(A/R\) with a precision of \(10^{-5}\) and its simulation time takes eight times longer than the non-linear flow model (in the case of the M89 experiment (Blanckaert and de Vriend, 2010)).
Figure 3.5: Numerical evaluation of (a) $f_{P1}(B/\pi \cdot A/R)$, (b) $f_{P2}(B/\pi \cdot A/R; B/R)$ and (c) $f_{P3}(U_{n0}/U)$.

Division of (3.24) by $GH \pi B$ and rearranging the terms gives:

\[
\frac{A}{R} + \frac{1}{GH} \frac{B^2}{\pi^2} f_{P1} \left( \frac{B}{\pi} \frac{A}{R} \right) \frac{\partial}{\partial s} \left( \frac{A}{R} \right) = \frac{Ae}{R} + \left( b - 1 \right) \frac{1}{GH} \frac{B^2}{\pi^2} f_{P1} \left( \frac{B}{\pi} \frac{\alpha_s}{R} \right) \frac{\partial}{\partial s} \left( \frac{\alpha_s}{R} \right) \\
+ \frac{1}{GH} \frac{B^2}{\pi^2} f_{P2} \left( \frac{B}{\pi} \frac{\alpha_s}{R}, \frac{B}{R} \right) \left[ \frac{\alpha_s}{R} \right] \left[ \left( b - 1 \right) \frac{U_{n0}}{U} - bGH \frac{Ae}{R} + bGH \frac{A}{R} \right] \\
- \frac{1}{GH} \frac{B^2}{\pi^2} f_{P2} \left( \frac{B}{\pi} \frac{A}{R}, \frac{U_{n0}}{U} \right) \left[ \frac{A}{R} \right] \\
- \left( b - 1 \right) f_{P3} \left( \frac{U_{n0}^2}{U^2} \right) \left[ \frac{U_{n0}^2}{U^2} \right] \frac{1}{GH} \frac{U_{n0}}{U} - \frac{Ae}{R} + \frac{A}{R},
\]

(3.28)
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which can be considered as a non-linear reduced order bed profile adaptation equation without curvature restrictions. The left hand side of the equation describes the transverse slope evolution of the bed subject to a lag effect. The first term on the right hand side represents the equilibrium profile for an infinitely long bend (Eq. (3.10)). The second term on the right hand side is related to accelerations and decelerations of the flow field. The second to fourth lines relate to sediment transport caused by higher-order flow variations.

The above equation in $A/R$ is combined with the non-linear flow equation in $\alpha_s/R$ by Blanckaert and de Vriend (2010). This yields a system of two coupled non-linear equations with two unknowns. As the functions $f_{P1}$ and $f_{P2}$ depend non-linearly on the variables $A/R$ and $\alpha_s/R$ and $f_{P3}$ depends on $U_{n0}/U$, the solution to this system must be obtained by numerical iteration.

3.2.2.3 Mild amplitude and mild curvature limits

Considering only the leading-order terms of the equations (3.25)-(3.27), which corresponds to a mild-amplitude assumption, transforms the non-linear bed adaptation equation (3.28) into

\[
\frac{A}{R} + \frac{1}{GH} \frac{B^2}{\pi^2} \frac{\partial}{\partial s} \left( \frac{A}{R} \right) = \frac{A_e}{R} + \left( b + 1 \right) \frac{1}{GH} \frac{B^2}{\pi^2} \frac{\partial}{\partial s} \left( \frac{\alpha_s}{R} \right) \\
\frac{1}{GH} \frac{B^2}{\pi^2} \left( \frac{8}{9} \frac{1}{R} \frac{\alpha_s}{R} \right) \left[ (b + 1) \frac{U_{n0}}{U} - bGHA_e \frac{A}{R} + bGHA \right] \\
\frac{1}{GH} \frac{B^2}{\pi^2} \left( \frac{8}{9} \frac{1}{R} \frac{\alpha_s}{R} \right) \left( b - 1 \right) \frac{U_{n0}}{U} - bGHA_e \frac{A}{R} + bGHA \\
\frac{b - 1}{4} \frac{U_{n0}}{U} \left[ 1 \frac{U_{n0}}{GH} \frac{A}{R} + A \right].
\]

(3.29)

With the mild-curvature assumption and $U_n << U$, Equation (3.29) reduces to

\[
\frac{A}{R} + \frac{1}{GH} \frac{B^2}{\pi^2} \frac{\partial}{\partial s} \left( \frac{A}{R} \right) = \frac{A_e}{R} + \left( b + 1 \right) \frac{1}{GH} \frac{B^2}{\pi^2} \frac{\partial}{\partial s} \left( \frac{\alpha_s}{R} \right),
\]

(3.30)

which is similar to the form found in Crosato (2008). The main difference is that the form presented above uses the geometric curvature, whereas Crosato (2008) uses streamline curvature. Blanckaert and de Vriend (2010), however,
Table 3.1: Hydraulic and Morphodynamic Conditions in the Three Experiments. $B$ is the channel width, $R_{ap}$ is the radius of curvature at the bend apex, $Q$ is the flow discharge, $H$ is the average water depth, $C_f$ is the friction factor as reported in the original article, $D_{50}$ is the 50-th percentile of the cumulative sediment diameter distribution, $Fr = U/\sqrt{gH}$ is the Froude number and $\theta$ is the dimensionless shear stress, $w_s$ is the settling velocity (van Rijn, 1984b) and $P = w_s/(\kappa\sqrt{C_fU})$ is the Rouse number.

<table>
<thead>
<tr>
<th></th>
<th>T3</th>
<th>M89</th>
<th>Kinoshita</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$ [m]</td>
<td>1.5</td>
<td>1.3</td>
<td>0.6</td>
</tr>
<tr>
<td>$R_{ap}$ [m]</td>
<td>12</td>
<td>1.7</td>
<td>0.72</td>
</tr>
<tr>
<td>arc length [$^\circ$]</td>
<td>140</td>
<td>193</td>
<td>-</td>
</tr>
<tr>
<td>$Q$ [l/s]</td>
<td>74</td>
<td>89</td>
<td>25</td>
</tr>
<tr>
<td>$H$ [m]</td>
<td>0.091</td>
<td>0.141</td>
<td>0.15</td>
</tr>
<tr>
<td>$C_f$ [-]</td>
<td>1.28e-002</td>
<td>1.26e-002</td>
<td>2.14e-02</td>
</tr>
<tr>
<td>$D_{50}$ [mm]</td>
<td>0.45</td>
<td>2</td>
<td>0.83</td>
</tr>
<tr>
<td>$Fr$ [-]</td>
<td>0.57</td>
<td>0.41</td>
<td>0.23</td>
</tr>
<tr>
<td>$s_{bs}$ [kg/s/m]</td>
<td>0.08</td>
<td>0.023</td>
<td>0.0024</td>
</tr>
<tr>
<td>$\theta$ [-]</td>
<td>0.52</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>$w_s$ [m/s]</td>
<td>0.06</td>
<td>0.17</td>
<td>0.1</td>
</tr>
<tr>
<td>$P$ [-]</td>
<td>2.6</td>
<td>7.8</td>
<td>6.2</td>
</tr>
</tbody>
</table>

showed that the difference between geometric curvature and streamline curvature is negligible, based on an order of magnitude analysis. Besides that, the model by Crosato (2008) includes higher-order sine and cosine approximations of the transverse bed level (cf. (3.18)) thereby allowing more complex shapes for instance including a mid-channel bar. These higher-order modes are not taken in to account here, as they do not conform with the hydrodynamic modelling component.

A linear model for the flow and the bed morphology is obtained by combining Eq. (3.30) with the linear formulation of Blanckaert and de Vriend (2010)’s hydrodynamic equation.

### 3.3 Model validation and analysis

#### 3.3.1 The experiments

Three different experiments are considered with three different flow conditions (cf. Table 3.1) and flow geometries (cf. Figure 3.6).

Figure 3.6 shows the mildly curved flume ($R_{ap}/B = 8$) at Delft Hydraulics (DHL). The 1.5 m wide flume consists of a straight inflow section, a bend
Figure 3.6: Experimental setups for the mildly curved DHL flume, the strongly curved flume at EPFL, and the strongly curved Kinoshita flume. The cross-sections indicated on the DHL flume and the M89 flume show the entrance and exit of the bend. The cross-sections indicated on the Kinoshita flume indicate the bend apices in the area where bed level measurements were performed.

with an arc length of approximately $140^\circ$ with a radius of curvature of 12 m, followed by a straight channel outflow section (Struiksma et al., 1985).

In experiment T3 the bed developed from an initially flat bed. The sediment was captured at the end of the flume in a sandtrap and reintroduced at the upstream boundary (Struiksma, 1983b). After the bathymetry reached its equilibrium, a transverse slope developed after the bend entrance (cf. Figure 3.7a). It reached its peak after some 5 m into the bend. Subsequently the transverse slope gradually decreases tending to a constant level. This feature at the beginning of the bend has been termed overshoot phenomenon (de Vriend and Struiksma, 1984). At the bend exit another slight increase in the transverse slope is observed.
Figure 3.7: Normalized transverse bed slope gradient (A/R) for (a) the T3 experiment (Struiksma et al., 1985), (b) the M89 experiment (Blanckaert, 2010) and (c) the Kinoshita flume (Abad and Garcia, 2009b) measured as well as simulated with the non-linear model. Non-linear model predictions are shown based on the three gravitational pull models (van Bendedeom type ($G \propto \theta^{-1}$), Ikeda type ($G \propto \theta^{-0.5}$) and Sekine and Parker type ($G \propto \theta^{-0.25}$)). The vertical lines indicated in the results of the T3 experiment and the M89 flume show the entrance and exit of the bend. The vertical lines indicated in results of the Kinoshita flume indicate the bend apices.
The Ecole Polytechnique Fédérale in Lausanne, Switzerland, has a sharply curved flume M89 \((R_p/B = 1.3)\). It is 1.3 m wide and consists of a 9 m straight inflow section, a 193° bend with a radius of curvature 1.7 m and a 5 m long outflow section (Blanckaert (2010), cf. Figure 3.6 and Table 3.1). At the upstream boundary sediment was fed at a constant rate.

The bed in the M89 experiment in this flume exhibits a zero transverse slope before the bend entrance (cf. Figure 3.7b). At the entrance of the bend a slightly negative slope is observed, after which the transverse slope increases until some 2 m into the bend. Subsequently, the slope decreases towards the bend exit. At the exit it increases slightly, after which it exhibits an oscillatory decay in the downstream straight reach.

The Kinoshita flume is located in the Ven Te Chow Hydrosystems Laboratory of the University of Illinois. The 0.6 m wide, sediment recirculating flume consists of a sequence of alternating bends, following a Kinoshita curve, which is an extension to a sine-generated curve including extra terms related to skewness and flatness (cf. Abad and Garcia, 2009b).

The results from one of the experiments in the Kinoshita flume are shown in Figure 3.7c. Just downstream of the bend apex \((s = 14.3 \text{ m})\) the largest transverse slope is found, after which it gradually decreases until \(s \approx 16 \text{ m}\). Subsequently, the slope exhibits a second, slightly lower peak at \(s = 17 \text{ m}\), after which it decays strongly towards a minimum just after the apex of the next (opposite) bend \((s = 19.3 \text{ m})\).

### 3.3.2 Assessment of the gravitational pull models

Figure 3.7a shows the results of the T3 experiment in the DHL-flume as simulated with the non-linear model. When using the van Bendegom type gravitational pull model \((G \propto \theta^{-1})\), the wave length of the bar oscillations seems to be reproduced correctly, but the amplitude is overestimated. The non-linear model combined with the Ikeda type gravitational pull model \((G \propto \theta^{-0.5})\) underestimates the wave length, but reproduces the bar amplitude better than the van Bendegom type model. The Sekine and Parker type model \((G \propto \theta^{-0.25})\) yields slightly better results than the Ikeda type model.

The non-linear model results for the M89 experiment in the EPFL-flume are shown in Figure 3.7b. At the entrance of the bend the non-linear model reproduces the slight decrease in the transverse slope. All of the models underestimate the position of peak transverse slope. The Sekine and Parker type model reasonably reproduces the magnitude of the transverse slope around the bend. The Ikeda and van Bendegom type models, however, underestimate the slope by as much as 40% and 75% respectively.

Figure 3.7c shows the non-linear model solution for the sharply curved Kinoshita flume for each of the three fitted gravitational pull models. The
Sekine and Parker type model shows the best agreement. The Ikeda and van Bendegom type models, however, significantly underestimate the transverse slope, again.

The evaluation of the performance of the non-linear model with the three gravitational pull models reveals that the results are quite sensitive to the choice of this pull model. All in all, the Sekine and Parker type gravitational pull model produces the best results for all three experiments.

### 3.3.3 Importance of non-linear effects

The importance of nonlinear effects will be analyzed by comparing the results of the linear and nonlinear models, both with the gravitational pull model of Sekine and Parker (1992).

The results of the linear model for the T3 experiment are almost the same as those of the non-linear model (cf. Figure 3.8a). This is not surprising as this is a case of mild curvature and mild amplitude. This is in line with our earlier observation that the non-linear model encompasses the linear model as a limit case for mild curvature and mild amplitude.

Figure 3.8b shows the results for the strongly curved M89 experiment. The two models show no difference in the upstream reach and until about 1.5 m into the bend. The non-linear model subsequently shows a decay in the normalized transverse slope $A/R$, whereas the linear model shows a slight increase and a levelling off of this quantity in the second half of the bend. Just before the bend exit both the linear and non-linear models show a slight increase in transverse slope, after which it decays rapidly towards the horizontal straight channel limit. The non-linear model results are clearly superior to those of the linear model, which strongly overestimates the transverse slope in the second half of the bend by as much as 92%.

The linear and non-linear model results for the Kinoshita experiment are shown in Figure 3.8c. The pattern of the transverse slope is approximately the same for both models, but the linear model overpredicts the magnitude of the transverse slope by as much as 100%.

The results for the M89 experiment and the Kinoshita flume reveal that non-linear effects play a significant role in the formation of the transverse bed slope in strongly curved bends. Furthermore, the computations for the T3 experiment show that non-linear effects are not important in mildly curved flumes with mild amplitude variations and confirm that the non-linear model reduces to the linear model in this limit case.
3.3. MODEL VALIDATION AND ANALYSIS

![Graphs of A/R (m⁻¹) vs. s-location (m) for linear and non-linear models from measurements.](image)

Figure 3.8: Normalized transverse bed slope gradient \((A/R)\) for (a) the T3 experiment (Struiksma et al., 1985), (b) the M89 experiment (Blanckaert, 2010) and (c) the Kinoshita flume (Abad and Garcia, 2009b), measured as well as simulated with the linear and non-linear models.

3.3.4 Dominant mechanisms in the formation of the transverse bed slope

The mechanisms underlying the development of the bed morphology in a bend were described in Section 3.2.2.2 and the corresponding terms in Equations (3.28) to (3.30) are indicated by roman numerals. These mechanisms are compared in further detail using the linear and nonlinear model results.

The forcing mechanisms for the mildly curved T3 experiment are shown in Figure 3.9a. The first forcing term starts at zero in the straight inflow reach, increases to the bend equilibrium value through the bend, and after the bend exit it decreases again to zero. There is no difference between the linear and the non-linear model. The second forcing term is related to flow field accelerations...
CHAPTER 3. REDUCED MEANDER FLOW AND BED MORPHOLOGY

\[ \text{T3} \]

\[ \text{linear} \]

\[ \text{non-linear} \]

\[ \text{M89} \]

\[ \text{non-linear} \]

\[ \text{Kinoshita} \]

\[ \text{non-linear} \]
and decelerations. At the bend entrance the abrupt change of curvature causes
the flow to accelerate near the inner bank and decelerate near the outer bank.
Flow redistribution by topographic steering and secondary flow subsequently
drives the downstream momentum towards the outer bank (cf. Blanckaert
and de Vriend (2010)). This effect is however too strong and an overshoot of
the equilibrium downstream velocity distribution for fully-developed bend flow
occurs. Subsequently, the velocity profile exhibits an oscillatory adaptation
to the equilibrium state (cf. Struiksma (1983a)). The linear and the non-
linear models (cf. Figure 3.9b) give comparable results. The third forcing
term (sediment transport caused by higher-order flow variations), which is
only included in the non-linear model, has a negligible effect on the transverse
bed slope.

For mildly curved bends and mildly sloping transverse slopes, the balance
of the secondary flow and the gravitational pull dominates the bed adaptation
behaviour throughout the domain. Local accelerations and decelerations of the
flow may lead to a local maximum in the point bar height, (e.g. to a navigation
bottleneck).

The forcing terms in the sharply curved M89 case are shown in Figure
3.9c,d. In the linear model case (Figure 3.9c) the first forcing term shows a
similar behaviour as in the mildly curved T3 experiment: zero in the straight
inflow and outflow reaches and constant around the bend. The non-linear
model results (Figure 3.9d) show a decay of the transverse slope in the second
half of the bend, leading to a value at the bend exit that is almost half the max-
imum. Clearly, this decay is due to the non-linear reduction of the secondary
flow (Figure 3.2a). The corresponding reduction of the upslope drag force
expresses itself in a smaller equilibrium bed slope. The second forcing term
(flow field accelerations and decelerations) clearly shows the effect of changes
in curvature at the bend entrance and exit in both the linear and non-linear
case. The third forcing term still has a negligible effect on the transverse bed
slope, be it that it is larger than in the T3 case.

The conclusion to be drawn from the M89 case is that the balance of the
secondary flow and the gravitational pull dominates the bed adaptation be-

Figure 3.9 (facing page): Forcing mechanisms for the T3 experiment (Struiksma
et al. (1985), top row), M89 experiment (Blanckaert (2010), middle row) and
the Kinoshita flume (Abad and Garcia (2009b), bottom row) as predicted by
the linear (left column) and the non-linear model (right column) using the
Sekine and Parker type ($G \propto \theta^{-0.25}$) gravitational pull model. The vertical
lines indicated in the results of the T3 experiment and the M89 flume show
the entrance and exit of the bend. The vertical lines indicated in the results
of the Kinoshita flume indicate the bend apices.
CHAPTER 3. REDUCED MEANDER FLOW AND BED MORPHOLOGY

haviour throughout the domain, but that the second forcing term can be locally important.

For the strongly curved Kinoshita flume, the magnitudes of the forcing terms according to the linear model are shown in Figure 3.9e. The forcing mechanism related to the equilibrium transverse slope is dominant, except at the transitions between the bends ($s \approx 12.8 \text{ m}$ and $s \approx 17.8 \text{ m}$), where the flow accelerations and decelerations are dominant. The non-linear model (cf. Figure 3.9f) shows a somewhat smaller contribution of the forcing term related to the equilibrium transverse slope, and behaviour of the second forcing term similar to that in the linear model. The third forcing term has a minor influence on the solution, again.

In summary, the first forcing term is dominant in all three cases, with local effects of the second forcing term. These results confirm the findings of Camporeale et al. (2007). In strongly curved bends a downstream decrease of the transverse slope occurs in the second half of the bend, due to the non-linear modelling of the secondary flow. The third forcing term plays a minor role in all cases investigated.

3.4 Conclusion

Based on the Exner conservation principle and earlier work on reduced order hydrodynamic modelling for sharp bends, a reduced order non-linear meander bed morphology model for strongly curved bends with large-amplitude bed level variations was derived. In its asymptotic formulation for mild curvature and small-amplitude bed level variations, the nonlinear model reduces to a previously proposed linear model.

The meander bed morphology model is quite sensitive to the parameterization of the downhill gravitational force on particles at a sloping bed. Three different gravitational pull models were considered, proposed by van Bemdegom (1947), Ikeda (1982) and Sekine and Parker (1992), respectively. The Sekine and Parker (1992) type gravitational pull model was the only one that accurately reproduced the bed morphology in all cases investigated.

The nonlinear model, without curvature restrictions, accurately reproduced the bed morphology in three investigated flume experiments: one with a mildly curved bend and two with strongly curved bends. The linear model significantly overestimated the transverse bed slopes, in one case by more than 100%.

The smaller transverse bed slope resulting from the nonlinear model must be attributed mainly to a weaker curvature-induced secondary flow strength, due to a nonlinear feedback mechanism. The weaker secondary flow causes a weaker transverse component of the drag force on the particles. This explains why scour factors (the coefficient of proportionality in the relationship
between the transverse bed slope in fully-developed bend flow and the depth-curvature ratio) in strongly curved channels are significantly smaller than in mildly curved channels. Further investigation is recommended to distinguish the role of the aspect ratio on the scour factor.

Overall, the balance between the uphill drag force exerted by the secondary flow on the sediment particles on the bed and the downhill gravitational force dominates the formation of the transverse bed slope. Only locally, e.g. near transitions in the channel curvature, flow accelerations and decelerations may be dominant.

The present paper clearly shows the importance of including non-linear effects when modelling the bed morphology in strongly curved channels. By coupling this model to a bank erosion model, a meander development model for the high-curvature range which allows sudden changes in curvature can be obtained.

Acknowledgments

This research was supported by the Dutch Technology Foundation (STW, applied science division of NWO under grant DCB.7780) and Deltares. The second author was partially supported by the Chinese Academy of Sciences Visiting Professorship for Senior International Scientists, grant number 2011T2Z24, by the Sino-Swiss Science and Technology Cooperation for the Institutional Partnership Project, grant number IP13_092911. Furthermore, the authors wish to thank Nicolette Volp for checking the mathematics.
Chapter 4

Quasi-3D hydrodynamic modelling of strongly curved bends

Abstract

The flow in curved open channels has a three-dimensional structure. Although detailed three-dimensional (3D) flow solvers exist, the associated computational resources and memory requirements are too large for most practical applications. Therefore river engineers and scientists often resort to depth- or cross-section-averaged models, complemented with parameterizations for the 3D processes. Such enhanced depth-averaged models are referred to as quasi-3D models whereas enhanced cross-section-averaged models are referred to as 1DH+ models. Such models have the advantage that they are faster than 3D models, but they are less generic.

The velocity perpendicular to the downstream velocity can be described in terms of two main contributions: its depth-averaged contribution (cross-flow) and the vertical deviations from it (secondary flow). The secondary flow is an important 3D flow process which redistributes the streamwise momentum and influences the direction of the bed shear stress which is turn important for determining the direction of the sediment transport. Parameterizations thereof have been proposed and implemented in quasi-3D flow models for many decades. The present chapter improves the existing parameterizations on two essential points.

Firstly, the validity range of existing parameterizations is limited to mildly curved bends, and they are known to considerably overestimate the effects of secondary flow in moderately to strongly curved bends. The present chapter implements a secondary flow parameterization without curvature limitations.

Secondly, existing parameterizations are based on hypotheses that are only
valid on the channel centerline. The effect of the secondary flow on the velocity redistribution is known to depend crucially on how it is parameterized over the width of the channel. The present chapter improves the width-distribution of the secondary flow parameterization.

Comparison of the flow field and the bed shear stress to measured data from laboratory experiments demonstrates that these improvements considerably enhance the predictive capabilities of quasi-3D flow models.

The present chapter also analyses the computational requirements versus the resolved flow processes of different types of flow solvers, including 1DH+, depth-averaged (2DH), quasi-3D, 3D RANS and 3D LES models.

4.1 Introduction

Assessing effectiveness and impacts of river engineering projects such as flood protection schemes, navigation improvement works, as well as the investigation of sediment transport, pollutant dispersion and morphodynamical processes often require long term predictions of the flow field over large domains. The three-dimensional (3D) flow structure in rivers can be solved by 3D computational flow solvers when considering relatively short time and length scales. To model phenomena at the scales typically encountered in river engineering projects, however, depth- and cross-section-averaged models are widely used. Although such models yield a less detailed image of the flow pattern, they are computationally at least an order of magnitude less expensive.

The depth-integration of the flow equations results in a 2DH model which causes a loss of information on 3D effects, for example on the secondary flow. This secondary flow, also called helical flow or spiral flow, is a characteristic feature of curved open-channel flow. It redistributes the velocities, influences the direction of the boundary shear stress, which in turn influences the bed load transport, shapes the channel bed and is an important agent in the spreading and mixing of heat, dissolved and suspended solids, biological species. To remedy this shortcoming of depth-averaged models, the solutions are enhanced by adding parameterizations of 3D effects. An enhanced 2DH model is referred to as a quasi-3D model, whereas an enhanced cross-section-averaged model is referred to as a 1DH+ model.

The parameterization of the secondary flow is generally obtained by considering a simplified momentum balance based on two essential assumptions:

- the curvature is mild; and
- the equations are considered at the channel centreline.

In the mild curvature limit, the parameterization of the secondary flow scales linearly with the ratio between the water depth at the centre-line and
curvature radius of that line. Due to this linear dependency, such parameterizations are also referred to as “linear”. Models including such a linear parameterization of the secondary flow have often been reported in scientific literature (e.g. (Kalkwijk and de Vriend, 1980; Johannesson and Parker, 1989b; Finnie et al., 1999; Lien et al., 1999; Hsieh and Yang, 2003; Duan and Julien, 2005; Duan and Nanda, 2006; Baek and Seo, 2009; Begnudelli et al., 2010; Duan and Julien, 2010; Baghlani, 2012; Song et al., 2012) and are widely applied in engineering software (e.g. Delft3D, iRIC-FastMECH, STREMR HySeD).

Some bend flow simulations have been reported by reduced-order models which neglect the secondary flow altogether (Ye and McCorquodale, 1998; Zarrati et al., 2005; Lai, 2010) and certain models (Jia and Wang, 1999; Duan et al., 2001; Kassem and Chaudhry, 2002; Dulal et al., 2010), iRIC-Nays, Mike 21C, and Telemac-2D/Sisyphe) neglect the influence of secondary flow on the hydrodynamics, but do include the effect of the secondary flow on the bed shear stress direction in the sediment transport by means of a linear parameterization.

In strongly curved channels such linear models tend to overestimate the secondary flow strength (de Vriend, 1981b; Yeh and Kennedy, 1993a; Blanckaert and de Vriend, 2003; Blanckaert, 2009). To resolve this problem a number of non-linear models have been proposed (Jin and Steffler, 1993; Yeh and Kennedy, 1993a,b; Ghamry and Steffler, 2002, 2005; Vasquez et al., 2011) which include non-linear feedback effects. Recently, Blanckaert and de Vriend (2003) developed a non-linear parameterization for the secondary flow at the channel centre-line based on a simplified momentum balance not limited by the mild curvature assumption. The non-linear parameterization can be expressed as a correction factor to the linear one by de Vriend (1977). This implies that reduced order models including the non-linear parameterization can be solved at roughly the same speed as those including a linear parameterization.

As the parameterization of the secondary flow is derived from the flow equation at the centreline, a distribution function is required to extend the it over the cross-section. Kalkwijk and de Vriend (1980), Finnie et al. (1999), Lien et al. (1999), Hsieh and Yang (2003), Duan and Julien (2005), Duan and Nanda (2006), Baek and Seo (2009), Begnudelli et al. (2010), Duan and Julien (2010), Baghlani (2012), Song et al. (2012), Delft3D, iRIC-FastMECH, and STREMR HySeD extend the secondary flow parameterization over the cross-section by substituting local flow quantities. This is not correct, because the assumptions on which the secondary flow parameterization is based, do not hold near the banks. Imposing such a lateral distribution in channels with steep banks leads to an unphysical representation of the secondary flow as momentum is transferred through the bank.

In the present chapter a novel quasi-3D model is presented which includes a non-linear parameterization of the secondary flow, and includes a more re-
alistic distribution over the width of the channel. The model’s performance (accuracy and computational time) are evaluated through a comparison with a range on models (the non-linear 1DH+ model by Blanckaert and de Vriend (2010), a 2DH model lacking a secondary flow closure, a linear quasi-3D model, a coarse Reynolds-averaged Navier Stokes (RANS) model with a linear k-ε closure, a detailed RANS model with a linear k-ε closure and a detailed Large eddy simulation (LES) by van Balen et al. (2010a,b)) and two strongly curved experiments performed by Blanckaert (2009, 2010); Blanckaert et al. (2012a) with a horizontal and developed bathymetry, respectively. Finally, it is discussed which of the above mentioned models is best suited for the spatial and temporal scales involved in long-term river modelling.

4.2 Model description

4.2.1 Depth-averaged hydrodynamic model

Integrating the Reynolds-averaged Navier Stokes equations from the bed, \( z_b \), to the water surface, \( z_s \), under the assumption of negligible vertical accelerations, yields the depth-averaged flow equations. In a cylindrical reference system (see Fig. 4.1), with the \( s \)-axis along the channel centerline, the \( n \)-axis perpendicular to it to the left and vertical \( z \)-axis, and when ignoring the lateral shear stress terms, they read (cf. Jin and Steffler (1993); Dietrich and Whiting (1989);
Blanckaert and de Vriend (2003):

$$\frac{\partial h}{\partial t} + \frac{1}{1 + n/R} \frac{\partial h}{\partial s} + \frac{\partial h}{\partial n} + \frac{1}{1 + n/R} \frac{h}{R} = 0 \quad (4.1)$$

$$\frac{\partial \langle v_s \rangle}{\partial t} + \frac{1}{1 + n/R} \frac{\partial h}{\partial s} + \frac{\partial h}{\partial n} + \frac{1}{1 + n/R} \frac{h}{R} \langle v_n \rangle = \frac{gh}{1 + n/R} \frac{\partial z_s}{\partial s} - \tau_{bs} \rho \quad (4.2)$$

$$\frac{\partial \langle v_n \rangle}{\partial t} + \frac{1}{1 + n/R} \frac{\partial h}{\partial s} + \frac{\partial h}{\partial n} + \frac{1}{1 + n/R} \frac{h}{R} \langle v_s v_n \rangle = \frac{gh}{1 + n/R} \frac{\partial z_n}{\partial n} - \tau_{bn} \rho \quad (4.3)$$

where $\langle \rangle$ denotes the depth averaging operation and an overbar denotes the ensemble averaging operation. The centreline curvature $R(s)$ is negative for left turning bends and positive for right turning bends; $h = z_s - z_b$ is the local flow depth; The bed shear stresses in transverse and streamwise direction are given by $\tau_{bn}$ and $\tau_{bs}$.

The local instantaneous velocity can be decomposed as follows:

$$v_j(t) = U_j + v_j^* + v'_j(t) \quad (j = (s, n)) \quad (4.4)$$

where $U_j = \langle v_j \rangle$ represents the depth averaged velocity, $v_j^*$ the vertical deviations from it and $v'_j$ the difference of the velocity with the ensemble-averaged velocity component. Inserting the velocity decomposition (4.4) into the equations (4.1)-(4.3) yields (cf. Jin and Steffler (1993); Dietrich and Whiting (1989); Blanckaert and de Vriend (2003)):

$$\frac{\partial h}{\partial t} + \frac{1}{1 + n/R} \frac{\partial hU_s}{\partial s} + \frac{\partial hU_n}{\partial n} + \frac{1}{1 + n/R} \frac{hU_n}{R} = 0 \quad (4.5)$$
\[
\begin{align*}
\frac{\partial U_s}{\partial t} + \frac{1}{1 + n/R} \frac{\partial hU_s^2}{\partial s} + \frac{\partial hU_sU_n}{\partial n} + \frac{1}{1 + n/R} \frac{hU_sU_n}{R} &= \frac{gh}{1 + n/R} \frac{\partial z_s}{\partial s} - \frac{\tau_{bs}}{\rho} \\
&- \frac{1}{1 + n/R} \frac{\partial h\langle v_s^* v_n^* \rangle}{\partial s} - \frac{\partial h\langle v_s^* v_n^* \rangle}{\partial n} - \frac{1}{1 + n/R} \frac{h\langle v_s^* v_n^* \rangle}{R} \\
&- \frac{1}{1 + n/R} \frac{\partial h\langle v_s^2 \rangle}{\partial s} - \frac{\partial h\langle v_s^2 \rangle}{\partial n} - \frac{1}{1 + n/R} \frac{h\langle v_s^2 \rangle}{R} \\
\frac{\partial U_n}{\partial t} + \frac{1}{1 + n/R} \frac{\partial hU_sU_n}{\partial s} + \frac{\partial hU_n^2}{\partial n} + \frac{1}{1 + n/R} \frac{h(U_n^2 - U_s^2)}{R} &= \frac{gh}{1 + n/R} \frac{\partial z_n}{\partial n} - \frac{\tau_{bn}}{\rho} \\
&- \frac{1}{1 + n/R} \frac{\partial h\langle v_n^* v_n^* \rangle}{\partial s} - \frac{\partial h\langle v_n^* v_n^* \rangle}{\partial n} - \frac{1}{1 + n/R} \frac{h\langle v_n^* v_n^* \rangle - \langle v_s^* v_n^* \rangle}{R} \\
&- \frac{1}{1 + n/R} \frac{\partial h\langle v'_s v'_n \rangle}{\partial s} - \frac{\partial h\langle v'_s v'_n \rangle}{\partial n} - \frac{1}{1 + n/R} \frac{h\langle v'_s v'_n \rangle - \langle v'_s v'_n \rangle}{R}
\end{align*}
\] (4.6) (4.7)

The cross terms \(\langle v_s^* v_n^* \rangle\), \(\langle v_n^* \rangle\) and \(\langle v_s^* \rangle\) are known as dispersion terms. In the 2DH shallow water equations these terms are neglected. \(\langle v_s^* v_n^* \rangle\) describes the advective transport of downstream momentum by secondary flow and it is the dominant 3D effect with respect to the redistribution of the velocity (Johannessen and Parker, 1989b; Blanckaert and Graf, 2004). \(\langle v_s^2 \rangle\) is a measure of the secondary flow strength. \(\langle v_n^* \rangle\) is about constant and its variation can generally be neglected (Olesen, 1987). The parameterization of these dispersion terms will be discussed in the following section.

The terms \(\langle v_s^2 \rangle\), \(\langle v'_s v'_n \rangle\) and \(\langle v_n^2 \rangle\) represent the depth-averaged Reynolds stresses. These are often also neglected or parameterized in the 2DH shallow water equations. In the present chapter, they are modelled using a Boussinesq hypothesis based on gradients in \(U_n\) and \(U_s\). The eddy viscosity is taken as \(\nu_t(s,n) = \kappa u_s h/6\). The depth-averaged Reynolds stresses are often neglected as their contribution is generally less important than that of the dispersion terms (Flokstra, 1976; Blanckaert, 2010).

### 4.2.2 Terms requiring parameterization

The dispersion terms \(\langle v_s^* v_n^* \rangle\), \(\langle v_n^* \rangle\) and \(\langle v_s^* \rangle\) and the bed shear stress components \(\tau_{bn}\) and \(\tau_{bs}\) need to parameterized, i.e. they need to be expressed as a function of the dependant variables \(U_s\), \(U_n\) and \(h\) in order to make the system of three equations solvable. There are basically two ways in which the dispersion terms and the bed shear stresses can be modelled. One way is to derive extra
4.2. MODEL DESCRIPTION

equations from the Navier-Stokes equations from which these quantities can be computed. Jin and Steffler (1993); Yeh and Kennedy (1993a,b); Ghamry and Steffler (2002, 2005); Vasquez et al. (2011) for example, take the first moment of the Navier-Stokes in order to derive a set of extra equations. The other way is to use a 1D vertical model in the channel axis, from which velocity profiles can be computed based on a simplified momentum balance. Engelund (1974); Kikkawa et al. (1976); de Vriend (1977); Johannesson and Parker (1989a); Blanckaert and de Vriend (2003) follow this approach. It allows computing the velocity profiles in advance and not during the simulation. The terms requiring parameterization are subsequently expressed as analytical functions of the dependent variables. For this reason, the second method is expected to be computationally less expensive as it does not require the solution of a larger number of coupled equations. Our investigation will therefore focus on the second method.

The spatial variations of the velocity from the mean velocity can be written as:

\[ v^*_s = U_s(s, n) f_s(z) \]  
\[ v^*_n = U_n(s, n) f_s(z) + U_s(s, n) \frac{H(s, n_c)}{R(s, n_c)} f_n(z) f_w(n) \]  

where \( R \) is the geometric radius of curvature \( f_s \) describes the normalized vertical profile of the downstream velocity, \( f_n \) describes a normalized secondary profile function (cf. Jansen et al., 1979), \( f_w \) describes a distribution function extending the strength of the secondary flow over the width of the channel. When setting \( f_n = 0 \) and \( f_s = 0 \), equations (4.5)-(4.7) become the 2DH shallow water equations. The terms needing parameterization can subsequently be evaluated as:

\[ \langle v^*_s^2 \rangle = U_s^2 \langle f_s^2 \rangle = U_s^2 I_{ss}, \]  
\[ \langle v^*_s v^*_n \rangle = U_n U_s \langle f_s^2 \rangle + U_s^2 \frac{H}{R} \langle f_s f_n \rangle f_w, \]  
\[ \langle v^*_n^2 \rangle = U_n^2 \langle f_s^2 \rangle + 2 U_s U_n \frac{H}{R} \langle f_s f_n \rangle f_w + U_s^2 \frac{H^2}{R^2} \langle f_n^2 \rangle f_w^2, \]  
\[ \frac{\tau_{bn}}{\tau_{bs}} = \frac{\tau_{bs}^*}{\tau_{bs}} + \frac{U_n}{U_s} = \alpha \frac{H}{R} f_w + \frac{U_n}{U_s} = I + \frac{U_n}{U_s} \]  
\[ \|\tau_{bs}\| = \psi C f_0 U_s^2 \]  

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The parameterization of the dispersion terms and the bed shear stress direction and magnitude now requires evaluation of the terms $I_{ss}$, $I_{sn}$, $I_{nn}$, $I_{\tau}$ and $\psi$. They are computed in three basic steps:

- the quantities are computed on the centreline for flow of an infinite bend of constant curvature, thus ignoring the distance required for the flow profiles to adapt to changes in curvature;
- a distribution function $f_w(n)$ is prescribed, which extends the computed values from the centreline over the width of the channel; and
- the streamwise adaptation of the quantities is included by means of a scalar transport equation.

4.2.2.1 Evaluation at the centreline for infinitely long bends

The streamwise and secondary flow profiles at the centreline can be computed by considering a simplified momentum balance at the centre of the channel. When taking a mild-curvature approximation, this leads to a vertical profile for the streamwise velocity similar to that found in straight channel flow (e.g. logarithmic, $1/7^{th}$ power; cf. 1987). The secondary flow results from the basic mechanism shown in Figure 4.2a,b. Whereas the outward centrifugal acceleration and the centripetal pressure gradient due to the tilting of the water surface (superelevation) are on average in equilibrium, their local disequilibrium - schematically indicated in gray in Figure 4.2 - generates the secondary flow. Mild curvature parameterizations for infinitely long bends of constant curvature (van Bendegom, 1947; Rozovskii, 1957; Engelund, 1974; Ikeda, 1975; de Vriend, 1977; Johannesson and Parker, 1989a; Odgaard, 1989) are referred to as linear and will be identified with the subscript 0.

In the present study the linear parameterization proposed by de Vriend (1977) is considered in which the quantities $I_{ss,0} = f(C_f)$, $I_{sn,0} = f(C_f)H/R$, $I_{nn,0} = f(C_f)H^2/R^2$, $I_{\tau} = f(C_f)H/R$ uniquely depend on the friction factor $C_f$, and on the curvature ratio $H/R$. (cf. normalized solutions shown in Figure 4.3a). The linear parameterization does not model the increase of the bed shear stress (i.e. $\psi_0 = 1$). Blanckaert and de Vriend (2003) presented a non-linear parameterization not limited by mild curvature assumptions. It includes a non-linear feedback mechanism between the streamwise and transverse velocity profiles. Due to advective momentum transport by the secondary flow the downstream velocity profile flattens, which means that $f_s$ decreases in the upper part of the water column and increases in the lower part of the water column. This reduces the non-uniformity of the centrifugal acceleration and limits the strength of the secondary flow (de Vriend, 1981b; Jin and Steffler, 1993; Yeh and Kennedy, 1993a; Blanckaert and Graf, 2004). This effect is schematically indicated in Figure 4.2c. Blanckaert and de Vriend (2003)’s
4.2. MODEL DESCRIPTION

**Linear closure submodel**

\[ \frac{v_t^2}{R} + \frac{1}{\rho} \frac{\partial p}{\partial n} = \frac{\partial}{\partial z} \left( v_t \frac{\partial v}{\partial z} \right) \]

**Non-linear closure submodel**

Figure 4.2: Basic forcing mechanisms underlying the centre region-cell at the centreline (n=0), and its modelling based on a) a simplified momentum balance, showing that the local imbalance between the centrifugal force and the hydrostatic pressure gradient gives rise to a shear stress gradient which leads to a secondary flow velocity. This forcing mechanism is displayed graphically according to the b) linear and c) non-linear closure submodels.
CHAPTER 4. QUASI-3D HYDRODYNAMICS OF STRONGLY CURVED BENDS

Figure 4.3: a) Linear model solution of the closure variables $\langle f_s f_s \rangle_0$, $\langle f_s f_n \rangle_0$, $\langle f_n^2 \rangle_0$ and $\alpha_{\tau,0}$, b) non-linear-model correction factors $\langle f_s f_s \rangle_\infty / \langle f_s f_s \rangle_0$, $\langle f_s f_n \rangle_\infty / \langle f_s f_n \rangle_0$, $\langle f_n^2 \rangle_\infty / \langle f_n^2 \rangle_0$ and $\alpha_{\tau,\infty} / \alpha_{\tau,0}$ to apply to the linear model solutions indicating the bend averaged values c) the shear stress amplification factor $\psi_\infty$ (courtesy Blanckaert and de Vriend (2003)).
4.2. MODEL DESCRIPTION

Figure 4.4: Different width distribution functions \( f_w \) depending on the coefficient \( n_p \) for a) a horizontal and b) a developed bathymetry for \( B/R \approx 0.76 \).

non-linear model, denoted by the \( \infty \) subscript, yields the deformed vertical profiles of \( f_s \) and \( f_n \), which enable the computation of the quantities \( I_{ss,\infty} \), \( I_{sn,\infty} \) and \( I_{nn,\infty} \), \( I_{\tau,\infty} \) and \( \psi_{\infty} \). The deviation of \( I_{ss,\infty}, I_{sn,\infty} \) and \( I_{nn,\infty}, I_{\tau,\infty} \) from their linear equivalents can be reduced to semi-heuristic correction factors based on the ratio of the non-linear solutions and the linear ones (e.g. \( I_{sn,\infty} = I_{sn,0}\langle f_sf_n\rangle_{\infty}/\langle f_sf_n\rangle_0 \) for infinitely long bends of constant curvature. These corrections factors (e.g. \( \langle f_sf_n\rangle_{\infty}/\langle f.sf.n\rangle_0 \), cf. Figure 4.3b) depend uniquely on the so-called bend parameter defined by:

\[
\beta = (C_f)^{-0.275}(H^2/R)^{0.25}(\alpha_s/R + 1/R)^{0.25}.
\]  

(4.15)

The parameter \( \beta \) combines the influences of the friction factor \( C_f \), the curvature ratio \( H/R \) and the parameter

\[
\alpha_s/R = [\partial U_s/\partial n/(U_s)]_{n_c}
\]

(4.16)

where \( n_c \) is the centreline \( n \)-coordinate. This parameter is representative of the cross-stream distribution of \( U_s \). The solution of the shear stress amplification factor \( \psi_{\infty} \) which parameterizes the increased energy losses is shown in Figure 4.3c. The correction factors are evaluated in each cross-section from the values of \( C_f, H/R \) and \( \alpha_s/R \), evaluated at the centerline. The parameter \( \alpha_s/R \) is especially important: it depends on the to-be-computed flow field, and thus provides for a dynamical coupling between the 2DH flow model and the parameterization for 3D effects.

4.2.2.2 Extension over the width of channel

Most quasi-3D models (e.g. (Koch and Flokstra, 1980; Jin and Steffler, 1993) extend the secondary flow \( v_n^s \) distribution (4.9) simply by replacing the values
for the flow depth $H$ and the radius of curvature on the centerline $R$ by their local values, $h(s, n)$ and $R + n$. This yields the following distribution over the width:

$$f_w(n) = \frac{1}{1 + \frac{n}{R}} \frac{h(n)}{H}$$

These extensions do not represent the secondary flow strength distribution correctly as the assumptions underlying the secondary flow closure model are no longer satisfied away from the centreline (e.g. $v_z \ll v_n$). In channels with steep banks, for example, they would erroneously predict that momentum be transferred through the banks, which would violate the principle of momentum conservation. Johannesson and Parker (1989b) have pointed at the importance of the width-distribution of the closure variables with respect to the velocity (re)distribution and especially to the importance of having $f_w$ tend to zero at the banks. It is not the value of $\langle v^*_s v^*_n \rangle$, predicted by the closure submodel, that redistributes the velocity, but rather its transversal gradient, $\partial \langle v^*_s v^*_n \rangle/\partial n$, which is the principal redistribution term in the 2DH momentum equations (cf. Eq. (4.6)). In addition, the experiments by (Blanckaert, 2009, 2010) reveal that the secondary flow strength tends to zero near the banks and has its maximum in the centre of the channel. For this reason, a power law extension to equation (4.17) is presented:

$$f_w(n) = \frac{1}{1 + \frac{n}{R}} \frac{h(n)}{H} \left[ 1 - \left( \frac{2n}{B} \right)^{2n_p} \right]$$

where $n_p$ is a positive integer. When setting $n_p = 0$, the erroneous distribution (4.17) is found. All positive values of $n_p > 0$ provide a width distribution with a maximum in the centre of the cross-section and zero at both banks (cf. Figure 4.4). These width-distribution functions lead to a decrease of the velocities in the inner part of the cross-section and an increase in the outer part. Varying the value of $n_p$ allows investigating the sensitivity to the shape of the distribution function.

### 4.2.2.3 Streamwise adaptation

Neglecting the streamwise lag-effect of the parameterizations is equivalent with assuming that the secondary flow adapts instantaneously to changes in curvature. In reality, this requires a certain distance, and flow in equilibrium with the imposed curvature only occurs in sufficiently long bends of constant curvature. The streamwise adaptation of the vertical flow structure to curvature changes is described by means of a scalar transport model, in which the secondary flow (and other related quantities) is advected by the depth-averaged
velocity (cf. Jagers (2003)). In the cylindrical reference system this transport equation is written as:

\[
\frac{1}{1 + n/R} \frac{\partial hU_s Y}{\partial s} + \frac{\partial hU_n Y}{\partial n} + \frac{1}{1 + n/R} \frac{hU_n Y}{R} = \frac{hU}{\lambda} (Y_e - Y),
\]

where \(Y\) denotes each of the quantities \(I_{ss}, I_{sn}, I_{nn}, I_\tau,\) or \(\psi\). The value \(Y_e\) is the source term of this equation, \(\lambda\) is the adaptation length as described by Johannesson and Parker (1989a). The source term \(Y_e\) is taken equal to the value of \(Y\) in fully developed flow, i.e. \(I_{sn,0}\) when considering the linear model by de Vriend (1977) and \(I_{sn,\infty}\) when the non-linear parameterization by Blanckaert and de Vriend (2003) is used. This streamwise adaptation model expresses that the solution \(Y\) lags behind its equilibrium value \(Y_e\), whereby the adaptation length determines how strong this spatial lag effect is. de Vriend (1981b); Kalkwijk and Booij (1986) report that the bed shear stress adapts faster to changes in curvature than the secondary flow, however assuming the same adaptation length for both has hardly any influence on the adaptation of the secondary flow. As the secondary flow is the dominant included quantity, in this chapter the same adaptation length is assumed for all of the quantities.

Similar models have been proposed in the literature, mainly differing in their definition of the adaptation length (Rozovskii, 1957; de Vriend, 1981b; Kalkwijk and Booij, 1986; Ikeda and Nishimura, 1986; Olesen, 1987; Odgaard, 1989; Johannesson and Parker, 1989a). Neglecting the streamwise adaptation of 3D effects (Lien et al., 1999; Hsieh and Yang, 2003; Duan and Julien, 2005; Duan and Nanda, 2006; Begnudelli et al., 2010; Duan and Julien, 2010; Song et al., 2012, CCHE2D, and iRIC-FastMECH), implies an overestimation of the secondary flow effects in regions of increasing absolute curvature and vice-versa in regions of decreasing absolute curvature.

In summary, the key property of our quasi-3D model is that is not limited to mildly curved bends (contrary to most existing models) and it improves a commonly used erroneous distribution of the parameterization quantities over the cross-section.

### 4.3 Experiments and model setup

#### 4.3.1 Investigated strongly-curved flume experiments

Typically, bend curvature is expressed by the ratio of the radius of curvature at the bend apex to the channel width \(R_{ap}/B\) (e.g Hickin and Nanson, 1975; Blanckaert, 2011). Meandering channel bends with \(R_{ap}/B\) smaller than approximately 2 are considered to be strongly curved. Blanckaert (2011) reported naturally occurring curved meander bends having \(R_{ap}/B\) values ranging be-
Figure 4.5: Laboratory flume and developed bed topography, referred to average bed level [cm]; (b) width averaged bed-elevation [cm] and (c) transverse bed slope angle [°].
Table 4.1: Hydraulic and geometric properties of the measured bends. $\bar{H}$ is the flume averaged water depth, $Q$ is the flow discharge, $B$ is the channel width, $R_{ap}$ is the radius of curvature at the bend apex, $\bar{U}$ is the flume averaged bulk velocity, $C_f$ is the dimensionless friction factor, $R_{ap}/B$ represents the radius to width ratio, $R_{ap}/\bar{H}$ represents the radius to depth ratio.

| Case            | $\bar{H}$ [m] | $Q$ [$m^3s^{-1}$] | $B$ [m] | $|R_{ap}|$ [m] | $|\bar{U}|$ [ms$^{-1}$] | $C_f^{-0.5}$ | $B/|R_{ap}|$ [-] | $H/|R_{ap}|$ [-] |
|-----------------|---------------|------------------|---------|---------------|-----------------|-------------|----------------|----------------|
| Horizontal bed  | 0.159         | 0.089            | 1.3     | 1.7           | 0.43            | 13.2        | 0.75           | 0.094          |
| Developed bed   | 0.141         | 0.089            | 1.3     | 1.7           | 0.49            | 8.9         | 0.75           | 0.083          |

between 0.8 and 1.7. In our investigation two experiments are chosen with an $R_{ap}/B$ value of 1.3, so falling within this range.

Secondary flow is known to be an important agent in the redistribution of the streamwise velocity in curved open channels [Blanckaert and de Vriend (2010), Chapter 2]. The first experiment by Blanckaert (2009) (also see Blanckaert et al., 2012a) was done in a rectangular cross-section. In this case secondary flow is even the dominant mechanism for streamwise momentum redistribution. The experiment can therefore be considered as an extreme testcase for the developed numerical model. A similar configuration could be encountered in a man-made channel. The flume consists of a 9 m long straight inflow reach followed by a 193° bend with a constant centerline radius of curvature of $R = 1.7$ m and a 5 m long straight outflow reach (cf. Figure 4.5a). The width is $B = 1.3$ m and the vertical banks are made of Plexiglass. The bed was composed of nearly uniform sand with diameters in the range $1.6 \text{ mm} < D < 2.2 \text{ mm}$. The horizontal bed was fixed by spraying paint on it, in such a way that grain roughness was preserved. At the entrance of the flume a constant water discharge of 89 l/s was imposed and at the downstream end a tail gate was used to keep the water level constant.

The second experiment concerned flow over a developed bed topography Blanckaert (2010), with similar hydrodynamic conditions. This experiment is considered representative of strongly curved open channel bends and is therefore a suitable test case to test the model’s ability to reproduce the flow in strongly curved naturally occurring channels. The experiment was performed in the same flume as the previous one, only in this case the bed was no longer horizontal. Starting from the horizontal (unfixed) bed, a constant water discharge of 89 l/s and constant sediment discharge of 0.023 kg/(ms) were fed into the flume, while keeping the water level at the tailgate constant. After the bed level had reached a dynamic equilibrium state, in which the large scale bed features were steady and small migrating dunes passed through the domain, the bed level was fixed by spraying paint on it. A pronounced point bar at the inner bend and pool at the outer bend are characteristic features of the result-
Table 4.2: Description of included processes and grid details of the various numerical models. The superscript * indicates that the process is included by 3D calculation.

<table>
<thead>
<tr>
<th>Model description</th>
<th>(I_{ss}) included</th>
<th>(I_{sn}) included</th>
<th>(I_{nn}) included</th>
<th>Width dist.</th>
<th>Grid details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1DH+</td>
<td>N</td>
<td>Y, non-linear</td>
<td>N</td>
<td>–</td>
<td>260 1 1</td>
</tr>
<tr>
<td>2DH</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>–</td>
<td>155 23 1</td>
</tr>
<tr>
<td>Q3D-L (A1)</td>
<td>N</td>
<td>Y, linear</td>
<td>N</td>
<td>1</td>
<td>155 23 1</td>
</tr>
<tr>
<td>Q3D-NL (A0)</td>
<td>N</td>
<td>Y, non-linear</td>
<td>N</td>
<td>0</td>
<td>155 23 1</td>
</tr>
<tr>
<td>Q3D-NL (A1)</td>
<td>N</td>
<td>Y, non-linear</td>
<td>N</td>
<td>1</td>
<td>155 23 1</td>
</tr>
<tr>
<td>Q3D-NL (A2)</td>
<td>N</td>
<td>Y, non-linear</td>
<td>N</td>
<td>2</td>
<td>155 23 1</td>
</tr>
<tr>
<td>Q3D-NL (A3)</td>
<td>N</td>
<td>Y, non-linear</td>
<td>N</td>
<td>3</td>
<td>155 23 1</td>
</tr>
<tr>
<td>Q3D-NL (B1)</td>
<td>N</td>
<td>Y, non-linear</td>
<td>Y, non-linear</td>
<td>1</td>
<td>155 23 1</td>
</tr>
<tr>
<td>Q3D-NL (C1)</td>
<td>Y, non-linear</td>
<td>Y, non-linear</td>
<td>Y, non-linear</td>
<td>1</td>
<td>155 23 1</td>
</tr>
<tr>
<td>RANS (k-\epsilon) (4)</td>
<td>Y*</td>
<td>Y*</td>
<td>Y*</td>
<td>*</td>
<td>155 23 4</td>
</tr>
<tr>
<td>RANS (k-\epsilon)</td>
<td>Y*</td>
<td>Y*</td>
<td>Y*</td>
<td>*</td>
<td>155 23 20</td>
</tr>
<tr>
<td>LES (horizontal)</td>
<td>Y*</td>
<td>Y*</td>
<td>Y*</td>
<td>*</td>
<td>1260 192 24</td>
</tr>
<tr>
<td>LES (developed)</td>
<td>Y*</td>
<td>Y*</td>
<td>Y*</td>
<td>*</td>
<td>1248 192 72</td>
</tr>
</tbody>
</table>

The hydraulic conditions of both experiments have been summarized in Table 4.1. \(H\) and \(\bar{U} = Q/(B\bar{H})\) are the flume-averaged water depth and bulk velocity. The friction factor has been estimated from the elevation of the water surface above the averaged bed level measured at the centerline, as \(C_f^{0.5} = \bar{U}/(gR_hS_s)^{0.5}\), where \(R_h\) is the overall mean hydraulic radius.

### 4.3.2 Model set-up

To simulate and analyse the flow in the two strong curvature bend setups described in Section 4.3.1 different numerical models of varying detail were set up (see Table 4.2):

- the cross-section averaged model developed by Blanckaert and de Vriend (2010) (1DH+) including a non-linear parameterization of the secondary flow,
- a depth averaged model (2DH)
- a linear quasi-3D model (Q3D-L)
- various non-linear quasi-3D models (Q3D-NL)
4.3. EXPERIMENTS AND MODEL SETUP

- two RANS models, and
- two large eddy simulations performed by van Balen et al. (2010a,b)

The 1DH+ model consisted of a one-dimensional grid of 260 cells. The depth-averaged 2DH, Q3D-L and Q3D-NL and the RANS simulations were all run using a research version of Delft-3D on the same horizontal grid. The detailed RANS models consisted of 20 layers in the vertical following a sigma layer approach (Stelling and van Kester, 1994). The coarse RANS k-\( \epsilon \) (4) model used 4 layers in the vertical and the large eddy simulation used a much finer grid. An overview of the grid dimensions is given in Table 4.2. In the quasi-3D models the letter in brackets A,B,C refers to the level of inclusion of the dispersion stresses (cf. Table 4.2), whereas the number refers to the choice of \( n_p \) in the cross-stream distribution of the secondary flow \( f_w \).

The bed roughness is expressed as a Nikuradse roughness height of 6 mm, which is roughly three times the mean sediment diameter at the bed (van Rijn, 1984a). The same roughness was also used by Zeng et al. (2008a) and van Balen et al. (2010a,b) The smooth Plexiglas sidewalls were modelled as a free-slip boundaries in the 2DH, quasi-3D and RANS k-\( \epsilon \) models as the chosen grid resolution was not fine enough to capture the boundary layer. At the downstream boundary a water level was prescribed and at the upstream boundary a total discharge of 89 l/s was imposed.

4.3.3 Model sensitivity

Before proceeding to the comparison of the different models and their accuracy when applied to strongly curved bend flow, two sensitivity tests were performed. These tests show the sensitivity of the redistribution of the stream-wise velocity in the non-linear quasi-3D model to (i) the distribution of the secondary flow over the cross-section and to (ii) the level of inclusion of the dispersion stresses. This section solely considers the different possible settings in the quasi-3D model. The flow redistribution and the underlying mechanisms will be further discussed in sections 4.4.1 and 4.4.2.

Figure 4.6a,b show the sensitivity of the flow redistribution of the quasi-3D model to the choice of \( n_p \) in distribution function of the secondary flow \( f_w \) (cf. (4.17) for the horizontal bed and developed bathymetry experiments. The number in the legend of each panel denotes the value of \( n_p \) (e.g. in Q3D-NL (A1), \( n_p = 1 \) which corresponds to an almost parabolic distribution function over a horizontal bathymetry, see also Figure 4.4 and Table 4.2). The results show that for \( n_p = 0 \) a different behaviour can be observed compared to when \( n_p > 0 \), but also that the model is not very sensitive to the choice of \( n_p \) once this parameter has a non-zero value. The difference between the model predictions when considering \( n_p = 0 \) or \( n_p > 0 \) will be discussed in further detail in the next sections.
Figure 4.6: Sensitivity of the flow redistribution, as parameterized by the normalized transverse gradient of the depth-averaged streamwise velocity $\alpha_s/R$ (cf. Eq. (4.16)), (i) to the width distribution function $f_w$ over the a) horizontal bed and the b) developed bed topography; and (ii) to the level of inclusion of the parameterizations $I_{ss}, I_{sn}$ and $I_{nn}$ over the c) horizontal bed and the d) developed bed topography.
4.4. RESULTS

In mildly curved channels it is known that the cross-term quantity $I_{sn}$ is dominant in the streamwise momentum redistribution (Olesen, 1987; Johannesson and Parker, 1989b). As the flow in strongly curved bends is considered, the sensitivity of the streamwise momentum redistribution to the level of inclusion of the dispersion stresses $I_{ss}$, $I_{sn}$ and $I_{nn}$ in the quasi-3d model is evaluated in Figure 4.6c and d, for the horizontal and developed bathymetry respectively. The simulation Q3D-NL(A1) considers $I_{sn}$ only, Q3D-NL(B1) $I_{sn}$ and $I_{nn}$, and Q3D-NL(C1) all three dispersion stress terms $I_{ss}$, $I_{sn}$ and $I_{nn}$ (cf. Table 4.2). Figure 4.6c,d shows that the flow redistribution in strongly curved bends is not sensitive to the inclusion of $I_{ss}$ and $I_{nn}$ and therefore only the option including $I_{sn}$ will be considered for the remainder of the chapter.

4.4 Results

4.4.1 Flow redistribution

4.4.1.1 Horizontal bed

Figure 4.7a,b shows the evolution of the depth-averaged streamwise velocity over the horizontal bed as measured (Blanckaert, 2009; Blanckaert et al., 2012a) and as resulting from the large eddy simulation (LES) by van Balen et al. (2010a). The results agree well with one another. Both the ADVP-data and the LES results are considered, since the measured data are not available over the whole flume and ADVP-measurements are known to show errors close to the boundaries (free surface, side walls and bed, see Blanckaert, 2010). Upstream of the bend the depth-averaged velocity is uniform over the width. At the entrance of the bend, the fluid accelerates near the inner bank and decelerates the near the outer, yielding to the so-called potential vortex distribution (e.g. Vardy, 1990). Passing through the bend the streamwise momentum is redistributed in outward direction. At the bend exit, a behaviour similar but opposite to that at the bend entrance can be observed: the fluid near the outer bank accelerates and decelerates near the inner bank.

Figure 4.7c,d show the depth-averaged streamwise velocity at the measured cross-sections and the streamwise evolution of the normalized transverse gradient in the streamwise velocity over the horizontal bed $\alpha_s/R$ (cf. Eq. (4.16)). The detailed RANS simulations agree well with the measured values and the LES. The RANS simulation with only four sigma layers in the vertical slightly underestimates the outward distribution of streamwise momentum. The depth averaged model (2DH) underestimates the streamwise development of the transverse gradient through the flume. The linear quasi-3d model (Q3D-L(A1)) with a parabolic secondary flow distribution ($n_p = 1$) overestimates the outward transport of streamwise momentum through the flume, whereas the
CHAPTER 4. QUASI-3D HYDRODYNAMICS OF STRONGLY CURVED BENDS

Figure 4.7: Depth-averaged streamwise velocities over the horizontal bed from a) measurements, b) large eddy simulation (LES), c) evaluated at the cross-sections and d) streamwise evolution of the normalized transverse gradient of the depth-averaged streamwise velocity $\alpha_s/R$ (cf. Eq. (4.16)).
non-linear quasi-3d model Q3D-NL(A1) with a parabolic secondary flow distribution ($n_p = 1$) and the non-linear 1DH+ model by Blanckaert and de Vriend (2010) show reasonable agreement with the measurements. The non-linear quasi-3d model Q3D-L(A0) with an erroneous distribution function of the secondary flow ($n_p = 0$) shows that most of the streamwise momentum stays near the inner bank as it passes through the bend.

4.4.1.2 Developed bed

Figure 4.8a,b shows the evolution of the depth-averaged streamwise velocity over the developed as measured by Blanckaert (2010) and as computed with LES by van Balen et al. (2010b), respectively. For the greater part of the flume the LES-results and ADVP-measurements agree well, although locally some differences occur. Upstream of the bend the depth-averaged velocity is uniform over the width. At the entrance of the bend, the fluid accelerates near the inner bend and decelerates near the outer bank. Subsequently at the 60° section the flow decelerates over the shallow point bar near the inner bank and accelerates over the deep pool. Over the shallow zone at the 90° cross-section, the ADVP measurements do not reveal a horizontal recirculation zone, however Blanckaert (2010) showed its presence by means of floating strings and explained that the error in ADVP measurements is likely to be caused by the housing of the ADVP velocimeter which touches the water surface thereby influencing the mean velocity in shallow zones. The presence of the horizontal recirculation zone is corroborated by the LES simulation. Between the 90° and the 150° section the streamwise momentum is gradually redistributed towards the inner bank, after which it gradually moves towards the outer bank between the 150° and the 193° section. At the bend exit, the behaviour is similar to the case with the horizontal bed: the fluid near the outer bank accelerates and the near the inner bank it decelerates. A second horizontal recirculation zone at the bend exit is visible in the LES-results, but not in the measured data. In the straight channel outflow the flow gradually tends to a uniform distribution of the streamwise velocity, i.e. $\alpha_s/R = 0$.

Figure 4.8c,d show the depth-averaged streamwise velocity at the measured cross-sections and the streamwise evolution of the normalized transverse gradient in the streamwise velocity over the developed bed (cf. Eq. (4.16)) as computed by different hydrodynamic models. The value $\alpha_s/R = 2/B$ shown in Figure 4.8d corresponds to a velocity distribution in which the velocity is zero near the inner bank and $2U$ near the outer bank. When $\alpha_s/R > 2/B$ a recirculation zone is expected (cf. Blanckaert and de Vriend, 2010)). The 1DH+ non-linear model overpredicts the inner bank velocity at the 60° and the 90° cross-sections, and is unable to model horizontal recirculations. The decay towards the straight channel limit is not well represented, either. The 2DH model, which does not contain any secondary flow information, shows a good
Figure 4.8: Depth-averaged streamwise velocities over the developed bathymetry bed from a) measurements, b) large eddy simulation (LES), c) evaluated at the cross-sections and d) streamwise evolution of the normalized transverse gradient of the depth-averaged streamwise velocity $\alpha_s/R$ (cf. Eq. (4.16)). The dashed lines indicate the horizontal recirculation zones.
agreement until the $60^\circ$ section, where the transverse bed slope reaches its maximum. The size of horizontal recirculation zone is strongly overestimated. Until the $150^\circ$ section the streamwise velocity is gradually redistributed towards the inner bank. The streamwise flow velocities in Figure 4.8c indicates that the 2DH model still shows negative velocities near the inner bank at this cross-section. Subsequently the outward redistribution of streamwise momentum and the second horizontal recirculation zone after the bend are also overestimated by the 2DH model. In the straight channel outflow the model shows reasonable agreement with the LES simulation. The linear quasi-3D model including a parabolic width distribution (Q3D-L(A1), $n_p = 1$) reasonably captures the strength of the first horizontal recirculation zone. It subsequently underestimates the redistribution towards the inner bank, and finally reasonably captures the transverse gradient near the second recirculation zone. The transverse distribution of the streamwise velocity in Figure 4.8c reveals a zone of high momentum near the outer bank, which deviates strongly from the LES-results. The non-linear model with a parabolic lateral distribution (Q3D-NL(A1), $n_p = 1$) shows good agreement until the first horizontal recirculation zone. It underestimates the redistribution towards the inner bank, yet performs better than the linear model. The strength of the second horizontal recirculation zone is well captured and the subsequent decay in the straight channel outflow shows good agreement with the LES simulation. The non-linear quasi-3D model with an erroneous lateral distribution (Q3D-NL(A0), $n_p = 0$) underestimates the size of the horizontal recirculation zone at the $90^\circ$ section, but reasonably captures the behaviour for the rest of the flume. The 4-layer RANS simulation slightly overestimates the outward transport of streamwise momentum at the $90^\circ$ section and subsequently transports more momentum through the outer half of the bend. The detailed RANS result shows good agreement with the LES simulation. The redistribution towards the inner bend in the second half of the bend is somewhat less than in the LES.

4.4.2 Dispersion stress

The dispersion stress $\langle v_s^* v_n^* \rangle$ is known to be important for the redistribution of streamwise momentum (Johannesson and Parker, 1989b; Blanckaert and de Vriend, 2010). When the cross-stream gradient of the dispersion stress is positive, the streamwise velocity slows down and when it is negative the streamwise velocity increases (cf. Eq. (4.6) and Kalkwijk and de Vriend, 1980). As the cross-stream gradient in the inner and outer halves of the channel should be opposite in sign, this leads to a redistribution of streamwise momentum. Roughly speaking, a negative value of the dispersion stress at the centreline leads to a redistribution of streamwise momentum towards the right bank and
4.4.2.1 Horizontal bed

The distribution of the dispersion stresses over the horizontal bed is given in Figure 4.9a and the value at the centreline is given in Figure 4.9b. The experimentally measured dispersion stress is equal to zero in the straight inflow. At the bend entrance \(\langle v_s^* v_n^* \rangle\) comes into existence, together with the secondary flow \(v_n^*\). Its overall magnitude increases until the 60\(^{\circ}\) cross-section, and subsequently it decreases towards the end of the bend and the straight channel outflow.

Figure 4.9: Dispersion stress \(\langle v_s^* v_n^* \rangle\) (multiplied by -1) distribution and centreline value for horizontal bed topography (a,b) and developed bed topography (c,d).

A positive value leads to a redistribution of streamwise momentum towards the left bank.
4.4. RESULTS

The 2DH simulation has no dispersion stress as no secondary flow is included, which implies that no streamwise momentum is redistributed towards the outer bend. The linear quasi-3d model including a parabolic lateral distribution (Q3D-L(A1), \( n_p = 1 \)) predicts an increase in the magnitude of the dispersion term \( \langle v_s^* v_n^* \rangle \) along the centreline through the bend until the equilibrium value for mildly curved bends is reached. However, since this is a strongly curved bend the magnitude of the dispersion stress is overestimated. The resulting outward transport of streamwise momentum, related to the cross-stream gradient of \( \langle v_s^* v_n^* \rangle \) is thereby also overestimated. The non-linear quasi-3d model including a parabolic lateral distribution (Q3D-NL(A1), \( n_p = 1 \)) shows an increase of the dispersion stress until the 60° section, but as it includes a non-linear feedback mechanism, the dispersion stress decreases in the latter half of the bend. The magnitude at the 60° section is slightly underestimated, but in the rest of the bend the values are similar to those obtained from measurements. As a consequence, the resulting outward transport of streamwise momentum is also slightly underestimated. The non-linear 1DH+ model also slightly underestimates the dispersion stress magnitude at the 60° section, but reasonably captures the non-linear feedback mechanism in the second half of the bend thereby showing a similar trend in outward streamwise momentum redistribution through the bend. The non-linear quasi-3d model with an erroneous lateral distribution (Q3D-NL(A0), \( n_p = 0 \)) tends to overestimate the dispersion stress at the centreline after the 60° section. Considering the cross-stream distribution of the dispersion stress in Figure 4.9a, the distribution is clearly incorrect as it shows large magnitudes near the flume walls. This erroneous cross-stream distribution of dispersion stress in Q3D-NL(A0) also leads to an erroneous redistribution of the streamwise momentum (cf. Figure 4.7d) in which the high streamwise momentum remains at the inner bank. The RANS model shows good agreement with the measured data, but it slightly underestimates the magnitude at the 60° section. Overall the outward transport of streamwise momentum is well modelled by the RANS model, which also captures the non-linear feedback mechanism (cf. Table 4.2). The coarse RANS simulation slightly underestimates the dispersion stress, although the trend matches the behaviour observed in the measurements. The cross-stream transport of streamwise momentum due to secondary flow is thereby also underestimated.

4.4.2.2 Developed Bathymetry

The distribution of the dispersion stresses over the developed bed is given in Figure 4.9c and the value at the centreline is given in Figure 4.9d. The dispersion stress from the LES is equal to zero in the straight inflow. At the bend entrance \( \langle v_s^* v_n^* \rangle \) comes into existence, together with the secondary flow \( v_n^* \). Its overall magnitude increases until the 60° cross-section and its effect on the
momentum redistribution is towards the right (outer) bank and subsequently shows an opposite sign between the 75° and 120° section. Beyond this reach the dispersion stress shows its usual sign again and after the bend between 1 and 3 metres after the bend exit an opposite sign of the dispersion stress is observed again. In regions with a negative sign the momentum redistribution is towards the left (inner bank). This opposite sign of the dispersion stress can be explained by the streamwise velocities which have the major part of its momentum in the bottom half of the water column and the transverse velocities which have the usual orientation (cf. van Balen et al. (2010b)'s Figure 6).

The 2DH model does not include the dispersion stress and is therefore not included in Figure 4.9d. The 1DH+ model does not capture the oscillating trend of the dispersion stress through the bend. The linear quasi-3D model Q3D-L(A1) overestimates the dispersion stress from the entrance of the bend until the 60° section but does capture the oscillating trend of the dispersion stress through the bend. It is however not able to capture the opposite sign of the dispersion stress as the underlying velocity profiles do not have the freedom to shift their maximum to the lower half of the water column. The variation of the dispersion stress is linked to the variation of the streamwise velocity (e.g. the zero value at the 90° cross-section is linked to the zero streamwise velocity which is found at the horizontal recirculation zone). The non-linear quasi-3d model Q3D-NL(A1) shows roughly the same pattern as the linear quasi-3D model, but its magnitude is closer to the value in the LES, due to the non-linear feedback mechanism. The non-linear quasi-3D model is also unable to model the dispersion stress with opposite sign between the 75° and 120° section and between 1 m and 3 m after the bend. Although the parameterization by Blanckaert and de Vriend (2003) can capture to a certain degree the flattening of the streamwise profiles, it lacks the freedom in the velocity profiles to represent streamwise velocity profiles which have the bulk of the streamwise momentum in the bottom half of the water column. The non-linear quasi-3D model with an erroneous secondary flow distribution Q3D-NL(A0) shows roughly the same behavior as Q3D-NL(A1). The detailed RANS model shows roughly the same streamwise evolution of the dispersion stress as the LES and is able to capture the dispersion stress with the opposing sign. The coarse RANS model captures the trend well but overall it underestimates the magnitude of the dispersion stress through the bend.

The role of the dispersion stress is more difficult to read directly from the redistribution of streamwise momentum than in the horizontal bed case. This is because the transport of streamwise momentum is also influenced by variations of the topography, in addition to dispersion stresses and sudden changes in curvature (Blanckaert and de Vriend, 2010). The mechanism related to the flow redistribution caused by variations of the topography, also referred to as “topographic steering”, is responsible for the flow going around rather than
over the point bar. (cf. Nelson, 1988; Dietrich and Whiting, 1989; Blanckaert, 2010).

Between the 0° and the 75° section all models show roughly the same streamwise momentum redistribution (cf. Figure 4.8d). As this region is not influenced by the dispersion stress \( \langle v^*_s v^*_n \rangle \), we may conclude that topographic steering and curvature variations are the dominant mechanisms for streamwise momentum redistribution. Between the 75° section and the 120° section the redistribution of streamwise momentum by secondary flow can be observed. In order of increasing inward redistribution of streamwise momentum by secondary flow we find the LES, the detailed RANS, the coarse RANS, the Q3D-NL(A1) and the Q3D-L(A1) (cf. Figure 4.8d). These correspond to the distribution of the dispersion stress in this region (cf. Figure 4.9d). The non-linear quasi-3D model with the erroneous lateral distribution Q3D-NL(A0) shows about the same dispersion stress magnitude at the centerline through the bend. This is because the transverse gradient of the streamwise velocity parameterized by \( \alpha_s/R \) is mainly related to curvature changes and the topographic steering mechanism which feeds into the non-linear feedback mechanism. The streamwise redistribution shown in Figure 4.8d of the Q3D-NL(A0) is however different from Q3D-NL(A1) and further away from the LES predictions. From the 120° section until the end of the flume the variation of the dispersion stress cannot be clearly linked to the redistribution of streamwise momentum which shows curvature variations and topographic steering are the dominant mechanisms for the redistribution of streamwise momentum.

### 4.4.3 Increase of bed shear stress

Using the simulated and measured data we will now compare the bed shear stresses resulting from the different approaches by analyzing \( \psi \).

#### 4.4.3.1 Horizontal bed

Figure 4.10a shows the evolution of the width-averaged bed shear stress normalized by the width-averaged velocity magnitude squared. The measurements show that in the straight inflow section there is no increase of \( \psi \). Through the bend entrance \( \psi \) increases and finally in the straight outflow section \( \psi \) decreases again. The LES shows a similar behaviour to the measurements. All of the other models capture the behaviour. The RANS k-\( \epsilon \) (4) model and the 1DH+ model both tend to underestimate the increase in \( \psi \). The Q3D-NL(A0) captures the behaviour to certain a degree but it shows less increase than the Q3D-NL(A1) model, which is linked to the erroneous prediction of the redistribution of the streamwise momentum. The linear Q3D-L(A1) and the 2DH model do not include this behaviour.
Figure 4.10: Streamwise evolution of the increase of width-averaged bed shear stress calculated as $\psi = (\int \tau_{bs} dn/B)/(\int U_s U_{tot} dn/B)^2/C_f$ for the a) horizontal bed case and b) the developed bed case. The bed shear stress values $\tau_{bs}$ from measurements were approximated by fitting a logarithmic profile using $k_s = 6$ mm between 1-2 cm (white dots), 2-3 cm (grey dots) respectively. Streamwise evolution of the angle between the bed shear stress and the depth averaged flow direction at the centerline $\alpha \tau H/R$ over c) the horizontal bed and d) over the developed bed. The direction of the bed shear stress from measurements was estimated from measurements by extrapolating the measured velocities between 1-2 cm (white dots), 2-3 cm (grey dots) respectively.
4.4.3.2 Developed bathymetry

Figure 4.10b shows $\psi$ for the case of a developed bed. In this case the measured data show a strong increase of the $\psi$ at the 90° section and at 0.5 m after the bend exit. This local increase of $\psi$ is likely to be partly responsible for the increased depth at the 90° and 1.5 m after the bend (cf. Figure 4.5b). Over the developed bathymetry the LES-results and measured data show good agreement, although the LES shows a higher peak at the 90°. The detailed RANS k-ε model is able to capture this behaviour well. The coarse RANS k-ε (4) model shows a similar behaviour, but tends to underestimate the increase in bed shear stress through the bend. The Q3D-NL models show good agreement for the peak value of $\psi$ at the 90°, but tend to overestimate $\psi$ in the latter half of the bend and do not show the increase of $C_f$ after the bend exit, as $R = \infty$. A similar behaviour is found for the Q3D-NL(A0) and Q3D-NL(A1) models, despite the different distribution functions of the secondary flow, as the non-linear feedback mechanism is mainly determined by curvature changes and topographic steering. The 1DH+ model tends to underestimate the increase in the bed shear stress. The linear Q3D-L(A1) and the 2DH model do not include this behaviour.

4.4.4 Bed shear stress direction

4.4.4.1 Horizontal bed

The tangent of the angle between the mean flow direction and the bed shear stress over the horizontal bed is given in Figure 4.10c. A value of $a_\tau H/R$ equal to one corresponds to an angle between the bed shear and the depth averaged flow direction of 45° directed to the left bank. Both the LES simulation and the measured data show that in the straight inflow section both the bed shear stress and the depth-averaged velocity are in the same direction. After the bend entrance the angle between the two increases to a maximum at around 60° and it subsequently decreases again. This is related to the secondary flow strength. Beyond the bend there is still a residual secondary flow cell which slowly decays and induces a difference between the bed shear stress direction and the depth-averaged velocity direction.

The RANS models, the non-linear quasi-3D models and the 1DH+ model reasonably capture the streamwise evolution of the bed shear stress, but they underestimate the magnitude. The coarse RANS model tends to underestimate the angle of the bed shear stress, due to its lack of resolution in the boundary layer. The 2DH model has no secondary flow model and therefore $a_\tau H/R = 0$. The Q3D-L(A1) model overestimates the secondary flow strength through the bend and therefore also overestimates $a_\tau H/R$. 
4.4.4.2 Developed Bathymetry

The pattern of $\alpha_{r} H/R$ as found for the case of a developed bed is given in Figure 4.10d. A great deal of scatter is observed in the measured data and therefore the results from the LES are used for comparison. In the straight inflow section $\alpha_{r} H/R = 0$. After the bend entrance a slight increase of $\alpha_{r} H/R$ is observed and a strong increase is observed at the 75° section, which coincides with the peak zone of outward streamwise momentum redistribution (cf. Figure 4.8d). Towards the 120° section $\alpha_{r} H/R$ decays to almost zero. Towards the end of the bend an increase in $\alpha_{r} H/R$ occurs. After the bend exit $\alpha_{r} H/R$ increases even further. Subsequently the angle tends to the straight channel limit.

By definition the direction of the bed shear stress is the same as the depth-averaged flow direction in the 2DH model ($\alpha_{r} H/R = 0$). The Q3D-L(A1) model overestimates the angle between the bed shear stress and the secondary flow. The Q3D-NL models slightly overestimate the angle at the bend entrance. They subsequently underestimate the peak at the 75° section and after the bend exit. The 1DH+ model does not capture the behaviour of the bed shear stress angle at the centreline. The RANS k-$\epsilon$ as well as the RANS k-$\epsilon$ (4) show a similar oscillating behaviour as the LES. Both models exhibit a strong peak near 75°.

To analyse the bed shear stress angle over the developed topography in further detail we show the top view of the depth-averaged velocities and bed shear stresses in Figure 4.11a and Figure 4.11b respectively. The depth-averaged velocity field in Figure 4.11a shows that the Q3D-NL and the RANS k-$\epsilon$ models show similar behaviour. Figure 4.11b shows the bed shear stress and reveals that at the 75° section fluid which previously impinged on the outer bank crosses the centreline near the bed. The RANS models can capture this behaviour because they have a greater degree of freedom in the choice of velocity profiles. The near-bed velocity is predicted independently of the free surface velocities. The Q3D-NL model is predicted independently of the free surface velocities. The Q3D-NL model is limited in its choice of velocity profiles and is not able to capture the behaviour at the 75° section because of the strong reduction of secondary flow strength at this location ($\alpha_{s}/R > 2/B$ due to the horizontal recirculation (cf. Figure 4.8d)).

4.4.5 Simulation times

The simulation times of the different models have been summarized in Table 4.3. The comparison we chose was based on the time needed to complete ten minutes of physical time. One could argue that a comparison of the time needed to reach a steady state solution would be a better and fairer comparison, considering the time scales in morphological simulations, in which the hydrodynamics can often be regarded as steady-state and can be decoupled.
Figure 4.11: Top view of a) the depth averaged velocity and b) the bed shear stresses over the developed bed (every third vector is plotted). The flow is from right to left.
Table 4.3: Simulation times for the various models over a horizontal and developed bathymetry needed to simulate ten minutes of physical time over the horizontal and fully developed bed and comparison to time needed for the depth averaged simulation without secondary flow. The simulations were performed on a desktop computer with 1.75 Gb of RAM on a single 2.67 GHz processor.

<table>
<thead>
<tr>
<th>Model description</th>
<th>Horizontal bed</th>
<th>ratio to 2DH</th>
<th>Developed bed</th>
<th>ratio to 2DH</th>
</tr>
</thead>
<tbody>
<tr>
<td>2DH</td>
<td>35”</td>
<td>1.0</td>
<td>1’05”</td>
<td>1.0</td>
</tr>
<tr>
<td>Q3D-L (A1)</td>
<td>1’16”</td>
<td>2.2</td>
<td>2’24”</td>
<td>2.2</td>
</tr>
<tr>
<td>Q3D-NL (A1)</td>
<td>1’29”</td>
<td>2.5</td>
<td>2’48”</td>
<td>2.6</td>
</tr>
<tr>
<td>RANS k-(\epsilon) (4)</td>
<td>2’41”</td>
<td>4.6</td>
<td>5’00”</td>
<td>4.6</td>
</tr>
<tr>
<td>RANS k-(\epsilon)</td>
<td>22’07”</td>
<td>37.9</td>
<td>41’32”</td>
<td>38.3</td>
</tr>
</tbody>
</table>

from the morphodynamic model. Nevertheless, the simulation times as presented here should give an indication of the differences in simulation speed between the different models.

The depth-averaged flow simulations without secondary flow over the horizontal bed and developed bed took around 35 and 65 seconds, respectively. The flow over the developed bed took longer as the water depth was approximately 2.5 times larger in the deepest zone. The linear quasi-3D model took approximately 2.2 times longer than the 2DH simulations for the horizontal as well as the developed bed. The non-linear model took slightly longer than the linear model (2.5 times the time need for the 2DH for the horizontal and 2.6 times for the developed bed. RANS computations with four layers are approximately 4.6 times slower than their depth averaged counterparts. A detailed RANS simulation with 20 layers requires approximately 38 times longer than the depth-averaged model, the 1DH+ model took a matter of seconds and the LES simulation time was in the order of weeks. The trade-off between computational cost and accuracy will be further discussed in Section 4.5.

4.5 Discussion

Dispersion stresses form an important mechanism for the redistribution of streamwise momentum in both mildly and strongly curved channels (Kalkwijk and de Vriend, 1980; Johannesson and Parker, 1989b; Blanckaert and de Vriend, 2003; Blanckaert and Graf, 2004). For mildly curved bends it is known that the dominant dispersion stress is the redistribution of streamwise momentum by the secondary flow \(\langle u_s u_n^* \rangle\) (Johannesson and Parker, 1989b). It was shown by Olesen (1987) that the term related to the shape of the
streamwise flow profile $\langle v_s^2 \rangle$ is about constant in mildly curved bends and its variation may generally be neglected. The last term $\langle v_n^2 \rangle$ is also negligible in mildly curved bends as it scales with the depth to radius of curvature ratio squared. In Figure 4.6c,d it was shown that the prediction of the non-linear quasi-3D model was well captured by only including the $\langle v_s v_n \rangle$ term, showing the dominance of this dispersion stress in strongly curved bends, as well.

Furthermore it was shown that the commonly used erroneous lateral distribution of quantities related to the secondary flow, which allows non-zero values of the secondary flow in regions with steep banks, yields an unphysical distribution of the secondary flow in which momentum can be transferred through the banks, which subsequently leads to erroneous predictions of the streamwise velocity redistribution along the channel. It was shown that the exact distribution is not very important as long as the secondary flow distribution tends to zero at the banks and obtains a maximum near the centre of the channel. This confirms the findings by Kalkwijk and de Vriend (1980); Johannesson and Parker (1989b).

The secondary flow in mildly curved bends is a well studied feature, with Thomson (1876) as its pioneer. Subsequently different linear, mild-curvature parameterizations thereof were developed (e.g. Rozovskii, 1957; Engelund, 1974; de Vriend, 1977) to extend depth-averaged modelling in mildly curved channels. Blanckaert and de Vriend (2003, 2010) developed a parameterization of the secondary flow including a non-linear feedback mechanism not limited by mild curvature assumptions and showed that such linear models tend to overestimate the secondary flow strength in strongly curved bends and non-linear models perform better than their linear counterparts. The linear and non-linear quasi-3D models predictions confirm the findings by Blanckaert and de Vriend (2003, 2010).

Having performed different simulations of strongly curved bends and shown the importance of the lateral distribution of the secondary flow and its non-linear parameterization, an assessment will be made of what models are best suited for the modelling of the hydrodynamics and long-term evolution of rivers.

The choice of which kind of model best suits the job depends not only on its accuracy, but also on the time needed to complete a simulation. Overall, the detailed RANS and LES simulations are too expensive to be considered for long-term simulations. For this reason the presented non-linear quasi-3D model will be compared to its linear counterpart, the non-linear 1DH+ model and the coarse RANS k-ε model.

The non-linear quasi-3D model is only slightly more expensive than the linear one, but the results are much better. The non-linear model is therefore preferable over the linear quasi-3D model.

The non-linear quasi-3D model provides similar results to the 1DH+ model
over the horizontal bathymetry, however over the developed bed its results are much better, because it is able to model the horizontal recirculations, whereas the 1DH+ model is not. The best choice of model depends on what is expected from it. If one would like to obtain a general idea of the flow redistribution from the start to the end of the bend the 1DH+ model will suffice, but if one would like to capture the horizontal recirculation zones, the non-linear quasi-3D model is preferred.

The non-linear quasi-3D model provides better results over the horizontal bathymetry compared to the RANS k-ε (4) model. Its prediction of the secondary flow strength, the associated outward transport of streamwise momentum and the direction of the bed shear stress, are better over the horizontal bed. This can be attributed to the lack of vertical resolution of the coarse RANS model, whereas the non-linear 1DV model by Blanckaert and de Vriend (2003) does include a high vertical resolution. Over the developed bed RANS k-ε (4) model provides better predictions. In particular its higher degree of freedom of the streamwise velocity profiles allows it to capture the inward redistribution of the streamwise momentum by secondary flow. The non-linear quasi-3D model is unable to capture this behaviour as the computed velocity profiles by the non-linear 1DV model by Blanckaert and de Vriend (2003) does not include this type of velocity profiles.

For turbulence-induced flow processes such as an internal shear layer and the outer bank cell of turbulence (van Balen, 2010) an advanced turbulence resolving hydrodynamic model should be used (e.g. non-linear k-ε (Kimura et al., 2008), LES (van Balen, 2010), Detached Eddy Simulation (Constantinescu et al., 2011) or Reynolds stress RANS model (Kashyap et al., 2012)).

For morphodynamic simulations the non-linear quasi-3D model and the 1DH+ model are computationally the most efficient for predicting the initial morphologic development of a mildly varying bathymetry. In cases with strongly varying bathymetries the coarse RANS model is the best choice, be it that the model is likely to underestimate the transverse bed slopes. The non-linear 1DH+ model shows an incorrect bed shear stress distribution over the developed bed (cf. Figure 4.10d) which implies that this model is not likely to give correct morphological predictions for strongly varying bathymetries. Zeng et al. (2008a) set up a more detailed RANS simulation including morphological development and starting from a horizontal bed, achieved good predictions of the final bathymetry of the developed bed experiment. The secondary flow over the horizontal bed as predicted by RANS is known to underestimate the secondary flow strength found in measurements and in LES (van Sabben, 2010; van Balen, 2010). Over the developed bathymetry, however, van Balen et al. (2010b) showed that the turbulent stress gradients are of minor importance for the momentum balance. These findings suggest that the physical time needed to reach the dynamic equilibrium state of the bathymetry is longer in
4.6 Conclusion

This chapter presented a non-linear quasi-3D model which was validated based on two experiments in a strongly curved laboratory flume, with curvature ratios which correspond to those in naturally occurring sharp river bends.

Secondary flow is shown to be important for the redistribution of streamwise momentum, besides topographic steering and streamwise variations in curvature.

The non-linear quasi-3D model produces much better predictions of the dispersion stress than a linear quasi-3D model, which overestimates the secondary flow strength and the associated outward transport of streamwise momentum.

The present study also shows that it is essential to include a secondary flow distribution function over the channel width which tends to zero at the banks. The model results proved to be insensitive to the exact choice of this lateral profile, as long as it tended to zero near the banks and obtained a maximum near the centre of the channel. Neglecting such a distribution did not yield the correct redistribution of streamwise momentum through the channel.

An analysis of the computational accuracy against costs showed that overall the quasi-3D model and the coarse RANS k-ε model are the computationally most efficient tools to capture the flow processes in strongly curved bends when one is interested in the 2D flow field.

Over the horizontal bed the quasi-3D model gave better results for the dispersion stress than the coarse RANS k-ε model, which was computationally twice as expensive. This can be attributed to the lack of resolution in the coarse RANS k-ε model, whereas the parameterization of secondary flow underlying the non-linear quasi-3D model includes much more detail.

Over the strongly developed bathymetry the coarse RANS k-ε provided better predictions of the dispersion stress, which was locally even opposite to its usual orientation. This local behaviour could be attributed to streamwise flow profiles with most of their momentum in the lower half of the channel. Although the underlying secondary flow model of the non-linear quasi-3D model

the detailed RANS model than in the LES.

Overall the 1DH+ can be considered the computationally most efficient tool when the bathymetry is mildly varying and horizontal recirculations are not present, the quasi-3D model can be considered the computationally most efficient tools for the hydrodynamic and morphodynamic prediction as long as the bathymetry remains mildly varying and horizontal recirculations may be present, whereas the coarse RANS k-ε model can be considered the best option for the modelling of strongly varying bathymetries including horizontal recirculations.
is able to capture this behaviour to a certain degree, it lacks the degrees of freedom to model these profiles.

Overall the 1DH+ and the quasi-3D model can be considered the computationally most efficient tools for long term hydrodynamic and morphodynamic prediction as long as the bathymetry remains mildly varying, whereas the coarse RANS k-ε model can be considered the best option for the modelling of strongly varying bathymetries.

Acknowledgments

This research was supported by the Dutch Technology Foundation (STW, applied science division of NWO under grant DCB.7780) and Deltares. The second author was partially supported by the Chinese Academy of Sciences Visiting Professorship for Senior International Scientists, grant number 2011T2Z24, by the Sino-Swiss Science and Technology Cooperation for the Institutional Partnership Project, grant number IP13_092911.
Chapter 5

Parameter study on open channel bend hydrodynamics and boundary shear stresses using large eddy simulation

Abstract

Meandering rivers and streams are a common planform in the world's populated areas. Furthermore, the recent increased focus on renaturalization projects has lead policy makers to also consider the partial remeandering of previously trained rivers. Economic factors such as navigation, man-made infrastructure and valuable farm land set the boundary conditions for such rivers. Understanding the behaviour of the near bank flow in schematized open channel bends could help in understanding the behaviour of meandering rivers and predict locations of potential damage, as well as aid in the development of design criteria of stable river banks.

The interaction of the hydrodynamics with the bed and banks causes meandering river to change their course over time. Hickin and Nanson (1975) showed that the yearly migration rate in meandering rivers depends on the ratio of the river width to the radius of curvature $B/R$ of a river bend. They showed that there is a peak migration rate at $(B/R)_{max}$ between 0.5 and 0.33. For milder curvature $B/R < (B/R)_{max}$, but also for sharper curvature.

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A modified version of this chapter has been published as: W. Ottevanger, K. Blanckaert, W.S.J. Uijttewaal, A parameter study on bank shear stresses in curved open channel flow by means of large-eddy simulation, RCEM 2011: 7th IAHR symposium on River, Coastal and Estuarine Morphodynamics, X. Shao, Z. Wang & G. Wang (Eds.), 2011.
The yearly migration rate is lower. The explanation for this behaviour is lacking.

Curved open channel flows exhibit complex flow structures (such as the outer bank cell) near the outer bank. To model these flow structures requires flow solvers with advanced turbulence modelling capabilities. Large-eddy simulation is able to capture the complex flow structures occurring in curved open channel flows. Using a well-validated large-eddy simulation code, a large set of axi-symmetric simulations (infinite length bend) were performed. The simulations are based on a wide range of mildly and sharply curved bends (represented by the parameter space \( B/R \) and \( C^{-1} \)).

The results indicate that for \( B/R > 0.1 \) the magnitude of the bank shear stress decreases with increasing inverse aspect ratio \( H/B \). The magnitude of the bank shear stress was found to increase strongly for small increasing \( B/R \). For large increasing \( B/R \) the magnitude of the bank shear stress no longer increases, but even slightly decreases. The bed shear stress magnitude, however, still increases for large increasing \( B/R \), which suggests that sharply curved channels tend to deepen rather than migrate laterally.

Furthermore, considering the bank shear stress to depend quadratically on the sum of the velocity excess and the bulk velocity multiplied by a bank friction factor \( C_{f,\text{bank}} \), a correction factor \( \psi_{\text{bank}} \) is derived. The correction factor \( \psi_{\text{bank}} \), which depends on \( B/R \) and \( H/B \), represents the increase of bank friction factor compared to a straight channel flow due to the complex curved outer bank hydrodynamics.

### 5.1 Introduction

Meandering rivers are known to change their path over the course of time. Their lateral migration through the alluvial plain has intrigued many generations of scientists. River rehabilitation projects often intend to enhance habitat heterogeneity by meandering river reaches which were previously straightened. The distribution of the boundary shear stresses are the principal hydraulic variables with respect to the planform evolution and bank protection.

The flow in meandering channel bends is three dimensional and can be described in detail by the three dimensional velocity vector given by the components \( v_i \) with the subscript \( i \), which is either \( s \) for the streamwise, \( n \) for the transverse or \( z \) for the vertical direction (cf. Figure 5.1). The flow in downstream direction is also referred to as streamwise flow, whereas the flow normal to the streamwise flow is referred to as transverse flow.

A typical and well studied component of the transverse flow in a curved open channel is the so-called centre-region secondary flow cell (cf. Figure 5.1). It was first studied by Boussinesq (1868) and Thomson (1876) and forms due to the local imbalance between the transverse pressure gradient and the centrifugal
5.1. INTRODUCTION

Depth averaged downstream velocity: $U$

Transverse velocity: $v_n$

$\Delta U$

Secondary flow: $(v_n, v_z)$

Outer-bank cell

Center-region cell

Figure 5.1: Bend geometry with definition of variables (modified from Blanckaert and de Vriend (2003)'s Figure 1).
forcing. Besides its importance for the formation of the transverse slope in river bends (e.g. van Bendegom (1947); Rozovskii (1957); Engelund (1974); Olesen (1987)), it plays a vital role in the outward transport of the streamwise flow (Kalkwijk and de Vriend, 1980; Johannesson and Parker, 1989b) which leads to more water flowing through the outer part of the bend.

Near the outer bank a smaller second secondary flow cell often exists called the outer-bank cell (cf. Figure 5.1). Bathurst et al. (1977) was the first to report the outer bank cell in a natural river bend in the River Severn. Blanckaert and de Vriend (2004) and van Balen et al. (2009) analyzed the streamwise vorticity equation experimentally and numerically and showed that the turbulent stress distribution and the centrifugal forcing terms are important for the existence of the outer bank cell. Furthermore, Blanckaert and de Vriend (2004) stated that accurately modelling the outer bank cell requires a turbulence model which can transfer turbulent kinetic energy to the mean flow. A large-eddy simulation model is such a model which is able to capture the outer-bank cell, whereas a RANS model with a linear k-ε closure is unable to do so correctly (cf. Booij, 2003).

The adaptation of meandering rivers occur at large temporal and spatial scales which mean a full 3D modelling approach is unfeasible for predicting the evolution of natural river systems. Therefore, simplified 1D models are still widely used (e.g. Struiksma (1983a), de Vriend and Struiksma (1984), Odgaard (1989), Lancaster and Bras (2002), Abad and Garcia (2006) and Crosato (2008)). In such models the river bend is characterized by the following quantities: the average depth $H$, the average width $B$, the radius of curvature $R$ and the bulk velocity $U$ (cf. Figure 5.1). Moreover, the horizontal velocity distribution is often modelled using a linear approximation as shown in Figure 5.1. The transverse distribution of the water depth is described in a similar way (cf. Blanckaert and de Vriend (2010) for a review of commonly used approximations of the streamwise velocity and water depth in 1D models). Inspite of all the progress in describing meandering rivers (e.g. 2D approaches (Mosselman, 1992; Darby et al., 2002; Duan and Julien, 2005; Chen and Duan, 2008) and recently even a full 3D approach (Rüther and Olsen, 2007)), there is no model for meander evolution which includes enough detail to capture the complex outer bank hydrodynamics accurately.

Hickin and Nanson (1975, 1984) showed that the yearly migration rate, which is closely linked to the bank erosion rate, in meandering rivers depends on the ratio between the curvature radius $R$ and the river width $B$. Hickin and Nanson (1984) show that for small increasing $B/R$ the migration rate increases, subsequently reaches a maximum for a certain $B/R$ (typically $B/R \approx 0.33$ to 0.5) and then the migration rate decreases again for large increasing $B/R$ (see Figure 5.2).

Hickin and Nanson distinguish various phases in the migration rate in Fig-
Figure 5.2: Modified conceptual curve after Hickin and Nanson (1984) illustrating the normalized rate of yearly migration $M/M_{max}$ of meandering rivers as a function of $B/R$ (modified from Blanckaert (2011))

Figure 5.2: the initiation stage (for small $B/R$, mildly curved bends), growth stage (for intermediate $B/R$, moderately curved bends) and termination stage (for large $B/R$, strongly curved bends). Plotting the migration rate as dependent on $B/R$ shows huge scatter, particularly in the sharply curved range denoted by $B/R > 0.5$ ($R/B < 2$). This suggests that there are hydrodynamic and morphodynamic processes which are unaccounted for in this parameterisation, but are important for the migration rate.

Ikeda et al. (1981) postulated that the migration rate can be modelled as a migration coefficient multiplied by the velocity excess $\Delta U$. The migration model by Ikeda et al. (1981) can be interpreted as a linearization of an erosion formulation for cohesive sediment by Ariathurai en Arulanandan (1978) based on the boundary shear stress (cf. Mosselman (1992)). Pizzuto and Meckelnburg (1989) provided some evidence in favour of the formulation by Ikeda et al. (1981), yet this simple parameterization cannot account for the complex near bank hydrodynamics and its effect on the bank shear stress.

The effect of the outer bank cell on the outer bank shear stress is still subject to debate. Bathurst et al. (1977) hypothesized the outer bank cell endangers the outer bank stability as it transfers high momentum fluid near the free surface to the toe of the outer bank. Blanckaert and Graf (2004), however, argue that the outer bank cell forms a buffer layer between the flow core and the bank thereby limiting the influence of the centre region cell, which is agreed to be an important factor for the outward transport of streamwise momentum. By means of a Large-eddy simulation van Balen et al. (2009, 2010a) showed that the outer bank cell leads to an increased turbulence production in the cross sectional plane. This implies an increase of the bank shear stress caused by the outer bank cell. Crosato (2009) argues that since it is not necessary to
include the effect of the outer bank cell in a meander evolution model in order to reproduce the proper migration rate patterns shown in Figure 5.2, the outer bank cell is not important for the reduction of the migration rate in strongly curved meander bends. A similar analysis by a mild curvature model lacking even the center-region cell of secondary flow (valid for mildly sinuous channels only) by Sieben (2000) could provide a similar conclusion.

By means of a parameter study on open channel bend hydrodynamics based on the two control parameters \( B/R \) and \( C_f^{-1}H/B \) using a large eddy simulation code with advanced turbulence modelling capabilities the following research questions will be addressed:

1. How are the streamwise and secondary flow processes in the centre of the channel dependent on the curvature and how can their behaviour be explained?

2. How is the outer bank cell dependent on the depth to curvature ratio \( H/R \) and is it correlated to the centre-region cell strength?

3. How are the boundary shear stresses influenced by secondary flow (e.g. does the outer bank cell protect or endanger the bank stability)?

4. Can the hydrodynamics explain why strongly curved bends tend to migrate slower than mildly curved flows (Hickin and Nanson, 1975, 1984, Figure 5.2)?

5. Can this behaviour be parameterized in a 1D model for flow and bank shear stresses?

### 5.2 Parameter study

#### 5.2.1 Case descriptions

Blanckaert (2011) showed based on a 1D non-linear model that for axi-symmetric flow the scaling parameters \( C_f^{-1}H/B \) and \( B/R \) determine the distribution of the streamwise velocity. In line with these findings, various axi-symmetric LES simulations are performed based on the scaling parameters \( C_f^{-1}H/B \) and \( B/R \) (cf. Table 5.1). In contrast with developing flows, axi-symmetric flows correspond to a velocity field which lacks streamwise gradients (i.e. the flow which has fully adapted to the curvature forcing). Within the different considered cases, the cases preceded by a letter ‘A’ to ‘F’ are based on the experimental parameters of Blanckaert (2011) in Lausanne. The side walls were considered to be hydraulically smooth and the bottom has a Nikuradse roughness height of 6 mm or 20 mm. The water depth varied between \( H = 0.108 \) m, \( H = 0.159 \) m and \( H = 0.206 \) m. Furthermore the radius of curvature varied between
5.2. PARAMETER STUDY

Table 5.1: Overview of simulation cases. \( H \) is the water depth, \( B \) is channel width, \( R \) is the radius of curvature at the centerline, \( U \) describes the bulk velocity, \( k_s \) is the Nikuradse roughness height at the bed and \( Q \) is discharge. \( B/R \) denotes the width to radius of curvature ratio, \( H/B \) is the inverse aspect ratio, \( C_f^{-1} H/B \) is a parameter accounting for roughness and the shallowness of the simulation and \( Re \) is the Reynolds number. \( N_s, N_n, N_z \) denote the grid dimensions in streamwise, transverse and vertical direction, respectively.

<table>
<thead>
<tr>
<th>Axi-symmetric flow cases</th>
<th>( H ) [m]</th>
<th>( B ) [m]</th>
<th>( R ) [m]</th>
<th>( k_s ) [mm]</th>
<th>( U ) [m/s]</th>
<th>( Q ) [l/s]</th>
<th>( H/B )</th>
<th>( C_f^{-1} H/B )</th>
<th>( Re )</th>
<th>( N_s )</th>
<th>( N_n )</th>
<th>( N_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developing flow cases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case</td>
<td>( H ) [m]</td>
<td>( B ) [m]</td>
<td>( R ) [m]</td>
<td>( k_s ) [mm]</td>
<td>( U ) [m/s]</td>
<td>( Q ) [l/s]</td>
<td>( H/B )</td>
<td>( C_f^{-1} H/B )</td>
<td>( Re )</td>
<td>( N_s )</td>
<td>( N_n )</td>
<td>( N_z )</td>
</tr>
<tr>
<td>Q60</td>
<td>0.108</td>
<td>1.3</td>
<td>([\infty,1.7,\infty])</td>
<td>6</td>
<td>0.43</td>
<td>60.4</td>
<td>0.08</td>
<td>18</td>
<td>[0.076,0]</td>
<td>46440</td>
<td>1260</td>
<td>192</td>
</tr>
<tr>
<td>Q89</td>
<td>0.159</td>
<td>1.3</td>
<td>([\infty,1.7,\infty])</td>
<td>6</td>
<td>0.43</td>
<td>88.9</td>
<td>0.12</td>
<td>29</td>
<td>[0.076,0]</td>
<td>68370</td>
<td>1260</td>
<td>192</td>
</tr>
<tr>
<td>Q115</td>
<td>0.206</td>
<td>1.3</td>
<td>([\infty,1.7,\infty])</td>
<td>6</td>
<td>0.43</td>
<td>115.2</td>
<td>0.16</td>
<td>40</td>
<td>[0.076,0]</td>
<td>88580</td>
<td>1260</td>
<td>192</td>
</tr>
</tbody>
</table>
CHAPTER 5. BANK SHEAR STRESSES IN OPEN CHANNEL BENDS BY LES

$R = \infty$ m (i.e. a straight channel) and $R = 1.5$ m (i.e. a strongly curved channel). The corresponding range of the control parameters in Table 5.1 for $C_f^{-1}H/B$ is between 15 and 40. This range corresponds well to the range typically found in experimental setups. In natural river bends $C_f^{-1}H/B$ generally varies between 5 and 10 (Blanckaert 2011). The range of the parameter $C_f^{-1}H/B$ and the rectangular cross-sections in the axi-symmetric simulations mean that the chosen set of simulations is not representative of natural river bends. The simulations have been simplified such that the role of the main channel and near bank hydrodynamics on the outer bank shear stress can be identified clearly. The outer bank shear stress is chosen as the main focus as it is the location where the bulk of the bank erosion is to be expected. This study is an initial investigation into role of the hydrodynamics on the bank shear stress, and could provide clues to understanding the effect of the main channel and near bank hydrodynamics on the bank shear stress in naturally occurring rivers.

5.2.2 Large eddy simulation model description

The present study uses the large-eddy simulation model originally developed by Eggels et al. (1994) and Pourquié (1994), and further developed by van Balen (2010) in his study on strongly curved bends. The model solves the equation of mass conservation:

$$\frac{1}{1 + \frac{n}{R}} \frac{\partial v_s}{\partial s} + \frac{1}{(R + n)} \frac{\partial (R + n) v_n}{\partial n} + \frac{\partial v_z}{\partial z} = 0,$$

(5.1)

and conserves momentum in the streamwise, transverse and vertical directions as follows (Schlichting and Gersten, 2000):

$$\frac{\partial v_s}{\partial t} + \frac{1}{1 + \frac{n}{R}} \frac{\partial v_s^2}{\partial s} + \frac{1}{(R + n)^2} \frac{\partial (R + n)^2 v_n v_n}{\partial n} + \frac{\partial v_s}{\partial z} = - \frac{1}{1 + \frac{n}{R}} \frac{1}{\rho} \frac{\partial p}{\partial s}$$

$$+ \frac{1}{1 + \frac{n}{R}} \frac{\partial}{\partial s} [2(\nu_{mol} + \nu_{sgs}) S_{ss}]$$

$$+ \frac{1}{(R + n)^2} \frac{\partial}{\partial n} [2(R + n)^2 (\nu_{mol} + \nu_{sgs}) S_{sn}] + \frac{\partial}{\partial z} [2(\nu_{mol} + \nu_{sgs}) S_{sz}]$$

(5.2)

$$\frac{\partial v_n}{\partial t} + \frac{1}{1 + \frac{n}{R}} \frac{\partial v_n v_n}{\partial s} + \frac{1}{R + n} \frac{\partial (R + n) v_n^2}{\partial n} + \frac{\partial v_n}{\partial z} - \frac{v_n^2}{R + n} = - \frac{1}{\rho} \frac{\partial p}{\partial n}$$

$$+ \frac{1}{1 + \frac{n}{R}} \frac{\partial}{\partial s} [2(\nu_{mol} + \nu_{sgs}) S_{sn}] + \frac{1}{(R + n)} \frac{\partial}{\partial n} [2(R + n) (\nu_{mol} + \nu_{sgs}) S_{nn}]$$

$$+ \frac{\partial}{\partial z} [2(\nu_{mol} + \nu_{sgs}) S_{nz}] - \frac{2(\nu_{mol} + \nu_{sgs}) S_{ss}}{R + n}$$

(5.3)
\[
\frac{\partial v_z}{\partial t} + \frac{1}{1 + \frac{n}{R}} \frac{\partial v_z}{\partial s} + \frac{1}{R + n} \frac{\partial (R + n) v_n v_z}{\partial n} + \frac{\partial v_z^2}{\partial z} = -g - \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{1}{1 + \frac{n}{R}} \frac{\partial}{\partial s} [2(\nu_{mol} + \nu_{sgs}) S_{sz}] + \frac{1}{(R + n)} \frac{\partial}{\partial n} [2(R + n)(\nu_{mol} + \nu_{sgs}) S_{nz}] + \frac{\partial}{\partial z} [2(\nu_{mol} + \nu_{sgs}) S_{zz}] \tag{5.4}
\]

In the LES approach eddies up to the size of the grid are resolved, whereas the effect of smaller eddies is modelled. The resolved velocity field is given by \( v \) with its subscript indicating its direction (\( s, n \) or \( z \)), \( p \) is the resolved pressure field, \( \rho = 1000 \text{ kg/m}^3 \) represents the density of water, \( g \) is the gravitational acceleration and the components of the strain rate tensor \( S \) are given by:

\[
S_{ss} = \frac{1}{1 + \frac{n}{R}} \frac{\partial v_s}{\partial s} + \frac{v_n}{R + n}
\]

\[
S_{sn} = \frac{1}{2} \left( \frac{1}{1 + \frac{n}{R}} \frac{\partial v_n}{\partial s} + (R + n) \frac{\partial}{\partial n} \left( \frac{v_s}{R + n} \right) \right)
\]

\[
S_{sz} = \frac{1}{2} \left( \frac{1}{1 + \frac{n}{R}} \frac{\partial v_z}{\partial s} + \frac{\partial v_s}{\partial z} \right)
\]

\[
S_{nn} = \frac{\partial v_n}{\partial n}
\]

\[
S_{nz} = \frac{1}{2} \left( \frac{\partial v_z}{\partial n} + \frac{\partial v_n}{\partial z} \right)
\]

\[
S_{zz} = \frac{\partial v_z}{\partial z}
\]

(5.5)

As the radius of curvature \( R \) tends to infinity (i.e. the straight channel limit) the equations for an implementation in Cartesian coordinates are found (cf. Nabi et al. (2012)). In equations (5.2)-(5.4) the parameter \( \nu_{mol} = 10^{-6} \text{ m}^2/\text{s} \) represents the molecular kinematic viscosity whereas \( \nu_{sgs} \) is the instantaneous turbulent eddy viscosity and requires closure, which is resolved by the Smagorinsky-Lilly model (Smagorinsky, 1963):

\[
\nu_{sgs} = (C_s \Delta g)^2 \sqrt{2S_{ij}S_{ij}} \tag{5.6}
\]

in which \( \Delta g \) is the cubic root of the volume of the grid cell, the indices \( i \) and \( j \) cycle over \( n,s,z \) and \( C_s \) is the Smagorinsky constant. For this study \( C_s \) is set to 0.1.

To save computation time, a wall function approach is used at the solid boundaries. For the smooth side walls, the viscous sublayer, the buffer layer
and the log layer are captured as follows:

\[ v^+ = y^+ \quad \text{for} \quad y^+ \leq 5 \]
\[ v^+ = 5.0 \ln(y^+) - 3.05 \quad \text{for} \quad 5 < y^+ \leq 30 \]
\[ v^+ = 2.5 \ln(y^+) + 5.5 \quad \text{for} \quad 30 < y^+ \]

The left hand side \( v^+ = v/v_* \) is the dimensionless velocity and the dimensionless wall coordinate is given by \( v^+ = yv_*/\nu \) where \( v_* \) is the local, iteratively determined friction velocity and \( \nu \) is the kinematic viscosity. The distance to the wall is given by \( y \). For the rough bed boundary the following wall function is used:

\[ v^+ = 2.5 \ln(y \cdot \frac{30}{k_s}) \quad \text{for} \quad y > k_s/30 \]

where \( k_s \) is the Nikuradse roughness height typically expressed as a constant times the sediment diameter (see e.g. van Rijn (1984a)). Further details of the model, its numerical implementation and extensive validation with mildly and strongly curved experimental data may be found in van Balen (2010).

5.3 Analysis of curved open channel hydrodynamics

5.3.1 Main channel hydrodynamics

5.3.1.1 Stream function definition

Transverse flow is the fluid motion perpendicular to the streamwise flow direction (cf. Figure 5.1). For axi-symmetric flow, the equation which describes mass conservation in the transverse plane is

\[ \frac{1}{(R+n)} \frac{\partial (R+n)v_n}{\partial n} + \frac{\partial v_z}{\partial z} = 0 \]  

(5.9)

A useful variable which may defined is the normalized stream function (cf. van Balen (2010)):

\[ \frac{HR}{(R+n)} \frac{\partial \phi}{\partial n} = \frac{v_z}{U} \quad \text{and} \quad \frac{HR}{(R+n)} \frac{\partial \phi}{\partial z} = -\frac{v_n}{U}. \]

(5.10)

In the limit of a straight channel (i.e. \( R = \infty \)) the definition (5.10) reduces to the streamfunction definition for a 2D plane:

\[ H \frac{\partial \phi}{\partial n} = \frac{v_z}{U} \quad \text{and} \quad H \frac{\partial \phi}{\partial z} = -\frac{v_n}{U}. \]

(5.11)
Figure 5.3: The normalized streamfunction $\phi$ (cf. Eq. (5.10)) for cases C01, C03 and C09 respectively. The red zones indicate a flow pattern in clockwise direction whereas the blue zones show a flow pattern in anti-clockwise direction. The contour lines are equidistantly chosen with intervals of 0.002. The streamwise flow is directed into the page.

### 5.3.1.2 Stream function distribution

Figure 5.3 shows the normalized streamfunction $\phi$ for cases C01, C03 and C09 respectively. It shows for the straight channel case C01 there are two equally strong secondary flow cell zones with opposite orientation. These could be referred to as the inner and outer bank cell, where inner refers to the left bank as in the case of a curved flow. There is no centre-region cell as there is no centrifugal forcing at the centre of the channel. For the mildly curved C03 case the inner bank cell has merged with the centre-region cell which is present due to the geometric curvature of the channel and the equal sense of rotation. The outer bank cell is smaller in size, however its magnitude remains roughly the same as in the straight channel case. For the strongly curved bend C09 the centre region cell is stronger than the C03 case and the core of the secondary flow is located lower in the water column and closer to the inner bank than in the C03 case. The outer bank cell zone is no longer connected to the corner where the bed meets the outer bank, but it connects at roughly at a quarter water depth from the bed. Near the inner bank a pattern associated with an internal shear layer may be distinguished (cf. van Balen (2010) for details).
5.3.1.3 Centre region cell strength

The centre-region cell is a well studied phenomenon in mildly curved bends with Boussinesq (1868) and Thomson (1876) as its pioneers. The behaviour of the centre-region cell in strongly curved bends has received increasing attention in recent years in strongly curved bends a limiting behaviour of the secondary flow strength appears (see Section 5.1). Based on our simulated flow cases the dependence of the centre-region cell strength on the geometric curvature is investigated.

The centre region cell strength is characterized by the maximum of the streamfunction. Figure 5.4a) reveals that for weakly curved flow (i.e. $H/R << 1$) a linear dependency may be observed, whereas for strongly curved flow saturation takes place (cf. Blanckaert (2009)) and the maximum normalized streamfunction $\phi$ no longer increases. Blanckaert and de Vriend (2003)’s non-linear one-dimensional vertical (1DV) model shows a similar limiting trend in the strength of centre-region cell (cf. Figure 5.4b), although the overall centre-region strength is weaker than from the LES simulation. A comparison of secondary flow strength magnitudes by van Balen (2010) and van Sabben (2010) revealed that a RANS simulation, which contains more detail than a 1DV model, also underestimates the secondary flow strength in comparison a LES simulation. The linear 1DV model does not account for non-linear hydrodynamic feedback (i.e. flattening of the streamwise velocity profile and the associated reduction of centrifugal forcing, see Blanckaert and de Vriend (2003) for details) and therefore increases linearly with the curvature ratio $H/R$. 

Figure 5.4: Centre-region cell strength a) LES results, its empirical approximation function (cf. Eq. (5.12)) and b) Blanckaert and de Vriend (2003)’s 1DV model results as function of $H/R$ for the axi-symmetric flow cases.
For the purpose of a simple parameterization, using the MATLAB curve fitting toolbox, a fitting function for the normalized stream function maximum is found based on the depth to curvature ratio $H/R$, on the normalized roughness height $k_s/H$ and the aspect ratio of the channel $B/H$ (cf. Figure 5.4a):

$$
\phi_{CRC} \approx \phi_{CRC, \text{sat.}} \left(1 - e^{31.63 \left(\frac{B}{H}\right)^{0.7519}}\right)
$$

$$
\phi_{CRC, \text{sat.}} \approx 0.0185 \cdot \left(\frac{k_s}{H}\right)^{0.0907} \cdot \left(\frac{B}{H}\right)^{0.553}
$$

(5.12)

This indicates that normalized stream function maximum increases with increasing depth to curvature ratio $H/R$, aspect ratio $B/H$ and increasing relative roughness $k_s/H$. The relative roughness effect only plays a minor role, compared to the aspect ration or the depth to curvature ratio.

5.3.1.4 Streamwise velocity distribution in the centre-region

The depth averaged velocity distribution based on LES and Blanckaert and de Vriend (2010)’s linear and non-linear 1d hydrodynamics model are plotted for a straight, a mildly curved and a strongly curved case in Figure 5.5. The straight channel case ($B/R = 0$) LES result shows local dips in the streamwise velocity which accompany weak secondary circulation cells in the centre of the channel caused by turbulence anisotropy. The linear and non-linear 1d model are both uniform over the cross-section. For the mildly curved case ($B/R = 0.07$) a much smoother pattern is visible than in the straight channel case. Due to the outward transport of streamwise momentum by secondary flow, the bulk of the fluid now moves through the right half of the channel. The linear and non-linear models show similar results, however the linear performs slightly better than the non-linear model. The strongly curved case ($B/R = 0.07$) shows a more uniform distribution of the depth-averaged streamwise velocity at the centre of the channel. Near the inner bank a slight dip is visible which is related to the internal shear layer (cf. van Balen et al. (2011)). Near the outer bank a local maximum of the depth-averaged streamwise velocity is found. The non-linear 1d model shows a reasonable approximation to the LES result, whereas the linear 1d model strongly overestimates the outer bank velocity.

To analyze the centre region velocity distribution, a linear least squares fit of the depth averaged velocity distribution $U_s$ from the LES over the width of the channel was performed (cf. Figure 5.5):

$$
U_s \approx U + \Delta U \cdot \frac{2n}{B}
$$

(5.13)
Figure 5.5: Streamwise flow distribution $U_s/U$ for the cases C01, C03 and C09 as computed by LES and by b) Blanckaert and de Vriend (2010)’s linear and non-linear 1d hydrodynamics model prediction.
where $U$ is the bulk velocity and $\Delta U$ is the velocity excess (cf. Ikeda et al. (1981)).

The normalized velocity excess based on LES and based on Blanckaert and de Vriend (2010)’s non-linear 1d hydrodynamic model are shown in Figure 5.6a and 5.6b) respectively. For weakly curved bends the velocity excess increases as $B/R$ increases, and for strongly curved bends the velocity excess decreases as $B/R$ increases. The dashed lines in Figure 5.6b) describe the normalized velocity excess as computed by a linear 1d hydrodynamic model and it is shown to increase proportionally to $H/R$. Linear models do not account for non-linear feedback mechanisms which are important when considering the hydrodynamics in strongly curved bends.

The model by Blanckaert and de Vriend (2010), given in axi-symmetric form in Blanckaert (2011), adequately succeeds in connecting the main channel processes such as momentum redistribution by the centre-region cell, topographic steering and superelevation of the free-surface to predict the main channel streamwise flow distribution. The near bank flow distribution is however not accurately captured by the 1D models due to a lack of resolution.

### 5.3.2 Outer bank hydrodynamics

#### 5.3.2.1 Outer bank cell strength

Figure 5.7a) shows the curvature dependence of the centre-region cell. It displays a region where the outer bank cell remains constant or even decreases slightly for increasing curvature. In the subsequent region $H/R \approx 0.01$ the
outer bank cell strength increases. For very sharp cases $H/R \approx 0.1$ the outer bank cell is roughly constant and even decreases in the case $k_s = 0.006$ m and $H/B = 0.12$.

To further investigate the secondary flow in the near bank region the outer bank cell strength is compared to the centre region cell strength in Figure 5.7b). Figure 5.7b) reveals three distinct regions: The first is a region in which the centre-region cell strength increases and the outer bank cell remains roughly constant, the second is a region in which both the centre region cell and outer bank cell strength increase and the third zone where the centre-region cell strength remains constant and the outer bank cell generally increases. The first and third zones, indicating the mild and strong curvature cases respectively, are regions in which the outer bank cell and the centre region cells are independent of each another. The second zone (intermediate curvature) shows a clear correlation between the centre region cell and the outer bank cell strength, which could imply that they are dependent on each other, or that they are both driven by another process. The explanation of the strength of the outer bank cell in the three zones in Figure 5.7b) is as follows: in the first zone (I) the outer bank cell is driven by turbulence anisotropy; in the second zone (II) the outer bank cell is also influenced by momentum transfer from the centre-region cell; and in the third zone grows in size and receives more momentum from the centre-region cell.

### 5.3.2.2 Near bank streamwise velocity distribution

The near bank streamwise velocity distribution is shown in Figure 5.8. The isolines of the streamwise velocity illustrate that as the curvature increases (from top to bottom) the highest streamwise velocities tend to concentrate in the zone close to the bank. It can further be observed that the outer bank zone leaves a clear imprint on the velocity distribution near the bank as the maximum velocity occurs below the free surface in the zone demarcated by the outer bank cell. Figure 5.8 also shows the bank shear stress distribution. As the curvature increases the overall bank shear stress increases and the maximum occurs at increasingly lower positions on the outer bank. The bed shear stress also increases for increasing curvature with the maximum occurring closest to the outer bank.

### 5.3.3 Boundary shear stresses

Figure 5.9 shows the relative increase of boundary shear stresses compared to the straight channel cases shown for the three different regions of mild, intermediate and strong curvature (I, II and III) as well as the outer bank cell strength in zone III (where the centre region cell has reached its saturated limit).
5.3. ANALYSIS OF CURVED OPEN CHANNEL HYDRODYNAMICS

Figure 5.7: Outer bank cell strength a) depending on the curvature $H/R$, and b) comparison with the centre-region cell strength normalized by its saturated value and the outer bank cell strength, indicating three regions: (I) a region of increasing centre-region cell strength and constant outer bank cell strength, (II) a region with both increasing centre-region and outer bank cell strength and (III) a region where the centre-region cell is constant and the outer bank shear stress increases.

The inner bank shear stress decreases strongly in region I, and subsequently increases again in regions II and III. The overall bed shear stress increases as a function of $B/R$. Both upper and lower outer bank shear stresses increase strongly in region I. The upper outer bank shear stress increases until a maximum in region II. In zone III the upper outer bank shear stress hardly varies any more. The lower outer bank shear stress increases gradually in region II and finally to a maximum in region III. With the exception of the case with $k_s = 20$ mm, the lower outer bank shear stress grows larger than the upper outer bank shear stress. Based on Figure 5.7a) on average a growing trend of the outer bank cell is observed in zone III. The upper outer bank shear stress is roughly constant in this zone, where as the lower outer bank shear stress increases slightly. Locally this suggests a protective effect of the outer bank cell on the upper half of the bank and a negative effect of the outer bank cell on the lower half of the bank, in line with the hypothesis by Bathurst et al. (1977). The overall effect (i.e. positive or negative) of the outer bank cell on the bank stability can however not be determined as this would require a flow simulation lacking the outer bank cell. Therefore, these LES simulations cannot confirm or disprove the statement by Blanckaert and de Vriend (2004) of the protective effect of the outer bank cell on the bank. Although the lower outer bank shear stress is higher than the upper outer bank shear stress, this
Figure 5.8: Streamwise flow field $v_s/U$ for cases C01, C03 and C09 respectively (directed into the page). The contour lines display the normalized streamfunction $\phi$ in which contour lines are spaced by steps of 0.002. The outward facing arrows show the boundary shear stress, where the red lines indicate the maximum boundary shear stress for the straight channel case.
does not imply that the outer bank cell has a overall negative effect on the bank stability as hypothesized by Bathurst et al. (1977). The findings by van Balen et al. (2009, 2010a) who showed that the outer bank cell leads to an increased turbulence production in the cross-sectional plane do also not necessarily mean that without the presence of the outer bank cell the turbulence production and hence the shear stresses near the bank would be lower.

5.3.4 Bank shear stress parameterization for 1D models

5.3.4.1 Axi-symmetric flow

For straight channel flow, the general assumption is that the depth-averaged velocity is roughly uniform over the cross-section and the bank shear stress is generally approximated as

\[ \tau_{\text{bank}} = \rho C_{f,\text{bank}} U^2 \]  

(5.14)

where \( \rho \) is the density of water, \( U \) is the bulk velocity and \( C_{f,\text{bank}} \) is the parameterization of the bank roughness.

In curved channel flow approximating the outer bank shear stress is no longer trivial as extra complicating factors arise: firstly the flow is no longer uniform in the cross-section and secondly near the bank a complex near bank flow pattern exists. These two factors will undoubtedly play a role in the bank shear stress approximation. To account for the non-uniformity of the streamwise velocity we consider the near bank velocity rather than the bulk velocity in equation (5.14). In the context of a 1D model the near bank velocity may be approximated as the sum of the bulk velocity and the velocity excess \( (U_{\text{bank}} = U + \Delta U, \text{cf. (5.13)}) \). Due to the complexity of the outer bank hydrodynamics and the simplicity of the formulation a correction factor may be required. Therefore we introduce a correction factor \( \psi_{\text{bank}} \) which is determined on the basis of the various axi-symmetric simulations.

\[ \tau_{\text{bank}} = \rho \psi_{\text{bank}} C_{f,\text{bank}} U_{\text{bank}}^2 \]  

(5.15)

Figure 5.10a) shows the correction factor \( \psi_{\text{bank},\infty} \) as obtained from the variables in equation (5.15) which are all known. The index \( \infty \) as been added to stress that the correction factor is for axi-symmetric flow. The correction factor increases for increasing curvature \( B/R \) and aspect ratio \( B/H \). By means of MATLAB’s fitting toolbox the following empirical fit was determined:

\[ \psi_{\text{bank},\infty} \approx 2 - e^{-1.58 \left( \frac{B}{H} \right)^{0.7791}} + \left( -0.6473 + 0.25 \log \left( \frac{B}{H} \right) \right) \frac{B}{R} \]  

(5.16)

A comparison between the bank shear stress from LES and a reconstructed bank shear stress based on Blanckaert and de Vriend (2010)’s 1D non-linear
CHAPTER 5. BANK SHEAR STRESSES IN OPEN CHANNEL BENDS BY LES

$k_s = 0.006, H/B = 0.08$

$k_s = 0.02, H/B = 0.08$

$k_s = 0.006, H/B = 0.12$

$k_s = 0.02, H/B = 0.12$

$k_s = 0.006, H/B = 0.16$

$k_s = 0.02, H/B = 0.16$
model is given in Figure 5.10b). For the shallow cases the bank shear stress estimated from the non-linear model underestimates the bank shear stress as computed by the LES by as much as 30 percent. This is not surprising as the velocity excess from the 1D non-linear model in Figure 5.6 was also smaller than the velocity excess obtained from the large eddy simulation. Furthermore Figure 5.5 shows that the outer bank velocity distribution computed by the LES and the non-linear 1D model are quite different. The rest of the simulations remain within the 10 percent error margin.

5.3.4.2 Extension to developing flow

Real river flows are not axi-symmetric and therefore the parameterization requires extension to improve the modelling of bank shear stresses in spatially developing flows. To this end, an adaptation equation is adopted to extend the correction factor for axi-symmetric flow to one for a developing flow:

\[ \psi_{\text{bank}} - \frac{1}{2H} \frac{\partial \psi_{\text{bank}}}{\partial s^2} = \psi_{\text{bank},\infty} \]  \hspace{1cm} (5.17)

Figure 5.11 shows the evolution of the outer bank shear stress through the bend for three strongly curved experiments by Blanckaert (2011) in which the depths have been varied (cf. Table 5.1).

All three experiments show the same basic trend in which the outer bank shear stress has a constant level before the bend. From about half a metre before the bend entrance the outer bank shear stress reduces until about half a metre into the bend whence the outer bank shear stress increases gradually until the bend exit where it increases strongly. Half a metre into the straight outflow section the outer bank shear stress reduces again. In all three experiments the parameterization of the outer bank shear stress including the correction factor provides an improved result compared to parameterization lacking it, however still a significant underestimation is found.

5.4 Relevance for real rivers

Figure 5.9 (facing page): Bank and bed shear stress for the 6 considered flow cases. The left column and right columns have a roughness height of \( k_s = 6 \) mm and \( k_s = 20 \) mm respectively. From top to bottom the inverse aspect ratio increases from 0.08, 0.12 to 0.16. The roman numerals correspond to the zones of mild, intermediate and strong curvature as described in Figure 5.7
Hickin and Nanson (1975, 1984) first observed that in strongly curved bends the migration rate is smaller than in moderately curved bends (cf. Figure 5.2). This behaviour is still not fully understood in spite of improved computational modelling possibilities (Large eddy simulation (LES) (van Balen, 2010), Detached eddy simulation (DES) (Constantinescu et al., 2011), Reynolds averaged Navier Stokes (RANS) models with non-linear k-ε (Kimura et al., 2008) or RANS with a Reynolds stress model closure (Kashyap et al., 2012)) and evidence provided by strongly curved bend experiments. Due to time, memory and input information constraints, the modelling of meandering channels is performed by reduced 1D models.

These reduced 1D models have the advantage that they are faster, more insightful and allow for a quick analysis of the problem when data is lacking. These simplifications however do come at a price, as they imply a loss of generality and cannot be applied in all circumstances. Blanckaert and de Vriend (2003, 2010) developed a non-linear 1D model which is also valid for strongly curved bends (and encompasses existing linear mildly curved models). The model is non-linear as it, amongst other non-linear effects, includes the non-linear interaction between streamwise and secondary flow which capture the saturation of secondary flow (Blanckaert, 2009). The non-linear 1D model identified $C_f^{-1} H/B$ and $B/R$ as the dominant control parameters with respect
Figure 5.11: Comparison of the outer bank shear stress from LES and its parameterization determined by means of Blanckaert and de Vriend (2010)'s 1D non-linear meander flow model with and without the correction factor $\psi_{\text{bank}}$ the a) Q60 experiment b) Q89 experiment and the c) Q115 experiment and the correction factor determined by equations (5.16) and (5.17). The vertical lines mark the start and end of the bend.
to the flow redistribution in meandering rivers. The first parameter characterizes the roughness and the shallowness of a reach, whereas the second parameter describes the curvature of a single bend.

A LES parameter study based on variations of $C_{f}^{-1}H/B$ and $B/R$ revealed that the behaviour in the centre of the channel is adequately captured by Blanckaert and de Vriend (2003, 2010)’s non-linear model. Although $C_{f}^{-1}H/B$ generally varies between 5 and 10 in natural river bends and the range within the parameter study varies between 15 and 40, and in Chapter 2 it was shown that Blanckaert and de Vriend (2003, 2010)’s non-linear model adequately represents the flow in strongly curved naturally occurring river bends. This means that the mechanism which controls the flow at the centre of the channel is explained by advective momentum transport.

Although the behaviour in the centre of the channel is reasonably represented, the non-linear 1d model does not have enough detail to model the near bank hydrodynamics accurately. These near bank hydrodynamics undoubtedly play an important role in quantifying the hydrodynamic forcing on the bank, which in turn is expected to be important for bank erosion processes. Bathurst et al. (1977) hypothesized the outer bank cell endangers the bank stability as it diverts high momentum fluid from the free surface towards the toe of the bank. Blanckaert and de Vriend (2004) on the other hand argue that the outer bank cell has a positive effect on the bank stability as it reduces the influence of the centre region cell thereby decreasing the outward transport of streamwise momentum. The LES simulations show that the bank shear stress on the lower half of the bank is greater than that in the upper half in strongly curved bends, thereby showing a negative effect on the bank shear stress on the lower half of the bank and a positive effect of the bank shear stress on the upper half of the bank. The overall effect of the outer bank cell, however, cannot be confirmed or disproved by these LES simulations, as that would require a simulation lacking the outer bank cell. The findings by van Balen et al. (2009, 2010a) who showed that the outer bank cell leads to an increased turbulence production in the cross-sectional plane do also not necessarily mean that without the presence of the outer bank cell the turbulence production and hence the shear stresses near the bank would be lower. Sieben (2000); Crosato (2009) used their respective models to produce a similar pattern to Figure 5.2 (Hickin and Nanson, 1975, 1984). On the basis of these observations one might argue that the outer bank cell is not of any importance for the hydrodynamic forcing on the bank as the pattern can be reproduced and the model does not contain this mechanism. This is however an incorrect reasoning as it is known that linear models neglect non-linear feedback mechanisms which are important in strongly curved bends, and therefore the role of the outer bank cell cannot be negated based on a linear model prediction.

The explanation of the behaviour in Figure 5.2 (Hickin and Nanson, 1975,
1984) is not immediately apparent. Based on the LES simulations, it was observed that for increasing $B/R$ the bank shear stresses obtain a maximum, whereas the bed shear stresses continue to increase. Although the cross-section is over-simplified from what is found in naturally occurring rivers, and erosion formulations are closely linked to the bank shear stress (e.g. Mosselman (2005)), this suggests that sharp channel bends will have a preference for deepening instead of migrating laterally. Evidence in favour of this hypothesis is provided by bathymetries found in naturally occurring sharp river bends (e.g. Jackson (1975), Fong et al. (2009); Schnauder and Sukhodolov (2012), Upper Mississippi bathymetry data in umesc.usgs.gov/aquatic/bathymetry.html).

Ikeda et al. (1981) postulated that the migration rate can be modelled as a migration coefficient multiplied by the velocity excess. The migration model by Ikeda et al. (1981) can be interpreted as a linearization of an erosion formulation for cohesive sediment by Ariathurai and Arulanandan (1978) based on the boundary shear stress (cf. Mosselman (1992)). Pizzuto and Meckelburg (1989) provided some evidence in favour of the formulation by Ikeda et al. (1981), yet this simple parameterization cannot account for the complex near bank hydrodynamics and its effect on the bank shear stress. By means of a correction factor which implicitly accounts for near bank flow patterns an extension for the bank shear stress is provided. Although more investigations are required to extend the parameterization to naturally occurring rivers with irregular cross-sections, the examples considered show improvement of the modelling of the bank shear stress for rectangular cross-sections.

5.5 Conclusion

This chapter presents the findings of a parameter study by LES for the flow in curved channels. The focus of the chapter was on streamwise flow and secondary circulation processes at the centre of the channel and near the outer bank, as well as the boundary shear stresses.

In the centre of the channel the strength of the centre-region cell of secondary flow is found to be linearly dependent on the curvature for mildly curved flows and reaches a maximum for strongly curved channels. The streamwise flow distribution in the centre of the channel shows a strong increase for increasing curvature for mildly curved flow and a decreasing trend for strongly curved flow.

The strength of the outer bank cell can be characterized for three flow regimes which correspond to mild, intermediate and strong curvature: in the mild curvature region the outer bank cell strength remains constant whereas the centre region cell increases in strength, in the intermediate curvature region both the outer bank cell and centre region cell increase in strength and finally
in the strong curvature region the centre region cell is saturated while the outer bank cell strength varies.

The upper outer bank shear stresses reach a maximum in strong curvature range. The lower outer bank shear stresses vary in a similar manner to the outer bank cell strength in the strong curvature range. Overall, however, the bank shear stresses obtain a maximum as a function of the curvature. The bed shear stresses however continue to increase for strong curvature which suggest that sharp channel bends will have a preference for deepening instead of migrating laterally, thereby leading to the migration rate pattern observed by Hickin and Nanson (1975, 1984).

The bank shear stress is subsequently parameterized by means of variables available in 1D models for meander evolution and a correction factor. The correction factor improved the parameterization considerably. One should, however, keep in mind that the cases under consideration all involved a rectangular cross-section. Further investigation is required in order to extend the bank shear stress parameterization to naturally meandering flows.
Chapter 6

Synthesis

6.1 Introduction

Detailed modelling of the flow in meandering channels requires a three dimensional model. In general, however, reduced-order models are used to deal with the large spatial and temporal scales found in naturally meandering rivers. There are many variations of such models and therefore the hierarchy of the hydrodynamic models is discussed in this section (cf. Figure 6.1). Direct numerical simulation (DNS) solves the Navier Stokes equations directly at the turbulence scale. Filtering out the smaller eddies in the Navier Stokes equations and modelling them parametrically results in the Large Eddy simulation (LES). The effect of the smaller eddies is subsequently modelled. Ensemble-averaging the Navier Stokes equations yields the Reynolds-averaged Navier Stokes (RANS). RANS models require a closure for the ensemble-averaged contribution of the Reynolds’ stresses (e.g. linear or non-linear $k-\epsilon$). Detached eddy simulation (DES) is a hybrid eddy-resolving approach which solves most of the domain by LES and employs RANS near the boundaries. Reduced-order models are obtained by depth-averaging or cross-section-averaging operations. After depth-averaging a 2DH model is obtained, and after cross-section-averaging a 1DH model is found. By adding of parameterizations, the 2DH and 1DH models are able to include simplified representations of three-dimensional effects. Such parameterizations are frequently based on mild-curvature assumptions (linear model). Recently, however, Blanckaert and de Vriend (2003) developed a non-linear parameterization model, which is also valid for strong curvature. Adding these parameterizations to 2DH and 1DH models yields quasi-3D and 1DH+ models, respectively.

Figure 6.2 shows the streamwise velocities computed by different hydrodynamic models: a non-linear 1DH+ model (cf. Chapter 2), a depth-averaged
model without any secondary flow, the non-linear Quasi-3D model (cf. Chapter 4), a 3D RANS model with \( k-\epsilon \) closure and a very large eddy simulation (VLES) model. The results show differences between the simulations.

The 1DH+ model is able to capture the general momentum redistribution from the entrance to the exit of the bend, but it does not capture the horizontal recirculation zones. This model is therefore not suited to model local features of the flow. It is however suitable to model the large spatial and temporal scales that are relevant to meandering rivers. The linear solution of the model is not shown here as it gives roughly the same result as the non-linear model (cf. Figure 2.6).

The 2DH model shows behaviour which looks reasonable, but it lacks important physical processes such as the redistribution of streamwise momentum by secondary flow. The model gives the right results, but for the wrong reasons.

The quasi-3D model also captures the behaviour from the entrance to exit of the bend correctly and shows more detail of the horizontal flow structure.

Figure 6.1: Hierarchy of hydrodynamic models.

Figure 6.2 (facing page): Streamwise velocity in the Tollense bend as computed by a) the 1DH+ model, b) the 2DH model, c) the Quasi-3D model, d) a 3D RANS model with a linear \( k-\epsilon \) closure, e) a 3D VLES model, f,g) measurements and photograph (courtesy Ingo Schnauder). The black arrow indicates the positive streamwise direction of the flow.
6.1. INTRODUCTION

- 1DH+
- 2DH
- Quasi-3D
- 3D RANS $k-\varepsilon$
- 3D VLES (time and depth-averaged)
- Measurements

Measurements: 0.3 m/s (depth-averaged)
than the 1DH+ model. The model is therefore more accurate than the 1DH+ model. It is however also more time consuming than the 1DH+ model and can therefore be applied at intermediate time and length scales.

The RANS model with a linear k-ε closure shows much more detail and even captures the two mixing layers which form at the left and right banks. It is able to model the recirculation zone at the outer bank and also captures the weak recirculation zone at the inner bank. The strength of the horizontal recirculation zones is generally underestimated by RANS models linear k-ε closure (cf. Kang and Sotiropoulos (2012)). The RANS model should be used when one is interested in a short sequence of bends. The model should be used with caution when processes such as the rate of sediment trapping in horizontal recirculation zones and counter gradient diffusion (van Balen, 2010) are relevant to the study.

The eddy-resolving VLES model is able to model the horizontal recirculation zones and clearly shows the presence of the two mixing layers. The strength of the recirculation zone near the inner bank seems to be overestimated and further investigation is recommended into the use of the Smagorinsky model for large grid sizes. It can be viewed as a scaled-down LES-version of the river bend. Even though the model is rather coarse for a large eddy simulation it is able to capture the time-varying nature of the turbulent hydrodynamics in strongly curved bends. Due the computational effort of this model, its application domain is restricted to the modelling of a few consecutive river bends.

To analyse the performance of each of the models, a schematic figure displaying the main processes occurring in meander bends is given in Figure 6.3. The accuracy of the linear 1DH+, non-linear 1DH+ model (Chapters 2,3), linear quasi-3D model, non-linear quasi-3D model (Chapter 4), a coarse RANS linear k-ε (4) model (cf. Chapter 4), a detailed RANS linear k-ε model (cf. Chapter 4), and the LES model (cf. Chapter 5) concerning the different phenomena occurring in strongly curved open channel flow is quantified in Table 6.1 and will be further evaluated in the following sections (hydrodynamics in Section 6.2, bed morphodynamics in Section 6.3, bank morphodynamics in Section 6.4. Finally in Section 6.5 an outlook on the numerical modelling of meandering rivers is given.

## 6.2 Hydrodynamics

### 6.2.1 Centre-region secondary flow cell

The centre region cell is a well studied phenomenon in curved open channel flows with Boussinesq (1868) and Thomson (1876) as its pioneers. It forms because of two counteracting mechanisms, viz. the centrifugal acceleration and
Figure 6.3: Schematic overview of processes and features in meandering channels a) top view and b) cross-section A-A': 1 – centre region secondary flow cell, 2 – reduction of centre region cell strength as it passes through the bend, 3 – the centreline of the river (blue line) and the line which splits the streamwise discharge (thus describing the momentum redistribution), 4 – momentum redistribution caused by changes in curvature, 5 – point bar, 6 – horizontal recirculation zone, 7 – outer bank secondary flow cell, 8 – forces acting on a sediment particle, 9 – sediment trapping by horizontal recirculation zone, 10 – bank erosion, 11 – bank accretion, 12 – vegetation.
Table 6.1: Process number in Figure 6.3 and the ability of different models to reproduce it (‘-‘ = bad, ‘+’ = good, ‘++’ = excellent).

| process no. | reduction of the centripetal pressure gradient delivered by the cross-stream water surface slope. Feature 1 and 2 in Figure 6.3 show the secondary flow at the entrance of the bend and the subsequent reduction of the secondary flow strength as the flow passes through the bend, respectively. The detailed RANS k-ε and LES models succeed in reproducing both of these processes. The coarse RANS k-ε (4) model is able to capture the evolution of the secondary flow along the bend, but underestimates its strength. The 1DH and 2DH models are not able to capture these processes. By adding parametric models for the secondary flow to the 1DH and 2DH models are transformed to 1DH+ and quasi-3D models, respectively. The linear 1DH+ and quasi-3D models are able to model process 1, but not process 2. The non-linear ones, however, are able to represent process 2. The 1DH+ and quasi-3d models represent the secondary flow reasonably well over mildly varying bathymetries. Over strongly varying bathymetries the limited degrees of freedom in the streamwise and transverse velocity profiles limits the accuracy of the secondary flow prediction. The secondary flow is important for the redistribution of the streamwise momentum (Section 6.2.2) | process desc. | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| process no. | | reduction of momentum | redistribution by secondary flow | redistribution by curvature variations | point bar, topographic steering | horizontal recirculation zone |
| linear 1DH+ | + | + | + | + | + | + | + | + |
| non-linear 1DH+ | + | + | + | + | + | + | + | + |
| 2DH | - | - | - | + | + | + | + | + |
| linear quasi-3D | + | + | + | + | + | + | + | + |
| non-linear quasi-3D | + | + | + | + | + | + | + | + |
| coarse RANS k-ε (4) | ++ | ++ | ++ | ++ | ++ | ++ | ++ | ++ |
| detailed RANS k-ε | ++ | ++ | ++ | ++ | ++ | ++ | ++ | ++ |
| LES | ++ | ++ | ++ | ++ | ++ | ++ | ++ | ++ |

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<th>bank shear stress for bank erosion vegetation time</th>
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<td>detailed RANS k-ε</td>
<td>++2.3/++</td>
<td>++</td>
<td>+</td>
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<td>LES</td>
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</tbody>
</table>

1Not included, but it is a reasonable assumption in mildly curved channels.
2Slightly underestimated.
3Well predicted over mildly varying bathymetry, but not over strongly varying bathymetry.
4Well predicted over strongly varying bathymetry, but not over mildly varying bathymetry.
5Well predicted for mildly curved channels.
and the formation of the point bar (cf. Figure 6.3, process 5 and Section 6.3.1).

6.2.2 Redistribution of streamwise momentum

Streamwise momentum is redistributed as it passes through a meander bend. Feature 3 in Figure 6.3 shows the centreline of the river bend in blue and the line which splits the streamwise discharge in two equal parts in red. When the red line is nearer to the inner bank the streamwise flow is concentrated near the inner bank. The opposite holds true when red line lies closer to the outer bank.

At the beginning of the bend (process 4 in Figure 6.3) the abrupt change in curvature causes the water level to increase in the outer bend and to decrease in the inner bend. Due to the change in streamwise pressure gradient, which is favourable near the inner bend and unfavourable near the outer bend, the streamwise momentum is driven towards the inner bank. At the end of the bend the opposite occurs. All of the considered models are able to capture this process. Some linear 1DH+ models, however neglect this process, which is a reasonable assumption in channels with slowly varying curvature. In channels with strong changes in curvature, Blanckaert and de Vriend (2010) showed that this process is in fact a dominant process for streamwise momentum redistribution and should therefore not be neglected.

Point bars are typical of meandering channels (cf. Figure 6.3, feature 5) and their formation is discussed in Section 6.3.1. They play an important role in the redistribution of streamwise momentum. This occurs through the process of topographic steering, which forces the flow around rather than over the point bar (cf. Nelson, 1988; Dietrich and Whiting, 1989). All of the models are able to capture this behaviour correctly up to their specific resolution.

The centre-region secondary flow cell is also important for the redistribution of streamwise momentum. Generally the secondary flow is responsible for an outward transport of streamwise momentum. This behaviour is well captured by the detailed RANS k-ε, LES, non-linear 1DH+ and non-linear quasi-3D model over a mildly varying bathymetry. The coarse RANS k-ε (4) tends to underestimate this behaviour. Over a strongly varying bathymetry the secondary flow also redistributes streamwise momentum. Locally even an inward redistribution of streamwise momentum by the secondary flow could be observed in the studied strongly curved bend experiment by Blanckaert (2010), which is contrary to most experimental observations to date. This behaviour is well captured by the LES and detailed RANS models. The coarse RANS model is able to capture this behaviour, however underestimates the magnitude. The non-linear 1DH+ and quasi-3D models are not able to model this behaviour correctly (cf. Section 6.2.2). Over a strongly varying bathymetry the role of the secondary flow for momentum redistribution is less important.
for the hydrodynamics as the other two processes of topographic steering and curvature variations are dominant. This does not imply that the secondary flow is unimportant in such cases, as it also influences the direction of the bed shear stress (cf. Section 6.3.2).

6.2.3 Horizontal recirculation zone

In zones of strong width variations, or in shallow near-bank zones horizontal recirculation may occur. Its role in sedimentation processes is discussed in Section 6.3.5. Horizontal recirculation zones (cf. Figure 6.3, feature 6) form when the near bank boundary layer flow separates from the bank. A necessary condition for this is a reversal of the streamwise pressure gradient. In relation to the Blanckaert (2010) experiment with a varying bathymetry, van Balen et al. (2010) report that this occurs on the shallowest part of the point bar, and that this coincides with the change in sign of the streamwise bed level gradient. The 1DH+ model is unable to model the horizontal recirculation zones as it lacks the proper parameterization. The quasi-3D model is able to model them in certain cases, such as the experiment by Blanckaert (2010), but it is unable to capture the recirculation zone in the Tollense River bend (cf. Figure 6.2). The RANS k-ε model is better at resolving these zones, though recent numerical evidence by Kang and Sotiropoulos (2012) suggests that RANS k-ε models give less accurate predictions than LES.

6.2.4 Outer bank secondary flow cell

Near the outer bank often a smaller second secondary flow cell occurs, called the outer-bank cell, which rotates opposite to the centre-region cell (cf. feature 7 in Figure 6.3). It is a combination of a Taylor-Görtler type instability (de Vriend, 1981a) and turbulence anisotropy, slightly amplified by the motion of the centre-region cell (Blanckaert and de Vriend, 2004; van Balen, 2010). Furthermore, Blanckaert and de Vriend (2004) stated that accurately modelling the outer bank cell requires a turbulence model which can transfer turbulent kinetic energy to the mean flow. A large-eddy simulation model does, whereas a RANS model with a linear k-ε closure is unable to do so correctly. Kimura et al. (2008) recently showed promising results with a coarse non-linear k-ε model which was able to capture the outer bank cell correctly.

The strength of the outer bank cell can be categorized for three flow regimes which correspond to mild, intermediate and strong curvature. In the mild-curvature regime the outer-bank cell strength remains constant, whereas the centre-region cell increases in strength, in the intermediate-curvature regime both the outer-bank cell and centre-region cell increase in strength and finally in the strong-curvature regime the centre-region cell is saturated while the outer-bank cell strength still varies.
The outer bank cell was not observed in Blanckaert (2010)’s strongly varying bed experiment. In fact the outer bank cell was also not observed in numerical results in the LES by van Balen et al. (2010b) or in the DES (Detached eddy simulation) by Constantinescu et al. (2011). van Balen et al. (2010b) showed that the RANS and LES models showed good agreement for the larger part of the flow in that experiment, and concluded from this that turbulence-related momentum transport was of minor importance compared to the momentum transport induced by the strong curvature and the complex topography.

Although reduced-order and RANS k-ε models are unable to model the outer-bank cell correctly, the redistribution of streamwise momentum is hardly affected by this shortcoming, as the outer-bank cell is weaker and smaller in size than the centre-region cell. The effect on the bank shear stress in the 1DH+ and quasi-3d models in curved open channels is discussed in Section 6.4.1.

6.2.5 Internal shear layer

van Balen et al. (2010a) reported the occurrence of an internal near the inner bank, starting at the bend entrance. This shear layer is the result of fluid with high momentum streamwise which interacts with upwelling low-momentum fluid Blanckaert et al. (2012b). Only the LES model succeeds in modelling this flow phenomenon correctly. So far, this phenomenon has however only been observed in laboratory flumes with a rectangular cross-section, and its occurrence and importance in natural rivers remains to be investigated.

6.3 Bed morphodynamics

In meandering rivers the bed is generally erodible. The conservation of sediment mass can be described by the Exner balance which relates the evolution of the bathymetry in time to the spatial variation of the local transport rate. The local transport rate is related to the bed shear stress and a gravitational pull model which quantifies the relative influence of the force of gravity on a sediment particle. The direction of the shear stress in relation to the local morphology is therefore of importance for reproducing the morphodynamics. Although this goes to some extent for suspended load transport, we will focus in the following on the bed load transport.

6.3.1 Formation of the point bar

The point bar initially forms due to two counteracting mechanisms, namely the uphill drag force induced by the transverse component of the bed shear
stress and the downhill component of the gravitational force. Furthermore, point bar formation is also influenced by the low velocities over the point bar and the trapping of sediment by horizontal recirculations.

### 6.3.2 Direction of the bed shear stress

In fully developed straight open channel flow, the streamwise velocity profile can be approximated by a logarithmic function. In curved open channel flow the presence of secondary flow causes the bed shear stress to be rotated away from the streamwise flow in the direction of the inner bend. The LES and the detailed RANS k-\(\epsilon\) model accurately reproduce the bed shear stress direction in strongly curved bends over strongly varying bathymetries. In the rectangular cross-section in the experiment by Blanckaert (2009), the RANS model underestimated the angle of the bed shear stress. The RANS k-\(\epsilon\) (4) underestimated the transverse bed shear stress component. Over mildly varying bathymetries quasi-3D models are able to capture the distribution of the bed shear stress reasonably well. The quasi-3d model even outperforms the RANS k-\(\epsilon\) (4) model in strongly curved bends over mildly varying bathymetries as its representation of the boundary layer turns out better in the parameterised form. Over strongly varying bathymetries the RANS k-\(\epsilon\) (4) model outperforms the quasi-3d model as it has more degrees of freedom. In the 1DH+ model a cosine-shaped lateral distribution of the transverse bed shear stress is adopted, which complies with the imposed distribution of the secondary flow strength. The resulting transverse bed slope reasonably matches the equilibrium value obtained from experimental data, however a more detailed inspection 1DH+ model’s prediction of the bed shear stress direction shows it is erroneous. This suggests the 1DH+ model can only be used when the bathymetry is mildly varying.

### 6.3.3 Magnitude of the bed shear stress

The magnitude of the bed shear stress is well approximated by the RANS k-\(\epsilon\) model. The RANS k-\(\epsilon\) (4) model underestimates the local magnitude of the bed shear stress by up to 50%, which can be explained by the poor resolution of the boundary layer. The distribution of the magnitude of the bed shear stress in the RANS k-\(\epsilon\) (4) model is however similar to that according to the RANS k-\(\epsilon\) model. In the 3D models the increase of the bed shear stresses is included by definition. In the quasi-3D and the 1DH+ model the increase in magnitude of the bed shear stress is included by means of the non-linear secondary flow closure model. Over mildly varying bathymetries, the quasi-3D model shows good agreement, but over strongly varying bathymetries the magnitude is reasonably approximated, but the distribution is different from the RANS models and the measurements. The bed shear stress in the quasi-3D
and the 1DH+ model is also affected by the streamwise flow distribution. The quasi-3D model captures the streamwise flow distribution reasonably well. The 1DH+ model underestimates the outward flow redistribution over the strongly varying bathymetry, which implies that the effect of streamwise flow variations on the bed shear stress distribution is underestimated by the 1DH+ model.

6.3.4 Gravitational pull model

The gravitational pull model quantifies the relative influence of the force of gravity on a sediment particle. For weakly sloping streamwise slopes the gravitational model was refined by reanalysing experimental data by Zimmermann and Kennedy (1978) using the non-linear secondary flow model by Blanckaert and de Vriend (2003). The resulting gravitational model in combination with the 1DH+ model resulted in lower transverse slopes compared to the earlier findings by Olesen (1987) based on the linear secondary flow model by de Vriend (1977).

6.3.5 Sediment trapping by horizontal recirculation

Horizontal recirculation zones may occur in meandering rivers (cf. Section 6.2.3). These low-momentum recirculation zones are important for the trapping of sediments Wright and Kaplinski (2011); Schnauder and Sukhodolov (2012) and they form zones of refuge for aquatic fauna. Sediment trapping in these zones occurs due to the combination of the low velocities and the so-called “teacup-effect”. Due to the presence of secondary flow, directed inwards near the bed and outwards near the free surface, sediment tends to concentrate at the centre of the recirculation zone (cf. process 9 in Figure 6.3). The 1DH and 1DH+ models are unable to capture these horizontal recirculation zones and therefore do not include this sediment trapping mechanism. The 2DH and quasi-3d models are usually able to capture the horizontal recirculation zone. By lacking a secondary flow parameterization, necessary to have the right direction of the bed shear stress, the 2DH model is unable to capture this process. The quasi-3d models are able to do so, as they do include a secondary flow closure. The 3D models reasonably capture this process, although Kang and Sotiropoulos (2012) report that the onset of the horizontal recirculation zone in RANS k-ε models occurs for higher transverse slopes than in LES. This implies that the sedimentation trapping rate in horizontal recirculation zones may be underestimated by RANS and quasi-3D models.
6.4 Bank morphodynamics

Migration of meandering rivers occurs by erosion and accretion of the banks. Generally, bank erosion occurs at the outer bank and bank accretion occurs at the inner bank and the shear stresses on the bank are important agents.

6.4.1 Bank shear stress

The bank erosion rate (process 10 in Figure 6.3) depends partly on the bank shear stress. This hydrodynamic forcing on the bank is investigated by means of a parameter study using LES. The bank shear stress is influenced by the near-bank velocity and by the complex dynamics near the outer bank (e.g. the outer bank cell). The processes which influence the near-bank velocity are described in Section 6.2.2. The comparison of the LES model with the 1DH+ model shows that both models reveal a similar behaviour of the streamwise velocity near the outer bank. The outer-bank cell locally causes a reduction of the bank shear stress in the upper half of the water column and an increase of the bank shear stress in the lower half. This is caused by the transfer of high streamwise momentum to the toe of the bank. On the basis of the parameter study performed in Chapter 5 the overall effect (reduction or increase of the bank shear stress) cannot be confirmed or disproved as it requires a simulation without the presence of the outer bank cell in otherwise identical flow. To deal with the complex outer-bank hydrodynamics, a parameterized model for the outer-bank shear stress is developed, which will improve the representation of the hydrodynamic forcing on the bank. Appendix A shows a practical application of this parametric model included in the 1DH+ model.

6.5 Meander models: Possibilities, Uncertainties and Outlook

The future of numerical modelling of meandering rivers will depend on the future of modern computing and the ability to describe the physical processes involved. The present accuracy of the process description of the processes depends on different factors:

- the scale of the river system, the available computational resources and associated computational detail,
- parameterization and closure models used,
- variability in the river system (e.g. discharge, vegetation cover, sediment supply),
• and the scarcity of data (e.g. geotechnical bank properties, vegetation cover, bed (surface) sediment characteristics, discharge hydrograph).

6.5.1 Hydrodynamics

Three-dimensional hydrodynamic computations are possible at high levels of detail with the solution of the Navier Stokes by direct numerical simulation (DNS). The largest DNS simulation to date works with $4096^3$ grid cells with Reynolds numbers comparable to laboratory experiments (Ishihara et al., 2009). Coleman and Sandberg (2010) estimate that, if Moore’s law continues to hold, the Reynolds number for similar flows can only be doubled every five to six years, which implies that DNS of Reynolds number flows typical of engineering and geophysical applications are still far out of reach.

Reynolds-averaged approaches (RANS), large eddy simulations (LES) and detached eddy simulations (DES) model the smallest scales of turbulence by means of so-called closure models. These models have shown great promise in recent years for the modelling of schematized meandering flows (e.g. Stoesser et al., 2008; Zeng et al., 2008b; van Balen et al., 2009; Constantinescu et al., 2011). These models are frequently used to model sequences of bends at a constant discharge. To increase the capabilities of coarse 3D models it is recommended to improve the boundary layer treatment using an approach similar to Blanckaert and de Vriend (2003). For larger time and length scales frequently encountered in river meandering, the hydrodynamics are often modelled through 1DH, 2DH and quasi-3D models. 2DH and quasi-3D models often employ a sequence of steady state hydrographs whereas 1DH approaches may used a variable discharge hydrograph.

6.5.2 Bed morphodynamics

Recently Nabi (2012) succeeded in coupling a LES model to a spherical particle based model for the bed and suspended load transport and achieved good agreement to experimental data. The time needed to reach a quasi-steady state including sediment was roughly seven times the simulation time needed to reach a hydrodynamic quasi-steady state. Generally, however, sediment transport in 1DH, 2DH, quasi-3D and 3D models is described by means of empirically derived sediment transport formulae for the bulk transport combined with the Exner sediment balance equation. These models have often been derived for mildly sloping bathymetries and although results obtained with these models are often good, they are frequently used outside the range of the experimental data on which they have been calibrated. Recent work by Seminara et al. (2002) and Parker et al. (2003) presents a bed load transport model based on the force balance on a particle. This may bridge the gap between the
6.5.3 Bank migration processes

Bank erosion (cf. Figure 6.3, process 10) is commonly modelled with a bank erosion model, like the one first presented by Ikeda et al. (1981), which can be interpreted as a linearization of an erosion formulation for cohesive sediment by Ariathurai and Arulanandan (1978). This model is widely used for modelling bank erosion, but misses certain physical processes such as mass failure mechanisms. 2D bank erosion models like the one used by Darby et al. (2002) and Motta et al. (2012) allow greater detail as soil properties can also be specified in the vertical. Overall the accuracy of such models is hampered by the uncertainty about the bank soil properties and the space and time scales at which meander models are applied. The latter makes comprehensive data collection very expensive. Motta et al. (2012) found that the effect of horizontal floodplain heterogeneity is at least as important in describing the evolution of meandering rivers as a detailed description of the hydrodynamics.

Another mechanism which is usually unaccounted for is the fate of eroded bank material. Mosselman (1998) and more recently, Dulal et al. (2010) do include the effect of the slump block in a quasi-3D meander model, but most meander models neglect this effect, (implicitly) assuming that this block of soil is transported down the river as wash load. In reality it may remain for some time at the toe of the bank, and influence the flow and morphology in the channel.

While bank erosion can be linked to some extent to a physical mechanism, bank accretion (cf. Figure 6.3, process 11) is based mostly on the empirical finding that over long time scales meandering rivers tend towards a uniform width. The process of accretion relies on sedimentation processes near the inner bank, which are influenced by the establishment and survival of pioneer vegetation.

In meander evolution the occurrence of modelling of neck cut-offs can be described reasonably well, however the occurrence of chute cut-offs can only be included in a probabilistic manner, quite arbitrary and neglecting possible underlying physical mechanisms.

6.5.4 The role of vegetation

Vegetation (cf. Figure 6.3, process 12) on the river bed, on the banks and on the floodplain may influence the flow, erosion and accretion processes. This is a topic which has received some attention in previous years, but the processes are not yet fully understood. At present very detailed models exist of the flow around flexible rods which are claimed to represent vegetation. Although the
approach is not wrong, it remains simple compared to the complexity associated with actual vegetation. In 1DH, 2DH and quasi-3D models vegetation is often parameterized by means of a hydrodynamic roughness coefficient and an increased critical shear stress for bank erosion. The impact of vegetation on bed morphology and bank morphology remains uncertain, which is mainly due to the uncertainty of the vegetation height, width, density and the distribution of roots in the banks which can only be estimated.

6.5.5 Outlook

Having summarized the evolution of computational possibilities and the accuracy of process-based models at the large time and space scales of river meandering, it is expected that two lines of development are most likely to be followed in the future. As long as the uncertainty in underlying mechanisms such as bank erosion, bank accretion and the role of vegetation remain high, the presently used reduced-order models will still be used for the evolution of meandering rivers.

Two approaches are likely to be dominant in that case. The first is related to Monte-Carlo simulations, by which the role of variability and the uncertainty in the model predictions can be quantified (van Vuren, 2005; van Adrichem, 2013). The second combines the Monte-Carlo approach with continually assimilated data from rapidly improving data collection techniques which (e.g. water and bed level measurements by multibeam scanning, LIDAR-images, etc.), similar to approaches in weather modelling or flood forecasting.

As reduced-order models are likely to be widely used in the future, it is also expected that the research into the insufficiently resolved processes will continue. Important processes which require further understanding (and parameterization) are related to bank accretion and erosion, the role of vegetation and the bank strength. Furthermore it is expected that the research into eddy resolving techniques such as DES will continue until they can also be applied to the large temporal and spatial scales involved in river modelling.


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Appendix A

Practical application

In the framework of Room for the Vecht a side channel was attached to the Vecht (cf. Figure A.1) by the water board Velt and Vecht. Part of the newly created side-channel follows an old meander bend of the Vecht. The side channel, which only flows at high river discharges, has been created to increase the capacity of the River Vecht as well as to create variations in the landscape which are expected to be beneficial for the local ecology (Dienst Landelijk Gebied, 2012). Furthermore, it has been designed as a park with hiking paths and benches so that people can enjoy the views.

The present section shows a rapid assessment of the expected morphological evolution of the side channel of the River Vecht. The hydrodynamics and bed morphology are solved by the reduced order non-linear models for flow and bed morphology as presented in Chapters 2 and 3. Bank migration is computed by means of the formulation proposed by Ikeda et al. (1981). The expected migration of the channel is expressed as change of the centre line of the river:

$$\frac{\partial n_c}{\partial t} = E_u (\psi_{\text{bank}} U_{\text{bank}} - U)$$

(A.1)

where $U$ is the bulk velocity, $U_{\text{bank}}$ is the near bank velocity, and $\psi_{\text{bank}}$ is a correction coefficient accounting for the increase in bank shear stress caused by curvature and aspect ratios (Chapter 5). $E_u$ is the migration coefficient. $E_u$ is estimated using the empirically derived relation by Hanson and Simon (2001) (cf. Crosato (2008) and Darby et al. (2010)):

$$E_u = \frac{d}{\sqrt{\tau_{cr}}}$$

(A.2)

where $d = 2 \cdot 10^{-7}$ m $N^{-0.5}$. 
Figure A.1: a) Design sketch of the side channel of the Overijsselse Vecht near Junne overlaid on Google maps and b) finalized overview photograph (Dienst Landelijk Gebied, 2012). The red and yellow arrows show the direction of the flow in the main and side channels respectively.
A.1 Modelling assumptions

From the entrance to the end of the side channel there is a water level drop of around 1.5 m. In the main channel there is a weir located at the bridge to deal with the large water level difference. At the entrance to the side channel a cascade has been constructed, which reduces the water level slope in the remaining part of the side channel at low discharges. In the rapid assessment the water and bed level slopes are assumed to be constant over the length of the channel. The bed roughness height is estimated as 1.2 mm according to Vermeer and Struiksma (1978). Near the inflow section up to the bridge (Junnerweg) crossing the side channel the bank is protected by interlocking boulders. The outflow section is also protected by interlocking boulders. For the remaining part, the side channel the bed and banks have not been protected which allow natural processes such as erosion and sedimentation of the bed and migration of the river banks.

The discharge distribution has been schematised in two scenarios. The first scenario is similar to the currently implemented design. The discharge distribution is evaluated according to a simplified cross-section of the main channel and the adjustable weir at the entrance of the channel (cf. Figure A.2a). This results in two curves for the distribution of the total discharge between the main channel and the side channel (for the summer and winter, see Figure A.2b). The resulting discharge series for the first scenario is shown in Figure A.2c). The second scenario is a fictive approach in which the side channel admits discharges up to $30\text{m}^3/\text{s}$ (see Figure A.2c). The discharge in excess of $30\text{m}^3/\text{s}$ flows through the main channel.

A.2 Results

Spruyt et al. (2012) mention that Verheij et al. (1995) quantifies the critical shear stress on sand banks ($\tau_{cr} = 0.15N/m^2$), clay banks ($\tau_{cr} = 3N/m^2$) and vegetated banks ($\tau_{cr} = 95N/m^2$). In this rapid assessment, as we do not know the exact composition of the bank we compute 3 cases in which the banks are uniformly composed of either sand, clay or covered by vegetation. The simulations are run using the 28 year long discharge scenarios presented in Figure A.2c).

For the first scenario shown in Figure A.3a) there is hardly any bank erosion after 28 years according to the model for all three cases, which suggest that there will be almost no bank migration in the side channel.

For the second fictive scenario shown in Figure A.3b) it may be observed that sandy banks migrate faster than banks composed of clay and they in turn migrate faster than vegetated banks. These results show that managing the
Figure A.2: a) Schematic cross-section near the upstream entrance of the side channel indicating the summer and winter heights of the adjustable weir. b) Resulting side channel discharge as a function of the total upstream discharge for Scenario 1. c) Resulting side channel discharge distribution for Scenario 1 and 2.
vegetation cover or the cohesivity of the banks also allow a reduction of the migration velocity. Another important observation in Figure A.3b) is direction of the meander migration as indicated by the black arrows, which show that the sidechannel migrates away from the main channel. This indicates that the left bank does not require any further protection to prevent an intermediate connection between the side and the main channel.

Finally in Figure A.3c) a comparison of the non-linear model with its mild curvature is shown, linear mild amplitude equivalent. The results shows that the linear model overestimates the bank migration as it neglects the non-linear feedback mechanisms which reduce the secondary flow strength. Neglecting these feedback mechanisms results in a higher transverse bed slope and a higher outward transfer of streamwise momentum (by secondary flow and by increased topographic steering).

The presently used bank migration model (Eq. (A.1)) is a simple formulation which gives an impression of the expected meander behaviour. Currently in cooperation with the Chinese Academy of Sciences the reduced order non-linear models for flow and bed morphology is being combined with the Bank Erosion and Retreat Model (BERM), by Chen and Tang (2012) containing more advanced bank erosion mechanisms. Furthermore, the water board Velt and Vecht will continue to monitor the evolution of the channel. This information is vital to ensure safety from flooding and improve our understanding of the mechanisms of meander migration and the modelling thereof.

**Acknowledgements**

The author would like to express his gratitude to Paul Termes and Anne Wijbenga of HKV consultants for suggesting this interesting case and providing the necessary data to setup the model.
Figure A.3: a) Meander evolution for sand, clay and vegetated banks for Scenario 1. b) Meander evolution for sand, clay and vegetated banks for Scenario 2. c) Meander evolution for clay bank for Scenario 2 indicating the difference between the reduced order non-linear meander model and its mild curvature, linear mild amplitude equivalent.
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