Characterization of the Atmosphere in Jupiter's Great Red Spot

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CHARACTERIZATION OF THE ATMOSPHERE IN JUPITER’S GREAT RED SPOT

by

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An electronic version of this thesis is available at http://repository.tudelft.nl/.
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Polarimetry is a powerful remote sensing technique that can be used to characterize planetary atmospheres and potentially, to detect and characterize exoplanets. The degree of linear polarization as a function of wavelength and phase angle varies with the presence and composition of cloud, haze and gas particles. In particular, when in-situ measurements are difficult to obtain, this technique can be used to obtain an approximate atmospheric profile. This is the case of Jupiter. The aim of this thesis is to compute a detailed profile of the atmosphere in the Great Red Spot of Jupiter. Polarimetry observations for this planet at different wavelengths and a phase angle of 10.7° are used for the fitting process. The objective of the project is to obtain the cloud and haze particles present in the atmosphere, together with the altitude at which they are. For this purpose, a FORTRAN code is used to simulate the polarization values by means of a doubling-adding radiative transfer algorithm. Mie scattering theory is used to obtain the particle properties present in the atmosphere. In order to find the best match of the simulations to the observations, a Matlab fitting algorithm is developed. Once, the fitting is done, the main conclusions obtained are that in the Great Red Spot, higher and thinner clouds are present than in the surroundings. The atmospheric profile that best fitted the observations includes a base Water crystal cloud layer, followed by a higher Amonia cloud and finally, a top haze layer. It must be noted that at higher wavelengths, the presence of the GRS is clearer but also it was challenging to find a fitting of the highest polarization values. This fitting method can be also extrapolated to Jupiter-like exoplanets, therefore being able to, either simulate what a planet would look like if it existed by changing the geometry conditions, or characterize a already known exoplanet if polarization measurements are available. An additional analysis was done simulating how the Great Red Spot would look like if observed at different phase angles as if the measurements were taken with a satellite in-situ including a polarimeter on-board. The conclusion was that as the phase angle increased, so did the degree of polarization. In the case of the Great Red Spot, the regions of higher and lower polarization could still be identified. However, as the phase angle increased and so did the shadow region of the planet, the fitting became more difficult.
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In this thesis, the atmosphere of Jupiter and, in particular, the atmosphere in the Great Red Spot (hereinafter referred to as GRS) will be studied using as a baseline polarimetry data. Jupiter is one of the most interesting planets in the Solar System. It is a gaseous planet that has an atmosphere composed mostly by helium and hydrogen and a core that may contain heavier metals (Hubickyj et al., 2005, Militzer et al., 2008). Its atmosphere shows bands of different colors known as belts and zones (see 1.1). These bands and zones originate by dynamical processes in the atmosphere that induce convection and subsistence of the clouds and hazes forming the layers. Belts are the darker regions, formed by thin clouds at lower altitudes with still unknown composition and warmer temperature than zones. On the other hand, zones are the brighter regions, thought to be composed of dense ammonia ice clouds at higher altitudes, corresponding to upwelling of air. And above the ammonia cloud layer, there is a layer of haze particles (Ingersoll et al., 2004). In this thesis, we refer to haze as particles with sub-micron radii in optically thin layers and cloud denotes particles of larger size, in optically thicker layers.

In this work, we will focus on Jupiter’s Great Red Spot. The GRS is the most striking feature in Jupiter’s atmosphere, and, whether or not it is seen from a telescope, depends on the wavelength at which you are observing. It is probably more than 300 years old, it has been observed since 1878 but it was first observed by Cassini around 1666 (Flasar et al., 1981, Streett et al., 1971). The GRS is elliptical with a long axis of 40,000 km and a short axis of 28,000 km, but it is shrinking. The latitude of the long axis is 22° south to the Equator approximately (Hide, 1961, West et al., 1986). The latitude is constant but longitude varies a few degrees per year, in particular, it has a 3-month oscillation with an amplitude of 1 degree; and an 8 year oscillation with an amplitude of 8°. The clouds in the GRS rotates counterclockwise with a period of approximately 6 Earth days. In the spot, a photochemical production of red phosphorus particles, whose intensity is also related to the dynamical activity in the atmosphere, causes the red coloration and this explains why the GRS has a more intense color compared to the rest of the planet (Prinn and Lewis, 1975). This difference in the reflective properties of the GRS is more obvious in the ultraviolet and blue filters. In these filters, the GRS is darker than the ambient at-
mosphere (Muñoz et al., 1999).

Figure 1.1: Jupiter representation from Cassini mission, belts, zones and the GRS can be clearly distinguished. Source: NASA/Cassini.

Regarding the origin of the spot, there are two main theories that try to explain it. The first is related to Taylor Columns. This theory explains that a topographical feature in the surface (or inhomogeneity in the bottom of the atmosphere) with sufficiently large horizontal dimensions can create a disturbance in the atmosphere and, due to the rapid rotation of the planet, create a phenomenon such as the GRS (Hide and Ibbetson, 1966). The second theory is known as Cartesian Diver and explains that a mass of $H_2$-rich molecular solid floating within a fluid layer of helium and hydrogen would create such a kind of feature (Streett et al., 1971). The most accepted theory is the first one related to Taylor Columns.

The main objective of this project is to model the atmosphere in the Great Red Spot. There is considerably more information on Jupiter’s atmosphere as a whole, compared to the atmosphere in the GRS. These observations include both, ground-based measurements and Earth-orbiting telescope measurements. Also, there are many measurements coming from space missions, such as Pioneer 10 and 11 (West et al., 1986), Voyager 1 and 2 (Smith et al., 1979, West et al., 1986), Ulysses, Galileo (Vasavada et al., 1998), Cassini (Porco et al., 2003) and New Horizons. Currently, the Juno mission has been launched in 2011 and arrived to Jupiter in 2016 and its objective, among other, is to study the origin, interior structure and atmospheric composition and dynamics of the planet (Bolton, 2010, Matousek, 2007). This might bring some light to the existence of a solid core and in particular to the dynamics taking place in the Red Spot.
In West et al. (1986, 2004) a detailed description of the cloud structure and haze composition of the planet is given based on theory and observations from the different missions mentioned above (except for New Horizons and Juno) and some ground-based measurements. In particular, West et al. (2004) includes images of Jupiter and results obtained from the Cassini mission. These images were taken in 619 nm, 727 nm, and 750 nm wavelengths, which include the strongest methane and $H_2$ absorption bands. Some differences in composition between the southern and the northern half of the planet have also been noticed. Also between the upper half and the lower half of the spot. These differences were observed also in West et al. (2004) from the Cassini mission observations and it was concluded that they were due to seasonal effects. Also, it was concluded that the GRS, which is located in the southern hemisphere, is a region of elevated tropospheric haze. Another paper that focus in describing the atmosphere of Jupiter, mostly based on flux measurements is Flasar et al. (1981). In this paper, they suggest three distinct layers: an upper troposphere (from 100 to 500 mbar) in the $H_2$ absorption line, an upper stratosphere (3 to 10 mbar) in the $CH_4$ absorption line and a lower stratosphere (from 10 to 100 mbar) in the $CH_4 / H_2$ absorption line, but mainly unknown components present at longer wavelengths These layers have been derived from observations in those absorption lines.

In Fletcher et al. (2010), the GRS is described as an inhomogeneity in the core region present in all filters except in the 8.59 micrometers (most sensitive to tropospheric aerosol opacity) at altitudes from 80 to 600 mbar. This region is warmer than the surrounding cold vortex and located at 1-2° south from the geometric center. The size of the warm core was 8° in longitude and 3° in latitude. Finally, there are large contrasts present in the thermal field that have consequences on the wind field in the 100-500 mbar region. In this paper, also the thermal profile is described in detail.

However, all these findings were done based mainly on theory or flux measurements. Polarimetry a very powerful technique to characterize the atmosphere of a planet. Integrated over the stellar disk, direct starlight (i.e. light coming directly from the star) can be considered to be unpolarized (Kemp et al., 1987), while reflected starlight from the planet will generally be polarized. Whereas spectrometry focuses on measuring the flux reflected from the planet, independently of the polarization of this light. Polarimetry is also used in exoplanet detection (Seager et al., 2000). The degree of polarization of the reflected light depends on the characteristics and properties of the planetary atmosphere. Therefore, if measured, it gives information of the particles present and the altitude of the clouds. Most of the atmospheric characterization up to now was done based on flux measurements coming from spectrography. However, polarimetry, if successful, allows a more accurate fitting of the observations, since it is more sensitive to the atmospheric composition. This technique has already been demonstrated in Venus (Hansen and Hovenier, 1974) and in the Earth (Hansen and Travis, 1974).

In this project, the observations used for the fitting were obtained using polarimetry observations. As is was mentioned, these kind of measurements are more sensitive to the
atmospheric composition (i.e. aerosol and gas particle composition, cloud and haze altitude and thickness), therefore they will allow a more accurate atmospheric characterization of Jupiter compared to the results using flux measurements. Simulations of polarimetry data will be performed varying the atmospheric composition and optical properties to fit these observations for the Great Red Spot area, which has only been characterized using flux measurements. Taking this into account, it is clear that there are some areas that have not been covered and that can be studied to bring more light into the dynamics and the composition of the atmosphere in Jupiter and in particular, in the GRS. For this purpose, in the next section, the research questions that will be addressed in this thesis will be presented.

1.1. RESEARCH QUESTIONS
The main motivation of this thesis is to characterize the atmosphere in the GRS. In order to be able to do this, the research questions that will be addressed in this thesis are:

- What is the atmospheric profile in Jupiter’s Great Red Spot derived from polarimetric observations?
  - What is the composition of the clouds and haze present in the GRS atmosphere?
  - What is the altitude at which the clouds and haze can be found?

For this purpose, we use a numerical model to simulate the polarimetry curves for the whole planet and then fit these simulations to the observations. The algorithm has three distinct parts. First, the properties of the particles that form the clouds and the haze are computed. This is done using the Mie scattering algorithm, as presented in De Rooij and Van der Stap (1984), in order to obtain the scattering matrix coefficients of the cloud and haze particles. Then, a model of a layered atmosphere composed by the particles previously computed is assumed and the Stokes parameters are computed using a doubling-adding radiative transfer algorithm (De Haan et al., 1987). Finally, the Stokes parameters are obtained for each pixel together with the degree of polarization and the angle of polarization. This is all coded in FORTRAN. Once these results are obtained, the fitting is done using Matlab. The characteristics of the particles and the layers of the atmosphere were derived from models presented in McLean et al. (2017), Muñoz et al. (1999), Sromovsky et al. (2017), Stam et al. (2004) and values for the refractive index and depolarization factor were obtained from Penndorf (1957, 1962).
1.2. Outline

In this report, the theoretical background and the concept of polarimetry will be explained in Chapter 2. A description of the observations that have been used as a reference for the fitting of the simulations will be presented and explained in Chapter 3. A detailed explanation of the numerical model, including the equations used to simulate the polarimetric curves will be given in Chapter 4. A detailed description of the fitting algorithm is given in Chapter 5. The results that have been obtained, including the atmospheric profile that best fitted the observations, will be presented in Chapter 6. An additional study of the dependence of the polarization at different phase angles was performed and the conclusions will be presented in Chapter 7. Finally, the conclusions drawn from this project will be explained in Chapter 8. Some additional information regarding the validation of the code is included in Appendix A.
THEORETICAL BACKGROUND

In this chapter, the theoretical background of the polarimetry technique will be described. First, the concept will be explained, together with the main parameters that will appear in the model. Then, the process needed to compute the degree of polarization of a planet will be described. This process consists of three parts: first, the properties of the aerosol and cloud particles are obtained using the Mie Scattering theory. Then, these particles are included in the atmosphere profile, modelled as a set of horizontal layers composed by gas particles and that may or may not contain cloud or aerosol particles. This is used in the radiative transfer doubling-adding code, that computes the coefficients of the Stokes vector, which will be explained in this chapter. The final step is to obtain the local values of the degree of polarization, $P_L$, the angle of polarization, $\chi$, and other parameters that are relevant to the results.

2.1. Polarimetry

Polarimetry is a very powerful technique that can be used to characterize the atmosphere of a planet. Direct starlight can be considered to be unpolarized when integrated over the stellar disk (Kemp et al., 1987), while reflected starlight from the planet will generally be polarized. The degree of polarization of the reflected light depends on the characteristics and properties of the planetary atmosphere, therefore, if measured, it gives information of the particles present and the altitude of the clouds and hazes. This can also be used in exoplanet characterization, since some exoplanet detection techniques are based on measuring the reflected light from the star (Seager et al., 2000, Stam et al., 2004). For our Solar System, this technique has already been demonstrated in Venus (Hansen and Hovenier, 1974) and the Earth (Hansen and Travis, 1974).

Most of the methods for atmospheric planetary characterisation are based on flux measurements. These measurements, while being able to provide an accurate fit, are less sensitive to the particles properties than those obtained with polarimetry. This situation leads to several possible fits for the same atmosphere. Polarization avoids this situation since it strongly depends on the particles present, the composition and the altitude...
at which they appear, therefore imposing more constraints on the problem. The main challenge when fitting polarization observations, is that finding the atmospheric profile that best fits these observations is complex. It depends on many parameters such as size distribution and optical properties of the aerosol and gas particles, the model used for the layered atmosphere or the altitudes at which the haze and clouds are found. In this project, a first attempt to fit the GRS, starting from models for the atmosphere of the whole planet and focusing on cloud altitude, will be done.

The objective of using polarimetry is to obtain the degree of polarization of the reflected light. This value will depend on the aerosol and gas particles that are present in the atmosphere. Scattering is a physical phenomena where light is deviated from its straight trajectory by one or more paths due to localized non-uniformities through which they pass. Different particles at different altitudes will scatter the light coming from the star differently (see Figure 2.1). In the atmosphere, multiple and single scattering can be found. Single scattering occurs when the light is deviated only by one center. If there are many centers together, this light will scatter many times causing multiple scattering. This occurs specially in optically thick media. Regarding single scattering, depending on the size of the particles, different types of scattering can occur. Using the ratio of the radius of the particle versus the wavelength (see Eq. 2.1),

$$x = \frac{2\pi r}{\lambda}$$

(2.1)

if $x \approx 1$, light scatters according to the Mie scattering and they, if $x \gg 1$ light scatters according to their projected area, finally, if $x \ll 1$ Rayleigh scattering appears, and the light is equally scattered in all directions (see Figure 2.2). This applies to spherical particles, if there are other irregular shapes, other approximations need to be used.

Figure 2.1: Sketch of the types of scattering.
On the other hand, if the phase angle is low, the observed light crosses the atmosphere perpendicular to the local horizontal, whereas if the phase angle is large, the observed light will cross a larger area of the atmosphere until it reached the observed point, leading to more scattering of this light and a a higher degree of polarization. Therefore, if the phase angle is low, the relevant parameter to take into account is the altitude at which the particles are present, whereas for large phase angles, it is the variation of particle size across the layers of the atmosphere that is more relevant.

For example, an atmosphere with a layer of gas that absorbs the light at a certain wavelength will have a higher degree of polarization if a filter close to that wavelength is used than another profile that has more clouds in that area. On the other hand, Rayleigh scattered light (i.e. scattering of very small particles, compared to the wavelength, detected by short wavelengths) reach higher polarization values. Also, thicker clouds increase the multiple scattering, reducing the overall degree of polarization. All these phenomena depend on the optical properties, size and amount of particles in the atmosphere.

Therefore, the physical parameters to take into account when characterising the atmosphere are: the size distribution and the refractive index of the particles, the optical thickness of the aerosols in the cloud or haze (multiple and single scattering optical thickness, scattering coefficient and absorption coefficient, which depend on the optical properties of the scattering particles). Finally, other parameters to bear in mind are the wavelength and phase angle at which the observations were taken (which are given by those observations), and the number of layers and pressure level at which these layers are fixed (which is selected depending on the atmospheric model to be used). These layers are important for the radiative transfer algorithm and have a large influence in the results obtained. These parameters will be explained in detail in this chapter.

2.2. Stokes Vector and Polarization

In the next sections, the process and the expressions used to obtain the parameters above mentioned will be described. The objective is to obtain the state of polarization of the planetary radiation. This can be computed from the parameters of the Stokes vector, also known as intensity vector, \( F(\alpha, \lambda) \). This vector includes the Stokes parameters and
is a function of the wavelength $\lambda$ and phase angle $\alpha = 180^\circ - \Theta$, where $\Theta$ is the single scattering angle. Once this vector is obtained, the degree of polarization is a function of these parameters and the local geometry values that correspond to each pixel of the planet.

$$F(\alpha, \lambda) = \begin{bmatrix} F(\alpha, \lambda) \\ Q(\alpha, \lambda) \\ U(\alpha, \lambda) \\ V(\alpha, \lambda) \end{bmatrix}$$ \hspace{1cm} (2.2)$$

The first component of the Stokes vector, $F$, defines the total flux, the second and third components, $Q$ and $U$, define the linearly polarized flux and the last component, $V$, is the circularly polarized flux, which will be assumed to be negligible in this project. These parameters have units of $Wm^{-2}m^{-1}$, which integrated over the wavelength gives the units across the filter, and are defined with respect to a reference plane called the scattering plane. In this case, this plane is defined by the center of the star, the planet and the observer. In order to transform the output between two other reference planes, a rotation matrix, $R(\alpha, \lambda)$, such as the one presented in Eq. 2.3, is used, where $\beta$ is the angle between the two reference planes.

$$R(\alpha, \lambda) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\beta & \sin 2\beta & 0 \\ 0 & -\sin 2\beta & \cos 2\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ \hspace{1cm} (2.3)$$

With these components, the degree of polarization, $P$, and the angle of polarization, $\chi$, can be computed as presented in Eqs. 2.5 and 2.7, respectively. Therefore, the state of polarization of the whole planet would be defined. These two values are the output of the radiative transfer model which performs the simulations.

$$P(\alpha, \lambda) = \frac{\sqrt{Q^2(\alpha, \lambda) + U^2(\alpha, \lambda)} + V^2(\alpha, \lambda)}}{F(\alpha, \lambda)}$$ \hspace{1cm} (2.4)$$

which assuming that the circularly polarized flux, $V$, is zero for a Jupiter-like planet (Kemp and Wolstencroft, 1972), the degree of linear polarization can be further simplified to obtain the expression shown in Eq. 2.5.

$$P_L(\alpha, \lambda) = \frac{\sqrt{Q^2(\alpha, \lambda) + U^2(\alpha, \lambda)}}{F(\alpha, \lambda)}$$ \hspace{1cm} (2.5)$$
To add information about the direction of polarization and if the planet is mirror-symmetric with respect to the reference plane, \( U \) and \( V \) are zero when integrated over the disk, leading to Eq. 2.6,

\[
P_L(\alpha, \lambda) = -\frac{Q(\alpha, \lambda)}{F(\alpha, \lambda)} \quad (2.6)
\]

where the sign can be interpreted as follows: if \( P_L > 0 \) then the light is polarized perpendicular to the reference plane. On the other hand, if \( P_L < 0 \), then the light is polarized parallel to the reference plane.

The angle of polarization with respect to the reference plane can be computed using Eq. 2.7.

\[
\chi = \frac{1}{2} \arctan \left( \frac{U}{Q} \right) \quad (2.7)
\]

Now, the main question is how to obtain the parameters of the Stokes Vector. This is done from the reflected light or flux from the planet and it will be described in Chapter 4. Finally, for the observations, the instrument used is a polarimeter that includes a CCD (Charge-Coupled Device). In this device, the incoming light is separated into four beams: \( F + Q, F - Q, F + U, F - U \) and, from this the Stokes parameters can be computed combining these four beams. In the next chapter, the observations used as a baseline for the fitting will be presented and described and this process to obtain them will be explained more in detail.
In this section, the polarization observations used as a reference will be described. The observations used in this thesis as a baseline were taken on January 3rd of 2017. The instrument used is the Torino Polarimeter (ToPol), and consists of a double wedge Wollaston prism configuration that splits the incoming light into four beams (McLean et al., 2017). These observations were ground-based polarimetric observations taken in the Ultraviolet, Blue, Visible and Red filters (hereinafter referred to as U, B, V and R). The spectral shape of each filter can be seen in Figure 3.1. The phase angle at which the observations were taken was 10.7°, which is the angle between line observer-planet and planet-star. At this small phase angle, the degree of polarization is expected to be more sensitive to the altitude of clouds than to particle size, since the light will cross the atmosphere almost perpendicular to the local horizontal, as opposed to higher phase angles where the light will cross a larger area of the atmosphere. For this reason, the particle size will be fixed after a sensitivity analysis and the parameter to be varied to find the optimum fitting will be the cloud altitude. The observations are shown in Figure 3.4. These filters cover different wavelengths ($\lambda$), the ranges are shown in Table 3.1 and were obtained from Bessell (2005). In order to obtain a fit of these observations, the effective wavelength for each range was selected. This is also given in Bessell (2005). And to check whether these values were indeed the most effective, additional wavelengths were included in a preliminary analysis. However, the fitting obtained for these additional wavelengths was poorer than that obtained using the effective ones, for this reason, only the wavelengths shown in Table 3.1 were included in the simulations.

<table>
<thead>
<tr>
<th>Filter</th>
<th>$\lambda$ Range [nm]</th>
<th>$\lambda$ Effective [nm]</th>
<th>Observations Time (UTC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>320 - 400</td>
<td>366</td>
<td>14:29:48</td>
</tr>
<tr>
<td>B</td>
<td>400 - 500</td>
<td>436</td>
<td>20:28:54</td>
</tr>
<tr>
<td>V</td>
<td>500 - 700</td>
<td>544</td>
<td>21:32:05</td>
</tr>
<tr>
<td>R</td>
<td>550 - 800</td>
<td>640</td>
<td>22:03:09</td>
</tr>
</tbody>
</table>

Table 3.1: Filter wavelengths and observation times.
In order to derive \( F, Q, \) and \( U \) from the observations, special attention had to be paid. In McLean et al. (2017), the process followed to obtain the measurements in the correct format is explained. The ToPol instrument allows the simultaneous measurement of one CCD image read-out of \( F + Q, F - Q, F + U, F - U \) (see Figure 3.2). Therefore, in order to obtain the Stokes parameters, these four beams have to be combined.

This procedure leads to some errors, which vary with the signal-to-noise ratio of the data (The observed variation in \( P_Q \) and \( P_U \) is of the order \( 10^{-3} \) at most (McLean et al., 2017)). The different optical paths of the beams lead to two effects that need to be corrected for. These two errors were: different images on the CCD have to be aligned in order to be superimposed since the slightly inaccurate guiding of the telescope leads to misalignments of successive images taken; and different images on the CCD have a different level of sharpness, even when taken at the same time, since the four beams travel along a slightly different path and, therefore, focused at different distances from the CCD (McLean et al., 2017).

In Figure 3.3, Jupiter can be seen from flux data. The belts, zones and the GRS can be clearly identified and the GRS can be found close to the center of the planet below the equator. However, polarimetry data is more sensitive to the atmospheric composition than flux measurements, therefore it is expected that it is more complex to find a fitting due to the accuracy needed. In Figure 3.4, the observations used for the fitting in this project are shown. It can be observed that the bands, zones and the GRS are clearer in the U and B filter, which correspond to the shorter wavelengths. These two filters are more sensitive to higher clouds and smaller particles that reach higher polarization values, which correspond to smaller ratios \( 2\pi r/\lambda \) and the particles are more sensible to Rayleigh scattering and Mie scattering. The V and R filters are more sensitive to larger particles and lower clouds, making the observations to look more uniform because less types of particles are seen by the filter. Also, the methane absorption band is close to these two wavelengths, leading to the light being mainly absorbed by this gas. It can also be noticed that in the poles, there is a region that reaches the highest polarization, this is due to the polar haze which increases the degree of polarization. These four sets of data will be the ones used as a reference for the fitting and were obtained from McLean et al. (2017).
Figure 3.2: ToPol CCD image of Jupiter in the V filter (544 nm). From the top down, the four beams are proportional to: $F + Q, F - Q, F + U, F - U$.

Figure 3.3: Jupiter representation reconstructed from flux data. Source: NASA/ESA, and A. Simon (GSFC).
Figure 3.4: Polarization observations for the whole planet and the different filters U (366 nm) (top left), B (476 nm) (top right), V (544 nm) (bottom left) and R (640 nm) (bottom right). Note that in this figure, the rotation angle is opposite to that of 3.2,
In Figure 3.5, the angle of polarization of the observations can be seen together with the degree of polarization. The change in the angle of polarization is obvious between regions of maximum and minimum polarization values. This is specially true for the B filter, where large variations can be noticed in the region around the Great Red Spot, as opposed to the V and R filters, that only present large variations close to the edges of the planet.

The variations of the location of the GRS over the course of the observations are small since the observations were taken in a short timespan and the period of the GRS is around 6 days. Only a small variation can be noticed in the U filter observations that were done 6 hours before, this corresponds to around 50° shift in longitude. The latitude of the long axis is 22° south with a variation of the short axis position, this corresponds with the findings by Hide (1961). The position can be seen in Figure 3.6.

The region where the GRS is found was used as an input for the fitting algorithm. For the U and the B filter, the position was obvious; however, for the V and the R filters, the position was not clear (see Figure 3.4). The position was inferred from the time when the observations were taken. Since the observations for the B, V and R filters were taken 30 minutes away from each other, the variation of the position should be negligible, therefore, it was assumed that the GRS was in the same position for all three filters and only the position in the U filter had varied a few degrees in the longitudinal direction. In Figure 3.7, the angle of polarization of the GRS can be found. The change in the angle of polarization above mentioned is even more obvious when we focus on the GRS area.
Figure 3.5: Degree and angle of polarization of the observations for the whole planet and the different filters U (366 nm) (top left), B (476 nm) (top right), V (544 nm) (bottom left) and R (640 nm) (bottom right). Note that in this figure, the rotation angle is opposite to that of 3.2,
**U filter**
This filter covers the 320 to 400 nm range, with an effective wavelength of 366 nm. This is the shortest filter, in this range, Rayleigh scattering is very strong, increasing the degree of polarization. These observations were taken 6 hours before the ones with the other 3 filters, therefore, a slight shift in the location can be found. In this filter, the GRS can be clearly seen in the band below the equator (see Figure 3.6).

**B filter**
This filter covers the 400 to 500 nm band, with an effective wavelength of 436 nm. In this range, the bands and the zones can be clearly seen and it is where the GRS is more clearly seen compared to the other filters. The reason of this is, on one hand, Rayleigh scattering that is still very strong, increasing the degree of polarization and other factors that will be explained in more detail in the next sections (see Figure 3.6).

**V filter**
This filter covers the 500 to 700 nm band, with an effective wavelength of 544 nm. In this filter there is a decrease in the degree of polarization with respect to the U and B filters. The bands and zones, although they can be inferred, are no longer clear and the GRS can no longer be noticed since this filter is more sensitive to the lower clouds in the atmosphere. This may be due to the fact that this wavelength is close to the methane absorption band, which is one of the main gases present in Jupiter’s atmosphere (see Figure 3.6).

**R filter**
This filter covers the 550 to 800 nm band, with an effective wavelength of 640 nm. In this filter, large particles are detected, therefore, the degree of polarization decreases substantially with respect to the U and B filters. In this range, neither the bands and zones nor the GRS can be noticed, similarly to the V filter. As it was previously mentioned, this may be due to the fact that this wavelength is close to the methane absorption band (see Figure 3.6).

In the next chapter, the atmospheric model, together with the numerical approach used to simulated the polarization of the planet will be described.
Figure 3.6: Polarization observations zoom-in of the GRS for the U (366 nm) (top left), B (476 nm) (top right), V (544 nm) (bottom left) and R (640 nm) (bottom right) filters.
Figure 3.7: Angle of polarization of the observations for the GRS area and the different filters U (366 nm) (top left), B (476 nm) (top right), V (544 nm) (bottom left) and R (640 nm) (bottom right).
In this chapter, the numerical model used to simulate the polarization values will be explained. This model processes the particles that compose the clouds and hazes and computes the reflected light properties based on these particles. Once the properties of the reflected light are computed, the Stokes parameters for each point in the surface for a specific geometry are computed. Note that there are two parts in the process. The simulations are done with the polarization model that will be described in this section, which is coded in FORTRAN. The fitting of this data to the observations is done with a fitting algorithm coded in Matlab and described in Chapter 6.3.

The theoretical background of the polarization technique was explained in Chapter 2, where the equations for the degree of polarization, the angle of polarization and the Stokes parameters are included. The numerical model is divided in 3 different parts: computation of the particle properties using the Mie Scattering theory (De Rooij and Van der Stap, 1984), computation of the Fourier coefficients with a doubling-adding algorithm (DAP algorithm (De Haan et al., 1987)), and computation of the Stokes parameters locally for each pixel in the planet. In the next sections, a detailed description of the equations implemented in the code is given. This process can be seen in Figure 4.1.

In order to be able to implement the algorithm, the initial set up must be well understood. This includes the geometry of the problem, the gas and aerosol particles and the atmospheric model that is going to be used. Once these parameters have been chosen, the simulations can be done.

4.1. Geometry

The main parameter in the geometry is the phase angle, defined as \( \alpha = 180^\circ - \Theta \), where \( \Theta \) is the single scattering angle. This is the angle between the line from the observer to the planet and the line from the star to the planet. This angle is fixed by the observations, that are taken from a specific point at a specific time. Then, the angles defining the local geometry of each point of the planet also need to be taken into account. These angles
Figure 4.1: Simulations and fitting process steps sketch.
are the azimuthal angle, $\phi$, the illumination angle, $\theta_0$ and the reflection angle or viewing angle, $\theta$. These two last parameter are important for the fitting since they depend on the position of the Spot, therefore $\theta$ will be used as a constraint. A graphical representation of these angles can be seen in Figure 4.2 and are defined as follows: $\theta_0$ is the angle between incident sunlight and the local vertical, $\theta$ is the angle between the reflected sunlight and the local vertical and $\phi - \phi_0$ is the difference of the azimuthal angles, which are defined clockwise when looking upwards from the planet surface.

![Figure 4.2: Local angles.](image)

### 4.2. Atmospheric Model

In order to simulate the polarization measurements, an atmospheric model is needed. For this purpose, gas, cloud and haze particles are distributed vertically in Jupiter’s atmosphere. The profile model selected for the simulations includes 15 horizontally parallel layers which cover a range of pressure levels from 6.723 bars to 0.1 bars (Lindal, 1992, Sromovsky and Fry, 2002, Stam et al., 2004, West et al., 1986). These layers contain gas molecules and they may also include cloud or haze particles. The geometric albedo for Jupiter was set to 0.5 (Karkoschka, 1994, Stam et al., 2004) and the chosen depolarization factor was that of $H_2$ which is 0.022 (Hansen and Travis, 1974, Penndorf, 1957). In Jupiter, the main absorbing gasses are methane ($CH_4$) with a mixing ratio of 0.18% (McLean et al., 2017, Stam et al., 2004). These layers are bounded by a black, homogeneous surface. The presence of clouds and haze will be varied to obtain the best combination. According to West et al. (1986), it is expected to have a haze top layer around 0.25 bars and a base Amonia-Water cloud layer around 3 bars. Finally, a higher layer of Amonia ice particles can be seen around 0.5 bars or 2 bars level. This atmospheric profile will be used as a reference together with the iterations done to fit the observations.
4.3. AEROSOL PARTICLES

In this section, the aerosol particles that have been used in the atmospheric profile are described. For these particles, the papers by Stam et al. (2004) (polarization signals of EGPs at Jupiter-like distances), McLean et al. (2017) (polarimetry observations), Muñoz et al. (1999) (flux CCD ground-based observations), West et al. (1986) (color-enhanced Voyager and ground-based flux imagery), Sromovsky and Fry (2002) (flux measurements from Galileo mission). In Stam et al. (2004), they perform simulations of direct observations of extrasolar planets that orbit their star at Jupiter-like distances. Such observations can be carried out with adaptive optics systems on ground-based telescopes, such as CHEOPS. In McLean et al. (2017), they use ground-based polarimetric observations taken in the B, V, and R filters at five epochs, these same observations are the ones used in this project. Finally, in Muñoz et al. (1999), a log-normal distribution is used to compute the expansion coefficients. They observed the GRS on Jupiter at four dates: 1993 March 11; 1993 June 4; and 1994 July 18 and 21. The Jovian images were acquired on CCD detectors through narrow-band filters. The observations in 1993 were made at the f/8 focus of the 1.52 m of the Spanish Telescope at Calar Alto Observatory, and the observations of 1994 July were made at the Nasmyth focus of the 4.2 m William Herschel Telescope (WHT) at the Roque de los Muchachos Observatory (La Palma, Spain) during the week of the Shoemaker-Levy 9 impacts on Jupiter and wavelength ranging from 360-948 nm.

4.4. NUMERICAL MODEL

4.4.1. REFLECTED LIGHT

The objective of the radiative transfer code is to obtain the Stokes parameters reflected from the planet, shown in Eq. 2.2. If we assume a spherical planet with radius r, reflected by a planet and arrives at a distance d is assumed, the elements of the vector can be obtained using the following equation (Eq. 4.1).

\[ F(\alpha, \lambda) = \frac{r^2}{d^2} \frac{R^2}{D^2} \frac{1}{4} S(\alpha, \lambda) \pi B_0(\lambda) \]  

(4.1)

where R [m] is the radius of the star and D [m], the distance between the star and planet, \( B_0(\lambda) \) is the stokes vector previously described, \( B_0[1,0,0,0] \) and the term \( \pi B_0(\lambda) \) is the stellar flux received by the planet, which integrated over the stellar disk, can be considered to be unpolarized (Kemp and Wolstencroft, 1972). Finally, the planetary scattering matrix, \( S(\alpha, \lambda) \), describes the scattered light from the planetary atmosphere and that is reflected towards the observer.
In the next section, the Mie Scattering algorithm to compute the expansion coefficients of the scattering matrices of the molecules, and either the cloud or haze particles, is described. With these coefficients, the radiative transfer can be computed.

4.4.2. Mie Scattering
In this section, the Mie Scattering algorithm will be described. This algorithm was developed by De Rooij and Van der Stap (1984). This first part of the code computes the expansion coefficients corresponding to the particles used as input. The input files of the code include the refractive index and the size distribution of the particles. The size of the particles can be computed following several size distributions, the two size distribution used in this project are the two parameter gamma and the modified gamma, that will be described in this section. The particles used in this project are shown in Table 4.1. These particles are water, amonia and haze particles modelled as spherical particles. For validation purposes, all the particles were tested against the literature, this process is shown in Appendix A.

If the scattering particles are spherical, the elements of the scattering matrix (see Eq. 4.2) can be expanded in a series of Legendre polynomials for the polarized light to obtain the Mie coefficients \(a_n\) and \(b_n\). Then, the expansion coefficients needed for the Radiative Transfer code can be obtained from this values. However, this procedure includes only single scattering.

If we express the scattering matrix for a volume element, \(F_{m,n}(\cos \theta)\), as a function of the generalized spherical functions, \(P^l_{m,n}(\cos \theta)\) and the expansion coefficients, \(S^l_{m,n}\), we obtain the following expression:

\[
F_{m,n}(\cos \theta) = \sum_{l=\max(|m|,|n|)}^{\infty} S^l_{m,n} P^l_{m,n}(\cos \theta) \tag{4.3}
\]

The generalized spherical functions follow this relation:

\[
P^l_{n,m}(u) = P^l_{m,n}(u) = P^l_{-m,-n}(u) \tag{4.4}
\]

And knowing that the expansion coefficients follow the symmetry relations:
\[ S_{n,m}^l = S_{m,n}^l = S_{-m,-n}^l \]  

(4.5)

We can combine these relations with Eq. 4.2 to obtain the expressions for the coefficients \( a_n \) and \( b_n \).

\[
a_1(\theta) = \sum_{l=0}^{\infty} \alpha_{1}^{l} P_{0,0}^{l}(\cos \theta) \]  

(4.6)

\[
(a_2(\theta) + a_3(\theta)) = -\sum_{l=2}^{\infty} (\alpha_{2}^{l} + \alpha_{3}^{l}) \sqrt{\frac{(l-2)!}{(l+2)!}} P_{2,2}^{l}(\cos \theta) \]  

(4.7)

\[
(a_2(\theta) - a_3(\theta)) = -\sum_{l=2}^{\infty} (\alpha_{2}^{l} - \alpha_{3}^{l}) \sqrt{\frac{(l-2)!}{(l+2)!}} P_{2,-2}^{l}(\cos \theta) \]  

(4.8)

\[
a_4(\theta) = \sum_{l=0}^{\infty} \alpha_{4}^{l} P_{0,0}^{l}(\cos \theta) \]  

(4.9)

\[
b_1(\theta) = \sum_{l=2}^{\infty} \beta_{1}^{l} P_{0,2}^{l}(\cos \theta) \]  

(4.10)

\[
b_2(\theta) = \sum_{l=2}^{\infty} \beta_{2}^{l} P_{0,2}^{l}(\cos \theta) \]  

(4.11)

where the expressions for \( \alpha_{n}^{l} \) and \( \beta_{l}^{l} \) are given as a function of the expansion coefficients \( S_{n,m}^l \) (see De Rooij and Van der Stap (1984)). So, after working on the equations and using the initial conditions shown in Eqs. 4.12 to 4.15:

\[
P_{0,0}^{0}(u) = 1 \]  

(4.12)

\[
P_{0,2}^{2}(u) = -\frac{1}{4} \sqrt{6}(1 - u^2) \]  

(4.13)

\[
P_{2,\pm 2}^{2}(u) = -\frac{1}{2} \sqrt{6}(1 \pm u)^2 \]  

(4.14)

\[ P_{s,m}^{l}(u) = 0 \text{ if } l < \text{max}(s,|m|) \]  

(4.15)

it can be deduced that element \( a_1(\theta) \) is the expansion of the phase function in Legendre Polynomials and that, in the case of Mie scattering, we have that the rest of the expansion coefficients follow the properties presented in Eqs. 4.16 and 4.17.
\[ a_1(\theta) = a_2(\theta) \] (4.16)
\[ a_3(\theta) = a_4(\theta) \] (4.17)

Once these parameters are obtained, the objective of this algorithm is to get a value for \( \alpha_n \) and \( \beta_n \), the expansion coefficients, from equations 4.6 to 4.11. These coefficients are the input for the Radiative Transfer algorithm (see Section 4.4.3). In this project, the algorithm computes these coefficients following the procedure previously shown and receiving as an input the wavelength, \( \lambda \), the size distribution \( (n(r)) \) of the particles (i.e. aerosol particles) and the complex refractive index (which we assume to be zero in this project). The two size distributions used in this project are the following:

- **Two parameter gamma:**
  \[ n(r) = Cr^{\frac{1}{v_{eff}} - 3} e^{-\frac{r}{v_{eff}}} \] (4.18)
  where \( r_{eff} \) is the effective radius and it must be positive, \( v_{eff} \) is the effective variance and it must be positive and
  \[ C = \frac{1}{(v_{eff} r_{eff})^2 \Gamma \left( \frac{1}{v_{eff}} - 2 \right)} \] (4.19)

- **Modified gamma:**
  \[ n(r) = Cr^\alpha e^{-br^\gamma} \] (4.20)
  where \( r_c \) is the mode radius and must be positive, \( \alpha \) must be larger than -1 (and if it is negative, \( n(r) \) is singular at \( r = 0 \)), \( \gamma \) must be positive and
  \[ C = \frac{\gamma b^{\alpha + 1}}{\Gamma \left( \frac{\alpha + 1}{\gamma} \right)} \] (4.21)
  \[ b = \frac{\alpha}{\gamma r_c^\gamma} \] (4.22)

These expressions are implemented in the code that has been used to compute the expansion coefficients, \( \alpha_n \) and \( \beta_n \). The properties of the particles that were selected for the fitting process are the ones presented in Table 4.1.

Before selecting the particles, a sensitivity analysis of the effect of size, refractive index and effective variance variation was done. This is described in the next section.
### 4.4. Numerical Model

<table>
<thead>
<tr>
<th>Particle</th>
<th>λ [nm]</th>
<th>( n_r )</th>
<th>Effective Radius [microns]</th>
<th>Effective Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water crystals</td>
<td>700</td>
<td>1.33</td>
<td>( r_{eff} = 2.0 )</td>
<td>( v_{eff} = 0.045 )</td>
</tr>
<tr>
<td>Amonia Cloud</td>
<td>700</td>
<td>1.42</td>
<td>( r_{eff} = 1.0 )</td>
<td>( v_{eff} = 0.10 )</td>
</tr>
<tr>
<td>Haze particles</td>
<td>700</td>
<td>1.66</td>
<td>( r_{eff} = 0.5 )</td>
<td>( v_{eff} = 0.01 )</td>
</tr>
</tbody>
</table>

**Sensitivity to Particle Size Analysis**

Before including the particles in the atmosphere model, a preliminary analysis of the effect of particle size and particle properties was done to have a first approximation and decide which particles to be used. This analysis only includes singly scattered light. The results are shown in Figures 4.5 to 4.10, where the degree of polarization and the phase function were obtained as a function of the phase angle varying the refractive index, the effective radius and the effective variance. For \( n_r = 1.00 \), there is no scattering, therefore, no difference between the particle and its ambient environment.

In Figures 4.3 and 4.4 it can be noted that, as the refractive index, \( n_r \), approaches to 1, the curve approaches that of Rayleigh Scattering (see curves for \( r_{eff} = 0.01 \) in Figures 4.3 to 4.8 to see the Rayleigh Scattering curve), irrespective of the values of the effective radius and the effective variance. This situation is the same as \( r_{eff} \) decreases and approaches zero (Figures 4.5, 4.6, 4.3, 4.4, 4.7 and 4.8). Even for higher values of the refractive index, if the effective radius is small, it approaches the Rayleigh scattered light curve.

On the other hand, as we vary the refractive index while maintaining the same effective radius, even for low values of \( n_r \), the curves no longer approach that of the Rayleigh scattered light. When \( n_r \) is higher, the variation of the degree of polarization with the viewing angle tends towards a less monotonic behaviour (this could also be seen in Figures 4.7
4.4. Numerical Model

Figure 4.5: Degree of polarization as a function of $\theta$, varying the effective radius (Nominal case: $v_{\text{eff}} = 0.1$ and $n_r = 2.5$).

Figure 4.6: Phase function as a function of $\theta$, varying the effective radius (Nominal case: $v_{\text{eff}} = 0.1$ and $n_r = 2.5$).

and 4.8 for $r_{\text{eff}} > 0.2$). Finally, the variation of the effective variance has a smaller influence in the results as if can be seen in Figures 4.9 and 4.10.

Figure 4.7: Degree of polarization as a function of $\theta$, varying the effective radius (Nominal case: $v_{\text{eff}} = 0.1$ and $n_r = 1.42$).

Figure 4.8: Phase function as a function of $\theta$, varying the effective radius (Nominal case: $v_{\text{eff}} = 0.1$ and $n_r = 1.42$).
4.4. **Numerical Model**

Figure 4.9: Degree of polarization as a function of the phase angle, varying the effective variance (Nominal case: \( r_{\text{eff}} = 1.0 \) and \( n_r = 1.42 \)).

Figure 4.10: Phase function as a function of the phase angle, varying the effective variance (Nominal case: \( r_{\text{eff}} = 1.0 \) and \( n_r = 1.42 \)).

### 4.4.3. Radiative Transfer Code

Once the expansion coefficients shown in equations 4.6 to 4.11 are computed, the components of the Stokes vector can be obtained. In order to do this, an adding-doubling algorithm is used as described in De Haan et al. (1987). For this purpose, a vertically layered model for the atmosphere is defined. This atmosphere model needs to be described as layers which are locally parallel to the surface and horizontally homogeneous. And they contain different scattering and/or absorbing gaseous molecules and may contain aerosol, cloud and/or haze particles. They are bounded by a black surface below (i.e the light that reaches the bottom of the layered atmosphere is assumed to be absorbed). The output of the code is the Stokes vector for a given location on the planet, shown in Eq. 2.2.

The parameters that need to be included to compute the radiative transfer within the model atmosphere at a certain wavelength are the solar zenith angle, the viewing angle \( \theta \), the gometric albedo, and, for each atmospheric layer, the molecular scattering, \( b_{\text{m sca}} \), and absorption, \( b_{\text{m abs}} \), optical thickness, the aerosol scattering, \( b_{\text{a sca}} \), and absorption, \( b_{\text{a abs}} \), optical thickness, the single-scattering albedo \( a(\lambda) \) and the single scattering matrix \( S_{\text{sca}}(\lambda) \).

The molecular scattering optical thickness of an atmospheric layer, \( b_{\text{sca}} \), is calculated according to equation 4.23.

\[
 b_{\text{sca}}(\lambda) = \frac{24 \pi^3}{\lambda^4 N_L^2} \left( \frac{n^2(\lambda) - 1}{n^2(\lambda) + 1} \right)^2 6 + 7 \rho_n(\lambda) \frac{N_{av}}{R} \int_{z_i}^{z_{i+1}} \frac{p(z)}{T(z)} dz
\]

(4.23)

where \( n \) is the refractive index of dry air under standard conditions, \( N_L \) is the Loschmidt’s
number $2.686 \times 10^{25} m^{-3}$, $N_{av}$ is the Avogadro’s number $6.0221 \times 10^{23} mol^{-1}$, $\rho_n$ is the depolarization factor of the air, and $R$ is the gas constant per mole. The molecular absorption optical thickness of an atmospheric layer, $b_{abs}^m$, is calculated according to equation 4.24.

$$b_{abs}^m(\lambda) = \frac{N_{av} R}{\int_{z_i}^{z_{i+1}} \eta(z) \sigma_{sca}^m(\lambda, z) p(z) T(z) \, dz}$$  \hspace{1cm} (4.24)$$

where $\eta$ is the volume mixing ratio of the absorbing gas and $\sigma_{sca}^m$ is the absorption cross-section per molecule.

The single scattering albedo of the mixture of gas molecules, and either cloud or haze particles, is given by Eq. 4.25

$$a(\lambda) = \frac{b_{sca}^m(\lambda) + b_{sca}^a(\lambda)}{b_{sca}^m(\lambda) + b_{abs}^m(\lambda) + b_{abs}^a(\lambda)}$$  \hspace{1cm} (4.25)$$

and the scattering matrix of the mixture is shown below (where $S_{sca}^m(\lambda)$ and $S_{sca}^a(\lambda)$ are the scattering matrices of the particles given by the Mie Scattering algorithm).

$$S_{sca}(\lambda) = \frac{b_{sca}^m(\lambda) S_{sca}^m(\lambda) + b_{sca}^a(\lambda) S_{sca}^a(\lambda)}{b_{sca}^m(\lambda) + b_{sca}^a(\lambda)}$$  \hspace{1cm} (4.26)$$

Since the observations were done with filters at different wavelengths, the input parameters had to be adjusted to those values of $\lambda$. In the case of the $b_{sca}^m$, the relation is the one shown in Eq. 4.29.

$$b_{sca,1}^m(\lambda) = \frac{\lambda_1^4}{\lambda_2^4} b_{sca,2}^m(\lambda)$$  \hspace{1cm} (4.27)$$

However, for the aerosol optical thickness, the relation with wavelength is trickier. It is based on the particle scattering efficiency, $Q_{ext}$, the geometrical cross-section, $G$, and the single-scattering albedo following the relation shown in Eq. 4.28.

$$b_{sca}^a(\lambda) = Q_{ext}(\lambda) G a(\lambda) b_{aer}$$  \hspace{1cm} (4.28)$$

$$b_{abs}^a(\lambda) = Q_{ext}(\lambda) G (1 - a(\lambda)) b_{aer}$$  \hspace{1cm} (4.29)$$
Once the layered model is defined, a local reflection matrix $R$ is obtained as a function of various combinations of azimuthal angle, $\phi$, and cosine of the downward vertical, $\theta$ and $\theta_0$. These angles are shown in Figure 4.2. Therefore, the locally reflected flux vector can be computed as

$$F(\mu, \mu_0, \phi - \phi_0) = \mu_0 R(\mu, \mu_0, \phi - \phi_0) F_0$$

(4.30)

where $\mu = \cos \theta$ and $\mu_0 = \cos \theta_0$, $F_0$ is the incident or unpolarized flux described by $[F_0, 0, 0, 0]$ and $F_0$ divided by $\pi$ is the incident flux measured perpendicular to the direction of incidence, and $R(\mu, \mu_0, \phi - \phi_0)$ is the mentioned local reflection matrix.

Since the incident light is assumed to be unpolarized ($F_0 = [F_0, 0, 0, 0]$), only the first column of the matrix $R(\mu, \mu_0, \phi - \phi_0)$ is needed and, since circular polarization is ignored, only the first three elements of the vector are needed (i.e. $F$, $Q$ and $U$).

The atmospheric models used in this project are described in Section 4. The input parameters needed for each atmospheric layer are the optical thickness, the single scattering albedo, and the scattering matrix of the gaseous molecules, in addition to the cloud, haze and aerosol particles of each layer (Stam et al., 2004). These parameters are obtained by the expansion of the single scattering matrix in the Legendre polynomials (De Rooij and Van der Stap, 1984), as explained in the Mie Scattering theory section.

Once the particles are selected, the next step is to include these particles in a layered atmosphere model and to implement the adding-doubling algorithm to compute the Fourier coefficients of the cloud and haze layers. As an input, the files for the expansion coefficients given by the Mie code are used together with the file including the atmosphere layers including the scattering parameters and the particles to be used in each layer.

**4.4.4. Geometry code**

Finally, once the Fourier coefficients are computed, the Stokes parameters and therefore, the degree of polarization, along with the angle of polarization, $\chi$, can be computed locally for each location of the planet. First, the local reflection matrix is computed for each pixel. With this matrix, the parameters above mentioned can be obtained together with the local angles (see Figure 4.2). The pixels distribution is shown in Figure 4.11. The number of pixels for the whole planet can be selected, but the number of pixels used for the GRS region are the same as those of the zoom-in of the GRS in the observations.

In the next chapter, the fitting algorithm developed in order to fit the simulations for each atmospheric model will be explained.
Figure 4.11: Pixels across the whole planet (left) and the GRS (right).
In this chapter, a detailed description of the fitting algorithm will be given. Once the simulations are obtained, they need to be compared to the observations to find the best fit that meets the constraints. For this purpose, a Matlab code was developed. This fitting algorithm uses as an input the polarization and geometry data obtained from the Mie, DAP and geometry codes explained in Chapter 4 and compares it with the observations. This project is based on the influence of cloud altitude and the presence of different clouds and hazes in the atmosphere of Jupiter, and in particular of the Great Red Spot. Therefore, different input files are needed to cover a realistic sample of possible combinations. These files are generated prior to the fitting algorithm and used as an input. This process was divided into two different steps: a sensitivity analysis with different cloud altitudes and a simple model atmosphere and a final iteration to obtain a realistic profile that best fitted all filters. In this section, the algorithm used in these two steps will be explained.

5.1. DATA FITTING ALGORITHM

In order to study the effect of cloud altitude in the polarization values, one type of cloud was selected. According to the literature, the clouds that are present in more abundance in Jupiter’s atmosphere are $NH_4$ (Amonia) clouds. From the profiles described in McLean et al. (2017), Sromovsky and Fry (2002), West et al. (1986), a reasonable range of pressure levels was included that covers all the possible locations of the clouds, gas and haze particles. The findings of this analysis will be explained more in detail in Chapter 6.3. This single cloud particle profile was selected for the preliminary sensitivity analysis. For the final analysis, this profile was refined according to the findings from this analysis and additional particles were included. The composition and size distribution of the aerosol particles used for the haze and clouds are described in Tables 4.1.

The inputs needed for the fitting code are two different types of files: observations files and simulations files. For both, files for the degree of polarization and the angle of polarization per filter are needed and in the case of the simulations, also per altitude. Addi-
tionally, files to match the theta angle with each pixel are also needed, which was computed in the simulations. This means that in order to include the four filters (U, B, V and R filters) with seven different altitudes for the sensitivity analysis, 56 files for the simulations and 12 files for the observations are needed as an input.

Once these files are included in the code, the fitting algorithm is based on independent fittings per filter. This is equivalent to say that there is no optimization for all the filters together. This could be also included in the algorithm in future work. The working principle of the fitting algorithm is based on finding the best fit (i.e. the altitude that best matches the $P_L$) per filter for each pixel of the observations, always ensuring that the constraints are met. The fitting process can be summarised as follows:

1. A filter is selected and the corresponding files (observations and model results files for all the atmospheric models) for that filter are read.

2. The degree of polarization for each pixel of the Red Spot area of the observations is evaluated.

3. This value of $P_L$ of the observations is compared to all the elements of the simulations for each cloud altitude. The difference in $P_L$ and the viewing angle are checked. If these two conditions:
   - The viewing angle ($\theta$) is within the range $36 \pm 8^\circ$. This value was checked with the position of the Red Spot at the time of the observations and the range was included to be able to fit both, the higher and the lower values.
   - The difference between the degree of polarization of the simulations and the observations for each pixel is lower than a certain tolerance to be defined in the sensitivity analysis and the final fitting.
   - The difference between the angle of polarization of the simulations and the observations for each pixel is lower than a certain tolerance to be defined in the sensitivity analysis and the final fitting.

   are met, then the value of the $P_L$ is stored in a matrix that includes all the values that meet the condition for each pixel. If there is no match, a zero value is stored.

4. Once all the pixels are evaluated, the values stored in the $P_L$ matrix are evaluated for each pixel:
   - If a pixel has two or more entries in the matrix, the minimum $P_L$ value is selected (if there is only one entry, that value is selected) so that only one output (the optimum) per pixel is selected.
   - If a pixel has no corresponding $P_L$ value, this pixel is set to zero.

Several matrices are then created that have the same size as the observations: $P_L$, angle of polarization, atmospheric model used, $P_{L,\text{obs}} - P_{L,\text{model}}$ and viewing angle.
In the sensitivity analysis, the results may or may not follow a pattern related to the degree of polarization. This depends on the atmosphere models selected for the fitting. Therefore, an additional step was included to identify a pattern, if any, and identify possible modifications in the atmosphere models used as input. Also, some gaps in the fittings may be found for the most extreme values of the polarization, these gaps correspond to the zero valued pixels above mentioned.

The last part of the algorithm aims to remove these gaps present in the results. This is based on selecting the value of the pixel next to the blank value and finding the match in the results. If a pixel is found that has \( P_L = 0 \) (i.e. if in the atmosphere model matrix, the model number is 0, which corresponds to no fitting found), the consecutive pixels in the fitting matrix are checked (see Figure 5.1). The one that has lowest \( P_{L,\text{obs}} - P_{L,\text{model}} \) is selected (\( P_{L,j-1} \) in the figure). Then, this value is found in the simulations matrix (the one used as an input to the code) and the consecutive pixel (the one that would correspond to the gap, \( P_{L,i,j} \) in the figure) is assigned to this pixel. This procedure decreases the overall accuracy of the fitting, since the value assigned is not optimum. Therefore, the usage of this procedure to remove gaps is a trade-off between the need of a model with no gaps and the need of a high accuracy model.

**Figure 5.1: Gap removal procedure scheme.**

5.2. **Atmosphere Profile Optimization**

Once the algorithm described in the previous section was implemented, the fitting process was divided into two parts, as it was already mentioned. A first attempt was done to match the observations by using as an input the same atmosphere model but with 7 different cloud altitudes (with only one Ammonia cloud layer and haze layer per model), the conclusions of this process are presented in Chapter 6.3. These results were then...
evaluated find the relation between the cloud altitude and the degree of polarization. The next step was to use these findings to develop a more realistic atmosphere model with more cloud particles that best matched all the filters. This model together with the results and conclusions are presented in Chapter 6.

The constraints for these two different steps were varied to improve the accuracy. When a more realistic model was developed from the literature and the findings from the sensitivity analysis, the difference between the simulations and the observations was decreased and more accurate results were obtained. In the next section, the sensitivity analysis settings, results and conclusions will be presented.
In this chapter, the results obtained from the sensitivity analysis and the final fitting will be discussed. The algorithm to obtain these results has already been explained in the previous chapter. As it was explained in this chapter, before trying to fit a realistic atmosphere model, a sensitivity analysis of the degree of polarization with cloud altitude was done. Once the conclusions from this analysis were obtained, the next step was to select a realistic atmosphere structure. One single structure needs to be selected that combines different cloud and haze altitudes and fits all the data. This is a complex process since, as it could be seen in the observations in Chapter 3, the different filters show different profiles. Therefore, it is expected that the selected atmosphere profile will not fit all the filters with the same accuracy. In this chapter, these results are presented and discussed, together with the sensitivity analysis.

6.1. Comparison with Literature
According to the literature, the clouds and haze present in Jupiter, obtained from flux measurements and observations, correspond to a top haze layer around 0.2 bars pressure level, an $NH_3$ cloud around 0.5 bars and a base layer of $NH_3-H_2O$ cloud around the 2 bars level (Sromovsky and Fry, 2002, West et al., 1986). This profile is an approximation given for the whole planet, therefore the profile in the GRS is expected to be different with higher clouds, since a higher degree of polarization is reached. Also, note that most Jupiter’s atmospheric profiles found in the literature (the two paper above mentioned among them) are obtained from flux measurements, therefore a different fit could be found with polarimetry observations, since these are more sensitive to cloud and haze particles.
6.2. **Atmospheric Models Preparation**

The first step was to select the particles that would be used for the model. One single set of aerosols and haze particles were selected, these particles are shown in Table 4.1 and consist on Ammonia ice, Water crystals and haze particles. In this project, we would limit ourselves to study the influence of the altitude of the cloud and haze particles and their composition. The effect of the particle size is expected to be negligible at low phase angles such as the one where the observations were taken, since the light will cross the atmosphere almost perpendicular to the local horizontal, as opposed to higher phase angles where the light will cross a larger area of the atmosphere. In future work, a variation of particle composition present in the clouds and haze could be included to refine the fitting. The predominant gas present in the atmosphere is methane with a mixing ratio of 0.18 and $H_2$ with depolarization factor 0.2. These gas and aerosol particles will be included in different layers to form the atmospheric profile, and the altitude and presence of these particles will be varied to obtain the best combination.

Once the simulations for the selected atmospheric models were computed, a series of constraints needed to be selected to obtain the best solution but at the same time, a realistic solution. For this purpose, the next step was to study the geometry of the Great Red Spot to match the position and viewing angle of the observations with the results form the model. The area covered by the GRS is small compared to the whole planet, so the theta angle is not expected to vary a lot. After checking this assumption with the data, it was concluded that a single geometry configuration could be assumed, which is shown in Table 6.1. This simplifies the fitting since the theta angle is now assumed to be constant and it can be fixed in the fitting processed and used as a constraint, as it was mentioned in Chapter 5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Angle</td>
<td>10.7°</td>
</tr>
<tr>
<td>Theta Angle</td>
<td>36 ± 8°</td>
</tr>
</tbody>
</table>

This geometry constraint together with a tolerance for the degree of polarization value were included in the fitting code. With this setup, the sensitivity analysis can now be done. In the next section, the configuration used and the results drawn from this analysis will be described.

6.3. **Cloud Altitude Sensitivity Analysis**

The objective of the sensitivity analysis is to identify the effect of cloud altitude variation in the degree of polarization. Therefore, seven cloud altitudes were selected for the same cloud and haze particles profile. For this purpose, the radiative transfer code described in Chapter 4 was modified to include a specific wavelength and allow as an input several
files with different profiles. Therefore, a set of outputs could be given to the fitting code in order to compare different cloud combinations and check the agreement with different filters.

To study the effect of cloud altitude in the degree of polarization, a first run was performed and the results of several cloud altitudes were obtained to check this influence, the vertical profile set up for this analysis is shown in Figure 6.1 and the values used in each layer are included in Table 6.2. Note that each model includes an Amonia cloud (covering two layers) and the top haze (also covering two layers). Note that in Figure 6.1, all seven model are represented, therefore, there are some layers where two different models overlap. As a summary, Models H1 to H6 include haze layers in layers 13 and 14 and Model H7 has a haze layer in layers 14 and 15. Then, the Amonia cloud altitude increases two layers per model, except for Models H6 and H7 that only increase one layer.

The constraints included in the code for this sensitivity analysis were:

- The difference of the degree of polarization of the observations minus the model results had to be lower than $5\epsilon - 2$.
- The theta angle for a value to be valid had to be $36 \pm 8^\circ$.
- The difference of the angle of polarization of the observations and the simulations had to be within $10^\circ$.

Note that these constraints are more relaxed than the ones used for the final fitting in order to be able to obtain a fitting at this point. These constraints will be modified in the next step to improve the accuracy.

After several runs with the different filters, it was concluded that, as it was expected, the higher the clouds, the higher the value of the degree of polarization was (see Figure 6.7). With this conclusion, limits were set to the degree of polarization to select a certain cloud altitude. This profile was then fixed based on these results to facilitate a gradual fitting of the different models, the resulting limits can be found in Table 6.3. These limits are common to all filters and this was taken as a reference in order to build a more realistic atmosphere model based on these results together with the profiles described in the literature.

With these constraints, the fitting was redone in order to obtain the optimum cloud altitude profile for each filter. The results are shown in Figures 6.2, 6.3, 6.4 and 6.5, together with the accuracy of the results in Figure 6.6. It can be noted that for the U and B filters, even though there are some small areas where no matches could be found (in particular the areas with the highest polarization values) the degree of polarization matches the observations. In the case of the V filter, a match could be found for all the pixels. Finally, in the case of the R filter, the regions where no match could be found are larger, and these regions are the ones where a lower degree of polarization is found.
6.3. **Cloud Altitude Sensitivity Analysis**

![Graphical representation of the vertically layered atmosphere model used for the computations.](image)

**Table 6.2:** Pressure levels and scattering parameters for each layer (Top layer with $b_{aer} = 0.2$ corresponds to the haze layer and the rest are Amonia clouds).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Pressure [bar]</th>
<th>$b_{m,sc}$</th>
<th>$b_{aer,H1}$</th>
<th>$b_{aer,H2}$</th>
<th>$b_{aer,H3}$</th>
<th>$b_{aer,H4}$</th>
<th>$b_{aer,H5}$</th>
<th>$b_{aer,H6}$</th>
<th>$b_{aer,H7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.6230</td>
<td>7.6223</td>
<td>20.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>4.2170</td>
<td>3.8112</td>
<td>16.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>3.1620</td>
<td>1.9056</td>
<td>0.0</td>
<td>20.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>2.3710</td>
<td>2.0000</td>
<td>0.0</td>
<td>16.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
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<td>0.0</td>
<td>0.0</td>
<td>20.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>1.0000</td>
<td>0.6222</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>20.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>0.7499</td>
<td>0.5780</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>16.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>9</td>
<td>0.5623</td>
<td>0.4350</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>0.4217</td>
<td>0.3520</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>20.0</td>
<td>0.0</td>
</tr>
<tr>
<td>11</td>
<td>0.2371</td>
<td>0.1380</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>16.0</td>
</tr>
<tr>
<td>12</td>
<td>0.1778</td>
<td>0.0881</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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</tr>
<tr>
<td>13</td>
<td>0.1000</td>
<td>0.0191</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>14</td>
<td>0.0100</td>
<td>0.0029</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>15</td>
<td>0.0001</td>
<td>0.0038</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>
6.3. Cloud Altitude Sensitivity Analysis

Table 6.3: Selected atmosphere profile in the GRS.

<table>
<thead>
<tr>
<th>Degree of Polarization [%]</th>
<th>Pressure Level [bar]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low limit</td>
<td>High limit</td>
</tr>
<tr>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
<td>1.1</td>
</tr>
<tr>
<td>1.1</td>
<td>1.7</td>
</tr>
<tr>
<td>1.7</td>
<td>1.8</td>
</tr>
<tr>
<td>1.8</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>3.5</td>
</tr>
<tr>
<td>3.5</td>
<td>-</td>
</tr>
</tbody>
</table>

In Figure 6.6, the differences between the values in the observations and the ones obtained with the model can be observed. The greatest differences can be found in the regions where the values for the degree of polarization are most extreme. In particular, for the V and R filters these values are the lowest, and for the U and B filters values are the highest.

Finally, in Figure 6.7 the different atmosphere profiles used can be seen. The numbers correspond to the different altitude altitudes used (1 to the lowest clouds and 6 to the highest clouds). It can be seen that, the GRS shape for each filter can be inferred also when only the selected models are represented, meaning that there is a gradual cloud altitude change towards region of higher or lower degree of polarization. These results were studied to obtain a common cloud altitude that fit all filters.

Figure 6.2: Degree of polarization [%] using the U filter with the constraints set. Left: Observations. Right: Model. Note that the scale vary between filters.
6.3. **Cloud Altitude Sensitivity Analysis**

Figure 6.3: Degree of polarization [%] using the B filter with the constraints set. Left: Observations. Right: Model. Note that the scale vary between filters.

Figure 6.4: Degree of polarization [%] using the V filter with the constraints set. Left: Observations. Right: Model. Note that the scale vary between filters.
6.3. Cloud Altitude Sensitivity Analysis

Figure 6.5: Degree of polarization [%] using the R filter with the constraints set. Left: Observations. Right: Model. Note that the scale vary between filters.

Figure 6.6: Difference in degree of Linear Polarization of the observations against the model for the U (top left), B (top right), V (bottom left) and R (bottom right) filters.
6.3. Cloud Altitude Sensitivity Analysis

Figure 6.7: Cloud altitude for the U (top left), B (top right), V (bottom left) and R (bottom right) filters. Different colors represent the different atmospheric models used (M1 to M7), zero value represents no fitting.
As it was already mentioned, once the effect of cloud altitude variation was identified and in order to improve the fitting and obtain a more realistic atmosphere profile was developed. In this new profile, additional Water ice cloud particles were included. The final results are shown in the next section, where the refined atmospheric model and the results obtained will be described.

6.4. **REALISTIC ATMOSPHERE MODEL**

After studying the effect of varying the cloud altitude, a realistic atmosphere model that fitted the results obtained in the fitting was developed. In Figure 6.8 a graphical representation can be found and the corresponding optical properties and altitude equivalence is presented in Table 6.4. The area covering the center of the GRS has three cloud layers, and the outer area has lower clouds and it is present surrounding the whole GRS area. This final fitting was done based on the profile described in West et al. (1986) and Stromovsky and Fry (2002), together with the results from the sensitivity analysis. In these two papers, the profile presented for Jupiter’s atmosphere has a haze top layer around 0.25 bars and a base Amonia-Water cloud layer around 3 bars. Finally, a higher layer of Amonia ice particles can be seen around 0.5 bars or 2 bars level. Based on this data, four models were developed to fit the variations from the surroundings of the GRS to the center, these are shown in Table 6.4. This model presents two different areas that are linked with a gradient in the clouds, in the area around the center of the spot, where the polarization in not as high as in the center, the profile that best fit the observations was a base water cloud layer at 6 bars pressure level, then a higher Amonia cloud at 3-4 bars and finally a top haze layer around the 0.1-0.2 bars pressure level. In the center of the GRS, these clouds needed to be higher and thinner, therefore the water cloud is at the 3 bars pressure level, followed by an Amonia cloud at 1 bars and finally, the haze top layer is at the same pressure level, but it is a thinner layer.

The equivalence in altitude difference according to these pressure variations can be obtained from Jupiter’s scale height,

\[ H = \frac{kT}{Mg} \]  

(6.1)

where \( k \) is the Boltzmann constant \( 1.38 \times 10^{23} \text{JK}^{-1} \), \( T \) is the mean atmospheric temperature in kelvins, \( M \) is the mean mass of a molecule (units kg) and \( g = \text{acceleration due to gravity on planetary surface (m/s}^2) \). According to Jupiter Fact Sheet, Jupiter’s scale height is 27 km. Since

\[ \frac{dP}{P} = -\frac{dz}{H} \quad \Rightarrow \quad P = P_0 \exp\left(-\frac{z}{H}\right) \]  

(6.2)
Figure 6.8: Atmosphere profile in the GRS, the corresponding optical thickness is shown in Table 6.4. (Jupiter's image credit: NASA/ JPL/ Bjorn Jonsson/ Sean Doran/ Flickr (CC BY-NC-ND 2.0)).
Therefore, knowing the pressure at a reference altitude (we assume that the base layer is at zero altitude and represents the bottom of the atmosphere), we can obtain the altitude of the clouds in kilometres, in addition to the pressure levels.

Table 6.4: Pressure levels and scattering parameters for each layer (Top layer with $b_{aer} = 0.2$ corresponds to the haze layer and the rest are Ammonia clouds (upper) and water clouds (lower clouds)).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Pressure [bar]</th>
<th>Altitude [km]</th>
<th>$b_{m,sca}$</th>
<th>$b_{aer,M1}$</th>
<th>$b_{aer,M2}$</th>
<th>$b_{aer,M3}$</th>
<th>$b_{aer,M4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.7230</td>
<td>0</td>
<td>9.98</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>10.0</td>
</tr>
<tr>
<td>2</td>
<td>6.1230</td>
<td>2.52</td>
<td>8.3100</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>10.0</td>
</tr>
<tr>
<td>3</td>
<td>5.6230</td>
<td>4.82</td>
<td>6.8223</td>
<td>0.0</td>
<td>0.0</td>
<td>10.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>4.2170</td>
<td>12.59</td>
<td>3.8112</td>
<td>0.0</td>
<td>10.0</td>
<td>0.0</td>
<td>16.0</td>
</tr>
<tr>
<td>5</td>
<td>3.1620</td>
<td>20.37</td>
<td>2.1056</td>
<td>10.0</td>
<td>0.0</td>
<td>16.0</td>
<td>20.0</td>
</tr>
<tr>
<td>6</td>
<td>2.3710</td>
<td>28.14</td>
<td>1.6056</td>
<td>0.0</td>
<td>0.0</td>
<td>16.0</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>1.7780</td>
<td>35.91</td>
<td>1.0000</td>
<td>0.0</td>
<td>16.0</td>
<td>16.0</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>1.3340</td>
<td>43.67</td>
<td>0.7622</td>
<td>0.0</td>
<td>16.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>9</td>
<td>1.0000</td>
<td>51.45</td>
<td>0.6222</td>
<td>16.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>0.7499</td>
<td>59.22</td>
<td>0.5780</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>11</td>
<td>0.5623</td>
<td>66.99</td>
<td>0.4350</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>12</td>
<td>0.4217</td>
<td>74.76</td>
<td>0.3520</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>13</td>
<td>0.2371</td>
<td>90.31</td>
<td>0.1380</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
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<td>0.1778</td>
<td>98.08</td>
<td>0.0881</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>15</td>
<td>0.1000</td>
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<td>0.0191</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The new limits included in the code in order to improve the accuracy for this step of the fitting were:

- The difference of the degree of polarization of the observations minus the model results had to be lower than $5 \times 10^{-3}$.
- The theta angle for a value to be valid had to be $36 \pm 8^\circ$.
- The difference of the angle of polarization of the observations and the simulations had to be within $10^\circ$.

Note that the constraints here are less relaxed than the ones used for the sensitivity analysis in order to improve the accuracy. In the next sections, an analysis of the individual parameters will be presented, showing the results and conclusions of this final fitting.
6.4.1. Degree of Polarization

First, the degree of polarization will be shown. In Figures 6.9, 6.10, 6.11 and 6.10, a comparison between the observations (left plot in each figure) and the results obtained after the fitting with the atmospheric profile (right plot in each figure) is presented. It can be seen that the results resemble the observations, except for some pixels in the B filter for the most extreme values, where no fitting could be found. To fill these gaps if necessary, a gap removal algorithm was included in the fitting code, which was described in Chapter 5. The results and conclusions are shown in Section 6.5.

In Figure 6.13, the models that have been used in each pixel are shown. The identifier 1 corresponds to the Model 1 (M1) and 2 to the Model 2 (M2) in Table 6.4. It can be seen that, specially for the U and B filters, the center area of the spot can be clearly identified. However, for the V and the R filters, the GRS cannot be identified anymore. This was also true for the degree of polarization. This is because this filter is more sensitive to larger particles that have a lower polarization value, and also, at these wavelengths, the methane absorption band is present, which causes that the GRS cannot be identified anymore.

Figure 6.9: Degree of polarization [%] using the U filter with the constraints set. Left: Observations. Right: Model.
6.4. **REALISTIC ATMOSPHERE MODEL**

Figure 6.10: Degree of polarization [%] using the B filter with the constraints set. Left: Observations. Right: Model.

Figure 6.11: Degree of polarization [%] using the V filter with the constraints set. Left: Observations. Right: Model.
Figure 6.12: Degree of polarization [%] using the R filter with the constraints set. Left: Observations. Right: Model.

Figure 6.13: Models used for each pixel for the U (top left), B (top right), V (bottom left) and R (bottom right) filters.
6.4.2. **Angle of Polarization**

In this section, the Angle of Polarization ($\chi$) is presented. In Figure 6.14, the variation of this angle can be seen. Note that these polarization angles present huge variations when the polarization area changes from high to low polarization values and vice versa. It can also be seen that in the filters where the polarization reach higher values, i.e. the B and U filters, this angle tends to have more abrupt changes, since the $P_L$ reach more extreme values. This makes sense since in the shorter wavelengths, the presence of the GRS is clearer and these filter are less sensitive to larger particles in the clouds that decrease the polarization value, leading to higher values of $P_L$, therefore causing more changes in the angle of polarization. Note that, in order to be able to find a fitting, the accuracy of this value had to be adjusted, and was decreased to be able to meet the constraints for $P_L$ and $\theta$ angle values, therefore, a difference can be observed with respect to the observations although a similar behaviour can be observed (see Figure 3.7).

![Figure 6.14: Angle of Polarization for each pixel for the U (top left), B (top right), V (bottom left) and R (bottom right) filters.](image-url)
6.4.3. **Accuracy of the results: $P_L$ difference and $\theta$**

In Figure 6.15, the differences between the values in the observations and the ones obtained with the model can be observed and in Figure 6.16, the theta angle values corresponding to each pixel can be seen. It can be noted that the fitting meets the constraints. The largest differences can be found in the regions where the values for the degree of polarization are most extreme. In particular for the V and R filters are the lowest values and for the U and B filters, the highest values.

Note that this theta angle does not correspond to the physical viewing angle. Since this value was used as a constraint for the fitting, the viewing angle range was taken as the real value but the variation that can be seen in the results from the fitting, is only due to numerical effects of the fitting rather than to real variations.

On the other hand, if the gaps in the fitting need to be filled, these constraints may not be met. This can be seen in the case of the B filter with no gaps in Figure 6.19. The difference between the polarization values is no longer below the $5e^{-3}$ limit but it reaches a value of 0.3 for the most extreme degrees of polarization.

Figure 6.15: Difference in degree of Linear Polarization of the observations against the model for the U (top left), B (top right), V (bottom left) and R (bottom right) filters.
Figure 6.16: Theta angle for each pixel for the U (top left), B (top right), V (bottom left) and R (bottom right) filters.
6.5. **Gap Removal**

An additional option to fill the gaps in the fitting process was included. This procedure would imply a decrease in the accuracy of the results. The way these gaps are filled is based on the value of the degree of polarization that is in the cell next to the one with the gap, this is explained in more detail in Chapter 5. However, for the atmospheric profile selected in this project, the only filter that presented gaps was the B filter in the area where the polarization was the highest. The results after using this option can be seen in Figures 6.17, 6.18 and 6.19. It can be seen that the results resemble the observations in all the pixels; however, the cost of this process is a decrease in the accuracy of the results of the order of $1e^{-2}$.

![Figure 6.17: Degree of polarization [%] using the B filter with the constraints set applying the gap removal flag. Left: Observations. Right: Model.](image1)

![Figure 6.18: Viewing angle for the B filter after removing the gaps present in the fitting.](image2)

![Figure 6.19: $P_L$ difference for the B filter after removing the gaps present in the fitting.](image3)
6.6. **Discussion**

Once the final configuration is selected and the corresponding results are obtained, these need to be well understood. In this section, a discussion of the main conclusions obtained from the fitting process and the results obtained will be presented. As a remainder, the polarization technique is based in the fact that the direct starlight is unpolarized and it becomes polarized when crosses a planet’s atmosphere due to scattering and finally, it is reflected. This means that depending of the particles present in the atmosphere, the polarization will be different.

Some conclusions could already be identified from the cloud altitude sensitivity analysis. The models that had higher clouds were able to cover regions of higher polarization, but the $P_L$ range that they covered was smaller, therefore not being able to fit lower polarization regions. All of them included a haze layer at the top of the atmosphere, that single scattered the light, which increased the degree of polarization.

First, it must be noted that the profile in the GRS has not been previously studied from polarimetry data and the literature corresponds either to flux data, or to polarimetry data for the whole planet. The results obtained behave as expected and the cloud distribution around the GRS correspond to the findings presented in the literature, as it was explained in Chapter 4. However, the presence of thinner clouds and haze towards the core of the GRS, is a new feature that had not need previously identified in the literature. This can be explained by the fact that, in this area, the temperature is higher than in the surroundings of the core as they explain in (Fletcher et al., 2010). In this paper, they also fit a tropospheric haze at the 0.2-0.5 bar pressure level to fit the thermal-IR imaging they used as a reference. Therefore, this new finding would explain this difference in the temperature.

On the other hand, it could be already seen from the observations that the bluer wavelengths covered higher polarization values. This is due to an increase in the Rayleigh scattered light since it reaches higher values than light scattered by larger particles. Another factor to take into account is that the presence of clouds increases the multiple scattering which tends to lower the degree of polarization. Also, thicker clouds allow less light through, therefore causing a decrease in the polarization below. Finally, the presence of a haze layer causes single scattering of the light, which could increase the degree of polarization depending on the haze particles and the scattering angle.

As it was already mentioned, the final distribution includes a base layer of water crystal clouds, higher towards the center of the spot, followed by an Amnonia cloud upper in the atmosphere. Finally, a top haze layer is present up in the atmosphere. The clouds in the spot are thinner than the surrounding clouds. This can be explained by the fact that thinner clouds decrease the multiple scattering with respect to thicker ones and this allows the light to be scattered mainly by the methane and $H_2$ gas particles, therefore increasing the polarization. On the other hand, higher clouds allow the beam of light to go through more gas in the atmosphere, increasing the degree of polarization.
Another interesting study would be the variation of the degree of polarization with the phase angle. For higher phase angles, the $P_L$ is expected to be higher, since the light travels more distance through the atmosphere. On the other hand, the appearance of a shadow region of the planet would constraint the positions where the GRS can be studied. In the next section, this analysis will be shown.
In this section, a different scenario is simulated in order to see the effects when the phase angle varies. This kind of observations could be taken with a spacecraft orbiting or flying by Jupiter with a polarimeter on board. The value of 10.7 is close to zero, which means that the star is almost in line with the observer and the polarization is expected to be lower. Therefore, the differences in the degree of polarization for different phase angles will be studied.

In order to simulate how the observations would look like if the phase angle would have been higher, several simulations were done for 0°, 30°, 60° and 90° phase angle. The atmosphere profile obtained from the fitting was used for these new simulations. The different models used for the fitting in the GRS were saved to be used in this new study. Therefore, the input files obtained from the radiative transfer code are the same, and only the phase angle was increased when obtaining the polarization for each pixel.

Bear in mind that Model 4 corresponds to the outer region of the GRS and Model 1 corresponds to the central area of the GRS where the polarization reaches the highest values (see Figure 6.8). In Figures 7.1 and 7.2, the degree of polarization for the whole planet for Models 1 and 4 are shown (see Table 6.4). Recall that Model 1 included higher and thinner clouds than Model 4.

It can already be seen that the degree of polarization increases with the value of the phase angle, but it can also be noted that, as the phase angle increases, so does the shadow region of the planet. The shape evolves from a symmetrical profile for the 0° phase angle, where the Sun is just behind the observer. In this case the polarization reaches the lowest values since the light goes straight through the atmosphere and back to the observer. As expected from the definition of the phase angle, it can be noted that, as it increases, so does the shadow region until half of the planet is in shadow at 90°. In this case the polarization has extreme values, reaching its maximum in the limb. This may be due
to the fact that the light crosses a larger region of gas in the atmosphere until it reaches this point and then goes back to the observer in a straight line and light scattered by gas molecules has its highest degree of polarization at a scattering angle of 90°.

On the other hand, a variation in the degree of polarization can also be seen when the cloud altitude is changed, note that the polarization level is higher when Model 1 is used than for Model 4 (Model 1 has higher and thinner clouds than Model 4) for all the phase angles, but it is especially for the R filter, rather than the B filter. For the R filter it can be seen that this change in $P_L$ can be seen in both, the maximum value of $P_L$ and the regions of minimum polarization. However, for the case of the B filter, the maximum value remains unchanged but a variation in the lower polarization regions can be noted. Therefore, it is expected that the simulations using Model 1 reach higher polarization values than those using Model 4. This increase increase was already explained in Chapter 6 with the decrease in multiple scattering in the clouds and therefore, the larger influence of the gas layers that increase the $P_L$. 
Figure 7.1: Degree of Polarization for the whole planet simulating a value for the phase angle of 0, 30, 60 and 90 for the B filter and Model 1 (left figures) and Model 4 (right figures) for the atmosphere.
Figure 7.2: Degree of Polarization for the whole planet simulating a value for the phase angle of 0, 30, 60 and 90 for the R filter and Model 1 (left figures) and Model 4 (right figures) for the atmosphere.
The Great Red Spot was also studied with these phase angles. For this purpose, the results obtained with the final atmosphere profile were used as a reference for the new scenarios. From these results, the model used in each pixel was saved and this same model was used in that same pixel for the new phase angle scenario. The main challenge encountered was that in order to find the optimum fitting for the observations, any pixel, as long as it fitted the constraints, was a valid candidate to be included in the model. Therefore, when the phase angle is increased (and so does the shadow region of the planet), these pixels may be located in the zero polarization region. For this purpose, an algorithm similar to the gap removal algorithm was implemented. The zero \( P_L \) pixels were identified. The pixels next to these ones that already had a fitting were inspected and that value was identified in the simulations for that phase angle (similarly to the procedure shown in 5.1). Then, the corresponding \( P_L \) for the pixel being inspected was assigned to the fitting. However, there were still some limit cases were no match could be found.

The results for the B and the R filters can be seen in Figures 7.3 and 7.4. Note that the non-uniform shape of the plots are due to the fact that these results were optimum for the 10.7° phase angle, but for the new values, the \( P_L \) that was assigned to each pixel may not be the best match. The higher and lower polarization regions can still be identified, although as the phase angle increases, it is less clear, since the values of the \( P_L \) reach higher values but there are also large regions of low polarization. For the R filter, the results resemble more to the nominal 10.7° case than those for the B filter, this may be due to the fact that in the R filter, the gas is less important, so the scattering is dominated by the cloud particles, and the scattering by the cloud particles is very similar in the different filters.
Figure 7.3: Degree of Polarization for the GRS simulating a value for the phase angle of 0 (top left), 30 (top right), 60 (bottom left) and 90 (bottom right) degrees for the B filter. At the top, the observations taken at 10.7° are included as a reference.
Figure 7.4: Degree of Polarization for the GRS simulating a value for the phase angle of 0 (top left), 30 (top right), 60 (bottom left) and 90 (bottom right) degrees for the R filter. At the top, the observations taken at 10.7° are included as a reference.
CONCLUSIONS

In this chapter, the conclusions drawn from this thesis will be included. This project was based on the fitting of a series of polarization observations given at four different wavelengths (ultraviolet, blue, visible and red) with focus on the Great Red Spot of Jupiter. The objective was to characterize the atmosphere in the spot to obtain the gas and aerosol particles composing the clouds and hazes in the atmosphere, and the altitude at which they are present. For this purpose, FORTRAN and Matlab codes were used, that have been described in this report.

One of the challenges that appeared when running the simulations was that, as the number of files and layers was increased, the computational time also increased substantially. This made the iterative process of the fitting difficult. This was mitigated by adapting the radiative transfer code to process several input files and generate a single output.

Regarding the fitting algorithm, the objective of developing this algorithm was to find the optimum configuration and also make the process faster and more efficient. The results of the fitting highly depended on the polarization values obtained from the simulations. Therefore, a deep understanding of the physical processes causing the results to behave like they do was also needed to be able to provide reasonable simulations to the fitting algorithm. Otherwise, even if the fitting algorithm was optimized, it would not be able to find a proper fitting.

From the observations, it could be concluded that the shorter wavelengths were more favourable to identify the presence of the spot, due to the increase in the Rayleigh scattered light (that reaches higher values than light scattered by larger particles, which are better seen by these short wavelengths) and the presence of the methane absorption band near the R and V wavelengths. However, the B and U filters were more sensitive to higher polarization values, which was the main challenge in the fitting process, specially in the case of the B filter.
As it was mentioned in the discussion of the results, there are no previous attempts to fit the GRS atmospheric profile in the GRS using polarimetry data. The literature used as a reference was based on flux data from the GRS and polarimetry data for the whole planet. After preforming the fitting of the data, the atmosphere profile that best fitted the observations was described in Section 6. This final distribution includes a base layer of water crystal clouds, higher towards the center of the spot, followed by an Ammonia cloud upper in the atmosphere. Finally, a top haze layer is present up in the atmosphere. The clouds in the spot are thinner than the surrounding clouds. The thinner clouds decrease the multiple scattering which allows the light to be scattered mainly by the $H_2$ and methane gas particles, therefore increasing the polarization. Also, higher clouds allow the beam of light to go through more atmosphere, mainly more gas zones that also increase the degree of polarization. The results corresponding to the surroundings of the core were as expected from Jupiter's atmosphere profile and they correspond to the literature, since they should be similar to those of other areas of the planet. In the core of the GRS, thinner and higher clouds and haze are found. This has not been previously identified in the literature. However, this result is supported by the fact that towards the core, there is a rise in the temperature that would explain this need of thinner clouds and haze, as presented in Fletcher et al. (2010)

Finally, an additional analysis was carried out to see the effect of the variation of the phase angle. It could be seen that as the phase angle increased, so did the polarization. This was explained with the area of the atmosphere the ray of light had to cross. For higher values of the phase angle, the ray crossed a larger region of gas particles, which increased the degree of polarization due the absence of multiple scattering. In the limb, the maximum values for the $P_L$ could be found. Regarding the spot, it could be seen that as the phase angle increased, it became harder to find a fitting due to this more extreme values of the polarization values and the larger shadow region that led to more regions of zero polarization.

After performing these analysis and obtaining the results, there are some lessons to be learned and some aspects that can be further improved in future work. These recommendations will be addressed in the next Chapter.
In the future, there are some features of the algorithm and some additional analysis that can be done to improve the atmospheric model in the GRS. There are two main areas that could be improved, the parameters that are varied in the fitting process and the layered atmosphere model (i.e. the physical properties included as an input in the code), and the fitting algorithm.

On the one hand, an analysis to study the variation of the particle type (different compositions in addition to Amonia and Water ice particles) and the size distribution of the particles could be done to obtain a more accurate model for the atmosphere. Also, a refinement of the atmospheric model layers could be done. More layers could be included in the areas where the clouds are found. If more combinations are studied at the same time, a more accurate atmospheric model could be obtained. However, this refinement would imply an increase in the computational time. On the other hand, an improvement in the code could be to optimize the altitude at which the clouds are found automatically, this would imply to include a modification of the DAP FORTRAN code while running the fitting algorithm, instead of just using the simulation results. If this was implemented, it would reduce the effort needed to create all the combinations and the uncertainties that this manual selection implies. However, this would increase substantially the computational time and the effort needed to modify the code.

On the other hand, the fitting algorithm can also be optimized. At the moment, it is costly in terms of computational time, specially for the pixels where many fittings are possible. On the other hand, an optimization for all the filters together instead of for each filter independently could be included.
REFERENCES


Penndorf, R. Tables of the refractive index for standard air and the rayleigh scattering coefficient for the spectral region between 0.2 and 20.0 μ and their application to atmospheric optics. *Josa*, 47(2):176–182, 1957.


In order to use the radiative transfer code, it needed to be modified to include different wavelengths and allow different input files, in addition to be set up in the computer. For this purpose, the code was first validated by running different models with known results that could be replicated to ensure that the values obtained were the correct ones. The papers used as a reference for the validation were De Rooij and Van der Stap (1984), McLean et al. (2017), Stam et al. (2004). In this section, evidence of the similarity of the results will be shown. The figures used for the validation are presented in A.1, these are only for single scattering, not the multiple scattering by the whole atmosphere. Some figures are shown in this section, in particular the ones using Stam et al. (2004) and De Rooij and Van der Stap (1984).

### Table A.1: Mie scattering parameters

<table>
<thead>
<tr>
<th>Reference paper</th>
<th>Particle</th>
<th>λ</th>
<th>n_r</th>
<th>r_{eff}</th>
<th>v_{eff}</th>
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<td>v_{eff} = 0.01</td>
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<td>σ = 1.2</td>
</tr>
<tr>
<td></td>
<td>Aerosol 2</td>
<td>360</td>
<td>1.60</td>
<td>r_{eff} = 0.20</td>
<td>σ = 1.2</td>
</tr>
<tr>
<td></td>
<td>Aerosol 3</td>
<td>360</td>
<td>1.60</td>
<td>r_{eff} = 0.60</td>
<td>σ = 1.2</td>
</tr>
</tbody>
</table>

Regarding Stam et al. (2004), the parameters used for the cloud and haze particles and the molecules were shown in table A.1. In Figures A.1 and A.2, it can be seen that the model used here matches accurately the simulation presented in this paper.

The particles shown in these figures are the ones that will be used in the analysis, together with the ones described in Table A.1. These particles include molecules, cloud
particles and haze particles.

Regarding De Rooij and Van der Stap (1984), the results obtained with the model are plotted against those presented in the paper. This comparison is shown in Figures A.3 and A.4.

Figure A.1: Phase function validation.  
Figure A.2: Degree of Linear Polarization validation.
Figure A.3: Validation of the Mie Scattering code output using De Rooij and Van der Stap (1984).
Figure A.4: Validation of the Mie Scattering code output using De Rooij and Van der Stap (1984) (continuation).