Network Aggregation Effects upon Equilibrium Assignment Outcomes: An Empirical Investigation*

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The level of spatial detail (i.e. zone size and network detail) used in transportation analyses is commonly regarded as an important factor affecting the accuracy of the resulting estimates of the impacts of a transportation plan. The precise effects of the level of detail are, however, largely unknown. To investigate these effects empirically for the car traffic assignment module, an experiment was designed to allow especially the study of the individual as well as combined effects of the level of detail and the type of assignment model. It involves the application of various assignment models at different levels of detail. Three network models were developed for the road network of Eindhoven (population: 200,000): a fine, a medium and a coarse network model. In this article the results of the equilibrium assignments are presented, which are occasionally compared with all-or-nothing outcomes. Mainly load figures are dealt with here. The experiment indeed showed a significant effect of the level of detail on most assignment outcomes. This effect proved to be consistent but diminishing: an increase in the level of detail always yielded better results but only marginal improvement could be obtained beyond a certain level. Compared with all-or-nothing assignment results, equilibrium loads agree much better with the counts at all levels of spatial detail.

Transportation systems are usually very complex. In general, the analysis of such systems requires various kinds of simplifications because the resources in terms of time and money are limited. Spatially, a transportation system is simplified into what is called a network model.

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Many different network models of a particular transportation system can be constructed. An essential characteristic of a network model is its level of detail. It may be assumed that the level of detail greatly affects the time and costs needed for the analysis. Furthermore, it is expected that the level of detail has a considerable effect on the outcomes of the analyses, e.g. estimates of the plan's consequences.

An increase in the level of detail will probably yield an improvement in the outcomes at additional costs. Therefore, the transportation analyst has to determine the appropriate level of detail by trading off the accuracy of the analysis and the analysis effort in the light of the specific planning problem to be solved. In practice, however, the optimal level of detail is hard to determine because knowledge of its effects on accuracy and costs is lacking.

This article presents some results of empirical research into the effect of the level of spatial detail on car traffic assignment results. Outcomes of equilibrium and all-or-nothing assignments performed on network models having widely different levels of detail are presented and compared. The findings pertain also to the accuracy of both assignment models as a function of the level of detail. They assist the transportation analyst in selecting the optimal combination of network model and assignment technique, given certain requirements for the outcomes.

In this article mainly results on loads are presented. Emphasis is on the sensitivity of both models to the level of detail not on the direct comparison of the models.

Only limited research has been carried out previously on this subject. Some attempts were made to analyze the special aggregation problem theoretically.[1, 8] In view of the seemingly unsurmountable complexity of the problem, we preferred an empirical approach. Other researchers adopted the same methods.[7, 12, 13] Our research is somewhat different from theirs, however, as to the network models used and the assignment models applied. In addition, a more elaborate analysis of the outcomes is performed.

The objectives and the setup of the experimental work are described in Section 1. In particular, the network models used are explained in detail. Sections 2 and 3 present the findings on the effect of the level of detail upon various assignment outcomes, such as loads, trip characteristics, mileage, etc. Section 4 deals with the convergence of the equilibrium assignment model. Section 5 summarizes the main conclusions.

1. EXPERIMENTAL WORK

1.1. Experimental Design

We decided to investigate the effects of the level of detail on the assignment outcomes empirically, using real-world situations. The Dutch

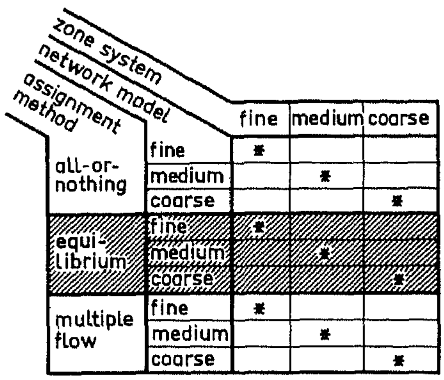

Fig. 1. Experimental design.

city of Eindhoven with its nearly 200,000 inhabitants has been chosen for a case study. The city is representative of a large group of medium-sized European towns. Larger cities having more congestion would involve prohibitive costs. Moreover, excellent trip data were available for Eindhoven.

We applied three assignment models at three network levels of the Eindhoven road system. Figure 1 presents the experimental design. It can be seen that the fineness of the zone system and the degree of network detail were varied in combination. Network models having this zone-network compatibility are assumed to be more efficient. The levels of detail selected are called: fine, medium, and coarse. In the next section these network models will be described. Only assignment models used in current practice have been applied: all-or-nothing, equilibrium, and multiple route assignment. Observed peak-period car trips were assigned.

Most of the results presented in this paper deal with equilibrium assignment. Some of these results are compared with corresponding all-or-nothing outcomes in order to show the relative effects of both the level of detail and the assignment model type. When judging the effects of the experiment we investigated various kinds of assignment results such as link loads, trip times, link travel times, impedances and routes. In this article mainly loads will be dealt with.

An equilibrium assignment model produces a load pattern that satisfies the well-known first principle of Wardrop: "The journey times on all routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route." Equilibrium assignment models derived this load pattern by solving an equivalent nonlinear minimization problem.[3, 4, 11]

We adopted the so-called Frank-Wolfe algorithm for our study and performed five iterations.

1.2. Assignment Network Models

The urban road system was simplified using the *reduction* method. This means that the real links of a network model were selected directly from the actual road network. The selection was based on the functional class of the real-world links. The characteristics of the real links such as length, capacity, etc. are identical to those of the corresponding real-world links.

The zone system was based upon the selected networks: they are the "holes" delimited by the real links selected. Consequently, zone boundaries generally coincide with the real links selected. Each zone is represented by a centroid which is linked to the real links by connectors.

An essential property of our network models is that they are strictly *hierarchical*. A link included in a network model of a lower level of detail is also included in every higher-level network model. Real links common to more than one network have the same characteristics at every level of detail. It is also true that a fine-level zone is always included completely in a coarse-level zone. These hierarchical relationships guarantee very consistent network models and thereby enable an easier tracing of the effects of the level of detail.

Three network models were developed: a fine, a medium and a coarse one. They are illustrated in Figure 2. The fine model is nearly identical to the actual road network. It includes almost all streets and has building blocks as zones. The medium level was chosen such that it corresponds with normal transportation planning practice. It includes all arterials and collectors. Our coarse network model represents only the arterials and may therefore be regarded as a sketch-planning network. The choice of

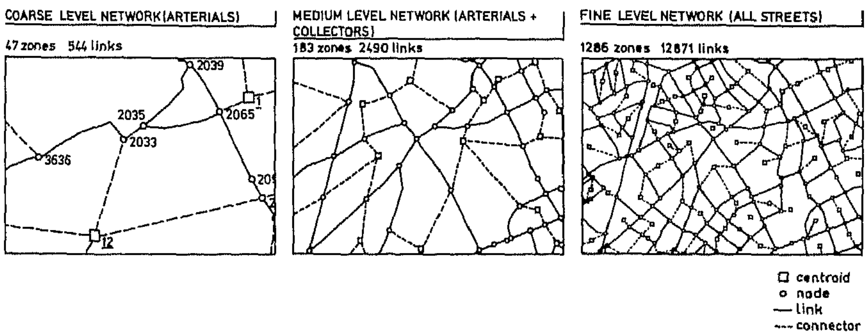

Fig. 2. The same section of the network at three levels of detail.

TABLE I

Overall Statistics of the Actual Network and Network Models Indicating the Level of Detail of the Models

Network Characteristics	Network Type[a]					
	Actual Network Abs.	Network model				
		Fine Abs.	Medium		Coarse	
			Abs.	%[b]	Abs.	%[b]
Total Model						
No. of directional links	10,018	12,871	2,490	19	544	4
No. of nodes	3,570	4,312	826	19	204	5
Link length (directional) (km)	1,245	1,348	648	48	275	20
Length·capacity (km·veh./h)		898,651	525,268	58	305,388	34
Mean free-flow speed (km/h)[c]		32	35		40	
Real Part						
No. of actual links (nondirectional)	5,009	4,338	1,599	37	487	11
No. of model links (directional)		8,283	1,548	19	342	4
No. of real crossings	2,692	2,525	553	22	111	4
No. of auxiliary nodes	878	501	90	19	46	9
Real link length (directional) (km)	1,245	969	425	44	166	17
Length·capacity (km·veh./h)	—	898,651	525,268	58	305,388	34
Mean free-flow speed (km/h)[c]	—	34	46		52	
Connector Part						
No. of centroids (= zones)	—	1,286	183	14	47	4
Mean no. of inhabitants per internal zone	—	150	1,130		5,300	
No. of connectors (directional)	—	4,588	942	21	202	4
Connector length (directional) (km)	—	379	223	59	109	29
Mean speed (km/h)	—	25	25		30	
Trip Table						
No. of assigned trips (total)	—	58,575	57,999		56,260	
No. of intrazonal trips	—	143	719		2,458	
% of zero interchanges (internal)	—	99.8	90.0		30.9	

[a] Abs. = Units as indicated in row heading.
[b] Relative to fine model.
[c] Mean free-flow speed = (∑ link length)/(∑ free-flow link travel time).

the levels of detail was made in such a way that the medium level has equal "distances," in terms of the degree of simplification, to both the fine and the coarse levels.

Special attention was paid to the development of travel time/volume relationships for the network model links. It is essential that the relationships for a common stretch of road at various levels of detail are consistent.[6]

Table I presents some overall statistics of the network models. It can be seen that the size of the medium and coarse networks, measured by the number of nodes and links, is about 20% and 4%, respectively, of the fine network model. Link length and capacity, however, appear to decrease much less: about 50% and 25% for the medium and coarse network, respectively. Thus, when reducing the size to one fifth, only half of the capacity is lost.

Unexpectedly, the number of intrazonal trips is very small at all levels of detail. This might be largely due to the rather widespread use of the bike for short trips in Holland. An important implication is that in our case intrazonal trips may be neglected as an important factor influencing the differences between the assignment outcomes at various levels of detail.

2. PREDICTION OF LINK LOADS

2.1. Network Load Totals

First, we will discuss network-wide results whereas information on individual links is given in the next section. Some network-wide results are given in Table II. The findings on *total load kilometers* show only small differences between the three levels of detail. It may be concluded

TABLE II

Some Equilibrium Overall Load Figures at Three Levels of Detail (2-Hour Volumes)

Load Characteristics		Level of Detail		
		Fine	Medium	Coarse
Total Model				
Load kilometers	veh·km/2 h	304,823	305,796	313,873
Load hours[a]	veh·h/2 h	7,308	7,575	10,869
Mean link load[b]	veh/2 h	226	472	1,141
Real Part				
Load kilometers	veh·km/2 h	279,706	272,211	256,456
Load hours[a]	veh·h/2 h	6,553	6,457	8,955
Mean link load[b]	veh/2 h	289	641	1,545
Load factor[c]	—	0.16	0.26	0.42
Connectors				
Load kilometers	veh·km/2 h	25,117	33,584	57,417
Load hours	veh·h/2 h	775	1,119	1,914
Mean connectors load[b]	veh/2 h	66	151	527

[a] Based on link times of penultimate iteration.
[b] Mean link load = (∑ load kilometers)/∑ link length).
[c] Load factor = (∑ load kilometers)/∑ length capacity).

TABLE III

Equilibrium Overall Load Figures of the Real Links by Comparable Functional Class Groups (2-Hour Volumes)

Load Characteristics[a]		Level of Detail		
		Fine	Medium	Coarse
Functional Class I:				
Load kilometers	veh·km/2 h	192,627	197,121	256,456
Load hours	veh·h/2 h	4,058	4,327	8,955
Mean link load	veh/2 h	1,155	1,185	1,545
Load factor	—	0.27	0.30	0.42
Mean link speed[b]	km/h	47	47	29
Functional Class II:				
Load kilometers	veh·km/2 h	68,517	75,090	—
Load hours	veh·h/2 h	1,866	2,130	—
Mean link load	veh/2 h	263	291	—
Load factor[c]	—	0.16	0.20	—
Mean link speed[b]	km/h	37	35	—
Functional Class III:				
Load kilometers	veh·km/2 h	18,562	—	—
Load hours	veh·h/2 h	609	—	–
Mean link load	veh/2 h	34	—	—
Load factor[c]	—	0.03	—	—
Mean link speed[b]	km/h	30	—	—

[a] Class I = primary roads (motorways, principal arterials, etc.) making up the coarse network; Class II = secondary roads (minor arterials, major collectors), only included at the medium and fine levels; and Class III = local roads (minor collectors, local streets), only included in the fine network.

[b] Weighted with loads.

[c] Load factor = ($\sum$ load kilometers)/($\sum$ length capacity).

therefore that the estimation of this variable is nearly insensitive to the level of detail used. Essential to this result are the small number of intrazonal trips as well as the carefully designed network models. *Total load hours* for the fine and medium level are nearly equal, but the coarse level estimate is substantially higher (nearly 50%). An explanation for this will be offered below. Examination of the load figures for the *connector system* shows an expected increase in the connector system's share when decreasing the level of network detail. This is in accordance with the function of the connectors: they represent the elements of the road system not included in the real part of the network model.

Since the real part of the network model represents a different selection of the road system at each level of detail, a comparison of load outcomes is more meaningful if they refer to identical parts of the road system. The functional classification defines such common parts of the various levels. Table III gives the corresponding findings for these link groups: primary, secondary and local roads. It can be seen that load hours, load kilometers as well as mean link load for *functional class I roads*, which are common to all levels of detail, are roughly the same for the fine and medium level, but show significantly higher values for the coarse level. In particular, a further explanation is required for the doubling of the load hours, when the coarse level is used. The explanation is important because it provides useful insights into the way network reduction affects the assignment process. Two effects will be distinguished: first, the increase of the route lengths and second, speed changes. Let us start the analysis with the elimination of functional class II roads when using the coarse level instead of the medium level network model. The kilometrage on these links (75,090; cf. Table III) is taken over by the connectors as well as by the functional class I links. Connectors get 54,417 − 33,584 = 23,833 additional load kilometers (cf. Table II), whereas functional class I get the remainder, i.e. 75,090 − 23,833 = 51,257 load kilometers. Thus, the elimination directly results in a 26% increase of the functional class I kilometrage. This additional amount of kilometrage causes a subsequent redistribution of all loads among functional class I links through the equilibration mechanism. Since the load kilometers on the links appear to increase by 256,456 − 197,121 = 59,335 or 30% (see Table III), equilibration accounts for an increase of 4%.

When assuming the same link speeds for both levels, this increase in load kilometers already would lead to a 30% increase in load hours at the coarse level. Load hours of functional class I appear to increase from 4327 to 8955 or 107%, however. The remaining 77% of the load hour difference must then be caused by (fictitious) congestion effects, as is indicated by the substantial drop in the mean link speed from 46 to 29 km/h. (*Note:* This implies that the coarse level loads are in the steep part of the travel time functions.) Figure 3 illustrates the effects.

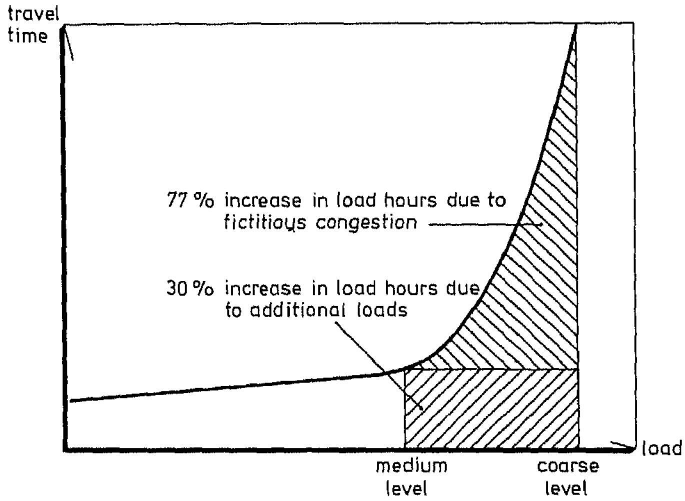

Fig. 3. Illustration of network reduction effects on load hours on a typical functional class I link.

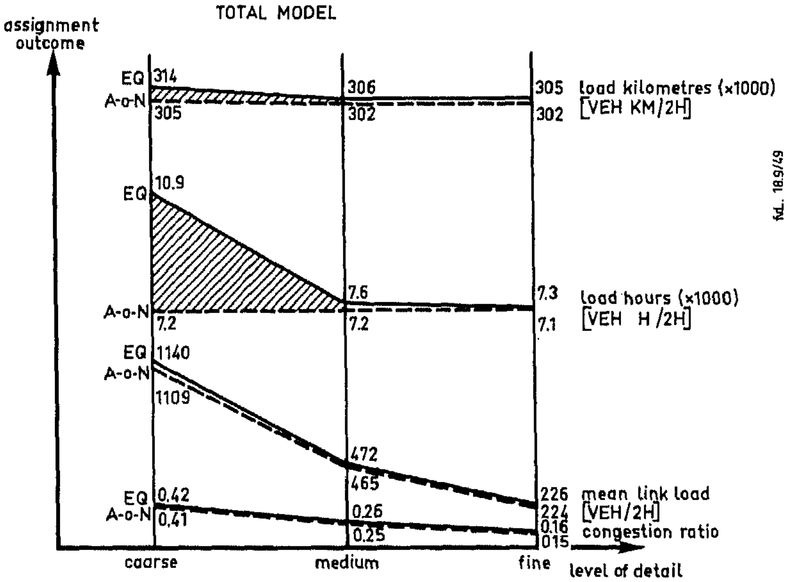

Fig. 4. Equilibrium and all-or-nothing assignment results (network load totals) at three levels of detail.

The small extra increase of load kilometers due to equilibration (4%) might suggest a limited spatial diversion of trips among equilibrium routes. This is not the case, however. An analysis of the equilibrium routes has revealed[6] that there exists a considerable amount of route spreading. The small increase in load kilometers follows from the small differences in lengths between these alternative routes.

When comparing equilibrium and *all-or-nothing* assignment outcomes (see Figure 4) very similar results are found at each of the levels. In general, the equilibrium model gives only slightly higher figures, except for the total load hours. At the coarse level this quantity is estimated nearly 50% higher with the model in question. The reason for this is clear: when going from the medium to the coarse network loads on the same links become much higher (see Table III). In the equilibrium method the link travel times increase correspondingly, whereas link travel times in the all-or-nothing method remain constant at all levels of network detail.

The strong similarity of the other total load findings stems primarily from the small differences in length between the equilibrium routes and the all-or-nothing path, despite clear differences in spatial pattern.

2.2. Individual Link Loads

Link load estimates are essential outcomes of the traffic assignment procedure. In this section we will deal with the differences between these estimates made at various levels of detail.

For this purpose, random samples of links were drawn in each functional class. We confine ourselves here to the primary roads where each link in the sample has a fine, a medium as well as a coarse-level estimate of the load. Differences in individual link load between levels are expressed in the root mean square error being an appropriate measure of disagreement between two series of outcomes:

$$\text{RMSE} = \sqrt{\sum (V_c - V_f)^2/(N-1)}$$

V_f = finer level volume (fine or medium); average value is $\bar{V}_f$

$$\text{RMSE (\%)} = (\text{RMSE}/\bar{V}_f)\cdot 100$$

V_c = coarser level volume (medium or coarse); average value is $\bar{V}_c$;
N = sample size

Table IV gives this measure for three comparisons. The RMSE (%)-value for the medium versus fine-level comparison is 15%, while coarse-level volumes show a RMSE-difference of 66% with fine-level estimates. These figures indicate that at the level of individual link volumes a much greater difference between levels appears than was found with the aggregate load measures.

Furthermore, since the fine level might be assumed to have no aggregation error, the above-mentioned RMSE-figures also roughly indicate the level of error due to spatial aggregation that might be expected in an assignment analysis.

Using a split-up of the RMSE into three components it is possible to gain further insight into the factors that affect the differences between the levels. The following expression holds:

$$\text{RMSE} = (N/N-1)\text{AE}^2 + \text{DSD}^2 + \text{CV}^2$$

with:

AE = average error = $\bar{V}_c - \bar{V}_f$,

DSD = difference between standard deviations = $\text{SD}_c - \text{SD}_f$

$= \sqrt{(V_c - \bar{V}_c)^2/(N-1)} - \sqrt{(V_f - \bar{V}_f)^2/(N-1)}$

CV^2 = covariation between series = $2(1-R)\cdot\text{SD}_f\cdot\text{SD}_c$

R = correlation coefficient

N = number of observations.

This relationship expresses that the total variance between two series is a sum of three components:

1. The (squared) difference between the means of both series, which is equal to the average difference or bias:

TABLE IV

Comparison of Individual Link Loads between Various Levels of Spatial Detail (Primary Roads, 144 links) (Equilibrium Assignment [veh/2 h])

Comparison	$\bar{V}_f$	$\bar{V}_c$	AE	DSD	CV	RMSE	% RMSE	%[a]		
								AE^2	DSD^2	CV^2
Coarse-Fine	1156	1668	+512	+384	420	767	66	45	25	30
Coarse-Medium	1206	1668	+463	+326	390	688	57	46	22	32
Medium-Fine	1156	1206	+49	+58	150	168	15	9	12	79

[a] % AE^2 is the ratio of AE^2 and $RMSE^2$ as a percentage; % DSD^2 is the ratio of DSD^2 and $RMSE^2$ as a percentage; and % CV^2 is the ratio of CV^2 and $RMSE^2$ as a percentage.

2. The (squared) difference between the standard deviations of both series;
3. The covariance between the series.

The first two components express that part of the differences that is caused by a change in the general level of the outcome. In our case this change stems from forcing the same trips over a smaller network when going from e.g. a medium to a coarse network. This introduces a systematic difference (bias) between the volumes.

The third component expresses the difference that follows from interchanging and redistributing volumes between links. In our case this stems from a change in routing possibilities due to network changes and equilibration respectively. We call such differences dispersion around the bias. From Table IV we can see that when switching from the fine to the medium level differences are generally small and for the most part (79%) unsystematic. However, when the coarse level is used the differences become very great where 70% is due to a general increase in volumes (high bias) and only 30% is caused by random variations.

A further refinement can be gained by establishing simple regression lines between the medium and coarse level loads respectively and the fine level loads using the primary road sample (see Figure 5). The lines show that the general increase found before is a strongly proportional increase: on the average, high volume links have a high increase and low volumes only a small increase. At the coarse level, the standard error of estimation after regression is relatively small: less than half of the total

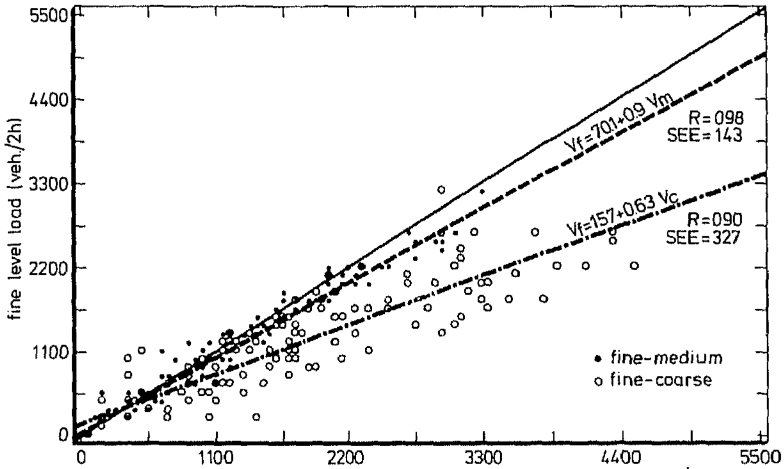

Fig. 5. Relationships between fine level and coarser level link loads (primary roads, $N = 144$).

difference between the series

$$(SEE/RMSE = 327/767 = 0.43).$$

In conjunction with the relatively small amount of covariation between levels this result offers possibilities for correcting the bias in the predictions. If one is able to relate the systematic difference to an independent factor, e.g. the amount of network capacity eliminated, one could partly correct for the error due to network simplification.

From this error analysis the following can be learned: assuming that the fine level outcomes are most accurate an increase in network simplification leads to increasing link load errors. The greater the deviation from the real network, however, the larger will be the systematic error component, and consequently the better the possibility of correction for the bias resulting from network simplification. It seems possible to correct nearly 60% of the error in coarse-level estimates.

Hence, there is evidence that it is worthwhile to develop rules of thumb for improving link load estimates from crude assignments.

If we look at the same figures (not presented here) of the *all-or-nothing assignment* two interesting findings can be revealed:

1. Equilibrium assignments show somewhat smaller differences between spatial levels;
2. The systematic component of these differences is much larger with the equilibrium than with the all-or-nothing model.

Hence, in the case of equilibrium assignment link load errors due to spatial aggregation are not only smaller but can presumably also corrected be much better.

2.3. Link Loads Versus Counts

Another, more powerful, means of studying the effect of spatial detail is a comparison of assigned volumes with ground counts on a link by link basis. In our study 190 directional counts were available.

At each level of detail the differences between volumes and counts will be analyzed separately for groups of links common to the levels (the three functional classes). Whereas in the preceding direct comparison of levels the only possible factor causing link load differences is the level of spatial detail, now in this indirect comparison of levels we also have to do with other error sources, such as:

—the assignment model
—trip data
—count data
—link characteristics.

We will unravel the total error using the same RMSE-decomposition as in the previous section.

Table V gives detailed RMSE-figures and their decompositions for the various relevant link groups. A first important finding from this table is the continuous improvement of load estimates, in every respect, when going from a coarse to a detailed network:

(i) For all links as well as specific link groups
(ii) For total error as well as error components
(iii) For the absolute as well as relative errors.

The analysis of the whole set of counted links at each level of detail shows that the fine level (complete network) gives the best estimates. Going from the coarse to the medium level a large shift in the figures can be observed. From the medium to the fine level, however, only a small but clear improvement can be achieved, notwithstanding the large difference in network size.

The higher precision at the fine and medium level may stem either from a better estimation of additional links or from an improvement at common links. Therefore, counted links are analyzed by functional class (Table V, Figures 6 and 7). For the primary roads (*I*) the relative error is substantially reduced when using a medium network instead of a coarse network: 87% versus 45% (Table V, column 8). Going from the medium to the fine level, however, only marginal improvement (45% versus 41%) can be observed, notwithstanding the large difference in network size.

For the secondary roads (*II*), which are included only in the medium and fine level, the relative error is improved slightly more: 81% versus 68%. The large reduction of the errors between the coarse and medium level and the small reduction between medium and fine level make up a very important finding for practice (Figure 8). The heavier the loads, the larger the absolute error and the smaller the relative error, a well-known phenomenon that also can be observed here. This explains the differences between the various link groups. The primary roads have therefore a much smaller percentage error (only 41% at the fine level) than the other road types. The differences between levels now look smaller than those found with the direct comparison in the previous section: the inclusion of other error sources hides the influence of the level of spatial detail.

Let us now have a look at the error components. There is a considerable difference between levels in this respect: at the coarse level 65% (Table V, columns 9 + 10) of the error stems from a systematic difference in the general level of the volumes whereas the medium and fine level estimates show only a relatively small contribution of systematic influences, i.e. 26 and 20%, respectively (primary roads only). At the two finer levels the contribution of random factors is thus predominant.

TABLE V

Load-Count Agreement at Three Levels of Detail for Various Link Groups (Definition of Variables in Section 2.2. and Table IV [veh/2 h])

Level	Functional Class	N (1)	$\bar{V}_{obs}$ (2)	$\bar{V}_{est}$ (3)	AE (4)	DSD (5)	CV (6)	RMSE (7)	% RMSE (8)	% AE^2 (9)	% DSD^2 (10)	% CV^2 (11)
Coarse	I	57	1107	1698	+591	+505	572	968	87	38	27	35
Medium	I	57	1107	1265	+158	+197	432	501	45	10	16	74
	II	83	342	386	+44	+115	250	278	81	3	17	80
	I + II	140	654	744	+90	+152	341	383	59	5	16	79
Fine	I	57	1107	1228	+121	+167	405	454	41	7	13	79
	II	83	342	373	+32	+82	216	233	68	2	12	86
	III	50	76	73	−3	+16	88	89	117	0	3	97
	I + II	140	654	721	+68	+121	310	340	52	4	13	83
	I + II + III	190	502	551	+49	+107	270	295	59	3	13	84

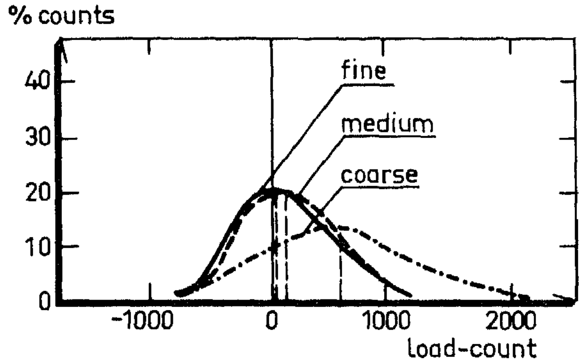

Fig. 6. Frequency distributions of volume-count differences at primary roads (functional class I) at three levels of detail ($N = 57$).

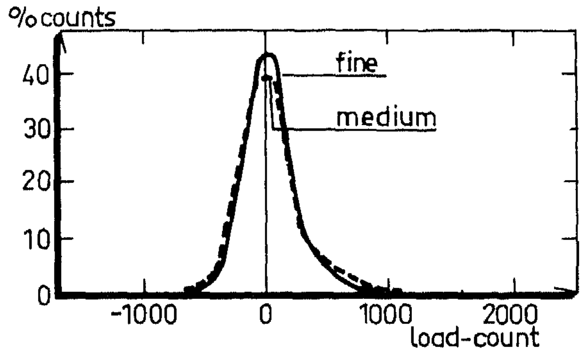

Fig. 7. Frequency distributions of volume-count differences at secondary roads (functional class II) at two levels of detail ($N = 83$).

Also when looking at the absolute error components there are noticeable differences: the level of the covariance is relatively constant compared to the level of the systematic differences. This absolute as well as relative increase in the systematic error component when coarser networks are used can only stem from spatial aggregation.

At this point it is interesting to know the contribution of spatial aggregation to the total error. Elsewhere a statistical analysis of the errors has been performed.[6] It showed that for the primary roads spatial aggregation accounts for 18% and 78% of the total error at the medium and coarse level of detail respectively. Thus, at the coarse level spatial aggregation is the major source of error.

Since we know that spatial aggregation primarily leads to a systematic bias (proportional increase) in the predictions it is worthwhile to study

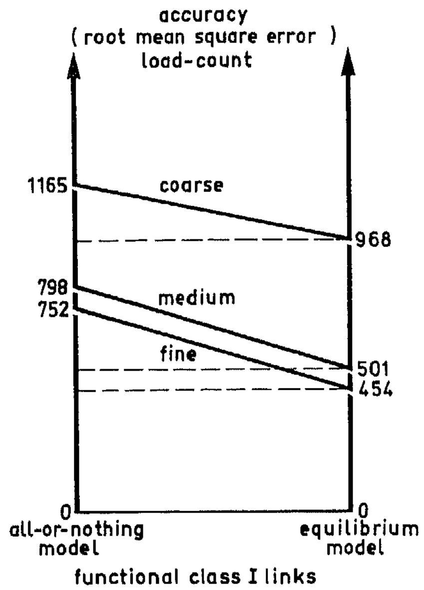

Fig. 8. Prediction errors of functional class I links by assignment model type and level of network detail (57 counts).

the possibility of correcting the estimates. By fitting correction formulas with observations a considerable improvement in the link load estimates might be possible. As is shown in the previous section on direct-level comparison simple regression lines could be established between the predicted and observed link volumes. Such a line fully accounts for the systematic error components, i.e. differences in means and in standard deviations of the series, and partly reduces the covariance between the series.

Compared to the all-or-nothing model the equilibrium results are far more accurate in every respect, especially at the finer levels of detail. This is indicated in Figure 8 for the primary roads.

In addition, the error is more systematic with the equilibrium model giving better possibilities for correction.

3. TRIP CHARACTERISTICS

ESTIMATES of trip characteristics, e.g. mean trip length and mean trip time, are useful indicators of the transportation system's performance. Furthermore, the trip length and trip time variables are an essential input to other transportation submodels, such as mode choice and distribution models. For these reasons it is interesting to know the sensitivity of estimates of trip characteristics to the level of spatial detail.

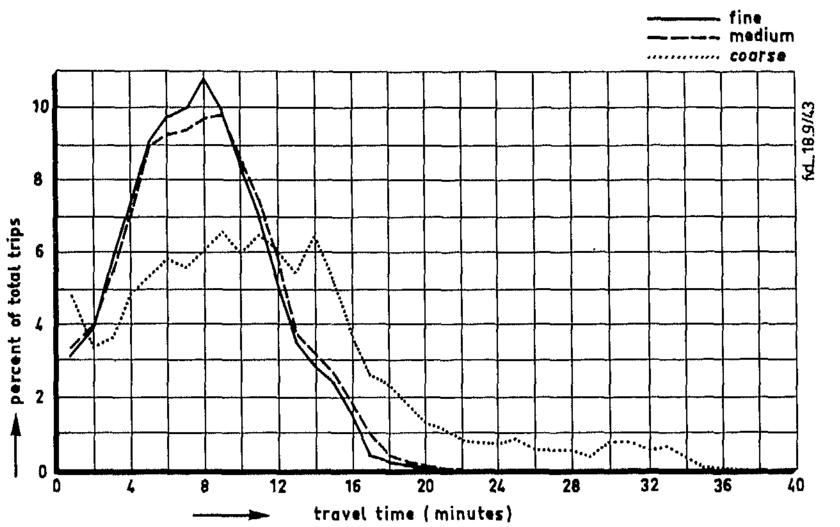

Fig. 9. Equilibrium trip time distributions at three levels of detail.

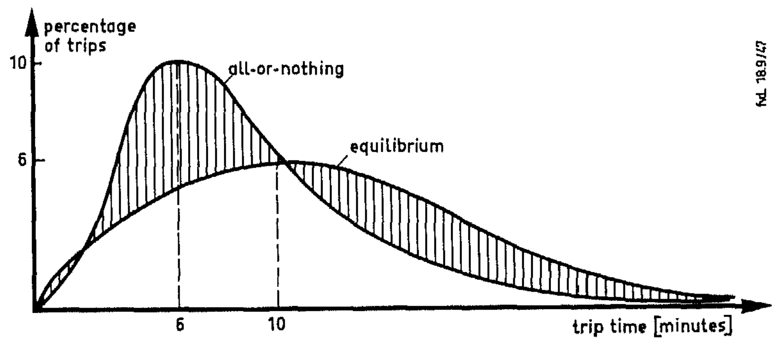

Fig. 10. A comparison of stylized coarse-level trip time distributions obtained with the equilibrium and the all-or-nothing assignments.

From the network-wide results on load kilometers and load hours (see Section 2.1) the sensitivity of the mean trip characteristics is already known. In addition, Figure 9 shows the trip time distributions at three levels.

As can be seen, coarse-level trip time estimates are substantially different: on the average nearly 50% higher than with the other levels.

The trip time distributions resulting from the *all-or-nothing* and equilibrium assignments are significantly different at the coarse level (see Figure 10), whereas only slightly different at the other levels. At every level, the equilibrium trip time distribution gives more trips with long

TABLE VI

Trip Time Frequency Statistics Estimated with the Equilibrium and All-or-Nothing Models, Respectively, at Three Levels of Detail

Trip Time Characteristics	Level of Detail					
	Fine		Medium		Coarse	
	Equilibrium	All-or-nothing	Equilibrium	All-or-nothing	Equilibrium	All-or-nothing
Mode (min)	8.5	7.5	9.0	7.5	10.0	6.0
Median (min)	7.6	7.1	7.8	7.2	10.2	7.2
Mean (min)	7.5	7.2	7.7	7.4	11.1	7.7
Mean speed (km/h)	41.7	42.7	40.4	42.1	28.9	42.5

trip times. Table VI summarizes a few frequency statistics of these distributions.

The small difference in mean link load between equilibrium and all-or-nothing assignments could not cause such large trip time deviations. The only reason is the flow-dependence of link travel times in the equilibrium model whereas all-or-nothing link times remain constant between levels, regardless of the volume (cf. Section 2.1).

Combining the findings on loads and trip times, it is reasonable to assume, in the absence of observations, that trip times obtained with the equilibrium model at a coarse network level greatly overestimate (by nearly 50%) the true values, and are worse than the all-or-nothing estimates. (The latter result stems from a compensation of errors in the all-or-nothing case.)

Coarse-level equilibrium trip time estimates as input to other travel models should therefore be used with caution.

4. CONVERGENCE

UNLIKE the usual procedure, we decided to apply a predetermined and equal number of iterations (five) at each level of detail. By comparing the degrees of convergence the impact of the level of detail on convergence speed can be established.

In this section outcomes on two meaningful convergence statistics will be presented. The first one is the relative duality gap δ_R which is related to the optimization process [10]:

$$\delta_R = \left(\textstyle\sum_a c_a \cdot f_a - \sum_k c^k \cdot f^k\right) / \left(\sum_a c_a \cdot f_a\right)$$

c_a = time travel on link a
f_a = flow on link a
c^k = time of shortest path between O-D pair k
f^k = number of trips between O-D pair k.

δ_R may be interpreted as the total assigned travel time less the total

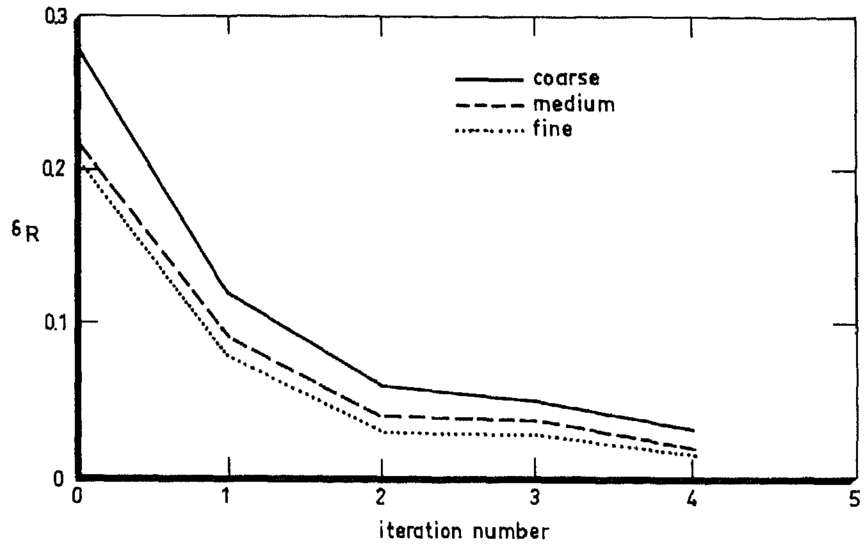

Fig. 11. Convergence as measured by the relative duality gap.

travel time that would be obtained if all traffic were to follow the shortest paths, divided by the total assigned travel time.[9] In case of equilibrium this excess travel time is zero. Figure 11 shows the convergence at three levels of detail as measured by this statistic. In general the largest reduction in δ_R took place in the first and second iteration. From then on, each additional iteration yielded only small reductions. This tailing off phenomenon of this algorithm has been noted previously by others.[5]

As to the effect of the level of detail on convergence the following can be observed. The finer the network model the better the load patterns satisfy the Wardrop equilibrium as measured by δ_R. This holds for every iteration.

In particular the coarse-level load patterns are clearly worse, whereas the medium and fine-level solutions are much more similar. It may be concluded fron this that, although the coarse-level convergence speed is somewhat higher, the coarse-level network requires more iterations to obtain an equally good solution in this respect than the other network models.

In contrast to this rather theoretical stopping criterion we also investigated convergence in a more practical manner. Therefore we compared the link loads computed after each iteration with ground counts. Figure 12 shows the root mean square error (RMSE) for 57 counted links of functional class I.

First of all it is evident that the error decreases with increasing level of

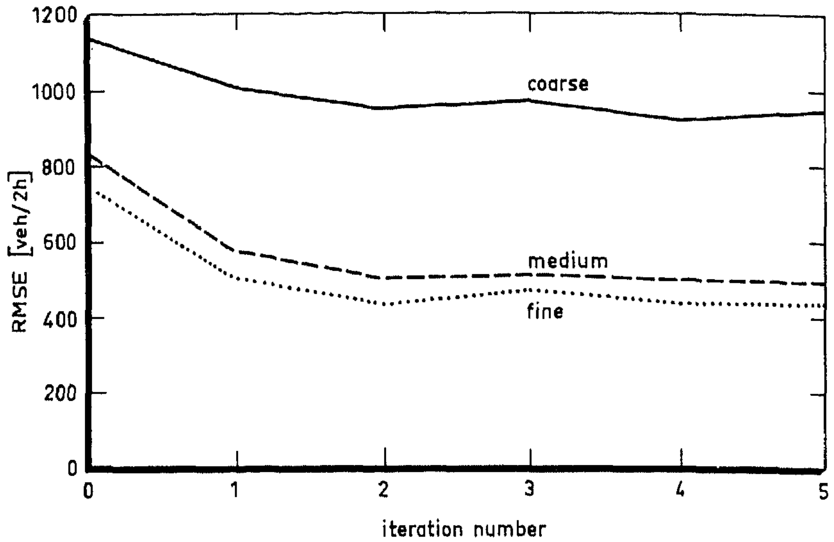

Fig. 12. Convergence as measured by the root mean square error of computed and counted flows on major links (functional class I).

detail. Especially the difference between the coarse and medium level is significant. This finding was already brought out in Section 2.3. The difference between the levels of detail remains more or less the same during the entire iterative process.

Furthermore it can be seen that the improvement of the solution, as compared with ground counts, is mainly obtained from the first and second iteration. Additional iterations do not yield any further improvement: the RMSE-values start oscillating.

In conclusion we may state that, although the assignment process keeps improving the solution in a Wardrop sense, from an empirical point of view improvement stops after a few iterations. It must be realized however, that both statistics have a network-wide significance whereas in many planning situations a judgment of local load patterns is more desirable.

5. CONCLUSIONS

THIS ARTICLE reports on the results of an empirical study into the effect of the level of detail in the network model upon car traffic assignment output quality. Two assignment models have been applied at three levels of detail: the equilibrium assignment and the all-or-nothing assignment model.

It was found that:

(i) The level of detail of the network model has a significant effect on the assignment output quality:
(ii) Refining the network and the zone system always improves assignment outcomes. Beyond a certain level, however, further refinement only yields marginal improvements;
(iii) At every level of detail investigated the equilibrium assignment model performs much better than the all-or-nothing model, even though the network was only slightly congested.

The findings presented on the effect of the level of spatial detail are purely experimental until now, referring to a single empirical study. There is a strong need for generalization of these results. To this end further work on this subject should try to establish a theoretical foundation from which such effects can be predicted mathematically in a variety of circumstances.

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