An Axial Array for Volumetric Intravascular Ultrasound Imaging

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An Axial Array for Volumetric Intravascular Ultrasound Imaging

PROEFSCHRIFT

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In 2008, a total of 135,136 people died in the Netherlands. Of this number, 40,868 (30.2 %) died of cardiovascular disease (CVD), equivalent to one person every 13 minutes, a number similar to that of people dying from cancer (40,750 or 30.2 %). From the CVD related deaths, 27.9 % can be attributed to (acute) myocardial infarction (AMI, a heart attack) or (acute) cerebrovascular attack (CVA, a stroke).

Apart from high death tolls, in total 345,830 people were hospitalized that year due to CVD related conditions. In reality this number is larger still as this number does not include hospitalizations of duration less than a day.

This number by itself does not necessarily lead to dramatic conclusions if effective treatment were available. However, 36 % of the people below 45 years of age hospitalized for (acute) myocardial infarctions or cerebrovascular attacks die within a single year after treatment from another AMI or CVA [1]. This fraction increases dramatically with time: more than 70 % of the hospitalized people over 70 years of age die within five years of the treatment [1].

The above numbers, taken from [2] unless stated otherwise, clearly indicate the need for effective diagnosis of cardiovascular diseases at an early stage, and for better guidance of the possible treatments. Even though several medical examinations are currently used to detect cardiovascular disease, none of these methods is able to diagnose the disease in its initial stage. Both diagnosis and treatment guidance will benefit from higher quality imaging techniques that can diagnose CVD in earlier stages.

## 1.1 Atherosclerosis

Lethal cardiovascular events include ruptures of aneurysms (local, blood-filled dilations of arteries), internal hemorrhages (bleedings) and thrombosis (blood flow restriction by a clot). Each of these conditions can be caused by a number of
cardiovascular diseases. However, the major CVD leading to these lethal conditions is atherosclerosis [3, 4]: thickening and/or hardening of blood vessel walls due to a build-up of fatty material.

Since it is the major contributor to cardiovascular events, much research is devoted to diagnosing atherosclerosis in as early a stage as possible. To understand the diagnosis strategies for atherosclerosis, first the mechanisms behind the development of atherosclerosis will be discussed along with measurable changes in the physiology of the arteries. All facts stated in the remainder of this section are taken from [3].

1.1.1 Formation
In a healthy artery, the lumen (the area through which blood flows) is separated from the arterial wall by the endothelium; a smooth, impermeable layer of only one cell thickness that reduces flow friction, see figure 1.1. If this endothelium is damaged, the change in physiology results in a ‘call for help’, aided by means of inflammation, which attracts monocytes from the bloodstream.

These monocytes, whose primary role is to neutralize foreign material or material indicated as ‘hostile’, will enter the intima, the innermost layer of the arterial wall. Once inside the intima they transform into macrophages and absorb all tissue that should not be present in the intima, among others LDL (Low-Density Lipoprotein) cholesterol. Upon absorbing LDL cholesterol, macrophages transform into foam cells, and if foam cells are accumulated, a fatty streak is formed - the first stage of atherosclerosis.

In theory this should not cause any threat since macrophages can move from the intima back into the bloodstream. However, inflammation also initiates the formation of smooth muscle cells (to repair the damage of the arterial wall) which may trap the foam cell. Due to ageing and entrapment, macrophages can die inside the intima, leaving a lipid (a localized body of fat too large for a single macrophage to absorb) behind.

The death of a macrophage leads to more inflammation since the remaining debris have to be cleaned up, and the attracted monocytes can introduce more LDL, get trapped, die, form a lipid and send out for even more monocytes to clean up. If this circle is allowed to continue long enough, a lipid pool (a collection of lipids contained in a pocket inside the artery wall) may be formed. This early atheroma is the second stage of atherosclerosis.

Once a lipid pool is formed, one of the scenarios depicted in figure 1.1 will happen. First, the smooth muscle cells, originally formed to repair the artery wall, will inevitably die. In some cases they calcify after dying, leading to a calcific plaque in and on the arterial wall which stiffens the blood vessel. Stiffening of the blood vessel makes it more prone to rupture, especially at non-stationary locations, as in the case of, e.g., coronary arteries.

Second, the lipid pool may stabilize if the covering layer of smooth muscle cells is thick enough to prevent more monocytes from being attracted. The lipid pool may even dissolve if monocytes can still enter and the newly formed
1.1. ATHEROSCLEROSIS

Figure 1.1: The various stages of atherosclerosis, together with the anatomy of an artery, and its possible developments. Image taken from [3].

macrophages can be transported to other parts of the body. In both cases there is little to no change in the stiffness and cross-section (lumen area) of the artery and thus no health risk.

Third, the lipid pool may continue to grow. At first the artery wall will extend outwards so that the lumen area remains virtually constant. This remodelling may weaken the arterial wall and can therefore cause an aneurysm. Even this situation can stabilize, leaving a relatively large lipid pool covered by a thick fibrous cap, a relatively safe situation. However, once this stage is reached, complete healing is rare [5].

If the atheroma continues to grow, at some point remodelling cannot continue and the atheroma extends inwards, thereby decreasing the lumen area (see figure 1.1). The narrowing of the lumen area is called stenosis. If the covering fibrous cap is sufficiently thick this situation is stable and only the bloodflow is decreased. This causes ischemia, a shortage of oxygen to organs further along the artery, causing, e.g., angina pectoris.

If the covering fibrous cap is thin, however, it is prone to rupture, especially if it is also calcified. This kind of atheromas is dubbed vulnerable plaque [6]. Upon rupture, lipids, foam cells and debris enter the lumen and attract blood platelets, and a thrombus or embolus (static or moving blood clot) may form inside the lumen, which acutely, partially or completely, occludes the blood vessel. This
inevitably leads to severe ischemia of the affected tissue and is the mechanism behind heart attacks and strokes. Usually part of the affected organ dies off, of which, should the victim survive, only a small fraction regenerates.

1.1.2 Treatment

Once atherosclerosis is detected, treatment can be aimed at lowering the LDL cholesterol level, widening the narrowed lumen, or a combination of the two. Usually the cholesterol level is lowered by putting the patient on a diet. This treatment has the lowest risk.

The lumen can be rewidened by introducing a balloon into the blood vessel at the location of the stenosis. By inflating the balloon, the artery wall is pushed outwards, a process called angioplasty, thereby opening up the lumen. However, this is usually only a temporary solution, as the artery wall tends to revert to its original, decreased diameter.

To fixate the new lumen area, a stent (a metal grating that, when expanded, does not retract inwards) is placed around the balloon and expanded upon inflation of the balloon. Unfortunately a stent by itself does not stop the accumulation of lipid bodies and actually acts as an effective substrate for lesion growth. To slow down the rate at which stenosis reforms, and hence recurrence of lumen narrowing, the stent can be imbibed with drugs that reduce inflammation.

Angioplasty can only be applied to non-vulnerable plaque, as the procedure might rupture the plaque and cause complete occlusion. In addition, during inflation of the balloon the artery is completely occluded, which introduces a risk of ischemia to organs further down the artery.

When a vulnerable plaque has already ruptured and completely occluded the artery, the only option is by-pass surgery, where the occluded section of the artery is by-passed by connecting a piece of artery, taken from somewhere else in the body, to points just before and just after the occlusion in the affected artery.

1.2 Detection Methods

Since its invention in 1927 by Egas Moniz [7], angiography has been the gold standard to scan arteries for plaques (atherosclerotic lesions). Back then it was required to submit patients not only to radiation, but also to probes introduced into the bloodstream designed to administer contrast agents in the vascular regions of interest. In 1953 the technique was significantly improved by Sven Seldinger [8] by introducing a guidewire along which catheters could be manoeuvred to previously unreachable places.

Angiography is a very useful tool to image the contours of the arteries. By injecting contrast agents into the blood, the blood will absorb much more radiation than its surrounding tissues and will therefore be clearly distinguishable. This way, narrowing of the blood vessel can be visualised as in figure 1.2.
Even though very clear images are obtained from which the amount of narrowing (stenosis) of the artery can be determined, this technique has several serious drawbacks. As the obtained images are transmission cross-sections, only projections are obtained and the image quality depends strongly on the imaging angle. This also brings the risk that improper stent placement, for instance when the stent is not completely in contact with the artery wall, goes unnoticed. In addition, a patient is subjected to a radiation dose.

These drawbacks, however, are small compared to the fact that only narrowing of the lumen is observed. As discussed in the section above, severe narrowing is the result of either ruptures of previously vulnerable plaques or the continuous growth of stable atheromas. The remodelling phase is not observed at all, as the lumen area remains the same. In short, angiography only detects very late stages of atherosclerosis [10] and is thus not sufficient for diagnosis of atherosclerosis in its initial stage. Moreover, the dangerous vulnerable plaques will not show up until after they have ruptured.

To overcome the latter limitation, numerous other imaging techniques, aimed at determining the structure of the arterial wall rather than the lumen, have been developed. In the remainder of this section an overview of current techniques will be given.

### 1.2.1 Non-Invasive Techniques

Cardiovascular imaging techniques can be divided into two main groups: extraluminal and intraluminal. The former group consists of techniques that image relevant bodyparts from outside the body, and are thus completely non-invasive.
1. INTRODUCTION

The latter group is composed of techniques where probes are introduced into the human body through various openings, either natural or artificial (by means of incisions).

From the patient’s perspective, extraluminal techniques are preferred as they require no surgery, anaesthetics or any other preparation. However, since most organs are covered by other tissues, imaging has to be performed through or around this tissue. Especially the lungs and bones present a challenge since their material properties are very different from soft tissue and therefore they generate strong reflections of, e.g., acoustic waves.

Nevertheless, successful extraluminal techniques have been developed. For superficial arteries, it is possible to obtain images with ultrasound. Carotid artery imaging, for example, is frequently performed and intima-media thicknesses can be obtained with reasonable accuracy. A high intima-media thickness (see figure 1.1) may indicate the presence of an atheroma.

For relatively stationary arteries, Magnetic Resonance Imaging (MRI) has been used to assess the state of artery walls. Utilizing the relaxation time of excited spins of atom cores, the hydrogen content of the measured area can be determined and then used to distinguish different media [6]. Unfortunately the resolution and signal-to-noise ratio are unsatisfactory [11, 12].

Angiographic images can be improved by using Computed Tomography (CT) rather than single X-ray images. Using CT, a higher resolution can be achieved than with angiography. However, as with angiography, only the lumen area is imaged rather than the arterial wall, so a higher resolution will not reduce the chance of missing atheromas.

1.2.2 Invasive Techniques

In order to image arterial walls of non-superficial arteries, several intraluminal techniques have been developed. To cope with non-stationary arteries, e.g. the coronary arteries which are subjected to severe heart or respiratory motion, intravascular methods have been designed: artery walls are imaged from within the artery itself by means of a catheter. These intravascular imaging modalities can be divided into two groups: fully invasive, where in addition to introducing probes into the bloodstream, the blood is flushed away by saline (a salt solution mimicking the salinity of blood), and so-called minimally invasive, where only a probe is inserted into the blood vessel and no flushing is performed.

Fully Invasive Modalities

The most logical means of inspection is simply looking at the arterial wall, which is exactly the aim of angioscopy. With angioscopy, a relatively large catheter is introduced in which illumination and optics are integrated. Using a miniature camera, real-time images of the arterial wall can be viewed directly without processing. Since visible light does not penetrate deep into tissue, angioscopy only gives an impression of the innermost layer of the arterial wall and will only
show thrombosis and fatty streaks rather than fibrous cap thickness or other indications of vulnerable plaque [6]. However, blood is non-transparent so it needs to be flushed by saline. As saline contains no red blood cells, no oxygen is transported by it and ischemia may occur if too much is used for longer durations.

Rather than visible light, Raman spectroscopy uses single-frequency infrared light to determine the composition of the arterial wall surface. In Raman scattering, absorbed photons are re-emitted at different wavelengths, and by measuring the resulting wavelengths, information on the scattering material can be obtained. As with visible light, flushing is required and penetration is limited [13]. Therefore, little to no geometrical information about the wall itself is obtained [14, 15].

Infrared light is also used in Optical Coherence Tomography (OCT). In OCT, the intensity and travel time of backscattered infrared light are used to generate an image of the arterial wall. Using this technique, very high resolutions can be achieved [16], but this technique as well has a very limited penetration depth [17] and flushing is required.

A final fully invasive imaging technique is thermography. In the section above it was stated that atherosclerosis is an inflammation process and thus gives rise to an increase in temperature. Thermography is a technique that measures the temperature of the arterial wall to localize possible regions of atherosclerosis. As elegant as this approach may seem, it has serious drawbacks.

The temperature difference between a healthy and diseased artery wall is small, and the temperature difference for stable and vulnerable plaque are the same. Due to the small temperature differences and noise, measurements are rarely reproducible and different studies show different results. Furthermore, bloodflow around the catheter strongly influences the thermal behaviour of the probe and artery wall, and this behaviour is not yet fully understood. Ideally the blood flow thus has to be stalled temporarily by deliberately occluding the lumen, which may cause ischemia or the formation of a thrombus [18].

Minimally Invasive Modalities

Only two imaging techniques exist that do not require saline flushing, and thus have a reduced risk of catheter-induced ischemia. The procedure is never risk-free: in case of lumen narrowing the catheter may block the already limited bloodflow, the catheter can rupture vulnerable plaque or the catheter may damage otherwise healthy parts of the endothelium and thereby initialize new atherosclerotic lesions.

The first is Near-InfraRed spectroscopy, where the absorption spectrum of infrared light is used to determine what tissue is being illuminated. Again, as penetration is limited, no geometrical information beyond the wall surface is obtained, and measurements are only performed in a single spot rather than a cross-sectional image [19]. Furthermore, no reference absorption spectra are yet available so tissue characterization is difficult [20].

The second technique that does not require flushing is IntraVascular Ultra-
Sound (IVUS), where ultrasound pulse-echo images are acquired by introducing ultrasonic transducers into the artery. No health issues originating from the use of ultrasound are known, and since no ionising radiation is used, patients are not subjected to a radiation dose. In addition, three-dimensional imaging is possible and common practice, large penetration depths of up to 1 cm can be achieved and high resolution can be obtained. Due to these benefits, in the remainder of this work the focus will be on IVUS.

1.3 IntraVascular UltraSound (IVUS)

Of the currently available imaging techniques, IVUS shows the most advantages. In this section, first the history and development of IVUS are discussed, most of which can be found in [21, 22], and then current commercial applications will be given.

1.3.1 History

IVUS is based on the pioneering work of Wild & Reid in 1957 [23]. In this work, a side-viewing, rotating endoluminal probe was used to rectally image the abdominal organs. The probe was about 2.5 cm in diameter, contained a single transducer element and operated at 15 MHz. By rotating the transducer element inside the probe, different angles could be illuminated so that a cross-sectional image of the tissue surrounding the probe could be obtained. Even though only a single element was used, images were clear enough to distinguish between various organs.

Only three years later, the first intravascular probe was presented [24]. In this work, the probe consisted of a single transducer element as well, which was rotated to obtain cross-sectional images of the artery. Even though the device was small enough to be introduced into major arteries, it could not be used for e.g. coronary arteries with diameter of 3 mm and below. The device was mainly aimed at imaging the heart rather than arteries, and is sketched on the left hand side of figure 1.3.

In 1972 the first endoluminal phased array was introduced [25]. It consisted of 35 elements transmitting and receiving at 5 MHz. By summing and delaying the individual signals, the beam could be focused in a specific location. This resulted in much sharper images, however more measurements and computations were required to obtain cross-sectional images. With an outer diameter of 3 mm, this probe could still not be introduced into coronary arteries.

After that, it took until the end of the 1980’s for the first commercial devices to arrive. In 1989 the first in vivo demonstration of IVUS was performed [26]. After further miniaturisation and optimisation, in 1995 a 64-element transducer operating at 20 MHz and with outer diameter of 1.2 mm was developed [27]. This device was small enough to be introduced in the smaller arteries. By employing clever algorithms, both the number of ultrasound firings and the amount of com-
putations required to obtain a cross-sectional image were significantly decreased. The design is schematically shown on the right hand side in figure 1.3.

Both the rotational and phased array designs have been used extensively since their development. In both situations cross-sectional images are obtained, and by pulling the device slowly through the artery, multiple slices of the artery are imaged. A three-dimensional model of the artery can be formed by stacking the successive cross-sectional images.

### 1.3.2 Current Commercial Applications

Rotating element and phased array IVUS catheters have been used since 1960 and 1972, respectively, and are nowadays both commercially available. Boston Scientific, for instance, produces a rotating single element transducer with a center frequency of 40 MHz [28], and Volcano stocks both a rotating single element transducer with a frequency of 45 MHz and a 64-element phased array probe operating at 20 MHz [29] (shown in figure 1.4). Even though the image quality is still being improved, mostly by shifting to higher frequencies, the basic designs have not been altered for decades.

### 1.4 Current research on IVUS

Over the past two decades, IVUS has developed from an experimental technique to a clinical standard that is widely used and commercially available. Much progress has been made in image quality over the years. Lately, however, the only improvement on current hardware has been to increase the frequency. Current IVUS research can be divided in three categories: (i) the development of a new IVUS modality, namely Forward Looking IVUS (FLIVUS), (ii) research on the
1.4.1 Forward Looking IVUS

One problem with IVUS is that images are only obtained perpendicular to the catheter since the traditional designs are all side-looking. In partially narrowed blood vessels, this introduces the risk of completely occluding the vessel with the catheter tip if the amount of stenosis in the arteries is not known a priori. Furthermore, severely narrowed or occluded vessels cannot be imaged and it is difficult to image the branching of vessels.

To overcome these problems, in 1997 a forward looking IVUS (FLIVUS) device was described [30]. FLIVUS, as the name suggests, is aimed at imaging the artery in the direction in which the catheter is fed into the blood vessel. Usually a phased array annular ring is used [31]. It is successfully applied during interventions to guide physicians during, e.g., stent placement.

1.4.2 Acoustical Aspects

A wide variety of acoustical aspects in IVUS are being studied, ranging from ultrasound generation and detection to nonlinear behaviour of ultrasound. The different subjects can be roughly divided into three categories: ultrasound gener-
ation, nonlinear behaviour and flow measurements. In the following paragraphs an overview of current research is given.

**Ultrasound Generation**

Traditionally, ultrasound transducers operating in the MHz range have been constructed from piezoelectric materials; materials that expand or contract upon applying a potential across its surfaces. Due to mechanical constraints they cannot be minimized infinitely, which limits their operating frequency. In addition, creating very small transducers requires much effort, resulting in high costs.

To overcome these limitations, research is carried out on Capacitive Micromachined Ultrasound Transducers (CMUTs) [32, 33, 34] to steer away from using piezoelectric materials. Numerous groups have designed and constructed CMUTs, but all designs consist of a vacuum cavity with a top and bottom electrode. Sound is generated by applying a potential difference between the electrodes which causes attraction or repulsion of the two electrodes. Acoustic waves hitting the transducer change the capacitance of the cavity, which can be accurately measured.

With techniques very similar to those used in the processor chip industry, very small designs can be built and large batches can be produced simultaneously, which may significantly decrease the cost of the end product [32, 35]. CMUTs have been demonstrated to have similar sensitivity as traditional piezoelectric transducers, though with a significantly higher bandwidth which may improve image quality [36]. Their small sizes make CMUTs ideal for applications in FLIVUS [37, 31].

Piezoelectric Micromachined Ultrasound Transducers (pMUTs) are based on a similar design, although sound is generated by a piezoelectric element attached to one of the electrodes [35, 38]. In principle, pMUTs should show the same advantages over piezoelectric elements as CMUTs, but this is yet to be demonstrated [39].

Finally, in photoacoustics, high-intensity light is employed to rapidly locally heat the tissue. The resulting thermal expansion generates an ultrasonic signal, which is registered by conventional ultrasound transducers. Since the absorption spectrum and mechanical properties of the tissue determine the amplitude and spectrum of the ultrasound generated, information on tissue composition can be obtained in addition to spatial information. Recently, the feasibility of photoacoustics using commercially available IVUS transducers was demonstrated in vitro [40].

**Nonlinear Behaviour**

When pressure amplitudes are relatively low and the medium is not strongly nonlinear, ultrasound propagates approximately linearly, i.e., pressure waveforms propagate undistorted. If the pressure is sufficiently high, however, the medium is locally compressed sufficiently to change the sound velocity. Propagation becomes
nonlinear, waveforms will be distorted and (sub)harmonics will be created.

The strongest harmonics are generated in the location with the highest acoustic pressure, usually the focus of a transducer array. Harmonics have higher frequencies and are thus attenuated more strongly in tissue due to the frequency dependence of the attenuation coefficient [22]. However, the harmonics are generated inside the tissue, and thus only need to propagate one way. Therefore, a large penetration depth can be achieved despite stronger attenuation. In addition, the array in transmit mode has a different radiation pattern than in receive mode. This fact can be exploited to reduce sidelobes and further improve the image quality. Imaging tissue with nonlinear pressure fields, so-called tissue harmonic imaging, applied to IVUS, has been described in e.g. [41, 42].

Usually, boundaries of organs and arterial walls are not clearly delineated in ultrasound images, due to low contrast between the media around the interface. To increase the delineation of separate tissues, use can be made of microbubbles that selectively enter certain media (usually blood). If the size of the bubbles is properly tuned to the center frequency of the ultrasound transducer, resonance occurs, which may cause buckling or rupture of the bubbles. If this occurs, harmonics, usually of lower frequency than the transmitted wave, are generated. By measuring this response, information on the targeted medium only can be obtained. Nonlinear contrast agents in combination with IVUS are currently being investigated [43].

Flow Measurements

Rather than actually imaging the arterial wall, information about lumen narrowing and wall friction, which changes with changing physiology, can also be obtained by measuring the flow profile of the blood. Flow of the blood is measured using the Doppler-effect: ultrasound scattering off moving blood particles changes in frequency. Doppler flow imaging originated in 1967 and has since been applied both extraluminally and intraluminally, and intravascularly as well [21].

1.4.3 Signal Processing

Besides the development of forward looking IVUS and studies on nonlinear effects, research is also carried out using traditional side-looking catheters operating in the linear regime. Much progress has been made in the (automatic) segmentation and classification of the tissues in ultrasound measurements by means of signal processing, structural modelling and combining IVUS with other techniques.

Autoregression, integrated backscatter and wavelet analysis are examples where the radio frequency signals and their spectra are used to determine the type of tissue responsible for the measured backscattered ultrasound [44]. Autoregression is currently implemented in commercial applications [29].

Elastography is a technique where two measurements taken under different blood pressures are compared to measure the strain of the artery. From this strain the elastic modulus of the arterial wall is then computed, and based on the
obtained modulus the type of tissue is determined [30, 45]. This approach is taken a step further in modulography, where strain images are used to optimize a finite element model of the artery to obtain the optimal fit to the measurements [46].

IVUS has also been combined with several other techniques [21]. One of these combinations is ANGUS, a combination of angiography and IVUS, where externally acquired X-ray angiography images are combined with IVUS images to form an accurate three-dimensional model of the artery [47]. Recently, a catheter prototype has been presented that demonstrates the co-registration of IVUS and OCT in vitro [48], which combines the high resolution of OCT with the penetration depth of IVUS.

1.5 IVUS Shortcoming

Even though CMUT research could potentially bring down the costs of an IVUS catheter dramatically, and nonlinear effects could significantly improve the image quality for some diagnoses, current side-looking IVUS catheter designs suffer from severe shortcomings.

1.5.1 Motion Artefacts

Traditional IVUS probes generate cross-sectional, two-dimensional images of the arterial wall. Cross-sectional images are generated either electronically in a phased array, or by rotating a single element, but with the exception of FLIVUS all catheters are side-looking. To obtain information in the artery direction, the catheter is gradually pulled back through the artery and cross-sectional images are continuously imaged. If the pullback velocity is known, it can be determined where each successive cross-sectional image was obtained, and a three-dimensional image of the artery can be generated.

However, the above only holds under a severe set of assumptions. In the case of a rotating single element transducer, it is assumed that the tip rotates at a constant speed so that all the lines in a cross-sectional image are equally spaced and at the proper locations. This, however, appears not to be true. Due to friction of the rotary shaft with its casing, curvature of the artery and occasional discontinuous pullback, the rotation speed is not always constant. This in turn causes Non-Uniform Rotation Distortion (NURD) and results in observable errors in the obtained images [49, 50].

The second assumption, applying to both rotating and phased array transducers, is that the position of the catheter with respect to the artery is known since the pullback rate is fixed. Besides the minor effect that the pullback rate is not always fixed due to friction, curvature and non-ideal pullback equipment, a large problem is caused by the motion of surrounding tissue. Especially in the case of coronary arteries, located directly on the heart muscle, this motion is severe: within one cardiac cycle the catheter tip can move as much as 6 mm back and forth along the artery axis [51].
Besides motion along the arterial axis, the catheter can also rotate around
the arterial axis due to curvature of the artery [52, 53], which will add to the
distortions caused by non-uniform rotation. Rotation around the other axes
is inhibited by the guidewire used to manoeuvre the catheter into the correct
artery. In addition, due to blood pressure variation over time, the blood vessel
will expand and contract [52, 53, 54]. Finally, respiratory motion significantly
contributes to catheter tip motion.

For two-dimensional images, only NURD plays a significant role. However, in the
case of three-dimensional imaging, both rotation and displacement of the catheter
lead to significant errors. Due to the significant motion of the catheter relative
to the artery within a cardiac cycle, only a single frame per cardiac cycle is used.
This frame is selected by analysing electrocardiograms that are measured during
the IVUS procedure, a technique called electrogating. To suppress respiratory
motion, usually the patient is asked to hold his or her breath during the pullback.

As a person at rest has a heart rate of about 60 beats per minute, electrogating
leads to a temporal sampling of roughly one frame per second. Current clinical
examinations are performed with a pullback rate of one millimeter per second,
which leads to a spatial sampling of one cross-sectional image per millimeter.
However, acoustic radiation patterns of current commercial probes are spatially
limited, as is clear from figure 1.5.

In this figure, simulated radiation patterns at two consecutive electrogated
positions are superimposed. Because the radiation pattern of a single element
is relatively narrow, no overlap between successive measurements on the two
positions is present, and an area with virtually no coverage is present. Due to
this spatial undersampling, large parts of the artery are not being imaged at all.

Furthermore, to obtain three-dimensional images, the motions of the catheter
have to be compensated for. Modelling the motions of the artery, e.g., heart
motion and rhythmic expansion due to blood pressure variation, is only partially
successful [56] due to the complexity of these motions [54].

Therefore, usually information from the (distorted) acoustical measurements
is used to compensate for motion artefacts [52, 57, 58, 59, 60]. However, even
these methods are only successful to a limited extent, as they assume consecutive
electrogated frames to contain certain landmarks (e.g., scattering events, arterial
branching) used in the motion compensation. Due to the severe spatial under-
sampling in the pullback direction, there is no guarantee that certain landmarks
used are indeed present in both frames, and in reality it indeed turns out that
often little to no similar information is present in two consecutive electrogated
cross-sectional images [52].

Reducing the pullback rate is not a practical solution, as there is a limit to
the amount of time a patient can hold his or her breath, and hence a limit to
how much time a pullback can take. In addition, the risk of complications caused
by the catheter increases with the pullback duration. In conclusion, motion
compensation is not possible with current hardware, so obtaining accurate three-
dimensional models of the artery under study using traditional, side-looking IVUS
1.5. IVUS SHORTCOMING

Figure 1.5: Simulation of the sensitivity pattern of an Eagle Eye Gold catheter from Volcano, in two different pullback positions spaced 1 mm apart. The transmitting element is 400 µm long in the artery direction and 27 µm wide. The two-way sensitivity patterns are computed using the FOCUS simulation software [55], using water for the surrounding medium (volume density of mass $\rho_0 = 1000$ kg m$^{-3}$, speed of sound $c_0 = 1500$ m/s). Patterns are shown on a logarithmic colour scale with a 25 dB dynamic range, and the contour indicates $-25$ dB. If the pullback rate is too high, severe spatial undersampling results, indicated by the lack of overlap of the radiation patterns at the two successive positions.

is not feasible.

1.5.2 Stent Imaging

Ideally, stent placement is an image guided procedure, where the clinician has a live visual aid during placement. This way, improper placement can be avoided and the chance of atherosclerosis recurrence is decreased. In principle, an IVUS catheter could be introduced along with the stent and balloon, or be used after angioplasty, to image the stent in the artery.

However, due to the large difference in material properties between the stent
and the artery wall, a very strong signal scattering off the stent is measured, which in practice saturates the signal amplifiers, causing saturated bright spots in the image. With conventional side-looking IVUS, either no stent wire is imaged because none is present, or only the stent wire is imaged as all acoustic energy of the pulse is reflected off the stent and no energy reaches the tissue behind it. Thus, imaging through or behind stents with conventional side-looking IVUS is hardly feasible.

1.6 Thesis Outline

Both the spatial undersampling and the inability to image through stents are due to measurements being performed in a single cross-sectional plane perpendicular to the pullback direction. In this work, the feasibility of imaging volumes rather than cross-sectional planes in each catheter position is studied. If these volumes are sampled densely enough, spatial undersampling is avoided and every part of the artery is imaged. In addition, if the electrogated consecutive volumes overlap, the overlap can be used to compensate for motion, and to generate a three-dimensional model of the entire scanned artery section.

Volume scans are acquired using a rotating linear transducer array placed in the pullback direction. This direction will be called the 'axial direction' throughout this thesis. In different fields of research, this direction is known as the elevational direction.

If the elements of the array are located closely to each other, acoustic energy can be focussed, either during the measurement or during post-processing, to yield high resolution images in the axial-radial plane. By rotating the array, a toroidal volume is imaged in every pullback location. Acoustic energy can also be focused in positions just behind stents wires, which will improve the imaging during stent placement.

The remainder of this thesis is divided into two parts. Part I - Design is aimed at simulating the performance of new prototype transducer arrays by first treating the forward scattering problem in Chapter 2, followed by incident field computation methods in Chapter 3. To avoid reflections off the numerical domain boundaries, in Chapter 4 frequency domain perfectly matched layers are introduced. Finally, in Chapter 5 the results of the previous chapters are combined to optimise the design of a linear array in the axial direction, and the fabrication process of the corresponding prototype is discussed.

In Part II - Prototype Characterisation, first a method is presented in Chapter 6 that reconstructs the motion of the transducer surface from far-field pressure measurements. Using this method, the quality of the prototype in terms of effective element size, cross-coupling and inter-element variation is assessed. Next, the imaging principles are discussed in Chapter 7, and these algorithms are applied to signals acquired with the prototype transducer array. Several tissue-mimicking phantoms are studied, and results are compared with measurements performed using a purely side-looking modality.
Part I

Design
When a sound source radiates sound, it will disturb the equilibrium of the surrounding medium by locally altering the particle velocity and pressure. The resulting motion of the surrounding medium will, in turn, disturb the equilibrium of the medium surrounding the excited area, and in this way the resulting wave front will propagate away from the source through the volume enclosing the sound source.

The rate at which the wave front propagates through the medium, referred to as the speed of sound, depends on the material properties, e.g., the compressibility and volume density of mass. If the material properties vary spatially, the speed of sound may become spatially dependent. If the speed of sound varies smoothly with space, refraction occurs.

However, when the speed of sound changes abruptly, for instance at interfaces between different media, an interesting phenomenon occurs. At the interface, the pressure and the particle velocity normal to the interface need to be continuous, as otherwise unphysical singularities would be created. To avoid these singularities, a second wave front is generated that ensures that, at the interface, the continuity requirements are met. Away from the interface, however, this additional wave front is observed as a reflection.

In intravascular ultrasound, this property is exploited to image the interfaces between different media. Here, an ultrasonic acoustic wave is transmitted into the surrounding blood, propagates radially away from the source and encounters a discontinuity in the acoustic material properties (e.g., the vessel wall, a calcific plaque, or a stent). At the discontinuity, part of the original incident wave is scattered or reflected back towards a receiver, where it is measured. If the speed of sound of the medium is known, spatial information can be extracted from the resulting signal, and after applying imaging algorithms an image of the artery wall is formed.

In this chapter, first the theory describing the propagation of acoustic wave fields in inhomogeneous media is derived along the lines of [61], followed by a
2. FORWARD SCATTER PROBLEM

2.1 Scatter Integral Equation

In the absence of acoustical sources and assuming time independent material properties, the medium will be in equilibrium. Within the medium, there will be a static ambient pressure \( P_0 \).

If the medium under studies is disturbed by an acoustical source, the total pressure within the medium will vary with location \( r \) and time \( t \), hence \( P(r, t) = P_0 + p_{\text{tot}}(r, t) \), where an acoustic pressure field \( p_{\text{tot}}(r, t) \) is superimposed on the static ambient pressure. Only this superimposed signal is of importance, as only this signal is caused by the acoustical sources. Therefore, only the additional acoustic pressure field \( p_{\text{tot}}(r, t) \) and particle velocity field \( v_{\text{tot}}(r, t) \) caused by acoustical sources or a propagating incident pressure field will be calculated. The use of the subscript \( _{\text{tot}} \) is explained in the next section.

2.1.1 Wave Equation in Inhomogeneous Media

Upon propagation through a medium, an acoustical wave field both translates and compresses the fluid. Consider the fluid element shown in figure 2.1, in which the possible motions of the element are illustrated. On the left, the fluid element is subjected to a force and thereby both translated and compressed, resulting in both pressure and particle velocity differences. On the right, the element is subjected to an outflow of mass which changes both the pressure and particle motion within the element. These motions already suggest an interaction between pressure and particle velocity, and indeed the equations for pressure and particle velocity will in general turn out to be coupled.

Translation and compression of a fluid element results in mass being transported in to and out of the element. Mass transportation is governed by the equation of motion,

\[
\nabla p_{\text{tot}}(r, t) + \dot{\Phi}(r, t) = f(r, t),
\]  

(2.1)
where $\nabla$ is the nabla operator, $\dot{\Phi}(\mathbf{r}, t)$ is the mass-flow density rate and $\mathbf{f}(\mathbf{r}, t)$ the volume source density of volume force. This equation relates the external force source to a change in pressure and density. Source $\mathbf{f}(\mathbf{r}, t)$ represents a dipole source.

An inflow of mass into a fluid element will cause the element to change in shape. This behaviour is governed by the deformation equation,

$$\nabla \cdot \mathbf{v}_{\text{tot}}(\mathbf{r}, t) - \dot{\Theta}_{\text{ind}}(\mathbf{r}, t) = q(\mathbf{r}, t),$$

(2.2)

where $\dot{\Theta}_{\text{ind}}(\mathbf{r}, t)$ is the induced part of the cubic dilatation rate, $q(\mathbf{r}, t)$ is the volume source density of injection rate and $\cdot$ signifies the inner product. Source $q(\mathbf{r}, t)$ represents a monopole source.

The functions $\dot{\Phi}(\mathbf{r}, t)$ and $\dot{\Theta}_{\text{ind}}(\mathbf{r}, t)$ depend on the type of fluid under study. For a linear, time-invariant, locally and instantaneously reacting and isotropic fluid, the constitutive relations for an inhomogeneous medium are [61, p. 66]

$$\dot{\Phi}(\mathbf{r}, t) = \rho(\mathbf{r}) D_t \mathbf{v}_{\text{tot}}(\mathbf{r}, t),$$

(2.3)

$$\dot{\Theta}_{\text{ind}}(\mathbf{r}, t) = -\kappa(\mathbf{r}) D_t p_{\text{tot}}(\mathbf{r}, t),$$

(2.4)

where $\rho(\mathbf{r})$ and $\kappa(\mathbf{r})$ are the volume density of mass and compressibility of the fluid, respectively, and $D_t$ the material time derivative, i.e. the time derivative as registered by an observer moving along with the fluid (with speed $\mathbf{v}$),

$$D_t = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla.$$ 

(2.5)

For low amplitude acoustic signals, the particle velocity $\mathbf{v}$ is small, and the material time derivative reduces to $\frac{\partial}{\partial t}$. Under this assumption, equations (2.3) and (2.4) reduce to linear equations. Inserting these linear forms of equations (2.3) and (2.4) in equations (2.1) and (2.2) yields the linear acoustic wave equation for inhomogeneous media,

$$\begin{cases}
\nabla p_{\text{tot}}(\mathbf{r}, t) + \rho(\mathbf{r}) \frac{\partial}{\partial t} \mathbf{v}_{\text{tot}}(\mathbf{r}, t) = \mathbf{f}(\mathbf{r}, t), \\
\n\nabla \cdot \mathbf{v}_{\text{tot}}(\mathbf{r}, t) + \kappa(\mathbf{r}) \frac{\partial}{\partial t} p_{\text{tot}}(\mathbf{r}, t) = q(\mathbf{r}, t).
\end{cases}$$

(2.6)

After applying the temporal Fourier transformation (cf. appendix A.1) and using the property $\mathcal{F}\{\frac{\partial}{\partial t} f(t)\} = i \omega \hat{f}$, the above equation (2.6) changes into

$$\begin{cases}
\nabla \hat{p}_{\text{tot}}(\mathbf{r}) + i \omega \rho(\mathbf{r}) \hat{\mathbf{v}}_{\text{tot}}(\mathbf{r}) = \hat{\mathbf{f}}(\mathbf{r}), \\
\n\nabla \cdot \hat{\mathbf{v}}_{\text{tot}}(\mathbf{r}) + i \omega \kappa(\mathbf{r}) \hat{p}_{\text{tot}}(\mathbf{r}) = \hat{q}(\mathbf{r}),
\end{cases}$$

(2.7)

where $\mathcal{F}\{\cdot\}$ indicates the forward temporal Fourier transformation, $\omega$ is the angular frequency, $i$ is the complex-valued unit, and the notation $\hat{f} \equiv f(\omega)$ is used to indicate a frequency domain variable.
2.1.2 Wave Propagation in Inhomogeneous Media

Wave equation (2.7) can be rewritten into a scalar equation in \( \hat{p}_{\text{tot}}(\mathbf{r}) \) only, which simplifies the solution of the equation. This derivation starts by rewriting the wave equation in terms of spatially invariant background material properties \( \rho_0 \) and \( \kappa_0 \), i.e.,

\[
\begin{align*}
\nabla \hat{p}_{\text{tot}}(\mathbf{r}) + i\omega \rho_0 \hat{v}_{\text{tot}}(\mathbf{r}) &= \hat{f}(\mathbf{r}) + i\omega (\rho_0 - \rho(\mathbf{r}))\hat{v}_{\text{tot}}(\mathbf{r}), \\
\nabla \cdot \hat{v}_{\text{tot}}(\mathbf{r}) + i\omega \kappa_0 \hat{p}_{\text{tot}}(\mathbf{r}) &= \hat{q}(\mathbf{r}) + i\omega (\kappa_0 - \kappa(\mathbf{r}))\hat{p}_{\text{tot}}(\mathbf{r}).
\end{align*}
\tag{2.8}
\]

Taking the gradient of the first line, multiplying the second line by \(-i\omega \rho_0\), and adding the results yields

\[
\nabla^2 \hat{p}_{\text{tot}}(\mathbf{r}) + \hat{k}_0^2 \hat{p}_{\text{tot}}(\mathbf{r}) = -\left( i\omega \rho_0 \hat{q}(\mathbf{r}) - \nabla \cdot \hat{f}(\mathbf{r}) \right) + \hat{k}_0^2 \frac{\kappa(\mathbf{r}) - \kappa_0}{\kappa_0} \hat{p}_{\text{tot}}(\mathbf{r}) + i\omega \nabla \cdot \{(\rho_0 - \rho) \hat{v}_{\text{tot}}(\mathbf{r})\},
\tag{2.9}
\]

where \( \hat{k}_0^2 = \frac{\omega^2 \kappa_0}{c_0^2} \) [61, p.79], with \( c_0 \) the speed of sound in the background medium and \( \hat{k}_0 \) is the wave number in the background medium.

The particle velocity \( \hat{v}_{\text{tot}}(\mathbf{r}) \) can be eliminated using the first line in equation (2.7). Together with the assumption that \( \hat{f}(\mathbf{r}) = 0 \) wherever \( \rho(\mathbf{r}) \neq \rho_0 \), the above equation (2.9) simplifies to

\[
\nabla^2 \hat{p}_{\text{tot}}(\mathbf{r}) + \hat{k}_0^2 \hat{p}_{\text{tot}}(\mathbf{r}) = -\hat{S}_{\text{pr}}(\mathbf{r}) - \hat{S}_{\text{scat}}(\mathbf{r}),
\tag{2.10}
\]

with primary source term

\[
\hat{S}_{\text{pr}}(\mathbf{r}) = i\omega \rho_0 \hat{q}(\mathbf{r}) - \nabla \cdot \hat{f}(\mathbf{r}),
\tag{2.11}
\]

which generates the incident pressure field, and contrast source term

\[
\hat{S}_{\text{scat}}(\mathbf{r}) = \hat{k}_0^2 \chi_{\kappa}(\mathbf{r}) \hat{p}_{\text{tot}}(\mathbf{r}) + \nabla \cdot \left[ \chi_{\rho}(\mathbf{r}) \nabla \hat{p}_{\text{tot}}(\mathbf{r}) \right],
\tag{2.12}
\]

where \( \chi_{\kappa}(\mathbf{r}) = \frac{\kappa(\mathbf{r}) - \kappa_0}{\kappa_0} \) and \( \chi_{\rho}(\mathbf{r}) = \frac{\rho_0 - \rho(\mathbf{r})}{\rho(\mathbf{r})} \) are the compressibility and density contrast functions, respectively. This scalar wave equation for inhomogeneous media is referred to as an inhomogeneous Helmholtz equation, where the inhomogeneities appear as source terms.

The inhomogeneous Helmholtz equation (2.10) is solved by convolving source term \( \hat{S}_{\text{pr}}(\mathbf{r}) + \hat{S}_{\text{scat}}(\mathbf{r}) \) with the Green’s function [62], i.e., with the solution of the wave equation for a dirac delta monopole source,

\[
\nabla^2 \hat{p}_{\text{tot}}(\mathbf{r}) + \hat{k}_0^2 \hat{p}_{\text{tot}}(\mathbf{r}) = -\delta(\mathbf{r}),
\tag{2.13}
\]

where \( \delta(\mathbf{r}) \) is the Kronecker delta distribution. The Green’s function is the spatial impulse response of the system in the absence of inhomogeneities. For a homogeneous medium, the Green’s function reads [61]

\[
\hat{G}(\mathbf{r}) = \frac{e^{-i\hat{k}_0 \|\mathbf{r}\|}}{4\pi \|\mathbf{r}\|},
\tag{2.14}
\]
where $||r||$ is the euclidean length of vector $r$ between the source and observation points. Using this Green’s function, equation (2.10) yields as solution

$$\hat{p}_{\text{tot}}(r) = \hat{G}(r) \ast_r \left\{ \hat{S}_{\text{pr}}(r) + \hat{S}_{\text{scat}}(r) \right\},$$  

where $\ast_r$ indicates spatial convolution.

In homogeneous media, $\hat{S}_{\text{scat}}(r) = 0$, and the total pressure field is generated only by the primary sources. In inhomogeneous media, however, the total pressure field will differ from the incident pressure field due to scattering, reflection and refraction, see figure 2.2. Denoting the incident pressure field by

$$\hat{p}_{\text{inc}}(r) = \hat{G}(r) \ast_r \hat{S}_{\text{pr}}(r),$$  

equation (2.15) can finally be written as

$$\hat{p}_{\text{tot}}(r) = \hat{p}_{\text{inc}}(r) + \hat{p}_{\text{scat}}(r),$$  

with scatter pressure field

$$\hat{p}_{\text{scat}}(r) = \hat{G}(r) \ast_r \hat{S}_{\text{scat}}(r).$$  

The above scatter integral equation (2.17)-(2.18) is referred to as a Fredholm integral equation of the second kind, where the integration is contained in the convolution.

## 2.2 Solution Methods

In principle, for known incident field and known contrast source, equation (2.18) completely solves the scatter problem for the unknown total pressure field: by convolving the contrast source term with the Green’s function for a homogeneous
background medium, the scatter pressure field is obtained. Unfortunately, the total pressure field depends on the scatter pressure field, see equation (2.17)-(2.18), which in turn depends on the total field. Consequently, the scatter integral equation (2.17)-(2.18) is an inversion problem.

This inversion problem can be approximately solved by neglecting multiple scattering, a valid assumption in weakly scattering media. In this so-called Born approximation, equation (2.18) reduces to

$$\hat{p}_{\text{scat}}(r) = \hat{G}(r) \ast_r \left\{ \hat{k}_0^2 \chi_\kappa(r) \hat{p}_{\text{inc}}(r) + \nabla \cdot [\chi_\rho(r) \nabla \hat{p}_{\text{inc}}(r)] \right\},$$

(2.19)

which is solved by performing only a single spatial convolution. In this approximation, no internal scattering and no refraction are included, and incident, scatter and total pressure fields all propagate with a spatially invariant speed of sound.

Especially when acoustic travel times are important, as is the case in intravascular ultrasound, the Born approximation is not sufficient. Hence, in this work the full scatter problem will be solved by means of inversion. The scatter integral equation (2.17)-(2.18) can be rewritten as

$$\hat{p}_{\text{inc}}(r) = \hat{p}_{\text{tot}}(r) - \hat{G}(r) \ast_r \left\{ \hat{k}_0^2 \chi_\kappa(r) \hat{p}_{\text{tot}}(r) + \nabla \cdot [\chi_\rho(r) \nabla \hat{p}_{\text{tot}}(r)] \right\},$$

(2.20)

or, in operator notation,

$$\hat{p}_{\text{inc}}(r) = \hat{L} \hat{p}_{\text{tot}}(r),$$

(2.21)

where $\hat{L}$ is the operator governing the scatter problem.

This equation is solved by inverting the operator,

$$\hat{p}_{\text{tot}}(r) = \hat{L}^{-1} \hat{p}_{\text{inc}}(r),$$

(2.22)

where $\hat{L}^{-1}$ is the inverse of the scatter operator $\hat{L}$. However, in practice the inverse of the scatter operator $\hat{L}$ cannot directly be found, either analytically or numerically, and inversion is performed iteratively. A very efficient inversion scheme, with guaranteed convergence within a finite number of iterations, is the conjugate gradient (CG) scheme [63].

Unfortunately, the CG scheme can only iteratively invert operators that are invertible. However, the scheme can be applied to operators that are both self-adjoint and positive definite to yield a least-squares inversion of the operator. Unfortunately, operator $\hat{L}$ does not meet these requirements. However, by applying the adjoint of an operator to the operator working on an input, the resulting combined operator is both symmetric and positive definite [63],

$$\hat{L}^{\ast} \hat{p}_{\text{inc}}(r) = \hat{L}^{\ast} \hat{L} \hat{p}_{\text{tot}}(r),$$

(2.23)

where $\hat{L}^{\ast}$ is the adjoint of the scatter operator, which is generated by solving $b \cdot \hat{L}c = \hat{L}^{\ast}b \cdot c$ for $\hat{L}^{\ast}$. Equation (2.23) is called the normal equation.

Note that CG on the normal equations only yields the actual solution in case the original operator $\hat{S}$ is invertible, otherwise only the solution with minimum
residual norm is obtained. An operator is invertible if and only if the kernel of the operator consists of only the zero-element. The kernel of the scatter operator is found by setting $\hat{L}\hat{p}_{tot}(\mathbf{r}) = \hat{p}_{inc}(\mathbf{r}) = 0$, and from a physical argument it follows that only $\hat{p}_{tot}(\mathbf{r}) = 0$ satisfies this requirement, as otherwise there would be a non-zero total field in the absence of an incident field. Thus, CG on the normal equations applied to the scatter operator will yield the actual total pressure field.

Even though CG on the normal equations applied to the scatter operator is guaranteed to yield the solution to the scatter problem, it requires the derivation and implementation of the adjoint operator, a cumbersome and time consuming task that has to be performed each time the operator is modified. To avoid this, the Bi-CGSTAB scheme, an adjoint-free CG scheme that, in addition, converges significantly faster than CG on the normal equations for most cases, is applied [64]. Both the standard CG and Bi-CGSTAB schemes can be found in appendix A.4.

### 2.3 Numerical Implementation

In order to enable simulation of transient acoustic scattering in an arbitrary inhomogeneous three-dimensional medium, software was written in Fortran 90 in which the scatter integral equation (2.20) is solved for $\hat{p}_{tot}(\mathbf{r})$ using the Bi-CGSTAB scheme. The scatter problem is solved individually for each frequency. To limit computation time, MPI [65] is used to solve for multiple frequencies simultaneously. To obtain the highest possible accuracy, certain operations should be treated with care.

#### Spatial Convolution

The spatial convolution between the Green’s function and the source term in equation (2.18) can be computed using a discrete sum. However, this computation is inordinately expensive. Therefore, spatial convolutions will be computed in the spatial Fourier domain, which reduces the convolution to inexpensive multiplications at the expense of an increase in memory load.

#### Spatial Derivatives

The spatial derivatives in the source term in equation (2.18) can be computed in the spatial Fourier domain. However, for discontinuous $\chi_\rho(\mathbf{r})$ the term $\nabla \cdot (\chi_\rho(\mathbf{r})\nabla \hat{p}_{tot}(\mathbf{r}))$ suffers from Gibbs’ phenomena. This is clearly visible in figure 2.3, where this density contrast source term is computed, in the wave number domain, for a one-dimensional situation using a 2.5 MHz continuous wave for $p_{tot}(x)$ and $\chi_\rho(x) = 0.1$ for $1.6 \text{ mm} \leq x \leq 4.7 \text{ mm}$, and $\chi_\rho(x) = 0$ otherwise. Significant differences between the analytical expression and the wave number domain result are observed throughout the spatial domain, especially outside the contrast region.
Figure 2.3: Contrast source term $\nabla \cdot (\chi \rho \nabla \hat{p}_{\text{tot}})$ for a one-dimensional case, using a 2.5 MHz continuous wave for $\hat{p}_{\text{tot}}(x)$ and $\chi(x) = 0.1$ for $1.6 \text{ mm} \leq x \leq 4.7 \text{ mm}$, and $\chi(x) = 0$ otherwise. The heavy solid line is the analytical result, consisting of a sinusoid and two delta spikes at the edges of the density contrast. The dotted line is the result when spatial derivatives are computed in the wave number domain, and strong abberations are observed, even outside the contrast region. The result obtained using finite difference with a 17-point stencil yields a source term that is very similar to the analytical expression, and mainly differs around the boundaries of the contrast region.

To avoid these errors, the spatial derivatives are computed in the spatial domain instead. Using Taylor expansions, the first order spatial derivative of generic function $f(x)$ can be shown to equal

$$\frac{\partial f(x)}{\partial x} = \frac{1}{2\Delta x} \sum_{j=1}^{J} \gamma_{j} [f(x + j\Delta x) - f(x - j\Delta x)] + O(\Delta x^{2\gamma}), \quad (2.24)$$

where $\Delta x$ is the grid spacing, coefficients $\gamma_{j}$ are determined by the method described in appendix A.3, and $O(\Delta x^{2\gamma})$ indicates that the approximation error is proportional to $\Delta x^{2\gamma}$. The symmetric three-point derivative is obtained by
setting $J = 1$.

For a 17-point stencil size, i.e., when $J = 8$, the resulting density contrast source term is shown in figure 2.3 by the dashed line. It is clear that the errors are significantly reduced. In experiments not treated here it was shown that a 17-point stencil yielded the highest numerical accuracy for a given spatial discretisation density.

To compute the spatial derivatives in a point, function values are required on $J$ points on all sides of the point of interest. However, these points do not exist on or near the numerical domain boundaries. To enable computation of derivatives on all points on the grid, additional points are appended with $\chi_r(r) = \chi_\rho(r) = 0$ in those points.

**Weak Form Green’s Function**

The Green’s function in equation (2.14) contains a simple pole, which complicates numerical implementation as the function goes to infinity at the pole. In appendix A.2 it is shown, however, that the local spherical average of the Green’s function, i.e., the spatial average in a sphere with volume equal to that of a grid element, around each point $r$ is finite. This spatially averaged Green’s function, called the weak form Green’s function [66], reads

$$
\hat{G}(r) = \begin{cases} 
\frac{6e^{-ik_0r}}{8k_0^3\pi a^3r} \left[ \sin(\hat{k}_0a) - \hat{k}_0a \cos(\hat{k}_0a) \right] & \text{for } r \neq 0 \\
\frac{6}{8k_0^2\pi a^3} \left[ (1 + ik_0a) e^{-i\hat{k}_0a} - 1 \right] & \text{for } r = 0
\end{cases}
$$

(2.25)

where $a = \sqrt[3]{\frac{3\Delta x^3}{4\pi}}$ is the radius of the averaging sphere.

**2.4 Accuracy**

The above implementation details were decided upon mainly based on limitations of machine precision, memory restrictions and Fourier transformation properties. The accuracy of the method was not taken into account. Therefore, in this section results obtained from simulations are compared to analytical solutions. First, a plane incident wave propagating perpendicular to a contrast boundary is modelled, followed by a plane wave propagating through a homogeneous penetrable sphere.

In this entire work, two error norms are used. The $L_1$-norm is computed by

$$
\epsilon_1(f, g; a, b, \ldots) = \frac{\sum |f(a_i, b_j, \ldots) - g(a_i, b_j, \ldots)|}{\sum |g(a_i, b_j, \ldots)|},
$$

(2.26)
Blood
\[ \rho = 1050 \text{ kg m}^{-3} \]
\[ \kappa = 3.91 \times 10^{-5} \text{ Pa}^{-1} \]

Fat
\[ \rho = 960 \text{ kg m}^{-3} \]
\[ \kappa = 4.82 \times 10^{-5} \text{ Pa}^{-1} \]

Observation point

Figure 2.4: A slice of the three-dimensional volume used to compare simulation results with the analytic solution of a planar pressure wave propagating perpendicular to the contrast boundary. Half of the volume consists of water as the background medium, the other half consists of fat, with material properties as indicated in the figure. The volume has dimensions 1.014 mm × 1.014 mm × 1.014 mm.

where \( f(\ldots) \) and \( g(\ldots) \) are arbitrary functions of at least the variables \( a, b, \ldots \), and summations run over all discrete values of the variables \( a, b, \ldots \). The \( L_2 \)-norm is computed in a similar manner,

\[
\varepsilon_2(f, g; a, b, \ldots) = \sqrt{\frac{\sum_{i,j,\ldots} |f(a_i, b_j, \ldots) - g(a_i, b_j, \ldots)|^2}{\sum_{i,j,\ldots} |g(a_i, b_j, \ldots)|^2}}. \quad (2.27)
\]

2.4.1 Plane Wave at Normal Incidence

Consider the situation sketched in figure 2.4. A pulsed planar wave \( \hat{p}_{\text{inc}}(x) \) is propagating through water from left to right, and hits the interface between water and blood under normal incidence. Part of the wave will reflect off the interface, \( \hat{p}_{\text{refl}}(x) \), and part of the energy will propagate through the contrast medium, \( \hat{p}_{\text{trans}}(x) \).

By requiring both the pressure and the particle velocity normal to the interface to be continuous on the contrast interface, for the incident, reflected and transmitted pressure fields can be derived that [67]

\[
\hat{p}_{\text{inc}}(x) = \hat{A}_{\text{inc}} e^{-i \hat{k}_0 x}
\]
\[
\hat{p}_{\text{refl}}(x) = \hat{A}_{\text{refl}} e^{i \hat{k}_0 x}
\]
\[
\hat{p}_{\text{trans}}(x) = \hat{A}_{\text{trans}} e^{-i \hat{k}_{\text{scat}} x},
\]

in which \( \hat{k}_{\text{scat}} \) is the wave number in the scatter body, and \( \hat{A}_{\text{inc}}, \hat{A}_{\text{refl}} \) and \( \hat{A}_{\text{trans}} \) are the amplitudes of the incident, reflected and transmitted pressure fields, re-
2.4. ACCURACY

respectively. The reflected and transmitted pressure field amplitudes are given by

\[ \hat{A}_{\text{refl}} = \hat{A}_{\text{inc}} \left[ \frac{1 - \frac{\rho_0 \kappa_{\text{fat}}}{\rho_{\text{fat}} \kappa_0}}{1 + \frac{\rho_0 \kappa_{\text{fat}}}{\rho_{\text{fat}} \kappa_0}} \right] \]  

(2.29)

\[ \hat{A}_{\text{trans}} = \hat{A}_{\text{inc}} + \hat{A}_{\text{refl}}, \]  

(2.30)

where \( \rho_{\text{fat}} \) and \( \kappa_{\text{fat}} \) are the volume density of mass and compressibility of the fat contrast, respectively.

The reflected and transmitted pressure field amplitudes are given by

\[ \hat{A}_{\text{refl}} = \hat{A}_{\text{inc}} \left[ \frac{1 - \frac{\rho_0 \kappa_{\text{fat}}}{\rho_{\text{fat}} \kappa_0}}{1 + \frac{\rho_0 \kappa_{\text{fat}}}{\rho_{\text{fat}} \kappa_0}} \right] \]  

(2.29)

\[ \hat{A}_{\text{trans}} = \hat{A}_{\text{inc}} + \hat{A}_{\text{refl}}, \]  

(2.30)

where \( \rho_{\text{fat}} \) and \( \kappa_{\text{fat}} \) are the volume density of mass and compressibility of the fat contrast, respectively.

The effects of discretisation density and spatial derivative stencil size on the accuracy of the resulting pressure field are tested using the geometry of figure 2.4. In this simulation, a Gaussian modulated plane wave is used. The Gaussian modulated excitation pulse \( \epsilon_{\text{Gauss}} \) is defined as

\[ \epsilon_{\text{Gauss}}(t) = A_0 \cos(2\pi f_0 t) e^{-(t-\mu)^2 / 2\sigma^2} \sqrt{2\pi\sigma}, \]  

(2.31)

where \( f_0 \) is the center frequency, \( A_0 \) is the pulse amplitude, \( \mu \) is the temporal offset, and \( \sigma \) is the temporal width of the pulse.

The center frequency is 20 MHz, and a temporal sampling rate of 100 MHz is used. The temporal width and offset are \( \sigma = 30 \) ns and \( \mu = 200 \) ns, respectively, which yields a fractional bandwidth of 62%. A temporal extent of 1.34 ps is simulated, which is twice the amount of time required for the incident plane wave to propagate through the volume. The Bi-CGSTAB scheme is stopped when an accuracy of \( 10^{-3} \) is reached. The spatial grid consists of cubic elements of dimensions 4.875 \( \mu m \times 4.875 \mu m \times 4.875 \mu m \), which amounts to 16 points per wavelength at the center frequency. The spatial derivatives are computed using \( J = 8 \).

In figure 2.5, the analytic solution of equation (2.28) to a plane wave reflecting off a blood-fat contrast interface at normal incidence is shown, together with the solution obtained using the numerical implementation. The figure shows the time trace of the total pressure \( p_{\text{inc}}(x, t) + p_{\text{refl}}(x, t) \) in the observation point indicated in figure 2.4.

The incident pressure is visible around \( t = 0.2 \) ps, and the reflection off the blood-fat interface is visible around \( t = 0.85 \) ps. At these two times, the numeric result is very close to the analytic solution. However, the numeric solution shows additional arrivals at around \( t = 0.3 \) ps and \( t = 1.0 \) ps. These arrivals are caused by reflections off the numerical domain boundaries, as outside the defined volume, the contrast functions \( \chi_\rho(r) \) and \( \chi_\kappa(r) \) equal zero, and hence additional contrast interfaces are present.

Even though the additional reflections can be explained, they do complicate quantitative comparison of analytic and numeric results. Therefore, quantitative analysis is limited to the time interval of the arrival of the reflected wave, i.e., to \( 0.77 \) ps \( \leq t \leq 0.94 \) ps. The error, in this time interval, between analytic and numeric results is compared using \( \epsilon_1(p_{\text{tot, num}}(t), p_{\text{tot, ana}}(t); 0.77 \mu s \leq t \leq 0.94 \mu s) \),
Figure 2.5: Time trace of the total pressure field in the observation point indicated in figure 2.4. The strong arrival around $t = 0.2 \, \mu s$ is the incident Gaussian modulated plane wave, the arrival around $t = 0.85 \, \mu s$ is the reflection off the only physical contrast interface. The differences between the numerical and analytic results at $t = 0.3 \, \mu s$ and $t = 1.0 \, \mu s$ are caused by reflections off the numerical domain boundaries.

the $L_1$-norm of equation (2.26), where $p_{\text{tot,ana}}(t)$ is the transient analytic total pressure field in the observation point, $p_{\text{tot,num}}(t)$ is the numeric total pressure field, and the summation runs over all time samples in the interval $0.77 \, \mu s \leq t \leq 0.94 \, \mu s$. For the situation shown in figure 2.5, this error amounts to 3.93 %.

From experiments not shown here it was found that the error decreases for increasing grid density. However, at 16 points per wavelength and $J = 8$, the transient three-dimensional problem already contained 900 million unknowns, and further decreasing the grid element size was impractical. Nevertheless, an error below 4% is found, for realistic contrast values, $J = 8$ and 16 points per wavelength $\lambda_0$, using the numerical implementation of the scatter integral equation (2.17)-(2.18).
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Figure 2.6: Sketch of the geometry used when modelling scattering of a plane wave off a homogeneous penetrable sphere. A plane wave propagates left to right through the background medium mimicking blood. A sphere with radius \( a = 256 \, \mu m \) consisting of fat is centered in the numerical domain. The dashed lines indicate the plane in which numeric results will be compared to the analytic solution.

2.4.2 Homogeneous Penetrable Sphere

The previous experiment showed that, using dense spatial sampling and \( J = 8 \) for the stencil size, accurate results could be obtained. However, the problem effectively was a one-dimensional one, which could have been solved much more efficiently by a one-dimensional treatment. In the following experiment, a full three-dimensional situation is modelled.

Consider the geometry in figure 2.6. A Gaussian modulated planar wave propagates through a cubic volume mimicking blood, of dimensions 1.024 mm \( \times 1.024 \, \text{mm} \times 1.024 \, \text{mm} \). Centered in the numerical volume, a sphere of radius \( a = 256 \, \mu m \) is placed with the material properties of blood. The material properties can be found in figure 2.4.

For a soft fluid sphere, i.e., a sphere consisting of materials incapable of sustaining shear waves, the solution to this scattering problem can be found in [68]. The analytic solution is compared to the result obtained with the discretised version of the scatter integral equation (2.17)-(2.18) in figure 2.7. In this figure, time slices of the total pressure fields obtained in the plane indicated by the dashed line in figure 2.6 are shown.

The numeric result is obtained on a grid consisting of \( 256 \times 256 \times 256 \) points positioned 4 \( \mu m \) apart. The temporal sampling frequency is set at 100 MHz, and the Gaussian modulated pulse of equation (2.31) is chosen as incident field, with temporal width and offset \( \sigma = 35 \, \text{ns} \) and \( \mu = 250 \, \text{ns} \), respectively, and a center frequency \( f_0 = 20 \, \text{MHz} \). A temporal duration of 2.56 \( \mu s \) is simulated, and the Bi-CGSTAB scheme is stopped when an accuracy of \( 10^{-5} \) is reached.

In figure 2.7, the incident plane wave can be seen propagating through the volume, and is both scattered and refracted by the spherical contrast. In addition, focussing inside the sphere is observed at \( t = 0.89 \, \mu s \). All six panels use the same
Figure 2.7: Time slices of the total pressure field for a plane wave scattering off a penetrable sphere, acquired at the plane indicated by the dashed line in figure 2.6. The top row shows the analytical solution, the bottom row the numerical result. All plots are shown on the same colour scale, which is clipped by a factor of ten to improve visibility of the scattered fields. The location of the sphere is indicated by the dotted lines.

Quantitative comparison is difficult, since the numeric solution is obtained on a cartesian grid, whereas the analytic solution is computed in a spherical coordinate system. However, the same amplitudes are found, the same amount of scattering and refraction occurs, and virtually no visible artifacts are present. Combined with the quantitative results of the plane wave scattering off a single plane interface at normal angles, this suggests that the solution method performs well for arbitrary three-dimensional contrasts. The small differences that are observable stem partly from the fact that the sphere is approximated by cubic volume elements, which gives rise to some minor artifacts.
2.5 Summary

A scalar integral equation has been derived that can, for contrasts in density and
compressibility of arbitrary strength and geometry, simulate transient, three-
dimensional forward scattering problems. By iteratively inverting the governing
operator, the full scattering problem is solved, including refraction and multiple
scattering.

The numerical implementation computes spatial convolutions in the wave
number domain, and utilises many-point stencils to compute spatial derivatives.
The singularity in the Green’s function is avoided by switching to the weak form
formulation.

Results obtained with the numerical implementation show good agreement
to analytical solutions, and errors in the range of several percent. These errors
appear to be the result of the combination of discretisation limitations, a lo-
cally spatially averaged Green’s function, and numerical difficulties introduced
by computations involving discontinuous functions. In addition, when contrast is
present at the boundaries of the numerical domain, reflections off the boundaries
are observed due to truncation of the computational domain.
2. FORWARD SCATTER PROBLEM
Chapter 3

Incident Pressure Field Computations

In the previous chapter, the incident pressure field was computed by convolving an arbitrary, three-dimensional distribution of sources $f(r, t)$ and $q(r, t)$ with the Green’s function in the background medium. In this chapter, significantly more efficient methods to compute the transient pressure field generated by planar vibrating transducer surfaces will be derived.

In the first section, the following analytically equivalent methods will be treated, in order of increasing numerical efficiency. First the Rayleigh integral of the first kind will be derived, followed by the spatial impulse response method for piston transducers. Next, the fast near-field method is discussed. This method is a much more efficient formulation of the impulse response method, where the gain in efficiency requires no further approximations or assumptions. If, in addition, the piston surface velocity meets certain requirements, the temporal dependence of the pressure field is decoupled and obtained at virtually no computational cost. This method is referred to as the time-space decomposition method.

However, this latter approach is only possible for a limited collection of function classes. Therefore, in this chapter the fast near-field method combined with time-space decomposition is extended to enable efficient computation of transient pressure fields for any discretised piston surface velocity. The resulting method is referred to as frequency domain time-space decomposition (FDTSD).

In the second section, the five methods given above will be applied to the same transient problem, and both computation time and memory load will be compared. In this chapter, the discussion will be limited to circular (piston) transducers as this simplifies the math. However, all methods discussed are also applicable to rectangular and triangular piston transducers.
3. INCIDENT PRESSURE FIELD COMPUTATIONS

3.1 Pressure Field Computation Methods

3.1.1 Rayleigh Integral of the First Kind

Historically, the incident pressure field \( \hat{p}_{\text{inc}}(r) \) generated by an arbitrary vibrating planar surface is computed using the Rayleigh integral of the first kind, which treats the vibrating surface as a collection of point sources. It is the most flexible method, because it allows for arbitrary geometry and spatially variant surface vibration.

Following [69], the derivation of the Rayleigh integral of the first kind begins with Green’s second theorem,

\[
\int_{V} \left[ \hat{F}(r) \nabla^{2} \hat{G}(r) - \hat{G}(r) \nabla^{2} \hat{F}(r) \right] \, dV(r) = \oint_{\partial V} \left[ \hat{F}(r) \nabla \hat{G}(r) - \hat{G}(r) \nabla \hat{F}(r) \right] \cdot \hat{n} \, dS(r), \tag{3.1}
\]

which relates two arbitrary twice differentiable functions \( \hat{F}(r) \) and \( \hat{G}(r) \), defined inside volume \( V \) and on the closed boundary \( \partial V \) enclosing \( V \). The unit vector \( \hat{n} \) normal to \( \partial V \) points outwards.

For \( \hat{F}(r) \) and \( \hat{G}(r) \) are chosen

\[
\hat{F}(r) = \hat{p}_{\text{inc}}(r) \tag{3.2}
\]

and

\[
\hat{G}(r) = \frac{e^{-ik_{0}\Delta r}}{4\pi\Delta r} + \hat{\Gamma}(r), \tag{3.3}
\]

respectively, where \( \Delta r = \|r - r_{A}\| \), \( r_{A} \) is any point inside \( V \cup \partial V \), and \( \hat{\Gamma}(r) \) and \( \hat{p}_{\text{inc}}(r) \) satisfy \( \nabla^{2} \hat{\Gamma}(r) + k_{0}^{2} \hat{\Gamma}(r) = 0 \) and \( \nabla^{2} \hat{p}_{\text{inc}}(r) + k_{0}^{2} \hat{p}_{\text{inc}}(r) = 0 \) inside \( V \cup \partial V \), respectively. In addition, volume \( V \cup \partial V \) is assumed to be free from sources.

Substituting equations (3.2) and (3.3) and the accompanying wave equations for \( \hat{p}_{\text{inc}}(r) \) and \( \hat{\Gamma}(r) \) into equation (3.1), changes the latter equation changes into

\[
\int_{V} \left[ \hat{p}_{\text{inc}}(r) \nabla^{2} \hat{G}(r) - \hat{G}(r) \nabla^{2} \hat{p}_{\text{inc}}(r) \right] \, dV(r) = \int_{\partial V} \left[ \hat{p}_{\text{inc}}(r) \frac{\partial}{\partial n} \left( \frac{e^{-ik_{0}\Delta r}}{4\pi\Delta r} + \hat{\Gamma}(r) \right) - \frac{\partial \hat{p}_{\text{inc}}(r)}{\partial n} \left( \frac{e^{-ik_{0}\Delta r}}{4\pi\Delta r} + \hat{\Gamma}(r) \right) \right] \, dS(r), \tag{3.4}
\]

where \( \partial/\partial n \) indicates a spatial derivative in the direction of the normal unit
3.1. PRESSURE FIELD COMPUTATION METHODS

Figure 3.1: Sketch of the geometry used to derive the Rayleigh integral of the first kind. The closed boundary \( \partial V \) enclosing volume \( V \) consists of \( S_0 \), a planar surface located in \( z = 0 \), and \( S_1 \), a hemisphere of radius \( a \). The normal unit vector \( \hat{n} \) points inwards, and point \( A \) is mirrored with respect to \( S_0 \) to yield \( A' \). The distance from a point on \( S_0 \) to point \( r_A \) or \( r_{A'} \) is denoted by \( \Delta r \) or \( \Delta r' \), respectively.

Vector \( \hat{n} \). Using equation (2.13), equation (3.4) reduces to

\[
\hat{p}_{\text{inc}}(r_A) = -\oint_{\partial V} \left[ \hat{p}_{\text{inc}}(r) \frac{\partial}{\partial n} \left( \frac{e^{-ik_0\Delta r}}{4\pi\Delta r} + \Gamma(r) \right) \right.
\]

\[
- \frac{\partial \hat{p}_{\text{inc}}(r)}{\partial n} \left( \frac{e^{-ik_0\Delta r}}{4\pi\Delta r} + \Gamma(r) \right) \left] dS(r), \quad (3.5) \right.
\]

which is commonly referred to as the Kirchhoff equation for a homogeneous medium.

The remaining surface integral is evaluated over the surface sketched in figure 3.1, where a planar surface \( S_0 \) and hemi-spherical surface \( S_1 \) form the closed boundary \( \partial V \) enclosing \( V \). Over this surface, the Kirchhoff equation is given by

\[
\hat{p}_{\text{inc}}(r_A) = -\oint_{S_0} \left[ \hat{p}_{\text{inc}}(r) \frac{\partial}{\partial n} \left( \frac{e^{-ik_0\Delta r}}{4\pi\Delta r} + \Gamma(r) \right) \right.
\]

\[
- \frac{\partial \hat{p}_{\text{inc}}(r)}{\partial n} \left( \frac{e^{-ik_0\Delta r}}{4\pi\Delta r} + \Gamma(r) \right) \left] dS_0(r) \right.
\]

\[
-\oint_{S_1} \left[ \hat{p}_{\text{inc}}(r) \frac{\partial}{\partial n} \left( \frac{e^{-ik_0\Delta r}}{4\pi\Delta r} + \Gamma(r) \right) \right.
\]

\[
- \frac{\partial \hat{p}_{\text{inc}}(r)}{\partial n} \left( \frac{e^{-ik_0\Delta r}}{4\pi\Delta r} + \Gamma(r) \right) \left] dS_1(r) \right. \quad \quad (3.6) \]

If all sources are located in or below the plane \( S_0 \) and the pressure in point \( r_A \) is computed for only a limited temporal extent, then radius \( a \) can always be chosen such that the contribution of \( S_1 \) to the pressure in point \( r_A \) has not yet reached the point of interest. Consequently, for \( a \to \infty \), the integral over \( S_1 \) in equation (3.6) equals zero.
By an appropriate choice for $\hat{\Gamma}(r)$, either the first or the second term in the remaining integral can be eliminated on $S_0$. Choosing

$$\hat{\Gamma}(r) = \frac{e^{-i k_0 \Delta r'}}{4\pi \Delta r'},$$

(3.7)

where $\Delta r' = \|r - r_A\|$ and $A'$ is obtained by mirroring point $A$ in the planar surface $S_0$, the first term vanishes. Note that on $S_0$, $\Delta r' = \Delta r$.

Using the first line of equation (2.7), together with the above choice for $\hat{\Gamma}(r)$, changes equation (3.6) into

$$\hat{p}_{\text{inc}}(r_A) = 2i \omega \rho_0 \int_{S_0} \hat{v}_\perp(r) e^{-i k_0 \|r - r_A\|} \frac{4\pi}{4\pi \|r - r_A\|} dS_0(r),$$

(3.8)

which is the Rayleigh integral of the first kind. This equation is a quantification of Huygens’ principle, where a pressure field is generated by a collection of individual virtual dipole point sources on $S_0$.

For a transducer placed in a locally, instantaneously reacting fluid, the medium particle velocity and the normal component of the transducer surface velocity are identical, and the Rayleigh integral of the first kind can thus be used to compute the pressure field generated by a planar transducer with known normal component of the surface velocity $\hat{v}_\perp(r)$.

Even though the Rayleigh integral is very flexible in terms of transducer geometry and spatially varying normal surface velocity $v_\perp(r, t)$, it requires, for transient problems, a spatio-temporal convolution in every point in time and space. Consequently, it is numerically a very expensive method that suffers from slow numerical convergence. Therefore, in practice faster methods are often applied.

### 3.1.2 Spatial Impulse Response Method

A significant reduction in computation time can be achieved if $\hat{v}_\perp(r)$ is independent of space, i.e., if the transducer vibrates as a piston. In this case, the integral in equation (3.8) changes into

$$\hat{p}_{\text{inc}}(r_A) = 2i \omega \rho_0 \hat{v}_\perp \int_{S_0} e^{-i k_0 \|r - r_A\|} \frac{4\pi}{4\pi \|r - r_A\|} dS_0(r)$$

$$= i \omega \rho_0 \hat{v}_\perp \hat{h}(r_A),$$

(3.9)

where $\hat{h}(r_A)$ is the spatial pressure response to an impulse surface velocity, or in short the spatial impulse response.

For certain transducer geometries, a closed form analytical expression is available. For instance, expressions exist for a triangular [70], rectangular [71] and circular piston [71, 72]. In these cases, transient pressure computations involve only the evaluation of the impulse response in a particular point, and are therefore...
3.1. PRESSURE FIELD COMPUTATION METHODS

Figure 3.2: A circular piston transducer of radius $a$ is situated in the plane $z = 0$. At an angle $\phi$, the distance between the point $(r, z)$ and the edge of the transducer is given by $R$.

much more efficiently performed than the two-dimensional spatial convolution contained in the Rayleigh integral.

For a circular piston of radius $a$, the impulse response reads [73], in cylinder coordinates,

$$\hat{h}(r, z) = \frac{a}{ik_0 \pi} \int_0^\pi M(r, \phi)e^{-ik_0 R} \, d\phi + \frac{1}{ik_0} e^{-ik_0 z} \times \begin{cases} 1 & \text{for } r < a \\ \frac{1}{2} & \text{for } r = a \\ 0 & \text{for } r > a \end{cases}, \quad (3.10)$$

where only the last term is multiplied with the conditional term, and where

$$R = \sqrt{r^2 + a^2 - 2ar \cos \phi + z^2} \quad (3.11)$$

and $\phi$ are as defined in figure 3.2, and the kernel $M(r, \phi)$ is given by

$$M(r, \phi) = \frac{r \cos \phi - a}{r^2 + a^2 - 2ar \cos \phi}. \quad (3.12)$$

Note that the kernel is independent of both $z$ and $\omega$, and thus needs to be evaluated only a limited number of times.

Thus, by assuming piston behaviour of the transducer and evaluating the spatial integral analytically, the spatio-temporal convolution for every point in time and space in the Rayleigh integral (3.8) is reduced to the evaluation of a one-dimensional integral for every point in space combined with, for transient problems, a temporal convolution.

3.1.3 Fast Near-Field Method

The spatial impulse response method significantly reduces the computational cost involved in computing incident pressure fields generated by piston transducers. However, the kernel $M(r, \phi)$ in (3.12) contains a pole in $(r, \phi) = (a, 0)$, which can not be accurately represented on a discrete grid.
3. INCIDENT PRESSURE FIELD COMPUTATIONS

In addition, kernel $M(r, \phi)$ has strong slopes, as is shown in figure 3.3. In this figure, $M(r, \phi)$ is plotted as a function of $r/a$ and $\phi$. The discontinuity around the pole is clearly visible, and the kernel has strong slopes in the vicinity of this discontinuity.

If an accurate incident field is required, the presence of a discontinuity and the accompanying strong slopes dictate dense spatial sampling in the $\phi$-domain. This limits the improvement in numerical efficiency. Fortunately, the spatial impulse response in equation (3.10) can be rewritten in an analytically equivalent expression that is numerically much more efficient.

The derivation of this scheme, referred to as the fast near-field method and presented in [74], starts by subtracting a term $M(r, \phi)e^{-ik_0z}$ from the integrand.
in equation (3.10), and subsequently correcting for this subtraction, i.e.,

\[ \hat{h}(r, z) = \frac{a}{ik_0\pi} \int_0^\pi M(r, \phi) \left( e^{-ik_0R} - e^{-ik_0z} \right) d\phi \]

\[ + \frac{a}{ik_0\pi} \int_0^\pi M(r, \phi) e^{-ik_0z} d\phi + \frac{1}{ik_0} e^{-ik_0z} \times \begin{cases} 1 & r < a \\ \frac{1}{2} & r = a \\ 0 & r > a \end{cases}. \quad (3.13) \]

The second right hand side correction term exactly cancels out the conditional term. Thus, by the simple act of subtracting and adding a complex exponential, the spatial impulse response in equation (3.10) simplifies to [74]

\[ \hat{h}(r, z) = \frac{a}{ik_0\pi} \int_0^\pi M(r, \phi) \left( e^{-ik_0R} - e^{-ik_0z} \right) d\phi, \quad (3.14) \]

where it is important to note that no assumption or approximation is introduced to yield this simplification.

In the above formulation, the same kernel is used. Hence, at first glance this alternative expression still suffers from the discontinuity and accompanying slopes. However, in the vicinity of the discontinuity \( R \) approximately equals \( z \), and hence \( \left| e^{-ik_0R} - e^{-ik_0z} \right| \approx 0 \). Thus, in the vicinity of the singularity, the kernel is multiplied by a complex number which is approximately zero, and therefore the contributions of both the singularity and the slopes are reduced. Using the terminology of [74], the singularity is effectively subtracted from the problem. A similar technique can be used to obtain a fast near-field method formulation of the spatial impulse response of a rectangular piston transducer [75].

Using the fast near-field method, a one-dimensional integral similar to that used in the spatial impulse response method is computed for every point in space. However, the fast near-field method allows for a significantly coarser sampling of the angular coordinate \( \phi \) to reach the same accuracy as the impulse response method.

### 3.1.4 Time-Space Decomposition

Switching from the spatial impulse response to the analytically equivalent fast near-field method formulation allows for coarser angular sampling and hence improved computational efficiency. However, transient problems still require a temporal convolution in every point in space.

This temporal convolution can be reduced to simple multiplications by decoupling the spatial and temporal components of \( v_\perp(t) \). This decoupling, called time-space decomposition, is only possible for certain classes of expressions for \( v_\perp(t) \), and is currently only published for circular pistons [76], but also implemented for rectangular transducers in the FOCUS software [55].
Combining equations (3.9) and (3.14) yields, after switching to the time domain,

\[ p_{\text{inc}}(r, z, t) = \frac{ac_0 \rho_0}{\pi} \int_0^\pi M(r, \phi) \left[ v_\perp \left( t - \frac{R}{c_0} \right) - v_\perp \left( t - \frac{z}{c_0} \right) \right] d\phi. \]  

(3.15)

The term \( v_\perp \left( t - \frac{z}{c_0} \right) \) is independent of \( \phi \) and hence can be taken out of the integral, i.e.,

\[ p_{\text{inc}}(r, z, t) = -\frac{ac_0 \rho_0}{\pi} v_\perp \left( t - \frac{z}{c_0} \right) \int_0^\pi M(r, \phi) d\phi + \frac{ac_0 \rho_0}{\pi} \int_0^\pi v_\perp \left( t - \frac{R}{c_0} \right) M(r, \phi) d\phi. \]  

(3.16)

Certain time-delayed velocity signals \( v_\perp(t - \tau) \) can be written as a finite sum

\[ v_\perp(t - \tau) = \text{rect}\left( \frac{t - \tau}{W} \right) \sum_{n=1}^{N} f_n(\tau) g_n(t), \]  

(3.17)

where \( W \) is the temporal width of the pulse, \( \text{rect}(x) = 1 \) for \( 0 \leq x \leq 1 \) and zero otherwise, \( f_n(\tau) \) is a function depending solely on spatial variables, and \( g_n(t) \) is a function of time \( t \) only. For instance, the time-delayed tone burst

\[ e_{\text{toneburst}}(t - \tau) = \text{rect}\left( \frac{t - \tau}{W} \right) \cos(2\pi f_0(t - \tau)), \]  

(3.18)

where \( f_0 \) is the center frequency, can be written as

\[ e_{\text{toneburst}}(t - \tau) = \text{rect}\left( \frac{t - \tau}{W} \right) \left\{ \cos(2\pi f_0 \tau) \cos(2\pi f_0 t) + \sin(2\pi f_0 \tau) \sin(2\pi f_0 t) \right\}. \]  

(3.19)

For this excitation signal, the number of time-space decomposition terms \( N = 2 \). If condition (3.17) is met, equation (3.16) can be written as

\[ p_{\text{inc}}(r, z, t) = -\frac{ac_0 \rho_0}{\pi} v_\perp \left( t - \frac{z}{c_0} \right) \int_0^\pi M(r, \phi) d\phi + \frac{ac_0 \rho_0}{\pi} \sum_{n=1}^{N} \left[ \int_0^\pi \text{rect}\left( \frac{t - \frac{R}{c_0} W}{c_0} \right) \frac{R}{c_0} M(r, \phi) d\phi \right] g_n(t). \]  

(3.20)
3.1. PRESSURE FIELD COMPUTATION METHODS

or

\[ p_{\text{inc}}(r, z, t) = -\frac{ac_0\rho_0}{\pi} v_\perp \left( t - \frac{z}{c_0} \right) D(r) + \frac{ac_0\rho_0}{\pi} \sum_{n=1}^{N} E_n(r, z, t) g_n(t), \]  

(3.21)

where \( D(r) \) and \( E_n(r, z, t) \) are the so-called direct and edge wave contributions, respectively [76]. Note that \( D(r) \) depends only on radius \( r \) and is computed only once, and \( E_n(r, z, t) \) depends only on the spatial coordinates \( r \) and \( z \), with the exception of a temporal windowing.

When time-space decomposition is combined with the fast near-field method, the temporal dependence of the pressure field generated by a piston transducer is reduced to a computationally inexpensive multiplication. In every point in space, only \( N \) one-dimensional integrals have to be computed, whereas the temporal behaviour is obtained by multiplication.

3.1.5 Frequency Domain Time-Space Decomposition

Even though time-space decomposition yields a significant improvement in efficiency, it does suffer from a serious drawback. In the formulation presented in the previous section, only analytical expressions for \( v_\perp(t) \) that satisfy requirement (3.17) can be used with time-space decomposition.

Unfortunately, only a few classes of functions can be analytically time-space decomposed, e.g., polynomials, sinusoids and exponentials with linear arguments. The widely used Gaussian modulated pulse of equation (2.31) contains an exponent with quadratic argument, and therefore cannot be time-space decomposed.

In this section, time-space decomposition is extended to arbitrary piston surface velocities by switching from continuous analytic functions \( v_\perp(t) \) to discretely sampled signals \( v_\perp(t_k) \). Using the discrete Fourier transform in appendix A.1, the piston surface velocity \( v_\perp(t_k) \) is transformed to the frequency domain, yielding \( \hat{v}_\perp(\omega_k) \). The inverse discrete Fourier transform applied to this spectrum reads

\[ v_\perp(t_k) = \text{rect} \left( \frac{t_k}{W} \right) \frac{1}{N_t} \sum_{n=1}^{N_t} \hat{v}_\perp(\omega_n)e^{i\omega_n t_k}, \]  

(3.22)

where \( N_t \) is the number of time samples, and \( W \) is the temporal extent of the sampled signal \( v(t_k) \). Since the discrete Fourier transform assumes periodicity outside the temporal window \( 0 \leq t_k \leq W \), the rect function is introduced to avoid errors introduced by time-delaying \( v_\perp(t_k) \).

By extending equation (3.22) back to continuous time, an analytic expression for \( v_\perp(t) \) is found that is identical to the sampled version in all \( t = t_k \), and is an interpolation of the sampled signal at all other times. Time-delaying this analytic
expression yields

\[ v_{\perp} (t - \tau) = \text{rect}\left( \frac{t - \tau}{W} \right) \frac{1}{N_t} \sum_{n=1}^{N_t} \hat{v}_{\perp} (\omega_n) e^{i\omega_n (t - \tau)} \]

\[ = \text{rect}\left( \frac{t - \tau}{W} \right) \sum_{n=1}^{N_t} f_n(\tau) g_n(t), \quad (3.23) \]

with

\[ f_n(\tau) = e^{-i\omega_n \tau}, \quad (3.24) \]

and

\[ g_n(t) = \frac{1}{N_t} \hat{v}_{\perp} (\omega_n) e^{i\omega_n t}. \quad (3.25) \]

Comparing equation (3.23) with (3.17) reveals that by simply numerically transforming the discretised piston surface velocity \( v_{\perp} (t_k) \) to the frequency domain using a discrete Fourier transform, followed by an analytical inverse discrete Fourier transform, a time-space decomposed representation is found for arbitrary \( v_{\perp} (t_k) \). This technique is called frequency domain time-space decomposition (FDTSD).

Using the FDTSD method, an efficiency similar to that obtained with the regular time-space decomposition method combined with the fast near-field method can be achieved. However, rather than a fixed number of terms for a given piston surface velocity function \( v_{\perp} (t) \), FDTSD requires \( N_t \) terms and hence the efficiency is dependent on the temporal extent of the simulation. In addition, the number of time samples \( N_t \) is significantly larger than the number of terms required in the time-space decomposition method, and hence the method is significantly slower than time-space decomposition.

To limit the number of terms in FDTSD, use is made of two properties of typical transducer surface velocities. First, piston surface velocities \( v_{\perp} (t_k) \) are real-valued. This implies that the discrete Fourier transform \( \hat{v}_{\perp} (\omega_k) \) is conjugate symmetric, i.e., \( \hat{v}_{\perp} (\omega_k) = \overline{\hat{v}_{\perp} (-\omega_k)} \), where \( \overline{\cdot} \) denotes the complex conjugate. The negative half of the frequency axis is thus found by simply taking the complex conjugate of the positive frequencies, and the number of required terms for an accurate time-space decomposition is approximately halved.

Second, due to limitations in production techniques and materials, ultrasound transducers usually exhibit limited bandwidths. As a consequence, a significant number of frequencies are of very low power, and therefore hardly contribute to the piston surface velocity. By omitting the frequency components of power lower than a certain threshold, a significant reduction in the number of FDTSD terms can be achieved at the cost of a limited reduction in accuracy.

The efficacy of this spectral clipping method is shown in figure 3.4. In this figure, the Gaussian modulated pulse of equation (2.31) is shown (\( \mu = 400 \) ns, \( \sigma = 133 \) ns), for 257 time values sampled at 100 MHz, in the top left panel. In the top right panel, the normalised power spectrum \( \hat{v}_{\perp} (\omega_k) \) is plotted, showing a narrow bandwidth.
In the lower left panel, spectral clipping is applied by omitting frequency components of power below a varying threshold, and the accuracy of the resulting spectrally clipped signal $v'_\perp(t)$ is computed using the normalised $L_1$-norm $\epsilon_{1,v}(v'_\perp(t), v_\perp(t); t)$ of equation (2.26). By varying the power threshold level, accuracies ranging from $\epsilon_{1,v} = 10^{-5}$ to $10^0$ can be achieved. Finally, in the lower right panel the result for spectral clipping down to an error $\epsilon_{1,v} = 10^{-2}$ is shown, corresponding to the $-42$ dB level indicated in the top right panel. No differences between the original and reconstructed signal are discernible using only 21 of the 257 frequency components, while the number of FDTSD terms, and hence the computation time, is reduced by 92%.
Figure 3.4: In the top left panel, a Gaussian modulated pulse with a center frequency of 5 MHz is displayed for 257 time points sampled at 100 MHz, together with its spectrum in the top right panel. In the bottom left panel, the effect of spectral clipping on the accuracy of the inverse Fourier transform is shown. Every frequency component of power below a varying threshold is zeroed out, and the resulting normalised error $\epsilon_{1,v}(v'_{\perp}(t), v_{\perp}(t); t)$ is plotted against the threshold. In the bottom right panel, the reconstruction result when all frequency components of power less than $-42$ dB, i.e., all frequency components below the dashed line in the top right panel, is shown. In this case, only 21 frequency components were required to reconstruct $v_{\perp}(t_k)$ to within an error of $\epsilon_{1,v} = 10^{-2}$. 
3.2 Efficiency Comparison

In the above section, four efficient alternatives for transient pressure field computations using the Rayleigh integral of the first kind are presented. In this section, the efficiency of these methods is studied in terms of computation time and accuracy.

In all simulations, a circular piston transducer of radius \( a = 1 \) mm is driven by a piston surface velocity \( v_\perp(t_k) \) that is given by the Hanning weighted pulse,

\[
e_{\text{Hanning}}(t_k) = \text{rect} \left( \frac{t_k}{W} \right) \frac{A_0}{2} \left( 1 - \cos \left( \frac{\pi t_k}{W} \right) \right) \sin \left( 2\pi f_0 t_k \right),
\]

(3.26)

where center frequency \( f_0 = 5 \) MHz, temporal width \( W = 800 \) ns to allow for exactly four cycles, and \( A_0 = 1 \) m/s. The temporal sampling rate \( f_s = 100 \) MHz, and \( N_t = 241 \).

At a distance \( z = 3 \) mm, the transient pressure field is computed in \( 21 \times 21 \) grid points spaced 200 \( \mu \)m apart, with the piston located centrally in the grid. The surrounding medium is water, with \( c_0 = 1500 \) m/s and \( \rho_0 = 1000 \) kg/m\(^3\). Pressure fields are computed using version 0.282 of the FOCUS software package [55], a package written in C++ and called from MATLAB R2010b that contains efficient transient implementations of the Rayleigh integral, the fast near-field method and time-space decomposition.

The impulse response method is efficiently implemented in MATLAB in the temporal frequency domain, using the expression in equation (3.10). Numerical integration is performed using Gaussian quadrature [77]. Frequency domain time-space decomposition, both with and without spectral clipping, is implemented separately in C++ and compiled into mex-files called from MATLAB.

The memory load of the five methods is, for the situations and implementations studied, approximately the same, and is dominated by the memory required to store the four-dimensional transient pressure field.

Computation Time

By varying the spatial discretisation density of the integration grids, the relation between achieved accuracy and required computation time is established for the various methods. To determine the accuracy of a computed pressure field, all transient pressure fields obtained with the Rayleigh integral, impulse response method, fast near-field method, time-space decomposition and FDTSD are compared to a reference pressure field \( p_{\text{ref}}(r, t) \).

The reference field is computed with the impulse response method using 10,000 abscissas. In experiments not shown here, this number of abscissas is found to be more than sufficient to achieve numerical convergence. In comparison, the most densely sampled fast near-field method simulation is performed using 126 abscissas.

The error in the transient pressure fields \( p(r_j, t_k) \) obtained with the methods discussed above are evaluated using the \( L_1 \)-norm \( \epsilon_{1,p}(p_{\text{inc}}(r, t), p_{\text{ref}}(r, t); r, t) \),
where the summations run over all discrete points in time and space. This error is plotted, as a function of computation time and for the various methods described above, in figure 3.5. In all methods, the number of \( \phi \)-abscissas, obtained with Gaussian quadrature [77], ranges from 2 to 126.

From this figure it is clear that the Rayleigh integral has the lowest convergence rate of the five methods studied. The spatial impulse response method converges significantly faster, and achieves errors below \( \epsilon_{1,p} < 10^{-3} \) significantly
faster than the Rayleigh integral. The fast near-field method is two orders of magnitude faster still, and when combined with time-space decomposition yet another factor of ten in computation time reduction is achieved. The FDTSD method is slower than the fast near-field method in this experiment. However, the effect of spectral clipping on the computation time is clearly visible; by allowing for a larger error $\epsilon_{1,v}$, the computation time is significantly reduced. Note that the spatial impulse response method is implemented in Matlab and therefore not as efficient as the other methods. In general, upon switching from MATLAB code to precompiled code, an additional factor of five to ten in computation time reduction is expected.

For this particular piston face velocity and configuration, FDTSD is slower than the fast near-field method and conventional time-space decomposition. This is caused by the number of terms required in the FDTSD representation of the Hannings weighted pulse, and by the computational overhead introduced by spectral clipping. For an error $\epsilon_{1,v} = 10^{-2}$, 22 terms are required in FDTSD, whereas the traditional time-space decomposition representation contains only six terms, while the computation time per term is the same in both methods. However, when a larger grid of 101 $\times$ 101 points placed 40 $\mu$m apart is used, and the spectral clipping overhead becomes negligible, the computation time of FDTSD is increased to 24.5 s, whereas the fast near-field method requires 33.4 s for this denser grid. For even larger grids, this difference increases further.

Another important observation is that the spectral clipping error $\epsilon_{1,v}$ is found to be an upper limit to the transient pressure field error $\epsilon_{1,p}$ when the discretisation is dense enough to achieve numerical convergence. This observation also holds for experiments not reported here, including different piston face velocities and grid geometries. However, this trend only holds for desired accuracies several orders of magnitude lower than the maximum accuracy achieved without spectral clipping. In the vicinity of this maximum accuracy, the errors introduced by numerical integration, finite machine precision and discretisation contribute to the error $\epsilon_{1,p}$ to a similar extent as the error $\epsilon_{1,v}$ in the piston face velocity. Thus, by applying spectral clipping, the computation time of FDTSD is reduced at the expense of a limited accuracy which can be set in advance.

### 3.3 Summary

Four analytically equivalent alternatives to the Rayleigh integral of the first kind are presented for computations of transient pressure fields generated by circular piston transducers. Each of these alternative formulations is exact, and only piston behaviour is assumed.

By assuming piston behaviour, the spatial dependence of the transducer surface vibration can be taken out of the integral, yielding the spatial impulse response method. By subtracting the singularity from the impulse response expression, the much more efficient fast near-field method is obtained. For certain classes of analytic expressions for piston face velocity signals, the entire temporal
dependence is removed from the impulse response, yielding the time-space decomposition method that involves multiplications rather than temporal convolutions. Finally, by switching to discrete signals, any arbitrary piston face velocity signal can be treated using the frequency domain time-space decomposition (FDTSD) method.

All four treated methods assuming piston behaviour reach a high accuracy significantly faster than the Rayleigh integral method. The impulse response method in this chapter is the only method that does not use precompiled code and therefore its performance is difficult to compare. The fast near-field method when combined with time-space decomposition is by far the fastest method. However, its frequency domain formulation, enabling treatment of arbitrary signals, is slower than the fast near-field method without applying time-space decomposition for the signal and geometry studied. On larger grids, however, FDTSD indeed is faster than the fast near-field method.

Spectral clipping is applied in combination with FDTSD to reduce the computation time at the expense of a reduction in accuracy. The error introduced in the piston face velocity translates into an upper bound of the error in the transient pressure field, and hence the desired accuracy is set a priori. However, even with spectral clipping, FDTSD in general requires more terms, and hence computation time, than conventional time-space decomposition. In addition, the number of terms depends on the number of time samples in the simulation. To further reduce the computation time of a time-space decomposition method for arbitrary signal, an alternative to the FDTSD method should be sought.
Chapter 4

Perfectly Matched Layers

The integral describing the forward problem, equation (2.17)-(2.18), may be solved numerically using an iterative scheme. Even though the numerical implementation of this scheme, as presented in Chapter 2, yields pressure fields that are in agreement with analytical solutions, errors may be introduced by spurious, unphysical reflections off the numerical domain boundaries in case the contrast function extends the spatial numerical domain.

These reflections are shown in figure 4.1. Here, the total pressure field $p_{\text{tot}}(r,t)$ has been computed for a Gaussian plane wave propagating at normal incidence towards a contrast interface, in a geometry identical to that of figure 2.4. The two time slices shown are taken at half-height in the cubic volume. On the left, the time slice at $t = 0.8 \mu$s is shown, where the incident field propagates from left to right and can be seen at around $x = 0.4$ mm. The reflection off the contrast interface is visible at around $x = -0.4$ mm, and reflections off the upper and lower sides of the contrast domain can be seen in between. The colour axis is clipped to improve the visibility of the scattered field. On the right in figure 4.1, the time slice at $t = 1.1 \mu$s is shown. The incident field has propagated out of the numerical domain, and the entire pressure field present in this time slice is generated by reflections off the numerical domain boundaries. Both time slices are shown on the same colour axis.

Several techniques exist to suppress the above unwanted reflections off the spatial boundaries of the numerical domain. The conceptually easiest technique is tapering: gradually reducing the contrast functions towards the domain boundaries. If this reduction is gradual enough with respect to wavelength, no reflection will occur as no discontinuities are present. Unfortunately, this requires thick layers which significantly increase the memory load and the computation time.

A different technique, commonly applied to finite-difference time-domain simulations (FDTD), is based on the absorbing boundary condition (ABC) [78]. The major problem with this technique, however, is that the attenuation is strongly
angle-dependent. Consequently, the boundary is not reflectionless for most angles.

Instead of changing the contrast distribution by tapering, or the boundary conditions by applying an ABC, the material properties can also be locally changed to obtain an absorbing layer. This method has the advantages that the angular dependence problem of the ABC can be reduced and, if applied properly, a significantly thinner layer as compared to tapering can be used. However, care has to be taken that no reflections off the boundaries of the absorbing layer occur instead. In literature, this reflectionless absorbing layer is referred to as a perfectly matched layer (PML).

4.1 PMLs for the Scatter Integral Equation

Formulations for perfectly matched layers were originally developed for two dimensional electromagnetic wave problems [79] and later on expanded to three dimensional problems [80, 81]. Others have applied the method to elastodynamic wave fields [82, 83, 84] and FDTD simulations of acoustic wave fields [85, 86].
Here, a PML will be derived and implemented for the scatter integral equation.

In figure 4.2, the various spatial domains used in this chapter are defined. A scattering object is confined to $D_{\text{scat}}$ and located inside the infinitely extending total volume $D_{\text{tot}}$. The numerical domain of interest $D$ is surrounded by a PML domain $D_{\text{PML}}$, which is bounded on the inside by the surface $\partial D_{\text{PML}}$ and on the outside by the surface $\partial D_{\text{num}}$. The total numerical domain consists of the union of the domain of interest and the PML domain, i.e., $D_{\text{num}} = D \cup D_{\text{PML}}$.

### 4.1.1 Analytic Continuation

In all the preceding derivations, coordinates are implicitly assumed to be real-valued. Although this is a natural choice, the mathematics used are by no means limited to this assumption. Consider the one-dimensional homogeneous Helmholtz equation

$$\frac{\partial^2}{\partial x^2} \hat{f}(x) + \hat{k}_0^2 \hat{f}(x) = 0,$$

which has as solutions the plane waves

$$\hat{f}(x) = A_0 e^{i k_0 x}, \quad \hat{k}_0^2 = \hat{k}^2_0 + \hat{k}^2_0 h(x),$$

with real-valued coordinate $x$. By analytic continuation of equations (4.1) and (4.2), their validity is extended to complex coordinates \[87, p. 208\]

$$\hat{x} = x + i h(x),$$

where $h(x)$ is a real-valued function. In this coordinate system, the Helmholtz equation changes into

$$\frac{\partial^2}{\partial \hat{x}^2} \hat{g}(\hat{x}) + \hat{k}_0^2 \hat{g}(\hat{x}) = 0,$$

with solutions

$$\hat{g}(\hat{x}) = A_0 e^{i k_0 \hat{x}} = A_0 e^{i k_0 x} e^{-\hat{k}_0 h(x)}.$$
Due to the uniqueness of analytical continuation \[87\], it is guaranteed that \( \hat{g}(\tilde{x}) = \hat{g}(x) = \hat{f}(x) \) whenever \( h(x) = 0 \). Consequently, the solution \( \hat{g}(\tilde{x}) \) of the analytically continued Helmholtz equation only differs from \( \hat{f}(x) \) where \( h(x) \neq 0 \). Hence no reflections are generated, and the solution exponentially decays wherever \( \hat{k}_0 h(x) \) is real-valued and greater than zero.

### Attenuative Waves

In the above subsection, a reflectionless attenuative layer is obtained, for propagating waves, using the coordinate transformation of equation (4.3). Even though attenuative waves, i.e., waves with complex-valued wave number \( \hat{k}_0 = \hat{k}_r + i\hat{k}_i \), decay exponentially on their own accord, PMLs can be used to increase their decay rate by allowing \( h(x) = m(x) - i n(x) \) to become complex-valued, i.e.,

\[
\tilde{x} = x + i h(x) = x + i m(x) + n(x),
\]

where \( m(x) \) and \( n(x) \) are real-valued functions of the real-valued coordinate \( x \). The imaginary part of \( h(x) \) amounts to a coordinate stretching wherever \( n(x) \neq 0 \).

By introducing coordinate stretching, the solution to the analytically continued Helmholtz equation (4.4) becomes

\[
\hat{g}(\tilde{x}) = A_0 e^{i \hat{k}_0 \tilde{x}} = A_0 \exp \left[ i \left( \hat{k}_r + \hat{k}_r n(x) - \hat{k}_i m(x) \right) x \right] \times \exp \left[ - \left( \hat{k}_r \frac{m(x)}{x} + \hat{k}_i + \hat{k}_i n(x) \right) x \right],
\]

which consists of a propagating part with effective wave number \( \hat{k}_r + \hat{k}_r \frac{n(x)}{x} - \hat{k}_i \frac{m(x)}{x} \), and an attenuative part wherever \( \hat{k}_r \frac{m(x)}{x} + \hat{k}_i + \hat{k}_i \frac{n(x)}{x} > 0 \).

Thus, by applying coordinate stretching, i.e., by setting \( n(x) \neq 0 \), the attenuation rate of attenuative waves can be increased. Unfortunately, in the case of coordinate stretching, the effective wave number may become larger inside the PML than outside the absorbing region. As a consequence, attenuative waves are represented less accurately on the numeric grid inside the absorbing region. Due to the corresponding decrease in accuracy, slight reflections off the PML are expected in the presence of coordinate stretching.

### 4.1.2 PML Implementation

By locally switching from real-valued to complex-valued coordinates, a locally attenuating, reflectionless region is obtained. However, solving differential equations in complex-valued coordinates is more involved from a numerical point of view than in real-valued coordinates. Therefore, the analytically continued Helmholtz equation (4.4) is rewritten into an equivalent equation in real-valued coordinates.
Starting with the coordinate transformation of equation (4.3), where $h(x)$ is allowed to be complex-valued, the partial derivative with respect to $\hat{x}$ can be written as

$$\frac{\partial}{\partial \hat{x}} = \frac{1}{1 + i \frac{\partial h(x)}{\partial x}} \frac{\partial}{\partial x}. \quad (4.8)$$

The particular choice of

$$\frac{\partial h(x)}{\partial x} = \frac{\sigma(x)}{\omega}, \quad (4.9)$$

with PML strength function $\sigma(x) \neq i\omega$, ensures that the same attenuation per unit distance is achieved for all frequencies. Hence, the frequency independent decay factor $\Delta$ inside a PML of thickness $d$ equals

$$\Delta = \left| e^{-\hat{k}_0 h(d)} \right| = \left| \exp \left( -\frac{1}{c_0} \int_0^d \sigma(x) \, dx \right) \right|. \quad (4.10)$$

Combining equations (4.8) and (4.9) yields, for $\sigma(x) \neq i\omega$,

$$\frac{\partial}{\partial \hat{x}} = \frac{1}{1 + i \frac{\sigma(x)}{\omega}} \frac{\partial}{\partial x} = \hat{X}_x^\sigma(x) \frac{\partial}{\partial x}. \quad (4.11)$$

Note that the particular case $\sigma(x) = i\omega$ corresponds to pure coordinate stretching which does not attenuate propagating waves, and is therefore disregarded.

To simplify the derivations and implementation, the material properties inside $\mathbb{D}_{\text{PML}}$ will be assumed spatially invariant in the direction of the spatial derivatives in all the succeeding.

The derivation of a PML formulation for the one-dimensional homogeneous Helmholtz equation (4.1) starts with substituting equation (4.11) into the analytically continued Helmholtz equation (4.4),

$$\frac{\partial^2}{\partial \hat{x}^2} \hat{g}(\hat{x}) + \hat{k}_0^2 \hat{g}(\hat{x}) = \hat{X}_x^\sigma(x) \frac{\partial}{\partial x} \left[ \hat{X}_x^\sigma(x) \frac{\partial \hat{g}(x)}{\partial x} \right] + \hat{k}_0^2 \hat{g}(x) = 0. \quad (4.12)$$

After applying the chain rule, equation (4.12) can be written as

$$\frac{\partial^2 \hat{g}(x)}{\partial x^2} + \hat{k}^2(x) \hat{g}(x) = -\frac{1}{\hat{X}_x^\sigma(x)} \frac{\partial \hat{X}_x^\sigma(x)}{\partial x} \frac{\partial \hat{g}(x)}{\partial x}. \quad (4.13)$$

with effective, complex-valued wavenumber

$$\hat{k}(x) = \frac{\hat{k}_0}{\hat{X}_x^\sigma(x)}. \quad (4.14)$$

The above equation (4.13) governs the wave propagation through a spatially varying lossy medium, where the contrast source term on the right hand side cancels out any reflections off $\mathbb{D}_{\text{PML}}$. 
In three dimensions, the substitution for the spatial derivatives reads

\[
\nabla \rightarrow \begin{pmatrix} \hat{X}_x^\sigma & 0 & 0 \\ 0 & \hat{X}_y^\sigma & 0 \\ 0 & 0 & \hat{X}_z^\sigma \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \nabla^\sigma, \quad (4.15)
\]

where \( \hat{X}_x^\sigma(r) \) is the PML contrast function in the \( x \)-direction, and likewise for \( y \) and \( z \). Note that the medium is invariant in the direction of the spatial derivative inside \( D_{\text{PML}} \). The shorthand notation \( \nabla^\sigma \) will be used hereafter for better readability.

### 4.1.3 PML Formulation

Observing the scatter integral equation (2.20), deriving a PML formulation of the forward scatter problem seems trivial. By replacing the spatial derivatives in equation (2.20) with those of equation (4.15), the resulting integral equation reads

\[
\hat{p}_{\text{inc}}(r) = \hat{p}_{\text{tot}}(r) - \mathcal{G}(r) * \left\{ \hat{k}_0^2 \chi_{\kappa}(r) \hat{p}_{\text{tot}}(r) + \nabla^\sigma \cdot [\chi_{\rho}(r) \nabla^\sigma \hat{p}_{\text{tot}}(r)] \right\}, \quad (4.16)
\]

where the total pressure field \( \hat{p}_{\text{tot}}(r) \) is attenuated wherever \( \sigma(r) > 0 \).

Even though this method is effective, it is not the most efficient approach. Both the incident and the scattered field are attenuated inside the PML, which requires strong attenuation and hence strong PML contrasts. Consequently, the convergence rate of the iterative solver is relatively low.

Alternatively, a PML formulation may be derived that operates only on the scatter pressure field \( \hat{p}_{\text{scat}}(r) \). Since the scattered field has a significantly lower amplitude than the incident field, the convergence rate of the iterative solver will improve when only the scattered field is attenuated.

Starting with the scalar wave equations (2.10) - (2.12), the inhomogeneous Helmholtz equation for \( \hat{p}_{\text{scat}}(r) \) reads

\[
\nabla^2 \hat{p}_{\text{scat}}(r) + \hat{k}_0^2 \hat{p}_{\text{scat}}(r) = - \left\{ \hat{k}_0^2 \chi_{\kappa}(r) \hat{p}_{\text{tot}}(r) + \nabla \cdot [\chi_{\rho}(r) \nabla \hat{p}_{\text{tot}}(r)] \right\}. \quad (4.17)
\]

The PML formulation operating on the scattered field is obtained by substituting the spatial derivatives of equation (4.15) into equation (4.17), i.e.,

\[
\nabla^\sigma \cdot [\nabla^\sigma \hat{p}_{\text{scat}}(r)] + \hat{k}_0^2 \hat{p}_{\text{scat}}(r) = - \left\{ \hat{k}_0^2 \chi_{\kappa}(r) \hat{p}_{\text{tot}}(r) + \nabla^\sigma \cdot [\chi_{\rho}(r) \nabla^\sigma \hat{p}_{\text{tot}}(r)] \right\}. \quad (4.18)
\]

The above equation (4.18) is rewritten into an inhomogeneous Helmholtz equation, and solved for the scatter pressure field. The resulting integral equation for
the scatter pressure field reads

\[ \hat{p}_{\text{scat}}(r) = \hat{G}(r) \ast_r \left\{ \hat{k}_0^2 \chi_\kappa(r) \hat{p}_{\text{tot}}(r) + \nabla^\sigma \cdot [\chi_\rho(r) \nabla^\sigma \hat{p}_{\text{tot}}(r)] + \nabla^\sigma \cdot [\nabla^\sigma \hat{p}_{\text{scat}}(r)] - \nabla^2 \hat{p}_{\text{scat}}(r) \right\}. \] (4.19)

Finally, by applying equation (2.17), i.e., \( \hat{p}_{\text{tot}}(r) = \hat{p}_{\text{inc}}(r) + \hat{p}_{\text{scat}}(r) \), an alternative PML formulation, in which only the scattered field is attenuated, is obtained, which reads

\[ \hat{p}_{\text{inc}}(r) - \hat{G}(r) \ast_r \left\{ \nabla^\sigma \cdot [\nabla^\sigma \hat{p}_{\text{inc}}(r)] - \nabla^2 \hat{p}_{\text{inc}}(r) \right\} = \hat{p}_{\text{tot}}(r) - \hat{G}(r) \ast_r \left\{ \hat{k}_0^2 \chi_\kappa(r) \hat{p}_{\text{tot}}(r) + \nabla^\sigma \cdot [(\chi_\rho(r) + 1) \nabla^\sigma \hat{p}_{\text{tot}}(r)] - \nabla^2 \hat{p}_{\text{tot}}(r) \right\}. \] (4.20)

Observe that a correction to the incident field is introduced that depends only on the known incident field itself, which can therefore be computed before initiating the iterative solver scheme, while the right hand side still only contains a single convolution. Consequently, the increase of the computational cost per iteration is negligible.

To demonstrate the difference in convergence of the BiCGSTAB scheme for a PML formulation operating on the scattered or on the total pressure field, the geometry shown in figure 4.3 is used. Here, a 1.5 MHz monochromatic pressure field is generated by a rectangular piston of dimensions 400 μm × 27 μm. The piston is located directly outside the numerical domain. This incident field is scattered by a cubic fat-mimicking contrast placed in a blood-mimicking background medium. The numerical domain has dimensions 512 μm × 512 μm × 512 μm, and is divided into cubic elements of 8 μm × 8 μm × 8 μm. Surrounding the numerical domain, a PML of thickness \( d = 128 \) μm is present.

The residual after the \( i \)-th iteration, \( \hat{r}_i(r) = \hat{p}_{\text{inc}}(r) - \hat{L}\hat{p}_{\text{tot},i}(r) \), is a measure of how far the solver is from solving the scatter integral equation, and ideally decreases rapidly with increasing \( i \). For varying iteration number, the L2-norm of

**Figure 4.3:** A cubic contrast is located centrally in a cubic numerical domain, and is surrounded by a perfectly matched layer of thickness \( d \). A continuous wave source is located directly outside the numerical domain.
Figure 4.4: The $L_2$-norm of the residual as a function of the BiCGSTAB iteration number, using PML formulations affecting either $\hat{p}^{\text{scat}}(r)$ (solid line) or $\hat{p}^{\text{tot}}(r)$ (dashed line).

This residual, $\epsilon_2(\hat{\mathcal{L}}\hat{p}^{\text{tot},i}(r),\hat{p}^{\text{inc}}(r); r \in \mathbb{D}_{\text{num}})$, is computed using equation (2.27) and shown in figure 4.4. It is clear from this figure that the residual norm decreases more rapidly when only the scattered field is attenuated instead of the total field. This observation also holds for other experiments not reported here, where different geometries, contrast distributions, contrast strengths and discretisations have been tested. Therefore, from here on only the PML formulation of equation (4.20), operating on the scattered field, will be considered.

4.2 PML Validation

In this section, the perfectly matched layer defined by equation (4.20) is validated in terms of attenuation of spurious reflections and accuracy of the obtained pressure field. The same two test configurations as in Chapter 2, i.e., a plane wave impinging at normal incidence on a planar contrast interface and a plane wave propagating through a homogeneous penetrable sphere, are used. In this section, a step function is used for the PML strength function $\sigma(x)$. 
4.2.1 Plane Wave at Normal Incidence

As was shown in figure 4.1, truncation of the numerical domain in areas of non-zero contrast results in unwanted reflections off the domain boundaries. Ideally, a perfectly matched layer suppresses these spurious reflections without affecting the reflections off the actual contrast.

Consider the situation in figure 2.4, where a plane wave propagates at normal incidence towards a planar contrast interface. In figure 2.5, a single time trace obtained by solving the scatter integral equation (2.17)-(2.18) is compared with the analytical solution of equation (2.28), and additional reflections are clearly visible.

The same experiment is repeated here, where now the cubic numerical domain is surrounded by a PML with thickness $d = 78 \, \mu\text{m} (= \lambda_0)$ and PML strength $\sigma_0 = 50 \, \text{Mrad/s}$. The analytic solution and the solutions obtained with and without PML are shown in figure 4.5. The efficacy of the applied PML is clearly visible. The additional reflections are strongly suppressed, and the scattering off the actual contrast remains unchanged. In this particular example, the amplitude of the pressure field scattered off the domain boundary $\partial \mathbb{D}_{\text{num}}$ is suppressed by a factor of 30, and virtually no energy is reflected off $\partial \mathbb{D}_{\text{PML}}$.

4.2.2 Homogeneous Penetrable Sphere

In Chapter 2 it is shown that the propagation of a plane wave through a homogeneous penetrable sphere, depicted in figure 2.6, is accurately modelled using the scatter integral equation, see figure 2.7.

To demonstrate the effects of truncating the numerical volume, the same geometry is used, but the distal half of the spherical contrast is omitted. By truncating the spatial domain, the scattering effects caused by the omitted contrast will not be taken into account, and the solution will thus not be complete. In addition, strong reflections off the introduced contrast truncation interface are expected.

In figure 4.6, the numerical solution of the scatter integral equation is shown for the complete sphere in the top row. In the middle row, the solution for the truncated sphere is shown in the absence of PMLs. The reflection generated by the domain boundary is clearly visible in addition to the incident field and the actual scattering events. In the bottom row, a PML of thickness $d = 256 \, \mu\text{m}$ and strength $\sigma_0 = 80 \, \text{Mrad/s}$ surrounds the numerical domain containing the truncated sphere. As in the previous example, the additional reflection is strongly suppressed, and the actual scattering events remain unchanged. All panels use the same, clipped colour scale as used in figure 2.7.
Figure 4.5: Time trace of the total pressure field in the observation point indicated in figure 2.4. The analytical solution and the solution obtained without a PML are identical to the results shown in figure 2.5. When a PML is applied, the additional, unwanted reflections are removed, while the incident Gaussian plane wave around $t = 0.2 \, \mu s$ and the reflection off the contrast interface around $t = 0.85 \, \mu s$ remain unchanged.
Figure 4.6: Time slices of the total pressure field for a plane wave scattering off a penetrable sphere, acquired at the plane indicated by the dashed line in figure 2.6. The top row shows the numerical result obtained for a complete sphere, and the middle row shows the same simulation on a truncated domain in the absence of a PML. The bottom row shows the simulation on a truncated domain when a PML is present. The erroneous reflections are strongly suppressed by the PML, while the scattering off the part of the sphere inside the numerical domain is accurately computed. The location of the sphere is indicated by the dotted circle, and the boundary of the truncated domain is indicated by the dashed line.
4.3 PML Effectiveness

The perfectly matched layer formulation for the frequency domain scatter integral equation, equation (4.20), is based on analytic continuation of the governing inhomogeneous Helmholtz equation to complex coordinates. This continuation is achieved by substituting the nabla operator by the expression in equation (4.15), which contains the PML strength functions \( \sigma(x), \sigma(y) \) and \( \sigma(z) \).

Since this strength function determines the attenuation rate inside the PML, ideally this function is as large as possible, so that the PML is as thin as possible. However, the larger its amplitude, the slower the convergence of the iterative scheme, as will be demonstrated below. In addition, PMLs are only reflectionless in the analytical case. In a numerical implementation, small reflections off \( \partial \mathbb{D}_{PML} \) will occur. For larger \( \sigma(x) \), the discontinuity between the two domains is stronger, and hence more energy is reflected off the PML.

In this section, the PML strength function \( \sigma(x) \) is studied. First, the shape of the PML strength function is varied to minimise the reflection off the PML. Next, the amplitude of this function is varied in order to quantify the achieved attenuation and the strength of the reflection off the PML. Finally, \( \sigma(x) \) is allowed to become complex-valued to increase the attenuation rate of attenuative waves. Note that the numerical domains of all following experiments are surrounded by an isotropic PML on all sides, and thus that PMLs operating on the \( y \)- and \( z \)-direction are identical to the \( x \)-direction.

4.3.1 PML Strength Function

Consider the step PML strength function in the \( x \)-direction,

\[
\sigma(x) = \sigma_0 \times \begin{cases} 
1 & \forall \ x \in \mathbb{D}_{PML} \\
0 & \forall \ x \in \mathbb{D}
\end{cases},
\]

where \( \sigma_0 \) is the PML strength amplitude. This function contains a strong discontinuity at \( \partial \mathbb{D}_{PML} \), and might therefore give rise to reflections off this discontinuity. These reflections may be suppressed by switching to a smooth PML strength function, for instance the one shown in figure 4.7, where \( \sigma(x) \) increases gradually from 0 to \( \sigma_0 \). This tapering is achieved by replacing the step function in equation (4.21) with the smoothly varying function

\[
\sigma(x) = \sigma_0 \times \begin{cases} 
1 & \forall \ -d < x < -w \\
\frac{1}{2} - \frac{1}{2} \cos \left( \frac{\pi x}{w} \right) & \forall \ -w \leq x \leq 0 \\
0 & \forall \ 0 < x < x_{\text{max}} \\
\frac{1}{2} - \frac{1}{2} \cos \left( \frac{\pi (x_{\text{max}} - x)}{w} \right) & \forall \ x_{\text{max}} \leq x \leq x_{\text{max}} + w \\
1 & \forall \ x_{\text{max}} + w < x < x_{\text{max}} + d
\end{cases},
\]

where \( w \) is the spatial extent of the transition region, and \( x_{\text{max}} \) is the spatial extent of domain \( \mathbb{D} \) in the \( x \)-direction. Note that the area under the curve,
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Figure 4.7: Example of a smoothly increasing PML strength function $\sigma(x)$, of which the amplitude varies over a transition region of width $w$.

![Figure 4.7: Example of a smoothly increasing PML strength function $\sigma(x)$, of which the amplitude varies over a transition region of width $w$.](image)

and hence the amount of attenuation inside the PML, decreases for increasing width $w$. To compensate, $\sigma_0$ should be increased, which influences the reflection strength and convergence rate of the iterative solver.

Consider the situation sketched in figure 4.8, which is composed of the same cubic volume as in figure 4.3, surrounded by a PML of varying thickness $d$. This time, the piston source of $400 \mu m \times 27 \mu m$ is placed centrally in the volume, with the surface normal in the $x$-direction. The source transmits a monochromatic wave at 10 MHz, and the PML thickness $d$, including the transition region of width $w$, is $176 \mu m (= 1.13\lambda_0)$. PML strength amplitude $\sigma_0$ is a function of width $w$ to yield a constant attenuation factor of $\Delta = 282$, which is computed using equation (4.10). No contrast is present, and hence the difference between the total and incident field is caused only by reflections off $\partial D_{\text{PML}}$. Using this geometry, the strength of the reflection off the PML due to the amplitude and shape of the PML strength function $\sigma(x)$ is thus readily determined.

To quantify the amount of reflection, the $L_2$-norm of the scatter pressure field $\epsilon_2(\hat{p}_{\text{tot}}(r), \hat{p}_{\text{inc}}(r); r \in D_{\text{num}})$ is computed. Intuitively, it seems more logical to
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Figure 4.9: The normalised energy content $\epsilon_2(\hat{p}_{\text{tot}}(r), \hat{p}_{\text{inc}}(r); r \in D_{\text{num}})$ of the scatter pressure field, together with the number of iterations required to reach a residual norm of $10^{-4}$. No actual contrast is present, hence the entire scattered field is caused by reflections off the PML. For increasing $w$, the decrease in $\epsilon_2$ is negligible, whereas the increase in the number of iterations, and hence the total computation time, is significant.

sum over $D$ only, as the scatter pressure field inside $D_{\text{PML}}$ is irrelevant. However, the size of $D$ varies with $d$, so quantitative comparison between different situations is less straightforward.

In figure 4.9, the normalised energy content $\epsilon_2(\hat{p}_{\text{tot}}(r), \hat{p}_{\text{inc}}(r); r \in D_{\text{num}}$ of the scattered field is shown for varying width $w$, together with the number of iterations required to yield a residual norm of less than $10^{-4}$. As a general trend, for increasing width $w$ the energy content $\epsilon_2$ decreases, but only marginally, while the number of iterations required by the iterative solver increases significantly. Since PML strength amplitude $\sigma_0$ is varied to achieve the same attenuation, the PML contrast term becomes stronger for increasing width $w$. As a result, the convergence rate drops, and the computation time increases.

Fortunately, for $w = 0$, i.e., for a step PML strength function, the fastest convergence is achieved, while the energy content of the scattered field is approx-
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approximately 2000× smaller than the energy of the incident field. Thus, reflections off ∂\(D_{\text{PML}}\) are negligible in any case. Therefore, from here on only the step PML strength function will be considered.

4.3.2 Real-Valued PML Strength Function

In this subsection, the effect of PML thickness \(d\) and PML strength amplitude \(\sigma_0\) on the efficacy of the PML is studied, in terms of the achieved attenuation of the scattered field, the energy reflecting off the PML, and the convergence rate of the iterative solver. In all cases, the step PML strength function of equation (4.21) is used. Two situations are studied:

1. The situation sketched in figure 4.3 is used to quantify the attenuation achieved inside the PML. The domain consists of a cube mimicking fat placed centrally in a cube mimicking blood. Surrounding the entire domain, a PML of varying thickness \(d\) is present, and the continuous wave source is located directly outside the cube.

2. The homogeneous cube of figure 4.8, mimicking blood, is used to study the strength of the field scattering off the PML. Here the source is placed centrally inside the volume. In the absence of actual contrast, all scattered energy is generated by the PMLs.

Note that both situations do not require the use of PMLs, since no physical contrast is present on the boundary \(\partial D_{\text{num}}\) of the numerical domain. However, these situations simplify the quantification of the attenuation in and the reflectivity of the PML.

Attenuation Using situation 1, the L₂-norm of the scattered field, denoted by \(\varepsilon_2(\hat{p}_{\text{tot}}(\mathbf{r}), \hat{p}_{\text{inc}}(\mathbf{r}); \mathbf{r} \in \partial D_{\text{num}})\), is computed for varying PML thickness \(d/\lambda_0\) and varying PML strength amplitude \(\sigma_0\). For two frequencies, \(f_0 = 10\ \text{MHz}\) and \(f_0 = 20\ \text{MHz}\), the results are shown in the left column of figure 4.10.

From these results, it is clear that the attenuation increases for stronger and/or thicker PMLs. For the strongest and thickest PMLs, the scattered field amplitude is decreased by more than a factor 100. For both frequencies, the dependencies on \(d\) and \(\sigma_0\) are similar, and similar attenuation is achieved.
Figure 4.10: In the left column, the attenuation of PMLs for varying thickness $d/\lambda_0$ and strength $\sigma_0$ is shown for two different frequencies. The norm $\epsilon_2(\hat{p}_{\text{tot}}(r), \hat{p}_{\text{inc}}(r); r \in \partial D_{\text{num}})$ decreases significantly for PMLs of increasing thickness and/or strength, and for both frequencies the scattered field amplitude is decreased by more than two orders of magnitude. For these two experiments, situation 1 is used. In the middle column, the amount of energy reflecting off the PML is determined, for the same two frequencies, using situation 2. The norm $\epsilon_2(\hat{p}_{\text{tot}}(r), \hat{p}_{\text{inc}}(r); r \in D_{\text{num}})$ increases for increasing $d/\lambda_0$ and/or $\sigma_0$, which indicates that for thicker and/or stronger PMLs more energy is scattered by the PML. In the right column, the number of iterations required to achieve convergence of the iterative solver is shown, using situation 2. The number of iterations increases both with increasing PML thickness and with increasing strength, and the increase is more pronounced for the lower frequency. In addition, the effect of the PML strength on the convergence rate is dominant over that of the PML thickness.
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Reflections  In the middle column of figure 4.10, \( \epsilon_2(\hat{p}_{\text{tot}}(r), \hat{p}_{\text{inc}}(r); r \in D_{\text{num}}) \) is shown for varying PML thickness \( d/\lambda_0 \) and strength \( \sigma_0 \), where summations run over all \( r \in D_{\text{num}} \). In this case, situation 2 is used to determine how much energy is scattered by the PML itself.

In these two panels it is clearly visible that, for both frequencies, the energy contained in the scattered field increases for increasing \( d \) and/or \( \sigma_0 \). However, even for the thickest and strongest PMLs used in this experiment, the scattered field is at least two orders of magnitude smaller than that of the field scattering off typical biomedical ultrasound contrasts. Thus, the PMLs used in this research are effectively reflectionless.

Convergence  In the right column of figure 4.10, the number of iterations required to reach a residual \( L_2 \)-norm of \( 10^{-4} \) using situation 2 is shown. In situation 2, no physical contrast is present, and hence all effects on convergence are caused by the PML.

For both frequencies, as a general trend the number of required iterations increases for increasing PML thickness \( d \) and/or strength \( \sigma_0 \). However, \( \sigma_0 \) has a greater influence on the convergence rate than \( d \). In addition, the decrease in convergence rate is more pronounced for lower frequencies, since the fraction \( \sigma_0/\omega \) of equation (4.11), and hence the strength of the contrast source terms in equation (4.20), increases for decreasing frequency.

4.3.3 Complex-Valued PML Strength Function

In section 4.1.1 it is reasoned that the attenuation rate of evanescent and attenuative waves remains unchanged by PMLs with real-valued PML strength function \( \sigma(x) \), and that the rate could be increased by allowing for complex-valued PML strength functions. In this final experiment, the step function of equation (4.21) is used, but with complex-valued PML strength amplitude

\[
\sigma'_0 = (1 + i\alpha)\sigma_0. \tag{4.23}
\]

The same two situations as above are studied, using a frequency \( f_0 = 10 \) MHz and a PML strength amplitude of \( \sigma_0 = 50 \) Mrad/s.

The attenuative waves are generated by using complex medium parameters for the background medium, mathematically represented by a complex wavenumber \( k = k_0(1 + i\beta) \). In this experiment, \( \beta = 0.1 \) is chosen, which corresponds to an attenuation rate of 175 dB MHz\(^{-1} \) cm\(^{-1} \) which is much higher than is typically found in biomedical tissue. This ensures that any effects found here will be much less pronounced in practical biomedical applications.

Attenuation  The energy norm \( \epsilon_2(\hat{p}_{\text{tot}}(r), \hat{p}_{\text{inc}}(r); r \in \partial D_{\text{num}}) \) is computed for situation 1, with varying PML thickness \( d/\lambda_0 \) and strength amplitude \( \sigma_0 \), and shown on the left in figure 4.11. It is clear from this figure that a real-valued PML strength amplitude, i.e., \( \alpha = 0 \), yields the strongest attenuation even for attenuative waves.
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Figure 4.11: Attenuation by and reflection off PMLs with complex-valued strength amplitude $\sigma'_0$, for waves propagating through an attenuative medium. In the left panel, the $L_2$-norm of the scatter pressure field on $\partial D_{num}$ is given for varying PML strength and thickness, when situation 1 is used. On the right, the $L_2$-norm of the scattered field generated by only the PML is given when situation 2 is used. In these experiments, $\sigma_0 = 50 \text{ Mrad/s}$ and $f_0 = 10 \text{ MHz}$. The strongest attenuation and weakest reflections off the PML are obtained for real-valued $\sigma'_0$, i.e., for $\alpha = 0$.

For $\alpha < 0$, the attenuative waves are amplified rather than attenuated, see equation (4.7), and hence the energy at the domain boundary increases. For $\alpha > 0$, the effective wave number of the propagating waves is larger inside the PML than outside, as was reasoned in section 4.1.1. Consequently, this reduces the attenuation rate inside the PML, and hence more energy remains on the boundary $\partial D_{num}$ of the numerical domain.

Reflections On the right in figure (4.11), the normalised energy of the scatter pressure field generated by the PMLs employed in situation 2, is shown. It is clear from this figure that for $\alpha = 0$ the least amount of energy is reflected off the PML.

In experiments not treated here it was found that the convergence rate of the iterative solver is highest for real-valued PML strength function $\sigma(x)$. Thus, in terms of attenuation, reflectivity and convergence rate, using a real-valued PML strength function yields the best results.
4.4 Summary

To avoid unwanted reflections off the numerical domain boundaries, a perfectly matched layer (PML) formulation of the scatter integral equation of Chapter 2 is derived and implemented. With the implementation, scatter pressure fields are attenuated by more than a factor of 100 in amplitude by PMLs of less than a wavelength thick, and virtually no reflections off the PML occur.

The accuracy of the implementation is tested using two analytic solutions: a monochromatic plane wave impinging at normal incidence on a contrast interface, and a monochromatic plane wave propagating through a truncated penetrable sphere. In both cases, unphysical reflections off the domain boundaries are strongly suppressed, while the scattering off the actual contrast remains unaffected.

Numerical experiments show that, to achieve the highest convergence rate of the iterative solver, a discontinuous step function rather than a smooth function should be used for the PML strength function. The reflections caused by the discontinuity in this PML strength function are negligible, while a smaller PML strength amplitude, and hence smaller contrast source terms, are required to achieve the same attenuation. In addition, taking the PML strength function purely real-valued yields the best results in terms of attenuation inside the PML, reflectivity of the PML, and convergence of the solver.

Unfortunately, due to the introduction of extra contrast terms, the iterative solver converges significantly slower when strong and/or thick PMLs are applied, an effect which is more pronounced for the lower frequencies. The highest convergence rate for the iterative solver is achieved when only the scattered field rather than the total field is attenuated inside the PML, and hence the implementation used in this chapter is based on attenuating the scattered field only. In addition, by adding a PML, the numerical domain grows from $\mathbb{D}$ to $\mathbb{D}_{\text{num}}$ and additional grid points are introduced. This results in a higher memory load, and hence the optimal thickness of the PML is a compromise between convergence rate and memory load.
Current commercial IVUS catheters use either a rotating single element transducer or a circumferential phased array to generate cross-sectional images, and by performing a pullback the third dimension is imaged. The element dimensions are chosen such that the desired volume is insonified; in the case of a rotating single element the radiation pattern is narrow in the circumferential direction, in the case of the phased array a wide radiation pattern is preferable. In the axial direction, both designs use a narrow radiation pattern. However, due to the narrow radiation pattern, in combination with a relatively high pullback rate, consecutive measurements contain no common information, as is shown in figure 1.5.

To demonstrate the effects of this shortcoming, a pullback over a distance of 3 mm is simulated. The pullback consists of six measurements acquired every 0.5 mm, emulating a pullback rate of 0.5 mm/s. The results are shown in figure 5.1, and were obtained using the simulation parameters listed below. In each position, a single pressure trace (A-scan) is obtained, and the six A-scans are combined into the B-scan shown on the right in figure 5.1. The actual contrast is shown on the left, and is composed of a fat inclusion, covered by a fibrous cap, in a blood background medium.

From this simulation, two important observations can be made. First, using a conventional IVUS catheter, it is possible to image the fatty region. However, the boundary of this region is strongly discontinuous in the B-scan. Second, the majority of the line scatterers is absent in the image, which demonstrates the low axial resolution and the severe spatial undersampling occurring during a pullback using a conventional IVUS catheter.

Most of the motion compensation techniques mentioned in Chapter 1 are based on the assumption that consecutive electrogated cross-sectional images contain common features. However, the simulation performed here shows that this assumption is not always valid: tissue boundaries appear discontinuously in images, and local events are either absent or only visible in single cross-sectional
images. For this reason, many of the motion compensation techniques fail to yield the desired improvements.

In conclusion, current IVUS catheters are incapable of creating accurate three-dimensional images of the arterial wall structure. In clinical practice, where a higher pullback rate of 1 mm/s is used and where motion artefacts further deteriorate the image quality, the demonstrated limitations are even more severe.

When instead of two-dimensional cross-sections, three-dimensional volumes are imaged in each measurement position, these limitations can be avoided. In addition, when the volume is large enough, a significant overlap between measurements taken in consecutive pullback positions can be achieved. Due to this overlap, motion compensation can be applied to combine separate volumes into one accurate three-dimensional image. Therefore, in this chapter an axial IVUS array is designed that meets the design criteria specified in the next section. In addition, the fabrication process leading to an array prototype is discussed.

**Simulation Parameters**

Transducer dimensions $400 \, \mu m \times 27 \, \mu m$. 

---

**Figure 5.1**: Simulation of a conventional IVUS pullback using a single ultrasound transducer of dimensions $400 \, \mu m \times 27 \, \mu m$. The pullback rate is $0.5 \, mm/s$, resulting in one measurement every $0.5 \, mm$. On the left the actual contrast is shown. On the right, the envelopes of the six A-scans are shown. The positions and size of the transducer element are indicated by the gray rectangles, and the dashed lines represent the borders of imaged slices. Observe that most of the clusters of line scatterers are absent in the imaging result, which demonstrates the spatial undersampling. In addition, the image is discontinuous, and small features imaged in one A-scan are not present in neighbouring A-scans.
Incident field

Gaussian modulated pulse, $f_0 = 20$ MHz, $\sigma = 30$ ns, $\mu = 200$ ns, $A_0 = 1$ m/s, computed with the fast near-field method using 15 abscissas.

PML

A step PML strength function is used, with an amplitude of $\sigma_0 = 50$ Mrad/s, and a thickness $d = 80 \mu m$.

Iterative scheme

The BiCGSTAB scheme is stopped when the normalised $L_2$-norm of the residual drops below $10^{-3}$.

Signal acquisition

A-scans are obtained by computing the envelope of the pressure field integrated over the finite sized transducer element.

Numerical domain

Shoe box shaped domain of dimensions 3 mm $\times$ 3 mm $\times$ 200 $\mu m$, divided into cubic elements of dimensions 10 $\mu m$ $\times$ 10 $\mu m$ $\times$ 10 $\mu m$. Spatial derivatives are computed using $J = 8$, and a total temporal extent of 4.1 ps is simulated using a sampling rate of 100 MHz.

Medium parameters

The medium is homogeneous in the $x$-direction, and a section of an artery wall is simulated in the $r$-$y$-plane. This numerical phantom is composed of, for increasing $r$,

- blood as background medium: $c_0 = 1560$ m s$^{-1}$, $\rho_0 = 1051$ kg m$^{-3}$ [88], $\kappa_0 = 3.91 \cdot 10^{-10}$ Pa$^{-1}$ [89],
- a fibrous cap: $c = 1567$ m s$^{-1}$, $\rho = 1100$ kg m$^{-3}$ [90], $\kappa = 3.70 \cdot 10^{-10}$ Pa$^{-1}$ [91],
- a fat layer: $c = 1470$ m s$^{-1}$, $\rho = 960$ kg m$^{-3}$ [92], $\kappa = 4.82 \cdot 10^{-10}$ Pa$^{-1}$ [92],
- a healthy arterial wall, simulated using the material properties of blood,
- and line scatterers to simulate clutter and local contrasts: $c = 1651$ m s$^{-1}$, $\rho = 1000$ kg m$^{-3}$, $\kappa = 3.67 \cdot 10^{-10}$ Pa$^{-1}$.

5.1 Design

To scan a three-dimensional volume, either a two-dimensional or a rotating one-dimensional transducer array should be used. Due to the dimensions and geometry of the arteries, space is limited and the catheter tip should remain flexible. Consequently, the number of connecting cables should be small, and the catheter tip has only limited room to house electronics. As a result, the total number of elements is limited, and hence a two-dimensional array is not feasible given the prototyping expertise present in our laboratory. Since the rotation is well controlled in commercial IVUS systems, in this work a rotating linear array in the axial direction is investigated.

To compete with current IVUS catheters and to achieve optimal image quality,
the design should meet the following criteria:

- To achieve a radial resolution in the order of 100 µm and a penetration depth of about 6 mm, the array should operate at a frequency of at least 20 MHz and have a high sensitivity. In addition, the array needs to be small enough to fit into a coronary artery with a diameter of 2 mm. Thus, the width of the array, including housing and connections, cannot exceed 1 mm.

- The most relevant tissue interfaces in IVUS are located between 1.5 mm and 3 mm in the radial direction. Hence, the insonified volume should, for radial distances of 1.5 mm and above, be at least 1.5 mm wide in the axial direction to allow for significant overlap between consecutive volumes at a pullback rate of 1 mm/s. In practice, the first 1.5 mm in the radial direction cannot be imaged due to saturation of the preamplifiers, delays in the switching electronics and “ring-down”; the vibration of the transducer surface due to inertia after the excitation pulse is transmitted [93].

- In the circumferential direction, at least 64 lines should be present in the three-dimensional image, to match the number of lines obtained with conventional catheters.

- For the highest signal-to-noise ratio, the transducer elements should be as large as possible within the given constraints.

- To maintain flexibility of the catheter, the array cannot exceed 2 mm in length in the axial direction.

The array design is optimised for use in water (\(\rho_0 = 1000 \text{ kg m}^{-3}, \kappa_0 = 4.44 \cdot 10^{-10} \text{ Pa}^{-1}, c_0 = 1500 \text{ m s}^{-1}\)), since all successive measurements will be performed in water. For intravascular implementations, the array should be redesigned for use in blood.

### 5.1.1 Array Dimensions - Circumferential

In the circumferential direction, the image quality is mainly determined by the width \(b\) of the element. To determine the optimal width \(b\), the two-way sensitivity patterns for rectangular transducers of dimension 100 µm × \(b\), operating at a center frequency \(f_0 = 20 \text{ MHz}\) and placed in water, are computed for varying element width \(b\) using the fast near-field method. The most relevant radial distance \(r\) is considered to be between 1.5 mm and 3.0 mm.

For four widths, the sensitivity patterns, given by \(|\hat{p}(r)|^2\), are shown in figure 5.2 on a logarithmic scale. The desired insonified area is indicated by the grey lines, and is chosen such that 64 independent line scans are obtained per rotation. All four plots use the same colour scale covering a 20 dB dynamic range, indicated by the black contour lines.

Clearly, the width of the main lobe decreases for increasing element width, while the strength of the side lobes increases. However, if the element width
is increased beyond \( b = 600 \text{ µm} \), the main lobe increases in width again. If
the width is chosen smaller than \( b = 250 \text{ µm} \), the side lobes disappear but the
directionality disappears as well. Judging from the results shown in figure 5.2,
an element width of \( b = 350 \text{ µm} \) is the best compromise between the width of the
main beam and the strength of the side lobes at radial distances between 1.5 mm
and 3 mm. Therefore, an element width of \( b = 350 \text{ µm} \) is chosen.

### 5.1.2 Array Dimensions - Axial

In the previous section, it was shown that the radiation pattern is wider for narrow
transducers. However, the sensitivity of narrow transducers is low. Therefore, in
the axial direction a trade-off between the size of the insonified volume and the
sensitivity of the transducer array has to be made.
In this work, array designs are limited to linear, equally spaced arrays consisting of a number of elements separated by an inter-element kerf. By reducing the kerf, more elements can be positioned in the same space, and the two-way cross-sensitivity, when transmitting and receiving with two different elements, is increased. Thus, the kerf ideally is as small as possible. In the design presented in this chapter, a fixed kerf of 100 μm was chosen due to fabrication limitations.

The element size in the axial direction, i.e., the height $h$, is optimised to yield a sensitivity pattern containing a single lobe beyond a radial distance $r = 1.5$ mm that is at least 1.5 mm wide when assuming a 20 dB dynamic range. In addition, the cross-sensitivity pattern with the fourth next element, i.e., the sensitivity pattern when transmitting with one element and receiving with the fourth next element, should meet the same conditions. This latter requirement is dictated by the fact that imaging using five elements yields significantly better results than when using a lower number, as is demonstrated below.

In figure 5.3, the sensitivity patterns (top row) and cross-sensitivity patterns (bottom row) for three elements of width $b = 350$ μm and heights $h = 75$ μm, $h = 100$ μm and $h = 150$ μm are shown in a 20 dB dynamic range. It is clear from this figure that increasing the element height $h$ increases the sensitivity. However, the main lobe width reduces and the side lobe strength increases for increasing element height. Due to these side lobes, the cross-sensitivity patterns for large $h$ consist of multiple lobes, which deteriorates the image quality and therefore should be avoided. Judging from figure 5.3, and simulations for values of $h$ not reported here, an element height $h = 100$ μm is considered to be optimal. Thus, the axial IVUS array will consist of elements of dimensions $350 \times 100$ μm, with a kerf of 100 μm, operating at a center frequency $f_0 = 20$ MHz.

### 5.1.3 Number of Elements

In order to optimise the number of elements within the constraints and requirements, the image quality of the prototype array will be assessed numerically. The simulated artery is the same as in figure 5.1, and the same discretisations, excitation and perfectly matched layer are used. The transmitting and receiving elements measure $350 \times 100$ μm, and the kerf is 100 μm.

The total pressure field generated by each transmitting element is simulated separately, and the scattered pressure field is recorded in a number of element positions by integrating the pressure field over the surface of the element. The obtained A-scans are processed into an image of the artery phantom using the imaging method described in Chapter 7.

For three, five and eight elements, the resulting images are shown in figure 5.4. It is clear that increasing the number of elements improves the image quality, in terms of both field of view and clarity of details. The increase in field of view is caused by the increase in length of the array and hence in the length of the insonified volume. Further increasing the number of elements will increase the field of view, but not the clarity since the radiation pattern of an element, and hence the cross-sensitivity, is spatially limited.
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![Graphs showing two-way sensitivity patterns](image)

Figure 5.3: Two-way sensitivity patterns when transmitting and receiving with the same element (top row) and when transmitting with one element and receiving with the fourth next element (bottom row), on a logarithmic scale with a dynamic range of 20 dB. The contours show the −20 dB level. All six plots are shown on the same color scale. The dashed lines at $r = 1.5$ mm indicate where the main lobe width is assessed. Elements of height $h = 75$ µm and $h = 100$ µm both yield a main lobe of sufficient width, and the cross-sensitivity patterns are wide and contain only a single lobe for $r > 1.5$ mm. A larger element of $h = 150$ µm yields a main lobe which is too narrow, and in addition generates side lobes beyond $r > 1.5$ mm in the cross-sensitivity.

Judging from figure 5.4, the best image quality is obtained using an array consisting of at least eight elements. Due to limitations in the fabrication process, the axial IVUS array will consist of eight elements and will have a total length of 1.5 mm, which is well below the maximum catheter tip length of 2 mm.
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Figure 5.4: Synthetic images obtained with arrays of three, five or eight elements of dimensions 350 μm × 100 μm, indicated by the gray rectangles. The image quality increases significantly with increasing number of elements, both in field of view (indicated by the dashed lines) and in clarity of the details. In specific, observe the image of the line scatterers at \( y = 1 \) mm, \( r = 1.5 \) mm.

Figure 5.5: The design of the axial IVUS array, consisting of eight elements of dimensions 350 μm × 100 μm, with an inter-element kerf of 100 μm.

5.2 Fabrication

The optimal array consists of eight elements of dimensions 350 μm × 100 μm, with a kerf of 100 μm, and is shown in figure 5.5. Due to the size of the elements, the fabrication of this array is non-trivial, and specialised techniques are required. In this section, the steps required to fabricate the array are outlined, and electrical and acoustical measurements on the resulting prototype are given.

5.2.1 Concept

Due to the small dimensions of the elements, it is not possible to fabricate, place and connect the elements individually. Therefore, the array is built from a single slab of piezo-electric material that contains the entire array, to which first all electrical connections are made before the slab is diced to produce the individual elements. Each element requires two electrical connections, one individual driving
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Figure 5.6: Sketch of a single element. A slab of piezo-electric material is glued to the housing by conducting glue. Underneath the element, a chamber filled with conducting glue is located, in which a wire is placed. The element is diced from the slab after placement, and the saw cuts are filled with a dielectric to electrically insulate the separate elements. The ground electrode spans the entire top surface.

connection and one shared ground connection, which is located on the sound radiating side and spans the entire piezo-electric slab. This reduces the number of wires, and in addition shields the array from electromagnetic interference.

Due to the small dimensions of the elements, soldering a wire to the element is not feasible. In addition, the required heat will depolarise the piezo-electric material and thereby reduce the performance of the array. Therefore, the wires are glued to the elements using conducting glue. In order to create the necessary volume to make all the connections, every element is placed on top of a chamber filled with conducting glue, into which a wire is inserted. The resulting construction is shown schematically in figure 5.6.

5.2.2 Fabrication Steps

The first step in fabricating the array is the fabrication of the piezo-electric material, including matching and (pre-)backing layers. The bulk piezo-electric material, visible in figure 5.7a, is sold in slabs of a certain thickness, including electrodes on both sides, and has a thickness resonance frequency of around 8 MHz. To tune this layer to resonate at a frequency of 20 MHz, the material is ground down from one side to a thickness of \(107/F_1\) (figure 5.7b). The electrode removed during grinding is replaced by first sputtering a layer of nickel/chrome, followed by a layer of gold onto the material.

On top of this electrode, a layer of conducting epoxy resin is applied to the sound radiating side of the piezo-electric slab (figure 5.7d). This layer serves as both a shared ground connection and a matching layer. The matching layer is designed to yield the highest possible bandwidth when the array is coupled to water, and its thickness is determined using the KLM-model [94].

On the opposite side of the piezo-electric slab, a layer of backing material is applied to increase the resistance to mechanical stresses induced during dicing (figure 5.7c). This material is also used to fill the connection chambers and to glue the slab to the housing, and consists of another conducting epoxy resin.
5. PROTOTYPE DESIGN AND FABRICATION

Figure 5.7: Photographs of the piezo-electric material during various stages of the fabrication process. Image *a* shows the bulk material. In image *b* the slab has been ground to a thickness of 107 μm from one side. Image *c* shows the slab after the pre-backing is applied on one side. It is clear from the image that the material is very fragile at this thickness. Image *d* shows the same slab, but with the matching layer applied on the opposite side. In images *e* and *f* the rectangles used for the array are shown.

After placing the matching and pre-backing layers, the circular slab is cut by a laser cutter into rectangles of 2.5 mm × 1.2 mm (figure 5.7e-f). The arrays will be fabricated from these rectangles.

Next, the housing containing the chambers is fabricated from a dielectric polymer, where a hole is drilled in the bottom of each chamber, see figure 5.8a-b. Through each hole, the core of a coaxial cable is inserted and glued into each chamber using the backing material (figure 5.8c). The braidings are connected to a shared ground electrode.

After fabricating the housing, the rectangular piezo-electric slab is glued onto the housing (figure 5.8e), and the individual elements are diced. The excessive
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Figure 5.8: Photographs of the prototype fabrication at various stages. In images a and b the bottom and top of the housing are shown, respectively, and the wire holes and chambers are visible. The photograph has been retouched in order to improve the visibility of the holes. Image c shows the housing including the wiring. In each chamber, the core of a coaxial cable is fed through the hole, and glued in place with the backing material. In image d, the position of the housing with respect to the metal outer tube is shown. Images e and f show the housing with piezo-electric material glued on top, before and after cutting the elements, respectively. Image g shows the final prototype, where the entire array is covered by a gold electrode.

Piezo-electric material is then removed, and the saw cuts are filled with a dielectric epoxy resin (figure 5.8f).

Finally, a gold electrode is sputtered over the entire array, and connected to the braidings of the coaxial cables. The housing containing the electrically connected array is then placed inside a metal tube to improve the handleability of the prototype, and a connector is installed (figure 5.8d and g).

5.2.3 Results

To gain insights into the effect of various fabrication steps on the electro-mechanical behaviour, in figure 5.9 the modulus of the electrical impedance of the piezo-electric material measured in air is plotted. The measurements are made using an Agilent 4294A impedance analyzer. The solid line corresponds to the piezo-
electric bulk material, and shows a resonance frequency of 8 MHz. After grinding this material to a new thickness (dotted line), the resonance frequency has shifted to just above 21 MHz, while the mechanical performance remains approximately the same.

After dicing the individual elements (dashed lines), the center frequency decreases slightly to 21 MHz due to the addition of mass in the matching and backing layers. The change in impedance magnitude is more pronounced, and the increase in impedance is caused by the decrease in electrode area. It is clear that the prototype is poorly matched to the acquisition hardware, which is terminated at 50 Ω. The overall performance can thus be significantly improved if proper electrical matching filters are used.

To assess the acoustical qualities of the elements submerged in water, the pressure generated by an individual element is measured at a distance of approximately 1.3 mm directly above the center of the element. An electric step function with an amplitude of 5 V and a duration of 8 ns, generated by an Agilent 33250A arbitrary waveform generator, is presented to the transducer, and a Precision Acoustics calibrated needle hydrophone measuring 200 μm in diameter is used to record the pressure.

The resulting pressure traces are shown, for three different elements of the same array, in the top panel of figure 5.10. Observe that the measured acoustic pulses are virtually identical. The corresponding spectra are shown in the bottom panel. For all elements, the spectrum has a local maximum around $f_0 = 21$ MHz, and the fractional bandwidths of elements 2, 5 and 8 are 84 %, 83 % and 85 %, respectively. The electromechanical behaviour is comparable for the remaining elements on this array and between different array prototypes.

In figure 5.10, the same measurements on the same prototype are shown after approximately 250 hours of operation under water. The electrical excitation pulse was generated by an in-house built pulser, and had a temporal width of 30 ns and an amplitude of $-80$ V. Compared to element 8, the performance of elements 2 and 5 has decreased. In addition, element 2 generates an acoustic pulse of opposite sign. Wherever imaging results are presented in this thesis, the minus sign and amplitude difference have been corrected for prior to imaging.

5.3 Summary

A linear axial IVUS array operating at a center frequency $f_0 = 20$ MHz is designed to overcome the limitations of current IVUS catheters. Taking imaging requirements and limitations in the fabrication process into consideration, the element dimensions, inter-element kerf and number of elements were optimised, resulting in the design shown in figure 5.5. The array consists of eight elements of dimensions 350 μm × 100 μm, with a kerf of 100 μm.

The image quality of the prototype is assessed in numerical simulations, and the improvement in image quality achieved by using an axial array is clearly visible in figure 5.12. In this figure, the image obtained by a pullback of a con-
5.3. SUMMARY

The modulus of complex-valued electrical impedances of the piezo-electric material measured in three stages of the fabrication process. In every curve, the minimum corresponds to the thickness-mode resonance peak, whereas the maximum is the anti-resonance. The center frequency is found by averaging these two frequencies. As is visible from the solid line, the bulk material has a center frequency of 8 MHz. By grinding this material down (dotted line), a piezo-electric slab is obtained with a center frequency of 21 MHz and mechanical performance similar to that of the bulk material. After fabricating the individual elements (dashed lines), in the presence of the backing and matching layers, the center frequency remains unchanged. The increase in impedance is caused by the decrease in electrode area. Note that the impedances of elements 1 and 8 are virtually identical.

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In addition, a method to fabricate a linear array consisting of eight small ultrasound transducers of dimensions 350 μm × 100 μm operating at a center conventional IVUS catheter is shown in the middle, and on the right the image acquired with the axial array prototype, in a single pullback position, is shown. The original artery phantom is shown on the left. Observe that the image obtained with the new axial array contains details that are absent in the image of the conventional catheter, and in addition shows continuous interfaces.

In addition, a method to fabricate a linear array consisting of eight small ultrasound transducers of dimensions 350 μm × 100 μm operating at a center
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![Pressure trace and Power spectrum graphs](image)

**Figure 5.10:** Pressure trace (top) and corresponding spectrum (bottom) generated by three different elements of the same array prototype. The driving signal was an electric step function with an amplitude of 5 V and a temporal extent of 8 ns. Pressure measurements were performed directly in front of the elements, at a distance of 1.3 mm. Element 2 (solid curve) corresponds to the second element from the left in figure 5.8, element 8 (dashed curve) is the element placed furthest to the right in figure 5.8. Pressure measurements are corrected for the hydrophone sensitivity. The resonant frequency for all elements is 21 MHz, and a second resonance mode is present around 12 MHz. The fractional bandwidth is determined by the $-6$ dB threshold, indicated by the dashed line, and amounts to 84% for element 2, 83% for element 5, and 80% for element 8.

A frequency of 20 MHz is presented. Instead of fabricating, placing and connecting the elements separately, a larger piece of piezo-electric of the correct thickness including matching and backing layers is glued onto a structure that contains all electrical connections, and subsequently diced to the proper dimensions to fabricate the entire array at once.

Acoustical and electrical measurements on the array showed that, using this method, an array was produced of which the elements operate at a center fre-
Figure 5.11: Pressure trace (top) and corresponding spectrum (bottom) generated by three different elements of the same array prototype after 250 hours of operation. The driving signal was an electric step function with an amplitude of $-80 \text{ V}$ and a temporal extent of 30 ns. Pressure measurements were performed directly in front of the elements, at a distance of 1.5 mm. Pressure measurements are corrected for the hydrophone sensitivity. Due to the limited bandwidth of the excitation pulse, the 21 MHz resonance is only weakly excited and hence the bandwidth is reduced. Note that elements 2 and 5 generate an acoustic pulse of significantly reduced amplitude, and that element 2 generates an acoustic pulse of opposite sign when the same electrical excitation is presented.
Figure 5.12: Comparison of the image quality of a conventional IVUS catheter (middle) and the axial array prototype (right), for the synthetic artery shown on the left. Element positions are indicated by the gray rectangles. Note that the image of the conventional IVUS catheter is acquired from six different measurement positions, whereas the image of the array prototype is acquired in a single measurement.
Part II

Prototype Characterisation
Comparing the array design presented in figure 5.5 and the actual prototype in figure 5.8g, it is reasonable to assume that due to the fabrication process the array might not behave ideally. Especially since the elements are not physically separated but rather the kerfs are filled with a dielectric epoxy resin and the ground electrode spans the entire array. Consequently, mechanical cross-talk between elements might be introduced, which will deteriorate the image quality.

To assess the amount of cross-talk in the array, and to improve future designs, the velocity distribution of the transducer surface needs to be measured [95, 96]. Direct velocity measurements on the scale of the prototype are complicated. For instance, accelerometers significantly influence the motion of the transducer [97], and laser-Doppler vibrometry [98] requires specialised and expensive hardware to achieve a spot size of 25 μm or less. In addition, using direct methods it is non-trivial to measure under operating conditions, e.g., submerged in water or at appropriate temperatures.

In the current chapter, an indirect method is presented that reconstructs the normal component of the velocity distribution of the transducer surface \( \hat{v}_\perp(r) \), from here on referred to as the “velocity distribution”, from pressure field measurements. Here, the pressure measurements are inversely extrapolated (“back-propagated”) to the transducer surface. In other research areas, this technique is referred to as (near-field) acoustic holography [99, 100].

In Chapter 3, the Rayleigh integral of the first kind of is derived, see equation (3.8), which equals

\[
\hat{p}_{\text{inc}}(r_p) = 2i\omega \rho_0 \int_{S_v} \hat{v}_\perp(r_v) \frac{e^{-i\hat{k}_0 \|r_v - r_p\|}}{4\pi \|r_v - r_p\|} dS_v(r_v),
\]

where \( r_v \in S_v \) and \( r_p \in S_p \) are points lying in the arbitrary velocity distribution and pressure measurement domains, respectively, as indicated in figure 6.1. This equation governs the forward problem, i.e., it dictates the pressure field generated
by a distribution of monopole sources, and is essentially a spatial convolution of
the velocity distribution with the Green’s function of the medium. In operator
notation, the above equation reads

$$\hat{p}_{\text{inc}}(r_p) = \hat{R}\hat{v}_\perp(r_v),$$

(6.2)

where $\hat{R}$ is the forward operator computing the Rayleigh integral of the first kind.

In the present chapter, the inverse problem is studied, where $\hat{v}_\perp(r_v)$ is re-
constructed from pressure field measurements $\hat{p}_{\text{inc}}(r_p)$. Two approaches can be
used: analytic deconvolution of the Green’s function or (iterative) inversion. In
this chapter, both methods will be studied using synthetic data, and both meth-
ods will be applied to experimental data obtained from radiation measurements
on the fabricated prototype.

### 6.1 Analytic Deconvolution

Consider the situation depicted in figure 6.1, where the source and pressure field
measurement domains are parallel planes. For this geometry, equation (6.1) can
be spatially Fourier transformed with respect to $x$ and $y$ (see appendix A.1) to yield

$$\tilde{p}_{\text{inc}}(\hat{k}_x, \hat{k}_y, z) = 2i\omega\rho_0 \tilde{v}_\perp(\hat{k}_x, \hat{k}_y, z = 0)\tilde{G}(\hat{k}_x, \hat{k}_y, z),$$

(6.3)

where $\tilde{v}_\perp(\hat{k}_x, \hat{k}_y, z)$ is trivial, viz.,

$$\tilde{v}_\perp(\hat{k}_x, \hat{k}_y, z = 0) = \frac{\hat{p}_{\text{inc}}(\hat{k}_x, \hat{k}_y, z)}{2i\omega\rho_0 G(k_x, k_y, z)}.$$  

(6.4)

For a homogeneous background medium, the double spatial Fourier trans-
form of the Green’s function can be computed analytically. First, the Green’s

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**Figure 6.1:** Sketch of the geometry used in the deconvolution approach to inverse
extrapolation. A planar domain $S_v$ contains an arbitrary distribution of
monopole sources, indicated in gray, and the pressure field is measured in
the planar domain $S_p$ a distance $d$ away from the source plane. Points in
the source and measurement plane are denoted by $r_v$ and $r_p$, respectively.
function $\hat{G}(r)$ is transformed, using a three-dimensional spatial Fourier transform, to the $(\omega, \hat{k}_x, \hat{k}_y, \hat{k}_z)$-domain, and subsequently the inverse spatial Fourier transform with respect to $\hat{k}_z$ is taken. Using Cauchy’s residue theorem, Weyl’s representation of the Green’s function [101, 102, 103],

$$\hat{G}(\hat{k}_x, \hat{k}_y, z) = \frac{e^{-i\hat{k}_z |z|}}{2i\hat{k}_z},$$  \hspace{1cm} (6.5)

with

$$\hat{k}_z = \begin{cases} \sqrt{\frac{\omega^2}{c_0^2} - \hat{k}_x^2 - \hat{k}_y^2} & \text{for } \frac{\omega^2}{c_0^2} \geq \hat{k}_x^2 + \hat{k}_y^2 \\ -i\sqrt{\frac{\omega^2}{c_0^2} - \hat{k}_x^2 - \hat{k}_y^2} & \text{for } \frac{\omega^2}{c_0^2} < \hat{k}_x^2 + \hat{k}_y^2 \end{cases},$$  \hspace{1cm} (6.6)

is obtained. The latter case for $\hat{k}_z$ represents the evanescent waves which decay exponentially upon propagation. Upon inverse extrapolation, however, the evanescent waves are amplified, and with them the measurement and numerical noise [104]. To avoid the resulting numerical instability, the evanescent waves are omitted.

Unfortunately, analytic deconvolution using Weyl’s representation of the Green’s function is limited to parallel planar pressure measurement and velocity distribution domains. Due to limitations in the acquisition time and sensitivity of the pressure measurements, the pressure measurement domain, and hence the measurement aperture, is necessarily finite. However, for transducers that are small compared to the wavelength of the radiated field, as is typically the case in imaging arrays, the acoustic power is radiated in all directions, including large opening angles. This is demonstrated in figure 6.2, where the quantity $P$ given by

$$P(r_p) = \frac{\sum_i p_{\text{inc}}^2(r_p, t_i)}{\max_{r_p} \left[ \sum_i p_{\text{inc}}^2(r_p, t_i) \right]}$$  \hspace{1cm} (6.7)

is shown. In the far-field, this quantity approximately equals the normalised acoustic power. This figure clearly demonstrates that even at large aperture angles, i.e., for small $z$, a significant amount of power is radiated. Consequently, for source elements that are comparable to the wavelength or smaller, only a limited fraction of the transmitted power can be captured using a planar measurement grid. As a result, the inverse extrapolation can only partially be computed and the result will be spatially smoothed.

## 6.2 Iterative Inversion

The finite aperture used in the section above limits the accuracy of the reconstructed velocity distribution. Increasing the aperture size is impractical, and thus different geometries should be used. However, the deconvolution method
above cannot handle arbitrary geometries. Instead, in this section equation (6.2) is solved for \( \hat{\nu}_\perp (r) \) using the conjugate gradient scheme of appendix A.4 applied to the normal equation.

Simply applying an iterative scheme rather than analytic deconvolution to the inverse extrapolation problem does not improve the reconstruction result, as no new information is added. In fact, iterative inversion is computationally significantly more expensive, and, as will be shown lateron, significantly more sensitive to measurement noise.

### 6.2.1 Geometry

Despite these drawbacks, the iterative inversion approach is much more flexible than analytic deconvolution, as it allows for arbitrary geometries of both \( S_v \) and
6.3 SYNTHETIC RESULTS

$S_p$. Consequently, also curved elements and transducer arrays can be treated. In this work, however, the velocity distribution domain $S_v$ is taken to be planar, whereas the pressure measurement domain $S_p$ remains arbitrary. Preferably a domain enclosing the entire half-space $z > 0$ above the source is used. In this way, all acoustic energy is captured by the aperture, and a more accurate inverse extrapolation result is expected. In this work, only a spherical section is studied.

6.2.2 Regularisation

In addition, the iterative solution scheme allows for the application of regularisation techniques. By means of regularisation, which is required to avoid noise over-fitting [105, 106], constraints can be introduced to the iterative scheme to force the solution to assume a certain shape. For example, techniques can be applied that decrease the noise sensitivity by requiring a spatially smooth velocity distribution, or spatial features like, e.g., holes in the transducer surface can be set a priori. One such approach will be investigated in more detail.

Typically, ultrasound transducers are designed to move piston-like, i.e., the entire sound radiating surface moves in phase and with spatially invariant amplitude. This feature is exploited by assuming every point in $S_v$ to vibrate in phase, i.e.,

$$v_\perp (r_v, t) = S(t)\hat{v}_0(r_v),$$

where $\hat{v}_0(r_v)$ is the spatially varying amplitude and $S(t)$ is the source signature. By splitting the temporal and spatial components of $\hat{v}_\perp (r_v)$, equation (6.2) changes into

$$\hat{p}_{inc}(r_p) = \tilde{R}\hat{S}v_0(r_v),$$

which is solved for $v_0(r_v)$ only. Source signature $\hat{S}$ can be estimated either from the electrical signal fed to the transducer, or from another inverse extrapolation method.

Apart from reducing the number of degrees of freedom, and hence the computation time, assuming a constant phase across the transducer surface further reduces the sensitivity to noise. As noise is stochastic, it has a spatially varying phase and is therefore suppressed by the regularisation. In addition, by enforcing (apodised) piston behaviour on the velocity distribution reconstruction, potentially a more accurate reconstruction is achieved for piston-like transducers.

6.3 Synthetic Results

Analytic and iterative inversion are compared in this section for a single transducer element of dimensions $350 \mu m \times 100 \mu m$ operating at a center frequency of $20$ MHz, and the influence of the measurement geometry is explored. In addition, the effect of regularisation on the result obtained by iterative inversion is shown using the same transducer element. Finally, the robustness of the constant phase regularisation is studied using an array consisting of multiple elements of dimensions $350 \mu m \times 100 \mu m$. 
In all synthetic experiments, the planar velocity distribution domain $S_v$ is discretised using $41 \times 41$ grid points spaced $50 \mu m$ apart. The parallel planar pressure measurement domain $S_p$ has an identical discretisation and is located at a distance $d = 2$ mm. The hemi-spherical domain has a radius of $a = 2$ mm and is approximated by 2247 triangular elements of equal area. Source signature $S(t)$ is a Gaussian modulated pulse with a center frequency of 20 MHz, a temporal width $\sigma = 35$ ns and unit amplitude, and a temporal sampling rate of 200 MHz is used. Pressure fields are computed with the fast near-field method using 15 abscissas and water as background medium, and white noise with an amplitude of 1% of the maximum amplitude of the pressure field is added in the time domain. The iterative solver is stopped when the $L^2$-norm of the residual drops below $10^{-2}$. Experiments not treated here showed that further decreasing the residual norm only results in an increase of the noise content in the reconstruction.

### 6.3.1 Planar Pressure Measurement Domain

In figure 6.3, inverse extrapolation results obtained from pressure measurements along a planar domain are shown. On the left, a time slice of the original velocity distribution is shown. In the middle, the result obtained with analytic deconvolution is shown, where the transducer surface motion is localised around the actual element and has an accurate amplitude, but significant spatial smoothing is observed due to the finite aperture. Consequently, the reconstruction result suggests the presence of cross-talk for this single piston transducer.

Due to the omission of the evanescent waves, and thus the high frequencies, analytic deconvolution is relatively insensitive to noise. On the right, the result obtained using iterative inversion is shown. A similar amount of spatial smoothing is present, but due to the lack of regularisation the noise is over-fitted to the data, and the result contains significantly more noise. In addition, the computation time for iterative inversion is significantly higher. Thus, using a planar pressure measurement domain, analytic deconvolution performs best.

### 6.3.2 Hemi-Spherical Pressure Measurement Domain

When instead of a parallel planar domain, a hemi-spherical pressure measurement domain is used, the transducer is enclosed and all acoustic power can be captured. To demonstrate the gain in reconstruction accuracy by switching to a hemi-spherical domain, in figure 6.4 the reconstruction error obtained with unregularised CG inversion is plotted against the opening angle of a spherical section by the dashed curve, where an angle of 90° corresponds to a hemi-sphere. The error in the reconstructed velocity distribution $v_\perp(r_v, t)$ is assessed by computing the normalised $L^2$-norm $\epsilon_2(v_\perp(r_v, t), v_\perp, \text{orig}(r_v, t); r_v, t)$, where $v_\perp, \text{orig}(r, t)$ is the true velocity distribution.

By increasing the opening angle, the aperture size is increased and more acoustic power is captured. Consequently, for larger opening angles, the reconstruction error decreases down to a limiting value. This limit is dictated by the sampling
6.3. SYNTHETIC RESULTS

Figure 6.3: Inverse extrapolation of the pressure field generated by a rectangular transducer element of dimensions 350 \(\mu m \times 100 \mu m\), in the presence of 1 \% white noise. Pressure measurements are performed in a plane parallel to the transducer surface at a distance \(d = 2 \text{ mm}\). In the left panel, a time slice of the original velocity distribution \(v_{\perp}(r, t) \text{ [m/s]}\) is shown. In the middle panel, the same time slice of the reconstructed velocity distribution obtained using analytic deconvolution is shown, and in the right panel the result obtained using unregularised CG inversion is shown. The results exhibit similar spatial behaviour in terms of amplitude, localisation and spatial smoothing. Due to the absence of regularisation in the iterative scheme, the noise is fitted to the data and therefore the velocity distribution obtained using CG inversion is significantly more noisy. The transducer element is indicated by the white dashed box.

In addition, the sampling density in \(S_v\) also plays a significant role in the accuracy of the reconstructed velocity distribution. In the synthetic experiments, the measured pressure field is accurately computed using the fast near-field method. Conversely, the forward operator \(\tilde{R}\) computes the Rayleigh integral of the first kind, assuming the transducer surface to be divided into point sources. However, in the current implementation, each grid element is assumed to act as a point source, whereas its dimensions (50 \(\mu m \times 50 \mu m\)) are in the order of the wavelength (\(\lambda_0 = 75 \mu m\)). Hence the grid elements are not point-like. Consequently, a perfect reconstruction cannot be achieved, neither using experimental nor synthetic data.

Note that, when the same fraction of the acoustic power is captured, the iterative inversion of pressure measurements taken over a spherical section is more accurate than that computed from pressure measurements along a parallel plane. This is due to the fact that, in the far-field, all points on the spherical aperture are equi-distant from the transducer element. For a planar aperture, the distance increases with increasing angle, and hence the signal strength decreases.
As an example, in figure 6.5 a time slice of iterative inversion results obtained from synthetic pressure measurements along a planar (middle) and a hemispherical domain (right) are shown, together with the original velocity distribution (left). Using a spherical domain, the noise sensitivity is slightly decreased, and the spatial smoothing is significantly reduced. Moreover, the vibrating area exhibits more piston-like behaviour.

### 6.3.3 Regularisation

By enforcing conditions on the inverse extrapolation result by means of regularisation, the accuracy of the result can be further improved. Here, the constant phase
regularisation introduced in equation (6.8) is tested. First, the improvement will be demonstrated using a single transducer element. Then, the robustness of this type of regularisation is tested on an artificial array.

**Single Transducer Element**

To demonstrate that constant phase regularisation does indeed improve the reconstruction accuracy, the regularisation is applied to pressure measurements acquired over a spherical section, and the $L_2$-norm of the reconstruction error is plotted as a function of the opening angle in figure 6.4 by the solid line. For most opening angles, the reconstruction error is drastically decreased using regularisation.

As a general trend, the error for the regularised case decreases for increasing opening angle. However, beyond an angle of $57^\circ$ the error increases slightly again. The limiting value is determined by the same factors as in the unregularised case, i.e., discretisation density in both $S_v$ and $S_p$, the transducer dimensions and the radius of the spherical domain $S_p$. The slight increase beyond an angle of $57^\circ$ is caused by the spatial undersampling in the Rayleigh integral. Note that the same behaviour is less noticeably observed in the unregularised case.

To visualise the improvement in reconstruction accuracy when using constant
phase regularisation, in figure 6.6 the quantity

\[ K(\mathbf{r}_v) = \frac{\sum v_i^2(\mathbf{r}_v, t_i)}{\max_{\mathbf{r}_p} \left[ \sum v_i^2(\mathbf{r}_v, t_i) \right]} \]  

(6.10)

is shown. This normalised quantity is proportional to the kinetic energy distribution of the transducer surface and hence ideally suited to visualise the energy localisation. In the top row of figure 6.6, \( K(\mathbf{r}_v) \) is shown for two reconstructions using pressure measurements taken along the plane parallel to the transducer. In the bottom row, pressure measurements were performed along the hemi-sphere. In the left column, a standard CG scheme, i.e., in the absence of regularisation, is used. In the right column, a constant phase across the transducer surface is assumed.

For both geometries, the kinetic energy is better localised around the transducer element when constant phase regularisation is applied. In addition, as noise has a spatially varying phase and is therefore not reconstructed, the noise sensitivity is greatly reduced. For both inversion methods, the hemi-spherical pressure measurement domain yields a better energy localisation. The slight asymmetry observed in the reconstructions using hemi-spherical pressure measurement domains is caused by an asymmetric distribution of grid points over the hemi-sphere.

### 6.3.4 Transducer Array

The constant phase regularisation applied to an array assumes that all elements in the array oscillate piston-like and in phase. However, minute differences in the construction of the separate elements can cause slight differences in the amplitude, center frequency, bandwidth and temporal offset of the oscillations of the elements. Here, the effect of such differences on the velocity distribution reconstruction accuracy is studied.

For this study, a somewhat artificial array consisting of four piston elements of dimensions 350 µm × 100 µm is used. One of these elements, denoted element \( \mathbf{a} \), behaves identically as the element studied in the experiments on a single transducer element, i.e., it is excited by a Gaussian modulated pulse with a center frequency of 20 MHz and unit amplitude. The same discretisation grid for \( S_v \) is used, as well as the same hemi-spherical pressure measurement domain \( S_p \).

In addition to this element, the following elements are present. Element \( \mathbf{b} \) oscillates with the same source signature, but with half amplitude. Element \( \mathbf{c} \) oscillates with the same source signature and unit amplitude, but has a temporal delay of 1.245 µs. Element \( \mathbf{d} \) oscillates with unit amplitude and zero temporal delay, but with a Gaussian modulated pulse with a center frequency of 18 MHz as source signature.

In figure 6.7, the quantity \( K(\mathbf{r}_v) \) is shown for the velocity distribution reconstructions from measurements of the pressure field generated by this array. No
6.3. SYNTHETIC RESULTS

Figure 6.6: The kinetic energy $K(r_v)$ in velocity distribution reconstructions, on a logarithmic scale, using planar (top row) and hemi-spherical (bottom row) pressure measurement domains, and using unregularised CG (left column) and constant phase regularisation (right column). For both geometries, the kinetic energy is better localised around the transducer element when regularisation is applied, and the noise sensitivity is strongly reduced. For both inversion methods, the hemi-spherical pressure measurement domain yields a better energy localisation.

Noise was added to this pressure field. In the left panel, the true kinetic energy distribution is shown. For the result shown in the middle panel of this figure, the unregularised CG scheme is applied. The result in the right panel is obtained by applying constant phase regularisation, where the phase is taken from element a.

Observe that the unregularised CG scheme is able to reconstruct all four elements, with proper dimensions, energy and, not visible in figure 6.7, temporal behaviour. The correct center frequencies, amplitudes and temporal offsets are recovered for all elements. When the constant phase assumption is made, naturally only the elements that vibrate in phase, i.e., a and b, are accurately reconstructed. Element d is recovered, but with the wrong amplitude. In addition, the reconstructed oscillation of element d has a center frequency of 20 MHz. Element c is completely absent from the reconstruction.
Figure 6.7: The quantity $K(r_v)$ in velocity distribution reconstructions of a transducer array consisting of four elements of dimensions 350 μm $\times$ 100 μm. Pressure fields are measured over the hemi-spherical domain. On the left, the unregularised CG scheme is applied, on the right the constant phase regularisation is applied. Element a is identical to the element used in the previous experiments, and is excited by a Gaussian modulated pulse with a center frequency of 20 MHz and unit amplitude. Element b oscillates with the same Gaussian modulated pulse, but at half amplitude. Element c oscillates with unit amplitude and a center frequency of 20 MHz, but with a temporal offset of 1.245 μs. Element d oscillates with unit amplitude and zero temporal offset, but the Gaussian modulated pulse has a center frequency of 18 MHz. The unregularised CG scheme is able to reconstruct all four elements with the proper amplitudes and temporal behaviour, whereas CG using constant phase regularisation, where the phase from element a is taken as input, only accurately reproduces elements a and b. Element c is completely absent in the reconstruction result, and element d is reconstructed with an erroneous amplitude.

From these results it can be concluded that the constant phase assumption should be made with care, and is only valid when it is known a priori that either a single element oscillates or that all elements move in phase. Consequently, the velocity distribution of the surface of phased arrays cannot be accurately reconstructed using a constant phase assumption, and different regularisation methods are required for this case.

Furthermore, the constant phase assumption yields the best results when the separate elements in an array are as similar as possible in behaviour. For arrays consisting of dissimilar elements, or for transducers of which the structure allows for surface waves, either a different regularisation technique should be developed, or the unregularised CG scheme should be applied.
6.4 Experimental Results

In this section, the methods discussed above are used to reconstruct the velocity distribution of the surface of the actual prototype depicted in figure 5.8. Unfortunately, the construction of the prototype does not allow for a scan over a full hemisphere of radius 2 mm. As is visible in figure 5.8g, a half-sphere would traverse part of the metal tube, and a maximum achievable opening angle of only 50° can be achieved. However, figure 6.4 shows that such an angle actually results in the highest possible accuracy.

The experiments were performed with a Gaussian modulated electric excitation signal with a center frequency of 21 MHz, a temporal width of 35 ns and an amplitude of 5 V, generated using an Agilent 33250A arbitrary waveform generator. The pressure fields are measured using a Precision Acoustics needle hydrophone with a diameter of 200 μm, which is read out using an Agilent DSO7054A oscilloscope. The three-axis positioning system by Modulynx has a step size of 2.5 μm in all three directions. Pressure field measurements are taken on the same grid as that used in the synthetic experiments, but with a temporal sampling rate of 333 MHz.

In figure 6.8, inverse extrapolation results are shown for four methods: analytic deconvolution, regularised and unregularised CG inversion using a planar pressure measurement domain at a distance $d = 2.0$ mm, and unregularised CG inversion using a spherical section with a radius $a = 2.0$ mm.

There is virtually no difference between the results obtained from pressure measurements on a planar domain. From the measurements taken along a parallel plane, it is apparent that the motion is spatially limited to a region just extending beyond the physical boundaries of the element. In addition, the amplitude is almost constant throughout the oscillating region. Thus, the element effectively oscillates as a piston, and cross-talk to the surroundings is limited. Similar results, in terms of localisation, oscillation amplitude and piston-like behaviour, are obtained for all eight elements, and for different arrays. Thus, using the construction method described in Chapter 5, arrays are fabricated of which the elements oscillate independently and identically.

Figure 6.8 also shows the inverse extrapolation result for unregularised CG inversion of pressure field measurements taken along a spherical section with an opening angle of 50°. Even though synthetic experiments predict this angle to yield the velocity distribution reconstruction with the highest accuracy (figure 6.4), the results obtained from measurements are significantly less accurate than those obtained from measurements along a parallel plane. Using regularised CG inversion yields the same conclusion.

The main contribution to this loss in accuracy is the diameter of the hydrophone of 200 μm. At the center frequency of $f_0 = 21$ MHz, the wavelength $\lambda_0 = 72$ μm, and hence significant spatial averaging is introduced. In addition, the hydrophone exhibits a strong directivity at this frequency. However, in the measurement setup used in these experiments, it was not possible to vary both $x, y, z$ and the angle of the hydrophone accurately, and hence the hydrophone
Figure 6.8: Time slice of velocity distribution reconstructions using four different measurement or reconstruction methods. Only a single element of the array prototype is excited. The top left panel shows the result of analytic deconvolution of pressure field measurements taken along a parallel planar domain. The top right panel shows the unregularised CG inversion result from measurements taken along a spherical section. In the bottom left panel, the result using unregularised CG applied to data acquired on a parallel plane is shown. The bottom right panel shows the result obtained when the constant phase regularisation is applied to CG inversion of data acquired on a parallel plane. All panels are on the same, linear color scale, which is in arbitrary units. The white dashed boxes indicate the location and actual dimensions of the transmitting element.

was fixed perpendicular to the transducer surface. This introduces further inaccuracies in the measurements.

In [107] it has been demonstrated that the above problems are much less pronounced for a transducer of dimensions 700 μm × 200 μm operating at a center frequency of 12.2 MHz, and that using a hemi-spherical measurement geometry
does improve the reconstruction result for that particular transducer. Note that problems associated with the directivity of the hydrophone can be avoided by limiting experiments to elements operating at lower center frequencies, by using smaller hydrophones or by properly changing the angle of the hydrophone during the pressure field measurement.

6.5 Summary

Two methods to extrapolate inversely ("back-propagate") far-field pressure measurements to the transducer surface velocity distribution are presented, namely analytic deconvolution of the Green’s function and iterative inversion using a conjugate gradient (CG) scheme. The latter technique is more flexible in terms of transducer and pressure measurement domain geometries, and allows for non-planar velocity distribution and measurement domains. Using a hemi-spherical enclosing domain, all acoustic energy can be captured, and in addition the distance from the transducer surface to the pressure measurement point can be reduced compared to the case of a parallel planar pressure measurement domain. Consequently, the resulting velocity distribution reconstructions are significantly more accurate.

Using the methods described in this chapter, accurate velocity distribution reconstructions are obtained, for elements of dimensions $350 \text{ pm} \times 100 \text{ pm}$ operating at a center frequency of 20 MHz, from far-field pressure measurements taken a distance of 2 mm (27 wavelengths) away from the transducer surface. Both synthetic and experimental data were considered, and in the experiments presented the transducer surface was limited to a planar domain.

Both analytic deconvolution and iterative inversion yield imperfect reconstructions and introduce spatial smoothing. Therefore, the amount of cross-talk between the elements and their surroundings is over-estimated. Fortunately, the iterative inversion scheme allows regularisation techniques to be applied to enforce certain conditions on the solution. In this chapter, a constant phase of all points in the transducer surface domain is imposed by means of regularisation, as transducer elements are typically designed to act as piston transducers. For single element transducers and non-steered and non-focused transducer arrays where all elements are identical, this regularisation technique results in a significant improvement in the reconstruction accuracy.

The methods are used to validate the fabrication of the array prototype, and the results show that transducer arrays were fabricated of which the elements oscillate independently with virtually no cross-talk, and that the eight elements of an array show almost identical behaviour. Thus, based on these findings, imaging artifacts due to cross-talk or inter-element variability are expected to be negligible.
Chapter 7

Imaging Results

In Chapter 5 and 6 it is shown that the linear IVUS array prototype consists of eight virtually identical elements, which vibrate piston-like and exhibit no significant cross-talk. In the current chapter, first two imaging methods will be presented that take advantage of the array structure. Next, the image quality of the array in the axial direction will be assessed using phantoms, and compared to that of an emulated conventional single element catheter. Finally, a rotational pullback measurement through an ex vivo bovine carotid artery is made, and again the images obtained with the array are compared to those obtained with an emulated single element catheter.

7.1 Image Generation

Ultrasonic images of a volume are obtained by measuring the reflected, refracted and scattered pressure field over an aperture. Due to technical, topological and economical limitations, the aperture is necessarily finite, and hence should be chosen such that the scattering phenomena that are most important for the application are captured.

Several methods to capture the scatter pressure field over the desired aperture are possible. First, the aperture can be fully covered by a transducer array, resulting in a so-called “real aperture” [108]. However, such arrays are costly and complex, and therefore the size of real apertures is limited. Consequently, real aperture arrays have a limited resolution.

Second, the aperture could be scanned by translating a single transducer element. This approach, referred to as “synthetic aperture” [108], is the cheapest as only a single element has to be fabricated. However, the position of the transducer needs to be precisely controlled to avoid imaging artefacts. In addition, this approach requires repeated measurements and thus a significantly longer acquisition time.
A third method is a combination of both real and synthetic aperture. This way, the number of elements and the dimensions of the array are limited, and localisation errors can be corrected for using redundant information in the data.

The array presented in Chapter 5 is designed to generate a mixed aperture in the axial direction. In every pullback position a real aperture is used, and a synthetic aperture could be created in overlapping regions during pullback. To increase the angular resolution, ideally a real or synthetic aperture is used in the circumferential direction as well. However, a real aperture requires a complicated and expensive two-dimensional transducer array, and a synthetic aperture is sensitive to non-uniform rotation distortion (NURD) and requires significantly more acquisition and computation time. Therefore, in this work a single element with a narrow sensitivity pattern is used in the circumferential direction.

7.1.1 B-scan

In conventional commercial IVUS catheters, in the axial direction only a single element with a narrow radiation pattern is used. With this approach, independent consecutive A-scan measurements are obtained, which are plotted next to each other to obtain a B-scan image. In IVUS, the axial B-scan image has a very low axial resolution.

7.1.2 Focussing

To improve on the axial resolution, consider the situation sketched in figure 7.1, where source $i$ generates a propagating incident field. Upon reaching a contrast interface in $r$, part of the incident field will be scattered, and the scatter pressure field is recorded by receiver $j$. When the source and receiver are assumed to be point-like and the medium is homogeneous, the arrival time of the scatter pressure field at the receiver is given by

$$\Delta t = \frac{1}{c_0}[\Delta r_i(r) + \Delta r_j(r)],$$

(7.1)

where the distances $\Delta r_i(r)$ and $\Delta r_j(r)$ are the distances between the contrast point $r$ and source $i$ and receiver $j$, respectively.

Inverting this logic, the travel time $\Delta t$ can also be used to assess whether a scattering interface is present in point $r$. When a non-zero pressure is recorded in $r_j$ at time $\Delta t$, this is an indication that a scattering object is present at a distance $\Delta r_j(r)$. Thus, using this time-picking approach for multiple sources and receivers, from here on referred to as “focussing”, an image is obtained by computing

$$a(r) = \sum_{i,j} p_{\text{scat},i}(r_j, \Delta t),$$

(7.2)

where $a(r)$ is the image amplitude and $p_{\text{scat},i}(r_j, \Delta t)$ is the scatter pressure field measured at the receiver location $r_j$ when source $i$ is excited.
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Figure 7.1: Sketch defining the relevant quantities for focusing and imaging. A scattering object is located in $\mathbf{r}$, which is a distance $\Delta r_i(\mathbf{r})$ away from source $i$ indicated by the star. The scatter pressure field is recorded by a (number of) receiver(s) $j$, indicated by the rectangle, separated from the scattering object by a distance $\Delta r_j(\mathbf{r})$.

The procedure of picking appropriate time samples from pressure traces is analogous to deliberately setting time delays to the separate elements in a focused or steered array. Consequently, the method outlined above is commonly referred to as the “(synthetic) aperture focusing technique” [109, 110], and is adapted from radar techniques [111]. When multiple transmitters and receivers are used, the sum in equation (7.2) effectively averages out noise, and hence a very sensitive scheme is obtained.

Unfortunately, due to geometrical spreading and attenuation, the image amplitude depends on the distance to the source and generally decreases with increasing depth. Typically, this artefact is suppressed by a technique called “time-gain compensation” [112], where the appropriate time sample of the pressure trace is multiplied by $\Delta r_i(\mathbf{r}) \times \Delta r_j(\mathbf{r})$. However, this compensation is only accurate for point sources and receivers, and in addition amplifies noise.

Furthermore, finite sources and receivers and an excitation pulse of finite duration all contribute to a deterioration of the image quality, as the arrival time $\Delta t$ is not sharply defined. This problem is commonly avoided by showing the envelope of the computed image amplitude [113]. In addition, finite sources and receivers have non-spherical radiation patterns and thus do not homogeneously insonify the volume, which causes errors in the image amplitude $a(\mathbf{r})$.

Finally, arrival time $\Delta t$ only rarely coincides with a point on the discretely sampled temporal grid. To avoid errors caused by the rounding off of time samples, temporal interpolation is performed by zero-padding in the temporal frequency domain to obtain a sampling frequency of 1 GHz.

7.1.3 Imaging

To account for the finite dimensions of the transducers, a different approach is used, which from here on is referred to as “imaging”. Again consider the situation sketched in figure 7.1. A source $i$ generates an incident pressure field $p_{inc,i}(\mathbf{r}, t)$ in point $\mathbf{r}$. If a scattering interface is present in $\mathbf{r}$, a fraction of the incident field will be scattered towards the aperture and recorded by receiver $j$. 
The fraction of the incident field that is scattered by a contrast interface is approximately proportional to the change in material properties at that interface. Therefore, in imaging this fraction is taken as image amplitude, i.e.,

$$b(r) = \sum_i \frac{1}{2\pi} \int \frac{\hat{p}_{\text{backscat},i}(r)}{\hat{p}_{\text{inc},i}(r)} d\omega,$$  

(7.3)

where the integration performs a temporal deconvolution of the source signature from the back-scatter pressure field, and back-scatter pressure field $\hat{p}_{\text{backscat},i}(r)$ is the part of the scatter pressure field that propagates back towards the real aperture. To stabilise this spectral division, a small stabilisation factor $\nu$ is added, yielding

$$b(r) = \sum_i \frac{1}{2\pi} \int \frac{\hat{p}_{\text{backscat},i}(r)\hat{p}_{\text{inc},i}(r)}{|\hat{p}_{\text{inc},i}(r)|^2 + \nu^2} d\omega.$$  

(7.4)

Equation (7.4) contains two quantities, $\hat{p}_{\text{backscat},i}(r)$ and $\hat{p}_{\text{inc},i}(r)$, that cannot directly be measured, but have to be obtained from indirect measurements or simulations. As these quantities take into account the sensitivity patterns of the transducers and the excitation source signature, contrast interfaces can in theory be imaged more sharply than in the case of focusing.

In Chapter 6, it is shown that the elements of the prototype array vibrate independently and piston-like, and that motion is limited to the electrically excited area. Therefore, in this chapter the incident pressure field $\hat{p}_{\text{inc},i}(r)$ is computed using the fast near-field method presented in Chapter 3, and a pressure measurement acquired 2.0 mm above the center of the piston transducer is used to obtain the correct source signature $S(t)$.

The back-scatter pressure field $\hat{p}_{\text{backscat},i}(r)$ is recovered from measurements of $\hat{p}_{\text{scat},i}(r_j)$ using the Rayleigh integral of the second kind [114],

$$\hat{p}(r) = \frac{z}{2\pi} \int_{S_0} \hat{p}(x', y', z = 0) \left[ 1 + i\hat{k}_0 \|r' - r\| \right] \frac{e^{-i\hat{k}_0 \|r' - r\|}}{\|r' - r\|^3} dS_0(r'),$$  

(7.5)

where $S_0$ is the infinitely extending planar domain at $z = 0$ m, see figure 3.1. With this integral, the pressure in point $r$, with $z \geq 0$ m, can be computed from a collection of individual virtual dipole sources located along the plane $z = 0$ m.

However, to obtain $\hat{p}_{\text{backscat},i}(r)$, the opposite problem needs to be solved, i.e., that of a source located in $r_i$ and receivers on $S_0$. Fortunately, in the absence of sources between $0 \leq z \leq z_i$, the propagation direction can be reversed due to reciprocity. Using the anti-causal Rayleigh integral of the second kind,

$$\hat{p}(r) = \frac{z}{2\pi} \int_{S_0} \hat{p}(x', y', z = 0) \left[ 1 - i\hat{k}_0 \|r' - r\| \right] \frac{e^{i\hat{k}_0 \|r' - r\|}}{\|r' - r\|^3} dS_0(r'),$$  

(7.6)

the back-scatter pressure field can be computed from scatter pressure field recordings along an infinitely extending planar aperture [114].
Thus, for a finite receiver aperture $S_{rec}$ formed by all the elements of the array transducer, an approximation to $\hat{p}_{\text{backscat},i}(r)$ is obtained by computing the anti-causal Rayleigh integral of the second kind over the finite receiver aperture, i.e.,

$$\hat{p}_{\text{backscat},i}(r) \approx \frac{z}{2\pi} \int_{S_{rec}} \hat{p}_{\text{scat},i}(x', y', z = 0) \left[ 1 - i\hat{k}_0 ||r' - r|| \right] e^{i\hat{k}_0 ||r' - r||} \frac{1}{||r' - r||^3} dS(r').$$

(7.7)

However, in reality only a single electrical signal, and hence a single measurement of $\hat{p}_{\text{scat}}(x', y', z = 0)$, is obtained from each finite sized transducer in the aperture. To account for the finite dimensions of the transducers, the piezoelectric transducers are assumed to vibrate as pistons in reception as well as in transmission.

Imaging using equation 7.4 requires accurate estimates or measurements of the incident field throughout the imaging plane. However, as these are non-trivial to acquire, an approximate imaging scheme is typically applied, where the sources are assumed to be point-like and the excitation is a delta spike. It is easily shown that, under these assumptions, equation 7.4 reduces to [114]

$$b(r) = 4\pi |r - r_i| \hat{p}_{\text{backscat},i} \left( r, t = \frac{\Delta r_i}{c_0} \right),$$

(7.8)

where $r_i$ is the location of source $i$. This technique will be referred to as ‘approximate imaging’.

### 7.1.4 Comparison

#### Synthetic Comparison

To demonstrate the image quality of focusing and imaging, an array measurement is simulated using the same geometry, discretisation, excitation and solution scheme as in Chapter 5. The contrast distribution, consisting of a fat body covered by a fibrous cap placed in a blood background medium, is shown in the top row of figure 7.2. A-scans are obtained by integrating the scatter pressure field over the surface of a receiving element. In this way, the finite dimensions of the receiving elements are taken into account.

B-scan images are obtained by simulating a pullback at a rate of 0.5 mm/s of a single element of dimensions 400 $\mu$m $\times$ 27 $\mu$m, and computing the A-scans for each position. The resulting B-scan image is shown in the bottom left of figure 7.2, and demonstrates the low axial resolution achieved with conventional IVUS catheters. The contrast interfaces show up discontinuously, and the line scatterers are mostly absent. Note that measurements taken in six different positions were required to obtain this image.

In the bottom center, focusing is applied to simulated measurements acquired with the array prototype in a single position. No time-gain compensation is applied. The same A-scans are used to compute the imaging amplitude $b(r)$, which is shown in the bottom right of figure 7.2. The incident field in equation (7.4) is computed with the fast near-field method using 15 abscissas, and
stabilisation term \( \nu \) is set to 1 \% of the maximum amplitude of the incident field. Equation (7.7) is used to compute the back-scatter pressure field, and each of the eight receiving transducers is discretised into sub-elements of 12.5 \( \mu \text{m} \times 12.5 \mu \text{m} \), which is equivalent to six points per wavelength at the center frequency of 20 MHz.

In the case of focusing, the strong interfaces are accurately reconstructed, but significant artefacts are also present, especially in the near-field region. In addition, the interface between blood and fibrous tissue is absent. Using imaging, the artefacts are reduced, and the interface between blood and fibrous tissue becomes visible. These improvements are due to the inclusion of the finite dimensions of the elements. In addition, the interfaces are imaged more sharply as the source signature is deconvolved from the back-scatter pressure field. Finally, with imaging the two strong interfaces are imaged with approximately the same amplitude, whereas the amplitude decreases with depth when focusing was applied. Focusing and imaging yield a clear improvement over the conventional situation.

Unfortunately, the imaging approach is computationally significantly more expensive. For the examples shown in this chapter, imaging required roughly a factor of 600 more computation time. This increase in computational cost will be a challenge in real-time, high frame rate applications, which currently already require powerful workstations for the computationally inexpensive focusing technique.

**Experimental Comparison**

The above comparison was performed on synthetic data, for which the incident pressure field was accurately known and which was free from noise. To compare B-scan, focusing and imaging results in a real situation, a pullback measurement was performed on the aluminium “staircase” shown in the top row of figure 7.3. Each step on the staircase is approximately 3.5 mm wide and 0.5 mm deep, and the portion that is imaged is indicated by the three lines. Five A-scans were taken 1 mm apart to simulate a pullback rate of 1 mm/s. The array was positioned approximately 4 mm away from and parallel to the object. An in-house built pulser was used to excite one element at a time by an electrical step excitation with a duration of 30 ns and an amplitude of \(-80\, \text{V}\), and the scatter pressure field measured by the remaining seven elements was recorded using two Agilent DSO7054A oscilloscopes set to a sampling frequency of 100 MHz. Recorded pressure traces were the average of eight measurements to improve the signal-to-noise ratio.

In each of the five pullback positions, an A-scan measurement is acquired from an emulated single element of 400 \( \mu \text{m} \times 350 \mu \text{m} \) by transmitting the pulse described above with element 1 of the array and receiving the scatter pressure field with element 2 only. The five A-scans are combined into the B-scan shown in the bottom left of figure 7.3, which contains discontinuous interfaces, and in which the edges of the steps are not properly localised.

In the case of focusing, all eight elements are used both in transmission and
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**Figure 7.2:** Comparison of the image quality between B-scan, focussing and imaging for synthetic measurements. In the top panel, the original contrast distribution is shown, consisting of a fatty body covered by a fibrous cap. In the bottom left, the B-scan image obtained with a conventional catheter with an element of 400 μm × 27 μm is shown, where a pullback rate of 0.5 mm/s is simulated. In the bottom center, the envelope of the focussing result $a(r)$ is shown, see equation (7.2), for an array measurement in a single position. In the bottom right, imaging is applied, and the envelope of $b(r)$ of equation (7.4) is shown, again for measurements performed in a single position. The transducer element positions are indicated by the gray rectangles. Simulations were performed using the discretisation grids and pulse discussed in Chapter 5. Observe that the interfaces are reconstructed more sharply using imaging, and that artefacts are reduced, especially in the near-field. In addition, the weak interface between blood and fibrous tissue is reproduced using imaging, whereas this interface is lost in the case of focussing. Both imaging and focussing result in a significantly higher axial resolution, and both methods yield continuous interfaces.

reception, and in each pullback position an image with a width of 1 mm is generated. No time-gain compensation is applied, and only the real aperture is used, i.e., no information from neighbouring measurement positions is used. The resulting image, shown in the bottom center of figure 7.3, shows clear improvements over the conventional B-scan image. The contrast interface is imaged significantly
more sharply, and the step corners are properly localised in the correct positions.

In the bottom right of figure 7.3 the image obtained with imaging is shown. In this case, data collected in each position is used to generate an image of the entire domain. It is clear from this figure that the result obtained by means of focussing is significantly more clear than that obtained using imaging: the contrast interface appears sharper, and much less artefacts are present. This is mainly caused by the scale of the prototype, the high frequency and the size of the hydrophone, which all complicate accurate measurements and computations of the incident and the back-scatter pressure field.

To suppress the artefacts introduced in imaging, approximate imaging is applied to the data. The resulting image is shown in the top right of figure 7.3, and exhibits far fewer artefacts and an image quality similar to that obtained with focussing. However, the computation times for imaging and approximate imaging are significantly longer than for focussing. Therefore, in the remainder of this work only focussing and B-scan images are considered.

**Emulated Conventional Single Element Transducer**

In the experimental comparison above, A-scans obtained with a single element of 400 μm × 350 μm are emulated by transmitting a pulse with one element of the array and receiving the scatter pressure field with only its neighbouring element. That this is indeed a correct emulation is concluded from figure 7.4, where on the left the two-way sensitivity pattern of the actual single element located in the origin is shown at a frequency of 20 MHz on a logarithmic scale. The three contour lines plotted correspond to, from innermost to outermost, −6.7 dB, −13.3 dB and −20 dB. In the middle, the cross-sensitivity between a transmitting element of 100 μm × 350 μm centered around \( y = -100 \) μm and a receiving element centered around \( y = 100 \) μm is shown, on the same logarithmic scale. On the right in figure 7.4, the contour lines of the sensitivity patterns of both transducers are shown, and for a radial distance \( r \geq 1.5 \) mm the sensitivity patterns are virtually identical. Thus, by using only two neighbouring elements of the eight-element array, a single element of dimensions 400 μm × 350 μm, representative for a conventional single element IVUS catheter, can be accurately emulated.

**7.2 Axial IVUS Array Measurements**

In this section, the array designed in Chapter 5 is used to image several biological objects. First, two reference phantoms will be studied: a layered structure composed of ten stacked slices of smoked bacon of 1 mm thickness, and a stent phantom consisting of two bacon slices sandwiched between two brass meshes. Finally, a rotational pullback measurement is performed on an *ex vivo* bovine carotid artery. All measurements are performed in fresh water at room temperature.
Figure 7.3: Comparison of the image quality between B-scan, focussing and imaging for measurements during a pullback with the actual array prototype. A pullback rate of 1 mm/s is emulated, resulting in one array measurement per millimeter. In the top center panel, the aluminium staircase contrast is shown. The pullback traverses three steps, indicated by the three lines. In the bottom left, a B-scan image obtained from an emulated single element of dimensions 400 \( \mu \text{m} \times 350 \mu \text{m} \) is shown. In the bottom center, the image obtained using focussing is shown. In each pullback position, an image with a width of 1 mm is generated, and no information of neighbouring measurements is included. In the bottom right, the result obtained with imaging is shown. As in the synthetic experiment of figure 7.2, both focussing and imaging show a significantly higher axial resolution and continuous interfaces. In addition, local events like the corner of a step are properly localised. However, contrary to the synthetic results, the imaging result contains significantly stronger artefacts compared to the focussing result and contrast interfaces are reconstructed less sharp. This indicates that the simulated radiation pattern and the measured source wavelet are not accurate enough for proper imaging. In the top right panel, the imaging result obtained assuming point-like sources and delta spike excitation is shown. An image quality similar to that of focussing is obtained, and the image amplitude at the contrast interface is more homogeneous. However, as is visible at \((y, r) = (4 \text{ mm}, 4 \text{ mm})\), the individual steps are less clearly separated.

For these measurements, an in-house built pulser was used to generate an electrical step function with a width of 30 ns and amplitude of \(-80 \text{ V} \) which
Two different setups were used to record the pulse-echo measurements, namely:
1. The received signals were recorded by two Agilent DSO7054A oscilloscopes directly, without further amplification. A box containing eight high-frequency relays was used to easily switch between transmit and receive mode for each element. However, due to the lack of amplification in reception, the signal-to-noise ratio is poor and hence pressure traces are averaged over 512 measurements before being recorded.

2. The received signals are first amplified by +27 dB using an in-house built high-frequency amplifier, before being recorded by an Agilent DSO7054A oscilloscope. Only eight pressure measurements were averaged to yield a sufficiently noise-free measurements. Due to amplifier saturation and increased sensitivity to electro-magnetical interference, this measurement setup is unable to distinguish acoustical signals from noise in the first 1.5 µs and consequently a blind spot is created. In addition, for each combination of transmitting and receiving elements, a reference measurement is taken in the absence of contrast to compensate for the transient offset resulting from amplifier saturation. Only a single amplifier was used in reception, and hence only a single recording channel could be used. Consequently, each transmit-receive combination had to be connected by hand, which significantly increased the measurement time.

In both setups, the oscilloscopes were set to a temporal sampling rate of 100 MHz.

Pullbacks were performed using a Modulynx positioning system with a step size of 2.5 µm. Five measurement positions spaced 1 mm apart were used. The rotational pullback measurement traverses only a quarter circle to limit the acquisition time, and was achieved by rotating the bovine artery around a stationary array prototype in 16 discrete steps of 5.625°.

As is shown in figure 5.11, after prolonged usage of the array prototype, the performance of the individual elements changed. Three of the eight elements generated an acoustic pulse of opposite sign, and three other elements generated acoustic pulses of lower amplitude. These effects are compensated for by multiplying measurements involving these elements by a constant factor. This improves the image quality, but also amplifies noise.

The focussing algorithm discussed above is used to generate the pullback image, and a single element measurement is emulated using only two elements of the array to compare the image obtained from array measurements to that obtained with a conventional single element IVUS catheter. Only real aperture focussing is applied, and in each pullback position an image of 1 mm wide is generated. No information from consecutive pullback positions is used, and as a result pullback measurements show slight discontinuities. If synthetic aperture focussing is applied, these discontinuities will disappear and a better image results. However, compensating for slight misalignment errors and imaging artefacts introduced with synthetic aperture focussing requires further research.
7.2.1 Reference Phantoms

Layered Biological Tissue

Using measurement setup 2, a pullback measurement of a stack of ten slices of smoked bacon is performed. Each slice is approximately 1 mm thick. The layered structure is shown on the left in figure 7.5, and the black line indicates the pullback path traversed during the measurement. In the middle, the result obtained with an emulated conventional single element transducer is shown. The first water-bacon interface is visible around \( r = 3 \text{ mm} \), and three additional interfaces can be distinguished around \( r = 4 \text{ mm}, r = 5 \text{ mm} \) and \( r = 6 \text{ mm} \). However, the interfaces show up discontinuously, and in addition have a spatially varying amplitude. Furthermore, the interface around \( r = 5 \text{ mm} \) is barely visible.

On the right of figure 7.5, the image resulting from focusing is shown, clearly showing the first bacon slice starting at around \( r = 3 \text{ mm} \) and the continuous interfaces between the first four slices. The interface between the second and third slice, at \( r = 5 \text{ mm} \), appears slanted, and close inspection of the phantom revealed that the second and third slices indeed had a spatially varying thickness. It is clear from this figure that, for this phantom, the array prototype yields clearer images that are closer to reality.

Stent Phantom

Using measurement setup 2, a pullback measurement of the stent phantom shown on the left in figure 7.6 is performed. The phantom consists of two slices of bacon sandwiched between two brass meshes, where the meshes consist of wires 300 \( \mu \text{m} \) in diameter, with a pitch of 1 mm between the wires. As these mesh dimensions are close to the dimensions of a typical stent [115], this phantom emulates the imaging of a stent placed in an artery. The white line indicates the pullback path traversed during the measurement.

In the middle, the result obtained with an emulated conventional single element transducer is shown. At a depth of \( r = 2.5 \text{ mm} \), the mesh is visible. However, the wires are not properly localised due to the low axial resolution. In addition, the bacon interfaces are not recognisable. Effectively, the conventional single element transducer cannot see through the mesh, which is also observed in clinical practice.

On the right of figure 7.6, the image resulting from focusing is shown, which clearly shows the individual wires of the mesh at a distance \( r = 2.5 \text{ mm} \). In addition, at a depth of \( r = 4 \text{ mm} \) and \( r = 5.2 \text{ mm} \), the interface between the bacon slices and the outermost interface of the second slice are visible. With the array prototype, looking through stents is thus possible, which facilitates proper stent placement.
7.2.2 Bovine Artery

On the left of figure 7.7, the setup used to perform a rotational pullback measurement on the bovine artery is shown. The artery is fixed in a cylindrical shape using two leaf springs, and suspended vertically using a clip. Measurements were performed in fresh water, but the artery was stored in a salt water solution of 0.62 mass percent table salt to mimic bovine saline [116]. The artery had a diameter of about 2 cm, and seemed to be free from atherosclerotic plaque. Due to the large diameter and the accompanying geometric spreading and attenuation, pulse-echo measurements had a low signal strength.

Due to measurement time considerations, measurement setup 1 had to be used and the acoustic measurements are thus not amplified in reception. Combined with the large diameter of the vessel, a low signal-to-noise ratio was obtained. To partially compensate for this, the average of 512 measurements was recorded.

For a pullback in one angle, the images obtained with an emulated single element transducer and focussing are shown in the middle and on the right in figure 7.7, respectively. The image obtained with the emulated single element transducer does show the artery wall, but is strongly distorted. Notice how the artery wall is, even in the absence of artery motion, discontinuously imaged and how the amplitude of the image varies spatially. Thus, even in stationary arteries, current motion compensation techniques based on common features in successive cross-sectional images will introduce artefacts.

In the case of focussing, the artery wall is clearly visible at a depth of around $r = 10$ mm, and the inner edge of the wall is continuously imaged. The outer edge of the artery wall is less sharply defined as the tissue has a tendency to fan out when submerged in water, which is visible in the bottom left corner of the left panel. The signal strength was insufficient to determine whether the events occurring in front of the artery wall are due to noise or due to actual contrast.

In figure 7.8, a cross-sectional image obtained in every pullback position is shown, obtained using an emulated single element transducer (left) and focussing (right). In both cases, the artery wall is visible at a depth of about $8 - 10$ mm, with approximately the same image quality. Using focussing, the artery wall is only marginally clearer, and both methods show only a very limited correlation between cross-sectional images obtained in successive pullback positions.

7.3 Summary

To generate images from pulse-echo measurements, both focussing and imaging are applied to obtain images of a phantom. Focussing assumes point-like sources and receivers and a delta spike acoustical excitation, whereas imaging accounts for the finite dimensions of the transducer and the finite pulse duration. In synthetic studies, imaging therefore yields a superior image quality. However, experimental results show that applying the imaging algorithm to the prototype array results in lower image quality. This is due to the scale of the array, the high frequency and the dimensions of the hydrophone used to measure the acoustic
pulse, all of which complicate accurate computation of the incident and scatter pressure fields throughout the volume.

Using both reference phantoms created from biological tissue and an \textit{ex vivo} bovine artery, the quality of images obtained with the array prototype is compared to that of images obtained with a conventional single element IVUS catheter. In the circumferential direction, a similar image quality is achieved, which was one of the design criteria. Rather than using an actual commercial catheter, a conventional single element transducer is emulated using only two of the eight elements of the array prototype. This way, an identical positioning with respect to the objects and identical frequency responses of the transducers can be achieved, which results in the most fair comparison.

In the radial and axial direction, however, the array prototype yields significant improvements in the image quality. Despite limitations in the acquisition hardware, signal generator, array prototype, and dimensions of the bovine artery, in all measurements continuous interfaces are imaged during pullback, and local events are properly localised. In addition, with the array prototype it is possible to image both the wires of a stent phantom and the structures behind it, whereas the conventional single element catheter yields only the stent itself, smeared out in the axial direction.
Figure 7.5: Comparison of the image quality obtained from a pullback measurement with the axial IVUS array prototype and a conventional catheter. On the left, the contrast composed of stacked bacon slices is shown, and the part of the contrast that is imaged is indicated by the dashed line. In the middle, the result obtained with an emulated single element transducer is shown, where the contrast interfaces show up discontinuously, and one of the interfaces is only weakly present. On the right, the result obtained with the axial array prototype is shown, where focussing is applied to generate the image. The image shows four bacon-bacon interfaces, one of which is under a different angle. Close inspection showed that these slices were indeed cut non-parallel.
Figure 7.6: Comparison of the image quality obtained from a pullback measurement with the axial IVUS array prototype and a conventional catheter. On the left, the contrast composed of two slices of bacon sandwiched between brass meshes is shown, and the part of the contrast that is imaged is indicated by the white line. In the middle, the result obtained with the axial array prototype is shown, where focussing is applied to generate the image. Observe that at a depth of about 2.5 mm the wires of the mesh are visible, and that two additional interfaces are present at depths corresponding to bacon interfaces. On the right, the result obtained with an emulated single element transducer is shown. With a single element transducer, the high echogenicity of the mesh wires dominates the image and as a consequence the bacon interfaces are not distinguishable from the response to scattering off the mesh.
Figure 7.7: Comparison of the image quality obtained from a pullback measurement with the axial IVUS array prototype and a conventional catheter. On the left, the bovine artery is shown in the actual measurement setup, and the pullback direction is indicated by the white arrow. In the middle, the result obtained with an emulated single element transducer is shown. With a single element transducer, the artery wall present around $r = 10$ mm appears discontinuous and the artery wall boundary are not sharply localised. On the right, the result obtained with the axial array prototype is shown, where focussing is applied to generate the image. Observe that both the inner- and outer boundaries of the artery wall are visible and that especially the inner side appears continuously. The outer boundary is less clearly defined as the tissue has a tendency to fan out when submerged in water, which is visible in the bottom left corner of the left panel.
Figure 7.8: Comparison of side-looking cross-sectional images obtained with an emulated single element transducer (left) and the axial array (right). With the axial array, the visibility of the artery wall is slightly better, but the overall image quality remains approximately the same. In both cases, the successive cross-sectional images show only a limited similarity.
Conclusion and Discussion

Chapter 1

Intravascular ultrasound (IVUS) is a medical imaging modality aimed at imaging blood vessel walls from within the artery. Current commercial IVUS catheters are designed to yield two-dimensional cross-sectional images perpendicular to the vessel wall. By pulling the catheter back through the artery (in the ‘axial direction’), and stacking the resulting cross-sectional images, a three-dimensional image of the artery can be obtained.

However, in non-stationary blood vessels like, e.g., the coronary arteries, additional motion besides the pullback is present and as a result consecutively obtained cross-sectional images are not necessarily acquired in neighbouring positions. Therefore, IVUS measurements are electrogated in order to minimise motion artifacts due to the cardiac cycle and to ensure proper localisation of the consecutive cross-sectional images. Unfortunately, the radiation pattern of commercial catheters is 400 μm wide or less in the axial direction, which, combined with a pullback rate of 1 mm/s used in clinical practice, leads to severe spatial undersampling. Consequently, the dataset contains large gaps, and motion compensation attempts are unsuccessful.

To overcome this limitation, in this work the design and fabrication of an axial IVUS array are presented. With the array, three-dimensional volumes rather than cross-sectional planes are imaged in each pullback position. If the imaged volume is large enough, overlap between consecutive electrogated measurements can be achieved. This enables motion compensation techniques, as well as the acquisition of continuous three-dimensional images of artery sections.

Due to the small diameter of the targeted coronary arteries ranging from 2 mm to 5 mm, the dimensions of the array are limited. In addition, image acquisition needs to be performed real-time, and the image quality in the radial direction should be similar or better than in current catheters. In this thesis, an array that meets these requirements is designed, fabricated and tested.
Part I

Chapter 2

The frequency domain scatter integral equation, governing linear acoustic propagation through inhomogeneous soft tissue, is derived and implemented in Fortran. Using parallelisation, the software can efficiently and accurately handle arbitrary transducer geometries, and arbitrary contrasts in both compressibility and volume density of mass. The integral equation is solved, for known incident pressure field and contrasts, and unknown total pressure field, using the iterative BiCGSTAB scheme. The resulting total pressure field includes multiple scattering. The spatial derivatives involved are computed using finite differences, and the corresponding stencil size has a strong influence on the accuracy of the method.

The method could be extended to different problems. For instance, attenuative media can be treated, one such example is given in Chapter 4. In addition, since all frequencies are solved independently, transient contrasts could be modelled. However, this has not been attempted.

The efficiency of the method could be further improved by using different solution schemes. For instance, BiCGSTAB(l) [117] could be used rather than BiCGSTAB. Experiments not reported here showed that an increase in convergence rate of five to ten times can be achieved at the expense of a higher memory load. In addition, spatial filtering of the Green’s function and contrast sources [118] could be applied to drastically reduce the spatial sampling density, and hence the memory load. Furthermore, using spatial filtering, the spatial derivatives can be computed in the wave number domain, which reduces the complexity of the code and increases the accuracy.

Chapter 3

An overview is presented of analytically equivalent methods that compute the transient pressure field generated by a circular piston transducer. Starting with the Rayleigh integral, a major improvement in efficiency is achieved by switching to the impulse response method, where the temporal dependency is taken outside the convolutional integral and a one-dimensional integral remains which accounts for the spatial dependency.

The impulse response contains a singularity and therefore requires dense spatial sampling to obtain accurate pressure fields. By subtracting this singularity, a technique referred to as the fast near-field method, a much coarser grid suffices to reach the same accuracy, and hence the computational cost is greatly reduced. If, in addition, the piston face velocity signal meets certain requirements, the temporal dependence of this coarsely sampled one-dimensional integral reduces to simple multiplications. This technique, called time-space decomposition, further reduces the computational cost.

However, time-space decomposition can only be applied to a limited set of
piston face velocity signal classes. To extend its validity, a frequency domain formulation of time-space decomposition is derived and implemented, and all methods are compared in efficiency. When applicable, the original time-space decomposition method is the fastest method, followed by its frequency domain formulation. The fast near-field method is slower, but still significantly faster than the impulse response method, the Rayleigh integral and the Field II method which uses an approximation to the impulse response method.

All methods, except for the Rayleigh integral, are given for a circular piston. Even though other geometries can be treated using the fast near-field method and time-space decomposition (e.g., rectangular and triangular), the faster methods are limited to (apodised) pistons only. Arbitrary planar transducer geometries can only be treated using the Rayleigh integral. In addition, the time-space decomposition method is limited to certain signal types, and the efficiency of its frequency domain formulation depends on the temporal extent of the transient problem. To avoid this dependency, a different method to extend the validity of time-space decomposition should be investigated. One candidate is the matrix pencil method [119].

Chapter 4

The method presented in Chapter 2 accurately models scattering in arbitrary inhomogeneous media. However, where contrasts extend over the boundaries of the finite numerical domain, spurious reflections are generated at these domain boundaries. These reflections are effectively suppressed using a perfectly matched layer (PML), derived for the frequency domain scatter integral equation. The amplitude of the spurious reflections is reduced by more than a factor of 100, while the PML itself is virtually reflectionless. Experiments showed that the simplest approach, i.e., a real-valued step function as PML strength function, yields the best results in terms of attenuation, reflectivity and convergence rate.

To obtain the fastest convergence, the PML is designed to attenuate only the scatter pressure field rather than the total field. However, due to the iterative nature of the solution scheme, the integral equation is solved for the total pressure field down to a limited accuracy only, and hence some reflections off the PML itself can occur. These reflections are significantly stronger than those typically generated by the PMLs used in finite difference time domain simulations. Although not tested, it is expected that the reflections can possibly be weaker when the PML operates on the total rather than the scattered field.

Chapter 5

An axial array for volumetric IVUS was designed and fabricated. The array consists of eight elements of dimensions 100 μm × 350 μm, with an inter-element kerf of 100 μm, and operates at a center frequency of 20 MHz. The element dimensions and spacing were optimised to obtain the highest possible image quality given
the constraints, using simulations of radiation patterns and acoustic scattering of the probing wave field in a numerical model of an artery.

A fabrication process for the array has been outlined, and the resulting prototype consists of eight virtually identical elements operating at a center frequency of 21 MHz. After 250 hours of operation, the acoustical output has significantly decreased, and the sign of the acoustic response of three of the eight elements has been inverted. Microscopic inspection of the monolithic gold layer spanning the transducer face showed that this electrode has mostly disappeared, which allows water to seep through. This influences both the mechanical and electrical behaviour. Ideally, a layer is added that waterproofs the construction and does not influence the acoustic behaviour of the array. Preliminary results show that a thin layer of the dielectric epoxy resin used to electrically insulate the individual elements suffices.

The prototype is fabricated using common techniques and from common materials. The most difficult step is inserting the connecting wires into chambers backing the elements, which is currently performed by hand. However, using different methods, this step could be automated as well and hence the array can be mass-produced.

The elements have a wide aperture in the axial direction, and a narrow aperture in the circumferential direction. Ideally, a wide aperture would be used in the circumferential direction as well, together with angular synthetic aperture imaging techniques. This approach would yield approximately the same signal to noise ratio, but should yield clearer images. However, due to the increased amount of data this approach would require clever data acquisition and processing schemes.

The design of the prototype is currently overdimensioned, i.e., the housing is currently larger than required to construct the elements. Thinner coaxial cables than the ones used exist, and using multiplexers (common practice in commercial catheters) the number of wires can be reduced. Consequently, either the external dimensions of the array including its housing can be reduced, or the number of elements could be increased.

During the design, no consideration was given to the electrical impedance of the array, and the resulting elements are severely mismatched to the 50 Ω terminated sources and receivers used in the measurement setups. In fact, the elements are completely capacitive with a phase angle of $-90^\circ$. This mismatch is not compensated for in this work. Fortunately, computations show that the power output could be increased by a factor of one hundred if electrical matching is applied to the current prototype.

**Part II**

**Chapter 6**

The elements of the prototype are designed to vibrate independently and as a piston. However, due to the complexity of the prototype construction, it is
reasonable to assume that some mechanical coupling between the elements is present. In order to assess the amount of this cross-talk, a method is presented with which the transient velocity distribution of the transducer surface can be reconstructed from far-field pressure measurements.

The velocity distribution can be obtained using either analytic deconvolution or iterative inversion techniques. Unfortunately, analytic deconvolution is limited to planar and parallel velocity and pressure measurement domains, which are necessarily finite. Therefore, for transducers that are small compared to the wavelength, a significant amount of energy propagates outside the aperture, and the resulting velocity reconstruction is severely spatially smoothed.

Iterative inversion allows for arbitrary spatial domains, and in addition for the application of regularisation schemes. When measuring the pressure along a domain enclosing the transducer, all acoustic energy is captured and available upon inversion, and the resulting velocity distributions are significantly sharper than in the case of analytic deconvolution. Using the method developed, it was shown that the elements of the array prototype vibrate independently, and that motion is spatially limited to only the actively excited areas.

Assuming the transducer surface to move in-phase, but with spatially varying amplitude, further improves the reconstruction. However, the constant phase regularisation method has a very limited validity. For instance, focussed or steered arrays cannot be treated under this assumption. Instead, a spatially varying amplitude with spatially varying time delay should be imposed through regularisation.

The accuracy of the method is limited by the accuracy of the Rayleigh operator, which is computed on a very coarse grid of $2.5$ points per wavelength. Ideally, a denser grid (or an alternative method like, e.g., the fast near-field method) should be used, but this is computationally intractably expensive.

In addition, the hydrophone and its usage have a strong influence on the accuracy of the method. In this work, a $200 \mu \text{m}$ hydrophone was used to image the motion of an element of dimensions $100 \mu \text{m} \times 350 \mu \text{m}$, vibrating at a frequency corresponding to a wavelength of $75 \mu \text{m}$. This introduces spatial averaging in the pressure measurements, and in addition the hydrophone has a strong directivity. When measurements are performed over an enclosing surface, the hydrophone should be positioned such that the highest sensitivity is in the direction of the element to ensure homogeneous sensitivity throughout the grid. Ideally, as small a hydrophone as possible should be used, or the sensitivity pattern of the hydrophone should be included in the reconstruction. However, determining an accurate model of the hydrophone is non-trivial, and even when available, accurately incorporating the model is complicated.

Finally, reducing the measurement distance significantly improves the reconstruction accuracy, especially in the case of a parallel planar pressure field measurement domain. However, due to amplifier saturation and electromagnetic interference, the hydrophone only produces clean signals beyond a certain temporal offset. It is therefore of importance to minimise the interference and saturation, so that measurements can be performed as close to the surface as possible.
Chapter 7

The performance of the IVUS array prototype is tested on various objects. Using both reference phantoms created from biological tissue and an *ex vivo* bovine artery, the quality of images obtained with the array prototype is compared to that of images obtained with a conventional single element IVUS catheter. Rather than using an actual commercial catheter, a conventional single element transducer is emulated using only two of the eight elements of the array prototype. This way, an identical positioning with respect to the objects and identical frequency responses of the transducers can be achieved, which results in the most fair comparison.

In the circumferential direction, a similar image quality is achieved. In the radial and axial direction, however, the array prototype yields significant improvements in the image quality. In all measurements, interfaces are imaged continuously during pullback, and local events are properly localised. In addition, with the array prototype it is possible to image both the wires of a stent phantom and the structures behind it, whereas the conventional single element catheter images only the stent itself, which is smeared out in the axial direction.

Both focussing and (approximate) imaging are applied to obtain images of phantoms. Focussing assumes point-like sources and receivers and a delta spike acoustical excitation, whereas imaging accounts for the finite dimensions of the transducer and the finite pulse duration. In approximate imaging, only the sources are assumed point-like and the excitation is a delta spike, whereas finite receivers are allowed. In synthetic studies, imaging yields a superior image quality. However, experimental results show that applying the imaging algorithm to actual measurements results in lower image quality as compared to focussing. This is due to the scale of the array, the high center frequency of the elements and the dimensions of the hydrophone used to measure the acoustic pulse, all of which complicate the accurate computation of the incident and scatter pressure fields throughout the volume. Approximate imaging yields an image quality similar to focussing, but requires significantly more computation time. Therefore, only focussing is used to generate images with the array.

The results obtained with imaging could be improved if a smaller hydrophone is used. With a smaller hydrophone, the incident pressure field could be measured more accurately and consequently the finite dimensions of the transducer elements and the source signature can be deconvolved more precisely. However, more research is required on how sensitive the method is to the minute differences between different array prototypes, as calibrating each catheter individually is impractical. In addition, the increased computation time complicates a real-time implementation of the algorithm. It is unlikely that the limited increase in image quality observed in synthetic experiments will justify the additional effort required.

All measurements presented were performed on stationary objects. Similar to conventional side-looking IVUS catheters, the array prototype will suffer from artery motion. However, using focussing a smooth, continuous image of the artery
wall is obtained, which enables the application of motion compensation techniques due to the overlap between measurements taken in consecutive pullback positions.

The results obtained with focussing are generated from pressure measurements averaged over either eight or 512 measurements. While averaging over eight measurements is feasible, averaging over 512 measurements is unrealistic in terms of acquisition time. Therefore, the signal strength should be increased by using amplifiers in reception or by applying electrical matching filters. In addition, while images with a high image quality were obtained, the results can be easily improved if a better measurement setup is used. For instance, if a pulser is used that generates a shorter pulse, the full bandwidth of the transducer elements can be used, which will result in sharper images.

Furthermore, the image quality achieved with focussing can be improved if the synthetic aperture formed by the overlap between consecutive measurements is incorporated as well. Even though more research is required on how to correct effectively for slight misalignments and how to combine the real and synthetic aperture data, it is expected that the slight discontinuities observed in pullback images from biological tissue will diminish.

Finally, the performance of the elements of the array deteriorated after prolonged use, and correcting for the change in behaviour increases the effect of noise. In clinical practice, however, IVUS catheters are disposed of after a single use for hygienic reasons, and the limited life span of the prototype forms no limitation.
Summary

An Axial Array for Volumetric Intravascular Ultrasound Imaging

Intravascular ultrasound (IVUS) is a medical imaging modality aimed at imaging blood vessel walls from within the vessel. Current commercial IVUS catheters are designed to yield two-dimensional cross-sectional images perpendicular to the vessel wall. By pulling the catheter back through the artery (in the ‘axial direction’), and stacking the resulting cross-sectional images, a three-dimensional image of the artery can be obtained. However, in non-stationary blood vessels like, e.g., the coronary arteries, artery motion is added to the pullback motion, and as a result consecutively obtained cross-sectional images are not necessarily acquired in neighbouring positions. Therefore, IVUS measurements are electro-gated to ensure proper localisation of the consecutive cross-sectional images. Unfortunately, the radiation pattern of commercial catheters is very narrow, which, combined with a high pullback rate, leads to severe spatial undersampling in the axial direction. Consequently, the data is incomplete and motion compensation attempts are unsuccessful. Instead, in this work the design of an axial IVUS array is presented. With the array, three-dimensional volumes rather than cross-sectional planes are imaged in each pullback position. If the imaged volume is large enough, overlap between images from consecutive electro-gated measurements can be achieved. This enables motion compensation techniques, as well as the acquisition of continuous, correct three-dimensional images of artery sections.

In order to assess the image quality of the resulting prototype design, software has been developed which models linear acoustic propagation through inhomogeneous soft tissue. The resulting scattering is not limited to the Born approximation. The governing frequency domain integral equation is solved iteratively using a parallelised Fortran implementation, which is accurate to within several percent when compared to analytical solutions. Arbitrary contrasts in both compressibility and volume density of mass, as well as arbitrary transducer configurations can be treated. Unfortunately, where contrasts extend the numerical domain boundaries, spurious reflections are generated. These reflections are
suppressed by a factor of one hundred or more using a frequency domain scatter integral equation formulation of a perfectly matched layer, while virtually no reflections occur off this attenuative layer.

Both the scatter simulation software and the design of an ultrasound array require the accurate and efficient computation of incident pressure fields generated by a (piston) transducer. Five methods are compared in efficiency and accuracy, viz. the Rayleigh integral, the impulse response method, the fast near-field method, the fast near-field method combined with time-space decomposition, and a frequency domain formulation of the latter time-space decomposition method. The frequency domain formulation has been derived to extend the validity of the time-space decomposition method. To reach the same accuracy, the fast near-field method and time-space decomposition easily yield a speed-up factor of more than one hundred when compared to the Rayleigh integral, and the maximum attainable accuracy is significantly higher for these methods. Note that, apart from the Rayleigh integral, all methods assume piston behaviour of the transducer.

The fast near-field method was used to design an optimal axial IVUS array given the constraints in dimensions, frequency, penetration depth and sensitivity. The resulting array consists of eight elements of dimensions 100 μm by 350 μm, with an interelement kerf of 100 μm, and operates at 20 MHz. Using the scatter simulation software, the expected image quality is simulated, and significant improvements over the image quality of a current commercial catheter are achieved.

The array prototype was fabricated in-house by first fabricating piezo-electric wafers including matching and backing layers. Next, the electrical connections were made, and finally the separate elements were diced from the wafer. The resulting array contains almost identical elements operating at 21 MHz, with a fractional bandwidth of 80 % or more. Using far-field pressure field measurements, the transducer surface velocity distribution was computed using an iterative inversion scheme. From this measurement it followed that, despite the complicated construction, the elements vibrate independently, and that motion is spatially confined to the actively driven regions.

Using both reference phantoms and a bovine artery, the image quality of the array prototype was compared to that of a conventional single element IVUS catheter. In the radial and axial direction the array yields a significantly higher image quality, while in the circumferential direction a similar image quality was obtained. With the array prototype, contrast boundaries are smoothly and continuously imaged in the axial direction, and local events are properly localised. In addition, using the array prototype both the wires of a stent phantom and the structure behind the wires can be imaged, whereas using a conventional single element transducer only the wires are visible, and smeared out in the axial direction.
Samenvatting

Een Axiaal Array voor Volumetrisch Intravascular Ultrageluid

Intravascular ultrageluid (“intravascular ultrasound”; IVUS) is een medische afbeeldingstechniek gericht op het afbeelden van de wanden van een bloedvat, vanuit het bloedvat zelf. Huidige commerciële IVUS-katheters zijn zo ontworpen dat tweedimensionale afbeeldingen van dwarsdoorsneden loodrecht op de aderwand worden gemaakt. Door de katheter terug te trekken in de richting van de ader (de “axiale richting”), en de opeenvolgende dwarsdoorsneden samen te voegen, wordt een driedimensionale afbeelding van de ader verkregen. Echter, in niet-stationaire aderen, bijvoorbeeld in de kransslagaders, treedt, bovenop de terugtrekbeweging van de katheter, aderbeweging op. Als gevolg hiervan zijn opeenvolgende afbeeldingen niet noodzakelijkerwijs verkregen in naburige posities. Om ervoor te zorgen dat de axiale coördinaat van een afbeelding goed bepaald is, wordt met behulp van electrocardiografie steeds op hetzelfde moment in de hartcyclus een afbeelding gemaakt, een techniek genaamd “electro-gating”. Helaas is het uitstralingsprofiel van commerciële katheters in de axiale richting erg smal, en wordt de katheter per hartslag meer dan de breedte van het uitstralingsprofiel verplaatst. Hierdoor wordt de ader axiaal zeer grof bemonsterd en worden delen van de aderwand niet afgebeeld, en als gevolg daarvan zijn algoritmen voor aderbewegingen te compenseren niet succesvol. Dit werk draait om het ontwerp van een axiaal IVUS array. Met dit array kan in iedere positie een driedimensionaal volume in plaats van een tweedimensionale dwarsdoorsnede worden afgebeeld. Als het afgebeelde volume groot genoeg is, dan kunnen twee afbeeldingen vekregen in twee opeenvolgende metingen elkaar deels overlappen. Hierdoor wordt bewegingscompensatie mogelijk, en kunnen goed bemonsterde, complete driedimensionale afbeeldingen van adersecties verkregen worden.

Om de beeldkwaliteit van het resulterende prototype te voorspellen, is software ontwikkeld waarmee de lineaire geluidsvoortplanting door zacht inhomogene weefsel gesimuleerd kan worden. De gesimuleerde verstrooiing is niet beperkt tot de Born benadering. De onderliggende frequentiedomein integraalvergelijking
wordt iteratief opgelost met behulp van geparalleliseerde Fortran code, waarmee een fout van enkele procenten behaald wordt wanneer het resultaat vergeleken wordt met analytische oplossingen. De code laat willekeurige contrasten in massadichtheid en compressibiliteit toe, alsmede een willekeurige transducentenconfiguratie. Helaas treden hinderlijke reflecties op op plaatsen waar de contrasten zich uitstrekken tot voorbij de spatiële grenzen van het numerieke domein. Door een reflectievrije absorberende laag ("perfectly matched layer", PML) te formuleren voor de frequentiedomein verstrooingsintegraalvergelijking, kunnen deze reflecties met een factor honderd of meer onderdrukt worden, terwijl vrijwel geen reflecties van de absorberende laag optreden.

Zowel de software om verstrooing te simuleren als het ontwerpen van een ultrageluid array vereisen snelle en nauwkeurige berekeningen van het drukveld dat gegenereerd wordt door een transducer. Derhalve worden vijf rekenmethoden vergeleken in nauwkeurigheid en efficiëntie, te weten: de Rayleigh integraal, de impulsie respons methode, de "fast near-field method", de fast near-field method in combinatie met ruimte-tijd decompositie en een frequentiedomein formulering van de laatstgenoemde. De frequentiedomein formulering breidt de geldigheid van de ruimte-tijd decompositie uit. De fast near-field method en haar ruimte-tijd decompositie bereiken dezelfde nauwkeurigheid ruim honderd keer sneller dan de Rayleigh integraal, en de hoogst bereikbare nauwkeurigheid is significant hoger voor deze methoden. Afgezien van de Rayleigh integraal, gaan alle methoden uit van "piston" transducers, waarvan het gehele transducentoppervlak in fase beweegt.

Met behulp van de fast near-field method is een axiaal IVUS array geoptimaliseerd in termen van dimensies, frequentie, penetratiediepte en gevoeligheid. Het resulterende array bestaat uit acht elementen met dimensies 100 μm bij 350 μm, met een steek van 100 μm tussen de elementen, welke opereren rond een frequentie van 20 MHz. De verwachte beeldkwaliteit is gesimuleerd met de verstrooings simulatiesoftware, en significante verbeteringen in beeldkwaliteit in vergelijking met die van huidige commerciële katheters worden verwacht.

Het array prototype is intern gefabriceerd door eerst een piezo-elektrische plak te vervaardigen, inclusief aankoppel- en "backing"-lagen om de vermogensoverdracht aan het omringende medium te maximaliseren. Vervolgens zijn de elektrische aansluitingen op de plak gemaakt, en tenslotte zijn de individuele elementen uit de plak vrijgezaagd. De resulterende arrays bestaan uit vrijwel identieke elementen opererend op 21 MHz, met een fractionele bandbreedte van ten minste 80 %. De snelheidsverdeling op het transducentenoppervlak is afgebeeld, gebruik makende van drukveldmetingen in het verre veld, met behulp van een (iteratief) inversieschema. Uit deze berekeningen is gebleken dat, ondanks de ingewikkelde constructie, de elementen onafhankelijk bewegen, en dat de beweging spatiaal beperkt is tot enkel de aangestuurde gebieden.

De beeldkwaliteit van het array prototype is vergeleken met een conventionele IVUS transducer, bestaande uit een enkele transducer, voor zowel gecontroleerde objecten als voor een ader van een rund. In de radiële en axiale richting bereikt het array een significant hogere beeldkwaliteit, terwijl de beeld-
kwaliteit in de omtreksrichting gehandhaafd blijft. Contrastgrenzen worden met het array prototype continu afgebeeld in de axiale richting, en lokale contrasten worden in de juiste locatie afgebeeld. Bovendien worden met het array zowel de metaaldraden van een nagebootste stent als het achterliggende weefsel correct afgebeeld, terwijl met een conventionele transducer enkel de metaaldraden, uitgesmeerd in de axiale richting, zichtbaar zijn.
Appendices

A.1 Notation, Definitions and Symbols

Notation

\( a \) \hspace{1cm} \text{scalar number}
\( \bar{a} \) \hspace{1cm} \text{complex conjugate of } a
\( b \) \hspace{1cm} \text{vector}

\( V \) \hspace{1cm} \text{closed volume}
\( \partial V \) \hspace{1cm} \text{boundary of closed volume } V
\( \hat{n} \) \hspace{1cm} \text{unit vector normal to } \partial V

\( \dot{d}(t) \) \hspace{1cm} \text{time domain variable}
\( \dot{d}(\mathbf{r}) \) \hspace{1cm} \text{spatially varying variable}
\( \dot{d}(\mathbf{k}) \) \hspace{1cm} \text{wave number domain variable}
\( [\dot{d}](\mathbf{r}) \) \hspace{1cm} \text{locally spatially averaged variable}

\( x \) versus \( \bar{x} \) \hspace{1cm} \text{real-valued versus complex-valued coordinate}

\( \dot{d}(e, f, \ldots) \) \hspace{1cm} \text{function in variables } e, f, \ldots
\( \dot{d}(e) \) versus \( \dot{d}(e_k) \) \hspace{1cm} \text{continuous versus discretely sampled function}

\( \hat{d}(e, f, \ldots) = \dot{d}(\omega, e, f, \ldots) \) \hspace{1cm} \text{function defined in temporal frequency domain}
\( \hat{d}(e, f, \ldots) = \dot{d}(\omega, k_x, k_y, z, e, f, \ldots) \) \hspace{1cm} \text{function defined in spatial and temporal frequency domain}

\( \frac{\partial \dot{d}(e, f, \ldots)}{\partial e} \) \hspace{1cm} \text{partial derivative of function } \dot{d}(e, f, \ldots) \text{ with respect to } e
\( \frac{\partial \dot{d}(e, f, \ldots)}{\partial n} \) \hspace{1cm} \text{spatial derivative of function } \dot{d}(e, f, \ldots) \text{ in the direction of normal vector } \hat{n}
\[ d^{(n)}(e) \]

\[ \nabla = \left( \frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \right)^T \]

\[ \nabla^{\sigma} = \left( \begin{array}{ccc} \hat{X}_{x}^{\sigma} & 0 & 0 \\ 0 & \hat{X}_{y}^{\sigma} & 0 \\ 0 & 0 & \hat{X}_{z}^{\sigma} \end{array} \right) \left( \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{array} \right) \]

\[ \sum_{i} a_{i} \sum_{e,f,\ldots} d(e,f,\ldots) \]

\[ \mathbf{b} \cdot \mathbf{c} = \sum_{i} b_{i}c_{i} \]

\[ A \quad \text{matrix} \]

\[ A^T \quad \text{matrix transpose} \]

\[ A^* \quad \text{conjugate transpose of matrix } A \]

\[ \mathbf{B} \quad \text{operator} \]

\[ \mathbf{B}^* \quad \text{operator adjoint} \]

\[ |d(e,f,\ldots)|^2 = \frac{d(e,f,\ldots)d(e,f,\ldots)}{d(e,f,\ldots)d(e,f,\ldots)} \]

\[ \|\mathbf{b}\| = \sqrt{\sum_{i} b_{i}b_{i}} \]

\[ \in_{1}(d,g;e,f,\ldots) \quad \text{L}_1\text{-norm between } d(e,f,\ldots), g(e,f,\ldots) \]

\[ \in_{2}(d,g;e,f,\ldots) \quad \text{L}_2\text{-norm between } d(e,f,\ldots), g(e,f,\ldots) \]

\[ \mathcal{F}\{d(t)\} \quad \text{forward temporal Fourier transform} \]

\[ \mathcal{F}^{-1}\{d(\omega)\} \quad \text{inverse temporal Fourier transform} \]

\[ \mathcal{F}_{k}\{d(\mathbf{r})\} \quad \text{forward spatial Fourier transform} \]

\[ \mathcal{F}^{-1}_{k}\{d(\mathbf{k})\} \quad \text{inverse spatial Fourier transform} \]

\[ \delta(\mathbf{r}) \quad \text{spatial Dirac delta distribution} \]

\[ e_{*} \quad \text{convolution in variable } e \]

\[ \mathcal{O}(\Delta x^n) \quad n\text{-th order approximation error} \]
Definitions

Temporal Fourier Transforms

\[
    d(\omega) = \mathcal{F} \{ d(t) \} = \int_{-\infty}^{\infty} d(t)e^{-i\omega t}dt \\
    d(\omega_n) = \mathcal{F} \{ d(t_k) \} = \sum_{k=1}^{N_t} d(t_k)e^{-i\omega_n t_k} \\
    d(t) = \mathcal{F}^{-1} \{ d(\omega) \} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d(\omega)e^{i\omega t}d\omega \\
    d(t_k) = \mathcal{F}^{-1} \{ d(\omega_n) \} = \sum_{n=1}^{N_t} \frac{d(\omega_n)}{N_t}e^{i\omega_n t_k}
\]

Spatial Fourier Transforms

\[
    d(k_x) = \mathcal{F}_k \{ d(x) \} = \int_{-\infty}^{\infty} d(x)e^{ik_x x}dx \\
    d(k_{x,n}) = \mathcal{F}_k \{ d(x_k) \} = \sum_{k=1}^{N_t} d(x_k)e^{ik_{x,n} x_k} \\
    d(x) = \mathcal{F}^{-1} \{ d(k_x) \} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d(k_x)e^{-ik_x x}dk_x \\
    d(x_k) = \mathcal{F}^{-1} \{ d(k_{x,n}) \} = \sum_{n=1}^{N_t} \frac{d(k_{x,n})}{N_t}e^{-ik_{x,n} x_k}
\]

Convolution

\[
    (d(e)*_eg(e)) (e) = \int_{-\infty}^{\infty} d(e')g(e-e')de' \\
    (d(e_n)*_eg(e_n)) (e_k) = \sum_{n=-\infty}^{\infty} d(e_n)g(e_k-e_n)
\]

Inner Product

\[
    b \cdot c = \sum_{i} b_i c_i
\]
L₁- and L₂-norms

\[
\epsilon_1(d, g; e, f, \ldots) = \frac{\sum_{i,j,\ldots} |d(e_i, f_j, \ldots) - g(e_i, f_j, \ldots)|}{\sum_{i,j,\ldots} |g(e_i, f_j, \ldots)|}
\]

\[
\epsilon_2(d, g; e, f, \ldots) = \sqrt{\frac{\sum_{i,j,\ldots} |d(e_i, f_j, \ldots) - g(e_i, f_j, \ldots)|^2}{\sum_{i,j,\ldots} |g(e_i, f_j, \ldots)|^2}}
\]

rect-function

\[
\text{rect}(x) = \begin{cases} 
1 & \text{for } 0 \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]

Symbols

**Greek**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>complex part of PML strength function</td>
</tr>
<tr>
<td>β</td>
<td>complex part of wave number in attenuative medium</td>
</tr>
<tr>
<td>Γ</td>
<td>spatial derivative stencil matrix</td>
</tr>
<tr>
<td>γᵢ</td>
<td>spatial derivative weight factor</td>
</tr>
<tr>
<td>∆</td>
<td>PML decay factor</td>
</tr>
<tr>
<td>Δₓ [m]</td>
<td>increment/difference in spatial coordinate x</td>
</tr>
<tr>
<td>Δᵣ [m]</td>
<td>increment/difference in position r</td>
</tr>
<tr>
<td>Δᵣᵢ, Δᵣⱼ [m]</td>
<td>distance between point r and source i or receiver j</td>
</tr>
<tr>
<td>Δᵗ [s]</td>
<td>increment/difference in time t</td>
</tr>
<tr>
<td>δ</td>
<td>Dirac delta distribution</td>
</tr>
<tr>
<td>∂Dₜₜ</td>
<td>boundary enclosing total numerical domain</td>
</tr>
<tr>
<td>∂Dₚₚ</td>
<td>boundary enclosing numerical domain outside PML</td>
</tr>
<tr>
<td>ℰ₁, ℰ₂ [-]</td>
<td>L₁-, L₂-norm</td>
</tr>
<tr>
<td>Θₘₚ [N m⁻³]</td>
<td>induced part of the cubic dilation rate</td>
</tr>
<tr>
<td>κ, κ₀, κₚ [Pa⁻¹]</td>
<td>compressibility: general, in background medium, in fat contrast</td>
</tr>
<tr>
<td>λ, λ₀ [m]</td>
<td>wavelength: general, in background medium</td>
</tr>
<tr>
<td>μ [s]</td>
<td>temporal offset Gaussian pulse</td>
</tr>
<tr>
<td>ν [Pa]</td>
<td>imaging stabilisation factor</td>
</tr>
<tr>
<td>ρ, ρ₀, ρₚ [kg m⁻³]</td>
<td>volume density of mass: general, in background medium, in fat contrast</td>
</tr>
<tr>
<td>σ [s]</td>
<td>temporal width Gaussian pulse</td>
</tr>
<tr>
<td>σ(x) [rad s⁻¹]</td>
<td>PML strength function</td>
</tr>
<tr>
<td>σ₀, σ₀’ [rad s⁻¹]</td>
<td>real-valued, complex-valued PML strength amplitude</td>
</tr>
</tbody>
</table>
A.1. NOTATION, DEFINITIONS AND SYMBOLS

\[ \tau \ [s] \]

Temporal shift caused by spatial offset in fast near-field method representation of normal component of piston surface velocity \( v_{\perp}(t - \tau) \)

\[ \phi \ [\text{rad}] \]

Spherical, cylindrical coordinate

\[ \chi_{\kappa}, \chi_{\rho} \ [-] \]

Compressibility, density contrast function

\[ \omega \ [\text{rad s}^{-1}] \]

Angular frequency

Roman

\[ A_0 \ [\text{m s}^{-1}] \]

Piston face velocity amplitude

\[ A_{\text{inc}}, A_{\text{ref}}, A_{\text{trans}} \ [\text{Pa}] \]

Incident, reflected, transmitted plane wave amplitude at a contrast interface

\[ a(r) \ [\text{Pa}] \]

Focussing image amplitude

\[ a \ [\text{m}] \]

Radius sphere, circle

\[ b(r) \ [\text{rad s}^{-1}] \]

Imaging image amplitude

\[ b \ [\text{m}] \]

Prototype element width

\[ c, c_0 \ [\text{m s}^{-1}] \]

Speed of sound: general, in background medium

\[ D(r) \ [\text{m}^{-1}] \]

Direct wave contribution to radiated pressure field

\[ D_t \ [\text{s}^{-1}] \]

Material time derivative

\[ \mathbb{D}, \mathbb{D}_{\text{num}}, \mathbb{D}_{\text{tot}} \ [-] \]

Spatial domain of interest, total numerical domain, infinitely extending domain

\[ \mathbb{D}_{\text{PML}}, \mathbb{D}_{\text{scat}} \ [-] \]

PML domain, domain of non-zero contrast

\[ E_{n}(r, z) \ [\text{m}^{-\frac{3}{2}} \text{s}^{-\frac{1}{2}}] \]

Edge wave contribution to radiated pressure field

\[ e_{\text{Gauss}}(t) \ [-] \]

Gaussian modulated pulse

\[ e_{\text{Hanning}}(t) \ [-] \]

Hanning weighted pulse

\[ e_{\text{toneburst}}(t) \ [-] \]

Toneburst

\[ f(r, t) \ [\text{N m}^{-3}] \]

Volume source density of volume force

\[ f_0 \ [\text{Hz}] \]

Center frequency excitation pulse

\[ f_n(\tau) \ [\text{m}^\frac{1}{2} \text{s}^{-\frac{1}{2}}] \]

Spatial dependency in \( v_{\perp}(t - \tau) \) in time-space decomposition

\[ \hat{G}(r) \ [\text{m}^{-1}] \]

Green’s function

\[ g_n(t) \ [\text{m}^\frac{1}{2} \text{s}^{-\frac{1}{2}}] \]

Temporal dependency in \( v_{\perp}(t - \tau) \) in time-space decomposition

\[ h \ [\text{m}] \]

Prototype element height

\[ h(x) \ [\text{m}] \]

Complex-valued coordinate extension for analytic continuation

\[ i \ [-] \]

Complex-valued unity, \( i^2 = -1 \)

\[ J \ [-] \]

\( 2J + 1 \) is stencil size in spatial derivative

\[ k, k_0, k_{\text{scat}} \ [\text{m}^{-1}] \]

Wave number: general, in background medium, in non-zero contrast region

\[ k_r, k_i \ [\text{m}^{-1}] \]

Real and imaginary part of complex-valued wavenumber

\[ k_x, k_y, k_z \ [\text{m}^{-1}] \]

Component of wave number in \( x-, y- \) and \( z- \) direction

\[ \mathcal{L} \ [-] \]

Scatter operator

\[ M(\phi, r) \ [\text{m}^{-1}] \]

Fast near-field method integral kernel
\( N \) [-] number of time-space decomposition terms
\( N_t \) [-] number of time samples
\( P_0, P(r, t) \) [Pa] static ambient pressure, total pressure (ambient + excess)
\( p(r, t) \) [Pa] excess acoustic pressure field: general
\( p_{tot}(r, t) \) [Pa] total pressure field
\( p_{inc}(r, t) \) [Pa] incident pressure field
\( p_{scat}(r, t) \) [Pa] scatter pressure field
\( p_{ref}(r, t) \) [Pa] reference pressure field
\( p_{backscat}(r, t) \) [Pa] part of the scatter pressure field scattered back towards the receive aperture
\( q(r, t) \) [s\(^{-1}\)] volume source density of injection rate
\( R \) [m] smallest distance from observation point \( r \) to the edge of a circular transducer
\( \hat{R} \) [-] forward Rayleigh-1 operator
\( r \) [m] observation point
\( r' \) [m] source point
\( S_0, S_1, S_{rec} \) [-] domains involved using Rayleigh integrals: infinitely extending planar domain, hemispherical domain at infinite radius, spatially limited aperture
\( S(t) \) [m s\(^{-1}\)] piston face velocity source signature
\( \hat{S}_{pr}(r) \) [kg m\(^{-1}\) s\(^{-1}\)] source term generating incident pressure field
\( \hat{S}_{scat}(r) \) [kg m\(^{-1}\) s\(^{-1}\)] contrast source term generating scattering effects
\( t \) [s] time
\( v(r, t) \) [m s\(^{-1}\)] particle velocity
\( v_{\perp}(r, t), v'_{\perp}(t) \) [m s\(^{-1}\)] piston face velocity, spectrally clipped representation
\( W \) [s] temporal width piston face velocity pulse
\( w \) [m] width of smooth PML transition region
\( x \) [m] \( x \)-coordinate
\( y \) [m] \( y \)-coordinate
\( z \) [m] \( z \)-coordinate
A.2 Weak Form Green’s Function

The local spatial average of the Green’s function for a homogeneous background medium,

$$\hat{G}(\mathbf{r}) = \frac{e^{-ikr}}{4\pi r},$$  \hspace{1cm} (1)

with \( r = ||\mathbf{r}|| \), is computed by averaging \( \hat{G}(\mathbf{r}) \) over a spherical volume centered at \( \mathbf{r} \), i.e.,

$$\left[ \hat{G}(\mathbf{r}) \right] = \frac{1}{4\pi a^3} \int_{||\mathbf{r}'|| \leq a} \hat{G}(\mathbf{r} + \mathbf{r}') dV(\mathbf{r}'),$$  \hspace{1cm} (2)

where the radius of the sphere \( a \) is chosen such that the volume of the sphere equals the volume of a grid element.

Since the Green’s function is spherically symmetric around the origin, the weak form Green’s function is symmetric as well, and only dependent on \( r \). Therefore, the spatial average can be computed for all space using only a single direction, and thus \( \mathbf{r} \) can be chosen freely. For simplicity, in this derivation the vector \( \mathbf{r} \) is taken to be parallel to the \( z \)-axis, which eliminates two angular dependencies. Using this direction for the derivation, and defining

$$Q = ||\mathbf{r} + \mathbf{r}'|| = \sqrt{r^2 + r'^2 + 2rr'\cos\theta'},$$  \hspace{1cm} (3)

for the locally spatially averaged Green’s function is found

$$\left[ \hat{G}(\mathbf{r}) \right] = \frac{3}{8\pi a^3} \int_0^a \int_0^\pi \int_0^{2\pi} e^{-ikQ} r'^2 \sin\theta' d\phi' d\theta' dr',$$  \hspace{1cm} (4)

where the integration with respect to \( \phi' \) is trivial.

Using the change of variables

$$\frac{\partial Q}{\partial \theta'} = -\frac{r'r'}{Q} \sin\theta' \Rightarrow d\theta' = -\frac{Q}{r'r'\sin\theta'} dQ,$$  \hspace{1cm} (5)

equation (4) changes, for \( r \geq a \), into

$$\left[ \hat{G}(\mathbf{r}) \right] = \frac{-3}{8\pi a^3 r} \int_0^a \int_{\sqrt{r^2 + r'^2 - 2rr'}}^{\sqrt{r^2 + r'^2 + 2rr'}} e^{-ikQ} dQ dr'.$$  \hspace{1cm} (6)

Since \( \mathbf{r} \) was chosen parallel to the positive \( z \)-axis, the lower and upper limits for
the integration over $Q$ simplify to $r + r'$ and $r - r'$, respectively, so that

$$
\hat{G} (r) = \frac{3}{8ik\pi a^3r} \int_0^a r'[e^{-ikr} - e^{-ikr'}] \, dr' = \frac{3e^{-ikr}}{4k\pi a^3r} \int_0^a r' \sin(kr') \, dr'.
$$

Using integration by parts, the weak form Green’s function is found to be

$$
\hat{G} (r) = \begin{cases} 
3e^{-ikr} \left[ \sin(ka) - ka \cos(ka) \right] & \forall r \geq a \\
\frac{3}{4k^2 \pi a^3} \left[ (1 + ika) e^{-ika} - 1 \right] & \text{for } r = 0
\end{cases},
$$

where the expression for $r = 0$ is found by integrating equation (4) directly, i.e., without the change of variable. This expression is identical to that in [66].
A.3 Spatial Derivatives

Approximations to the first order spatial derivative of a function \( f(x) \) can be obtained by computing the Taylor expansion [120] of function \( f(x) \) about the point \( x \), i.e.,

\[
f(x \pm jh) = \sum_{n=0}^{N} \frac{(\pm j)^n}{n!} h^n f^{(n)}(x) + O(h^{N+1}),
\]

where \( j \) is an integer determining the stencil size \( J = 2j + 1 \), \( f^{(n)} \) denotes the \( n \)-th order derivative with respect to \( x \), \( h \) is a step size and \( O(h^{N+1}) \) is the \( N + 1 \)-th order approximation error. The difference \( f(x + jh) - f(x - jh) \) is given by

\[
f(x + jh) - f(x - jh) = 2 \sum_{n=1,3,5,...}^{N} \frac{j^n}{n!} h^n f^{(n)}(x) + O(h^{N+1}).
\]

For an \( N + 1 = 2J \) approximation order, the following set of equations is required:

\[
\begin{pmatrix}
  f(x + h) - f(x - h) \\
  f(x + 2h) - f(x - 2h) \\
  \vdots \\
  f(x + Jh) - f(x - Jh)
\end{pmatrix} =
\begin{pmatrix}
  1 & \frac{1}{J!} & \cdots & \frac{(2J-1)!}{(2J-1)!} \\
  2 & \frac{1}{J!} & \cdots & \frac{(2J-1)!}{(2J-1)!} \\
  \vdots & \vdots & \ddots & \vdots \\
  J & \frac{1}{J!} & \cdots & \frac{(2J-1)!}{(2J-1)!}
\end{pmatrix}
\begin{pmatrix}
  2hf^{(1)}(x) \\
  2h^3f^{(3)}(x) \\
  \vdots \\
  2h^{2J-1}f^{(2J-1)}(x)
\end{pmatrix} + O(h^{2J}).
\]

Denoting the matrix in equation (11) by \( \Gamma^{-1} \) and omitting the approximation order term, equation (11) can be written as

\[
\begin{pmatrix}
  f(x + h) - f(x - h) \\
  f(x + 2h) - f(x - 2h) \\
  \vdots \\
  f(x + Jh) - f(x - Jh)
\end{pmatrix} \approx \Gamma^{-1}
\begin{pmatrix}
  2hf^{(1)}(x) \\
  2h^3f^{(3)}(x) \\
  \vdots \\
  2h^{2J-1}f^{(2J-1)}(x)
\end{pmatrix},
\]

which after multiplying both sides by \( \Gamma \), the inverse of matrix \( \Gamma^{-1} \), yields

\[
\Gamma
\begin{pmatrix}
  f(x + h) - f(x - h) \\
  f(x + 2h) - f(x - 2h) \\
  \vdots \\
  f(x + Jh) - f(x - Jh)
\end{pmatrix} \approx
\begin{pmatrix}
  2hf^{(1)}(x) \\
  2h^3f^{(3)}(x) \\
  \vdots \\
  2h^{2J-1}f^{(2J-1)}(x)
\end{pmatrix}.
\]

Thus, by taking the inner product of the first row of \( \Gamma \) and the differences \( f(x + jh) - f(x - jh) \), a scaled approximation to the first order derivative of \( f(x) \) is
obtained, with an approximation order $h^{2J}$. The first row of $\Gamma$ contains the $\gamma_j$ of equation (2.24). In the implementation discussed in Chapter 2, the inversion of matrix $\Gamma^{-1}$ is performed numerically. Especially for large $J$, this may lead to loss of accuracy.
A.4 Conjugate Gradient Schemes

Conjugate Gradient

The conjugate gradient (CG) method is an iterative solution method to solve problems of the form

$$Ax = b,$$  \hspace{1cm} (14)

where matrix $A$ and vector $b$ are known, and the equation is solved for the unknown vector $x$. In this appendix, the discussion is limited to symmetric, positive definite matrices $A$, which are always invertible.

In the CG method, an initial solution vector $x_0$ is picked. In this entire work, the initial guess for the solution is the zero vector. In every iteration, the solution vector $x_i$ is updated by taking a step in direction $d_i$ with stepsize $\alpha_i$,

$$x_{i+1} = x_i + \alpha_i d_i.$$  \hspace{1cm} (15)

By enforcing conjugacy on the search directions, i.e.

$$d_i^* A d_j = 0 \quad \forall i \neq j,$$  \hspace{1cm} (16)

convergence in exactly $n$ steps is guaranteed [63], where $n$ is the dimensionality of $b$, and $^*$ indicates the conjugate or Hermitian transpose operator. By further requiring that the residuals

$$r_i = b - A x_i$$  \hspace{1cm} (17)

of all iterations are conjugate, only the last update direction is required to compute the next [63], which yields an efficient scheme in terms of memory load. The complete scheme, of which the derivation can be found in [63], is given in algorithm 1.

\textbf{Algorithm 1} The conjugate gradient scheme

\begin{verbatim}
x_0 = 0
d_0 = r_0 = b - A x_0
i = 0
while $i \leq i_{\text{max}}$ and $\frac{r_i^* r_i}{b^* b} \geq \epsilon^2$ do
    $\alpha_i = \frac{r_i^* r_i}{d_i^* A d_i}$
    $x_{i+1} = x_i + \alpha_i d_i$
    $r_{i+1} = r_i - \alpha_i A d_i$
    $\beta_{i+1} = \frac{r_{i+1}^* r_{i+1}}{r_i^* r_i}$
    $d_{i+1} = r_{i+1} + \beta_{i+1} d_i$
    $i = i + 1$
end while
\end{verbatim}
BiCGSTAB

The above scheme can only be directly applied to symmetric, positive definite matrices $A$. However, by solving the so-called normal equations,

$$A^*Ax = A^*b,$$  \hfill (18)

in principle any matrix $A$ can be inverted in the least squares sense, even non-square matrices. However, this requires the computation of the conjugate transpose of $A$. In addition, convergence of the CG scheme applied to the normal equations is slow.

To avoid these issues, the BiCGSTAB variant [64] on the above CG scheme is used. This scheme, see algorithm 2, does not require the computation of the conjugate transpose, and in addition converges significantly faster than the standard CG scheme in most cases at the expense of a higher memory load.

Unfortunately, convergence is not guaranteed, and for strong and large contrasts the convergence rate decreases or divergence is even observed. However, for all simulations performed in this work, the scheme converged. In all comparative studies, BiCGSTAB converged significantly faster than CG for clinically relevant contrast functions.

**Algorithm 2** The BiCGSTAB scheme

\[
\begin{align*}
 x_0 &= 0 \\
r_0 &= b - Ax_0 \\
\hat{r}_0 &= r_0 \\
\rho_0 &= \alpha = \omega = 1 \\
v_0 &= p_0 = 0 \\
i &= 0 \\
\text{while } i \leq i_{\text{max}} \text{ and } \frac{|r_i|}{|b|} \geq \epsilon^2 \text{ do} \\
\rho_{i+1} &= \hat{r}_0 \cdot r_i \\
\beta &= \frac{\rho_{i+1} \alpha}{\rho_i \omega} \\
p_{i+1} &= r_i + \beta(p_i - \omega v_i) \\
v_{i+1} &= Ap_{i+1} \\
\alpha &= \frac{\rho_{i+1}}{\hat{r}_0 \cdot v_{i+1}} \\
s &= r_i - \alpha v_{i+1} \\
t &= As \\
\omega &= \frac{t \cdot s}{t \cdot t} \\
x_{i+1} &= x_i + \alpha p_{i+1} + \omega s \\
r_{i+1} &= s - \omega t \\
i &= i + 1 \\
\text{end while}
\]
Bibliography


Curriculum Vitae

Erwin Jozef Alles was born in Haarlem, the Netherlands, on May 17th, 1985. He attended secondary school at Atheneum College Hageveld in Heemstede and received his ‘VWO diploma’ in 2002. In 2005, he received his Bachelor of Science degree in Applied Physics at Delft University of Technology, the Netherlands, with a thesis on seismic multiple removal using a genetic algorithm.

The next two years he studied for his master’s degree in Applied Physics at Delft University of Technology, and in 2007 he received his Master of Science degree with a thesis on the development and testing of a finite element model for a multi-actuator loudspeaker panel. From June 2007 through October 2007, he held an internship at the Netherlands Forensic Institute, where he developed a method to extract a photo-sensitivity fingerprint from a digital camera and determine the source camera of a set of digital photographs.

Following an invitation by his later supervisor, he joined the Laboratory of Acoustical Wavefield Imaging at Delft University of Technology in November 2007, where he conducted the research that led to this PhD thesis. From February 2011 to August 2011, he was invited to perform research at Michigan State University as a visiting scholar. The research concerned efficient computation of the transient pressure field generated by a piston transducer.

Currently, he is a Post-doctoral training fellow in medical photoacoustic imaging at the Institute of Cancer Research, Sutton, United Kingdom. Here he works on the optimisation of a prototype photoacoustic system and reduction of the clutter of this system.
Publications


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