The Residual Strength Failure Sequence in Fibre Metal Laminates
The Residual Strength Failure Sequence in Fibre Metal Laminates

Proefschrift

der verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus prof. ir. K.C.A.M. Luyben,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen op maandag 19 maart 2012 om
10.00 uur

door

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This research was carried out under project number MC4.06268 in the framework of the Strategic Research Programme of the Materials Innovation Institute M2i (www.m2i.nl), the former Netherlands Institute for Metal Research.


Keywords: Fibre Metal Laminates, Glare, Residual Strength, Damage Tolerance

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To my Family
Summary

“ The residual strength failure sequence in Fibre Metal Laminates”
Riccardo Rodi

The concept of damage tolerance is a key aspect in ensuring and maintaining safety of an airframe structure over its design life. Developments in materials and structural design have both contributed to improvements in the damage tolerance of modern aircraft structures. Indeed, new developments in metal alloys, composite materials, and hybrid materials such as the Fibre Metal Laminates (FMLs) have all resulted in structures less sensitive to damage and capable to withstand more severe loading conditions. Among other materials, FMLs represent a clear example of damage tolerant hybrid materials, made by bonding thin metal sheets together with fibres embedded in epoxy.

Exploiting the damage tolerance capability of FMLs is strictly related to the ability to firstly understand the occurring failure mechanisms, and secondly to be able to accurately describe those mechanisms. In this light, the present dissertation describes the investigation on the residual strength failure sequence in FMLs, and presents the development of an accurate analytical prediction method. The failure sequence is studied in particular for standard Glare laminates, which are relevant laminates for applications in aircraft pressurized fuselages. The developed analytical method has been implemented into two numerical models, considering both through-the-thickness crack and fatigue crack configurations. The developed models are validated against a large number of experimental data, which are also presented in this thesis. The main concept in this dissertation is that the crack growth process in the metallic layers of an FML can be described with the Crack Tip Opening Angle concept (CTOA). This approach includes the contribution of the fibre layers (e.g. fibre failure and fibre bridging) and the associated quasi-static delamination growth.

An introduction to FMLs and to all various Glare grades, lay-ups, and manufacturing processes is provided in chapter 2. Some current and future applications for aircraft structures are also discussed in that chapter.

A qualitative description of the principal failure mechanisms occurring during the residual strength failure sequence is presented in chapter 3. Based on experimental observations, the metal crack growth mechanisms, permanent plastic deformation,
fibre failure and static delamination growth are discussed. All these mechanisms are related to each other, and all contribute to the residual strength of the laminate.

The development of the prediction models aimed to be a step forward with respect to previous relevant prediction models available in literature. Therefore, both empirical and analytical prediction models available in literature are presented and discussed in chapter 4. A critical evaluation of those models has pointed out their limitations in applicability and versatility towards a “generic FML” concept. From this chapter, some guidelines have been defined to address the subsequent model development.

Two types of experimental activities were carried out. The first type consisted in experiments to gain understanding of the deformation behaviour of both metallic and fibre layers. Extensive use of Digital Image Correlation technique enabled to observe and measure the deformation field of both metal and fibre layers, and their interaction. Further insight into the fibre bridging mechanism and into the metal-fibre interaction was obtained. These experimental activities are discussed in chapter 5.

The second type of experimental activities aimed to generate input data for the prediction model, and to validate the CTOA approach. These are discussed in chapter 6. A large amount of experimental CTOA tests were conducted on several FML grades to evaluate the CTOA as failure criterion for FML. This included the investigation of the effect of metal sheet thickness, crack length-to-panel width ratio and the effect of bridging fibres. Static delamination growth tests were conducted to obtain the critical Strain Energy Release Rate. This parameter was subsequently used as input in the prediction model to define the critical condition for the delamination growth. Furthermore, in the same chapter, it is also discussed the complex interaction between static delamination growth and plastic deformation of the metallic layers.

The core aspect of the present thesis concerns the modelling of the residual strength failure sequence, which is presented in chapter 7. Two models are described: one for the through-the-thickness crack and one for the fatigue crack. Both models are based on the same method, which uses the CTOA as crack growth driving parameter. The method is based on the idea that crack extension in the metallic layers occurs when the calculated CTOA reaches the critical value obtained from CTOA experiments on metal laminates containing the same metal layers used in the FML. The calculated CTOA is a function of the contribution due to the far-field stress in the aluminium layers, and the contribution of the fibres. The fibre can contribute either in terms of crack opening contribution (broken fibres) or crack closing contribution (bridging fibres present in the fatigue crack configuration). Plastic deformation ahead and behind the metal crack tip is accounted and implemented into the calculation.
In addition, in the case of fatigue crack configurations, the bridging stress is calculated by solving the deformation compatibility equation, accounting for the plastic zone ahead of the crack tip and fibre failure in the bridging area. The bridging stress is subsequently used to calculate the quasi-static delamination growth occurring at the fibre-metal interface using the Strain Energy Release Rate approach.

The model for through-the-thickness crack showed a very good agreement with the experimental data, while the model for fatigue crack configuration showed sufficient agreement with experimental data. The modelling of the fatigue crack configuration presents higher degree of complexity, which required a number of simplifications and assumptions, making the model less robust than the one for through-the-thickness crack.

Chapter 8 summarises the conclusions of the investigations. It can be concluded that with the proposed models, the mechanisms related to the residual strength failure sequence are fully described and characterized. The model for through-the-thickness crack is robust and validated, and can be extended to other material and geometrical configurations. The model for fatigue crack is not robust enough, but further improvements are possible.
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>$a$</td>
<td>Half crack length</td>
<td>[mm]</td>
</tr>
<tr>
<td>$a_{\text{fibre}}$</td>
<td>Half crack in the fibre layer</td>
<td>[mm]</td>
</tr>
<tr>
<td>$\Delta a$</td>
<td>Half crack extension</td>
<td>[mm]</td>
</tr>
<tr>
<td>$\Delta a_t$</td>
<td>Half crack extension at the end of the transition phase</td>
<td>[mm]</td>
</tr>
<tr>
<td>$a_s$</td>
<td>Half saw-cut length</td>
<td>[mm]</td>
</tr>
<tr>
<td>$a_f$</td>
<td>Half fatigue crack extension</td>
<td>[mm]</td>
</tr>
<tr>
<td>$a_0$</td>
<td>Half initial notch length $(a_s+a_f)$</td>
<td>[mm]</td>
</tr>
<tr>
<td>$a_{\text{eff}}$</td>
<td>Half effective crack length</td>
<td>[mm]</td>
</tr>
<tr>
<td>$a_{\text{phy}}$</td>
<td>Half physical crack length</td>
<td>[mm]</td>
</tr>
<tr>
<td>$b$</td>
<td>Half delamination length</td>
<td>[mm]</td>
</tr>
<tr>
<td>$b_0$</td>
<td>Initial delamination length ahead of the saw-cut tip</td>
<td>[mm]</td>
</tr>
<tr>
<td>$\Delta b$</td>
<td>Half delamination extension</td>
<td>[mm]</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance for CTOA calculation</td>
<td>[GPa]</td>
</tr>
<tr>
<td>$E_{11}$</td>
<td>Longitudinal Young’s modulus</td>
<td>[GPa]</td>
</tr>
<tr>
<td>$E_{22}$</td>
<td>Transversal Young’s modulus</td>
<td>[GPa]</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Young’s modulus of the metallic layer</td>
<td>[GPa]</td>
</tr>
<tr>
<td>$E_f$</td>
<td>Young’s modulus of the fibre layer</td>
<td>[GPa]</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>Shear modulus</td>
<td>[GPa]</td>
</tr>
<tr>
<td>$G_f$</td>
<td>Shear modulus of the fibre layer</td>
<td>[GPa]</td>
</tr>
<tr>
<td>$E$</td>
<td>Strain energy release rate</td>
<td>[MPa·m]</td>
</tr>
<tr>
<td>$G_c$</td>
<td>Critical strain energy release rate for delamination</td>
<td>[MPa·m]</td>
</tr>
<tr>
<td>$J_c$</td>
<td>Critical J-integral</td>
<td>[MPa·m]</td>
</tr>
<tr>
<td>$j$</td>
<td>Number of metal/fibre interfaces</td>
<td>[-]</td>
</tr>
<tr>
<td>$K$</td>
<td>Stress Intensity Factor</td>
<td>[MPa·mm$^{1/2}$]</td>
</tr>
<tr>
<td>$K_{IC}$</td>
<td>Plane strain fracture toughness</td>
<td>[MPa·mm$^{1/2}$]</td>
</tr>
<tr>
<td>$K_C$</td>
<td>Plane stress fracture toughness</td>
<td>[MPa·mm$^{1/2}$]</td>
</tr>
<tr>
<td>$L$</td>
<td>Panel length</td>
<td>[mm]</td>
</tr>
<tr>
<td>$n_m$</td>
<td>Number of metallic layers</td>
<td>[mm]</td>
</tr>
<tr>
<td>$n_{\text{f00}}$</td>
<td>Number of layers perpendicular to the loading direction</td>
<td>[mm]</td>
</tr>
<tr>
<td>$n_{\text{f0}}$</td>
<td>Number of layers parallel to the loading direction</td>
<td>[mm]</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Point load per unit of width</td>
<td>[N/mm]</td>
</tr>
<tr>
<td>$P_k$</td>
<td>External remote step-load</td>
<td>[N]</td>
</tr>
<tr>
<td>$P_{\text{ext}}$</td>
<td>Load in correspondence of crack extension</td>
<td>[N]</td>
</tr>
</tbody>
</table>
\( \Delta P_{up} \) Load increase [N]
\( \Delta P_{down} \) Load decrease [N]
\( \Delta \text{error} \) Error in the DIC [pixels]
\( \Delta \theta \) Initial distance between grid points [pixels]
\( R \) Stress ratio [pixels]
\( r_p \) Plastic zone size [-]
\( t_{tot} \) Laminate thickness [mm]
\( t_m \) Thickness of the metallic layer [mm]
\( t_f \) Thickness of the fibre layer [mm]
\( t_{pp} \) Thickness of the prepreg [mm]
\( t_{lam} \) Thickness of the laminate [mm]
\( t_{nom} \) Nominal thickness [mm]
\( v \) Vertical displacement [mm]
\( v_f \) Vertical displacement due to fibre failure [mm]
\( v_{br} \) Vertical displacement due to fibre bridging [mm]
\( v_{eff} \) Effective vertical displacement [mm]
\( W \) Panel width [mm]
\( w_k \) Variable width of the bar-element [mm]
\( w_{fix} \) Fixed width of the bar element [mm]

\( \alpha \) Plane stress/plane strain parameter [-]
\( \sigma_{ys} \) Yield strength [MPa]
\( \sigma \) Remote stress [MPa]
\( \sigma_{ult} \) Ultimate Strength [MPa]
\( \sigma_{coh} \) Cohesive Strength [MPa]
\( \sigma_{f,far\ field} \) Far field stress in the metallic layers [MPa]
\( \sigma_{f,failure} \) Far field stress in the fibre layers [MPa]
\( \sigma_{fmetat,f} \) Fibre failure stress [MPa]
\( \sigma_{br} \) Bridging stress [MPa]
\( \sigma_{f,tot} \) Total stress in the fibre layer (far field + bridging) [MPa]
\( \sigma_{net} \) Net-section stress [MPa]
\( \sigma_{lam} \) Laminate stress [MPa]
\( \sigma_{ylam} \) Laminate Yield strength [MPa]
\( \nu \) Poisson’s modulus [-]
\( \varepsilon_{break} \) Elongation to break [%]
\( \varepsilon_{0.2} \) Yield strain [%]
\( \varepsilon_f \) Fibre elongation [%]
\( \gamma \) Shear strain [-]
\( \alpha \) Coefficient of Thermal Expansion [°C⁻¹]
\( \rho \) Density [g/cm³]
\( v \) Vertical displacement [mm]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Vertical crack flanks separation</td>
<td>[mm]</td>
</tr>
<tr>
<td>$\delta_{sc}$</td>
<td>Critical crack opening displacement for CTOA calculation</td>
<td>[mm]</td>
</tr>
<tr>
<td>$\delta_{pp}$</td>
<td>Displacement due to prepreg shear deformation</td>
<td>[mm]</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>Displacement due to fibre elongation</td>
<td>[mm]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angular parameter</td>
<td>[deg]</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Plastic zone angle</td>
<td>[deg]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Orthotropic components</td>
<td>[-]</td>
</tr>
</tbody>
</table>
Abbreviations

ARALL  Aramid Reinforced Aluminium
CARALL  Carbon Reinforced Aluminium
CentrAl  Centre Reinforced Aluminium
CFRML  Carbon Reinforced Metal Laminate
CCT  Centre Crack Tension
COD  Crack Opening Displacement of the FML
COD\(_m\)  Crack Opening Displacement of the metallic layers
COD\(_f\)  Crack Opening contribution due to fibre failure
COD\(_{br}\)  Crack Opening contribution due to bridging fibres
COD\(_{pl}\)  Elastic-plastic Crack Opening Displacement
CTOA  Crack Tip Opening Angle
CTOA\(_{0.5}\)  Critical Crack Tip Opening Angle calculated 0.5 mm behind the crack tip
CTOA\(_1\)  Critical Crack Tip Opening Angle calculated 1 mm behind the crack tip
CTOA\(_{1.5}\)  Critical Crack Tip Opening Angle calculated 1.5 mm behind the crack tip
CTOA\(_c\)  Critical Crack Tip Opening Angle
CP  Cross-ply
CTE  Coefficient of Thermal Expansion
C(T)  Compact Tension Specimen
DIC  Digital Image Correlation
EGCM  Effective Crack Gr
EPFM  Elastic-Plastic Fracture Mechanics
FCG  Fatigue Crack Growth
FEM  Finite Element Model
FEA  Finite Element Analysis
FOD  Foreign Object Damage
FML  Fibre Metal Laminate
FVF  Fibre Volume Fraction
FVF\(_{LD}\)  Fibre Volume Fraction in loading direction
ML  Metal laminate
MVF  Metal Volume Fraction
M(T)  Middle crack tension specimen
GLARE  Glass Reinforced Aluminium
IFC  Inherent Flaw Criterion
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEFM</td>
<td>Linear Elastic Fracture Mechanics</td>
</tr>
<tr>
<td>LT</td>
<td>Longitudinal Transverse</td>
</tr>
<tr>
<td>L</td>
<td>Longitudinal direction</td>
</tr>
<tr>
<td>L-T</td>
<td>Rolling direction $\parallel$ to the applied load direction, crack $\perp$ to the applied load direction</td>
</tr>
<tr>
<td>MNS</td>
<td>Modified Net Section</td>
</tr>
<tr>
<td>T-L</td>
<td>Rolling direction $\perp$ to the applied load direction, crack $\parallel$ to the applied load direction</td>
</tr>
<tr>
<td>TiGr</td>
<td>Titanium Carbon Laminate</td>
</tr>
<tr>
<td>T-FML</td>
<td>Thick FML</td>
</tr>
<tr>
<td>SERR</td>
<td>Strain Energy Release Rate</td>
</tr>
<tr>
<td>SIF</td>
<td>Stress Intensity Factor</td>
</tr>
<tr>
<td>UD</td>
<td>Unidirectional</td>
</tr>
<tr>
<td>VARTM</td>
<td>Vacuum Assisted Resin Transfer Moulding</td>
</tr>
</tbody>
</table>
Developments in materials and structural design have both contributed to improvements in the damage tolerance of modern aircraft structures. New developments in metal alloys, composite materials, and hybrid materials such as the Fibre Metal Laminates (FMLs) [1] have all resulted in structures less sensitive to damage and with slower damage growth rates. The concept of damage tolerance is a key aspect in ensuring and maintaining safety of an airframe structure over its design life. Indeed, damage tolerance can be defined as the ability of a structure to sustain sufficient levels of damage, resulting from fatigue, corrosion, and incidental sources such as impact, in a way that the damage can be detected and repaired through regular inspections before it reaches a critical level.

FMLs represent a clear example of damage tolerant hybrid materials, made by bonding thin metal sheets together with fibres embedded in epoxy. The most known FML is Glare (aluminium alloy 2024-T3 with S2-glass fibre/epoxy resin), for which an extensive investigative work [1-5] has been carried out on the fatigue crack growth and damage tolerance behaviour, such that Glare has been successfully applied in the upper section of the fuselage and in the leading edge of the vertical and horizontal tails of the Airbus A380. Thanks to its composite nature, Glare exhibits excellent fatigue and damage tolerance behaviour. Indeed, the fibres present in the material do not suffer of fatigue at the relevant fatigue loading and bridge the cracks in the metal layers thus reducing the stress intensity at the crack tip in the metal layers [2], see Figure 1.1.

The composite nature of such hybrid material raises some challenges concerning the possibility to predict the material/structure behaviour under the operational loads.
The complex mechanisms involved in Glare during fatigue loading were investigated by many researchers [2-4] in the past and are now fully understood with the development of a number of analytical and physical sound prediction models [3-6]. The main conclusion from previous research is that an FML cannot be dealt with as a monolithic material, but have to be treated as a composite were metal and fibre reinforced polymer layers interact with each other. It is known that during fatigue loading cracks nucleate and propagate in the metal layers, while delamination occurs between the metal and fibre layers [2]. In the wake of the fatigue crack the intact fibres bridge the crack restraining the crack opening in the metal layers. Thanks to that, the stress intensity at the crack tip is much lower compared to an equivalent monolithic structure, and crack growth rate reduces accordingly.

In addition to the very good fatigue behaviour, Glare laminates show also higher specific residual strength than an equivalent, in thickness, monolithic aluminium structure [7]. Indeed, also in this case the interaction between the metallic- and fibrous constituents is beneficial to obtain relative high residual strength.

The importance of the residual strength evaluation lays in the fact that airworthiness regulations require that for each part of the structure, a residual strength analysis must be performed to show that in case of damage (due to fatigue phenomena or to accidental events, such as impacts) the remaining structure is able to withstand the operational loads. The residual strength evaluation can be performed using either analytical or numerical prediction models, or by generating sufficient experimental data. As the experimental data can only be obtained by performing costly and time-
consuming residual strength tests, the manufacturers are driven to develop accurate prediction tools to decrease the amount of tests.

Much research has been carried out in the last two decades [9-13] and the complex non-linear failure mechanisms involved during the residual strength failure sequence were investigated in details. Despite the effort spent in understanding the failure mechanisms (such as metal cracking, static delamination, plastic deformation of the metal layers and fibre failure), a few number of predictive models were developed for residual strength in FMLs. Despite the composite nature of FMLs and the complexity of the failure mechanisms, many investigations have been reported where the residual strength was treated in a similar simple manner as is engineering practice for monolithic aluminium. Castrodeza and his colleagues [8, 9] have tried to link the residual strength behaviour to the standard fracture toughness properties, which are determined with C(T) specimen configurations, standardized for monolithic metals [10]. The fracture toughness test is not well defined for FMLs, unless treated as a monolithic material, which makes the approach inapplicable for the residual strength evaluation of fatigue crack configurations. Jim and Batra [11] have attempted to derive equations by simplifying the general composite Classical Laminate Theory towards a set of relations that could be solved numerically, while a model based on cohesive forces was proposed in [12-16]. Although these methods included the ductile behaviour of the metal layers in the laminate using, for example, the Dugdale strip yielding zones, their approach did not account for the bridging stress distribution and the related delamination extension. Several researchers have attempted to calculate the residual strength of FMLs using the R-curve approach, normally applied to monolithic aluminium [17]. De Vries [7] developed a model that describes the residual strength based on the R-curve, in which he correlates the R-curves derived from the various tested Glare types to the aluminium R-curve using an empirical fitting relation. The R-curve method is the current approach used by Airbus to comply with the regulations and prove that the designed Glare structure is able to operate safely without failing even in presence of severe damage. Despite understanding the significance of the R-curve for the monolithic metallic constituent, this approach does not enable extension towards a generic model for any Fibre Metal Laminate. In addition, being based on curve fitting, the R-curve method requires a number of expensive and time consuming experimental tests.

To enable the step towards a comprehensive damage tolerance assessment of FMLs, a physical sound model of the residual strength failure sequence is necessary. Based on the constituent materials, rather than on empirical curve fitting as the R-curve is, a generic prediction model for residual strength in FMLs would be beneficial during the material qualification phase. The driving idea behind this thesis is that the analytical description of the failure process of an FML under quasi-static loading can be achieved by “separating” the behaviour of each constituent, accounting for
the interaction between them. A similar approach has been already adopted for fatigue crack growth modelling of Arall by Marissen [18], and for Glare by Alderliesten [2]. In his method, Alderliesten assumed that the stress intensity factor (SIF) at the crack tip in the metal layer of an FML is the determining factor for the extension of the crack under cyclic loading. This means that the SIF should be determined as a function of all the fatigue crack propagation mechanisms occurring in FMLs (delamination, bridging stress, adhesive shear deformation and metal crack growth) that directly affect the effective stress intensity at the crack tip in the metal layers [2].

In a similar manner, the major concept of this thesis is that the crack-tip-opening angle (CTOA), which characterizes the fracturing process of the metal constituent, can be used to describe the quasi-static fracturing process of an FML containing the same metal constituent. Indeed, the CTOA is defined as the angle between the crack’s flanks measured at a specific distance. Therefore, the quasi-static crack growth of the metallic constituent can be described in terms of “critical-CTOA” vs. crack extension curve, where the “critical-CTOA” is the maximum angle measured at the moment of crack extension. It has been proven [19] that the CTOA is a material parameter independent on the specimen geometry and, despite the presence of fibres, the thin metal sheets of an FML present a CTOA vs. crack extension behaviour similar to an equivalent metal laminate, or monolithic metal with comparable thickness [20]. This means that, the CTOA approach can be implemented into an analytical prediction model for the calculation of the residual strength in FML, accounting for the interaction with the fibre layers.

The aim of this thesis is to develop an analytical model to accurately predict the residual strength of these structural materials based on the properties of the constituent materials and their interfaces. The development of the prediction model is based on the description and understanding of the main failure mechanisms, such as static delamination, metal cracking, fibre bridging and fibre failure.

Two damage scenarios are considered in this thesis, and both are subjected to quasi-static increasing load:

- Through the thickness damage (or accidental damage- represented by a saw-cut),
- Fatigue damage (represented by cracks in the metal layers only obtained by cyclic loading).

The two damage scenarios are fundamentally different from each other. An FML containing an accidental damage behaves, to some extent, similarly to an equivalent monolithic metal, although the fibres are still responsible for carrying part of the load until they fail. The case of fatigue crack presents many aspects that differentiate
the residual strength behaviour of FMLs from monolithic metals. In this case, delamination extension and fibre failure in the wake of the fatigue crack are paramount mechanisms which interact together with the plastic deformation of the metal layers.

The analytical model has been developed based on assumptions which have been supported by experimental evidence. In addition, to gain substantial experimental proof of the predictive capability of the developed model, a large number of residual strength tests were conducted on large-scale panels, covering both accidental and fatigue damage scenarios. The model has been validated considering several FML lay-ups, panel dimensions and crack lengths.

Chapter 2 introduces FMLs as family of hybrid materials focusing mainly on the manufacturing process of Glare and on its application as structural material in the aircraft industry. Chapter 3 provides a detailed description of the main failure mechanisms involved during the quasi-static failure process, pointing out the crack initiation, crack propagation, static delamination, and fibre failure. Chapter 4 reviews the most relevant methods available in literature to predict the residual strength in FMLs, focusing the discussion mainly on the assumptions, boundary conditions and limitations of such methods. Chapter 5 discusses the experimental tests that were carried out to generate the understanding of the failure mechanisms, while in chapter 6 the experimental tests necessary to produce input data, such as delamination resistance and CTOA curve, are presented and discussed. The use of the CTOA as fracture parameter is discussed in detail and an extensive experimental test campaign provides a solid justification to the use of such parameter in FMLs. Parts of the results are also discussed in this chapter.

The analytical method is presented in chapter 7, covering both accidental and fatigue damage scenarios. The final chapter discusses the conclusions and outlines the further developments. The results and validation of the analytical model are presented in appendix E.

References


1. Introduction


[16] Zerbst U, Heinimann M, Donne CD, Steglich D. Fracture and damage mechanics modelling of thin-walled structures - An overview. Integrity of


Fibre Metal Laminates

Abstract – This chapter describes the most important aspects related to fibre metal laminates and their applicability to actual aircraft structures. Description of the manufacturing process and recent developments are provided together with an overview of the current and potential applications of such hybrid materials.

2.1 Introduction

The Fibre Metal Laminate concept was developed in the late eighty’s at Delft University of Technology to improve the fatigue performance of laminated metal structure. The development of FMLs was preceded by fatigue investigations on laminates of bonded aluminum alloy sheets, which already showed better fatigue performance than an equivalent thick monolithic metallic structure. The step forward for a further improvement of the fatigue behaviour was possible by introducing fibres into the bond-line between metal layers, see Figure 2.1. Indeed, after a crack has nucleated, the fibres constrain the opening of the crack – thus reducing the stress intensity of the tip of the crack in the metallic layers. This induces a decrease in the crack growth rate. The first FML introduced was ARALL (aluminium layers with aramid fibres), subsequently replaced by Glare (aluminium layers with glass fibres). Although other variants of FMLs were developed [1] (CARALL, TiGr), Glare has been the most successful for application in aircraft fuselage panels. Since the manufacturing process for FML is still rather expensive, the effort is currently focused on potential out-of-autoclave process, which would drastically reduce the manufacturing costs.
FMLs were developed two decades ago mainly for fuselage skin applications, but research to explore the applicability of the FML concept for wing skin panels started already in 2005-2007 and is still going on [2-4]. The present thesis is mainly focused on standard Glare and, to some extends, on the variant containing carbon fibre (CARALL). Nevertheless, the concepts developed in this thesis are potentially applicable to other variants of FMLs.

![Example FML lay-up](image)

2.2 Manufacturing

At the current state the standard manufacturing process of FMLs requires the use of an autoclave [1], although research is being carried out on the development of the out-of-autoclave process [25-27]. For standard Glare, the typical metal sheet thickness range is 0.3-0.5 mm, while the nominal thickness of the S2-glass/FM94-epoxy prepreg is 0.133 mm (single layer). To ensure a proper bond quality, the aluminium sheets are subjected to a surface pre-treatment consisting of de-greasing and chromic acid (or phosphoric acid) anodizing, followed by priming with BR-127 corrosion inhibiting bond primer. The aluminium and prepreg layers are bonded together in an autoclave curing cycle where temperature and pressure follow specific gradients during the whole process. For standard Glare, the bonding between aluminium and prepreg consolidates at 120 °C at maximum pressure of 6 bar [2]. A general overview of the most important mechanical characteristics of Glare constituents are provided in Table 2.1.
2.2.1 Post-curing effects

At the end of the curing process, assuming rigid bond between aluminium and prepreg, residual tensile stresses are present in the aluminium layers and compressive residual stresses in the prepreg layers. This results from the different coefficients of thermal expansion of aluminium (2.2 $\times 10^{-6}$ C°$^{-1}$) and S2-glass fibre (6.1 $\times 10^{-6}$ C°$^{-1}$ in L direction and 2.6 $\times 10^{-6}$ C°$^{-1}$ in LT direction). These tensile residual stresses are responsible for the shorter fatigue initiation life in Glare laminates compared to equivalent monolithic aluminium [3], because the tensile residual stress in the aluminium layers superimpose to the far-field stress, thus increasing the actual stress in the metal layers. The residual stress influences also the delamination process, being this one dependent on the amount of load transferred through the metal/fibre interface [4, 5]. The negative effect of residual tensile stresses in the metal layers is negligible for the case of quasi-static increasing load. Indeed, the large plastic strains which mainly develop around the crack tip redistribute the stresses through the thickness among the constituents, changing the stress system established after curing. Post-stretching Glare laminates is also a method to reverse this unfavourable residual stress system in FMLs [1, 6].

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbol</th>
<th>UD S2 Glass / FM94</th>
<th>AA2024-T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus [GPa]</td>
<td>$E_{11}$</td>
<td>48.9</td>
<td>72.4</td>
</tr>
<tr>
<td></td>
<td>$E_{22}$</td>
<td>5.5</td>
<td>72.4</td>
</tr>
<tr>
<td>Shear modulus [GPa]</td>
<td>$G_{12}$</td>
<td>5.55</td>
<td>27.6</td>
</tr>
<tr>
<td>Yield strength [GPa]</td>
<td>$\sigma_{y}$</td>
<td>-</td>
<td>345</td>
</tr>
<tr>
<td>Ultimate strength [MPa]</td>
<td>$\sigma_{ult}$</td>
<td>2640</td>
<td>470</td>
</tr>
<tr>
<td>Poisson’s modulus [-]</td>
<td>$\nu$</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>Elongation at break [%]</td>
<td>$\varepsilon_{break}$</td>
<td>4.5</td>
<td>15±18</td>
</tr>
<tr>
<td>CTE [-]</td>
<td>$\alpha$</td>
<td>6.1 $\times 10^{-6}$</td>
<td>22 $\times 10^{-6}$</td>
</tr>
<tr>
<td>Density [g/cm$^3$]</td>
<td>$\rho$</td>
<td>1.98</td>
<td>2.78</td>
</tr>
</tbody>
</table>

2.3 Current developments in manufacturing

Recently, exploratory investigations have focused on the development of a Vacuum Assisted Resin Transfer Moulding (VARTM) as an alternative out-of-autoclave process [8, 9]. The main idea behind this process is to create pathways for the resin to be infused and homogenously distributed. Two methods providing for through the thickness infusion were studied:

- Insertion of resin flow pathways by drilling micro-holes in either the metal sheets and prepreg layers
- Utilizing a metal deposited layer with porosity.
To be effective for FMLs, the VARTM method requires the creation of pathways for the resin, and this inevitably would complicate the manufacturing process and create overall distributed stress concentrations. Although the potential cost benefit of VARTM process are interesting, the current process does not provide FMLs with certified quality for application as structural material for airframes.

Recently, promising results have been obtained during preliminary investigations [10] on the double-vacuum technique as an out-of-autoclave process, alternative to the more expensive in-autoclave process. The idea behind this alternative process is to perform the curing cycle inside an oven with the double-vacuum technique. Fatigue crack growth tests of samples manufactured in this way showed similar results of Glare panels manufactured in-autoclave. This process, when fully developed, might potentially reduce the production cost of FMLs.

For what concerns safety and environmental control, the new OSHA PEL [11] for Cr+6 and growing restrictions on the use of chromates around the world are driving manufacturers to find less harmful alternatives. Chromium-based pre-treatments are among the most efficient and successful systems for aluminium and its alloys. However, due to their carcinogenic nature (Cr+6), toxicity hazards, and associated costs, chromium- based compounds pose serious challenges in their usage and are a target for replacement with alternative processes [12].

2.4 Material definition

The constituents of FMLs can be configured in various ways to maximize the laminates performance for a given application (e.g. fatigue, strength, impact, shear). Table 2.2 lists the most common available Glare grades. The prepreg layers in Glare 2, 4 and 5 are stacked symmetrically. The prepreg layers in Glare 3 are stacked so that the fibres in the layer closest to the outer aluminium layer are orientated in the aluminium rolling direction. The rolling direction of the aluminium is defined as 0°, and the transverse rolling direction is defined as 90°.

So far only Glare has been certified, for which a clear coding system was defined to identify the grade and lay-up. For example, the lay-up for Glare 3-3/2-0.4 and Glare 2-3/2-0.3 is coded as follow:

Glare 3-3/2-0.4  [2024-T3 / 0°glass / 90°glass / 2024-T3 / 90°glass / 0°glass / 2024-T3]
Glare 2-3/2-0.4  [2024-T3 / 0°glass / 0°glass / 2024-T3 / 0°glass / 0°glass / 2024-T3]
Table 2.2 Standard Glare grades [1]

<table>
<thead>
<tr>
<th>grade</th>
<th>Sub</th>
<th>Prepreg orientation</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glare 1</td>
<td>-</td>
<td>0/0</td>
<td>fatigue, strength, yield stress</td>
</tr>
<tr>
<td>Glare 2</td>
<td>Glare 2A</td>
<td>0/0</td>
<td>fatigue, strength</td>
</tr>
<tr>
<td></td>
<td>Glare 2B</td>
<td>90/90</td>
<td></td>
</tr>
<tr>
<td>Glare 3</td>
<td>-</td>
<td>0/90</td>
<td>fatigue, impact</td>
</tr>
<tr>
<td>Glare 4</td>
<td>Glare 4A</td>
<td>0/90/0</td>
<td>fatigue, strength in 0° direction</td>
</tr>
<tr>
<td></td>
<td>Glare 4B</td>
<td>90/0/90</td>
<td>fatigue, strength in 90° direction</td>
</tr>
<tr>
<td>Glare 5</td>
<td>-</td>
<td>0/90/90/0</td>
<td>impact</td>
</tr>
<tr>
<td>Glare 6</td>
<td>Glare 6A</td>
<td>+45/-45</td>
<td>shear, off-axis properties</td>
</tr>
<tr>
<td></td>
<td>Glare 6B</td>
<td>-45/+45</td>
<td></td>
</tr>
</tbody>
</table>

For what concerns experimental testing, it is normally adopted the nomenclature L-T and T-L to indicate respectively if the rolling direction of the aluminium sheets is parallel (L-T) or perpendicular (T-L) to the loading direction. Figure 2.2 shows an example of material designation for laboratory experiments purpose. For example, with respect to Figure 2.2, the double amount of fibres in loading direction of panel 2 increases the residual strength of about 14% respect to panel 1, which contains only one ply of fibre in loading direction [13, 14]. In the present thesis the mentioned terminology will be used.

Figure 2.2  Example of Glare 4 designation for laboratory testing
The Residual Strength Failure Sequence in Fibre Metal Laminates

2.5 Structural application of FMLs and damage scenarios

The aircraft industry is very conservative in the adoption of new designs and technologies. Significant risks and low profit margins provide little incentive to change. Even when new aircraft are introduced, they tend to build heavily upon past designs, introducing only incremental updates in technology [1,13,15]. Significant changes can occur, but the process to implement and optimize them into actual structures may be very slow and associated risks are high.

Although FMLs are composite materials, the material properties and the way they are machined and implemented in real structures are very similar to bulk monolithic metal sheets. They have far less in common with composite structures concerning design, manufacture, inspection or maintenance. These aspects facilitated the introduction of FMLs as structural materials for aircraft application.

Although it is possible to bond together several types of metal (aluminium, titanium, steel) and fibres (glass fibre, carbon fibre, aramid fibre, Zylon, etc.) not all combinations make sense for actual structural applications. As known the best metal/fibre combination for aircraft fuselage skin resulted in the creation of GLARE (aluminium with S2-glass fibres). Parts made of Glare are constructed and repaired using mostly conventional metal material techniques. These aspects configure Glare as a perfect candidate for applications in those parts of an airplane where high fatigue resistance and high damage tolerance are required, such as fuselage panels, leading edges and lower wing skin. In addition Glare shows better impact resistance, corrosion resistance and flame resistance than monolithic aluminium alloys [1,34,35]. Glare has been certified as a metal considering the following characteristics:

- Ductility (energy absorption and predictable strength degradation)
- Oxidation (relates to the metal used)
- Fatigue (relates to the metal used)
- Electrical conductivity (similar to metals)
- Machining (similar to metals, but not equal)
- Environmental aspects

Glare and FMLs in general can be tailored for specific applications, providing an optimized structure. For example, the top part of the A380 fuselage is made with different Glare grades panels, and the location of a specific Glare grade depends on specific fatigue, damage tolerance and stiffness requirements. The geometrical continuity between Glare panels with different lay-ups is guaranteed by means of splices at the transition area [1, 15].
Figure 2.3 illustrates a schematic example of hypothetical application of Glare 4B panels in the front and rear sections of an airplane fuselage. A typical fuselage section is illustrated at the centre of the figure, here two damage scenarios are represented: a two-bay longitudinal crack and a two-bay circumferential crack. Considering the pressurization loads only, it is known that the hoop stress is about two times larger than the longitudinal stress. For this reason, higher strength in circumferential direction might be obtained using Glare 4 with double amount of fibres in circumferential direction (green lines). To compensate for the extra longitudinal stresses in the fuselage skin induced by bending forces generated by the horizontal tail planes and payload weight, the rolling direction of the aluminium sheets (blue arrows) of Glare panels can be oriented in longitudinal direction. This is just an example, but the important message is that the composite nature of FMLs enables a certain degree of freedom to exploit the benefit of both metallic and fibre constituents.

Airworthiness regulations (EASA 25.571) require that for a primary structure (e.g. fuselage, wings, tails, etc.) residual strength evaluations are performed to show that, in case of damage, the remaining structure is able to withstand the operational loads. The strength evaluation of fuselage skin panels made of Glare is often performed by residual strength tests on large flat Glare panels containing a saw-cut, as illustrated in Figure 2.3. This is a reasonable representation of a trough-the-thickness Foreign Object Damage (FOD) [14]. As the experimental data can only be obtained by performing large and costly residual strength tests, the manufacturers are driven to develop accurate prediction tools to decrease the amount of tests. Prediction tools can provide design support during the material and structure development phase, with subsequent cost reduction and design optimization.

Glare sheets were primarily developed as sheet material for pressurized transport aircraft. However, the tension skin of a wing structure may also suffer from fatigue. The design step to move from fuselage structures to semi-monocoque wing structures is not a trivial problem. Heavily loaded wing structures, such as those found on a cargo or passenger aircraft, must withstand a much higher load intensity than fuselage structures, thereby necessitating much thicker skins. Standard fuselage skins are generally one to two millimetres in thickness (or possibly 5mm for local reinforcements), which for a standard FML grade would equate to 3-5 thin (0.3-0.5mm thick aluminium sheets for GLARE) layers of aluminium and 2-4 layers of fibreglass prepreg. Conversely, wing skins for large passenger or cargo aircraft may be 20 mm or thicker in the root area. If manufactured from standard FML grades, it would require 20 or more metal layers, with the associated additional cost for manufacturing. These manufacturing and cost related issues have led the development of potential alternative solutions such as CentrAl [16] and T-FMLs [17]. CentrAl combines Glare sheets with thicker monolithic aluminium plates. The
Glare sheet is bonded internally to two or more aluminium plates using a low fibre density epoxy adhesive denoted as Bond-preg® [16]. Fatigue crack growth tests of CentrAl specimens showed lower fatigue crack growth rate compared to equivalent thick monolithic aluminium plates.

Parallel to the CentrAl development, the T-FML (Thick FML) solution for lower wing skin application of FMLs is being explored. This new variant is made by bonding thicker aluminium sheets (with a thickness ranging from 0.6 mm to 1 mm) to S2-glass fibre/epoxy prepreg. Fibres can be oriented both in wing-span direction, to maximise the bending stiffness and strength, and in off-axis direction (e.g. ± 45°) to provide stiffness and strength in planar shear. The application of metal sheets in this considered thickness range of 0.6-1.0 mm requires additional considerations compared to the thickness range used in current FML applications. These considerations, including variations in material performance, manufacturing methods, design approaches, and structural performance are discussed in [17]. Figure 2.4 illustrates the schematic cross-section of both CentrAl and T-FML.

Even if concepts and analytical models discussed in the present thesis are mainly addressed to standard FML configurations, and in particular Glare, their extension towards either CentrAl or T-FMLs might be possible.

![Figure 2.3 Example of Glare application in the A380 skin fuselage. During the material qualification phase, flat Glare panels were tested for strength justification of that specific location in the fuselage.](image)
2. Fibre Metal Laminates

![Cross-section of CentrAl and T-FML configurations](image)

**CentrAl**

**T-FML**

*Figure 2.4 Cross-section of CentrAl and T-FML configurations*

## References


Fracture mechanisms in FMLs

Abstract — This chapter provides a discussion about the most relevant failure mechanisms involved during the residual strength failure sequence. Mechanisms such as crack initiation and propagation, static delamination extension and fibre bridging are here discussed. The most important understandings in the view of the analytical modelling of the residual strength failure sequence are pointed out.

3.1 Introduction

The main source of damage in the service life of current flying aircraft is fatigue, followed by corrosion- and impact [1]. Regardless its nature, the presence of damage induces a reduction of the static strength of the structure, if compared to the undamaged structure. For example, the strength of a structure containing a fatigue crack is substantially lower than the strength of the undamaged structure. The residual static strength is defined as the amount of static strength available at any time during the service period considering that damage grows as a function of service loads and time [2]. To satisfy safety requirements and to ensure sufficient aircraft service performance, the above mentioned damage sources must be considered during the design phase. Therefore, for example, predicting the fatigue crack initiation and growth, and the related reduction in strength (residual strength) is a key aspect in ensuring and maintaining safety of an airframe over its design life. This is possible only if the failure processes and the related mechanisms which cause the reduction of static strength are known, well understood, and correctly addressed in terms of failure criteria. The knowledge generated from this understanding can be subsequently used to develop predictive models for the crack growth in real structures.
The quasi-static fracture process in ductile metals, such as aluminium alloys is normally characterized by the development of a certain amount of plastic deformation which precedes crack extension. From a macroscopic point of view, the fracture process can be schematically divided in four phases: crack-tip blunting accompanied by large plastic deformations, crack initiation, stable and unstable crack extension. The metallic constituent of an FML shows the same fracture mechanisms present in monolithic metals, while the composite constituent, fibre/epoxy prepreg, behaves according to the typical composite characteristics, showing an elastic behaviour until failure [2-4]. Metal and fibre/prepreg layers are bonded together sharing an interface which can be subjected to delamination when the critical shear stress is exceeded locally. This phenomenon is called “static delamination” which differs from the “fatigue delamination” [5].

The assumption that crack initiation and crack growth can be described with Linear Elastic Fracture Mechanics (LEFM) applies mainly to fatigue crack propagation [5, 6]. In the case of increasing load, the large amount of plastic deformation that develops during the failure process dictates the use of Elastic-Plastic Fracture Mechanics (EPFM) [7]. Among the available fracture mechanics descriptions, the Crack-Tip-Opening Angle (CTOA) is one of the suitable approaches for monolithic metals, enabling an elastic-plastic fracture mechanism description [6-9]. This approach is the core aspect of the present thesis and is described in details in chapter 6 and its implementation into the analytical model is presented in chapter 7.

The following sections describe in details the dominant fracture mechanisms present in FMLs during quasi-static loading conditions, and highlight the differences when both fatigue cracks and saw-cuts are subjected to an increasing load. The understanding of the fracture mechanisms is paramount to properly describe the whole failure process and to generate proper assumptions and simplifications for the development of the prediction model.

### 3.2 Crack initiation and crack-tip blunting

The term crack initiation defines that particular phase of the failure process where a new cracked surface forms from an existing damage or geometrical notch within the material or structure. Crack initiation in monolithic metals containing a notch and subjected to a tensile load is normally preceded by a large amount of plastic deformation around the tip of the initial notch. Plastic deformation is often accompanied by crack-tip blunting and both mechanisms precede the actual crack formation.
Crack initiation in FMLs is similar to monolithic metals, although the deformation field may be influenced by the presence of fibres [8], and the amount of crack-tip blunting depends on the ductility of the metal constituent and thus on the yield strength. The crack-tip blunting is accompanied by large deformation ahead of the crack tip, such that a plastic zone is created. In monolithic metals, the shape and size of the plastic zone depends mainly on the ductility, while in FMLs the shape of the plastic zone is influenced also by the presence of fibres. Indeed, the typical “butterfly wing” shape of the plastic zone, that represents the area of maximum shear deformation, has a specific inclination in a monolithic metals, while in a FML containing the same metallic constituent the shape and size of the plastic zone is altered. This aspect is discussed in detail in chapter 5 of this thesis.

Figure 3.1 illustrates an example of crack tip blunting (white arrows) together with plastic deformation (black arrows) occurring in a crack initially obtained with cyclic loading, and subsequently subjected to increasing load. Both are visible ahead of the crack tip. In the sequence from A to D the increasing opening of the fatigue crack due to the increasing load can be observed. Figure 3.1-D shows the subsequent creation of a stable tear from the tip of the fatigue crack.

After the crack has initiated, it extends for few tens of millimetres following one of two branches of the plastic zone, then the extension proceeds mainly perpendicular to the applied load. The initiation phase is therefore completed and the propagation phase takes place.
3.3 Crack propagation

After some millimetres of crack extension, the mechanisms change. The blunting reduces drastically, while the plastic zone keeps expanding according to the increasing load and increasing crack length. In absence of delamination at the metal/fibre interface, the crack propagation in the metal layers is always accompanied by fibre failure [4]. Since plastic deformation develops mainly in front of the crack tip, the crack propagates into a highly deformed material, thus residual plastic deformations are present in the wake of the propagating crack. During propagation, complex stress re-distribution mechanisms take place between the yielded metal and the intact fibres underneath. Examples of features typical of the crack propagation phase are illustrated in Figure 3.2.

Like monolithic metals, also the thin metallic sheets in an FML show the typical features of a statically extended crack. As for monolithic aluminium alloys [9], the fractured surfaces of thin metal sheets of an FML are characterized by three main regions:

- Flat surface (fatigue crack propagation)
- Flat-to-slant transition (quasi-static propagation)
- Slant surface (quasi-static propagation)

These features are visible in Figure 3.3 as result of fractographic analysis on Glare panels subjected to increasing load until failure. The observed changes in fracture features seem to be consistent with what is reported in [13,14] where the authors stated that in monolithic aluminium alloys the extension of the “flat” stable tearing area, which follows the fatigue crack, is thickness dependent, and reduces by reducing the thickness. In the case of FMLs, the fracture surface characterized by the “flat” surface is not present or is reduced to a very small portion. This is due to the small thickness of the metal layers (0.2-0.5 mm), which reduces the constrains in thickness direction and enables to establish almost immediately a plane stress state.

In Figure 3.3 the transition area is marked with black arrows and extends for a distance equivalent to the thickness of the metal sheet; after the transition, the crack slants and the characteristic shear lips are formed (visible also in Figure 3.2).

Understanding both crack initiation and propagation in FMLs is important to relate these failure mechanisms to the final failure of the laminate. Chapter 6 of this thesis provides an extensive discussion on the relationship between crack initiation/propagation and the CTOA. The justification for choosing the CTOA as fracture parameter for FMLs, and its implementation into the prediction model is described in chapter 6 and 7 respectively.
3. Fracture Mechanisms in FMLs

Figure 3.2  Sequence of crack propagation in Glare3-3/2-0.4 under quasi-static increasing load. The residual plastic deformation is visible in the wake of the crack (black arrows) together with the shear lips (white arrows).

Figure 3.3  SEM photographs of the fracture surface of Glare 3-3/2-0.4 containing a fatigue crack of 1 mm [9]
3.4 Static delamination

Understanding the delamination mechanism is very important to assess the failure sequence in FMLs, indeed delamination can occur in different ways and under different loading conditions. For example, during fatigue crack growth, fatigue delamination occurs at the prepreg/metal interface as result of the load transferred via shear stress from the cracked metal into the bridging fibres; this happens due to the cyclic shear stress generated at the interface. The resistance to delamination growth under fatigue loading is therefore related to the applied cyclic load and to the fatigue delamination resistance of the interface [5, 6].

On the other hand, during the residual strength failure sequence, delamination occurs as a quasi-static process (static delamination) where the high applied load generates high shear stress at the fibre metal interface. If the shear stress reaches the critical value, delamination occurs. In both cases (fatigue delamination and static delamination), the disbonding process takes place at the interface between metal and prepreg without fibre failure.

The resistance to static delamination is, as for the fatigue case, related to the characteristics of the interface. Another important aspect to point out is the effect of plastic deformation of the metal layers; indeed, due to the high applied load the metal layers may develop plastic deformation at the boundary of the delamination (where metal and fibre are still intact). This causes a delay in the delamination growth because part of the energy that would be introduced at the interface in the form of shear stress is dissipated in the form of permanent plastic deformation. This aspect is discussed in detail in chapter 6 and 7.

The delamination process is a complex mechanism influenced by many parameters, such as: thickness and stiffness of the individual layers, lay-up, fibre orientation, and applied load. In addition, given the just mentioned parameters, the delamination resistance depends on the type of failure occurring at the interface. Indeed, it is known that in general cohesive failure (failure in the epoxy rich layer of the prepreg) depends only on the type of resin, while the adhesive failure (failure at the either metal/epoxy or fibre/epoxy interfaces) depends on the surface pre-treatments of both interfaces. Therefore, It is general practice to have a surface treatment such that adhesive failure is less critical, so that the energy required for adhesive failure relative to cohesive failure is effectively increased. The transition between cohesive and adhesive debonding is strongly affected by the surface pre-treatment of the metallic constituent [2, 3], and the surface treatment of the fibres (sizing). Indeed, the standard metal surface pre-treatment, consisting of de-greasing, chromium anodising and priming, induces a high delamination resistance as result of a cohesive-type of debonding [2,16,17]. On the other hand, a poor surface pre-
treatment results in an adhesive-type of debonding, this reduces the delamination resistance and facilitates the occurrence of static delamination even ahead of the crack tip [2].

Vermeeren [3] observed that static delamination in front of the crack tip occurs when the pre-treatment of the aluminium sheets consists of degreasing and pickling only. He concluded that no static delamination occurs in residual strength tests on standard material. The latter statement may be questionable because further researches performed by de Vries [2] showed that some static delamination may occur in front of the crack tip also in standardized materials. In [2] the author pointed out the beneficial effect of the delamination on the residual strength obtained with different metal surface pre-treatments. Indeed, the larger the delamination extension the lower the probability of fibre failure. If delaminated, the fibres are de-attached from the metal layers and are free to strain along a longer length, thus the fibre stress reduces delaying the occurrence of fibre failure [2, 3, 5].

Further research on this topic has been carried out by Lawcock et al. [10] who investigated the influence of the metal pre-treatment on the mechanical properties in carbon fibre reinforce laminates (CFRMLs) specimens. They induced differences in the interfacial adhesion between the aluminium alloy and the composite layer by using different methods of surface preparation. They concluded that no substantial differences can be found in the laminates’ in-plane mechanical properties such as strength and stiffness, while differences in residual strength have been observed between the specimens with different pre-treatments, due to the different delamination resistance.

An experimental approach [4] to observe in detail the delamination phenomenon in Glare panels consists of removing the outer aluminium layer by chemical etching. I properly done, this can reveal the fibre/epoxy substrate. Alternatively, “inverted lay-up” [11] can be manufactured where the prepreg layers are bonded directly on the outer metal layers. This enables the observation of the delamination directly on the fibre layers.

3.4.1 Static delamination at the boundary of the fatigue delamination

Fatigue cracks are characterized by the presence of bridging fibres which act as second load path. The load from the cracked metal layers is transferred into the fibres via shear stresses generated at the metal/fibre interface. Therefore, an existing fatigue delamination can further extend statically if the increasing load that is transferred through the interface generates enough shear stress to reach the critical shear stress of the interface. As mentioned in the previous section, a way to directly observe the delamination growth during a test is to use an “inverted lay-up” where
the fibre/epoxy layers are bonded externally to the aluminium layers enabling a direct observation of the failure mechanisms at the metal/fibre interface. When delamination occurs the prepreg layer separates from the aluminium sheet letting the air come in-between, this changes the colour enabling a direct observation.

An example of static delamination (light shade) extended from an existing fatigue delamination (dark shade) is provided in Figure 3.4. Static delamination is in general a mechanism where the remotely applied energy is locally dissipated by creating a crack along the bond-line. The elastic Strain Energy Release Rate (SERR) is a common fracture mechanics approach adopted to describe the delamination process [5-7]. If plastic deformation occurs the SERR can be still applied but its analytical formulation must be based on an elastic-plastic approach, rather than an elastic approach. This aspect is discussed in detail in chapter 7 and in appendix D.

3.4.2 Static delamination ahead of the crack tip

Static delamination can occur also ahead of the crack tip. Vermeeren [3] and de Vries [2] both refer to the static delamination ahead of the crack tip as a disbonding mechanism caused by the superposition of shear stresses due to the crack tip blunting and due to the plastic deformation.

![Figure 3.4 Example of fatigue delamination (dark shade) extended statically (light shade) due to quasi-static increasing load. Inverted laminate with standard surface pre-treatment.](image)

The occurrence of static delamination ahead of the crack tip might be beneficial for the residual strength of the material being considered. Indeed, if static delamination occurs, the fibres can strain along the delamination length, independently from the
metal deformation. This would reduce to some extent the strain in the fibres, thus delaying fibre failure.

Vermeeren [3] stated that static delamination ahead of the crack-tip is hardly observable in Glare 3, even if the pre-treatment of the aluminium was pickled only. On the other hand, Glare 2 laminates may show higher tendency to develop static delamination ahead of the crack tip. Apparently the lower shear stiffness of the prepreg layers, in the case of cross ply configuration (Glare 3), prevents the creation of sufficiently high shear stresses to initiate the static delamination.

A practical way to determine whether or not static delamination has occurred is the presence of protruding fibres after the final failure of the sample. Delaminated fibres can strain independently form the metal layers and can elongate more than the undelaminated fibres; fibre failure occurs at the boundary of the delamination, where the stress concentration is higher resulting then in protruded fibres after final failure.

As illustrated in Figure 3.4, the “inverted lay-up” approach can be used to observe the delamination during a test. In the example provided in Figure 3.5 the delamination ahead of the crack tip is visible (the black arrow indicates the disbonded area in front of the fatigue crack tip). As for the case of Figure 3.4, the already existing fatigue delamination extended statically under increasing load. The difference between the examples provided in Figure 3.4 and Figure 3.5 is the different pre-treatment used for the surface of the metal. In Figure 3.4 the standard pre-treatment was used, this provided higher delamination resistance; indeed no delamination ahead of the crack tip is visible. In the case of Figure 3.5, the pre-treatment consisted only in de-greasing and sandblasting; the resulting delamination resistance is lower than for the case of a standard pre-treatment.

![Figure 3.5](image.png)

*Figure 3.5* Example of static delamination ahead the crack tip due to quasi-static increasing load. Inverted laminate with only de-greasing and sandblasting as surface pre-treatment.
3.5 Dynamic delamination and fibre failure

Dynamic delamination is a mechanism related to fibre failure. When the strain in the fibres reaches the strain to failure, the fibres break and the energy is released partially into the rest of the intact material and partially into the metal/fibre interface.

Some authors have reported studies related to the blunt notch resistance of FMLs [12] and hybrid-FMLs [13], where they performed post-mortem analysis based on visual observation of the dynamic delamination. The authors concluded that observation of the dynamic delamination might provide qualitative information regarding the relation between delamination resistance and surface pre-treatment. Indeed, based on the amount of dynamic delamination observed after the test, it is possible to compare different surface pre-treatments in terms of delamination resistance. This makes sense as long as the comparison is based only on the surface pre-treatment. Indeed, if a stiffer prepreg system is used (e.g. same epoxy resin, but with stiffer fibres), the amount of energy released after fibre failure is higher and therefore the dynamic delamination will be larger as well. This observation would not provide an indication of the actual static delamination resistance, but simply a qualitative observation of the effect of fibre failure.

Dynamic delamination as failure mechanism does not have much importance in assessing the damaging sequence in FMLs because it is just the consequence of the more important fibre failure.

3.6 Bridging stress

During fatigue crack propagation, delamination between metal and fibre layers occurs. In presence of a crack, fibres have to match the crack opening therefore they are subjected to relatively high strains. In absence of delamination the fibre must elongate and match the crack opening displacement, the strains in the fibres would be so high that fibre failure would occur immediately [5, 14]. Thanks to delamination, fibres can strain over a longer length so they are capable to absorb the crack opening displacement and distribute the strain along a longer length (delamination length). This is beneficial to keep the stress in the fibres below the failure stress.

Under fatigue loading, the fibre bridging acts as a second load path and constrains the crack opening in the metal layers, reducing the crack growth rate. The bridging mechanism has been studied and fully understood for the fatigue crack growth case and although it is not easy to measure the bridging stress directly during the experiments, it is however possible to calculate it analytically [5].
In general, the shape of the bridging stress distribution is strictly related to the shape and size of the delaminated area, size of the initial notch and stiffness of the prepreg [5]. As illustrated in Figure 3.6, for fatigue cracks growing under constant amplitude load, the bridging stress distribution does not increase much and maintains a quasi-constant value in the wake of the crack. On the other hand, in the case of quasi-static increasing load applied to a fatigue crack, the bridging stress increases according to both the applied load and the current delamination size and shape [11]. Also in this case delamination can occur if the critical shear stress of the fibre/metal interface is reached locally.

In general, the bridging effect is a phenomenon common in both fatigue crack growth and static crack growth. The main difference between the two cases is the amount energy related to the bridging mechanism and the subsequent delamination growth (either due to cyclic load or increasing load). Prediction models for both fatigue crack growth and static crack growth cannot be developed without the understanding of the fibre bridging phenomenon.

![Figure 3.6](image)

*Figure 3.6* Bridging stress derived from the experimentally measured strain on the external prepreg along three fatigue cracks in a 2024-T3 /S2-glass inverted laminate [11]. Results are compared with analytical calculations based on the analytical model developed in [5].
3.7 Conclusions

The topics discussed in this chapter are fundamental to understand the complex mechanisms occurring during the residual strength failure sequence.

Crack initiation and subsequent crack propagation are the main mechanisms characterizing the failure in the metal layers. Due to the relative high loads applied, permanent plastic deformation occurs ahead of the crack tip which enables a redistribution of the load paths. When the crack extends residual plastic deformation is left in the wake of the crack, this is an important aspect to take into account for predictive modelling.

Delamination extension is one of the most important parameter that determines the bridging stress, it generates at the metal/fibre interface as results of the load transfer via shear stress. A larger delamination reduces the bridging stress, while a smaller delamination increases the bridging stress; it is therefore important to understand how the delamination generates and what is its influence on the failure process.

Saw-cut and fatigue crack configurations are very dissimilar from each other in terms of residual strength. In an FML panel containing a saw-cut crack initiation, stable crack growth and unstable crack growth are similar to a monolithic metal panel, while the same FML containing a fatigue crack behaves differently. Higher crack initiation stress and shorter stable crack growth are characteristic for configuration containing bridging fibres.

The analytical model described in Chapter 7 is based on a number of assumptions and simplifications based on the understanding of the actual failure mechanism of FMLs.

References


3. Fracture Mechanisms in FMLs


The Residual strength in FML: approaches from literature

Abstract — This chapter discusses the most relevant modelling approaches for residual strength in Fibre Metal Laminates available in the literature. The approaches are discussed with respect to their applicability on residual strength in FMLs, and categorized in empirical, phenomenological, analytical and Finite Element methods.

4.1 Introduction

Since the development of FMLs, several authors [1-15] have developed and published their research on residual strength of FMLs, especially for Glare and Arall. Within The Netherlands large research programs were conducted during the last two decades with the objective to generate enough experimental data to support the residual strength justification for structures made of Glare [2, 3, 16, 17].

Although experimental studies on the residual strength of FMLs have been intensively carried out, very few models have been specially developed to evaluate the fracture behaviour of these laminates, including analyses based on finite elements. Prasad Kadiyala et al.[18] showed that the behaviour of notched ARALL laminates cannot be described simply by a linear elastic fracture analysis, but rather requires an elastic-plastic approach. Macheret et al.[9] suggested that the residual strength can be determined by a modified net section (MNS) criterion for laminates with small ratio of notch size to specimen width. Successively, an R-curve approach based on the concepts of fracture mechanics was proposed to evaluate the residual strength of ARALL laminates [19]. Macheret and Teply [10] showed that the
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The inherent flaw criterion (IFC) [20] can be modified to predict the residual tensile strength of ARALL laminates on the basis of the tensile strength of composite layers. Rijn [21] has reviewed the applicability of various models of polymer composite laminates for FMLs.

In the biennium 1996-97, a new model was proposed by Afaghi-Khatibi et al.[22-25]. This model, called Effective Crack Growth Model (ECGM), is based on the Damage Zone Criterion [26] and was initially developed for the prediction of residual strength for composite laminates. Afterwards it was modified towards FMLs applications. One of the most important methods for evaluating the residual strength of FMLs, proposed by Vermeeren [27], and afterwards reprised by de Vries [2], is a method based on the R-Curve. Jin et al. [28] proposed a prediction model assuming that both a Dugdale strip yielding zone in the aluminium layers and a strip damage zone in the fibre composite layer are developed concurrently at the crack tip.

The most important prediction models available in literature are selected and presented in this chapter. Discussions are provided based on a critical review of the methods, pointing out merits and limitations of the available models to provide a reference framework for the research described in chapters 5, 6 and 7.

4.2 Effective Crack Growth model (Afaghi-Khatibi et al.)

The EFCGM [22-25] has been developed in 1996 to predict the residual strength of composite laminates with both circular holes and sharp cracks. The authors thought that the same approach could be extended towards FMLs implementing the metal layers contribution. In this approach, the damage is simulated by a fictitious crack with cohesive stresses acting on its surfaces, and damage development is modelled for both metal and composite layers separately.

4.2.1 Damage initiation in both metallic and composite layers

In the metallic layer, damage is assumed to initiate when the local normal tensile stress at the edge of the hole, or crack tip, reaches the yield strength of the metal $\sigma_y^M$. The extension of the plastic zone ahead of the notch is modelled using cohesive stresses, $\sigma_{coh}^M$, acting on crack surfaces. Cohesive stress profile is correlated with the crack opening displacement, $v(x)$, using the apparent fracture energy, $G^M$. Indeed, the fracture energy of the metal $G_c^M$ can be used to define the critical crack opening, $v_c^M$, associated with the yield strength, $\sigma_y^M$. When the calculated crack opening displacement reaches the critical value, $v_c^M$, the crack extends and propagates stably.
A similar approach is used to define the composite behaviour. In this case the damage in the composite layer is assumed to start when the normal tensile stress at the hole edge reaches the ultimate tensile strength, \( \sigma_{uts}^C \). As for the metallic constituent, the composite layer degradation is simulated by a fictitious crack with cohesive stresses, \( \sigma_{coh}^C \), acting on the crack surfaces.

An illustration of the cohesive stress distribution is shown in Figure 4.1, while the relationships between fracture toughness and crack opening for both the metallic and composite constituents are illustrated in Figure 4.2.

### 4.2.2 Artificial crack opening displacement

The crack opening displacement is related to the applied force and the cohesive force. The total crack opening of the damage increment is described by

\[
v(i) = v_{app}(i) - v_{coh}(i)
\]  

(4.1)

where \( v_{app} \) and \( v_{coh} \) are the crack opening displacement due to the applied stress and due to the cohesive stress respectively. Since the cohesive stress depends on the crack opening and vice versa, iterative calculation of \( \sigma_{coh} \) is needed by relating the critical crack opening \( v_c \) to \( \sigma_{coh} \) by using the apparent fracture energy, \( G_c \). This procedure is done for both metallic and composite layers separately. In Figure 4.1, the regions where \( v > v_c \) and \( v < v_c \) represent respectively the zone where there is no cohesive (failure occurred) and the area where cohesive stresses are present.

![Figure 4.1](image)

**Figure 4.1** Cohesive forces distribution along the fictitious crack length [22].

### 4.2.3 Damage growth

The damage growth is simulated by increasing the damage size \( \Delta a \) and by calculating the equilibrium between the applied force, the cohesive forces and the resultant of the stress distribution ahead of the damage zone, see equation (4.2). The local stress distribution, \( \sigma_{y*} \), ahead of the damage zone in the undamaged ligament
of the laminate is described by a modified linear elastic stress distribution, as described in [22]. In Equation (4.2) \( R \) is the radius of the initial notch, \( W \) is the width, \( t \) is the thickness

\[
\sum_{n=1}^{i} F_{n}^{coh} + \int_{R+a_i}^{W/2} \sigma_{y}^{*} \cdot t \cdot dx = \sigma_{app(i)} \frac{W}{2} t
\]

The index \( i \) represents the number of load steps, which are cumulated. An example of applied stress vs. fictitious crack length is given in Figure 4.3.

![Figure 4.2](image1)

**Figure 4.2** Relation between fracture toughness and crack opening for the metal constituent (left) and for the composite constituent (right) [22].

![Figure 4.3](image2)

**Figure 4.3** Applied stress vs. fictitious crack length [22]. Here \( c \) represents the crack length.
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4.2.4 Comments

The Effective Crack Growth Model was developed with the intent to progress forward in the modelling of the residual strength of composite laminates. In other approaches [9, 10, 19], models were developed to obtain the stress state and/or corresponding damage at a given applied load. In the above described method, a reverse route was applied, i.e. to find the applied load corresponding to a given damage.

The model describes the failure mechanisms in FMLs in a simplified way, without taking the delamination at the interface into account. The main drawbacks of the model are the use of fictitious crack length increments and fictitious crack opening profiles, which cannot be easily related to physical mechanisms and material properties. Moreover, The Apparent Fracture Energy, $G_c^*$, has to be obtained separately either via experimental testing or by using FEA. There exist two models: one for circular notch [22] and one for sharp notch [23]; both models are based on the same approach, and only the definition of the stress distribution in the undamaged ligament is different.

Although this model is one of the most detailed including failure mechanisms of both constituents, its validation is limited to a relative small number of ARALL laminates, with width of 50.8 mm. Given the relatively small width, the residual strength predicted by the model is mainly related to net-section yielding failure and cannot be relevant for large width panels.

4.3 Jin, Mai and Batra approach

Jin et al. [28] have attempted to derive equations by simplifying the general composite Classical Laminate Theory towards a set of relations that could be solved numerically. Although they included the ductile behaviour of the metals layers in the laminate using the Dugdale strip yielding zones, their approach did not account for the delamination process at the metal/fibre interface, neither did it include the progressive nature of the residual strength failure.

Jin and Mai stated that the fracture of metal layers may not be simply described by either the Modified Net Section criterion [9] or LEFM. In general elastic-plastic fracture mechanics has to be invoked. By considering both damage in the composite layer and plastic deformation in the aluminium layers modeled by the Dugdale strip yielding zones [29], the residual strength of ARALL was determined.

4.3.1 Model and formulation

The approach followed is based on the idea that in both metal and composite layers a damage area is present. They assumed that the Dugdale zone for metal layers and the Damage zone for composite layers have the same length. It was also assumed that
there is perfect bonding between the layers. The model is based on boundary conditions expressed as

\[
\begin{align*}
\sigma_{11} &= -\sigma_\infty + H(\{ x_2 \} - a_y)(1-V_p)\sigma^A + H(\{ x_2 \} - a_d)V_p\sigma^P, x_1 = 0, \{ x_2 \} \leq a \\
\sigma_{12} &= 0, \quad x_1 = 0, \{ x_2 \} \leq b \\
u_1 &= 0 \quad x_1 = 0, a < \{ x_2 \} \leq b
\end{align*}
\]  

(4.3)

In equation (4.3) \( a_y \) and \( a_d \) represent the damage and yielding zone respectively, while \( \sigma^A \) and \( \sigma^P \) are the stress in the Dugdale zone and the bridging stress respectively, \( V_p \) is the prepreg volume fraction in the laminate and \( \sigma_{11} \) is the stress distribution in loading direction. \( H(\cdot) \) is the Heaviside step function.

![Figure 4.4 FML specimen with a centre crack [28].](image)

Referring to Figure 4.5, the following relationship are defined

\[
\begin{align*}
\sigma^A &= \sigma_{ys} \quad \text{for} \quad \delta < \delta_c \\
\sigma^A &= 0 \quad \text{for} \quad \delta \geq \delta_c
\end{align*}
\]  

(4.4)

(4.5)

Where \( \delta_c \) is the critical separation displacement, related to the yielding stress and the critical value of J-integral by
\[ \delta_c = \frac{J_c}{\sigma_{ys}} \]  
(4.6)

For this method, the bridging stress \( \sigma_P \) is calculated for three different stages:

- fibres are unbroken and perfectly bonded to the matrix \( \rightarrow 0 \leq \delta \leq \delta_1 \)
- fibres debonded from the matrix, but still intact \( \rightarrow \delta_1 \leq \delta \leq \delta_2 \)
- fibres are broken and pulled out from the matrix \( \rightarrow \delta \geq \delta_0 \)

For each of these stages, a relationship between \( \delta \) and \( \sigma_P \) is defined [28]. \( \delta_1 \) and \( \delta_2 \) are the separation displacements at which fibre debonding and fibre breakage occurs respectively, while \( \delta_0 \) is the maximum separation displacement, measured experimentally, while \( \delta_1 \) and \( \delta_2 \) are analytically defined in [28].

By assuming the total bridging law \( \sigma_P \) is a continuous function, the parameters \( \delta_1 \) and \( \delta_2 \) are determined in close form formulations, described in [28]. Combining equations and boundary conditions, Jin et al. [28] obtained a singular nonlinear integral equation, which has been solved numerically. The authors developed the following equation describing the critical value of \( K_c \).

![Crack geometry showing the fully developed yielding and damage zones in a FML [28].](image-url)
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\[ K_e = \sqrt{\frac{2\lambda E_0 V_p (1-V_f)}{E_m \sqrt{2(1+x)}}} K_{mc} \quad (4.7) \]

Where \( V_p \) is the prepreg volume fraction, \( V_f \) is the fibre volume fraction, \( K_{mc} \) is the fracture toughness of the metallic layers. \( E_0, E_m \) and \( \lambda \) are constants defined in [28].

### 4.3.2 Comments

The described method presents some difficulties because the solution of the non-linear integral equation requires a numerical calculation. Moreover, the authors pointed out that improvements of the model may be required to include unequal lengths of the damage zone in the fibre composite layer and the yielding zone in the metal layers. This method does not include the delamination process at the interface, which plays a fundamental role for residual strength behaviour of a FML. Indeed, in the reality, the damage progression ahead of the crack tip is a combination of several mechanisms, such as plastic deformation of the metallic layers, static delamination, metal cracking and fibre failure. All these mechanisms are here addressed with two parameters \( \delta_1 \) and \( \delta_2 \) implicitly obtained with experimental tests.

### 4.4 The KR-curve approach (De Vries)

The \( K_R \)-curve (or R-curve) approach, first introduced by Irwin [30, 31], is a method derived from the Griffith energy criterion for crack growth in plane stress condition. The R-curve is based on the condition that crack growth occurs if the crack driving energy becomes larger than crack resistance energy, as described by equation (4.8).

\[ \frac{dG}{da} \geq \frac{dR}{da} \quad (4.8) \]

Irwin, postulated that the resistance to crack extension is equal to the sum of the elastic surface energy, \( \gamma_e \), and the plastic strain work, \( \gamma_p \), accompanying crack extension. The crack driving energy is therefore defined as

\[ G = \frac{\pi \sigma^2 a}{E} = G_c = 2(\gamma_e + \gamma_p) = R \quad (4.9) \]

For plain strain condition \( R \) is constant, because the plastic strain contribution, \( \gamma_p \), is negligible compared to the elastic term, \( \gamma_e \). Therefore, there is only one value defining the toughness of the material, \( G_{lc} \). In case of plane stress condition (e.g.
small thickness), $R$ is not constant because every next crack extension $da$ requires more energy until the situation of instability occurs [30]. Figure 4.6 schematically shows the instability point after the increasing crack length reaches a critical value [30].

![Figure 4.6](image.png)

*Figure 4.6 The R-curve showing the instability point [30].*

The R-curve deals both with stable crack extension and limited plastic deformations at the crack tip. In addition, it was also observed that the form of the rising part of the R-curve is also independent on the initial crack length [2, 27, 30, 32]. The R-curve can be either based on a physical or an effective crack extension. However, according to the ASTM standard E561[33], instability predictions can be made only from the effective crack extension graph. To correct the physical crack size for the effects of crack-tip plastic deformation, the fracture toughness is described as a function of a modified crack length: the effective crack length $a_{eff}$. Two correction methods are proposed by the ASTM standard E561[33]: the *Irwin correction* and the *Compliance correction*.

In case of the *Irwin method* for correction, the effective crack size, $a_{eff}$, is derived from the measured physical crack size, $a_{phys}$, by adding a calculated value of a plastic zone adjustment, $r_Y$, based on the actual value of the stress intensity factor, i.e. the one based on the effective crack length. Therefore, the effective crack length is defined as $a_{eff} = a_{phys} + r_Y$.

The *compliance method* for correction uses an analytical expression to calculate the elastic compliance of the CCT specimen based on the measured Crack Tip Opening Angle, COD.
Several researchers investigated whether the R-curve concept as applied for metallic sheets could also have been used for FMLs [1-4, 16, 27]. Vermeeren [27] and De Vries et al. [2, 32] investigated the applicability of the R-curve concept for Glare. They all stated that the R-curve could function as a material parameter but within certain limitations of the specimen width and the initial crack length. Small values of the width (W) and large values of initial notch (2a₀) should be avoided. De Vries [2] reported that the R-curve can be considered as a “material property”, even if it shows a dependency from the thickness and panel width, but not from the initial crack length. De Vries and Holleman [34] showed that the major part of the effective crack length at the onset of net section yielding (NSY) consists of the plastic zone correction while the physical crack extension is relatively small.

In [2, 27, 34] several R-curves for different Glare laminates are presented. It is shown that there might be differences between an R-curve obtained with the *compliance* method and with the *Irwin’s correction* method. This is due to the fact that the *compliance* curves are based on all available experimental data points while the *Irwin’s correction* based curves are only valid before NSY occurs. Indeed the *Irwin’s correction* is based on LEFM, therefore making the correction invalid for NSY. On the other-hand, the *compliance* method is based on an experimental measurement of the COD, which accounts for the large plastic deformation ahead of the crack tip.

Vermeeren, Pacchione and De Vries [2, 27, 34, 35] have proposed several curve fits to represent the $R$-curves for different laminates that are obtained both with the *Irwin*- and the *compliance* method. However, most of these curves were based on specimens not wider than 400 mm. To use these curves for residual strength predictions of wider panels or larger relative crack lengths, the curves must be extrapolated and are no longer supported by experimental data. For that reason $R$-curves should be based on specimens that are as wide as possible to avoid extrapolation [2].

Based on the experimental results obtained during his study, De Vries concluded that the $R$-curve of a Fibre Metal laminate is a function of:

- the amount of aluminium in the laminate, represented by the MVF,
- the material properties of the metal layers,
- the rolling direction of the metal sheets compared to the loading direction,
- the amount of fibres in the laminate in loading direction,
- the material properties of the fibre layers, and
- the characteristics of the interfaces between the different layers
More than two decades of studies on the residual strength of fibre metal laminates have culminated in the development of a semi-empirical prediction tool, as described in the following sections.

4.4.1 Prediction tool

The $R$-curve curves of flat CCT panels made of a monolithic metal, e.g. aluminium alloy, can be considered as a material "parameter" since they are independent on the panel geometry [33]. On the other hand, every Glare variant (Glare 2, Glare3, etc.) has its own $R$-curve, that needs to be determined experimentally. Therefore, to avoid the execution of expensive residual strength tests for each Glare grade, lay-up, and width, it was necessary that the $R$-curve of a given laminate could have been generated upon the constituent properties [2].

Based on experimental observations, it was understood that fibres perpendicular to the load direction do not add any strength in loading direction. Because of that, a modified fibre volume fraction term was defined as $FVF_{LD}$, defined as the FVF of the fibres in loading direction only. According to Figure 4.7, De Vries stated that the residual strength ranking for those laminate grades is based only on the $FVF_{LD}$, because the laminates have the same type of fibre and pretreated aluminium layers.

Based on these results, a new parameter $T$ was introduced representing the effective load-carrying fraction in the laminate. It is defined as:

$$T = \alpha \cdot FVF_{LD} + MVF$$  \hspace{1cm} (4.10)

where $\alpha$ is a constant that represents the contribution of fibre layers to the residual strength in comparison to the contribution of the metal layers. With a fitting procedure De Vries was able to obtain a value for $\alpha$ for L-T direction and an estimation for T-L direction.
The prediction tool is simply based on a factor that relates the residual strength of a specific Glare laminate to the residual strength of the aluminium alloy ($T=1$). Therefore, a new variable $M(T)$ was introduced:

$T = \alpha FVF_{LD} + MVF$
4. The Residual Strength in FMLs: Approaches from Literature

\[ M(T) = \left( \frac{a \cdot T^2 + b \cdot T + c}{a + b + c} \right) \]  

This factor has to be multiplied by the standard R-curve of the aluminium alloy contained in the laminate, for example 2024-T3 if standard Glare is considered.

\[ K_R(T) = M(T) \cdot K_{R_{2024-T3}} \]  

The prediction tool coefficients \((a, b, \text{and } c)\) calculated for Glare laminates are reported in [2], where the 2024-T3 aluminium alloy was used. In Figure 4.9 an example of good correlation between experimental and predicted \(K_R\)-curve is given.

\[ \text{Figure 4.9} \quad \text{Examples of R-curve predictions, compared with experimental results for different Glare grades [2].} \]

4.4.2 Comments

The prediction model based on the R-curve has been used for evaluation of the residual strength during the certification program of the fuselage Glare panels of the A380. However, this method is based on the assumption that \(\alpha\) is known for the aluminium constituent. When either a new metal or a new fibre is introduced, residual strength tests on large CCT panels made of the “new FML” must be performed for different lay-ups that cover a large range of possible T-values. With this data a new \(\alpha\) can be derived that is related to the new metal (or fibre) used. Although very good agreement between predictions and experimental results has
been obtained, the empirical nature of this method does not enable extension towards a generic model for any FML. The large amount of residual strength tests needed to generate a new curve fit would be extremely expensive and time consuming.

4.5 The CTOA method (De Vries)

In the past, the phenomenon of stable crack growth in metallic materials under mode I (tensile) loading has been studied extensively using elastic–plastic finite-element methods [36]. These studies were conducted to develop efficient techniques to simulate crack extension and to study various local and global fracture criteria. Some of these criteria were crack-tip stress or strain [37], crack-tip-opening displacement or angle [36], crack-tip force, energy-release rates, $J$-integral [38], and the tearing modulus [39]. Of these, the critical crack-tip-opening angle (CTOA) or displacement (CTOD) at a specified distance from the crack tip was shown to be suitable for modelling stable crack growth and instability during the fracture process [36]. As defined in [40], the CTOA is the angle formed by the crack flanks measured at a specific distance from the tip. It is then possible to define the critical CTOA, $\text{CTOA}_c$, as the maximum angle reached before crack extension occurs. Therefore this parameter provides an indication of the toughness of the material being tested. Each material is therefore characterized by a CTOA vs. crack extension curve, where this curve is dependent on the sheet thickness [41]. A detailed study of the CTOA criterion applied to FMLs is provided in chapter 6 of this thesis.

Many studies [36, 39, 42-50] have demonstrated that the CTOA is nearly constant after a small amount of crack extension. The non-constant CTOA region has been shown to be related to the transition of the fractured surface from flat to slant and to the tunnelling of the crack during the initiation of the stable tearing [36]. Several authors have used the CTOA approach in FE analysis to predict the residual static strength of materials or structures [46, 51, 52]. The good agreement observed between the predicted and experimental data demonstrated that the CTOA approach is attractive for investigating structural integrity of thin-walled structures [53].

The first exploratory investigation on the use of the CTOA criterion for FMLs is reported by Longhi in [11, 54]. CTOA measurements were performed for 400 mm wide panels made of Glare 3-3/2-0.3 containing a saw-cut. Test results indicated an average CTOA of 7.4° (constant part only). Based on these results a more intensive investigation on the use of the CTOA criterion for Glare is reported by de Vries in [2, 3]. CTOA measurements were performed for different Glare grades, from relative thin Glare 3-3/2-0.3 to relative thick Glare 4B-8/7-0.5. It has been reported that the CTOA presents a more or less constant value (between 4 and 8°) but with a
large scatter bandwidth. The variation of CTOA was related to the different lay-ups used for that study. The author concluded that the CTOA method might have potential application as material parameter for FMLs, but further investigations were desirable.

De Vries [2] proposed the use of the CTOA method as an alternative to the R-curve, indeed he assumed that the CTOA maintains a constant value during stable crack growth. This enabled him to use one single parameter as fracture criterion. The strength of this criterion lies in the fact that it is a very local parameter and is therefore easily applicable to predict the residual strength of more complex structures [46, 48, 51, 52]. De Vries calculated the CTOA values from different tests carried out on Glare 3 and Glare 4B specimens. He observed that the CTOA decreases from a high value to an approximately constant value; However, large scatter was observed for larger crack length. To investigate the applicability of critical CTOA values to predict residual strength of Glare panels, a finite element analysis has been carried out, see Figure 4.10. It was concluded that the CTOA approach as used by De Vries showed potential, but was not fully exploited due to the large amount of scatter of the experimental data.

![Figure 4.10 Boundary conditions and mesh for finite element calculations [2]](image)

**4.5.1 Comments**

From the FEA results illustrated in Figure 4.11, it is visible that none of the predictions fully agrees with the experimental results. This is due to the fact that only a fix value of the CTOA was used as critical value, rather than using the whole CTOA curve. Indeed, the comparison between predictions and experimental results in Figure 4.11 shows that:
If a small angle is used, e.g. 6 deg., the prediction is below the experimental curve.

If a high angle is used, e.g. 8 deg., the prediction over estimates the residual strength and critical crack length.

None of the curves follow the trend of the experimental data.

In reality, it has been pointed out in [55] that the non-constant part of the CTOA curve must be taken into account for residual strength predictions because it accounts for the crack tip blunting and plastic zone development. An example of CTOA curve obtained from 2024-T3 metal laminate is shown in Figure 4.12; here three areas are highlighted:

- Part 1: High CTOA values, where crack-tip blunting, large plastic zone increments and crack initiation take place.
- Part 2: Transition part, where the amount of plastic zone increment reduces and the strained material is hardening.
- Part 3: Steady CTOA, where stable crack growth and successively unstable crack growth occurs, with limited increments of plastic zone.

![Figure 4.11](image)

**Figure 4.11** *Applied force versus physical crack extension, calculated for different CTOA values for Glare 3-3/2-0.4 [2]*

The CTOA method has intrinsically much potential as tool for prediction of the residual strength in FMLs, especially if implemented in a FE model. Indeed, the CTOA is a local parameter, which is independent on the geometry and on the presence of external stiffening elements (e.g. stringers, frames, doublers, etc.).
therefore possible to use the CTOA method to predict the stable crack growth of complex structures [53].

Figure 4.12 Example of CTOA curves obtained experimentally on three different 2024-T3 metal laminates.

### 4.6 Critical fracture toughness method (Castrodeza et al.)

Castrodeza et al. [5-8, 56] have tried to link the residual strength behaviour to the standard fracture toughness properties, which are determined with C(T) specimen configurations, standardized for monolithic aluminium [57]. They calculated critical toughness values of unidirectional FMLs using $J_c$ and $\delta_{sc}$ [58]. Experimental values of $J_c$ are obtained and then related to $K_{lc}$ [59]. The residual strength of cracked sheets of finite width is calculated using the $K_I$ expression and including the length correction due to plastic deformation at the crack tip [33]. So, the residual strength is calculated with:

$$\sigma_{res} = \frac{K_{lc}}{\sqrt{\pi a_{eff} sec \frac{\pi a_{eff}}{W}}}$$  \hspace{1cm} (4.13)

where
\[ a_{\text{eff}} = a_{0} + \frac{1}{2\pi} \left( \frac{K_{\text{tc}}}{\sigma_{\text{YS}}} \right)^2 \] (4.14)

Validation of the above described method has been performed with respect to M(T) specimens made of ARALL 2-3/2-0.3 and ARALL 3-3/2-0.3, with widths ranging from 152 mm to 406 mm. The comparison has been based only on the residual strength values, as defined in equation (4.13) and not on the crack growth curve. Although good agreements between predictions and test results have been reported by the authors, some differences are present for the largest panels tested, see Table 4.1.

<table>
<thead>
<tr>
<th></th>
<th>2a (mm)</th>
<th>W (mm)</th>
<th>Prediction (MPa)</th>
<th>Experiments (MPa)</th>
<th>Experiments/prediction ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arall 3</td>
<td>12.7</td>
<td>152.4</td>
<td>414</td>
<td>413</td>
<td>1.00</td>
</tr>
<tr>
<td>3/2-0.3</td>
<td>38.1</td>
<td>152.4</td>
<td>253</td>
<td>262</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>101.6</td>
<td>406.4</td>
<td>161</td>
<td>179</td>
<td>1.11</td>
</tr>
<tr>
<td>Arall 2</td>
<td>12.7</td>
<td>152.4</td>
<td>351</td>
<td>331</td>
<td>0.94</td>
</tr>
<tr>
<td>3/2-0.3</td>
<td>50.8</td>
<td>152.4</td>
<td>208</td>
<td>220</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>25.4</td>
<td>203.2</td>
<td>292</td>
<td>289</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>50.8</td>
<td>203.2</td>
<td>221</td>
<td>241</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>101.6</td>
<td>406.4</td>
<td>171</td>
<td>213</td>
<td>1.26</td>
</tr>
</tbody>
</table>

### 4.6.1 Comments

Beside the simplicity of the method and the good results obtained, the fracture toughness test is not well defined for Fibre Metal laminate, unless treated as a monolithic material. This makes the approach inappropriate for the residual strength evaluation of fatigue crack configurations for example. Moreover, the progressive failure process is not taken into account because all mechanisms are related to the experimentally calculated \( J_c \). Plastic deformation, fibre failure, static delamination, metal crack growth are mechanisms not described within this method and therefore the predictive capability is limited to the calculation of \( J_c \) specifically for a given lay-up.

### 4.7 Conclusions

A number of prediction methods for the residual strength of FMLs have been presented and evaluated. Despite good agreement with experimental results, the validation of those methods was limited to a relative small amount of configurations
and lay-ups. Another common limitation of the described models is the limited or absent amount of physical sound description of the failure mechanisms occurring during the residual strength failure sequence in FMLs.

Two analytical models described in [22, 23, 28] have the potential for further development and refinement. The Effective Crack Growth Model of Afaghi-Kathibi et al. [22, 23] seems to be somehow the most complete model among those presented in this chapter. Both fibre and metal failures are described separately using an energy parameter, and the crack growth model is based on global equilibrium equations.

Jim and May [28] proposed a prediction model based on EPFM. They used a geometrical parameter, related to the CTOD, to define three stages of failure. Indeed, plastic deformation, local fibre delamination and fibre failure are all included in the single parameter δ. In addition, the assumption that the length of the damage is the same in both composite and metallic layers does not reflect the reality.

The CTOA approach described by De Vries has not been fully exploited because of the missing understanding of the CTOA as fracture parameter. Based on the literature review the author of the present dissertation believes that the use of the non-constant part of the CTOA, as described in [55], can make the CTOA eligible as fracture parameter for FMLs. This is experimentally investigated in chapter 6 and the implementation of the CTOA into a prediction model for residual strength in FML is described in chapter 7.

References


4. The Residual Strength in FMLs: Approaches from Literature


[21] Rijn JCFN. The use of composite fracture models to describe the blunt notch behaviour. NLR TP 94058 L. National Aerospace Laboratory. 1994


The Residual Strength Failure Sequence in Fibre Metal Laminates

[34] de Vries TJ, Holleman E. An R-curve approach to fracture toughness analysis of some Glare grades (Memorandum M-687). 1995


4. The Residual Strength in FMLs: Approaches from Literature


Understanding the failure mechanisms in FMLs

Abstract – This chapter provides an overview of the experimental studies performed to understand the fibre/metal interaction. Tests comprise both static and fatigue loading of several FML types, for which strain measurements were performed using digital image correlation. The first investigation aimed to evaluate the variations of the shape and size of the plastic zone in the metallic layers as function of fibre stiffness. The second type of experiments investigate the interaction between fibres and metal by testing laminates with the prepreg on the outside, this configuration enables the measurement of strains directly on the prepreg surface. Results and discussions are provided.

5.1 Introduction

To develop a prediction model for the residual strength of FMLs, a detailed study has been performed on the important fracture mechanisms. This study primarily consisted of experiments that generated the understanding of specific failure mechanisms within the laminate, providing a support for the model development.

The experiments presented in this chapter and in chapter 6 are mainly based on the use of Digital Image Correlation (DIC), as a strain field measurement technique, which is briefly described in section 5.2. DIC has been widely used for a number of investigations and it has proven to be a powerful method to investigate specific failure mechanisms within FMLs in a quantitative manner.
The experiments are discussed in section 5.3 and section 5.4, where the crack-tip behaviour of FMLs and a study on the compatibility between fibre and metal layer deformation is presented respectively. Both analyses have been performed using DIC. Each section contains a description of materials, geometries, measurement techniques and results for all experiments performed. At the end of each section results are presented and discussed. Section 5.5 provides some concluding remarks.

5.2 Digital Image Correlation

DIC is an experimental technique capable to provide information on deformation fields on an object surface [1-3]. DIC has advantages with respect to other strain measurement systems like strain gauges, extensometers etc., simply because it is one of the few methods which can provide local quantitative strain data on large surface areas. The DIC has been performed using a tool developed at the Delft University of Technology by Lemmen [1, 4]. It uses digital grid points to mark specific locations (pixels) on the specimen surface when no deformation is present (source image). These locations are traced back in each subsequent image (target image) during the test, thus defining the current displacement field. From the displacement fields it is then possible to obtain the strain field. In his PhD thesis [4], Lemmen shows how he used the developed DIC tool for specific strain measurements in friction-stir-welded specimens. In addition, he provides a detailed description of the main characteristics of the DIC tool, covering specific topics such as:

- Single pixel recognition technique
- Grid recognition technique
- Fault tracking system

The DIC tool described in [4] has been slightly modified by the author of the present thesis to make it suitable for measuring specific damage mechanisms within FMLs. In particular, DIC has been paramount to quantify:

- Plastic deformation ahead and behind the crack tip
- Fatigue delamination growth under constant amplitude loading
- Quasi-static growth of the fatigue delamination under increasing loads
- Fibre layer deformation
- Stress (or strain) in the bridging fibres.

5.2.1 Surface preparation

In order to obtain an effective correlation between the source and target image, a speckle pattern has been applied to the surface of the specimen using black and white paint, see Figure 5.1. Cleaning and de-greasing the specimen surface is
important to ensure a perfect bonding between the paint and the surface. The importance of using a speckle pattern is two-fold: first, as result of its high contrast, it ensures a higher correlation between source and target image, which improves the accuracy of the measurement. Second, it prevents changes in the surface light reflection when high deformations develop.

**5.2.2 Sensitivity analysis and optimization**

As described in [1,4], the error in the DIC tool is in the order of 0.02 to 0.1 pixels, depending on the quality of the image and surface of the specimen. The effect of this error on the strain measurement can be influenced by changing the distance between the grid points, and thus the gauge length. The correlation process is made using a “correlation square”, see Figure 5.1 which is compared with all pixels in the target image, returning a correlation value for each pixel [4]. The location in the target image that corresponds best to the correlation square is recognized by the highest correlation value. However, to perform detailed strain measurements, it is requested sub-pixel resolution; therefore, a method was developed by Lemmen [4] which uses a 6th degree polynomial surface fitted through the correlation values around the peak, by a least square approach. The point with the highest value is the new coordinate of the grid point. This approach provides a resolution for the coordinates of 0.01 pixels [4].

Strains between two grid points are obtained by measuring the relative displacement and comparing it to the original distance. The inherent error in DIC can induce some
issues when displacement fields are transformed into strain fields. As schematically shown in Figure 5.2, the inherent error can be obtained by correlating the source image with itself.

\[
\varepsilon_{\text{error}} = \frac{\Delta^* - \Delta_0}{\Delta_0} = \frac{\Delta_{\text{error}}}{\Delta_0}
\]  

(5.1)

The inherent error can be the cause of scatter in the measurements, especially for relative small value of strains ($\varepsilon < 0.1\%$), this happens when the scatter band is of the same order of the measured strains.

Being aware of the accuracy of the DIC measurements, and how the initial distance between grid points can play a role in that, see equation (5.1), a sensitivity analysis was performed on the strain gradient ahead a saw-cut in a metal laminate panel. This was done by varying the vertical distance between the grid points from 10 to 60 pixels, see Figure 5.3. From this analysis, it can be concluded that:
5. Understanding the Failure Mechanisms in FMLs

- A reduction of the initial distance between the grid points increases the scatter, see Figure 5.3-b. If there is a high strain gradient between the grid points, the obtained value is an average of the strain along the distance between the grid points and it results in a less averaged value, see Figure 5.3-a. (example is the measurement made with 10px distance).

- An increment of the initial distance between the grid points reduces the scatter, see Figure 5.3-b. If there is a high strain gradient between the grid points, the obtained value is an average of the strain along the distance between the grid points and it results in a more averaged value, see Figure 5.3-a. (example is the measurement made with 60px distance).

The sensitivity analysis enabled to optimize the grid points distribution such to minimize the scatter and increase the accuracy of the measurements.

5.2.3 Validation of the DIC approach

To prove the validity of the DIC measurements, a comparison was made with the data obtained with strain gauges. As illustrated in Figure 5.4, four strain gauges were bonded in front of the crack tip at one side of the panel at a distance of 15 mm from each other. DIC measurements were performed on the other side of the panel following the procedure previously explained. Figure 5.4 illustrates a very good agreement between DIC and strain gauges results for two different stress levels. The DIC technique has even more resolution than the strain gauges in those areas were high strain gradients are present, e.g. ahead of the crack tip, because the strain gauges provide an averaged value of the strains underneath the gauge area, while DIC is able to measure the strain over a very small distance.

5.2.4 Exploiting DIC for investigations on FMLs

DIC has been used in a large number of experiments conducted during the research. Some of these applications will be illustrated and discussed in sections 5.3 and 5.4 and in chapter 6. In particular, DIC has been successfully used for:

- Measuring size and shape of the plastic zone
- Measuring the deformation of the prepreg layers
- Monitoring the fatigue delamination shape and growth
- Monitoring the plastic strains development below and above the delaminated area.

All these applications are discussed in the following sections and chapters.
Figure 5.3  Sensitivity analysis of strain gradients obtained with DIC for different grid points distances.

Figure 5.4  Comparison of strain values obtained with DIC and with strain gauges
5.3 Fibre/metal interaction in FMLs: metal deformation

The failure mechanisms described in chapter 3 (metal deformation and cracking, delamination, fibre failure, etc.) are strongly influenced by the strain field in front of the crack tip. For high static loads, the plastic part of the strain field in the metal layers plays a fundamental role because stress redistribution mechanisms will take place in the vicinity of the yielded metal. In this case, the yielded metal loses capability to carry load and part of the load is transferred from the yielded metal to both the elastic fibre layers and the remaining elastic part of the metal layers. In addition to that, in case of bridging fibres, the deformation field of the metal layers may change significantly both in magnitude and shape. Indeed, the bridging fibres attract load from the metal layers, reducing the amount of stress at the metal crack tip and subsequently the size of the plastic zone.

5.3.1 Objective of the investigation

The experimental study presented in this section aims to gain understanding on how the presence of fibres can influence the deformation of the metal layers in FMLs. In particular two kind of investigation were carried out:

- Investigating the effect of the fibre stiffness on the strain field (saw-cut configuration)
- Investigating the effect of bridging fibres on the strain field (fatigue crack configuration)

Both experimental investigations were carried out using DIC following the approach described in section 5.3.3.

5.3.2 Materials, geometry and test matrix

Several tests have been performed on Centre-Crack-Tension (CCT) specimens made of different FML types, see Table 5.1. In order to obtain a significant variation in fibre properties, like stiffness, elongation to break, and coefficient of thermal expansion (CTE), three different fibre/epoxy systems were used: M30-carbon/DT120, S2-glass/FM94 and Zylon/FM94.

The geometry depicted in Figure 5.5 has been used for all specimens discussed here. The saw-cut was realised with a milling process; in this way, all specimens contained similar notch lengths. Except for specimen 6, which contained a 5 mm saw-cut subsequently extended to 20 mm by fatigue loading, all other specimens contain a 20 mm saw-cut.
Table 5.1 Test matrix

<table>
<thead>
<tr>
<th>Spec</th>
<th>Lay-up</th>
<th>Materials</th>
<th>$t_{tot}$ [mm]</th>
<th>MVF [-]</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FML 3-3/2-0.4</td>
<td>2024-T3/FM94/S2-glass</td>
<td>1.75</td>
<td>0.69</td>
<td>Saw-cut</td>
</tr>
<tr>
<td>2</td>
<td>FML 3-3/2-0.4</td>
<td>2024-T3/FM94/Zylon</td>
<td>1.82</td>
<td>0.65</td>
<td>Saw-cut</td>
</tr>
<tr>
<td>3</td>
<td>FML 3-3/2-0.4</td>
<td>2024-T3/DT120/M30SC</td>
<td>1.82</td>
<td>0.65</td>
<td>Saw-cut</td>
</tr>
<tr>
<td>4</td>
<td>FML 2-3/2-0.4</td>
<td>2024-T3/FM94/S2-glass</td>
<td>1.75</td>
<td>0.69</td>
<td>Saw-cut</td>
</tr>
<tr>
<td>5</td>
<td>ML 3-3/2-0.4</td>
<td>2024-T3/FM94</td>
<td>1.45</td>
<td>~1</td>
<td>Saw-cut</td>
</tr>
<tr>
<td>6</td>
<td>FML 3-3/2-0.4</td>
<td>2024-T3/FM94/S2-glass</td>
<td>1.75</td>
<td>0.69</td>
<td>Fatigue</td>
</tr>
</tbody>
</table>

The mechanical properties of the selected materials are presented in Table 5.2. Only aluminium alloy 2024-T3 was selected as metallic constituent. Although more fibre/metal combinations are possible, not all configurations make sense for the current study. All laminates were manufactured at the Delft University of Technology. The standard Glare curing cycle was used for all laminates [5], because both DT120 [6] and FM94 [7] resin systems require the same temperature and pressure profile. The 2024-T3 aluminium sheets were pre-treated based on the standard surface pre-treatment, described in chapter 2.

Table 5.2 Mechanical properties of UD fibre/epoxy systems

<table>
<thead>
<tr>
<th>Properties</th>
<th>M30SC / DT120 (a)</th>
<th>Zylon / FM94 (b)</th>
<th>S2 glass / FM94 (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$ [GPa]</td>
<td>155</td>
<td>120</td>
<td>48.9</td>
</tr>
<tr>
<td>$E_{22}$ [GPa]</td>
<td>7.8</td>
<td>2.84</td>
<td>5.5</td>
</tr>
<tr>
<td>$G_{12}$ [GPa]</td>
<td>5.5</td>
<td>1.04</td>
<td>5.55</td>
</tr>
<tr>
<td>$\sigma_{ult}$ [MPa]</td>
<td>1800</td>
<td>2261</td>
<td>2640</td>
</tr>
<tr>
<td>$v_{12}$ [-]</td>
<td>0.27</td>
<td>0.22</td>
<td>0.33</td>
</tr>
<tr>
<td>$\varepsilon_{ult}$ [%]</td>
<td>1.6</td>
<td>2</td>
<td>4.5</td>
</tr>
<tr>
<td>CTE [-]</td>
<td>-0.45·10^{-6 (a)}</td>
<td>-6·10^{-6 (a)}</td>
<td>6.1·10^{-6}</td>
</tr>
<tr>
<td>$\rho$ [g/cm$^3$]</td>
<td>1.76</td>
<td>1.54</td>
<td>1.98</td>
</tr>
<tr>
<td>$t_{nom}$ [mm]</td>
<td>0.156</td>
<td>0.155</td>
<td>0.133</td>
</tr>
</tbody>
</table>

(a), (b) data reported in [9]
(c) data reported in [10]
(*) data reported in [11]

5.3.3 Measurements and test execution

A digital camera system with electronically controlled x-y-z table was used to perform the digital image acquisition of the area in front of the crack tip, see Figure 5.6. The images were captured at different static loads and subsequently stored. The load increment step was performed under force control, while the images where captured under displacement control once a prefixed load level was reached. This procedure enabled to keep control of the crack tip deformation, avoiding undesired
crack propagation especially when high loads were applied. The captured images were processed using the DIC tool described in the previous section.

![Geometry of the panels](image1)

**Figure 5.5**  *Geometry of the panels*

![Test set-up for DIC analysis](image2)

**Figure 5.6**  *Test set-up for DIC analysis*
5.3.4 Finite element model

Beside the experimental activity, a simple FE model has been developed to study the effect of the metal plasticization on the shear stresses distribution at the fibre/metal interface. The FE model was made with the commercial code ABAQUS v6.8 with the intent to get an insight of the interlaminar behaviour in relation to increasing applied load. A validation of the model has been made by comparing the calculated strain fields with those experimentally measured with DIC. In addition, a convergence analysis and a mesh refinement study were performed, obtaining the convergence of the results.

FE models were created for specimens 1, 2, 3 and 5 in Table 5.1, using the same geometry, loads and constraints adopted during the experiments. Due to the three symmetry planes, the geometry has been simplified by modelling only one-eighth of the full specimen and assigning a proper set of symmetry constrains along the planes, as shown in Figure 5.7. The performed static analysis included six load-steps with in addition an extra initial temperature-step to take into account the residual curing stresses within the laminate due to the cooling process from 120°C to 20°C. The CTE assigned for the three directions of the prepreg layers are presented in Table 5.3. The only stress induced in the laminate after the initial temperature-step is due to the thermal contraction, without applying external load.

All mechanical properties of the 2024-T3 aluminium alloy, derived from the experimental stress/strain curve, were converted from engineering stress/strains into true stress/strains, as required by the ABAQUS pre-processor [12]. The prepreg layers use orthotropic properties, according to the local coordinate system where the first axis is the principal axis. Mechanical properties used to characterize the selected materials are reported in [6, 9, 10, 14].

In total 50145 linear hexahedral elements of type C3D8R were used for the complete model, with one element through the thickness for each prepreg layer and two for the metal layers. The FE model has been validated by comparing the calculated plastic zone size and shape with those experimentally measured using DIC.

| Table 5.3 Coefficients of thermal expansion |
|-------------------------------|----------------|----------------|----------------|
| Ref              | $\alpha_{11}$ (C°$^{-1}$) | $\alpha_{22}$ (C°$^{-1}$) | $\alpha_{33}$ (C°$^{-1}$) |
| S2-glass / FM94 | 0.61 $\cdot$ 10$^{-5}$ | 2.22 $\cdot$ 10$^{-5}$ | 2.22 $\cdot$ 10$^{-5}$ |
| M30SC / DT120   | -0.04 $\cdot$ 10$^{-5}$ | 3.7 $\cdot$ 10$^{-5}$  | 3.7 $\cdot$ 10$^{-5}$  |
| Zylon / FM94    | -0.6 $\cdot$ 10$^{-5}$  | 4.1 $\cdot$ 10$^{-5}$  | 4.1 $\cdot$ 10$^{-5}$  |
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5.3.5 Results and discussion

Variation of the fibre/prepreg constituent

To identify the effect of the prepreg type on the crack tip strain field, different specimens were tested each containing a different prepreg (specimens 1, 2, 3 and 5 in Table 5.1). The left-hand side of Figure 5.8 illustrates examples of strain field obtained with the DIC. The colours of the images define the strain values, which are scaled from 0% to 1%.

The effect of the stiffness of the fibre/prepreg system is visible in the extension and shape of the strain field ($\varepsilon_{yy}$), which strongly depend on the laminate stiffness. The right-hand side of Figure 5.8 shows the relative plastic zone shape, defined as the area where $\varepsilon_{yy} \geq \varepsilon_{0.2}$, where $\varepsilon_{0.2}$ represents the yield strain of the metallic constituent ($\varepsilon_{yy} = \varepsilon_{0.2}$ when $\sigma_{yy} = \sigma_{0.2}$). The effect of the fibre/prepreg is evident when the strain fields are compared to each other and especially with the strain field of the 2024-T3 laminate (specimen 5). In a 2024-T3 laminated specimen, as well as in a monolithic, the characteristic “butterfly wings” shape of the strain field develops along $\pm 50^\circ$ angle with respect to the crack line (x-direction). This characteristic behaviour is related to the isotropic nature of the 2024-T3 where the plastic deformation develops mainly along the direction of maximum shear stress [17].
When an FML is considered, the presence of orthotropic material, like fibre/prepreg layers, induces a change in the strain field in the metallic layers. This is due to the presence of fibres in loading direction that induces, to some extent, a change of the maximum shear stress direction; therefore the inclination of plastic zone shape is affected accordingly.

Figure 5.8  Strain fields measured experimentally scaled from 0% to 1% (left) and related plastic zone shape (right). For all cases the applied stress was 200 MPa.
In general the plastic zone develops starting from a circular area, gradually changing towards a “wing shape” with the load increments. This change in the shape of the plastic zone occurs also in FMLs and it is dependent on the stiffness of the fibre/prepreg system used. The plastic zones calculated with FEA and measured experimentally with DIC have been compared, as illustrated in Figure 5.9. The comparison shows a good agreement concerning size and shape of the plastic zone during its evolution. Some differences are present in those configurations where fibres with high stiffness induce large changes in the deformation field of the metal constituent.

![Figure 5.9 Development of plastic zone contours at different stress levels measured with DIC and comparison with FE analysis.](image)

These differences are clearly visible in Figure 5.11 where the variation of the wing angle is plotted against the applied stress (definition of the wing angle $\theta_w$ and plastic zone extension $r_p$ is provided in Figure 5.10). It can be seen that for 2024-T3 and 2024-T/S2-glass configurations FEM results and DIC measurements correspond with each other. On the other hand, the 2024-T3/Zylon and 2024-T3/M30-carbon configurations show somehow some differences with the FEA results. For low applied stresses the wing angle $\theta_w$ measured on the 2024-T3/Zylon configuration is similar to the one measured on the 2024-T3/S2-glass configuration (at 120MPa $\theta_w \approx$...
50°). This means that apparently, despite the different fibre stiffness, the orthotropy introduced by the fibres is similar in both cases.

When high loads are applied, the measured angle in the 2024-T3/Zylon configuration increases, while in the 2024-T3/S2-glass configuration decreases. This means that the stiffness of the laminate with Zylon fibres “changes” during the loading sequence, and seems to increase until the nominal higher stiffness of Zylon fibres is fully available, see Table 5.2. Similar behaviour is visible in Figure 5.12 where the plastic zone extension, \( r_p \), is plotted against the laminate remote stress. Also in this case there is a good agreement between DIC measurements and FEA results, except for the 2024-T3/Zylon laminate. After 220 MPa, the trend of the laminate with Zylon fibres deviates from the trend of the other laminates (S2-glass and 2024-T3 laminate), behaving thus as a stiffer laminate.

![Figure 5.10 Definition of the plastic zone size \( r_p \) and butterfly-wing angle \( \theta_w \)](image)

The differences observed in the test data concerning the 2024-T3/Zylon laminate can be explained by a change of tensile behaviour of Zylon fibres due to the residual stresses after the curing cycle. Once cured, the fibre layers within the laminate undergo to a compression state (due to the difference in CTE). As reported in [9], and shown in Table 5.4, the measured Young’s modulus of the Zylon prepreg is 120 GPa, while the compression modulus is about half of the tensile modulus. If an FML is made using Zylon fibres and 2024-T3 thin sheets, after the curing cycle residual stresses are present within the laminate: compressive stresses in the fibre layers and tensile stresses in the metallic layers. The tensile modulus, \( E_T \), of an FML with lay-up 3-3/2-0.4 containing Zylon fibres is 57.1 GPa, while the compressive modulus, \( E_c \), is 51.9 GPa, see Table 5.4. The comparison between the stiffness of the 2024-T3/S2-glass and the 2024-T3/Zylon reveals that, despite the higher stiffness of Zylon fibres, the laminate containing S2-glass fibres is stiffer than the laminate containing Zylon.
If an increasing load is applied to the laminate containing Zylon, the laminate initially reacts with a lower stiffness, similar to the stiffness of Glare. This is due to the fact that Zylon fibres are initially compressed due to the residual stresses, and the contribution of the fibres to the stiffness of the laminate is about half of the tensile modulus of the fibres alone.

When the applied load increases further, the actual stress in the fibre layers changes from compression to tension, increasing thus the stiffness of the laminate. This behaviour is not reproduced with the FE analysis because the compression modulus was chosen equal to the tensile modulus.

In addition to what stated above, one additional reason for the differences observed between FEA and DIC in Figure 5.11 (M30-carbon and Zylon laminates) might be due to the use of CTEs parameters obtained from literature [15, 16]. CTEs obtained experimentally might have led to a better characterization of the thermal behaviour of the material within the FEM. Anyhow, a better formulation of the constitutive model, accounting for the different stiffness moduli, might have been more appropriate.

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{figure5.11.png}
\caption{Comparison between measured and predicted plastic zone wing angle for all configurations}
\end{figure}
Table 5.4 Comparison between tensile and compressive modulus [9]

<table>
<thead>
<tr>
<th>FML</th>
<th>( E_T \text{ prepreg} ) [GPa]</th>
<th>( E_T \text{ laminate} ) [GPa]</th>
<th>( E_C \text{ laminate} ) [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3 / S2-glass</td>
<td>50</td>
<td>58.2</td>
<td>60.4</td>
</tr>
<tr>
<td>2024-T3 / Zylon</td>
<td>120</td>
<td>57.1</td>
<td>51.9</td>
</tr>
</tbody>
</table>

Figure 5.12  Evolution of the plastic zone size for all four configurations.

The FE model has been developed to provide an insight on the interlaminar behaviour when plastic deformation occurs. The idea behind this simplified FE model was to observe potential changes in the strain distribution in both fibre and metal layers when plastic deformation occurs. It has been observed that the occurrence of local metal plasticization determines a load transfer to the elastic fibre layer. This was already pointed out by de Vries [18]. Although the present FE model does not include any failure mechanisms, the observed behaviour provides an indication of what the actual interlaminar behaviour can be.

Effect of the bridging fibres

In an FML with a saw-cut, the strain field shape and size is determined by the interaction between metal and fibre layers. The orthotropic contribution of the fibre layers induces a re-distribution of the stress/strain such that the ‘wing-angle’ and size of the plastic zone are affected by the stiffness of the fibre/prepreg system.
If a fatigue crack configuration is considered, the bridging fibres in the wake of the crack introduce a larger orthotropic contribution inducing a further change of the deformation field. In Figure 5.13 a comparison between the strain fields of a saw-cut and a fatigue crack, under the same applied load, is provided. The images refer to two panels of Glare 3-3/2-0.4 with a 20 mm saw-cut and 20 mm fatigue crack (2a₀=5mm), respectively.

![Figure 5.13](image)

**Figure 5.13** Comparison of the strain field ahead of a 20 mm saw-cut (a) and a 20 mm fatigue crack (b) in Glare 3-3/2-0.4 under an applied stress of 200 MPa (Scale in %)

The applied load was increased quasi-statically and images were captured at different load levels. Figure 5.13 shows two examples referring to an applied load of 200 MPa, where the colours represent the strain measured on the surface. The scale in Figure 5.13 is limited to 0.5% strain in y-direction. The images cover a relatively small area in vicinity of the crack tip, including the last 5mm of the crack length.

Figure 5.14 illustrates the full sequence of the plastic zone development due to the increasing load, comparing both saw-cut and fatigue crack. The plastic zone is defined as the part of the strain field were εᵧᵧ ≥ ε₀.2, where ε₀.2 represents the strain at yield stress of the metal.

When a fatigue crack in FML is loaded with increasing load, the bridging fibres carry part of the applied load and reduce the amount of stress in the metallic layers. The stress intensity at the metal crack tip reduces as well, and therefore the plastic zone is smaller than in the case of a saw-cut loaded at the same stress level. In addition to that, plastic deformation may also occur behind the fatigue pre-crack tip. This is due to the bridging mechanism which attracts, load above and below the delamination boundary; this phenomenon does not occur in absence of bridging fibres because a large stress concentration takes place ahead of the crack tip. The
plastic deformation induced in the wake of the fatigue crack by the bridging fibres mechanisms and its effect on the delamination growth is described in details in chapter 6.

Figure 5.14 Comparison of the plastic zone between a 20 mm saw-cut and a 20 mm fatigue crack in Glare 3-3/2-0.4 under quasi-statically increased loads, measured using DIC

5.4 Fibre/metal interaction in FMLs: prepreg deformation

Metal and fibre layers are supposed to respect the compatibility of deformations. This means that in absence of delamination, both fibre and metal layers should show the same deformation pattern. Understanding the deformation mechanisms that occur during the load increase sequence is paramount to make the right simplifications and assumptions for developing physical sound prediction models. In standard FMLs, the metal layers are at the outside, covering the fibre layers within the laminate, it is therefore impossible to observe the fibre layer directly. A solution to observe and measure the deformation of the fibre layers is to manufacture specimens with prepreg at the outer layers. Direct observation of the prepreg layer is therefore possible, enabling measurements with DIC of the strain field, and visual observation of the delamination growth and fibre failure.
5.4.1 Objective of the investigation

The objective of this investigation is two-fold:

- Verify the deformation compatibility between fibre and metal layers and identify possible effects of the plastic deformation of the metal layers on the strain field of the fibre layers.
- Understand the strain (or stress) distribution in the fibre layer in presence of bridging fibres.

5.4.2 Materials, geometry and test matrix

The manufacturing of the specimens was done following the procedure for standard Glare, except for the fact that the prepreg was placed at the outside. Three configurations were therefore manufactured:

- “one-to-one” configuration, where one layer of UD fibre/prepreg was bonded to one sheet of 0.4 mm 2024-T3
- “inverted lay-up” configuration, where the specimens were made by bonding together two 0.4 mm thick 2024-T3 sheets with adhesive film FM94 in-between and one layer of UD S2-glass fibre/FM94 prepreg on each external sides.

Figure 5.15 illustrates both the one-to-one and inverted lay-up configurations compared to the standard laminate. The choice of this specific inverted lay-up enables a direct comparison with the standard Glare 2-2/1-0.4 configuration, because the MVF and the number of interfaces have been kept the same. The geometry and notch dimensions used for these tests are the same as depicted in Figure 5.6. An overview of the test matrix is given in Table 5.5. In addition to the laminates with prepreg at the outside, a full composite UD laminate was manufactured using M30 carbon/DT120 prepreg. This specimen has been used to investigate the strain field of a full composite laminate, in comparison with the strain fields related to the other configurations.

5.4.3 Measurements and test execution

The tests were performed on a 60 kN fatigue testing machine. Two digital cameras were used to perform the digital image acquisition of the specimen surface. The test set-up is the same shown in Figure 5.6. For the one-to-one configurations, DIC analysis was performed on both sides of the specimen, while for the inverted lay-up configurations DIC was employed on one side only because at the other side there was no spackle pattern applied. In this way, a direct observation of the delamination at the side without spackled pattern of the specimen was possible.
The Residual Strength Failure Sequence in Fibre Metal Laminates

**Inverted lay-up**
IFML-2/1/2-0.4

**Standard lay-up**
FML 2-2/1-0.4

**active interfaces = 2**

MVF = 0.75

*Figure 5.15  Illustration of the 'one-to-one, ‘inverted’ lay-up and ‘standard’ lay-up*

**Table 5.5 Test matrix**

<table>
<thead>
<tr>
<th>Spec</th>
<th>Lay-up</th>
<th>Materials</th>
<th>t&lt;sub&gt;metal&lt;/sub&gt; [mm]</th>
<th>t&lt;sub&gt;pp&lt;/sub&gt; [mm]</th>
<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0°/Al/0°</td>
<td>2024-T3/S2-glass</td>
<td>1 X 0.8</td>
<td>0.133</td>
<td>fatigue</td>
</tr>
<tr>
<td>2</td>
<td>0°/Al/Al/0°</td>
<td>2024-T3/S2-glass</td>
<td>2 X 0.4</td>
<td>0.156</td>
<td>fatigue</td>
</tr>
<tr>
<td>3</td>
<td>0°/Al/Al/0°</td>
<td>2024-T3/M30SC</td>
<td>2 X 0.4</td>
<td>0.156</td>
<td>fatigue</td>
</tr>
<tr>
<td>4</td>
<td>Al/0°</td>
<td>2024-T3/S2-glass</td>
<td>1 X 0.4</td>
<td>0.133</td>
<td>saw-cut</td>
</tr>
<tr>
<td>5</td>
<td>Al/0°</td>
<td>2024-T3/M30SC</td>
<td>1 X 0.4</td>
<td>0.156</td>
<td>saw-cut</td>
</tr>
<tr>
<td>6</td>
<td>10X0°</td>
<td>M30SC</td>
<td>-</td>
<td>0.156</td>
<td>saw-cut</td>
</tr>
</tbody>
</table>

With reference to Table 5.5, the tests were performed using two load sequences: quasi-static load increments for the one-to-one configurations (specimen 4 and 5), and fatigue loading plus quasi-static load increments for the inverted lay-up configurations (specimen 1, 2, and 3). The fatigue crack was extended under constant amplitude load ($\sigma =120$ MPa and $R =0.05$) until a total length of 20 mm was reached, subsequently an increasing static load was applied. Specimen 6 was loaded statically at several load levels.

For all specimens, images were taken under specific static loads. To monitor the fatigue delamination growth the images were captured and stored while the specimen was under static load equal to the maximum applied fatigue load.
5.4.4 Results and discussion

Compatibility of deformations

Metal and fibres are supposed to deform in the same way, as long as delamination does not occur. This aspect is illustrated in Figure 5.16 and Figure 5.17 for “one-to-one” combinations. Figure 5.16 shows the 2024-T3/S2-glass fibre laminate loaded at 200 MPa, while the images in Figure 5.17 show the 2024-T3/M30SC combination loaded at 238 MPa. Both figures illustrate the images taken at the metal and composite side. As discussed in section 5.3.5, and illustrated in Figure 5.8, the strain field of a metal laminate containing a crack is characterized by its butterfly shape. The strain field of a full UD composite laminate, on the other hand, is characterized by concentrated strains in the fibre bundle in front of the notch, see Figure 5.18. In a FML there is an interaction between the deformation of the metal layers and the deformation of the prepreg in a way that they influence each other. Indeed, in the right-hand side of Figure 5.16 and Figure 5.17, the deformation measured with DIC on the prepreg side shows features characteristic of the metal layer, the butterfly shape. This is an important aspect which proves that the displacement compatibility between metal and prepreg is not violated.

![Figure 5.16](image.png)

**Figure 5.16** Comparison of the experimentally measured strain fields of the 2024-T3 and S2-glass fibre/epoxy layers. Scale from 0 to 0.5% and σ = 200 MPa
Further proof of the compatibility of the deformation is given in Figure 5.19 where the strain gradients ahead of the crack tip are shown for both metal and composite constituents (2024-T3 and S2-glass fibre). The two strain distributions are very similar to each other, except at the location of the crack tip where small out-of-plane displacement due to crack-tip buckling have introduced small differences in the measurements.

Figure 5.17  Comparison of the experimentally measured strain fields of the 2024-T3 and M30SC -carbon fibre/epoxy layers. Scale from 0 to 0.5% and $\sigma = 238$ MPa

Figure 5.18  Experimentally measured strain fields of the full composite M30SC -carbon fibre/epoxy specimen. Scale from 0 to 0.5% and $\sigma = 250$ MPa
5. Understanding the Failure Mechanisms in FMLs

Figure 5.19  Experimentally measured strain field gradients in front of the crack tip for both the metallic and composite layers of one-to-one layup (2024-T3/S2-glass).

Furthermore, it has been observed in almost all tests that the deformation field on both metal and fibre layers presents two or more areas, away from the crack tip, where the strain concentrates and develops more than in other areas (see black arrows in Figure 5.16 and Figure 5.17 and see Figure 5.13). This represents the characteristic plane-stress type of deformation illustrated in Figure 5.20-a. Plastic deformation, visualized as slips, is a result of shear stresses. Consequently the different planes of maximum shear stress result in different patterns of deformation. Slips on planes through the X axis and at 45 to the sheet surface results in the 45 shear type of deformation typical for plane stress, see Figure 5.20-a. Slips on planes through the Z-axis gives rise to the hinge-type deformation, typical for plane strain [17]. It is therefore believed that the occurrence of such deformation patterns, highlighted in Figure 5.16 and Figure 5.17 with arrows, is related to the plane stress condition of the thin the metallic sheets of an FML.

Figure 5.20  a. 45° shear deformation in plane stress, b. Hinge type deformation in plane strain[17]
Strain distribution in fibre layers in presence of bridging

Fatigue tests were performed on laminates with inverted lay-up to measure the deformation of the fibre/prepreg layer in presence of a fatigue crack, see section 5.4.2. Figure 5.21 shows the strain field of the prepreg layer of an inverted 2024-T3/S2-glass lay-up laminate with a fatigue crack in the metal layers. The delamination has a triangular shape and the strain values in the disbonded area present on average a constant value, except in the vicinity of the crack tip where a peak is present. The calculation of the stress gradient is possible if the measured strain is related to the stress with the stress-strain curve of the prepreg. The right-hand side of Figure 5.21 illustrates qualitatively the stress distribution in the fibre layer, both ahead and behind the crack tip. Several images were captured at different fatigue crack lengths while applying the maximum fatigue load. Taking the stress values along the crack line enables to obtain the actual stress distribution in the fibre layer. This is depicted in Figure 5.22 where the evolution of the stress distribution in the fibre layer due to the growing fatigue crack is shown. The shape of the measured strain, recalculated in stress, along the fatigue crack is in agreement with the shape of the bridging stress analytically calculated by Alderliesten in [13, 19].

The quasi-constant value of the stress in the wake of the crack is the result of a balance between crack opening and delamination extension. In the vicinity of the crack tip, due to the small amount of delamination, a peak in the stress is present. This stress peak represents the link between the stress behind and ahead of the crack tip. As visible, a stress peak is also present at the tip of the saw-cut (or the beginning of the fatigue crack extension), which is due to the stress concentration at the saw-cut tip. Superposition of both bridging stress and far field stress generates a total stress distribution in the bridging fibres containing two stress peaks.

The observations concerning the double stress peak reveal new insight for what concern the damage evolution in fatigue crack in FMLs. Indeed, in [13, 19] the author stated that fibre failure in a fatigue crack may initiate in correspondence of the stress peak behind the crack tip. This statement is based on the calculated stress peak of the bridging stress which indeed shows such a peak [13, 19]. The occurrence of fibre failure is related not only to the local bridging stress, but to the local total stress, which accounts also for the stress distribution related to the far-field stress and initial notch geometry.

In general it is not always possible to predict exactly on which side of the fatigue crack length fibre failure may occur because there are a number of parameters which influence the total stress distribution, such as:

- local delamination length -and resistance
- ratio between saw-cut and fatigue crack lengths
- amount of plasticity in the metal layers
All residual strength tests performed on large scale panels containing a fatigue pre-crack showed that fibre failure occurred at the saw-cut tip rather than at the fatigue pre-crack tip. This aspect will be discussed in detail in chapter 6.

Figure 5.21 Illustration of the strain field of the fibres layer measured with DIC in a inverted lay-up laminate (2024-T3/S2-glass fibre) with containing a fatigue crack of 17.5 mm (left). Qualitative illustration of the stress distribution in the fibres layer (right)

In Figure 5.22, when the fatigue crack reached a length of 21 mm, the load was increased quasi-statically and images where taken at fixed load increments. Figure 5.23 illustrates the evolution of the stress in the fibres layer when high static loads have been applied. Despite the absence of crack propagation in the load range between 110 MPa and 230 MPa, the stress peak shifts slightly to the right. This is explained with the high deformation developed in front of the crack tip, which increases the strain of the fibres layer ahead of the tip.

Crack propagation in the metal layers was observed when the load reached 230 MPa. This induces the stress peak to shift further to the right, following the crack extension. The stress peaks at the left-hand side of the curves, see for example Figure 5.22, have been omitted in Figure 5.23 for clarity of illustration. Indeed, because of fibre splitting occurred ahead of the saw-cut tip, the strain value obtained with DIC were not trustable because changes in the spackle pattern occurred resulting in a poor correlation.

The bridging mechanism is a rather complex system of interactions between metal and fibre constituents through their interfaces. It has been experimentally observed that fibre failure occurs at the location of the highest stress peak. For residual strength tests performed on large Glare panels containing a fatigue crack, obtained from large saw-cut, fibre failure often occurs at the tip of the saw-cut. This is an important aspect and has been considered and implemented in the analytical prediction model described in chapter 7.
Figure 5.22 Fibre stress measured with DIC on the external prepreg along three fatigue cracks in a 2024-T3 /S2-glass laminate with inverted lay-up. $S_{am}=110$ MPa, $R=0.05$.

Figure 5.23 Fibre stress measured with DIC on the external prepreg along a fatigue crack in a 2024-T3 /S2-glass inverted laminate under increasing static loads.
5.5 Conclusions

DIC showed to be a powerful strain measuring technique, able to provide information related to plastic zone size and shape, delamination shape and growth, fibre layer deformation.

The presence of fibre layers (orthotropic material) influences the behaviour of the metal constituent (isotropic material). Indeed, the strain field measured on the external metallic layer of an FML changes, if compared to a metal laminate, because the fibre layers interact with the metal layers. Indeed, it was observed that the plastic zone changes in shape and size. The angle and the extension of the plastic zone, measured with DIC on the outer metal layer, changes and tends to increases if higher stiffness higher fibres are used.

Measuring the strain in configurations with prepreg at the outside provided a further understanding on how fibre and metal influence each other during the deformation process. Once bonded to a metal layer, fibres do no longer deform as an orthotropic layer, but rather tend to follow the deformation of the metal layer. On the other hand, the metal layer is influenced in its deformation by the orthotropic effect induced by the fibre layer.

Bridging fibres behave as a second load path, this results in a consistent change in the deformation field of the metal layer being bridged. The plastic zone in presence of bridging fibres is drastically smaller than in the case without bridging fibres. This is attributed to the load transfer to the bridging fibres which also determines potential plastic deformation above and below the delaminated area.

References


[6] [www.delta-tech.it](http://www.delta-tech.it).


Experimental data for failure criteria and model validation

Abstract – This chapter provides a detailed discussion on the results obtained during the experiments related to the failure criteria. Three types of investigations are discussed: first, experiments to prove that the CTOA can be used as failure criterion for the metal constituent, and to support the model development. Second, characterization of the static delamination growth under increasing load, together with the generation of input parameters for the prediction model. Third, experimental results on large-scale residual strength tests further used for the validation of the model.

6.1 Introduction

As discussed in chapter 3, metal cracking and delamination growth are two of the main failure mechanisms involved during the residual strength failure sequence in FMLs, and their analytical description requires a deep understanding of the actual phenomena. The CTOA has been employed as failure criterion for elastic-plastic failure of monolithic metal subjected to increasing load [1]. It is therefore believed that similar approach might be used for FMLs to characterize the failure sequence of the metallic constituent.

A large number of CTOA measurements therefore have been performed on several FML configurations, varying parameters such as lay-up, grade, type of fibre, type of metal, to investigate the potential effects of such changes on the CTOA. Results are presented in the form of CTOA vs. crack extension curve.
In a similar way, static delamination growth tests were performed on several FMLs configurations. It is already known that static delamination growth in FMLs can be affected by the occurrence of plastic deformation in the metallic layers [2].

To develop a prediction model for residual strength in FMLs, it is important to properly address both failure mechanisms and to simplify their analytical descriptions.

### 6.2 Crack-tip-opening angle in FMLs

Based on the studies reported in the literature, see section 4.5, it was decided to evaluate the potential benefits of using the CTOA as fracture parameter for predicting the residual strength in FMLs.

#### 6.2.1 Objective

The objective of the investigation presented in this section is to study and prove the benefit of using the CTOA in FMLs. The driving idea is to evaluate the applicability of CTOA as fracture parameter for FMLs. The effect of bridging fibres on the CTOA vs. crack extension curve has been investigated considering different lay-ups, fatigue pre-crack lengths, geometrical dimensions and type of fibre. It has been reported in [1,6,14-19] that the CTOA is considered a material parameter independent of the specimen geometry, therefore it is plausible to assume that the presence of fibres, both ahead and behind the crack tip, should not strongly affect the critical CTOA of the metal layers. This means that, despite the presence of fibres, the thin metal sheets of an FML should exhibit a CTOA vs. crack extension curve similar to an equivalent metal laminate, or monolithic metal with comparable thickness.

Results generated in this test campaign are further used as support for the development of the prediction model and as input parameter for the failure criterion.

#### 6.2.2 Materials

The experiments were carried out on three types of materials: monolithic aluminium 2024-T3, laminated aluminium 2024-T3 (with metal sheets 0.4 mm thick) and several Glare 2 and Glare 3 laminates. The panels were manufactured at Delft University of Technology following the standard manufacturing process for Glare, explained in chapter 2. Next to the standard Glare configurations some non-standard configurations were tested, including Zylon fibre-based FML and carbon fibre-based FML.
6. Experimental Data for failure criteria and model validation

6.2.3 Geometry and test matrix

The specimens used in the CTOA tests were CCT panels with widths of 170, 400 and 800 mm. Figure 6.1 illustrates the geometry of the panels, while dimensions are provided in Table 6.1 for the 170 mm wide panels, in Table 6.2 for the 400 mm wide panels and in Table 6.3 for the 800 mm wide panels.

**Table 6.1 Test matrix for CTOA measurements in 170 mm wide panels**

<table>
<thead>
<tr>
<th>Test</th>
<th>Lay-up</th>
<th>Width</th>
<th>Length</th>
<th>Thickness</th>
<th>Saw-cut</th>
<th>Pre-crack</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2024-T3 3/2-0.3</td>
<td>170</td>
<td>280</td>
<td>1.15</td>
<td>7</td>
<td>8</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>2024-T3 3/2-0.4</td>
<td>170</td>
<td>280</td>
<td>1.7*</td>
<td>7</td>
<td>8</td>
<td>0.04</td>
</tr>
<tr>
<td>3</td>
<td>2024-T3 3/2-0.4</td>
<td>170</td>
<td>280</td>
<td>1.7*</td>
<td>7</td>
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<td>0.10</td>
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<tr>
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<td>2024-T3 3/2-0.4</td>
<td>170</td>
<td>280</td>
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<td>36</td>
<td>0.21</td>
</tr>
<tr>
<td>5</td>
<td>Glare 3-3/2-0.4</td>
<td>170</td>
<td>280</td>
<td>1.75</td>
<td>7</td>
<td>8</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>Glare 3-3/2-0.4</td>
<td>170</td>
<td>280</td>
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<td>0.16</td>
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<tr>
<td>7</td>
<td>Glare 2-3/2-0.4</td>
<td>170</td>
<td>280</td>
<td>1.75</td>
<td>7</td>
<td>18</td>
<td>0.10</td>
</tr>
<tr>
<td>8</td>
<td>Glare 2-3/2-0.4</td>
<td>170</td>
<td>280</td>
<td>1.75</td>
<td>7</td>
<td>36</td>
<td>0.21</td>
</tr>
<tr>
<td>9</td>
<td>2024-T3 3/2-1</td>
<td>170</td>
<td>280</td>
<td>3.25</td>
<td>7</td>
<td>8</td>
<td>0.04</td>
</tr>
</tbody>
</table>

* 2 plies of 0.1 mm thick FM94 epoxy resin film in each adhesive layer (=four in total)

**Table 6.2 Test matrix for CTOA measurements in 400 mm wide panels**

<table>
<thead>
<tr>
<th>Test</th>
<th>Lay-up</th>
<th>Width</th>
<th>Length</th>
<th>Thickness</th>
<th>Saw-cut</th>
<th>Pre-crack</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>2024-T3 3/2-0.4</td>
<td>400</td>
<td>600</td>
<td>1.45*</td>
<td>35</td>
<td>47</td>
<td>0.23</td>
</tr>
<tr>
<td>10</td>
<td>Glare 3-3/2-0.4</td>
<td>400</td>
<td>600</td>
<td>1.75</td>
<td>46</td>
<td>47</td>
<td>0.23</td>
</tr>
<tr>
<td>11</td>
<td>Glare 3-3/2-0.4</td>
<td>400</td>
<td>600</td>
<td>1.75</td>
<td>38.5</td>
<td>47</td>
<td>0.23</td>
</tr>
<tr>
<td>12</td>
<td>Glare 2-3/2-0.4</td>
<td>400</td>
<td>600</td>
<td>1.75</td>
<td>38.5</td>
<td>47</td>
<td>0.23</td>
</tr>
<tr>
<td>13</td>
<td>Glare 2-3/2-0.4</td>
<td>400</td>
<td>600</td>
<td>1.75</td>
<td>38.5</td>
<td>47</td>
<td>0.23</td>
</tr>
<tr>
<td>14</td>
<td>Glare 3-5/4-0.4</td>
<td>400</td>
<td>600</td>
<td>1.15*</td>
<td>38.5</td>
<td>47</td>
<td>0.23</td>
</tr>
<tr>
<td>15</td>
<td>2024-T3 3/2-0.3</td>
<td>400</td>
<td>600</td>
<td>1.55</td>
<td>38.5</td>
<td>47</td>
<td>0.23</td>
</tr>
<tr>
<td>16</td>
<td>Zylon 3-3/2-0.3</td>
<td>400</td>
<td>600</td>
<td>1.52</td>
<td>38.5</td>
<td>47</td>
<td>0.23</td>
</tr>
<tr>
<td>17</td>
<td>Carbon 3-3/2-0.3</td>
<td>400</td>
<td>600</td>
<td>1.52</td>
<td>38.5</td>
<td>47</td>
<td>0.23</td>
</tr>
</tbody>
</table>

* 1 ply of 0.1 mm thick FM94 epoxy resin film in each adhesive layer (=two in total)

**Table 6.3 Test matrix for CTOA measurements in 800 mm wide panels**

<table>
<thead>
<tr>
<th>Test</th>
<th>Lay-up</th>
<th>Width</th>
<th>Length</th>
<th>Thickness</th>
<th>Saw-cut</th>
<th>Pre-crack</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Glare 4B-3/2-0.4</td>
<td>800</td>
<td>1000</td>
<td>2</td>
<td>103</td>
<td>200</td>
<td>0.25</td>
</tr>
<tr>
<td>19</td>
<td>Glare 4B-3/2-0.4</td>
<td>800</td>
<td>1000</td>
<td>2</td>
<td>103</td>
<td>300</td>
<td>0.37</td>
</tr>
<tr>
<td>20</td>
<td>Glare 4B-3/2-0.4</td>
<td>800</td>
<td>1000</td>
<td>2</td>
<td>103</td>
<td>400</td>
<td>0.5</td>
</tr>
</tbody>
</table>
All specimens contain a saw-cut ($2a_0$) and fatigue cycles were applied to extend the crack up to a prefixed length ($2a_f$). In this way it is possible to study the effect the amount of bridging fibres has on the CTOA in FML. Anti-buckling guides were used in all tested panels to prevent crack-tip buckling, which would negatively affect the measurements and the residual strength results [11-13].

Because of the large amount of data generated in this test campaign, only selected results will be discussed in the next sections to point out the most important findings.

6.2.4 Measurements and test execution
The tests were performed in the 250 kN (test 1-8), 500 kN (test 9-17), and 1000 kN (test 18-20) computer controlled servo-hydraulic testing machines. The load was increased under cross-head displacement control at a constant rate varying from 0.5 to 2 mm/min until a stable tearing event occurred, the displacement increase was then paused.

Several publications report different available techniques for CTOA measurements (Digital Image Correlation [3, 4], Optical Measurement [5-8] and $\delta$ [9]). To obtain the CTOA value for each stable tearing event the Optical Measurement method was used. Just before the occurrence of crack extension, and during crack propagation, a set of images was captured using a digital camera with a resolution of 2048 X 2048 pixels. The fixed focal distance lens adopted to magnify the images provided a field of view of 5 x 5 mm which, together with the high resolution of the digital images, provided a high detailed visualization of the area surrounding the crack tip. The camera was mounted on a computer controlled translation stage which enabled
controlling the position of the camera along the three axes of translation with an accuracy of 0.001 mm. Each image was stored on a hard disk and subsequently post processed with a dedicated algorithm developed at Delft University of Technology by the author of the present thesis. Both applied force and cross-head displacement were constantly recorded every second.

6.2.5 Definition of CTOA curve

The CTOA is defined as the angle between two straight lines, one traced by connecting the crack tip to a point selected on the upper flank of the crack, and the other line traced from the crack tip to a point on the lower flank. The selected points result from the interception between the crack flanks and a circle with the centre at the crack tip and radius equal to \( d \).

The measurements were performed on one side of the specimen only, assuming that small differences between each metal layer are present. As described in [10], to ensure consistency in the measurements, the CTOA was measured at three distances behind the crack tip: 0.5, 1 and 1.5 mm, as illustrated in Figure 6.2. The final value of the CTOA results from the average of the three measurements.

As illustrated in Figure 4.12, the measurements of the CTOA are plotted versus the crack extension \( \Delta a \), which is the distance measured from the tip of the fatigue pre-crack to the tip of the quasi-statically extended crack. The constant part of the CTOA curve is denoted in literature [1] as “CTOA_c”.

![Figure 6.2 Example of CTOA measurement illustrating three measurement distances](image-url)
It is important to point out here that for the present study, the term CTOA_c means the “critical CTOA” and not the constant part only. Indeed, the whole CTOA curve is the collection of those CTOA values for which crack extension occurred. The following definitions apply through the whole thesis:

- The CTOA is the generic term indicating the angle formed between the two crack flanks and measured at a fixed distance, \( d \).
- The CTOA_c is the “critical CTOA” related to a given crack extension, \( \Delta a \). This means that for a given crack length, further crack extension occurs if the CTOA equals the CTOA_c.
- The CTOA curve is the graph obtained by plotting the CTOA_c vs. \( \Delta a \), see Figure 6.3.

The CTOA curve in Figure 6.3 has been fitted with an exponential trend line of the type \( y = ae^{-bx} + c \) following a non-linear least squares method, where the \( c \) value represents the constant value of the CTOA curve [11]. The trend line equation is used in the prediction model as input parameter for the CTOA failure criterion, as described in chapter 7.

![Figure 6.3](Image)  
**Figure 6.3**  
Example of CTOA vs. crack extension curve, used as input parameter. 2024-T3 metal laminate \( t=0.4 \text{ mm} \) (single metal sheet)
6.2.6 Results and discussion

The large amount of results obtained within the experimental programme cannot be presented here, however, with the help of some examples, the most important observed effects of material- and geometrical parameters on the CTOA curve will be discussed.

The effect of some parameters, such as the fatigue crack length, the amount of bridging fibres, the CTOA measurement distance, and the metal thickness will be discussed in the following sections. This approach is important to prove that the CTOA is eligible as failure criterion for FMLs.

Effect of the ratio fatigue crack length

Three metal laminates 2024-T3 3/2-0.4 containing three different fatigue pre-cracks were tested (Test 2, 3 and 4). As shown in Figure 6.4, the CTOA curve of the tested laminate does not show dependency on the $2a_f/W$ ratio. In all three ML configurations, the CTOA decreases from high values, $10^\circ\sim12^\circ$, to a constant value of about $5^\circ\pm0.6^\circ$. This transition is completed when the crack extends up to a value between 8 and 12.5 mm.

![Figure 6.4](image_url)

CTOA measured on three metal laminates 2024-T3 3/2-0.4 with different pre-crack length, and therefore different $2a_f/W$ ratios.

The results for Glare 3-3/2-0.4 and Glare 2-3/2-0.4 are illustrated in Figure 6.5. Also for these configurations the CTOA curve follows the same trend: high values at the beginning which decrease towards a constant value of about $5.2^\circ\pm0.65^\circ$.  

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Similar trends have been obtained in other tests, where different panel’s width were used, 400 and 800 mm. From these observation it emerges that the $2a_f/W$ ratio does not have a significant influence on the CTOA curve of FMLs. This statement is supported by a large number of tests where the $2a_f/W$ ranged from 0.04 to 0.37, see Table 6.1, Table 6.2 and Table 6.3.

**Comparison between ML and FML**

To study the effect the bridging fibres may have on the CTOA curve, a comparative analysis between ML and FML has been made. In Figure 6.6 to Figure 6.8 the metal laminate 2024-T3 3/2-0.4 is compared with Glare 2-3/2-0.4 considering different pre-crack lengths of 8, 18 and 36 mm.

In a FML, a longer fatigue pre-crack is accompanied by a larger number of bridging fibres which act as a second load path. As visible from the comparison between ML and FML, bridging fibres induce some small differences in the first part of the CTOA curve. This difference decreases and eventually disappears as the crack extends away from the pre-crack tip, where the influence of the bridging fibres is higher. Indeed, when the crack extended statically fibre failure occurred ahead of the crack, but not in the wake of the fatigue pre-crack where static delamination took place.

Although the bridging fibres stayed intact for large part of the stable crack growth, their effect on the CTOA$_c$ reduced with crack extension. To point out this
behaviour, exponential trend curves of the type $y = a \cdot e^{-bx} + c$ are plotted over the measurement points in Figure 6.6 to Figure 6.8.

**Figure 6.6**  CTOA vs. crack extension, comparison between ML and FML with a pre-crack of 8 mm

**Figure 6.7**  CTOA vs. crack extension, comparison between ML and FML with a pre-crack of 18 mm
What discussed so far has been observed also in other configurations containing different prepregs.

It has been observed that relative large differences between ML and FML occur in configuration containing unidirectional prepreg (e.g. Glare 2), while differences reduce if a cross-ply grade is considered, see from Figure 6.6 to Figure 6.8.

The analysis of the results provided so far would not be fully satisfactory in the view of using the CTOA as fracture parameter for FMLs. Indeed, the relative large differences in the first part of the CTOA curve denote an influence of bridging fibres on the measurements of the CTOA, see Figure 6.6 to Figure 6.8 and this means that a CTOA curve generated with a test on ML might not be fully representative of the CTOA curve of a FML containing the same metal constituent. This consideration applies especially for configurations containing fatigue pre-crack, thus containing bridging fibres.

Based on the above considerations, a further analysis of the CTOA curve has been performed aiming to understand the potential effect of the measurement distance on the CTOA, for example the distances illustrated in Figure 6.2. It was considered the opportunity to compare the CTOA curves between ML and FML using the measurements relative to three measurement locations separately. This is discussed in the next section.
**Effect of the CTOA measurement distance from the crack tip**

As described in section 6.2.5, the CTOA$_c$ has been measured at three different locations behind the crack tip for each crack extension and the average values was used to obtain the actual CTOA$_c$ value. Although the three measurements belong to the same crack under the same critical conditions, some differences can be pointed out. Indeed, the three measurements taken at 1.5, 1 and 0.5 mm behind the crack tip are, in this section, respectively denoted as CTOA$_{1.5}$, CTOA$_1$ and CTOA$_{0.5}$. The CTOA$_c$ measurements made at each location are illustrated for MLs (Test 3) and FMLs (Test 8) in Figure 6.9 to Figure 6.11.

If the measurements are performed closer to the crack tip, see Figure 6.9 to Figure 6.11, the difference between the CTOA curves of ML and FML decreases. In particular, in Figure 6.9 and Figure 6.10 the difference between the two laminates is larger for higher values of the CTOA$_c$, while it reduces with the crack propagation. Indeed, in the constant part of the curve, the CTOA$_c$ values of both ML and FML are roughly the same.

![Figure 6.9 CTOA$_{1.5}$ curve. Comparison between ML and FML, 2a$_0$/W=0.1](image)

On the other hand, the difference between ML and FML seems to be very small if the CTOA$_{0.5}$ curve is considered, see Figure 6.11. Although the CTOA$_{0.5}$ curve presents higher scatter, the measurements of the CTOA made at 0.5 mm behind the crack tip seem to be less affected by the bridging fibres. Indeed, it is reasonable to consider that the larger difference at the beginning of the CTOA curves illustrated in Figure 6.9 and Figure 6.10 is due to the presence of bridging fibres which somehow locally influence the crack opening shape of the metal layers. This effect reduces
The Residual Strength Failure Sequence in Fibre Metal Laminates

when the crack extends and moves away from the bridging fibres. The crack opening shape is important because the CTOA is the result of geometrical considerations, see equation 7.2, based on the crack opening displacement (COD) at specific locations.

![Figure 6.10 CTOA\(_1\) curve. Comparison between ML and FML, 2a_0/W=0.1](image1)

![Figure 6.11 CTOA\(_0.5\) curve. Comparison between ML and FML, 2a_0/W=0.1](image2)
Figure 6.12 shows the crack opening shapes at 0 and 2 mm of crack extension for four different 400 mm wide laminates. Each of these curves represents the COD at the moment prior crack extension (e.g. critical opening).

As illustrated, the arrows indicate the location 1.5, 1 and 0.5 mm behind the crack tip. Despite the different load levels needed to reach the critical condition, the displacements of the crack flanks at 0.5 mm behind the tip of the different laminates do not differ from each other. Larger differences are visible in the measurements taken further behind the crack tip.

Although a similar displacement is measured at 0.5 mm behind the crack tip (1.5 mm in Figure 6.12), the applied load necessary to reach the critical opening for that specific crack length varies significantly from 181 MPa (metal laminate) to 492 MPa (Glare 2). The amount of force necessary to reach the critical condition, and thus crack propagation, changes in relation to the different lay-ups and number of fibre layers in loading direction. The energy required to extend the crack in configurations with unidirectional lay-up is higher than in cross-ply configurations, but the CTOA_c is about the same. This aspect has been observed in all tested specimens, independently on the dimension of the panels.

Based on the above discussion, it can be concluded that the CTOA curve measured at 0.5 mm behind the crack tip is the most representative to be used as fracture criterion for the prediction of the residual strength in FMLs containing bridging fibres.
Although, measuring the CTOA at 0.5 mm behind the crack tip is a source of uncertainties in determining the CTOA\(_{c}\), and is a challenging operation that requires high precision equipment for positioning camera systems. The area behind the crack tip requires either relative high magnification lenses, or high resolution cameras. This is necessary to perform a proper CTOA\(_{c}\) measurement, because uncertainties in the location in the actual location of the crack flanks can is one of the sources of scatter in the measurement.

**Effect of the thickness**

It has been reported in [1, 7] that the CTOA curve, and in particular its constant value (CTOA\(_{c}\)), is dependent on the specimen thickness. This agrees with the thickness dependency of the R-curve [3, 7, 12-15] because, although in a different way, both the CTOA curve and the R-curve characterize the toughness of the material being considered.

Based on this consideration it is expected to observe a metal thickness dependency of the CTOA curve also for the relative small thicknesses range typical of FMLs. This aspect is illustrated in Figure 6.13, where CTOA curves of 2024-T3 3/2 metal laminate specimens with single sheet thickness of 0.3, 0.4 and 1 mm are shown. The CTOA\(_{c}\) reduces with the reduction of the thickness as indicated by the constant values of the CTOA for the three thicknesses. This trend disagrees with the results reported in [1,7], where the authors described how the CTOA\(_{c}\) decreases by increasing the thickness. A potential explanation of this apparent contradiction can be based on references [15-17].

In the cases reported in [16], the investigation of the effect of the thickness on the ductile fracture toughness was experimentally carried out on C(T) plates made of steel alloy with thicknesses of 1.25, 1.64 and 4.06 mm. In this investigation the authors showed that both the trend of the measured toughness’s (\(J_c\)) and those calculated by means of FEA present a peak. While in [15] a similar study was carried out on Al 7075-T6 with thicknesses of 0.5, 1, 2 and 3.4 mm. It was observed that a reduction in thickness down to about 2 mm gives an increase in the plane-stress fracture toughness \(K_c\), while for thicknesses less than 2mm the \(K_c\) reduces again, see Figure 6.14.
Figure 6.13  Influence of the metal sheet thickness on the CTOA curve. Laminated 2024-T3 3/2, thickness 1, 0.4 and 0.3 mm. All test specimens are metal laminates.

The relationship between thickness and toughness, qualitatively illustrated in Figure 6.14, should apply also to the metal layers of FMLs. Indeed, the CTOA curves
depicted in Figure 6.13 show a dependency of the CTOA from the metal thickness which correlates with the residual strength curves of the same specimens shown in Figure 6.15. Indeed, the CTOA curve with the highest values is the one of the 1mm thick metal laminate, which is also the laminate which shows the highest residual strength in Figure 6.15.

The effect of the metal sheet thickness on the fracture behaviour of FMLs was also studied through fractographic analysis [11]. Three main areas characterize the fractured surface: flat surface (fatigue pre-crack), flat-to-slant transition (quasi-static propagation) and slant surface (quasi-static propagation). This seems to agree with what is reported in [1, 5, 7, 19, 20] where the authors stated that the extension of the “flat” stable tearing area, which follows the fatigue pre-crack, is thickness dependent, and reduces with reducing thickness.

In case of FMLs, due to the very small thickness of the metal layers, this stable tearing area characterized by a “flat” surface is not present or is reduced to a very small portion.

It can be concluded that the CTOA depends on the metal thickness and a proper thickness-related CTOA curve has to be used as fracture parameter. The curve can be obtained experimentally from a metal laminate CCT specimen containing metal sheets with the same thickness of the metal sheets used in the FML.

![Figure 6.15](image-url)  
*Residual strength curves for three different thicknesses. Laminated 2024-T3 3/2, thickness 1, 0.4 and 0.3 mm*
6.3 Static delamination characterization

Static delamination is an important mechanism present during the residual strength failure sequence, as described in chapter 3. In particular, this mechanism is fundamental for residual strength of fatigue crack configurations, where the pre-existing fatigue delamination is subjected to quasi-static increasing load, potentially inducing static delamination extension.

To fully understand this mechanism and to obtain the input data for the analytical model describing the delamination growth, quasi-static delamination growth tests were performed on several lay-ups, considering two different prepreg systems and two different aluminium alloys.

6.3.1 Objective of the investigation

The objective of this programme is to generate input data for delamination growth in FMLs. A relationship between the quasi-static delamination growth and the strain energy release rate for delamination (SERR) is therefore formulated based on previous research [21-24]. In addition, the effect of metal plasticization on the delamination growth process is investigated as well.

The understanding generated in this investigation is meant to provide quantitative data related to the static delamination growth, but also to address the modelling of the static delamination growth of fatigue cracks subjected to increasing load.

6.3.2 Materials and geometry

In [25], Alderliesten has demonstrated that the fatigue delamination growth rate in FMLs is not affected by the lay-up. Indeed no differences were observed in fatigue delamination growth rate between mode II (5/4 lay-up with the interrupted metal layer in the middle) and mode mix tests (2/1 lay-up, with interrupted metal layers at the outside). Based on that, only the 2/1 lay-up was selected for this investigation, for which an illustration is depicted in Figure 6.16. It is important to point out here that the 2/1 lay-up does not provide an actual mixed mode delamination growth. Indeed the out-of-plane displacement of the interrupted external metallic layers, induces a not-driving mode I opening. This mode I opening is therefore negligible compared to the mode II driven delamination growth.

The specimens were manufactured at Delft University of Technology following the standard manufacturing process described in chapter 2. The artificial pre-crack was created during laminating, by positioning two aluminium sheets head-to-head. The edges of the aluminium sheets were deburred to avoid any unwanted damage to the adjacent fibre layer.
For this specific test programme S2-glass/FM94 and M30SC-carbon/DT120 prepreg systems were considered, while 2024-T3 and 7475-T761 aluminium alloys, both 0.4 mm thick, were selected. Each lay-up was tested in two configurations: one containing thin aluminium sheets with a thickness of 0.4 mm, and the other one containing the same 0.4 mm thick aluminium layer plus 1 mm aluminium layer bonded on the thinner aluminium sheets, as illustrated in Figure 6.16. The thicker configuration has been used to prevent the aluminium layers to undergo potential plastic deformation, which would not have been compatible with the definition of elastic strain energy release rate.

### 6.3.3 Measurements and test execution

In total 26 delamination specimens were tested to investigate the delamination behaviour: 10 with thick configuration (denoted with the term \( T \)) and 16 with thin configuration (denoted with the term \( t \)). An overview of the test matrix is provided in Table 6.4.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Materials</th>
<th>Lay-up</th>
<th>Type</th>
<th>Metal orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S2 / 2024-T3</td>
<td>0/90</td>
<td>T</td>
<td>L</td>
</tr>
<tr>
<td>2</td>
<td>S2 / 2024-T3</td>
<td>0/0</td>
<td>T</td>
<td>L</td>
</tr>
<tr>
<td>3</td>
<td>M30/ 7475-T761</td>
<td>0/0</td>
<td>T</td>
<td>L</td>
</tr>
<tr>
<td>4</td>
<td>S2 / 2024-T3</td>
<td>0/90</td>
<td>t</td>
<td>L</td>
</tr>
<tr>
<td>5</td>
<td>S2 / 2024-T3</td>
<td>0/0</td>
<td>t</td>
<td>TL</td>
</tr>
<tr>
<td>6</td>
<td>S2 / 2024-T3</td>
<td>0/0</td>
<td>t</td>
<td>L</td>
</tr>
<tr>
<td>7</td>
<td>M30 / 2024-T3</td>
<td>0/0</td>
<td>t</td>
<td>L</td>
</tr>
<tr>
<td>8</td>
<td>M30/ 7475-T761</td>
<td>0/0</td>
<td>t</td>
<td>L</td>
</tr>
</tbody>
</table>

*The symbols “T” and “t” denote respectively thick and thin configurations.*

The specimens were tested at the 100 kN servo-hydraulic, close loop mechanical and computer controlled testing machine. To replicate the actual condition in a fatigue delamination configuration, fatigue loading (\( \sigma_{\text{max}}=100 \text{ MPa}, R=0.1 \)) was applied to all specimens such to create a fatigue delamination of a length of 10 mm. Subsequently, increasing force was applied in displacement control until delamination extension occurred, the test was then paused. When the displacement was hold, the delamination grew until the SERR dropped below the critical value \( G_c \). The reduction of the SERR, for the tested configuration, was always accompanied by a drop of the applied force. After the SERR dropped below the critical value, the delamination growth arrested and measurements of the delamination length were performed.
Applied force and cross-head displacement were recorded and delamination length measurements were taken in each interface using a high magnification digital camera system. An example of measurement is illustrated Figure 6.17, where two delaminations are visible.

6.3.4 Results and discussion

Due to the large amount of tests performed, only the most important aspects related to the strain energy release rate and delamination growth are here pointed out.

Delamination growth

Mode II static delamination occurs in the form of separation between the fibre layers and metallic layers due to the increasing shear loads at the interface. The accumulation of strain energy is a progressive process which results in delamination initiation and delamination growth. The initiation phase is characterized by an increasing force related to moderate increase in delamination length, while the steady-state growth is characterized by a constant force associated to relative large extension of delamination. The steady-state growth phase is representative of the critical condition; indeed, any increment of force is accompanied by static delamination extension until the a new equilibrium is reached. When delamination occurs under fixed displacement, the force reduces due to the loss of stiffness in the laminate.

Figure 6.16 Illustration of delamination specimen geometries for delamination growth tests.
The delamination growth can be studied using the SERR. This means that the delamination growth is related only to the strain energy stored into the laminate and made available for generating a delamination extension when the critical condition is reached. An example of remote stress vs. delamination length is shown in Figure 6.18, where the curve is an average of the four measurements performed as described in section 6.3.3. Given a fixed geometry, the amount of force that may generate a delamination extension is related to a number of variables, such as stiffness of the constituents, MVF and interfaces properties.
Strain Energy Release Rate

The SERR is an elastic fracture mechanics parameter which has been successfully used for characterizing the fatigue delamination in FMLs [5,81] and is calculated based on the total stress cumulated in the bridging fibres. Using the derivation given in [25, 26] the total elastic strain energy release rate is calculated as

\[
G = \frac{n_f t_f}{2 \gamma E_f} \left( \frac{n_m t_m E_m}{n_m t_m E_m + n_f t_f E_f} \right) \left( \sigma_{br} + \sigma_f \right)^2
\]

(6.1)

Equation (6.1) represents the amount of elastic energy stored inside the laminate and available for mode II delamination. If the stored strain energy reaches the critical level, \( G_c \), static delamination occurs; \( G_c \) is therefore a parameter that needs to be determined by experimental testing.

It has been experimentally observed that \( G_c \) strongly depends on the relative stiffness of constituents, MVF, surface pre-treatment and yielding strength of the metal; some of these aspects are therefore addressed in the following discussion.

To investigate the effect of the yield strength on \( G_c \), tests were performed considering the rolling direction perpendicular of the applied load. Indeed, the yield strength of a 0.4 mm thick 2024-T3 aluminium sheet tested perpendicular to the rolling direction (LT) is 15% less than the yield strength in rolling direction (L). The difference in yield strength reflects on the delamination growth because of the occurring plastic deformation; indeed, the elastic strain energy available for delamination reduces and transforms in plastic strain energy, due to permanent plastic deformation of the metal sheets. This is visible in Figure 6.19, where the curves of the SERR for both L and LT configurations are plotted together. In both cases the curve sets to a steady value, \( G_c \), after an increasing part related to delamination initiation. The higher \( G_c \) for the LT configuration is due to the higher delamination resistance obtained in LT configuration because of the plastic strain.

Using a stiffer fibre, such as M30 carbon fibre, induces a drastic change in the delamination behaviour. Tests performed on such a laminate (2024-T3 L with M30/DT120) show an increasing value of the SERR related to delamination extension, as shown in Figure 6.20. The reason for this behaviour might be two-fold:

- The higher shear resistance of the DT120 epoxy resin provide extra strength at the interface, delaying the occurrence of static delamination. However, this aspect has not been evaluated with any experimental evidence.
The high stiffness fibres attract more load than S2-glass fibres. This means that in the far-field of two identical FMLs containing different fibre systems and subjected to the same load, the high stiffness fibres carry more load than the low stiffness fibres. Therefore, the amount of load that has to be transferred from the interrupted metal layers to the intact fibres through the interface is less than in the case of low stiffness fibres. As consequence, the delamination growth requires higher applied load.

In Figure 6.20 the curve related to the M30 carbon fibre configuration does not show any plateau, because specimen failure occurred. After failure the metal layers were plastically deformed over the entire length.

To avoid plastic deformation, some specimens contained a thicker aluminium layer bonded on top of the external interrupted thin layer. This enables to increase the MVF and therefore increase the amount of load transferred through the interface by shear. This prevents the formation of residual plastic strain in the metal layer and stimulates delamination extension.

An example is provided in Figure 6.21 where 7475-T761 alloy sheets were bonded with M30 carbon fibres. The sample containing thicker metal sheets behaves differently from the one containing only thin sheets. Even if the prepreg/metal interface is exactly the same, the occurrence of metal plasticity in the thin specimen delayed delamination extension.

![Figure 6.19 Effect of the rolling direction on the SERR.](image-url)
The amount of load transferred through the interface is higher in the thicker configuration and delamination extension occurred before any potential plasticization of the metal layers. The results of the delamination tests are summarized in Table 6.5.
Table 6.5 Critical SERR results

<table>
<thead>
<tr>
<th>Test</th>
<th>Materials</th>
<th>Lay-up</th>
<th>type</th>
<th>Metal orientation</th>
<th>G_c [MPa mm]</th>
<th>Plasticity observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S2 / 2024-T3</td>
<td>0/90</td>
<td>T</td>
<td>L</td>
<td>0.7</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>S2 / 2024-T3</td>
<td>0/0</td>
<td>T</td>
<td>L</td>
<td>0.67</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>M30/ 7475-T761</td>
<td>0/0</td>
<td>T</td>
<td>L</td>
<td>0.65</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>S2 / 2024-T3</td>
<td>0/90</td>
<td>t</td>
<td>L</td>
<td>0.7</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>S2 / 2024-T3</td>
<td>0/0</td>
<td>t</td>
<td>TL</td>
<td>0.8</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>S2 / 2024-T3</td>
<td>0/0</td>
<td>t</td>
<td>L</td>
<td>0.76</td>
<td>No</td>
</tr>
<tr>
<td>7</td>
<td>M30 / 7475-T761</td>
<td>0/0</td>
<td>t</td>
<td>L</td>
<td>n.a.</td>
<td>Yes</td>
</tr>
<tr>
<td>8</td>
<td>M30 / 7475-T761</td>
<td>0/0</td>
<td>t</td>
<td>L</td>
<td>n.a.</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Effect of the plastic zone behind the fatigue crack tip

The major difference between an FML panel containing a saw-cut and a similar FML panel containing a fatigue crack, for what concerns their behaviour when subjected to monotonically increasing load, is attributed to the presence of bridging fibres. The second load path created by the bridging fibres induces deformation of the metal layers above and below the delaminated area which may result in plastic deformation of the metal layers. This mechanism is illustrated in Figure 5.14 and Figure 6.22.

For a given applied load, the configuration containing a fatigue crack develops a smaller plastic zone ahead of the crack tip, if compared to the saw-cut case. In addition, plastic deformation occurs also behind the crack tip in the region where the crack has grown from the artificial notch. This is due to the bridging mechanism which attracts, load above and below the delamination boundary; this phenomenon does not occur in absence of bridging fibres.

It is important to point out here that the static delamination process occurring in a flat panel containing a fatigue pre-crack is rather different from the delamination process observed during delamination tests, as illustrated in Figure 6.16. If plastic deformation does not occur or is negligible, as for example in fatigue crack growth tests, there exist a direct relationship between the state of stress in the far field and the state of stress in the vicinity of the delamination front. Based on this observation, Marissen [26] developed the previously described delamination test method based on the SERR which was further exploited by Alderliesten [25, 27] and Khan [28].

If plastic delamination occurs, as in the case of increasing load, there is no longer a direct relationship between the far-field stress state and the near field stress state. On the other hand, in case a fatigue crack is subjected to fatigue loading, the far-field stress state is very similar to the near-field stress state. This is schematically illustrated in Figure 6.23 and Figure 6.24.
In the case of increasing load applied to an FML panel containing a fatigue crack, the amount of total strain energy (elastic) introduced into the laminate in the far field partially transforms in plastic strain energy due to the presence of a plastic zone surrounding the crack tip, see Figure 6.24. The local loss of stiffness due to the presence of a plastic zone redistributes load both ahead and behind the crack tip. The bridging fibres attract load and strain the metal layers in the area above and below the delamination front. If the strained metal layers undergo plastic deformation, part of the energy remains stored into the plastically deformed metal layers. Therefore, the available elastic strain energy reduces and this may delay the occurrence of delamination. This means that even if the strain state in the far-field is still elastic, the near-field strain state may be plastic, see Figure 6.24-left.

In the delamination test sample in Figure 6.24-right, the far-field strain state is always equal to the near-field strain state, as result of both the through-the-width interruption of the metallic layers and the simplified specimen geometry. Indeed, in the delamination specimen both stress and strain develop in loading direction only, and the current stress state in the far-field is the same as in the near-field state. Therefore, the delamination test specimen represented in Figure 6.24-right, does not represent the local state of stress in the actual panel. Even if the delamination test sample depicted in Figure 6.24-right does not reflect the state of stress of the actual panel, the delamination mechanisms are the same as in the actual panel. For what concerns the modelling of the delamination resistance, equation (6.1) applies only if the strain field is elastic. To account for the elastic-plastic condition, a modification of equation (6.1) accounting for plastic deformations is described in Appendix D.
The Residual Strength Failure Sequence in Fibre Metal Laminates

Figure 6.23  Qualitative illustration of the far-field and near-field of a panel containing a fatigue crack subjected to fatigue load. The highlighted strip represents the generic area of the panel that is normally characterized with a delamination test sample, as illustrated in Figure 6.16.

Figure 6.24  Qualitative illustration of the far-field and near-field of a panel containing a fatigue crack subjected to increasing load. The highlighted strip represents the generic area of the panel that is normally characterized with a delamination test sample, as illustrated in Figure 6.16.
6.4 Residual strength on large Glare panels

The behaviour of an FML panel containing fatigue cracks and saw-cuts subjected to increasing load may differ substantially. Although the failure mechanisms described in chapter 3 are present in both damage scenarios, the mutual interaction between the FML’s constituents may change drastically.

Several residual strength tests on 800 mm wide Glare panels containing either a saw-cut or a fatigue crack were performed, and because of the large amount of data, see Table 6.6 and Table 6.7, only few tests are described in the following sections. This is meant to provide the reader with the most important observations related to the two damage scenarios. A first comparison is provided in Figure 6.25 where the residual strength curves of two 800 mm wide Glare 3-3/2-0.4 panels are shown. One panel contains a 200 mm saw-cut, the other one a 103 mm saw-cut plus a 97 mm fatigue crack. In this figure it is evident that, the stress at initiation is two times higher for the fatigue crack (200 MPa) than for the saw-cut (100 MPa), and the gradient of the curves is also very different from each other. The saw-cut configuration behaves similar to a monolithic panel, showing a relative low crack initiation stress and high ductility. Fibres ahead of the crack tip do not provide any bridging effect, but only support the metal layers in carrying the remote load. On the other hand, a panel containing a fatigue pre-crack shows a more “brittle-like” behaviour. Bridging fibres in the wake of the fatigue pre-crack are responsible for carrying a large part of the remote load, and when fibre failure occurs the load is released immediately into the rest of the material. This results in a much smaller amount of stable crack growth, but higher residual strength, if compared to a saw-cut equivalent in length.

![Figure 6.25 Stress vs. crack extension curve of Glare 4B-3/2-0.4 TL, W=800 mm. Comparison between a configuration containing a 200 mm saw-cut and a configuration containing a 103 mm saw-cut with 97 mm fatigue crack.](image)
The prediction models described in chapter 7 have been validated against a large number of residual strength tests performed on several Glare configurations and lay-ups. An overview of the test matrices for both saw-cut and fatigue crack configurations is provided in Table 6.6 and Table 6.7, while dimensions of the panels are illustrated in Figure 6.26.

### Table 6.6 Test matrix for residual strength evaluation of Glare panels containing a saw-cut

<table>
<thead>
<tr>
<th>Test</th>
<th>Ref</th>
<th>Lay-up</th>
<th>W [mm]</th>
<th>L [mm]</th>
<th>( t_{\text{nom}} ) [mm]</th>
<th>( a_0 ) [mm]</th>
<th>( 2a_0/W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[29]</td>
<td>3-3/2-0.3</td>
<td>800</td>
<td>1100</td>
<td>1.45</td>
<td>100</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>[29]</td>
<td>3-4/3-0.4</td>
<td>800</td>
<td>1100</td>
<td>2.39</td>
<td>100</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>[29]</td>
<td>3-8/7-0.4</td>
<td>800</td>
<td>1100</td>
<td>3.73</td>
<td>100</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>[29]</td>
<td>3-4/3-0.5</td>
<td>800</td>
<td>1100</td>
<td>2.79</td>
<td>100</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>[29]</td>
<td>4B-4/3-0.4</td>
<td>800</td>
<td>1100</td>
<td>2.79</td>
<td>100</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>[29]</td>
<td>4B-4/3-0.5</td>
<td>800</td>
<td>1100</td>
<td>3.2</td>
<td>100</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>[12]</td>
<td>3-3/2-0.3</td>
<td>400</td>
<td>700</td>
<td>1.45</td>
<td>50</td>
<td>0.25</td>
</tr>
<tr>
<td>8</td>
<td>[12]</td>
<td>3-3/2-0.3</td>
<td>400</td>
<td>700</td>
<td>1.45</td>
<td>66</td>
<td>0.33</td>
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<tr>
<td>9</td>
<td>[12]</td>
<td>3-3/2-0.3</td>
<td>400</td>
<td>700</td>
<td>1.45</td>
<td>66</td>
<td>0.33</td>
</tr>
<tr>
<td>10</td>
<td>[12]</td>
<td>2-3/2-0.3</td>
<td>400</td>
<td>700</td>
<td>1.45</td>
<td>50</td>
<td>0.25</td>
</tr>
<tr>
<td>11</td>
<td>[12]</td>
<td>2-3/2-0.3</td>
<td>400</td>
<td>700</td>
<td>1.45</td>
<td>66</td>
<td>0.33</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>3-3/2-0.4</td>
<td>400</td>
<td>700</td>
<td>1.75</td>
<td>47</td>
<td>0.23</td>
</tr>
</tbody>
</table>

### Table 6.7 Test matrix for residual strength evaluation of Glare panels containing a fatigue crack

<table>
<thead>
<tr>
<th>Test</th>
<th>Lay-up</th>
<th>W [mm]</th>
<th>( t_{\text{tot}} ) [mm]</th>
<th>( a_s ) [mm]</th>
<th>( \Delta a_f ) [mm]</th>
<th>( a_0 ) [mm]</th>
<th>( 2a_0/W )</th>
<th>( 2a_0/a_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4B-3/2-0.4 TL</td>
<td>800</td>
<td>2</td>
<td>51.5</td>
<td>48.5</td>
<td>100</td>
<td>0.128</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>4B-3/2-0.4 TL</td>
<td>800</td>
<td>2</td>
<td>51.5</td>
<td>98.5</td>
<td>150</td>
<td>0.128</td>
<td>0.37</td>
</tr>
<tr>
<td>3</td>
<td>4B-3/2-0.4 TL</td>
<td>800</td>
<td>2</td>
<td>51.5</td>
<td>148.5</td>
<td>200</td>
<td>0.128</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>4B-3/2-0.4 TL</td>
<td>800</td>
<td>2</td>
<td>51.5</td>
<td>54.5</td>
<td>105</td>
<td>0.187</td>
<td>0.256</td>
</tr>
<tr>
<td>5</td>
<td>4B-3/2-0.4 TL</td>
<td>800</td>
<td>2</td>
<td>75</td>
<td>125</td>
<td>200</td>
<td>0.187</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>3-5/4-0.4 TL</td>
<td>800</td>
<td>3</td>
<td>51.5</td>
<td>48.5</td>
<td>100</td>
<td>0.128</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>3-5/4-0.4 TL</td>
<td>800</td>
<td>3</td>
<td>51.5</td>
<td>98.5</td>
<td>150</td>
<td>0.128</td>
<td>0.37</td>
</tr>
<tr>
<td>8</td>
<td>3-5/4-0.4 TL</td>
<td>800</td>
<td>3</td>
<td>51.5</td>
<td>25.5</td>
<td>77</td>
<td>0.128</td>
<td>0.5</td>
</tr>
<tr>
<td>9</td>
<td>2A-3/2-0.3 LT</td>
<td>800</td>
<td>1.43</td>
<td>51.5</td>
<td>48.5</td>
<td>100</td>
<td>0.128</td>
<td>0.25</td>
</tr>
</tbody>
</table>

### 6.4.1 Through-the-thickness crack configuration

The through-the-thickness crack is a configuration extensively studied and evaluated in the past [30, 31]. A lot of data is available about the residual strength of flat panels made of Glare containing a saw-cut [12, 29], and the shape of the related stress vs. crack extension curve is very similar to the curve of an equivalent monolithic aluminium alloy. The actual shape of the curve is mainly related to the panel dimensions and crack size, but, in all cases, the ductility of the metallic
constituent (2024-T3 for standard Glare panels) is responsible for the amount of stable crack growth.

6.4.2 Fatigue crack configuration

If bridging fibres are present in the wake of the fatigue pre-crack, the failure sequence may change drastically compared to the saw-cut configuration. Indeed, it has been observed during the experiments on wide Glare panels that fibre failure begins normally from the side of the initial saw-cut, and the damage propagates in the direction of the fatigue crack tip. In addition, for relative large fatigue cracks length to saw-cut length ratio (e.g. $a_f > 2a_s$), the residual strength may be reached without stable crack propagation. This means that, depending on the length of the fatigue pre-crack, the bridging fibres may store a large part of the applied load. The capability to withstand higher loads due to the bridging fibres can cause the panel to reach the condition of net-section yielding. If this condition is reached, the loss of stiffness in the net-section further overloads the bridging fibres, causing progressive fibre failure and potential final failure without stable crack extension.

An example of progressive failure is illustrated in Figure 6.27 where the experimental residual strength curve is shown for an 800 mm wide panel made of Glare 4B-4/3-0.4 and containing a fatigue pre-crack length ($a_0$) of 100 mm (saw-cut, $a_s = 51.5$ mm). In this example, the residual strength is reached after almost four millimetres of stable crack extension. Four images illustrate how the fibre failure propagates from the saw-cut towards the fatigue crack tip in the metal layers.

If the bridging fibres-to-panel width ratio is high enough (e.g. $a_f > 2a_s$), the final failure of the panel can occur without stable crack propagation. Indeed, if the
amount of bridging fibres is increased (e.g. due to a longer fatigue crack or due to a unidirectional fibre configuration) the brittle characteristic of the composite layer becomes dominant. This is shown in Figure 6.28 where stress vs. crack extension curves of three Glare 4B-4/3-0.4 panels are shown.

**Figure 6.27** Progression of the fibre failure during an experimental residual strength test on a Glare 4B-4/3-0.4 panel, $W=800$ mm and $2a_0=200$ mm. Initiation of fibre failure on the side of the saw-cut (1), followed by a progressive failure towards the fatigue crack tip (2), until fibre failure reaches the location of fatigue crack tip together with stable crack extension(3) and (4).

**Figure 6.28** Applied stress vs. crack extension curve for Glare 4B-4/3-0.4 panels containing fatigue crack lengths:100, 150 and 200 mm
The panels were 800 mm wide and all contained a saw-cut of $2a_s = 103$ mm and in each of them a fatigue crack was created by applying cyclic load of constant amplitude ($\sigma = 120$ MPa, $R = 0.05$). This resulted in three cracks with total length respectively of $a_1 = 100$ mm, $a_2 = 150$ mm and $a_3 = 200$ mm, where the subscripts refer to the test name as illustrated in Figure 6.28. The panel with the longest fatigue crack failed without any stable crack extension, while 0.5 and 3.5 mm stable crack extensions were obtained in the other two panels.

6.5 Conclusions

Several tests were performed on different FMLs configurations, and it can be concluded that:

- The CTOA does not depend on the $2a/W$ ratio for saw-cut configuration both in monolithic aluminium alloys and in laminates, such as ML or FML.
- The CTOA measured on a FML is equal to the CTOA measured on a metal laminate containing the same metal constituent.
- Bridging fibres affect the CTOA because they locally change the deformation field and in particular the COD. If the CTOA is measured at 0.5 mm behind the tip of the fatigue pre-cack, the difference in the CTOA between saw-cut and fatigue crack configuration reduces.
- The CTOA can be interpreted as a fracture toughness parameter. It exhibits the same thickness dependency trends as the more commonly used fracture toughness parameter $K_{lc}$.

Static delamination growth tests were conducted on several FML configurations, and it can be concluded that:

- Static delamination is strongly influenced by the occurrence of permanent plastic deformation, which reduces the available elastic energy for delamination growth.
- Static delamination samples provided values for the critical SERR, based on an elastic formulation.

It has been understood that, the static delamination tests do not represent correctly the stress state of stress in a real FML panel containing a fatigue pre-crack and subjected to increasing load. A simplified elastic-plastic formulation of the SERR is described in Appendix D.
References


Abstract – This chapter presents the analytical method developed for the prediction of the residual strength in fibre metal laminates. Two damage scenarios are considered: through-the-thickness crack and fatigue crack. The metal crack growth under quasi-static increasing load is modelled using the Crack Tip Opening Angle, while the static delamination process is treated using the Strain Energy Release Rate approach. The interaction between the constituents, including plastic deformation of the metallic layers, fibre bridging and fibre failure, are implemented into a numerical program, validated with a wide range of test data. Good correlation between predictions and experimental results is obtained.

7.1 Introduction

It has been explained in the previous chapters that the residual strength failure is a rather complex phenomenon, which involves multiple failure mechanisms such as plastic deformation, static delamination, metal crack growth and fibre failure. Developing an analytical prediction model accounting for all these failure mechanisms and their interactions requires a number of assumptions and simplifications. However, the method should remain sufficiently accurate and robust compared to the experimental data.

The main assumptions adopted in the present method are that Elastic Plastic Fracture Mechanics may be applied, and that the CTOA curve of a generic thin metal sheet is valid also for the same thin metal sheet used in the FML. It follows that the quasi-static crack growth in FMLs is described using an “effective CTOA” calculated at a
prefixed distance behind the crack tip and geometrically derived from the “effective COD”. The COD is here defined as the Crack Opening Displacement along the crack length. Each failure event, such as delamination extension, fibre failure or plastic zone extension affects the COD of the metal layers in the FML. Therefore, to account for all mechanisms involved, the superposition principle is employed.

The main idea behind the developed method is that crack growth occurs when the calculated CTOA of the FML reaches the critical value related to the metallic constituent.

The method presented in this chapter consists of several steps, where the above mentioned failure mechanisms are modelled accounting for their interaction. Two damage scenarios are hereby considered: through-the-thickness crack and fatigue crack. Despite the generic approach of the method, these two damage scenarios are significantly different from each other leading to the development of two separate analytical models, however based on the same method.

7.1.1 Definitions of important terms

In the next sections the analytical method is explained in detail, and a large number of terms and variables are used. In the following, a brief explanation of the most important parameters is provided.

- The $\text{COD}(x)$ is the Crack Opening Displacement along the crack length. It represents the vertical displacement of the crack flanks, see Figure 7.1. The $\text{COD}(x)$ can be calculated also considering the plastic zone, $r_p$, ahead of the crack tip, in this case it is defined as $\text{COD}_{pl}(x)$, see Figure 7.2.
- The CTOA is the angle calculated between three points: the crack tip, and two points located on the $\text{COD}(x)$ at a distance $d$ behind the crack tip, see Figure 7.1. However, in this thesis the CTOA is calculated considering the $\text{COD}_{pl}(x)$. The reasoning of that is provided in the coming sections.
- The term $v(x,y)$ represents the vertical displacement of a generic point of coordinates $(x,y)$. If $y=0$, the vertical displacement corresponds to half of the $\text{COD}(x)$, see Figure 7.1.
- The CTOD represents the vertical displacement at the crack tip when the $\text{COD}_{pl}(x)$ is considered, see Figure 7.2.

For clarity, from this point forward the term $v(x,y)$ will indicate the vertical displacement of a generic point of the field containing a centre crack and the plastic zone ahead of the crack tip, as illustrated in Figure 7.2. Therefore, this means that all calculations presented in the following sections are made considering an “effective crack length”, defined as
\[ a_{eff} = a + r_p = a + \frac{1}{2\pi} \left( \frac{K}{\sigma_{ys}} \right)^2 \] (7.1)

where \( a \), \( \sigma_{ys} \) and \( K \) are respectively the physical crack length, the yield strength and the stress intensity factor at the metal crack tip. Defining the effective crack length is equivalent to deal with a longer crack and a larger COD. This is a simplification of the local loss of stiffness due to the development of plastic strain ahead of the crack tip.

*Figure 7.1 Illustration of the crack geometry*

*Figure 7.2 Illustration of the crack geometry accounting for the plastic zone correction*
7.1.2 Criteria for the validation of the models

Both models have been evaluated and validated with experimental results. In addition to that, the validation of the models is also based on the three following criteria:

*Physical realism:* The physical mechanisms involved during the failure sequence must be described based on the constituent’s characteristics, without the need of fitting parameters.

*Accuracy:* The model must be sufficiently accurate and verified against several geometries, lay-up, and initial damage size.

*Robustness:* The results must be coherent with the initial conditions imposed on the saw-cut length, width and constituents mechanical properties.

These three criteria are met in the following discussion with the help of examples and reference test data to compare the prediction with.

7.2 General approach

The analytical method presented in this thesis is mainly based on the superposition principle, and the following general assumptions are valid for both configurations:

- Linear elastic-plastic fracture mechanics can be applied.
- The metal crack growth occurs in mode I only and is governed by the CTOA parameter.
- Metal and composite constituents are equally strained if no delamination is present.
- The CTOA curve for the metal constituent alone is also valid for the metal constituent of the FML being considered.
- When fibre failure occurs all the energy stored inside the broken fibres transfers to the intact part of the laminate. Fibre failure occurs when the strain-to-failure of the fibre is reached locally.

The general approach consists in treating the FML as a build-up material, where metal and fibres carry part of the load and interact with each other. The first step consists in defining the far-field stress acting in each metal and fibre layer. The Classical Laminate Theory (CLT) is used as described in [1, 2] to define the amount of stress acting in each layer based on the relative stiffness and on the compatibility of the deformations.
The CTOA is derived directly from the calculated COD(\(x\)), and results from the superposition of three contributions:

- the opening contribution from the far-field load, including plastic deformation, COD\(_m\)(\(x\)).
- the closing contribution from the fibre bridging stress, COD\(_{br}\)(\(x\)) (only for fatigue crack configurations).
- the opening contribution from the broken fibres, COD\(_f\)(\(x\)).

The COD(\(x\)) equation is then written as

\[
\text{COD}(x) = \text{COD}_m(x) + \text{COD}_f(x) - \text{COD}_{br}(x)
\]  

(7.2)

It is therefore possible to calculate the current CTOA value directly from the COD(\(x\)) by means of geometrical considerations, as illustrated in Figure 7.3. The CTOA is calculated as

\[
\text{CTOA} = \tan^{-1}\left(\frac{\text{COD}(a-d)}{d}\right)
\]  

(7.3)

where COD(\(a-d\)) represents the local value of the crack opening displacement calculated at a distance \(d\) behind the crack tip. Considering the relationship between \text{COD}(\(x\)) and \(v(x,y)\), equation (7.3) can be written also as

\[
\text{CTOA} = \tan^{-1}\left(\frac{2v(a-d,0)}{d}\right)
\]  

(7.4)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure7_3.png}
\caption{Schematic illustration of the geometrical relationship between COD and CTOA}
\end{figure}

The general approach to define the contributions of both metal and fibre layers to the COD(\(x\)) is presented in the following sections. It is important to point out that from this point forward, the term \(v(x)\) indicates half of the crack opening displacement, and it is equivalent to \(v(x,y)\) with \(y=0\).
7.2.1 Crack opening due to far-field stress

The crack opening due to the applied far-field stress in a homogeneous and isotropic material is calculated using equation (7.5) for \( y=0 \), which can be solved by employing the Westergaard stress function \( Z \) [3].

\[
v_m(x) = \frac{1}{2G} \left( \frac{k+1}{2} \Im Z - y \Re Z - \frac{k-3}{2} \frac{\sigma_m}{2} \right) \quad \text{with } y=0
\]  

(7.5)

where \( k=(3-\nu)/(1+\nu) \) for plane stress (\( \nu \) is the Poisson’s ratio). In equation (7.5) the terms \( G \), \( y \) and \( \sigma_m \) represent respectively the shear modulus, the position along the \( y \)-axis and the remote load in the metallic layers. The third term of equation (7.5) is the result of a correction imposed on the original equation because of the uniaxial load condition, as described in [4]. The definition of terms \( Z \) and \( \overline{Z} \) for centre crack configurations is provided in appendix A.

Equation (7.5) accounts for the plastic zone ahead of the crack tip; this means that an effective crack length is considered. Calculation of \( a_{eff} \) requires an iterative procedure on \( K \), as described in Appendix C.

7.2.2 Crack opening due to fibre failure

If fibre failure occurs, the amount of stored load is released into the rest of the material, thus inducing an opening effect. This behaviour is modelled by defining the second term of equation (7.2), \( COD_f \), as the crack opening displacement due to point loads at the location of the broken fibres. The amount of load released into the metal layers, \( \sigma_{metal,f} \), is equivalent to the load per unit of width stored in the broken fibres at the moment of failure, as described by equation (7.6).

\[
\sigma_{metal,f} = \sigma_{f, failure} \cdot \frac{t_{fibre} \cdot h_{fibre}}{t_{metal} \cdot n_{metal}}
\]  

(7.6)

This equation assumes that there is no variation through the thickness.

Fibre failure is reached in those locations where the calculated fibre strain exceeds the ultimate strain of the fibre. This normally happens where the strain gradient is the highest, and in particular:

- in saw-cut configurations, fibre failure occurs in front of the crack tip
- in fatigue crack configurations, fibre failure can occur both in front and behind the crack tip (e.g. bridging fibres).
The approach used to calculate $\sigma_{f,\text{failure}}$ is described in detail in section 7.3 for the saw-cut configuration, and in 7.4.1 for the fatigue crack configuration.

The opening contribution, $v_f(x)$, is modelled using a four-point load configuration distributed symmetrically at the location of fibre failure, as shown in Figure 7.4 and Figure 7.5. In the generic four-point load configuration of Figure 7.4, $x_i$ and $y_i$ represent the coordinates of the location of fibre failure, and $P_i$ is the point-force per unit of width, defined multiplying equation (7.6) by the total metal thickness. This represents the load transmitted to the metallic layers as results of either fibre failure or fibre bridging.

The location of failure and the amount of force released depend on the crack configuration:

- for saw-cut configurations, it is assumed that failure occurs ahead of the crack tip. Therefore, it follows that $a < x_i < W/2$ and $y_i = 0$, where $W$ is the panel width.
- for fatigue crack configurations, fibre failure can occur both ahead and in the wake of the fatigue crack. Therefore, it follows that $a_s < x_i < W/2$ and $y_i = b_i$. In fatigue crack configurations, $b$ represents the delamination length and $a_s$ is half of the initial saw-cut length.

![Symmetric four-points load distribution for saw-cut configurations](image)

The method to calculate $v(x)$ due to four-point load configuration is provided in appendix A. The total crack opening displacement due to fibre failure is obtained by the summation of each single contribution of each couple of load-point, as described by equation (7.7).
\[ v_j(x) = \sum_{i=1}^{n} v_{j_i}(x) \] (7.7)

Equation (7.7) represents the total metal crack opening due to the broken fibres.

### 7.2.3 Crack closing due to fibre bridging

The procedure to calculate the crack closing contribution related to the bridging stress is similar to the procedure described in section 7.2.2. The main difference consists in the calculation of the fibre stress level in equation (7.6), which derives directly from the local bridging stress level. The description to calculate the bridging stress is provided in section 7.4.1.

Considering Figure 7.5, the location of the bridging fibres is denoted by the coordinates \( x_i \) and \( y_i \), and in particular \( a_s < x_i < a_f \) and \( y_i = b_i \), where \( a_s \) is the saw-cut tip and \( a_f \) is the fatigue crack tip.

Also for the case of bridging fibres, the method to calculate \( v(x) \) due to four-point load configuration is provided in appendix A. The total crack closing displacement is obtained by the summation of each single contribution of each couple of load-point, as described by equation (7.8).

\[ v_{br}(x) = \sum_{i=1}^{n} v_{br_i}(x) \] (7.8)

Equation (7.8) represents the total crack closing due to the bridging fibres.

![Symmetric four-points load distribution for fatigue crack configurations](image)
7.2.4 Residual plastic strain and crack propagation

The presence of a plastic zone ahead of the crack-tip generates crack-tip blunting, making the crack-tip in the metal layers no longer sharp. The CTOD is the parameter that can be used to represent this behaviour. It is defined by the displacement between the crack flanks in correspondence of the physical crack tip, as illustrated in the top-left part of Figure 7.6. This parameter is taken into account to define the length of the yielded-bar-element in the wake of the propagating crack. When the critical CTOA is reached, see top-right part of Figure 7.6, the crack is extended of a fixed length $\Delta a$. The new formed crack flank is obtained by subtracting the length of the “critical CTOD” to the COD, see the bottom-left part of Figure 7.6. An “effective COD” is then obtained which accounts for the residual plastic strain in the wake of the crack. The CTOA is now calculated based on the “effective COD. Further crack extension occurs when the calculated CTOA reaches the CTOA$_c$ as result of the increasing applied load. The new CTOA$_c$ is the value of the CTOA curve, provided as input curve, for the current crack extension.

![Diagram of yielded bar distribution in the wake of the propagating crack](image_url)

Figure 7.6  Illustration of the yielded bar distribution in the wake of the propagating crack
7.2.5 Modelling approaches

A panel containing a through-the-thickness crack behaves similarly to an equivalent monolithic sheet, no bridging fibre are present and therefore no crack opening restrain mechanisms need to be implemented. On the other hand, fibre failure may occur locally ahead of the crack tip, where high strain levels are present in the fibre layer due to the stress concentration. Fibre failure is therefore modelled as an opening contribution to be superimposed to the opening term as result of the far field load acting on the metallic layers. Plastic deformation ahead of the crack tip is also present due to the high applied loads, and it influences the COD of the FML. The Irwin’s plastic zone correction is used to describe this phenomenon. In addition, when the crack extends statically, it grows through an highly plasticized area, the plastic zone. This means that the CTOA calculated from the COD must account for the residual plastic strain in the wake of the propagating crack. This is done by modelling the residual plastic strain by means of yielded-bar-elements.

Configurations containing a fatigue crack present more complexity. The bridging fibres in the wake of the crack are responsible for carrying part of the load and thus act as a second load path, restraining the crack opening in the metal layers. It is therefore important to accurately calculate the crack closing contribution provided by the bridging fibres which has to be superimposed to the opening contribution due to the far-field load. In addition, fatigue delamination is present as result of the cycling shear load at the fibre/metal interface. If an increasing load is applied, the delamination is assumed to growth quasi-statically in direction perpendicular to the fatigue crack in the aluminium layers, thus in loading direction, and can be approximated as one-dimensional mechanism. The Energy Release Rate approach is used to describe the delamination growth process. Also for this configuration the Irwin’s plastic zone correction and the yielded-bar-elements are used to include respectively the presence of the plastic zone ahead of the crack tip and the presence of the residual plastic strains in the wake of the crack.
7.3 Through-the-thickness crack configurations

The general assumptions presented in section 7.2 are here specialized for the saw-cut configuration, and in particular:

- The plastic zone develops only ahead of the crack tip and is modelled using the Irwin’s correction.
- The static delamination does not occur in any part of the laminate, thus compatibility of the deformation between metal layers and adjacent fibre layer is imposed.

Figure 7.7 schematically illustrates the crack geometry with both the far-field and fibre failure stress systems superimposed to each other, and a reference system $x-y$ is here used centred in the middle of the crack. The crack length is divided into bar-elements with a fixed width, $w$, and the position of the middle line of each bar-element is defined by $x_i$.

7.3.1 Calculation of the fibre failure stress and related crack opening

The remote stress acting in the composite layer is calculated using CLT [1], and therefore the Stress Intensity Factor (SIF) of the orthotropic composite layer is obtained according to the procedure described in [7], and here briefly summarized.

The crack in the composite layer is assumed to be equal to the one in the metal layer, therefore the strain gradient ahead of the crack tip is defined by

$$\sigma_f(x) = \frac{K_f}{\sqrt{2\pi(a-x)}} \cdot f(\theta, \mu_1, \mu_2), \quad \text{with } x \geq a$$

(7.9)

where $x$ is the coordinate with origin in the centre, $\theta$ is the angular coordinate, $\mu_1$ and $\mu_2$ are orthotropic components [7]. If the stress gradient is calculated for $\theta = 0$ and there is no mode II involved, equation (7.9) simplifies and $f(\theta, \mu_1, \mu_2)$ becomes equal to 1, as for isotropic materials. In equation (7.9) $K_f$ is defined as

$$K_f = \beta \sigma_{f,\text{farfield}} \sqrt{\pi a}$$

(7.10)

where $\sigma_{f,\text{farfield}}$ is the far-field stress acting on the composite layers calculated with CLT [1]. The amount of load transferred to the rest of the intact material is defined using equation (7.6), where $\sigma_{f,\text{failure}}$ is defined with equation (7.9).
The distribution of the force-points is schematically illustrated in Figure 7.7, where for simplicity the force-points are distributed along the crack line. This results, considering the generic scheme in Figure 7.4, in imposing that $y_i = 0$ and $x_i$ are the locations where the strain-to-failure of fibres is reached. Using equation (7.7) it is now possible to calculate the crack opening contribution due to fibre failure ahead of the crack tip.

\[
\sigma_i(x) = \frac{K_y}{\sqrt{\pi(a-x)}}
\]

Figure 7.7 Modelling of the fibre failure effect ahead of the crack tip

7.3.2 Plasticity in the wake of the crack

As described in section 7.2.4, crack extension, $\Delta a$, occurs when the calculated CTOA reaches the critical CTOA for the given crack length. The crack extends of a length equal to $\Delta a = w$, where $w$ is the width of the bar-element, and the calculated CTOD at the critical condition is subtracted to the newly formed crack surface, as schematically illustrated in Figure 7.8. It is important to point out that the bar-element width $w$ is divided in $n$ yielded bar-elements representing the permanent plastic deformation in the wake of the crack. The CTOD is calculated as

\[
CTOD = v(a)
\]  
(7.11)

where the $v(x)$, with $x=a$, is calculated with equation (7.2) divided by 2. The critical value $CTOD_c$ is defined as in equation (7.11) when the critical condition is reached, thus when $CTOA \geq CTOA_c$.

The “effective $v(x)$” of a generic crack length $a_i$ is calculated in equation (7.12). The critical $CTOD_c$, calculated with equation (7.11) when the critical condition is reached with crack length $a_{i-1}$, is subtracted to the elastic-plastic definition of the new $v(x)$. The subtraction is done only along the length of the crack extension $\Delta a$, which is divided in $n$ yielded bar-elements with a width equal to $w/n$.

\[
\begin{align*}
\left[ v_{eff}(x) \right]_{a_i} &= v(x)_{a_i} - CTOD_c \quad \text{for} \quad x = [a_i - \Delta a, a_i] \\
\left[ v_{eff}(x) \right]_{a_i} &= v(x)_{a_i} \quad \text{for} \quad x = [0, a_i - \Delta a]
\end{align*}
\]  
(7.12)
The meaning of the terms in equation (7.12) is the following:

- \( \nu_{\text{eff}}(x) \) is the effective \( \nu(x) \) which accounts for the permanent plastic deformation in the wake of the propagating crack.
- \( \nu(x)|_{a_i} \) is the crack opening displacement obtained for \( x \) ranging from \( a_i - \Delta a \) to \( a_i \), where \( a_i \) is the current crack length and \( \Delta a \) is the crack extension.
- \( CTOD|_{a_{i-1}} \) is the term that represents the length of the yielded bar-element. This is calculated when the generic crack length \( a_{i-1} \) reaches the critical condition, such that the crack extends from length \( a_{i-1} \) to length \( a_i \).

![Figure 7.8 Modelling of the residual plastic strain in the wake of the propagating crack](image)

The total \( \nu(x) \) of the laminate is then obtained by superposition of equations (7.12), (7.7) and (7.8). Finally the CTOA for the given configuration, applied load and crack length is obtained with equation (7.3) or (7.4).

An example of COD calculation during the crack propagation is provided in Figure 7.9. Here the elastic-plastic COD and the effective COD are plotted for an extended crack with a total length of 108 mm. In the example of Figure 7.9, the crack has extended for 8 mm with a total of 16 extensions of 0.5 mm (\( \Delta a \)), and the bar-element of extension number 4, \( CTOD_{c4} \), is highlighted together with the current crack-tip-opening-displacement (\( CTOD_a \)).

Figure 7.10 illustrates the detail of the bar-element width at the 4\(^{th} \) crack extension, which is divided in \( n \) yielded bar-elements representing the residual plastic strain in the wake of the crack. Figure 7.11 illustrates the progressive calculation of the CTOA related to the increasing remote load and the growing crack. At the
beginning, the calculated CTOA increases without crack propagation due to fixed load increments, $\Delta P_{\infty}$, until the moment when it intercepts the input CTOA curve for $\Delta a=0$. At this point crack extension occurs and the new crack geometry is calculated according to equation (7.2), as illustrated in Figure 7.8.

Figure 7.9  Example of COD calculation. Saw-cut at 100 mm and crack extended to 108 mm

Figure 7.10  Example of COD calculation. Saw-cut at 100 mm and crack extended to 108 mm
Due to the calculated effective $v(x)$, the current value of the calculated CTOA is lower than the previous one (given the same applied load). From this point on, the load is incremented step wise and the calculated CTOA increases as well. Crack extension does not occur until the calculated CTOA exceeds again the input curve for $\Delta a=0.5\text{mm}$. This procedure repeats for each crack extension and it is represented in Figure 7.11 by the dashed arrows.

![Figure 7.11 Correlation between calculated CTOA and input CTOA curve. Crack extension occurs when the increasing CTOA intercepts the input curve.](image)

### 7.3.3 Numerical calculation approach

The method presented in the previous section is implemented in a numerical calculation model. The structure of the model is shown in Figure 7.12 and Figure 7.13. The model requires input parameters such as geometry, mechanical properties of the materials, loading parameters, CTOA curve and fibre failure parameters, which need to be defined by the user.

The model starts with a given initial small applied load, from which the stress levels acting in each layer are calculated using CLT, see Figure 7.13. Subsequently the $\text{COD}_m(x)$ and the $\text{COD}_f(x)$ are calculated as described in sections 7.2 and 7.3.1. This takes into account fibre failure, plastic zone ahead of the crack tip and permanent plastic deformation in the wake of the crack (if crack extension has occurred). The $\text{COD}(x)$ is then calculated from which the CTOA is obtained. The comparison of the calculated CTOA with the value provided by the input curve for the current crack extension defines whether a new crack extension occurs, see Figure 7.12.
In case the criterion is not met, the load is increased of a fixed amount $\Delta P_{up}$ and the length of the yielded bar-element, CTOD$_c$, is set equal to zero. The procedure repeats with the increased load. If the criterion is met, the current CTOD is stored as CTOD$_c$ and the crack extended of a fixed length $\Delta a$. The stored CTOD$_c$ is used in the next loop to account for permanent plastic deformation in the wake of the propagating crack. In this case, the current applied force $P_k$ is stored as $P_{ext_i}$, where $P_{ext}$ represents the vector containing the load step at which crack extension occurs. After each crack extension the load is reduced of a quantity equal to $\Delta P_{down}$. This procedure enables to reduce the applied load when the residual strength is reached.

\begin{center}
\textbf{Figure 7.12} Flow diagram of the residual strength prediction model for saw-cut configurations
\end{center}

Subsequently, the loop is performed again with the new crack geometry; the condition CTOA $\geq$ CTOA$_c$ is then evaluated again to check if for the given applied load the crack can be extended again. This is somehow similar to what occurs during a residual strength test performed in displacement control.

Data is stored at each load increment (index $k$) and at each crack extension (index $i$). The program stops when the current $P_{ext}$ is 2% lower than the maximum value, $P_{ext\_max}$, which represents the residual strength.
Output data consists in:

- Applied stress vs. physical crack extension curve
- Applied stress vs. effective crack extension curve
- R-curve

From this data is therefore possible to calculate:

- Stress intensity at crack initiation, $K_i$
- Engineering fracture toughness, $K_e$
- Fracture toughness, $K_c$

![CTOA diagram]

*Figure 7.13 Flow diagram of the CTOA calculation model illustrated in Figure 7.11.*

### 7.3.4 Results and validation

The developed model has been validated by means of comparison based on the three criteria described in section 7.2.5. The physical realism criterion is discussed in section 7.5.
**Model accuracy**

The accuracy of the model is validated against test data provided in [8-10], and presented in appendix E. The following reference tests of standard Glare panels are used to compare predictions with experimental results. The input parameters for 2024-T3 and S2-glass/FM94 prepreg have been used, and are listed in Figure 7.14 and Figure 7.15.

Glare 3-4/3-0.4 and Glare 3-8/7-0.4 flat panels are here considered for validation of the developed method. Both panels have a width of 800 mm and contain a saw-cut of 200 mm.

Figure 7.16 and Figure 7.17 illustrate the remote stress vs. physical crack extension curve of both prediction and experimental results. In both cases the model is predicting the overall trend in accurate way. The model seems to underestimate the initial part of the curve and crack initiation but, on the other hand, the model predicts quite well the residual strength of both configurations.

In Figure 7.18 and Figure 7.19 the remote stress is plotted against the effective crack length, which includes the plastic zone size. The comparison is made with the experimental results where the plastic zone was calculated using the Irwin’s correction [9]. The predictions match well with the experimental results. In addition, predicted R-curves in Figure 7.20 and Figure 7.21 show that the model is capable to predict the R-curve accurately.

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Aluminium 2024-T3</th>
<th>S2-glass fibre/FM94 prepreg</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$</td>
<td>72400</td>
<td>MPa</td>
</tr>
<tr>
<td>$E_y$</td>
<td>72400</td>
<td>MPa</td>
</tr>
<tr>
<td>$\sigma_{sy}$</td>
<td>350/320 (L/TL)</td>
<td>MPa</td>
</tr>
<tr>
<td>$G_{xz}$</td>
<td>27600</td>
<td>MPa</td>
</tr>
<tr>
<td>$\nu_{xy}$</td>
<td>0.33</td>
<td>-</td>
</tr>
<tr>
<td>$\nu_{yx}$</td>
<td>0.33</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>$2.2 \cdot 10^{-5}$</td>
<td>$1/C^\circ$</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>$2.2 \cdot 10^{-5}$</td>
<td>$1/C^\circ$</td>
</tr>
<tr>
<td>$n$</td>
<td>number of layers</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>0.3-0.5</td>
<td>mm</td>
</tr>
<tr>
<td>$t_{0}$</td>
<td>0.133</td>
<td>mm</td>
</tr>
<tr>
<td>$t_{90}$</td>
<td>0.133</td>
<td>mm</td>
</tr>
</tbody>
</table>

*Figure 7.14  Input parameter for material properties corresponding to the flow diagram in Figure 7.10.*
7. Analytical Modelling of the Residual Strength Failure Sequence in FMLs

<table>
<thead>
<tr>
<th>Geometry and loading parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
</tr>
<tr>
<td>$W$ ... mm</td>
</tr>
<tr>
<td>$2a_0$ ... mm</td>
</tr>
<tr>
<td>$\Delta a$ 1 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Failure criteria parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTOA for 2024-T3 $t = 0.4$mm</td>
</tr>
<tr>
<td>$CTOA = a\cdot e^{bt} + c$</td>
</tr>
<tr>
<td>$a$ 8.4</td>
</tr>
<tr>
<td>$b$ 0.4</td>
</tr>
<tr>
<td>$c$ 5</td>
</tr>
<tr>
<td>Calculation distance behind the crack tip</td>
</tr>
<tr>
<td>$d$ 1 mm</td>
</tr>
</tbody>
</table>

Figure 7.15 Input parameter for geometry, loading conditions and failure criteria corresponding to the flow diagram in Figure 7.10.

Figure 7.16 Remote stress vs. physical crack extension: comparison between analytical prediction and experimental test for a Glare 3-4/3-0.4 panel with $W=800$ mm and $2a_0=200$mm. The experimental curves represent three different panels.
Figure 7.17 Remote stress vs. physical crack extension: comparison between analytical prediction and experimental test for a Glare 3-8/7-0.4 panel with $W=800$ mm and $2a_0=200$ mm. The experimental curves represent two different panels.

Figure 7.18 Remote stress vs. effective crack extension: comparison between analytical prediction and experimental test for a Glare 3-4/3-0.4 panel with $W=800$ mm and $2a_0=200$ mm. Same panels as in Figure 7.16.
7. Analytical Modelling of the Residual Strength Failure Sequence in FMLs

Figure 7.19   Remote stress vs. effective crack extension: comparison between analytical prediction and experimental test for a Glare 3-8/7-0.4 panel with $W=800$ mm and $2a_0=200$mm. Same panels as in Figure 7.17.

Figure 7.20   R-curve: comparison between analytical prediction and experimental test for a Glare 3-4/3-0.4 panel with $W=800$ mm and $2a_0=200$mm. Same panels as in Figure 7.16.
The Residual Strength Failure Sequence in Fibre Metal Laminates

Figure 7.21  R-curve: comparison between analytical prediction and experimental test for a Glare 3-8/7-0.4 panel with W=800 mm and 2a₀=200 mm. Same panels as in Figure 7.17.

Model robustness

The third criterion to validate the model is based on the robustness of the method. The question to be answered in the current section is whether the model is physically correct and robust, which means that by changing somewhat arbitrarily the chosen crack extension length, Δa, and distance for the CTOA calculation, d, the results do not change significantly. Indeed, in chapter 6 it has been discussed that the CTOA curve changes depending on the distance, d, considered for CTOA measurements.

This is illustrated in detail in Figure 7.22 where three CTOA curves have been obtained measuring the CTOA at 0.5, 1 and 1.5 mm behind the crack tip. Superimposed to the experimental data are the exponential trend curves, based on equation \( y = a \cdot e^{b} + c \), which are used as input parameter in the model. Table 7.1 provides an overview of the a, b and c parameters for the trend curves given in Figure 7.22.

<table>
<thead>
<tr>
<th>CTOA d [mm]</th>
<th>a [deg]</th>
<th>b [mm⁻¹]</th>
<th>c [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>6.74</td>
<td>0.3</td>
<td>6.15</td>
</tr>
<tr>
<td>1</td>
<td>6.52</td>
<td>0.34</td>
<td>5.34</td>
</tr>
<tr>
<td>1.5</td>
<td>7.37</td>
<td>0.32</td>
<td>4.84</td>
</tr>
</tbody>
</table>
Using the three experimentally generated CTOA curves, three predictions have been made on the same configuration where the calculation of the CTOA has been made at 0.5, 1 and 1.5 mm behind the crack tip. The results in Figure 7.23 show that, despite small differences in the crack initiation, the stress versus crack extension curves are similar and converge to the a similar residual strength value. This is a positive indication about the robustness of the model, which shows to be independent of the CTOA curve provided as input. This is true as long as the calculation of the CTOA is performed at the same distance, $d$, of the CTOA curve.

Further proof of the robustness of the developed method is provided in Figure 7.24, where three predictions are shown and compared with each other. The prediction has been performed on the same laminate, Glare 3-3/2-0.4, containing a saw-cut with total length of 150, 200 and 250 mm. The residual strength reduces according to the increasing damage size, while the shape of the curves maintains the same. Despite the differences in residual strength between the configurations, the calculated R-curves are very similar to each other, see Figure 7.25. This is a further proof of the validity of the model, indeed the R-curve is supposed to be a material parameter, which does not change with the damage size [10, 11].

Figure 7.22  CTOA curves experimentally obtained at 0.5, 1 and 1.5 mm behind the crack tip in 2024-T3 $t=0.4$ mm. Exponential trend lines are illustrated in full line, based on equation $y=a\cdot e^{bx}+c$ (parameters $a$, $b$ and $c$ are presented in Table 7.1).
Figure 7.23  Calculated remote stress vs. crack extension curve. Comparison between predictions performed at 0.5, 1 and 1.5 mm behind the crack tip and using related CTOA curves (measured at 0.5, 1 and 1.5 mm). Glare 3-4/3-0.4, W=800 mm, 2a₀=200 mm.

Figure 7.24  Residual strength predictions of a Glare 3-3/2-0.4, W=800 mm, containing a saw-cut 2a₀ of 150, 200 and 250 mm.
Figure 7.25  R-curve predictions of a Glare 3-3/2-0.4, W=800 mm, containing a saw-cut $2a_0$ of 150, 200 and 250 mm. (prediction based on Irwin’s correction)
7.4 Fatigue crack configurations

The behaviour of a FML panel containing a fatigue crack and subjected to increasing load is different from the case of an identical FML containing a saw-cut. The bridging fibres in the wake of the fatigue crack play an important role in carrying part of the applied load; it is therefore paramount to describe the effect of the bridging fibres on the COD(x) of the metal layers. The development of a prediction model for residual strength of an FML panel containing a fatigue crack requires additional assumptions and simplifications to those already stated in sections 7.2 and 7.3. In particular:

- Static delamination growth is a one-dimensional process and it occurs only in the fatigue delaminated area. (assumption)

- An elastic-plastic formulation of the Strain Energy Release Rate (SERR) approach is employed to describe the delamination process in presence of plastic deformation. (simplification)

- Fibre failure initiates in the wake of the fatigue crack where bridging fibres are present. This assumption is based on the observation of fibre failure during experimental tests on large panels.

The bridging stress is the stress that restrains the crack opening and reduces the stress intensity factor at the crack tip in the metal layers. Alderliesten [12, 13] relates the shape and magnitude of the bridging stress to the size and shape of the fatigue delamination and the crack opening contour. Furthermore, he has described a method to calculate the bridging stress distribution in the wake of a fatigue crack based on the solution of the displacement compatibility equation between the cracked metal and the bridging fibres. That method has been implemented into a fatigue crack growth prediction model, accounting also for fatigue delamination growth [12, 13].

A similar approach has been followed in the present study to compute the bridging stress acting along a fatigue crack, accounting for the plastic deformation at the crack tip. In this study the bridging stress at each location along the fatigue pre-crack increases in magnitude due to the increasing applied load. This implies that fibre failure may occur in those locations where the strain reaches the strain-to-failure of the fibres.

In the case of fatigue crack growth, the delamination growth is a uni-directional mechanism coupled with the metal crack growth; this means that the delamination
grows in loading direction due to the cyclic applied load, but extends in crack direction as consequence of crack growth.

In the case of a FML containing a fatigue crack and subjected to an increasing load, the delamination grows statically in loading direction only and it is not coupled with the metal crack growth. Indeed, if stable crack growth occurs the existing fatigue delamination extends statically only in the loading direction and the extension is restricted to the fatigue crack area only. These mechanisms are qualitatively illustrated in Figure 7.26.

7.4.1 Displacement compatibility equation

The bridging stress distribution is determined by solving the displacement compatibility equation in the wake of the fatigue crack. Indeed, the vertical displacement of the metal layers calculated at the boundary of the delamination has to be equivalent to the elongation of the fibres over the delaminated length. In addition to that, the deformation of the prepreg must be taken into account in the compatibility equation [12]. Here, the method is described considering a given crack with an effective length \( a_{\text{eff}} \), quasi-statically extending from a fatigue crack with a length \( a_f \). In addition, the presence of broken fibres is implemented into the method.

![Fatigue crack growth](image)

![Static crack growth](image)

*Figure 7.26  Illustration of the differences between delamination growth due to fatigue load (left) and delamination growth due to increasing load.*

The vertical displacement of the metal layers along the boundary of the delamination, \( b(x) \), at any location along the fatigue crack, illustrated in Figure 7.27, can be written as

\[
v(x, y) = v_m(x, y) + v_f(x, y) - v_{br}(x, y)
\]

(7.15)

where \( v_m(x) \) represents the vertical displacement of the metal layers calculated using equation (7.5) for \( y = b(x) \). The second term, \( v_f(x) \), represents the opening contribution due to fibre failure, while the third term, \( v_{br}(x) \), represents the crack
closing contribution due to bridging fibres. Both the second and third term can be calculated with the method of the four-points load configuration, as described respectively in section 7.2.2 and 7.2.3.

Considering Figure 7.5, it is imposed that the point-force $P_i$ represents the bridging force acting at a distance $y_i=b(x_i)$. Assuming that the bridging stress can be simplified as a distributed number of infinite point loads, the vertical displacement due to a given bridging stress is given by

$$v_{br}(x,y) = \int_0^{a_f} \frac{v_{br}(x,y)}{dx}$$  (7.16)

For the case of fibre failure in the wake of the fatigue crack, see Figure 7.4, it is assumed that the force stored into the broken fibres is released into the rest of the intact material. The point-forces, $P_i$, are for simplicity located at the boundary of the delamination, $y_i=b(x_i)$. In case fibre failure does not occur, $P_i=0$ and therefore $v_f(x,y)=0$.

$$v_f(x,y) = \int_0^{a_f} v(x,y)dx$$  (7.17)

The definition of $v(x,y)$ as function of the coordinates $x,y$ relates to the vertical displacement of generic points $(x,y)$ of the field. For the compatibility equation, this points corresponds to the boundary of the delamination, therefore $(x,b(x))$. The meaning of equation (7.15) is schematically illustrated in Figure 7.27. For simplicity of illustration, the vertical displacement of the crack flanks is shown, therefore for $y=0$.

Equation (7.11) can be defined also in terms of elongation of the prepreg over the delaminated length, $\delta_f$, and the deformation of the prepreg layer, $\delta_{pp}$, as written in equation (7.14)

$$v(x,y) = \delta_f(x) + \delta_{pp}(x)$$  (7.18)

The mechanisms modelled with equation (7.18) are illustrated in Figure 7.28. Indeed, for the elongation of fibre layers one can write

$$\delta_f(x) = \varepsilon_f(x)b(x) = \frac{\sigma_{f,\text{Far}}(x)}{E_f}b(x) = \frac{\sigma_f(x) + \sigma_{br}(x)}{E_f}b(x)$$  (7.19)

where the term $\sigma_f(x)$ represents the stress distribution due to the far field stress, while $\sigma_{br}(x)$ represents the bridging stress caused by the load transfer from the
delaminated metal layer into the adjacent fibre layer. The second term of equation (7.14) represents the deformation of the prepreg layer written as

\[ \delta_{pp}(x) = \gamma_{f} = \frac{t_{f}}{G_{f}} \]  

(7.20)

where \( \gamma \) is the shear strain, \( \tau_{f} \) the maximum shear stress at the delamination tip, \( G_{f} \) the shear modulus of the prepreg layer and \( t_{f} \) the prepreg layer thickness.

In [12, 14] Alderliesten has derived an analytical formulation that relates the prepreg deformation to the applied load in the metal layer and to the delamination length. Although that model was applied for fatigue load conditions, the same approach has been used here. For quasi-static loading, this implies a simplification of the actual prepreg deformation, because plastic deformation of the metal layers may influence the deformation of the prepreg.

**Calculation of the bridging stress distribution**

The bridging stress distribution can be now determined by equalling equations (7.15) and (7.18).

\[ v_{m}(x, y) - v_{br}(x, y) + v_{f}(x, y) = \delta_{f}(x) + \delta_{pp}(x) \]

(7.21)

There is no closed-form solution available for the unknown term, \( \sigma_{br}(x) \), which is present on both sides of equation (7.21). Therefore, the solution of this equation can be obtained only numerically.

The crack length is divided in \( N \) bar-elements with a variable width, \( w_{i} \), where the position of the centre line of each bar element is defined by \( x_{i} \). The method used to define the bar-elements with variable width is described in appendix B. The crack
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graphology is illustrated in Figure 7.29, where the terms $x_k$ and $x_j$ denote respectively the location of the broken fibres and of the bridging fibres, while the term $x$ denotes the generic location along the crack length.

\[ \sigma_{f,tot} = \sigma_f(x_k) \cdot w_k + \sigma_f(x_j) \cdot w_j + \sigma_{br}(x_j) \cdot w_j \]

**Figure 7.28** Illustration of the vertical displacement occurring at a generic location along an arbitrary fatigue crack. The figure on the right represents the prepreg elongation and deformation due to an applied load.

Equations (7.16) and (7.17) are discretized and written as summation of the individual crack closing (opening) contributions due to the bridging forces, $P_j$, and to the opening forces, $P_k$, in each bar element.

**Figure 7.29** Fatigue crack geometry divided in bar-elements. Illustration of the stress distributions due to fibre failure and fibre bridging.
\[ v_{br}(x,y) = \sum_{j=1}^{N} v_{br}(x_j, y) \quad \text{for } y = b(x_j) \] (7.22)

\[ v_f(x,y) = \sum_{k=1}^{N} v_f(x_k, y) \quad \text{for } y = b(x_k) \] (7.23)

The definition of the terms \( v_{br}(x) \) and \( v_f(x) \) is provided in appendix A.

Equation (7.21) contains the unknown term, \( \sigma_{br}(x) \), and can be re-written in two equations

\[ Q = v_m(x,y) + Q_2 \] (7.24)

where

\[
\begin{align*}
Q_2 &= \sum_{k=1}^{N} v_f(x,y) - \frac{\sigma_f(x)}{E_f} b(x) - \delta_{pp}(x) \\
&\quad \text{for } x \neq x_k \\
Q_2 &= 0 \\
&\quad \text{for } x = x_k
\end{align*}
\] (7.25)

and

\[ H = \sum_{j=1}^{N} \frac{v(x,b)}{\sigma_{br}(x_j)} - \frac{b(x)}{E_f} \] (7.26)

In equation (7.26) \( \sigma_{br} \) is defined as \( P_j = \sigma_{br}(x_j) \cdot w_j \cdot t_{f,tot} / t_{m,tot} \). The condition \( Q_2 = 0 \) for \( x = x_k \) is necessary because in those locations where fibre failure occurs there is no fibre elongation and prepreg deformation, therefore there is no displacement compatibility.

From the definition of \( H \) given in equation (7.22) it is possible to write,

\[ H \sigma_{br}(x) = Q \] (7.27)

where the bridging stress can be calculated inverting the matrix \( H \).

\[ \sigma_{br}(x) = H^{-1}Q \] (7.28)
**Definition of the fibre failure induced stress**

Fibre failure occurs in those locations where the strain reaches the strain-to-failure of the fibres. Therefore, the total fibre strain in the wake of the crack must be calculated by superimposing the bridging strain (or stress, $\sigma_{br}$) to the far-field fibre strain (or stress, $\sigma_f$).

From experimental observations, it is understood that fibre failure occurs in correspondence of the stress peaks. In a FML panel containing a fatigue crack and subjected to increasing load, fibre failure can therefore occur at the tip of the initial notch or at the tip of the fatigue crack, see Figure 7.30.

The approach followed to model fibre failure consists in considering the damage in the fibre layers as a “crack-like” damage, defined as $a_{\text{fibre}} = a_s + \sum w_k$, where $a_s$ is the initial saw-cut length and $\sum w_k$ is the cumulative extension of the broken fibres.

This is done calculating first $K_f$ at the tip of current crack length in the fibre layer, $a_{\text{fibre}}$, using equation (7.10). Subsequently, the stress distribution $\sigma_f(x)$ is calculated with equation (7.29).

\[
\sigma_f(x) = \frac{K_f}{\sqrt{2\pi x}}
\]  

(7.29)

After $\sigma_f(x)$ is calculated, the total fibre stress in the wake of the fatigue crack is given by superimposing the bridging stress to the stress distribution due to the far field stress. This results in a total stress distribution containing two stress peaks: one due to the stress concentration ahead of the initial notch, and the other one behind the fatigue crack tip, as shown as example in Figure 7.31.
The modelling approach for the failure sequence and extension of the crack in the fibre layers is schematically illustrated in Figure 7.32. Here the strain-to-failure of the fibres is represented with a horizontal line, and fibre failure occurs when the curve of the total strain in the fibres (derived from the total stress) intercepts the horizontal line. The location where fibre failure occurs is highlighted with a bar-element and the location of the boundaries of the area of intact fibres is represented with two triangles. The large triangle represents the boundary at the side of the initial notch (from where the fatigue crack has been generated), while the small triangle represents the boundary at the side of the fatigue crack tip. As described, the presence of two stress peaks can induce failure at both sides, as illustrated in Figure 7.32.

Equation (7.23) represents the vertical displacement due to fibre failure and is independent on the location of failure. This means that multiple failures are possible at either side of the bridging fibres.

Figure 7.31  Example of total stress in the fibre layer obtained by superposition of bridging stress to far-field stress distributions, calculated for Glare 3 4/3-0.4 at an applied stress of 180 MPa.

Convergence of the plastic zone and bridging stress

There is a relationship between the extension of the plastic zone ahead of the crack tip and the magnitude of the bridging stress. Indeed, the presence of bridging fibres reduces the stress intensity factor at the metal crack tip and thus the plastic zone size. On the other hand, a reduction of the plastic zone decreases the vertical displacement of the metal layers, causing a reduction of bridging stresses. These parameters are thus intrinsically related to each other. This implies that a
convergence on the stress intensity factor is necessary to correctly model both the plastic zone size and the bridging stress. Therefore, the method described in the previous sections includes the convergence of the plastic zone and bridging stress, and a description of the convergence approach is provided in appendix C.

\[ \epsilon_{\text{ult}} \]

Load is increased and damage extended

\[ \Delta \]

Position of the damage in the fibre layer starting from the fatigue crack tip

\[ \Delta \]

Position of the damage in the fibre layer starting from the saw-cut

**Figure 7.32**  
Schematic illustration of the modelling approach for fibre failure and fibre damage extension

### 7.4.2 Static delamination growth model

The delamination growth is modelled using the SERR, following an approach similar to the one used by previous authors [12, 15] for fatigue delamination growth. In the case of fatigue delamination growth, the delamination model is based on a
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Paris type of relation [12, 15], but in the case of static delamination growth the critical value for the SERR, $G_c$, is used. The complex interaction between static delamination and plastic deformation in the metallic layers, described in section 6.3.7, has been modelled in a simplified manner.

As described in section 6.3.7, the plastic zone initially grows in front of the crack tip, but extends backwards due to the presence of bridging fibre when the load increases, see Figure 6.23. This mechanism cannot be described easily with the available LEFM models. Therefore, a simplified elastic-plastic formulation of the SERR has been developed, see appendix D, and used together with the elastic formulation of equation (7.30). The method to combine the elastic and elastic-plastic formulations of the SERR is described in appendix D.

Considering Figure 7.29, at the centre line of each bar element the elastic SERR is calculated with equation 7.30 and with equation (D.16), it is subsequently compared with the critical SERR value obtained from experimental tests, see chapter 6. If the calculated SERR exceeds the critical value, the delamination extends of a fixed length $\Delta b$ locally. The stress re-distribution is subsequently calculated re-solving the compatibility equation with the given extended delamination at the given applied load, see Figure 7.33.

$$G(x) = \frac{n_f t_f}{2 j E_f} \left( \frac{n_m t_m E_m}{n_m t_m E_m + n_f t_f E_f} \right) \left( \sigma_{f,\text{tot}}(x) \right)^2 \quad (7.30)$$

This procedure is repeated for each location until the condition of $G < G_c$ is reached. When delamination extension occurs locally, the local fibre stress reduces and therefore the local SERR reduces. The local delamination extension is therefore a mechanisms that reflects also to the state of stress in the remainder of the crack. Indeed, if the applied load remains unchanged, the local reduction of the bridging stress induces a re-distribution of the stress among the rest of the material. This stress re-distribution is accounted for by solving the compatibility equation with the new delamination shape as input. When all locations along the delamination contour are satisfying the condition $G < G_c$, the applied load is increased again and a new loop is executed in the model.


7.4.3 Crack opening displacement and crack growth model

The procedure described in section 7.4.1 is used to solve the compatibility equation along the boundary of the delamination; the solution provides the bridging stress. After the bridging stress is calculated, all contributions to the actual COD(x) are known. Indeed, to calculate the COD(x) of the laminate the superposition principle is employed accounting for fibre bridging and fibre failure, as written in equation (7.2).

The term $COD_{m}(x)$ is calculated using equation (7.5) for $y=0$, while the terms $COD_{br}(x)$ and $COD_{f}(x)$ are defined using equations (7.22) and (7.23) with $y=0$. It follows,

$$COD_{m}(x) = \sum_{j=1}^{N} v_{br}(x,y) \quad \text{with } y=0$$

(7.31)

$$COD_{f}(x) = \sum_{k=1}^{N} v_{f}(x,y) \quad \text{with } y=0$$

(7.32)

The CTOA is then calculated with equation (7.3). When the condition $CTOA \geq CTOA_{c}$ is reached, the crack extends a length equal to $\Delta a$, and the CTOD is calculated with equation (7.11). The calculation of the effective COD(x) is done as described in equation (7.12).

In experiments, the residual strength of a flat panel made of Glare can be reached also due to net-section yielding [9]. To account for this, the net-section stress, $\sigma_{net}$, is calculated at each load step and compared to the yield stress of the laminate, $\sigma_{ys \ lam}$.
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\[
\sigma_{\text{net}} = \frac{P}{(W-2a) \cdot t_{\text{tot}}} \tag{7.33}
\]

If \(\sigma_{\text{net}} \geq \sigma_{\text{ys lam}}\) the intact ligament yields and the failure of the panel is assumed to be reached due to net-section yielding.

### 7.4.4 Numerical calculation approach

The method described in the previous sections is implemented into a numerical model similar to the one illustrated in Figure 7.12. In addition to the input parameters described in section 7.3.4, the model requires the following input parameters:

- The critical strain energy release rate, \(G_c\).
- The initial fatigue delamination shape.
- The saw-cut length and the initial fatigue crack length.

The flow diagram illustrated in Figure 7.12 applies also to the fatigue crack configuration, but in this case, the "CTOA block" is described in

![Flow diagram of the residual strength prediction model for fatigue crack configurations](image)

Figure 7.34  \textit{Flow diagram of the residual strength prediction model for fatigue crack configurations}
The model requires a given initial applied load, from which the stress levels acting in each layer are calculated by means of CLT. Subsequently, the compatibility equation is solved. This is done including the convergence procedure to define the actual plastic zone size and bridging stress distribution. The solution of the compatibility equation accounts also for the delamination growth model. Subsequently, the contributions $COD_{br}(x)$ and $COD_{f}(x)$ are computed and superimposed to $COD_{m}(x)$. The CTOA is therefore calculated from the effective COD$(x)$, resulting from the superposition of $COD_{br}(x)$, $COD_{f}(x)$ and $COD_{m}(x)$. Comparison of the calculated CTOA with the value provided by the input curve defines whether crack extension occurs or not. In case the criterion is not met, the load is increased by a fixed amount $\Delta P$ and the length of the yielded bar-element, $CTOD_{c}$, is set equal to zero; the procedure is repeated with the increased load. If the criterion is met, the current CTOD is stored as $CTOD_{c}$ and the crack extends a fixed length $\Delta a$. The stored $CTOD_{c}$ is used in the next loop to account for permanent plastic deformation in the wake of the crack. In this case, the current applied force $P_{k}$ is stored as $P_{ext}$, where $P_{ext}$ represents the vector containing the load step at which crack extension occurs. After each crack extension the load is reduced by a quantity equal to $\Delta P_{down}$. This procedure enables to reduce the applied load when the residual strength is reached. This is somehow similar to what occurs during a residual strength test performed in displacement control. After each crack extension, the CTOA is calculated with the new crack geometry, and the condition $CTOA \geq CTOA_{c}$ is evaluated again.

7.4.5 Results and validation

The developed model has been validated based on the three criteria described in section 7.2.5. The physical realism is discussed in section 7.5, while the accuracy and robustness are discussed in the following.

Model accuracy

The accuracy of the model is validated against experimental test data performed at Faculty of Aerospace Engineering of TUDelft, see Table 6.5 and presented in appendix E. Here, two reference tests on standard Glare panels are used to compare prediction with experimental results. The input parameters for 2024-T3 and S2-glass/FM94 prepreg are listed in Figure 7.14, while parameters related to geometry, loading conditions and failure criteria are listed in Figure 7.35. The input curve for the CTOA has been selected based on the discussions provided in chapter 6, thus calculated at 0.5 mm behind the fatigue crack tip. Flat Glare 4B-3/2-0.4 and Glare 3-5/4-0.4 (test 16 and 18 in Table 6.6 respectively) panels are here considered for validation of the developed method. Both panels have a width of 800 mm and contain a saw-cut, $2a_{s}$, of 100 mm.
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<table>
<thead>
<tr>
<th>Geometry and loading parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
</tr>
<tr>
<td>$W$  ...  mm</td>
</tr>
<tr>
<td>$2a_0$  ...  mm</td>
</tr>
<tr>
<td>$\Delta a$  0.5  mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Failure criteria parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTOA for 2024-T3 t = 0.4mm</td>
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<tr>
<td>$CTOA = a e^{-b(a-a_0)} + c$</td>
</tr>
<tr>
<td>a 10</td>
</tr>
<tr>
<td>b 0.4</td>
</tr>
<tr>
<td>c 6</td>
</tr>
<tr>
<td>Calculation distance behind the crack tip</td>
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<tr>
<td>d 0.5  mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strain-to-failure S2-glass/FM94</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{\text{ult}}$ 0.45%</td>
</tr>
<tr>
<td>Critical Energy release rate, $G_c$ 0.7  MPa-mm</td>
</tr>
</tbody>
</table>

Figure 7.35  *Input parameter for geometry, loading conditions and failure criteria corresponding to the flow diagram in Figure 7.35.*

Figure 7.36 and Figure 7.37 show the remote stress versus physical crack extension curve of both prediction and experimental results. In both cases, the model is predicting the overall trend. Noticeable is the fact that the residual strength is reached after few millimetres of stable crack extension in the case of Figure 7.36, and after half millimetre in the case of Figure 7.37.

**Figure 7.36**  *Remote stress vs. physical crack extension: comparison between analytical prediction and experimental test for a Glare 4B-3/2-0.4 T-L,*
panel with \( W = 800 \text{ mm} \) and \( 2a_s = 103 \text{ mm}, \ 2a_0 = 200 \text{ mm} \) (test 1 in Table 6.6).

Figure 7.37 Remote stress vs. physical crack extension: comparison between analytical prediction and experimental test for a Glare 4B-3/2-0.4 T-L, panel with \( W = 800 \text{ mm} \) and \( 2a_s = 103 \text{ mm}, \ 2a_0 = 300 \text{ mm} \) (test 2 in Table 6.6).

Model robustness

As for the model relevant to saw-cut configurations, the third criterion to validate the current model for fatigue crack configurations is based on the robustness of the method. For the saw-cut configuration, the model robustness was based on the variation of both \( d \) and \( \Delta a \). The model for saw-cut configurations showed to be robust, and therefore it is assumed that same robustness applies also to the current model for fatigue crack configuration. This assumption lies on the fact that both models are based on the same method to calculate the CTOA, and related dependency on both \( d \) and \( \Delta a \). The main difference in modelling saw-cut and fatigue crack configurations lies in the presence of bridging fibres in the wake of the fatigue crack. Therefore, the question to be answered in the current section is whether the model is robust relatively to the width of the bar element \( w_i \), defined in the fatigue crack length as described in appendix B.

The occurrence of local fibre failure is an important fact that may change the evolution of the subsequent damage, because the load is redistributed and further damage may occur in the adjacent material. As described in appendix B, the convergence of the procedure is based on a fixed value for the smallest bar-element, \( w_{fix} \). The width of this bar-element, which corresponds to the bar-element at the two sides of the fatigue crack, determines the calculated local peak in the bridging stress.
Four values for $w_{\text{fix}}$ were selected: 0.1, 0.25, 0.5, and 1mm. Predictions are illustrated in Figure 7.38.

The current model seems to be affected by the length of the bar-element width $w_{\text{fix}}$. It is important to remind that the stress distribution in the bridging fibres is the results of superposition of the stress gradient due to the far-field load and the bridging stress, see Figure 7.31. The sum of these two stress systems generates two peaks in the total stress distribution: the stress peak at the saw-cut tip and the stress peak at the fatigue crack tip.

The stress peak at the saw-cut tip is calculated using equation 7.29, where the term $x$ is substituted with $x_i$ as defined in appendix B. For a small value of $w_{\text{fix}}$ the calculated value of the stress in the fibre layers increases, and vice-versa.

The stress peak at the fatigue crack tip is calculated by solving the displacement compatibility, see equation (7.28), and it is strongly dependent on the local delamination length, $b(x)$. The equation defining the delamination length is based on the coordinate $x$, and can be expressed with a polynomial equation of the third order, see equation 7.34.

$$b(x) = a_3(x - a_2)^3 + a_2(x - a_1)^2 + a_1(x - a_0) + a_0$$  \hspace{1cm} (7.34)
Considering the description provided in appendix B, if a small value of $w_{\text{fix}}$ is used, the calculated delamination length at the fatigue crack tip results to be very small and therefore the calculated bridging stress is very high (the bridging stress is inversely proportional to the delamination length).

The developed model is very sensitive to the width of the bar-element $w_{\text{fix}}$, because its value may accelerate or delay local fibre failure and subsequent stress redistribution. Premature local fibre failure may be the cause of further fibre failure with subsequent reduction of the residual strength. This sensitivity to the parameter $w_{\text{fix}}$ affects the robustness of the model, and makes necessary to perform a tuning of $w_{\text{fix}}$ to optimize the prediction.

At the current state the model is not robust enough and requires a number of adjustments to be make it reliable for residual strength predictions. On the other hand, the model describes all important failure mechanisms in a physical sound way. This anyway provides a solid base to further improve the robustness of the model.

**Bridging stress distribution**

The effect of the amount of bridging fibres on the shape of the residual strength curve of FMLs containing fatigue cracks has been discussed already in section 6.4.2. The bridging fibres are responsible for carrying part of the load, which depends on the length of the fatigue crack. Indeed, the larger the amount of bridging fibres (e.g. longer fatigue crack), the higher the probability of brittle-like failure of the panel without stable crack growth, see section 6.4.2.

If the applied load is increased, the fibres can stretch over the delaminated length, thus restraining the crack opening in the metal layers. If the strain reaches the ultimate value (strain-to-failure), fibres failure occurs and the stress is released to the remaining intact material.

An example of evolution of bridging stress distribution is presented in Figure 7.39, where the curves represent the increasing bridging stress due to the increasing applied load. Fibre failure occurred at both sides of the fatigue crack, and is illustrated in Figure 7.39 as missing points in several curves. Indeed, when fibre failure occurs, the absence of fibre stresses implies that the compatibility equation no longer needs to be solved. Figure 7.39 shows only the bridging stress, but fibre failure accounts also for the stress gradient due to the far-field stress, as schematically illustrated in Figures 7.31 and 7.32.
Figure 7.39 Example of prediction of bridging stress distribution and related increase as result of increasing applied load. Fibre failure denoted by missing points in the curves when the load increases. Glare 4B-3/2-0.4 T-L, panel with $W=800$ mm and $2as=103$ mm, $2a0=200$ mm.

**Delamination growth due to increasing load**

The fatigue crack in the prediction model is defined in terms of pre-crack length, $a_f$, and fatigue delamination length, $b(x)$. The delamination shape is provided as external input, and needs to be determined either with experimental measurements or with an analytical approximation. To perform a proper prediction, it is important to input an initial fatigue delamination shape based on experimental measurements. Therefore, the fatigue delamination shape has been experimentally measured using DIC at the end of the fatigue crack growth test. From the DIC analysis of the tested panels, a curve fit equation representing the actual delamination shape has been obtained. This curve fit is then put into the model as initial fatigue delamination shape.

An example of delamination growth due to increasing load is depicted in Figure 7.40, which relates to the bridging stress distribution in Figure 7.39. Initially, the delamination growth occurs behind the fatigue crack tip, where the peak in the bridging stress induces a local increment of the SERR above the critical value, $G\geq G_c$. The missing points in the curves in Figure 7.40 relate to the locations where fibre failure occurred, see Figure 7.39.
Effect of the delamination shape on the residual strength

It has been reported in [9,16] that the amount of fatigue delamination influences the residual strength of FMLs. In [16] Vermeeren tested several 200 mm wide Glare 2 and Glare 3 panels containing fatigue cracks of 50, 75 and 100mm. Each of these crack lengths were obtained with two fatigue stress levels, 150 and 120 MPa. The larger fatigue delamination areas were obtained testing at 150 MPa. Subsequently, residual strength tests were performed on these panels, and results showed that panels containing a larger delaminated area (but still with the same crack length) exhibit higher residual strength. This is due to the fact that if fibres are free to strain over a longer length (delamination length), the actual stress in the fibres is lower.

This aspect is shown in Figure 7.41, where three predictions related to a Glare 4B-3/2-0.4 with W=800 mm, 2a_s=103mm and 2a_0=200mm are presented. To simulate the effect of different delamination sizes, a fictitious delamination shape was defined as in equation (7.35), where \( b_\mathrm{s} = b_\mathrm{0}(a-a_s) \) and \( a \) is the fatigue crack length \( (a_s+a_f) \). Three different values for \( b_\mathrm{0} \) were selected in order to obtain three delamination sizes with parabolic shapes; the parabolic shape has been chosen based on considerations reported in [12]. Predictions are illustrated in Figure 7.41.

\[
b(x) = b_\mathrm{s} \sqrt{1 - \frac{x-a_s}{a-a_s}} \tag{7.35}
\]
These predictions, although not validated with experimental results, show a trend that is in agreement with the experimental evidence reported in [16]. The increased bridging stress due to a small delaminated area may induce premature fibre failure if static delamination does not occur. This can lead to a lower residual strength. On the other hand, if the delaminated area is relatively large, fibre failure is postponed.

All failure mechanisms, such as delamination growth, plastic deformation, fibre failure and metal cracking, are interconnected with each other and any small change in any of the related parameters may determine relatively large changes in the residual strength behaviour.

7.5 Physical realism of the developed method

The first of these criteria listed in section 7.1.2 is met based on the description of the modelling of the failure sequence illustrated in the previous sections of this chapter. Indeed, metal crack growth and fibre failure mechanisms have been described in a physically sound way, accounting for the crack-tip plastic zone, residual plastic strain and load re-distribution after fibre failure.

The developed analytical method has the benefit to be generic and based on the behaviour of the constituents, rather than the behaviour of the whole laminate. The
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generic nature of the method enables its potential application to different materials and configurations.

7.6 Validity range of the developed models

An important constraint on the prediction model is the validity range; the range in which the validity of the method is proven with experimental results. To validate the method, the predicted crack growth behaviour is compared with experimental results. All experimental results consist of remote stress versus physical crack length data. The material parameters can be divided into the FML type, the laminate lay-up and the thickness of the aluminium layers. The validation of the model is presented in appendix E.

| Table 7.2     Validity range of the developed models |
|----------------|-----------------|
| Lay-up     | Saw-cut| Fatigue Crack |
| Grade      | 3/2, 4/3, 8/7 | 3/2, 5/4 |
| Metal type | Glare 3 T-L, Glare 4B T-L | Glare 3 T-L, Glare 4B T-L |
| Prepreg type | 2024-T3 | 2024-T3 |
| $t_m$ [mm] | S2 glass/FM94 | S2 glass/FM94 |
| $W$ [mm]   | 0.3, 0.4, 0.5 | 0.4 |
| $a_s$ [mm] | 400, 800 | 800 |
| $\Delta a_f$ [mm] | 47, 50, 66, 100 | 51.5, 75 |
| $a_0$ [mm] | 77, 100, 150, 200 |
| $2a_s/W$   | 0.23, 0.25, 0.33 | 0.25, 0.37, 0.5 |

7.7 Summary

Two analytical crack growth models for predicting the residual strength of FML panels containing either a through-the-thickness or a fatigue crack have been developed based on the CTOA failure criterion.

The model for through-the-thickness crack configurations is based on the calculation of the CTOA in the metallic layers accounting for the opening effect due to fibre failure. Plastic deformation of the metallic layers has been implemented both ahead of the propagating crack (Irwin’s correction) and in the wake of the crack (yielded strip bar-elements).

The model for fatigue crack configurations is based on the same approach, with in addition the implementation of the effect of fibre bridging and static delamination growth. The bridging stress calculation is based on the solution of the displacement compatibility equation, which is related to the crack opening of monolithic
aluminium and to the elongation and deformation of the prepreg. Fibre failure and plastic deformation of the metallic layers have been implemented and accounted for in the solution of the compatibility equation. In addition, a simplified static delamination growth model has been included based on an elastic-plastic formulation of the SERR.

The models are substantiated with a wide range of experimental data, partially presented in appendix E.

References


Conclusions and future prospects

8.1 Conclusions

The investigation of the present thesis is concerned with the residual strength failure sequence in generic FMLs, but with a greater focus on the Glare variant. Two damage scenarios were investigated: the through-the-thickness crack and the fatigue crack. The failure mechanisms common to both damage scenarios are the crack initiation and propagation in the metallic layers and the fibre failure. In addition, static delamination occurs at the metal/fibre interface as result of the increasing shear load transferred through the interface. Static delamination is a dominant failure mechanism in fatigue cracks subjected to increasing load.

An analytical method was developed to predict the residual strength in FMLs. The method is based on the CTOA as a failure criterion for the metallic constituent and on the ultimate strain for the composite constituent. The static delamination growth has been modelled with a modified SERR, accounting for the plastic deformation of the metallic layers.

Two numerical procedures were implemented based on the developed method: one for the through-the-thickness crack scenario and one for the fatigue crack scenario. The assumptions, on which both models are based, are:

- Linear Elastic-Plastic Fracture Mechanics is applied to describe the failure mechanisms.
o Metal and composite constituents are equally strained if no delamination is present.
o Crack growth in the metallic constituent occurs under plane stress conditions.
o The crack growth under increasing load is related to the critical CTOA of the metallic constituent.
o The CTOA curve for the metal constituent alone is also valid for the FML containing the same metallic constituent.
o Plastic deformation ahead of the crack-tip is described using the plastic zone Irwin’s correction.
o When fibre failure occurs all the energy stored inside the broken fibres transfers to the intact part of the laminate. Failure occurs when the strain-to-failure of the fibre is reached locally.

From the current investigation, several conclusions can be drawn with respect to the crack growth mechanisms of FMLs subjected to increasing load and to the crack growth prediction models.

### 8.1.1 Residual strength failure sequence in FMLs

The residual strength failure in FMLs is characterized by large plastic deformations as result of the high applied loads. The main failure mechanisms occurring in the metallic layers consist of crack tip blunting, followed by crack initiation and crack growth. The large amount of plastic deformation ahead of the crack tip results in a growing crack containing a plastically deformed wake.

Deformation compatibility between metal and composite layers is present unless delamination occurs. Therefore, in absence of delamination, fibres and metal undergo the same strain field. If the strain in the fibres reaches the strain-to-failure, fibre failure occurs locally and the load stored in the fibres releases into the remaining material.

*Through-the-thickness crack configuration*

The shape of the stress vs. crack extension curve of a FMLs containing a saw-cut is very similar to the curve of an equivalent monolithic aluminium alloy. The actual shape of the curve is mainly related to the ductility of the metallic layers, which is responsible for the amount of stable crack growth.

Fibre failure occurs ahead of the crack tip, where high strain gradients are present due to the stress concentration.
**Fatigue crack configuration**

For panels containing fatigue cracks of much longer length than the initial saw-cut length (e.g. $a_f > 2a_s$), the failure condition may be reached without stable crack propagation.

Fibre failure can begin at either sides of the fatigue pre-crack, but for large saw-cuts fibre failure mainly initiates at the saw-cut tip. Subsequently, fibre failure propagates in the direction of the fatigue crack tip.

Static delamination occurs when the shear stress at the fibre/metal interface reaches the critical value. However, plastic deformation occurring in the metallic layers can delay the onset of static delamination.

The occurrence of static delamination is always beneficial for the residual strength of FMLs panels.

**8.1.2 Residual strength prediction models**

The residual strength prediction models described in this thesis lead to the following conclusions:

The CTOA is a suitable parameter to describe the crack growth in FMLs. The CTOA curve of the metallic constituent is used as failure criterion and input into the model.

The analytical calculation of the CTOA directly results from geometrical considerations obtained by superimposing the crack opening contours. For a realistic calculation of the CTOA, the residual plastic strain in the wake of the propagating crack has to be taken into account, using for example yielded bar-elements.

Fibre failure is described as a crack opening stress system acting on the metallic layers. On the other hand, the effect of bridging fibres is described as crack closing stress system acting on the metallic layers.

Static delamination growth is modelled using a simplified SERR approach, accounting for the plastic deformation of the metallic layers.

The analytical models are physically sound as the effects of plasticity, fibre failure, fibre bridging and delamination growth are properly addressed. The prediction model for the through-the-thickness crack is accurate and validated with a wide range of experimental data, including crack growth results and R-curves.
The prediction model for the fatigue crack configuration has been validated with a relative small amount of experimental data. The higher degree of complexity of such damage scenario required a lot of simplifications in the model. The predictive capability of such model is therefore limited if compared to the model developed for through-the-thickness crack.

8.2 Future prospects

The current residual strength prediction model has the potential to be extended to other material and geometrical configurations. Fields of interests into which the model can be extended are mentioned below.

Material conditions

The material input parameters of the individuals constituents are associated with the CTOA curve for the crack growth in the metallic layers, the strain-to-failure for fibre failure and the critical Strain Energy Release Rate for the static delamination. CTOA curves can be obtained experimentally on metal laminates based on other aluminium alloy series, such as 7000 for example. Other fibre systems can be considered, e.g. carbon-fibres or S2-glass with different resin system. The models can be adapted with available input data to predict the residual strength of HSS Glare for example (7475 with S2-glass/FM906).

Geometrical conditions

The current models cover CCT flat specimens with either a through-the-thickness crack or a fatigue crack loaded parallel to the fibre direction or perpendicular to the fibres. The model for through-the-thickness crack can be extended for the following geometrical conditions:

- Different metal thickness – The CTOA curves can be obtained experimentally for a wide range of metal sheet thickness, e.g. from 0.3 to 1.5 mm. The models can subsequently be validated with experimental results obtained through residual strength tests on large FMLs panels. This may validate the prediction models also for thick FMLs.

- Bonded stiffened panels – The models can be extended towards stiffened configurations. The effect of either intact or broken stiffeners can be implemented in terms of crack closing or opening contribution. In this case, it will be necessary to describe the static delamination and static failure of the bonded stiffeners.
Coupling to the FCG model developed by Alderliesten

The FCG model developed by Alderliesten can be coupled with the residual strength prediction model for fatigue crack presented in this thesis. The FCG provides the final fatigue delamination shape of a fatigue crack grown until a prefixed length under constant amplitude fatigue load. Theoretically, the fatigue delamination shape and fatigue crack length can be input from the FCG model into the residual strength model. This may permit to obtain the reduction of residual strength of an FML related to the growing fatigue crack.
Westergaard stress functions

Abstract – This appendix presents the derivation of the Westergaard stress functions used to model the vertical displacement due to the far-field load, bridging fibres and broken fibres.

A.1 Introduction

As described in chapter 7, both bridging and broken fibres strongly affect the COD of the metal layers: the bridging fibres are modelled as a system of closing forces, while the broken fibres as a system of opening forces. Both are applied at the boundary of the delamination. The calculation of the bridging stress is done by solving the displacement compatibility equation; therefore the definition of the displacement for each term of equation (7.11) is needed. In the following the definition of the displacement terms for the far-field load, bridging and broken fibres is provided.

A.2 Vertical displacement due to far-field stress

Using Westergaard stress functions, $Z_I$ and $\bar{Z}_I$, from [1,2] augmented with the correction for uniaxial loading given by [3], the displacement due to far-field stresses at any position, $(x,y)$, can be written:
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\[ v(x, y) = \frac{1}{2G} \left( \frac{\kappa+1}{2} \text{Im} Z - y \text{Re} Z - \frac{\kappa-3}{2} \frac{\sigma_x}{2} y \right) \]  

(A.1)

where \( \kappa = \left(\frac{3-\nu}{1+\nu}\right) \) for plane stress, and the Westergaard stress functions are defined:

\[ Z_i = \frac{\sigma_x}{\sqrt{1 - \left(\frac{a_{eff}}{z}\right)^2}} \]  

(A.2)

and

\[ \bar{Z}_i = \sigma_x \sqrt{z^2 - a_{eff}^2} \]  

(A.3)

with \( z = x + iy \), where \( x \) and \( y \) are the horizontal and vertical coordinates at which the displacement is calculated. Here, \( \sigma_x \) is the far-field stress in the metal layers, which includes the thermal residual stresses and the stress in the metal due to the applied loading, calculated with laminate plate theory. Equations (A.1) is the same used in equation (7.5).

### A.3 Vertical displacement due to bridging or broken fibres

The displacement, measured at point \((x, y)\), due to four-point load \( P_j \) at any given point, \((x_j, y_j)\), along the delamination boundary is given by the following equations, adapted from [1]

\[ v_j(x, y) = \frac{1 - \nu y}{G} \text{Im} \bar{Z}_i - \frac{y}{2G} \text{Re} Z_i \]  

(A.4)

where
\[ Z_i = \frac{P_i}{\pi} \left[ \left( \frac{\sqrt{a^2 - z_j^2}}{z^2 - z_j^2} + \frac{\sqrt{a^2 - z_j^2}}{z^2 - z_j^2} \right) - \alpha y_j \left( \frac{-i \cdot z_j}{(z^2 - z_j^2)\sqrt{a^2 - z_j^2}} + i \cdot 2z_j \cdot \frac{\sqrt{a^2 - z_j^2}}{(z^2 - z_j^2)} \right) \right] \]

\[ + \frac{P_i}{\pi} \left[ \frac{i \cdot z_j}{(z^2 - z_j^2)\sqrt{a^2 - z_j^2}} - \frac{i \cdot 2z_j}{(z^2 - z_j^2)\sqrt{a^2 - z_j^2}} \right] \left( \frac{1}{\sqrt{1 - (a/z)^2}} \right) \]

\[ + \frac{P_i}{\pi} \left[ \frac{-y_j}{(z-x_j)^2 + y_j^2} - \frac{y_j}{(z+x_j)^2 + y_j^2} + \alpha y_j \left( \frac{(z-x_j)^2 - y_j^2}{((z-x_j)^2 + y_j^2)^2} + \frac{(z+x_j)^2 - y_j^2}{((z+x_j)^2 + y_j^2)^2} \right) \right] \] (A.5)

and

\[ \overline{Z}_j = \frac{P_i}{\pi} \left[ \tan^{-1} \frac{\sqrt{z^2 - a^2}}{\sqrt{z^2 - z_j^2}} + \tan^{-1} \frac{\sqrt{z^2 - a^2}}{\sqrt{z^2 - z_j^2}} \right] \]

\[ - \alpha y_j \left( \frac{i \cdot z_j}{z^2 - z_j^2} \cdot \frac{\sqrt{z^2 - a^2}}{\sqrt{z^2 - z_j^2}} - \frac{i \cdot z_j}{z^2 - z_j^2} \cdot \frac{\sqrt{z^2 - a^2}}{\sqrt{z^2 - z_j^2}} \right) \]

\[ + \alpha y_j \left( \frac{z}{z_j^2 - z^2} + \frac{z}{z_j^2 - z^2} \right) \left( i \cdot \tanh^{-1} \frac{z}{z_j} - i \cdot \tanh^{-1} \frac{z}{z_j} \right) \] (A.6)

with \( z = x + iy \), \( z_j = x_j + iy_j \), and \( \overline{z}_j = x_j - iy_j \), \( a = a_{\text{eff}} \)

\[ \alpha = \begin{cases} \frac{1}{2} (1 + \nu) & \text{plane stress} \\ \frac{1}{2} \left( \frac{1}{1 - \nu} \right) & \text{plane strain} \end{cases} \]

Equation (A.4) is used in equations (7.18) and (7.19) for the bridging and broken fibres respectively.
References


Definition of the bar-element with variable width

Abstract – This appendix describes the procedure adopted to define the variable width bar element.

During the experimental tests on fatigue crack configurations it has been observed that fibre failure began mainly at the tip of the saw-cut; this is due to the stress concentration at that location. The other stress peak in the fibre bridging area is located behind the fatigue crack tip. To properly describe the stress gradient in those locations, the fatigue crack length has been divided in bar-elements with variable width, in a way that more dense distribution of bar-elements is present near the peaks. Illustration of the fatigue crack geometry is provided in Figure B.1.

The fatigue crack is divided into two parts, $W_L$ and $W_R$, and each of these is divided into $n$ bar-elements with variable widths defined as in equations (B.1) and (B.2). The total number of bar-elements, $n$, is defined based on an iterative procedure where the smallest bar-element is minimized to a given value $w_{fix}$. This means that it is imposed that the total number of bar elements in each half of the fatigue crack is such that the first (for $W_L$) and the last (for $W_R$) bar-elements have the width smaller or equal to $w_{fix}$.

$$w_i = \frac{a - a_x}{2} \cos\left(\frac{2N_i - 1}{2N_i} \cdot \frac{\pi}{2}\right) \cdot \frac{\pi}{2N_i}$$

with $N_i = 1..n$  

(B.1)
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\[ w_i = \frac{a - a_s}{2} \sin \left( \frac{2N_i - 1}{2N_i} \cdot \frac{\pi}{2} \right) \cdot \frac{\pi}{2N_i} \]

with \( N_i = 1..n \) \hspace{1cm} (B.2)

After the number of bar element is defined, all terms contained in both equation both equation (B.1) and (B.2) are grouped forming two vectors \( W_L \) and \( W_R \), equations (B.3) and (B.4).

\[ W_L = [w_{i_1}, w_{i_2}, ..., w_{i_j}, ..., w_{i_n}] \]

(B.3)

\[ W_R = [w_{r_1}, w_{r_2}, ..., w_{r_j}, ..., w_{r_n}] \]

(B.4)

These are subsequently grouped forming one single vector of length \( 2n \) denoted as \( W \), see equation (B.5).

\[ W = [W_L, W_R] \]

(B.5)

The position of each bar-element along the fatigue crack line is defined by the coordinate \( x_j \), defined as in equations (B.6) and (B.7).

\[ x_j = \frac{W_j}{2} \]

(B.6)
\[ x_j = \frac{W_1}{2} + \frac{(x_{j-1} + W_{j-1})}{2} \quad \text{with } j=2..n \] (B.7)

For the prediction discussed in this thesis a value of 0.5 mm was selected as \( w_{fix} \). It is important to define a proper value of \( w_{fix} \) because the stress peak at the saw-cut tip is defined based on equation (7.9), which depends on the coordinate in the denominator. If the centre line of the bar element, \( x_i \), is too close to the origin (e.g. too small value of \( w_{fix} \)) the stress peak might be too large with subsequently premature fibre failure. On the other hand, if the centre line of the bar element, \( x_i \), is too far from the origin (e.g. too large value of \( w_{fix} \)), the stress peak might be too low and fibre failure would be postponed.
Iterative process for convergence of the plastic zone size and bridging stress

Abstract – This chapter provides details on the iterative procedure for the convergence of both plastic zone and bridging stress.

Given the plastic zone size, \( r_p \), calculated on the metal constituent only, the compatibility equation is solved as shown in section 7.4.1. Following the approach described by Alderliesten in [1], the stress intensity factor at the metal crack tip can be calculated accounting for the presence of bridging fibres

\[
K_{FML} = K_{metal} - K_{br} \tag{C.1}
\]

The new SIF is then used to re-calculate the plastic zone extension, \( r_p \), which depends on the actual SIF at the tip of the crack in the metal layers. The new calculated plastic zone size is then used to define the new \( a_{\text{eff}} \) which will be included in the compatibility equation. The calculated bridging stress provides a new value of the SIF at the metal crack tip. The procedure is a close loop and it stops when the difference between the SIF calculated at step \( i \) and \( i-1 \) is less than 2%. The convergence procedure is illustrated in Figure C.1.

The stress intensity factor due to the bridging loads, \( K_{br} \), is approximated by combining the solutions for the stress intensity factor due to point loads above the crack flanks along the centreline and due to point loads along the flanks, symmetric
about the centreline. This approximation, see equation (C.2), is the same used by Alderliesten [1,2] to calculate the stress intensity factor due to the bridging fibres, except for the use of $a_{\text{eff}}$ instead of $a$.

\[
K_{\text{old}} = f\left(\frac{a}{W}\right)\sigma\sqrt{\pi a}
\]

\[
r_s = \frac{1}{2\pi} \left(\frac{K_{\text{old}}}{\sigma_{\text{ys}}}\right)^2
\]

\[
a_{\text{eff}} = a + r_s
\]

\[
K_{\text{new}} = f\left(\frac{a_{\text{eff}}}{W}\right)\sigma\sqrt{\pi a_{\text{eff}}}
\]

\[
K_{\text{current}} = K_{\text{new}} - K_{\text{br}}
\]

\[
K_{\text{br}} = \frac{2P}{\pi a_{\text{eff}}^2} \left(1 + \frac{1}{2} (1+\nu) \frac{b^2}{a^2 - x_p^2 + b^2}\right)
\]

(C.2)

Figure C. 1 Convergence procedure to calculate the actual plastic zone extension and bridging stress


Abstract – This chapter provides details on the simplified approach adopted to formulate the elastic-plastic strain energy release rate.

Many authors have described the mode II delamination growth by means of SERR [1, 2], especially for growth due to cyclic loading. In this appendix a simplified version of the SERR accounting for the permanent plastic deformation is described. The energy release rate for delamination is calculated from the energy balance of a delamination specimen, see Figure 6.16. For delamination over a distance \( b \), the elastic energies of both metal and fibres layers have changed, due to the change of the stress in those layers after delamination. In addition, also the external energy has changed due to the displacement of the external load \( P \).

The occurrence of permanent plastic deformation in the metallic layers results in a loss of elastic strain energy available for creating delamination, therefore only the elastic strain energy has to be taken into account. A generic bilinear stress-strain curve is represented in Figure D. 1, where both elastic and elastic-plastic curves are illustrated. \( E_{el} \) and \( E_{pl} \) are the elastic- and the plastic Young’s modulus. Given the deformation, \( \varepsilon_m \), the elastic stress is \( \sigma_{m,el} \), while the actual elastic-plastic stress is \( \sigma_{m,pl} \). When plastic deformation occurs, the elastic energy available is \( U_{el,pl} \), which is significantly smaller than the elastic energy in absence of plastic deformation \( U_{el} \). This is the energy that is stored inside the material and is released elastically when unloading.
Figure D. 1 Illustration of the elastic-plastic bilinear stress-strain curve.

Figure D. 2 Illustration of the change in stress and energy levels after delamination.

The behaviour described above applies also to delamination extension in FMLs, as shown in Figure D. 2 where the illustration shows the situation after delamination $b$ has occurred. Following the approach of Marissen [1], the total energy balance is given by

\[ U - F + W = 0 \]  \hspace{1cm} (D.1)
U is the increase of the elastic energy in the various layers during delamination over a distance \( b \). \( F \) is the work of the external forces \( P \) during the displacement \( \Delta y \). \( W \) is the “surface” energy which is consumed during delamination over a distance \( b \).

Considering the energy per unit of specimen width, the work of the external forces is defined as

\[
F = P \cdot \Delta y
\]

and

\[
\Delta y = b \cdot \Delta \varepsilon
\]

where \( \Delta \varepsilon \) is the strain difference between the delaminated and non-delaminated area.

\[
\Delta \varepsilon = (\sigma_{f,\text{tot}} - \sigma_f) / E_f
\]

Therefore, it follows that the work of the external forces is

\[
F = P \cdot b \cdot (\sigma_{f,\text{tot}} - \sigma_f) / E_f
\]

which can be written as

\[
F = \sigma_{f,\text{tot}} \cdot t_f \cdot b \cdot (\sigma_{f,\text{tot}} - \sigma_f) / E_f
\]

The change in elastic strain energy \( U \) is the difference between the increase of elastic energy in the fibre layers, \( U_1 \), and the decrease of elastic strain energy in the metallic layers, \( U_2 \).

\[
U = (U_{1f} + U_{1m}) - (U_{2f} + U_{2m})
\]

where
Given the equilibrium of the system, and that the condition of equal strain in all non-delaminated system applies, it follows,

\[
\begin{align*}
\varepsilon_f &= \varepsilon_m \\
\sigma_m t_m + \sigma_f t_f &= \sigma_{f_{\text{tot}}} t_f
\end{align*}
\]  
(D.10)

In eq. (D.10), the strain in the metal layers, \( \varepsilon_m \), accounts for the permanent plastic deformation, and it is defined as

\[
\begin{align*}
\varepsilon_m &= \varepsilon_y + \frac{(\sigma_m - \sigma_{ys})}{E_{pl}} \\
\varepsilon_f &= \frac{\sigma_f}{E_f} = \varepsilon_m
\end{align*}
\]  
(D.11)

Defining \( F_m = t_m E_m \), \( F_{pl} = t_m E_{pl} \), and \( F_f = t_f E_f \), and substituting eq. (D.11) in eq. (D.10) it follows

\[
\begin{align*}
\sigma_m &= \frac{\sigma_{f_{\text{tot}}} t_f E_{pl} + \sigma_{ys} F_f E_{pl} \left( \frac{1}{E_{pl}} - \frac{1}{E_m} \right)}{(F_f + F_{pl})} \\
\sigma_f &= \frac{\sigma_{f_{\text{tot}}} F_f + \sigma_{ys} F_{pl} E_f \left( \frac{1}{E_m} - \frac{1}{E_{pl}} \right)}{(F_f + F_{pl})}
\end{align*}
\]  
(D.12)

Substitution of eq. (D.12) into eq. (D.8), (D.9) and (D.6) and combining eq. (D.8), (D.9) and (D.11), results in
Finally eq.(D.1) can be rewritten as

\[
W = \sigma_{f_{\text{tot}}} \cdot t_f \cdot b \cdot (\sigma_{f_{\text{tot}}} - \sigma_f) / E_f + \frac{1}{2} t_f b \cdot \frac{\sigma_{f_{\text{tot}}}^2}{E_f} - \frac{1}{2} t_m b \cdot \sigma_m \cdot \varepsilon_m - \frac{1}{2} t_f b \cdot \frac{\sigma_f^2}{E_f}
\]

(D.14)

where the terms \( \sigma_f \) and \( \sigma_m \) are defined in eq.(D.12) as functions of \( \sigma_{f_{\text{tot}}} \).

Equation (D.14) represents the energy which is generated during delamination over a distance \( b \). The total strain energy release rate \( G \) follows from differentiation:

\[
G = \frac{t_f}{E_f} \left( \frac{3}{2} \sigma_{f_{\text{tot}}}^2 - \sigma_{f_{\text{tot}}} \sigma_f \right) - \frac{1}{2} \left( t_m \cdot \sigma_m \cdot \varepsilon_m + t_f \cdot \frac{\sigma_f^2}{E_f} \right)
\]

(D.15)

Equation (D.15) describes the total strain energy release rate for two fibre-aluminium interfaces. The strain energy release rate for a generic number of interfaces is given dividing \( G \) by \( j \), where \( j \) is the number of fibre/metal interfaces. It follows that, the strain energy release rate for delamination, per fibre-metal interface is given by

\[
G = \frac{t_f}{j E_f} \left( \frac{3}{2} \sigma_{f_{\text{tot}}}^2 - \sigma_{f_{\text{tot}}} \sigma_f \right) - \frac{1}{2} \left( t_m \cdot \sigma_m \cdot \varepsilon_m + t_f \cdot \frac{\sigma_f^2}{E_f} \right)
\]

(D.16)

Figure D.3 illustrates a plot of the calculated SERR using both the elastic \( G_{el} \), equation (7.30), and elastic-plastic \( G_{pl} \), equation (D.16). It is important to point out that in this case, \( G_{pl} \) represents the elastic portion of the total strain energy release rate. The method is used into the model in the following way:

- If \( G_{el} < G_{pl} \), it follows that \( G_{\text{driving}} = G_{el} \)
- If \( G_{el} > G_{pl} \), it follows that \( G_{\text{driving}} = G_{pl} \)
Alternatively, a criterion based on the applied stress can be obtained by equalling equation (D.16) with equation (7.30). This is a simplified way of accounting for the effect of plastic deformation on static the delamination growth.

![Illustration of the driving SERR.](image)

Figure D.3 Illustration of the driving SERR.


Abstract – This appendix present a selection of the residual strength curves in which the prediction of the models are compared with the measurements from experiments.

E.1 Introduction

The residual strength models developed in chapter 7, for both the through-the-thickness and fatigue crack, are validated with a large amount of test data from experimental tests, see Table 6.6 and Table 6.7. The predicted applied stress vs. crack extension curve is compared with the experimental crack growth curve. Because presenting of all validation graphs goes beyond this thesis, a selection has been made, which is presented in this appendix.

The figures presented in this appendix are complementary to those presented in chapter 7, and consist of applied stress vs. crack extension curves and R-curves.

The validation tests presented in Figure E.1 to Figure E.9 have been selected to give a complete overview of all material- geometrical and starter notch length experimentally investigated. Results are grouped for saw-cut and fatigue crack configuration.
E.2 Through-the-thickness crack

Figure E.1 Comparison between experimental and predicted remote stress vs. crack extension curve of a Glare 3-3/2-0.3 L-T panel containing a saw-cut. \( W=800\text{mm}, 2a_0=200\text{mm} \) (Test 1 in Table 6.6)

Figure E.2 Comparison between experimental and predicted remote stress vs. crack extension curve of a Glare 3-4/3-0.5 L-T panel containing a saw-cut. \( W=800\text{mm}, 2a_0=200\text{mm} \) (Test 4 in Table 6.6)
Figure E.3  Comparison between experimental and predicted remote stress vs. crack extension curve of a Glare 4B-4/3-0.4 L-T panel containing a saw-cut. $W=800\text{mm}, 2a_0=200\text{mm}$ (Test 5 in Table 6.6)

Figure E.4  Comparison between experimental and predicted remote stress vs. crack extension curve of a Glare 4B-3/2-0.3 L-T panel containing a saw-cut. $W=800\text{mm}, 2a_0=200\text{mm}$ (Test 7 in Table 6.6)
Figure E.5  Comparison between experimental and predicted remote stress vs. crack extension curve of two Glare panels containing a saw-cut. \(W=810\) mm with \(2a_0=200\) mm, and \(W=400\) mm with \(2a_0=100\) mm [1].

Figure E.6  Comparison between experimental and predicted remote stress vs. crack extension curve of two 400 mm wide Glare 3-3/2-0.3 L-T panels. \(2a_0/W=0.25\) and \(2a_0/W=0.33\) (Test 9 and 10 in Table 6).
Figure E.7  Comparison between experimental and predicted R-curve of two 800 mm wide Glare panels, Glare 3-3/2-0.3 L-T and Glare 4B-3/2-0.3 L-T. $2a_0/W=0.25$ mm (Test 1 and 7 in Table 6.6)

Figure E.8  Comparison between experimental and predicted R-curve of two 400 mm wide Glare 3-3/2-0.3 L-T panels, with $2a_0/W=0.25$ mm and $2a_0/W=0.33$ mm (Test 9 and 10 in Table 6.6)
E.3 Fatigue crack configuration

E.3.1 Measuring the fatigue delamination shape

The residual strength prediction of fatigue cracks requires to input the fatigue delamination shape, which can be obtained either experimentally or approximated analytically. DIC has been used to measure the fatigue delamination shape at the end of the fatigue cycles. When the fatigue delamination is developed, the strain difference between the delaminated and the non-delaminated area is measured with DIC to obtain a visualization of the delamination shape. An example of delamination shape is illustrated in Figure E.9.

The delamination shape is represented by the dark color, related to very low strain levels (about zero strain level). The front of the delamination is marked with dots and the position of each dot is related to the position of the saw-cut tip and fatigue crack tip. In this way, an average curve for the delamination shape is obtained for a selection of the tested specimens in Table 6.7.

Figure E.9 Illustration of Fatigue delamination shape obtained with DIC.

Figure E.10 illustrates the delamination curves for Glare 3 and Glare 4B specimens in Table 6.7. All specimens were tested at 120 MPa and R=0.05. Each delamination curve is fitted with a third order polynomial curve obtained with DIC, see Table E.1.
The curve fit equation is subsequently used as input delamination shape for prediction of the residual strength.

![Figure E.10](image)

Figure E.10  Comparison between fatigue delamination shapes obtained with DIC for Glare 3-3/2-043 and Glare 4B-3/2-0.4, related to different fatigue crack length. Tests relate to Table 6.7.

<table>
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<th>$a_3$</th>
<th>$a_4$</th>
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<tr>
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<td>0.0043</td>
<td>-0.2049</td>
<td>23.13</td>
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<tr>
<td>8</td>
<td>-0.0015</td>
<td>0.0329</td>
<td>-0.2958</td>
<td>11.97</td>
</tr>
</tbody>
</table>

Table E.1 Curve fitting parameters for fatigue delamination shape \( b(x)=a_1 x^3 + a_2 x^2 + a_3 x + a_4 \)

E.3.2 Comparison between predictions and experiments

In the following the residual strength predictions are compared with the experimental results. Unfortunately, not all results of the tests reported in Table 6.7 are available due to problems occurred during the measurements. However, the quasi-static crack growth curves reported in the following, together with those presented in chapter 7, provide enough data to evaluate the developed prediction model, within the tested range.
The Residual Strength Failure Sequence of Fibre Metal Laminates

**Figure E.11** Comparison between experimental and predicted of the remote stress vs. crack extension curve of a 800 mm wide Glare 3-3/2-0.4 panel, with \( a_s = 51.5 \text{ mm}, \ a_f = 25.5 \text{ mm} \) (Test 8 Table 6.7).

**Figure E.12** Comparison between experimental and predicted of the remote stress vs. crack extension curve of a 800 mm wide Glare 3-3/2-0.4 T-L panel, with \( a_s = 51.5 \text{ mm}, \ a_f = 98.5 \text{ mm} \) (Test 7 Table 6.7).
Figure E.13  Comparison between experimental and predicted of the remote stress vs. crack extension curve of a 800 mm wide Glare 4B-3/2-0.4 T-L panel, with $a_e = 51.5$ mm, $a_f = 148.5$ mm (Test 3 Table 6.7).

"Het reststerkte faalproces in Vezel Metaal Laminaten"
Riccardo Rodi

Het concept van de schade tolerantie is een belangrijk aspect bij het waarborgen en de handhaven van de veiligheid van een vliegtuigconstructie gedurende zijn levensduur. De ontwikkelingen in materialen en constructief ontwerp hebben beide bijgedragen aan verbetering van de schade tolerantie van moderne vliegtuigconstructies. Nieuwe ontwikkelingen in metaallegeringen, composietmaterialen, en hybride materialen, zoals Vezel Metaal Laminaten (VMLs) hebben allen geleid tot constructies die minder gevoelig zijn voor schade en in staat zijn om grotere belasting te weerstaan. VMLs zijn een duidelijk voorbeeld van schade tolerante hybride materialen, gemaakt door het verlijmen van dunne metalen platen met vezels ingebed in epoxy.

Het benutten van het schade tolerantie vermogen van VMLs is strikt gerelateerd aan de mogelijkheid om in de eerste plaats de optredende faalmechanismen te begrijpen, en ten tweede om deze mechanismen nauwkeurig te kunnen beschrijven. In licht hiervan onderzoekt dit proefschrift onderzoekt het reststerkte faalproces in VMLs, en heeft als doel de ontwikkeling van een nauwkeurige analytische voorspellingsmethode. Het faalproces wordt met name bestudeerd voor standaard Glare laminaten, die relevant zijn voor toepassingen in de vliegtuig drukrompen. De ontwikkelde analytische methode is geïmplementeerd in twee numerieke modellen, die zowel de door-de-dikte scheur en de vermoeiingsscheur configuraties beschouwen De ontwikkelde modellen zijn gevalideerd aan de hand van een groot aantal experimentele gegevens, die ook worden gepresenteerd in dit proefschrift Het belangrijkste concept in dit proefschrift is dat het scheurgroeiprocess in de metaallagen van een VML kan worden beschreven met de Scheur Tip Opening Hoek (STOH). Deze aanpak bevat de bijdrage van de vezellagen (bijv. vezel falen en de vezeloverbrugging) en de daarmee gepaard gaande quasi-statische delaminatie groei.

Een inleiding op VMLs en alle verschillende Glare klassen, lay-ups, en productieprocessen wordt gegeven in hoofdstuk 2. In dit hoofdstuk worden ook een aantal huidige en toekomstige toepassingen in luchtvaartconstructies besproken.
Een kwalitatieve beschrijving van de belangrijkste faalmechanismen die zich tijdens het reststerkte faalproces voordoen wordt gepresenteerd in hoofdstuk 3. Gebaseerd op experimentele waarnemingen worden hier besproken: de scheur groei mechanismen in het metaal, de permanente plastische vervorming, het falen van vezels en de statische delaminatie groei. Al deze mechanismen zijn aan elkaar gerelateerd en dragen bij aan de reststerkte van het laminaat.

De ontwikkeling van de huidige voorspellingsmodellen was bedoeld om een stap voorwaarts te zijn in vergelijking met de beschikbare modellen in de literatuur. Zowel bestaande empirische als analytische voorspellingsmodellen worden gepresenteerd en besproken in hoofdstuk 4. Een kritische evaluatie van deze modellen heeft gewezen op hun beperkingen in toepasbaarheid en veelzijdigheid op weg naar een "generiek VML" concept. In dit hoofdstuk zijn een aantal richtlijnen vastgesteld om de verdere ontwikkeling van een model aan te pakken.

Twee typen experimentele activiteiten werden uitgevoerd. Ten eerste experimenten om inzicht te krijgen in het deformatiegedrag van zowel metalen als vezellagen (hoofdstuk 5). Veelvuldig gebruik van de Digitale Beeld Correlatie techniek stond toe om het vervormingsveld van zowel de metaal- als de vezellagen te observeren en meten, alsmede de interactie tussen deze lagen. Er werd nader inzicht verkregen in het vezel-overbruggingsmechanisme en in de metaal-vezel interactie (hoofdstuk 5).

Het tweede type experimenten had als doel om input voor het voorspellingsmodel te genereren en om de STOH benadering te valideren. Deze worden behandeld in hoofdstuk 6. Een werd een grote hoeveelheid experimentele STOH experimenten uitgevoerd werden uitgevoerd op verschillende FML klassen om de STOH als faalcriterium voor VMLs te evalueren. Dit omvatte onderzoek naar het effect van de metaalplaatdikte, de verhouding van scheurlengte tot paneel breedte en het effect van overbruggende vezels. Statische delaminatie groeitests werden uitgevoerd om de kritieke rek energie afgiften snelheid te verkrijgen. Deze parameter werd vervolgens gebruikt als invoer voor het voorspellingsmodel om de kritieke voorwaarde te definieren voor delaminatie groei. Daarnaast wordt in hetzelfde hoofdstuk de complexe interactie tussen statische delaminatie groei en plastische vervorming van de metaallagen besproken.

Het kernaspect van dit proefschrift heeft betrekking op het modelleren van het reststerkte faalproces, en wordt gepresenteerd in hoofdstuk 7. Twee modellen worden beschreven: een voor de door-de-dikte scheur en een voor de vermoeingsscheur. Beide zijn gebaseerd op dezelfde methode, die de STOH als drijvende parameter voor de scheurgroei gebruikt. De methode is gebaseerd op het idee dat scheurverlenging in de metaallagen optreedt wanneer de berekende STOH de kritische STOH waarde bereikt die is verkregen uit experimenten op VMLs met
Samenvatting

dezelfde metaallagen in de VML. De berekende STOH is een functie van een bijdrage van de verre veld spanning in de aluminium lagen, en de bijdrage van de vezels. De vezel kan een bijdrage leveren in termen van scheur openingsbijdrage (gebroken vezels) of scheursluitingsbijdrage (overbruggende vezels aanwezig in de vermoeiingsscheur-configuratie). Er wordt rekening gehouden met plastische vervorming voor en achter de scheurtip in de metaal laag en dit wordt geïmplementeerd in de berekening.

Daarnaast wordt voor het geval van de vermoeiingsscheurconfiguratie de overbrugging spanning berekend door het oplossen van de vervormingscompatibiliteitsvergelijking, rekening houdend met het plastic gebied voor de scheurtip en vezel falen in het overbruggingsgebied. De overbruggingsspanning wordt vervolgens gebruikt voor het berekenen van de quasi-statische delaminatie groei die plaatsvindt op de vezel-metaal interface, door middel van de rek energie afgiftesnelheid methode.

De twee analytische modellen zijn geïmplementeerd in twee numerieke voorspellingsmodellen die zijn gevalideerd met een breed bereik aan test data. Het model voor de door-de-dikte scheur vertoonde een zeer goede overeenkomst met de test data. Het model voor de vermoeiingsscheur configuratie vertoonde voldoende overeenkomst met de test data. Het modelleren van de vermoeiingsscheur omvat een hogere mate van complexiteit, waardoor een aantal aannames en versimpelingen nodig waren. Als gevolg hiervan is dit model minder robuust dat het model voor de door-de-dikte scheur.

Hoofdstuk 8 vat de conclusies van dit onderzoek samen. Er kan worden geconcludeerd dat de voorgestelde modellen de mechanismen gerelateerd aan het reststerkte faal proces volledig omschrijven en karakteriseren. Het door-de-dikte scheurmodel is robuust en gevalideerd en kan worden uitgebreid naar andere materialen en geometrische configuraties. Het model voor vermoeiingsscheuren is niet robuust genoeg, maar verdere verbeteringen zijn mogelijk.
Curriculum vitae

The author was born in Erice, Italy, in 1980. After graduated from the high school in September 1999, he moved to Pisa to study at University of Pisa, at the Faculty of Engineering. The author graduated in February 2007 with a Master degree thesis titled: “the effect of external stiffening elements on the fatigue crack growth in Fibre Metal Laminate”. The thesis was partially the result of a 5 months internship period at the chair of Structural Integrity at the Faculty of Aerospace Engineering of TU Delft, under the daily supervision of Dr.Ir. R. Alderliesten and Prof. L.Lazzeri (University of Pisa). The master degree thesis has been also published as book Ed.by VDM Verlag Dr. Muller.

After graduation, in May 2007, the author was appointed as PhD researcher, by the Material Innovation Institute –M2i –, at the Structural Integrity Group of the Faculty of Aerospace Engineering of TU Delft. During this 4-years period, the author conducted several experimental and analytical modelling activities, which resulted in the present PhD dissertation. In addition, the author has been involved in a number of side projects related to the damage tolerance of Hybrid structures and FMLs.

From May 2011 the author received a 2-years research contract from the Structural Integrity Group of the Faculty of Aerospace Engineering of TU Delft. The main field of research concerns the damage tolerance of aircraft structures, hybrid and composite materials.
Publications

International conferences presentations


Scientific international journal publications


Book publications

Acknowledgements

Going throughout this PhD research has been challenging and sometimes difficult, but thanks to the support and enthusiasm of colleagues and friends I made it!

First of all, I would like to express my appreciation to my supervisors, that accompanied me along the way of my PhD research. Rene’ Alderliesten and Rinze Benedictus have trusted in me and provide me with valuable discussions and suggestions to approach the research path in the best way.

I would also thank Prof. Jaap Schijve for being so energetic and enthusiastic about research on materials and structures. Despite his no longer young age, Prof. Schijve has been always curious and stimulating about the results of my research.

I want to thanks Gemma van der Windt for being so amazing and efficient in keeping all things going on. But, the most appreciated quality is her happiness and positivisms, which sometimes I needed especially during the last part of my research period. Thanks a lot Gemma!

A big thank goes also to the office staff of the Materials Innovation Institute –M2i– which have been always kind and enthusiastic with me. I am very grateful to Bert van Haastrecht, programme manager of M2i, who followed my project.

I would like to express my gratitude to all committee members who have dedicated part of their time to review this thesis and have travelled to Delft to participate to the defence.

Half of the time I spent at the Faculty of Aerospace Engineering of TUDelft was dedicated to experimental testing. Therefore, I am very grateful to the technicians of the Aerospace Materials and Structure Laboratory for their support and assistance. Berthil, Bob, Hans, Frans, Kees, Ed, Johan, and the others dedicated a lot of their time to help me out with the setup of testing machines and measuring devices.

The time I spent in Delft has seen many people around me, friends and colleagues were always part of my working day. A special thank goes to current and past colleagues of TUDelft: Ricky, Amir, Rik-Jan, Gustavo, Ligeia, Milan, Patricio, Roger, Amin, Jos, Greg W., Greg R., Vincent, Alfonso, Sharif, Gianni, Alessandro, Cecilia, Freddy, Konstantin, Maria Luisa, Maria Barroso, Vanessa, Frederik, Nick,
Bas, Chris V., Arjan, Adrian L., Adrian F., Irene, Ilhan, Kees, Paola, Giorgio, Rita, Paolo.

The biggest thank goes to my family, my father, my mother and my sister, that supported me during this challenging adventure. They have always been enthusiastic and pushed me forward during the though moments.

A special thank goes to Antonella, she appeared in my life in the very final stage of my PhD study, but she has been able to motivate me more and more, and provide me with one more reason to be happy!! Antonella, does not belong to my PhD experience, but certainly belongs to what will be after it!