REEF BREAKWATER RESPONSE
TO WAVE ATTACK

by

John P. Ahrens, Aff. M., ASCE

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Résumé

Un brise-lames de type récif est un brise-lames en enrochements à crête basse qui ne comporte pas, en coupe transversale, les couches successives des ouvrages classiques. Ce type de brise-lames est essentiellement un entassement homogène de pierres individuelles d'une masse analogue à celle des pierres utilisées pour la carapace et la première couche des brise-lames classiques. En raison de leur grande porosité, les brise-lames de type récif résistent avec une stabilité surprenante à l'assaut des vagues tout en dissipant de manière efficace l'énergie des vagues.
Reef Breakwater Response To Wave Attack

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Abstract

A method of predicting stability of a class of low-crested rubble mounds referred to as reef breakwaters is presented. Findings are based on a study which included an extensive series of physical model tests conducted with irregular waves. Stability is measured in terms of reduction in crest height due to wave attack. Reduction in crest height has less scatter than damage measured by the number of stones displaced and provides a direct link between stability and primary performance characteristic of wave transmission. A stability model is developed which can accurately predict degradation of the reef from zero-damage to very severe levels and for a wide range of wave conditions. An example is presented which illustrates the ability of the model to predict damage, which in turn is used to estimate wave transmission and reflection characteristics and energy dissipated by the reef when it is in equilibrium with wave conditions.

Introduction and Background

A reef breakwater is a low-crested rubble-mound breakwater without the traditional multilayer cross section. This type of breakwater, in essence, is a homogeneous pile of stone with individual stone weights similar to those used in the armor and first underlayer of conventional breakwaters. Because of their high porosity, reef breakwaters are surprisingly stable to wave attack and, at the same time, can dissipate wave energy effectively.

It is anticipated reef breakwaters would be used primarily for beach stabilization or to protect eroding shorelines in a shore parallel or near parallel manner. Generally these functions can allow, or possibly benefit from, larger transmitted wave heights than can be tolerated in a harbor. This suggests that considerable cost savings could be achieved by using lower created structures. In a discussion of a variety of typical coastal erosion problems Fulford (1995) concludes reef breakwaters represent the most satisfactory general approach to shoreline stabilization.

One of the recurring problems faced by coastal engineers has been to demonstrate that a stable rubble-mound configuration can be achieved with stone available. The fact that this could not always be done led to the development of concrete armor units. However, it was realized by some engineers that the full potential of stone was not being utilized. It has been frequently noted that the stability of a rubble-mound increases as it adjusts to wave attack (Bruun and Johannesson, 1976 and Bruun, 1985). Bruun has shown how some mature breakwaters have reached an equilibrium profile with relatively small

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stone, considering the wave climate. A logical extension of Bruun's work has been developed by Baird and colleagues through the use of an extensive, highly permeable berm incorporated into the breakwater cross section (Baird and Hall, 1984 and Baird and Hall, 1987). A highly porous berm helps distribute wave forces and dissipate energy. This strategy allows the use of smaller stone than would be required for armor of a traditional design which often means that construction can be conducted with land based equipment in a dump and push mode. Because of the tendency to develop a breakwater design with a traditional cross section and profile, concrete armor units have been used in situations where they were not required (Baird and Hall, 1984). This tendency may increase the cost of a rubble-mound and at the same time reduce its reliability. Baird and colleagues have now refined the berm breakwater approach and have shown that substantial cost savings can be obtained without loss of functional performance or compromise on safety. Research by van der Meer and Pilarczyk (1987) has produced effective methods to parameterize the process where rubble mounds adjust to wave conditions. From this effort, a mathematical model was developed which can predict the equilibrium profile of a berm breakwater.

Since reef breakwaters would ordinarily be used to protect beaches and eroding shoreline, short periods of relatively high wave transmission could be accepted as part of a cost effective design. A logical approach, consistent with the findings of Bruun and Baird, to the design of reef breakwaters would be to allow or accept considerable adjustment of the profile so long as wave transmission characteristics fall within acceptable limits. Unfortunately, until recently the information necessary to adopt this strategy has not been available. A flexible approach to the design of reef breakwaters is presented in this paper which allows consideration of the response of the reef to wave action and its resulting influence on the reef's performance. This approach is based on a synthesis of the findings of many other investigators and on an extensive series of laboratory model tests conducted at the Coastal Engineering Research Center (CERC). From these tests, a conceptual model for the stability of reef breakwaters evolved which finally led to the development of a mathematical model to predict the stability to wave attack. It is commonly observed that low-crested rubble mounds respond to severe wave action and heavy overtopping by having their crest heights reduced and their side slopes flattened (Carver and Davidson, 1983). This characteristic is a conspicuous response of reef breakwaters to wave attack and the response noticeably increases resistance to further damage. The reef breakwater stability model has the ability to predict this response accurately over a wide range of wave conditions and structure configurations.

Using the stability model to predict an equilibrium crest height for the reef allows this information to be used to estimate hydraulic performance characteristics of wave transmission and reflection and energy dissipation. Prediction of these characteristics are quite dependent on the stability model since hydraulic performance is so sensitive to crest height. The stability model will be discussed and
its use for predicting hydraulic performance characteristics of reef breakwaters will be illustrated. Results from this research will help alleviate the problem noted by Dally and Pope (1986) about the lack of quantitative design guidance for detached breakwaters used to protect shorelines and beaches.

Laboratory Setup, Conditions, and Procedures

Reef breakwater model tests were conducted in a 61 cm wide channel within CERC's 1.2 m high by 4.6 m wide by 42.7 m long wave tank. Water depths at the reef were 25 cm for most tests and 30 cm for a few tests. Water depths at the wave generator were 25 cm greater than at the reef and waves shoaled over a 1 on 15 slope to reach the concrete platform the reef was built on. This setup insured that very severe wave conditions could be produced at the structure. The testing channel was open to a wave absorber area on the landward side of the reef so there could be very little ponding effect behind the breakwater to complicate evaluation of stability.

All tests were conducted with irregular waves. Spectra were JONSWAP type in the deeper portion of the wave tank prior to wave breaking. Incident and reflected spectra were resolved with three parallel wire resistance wave gages in front of the reef by using the method of Goda and Suzuki (1976). Period of peak energy density of the spectra, $T_p$, ranged from about 1.45 to 3.60 seconds and the range of incident zero-moment wave heights, $H_{mo}$, was about 1.1 to 18.2 cm. Two gages were used behind the reef to measure transmitted wave heights.

Two basic types of tests were conducted during the laboratory investigation. Most tests fell into a category where the primary objective was to determine stability of the reef to a specific wave condition and wave transmission data were obtained as a by product. These are referred to as stability tests. The other category was tests conducted at the completion of a stability test to determine transmission characteristics of a damaged structure to a variety of less severe wave conditions. These are called "previous damage" tests. Table 1 organizes the tests into subsets and gives important characteristics of each subset. In Table 1, stability tests have odd subset numbers and "previous damage" tests have even numbers. For stability tests, the stone was dumped in the dry testing channel and pushed around primarily by foot to conform to a template outlining the desired profile of the structure. This procedure was used to prevent overly careful placement of the stone, but at the same time to insure the initial profile of all reefs within a subset would be reasonably consistent. Desired initial profile for a stability test is a trapezoid with a crest width of three stone diameters and having both seaward and shoreward slopes of 1 on 1-1/2. When the profiles were surveyed, initial crest elevations of reefs within a subset could vary as much as ±1.5 cm from the desired height. Duration of wave action for stability tests was one and one-half hours for tests with $T_p$ of 1.45 sec and up to three and one-half hours for tests of $T_p$ of 3.60 sec. This duration gives a total of between 3500-4000 waves.

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Table 1
Basic Data For Each Subset

<table>
<thead>
<tr>
<th>Subset No.</th>
<th>No. of Tests</th>
<th>Water Depth, $d_w$ (cm)</th>
<th>Crest Height as Built, $h'_{c}$ (cm)</th>
<th>Median Stone Weight, $W_{50}$ (gr.)</th>
<th>Area of Breakwater Cross Section, $A_t$ (cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23</td>
<td>25</td>
<td>25</td>
<td>17</td>
<td>1170</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>25</td>
<td>NA</td>
<td>17</td>
<td>1170</td>
</tr>
<tr>
<td>3</td>
<td>29</td>
<td>25</td>
<td>30</td>
<td>17</td>
<td>1560</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>25</td>
<td>NA</td>
<td>17</td>
<td>1560</td>
</tr>
<tr>
<td>5</td>
<td>41</td>
<td>25</td>
<td>35</td>
<td>17</td>
<td>2190</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>25</td>
<td>NA</td>
<td>17</td>
<td>2190</td>
</tr>
<tr>
<td>7</td>
<td>38</td>
<td>25</td>
<td>32</td>
<td>71</td>
<td>1900</td>
</tr>
<tr>
<td>8</td>
<td>26</td>
<td>25</td>
<td>NA</td>
<td>71</td>
<td>1900</td>
</tr>
<tr>
<td>9</td>
<td>13</td>
<td>30</td>
<td>32</td>
<td>71</td>
<td>1900</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>30</td>
<td>NA</td>
<td>71</td>
<td>1900</td>
</tr>
</tbody>
</table>

NA denotes not applicable to "previous damage" test series.

Based on $T_p$, and the reefs appeared to be at equilibrium with wave conditions well before the completion of a test. At the completion of a stability test, there was a final survey to document the equilibrium profile of the reef.

After some stability tests, previous damage tests would be conducted. Previous damage tests always used wave conditions less severe than the preceding stability test so there was almost no change in the profile during this type of test. Previous damage tests were not used to develop the reef stability model. Figure 1 shows the initial and equilibrium reef profile for a series of tests with progressively higher wave heights.

Two sizes of stone were used during this study. Characteristics of stone and associated gradation are given in Table 2. Ahrens (1987) provides extensive information on laboratory setup, conditions, and procedures.

Discussion of Findings

During this study, it was found that the stability of the reef was strongly dependent on the wave period. The following stability parameter was identified as the best way to characterize severity of wave attack on a reef breakwater where $N_s^*$ is referred to as the spectral stability number (Ahrens 1987),

$$N_s^* = \left( \frac{H_{\text{mo}}^2}{L_p} \right)^{1/3} \left( \frac{W_{50}}{W_r} \right)^{1/3} \left( \frac{W_r}{W_w} - 1 \right)$$

(1)
Figure 1: Reef profiles before and after wave action
Table 2
Stone and Gradation Characteristics

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Quartzite</th>
<th>Diorite</th>
</tr>
</thead>
<tbody>
<tr>
<td>2% weight (gr)</td>
<td>7.0</td>
<td>14.0</td>
</tr>
<tr>
<td>Median weight, ( W_{50} ) (gr)</td>
<td>17.0</td>
<td>71.0</td>
</tr>
<tr>
<td>98% weight (gr)</td>
<td>28.0</td>
<td>139.0</td>
</tr>
<tr>
<td>Density (gr/cm³)</td>
<td>2.63</td>
<td>2.83</td>
</tr>
<tr>
<td>Porosity</td>
<td>44%</td>
<td>45%</td>
</tr>
</tbody>
</table>

where \( L_a \) is the Airy wave length calculated using \( d_a \) and \( l_a \), and \( w_a \) is the specific gravity of water. \( N_a^s \) is similar to the stability number developed by Hudson (1958) and used extensively to quantify the stability of traditional breakwaters (Hudson and Davidson, 1975). The need to account for the influence of wave period in the study of reef stability, as compared to earlier studies of traditional type rubble mounds, appears to be partly due to the use of a rather small wave height, \( H_{mo} \), to characterize the stability, (i.e. small compared to the maximum wave height). Period effect also seems to be due to the fact that magnitude and intensity of overtopping flow are proportional to \( H_{mo} L_p \) as noted in the study of wave overtopping of seawalls, Ahrens, et al. (1986). Gravesen, et al (1980) gave an equation which contains the spectral stability number in their paper discussing irregular wave tests of breakwater stability. During this study, it was found that there was no net stone displacement for stability tests where \( N_a^s < 6.0 \) but there was noticeable stone movement for \( N_a^s > 8.0 \).

Figure 1 shows reef profiles for 4 tests. For each test, the profile at the beginning of the test (before any waves have been generated) is compared to the profile at the end of the test. The profile at the end of the test is regarded as the equilibrium profile. Table 3 summarizes important information about the tests shown in Figure 1. Tests shown in Figure 1 are intended to provide a typical sequence of how a reef responds to progressively higher waves. \( T_p \) is about the same for all four tests. One way to characterize response of the reef to wave attack is by use of reef response slope parameter, \( A_t/h_0^2 \). As wave conditions become more severe, the reef responds by becoming ever lower. Using the height of a reef to characterize stability is convenient since it is not so sensitive to random variations as using the number of stones removed or an equivalent method. The height of the reef also relates directly to its functional performance characteristics of wave transmission and reflection and energy dissipation. In the sequence of tests shown in Figure 1, the reef response slope \( A_t/h_0^2 \) has an initial value of about 1.80 for all tests. It increases from 1.92 for the mild wave conditions of Test 40 to 4.21 for the severe conditions of Test 36. \( A_t/h_0^2 \) is a measure of the seaward and shoreward slope of the reef (analogous to the contangent of a traditional breakwater) and can also be regarded as a shape variable.
Table 3

Conditions for Damage Sequence Profiles Shown in Figure 1

<table>
<thead>
<tr>
<th>Subset/Test No.</th>
<th>$H_m$ (cm)</th>
<th>$T_p$ (sec)</th>
<th>$A_t/h_o^2$</th>
<th>$N_s^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/40</td>
<td>13.68</td>
<td>2.79</td>
<td>1.92</td>
<td>5.91</td>
</tr>
<tr>
<td>3/39</td>
<td>7.49</td>
<td>2.82</td>
<td>2.33</td>
<td>9.53</td>
</tr>
<tr>
<td>3/38</td>
<td>10.98</td>
<td>2.81</td>
<td>3.13</td>
<td>12.28</td>
</tr>
<tr>
<td>3/36</td>
<td>15.63</td>
<td>2.98</td>
<td>4.21</td>
<td>15.86</td>
</tr>
</tbody>
</table>

In Figure 2, $A_t/h_o^2$ is plotted versus $N_s^*$ for all 148 stability tests. A regression curve is fit to the data for $N_s^* \geq 6.0$; it is these tests that indicate an adjustment of the reef to wave conditions. The regression curve follows the trend of data for $N_s^* > 6.0$ surprisingly well. It is surprising because it is known that the relative height of the structure, $h_o/d_o$, and the bulk of the reef also play an important role in determining the amount of response of the reef to wave action (Ahrens, 1987). The equation of the curve shown in Figure 2 is given by

$$
\frac{A_t}{h_o^2} = \exp \left[ K N_s^* \right]
$$

\[ (2) \]

![Figure 2](image-url) "Figure 2. Reef breakwater response slope, $A_t/h_o^2$, versus the spectral stability number, $N_s^*$"
where the regression coefficient is \( K = 0.0945 \) and is similar to a stability coefficient. Rewriting Equation 2 gives

\[
\frac{1}{K} = \frac{N_s^4}{\ln(A_t/h_c^2)} \tag{3}
\]

which resembles the form of Hudson's (1958) equation, i.e.

\[
K_D = \frac{N_s^3}{\cot \theta} \tag{4}
\]

where a stability coefficient is given as the ratio of a stability number to a function of the slope of the structure. Another advantage of the form of Equation 3, in addition to fitting the data well, is that it approaches logical limiting values

\[
\frac{A_t}{h_c^2} \rightarrow 0, \text{ as } N_s^4 \rightarrow 0
\]

and

\[
\frac{A_t}{h_c^2} \rightarrow 1.0, \text{ as } N_s^4 \rightarrow 0
\]

The latter limit is due to the fact that the angle of repose of stone is about 45 degree and therefore the smallest stable pile of stone in the absence of waves would be

\[
\frac{A_t}{h_c^2} = 1.0
\]

With this perspective, the regression curve in Figure 2 can be regarded roughly as a boundary between stable and unstable reef configurations. The data to the left of the curve for \( N_s^4 < 7.0 \) are reefs which remained substantially in their original configuration due to the lack of sufficient wave action to displace stones, e.g. the reef in Figure 1a. Figure 2 indicates that, for these conditions, the reefs would have been stable at somewhat steeper slopes.

When Figure 2 is examined, it can be seen that for \( N_s^4 > 7.0 \) data from some subsets are consistently above the Equation 2 curve and data from other subsets are consistently below. For example, Subset 5 data are consistently above and Subset 1 data are consistently below the Equation 2 curve. To help explain these trends, Figure 3 shows the relative crest height of the reefs, \( h_c/d_s \), versus \( N_s^4 \). In Figure 3, hand drawn curves are given for each subset consistent with the data showing progressive deterioration of the reef with increasing severity of wave attack. Figure 3 shows that, for any given value of \( N_s^4 \), the reefs of Subset 5 are higher than those of Subset 1. The
Figure 3. Damage trend curves showing the relative crest height, $h_c/d_s$, versus the spectral stability number $N_s$. 

<table>
<thead>
<tr>
<th>SUBSET</th>
<th>$B_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>337</td>
</tr>
<tr>
<td>3</td>
<td>450</td>
</tr>
<tr>
<td>5</td>
<td>631</td>
</tr>
<tr>
<td>7</td>
<td>222</td>
</tr>
<tr>
<td>9</td>
<td>222</td>
</tr>
</tbody>
</table>
greater stability of Subset 5 reefs is due to their flatter slope and greater bulk. Figure 2 indicates that the reefs of Subset 1 are stable at a steeper slope than those of Subset 5, for a given severity of wave action, due to their greater depth of submergence, as shown in Figure 3. The greater the depth of submergence, the more the reef is sheltered from wave forces. By using both Figures 2 and 3, the various influences on reef stability can be observed.

The implications of Figures 2 and 3, relating to reef stability or the reef's response to wave attack, can be summarized by generalizing Equation 2 in the following manner.

\[
\frac{A_t}{h_c^2} = \exp \left[ N_s^* \left( K \left( \frac{h_c}{d_s}, \frac{h_c}{d_s}, B_n \right) \right) \right]
\]

(5)

\[K \left( \frac{h_c}{d_s}, \frac{h_c}{d_s}, B_n \right)\] indicates that the stability coefficient, K, is regarded as a function of the relative crest height of the reef, both "as built" and at equilibrium to wave conditions, and the bulk or size of the structure as defined by the bulk number, \( B_n \), where

\[
B_n = \frac{A_t}{d_{50}^2}
\]

(6)

The typical dimension of the stone in the reef is given by

\[
d_{50} = \left( \frac{W_{50}}{W_T} \right)^{1/3}
\]

(7)

Using regression analysis, a number of equations of the form of Equation 5 were obtained and investigated. The equation which fit the data best and reflected the physics, as currently understood, most satisfactory is given by

\[
\frac{A_t}{h_c^2} = \exp \left[ N_s^* \left( C_1 \left( \frac{h_c}{d_s} - \frac{h_c}{d_s} \right) + C_2 \left( \frac{h_c}{d_s} \right)^{3/2} - C_3 \left( \frac{h_c}{d_s} \right)^2 + C_4 \frac{B_n}{B_n} \right) \right]
\]

(8)

where the regression coefficients are

\[C_1 = 0.0460\]
\[C_2 = 0.2083\]
\[C_3 = 0.1440\]
\[C_4 = 0.4317\]
Equation 8 is implicit in the equilibrium crest height and was found to converge rapidly using the as built crest height, \( h_0 \), as the initial value of \( h_c \).

When the predicted equilibrium crest height, \( h_c \), is compared to the observed crest height, it is found that Equation 8 provides very accurate estimates. Figure 4 shows the ratio of observed to predicted crest heights as a function of \( N_s^\alpha \). For \( N_s^\alpha > 7.0 \) the predicted crest heights, with one exception, all are within \( \pm 10 \) percent of the observed value. For \( N_s^\alpha < 7.0 \) the reefs are not at the boundary of equilibrium with wave conditions and Equation 8 indicated that reefs somewhat higher or using somewhat steeper side slopes than the as built conditions would have been stable.

![Diagram](image)

Figure 4. Ratio of the predicted crest height Equation 8 versus the observed crest height as a function of the spectral stability number.

The purpose of the reef stability model is largely to be used to calculate wave transmission and reflection and energy dissipation characteristics by providing the equilibrium crest heights of the structure. In Ahrens (1987), a simple method was developed to estimate wave transmission and reflection which was especially convenient for graphical presentation. This method selected the "best" formulation for wave transmission and reflection using two variables for both processes with one variable common to both. The functional relations selected were

\[
K_t = f\left(\frac{h_c}{d_s}, B_r\right)
\]

and

\[
K_r = f\left(\frac{h_c}{d_s}, \frac{d_s}{L_p}\right)
\]
where $K_t$ and $K_r$ are the transmission and reflection coefficients, respectively. Regression analysis was used to develop equations having the functional form noted above. The equations are

$$K_t = \frac{1.0}{1.0 + 0.0294 \left( \frac{h_o}{d_s} \right)^{3.333} (B_n)^{0.5857}} \quad (9)$$

and

$$K_r = \exp \left[ 0.2899 \left( \frac{h_o}{d_s} \right) - 0.7628 \left( \frac{d_s}{h_o} \right) - 7.3125 \left( \frac{d_s}{L_p} \right) \right] \quad (10)$$

The predictive ability of Equations 9 and 10 is demonstrated in Figures 5 and 6, respectively. In Figures 5 and 6, the observed data are plotted on the ordinate and the predicted values on the abscissa.

Figure 7 demonstrates how the reef stability model, Equation 8, can be used with the transmission equation, Equation 9, to estimate the transmission coefficients. In Figure 7, it can be seen on the left how the height of the reef is reduced as the severity of the wave attack increases and the stability model follows this trend quite well. Using the estimated crest height from Equation 8 in the transmission equation, Equation 9, produces the predicted transmission trend curve on the right side of Figure 7. It can be seen that the predicted transmission trend follows the observed data very well.

Equations 9 and 10 can be used to construct a rough energy balance diagram for a reef. Using the reefs tested in Subsets 3 and 4 as the example, Figure 8 was constructed. For Figure 8, a value of $B_n = 450$ was used in Equation 9. In Equation 10, two wave reflection trends were generated using the largest and smallest relative depth tested during Subsets 3 and 4, i.e.

$$\frac{d_s}{L_p} = 0.045 \text{ and } 0.121$$

Figure 8 reflects the basic conservation of energy relation

$$K_t^2 + K_r^2 + \text{dissipation} = 1.0 \quad (11)$$

In Figure 8, the consequences of degradation of the reef can be seen. As the crest height is reduced by wave attack, transmission increases and reflection decreases. It can also be seen in Figure 8 that, as the reef is knocked down, the ability to dissipate wave energy is reduced. For a crest height of $h_0/d_s = 1.2$, the reef will dissipate about 80 percent of the incident short wave energy and about 60 percent of the longer wave energy. At a crest height of $h_0/d_s = 1.0$, the reef will dissipate about 70 percent of the incident short
Figure 5. Predicted wave transmission coefficient Equation 9 versus the observed transmission coefficient

Figure 6. Predicted wave reflection coefficient Equation 10 versus the observed reflection coefficient
Figure 7. The reduction in height of the reef due to wave attack and the associated change in wave transmission.
wave energy and about 57 percent of the longer wave energy. In Ahrens (1987), methods are presented which allow better estimates of $K_t$ and $K_r$ than given by Equations 9 and 10 but they are more complicated and do not lend themselves to convenient graphical presentation such as Figure 8.

In using the reef stability model or figures such as Figure 8, a logical question would be how severe the wave conditions can be. It appears from wave tank tests that the limiting value for zero-moment wave height can be estimated as follows:

\[
(H_{mo})_{max} \approx (0.1) L_p \tanh \left( \frac{2\pi d_s}{L_p} \right)
\]  

(Ahrens and Helsaugh, 1987). When Equation 12 is used in the equation for the spectral stability number, Equation 1, the following relation can be obtained

\[
(N_s^*)_{max} \approx \left( \frac{d_s}{d_{50}} \right) \left[ 0.1 \tanh \left( \frac{2\pi d_s}{L_p} \right) \right]^{2/3}
\]
For the conditions of Subset 3

\[
\left( N^*_s \right)_{\text{max}} = \frac{(25)}{(1.86)} \left[ 0.1 x (0.2784) \right]^{2/3} \left( 0.045 \right) (1.63)
\]

\[= 16.7\]

Referring to Figure 7 it can be seen that the maximum observed value of \( N^*_s \) is about 17.0. Since the laboratory tests were conducted in such a manner as to encourage severe conditions, it appears that Equation 13 provides a reasonable estimate of the limiting conditions. Applying this information to Figure 8 indicates that the lowest relative crest height of the reef which could evolve due to natural wave action from the original trapezoidal shape would be

\[
h_c \approx 0.7
\]

However, since it might not be convenient to build a reef with an initial configuration such as shown in Figure 1, it is interesting to extend the range of the relative height to values lower than could be obtained through natural evolution to evaluate the performance of an equivalent structure built, for example, by dropping stone from a barge.

Summary and Conclusions

It was found the stability of reef breakwaters was strongly dependent on the wave period such that longer period waves were more damaging than shorter period waves, other factors being equal. The most important variable influencing the stability of reefs was found to be the spectral stability number, \( N^*_s \), defined by Equation 1. \( N^*_s \) is a good measure of the relative severity of wave attack on a reef. Other factors which affect the stability are relative crest heights of the reef, both as built and at equilibrium to the wave conditions, and the size of the structure as measured by the bulk number, \( B_B \), defined by Equation 6. An analytic expression, Equation 8, was developed for the stability of reef breakwaters which includes all variables mentioned above. Equation 8 fits the observed data very well and reflects the physics of the interaction between waves and reef as it is currently understood.

The stability model provided by Equation 8 can be used to predict the expected stable crest height of the reef for a wide range of wave and structure conditions. With this information, the wave transmission and reflection characteristics can be estimated using Equations 9 and 10. Knowing these characteristics, the amount of energy being dissipated by the reef can be calculated using the conservation of wave energy relation, Equation 11. Finally, a method for estimating the most severe conditions for a low-crested rubble-mound is given by Equation 13.
Acknowledgments

The tests described, unless otherwise noted, were conducted under the Coastal Structures Evaluation and Design program of the United States Army Corps of Engineers by the Coastal Engineering Research Center. Permission was granted by the Chief of Engineers to publish this information.

Appendix I. - References


Appendix II – Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_t$</td>
<td>Cross sectional area of breakwater (cm$^2$)</td>
</tr>
<tr>
<td>$d_s$</td>
<td>Water depth at toe of breakwater (cm)</td>
</tr>
<tr>
<td>$d_{50}$</td>
<td>$(W_{50}/W_r)^{1/3}$, typical dimension of the median stone, (cm)</td>
</tr>
<tr>
<td>$h_c$</td>
<td>Crest height of breakwater as built (cm)</td>
</tr>
<tr>
<td>$h'_c$</td>
<td>Crest height of breakwater after wave attack (cm)</td>
</tr>
<tr>
<td>$H_c$</td>
<td>Zero moment wave height at transmitted gage locations with no breakwater in channel (cm)</td>
</tr>
<tr>
<td>$H_t$</td>
<td>Zero-moment transmitted wave height (cm)</td>
</tr>
<tr>
<td>$H_{mo}$</td>
<td>Incident zero-moment wave height (cm)</td>
</tr>
<tr>
<td>$K_r$</td>
<td>Reflection coefficient of breakwater as defined and calculated by method of Goda and Suzuki (1976)</td>
</tr>
<tr>
<td>$L_p$</td>
<td>Airy wave length calculated using $T_p$ and $d_s$ (cm)</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Wave period of peak energy density of spectrum (sec)</td>
</tr>
<tr>
<td>$W_r$</td>
<td>Density of stone, (gr/cm$^3$)</td>
</tr>
<tr>
<td>$W_w$</td>
<td>Density of water, tests conducted in fresh water $W_w = 1.0$ (gr/cm$^3$)</td>
</tr>
<tr>
<td>$W_{50}$</td>
<td>Median stone weight, subscript indicates percent of total weight of gradation contributed by stones of lesser weight (gr)</td>
</tr>
<tr>
<td>$F$</td>
<td>$(H_c - d_s)$, freeboard of structure which for reef can be either positive or negative (cm)</td>
</tr>
<tr>
<td>$X_t$</td>
<td>$H_t/H_c$, wave transmission coefficient</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Spectral stability number, defined by Equation 1</td>
</tr>
<tr>
<td>$B_n$</td>
<td>Bulk number, defined by Equation 6</td>
</tr>
<tr>
<td>$K$</td>
<td>Stability coefficient for reef breakwaters</td>
</tr>
<tr>
<td>$K_D$</td>
<td>Stability coefficient for conventional breakwaters</td>
</tr>
<tr>
<td>$h_c/d_s$</td>
<td>Relative crest height</td>
</tr>
</tbody>
</table>