Sediment characterization by geo-acoustic inversion of marine seismic data received on a towed hydrophone array

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Preface

This report is the accumulation of 8 months of work on my graduation project for the Geomatics master programme, marking the end of five years of study at Delft University of Technology.

The graduation project has been a very welcome learning experience for me, being the first time to work on a long-term project all by myself. I found the thesis to be very different from the group work and concise assignments that made up most of my study, therefore providing unique challenges and opportunities.

Although my thesis report has been written with students and researchers in mind that have some background in acoustic remote sensing, I have done my best to keep the content as simple as possible, so that the report can appeal to a wider audience.

Those who are interested in the theory behind inverting marine seismic data are referred to chapter 2, while the more practical issues with respect to the inversion can be found in chapters 3 and 5. Of special interest might be chapter 6, which discusses the use of the arrival times of the head wave to estimate the sound speed at the top of the sediment.

I’m especially grateful to my daily supervisor, Mirjam Snellen, and my professor, Dick Simons, for giving me much freedom during the project while assisting me when necessary. I would like to express my thanks as well to Guy Drijkoningen and Nihed El Allouche at the Faculty of Civil Engineering & Geosciences, for teaching me the basics of marine seismics. Finally, I would like to thank all my friends and in particular my parents, my sister and my girlfriend, for their support and understanding.

I wish everyone a pleasant reading,

Koen Duijnmayer

Delft, 10 December 2009
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Summary

For various applications, such as dredging, off-shore construction works and retrieving sea sand for concrete production, knowledge of underwater sediment layers is a necessity. The underwater environment is of interest as well to several users in the academic world.

Traditionally, information about the underwater environment is acquired by bottom sampling and drilling boreholes, which is time-consuming and expensive. Therefore, alternative methods are explored to collect the same information ‘from a distance’, i.e. by remote sensing.

Because electromagnetic waves are highly absorbed in water, they are not suited for remote sensing of the underwater environment. In contrast, acoustic waves can travel large distances while experiencing limited attenuation. Consequently, the remote sensing techniques used for obtaining information about the underwater environment are, in general, acoustic techniques.

The remote sensing system considered in this report is a (marine) seismic system, consisting of a low-frequency sound source and an array of receivers, both towed behind a ship. When sound produced by the source reflects off the water bottom, the amplitude and phase change according to the characteristics of the sediment. Hence, when recording this sound, information about the underwater environment is acquired. The challenge is however, to extract this information.

In this report, one particular method to extract the information, called geo-acoustic inversion, is studied. In this method, a model is employed that predicts how sound propagates, based on several environmental parameters. The goal of geo-acoustic inversion is to find the combination of parameter values for which the model output gives the best match with the measurements. This best match is assumed to correspond to the real environment.

For modelling the propagation of sound from the source to the receiver, the so-called normal-mode sound propagation model is used. The equations of this model are derived in the report for a range-independent environment with a single, fluid sediment layer and for a broad-band acoustic source. The input of the model, i.e. the parameters that are inverted for, are the measurement geometry, water sound speed, sediment sound speed, sediment density and sediment attenuation coefficient. For finding the best match between the model output and measurements, a global optimization algorithm called differential evolution is used.

The inversion process has been applied to a dataset that was acquired with a ship towing an acoustic source and an array of hydrophones in the North Sea, approximately 10 km off the coast from Hook of Holland. The results of the inversion were found to be consistent.
at most locations. Moreover, part of the dataset was inverted successfully for two sediment layers, by extending the normal-mode model.

In addition, the method was applied to a second dataset acquired in the Danube River near Kulcs, Hungary. Here a somewhat different approach was followed, where instead of the complete recorded signals, the arrival times of the head wave were inverted to estimate the sound speed at the top of the sediment.

It is concluded that knowledge of the shape of the source pulse and the measurement geometry is important for a successful inversion. Moreover, it is proposed to investigate the benefits of accounting for the elasticity in the sediment. For future surveys, it is strongly recommended that sufficient independent measurements of the unknown sediment parameters are taken, to be able to evaluate the accuracy of the inversion results.
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1 Introduction

For various applications, such as dredging, off-shore construction works and retrieving sea sand for concrete production, knowledge of underwater sediment layers is a necessity. The underwater environment is of interest as well to several users in the academic world, not only in engineering, but also to those working in environmental sciences (earth sciences, geology, ecology, etc.).

Traditionally, information about the underwater environment is acquired by bottom sampling and drilling boreholes. A disadvantage of these methods is that they result in point measurements; hence for a detailed characterization of an area, many samples are needed. The collection of these samples is a time-consuming and expensive task.

Therefore, new methods that are able to collect the same information by acoustic remote sensing, i.e. ‘from a distance’, are looked at. Although various approaches are described in literature, in this report a measurement configuration is considered that consists of a towed source and receiver. When sound that is produced by the source reflects off the water bottom (and possibly deeper sediment layers), the amplitude and phase change according to the characteristics of the sediment. Hence by recording these reflections, information about the underwater sediment is acquired. The challenge is however, to extract this information correctly and in a way that is useful for the applications mentioned earlier.

A particular method to extract the information, which is studied in this report, is called geo-acoustic inversion. In this method, a model is employed that predicts how sound propagates, based on several environmental parameters. The goal of geo-acoustic inversion is to find the combination of parameter values for which the model output gives the ‘best’ match with the measurements. Efficient inversion techniques are required to find this ‘best’ match, which is assumed to correspond to the real environment.

The objectives of the research presented in this report are threefold:

- The first objective is to give an overview of the steps necessary to perform geo-acoustic inversion.

- The second objective is to apply the inversion process to two datasets gathered in different environments. For this purpose, a dataset collected in the North Sea and a dataset collected in the Danube River are looked at.

- The third and final objective is to analyze the inversion results of both datasets, and to make recommendations for further processing and future surveys, based on this analysis.
To meet these objectives, the theory of geo-acoustic inversion is discussed in detail in the next chapter. Chapter 3 continues with an overview of the North Sea dataset, for which the inversion results are presented and analyzed in chapter 4. The dataset that was gathered in the Danube River is looked at in chapter 5, after which the results for this dataset are discussed in chapter 6. The final chapter contains the conclusions and recommendations resulting from the research.
2 Theory

In the introduction, the concept of geo-acoustic inversion was explained shortly. The basic idea behind this method is to find the best match between a measured acoustic field and many modelled fields, in order to arrive at a ‘best’ description of the real environment.

The theory of geo-acoustic inversion goes back to 1984, when Frisk et al. [4] proposed inverting a measured acoustic field to estimate the sound speed and attenuation coefficient of a sediment layer. Since then, the method has been developed further and applied to real acoustic data.

According to Siderius et al. [14], geo-acoustic inversion consists of the following six steps:

1. Measuring the acoustic field at a site of interest.
2. Choosing a propagation model that is suitable for the experimental conditions.
3. Defining a geo-acoustic model for the site.
4. Determining a cost function to quantify the agreement between the measurements and modelled data.
5. Using an efficient algorithm to find the global minimum of the cost function.

In the remainder of this chapter, each of these steps will be discussed, except for estimating the quality of the inversion. Instead, this last step will be looked at in chapter 4, when discussing the results of applying the inversion method to one of the two datasets.

2.1 Measuring the acoustic field

The very first step in the geo-acoustic inversion process is to measure the acoustic field at a site of interest. In this report, it is assumed that the measurements are performed with a low-frequency acoustic source in combination with an array of hydrophones towed behind a ship.

For this measurement configuration, Siderius et al. [14] have defined a number of guidelines in order for the inversion to be successful:
• Towing the source close to the bottom is preferred, because of better insonification.

• Longer receiver arrays generally perform better than short ones.

• Recordings over a broad band of frequencies contain more information than recordings at a single frequency. Hence a broad-band source and recording equipment is recommended.

• Most information is obtained from propagation angles around the critical angle, so make sure sound is recorded coming from this direction.

The analysis of the geo-acoustic inversion results of the two datasets available for the thesis has led to additional recommendations for data acquisition. An overview of these recommendations is given in the final chapter.

2.2 The normal-mode propagation model

The second step in the geo-acoustic inversion process, as defined at the beginning of this chapter, is to select a propagation model with which the acoustic field can be modelled. In this case, use is made of a normal-mode sound propagation model, such as described in references [8], [13] and [15].

In the following four subsections, a description is given of how the main equation of the normal-mode model is derived and how it can be solved numerically for a single acoustic frequency. This is done for a range-independent environment, i.e. an environment in which parameters are only allowed to vary with depth. In addition, broadband modelling is discussed in section 2.2.5.

2.2.1 The acoustic wave equation

The derivation starts with considering an isotropic point source in a cylindrically symmetric, horizontally stratified (i.e. range-independent) acoustic medium, as shown in figure 2.1. With the source at zero range and at a depth $z_s$, the acoustic wave equation governing this problem is:

$$\nabla \cdot \left( \frac{1}{\rho} \nabla P \right) - \frac{1}{\rho c^2} \frac{\partial^2 P}{\partial t^2} = -\frac{s(t)}{2\pi r} \delta(r) \delta(z - z_s)$$

(2.1)

where $P(r, z, t)$ is the acoustic pressure as a function of range $r$, depth $z$ and time $t$. In addition, $s(t)$ is the source wavelet, and $\rho(z)$ and $c(z)$ are the density and sound speed as a function of depth, respectively.

To simplify the problem, the system described by equation 2.1 is considered to be time-invariant. That is, only the response to a continuous ‘hum’ at a single radial frequency $\omega$
Figure 2.1: The range-independent geo-acoustic environment that is modelled.
is looked at.

In that case, the source wavelet can be written as a time series of the form:

\[ s(t) = e^{-i\omega t} \quad (2.2) \]

which leads to a pressure field with the same harmonic time-dependence, i.e.:

\[ P(r, z, t) = p(r, z)e^{-i\omega t} \quad (2.3) \]

With this simplification, equation 2.1 can be rewritten as follows:

\[
\frac{1}{r\rho} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\rho} \frac{\partial p}{\partial z} \right) - \frac{1}{\rho c^2} \frac{\partial^2 p}{\partial t^2} = -\frac{s(t)}{2\pi r} \delta(r) \delta(z - z_s)
\]

\[
\left[ \frac{1}{r \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \rho \frac{\partial}{\partial z} \left( \frac{1}{\rho} \frac{\partial p}{\partial z} \right) + \frac{\omega^2}{c^2} p \right] e^{-i\omega t} = -\frac{\rho(z_s)}{2\pi r} e^{-i\omega t} \delta(r) \delta(z - z_s)
\]

This finally results in:

\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} - \frac{1}{\rho} \frac{\partial p}{\partial z} \frac{\partial^2 \Psi}{\partial z^2} + k^2 p = -\frac{\rho(z_s)}{2\pi r} \delta(r) \delta(z - z_s) \quad (2.4)
\]

where \( k \) is the total wavenumber defined as:

\[ k(z) = \frac{\omega}{c(z)} \quad (2.5) \]

Equation 2.4 is a special case of the acoustic wave equation, commonly referred to as the Helmholtz equation or reduced wave equation.

**2.2.2 Derivation of the normal-mode solution**

By applying the technique of separation of variables, a solution of the form \( p(r, z) = R(r)\Psi(z) \) can be found for the homogeneous or unforced form of equation 2.4 (i.e. with the right-hand side equal to zero):

\[
\frac{1}{R} \left[ \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right] + \frac{1}{\Psi} \left[ \frac{d^2 \Psi}{dz^2} - \frac{1}{\rho} \frac{d\Psi}{dz} \frac{d^2 \Psi}{dz^2} + k^2 \right] = 0 \quad (2.6)
\]
Note that equation 2.6 can only be satisfied if both terms within square brackets are equal to a constant. Denoting this separation constant by $\mu$, one obtains the following two ordinary differential equations:

\[
\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \mu R = 0 \quad (2.7)
\]
\[
\frac{d^2 \Psi}{dz^2} - \frac{1}{\rho} \frac{d\rho}{dz} \frac{d\Psi}{dz} + (k^2 - \mu) \Psi = 0 \quad (2.8)
\]

The first of these two equations is the so-called range-dependent Helmholtz equation, while the second equation is called the depth-dependent Helmholtz equation or modal equation.

Now, it is assumed the sea surface is a pressure release boundary, i.e. where the acoustic pressure is zero, and that at some sufficiently great depth $H_t = H_w + H_s + H_b$ (see also figure 2.1) a perfectly rigid boundary exists:

\[
\begin{align*}
  p(r, 0) &= 0 \\
  \frac{dz}{d\rho} (r, H_t) &= 0
\end{align*} \quad (2.9)
\]

This leads to the following boundary conditions for equation 2.8:

\[
\begin{align*}
  \Psi(0) &= 0 \\
  \frac{d\Psi}{dz} (H_t) &= 0
\end{align*} \quad (2.10)
\]

Together with these boundary conditions, equation 2.8 is identified as a classical Sturm-Liouville eigenvalue problem, which has an infinite number of solutions $\Psi_n(z)$ (eigenfunctions or modes) for distinct real (eigen)values $\mu_n = k_n^2$.

Since the eigenfunctions form a complete orthonormal set, the solution of equation 2.6 can be written as:

\[
p(r, z) = \sum_{n=1}^{\infty} R_n(r) \Psi_n(z) \quad (2.11)
\]

where the coefficients $R_n(r)$ are given by:

\[
R_n(r) = \frac{i}{4\rho(z_s)} \Psi_n(z_s) H^{(1)}_0(k_n r) \quad (2.12)
\]

in which $H^{(1)}_0$ is the zeroth order Hankel function of the first kind.
Figure 2.2: The critical angle and its relation to the type of normal-modes that occur.

For \( k_n r > 2 \), the Hankel function in equation 2.12 can be approximated asymptotically by:

\[
H_0^{(1)}(k_n r) \approx \sqrt{\frac{2}{\pi k_n r}} e^{i(k_n r - \frac{\pi}{4})}
\]  

(2.13)

Hence equation 2.11 can be written as:

\[
p(r, z) \approx \frac{e^{iz}}{\rho(z_s)\sqrt{8\pi r}} \sum_{n=1}^{\infty} \Psi_n(z_s) \Psi_n(z) \frac{e^{ik_n r}}{\sqrt{k_n}}
\]  

(2.14)

Furthermore, a distinction can be made between the discrete and continuous part of the eigenvalue spectrum. In this distinction, the eigenvalues of the discrete modes are given by:

\[
\frac{\omega}{c_b} < k_n < \frac{\omega}{c_{\text{min}}}
\]  

(2.15)

where \( c_b \) is the sound speed in the subbottom and \( c_{\text{min}} \) is the lowest value in the total sound speed profile.

Note that according to equation 2.15, discrete modes only exist when \( c_b > c_{\text{min}} \).

In addition, the eigenvalues of the continuous modes satisfy:

\[
0 < k_n < \frac{\omega}{c_b}
\]  

(2.16)

The continuous modes correspond to sound rays travelling at grazing angles greater than the critical angle at the sediment-subbottom interface, as sketched in figure 2.2.

Because part of the energy carried by these modes leaks into the subbottom, their contribution to the pressure field is not taken into account by the normal-mode model. In fact,
only the contribution of the discrete modes is looked at, which means equation 2.14 can be rewritten to:

\[ p(r, z) \approx \frac{e^{\mu_1}}{p(z_0)\sqrt{8\pi r}} \sum_{n=1}^{L} \Psi_n(z_0) \Psi_n(z) e^{i\kappa_n r} \sqrt{k_n} \]

where \( L \) is the total number of discrete modes. Because of neglecting the continuous modes, the normal-mode solution is an approximation, assuming that the distance between the source and receiver is sufficiently large.

To a very good approximation, the total number of discrete modes is given by:

\[ L \approx \left\lfloor \frac{2 (H_w + H_s) f \ln \left( \sqrt{1 - \frac{c}{c_0}} + 1 \right)}{\bar{c} \ln 2} + \frac{1}{2} \right\rfloor \]

with \( \bar{c} \) the average sound speed in the water column and sediment.

Setting \( L = 1 \) in equation 2.18 yields the so-called cut-off frequency \( f_c \):

\[ f_c \approx \frac{\bar{c} \ln 2}{4 (H_w + H_s) \ln \left( \sqrt{1 - \frac{c}{c_0}} + 1 \right)} \]

Below this frequency no discrete modes exist, hence it is required that \( f \geq f_c \).

An example of two discrete modes is given in figure 2.3. In fact, the first mode shown in figure 2.3 is the mode which corresponds to the largest eigenvalue, and has the lowest number of oscillations in each of the modelled acoustic layers. Therefore, this mode is referred to as the lowest order mode.

Similarly, the mode that corresponds to the smallest eigenvalue which satisfies equation 2.15, has the highest number of oscillations in each layer and is called the highest order mode.

### 2.2.3 Including material absorption

At this point, equation 2.17 does not yet include any loss effects apart from losses due to geometrical spreading. Other loss effects, such as absorption in the water column, sediment and subbottom, are taken into account using first-order perturbation theory.

Consider an arbitrary perturbation to the total wavenumber such that:

\[ k^2(z) = k_0^2(z) + \varepsilon k_1^2(z) \]
in which $k_0^2(z)$ corresponds to the unperturbed sound speed profile.

Equation 2.8 then has the following solutions:

$$\Psi_m(z) = \Psi_{m_0}(z) + \varepsilon \Psi_{m_1}(z) \quad (2.21)$$

for eigenvalues:

$$k_m^2 = k_{m_0}^2 + \varepsilon k_{m_1}^2 \quad (2.22)$$

Substituting expressions 2.21 and 2.22 in equation 2.8, and collecting terms of the same order gives:

$$O(1) : \quad \frac{d^2 \Psi_0}{dz^2} - \frac{1}{\rho} \frac{d\rho}{dz} \frac{d\Psi_0}{dz} + (k_0^2 - k_{m_0}^2) \Psi_0 = 0 \quad (2.23)$$

$$O(\varepsilon) : \quad \frac{d^2 \Psi_1}{dz^2} - \frac{1}{\rho} \frac{d\rho}{dz} \frac{d\Psi_1}{dz} + (k_0^2 - k_{m_0}^2) \Psi_1 = - (k_1^2 - k_{m_1}^2) \Psi_0 \quad (2.24)$$

with boundary conditions:
\[
\begin{aligned}
\Psi_0(0) &= \Psi_1(0) = 0 \\
\frac{d\Psi_0}{dz}(H_t) &= \frac{d\Psi_1}{dz}(H_t) = 0
\end{aligned}
\]  

(2.25)

It can be shown that equation 2.24 only holds when:

\[
\int_0^{H_t} (k_1^2(z) - k_{m_1}^2(z)) \frac{\psi_0^2(z)}{\rho(z)} dz = 0
\]  

(2.26)

When the modes \(\psi_{m_0}(z)\) are normalized, this implies that:

\[
k_{m_1}^2 = \int_0^{H_t} \frac{k_1^2(z)\psi_0^2(z)}{\rho(z)} dz
\]  

(2.27)

Furthermore, it is known that material absorption can be modelled by introducing an imaginary component to the sound speed, which leads to a complex total wavenumber, i.e.:

\[
k(z) \rightarrow k(z) + i\alpha(z)
\]  

(2.28)

From equation 2.20, it then follows that:

\[
\varepsilon k_1^2(z) = i k^2(z) = 2i\alpha(z)k(z)
\]  

(2.29)

Similarly, the perturbation term of the eigenvalues in equation 2.22 becomes:

\[
\varepsilon k_{m_1}^2 = i k_n^2 = 2i\alpha_n k_n
\]  

(2.30)

where \(\alpha_n\) are the so-called total modal loss coefficients.

Substituting expressions 2.29 and 2.30 into equation 2.27 and rearranging terms, gives:

\[
\alpha_n = \frac{1}{k_n} \int_0^{H_t} \frac{\alpha(z)k(z)\psi_n^2(z)}{\rho(z)} dz
\]  

(2.31)

in which \(\psi_n(z)\) and \(k_n\) are the undisturbed eigenfunctions and eigenvalues, respectively, i.e. the solutions of equation 2.8.

For the environment that is modelled, the total modal loss coefficients can be split into three individual loss terms: 
\[ \alpha_n = \alpha_n^{(w)} + \alpha_n^{(s)} + \alpha_n^{(b)} \]  

(2.32)

with \( \alpha_n^{(w)} \), \( \alpha_n^{(s)} \) and \( \alpha_n^{(b)} \) the loss coefficients due to volume attenuation in the water column, sediment and subbottom, respectively.

Knowing that \( k(z) = \frac{\omega}{c(z)} \), it follows from equation 2.31 that:

\[ \alpha_n^{(w)} = \frac{\omega \alpha_{n w}}{k_n} \int_0^{H_w} \frac{\Psi_n^2(z)}{c_{n w} \rho_{n w}} \, dz \]  

(2.33)

\[ \alpha_n^{(s)} = \frac{\omega \alpha_{n s}}{k_n} \int_{H_w}^{H_w + H_s} \frac{\Psi_n^2(z)}{c_{n s} \rho_{n s}} \, dz \]  

(2.34)

and

\[ \alpha_n^{(b)} = \frac{\omega \alpha_{n b}}{k_n} \int_{H_w + H_s}^{H_t} \frac{\Psi_n^2(z)}{c_{n b} \rho_{n b}} \, dz \]  

(2.35)

in which \( \alpha_{n w}, \alpha_{n s} \) and \( \alpha_{n b} \) are the volume attenuation constants in neper/m for the water column, sediment and subbottom, respectively.

After inserting the total loss coefficients \( \alpha_n \) into equation 2.17, the normal-mode solution of the wave equation becomes:

\[ p(r, z) \approx \frac{e^{\frac{\Psi}{r}}}{\rho(z) \sqrt{8\pi r}} \sum_{n=1}^{L} \Psi_n(z_a) \Psi_n(z) e^{\frac{i(k_n - \alpha_n)r}{\sqrt{k_n}}} \]  

(2.36)

### 2.2.4 The numerical solution of the modal equation

Although the normal-mode solution of the wave equation has been found, the acoustic pressure field given by equation 2.36 cannot be computed without knowing the eigenvalues \( \mu_n = k_n^2 \) and the eigenfunctions \( \Psi_n(z) \) of the modal equation.

Therefore, the modal equation is solved numerically by transferring equation 2.8 into an algebraic eigenvalue problem, i.e. of the form:

\[ A \Psi = \mu \Psi \]  

(2.37)

This is done using a method called finite-difference discretization, which starts with constructing a mesh in each of the acoustic layers, see figure 2.4.
Figure 2.4: The finite-difference mesh corresponding to figure 2.1.
As shown in the figure, the water column, i.e. the interval $0 < z \leq H_w$, is divided into $N$ equal intervals to construct a mesh of equally spaced points:

$$z_j = jh_w, \quad j = 1, 2, \ldots, N$$  \hspace{1cm} (2.38)

where $h_w$ is the mesh width given by $h_w = \frac{H_w}{N}$.

The number of intervals $N$ should be chosen such that the highest order mode (see also figure 2.3) is adequately sampled. When requiring approximately $k$ samples per wavelength of this mode, it can be shown that it is necessary that:

$$h_w < \frac{\pi}{k \sqrt{\left(\frac{\omega}{\Omega_{\text{min}}}\right)^2 - \left(\frac{\omega}{c_s}\right)^2}}$$  \hspace{1cm} (2.39)

Similar to the water column, the interval $H_w < z \leq H_w + H_s$, i.e. the sediment layer, is divided into $M$ equal intervals by constructing the mesh:

$$z_j = H_w + (j - N)h_s, \quad j = N + 1, N + 2, \ldots, N + M$$  \hspace{1cm} (2.40)

Finally, the subbottom, i.e. the interval $H_w + H_s < z \leq H_w + H_s + H_b$, is divided into $K - 1$ equal intervals, resulting in the mesh:

$$z_j = H_w + H_s + (j - N - M)h_b, \quad j = N + M + 1, N + M + 2, \ldots, N + M + K - 1$$  \hspace{1cm} (2.41)

As a side note, it can be mentioned that the practical implementation of the model is such that there is also a requirement on the minimum number of samples of the eigenfunctions in each of the layers. This is to make sure a sufficient number of samples is available even for very thin layers.

To allow the use of efficient numerical methods for solving equation 2.37, it is preferred that matrix $A$ is symmetrical. This can be achieved by choosing $h_s$ and $h_b$ such that:

$$h_s = h_w \frac{\rho_s}{\rho_w}$$  \hspace{1cm} (2.42)

and

$$h_b = h_s \frac{\rho_b}{\rho_s}$$  \hspace{1cm} (2.43)

A correction to the thickness of the sediment layer is then necessary though, to get a new value $H_s' = Mh_s$ that is an integer multiple of $h_s$ (see also figure 2.4):
\[
M = \left[ \frac{H_n}{h_s} \right] = \min \left\{ n \in \mathbb{Z} \mid n \geq \frac{H_n}{h_s} \right\} 
\]  
(2.44)

The same holds for the subbottom thickness, which is adjusted to a new value \( H'_b \) equal to \( (K - 1)h_b \).

Now, it is assumed the density is constant in each acoustic layer, i.e. \( \frac{d}{dt} = 0 \), leading to the following simplification of the modal equation:

\[
\frac{d^2 \Psi}{dz^2} + \left[ k^2(z) - \mu \right] \Psi = 0 
\]  
(2.45)

Introducing the notation \( \Psi_j = \Psi(z_j) \) and \( k_j = k(z_j) \), the second derivative of \( \Psi \) with respect to \( z \) can be approximated by a Taylor series expansion, i.e.:

\[
\frac{d^2 \Psi_j}{dz^2} = \frac{\Psi_{j+1} - 2\Psi_j + \Psi_{j-1}}{h^2} 
\]  
(2.46)

Substituting this expression into equation 2.45 and rearranging terms gives:

\[
\frac{\Psi_{j-1}}{h^2} + \left( k_j^2 - \frac{2}{h^2} \right) \Psi_j + \frac{\Psi_{j+1}}{h^2} = \mu \Psi_j 
\]  
(2.47)

When replacing \( h \) with \( h_w, h_s \) and \( h_b \), respectively, the finite-difference equations for the water column, sediment layer and subbottom are found.

Going back to the matrix notation of equation 2.37, it then follows that:

\[
A = \begin{pmatrix}
  d_1 & e_1 & 0 & \cdots & \cdots & \cdots & 0 \\
  e_2 & d_2 & e_2 & 0 & \cdots & \cdots & 0 \\
  0 & e_2 & d_3 & e_3 & 0 & \cdots & \vdots \\
  \vdots & 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\
  \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
  \vdots & \ddots & \ddots & \ddots & \ddots & 0 & e_{S-2} \\
  0 & \cdots & \cdots & 0 & e_{S-1} & d_{S-1} & e_{S-1} \\
  0 & \cdots & \cdots & 0 & e_{S-1} & 0 & d_S
\end{pmatrix}
\]  
(2.48)
with

\[
\vec{d} = \begin{pmatrix}
-\frac{2}{\hbar^2} + k_{u,1}^2 \\
-\frac{1}{\hbar^2} + k_{u,2}^2 \\
\vdots \\
-\frac{2}{\hbar^2} + k_{w,N-1}^2 \\
-\frac{1}{\hbar^2} + k_{w,N}^2 + k_{s,1}^2 \\
-\frac{2}{\hbar^2} + k_{s,2}^2 \\
\vdots \\
-\frac{2}{\hbar^2} + k_{s,M}^2 \\
-\frac{1}{\hbar^2} + k_{s,M+1}^2 + k_{b,1}^2 \\
-\frac{2}{\hbar^2} + k_{b,2}^2 \\
\vdots \\
-\frac{2}{\hbar^2} + k_{b,K}^2 \\
-\frac{1}{\hbar^2} + k_{b,K}^2
\end{pmatrix}, \quad \vec{e} = \begin{pmatrix}
\frac{1}{\hbar^2} \\
\frac{1}{\hbar^2} \\
\vdots \\
\frac{1}{\hbar^2} \\
\frac{1}{\hbar^2} \\
\vdots \\
\frac{1}{\hbar^2} \\
\frac{1}{\hbar^2} \\
\frac{1}{\hbar^2}
\end{pmatrix}
\]

and \( S \) the number of diagonal elements equal to \( N + M + K - 1 \).

### 2.2.5 Broadband modelling

In the previous two sections, the normal-mode solution to the wave equation for a single (angular) frequency \( \omega \) was discussed. However, acoustic sources typically do not transmit all energy at a single frequency, but at several frequencies at the same time, i.e. they are said to transmit a broad-banded pulse (see for example figure 3.5).

The effect of the source pulse being broad-banded can be taken into account by so-called Fourier synthesis of single frequency results. That is, repeatedly computing the normal-mode solution for a number of frequencies and integrating the results after proper weighting.

At the start of section 2.2.1, it was explained that the pressure field resulting from an omni-directional source transmitting sound at a single angular frequency \( \omega \) is given by:

\[
P_\omega(r, z, t) = p_\omega(r, z)e^{-i\omega t}
\]

(2.49)
in which \( p_\omega(r, z) \) is equal to expression 2.36, with the frequency \( \omega \) now explicitly indicated.

The complex conjugate of equation 2.49, denoted by \( P^{*}_\omega(r, z, t) \), is equal to:

\[
P^{*}_\omega(r, z, t) = p^{*}_\omega(r, z)e^{i\omega t}
\]

(2.50)

Furthermore, it is known that a function \( X(\omega) \) in the frequency domain can be transformed into the time-dependent function \( x(t) \) by applying the inverse Fourier transform, i.e.:
\[ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega \] (2.51)

When considering expression 2.50 as the Fourier synthesis at a single frequency, it can be understood now that for a source that transmits a pulse with frequency spectrum \( S(\omega) \), it is possible to write correspondingly:

\[ P^*(r, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S^*(\omega) p^*_\omega (r, z) e^{i\omega t} d\omega \] (2.52)

To solve equation 2.52 numerically, its integral has to be truncated and discretized. This results in the inverse discrete Fourier transform, i.e. the discrete equivalent of equation 2.52:

\[ P^*(r, z, t_k) = \frac{1}{N} \sum_{n=0}^{N-1} S^*_n p^*_n (r, z) e^{i\omega_n t_k} \] (2.53)

where

\[ \omega_n = 2\pi f_s \frac{n}{N} \] (2.54)

and

\[ t_k = \frac{k}{f_s}, \quad k = 0, 1, \ldots, N - 1 \] (2.55)

In addition, \( f_s \) is the sampling rate of the source wavelet and \( N \) the total number of samples taken.

Although this method is conceptually simple, the necessary truncation and discretization of the integral in equation 2.52 might cause some problems with the representation of the source signal. These practical matters are discussed further in section 3.2.

### 2.3 Defining a geo-acoustic model

The third step in the inversion process consists of defining a geo-acoustic model for the site that has been surveyed. In the previous sections, a propagation model has been discussed for a range-independent, time-invariant medium such as shown in figure 2.1. This environment is characterized by 12 parameters, i.e. a thickness, sound speed, density and attenuation coefficient for three different layers (water column, sediment and subbottom).

In addition to these 12 parameters, it is necessary to include the measurement geometry in the inversion, to know at what locations the acoustic field should be modelled and compared with the recorded data. Three additional parameters are needed with the assumption that
an array of hydrophones with known geometry is horizontal in the water and straight behind the source at a particular distance. In that case, the source depth, receiver array depth and horizontal distance between the source and first hydrophone are sufficient to describe the measurement geometry, arriving at a total of 15 parameters.

At this point, it is important to realize that exact values for the source depth, receiver array depth, distance between the source and receiver, and the water depth are unknown, even though an approximation might be available from survey data. Therefore, the values of these four geometrical parameters will have to be estimated in the inversion process.

Other parameters of which the values need to be estimated, are those that are of main interest for characterization of the underwater environment, i.e. the water sound speed, sediment sound speed, sediment density and sediment attenuation coefficient. In accordance with the previous section, it is assumed that a single sediment layer is modelled, although the matrix in equation 2.37 can be expanded to include multiple sediment layers when necessary. Furthermore, it is assumed that the sound speed is constant in each layer, even though the model derived in section 2.2 allows the sound speed to vary with depth.

All remaining parameters do not need to be estimated during the inversion, because their values can either be kept fixed or calculated using an empirical expression. The simplest example of this is the density of the water, which is assumed to be 1.0 g/cm$^3$.

Consequently, the 15 parameters needed for modelling can be divided into three separate groups:

- **Unknown geometrical input parameters:**
  - Source depth;
  - Receiver depth;
  - Horizontal distance between the source and first hydrophone;
  - Water depth.

- **Unknown geo-acoustic input parameters:**
  - Water sound speed;
  - Sediment sound speed;
  - Sediment density;
  - Sediment attenuation coefficient.

- **Fixed parameters:**
  - Water density;
- Water attenuation coefficient;
- Sediment thickness;
- Subbottom sound speed;
- Subbottom density;
- Subbottom attenuation coefficient;
- Subbottom thickness.

To continue the discussion of the values of the fixed parameters, the attenuation coefficient in the water is calculated using the following empirical relation of Thorp [21]:

\[
\alpha_t = \frac{0.1 f^2}{1 + f^2} + \frac{40 f^2}{4100 + f^2}
\]  

(2.56)

where \(f\) is the frequency in kHz and \(\alpha_t\) the attenuation coefficient in dB/kyd.

Although Thorp’s relation is a fit to experimental data obtained for the western part of the Atlantic Ocean near Bermuda, it is assumed valid for other environments as well. To use the value of \(\alpha_t\) in equation 2.33 though, it first has to be converted from dB/kyd to neper/m, i.e.:

\[
\alpha_w = \frac{20 \alpha_t \log_{10} e}{914.4}
\]  

(2.57)

The remaining five parameters that are not inverted for, all relate to the (virtual) subbottom. The first of these parameters is the thickness of the sediment layer, which should be chosen such that reflections off the subbottom have no noticeable effect on the modelled acoustic field in the water column. This is because the subbottom is not considered to be present in the real environment, but is needed in the normal-mode model to provide one of the boundary conditions for the modal equation.

The next parameter is the sound speed in the subbottom. As mentioned in section 2.2.2, the subbottom sound speed has to be larger than the minimum sound speed in the water column and sediment layer to satisfy equation 2.15. However, as explained in the same section as well, the eigenvalues in equation 2.15 correspond to sound rays travelling at grazing angles smaller than the critical angle at the sediment-subbottom interface. Therefore, the need to take into account sound rays travelling at specific grazing angles could pose a more stringent restriction on the minimum value of the subbottom sound speed.

Consider for example the geo-acoustic model shown in figure 2.5. As can be seen in the figure, this model consists of a water column, sediment layer and a subbottom, each having a constant sound speed \(c_w\), \(c_s\) and \(c_b\), respectively. The critical angle \(\theta_c\) at the sediment-subbottom interface is given by:
Figure 2.5: Geo-acoustic model of a simple environment.

\[
\theta_c = \cos^{-1} \left( \frac{c_w}{c_b} \right) \tag{2.58}
\]

This angle is related to the angle \( \theta \) at the water-sediment interface through Snell’s law, i.e.:

\[
\frac{\cos \theta}{\cos \theta_c} = \frac{c_w}{c_s}
\]

Rewriting equation 2.59 and substituting equation 2.58 gives:

\[
\theta = \cos^{-1} \left( \frac{c_w}{c_s} \cos \theta_c \right) = \cos^{-1} \left( \frac{c_w}{c_s} \frac{c_s}{c_w} \theta_c \right) = \cos^{-1} \left( \frac{c_w}{c_b} \right) \tag{2.60}
\]

Now suppose an acoustic field has to be modelled, including at least all sound waves that have grazing angles smaller than 60° at the water-sediment interface. With a water sound speed of 1500 m/s, equation 2.60 gives the minimum required value for the subbottom sound speed:

\[
c_b = \frac{c_w}{\cos \theta} = \frac{1500}{\cos 60°} = \frac{1500}{0.5} = 3000 \text{ m/s}
\]

Clearly, this value is more restrictive than the requirement that the subbottom sound speed should be larger than the minimum sound speed in the water column and the sediment, which in this case is 1500 m/s or less.

Based on this comparison, it can be said that the normal-mode model is a long-range approximation, in the sense that modifications are needed (i.e. including the continuous modes) for pressure field calculations involving small source-receiver separations, where sound waves with large grazing angles need to be taken into account.
A problem that still remains though, is to decide up to what grazing angle sound rays should be included in the model. One way to make this decision is to see whether different travel paths can be distinguished in the seismic traces that have been recorded. This method works well for one of the datasets that is used, as will be shown in section 3.3. Another option is to use a simple ray tracing model to calculate how often a sound ray has to be reflected in the water column to lose for example 99% of its energy before reaching the first hydrophone, given a realistic survey geometry and taking into account bottom losses and geometrical spreading.

With respect to the subbottom density, it can be said that its value has hardly any effect on the acoustic field that is modelled. This is because a change in density is most notable in the reflection coefficient for sound rays that travel at grazing angles greater than the critical angle (see also section 3.4), which are excluded from the propagation model by only taking into account the discrete modes. A larger value for the subbottom density increases the value of $h_b$ in equation 2.41 though, which in turn means the computation time needed to solve equation 2.37 is reduced.

The two final parameters are the subbottom attenuation coefficient and the thickness of the subbottom. The attenuation coefficient of the subbottom is best set at a rather high value, to reduce the strength of the reflections off the sediment-subbottom interface as much as possible.

The subbottom thickness is ideally chosen to be infinite, but this is not possible when numerically solving the modal equation. Therefore, in practice the thickness of the subbottom should be chosen large enough for the perfectly rigid boundary assumed in the normal-mode model not to cause any noticeable effect on the modelled acoustic field in the water column. Experience has learned that $H_b = 20\lambda$ is often sufficient, with $\lambda = \frac{\omega}{k}$ the acoustic wavelength in the subbottom.

### 2.4 The cost function

As mentioned in the introduction of this chapter, the objective of the inversion process is to obtain a ‘best’ estimate of the unknown parameters, by minimizing the mismatch between model and measurements. To do this, the mismatch has to be quantified first using a so-called cost or energy function.

In this case, the average correlation between the model output and recorded signals over each of the channels is used. To start with, the Pearson product-moment correlation \([2]\) \(\rho_{x,y}\) is determined for each of the receiver channels using the following equation:

\[
\rho_{x,y} = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{(N-1)\sigma_x\sigma_y}
\]

(2.61)

in which \(x\) and \(y\) are the modelled and measured signals in a single channel, \(\bar{x}\) and \(\bar{y}\) are
the sample means of $x$ and $y$, $\sigma_x$ and $\sigma_y$ are the sample standard deviations of $x$ and $y$, respectively, and $N$ is the total number of samples.

It has to be noted here that the model output and the recorded signals are matched filtered (i.e. cross-correlated) with the source pulse before calculating $p_{x,y}$ in order to reduce the influence of noise in the recorded signals on the energy value. In addition, the signals are normalized, hence the energy value represents a difference in shape rather than a difference in absolute value.

After having determined the Pearson correlation for each of the recording channels, the energy value $E$ is calculated by:

$$E = 1 - \overline{p_{x,y}}$$  \hspace{1cm} (2.62)

where $\overline{p_{x,y}}$ is the average of the correlation coefficients of the separate channels.

### 2.5 The differential evolution algorithm

One problem with the energy function described in the previous section is that it has many local minima and it is therefore not obvious to find the combination of parameters that results in the lowest energy value within the search bounds of the inversion.

One way to find the global minimum of the energy function is to try very many combinations of parameter values in order to get a detailed picture of the whole energy landscape. Such a purely random search does require many forward model calculations though, and is therefore far from practical. A better option is to make use of a so-called global optimization algorithm.

A global optimization algorithm takes into account the fact that if a certain combination of parameter values results in a high energy value, similar combinations are likely to do so as well. In other words, attention is focused on the combinations that seem to provide lower energy values, and an attempt is made to improve on these.

A large number of global optimization algorithms exist, of which genetic algorithms such as simulated annealing are among the most commonly used. For this thesis, a method called differential evolution is used. Differential evolution was introduced in 1995 by Storn and Price [20] and can be seen as a variant of the genetic algorithms.

A flow diagram of the differential evolution algorithm is shown in figure 2.6. As most optimization algorithms, the first step is to define an initial set of $q$ parameter value combinations that are randomly spread over the search space. This set is called the initial population with $q$ members.

For each of these $q$ combinations or members, a forward model run is done and the energy value as described in section 2.4 is calculated. Then a so-called partner population is created by repeatedly multiplying the difference between two arbitrary members in the population.
by a factor $F$ and adding the result to another member. The value of multiplication factor $F$ can be chosen between 0 and 1, but should be high enough, otherwise it will prevent exploration of the full search space.

After the partner population has been created, crossover is applied between the initial population and the partner population with probability $p_c$. This means that some of the parameter values in the elements of the initial population are replaced by those of the partner set, resulting in $q$ descendants. A higher value of $p_c$ leads, on average, to more parameter values of the partner set to end up in the descendant members.

The fitness of the descendants is then evaluated by running the normal-mode model and calculating the energy value for each member. If a descendant has a lower energy value than its parent in the initial population, it will replace this parent and become part of a new population of $q$ members.

From here, the process is repeated starting with the creation of another partner population, up to the point where a maximum number of generations is reached. The combination of values that results in the lowest energy value is then stored as the outcome of the inversion process.

As for the settings of the inversion method, Snellen and Simons [17] found that with a population size of 16 members, the combination of $F = 0.6$ and $p_c = 0.55$ gives the best results for a typical geo-acoustic inversion problem. Because finding the optimum values of $F$ and $p_c$ for the data under study is outside the scope of this graduation project, these values are assumed to work as well for the inversion problem at hand.

The only parameter that was varied during the processing was the number of generations,
which was eventually chosen to be 70 based on the first results. Figure 2.7 shows how the minimum and maximum energy value of the population varies over the number of generations for a typical single sediment layer inversion. In the figure, it is clearly visible that the energy value starts relatively high and decreases when the number of generations increases. The energy value stabilizes after about 35 generations, which is where the combination of parameter values remains almost constant. In this case it would perhaps have been sufficient to stop the optimization here, but to make sure the stabilizing point is always reached, a maximum of 70 generations has been chosen.

What is important to realize though, is that this number of generations works well for a single sediment layer model with 8 parameters. When a second sediment layer is included in the inversions, the number of parameters to be inverted for increases from 8 to 12, and it was found that 70 generations are no longer sufficient to find the global minimum. Instead, around 100 generations are necessary and the probability of finding the global minimum is greatly increased by choosing a twice as large population.
3 North Sea dataset

The geo-acoustic inversion process described in the previous chapter has been applied to a dataset that was acquired while sailing in the North Sea. This chapter provides details of the North Sea survey, as well as a discussion on practical matters such as modelling the source pulse and choosing suitable subbottom properties. In addition, the expected results for this dataset are looked at.

3.1 Collected data

The survey with which the North Sea dataset was gathered took place on 4 June 2002, about 10 km off the coast from Hook of Holland. Four tracks were sailed with an airgun as acoustic source and an array of hydrophones towed behind a ship, see figure 3.1. Details of these tracks – which are somewhat arbitrarily numbered 7, 45, 70 and 71 – can be found in table 3.1.

The hydrophone array or streamer that was used consists of 18 groups of hydrophones, of which the centres are 3.125 m apart. Each group comprises 12 hydrophones and 1 amplifier, where the distance between the first and last hydrophones of a group is 1.16 m.

Data from the array was recorded in SEG-Y format, with a sampling rate of 3000 Hz and a total recording time of 1.1 s per shot. In total there are 18 recording channels, which correspond to the 18 hydrophone groups. It is observed however that channels 4 and 15 don't contain any data. Next to this, it is noted that an offset of 11.25 ms between the start of the recording and the moment at which the airgun was fired is corrected for.

Figure 3.2 shows the first 150 ms of the signals recorded with the first group of hydrophones. As can be seen in the figure, the direct signal from the source takes approximately 17 ms to arrive, after which four reflected signals and a copy of the source pulse (see the next section) are visible. After about 80 ms the recordings contain only noise, except near the very end of tracks 7 and 45.

<table>
<thead>
<tr>
<th>Track number</th>
<th>7</th>
<th>45</th>
<th>70</th>
<th>71</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start/end time (UTC)</td>
<td>11:13-11:58</td>
<td>06:54-07:46</td>
<td>10:22-10:37</td>
<td>09:34-09:51</td>
</tr>
<tr>
<td>Total number of shots</td>
<td>547</td>
<td>621</td>
<td>184</td>
<td>204</td>
</tr>
<tr>
<td>Length of track (m)</td>
<td>6063</td>
<td>6072</td>
<td>1637</td>
<td>1561</td>
</tr>
</tbody>
</table>

Table 3.1: Details of the four tracks sailed during the North Sea survey.
Next to the SEG-Y files, a set of navigation files is available for the North Sea survey. These files contain the GPS time & position of the boat in UTM zone 31 easting/northing, together with the water depth as measured by a single beam echo sounder mounted under the ship. The same positions are also stored in the SEG-Y files, but then as latitude/longitude in the WGS 84 coordinate system.

The GPS positions are used to derive the sailing speed, which is shown in figure 3.3. Knowing that the airgun was fired every 5 seconds, it can now be calculated that the distance between any two shots is 7 to 11 m.

Finally, several water sound speed profiles were measured with a conductivity-temperature-and-depth chain during the day of the survey. A plot of these sound speed profiles is shown in figure 3.4.

### 3.2 The source pulse

As can be understood from section 2.2.5, it is necessary to know the shape of the pulse transmitted by the airgun before broadband geo-acoustic inversion can be performed. Because the source signal was not measured during the survey, TNO-NITG made a reconstruction of the signal based on the recordings of the hydrophone array. Figure 3.5 shows this so-called source signature in both the time and frequency domain.
Figure 3.2: Signal power recorded in channel 1 for each of the four North Sea tracks.
Figure 3.3: Sailing speed for the North Sea tracks as derived from GPS measurements.
Figure 3.4: Water sound speed as measured during the North Sea survey.

Figure 3.5: The pulse transmitted by the airgun as reconstructed by TNO-NITG.
In figure 3.5, it can be seen that the airgun transmits most of its energy in the frequency band 90-450 Hz, with a peak frequency around 200 Hz. The duration of the pulse is about 8 ms. The signal that is visible roughly 40 ms after the main pulse was explained to be a bubble pulse, but might just as well be the result of a reflection off the back side of the ship.

The frequency-domain representation of the source signal in figure 3.5 is part of the discretized spectrum $S_n$ in equation 2.53. Because the computation time needed to find $p_n(r, z)$ in this equation increases quadratically with frequency, the modelling can be sped up by excluding all frequencies for which $S_n$ is close to zero; i.e. by only calculating $p_n(r, z)$ for the interval between 90 and 450 Hz. Doing so, it was found that the largest difference in the modelled acoustic field was less than 1.5% when compared to the same calculations for which all frequencies were included. Because of the faster computation time and the small difference, it was decided to limit all broadband modelling to the frequency band 90-450 Hz.

In addition, some remarks can be made about the discretization of the source signal in the frequency domain, i.e. the total number of samples $N$. This is because the number of samples is related to the so-called observation time $t_{obs}$, calculated by:

$$t_{obs} = \frac{N}{f_s}$$

(3.1)

The observation time is equal to the time length of the modelled signal. For example, with $N = 512$ and $f_s = 3000$ Hz, all modelled signals will have a length of 171 ms. Looking at figure 3.2, this is more than sufficient to model the recorded signals in channel 1, which contains only noise after about 80 ms\(^1\). In fact, this length is sufficient for the signals in all other channels as well.

Next to this, the step size of the frequency discretization of the source pulse, and hence also the frequency step for broadband modelling, is given by:

$$\Delta f = \frac{f_s}{N} = \frac{1}{t_{obs}}$$

(3.2)

What becomes clear from equation 3.2, is that a larger value for $N$ leads to more precise broadband modelling, again at the cost of a longer computation time. In this case, with $N = 512$, equation 3.2 gives $\Delta f = 5.86$ Hz. That means that in the frequency band of 90-450 Hz that was selected, 61 frequencies are used for modelling. In other words, for each acoustic field that is modelled, the normal-mode model is ran 61 times for a different frequency.

\(^1\)Except for the end of tracks 7 and 45, but as will be discussed in chapter 4, these signals cannot be modelled.
3.3 Selection of model parameter values

In section 2.3, it was explained that the values of three model parameters have to be chosen based on the environment to be modelled. These parameters are the sediment thickness, subbottom sound speed and subbottom density.

As mentioned previously, the subbottom sound speed is related to the maximum grazing angle of the sound rays taken into account by the normal-mode model through equation 2.60. To choose a subbottom sound speed, it is therefore necessary to know what the maximum grazing angle is under which sound is still received at the hydrophone array. This angle is estimated as follows.

While looking at figure 3.2, it was noted that at most four reflections are visible in the recorded signals. This situation corresponds to figure 3.6, where a sound ray travels through the complete water column three times before arriving at the first hydrophone of the receiver array. It is the angle $\theta$ that is of interest here, because all signals received by the hydrophone array reflect off the sediment-subbottom interface at a grazing angle smaller than $\theta$.

From the single beam echo sounder data, it is known that the water depth $H_w$ in the survey area is about 25 m. Next to this, it is logged that the airgun was attached to a floater with a cable length of approximately 2 m, and that the receiver array is kept more or less halfway in the water column by a weight of 35 kg. In addition, the horizontal distance between the source and first hydrophone $D$ is estimated to be 25 m.

With this information, the angle $\theta$ can be calculated as:

$$\theta = \tan^{-1} \left( \frac{2 + 3 \cdot 25 + 12.5}{25} \right) = 74.4^\circ$$

Assuming a water sound speed of 1500 m/s, equation 2.60 now gives the minimum subbottom sound speed necessary to take all sound rays that are received into account:

$$c_b = \frac{c_w}{\cos \theta} = \frac{1500}{\cos 74.4^\circ} = 5578 \text{ m/s}$$
Because the geometry used to calculate this number is of course not more than a first approximation, a subbottom sound speed of 8000 m/s is chosen to be on the safe side with the normal-mode calculations. This sound speed corresponds to a maximum grazing angle of 79.2° for the geometry mentioned above.

Now that a value for the subbottom sound speed has been selected, the last step is to get a value for the sediment layer thickness and subbottom density. In section 2.3, it was mentioned that the sediment thickness should be sufficient for the subbottom not to influence the modelled acoustic field in the water column, and that the subbottom density is preferably chosen very high to speed up computations.

To find a suitable combination of these two values, one forward model run has been done with a sediment thickness of 1 km and a subbottom density of 2.0 g/cm³. This model run yields an acoustic field in the water column that is assumed not to be influenced by the subbottom. Next, various lower values for the sediment thickness have been tried in combination with a higher subbottom density. It was found that a sediment thickness of 250 m together with a subbottom density of 250 g/cm³ provides a fast computation time while hardly affecting the modelled acoustic field in the water column. Therefore, these values are selected for modelling.

As an extra check to see whether the lower bound of the frequency band selected for broadband modelling makes sense, it is now possible to calculate the cut-off frequency $f_c$ with equation 2.19 for the environment described above:

$$
 f_c \approx \frac{1500 \ln 2}{4 (25 + 250) \ln \left( \sqrt{1 - \frac{1500}{8000}} + 1 \right)} = 1.47 \text{ Hz}
$$

Clearly, $f_c$ is much smaller than 90 Hz, hence no problems are expected while modelling the chosen frequencies.

### 3.4 Expected results

Before performing geo-acoustic inversion on the data described in the previous sections, an expectation of the outcome of the inversion process is given first. This is beneficial for a later discussion of the results, as it becomes easier to identify where unexpected values occur.

Earlier in this chapter, approximate values for the geometrical parameters have already been given. In section 3.3, it was mentioned that the length of cable between the airgun and its floater is approximately 2 m. Hence it is expected that the inversion results show an estimate for the source depth around this value.

In a similar fashion, the estimated receiver array depth is expected to be about half of the water depth. The horizontal distance between the source and first hydrophone is expected to be about 25 m, and the water depth should be close to the values measured by the single
beam echo sounder. Moreover, the estimates of the water sound speed are expected to be around 1500 m/s, as was measured during the survey (see figure 3.4).

For the other geo-acoustic parameters, i.e. the sound speed, density and attenuation coefficient of the sediment layer, it is more difficult to give a value that is expected to result from the inversion, as no information on these parameters is available from the survey.

Nevertheless, the characteristics of the inversion results for these parameters can be predicted. For example, it is expected that the estimated values are strongly correlated in the spatial domain, because from a geological point of view it does not make sense to expect a large variability of bottom properties between two subsequent shots.

In addition, it is possible to look at how well the sediment density and sediment attenuation can be inverted for, based on the angles under which sound rays are received by the hydrophone array.

Consider the geometry shown in figure 3.7. In this figure, the angle $\theta$ is the smallest grazing angle under which sound is still received by the hydrophone array, which has a length of $L = 17 \cdot 3.125 = 53.125$ m. Using the same values for the geometry as mentioned before, i.e. $z_s = 2$ m, $z_r = 12.5$ m, $D = 25$ m and $H_w = 25$ m, the angle $\theta$ can be calculated, i.e.:

$$\theta = \tan^{-1} \left( \frac{12.5}{25 + 53.125} \right) = 24.4^\circ$$

Assuming a water sound speed of 1500 m/s and a realistic sediment sound speed for the North Sea of 1600 m/s, the critical angle $\theta_c$ at the water-sediment interface is computed as:

$$\theta_c = \cos^{-1} \left( \frac{c_w}{c_s} \right) = \cos^{-1} \left( \frac{1500}{1600} \right) = 20.4^\circ$$

Hence it follows that all grazing angles under which signals are received by the array are greater than the critical angle of the water-sediment interface. Note that this is exactly opposite to the reflections at the sediment-subbottom interface, which necessarily take
Figure 3.8: Change of reflection coefficient with sediment density and attenuation coefficient.

place at angles smaller than the critical angle of that interface, as explained in section 2.2.2.

To show how this relates to the results of the inversion process, it is necessary to compute the reflection coefficient of the water-sediment interface using the following equation [8]:

\[ R = \frac{Z_s - Z_w}{Z_s + Z_w} \]  

(3.3)

in which \( Z_w \) and \( Z_s \) are the angle-dependent complex impedances in the water column and sediment layer, respectively.

Figure 3.8 shows how a change in density or attenuation coefficient affects this reflection coefficient of an interface between two layers having a sound speed of 1500 m/s and 1600 m/s, respectively. As can be seen in the figure, a change in attenuation coefficient has hardly any effect on the reflection coefficient for grazing angles larger than the critical angle of 20.4°.

Therefore, the signals that are received by the receiver array are expected to contain very little to no information about the sediment attenuation coefficient, which in turn should lead to random inversion results for this parameter. On the contrary, the estimates for the sediment density are expected to be rather precise for the same reason.
4 Inversion results for the North Sea dataset

The geo-acoustic inversion process described in chapter 2 has been applied to the North Sea dataset discussed in the previous chapter. The search boundaries that were used for global optimization of the cost function are listed in table 4.1.

During the inversion process, the acoustic field has been modelled at the locations of all individual hydrophones of the receiver array. These signals were then added over the hydrophone groups, similar to what happens in the real array. This way, the directivity caused by summation over the groups that takes place in reality is taken into account in the modelling as well.

The results of the geo-acoustic inversion of the North Sea dataset are shown in figures 4.1 and 4.2. As is visible in these figures, the results have been split into five different groups and labelled A through E, based on their characteristics. This is to aid the discussion of the results, which is the topic of the remainder of this chapter.

The results in series A through E are discussed in the first five sections, respectively. In section 4.6, the inversion results for the sediment sound speed are compared with the three means of validation that are available, i.e. two maps and a number of borehole logs.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source depth</td>
<td>1 m</td>
<td>3.5 m</td>
</tr>
<tr>
<td>Receiver array depth</td>
<td>5 m</td>
<td>15 m</td>
</tr>
<tr>
<td>Distance source - first hydrophone</td>
<td>15 m</td>
<td>30 m</td>
</tr>
<tr>
<td>Water depth</td>
<td>10 m</td>
<td>30 m</td>
</tr>
<tr>
<td>Water sound speed</td>
<td>1490 m/s</td>
<td>1525 m/s</td>
</tr>
<tr>
<td>Sediment sound speed</td>
<td>1500 m/s</td>
<td>1900 m/s</td>
</tr>
<tr>
<td>Sediment density</td>
<td>1.0 g/cm³</td>
<td>2.2 g/cm³</td>
</tr>
<tr>
<td>Sediment attenuation coefficient</td>
<td>0.1 dB/λ</td>
<td>3.0 dB/λ</td>
</tr>
</tbody>
</table>

Table 4.1: Global optimization search boundaries for the North Sea dataset.
Figure 4.1: Geo-acoustic inversion results for tracks 7 and 45.
Figure 4.2: Geo-acoustic inversion results for tracks 70 and 71.
4.1 Results series A

The first series of results consists of the first parts of tracks 7, 45 and 70, and all of track 71. Series A contains those shots for which the inversion results are consistent from shot to shot. The results in the other four series show a somewhat different behaviour and are therefore discussed separately.

To start off the discussion with the source depth, it can be seen in figures 4.1 and 4.2 that the estimated depth of the source varies between 1.0 and 2.5 m for most of series A. For track 71, the estimated values are somewhat higher, but apart from that no clear difference exists with the other tracks. The average value for the source depth is found to be 1.7 m, which corresponds to the known length of cable between the airgun and its floater of approximately 2 m.

Looking at the results for the receiver array depth, it is noted that the precision of the estimated values is similar to that of the source depth, i.e. in the order of 1 m. A closer look shows a quick decrease of the receiver array depth at the beginning of track 7. A similar effect is visible for the start of track 45 (when ignoring results series D). Although figure 3.3 shows that the ship was already sailing at a constant speed at the start of the measurements, it might be that the array was not pulled firmly by the ship yet at the start of these tracks.

The estimated horizontal distance between the source and the first hydrophone of the receiver array is found to be fairly constant for all tracks of the North Sea dataset. This is an expected result when both the source and receiver are towed at a fixed distance behind the ship. The only clear exception is found for track 71 between a distance of 400 and 500 m, which corresponds to the location where the ship made a starboard turn as shown in figure 3.1. The difference of approximately 2 m that is seen around this point also shows up in the recordings, i.e. the signal arrives about 2 ms later here than elsewhere during the track. This might be caused by the ship making a turn. Similar to the other geometrical parameters, the estimated values for the distance between the source and receiver have a precision of 1 to 2 m.

The fourth and last geometrical parameter to be discussed is the water depth. What sets this parameter apart from the other three geometrical parameters, is that independent measurements are available to validate the estimated values. This is because during the
survey in the North Sea, a single beam echo sounder was used to record the water depth under the ship, such as is shown in figure 4.3 for track 7.

Looking at figure 4.3, it is obvious that there is a horizontal offset between the single beam echo sounder measurements and the inversion results. This is because both measurements are tied together by the GPS time and coordinates as measured on the boat. However, the single beam echo sounder ‘looks’ directly under the ship, while the inversion results are valid for some (virtual) point between the source and end of the receiver array.

Because there is a small trough in the water bottom halfway track 7, this track is the most suitable to get an estimate of the offset. Figure 4.4 shows a comparison of the estimated and measured water depth for various offsets. From this figure, it is clear that the minimum difference between the two sets of data occurs at a horizontal offset of 178 m. Applying this offset to the single beam echo sounder measurements results in the plot shown in figure 4.5.

Having applied the correction, it is possible to calculate the difference between the estimated and measured water depth, see figure 4.6. Interestingly, the difference appears to be either around +1 m or −1 m. A jump between these values occurs on the boundary of series A & B, and B & C, respectively.

The depth difference of +1 m in figure 4.6 can be explained partly by looking at the results for the water sound speed. The average water sound speed found for series A is 1517 m/s, which is higher than the 1500 m/s that was measured during the survey (see figure 3.4). For a water depth of 25 m, this difference in sound speed can account for a depth difference of up to 30 cm. However, when taking this into account a bias of 70 cm still exists. A possible explanation for this bias is that the single beam echo sounder measurements are not fully corrected for the draught of the ship. In addition, part of the bias might be caused by frequency-dependent effects, because the single beam echo sounder transmits a pulse with a much higher frequency than the airgun. This leads for example to a different penetration depth, of which the exact consequences for the inversion results are unknown.

The depth difference of −1 m that is found for the remainder of track 7 will be discussed in the next section about results series B. As far as the other tracks are concerned, they show a similar difference for results series A, though the exact difference between the estimated and measured water depth cannot be determined. This is because both the source and the hydrophone array have been retrieved and redeployed during the time in between the
several tracks, leading to a different value for the horizontal offset. This value in turn cannot be estimated reliably, as for tracks 45, 70 and 71 no clear feature is present on the water bottom.

Although the above discussion about the water depth difference might be considered unimportant at first, it can be understood now that the depth values measured by the single beam echo sounder cannot be readily used to fix the water depth in the inversion process. Being able to do so would – at least in theory – yield more precise inversion results, as the search space for the global optimization algorithm is made smaller. Clearly, for the water depth this only works if both the horizontal as well as the vertical offset are corrected for, which is not possible when these values are unknown.

To continue with the water sound speed, it is observed that the results for this parameter are similar for all tracks and series. As can be seen in figures 4.1 and 4.2, the water sound speed is not determined very precisely – that is, the estimated values can easily differ 10 m/s between shots.

A first thought might be that this imprecision is linked to the variability of the estimated geometry, but this is not the case. For example, a larger distance between the source and the first receiver can be corrected for by choosing a higher sound speed in the model, but as the geometry of the receiver array is fixed this leads to a wrong arrival time of the modelled signal in all other channels. Indeed, when looking at the correlation between the inversion parameters in figure 4.7, it turns out that there is no obvious correlation between the water sound speed and any of the geometrical parameters. This means a different explanation is necessary for the variation that is observed in the estimated values.
Figure 4.7: Correlation between inversion parameters.
Apart from the fact that the differential evolution algorithm is not optimizing for the exact minimum value, a possible explanation for the variability of the water sound speed is provided by looking at the sampling rate of the recorded signals. With a sampling rate $f_s$ of 3000 Hz, a real peak in the signal can be as much as $\frac{1}{2} \cdot \frac{1}{f_s} = \frac{1}{6}$ ms off from the corresponding peak in the recording. At a distance of 25 m, this difference alone can cause an error in the estimated water sound speed of 15 m/s. Moreover, the moment at which the airgun fires can change slightly with respect to the start of the recording from shot to shot, introducing an additional random error.

A problem that still remains with the results of the water sound speed though, is that the estimated values tend towards the high end of the search window. As mentioned previously, the average water sound speed found for results series A is 1517 m/s, which is higher than the 1500 m/s measured throughout the day of the survey with a conductivity-temperature-and-depth chain (see figure 3.4). What is interesting, is that the arrival times of the direct wave in the subsequent hydrophone channels indicate a sound speed of 1510 m/s, i.e. which is higher than 1500 m/s as well. Since this difference is too large to be explained by a tilt of the array, a remaining explanation might be that the streamer is not straight in the water. If this is the case, that also means that the inversion results cannot be improved by fixing the water sound speed at the measured value of 1500 m/s, as the assumptions about the geometry of the array do not hold.

Looking at the results for the sediment sound speed, it can be said that for most of series A the estimated values lie between 1600 and 1650 m/s. For track 71, somewhat higher values are found around a distance of 450 m and 800 m. These two locations again correspond to where the ship made a turn to starboard and port, respectively; hence the estimated values are likely to be erroneous there.

The second last parameter to be discussed is the sediment density. For most of results series A, the value of the sediment density is estimated to be between 1.0 and 1.2 g/cm$^3$. This is much lower than what is expected based on the empirical relation of Hamilton and Bachman [7] that is shown in figure 4.8. According to them, a sediment layer with a sound speed of 1600 m/s should have a corresponding density of about 1.7 g/cm$^3$.

An explanation of the low density values that are found is provided by Snellen et al. [18], who ascribe the effect to the presence of shear waves, which are not taken into account by the normal-mode model. Because of neglecting shear waves, in fact an effective density value $\rho_{\text{eff}}$ is inverted for, which is equal to:

$$\rho_{\text{eff}} = \rho \left[ 1 - 2 \left( \frac{V_s}{c_w} \right)^2 \right]^2 \tag{4.1}$$

in which $\rho$ is the actual sediment density, $V_s$ is the shear wave speed and $c_w$ is the water sound speed.

Equation 4.1 can be rearranged to get the following expression:
which allows to calculate the shear wave speed necessary to correct for the low density values, knowing the following empirical relation between the actual density $\rho$ and the compressional wave speed $V_p$ of the sediment [1]:

$$\rho = -11.393 + 1.3778 \cdot 10^{-2} V_p - 3.5162 \cdot 10^{-6} V_p^2$$  \hspace{1cm} (4.3)$$

where $\rho$ is in $g/cm^3$ and $V_p$ in m/s.

Figure 4.9 shows the results of applying equations 4.2 and 4.3. In this figure, it can be seen that the shear wave speed of the sediment must be roughly 450 m/s to account for the low density values that are observed in the inversion results.

According to Hamilton [6], the shear wave speed in the sediment $V_s$ is related to the compressional wave speed $V_p$ as well, through the following empirical relation:

$$V_s = -1485 + 1.137 V_p$$  \hspace{1cm} (4.4)$$

where $V_s$ and $V_p$ are both in m/s.

Equation 4.4 gives a shear wave speed of 334 m/s for a sediment layer with a compressional wave speed of 1600 m/s, which is lower than the estimated value of approximately 450 m/s.
Instead, a value of 450 m/s for the shear wave speed would be expected for a sediment with a compressional wave speed of about 1700 m/s, which is in turn somewhat higher than the values resulting from the inversion.

With relations 4.3 and 4.4, it is also possible to calculate the empirical values of $\rho_{\text{eff}}$ that correspond to the inversion results for the sediment sound speed. For results series A, this gives an average value for $\rho_{\text{eff}}$ of 1.33 g/cm$^3$, which is slightly greater than the average density of 1.14 g/cm$^3$ found by the inversion. However, it must be mentioned that the empirical relations used above have been derived from sediment characteristics at 20 different locations, not including the North Sea. Therefore, it is possible that they don’t hold fully for the type of sediment found under the North Sea, hence a definite conclusion is hard to draw.

The final inversion parameter is the sediment attenuation coefficient. Looking at the inversion results, it is found that the estimated values of this parameter are almost random. As explained in section 3.4, this effect is caused by most of the recorded signals reflecting off the water-sediment interface at a grazing angle greater than the critical angle, which means they are hardly if at all influenced by the attenuation coefficient of the sediment.

### 4.2 Results series B

The second results series is distinguishable from series A because of the following characteristics:

- A sudden jump of 2 m in the source depth, receiver array depth and water depth. For the source depth, this jump leads to unrealistically high values.
- A random sediment sound speed.
- A larger variability in the estimated sediment density and sediment attenuation coefficient.

In an attempt to find out what causes these effects, the modelled and measured signals of results series A and B have been compared. The only difference that was found between the measured signals of the two series, is that for all of series B, the direct signal from the airgun interferes with the first reflections off the water surface and water bottom, which is not the case for series A. Although this effect is modelled by the normal-mode model, its output is not correct for series B. Because of the measurement geometry, this is most notable in channel 6, see figure 4.10.

![Figure 4.10: Comparison of modelled and measured signals for series A and B.](image)

What is visible in figure 4.10, is that for a typical shot in series A, the shape of the modelled signal matches that of the measured signal quite well. Therefore, the inversion results are consistent and reliable for this series. Looking at a typical result for series B however, it is obvious that there is a large difference between the modelled and the measured signal. As is indicated by the arrows, the interference pattern of the direct signal with the first reflections, and that of the third reflections is not modelled well.

To work around this problem, three possible solutions have been tried:

1. Inverting for the first three channels only, which don’t show the interference problem.

2. Using a time window when determining the value of the cost function, excluding the reflections for which the model output does not match the measurements.

3. Fixing the geometry at realistic values to simplify the optimization problem.

The first two options have been tried for tracks 7 and 45, and were found to result in correct estimates of the geometrical parameters – that is, the source depth, receiver array depth and the water depth all attained values that are in line with results series A. However, the values of the geo-acoustic parameters appear even more random than before, which is likely a result of too little information being available to arrive at a more precise estimate.

---

1The modelled signal for series B shown in figure 4.10 is valid for realistic parameter values, rather than the wrong values found after the inversion.
The third option was applied to track 7, with the geometrical parameters fixed to the values shown in table 4.2. In addition, the water depth was fixed to the values measured by the SBES, corrected for a horizontal offset of 178 m and a vertical offset of 1 m, such as discussed in the previous section.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source depth</td>
<td>1.7 m</td>
</tr>
<tr>
<td>Receiver array depth</td>
<td>11.0 m</td>
</tr>
<tr>
<td>Horizontal distance source - first hydrophone</td>
<td>22.5 m</td>
</tr>
</tbody>
</table>

Table 4.2: Fixed geometrical parameter values.

Limiting the search space of the optimization in this way, it was found that the estimates of the geo-acoustic parameters do not longer appear random, see figure 4.11. In fact, as a result of fixing the geometry, the estimated values for the properties of the sediment are even more precise than the values of results series A.

This does not necessarily mean that the estimates are accurate though, because the values that are used to fix the geometry are not more than an educated guess. Any change in the values of table 4.2 leads to different inversion results for the geo-acoustic parameters.

In addition, it is noticed that the estimated values of the water sound speed are now lower than they are for the other results series. Because this parameter is very sensitive to changes in geometry – e.g. an increase of 0.5 m in the total distance between the source and receiver results in an increase of about 25 m/s in sound speed – this difference may indicate that the fixed geometry is not fully correct.

Therefore, a suggestion to improve the results is to use the estimates of options 1 and 2 mentioned above to fix the geometrical parameters. However, for future surveys it can be kept in mind that knowing the exact measurement geometry would be even better in this case.

### 4.3 Results series C

The third results series consists of the results found for the end of tracks 7 and 45. Similar to series B, this series is characterized by a random sediment sound speed and a larger variability in the estimated sediment density and attenuation coefficient. However, the geometrical parameters are estimated correctly for series C.

Looking at figure 4.1, it is noted that the results of series C correspond to a clear difference in the recorded signal power. In contrast to all other results series, there are far more copies of the source signal to be seen.

In figure 4.12, the recorded signals for a typical shot in series A and C are compared for the first receiver channel. Apparently, for series C the signals that are received after the direct wave have a larger amplitude and attenuate much slower than those of series A. The amplitude of the received signals of series C is even stronger than what would be expected.
Figure 4.11: New inversion results for track 7, with fixed geometry for series B.
when there is no attenuation except for (spherical) geometrical spreading. In any case, the arrival time of the first two peaks can still be used to estimate the values of the geometrical parameters correctly.

The question that remains now, is what causes the change in the signals that are received. Possible explanations include:

1. A change in the amplitude of the source signal or recording settings.
2. The presence of two or more sediment layers.
3. An effect caused by the sound speed profile.
4. Scattering objects such as shells being present at the sea floor.
5. The presence of converted surface waves or shear waves.

The first hypothesis is considered unlikely, because the direct signal that is received from the source was found to be the same as it is for all other results series.

The second hypothesis is somewhat more likely, as an additional sediment layer causes additional reflections to be present in the recordings. However, the signals recorded for series C are too strong and too irregular to be explained by a second sediment layer. An inversion run including a second sediment layer did not give any meaningful results either (see section 4.5 for a discussion about dual-layer modelling).

The third hypothesis is that an effect of the sound speed profile, such as the existence of a sound channel acting as a waveguide, influences the recorded signals. However, this hypothesis cannot explain why the amplitude of the recorded signals is larger than would be expected with spherical spreading loss, as indicated in figure 4.12.

The fourth hypothesis comes from a line of text in borehole log Q16-13 (see appendix A).
Although this borehole is somewhat north of track 7, it does suggest that in or around the survey area a thin layer of seashells is present at the water bottom. These shells can have a large influence on the recorded signals, since they are good scatterers [19]. If the layer of shells is continuous, it is expected that the received signals are more constant than what is observed though. Besides, the problem that the received signals are too strong cannot be solved by this hypothesis either.

The fifth and last hypothesis is that elastic waves converted back to compressional waves are recorded by the hydrophone array. Upon interaction with the sediment, part of the energy of the acoustic signal coming from the source generates surface and shear waves in the sediment, which cannot be modelled with the normal-mode model described in section 2.2. These surface and shear waves can be converted back to compressional waves and propagate back into the water column, and can therefore be received by the hydrophone array. Because of the slower travel speed of surface and shear waves, these converted waves arrive later at the array than the (non-converted) compressional waves and might thus explain the shape of the recorded signals. However, for the amplitude of the elastic waves to be sufficiently large to influence the recorded signals, it is necessary that the bottom is harder near the end of tracks 7 and 45. The amplitudes of the first and second arrival complexes do hardly increase at the end of these tracks though, which might indicate that this is not the case.

4.4 Results series D

Results series D comprises the inversion results between 150 and 500 m from the start of track 45. This series is characterized by an incorrect geometry in combination with a quite precise but erroneous estimate of the sediment sound speed.

When comparing the recorded signals for series D with those of series A, it was found that in channels 2 through 18, the direct signal arrives earlier than expected with respect to the first arrival in channel 1. In the last channel, the direct signal arrives on average 2.4 ms earlier than for results series A, which means that with the same geometry and sound speed, the last hydrophone is 7% closer to the source than it should be. This indicates that the receiver array is not in a straight line behind the ship.

In addition, it was found that forcing the geometry to be correct makes the inversion results rather worse than better. Although the minimum energy values are not much different from series A, adding a correction for vertical tilt of the array does not improve the results either. Therefore, the most likely conclusion is that the assumption of the receiver array being straight behind the ship does not hold for results series D.

4.5 Results series E

The fifth and last results series contains all shots in the second half of track 70. The results of this series are almost identical to series A, with the exception that the sediment
sound speed appears to split into two distinct values, i.e. about 1600 m/s and 1700 m/s, respectively. This might indicate that the hydrophone array is receiving signals from two different sediment layers. In that case, the top-most sediment layer needs to be relatively thin however, as no additional reflections are visible in the recorded signals.

To explore this possibility further, matrix $A$ in equation 2.37 has been expanded to include a second sediment layer with a thickness of 1 m. A smaller sediment thickness might have been more realistic, but leads to a much longer computation time. This is because for a very thin sediment layer, the depth increment of the finite-difference discretization has to become very small to get sufficient samples of the normal modes in that layer. This in turn leads to a larger matrix $A$ of which the eigenvalues and eigenvectors have to be calculated.

To verify the correctness of the dual-layer model, two inversion runs have been done: one with a very thin first sediment layer and another with a very thick first sediment layer. For both of these runs, the influence of the second sediment layer on the pressure level in the water column should be negligible, and indeed the results were found to be the same as for the single-layer model. In addition, a run was done with the thickness of the first sediment layer set to 1 m, but with the same sound speed, density and attenuation coefficient in both sediment layers. The results of this run are identical to those of the single-layer model as well.

Next, the new model was used to invert about $\frac{1}{4}$ of the shots of track 70, while constraining the search space of the global optimization algorithm, by requiring that the sound speed and density in the second sediment layer are higher than that of the first layer. Nevertheless, it was found necessary to increase the population size of the differential evolution algorithm to 32 members and set the number of generations to 100 (see also section 2.5).

Because of the additional degrees of freedom in the model – that is, the number of parameters to be estimated is increased from 8 to 12 – the resulting energy values after the inversion are lower than for the single-layer model. It was observed however, that the differential evolution algorithm does only find the approximate location of the global minimum in the dual-layer case. To optimize the results further, a different but faster optimization algorithm called downhill simplex [17] was used to do a local search for the minimum value, starting from the results of the differential evolution algorithm.

After the inversion, it was found that the estimates of the geometrical parameters do not change, and neither do the water sound speed and the sediment attenuation. Differences with respect to the single-layer model are found for the sediment sound speed and sediment density though, for which the new results are shown in figure 4.13.

What is visible in figure 4.13, is that for the first half of track 70, the sediment sound speed and sediment density are about the same for both sediment layers that are modelled. However, for the second half of the track, at a number of shots there is a clear split in estimated properties of the two sediment layers. Here, the figure indicates that the top-most sediment layer has a sound speed between 1500 and 1600 m/s, with a corresponding effective density of $1.2 \text{ g/cm}^3$, while the second layer has a sediment sound speed of about 1700 m/s and a somewhat higher effective density of $1.3$ to $1.4 \text{ g/cm}^3$.

Although it is believed that the results can still be improved further by looking at different
Figure 4.13: Estimated sediment sound speed and sediment density with dual-layer model.
layer thicknesses and more shots, figure 4.13 provides at least some evidence that two sediment layers can be identified in the recordings.

### 4.6 Comparison with validation data

Now that the results in each of the series have been discussed, the validation of the inversion results is looked at. The following three types of validation data are available for the North Sea dataset:

- A map of the coarse sand percentage in the survey area;
- A sediment classification map;
- Three borehole logs.

All of this data was kindly provided by TNO-NITG.

Figure 4.14 shows the map with the coarse sand percentage. The different colours of the map indicate the average coarse sand percentage over the top-most 20 cm of the sediment, where an increase in coarse sand percentage signifies an increase in mean grain size and a higher expected value for the sediment sound speed. Therefore, the map is best compared to the inversion results for this parameter.

To allow for a direct comparison with the map, the estimated sediment sound speed is converted into an estimate of the mean grain size, applying the following empirical relation of Bachman [1]:

\[
M_z = 172.66 - 0.1832V_p + 4.927 \cdot 10^{-5}V_p^2 + 4.927 \cdot 10^{-5}V_p^2
\]  

where \( V_p \) is the compressional wave speed in the sediment in m/s and \( M_z = -\log_2 m \), with \( m \) the mean grain size in mm.

The newly calculated estimates of the mean grain size are shown in figure 4.14. Note that for track 7, the new inversion results with fixed geometry for series B have been used. Because new inversion results for track 45 are not available (see also section 4.2), the second half of this track shows a random estimate of the mean grain size. This also explains the difference between tracks 45 and 71 at the location where they cross.

Ignoring the second half of track 45, it can be seen in figure 4.14 that at almost all locations the mean grain size is estimated to be lower than 50 µm. When coarse sand is defined as having a mean grain size of 528.5 µm, such as is done by Hamilton and Bachman [7], the corresponding coarse sand percentage is less than 10%, no matter what the rest of the sediment is composed of. This means that if equation 4.5 holds, the background map should be coloured black almost everywhere. Figure 4.14 shows that this is clearly not the case.
Figure 4.14: Estimated mean grain size and relative coarse sand percentage in the survey area (coarse sand percentages: black: <10%, light yellow: 10-20%, dark yellow: 20-30%, orange: 30-40%, red: >40%).
There are some potential problems with this comparison though. First of all, the TNO-NITG map shows the average coarse sand percentage over the top-most 20 cm of the sediment, while the inversion results might be valid for a different layer thickness. Secondly, the map is interpolated from a relatively small number of boreholes (how many is not exactly known), and thirdly, the precision of the map is unknown, and it is not specified how the bottom properties might change over time. In short, this all means the inversion results are not directly comparable with the map.

This argument is strengthened by the fact that there is no clear trend in the estimated mean grain size values when compared to the coarse sand percentages of the map, see figure 4.15. This figure shows the average and 90% spread of the (reliable) mean grain size estimates that correspond to a particular colour on the coarse sand percentage map. It is expected that the values of the mean grain size increase with the coarse sand percentage, but this is not the case.

The second map of the survey area that is used to validate the inversion results is the sediment classification map shown in figure 4.16. The different colours of this map indicate different sediment classes based on the so-called Wentworth-Folk classification scale, see figure 4.17. When comparing the map with the estimated sediment sound speed in figure 4.16, some correlation is visible, especially for track 7.

The correlation between the inversion results and the classification map is shown even better when plotting the four sediment classes present on the map against all estimates of the sediment sound speed that are deemed reliable, see figure 4.18. In figure 4.18, the sediment classes have been ordered according to their mean grain size, and hence an increase in sediment sound speed is expected to be visible from left to right. In fact, this is exactly the case, so it can be concluded that the classification map agrees better with the inversion results than the map of the relative coarse sand percentage.
Figure 4.16: Estimated sediment sound speed and sediment classes in the survey area (see figure 4.17 for the map legend).

Figure 4.17: Wentworth-Folk classification scale used for the TNO-NITG map.
Comparison of estimated sediment sound speed with TNO–NITG map

Figure 4.18: Comparison of the estimated sediment sound speed with the classification map (the error bars indicate the spread of 90% of the values).

It must be noted however that the values for the first class shown in figure 4.18 might not be completely valid, as the estimates of the sediment sound speed correspond mostly to that part of track 7 for which the geometry has been fixed. In addition, the spread of the sediment sound speed estimates is quite large for each of the classes, which means it is hardly possible to assign a single class to a particular sound speed. For example, a sediment sound speed of 1590 m/s can correspond to any of the four classes within the 90% confidence interval that is shown.

The third and last validation method comes from the three borehole logs, which are included in appendix A. One of these boreholes (Q16-187) is near the crossing of tracks 45 and 71 as indicated in figures 4.14 and 4.16, while the other two bore holes (Q16-13 and Q16-155) are just outside the map to the north of track 7.

The smallest mean grain size that is mentioned in the borehole logs is 180 µm for borehole Q16-187, close to tracks 45 and 71. According to Bachman [1], this grain size corresponds to a sediment sound speed of 1763 m/s, which is higher than all estimated values that are considered reliable. This does not come as a total surprise though, as according to figure 4.16, the location of this borehole corresponds to the coarsest type of sediment found in the survey area. Therefore, the log is not likely to be representative for the majority of the inversion results.

So far, only the comparison between the estimated sediment sound speed and the classification map has shown reasonable agreement. To do a better quality estimation of the inversion results, it is necessary though to have more detailed validation data. Therefore, it is suggested that during a future survey, additional measurements are taken that can possibly be used for validation, and at least give a more detailed view of the environment that is modelled. Such measurements can for example be taken with a multi-beam echo sounder, which is a system that is demonstrated to perform well for sediment classification [16].
5 Kulcs dataset

The second dataset that was used for the thesis was gathered while sailing in the Danube River near Kulcs, Hungary. The author participated in the survey near Kulcs himself, and an extensive report about the equipment used and all data that was gathered is available separately [3]. In this chapter, those parts of the survey that are of interest for the current report are discussed.

5.1 Collected data

Similar to the North Sea survey, four tracks were sailed with an airgun as acoustic source, see figure 5.1. All tracks were sailed several times though, in order to record acoustic pressure and ground acceleration at different depths in a nearby borehole (a log of this borehole is included in appendix A). Of special interest for this report are tracks 1 and 4, as for these tracks a hydrophone array was towed behind the boat.

The hydrophone array that was used in Hungary is somewhat longer than that of the North Sea survey. Instead of 18 hydrophone groups, the array consists of 24 groups with the same spacing of 3.125 m. Each group comprises 4 hydrophones, placed 40 cm apart.

In total 21 lines were sailed with this array: 16 lines for track 1 and 5 lines for track 4. For the former lines, the airgun was at a depth of 1.3 m, while the latter were sailed with the airgun at a depth of 2.0 m.

The data was recorded in SEG-Y format, with a default sampling rate of 2000 Hz and a total recording time of 2 s per shot. However, for 5 lines of track 1, more detailed recordings are available with a sampling rate of 8000 Hz.

For time reasons, it was decided to limit the inversion of the Kulcs dataset to a single line. One of the lines of track 1 with the higher sampling rate of 8000 Hz does not show any irregularities in the survey log and was selected for processing. This particular line was recorded on 23 October and starts with record number 800. Therefore, it will be denoted as ‘line 800’ in the remainder of the report.

Figure 5.2 shows the first 300 ms of the signals recorded with the first group of hydrophones. The first arrivals are visible around 17 ms, after which many reflections are recorded. Part of these reflections are known to originate from deeper sediment layers.

Beyond 100 ms, signals with a different shape are visible. These signals have a propagation speed of 200 to 300 m/s and are therefore identified as surface waves that are generated
Figure 5.1: Map of the tracks sailed during the survey near Kulcs.
in the bottom and converted back to compressional waves. Although these waves are not modelled by the normal-mode model, they can be time separated from the earlier reflections and thus do not have to influence the inversion process.

Next to the streamer data, a vertical seismic profile (VSP) was acquired with an array of 12 hydrophones in the borehole, see figure 5.3. The full depth of the borehole, i.e. 82.5 m, is covered by 8 records at different depths. Because the source was kept at a fixed position approximately 20 m from the well, these records can readily be stacked, as is shown in figure 5.4.

The VSP data can be processed to make an estimate of the sediment sound speed and is thus useful to validate the inversion results (see also chapter 6). In addition, the VSP is useful for reconstructing the pulse transmitted by the airgun, as will be discussed in the next section.

Similar to the North Sea dataset, a set of navigation files is available. These files contain the position of a differential GPS receiver that was floating on the water at a distance of 3.7 m behind the airgun, as well as the positions logged by a handheld GPS device in a rubber boat behind the streamer. Because the latter device is not very precise, the measurement geometry is only roughly known.

The coordinates of the differential GPS receiver have been used to derive the sailing speed for line 800, see figure 5.5. As can be seen in the figure, the sailing speed is about 0.9 m/s for most of the line. With the airgun firing every 4.5 s, this results in a distance between two shots of 4.1 m.
5.2 The source pulse

During the survey in Hungary, the acoustic pressure close to the source was recorded with a near-field hydrophone. From these measurements, it is known that the airgun fires 8.9 ms after the start of the recordings. This time delay needs to be corrected for, before attempting any inversion of the streamer data.

In addition, it was expected that the shape of the pulse transmitted by the airgun could be extracted from the near-field hydrophone records. Because of the shallow water depth however, reflections off the water surface and the water bottom interfere with the direct wave from the source. Therefore, the shape of the transmitted pulse appears distorted in the recordings from the near-field hydrophone and cannot be used for the inversion process.

Instead, the source pulse was reconstructed from the least deep traces of the VSP, for which the interference effects are less pronounced. The resulting pulse is shown in figure 5.6.

In figure 5.6, it can be seen that the airgun used for the survey near Kulcs transmits almost all its energy in the frequency band from 20 to 370 Hz. For a better representation of the source pulse though, frequencies up to 650 Hz are included in the modelling.

The total number of samples for the source pulse was chosen to be 1024, resulting in an observation time of 128 ms and a frequency step of 7.8 Hz, according to equations 3.1 and 3.2, respectively. In total, 82 frequencies are modelled, i.e. a number that is comparable to that of the North Sea dataset.
Figure 5.4: The recorded vertical seismic profile.

Figure 5.5: Sailing speed for line 800 of the Kulcs dataset.
Figure 5.6: The reconstructed pulse of the airgun used in Hungary.

5.3 Selection of model parameter values

For the Kulcs dataset, values for the sediment thickness, subbottom sound speed and subbottom density have to be chosen as well. This has been done with a somewhat simpler method than used for the North Sea dataset, as is explained in this section.

While introducing figure 5.2, it was mentioned that all signals of interest for the inversion with the normal-mode model arrive within 100 ms after the airgun fires. With a water sound speed of 1500 m/s, that means that the recorded signals cannot have travelled further than 150 m to the first hydrophone. If the depth difference between the source and receiver is neglected, the maximum grazing angle $\theta$ from which sound is recorded can be calculated by:

$$\theta = \cos^{-1}\left(\frac{30}{150}\right) = 78.5^\circ$$

The subbottom sound speed that is necessary to take sound rays up to this grazing angle into account is then given by equation 2.60:

$$c_b = \frac{1500}{\cos 78.5^\circ} = 7500 \text{ m/s}$$

This value for the subbottom sound speed can be interpreted as being a maximum though, because the sound speed in cold, fresh water is expected to be less than 1500 m/s. Besides, the assumed travel distance of 150 m is really the maximum that is found in the recordings, and smaller distances require a lower subbottom sound speed.

For convenience, it has been decided to use the same parameter values as for the North Sea dataset. This is because a subbottom sound speed of 8000 m/s works as well, and an
efficient combination of values for the subbottom density and sediment thickness is already known for this sound speed (see section 3.3).

### 5.4 Expected results

In line with chapter 3, the expected inversion results of the Kulcs dataset are looked at. At this point, it has to be mentioned however, that it was found that an inversion using the normal-mode propagation model does not give satisfying results for this dataset. Therefore, a different inversion method was applied, as is discussed in the next chapter.

Nevertheless, the results that are expected when the inversion process of chapter 2 would work successfully are discussed in this section. This not only allows to compare the potential of the Kulcs and North Sea datasets, but is also useful as a basis for future inversions using an improved propagation model.

As was mentioned in section 5.1, the airgun was at a depth of 1.3 m for the line that was selected, hence an inversion is expected to yield estimates around this value. Because the streamer was floating just below the water surface, its depth can be fixed at e.g. 1 cm and does not have to be inverted for.

In addition, it is known that the horizontal distance between the source and first hydrophone is about 30 m, and that the water depth is 3.5 m at the location of the borehole. Therefore, similar values are expected as a result of the inversion process.

To get an estimate of the water sound speed, the streamer data was matched-filtered with the pulse shown in figure 5.6. Looking at the subsequent arrival times in the various hydrophones, the water sound speed was estimated to be 1448 m/s.

According to the following empirical relation of Marczak [11], which is valid for fresh (distilled) water, it holds that:

\[
c_w = 1402.385 + 5.038813T - 5.799136 \cdot 10^{-2}T^2 + 3.287156 \cdot 10^{-4}T^3 \\
- 1.398845 \cdot 10^{-6}T^4 + 2.787860 \cdot 10^{-9}T^5
\]  

(5.1)

where $T$ is the temperature in °C and $c_w$ the water sound speed in m/s. With equation 5.1, it can be computed that a sound speed of 1448 m/s corresponds to a temperature of about 10 °C. This is conceived as being a realistic value.

Furthermore, the sediment sound speed and sediment density are expected to be higher in the area near Kulcs than for the North Sea, because the sediment of the Danube River is more lithified.

To make an estimate of the sediment sound speed, the data of the VSP was inverted using a simple ray tracing model (see appendix B), looking at the first signal arrivals for each
hydrophone depth. With a source depth of 1.3 m and a water sound speed of 1448 m/s, this inversion gives an average value for the sediment sound speed of 1745 m/s.

Moreover, it was tried to do an inversion of the VSP data with the normal-mode model. It was found however that the signals recorded in the borehole are much stronger than those that are modelled, especially at greater depths. This is likely caused by reflections in the borehole itself, which acts as a kind of waveguide. Because this effect is not modelled, looking at the shape of the recorded signals does not give a reliable estimate for the sediment density and attenuation.

It is again possible however, to look at how well the sediment density and attenuation are expected to be estimated by inverting the streamer data. The smallest grazing angle (see figure 3.7) for the Kulcs dataset is calculated to be:

$$\theta = \tan^{-1} \left( \frac{2.2 + 3.5}{30 + 71.875} \right) = 3.2^\circ$$

With a water sound speed of 1448 m/s and a sediment sound speed of 1745 m/s, the critical angle at the water-sediment interface is:

$$\theta_c = \cos^{-1} \left( \frac{1448}{1745} \right) = 33.9^\circ$$

Hence it follows that sound rays are received well below and well above the critical angle. Again looking at figure 3.8, it is therefore expected that the recordings of the streamer contain information about both the sediment density and the sediment attenuation, and that these parameters can thus be inverted for well.
6 Inversion results for the Kulcs dataset

The normal-mode model described in section 2.2 was used to invert line 800 of the Kulcs dataset. It was found however that the inversion does not lead to precise parameter estimates, because the acoustic field cannot be modelled well. The cause of this is that the water column acts as an ‘echo chamber’ with very many interfering reflections, which are in turn caused by the shallow water depth and the presence of several reflectors in the bottom. Since the source pulse is only approximately known (see section 5.2) and effects such as the generation of shear waves and surface waves are neglected, the inversion process is not able to find a workable description of the environment.

Therefore, it has been decided to take a completely different approach for the Kulcs dataset. One particular feature that was identified in the streamer records, is the presence of a so-called head wave.

When the sound speed in the sediment is greater than that of the water column, a critical grazing angle exists under which sound gets critically refracted, i.e. parallel to the water-sediment interface, see figure 6.1. The passage of the critically refracted wave along the top of the sediment layer causes a perturbation in the water column. The wave resulting from this perturbation propagates into the water column under the critical angle and can be received by a towed array of hydrophones if the distance from the source is sufficiently large. Kearey et al. [9] compare the existence of this so-called head wave to an aerodynamic shock wave, which is created by an aircraft that travels at a velocity greater than the speed of sound in air.

Since the head wave consists purely of (non-converted) acoustic waves, it is also modelled by the normal-mode model. Figure 6.3 shows a few stills from a time-lapse animation.

![Figure 6.1: Sketch of the path that is travelled by the critically refracted wave.](image-url)
Figure 6.2: Synthetically generated streamer record showing the head wave.

created with the normal-mode model for a water sound speed of 1500 m/s and a sediment sound speed of 3000 m/s. Although the head wave is also present in the model output for smaller sediment sound speeds, the value of 3000 m/s was selected to make the wave better visible in the plots.

Figure 6.2 shows a hypothetical recording of an array of 24 hydrophones, created with the same synthetic dataset. As is visible in the figure, the head wave arrives at the hydrophones first and has a propagation velocity that is close to 3000 m/s, indicated by the dashed line. The direct wave arrives next, with a propagation velocity of 1500 m/s.

This property of the head wave, i.e. that it can arrive faster than the direct wave because it travels partly through the sediment, can also be exploited in the case of the Kulcs dataset. In fact, the arrival time of the head wave at any particular hydrophone uniquely defines the sediment sound speed, assuming the measurement geometry and sound speed in the water are known. This can be shown as follows.

With a horizontal distance $\Delta L$ between each of the hydrophone channels (see also figure 6.1), the arrival time $t_n$ of the head wave in the $n$-th channel is given by:

$$ t_n = \frac{d_1 + d_2}{c_w} + \frac{d_4 + (n - 1)\Delta L}{c_s} \tag{6.1} $$

in which $c_w$ and $c_s$ are the constant sound speed in the water column and at the top of the sediment layer, respectively.
The distance $d_1$ that is travelled by the critically refracted wave from the source to the water bottom can be calculated as:

$$d_1 = (H_w - z_s) \sin \theta_c + d_3 \cos \theta_c$$  \hspace{1cm} (6.2)$$

Similarly, the travel distance $d_2$ from the sediment to any of the hydrophones is equal to:

$$d_2 = (H_w - z_r) \sin \theta_c + d_5 \cos \theta_c$$  \hspace{1cm} (6.3)$$

Substituting expressions 6.2 and 6.3 into equation 6.1, it follows that:

$$t_n = \frac{(2H_w - z_s - z_r) \sin \theta_c}{c_w} + \frac{d_3 \cos \theta_c}{c_w} + \frac{d_5 \cos \theta_c}{c_w} + \frac{d_4 + (n - 1) \Delta L}{c_s}$$  \hspace{1cm} (6.4)$$

In addition, it is known from Snell’s law that the following relation holds for the critical angle $\theta_c$ at the water-sediment interface:

$$\cos \theta_c = \frac{c_w}{c_s}$$  \hspace{1cm} (6.5)$$

Knowing that $D = d_3 + d_4 + d_5$, substituting equation 6.5 into equation 6.4 and rearranging terms eventually gives:
\[
    t_n = \left( \frac{2H_w - z_s - z_r}{c_w} \right) \sin \theta_c + \frac{D + (n - 1)\Delta L}{c_s}
\] (6.6)

Now consider the following example. If the water sound speed is 1448 m/s and the sediment sound speed is 1745 m/s, it follows from equation 6.5 that \( \theta_c = 33.9^\circ \). With a source depth of 1.3 m, a receiver depth of 0 m, a water depth of 3.5 m and a source-receiver distance of 30 m, it follows from equation 6.6 that the arrival time of the head wave in the first hydrophone channel is equal to:

\[
    t_1 = \left( \frac{2 \cdot 3.5 - 1.3}{1448} \right) \sin 33.9^\circ + \frac{30}{1745} = 19.4 \cdot 10^{-3} \text{ s}
\]

This is more than one millisecond earlier than the arrival time of the direct wave, which is:

\[
    \frac{\sqrt{30^2 + 1.3^2}}{1448} = 20.7 \cdot 10^{-3} \text{ s}
\]

Note that this difference in arrival time between the head wave and the direct wave only increases for the subsequent hydrophones, as a larger distance is travelled through the sediment layer.

The above can be compared with the North Sea dataset. Because for the North Sea there is much less contrast between the sound speed in the water column and the sediment, the critical grazing angle is also much smaller. Therefore, the distance between the source and receiver needs to be much larger – at least about 100 m – to record the head wave.

This is indicated in figure 6.3 as well, where lines A and B show the relative position of the streamer for the North Sea and Kulcs datasets, respectively. As is visible in the figure, the head wave is not present at any position along line A. Even if it were, the head wave would still arrive later than the direct wave in the case of the North Sea dataset, because of the larger distance that has to be travelled through the water column. The different geo-acoustic environment of the North Sea and the different measurement geometry thus make it impossible to do a head wave inversion for that dataset.

However, the mere fact that the head wave arrives first at the hydrophones for the Kulcs dataset does not mean that it can be identified in the recordings, as the strength of the signal must be above the noise level as well.

It has been verified both theoretically and experimentally that the amplitude decay of the head wave is proportional to [12]:

\[
    \frac{1}{\sqrt{D d_4^{3/2}}}
\] (6.7)

Furthermore, it is known that the amplitude of the direct wave decreases proportionally to \( \frac{1}{R} \), where \( R \) is the total distance between the source and receiver.
This means that for the above example (i.e. with \(d_4 = 21.5\) m), it can be estimated that the head wave is 18 times weaker than the direct wave in the first hydrophone channel, and 89 times weaker than the direct wave in the last channel.

With the assumption that the strongest recorded signal is equal to the direct wave, these figures correspond to a signal-to-noise ratio of 37 and 12 dB, respectively. Hence it is concluded that it must be possible to identify the head wave in the recordings and to use its arrival times to invert for the sediment sound speed.

As a matter of fact, this method is not new, as for example Godin et al. [5] used the arrival times of a head wave at a vertical hydrophone array to invert for bottom properties near Vancouver Island. In addition, head wave inversion is common in (marine) exploration seismics – where it is known as travel-time tomography – for estimating the sound speed in relatively shallow layers [10].

For inverting the data of line 800, it has been assumed that the first time at which a signal rises above the noise level is equal to the arrival time of the head wave. These times have been extracted from the measurements for each of the hydrophone channels and for all shots, after which they are compared to the times modelled with equation 6.6.

The mismatch between the modelled and measured arrival times of the head wave is quantified by:

\[
E = \frac{1}{N} \sum_{i=0}^{N} (t_{i,\text{modelled}} - t_{i,\text{measured}})^2
\]

in which \(N\) is the number of hydrophone channels.

This energy function is minimized using the differential evolution algorithm discussed in section 2.5, applying the same settings as for the North Sea dataset – except for the number of generations, which is set at 250.

After running the optimization for the first time, it was found that expression 6.8 is rather insensitive to changes in the source depth, water depth and water sound speed, because the critically refracted rays travel mostly through the sediment. Therefore, it was decided to keep these parameters fixed at 1.3 m, 3.5 m and 1448 m/s, respectively.

Because of this, only the source-receiver distance and the sediment sound speed are left to be optimized. Since there is some coupling between these two parameters, it was decided to constrain the change in source-receiver distance to the change in geometry that was measured during the survey. As a result, the estimates of the sediment sound speed were found to become more consistent from shot to shot.

The final outcome of the inversion process is shown in figure 6.4. In this figure, it can be seen that the estimated values lie between 1600 and 1700 m/s for the first 430 m of the line, after which the sediment sound speed increases to values above 1800 m/s. Beyond 600 m, the estimated values decrease again.
Although the pattern visible in figure 6.4 can be caused by a change in sediment properties over the line, this is considered unlikely based on previous experience in the survey area.

Another possible explanation for the increase in estimated values between a distance of 430 m and about 700 m is a change in measurement geometry. However, for this part of the line the GPS data does not show any clear change in the distance between the source and receiver. Moreover, an inversion of 1 out of 5 shots of a line acquired one hour earlier shows exactly the same pattern for the sediment sound speed. Hence this hypothesis is effectively ruled out.

A third hypothesis is offered by the fact that for the derivation of equation 6.6, it has been assumed that the water bottom is flat. From an earlier survey in the summer of 2008, it is known however that there are some topographical changes in the area. Since a slope of the water bottom either increases or decreases the travel time of the critically refracted wave between subsequent hydrophones, not correcting for this slope causes the estimated sediment sound speeds to be either too low or too high.

Therefore, it is suggested for further research that the topography is extracted from the data of the earlier survey and that it is included in the inversion process. Although not exactly the same track was sailed, doing so is expected to give a more accurate estimate of the sediment sound speed.

For now, it can be concluded that because of a possible bottom slope, the more consistent estimates for the sediment sound speed found at the start and the end of the line are likely to give the best representation of the environment. These values also agree reasonably with the 1745 m/s found after inverting the VSP data, which was gathered approximately 100 m beyond the end of line 800.

Another remark that can be made about the inversion of the head wave, is that it follows from equation 6.6 that the difference in arrival time of the head wave between the hydrophone channels is only a function of the known distance $\Delta L$ and the sediment sound speed – that is, again with the assumption that the water bottom is flat. Theoretically, it is therefore possible to alternatively use this difference as a basis to invert for $c_s$, which alleviates the need to know the measurement geometry and the water sound speed.

In practice however, the finite sampling rate of 8000 Hz means that the sediment sound speed can only be solved for in discrete steps of about 10 m/s. In addition, some outliers
will be present, because the first signal arrivals are not always identified correctly. Even after removing these outliers, averaging the estimated sound speeds over the hydrophone channels is not expected to give very precise results. Indeed, this method was tried and it was found that the results show the same trend as is visible in figure 6.4, but with a much larger variation in the estimated values. Hence it is a better idea to minimize the cost function for all hydrophones at the same time using the differential evolution algorithm, rather than inverting for each of the hydrophone channels separately and averaging the results.

To summarize, it has been found that the arrival times of the head wave can be used to estimate the sediment sound speed for the Kulcs dataset. Although the results have not been corrected yet for topography, they agree quite well with the value of 1745 m/s found by inverting the VSP.

For future research, it is recommended that after correcting for the topography, the head wave inversion is extended to the other lines sailed near Kulcs. When comparing the results of these lines, it might be possible to assess the influence of noted disturbances, such as passing ships, on the inversion estimates. However, the main challenge remains to invert the Kulcs dataset using an improved normal-mode model that incorporates elastic sediment layers.
7 Conclusions & recommendations

In this final chapter, an overview is given of the conclusions that are made while analyzing the inversion results in chapters 4 and 6. The recommendations that can be given for further processing and future surveys are included as well, in a separate section.

7.1 Conclusions

Inverting the North Sea dataset and interpreting the results has led to the following conclusions:

- The inversion results of the North Sea dataset are consistent for a majority of the shots. However, several irregularities are noted as well, which means that the inversion results are not to be trusted blindly.

- It is difficult to estimate the quality of the inversion, because of a lack of suitable validation data. A map of the mean grain size and three borehole logs that are available, were found not to be directly comparable to the inversion results. This is because they both give the average sediment properties over a certain layer thickness, for which the inversion results might not be valid. A sediment classification map of the area agrees quite well with the results of the inversion though.

- The precision of the estimates for the geo-acoustic parameters is degraded by estimating the measurement geometry in the inversion process. The measurement geometry cannot be fixed however, because it is not known well enough.

- The shape of the pulse transmitted by the acoustic source needs to be known very well if interference effects need to be modelled correctly.

- The water sound speed that results from the inversion is on average 17 m/s higher than the sound speed that was measured during the survey. This might indicate that the hydrophone array was not fully straight behind the ship.

- The horizontal offset between the single beam echo sounder measurements and the (virtual) point for which the inversion results are valid, was found to be 178 m for track 7.

- There is an offset of 1 m between the estimated water depths and the depths measured by the echo sounder. 30% of this difference is accounted for by the higher value for
the estimated water sound speed; the other 70% is believed to result from the echo sounder not being calibrated correctly and/or frequency-dependent effects that occur.

- The estimated values of the sediment density were found to be much lower than what would be expected based on empirical relations with the sediment sound speed. The most likely explanation for this is that an effective density is inverted for, because of neglecting shear waves in the propagation model.

- Because the end of the hydrophone array is relatively close to the source, sound is only recorded from directions above the critical grazing angle. Therefore, the North Sea dataset contains hardly any information, if at all, about the sediment attenuation, causing the estimates of this parameter to be random.

- At the end of tracks 7 and 45, the recorded signals have an amplitude that is too strong to be modelled. This could be a result from the presence of surface waves that are converted back to compressional waves.

- The estimated values for the sediment sound speed tend to split for the second half of track 70. An inversion with a dual-layer model indicates that a second sediment layer might be present here.

- To find the global minimum of the energy function when using the dual-layer model, it was found necessary to double the population size of the differential evolution algorithm and increase the number of generations from 70 to 100. Even then, the optimization algorithm only roughly locates the global minimum. Therefore, the downhill simplex algorithm was applied to do a local optimization starting from the differential evolution results.

Additional conclusions are drawn from the inversion of the Kulcs dataset, i.e.:

- The Kulcs dataset potentially provides more information than the North Sea dataset, because acoustic waves are recorded under much more angles.

- An inversion with the normal-mode model was not found to be feasible however, because the source pulse is only approximately known and very many reflections interfere with each other in the water column. In addition, quite some shear waves and surface waves are generated in the sediment, which are not included in the normal-mode model.

- The recordings of the near-field hydrophone are not useful to extract the shape of the pulse transmitted by the source, because of interference caused by reflections off the water surface and water bottom.

- It has been found possible to invert the arrival times of the head wave to get an estimate of the sound speed at the top of the sediment. The best results are obtained when optimizing the arrival times in all hydrophone channels at the same time, rather than looking at each hydrophone separately and averaging the results. The results that have been obtained still have to be corrected for changes in topography though.
7.2 Recommendations

Next to the conclusions, recommendations are given for continuation of the research work and the planning of future surveys.

The recommendations that are given for further research are as follows:

- For this study, the normal-mode model has been applied ‘as-is’, except for including a second sediment layer. The signals at the end of tracks 7 and 45 of the North Sea dataset and the signals recorded in Hungary are likely modelled better when making the sediment layer(s) in the model elastic instead of fluid. In addition, elastic modelling should yield correct values for the sediment density, rather than the effective density that is found with the current model. It is therefore recommended, that the inversion is performed with an elastic normal-mode model as well.

- A second recommendation is to improve the results of North Sea series B, by fixing the geometry at the realistic values that were found when applying a time window for calculating the energy values.

- Similarly, the results for the second half of track 70 can be improved by looking at more shots and different thicknesses of the top-most sediment layer.

- The fourth recommendation is to get a more reliable estimate of the sediment sound speed for the Kulcs dataset, by correcting the results of the head wave inversion for changes in topography. The necessary information about the water depth can be extracted from data of another survey, carried out in the summer of 2008. If this recommendation is followed, the head wave inversion might very well be extended to other lines of the dataset.

- It is also an option to change the normal-mode model such that a range-dependent environment can be modelled. This makes it possible to invert the VSP data in the Kulcs dataset with this model, and to invert other recordings that were made in the borehole.

- Last but not least, it is recommended that the accuracy of the inversion method is looked at. This is only possible with a new dataset though, for which sufficient validation data is available.

For planning future surveys, the following recommendations are made:

- The most important recommendation for future surveys is to collect as much independent data about the environment as possible, to allow for a better validation of the inversion results than is possible for the two datasets discussed in this report. The independent data can for example be collected using a multi-beam echo sounder, taking bottom samples and measuring the water sound speed with a sound velocity profiler. In any case, it has to be kept in mind that all instruments must be calibrated correctly and that the data that is gathered must be comparable to the inversion results in order to be useful for validation.
• Apart from collecting validation data, it is recommended that the measurement geometry is measured very well. The better the measurement geometry is known, the more precise the inversion results will be.

• During a future survey, care should be taken that the assumptions made for modelling are satisfied, i.e. the hydrophone array must be horizontal and fully stretched behind the ship.

• Furthermore, it can be recommended to use a near-field hydrophone to measure the pulse transmitted by the source whenever possible – that is, in all environments where the direct wave will not be interfered by reflections.

• Finally, it is strongly recommended that signals are measured coming from directions well above and well below the critical angle, to get reliable estimates of both the sediment density and attenuation coefficient. In general, longer arrays are better for this purpose.
References


A Borehole logs

<table>
<thead>
<tr>
<th>LAAG</th>
<th>DIEPTE IN METERS</th>
<th>ONSCHRIJVING DER AANDACHTEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,53</td>
<td>23,65 m -10°</td>
<td>Zand: geel grijs, middelfijn tot matig fijn, met veel matig grof zand, iets uitbundend, vaak schelpresten aan de basis van laagje schelpen, o.m. Sphincter ciliatus, C4</td>
</tr>
<tr>
<td>1,85</td>
<td>29,10 m</td>
<td>Zand: licht grijs, middelfijn en matig grof, iets uitbundend, schelpresten, o.m. Cardium edule, gelaagd met enkele grijze matzwangers, naar de wand toe neemt van stukken middelfijn zand, C4</td>
</tr>
<tr>
<td>2,30</td>
<td></td>
<td>Zand: grijs, middelfijn, uitbundend, een enkel uiterst fijne schelpresten, C4</td>
</tr>
</tbody>
</table>

Figure A.1: North Sea borehole log Q16-13.
**Figure A.2: North Sea borehole log Q16-155.**

<table>
<thead>
<tr>
<th>LAAG Nr.</th>
<th>DIEPE ONDERVLAK</th>
<th>OMSCHRIJVING DER AARDLAGEN</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td><em>Zand: grijs, matig grof, hoekig afgerond, met enkele grovere korrels, vrij veel schelpgruis en schelpen, angulus fabulus, spina subtruncata, litorina littorea.</em></td>
<td>255</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td><em>Zand: lichtgrijs, zeer grof, hoekig afgerond en afgerond, vrij veel schelpen en schelpgruis, en enkel grindje, ostrea, macoma baltica, cardium edule.</em></td>
<td>350</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
<td><em>Zand: grijs, zeer grof, hoekig afgerond en afgerond, en enkel cardium edule.</em></td>
<td>350</td>
</tr>
<tr>
<td>4</td>
<td>4.00</td>
<td><em>Zand: grijs, zeer grof, hoekig afgerond, zeer weinig schelpgruis, donax vittatus, cardium edule, macoma baltica.</em></td>
<td>350</td>
</tr>
<tr>
<td>5</td>
<td>5.00</td>
<td><em>Zand: grijs, matig grof, hoekig afgerond, met enkele grovere korrels, enkelige kwartsietjes, enkele fragmenten van nya arenaria, ostrea.</em></td>
<td>350</td>
</tr>
<tr>
<td>6</td>
<td>6.00</td>
<td><em>Zand: grijs, matig grof, hoekig afgerond, met enkele grovere korrels, enkelige kwartsietjes, enkele fragmenten van nya arenaria, ostrea.</em></td>
<td>475</td>
</tr>
<tr>
<td>7</td>
<td>7.00</td>
<td><em>Zand: grijs, matig grof, hoekig afgerond, met enkele grovere korrels, vrij veel schelpen, witte kwarts en kwartsietjes, leisteen cardium edule, macoma balthica.</em></td>
<td>300</td>
</tr>
<tr>
<td>8</td>
<td>8.00</td>
<td><em>Zand: grijs, matig grof, hoekig afgerond, met vrij veel schelpen en schelpgruis, cardium edule.</em></td>
<td>300</td>
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<td>9</td>
<td>9.00</td>
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<tr>
<td>10</td>
<td>10.00</td>
<td><em>Zand: grijs, matig tot zeer grof, hoekig afgerond, met vrij veel schelpen en schelpgruis, cardium edule.</em></td>
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</table>
Figure A.3: North Sea borehole log Q16-187.
Figure A.4: Kulec borehole log (part 1/3).
Figure A.5: Kulcs borehole log (part 2/3).
Figure A.6: Kules borehole log (part 3/3).
B Inversion of VSP data

In this appendix, it is explained how the data of the vertical seismic profile (shown in figure 5.4) has been inverted.

Figure B.1 shows a schematic representation of the measurement geometry, with a sound ray going from the source towards a hydrophone in the borehole. It is assumed that the water column and the sediment have a constant sound speed $c_w$ and $c_s$, respectively, causing the ray to be refracted at the water-sediment interface according to Snell’s law, i.e.:

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{c_w}{c_s} \quad \text{(B.1)}$$

From the figure, it follows that the length of the travel path in the water column $d_1$ is equal to:

$$d_1 = \frac{H_w - z_s}{\cos \theta_1} \quad \text{(B.2)}$$

Similarly, the distance $d_2$ that the ray travels through the sediment before arriving at the hydrophone is given by:

$$d_2 = \frac{z_r - H_w}{\cos \theta_2} \quad \text{(B.3)}$$

Knowing distances $d_1$ and $d_2$, the time at which the signal arrives at the hydrophone can be computed by:

$$t = \frac{d_1}{c_w} + \frac{d_2}{c_s} \quad \text{(B.4)}$$

In addition, simple trigonometry leads to the fact that the horizontal distance $D$ between the source and hydrophone can be calculated as:

$$D = \frac{H_w - z_s}{\sin \theta_1} + \frac{z_r - H_w}{\sin \theta_2} = \sqrt{d_1^2 - (H_w - z_s)^2} + \sqrt{d_2^2 - (z_r - H_w)^2} \quad \text{(B.5)}$$
Figure B.1: Schematic drawing of the measurement geometry of the VSP.
The VSP data of the Kulcs dataset is inverted by numerically solving for equations B.1 through B.5 and minimizing the squared difference between the modelled arrival times and those that were measured. The parameters that are allowed to vary in the optimization process to find the best match are the sediment sound speed, horizontal offset and the water depth.

The other geometrical parameters, i.e. the depth of the source and the depth of all hydrophones are known from the survey. Moreover, the arrival time \( t \) does not change much when the water sound speed is varied, because of the small distance that the sound travels through the water column. Therefore, this parameter is kept fixed at a value of 1448 m/s, i.e. as was extracted from the streamer data (see section 5.4).

After performing the inversion, it was found that there is a best match between the modelled arrival times and the measurements for a sediment sound speed of 1745 m/s, a horizontal offset of 17.0 m and a water depth of 3.8 m.

When the water depth is kept fixed at 3.5 m, such as is indicated in the borehole log (see appendix A), the ‘best’ sediment sound speed becomes 1744 m/s for a horizontal offset of 17.1 m.

In both cases, the output of the model agrees almost perfectly with the measurements, from which it is concluded that the assumption of the sediment sound speed being constant is at least valid for the depth range that is looked at.