delft hydraulics laboratory

limitations of dispersion concept

R 895 part I

October 1974
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Limitations of dispersion concept

Abstract

Considering a tidal estuary with inflow of a sufficiently large river the dispersion coefficient for an instantaneous release of a constituent is not equal to the one for a continuous release into the same estuary. Further, the river flow being sufficiently large the dispersion coefficients for continuous releases from different locations into the same tidal estuary may be different. These restrictions are limitations of the dispersion concept of the first type described in this study.

The dispersion coefficient describing the dispersive transport of a constituent which is lost through the water surface (such as heat) or gained through the water surface (such as dissolved oxygen) is different from the one describing the dispersive transport of a constituent without transport through the water surface (such as salinity). Further the magnitude of the dispersion coefficient depends upon the decay characteristics of the constituent. These restrictions are limitations of the dispersion concept of the second type described in this study.

Introduction

The longitudinal one-dimensional distribution of the concentration of a given constituent varies with the type of release and with the type of receiving water. The longitudinal distributions may be characterized as indicated in Table 1, where $\bar{c}$ represents the concentration averaged over the cross-section of the water course and where $\frac{\partial \bar{c}}{\partial t}$ represents the variation of this concentration with time as observed when moving at a velocity equal to the velocity of flow, averaged over the cross-section. Only for a continuous release into a tidal estuary with inflow of a river $\frac{\partial \bar{c}}{\partial t}$ is not equal to zero. For the other cases listed in Table 1 the magnitude of $\frac{\partial \bar{c}}{\partial t}$ vanishes either sufficiently long after the moment of injection as for instantaneous releases or sufficiently far from the point of discharge as for continuous releases.

*) see page 17.
The magnitude of the concentration, averaged over the cross-section can be
determined on the basis of the dispersion concept, stating that the magnitude
of the dispersive transport can be expressed in local one-dimensional para-
meters (waterdepth, values of velocity and concentration averaged over the
cross-section of the considered watercourse, and derivatives of these quanti-
ties with respect to the longitudinal coordinate). However, when introducing
the dispersion concept Taylor (1954) showed it to be valid only if \( \partial \bar{c} / \partial t \rightarrow 0 \). Otherwise the magnitude of the dispersive transport depends upon longitudi-
nally integrated effects.

Accordingly the condition required for the dispersive transport to depend
upon local one-dimensional parameters is not satisfied for continuous
releases of constituents into a tidal estuary with inflow of a river.

From the above considerations the following conclusions can be derived:
- information about the dispersive transport of a constituent which is
  continuously released into a tidal estuary with inflow of a river may not
  be derived from experiments on the dispersive transport of a constituent
  which is instantaneously released into the same estuary
- computations about the magnitude of the dispersive transport in tidal
  estuaries as presented by Holley et. al (1970) assuming \( \partial \bar{c} / \partial t = 0 \) may
  not be applied for continuous release of the constituents.

These conclusions are substantiated in this study. As a by-product it is
shown that:
- the magnitude of the dispersion coefficient, defined by Eq 6, depends upon
  the magnitude of the transport of the constituent through the water sur-
  face and upon the decay characteristics of the constituent.

The study is structured as follows. The definition of the dispersive trans-
port is summarized in section 2. A summary of Taylor's (1954) derivation of
the dispersion concept is given in section 3. Taylor's derivation is presen-
ted such as suitable for explaining the consequences of the condition
\( \partial \bar{c} / \partial t \rightarrow 0 \) not being satisfied. These consequences are presented in sections
4, 5 and 6. The conclusions are summarized in section 7.

The study described in this report is performed by dr.ir. G. Abraham as part
of the basic research program F.O.W.
2 Effect of spatial averaging, definition of dispersion

The total transport of a constituent through a given plane is equal to the sum of the convective transport (associated with the time mean values of velocity and concentration) and the turbulent transport (associated with the turbulent eddies). In formula

\[ F_{total} = \int_A (u_n c + T_n) \, dA \]  \hspace{1cm} (1)

where \( F_{total} \): total transport through plane A,
\( dA \): element of plane having area A,
\( u_n \): time mean value of velocity in direction n
\( c \): concentration of constituent
\( T_n \): turbulent transport in direction n
\( n \): coordinate perpendicular to considered plane

From Eq (1) it can be seen that

\[ F_{total} = A \overline{u c} + \int_A T_n \, dA \]  \hspace{1cm} (2)

where a double overbar represents spatial averaging over plane A. Since

\[ \overline{u c} \neq \overline{u} \overline{c} \]  \hspace{1cm} (3)

the total transport may be divided into the following parts

\[ F_{total} = A \overline{u c} + A (\overline{u c} - \overline{u} \overline{c}) + \int_A T_n \, dA \]  \hspace{1cm} (4)

In literature the total transport is often divided as indicated by Eq (4). The second part of the total transport is referred to as dispersive transport. It represents the total transport through the considered plane as seen by an observer moving with the average velocity \( \overline{u} \). It may be written as

\[ F_{disp} = \int_A (u_n - \overline{u_n}) (c - \overline{c}) \, dA + \int_A T_n \, dA \]  \hspace{1cm} (5)
It is customary to express the magnitude of the dispersive transport in terms of a dispersion coefficient $D$, defined as

$$\frac{\nabla}{\nabla_t} = F_{\text{disp}}$$  \hspace{1cm} (6)

So far $A$ is an arbitrary area. In actual applications of the dispersion concept the area $A$ may be taken as the depth times a unit width (dispersion as a consequence of depth averaging; $E$ referred to as $E_v$, the index v (vertical) referring to the direction of averaging), as a width times a unit height (dispersion as a consequence of width averaging; $E$ referred to as $E_h$, the index h (horizontal) again referring to the direction of averaging) or as the total profile (dispersion as a consequence of averaging over the total profile; $E$ referred to as $E_p$, the index p (profile) referring to the plane of averaging).

3 Conditions for dispersion coefficient depending upon local one-dimensional parameters, outline of Taylor's (1954) derivation

The derivation of the conditions for the dispersion coefficient depending upon local one-dimensional parameters will be given for the special case of two-dimensional flows (i.e. flows without variations of velocity, concentration and depth in the lateral direction).

Nevertheless the conclusion given in section 3.3 can be applied generally.

3.1 Basic equations

For two-dimensional flows the continuity equation for a constituent carried by the flow reads

$$\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} + \frac{\partial vc}{\partial y} + \frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} = 0$$  \hspace{1cm} (7)

where $x$: horizontal coordinate in the longitudinal (main flow) direction
$y$: vertical coordinate
$u$: horizontal velocity component in the longitudinal direction
$v$: vertical velocity component
$T_x$: turbulent transport in direction of index
$t$: time
Indicating depth mean values by a single overbar the concentration can be divided into the following part

\[ c = \bar{c} + c' \quad \text{with} \quad \bar{c}' = 0 \quad (8) \]

where \( \bar{c} \): depth mean value of concentration
\( c' \): local difference between concentration and depth mean value of concentration.

Eq 8 into Eq 7 gives

\[ \frac{\partial \bar{c}}{\partial t} + \frac{\partial \bar{c}'}{\partial t} + \frac{\partial \bar{u} \bar{c}}{\partial x} + \frac{\partial \bar{u} c'}{\partial x} + \frac{\partial \bar{v} \bar{c}}{\partial y} + \frac{\partial \bar{v} c'}{\partial y} + \frac{\partial \bar{T}}{\partial x} + \frac{\partial \bar{T}'}{\partial y} = 0 \quad (9) \]

The waterdepth, \( h \), is given by

\[ h = b - a \quad (10) \]

if a: coordinate of bottom
\( b \): coordinate of water surface

Integrating Eq 9 with respect to \( y \) between limits \( y = b \) and \( y = a \) (i.e. integrating Eq 9 over the waterdepth) gives

\[ \bar{h} \frac{\partial \bar{c}}{\partial t} + \bar{h} \frac{\partial \bar{c}'}{\partial t} + \bar{h} \frac{\partial \bar{u} \bar{c}}{\partial x} + \bar{h} \frac{\partial \bar{u} c'}{\partial x} + \bar{h} \frac{\partial \bar{v} \bar{c}}{\partial y} + \bar{h} \frac{\partial \bar{v} c'}{\partial y} + \bar{h} \frac{\partial \bar{T}}{\partial x} + \bar{h} \frac{\partial \bar{T}'}{\partial y} - \left( \frac{\partial \bar{c}'}{\partial t} + \frac{\partial \bar{c}'}{\partial t} \right) \left( \frac{\partial \bar{c}'}{\partial t} - \frac{\partial \bar{c}'}{\partial t} \right) \left( \frac{\partial \bar{c}'}{\partial t} - \frac{\partial \bar{c}'}{\partial t} \right) \left( \frac{\partial \bar{c}'}{\partial t} - \frac{\partial \bar{c}'}{\partial t} \right) = 0 \quad (11) \]

where \( a \) and \( b \) as single or second index refer to \( y = a \) or \( y = b \).

The transport of the constituent through the water surface and bottom being zero the terms between brackets, containing gradients of \( a \) or \( b \) are equal to zero. Further, the continuity equation for water reads

\[ \frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} = 0 \quad (12) \]
Due to the above considerations and observing that $\overline{cT} = 0$ Eq 11 may be reduced to

$$\frac{\partial \overline{c}}{\partial t} + \overline{u} \frac{\partial \overline{c}}{\partial x} + \frac{1}{h} \left( \frac{\partial \overline{hcT}}{\partial x} + \frac{\partial \overline{hT}}{\partial x} \right) = 0$$

(13)

The velocity may be written as

$$\overline{u} = \overline{\tilde{u}} + \overline{u}' \quad \text{with} \quad \overline{uT} = 0$$

(14)

where $\overline{u}'$ : local difference between velocity and depth mean value of velocity. Since $\overline{cT} = 0$ substituting Eq 14 into Eq 13 gives

$$\frac{\partial \overline{c}}{\partial t} + \overline{u} \frac{\partial \overline{c}}{\partial x} + \frac{1}{h} \left( \frac{\partial \overline{hu'cT}}{\partial x} + \frac{\partial \overline{hT}}{\partial x} \right) = 0$$

(15)

The term between brackets of Eq 15 represents the longitudinal gradient of the dispersive transport as can be seen from Eq 5.

3.2 Derivation of restrictive conditions

The following assumptions are made in connection with Eq 9

$$\frac{\partial T_x}{\partial x} \ll \frac{\partial T_y}{\partial y}$$

(16)

$$\overline{c}' \ll \overline{c} \quad \text{and} \quad \frac{\partial \overline{c}'}{\partial x} \ll \frac{\partial \overline{c}}{\partial x}$$

(17)

Because of Eqs 16 and 17 the terms $\partial u'c'/\partial x$ and $\partial T_x/\partial x$ may be neglected in Eq 9. This implies that the longitudinal gradient of the dispersive transport may be neglected in Eq 15. This can be seen by integrating Eq 9 over the water depth neglecting the above-mentioned terms.

This integration yields

$$\frac{\partial \overline{c}}{\partial t} + \overline{u} \frac{\partial \overline{c}}{\partial x} = 0$$

(18)

or

$$\frac{\partial \overline{c}}{\partial t} = 0$$

(19)

where $\tau$: time for observer moving at velocity $\overline{u}$. 
The longitudinal gradient of the dispersive transport being a quantity which may be neglected in Eq 15 can be seen by comparing Eqs 15 and 18.

The assumptions represented by Eqs 16 and 17 imply Eqs 18 and 19 to be satisfied. The assumption represented by Eq 17 is compatible with the assumption

\[ \frac{\partial c'}{\partial t} \ll \frac{\partial c}{\partial t} \]  

Because of the continuity equation for fluid Eq 9 may be written as

\[ \frac{\partial c}{\partial t} + \frac{\partial c'}{\partial t} + u \frac{\partial c}{\partial x} + u \frac{\partial c'}{\partial x} + v \frac{\partial c}{\partial y} + \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = 0 \]  

Eqs 16, 17 and 20 imply that the terms \( \frac{\partial c'}{\partial t}, u \frac{\partial c'}{\partial x} \) and \( \frac{\partial T}{\partial x} \) may be neglected in Eqs 21. The error which is made by substituting Eq 18 into Eq 21 may be neglected, the difference between the terms \( \frac{\partial c}{\partial t} \) and \( - \bar{u} \frac{\partial c}{\partial x} \) being small in comparison with the absolute magnitude of each of these terms. Hence because of Eqs 16, 17, 18 and 20 Eq 21 may be written as

\[ (u - \bar{u}) \frac{\partial c}{\partial x} + v \frac{\partial c'}{\partial y} + \frac{\partial T}{\partial y} = 0 \]  

For uniform flow

\[ \frac{\partial u}{\partial x} = 0 \]  

\[ v = 0 \]  

\[ (u - \bar{u}) = f_1 (\bar{u}, h, y) \]  

\[ \varepsilon = f_2 (\bar{u}, h, y) \]  

where by definition

\[ T_y = \varepsilon \frac{\partial c}{\partial y} = \varepsilon \frac{\partial c'}{\partial y} \]  

where \( \varepsilon \) : turbulent diffusion coefficient

\( f_1 \) : functional relationship

Eqs 23, 24, 25 and 26 into Eq 22 gives

\[ c' = (c - \bar{c}) = \frac{\partial c}{\partial x} f_3 (\bar{u}, h, y) \]  

(28)
Eqs 16, 25 and 28 into Eqs 5 and 6 gives

\[ F_{\text{disp}} = \frac{\partial c}{\partial x} \int f_1 (\bar{u}, h, y) f_3 (\bar{u}, h, y) \ dy = \frac{\partial c}{\partial x} f_4 (\bar{u}, h) \]  

(29)

and

\[ F_v = f_4 (\bar{u}, h) \]  

(30)

Accordingly the dispersive transport and the dispersion coefficient can be expresses in local one-dimensional parameters if

- the flow is uniform (which implies Eqs 23, 24, 25 and 26 to be satisfied),
- the assumptions represented by Eqs 16, 17 and 20 are satisfied.

The latter conditions are only satisfied if after a sufficient long initial period the considered concentration fields satisfy Eqs 19. Then the longitudinal gradient of the dispersive transport may be neglected in Eq 15, which implies \( \frac{\partial^2 c}{\partial x^2} = 0 \) as can be seen from Eq 29.

3.3 Examples of dispersion coefficients expressed in local one-dimensional parameters

In the preceding section the assumptions represented by Eqs 16 and 17 were found to imply Eqs 18 and 19 to be valid. Accordingly Eqs 16 and 17 may be applied only provided that the condition represented by Eq 18 (or Eq 19) is satisfied. Hence Eqs 18 and 19 give a condition which must be satisfied in order to express the dispersive transport in local one-dimensional parameters.

Along the procedure outlined in the preceding sections for pipe flow Taylor (1954) derived

\[ E_p = 10.1 \ R_o \ u^* \]  

(31)

where \( R_o \) : hydraulic radius,

\( u^* \) : shear velocity

For flows in the central part of wide channels having rectangular cross-section by depth averaging Elder (1959) showed

\[ E_v = 5.9 \ h \ u^* \]  

(32)
For steady river flows in irregular channels measurements about the magnitude of $E_v$ for dispersion in the lateral direction are given by Holley et al. (1973). Approximate expressions for the magnitude of $E_p$ in rivers are given by Fischer (1973).

For the flows represented by Eqs 31 and 32 $v = 0$ and both $(u - \bar{u})$ and $\epsilon$ are approximately proportional to $\bar{u}$. Hence, as can be seen from Eqs 22 and 27 $c'$ does not depend upon $\bar{u}$. Accordingly $E$ is proportional to $\bar{u}$, as can be seen from Eqs 5 and 6, observing that for the considered flows the second term of Eq 5 may be neglected.

4 Dispersion coefficient depending upon longitudinally integrated effects

4.1 Implications of Eq 19 not being satisfied

Eqs 16 and 17 do not hold if Eq 19 is not satisfied (see section 3.3). Accordingly Eq 19 not being satisfied $c'$ must be solved from the equation

$$\frac{\partial c'}{\partial t} + \frac{\partial c}{\partial t} + (u - \bar{u}) \frac{\partial c}{\partial x} + (u - \bar{u}) \frac{\partial c'}{\partial x} + \frac{\partial c'}{\partial y} - \frac{\partial E}{\partial y} \frac{\partial c'}{\partial x} + \frac{\partial T}{\partial x} = 0 \quad (33)$$

The terms containing derivatives $\frac{\partial c'}{\partial x}$ and $\frac{\partial T}{\partial x}$ may not be neglected in Eq 33. Neglecting these terms would make the last term of Eq 15 equal to zero, which in itself implies Eq 19 to hold as can be seen from the derivation of this equation. Accordingly solving $c'$ from Eq 33 requires an integration in the longitudinal x-direction as a consequence of which Eq 19 not being satisfied the dispersion coefficient expresses longitudinally integrated effects. Further $\frac{\partial c}{\partial x}$ and $\nu$ may be a function of $x$. These facts may cause the dispersion coefficient to be a function of $x$. This result is completely different from the solution found for concentration fields satisfying Eq 19, which is characterized by dispersion coefficients depending upon local one-dimensional parameters only. Accordingly when studying the dispersive transport it is important to investigate whether Eq 19 is satisfied or not.
4.2 Examples of concentration fields for which Eq 19 does not apply

Under quasi-steady conditions the salinity distribution in a well mixed tidal estuary — such as the Delaware — may be characterized by a longitudinal salinity distribution curve, $\bar{c} = f(x)$, being displaced longitudinally while its shape does not change (See Fig. 1). As observed by Ippen and Harleman (1961) in first approximation the salinity distribution at high water slack is given by the low water slack distribution curve displaced longitudinally by a distance equal to the tidal excursion. After one complete tidal cycle the salinity distribution curve has returned to its original place. However, an observer moving at velocity $\ddot{u}$ has covered a distance $u_{riv} T$ during a tidal cycle, where $T$ is tidal period and

$$u_{riv} = \frac{Q_{riv}}{F}$$

where $Q_{riv}$: discharge of river issuing into sea  
$F$: section of estuary  
$u_{riv}$: contribution of river flow to total velocity in tidal zone of estuary

Accordingly for the two-dimensional flow considered in section 3, averaged over a tidal period

$$\frac{\partial \bar{c}}{\partial \tau} = - u_{riv} \frac{\partial \bar{c}}{\partial \xi} = - u_{riv} \frac{\partial \bar{c}}{\partial x}$$

(35)

where $\xi$: longitudinal coordinate in coordinate system moving with velocity $\ddot{u}$ (both $x$ and $\xi$ are taken positive in the direction from sea to land).

Eq 35 into Eq 15 gives

$$-u_{riv} \frac{\partial \bar{c}}{\partial \xi} + \frac{1}{h} \left\{ \frac{\partial h \nu^T c^T}{\partial \xi} + \frac{\partial h_T}{\partial x} \right\} = 0$$

(36)

which is the basic equation for the one-dimensional analysis of salinity intrusion (Ippen and Harleman, 1961). The last term of Eq 36 would vanish if Eqs 16 and 17 were applied notwithstanding the fact that Eq 19 is not satisfied.
Under quasi-steady conditions the longitudinal distribution of a constituent which is released at a constant rate into a tidal river is characterized by distinct concentration distribution curves at high water slack and low water slack (see Fig. 2). Accordingly also in this flow after one complete tidal cycle the concentration distribution curve has returned to its original place, while an observing moving at velocity $\bar{u}$ has covered a distance $u_{riv} T$ during a tidal cycle. Hence also for the concentration fields from continuous discharges into a tidal river ($u_{riv} \neq 0$) Eq 19 is not satisfied.

4.3 Main groups of concentration - fields (see Table 1, page 17)

In the preceding sections an explanation has been given of the basic difference between dispersion coefficients for concentration fields satisfying $\partial c/\partial t = 0$ and those for concentration fields not satisfying this condition.

In general the condition of $\partial c/\partial t \neq 0$ is satisfied
- for instantaneous releases
- for steady flows
provided that the concentration field developed sufficiently far. The condition is not valid during an initial period (Fischer, 1973).

In general the condition of $\partial c/\partial t \neq 0$ is satisfied
- for continuous releases into tidal rivers ($u_{riv} \neq 0$)
- for salinity intrusion into tidal rivers ($u_{riv} \neq 0$)

5 Dispersion coefficient for one constituent derived from observations on the dispersive transport of an other constituent

5.1 Effect of transport through water surface and decay

Eq 19 being satisfied $c'$ can be solved from Eq 22. For constituents which are not transported through the water surface $T_y = 0$ at $y = h$. For other constituents, such as dissolved oxygen and heat there is a transport through the water surface. This requires $T_y \neq 0$ at $y = h$. Accordingly for both groups of constituents different solutions $c'(y)$ are found. This implies the dispersion coefficients to be different for both groups of constituents. The same conclusion can be derived from Eq 33 for the situation of Eq 19 not being satisfied.
For non-conservative constituents an additional term enters into Eqs 22 and 33, because the decay is expressed as

$$\frac{dc}{dt} = -kc$$

(37)

where k: coefficient, defined by Eq 37, expressing the effect of decay.

Because of this additional term the solution c'(y) and hence the dispersion coefficient depend upon the decay characteristics of the constituent.

5.2 Eq 19 satisfied for one constituent only (see section 4.3)

Eq 19 being satisfied for one constituent only for the one constituent the dispersion coefficient depends upon local one-dimensional parameters while for the other constituent it depends upon longitudinally integrated effects.

5.3 Eq 19 not satisfied for both constituents (see section 4.3)

As explained in section 4.1 Eq 19 not being satisfied the dispersion coefficient may be a function of the longitudinal coordinate x when \(\frac{\partial c}{\partial x}, \frac{\partial c}{\partial \tau}\) and (or) v are functions of x. Accordingly two constituents may show different variations of the dispersion coefficient in the longitudinal direction when both constituents have different variations of \(\frac{\partial c}{\partial x}, \frac{\partial c}{\partial \tau}\) and v in the longitudinal direction.

6 Continuous release of constituent into tidal estuary through which river issues into sea

The one-dimensional dispersion in a tidal estuary has been studied by Holley, Harleman and Fischer (1970). These investigators found the one-dimensional dispersive transport to be reduced by tidal action.

A method to derive the reduction of the dispersion coefficient has been presented in their study. This method was based on Eq 19, as can be seen from Eq 18 of their paper. Accordingly, the findings of Holley et al may be applied only for steady releases of a constituent into a tidal estuary if \(u_{riv} = 0\), as can be seen from section 4.2 and Eq 35. Only then the magnitude of the dispersion coefficient is not influenced by longitudinally integrated effects.
7 Conclusions

1. The dispersion coefficient can be expressed in local one-dimensional parameters if amongst others the condition $\frac{\partial \bar{c}}{\partial \tau} = 0$ (Eq 19) is satisfied. Eq 19 not being satisfied the dispersion coefficient depends upon longitudinal integrated effects. Then its magnitude can be determined only by solving Eq 33, limiting the applicability of the dispersion concept.

2. In general after a sufficiently long initial period Eq 19 is satisfied for instantaneous releases (Fischer, 1973, Holley et al, 1970). In general Eq 19 does not hold for continuous releases into tidal rivers with $u_{riv}$ not being zero (see section 4.2). Hence both groups of releases must be treated differently in as far as the dispersive transport is concerned.

3. The considerations of Holley et al (1970) on the magnitude of the dispersion coefficient in homogeneous tidal flows are based on Eq 19. Hence for continuous releases they are only valid if $u_{riv}$ is equal to zero.

4. The dispersion coefficient for the one constituent can be derived from observations of the dispersive transport of an other constituent only if restrictive conditions are satisfied. This procedure may not be applied for constituents having a different transport rate through the water surface (such as salinity and dissolved oxygen) and for constituents having different decay characteristics. Considering a tidal estuary through which a river having a sufficiently large flow issues into the sea the dispersion coefficient for an instantaneous release of a constituent is not necessarily equal to the one for a continuous release into the same body of water. Further, the river flow being sufficiently large the dispersion coefficients for continuous releases from different locations into the same tidal estuary may be different.

8 Final remark

A quantitative evaluation of the different effects mentioned in this study requires amongst others a (numerical) solution of Eq 33 for different types of releases. This was beyond the scope of this study. It should be the subject of further research.
References

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Symbols

coordinates

x : longitudinal coordinate
y : vertical coordinate
n : coordinate perpendicular to plane A
t : time

ξ : longitudinal coordinate in coordinate system moving at velocity \( \ddot{u} \) or \( \ddot{u} \)
\( \tau \) : time in this coordinate system

symbols

A : plane of averaging
dA : element of plane A
a : vertical coordinate of bottom
b : vertical coordinate of water surface
c : concentration
\( \bar{c} \) : average concentration in plane A
\( \bar{c} \) : depth averaged concentration
c' : difference between c and \( \bar{c} \) or between c and \( \bar{c} \).
E : dispersion coefficient
\( E_x \) : dispersion coefficient due to averaging over the width
\( E_p \) : dispersion coefficient due to averaging over the profile
\( E_v \) : dispersion coefficient due to averaging over the depth
F : cross section of river
\( F_{\text{total}} \) : total transport through plane A
\( F_{\text{disp}} \) : dispersive transport
h : depth
k : coefficient, defined by Eq 37, characterizing decay
\( Q_{\text{riv}} \) : discharge of river
\( R_v \) : hydraulic radius
\( T_l \) : turbulent transport in direction of index
u : velocity in x-direction
\( \bar{u} \) : average value of u in plane A
\( \ddot{u} \) : depth averaged value of u
\( u' \): difference between \( u \) and \( \bar{u} \) or between \( u \) and \( \bar{u} \)
\( u^* \): shear velocity
\( u_n \): velocity in n-direction
\( u_{riv} \): velocity due to river flow (Eq 34)
\( v \): vertical velocity
\( \varepsilon \): eddy diffusion coefficient.
Table 1, characteristics of concentration distributions

<table>
<thead>
<tr>
<th></th>
<th>instantaneous release</th>
<th>continuous release</th>
</tr>
</thead>
<tbody>
<tr>
<td>river, steady flow</td>
<td>$\frac{\partial c}{\partial t} \rightarrow 0$</td>
<td>$\frac{\partial c}{\partial t} \rightarrow 0$</td>
</tr>
<tr>
<td>tidal estuary, no inflow of river</td>
<td>$\frac{\partial c}{\partial t} \rightarrow 0$</td>
<td>$\frac{\partial c}{\partial t} \rightarrow 0$</td>
</tr>
<tr>
<td>tidal estuary, with inflow of river</td>
<td>$\frac{\partial c}{\partial t} \rightarrow 0$</td>
<td>$\frac{\partial c}{\partial t} \neq 0$</td>
</tr>
</tbody>
</table>

1) sufficiently long after release
2) sufficiently far from point of discharge
FIG. 1. CONCENTRATION DISTRIBUTION CURVE $\tilde{c}(x)$ FOR SALINITY INTRUSION INTO ESTUARIES (AFTER IPPEN AND HARLEMAN 1961)
FIG. 2. CONCENTRATION DISTRIBUTION CURVE $\bar{c}(x)$ FOR CONTINUOUS DISCHARGE OF A CONSTITUENT FROM POINT SOURCE INTO TIDAL ESTUARY (AFTER HARLEMAN, 1971)