The initial calculation of range and mission fuel during conceptual design

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SUMMARY

Derivations are presented for the range of aircraft with gas-turbine propulsion systems, which cannot be characterized to have either constant specific fuel consumption or constant propulsive efficiency. The effects of different cruise techniques are investigated. It is found that for preliminary design purposes a very simple approximation of the fuel fraction can be used for all aircraft categories and various cruise techniques. This result has been used to compute the total mission and reserve fuel. A method is proposed to derive the range parameter $\eta L/D$ for existing aircraft from their payload vs. range diagram. Such statistical data of the range parameter may be used as input for the calculation of the fuel fraction. The method is intended for use during conceptual design studies for a first estimation of the take-off weight.
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SYMBOLS

$C_1$, $C_2$ - constants in equation for $n$ (2-15)

$C_D$ - drag coefficient

$C_{D0}$ - zero-lift drag coefficient

$C_L$ - lift coefficient

$C_p$ - specific fuel consumption (propeller a/c)

$C_T$ - specific fuel consumption (jet a/c)

$D$ - drag

$F$ - fuel weight flow per unit time

$g$ - acceleration due to gravity

$H$ - calorific value of fuel

$h$ - altitude

$h_e$ - energy height ($h_e = h + \frac{1}{2} V^2/g$)

$L$ - lift

$M$ - Mach number

$n$ - exponent of $M$ in eq. for $n$ (2-12)

$P$ - range factor ($P = n L/D$)

$p$ - atmospheric pressure

$p_o$ - $p$ at sea level ISA

$R$ - range, distance flown

$R_H$ - range-equivalent of $H$ ($R_H = H/g$)

$R_h$ - harmonic range (for max. payload, Fig. 4)

$S$ - gross wing area

$s$ - horizontal distance flown

$T$ - net thrust

$t$ - time

$V$ - true airspeed

$W$ - weight (no index: All-Up Weight)

$W_f$ - fuel weight

$W_p$ - payload weight

$\beta$ - induced drag factor ($\beta = dC_D/dC_L^2$)

$\gamma$ - ratio of specific heats of air ($\gamma = 1.4$)

$\Delta$ - reduced range factor defined by (4-3), increment

$\delta$ - relative pressure ($\delta = p/p_o$)
\( \eta \) - overall powerplant efficiency
\( \eta_M \) - d log \( \eta \)/d log \( M \)
\( \theta \) - relative atmospheric temperature

Indices

cl - climb
cr - cruise
div - diversion
f - fuel
h - harmonic
hold - holding flight
lost - lost fuel
MD - minimum drag condition
m - mission
p - payload
to - take-off
o - initiation of cruise flight
l - end of cruising flight

Acronyms

AUW - All-Up Weight
MTOW - Maximum Take-Off Weight
MZFW - Maximum Zero Fuel Weight
OEW - Operating Empty Weight
TOW - Take-Off Weight

All derivations and units comply with the SI system of units, except the data in Tables 1 and 2, which have been derived from manufacturers brochures.
1. Introduction

The conceptual aircraft design process requires estimation of an All-Up Weight at take-off and a weight breakdown in the early stages of the design process, where many details of the physical aircraft characteristics have not yet been settled. In particular the fuel weight fraction \((\frac{W_f}{W_a})\) constitutes a large contributor, which for a given range is mainly dependent on:
- the type of propulsion system, notably its specific fuel consumption or overall efficiency,
- the lift-to-drag ratio of the aircraft in cruising flight.

For a given aircraft, the latter is determined by the drag polar and by the lift coefficient, i.e. by the operational conditions (wing loading, altitude and speed).

In conceptual design, it can generally be assumed that the engine (and propeller) efficiency are approximately known, even if a final engine selection has not yet taken place. However, the drag polar is a much more complex item, since it depends on many geometric and aerodynamic parameters which are either not yet known or difficult to calculate without extensive effort. These efforts are justified at a certain stage of the design, but not always necessary when an estimate of the All-Up Weight is to be made for the first time. The present method aims at a simple but reasonably accurate - and therefore validated - procedure to calculate the fuel fraction required for a specified range/payload combination.

The basis of the method is a generalized and simple equation for the cruise range of aircraft with gasturbine propulsion systems, derived in chapter 2. This equation will be used in chapter 3 to approximate the total mission fuel. Since in all equations the range parameter \(\eta\ L/D\) plays an important role, a method is devised in chapter 4 to derive this parameter from data of existing aircraft. This result gives an idea of values that can be achieved with present-day transport aircraft. In a later stage of the design it has to be verified that the actual value of \(\eta\ L/D\) is not too far from the original guessimate. A summary of results is given in chapter 5.

2. The cruise range for given fuel fraction and flight conditions

2.1. General statement of the problem

A closed-form expression for the cruise range is ascribed to Bréguet (1880-1955), who derived for propeller aircraft the well-known formula:

\[
R = \frac{\eta_D}{C_D} \frac{C_L}{C_P} \ln \frac{W_0}{W_1}
\]

(2-1)

where 0 and 1 denote the initial and end conditions of the flight, resp. Later, it was found that a very similar equation applies also to the range of jet aircraft in a cruise/climb. Expressions have also been derived for several other cruise techniques; they are summarized in Ref. 1.

Generally different analytical methods have been derived for jet aircraft (with constant TSFC) and for propeller aircraft (with constant BSFC and propeller efficiency). However, in modern transport aircraft engines are installed with various bypass ratios and in the near future high-speed propfans or Unducted Fans will be applied. From the flight mechanics point of view these engines do not comply with these simplifying assumptions (cf. Ref. 2).
It has therefore become desirable to reconsider the various range equations. Moreover, a choice of the most suitable cruise technique will have to be made for the purpose of calculating the fuel fraction for a given range.

2.2. The basic range equation

The range in (quasi-)steady cruise flight can be obtained from integration of the specific range \( V/F \):

\[
R = \int_{W_1}^{W_0} \frac{V}{F} \, dW
\]  

(2-2)

The integration is dependent on the cruise technique, for which various schemes are known:

- cruise/climb e.g. with constant \( \alpha \) and \( M \) or engine rating;
- horizontal cruise with constant \( \alpha \), \( M \) or engine rating.

In principle, these techniques result in different ranges. They do not necessarily lead to a maximum range; for example cyclic flight with dolphin-type climb/descent profiles may increase the range appreciably (Refs. 3,4). However, since practical flying procedures require certain parameters to be kept constant, we will only analyse and compare the above mentioned cases. In Ref. 2 a derivation is given of conditions resulting in constrained and unconstrained maxima for the range parameter. It was shown that the overall efficiency \( \eta \) and \( C_{L}/C_{D} \) are both functions of two parameters:

\[
C_{D} = f(C_{L}, M)
\]

\[
\eta = \frac{TV}{R_{H} F} = f(T/\delta, M)
\]  

(2-3)

Since in steady cruising flight \( T = D \), we have also:

\[
T/\delta = \frac{1}{2} \gamma p_{o} M^2 C_{D} S = f(C_{L}, M)
\]  

(2-4)

The two control variables \( C_{L} \) and \( M \) will therefore be treated in various ways, thus defining different cruise techniques in quasi-steady flight.

2.3. The cruise/climb with constant \( C_{L} \) and \( M \)

The specific range is rewritten as follows:

\[
\frac{V}{F} = \eta \frac{R_{H}}{T} = \eta \frac{R_{H} C_{L}}{C_{D} W}
\]  

(2-5)

Substitution into (2-2) yields:

\[
\frac{R}{R_{H}} = \int_{W_1}^{W_0} \frac{C_{L}}{C_{D} W} dW
\]  

(2-6)

Since for constant \( C_{L} \) and \( M \) also \( W/\delta \) and \( T/\delta \) are constant, the engine rating is constant and \( \eta \) can be considered as approximately constant. With gradually decreasing AUW due to fuel consumption, \( \delta \) has to be decreased, so that the aircraft climbs according to \( \delta + W \). We will therefore assume \( \eta \) to have an
average value \( \bar{\eta} \) for a mean cruising altitude. Since for constant \( C_L \) and \( M \) we have: \( C_L/C_D = \) constant, the result is:

\[
\frac{R}{R_H} = \bar{\eta} \frac{C_L}{C_D} \frac{W_o}{\ln W_1}
\]

and with \( W_1 = W_o - W_f \):

\[
\frac{R}{R_H} = \bar{\eta} \frac{C_L}{C_D} \ln \left( 1 - \frac{W_f}{W_o} \right)^{-1}
\]

This generalized Bréguet range equation is therefore valid for all categories of airbreathing propulsion under the present conditions.

It has been found practical to approximate eq. 2-8 as follows:

\[
\frac{R}{R_H} = \bar{\eta} \frac{C_L}{C_D} \frac{W_f}{\bar{W}}
\]

where \( \bar{W} \) is a suitably chosen mean cruising weight, for example:

- **case A** \( \bar{W} = \sqrt{W_o(W_o - W_f)} \)
- **case B** \( \bar{W} = W_o - 0.5 W_f \)

The range according to approximation (2-9) differs from eq. 2-8 by the following amount (in percents)

\[
\frac{W_f}{W_o} = \begin{array}{cccccc}
0.1 & 0.2 & 0.3 & 0.4 & 0.5 \\
\text{case A} : & 0.046 & 0.207 & 0.528 & 1.079 & 1.974 \\
\text{case B} : & -0.092 & -0.414 & -1.058 & -2.165 & -3.974 \\
\end{array}
\]

where the minus sign denotes an underprediction. This result shows that case A results in an error which is 50% of the error in case B. For most practical cases the fuel fraction is less than 30% and the difference between eqs. (2-8) and (2-9) is then less than 0.5% for case A.

### 2.4. Horizontal cruising flight with constant \( C_L \)

During this flight the Mach number is decreasing with \( W \) according to:

\[
M = \sqrt{\frac{W^2 - 1}{S \gamma_D C_L}}
\]

As a consequence, the drag increment due to compressibility also changes, whereas the overall efficiency varies with \( M \) and \( T/\delta \). This case is therefore rather complex and simplifying assumptions will have to be made, e.g.:
- constant compressibility drag coefficient (hence \( C_D/C_L = \) constant); 
- no effect of \( T/\delta \) on \( \eta \); 
- generalized relationships between \( \eta \) and \( M \).

Various relationships between \( \eta \) and \( M \) have been proposed in the literature. It is generally found that plotted on a log-log basis \( \eta \) vs. \( M \) appears to be an
almost linear function for variations of $M$ up to about $\pm 20\%$. Therefore we may use, for example:

$$
\eta = \eta_0 \left( \frac{M}{M_0} \right)^n \quad (2-12)
$$

where it is to be noted that the exponent $n$ is identical to $\eta_M = \frac{\eta}{\log \eta / \log M}$, used in Ref. 2. Its value generally varies between 0 for $\eta = \text{constant}$ and 1 for $\eta$ proportional to $M$, the case of constant $C_T/\sqrt{\theta}$. 

In combination with eq. 2-11 this relationship can be substituted into eq. 2-2, which yields:

$$
\frac{R}{R_H} = \eta_0 \frac{C_L}{C_D} \frac{W_0 \int_{W_1}^W \left( \frac{W}{W_0} \right)^{n/2} \, dW}{W} \quad (2-13)
$$

and after integration we have:

$$
\frac{R}{R_H} = \eta_0 \frac{C_L}{C_D} \frac{2}{n} \left[ 1 - (1 - \frac{W_f}{W_0})^{n/2} \right] \quad (2-14)
$$

In principle this equation covers all powerplant categories, provided appropriate values are taken for $n$, which can be found readily from engine brochures or from Ref. 5 (see Fig. 1). For $n = 0$ eq. (2-13) becomes numerically identical to the Bréguet formula for propeller aircraft, for $n = 1$ the well-known result for pure jet aircraft is found.

Another possible representation of the overall efficiency is:

$$
\eta = \frac{C_L}{C_D} \frac{M}{C_2 M} \quad (2-15)
$$

For $C_2 = 0$ we have a constant efficiency, for $M \ll C_2$, the case of $n + M$ is obtained. Substitution of (2-15) into eq. (2-2) yields, after integration:

$$
\frac{R}{R_H} = 2\eta_0 \frac{C_L}{C_D} \ln \frac{C_2/M_0 + 1}{C_2/M_0 + \sqrt{1 - W_f/W_0}} \quad (2-16)
$$

This result clearly includes eq. (2-8) for $C_2 = 0$, which is the result for aircraft with $\eta = \text{constant}$, i.e. the original Bréguet range equation for propeller aircraft.

It can be shown that eqs. 2-14 and 2-16 give numerically identical results provided the values of $n$ and $C_2$ are selected so that for the initial and end conditions the same values for $\eta$ are found. Again both equations can be approximated as follows:

$$
\frac{R}{R_H} = \eta \frac{C_L}{C_D} \ln \frac{W_f}{\bar{W}} \quad (2-7)
$$

where $\bar{\eta}$ and $\bar{W}$ refer to a suitably chosen mean flight condition. Similar to par. 2.3 cases A and B have been investigated. For $n = 1$ (jet aircraft) there
was no difference in the error for the two cases, but for very small \( n \) (constant \( n \)) case B gave errors twice as large as case A. For case A the error is always below 0.5% for fuel fractions up to 30%.

Furthermore the effect of \( n \) on the range according to eq. 2-14 can be seen from the following table, for \( \frac{W_F}{W_o} = 0.30 \) and given initial conditions:

\[
\begin{array}{cccccc}
R & n = 0.2 & 0.4 & 0.6 & 0.8 & 1.0 \\
\frac{R}{R_{n=0}} & 0.982 & 0.965 & 0.948 & 0.932 & 0.916 \\
\end{array}
\]

These data clearly show the disadvantage of the present cruise procedure for jet aircraft, since for pure jets with \( n = 1 \) the loss in range is more than 8% relative to the cruise climb. This is caused by the reduction in \( n \) with decreasing Mach number.

2.5. Horizontal cruising flight with constant speed

In this case the overall efficiency varies only as a consequence of the decreasing thrust with engine rating. The thrust itself decreases less than in the previous case since the L/D ratio is reduced due to the weight variation. Eq. (2-2) may be written for this case as follows:

\[
\frac{R}{R_H} = \frac{W_o}{W_1} \int n \frac{dW}{C_L} = \frac{C_L}{C_D} \int n \frac{dC_L}{C_D} \quad (2-17)
\]

For a parabolic drag polar this works out as follows:

\[
C_D = C_D^0 + \beta C_L^2 = C_D^0 \left[ 1 + \left( C_L/C_L^{MD} \right)^2 \right] \quad (2-18)
\]

where \( C_L^{MD} = \sqrt{C_D^0/\beta} \) and \( C_D^{MD} = 2 C_D^0 \) \quad (2-19)

It is to be noted that both \( C_D \) and \( \beta \) can be functions of \( M \). Substitution of (2-18) into (2-19) yields after integration

\[
\frac{R}{R_H} = 2 \bar{n} \left[ C_L/C_D \right]_{max} \left[ \arctan \frac{C_L}{C_L^{MD}} - \arctan \frac{C_L}{C_L^{MD}} \left( 1 - \frac{W_F}{W_o} \right) \right] \quad (2-20)
\]

Similar to the previous cases a mean value of the weight may be used to define a mean \( C_L \) and \( C_D \), with the range approximated as follows:

\[
\frac{R}{R_H} = \frac{\bar{C}_L W_F}{\bar{C}_D W} \quad (2-21)
\]

Here again cases A and B, defined by eq. 2-10 were investigated. It was found that for fuel fractions up to 30% case A resulted in errors less than 1% of the range. However, case B resulted in an even better approximation, with errors less than 0.3% for \( W_F/W_o < 0.3 \) and a maximum error of 0.5% for \( W_F/W_o = 0.5 \) and \( C_L/C_L^{MD} = 1.0 \). Therefore the best approximation is here:
\[
\frac{R}{R_H} = \frac{-n \overline{C_L}}{\overline{C_D}} \frac{\dot{W}}{\dot{W}_0 - 0.5 \dot{W}_f}
\]  (2-22)

with \(\overline{C_L}\) and \(\overline{C_D}\) calculated for \(\dot{W} = \dot{W}_0 - 0.5 \dot{W}_f\).

2.6. Horizontal cruising flight with constant thrust

In this case the variation in TAS is determined by the equilibrium condition \(D = T = \text{constant}\). Assuming a parabolic drag polar (the same for all Mach numbers), the lift coefficient variation is obtained from:

\[
\overline{C_L} = \frac{T}{2\beta \dot{W}} \left[ 1 - \sqrt{1 - 4 \beta \overline{C_D} (\dot{W}/T)^2} \right]
\]  (2-23)

Assuming that similar to eq. (2-12) the overall efficiency can be approximated by a power law, we have:

\[
\frac{R}{R_H} = n \int \frac{\dot{W}}{W_1} \left( \frac{M}{\overline{M}} \right)^n d\dot{W}
\]  (2-24)

For small variations in \(M\) the integrand can be expanded as follows:

\[
\left( \frac{M}{\overline{M}} \right)^n = \left( 1 + \frac{\Delta M}{\overline{M}} \right)^n = 1 + n \frac{\Delta M}{\overline{M}}
\]  (2-25)

From the equilibrium \(T = D\) we have:

\[
\frac{\Delta M}{\overline{M}} = -\frac{1}{2} \frac{\Delta C_D}{\overline{C_D}} = - \frac{d \log \overline{C_D}}{d \log (\dot{W}_f / \dot{W})} \frac{\Delta \dot{W}}{\dot{W}}
\]  (2-26)

Substitution into eq. (2-24) yields:

\[
\frac{R}{R_H} = n \int \left[ \dot{W}_f - n \frac{d \log \overline{C_D}}{d \log (\dot{W}_f / \dot{W})} \frac{\dot{W}}{\dot{W}_f} \right] dW
\]  (2-27)

If we now assume: \(\dot{W} = \dot{W}_0 - 0.5 \dot{W}_f\) (case B), then it is found that the integral in eq. 2-27 is equal to zero. Since we also know that \(T/\dot{W} = \overline{C_D}/\overline{C_L}\), the final result is that eq. 2-22 is the exact answer for the present case, provided \(\overline{C_L}\) is evaluated from eq. 2-23 for \(\dot{W} = \dot{W}_0 - 0.5 \dot{W}_f\).

2.7. Evaluation of results

The following observations can be made when the four cruise techniques are compared.

a) For the same initial cruise conditions the cruise/climb technique generally results in the longest range. The exception here is an initial condition where the TAS is below the minimum drag speed. In that case
application of (2-20) will indicate that a horizontal cruising flight with constant $M$ can lead to a slightly longer range due to a higher mean value of $C_L/C_D$.

b) For the same mean cruise conditions the differences between the various cruise techniques are not very large, dependent on the definition of that mean condition. Taking, for example, $\bar{W} = W_o - 0.5 W_f$, the difference between the 'exact' solution and the approximation according to eq. 2-9 is as follows, for $W_f = 0.30 W_o$:

const. $C_a$ and $M$ : difference $= -1.06 \%$
const. alt. and $C_a$ : $= -0.40 \%$ (n = 1)
const. alt. and $M^*$ : $= +0.08 \%$ ($C_o = 0.8 C_{LMD}$)
const. alt. and $T$ : $= 0$

c) Cruise techniques with constant $C_a$ (or $a$) are not practical from the ATC point of view, since they result in a gradually varying altitude or Mach number. However, an approximation of the cruise/climb by means of a step-climb is possible as soon as the AUW has decreased to a value where an increase in altitude of 4000 ft typically is possible.

d) Flying at constant altitude with constant $C_a$ is unfavourable for jet aircraft due to the reduction in $n$ during the flight, unless the initial Mach number is in the drag rise. The difference with the maximum range in cruise/climb, however, is reduced when $n$ is between 0 and 1, as argued in section 2.4. Therefore, aircraft with propeller or possibly propfan propulsion do not suffer in this respect. However, the range obtained with this technique is only slightly higher compared with the constant-$M$ or constant-$T$ techniques, which are more practical to fly.

e) Cruising with constant $T$ will result in a gradually increasing $M$ during the flight. For high-subsonic aircraft the drag rise will be penetrated and the derivation in section 2.6 is no longer accurate. This cruise technique will be useful primarily for short-range flights, where flying at max. cruise rating can yield an important gain in block time.

f) Cruising at constant altitude and Mach number is a practical procedure for medium- and long-range flights. It can be shown (Ref. 2) that for given altitude the unconstrained optimum cruise speed is mainly defined by $d \log C_P/d \log M$, which corresponds to a certain optimum Mach number for high-subsonic aircraft. However, using eq. 2-20, it can be shown that a single step climb will increase the range by about 5% typically for a fuel fraction of 30% and $C_L = 0.7 C_{LMD}$. More steps do not improve the range considerably and are not very often used in practice. We can therefore conclude that all range equations can be summarized adequately as follows:

$$\frac{R}{R_H} = \frac{C_L}{C_D} \frac{W_f}{W_o - 0.5 W_f}$$

(2-22)
where $\bar{n}$, $\bar{C_L}$ and $\bar{C_D}$ are defined for mean conditions of speed and $C_{w}$, derived from $\bar{W} = \bar{W_o} - 0.5 \bar{W}$. The amount of fuel required for a specified cruise range is then simply:

$$\frac{W_f}{W_o} = \frac{R/R_H}{\bar{p} + 0.5 \frac{R}{R_H}}$$

(2-28)

where $\bar{p} = \frac{\bar{n}}{\bar{C_L}/\bar{C_D}}$ is the mean value of the range parameter for the cruising flight.

3. Approximate calculation of the mission fuel

3.1. Mission and reserve fuel breakdown

A typical mission profile for an international flight is shown on Fig. 2. Total fuel is subdivided into
- block fuel, required to fly the complete mission, including fuel to taxi-out before take-off and taxi-out after landing
- reserve fuel, in accordance with the pertinent rules for the operation under consideration.

For the determination of the TOW the taxi fuel is not included, since the TOW is defined at the beginning of the take-off runway, and taxi fuel after landing is part of the reserve fuel. Mission fuel therefore includes thus
- take-off, acceleration and climb to cruise alt./speed;
- cruise (with or without steps);
- descent, approach and landing.
Reserve fuel will include items such as:
- one extra hour of cruising;
- a diversion flight (e.g. 200 nm);
- a 30 or 45 min. hold;
- a percentage of the block fuel (e.g. 5%).
A reserve policy will combine several of these items, in accordance with FAR rules or company policy, dependent on the type of operation.

The aim of the present method is to estimate mission and reserve fuel quantities, usually as a fraction of the TOW, in the design stage where detailed performance calculations cannot yet be made due to lack of reliable design data. The total fuel will therefore be estimated on the basis of cruise fuel, with semi-statistical allowances for climb, descent, etc.

3.2 Breakdown of the All-Up Weight (AUW)

The All-Up-Weight of a transport aircraft is split up into
- Operating Empty Weight (OEW) or Weight Empty Equipped;
- Payload Weight;
- Fuel Weight.

The payload plus fuel load is referred to as the Useful Load.
For a given aircraft, the OEW is considered independent of the range. The payload is limited (Fig. 3) by the Volumetric Payload - determined by the number of seats and volume of the cargo holds - or by the Maximum Zero Fuel Weight (MZFW), which is a structural limit. In the early design stage maximum payload is determined directly by the design specifications.
The fuel load is split up into reserve fuel - which is for a given type of operation almost independent of range - and mission fuel. The amount of mission fuel is determined by the TOW, which in turn is determined by the payload. Hence, the mission fuel vs. range is located within an envelope, initially determined by the case of zero payload and by the MZFW, assuming that this forms the critical payload limit. The AFW increases with range until it reaches a limiting value, assumed here as the MTOW. The maximum range with max payload is referred to as the harmonic range (point A). For increasing ranges the fuel required is determined directly by a MTOW, which is independent of the range. The allowable payload is obtained by subtracting the fuel load and the OEW from the MTOW.

A further increase in range and fuel is limited by the fuel tank capacity corresponding to the maximum useful range (point B). A further increase to the maximum range (point C) is obtained by a decrease in MTOW, since the fuel fraction can be increased only by reducing the initial weight. In the absence of a fuel tank capacity limit, the ultimate range (point D) could be reached, where the total fuel load is equal to the MTOW minus the OEW.

It must be mentioned that for a given aircraft the payload-range and fuel-range diagrams are dependent upon the cruise technique. In particular there is a considerable difference between Long-Range Cruise (LRC) and High-Speed Cruise (HSC) techniques. In the first case the altitude and speed are selected so that 98% or 99% of the maximum specific range (V/F) is obtained. The second case usually corresponds to max. cruise rating at a lower altitude. Furthermore there are effects of atmospheric conditions, assumptions w.r.t. wind, climb and descent flight profiles, reserve fuel policy and various payload configurations of the aircraft, which make it difficult to compare aircraft directly on a basis of their payload-range diagrams. However, it will be shown in Chapter 4 that some useful information can be derived from the payload-range diagram.

3.3. Estimation of the mission fuel

Accurate calculation of the contributions to the fuel for the various flight segments can only be done when sufficiently detailed data are available in the form of drag polars, engine thrust and fuel flow diagrams, design weights, etc. However, in the conceptual design stage the AFW is to be estimated from elementary information such as the maximum payload, the range, some preliminary engine data and basic aircraft dimensions. In that stage it will be acceptable to estimate the fuel weight fraction \( W_f/W_{to} \) on the basis of primarily the cruise fuel requirements. Added to this allowances can be given for additional effects (e.g. climb and descent). The only major unknown factor to be used as input for such a method will be the range factor \( P = \eta L/D \) in cruising flight.

The computational procedure to be followed here is as follows:

a) The amount of fuel required to cover the complete range (i.e. the block distance) in cruising flight is calculated first.

b) For take-off, climb and acceleration to cruise altitude and speed an extra quantity of fuel is required. From an energy balance it can be seen that this "lost fuel", which does not contribute to the range is found from:

\[
\Delta W_f + \frac{m_{cl} R_H}{W_{to}} = \frac{W_{to}}{2 g \frac{V_{cr}^2}{2}}
\]

(3-1)
For a mean value of the overall engine efficiency during climb ($\bar{n}_{ecl}$) equal to 70% of the value during cruise, this result is in accordance with the lost fuel method in Ref. 6 and it resembles Ref. 7. We may therefore take:

$$\frac{AW_f}{W_{to}} = 1.4 \frac{(h_e)_{cr}}{\bar{n}_{cr} R_H}, \quad \text{with} \quad (h_e)_{cr} = \frac{A}{h_{cr} R_H} + \frac{V_{cr}^2}{2g}$$  \hspace{1cm} (3-2)

A more detailed derivation and discussion are given in the Appendix.

c) In accordance with procedures sometimes adopted in airline evaluations of proposed aircraft, the fuel consumed during descent, holding, approach and landing is assumed to be equal to the fuel used during cruising flight over the same distance. It is thus assumed that a hypothetical cruising flight extension up to the field of destination is made without any further allowances for the actual fuel used.

The mission fuel fraction obtained is thus equal to the contributions of eqs. 2-28 and 3-2:

$$\frac{W_f}{W_{to}} = \frac{R/R_H}{(\bar{n} L/D)_{cr}} + 0.5 \frac{R/R_H}{R_H} + 1.4 \frac{(h_e)_{cr}/R_H}{\bar{n}_{cr}}$$  \hspace{1cm} (3-3)

The second term depends on the cruise conditions (altitude and speed) and for jet transports, cruising at high altitude and speed, will be of the order of 5%, for long-range flight, to 15% of the first term for short ranges.

3.4. Estimation of the reserve fuel

There are important differences in reserve fuel policy between, e.g., domestic and international flights on the one hand, and US or European conditions on the other hand. For example, the Association of European Airlines (AEA) specifies a 200 nm diversion flight, 0.5 hr holding at 1,500 ft and 5% trip fuel for reserves on short/medium haul flights. For US flights typical reserves are less, for example 130 nm diversion and 30 min. holding at cruise altitude. For business aircraft the NBAA usually specifies the reserves to be equivalent to 3/4 hr extension of the cruising flight.

It is desirable to devise a method where the user is free to insert the actual reserve policy in the case under consideration. This results in the appropriate use of several of the following allowances, at the discretion of the user, and in accordance with the design specification:

a) An extension of the cruising flight by $\Delta t_{cr}$, equivalent to an increase of $R$ by

$$\Delta R = \Delta t_{cr} V_{cr}$$  \hspace{1cm} (3-4)

b) A diversion distance equal to $R_{div}$, with a typical deterioration in $V/F$ of 20% due to flying at low altitude and speed, resulting in:

$$\frac{AW_f}{W_{f_m}} = 1.20 \frac{R_{div}}{R_H}$$  \hspace{1cm} (3-5)
where $W_f$ is the mission fuel corresponding to the harmonic range $R_h$, given by (3-3)$^m$.

c) An allowance for a holding period of $t_{\text{hold}}$ (hr) assumed to be independent of the actual holding altitude and on a basis of statistics determined by

$$\frac{\Delta W_f}{W_{f,m}} = 0.20 \ t_{\text{hold}} \ \frac{R_h}{R_m} \ (1 - \frac{W_{f,m}}{W_{to}}) \quad (3-6)$$

d) A fuel reserve expressed as a fraction (e.g. 0.05 or 0.10) of the mission fuel.

Using this procedure we find for the reserve fuel:

$$\frac{W_f}{W_{f,m}} = \frac{\Delta t_{cr} \ V_{cr}}{R_h} + 1.20 \ \frac{R_{div}}{R_h} + 0.20 \ t_{\text{hold}} \ \frac{R_h}{W_{to}} \ (1 - \frac{W_{f,m}}{W_{to}}) + \frac{\Delta W_f}{W_{f,m}} \quad (3-7)$$

where usually at least one of these terms is omitted, dependent on the reserve fuel policy.

4. Evaluation of the range parameter for existing aircraft

The input required for the present method of estimating the fuel load is:

a) a combination of payload and range;

b) operational cruise conditions: altitude and TAS or Mach number;

c) basic information about reserve policy;

d) the range parameter during cruising flight:

$$P = \left[ n \ L/D \right]_{cr}$$

The optimization of this important parameter is primarily a matter of:

- selecting the appropriate propulsion system;
- optimizing the geometry of the aircraft;
- selecting the best cruise conditions.

The first two subjects are very complex and will not be considered in this report. A treatment of the third subject is found in Ref. 2.

It is considered useful that a designer has a good feeling for the value of $P$ that can be obtained in practice. It is therefore proposed that data of existing aircraft are used to derive actual range parameters for these aircraft, which can then be used as a basis for comparison.

The most significant part of the fuel vs. range diagram (Fig. 3) is sector AB, corresponding to take-off with MTOW. At point A - the harmonic range $R_h$ - the slope of this line is equal to minus the increase in cruise fuel required for a small range increment. This is found from differentiation of eq. 2-22:

$$d\left(\frac{R}{R_h}\right) = \bar{P} \ \frac{d(W_f/W_{to})}{(1 - 0.5 \ W_f/W_{to})^2} \quad (4-1)$$

Substitution of $W_f/W_{to}$ according to eq. 2-28 yields a quadratic equation in $\bar{P}$, for which an approximate solution is (for $\bar{P} \geq 2.0$):
\[ \bar{P} = \sqrt{\Delta (A - 2 \frac{R_h}{R_H})} \]  
(4-2)

where \[ \Delta = \frac{d(R/R_H)}{d(W_p/W_{to})} = - \frac{W_{to}}{R_h} \frac{dR}{dW_p} \]  
(4-3)

The slope \( dR/dW \) (which is negative) can be measured in the payload vs. range diagram (Fig. \( \bar{P} \)). It can be shown that eq. (4-2) is also a good approximation when the cruise fuel is derived from the Bréguet range equation.

The method derived in this paragraph has been used to obtain a mean value of the range parameter for a number of existing aircraft, for which a reliable payload vs. range diagram was available. The result is found in Tables 1 and 2. The following observations are made:

a) Large long-range turbofan-powered transport aircraft achieve the highest range factors; approximately 5.5 for LRC and about 10% less for HSC.

b) The most efficient propeller aircraft reach their limit at about 4.5 for LRC conditions, and 15-20% below this value for HSC.

c) Short-range aircraft are considerably less efficient in fuel usage due to their low range factor. This is caused mainly by flying at lower altitudes and relatively short cruise sectors.

d) The range factor is also affected by the size of the aircraft: large aircraft have higher values.

e) For several aircraft types the effect of improvements in the state-of-the-art can be clearly seen. For example, for the Boeing 707/320 \( \bar{P} \approx 4.5 \) for LRC, while the 757 achieves 5.2 and the 767 even 5.7.

A realistic value for the range parameter can be obtained by

a) using engine SFC information available in the engine brochure with proper allowance for installation effects (e.g. 2% decrease in \( \eta \));

b) an estimation of the drag polar, for which various possibilities exist (Ref. 8);

c) comparison with data in Table 1 or 2.

5. References

1. An.  
"Introduction to estimation of range and endurance".  

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"Generalized maximum specific range performance".  

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Christodoulou, T.  
"Reducing fuel consumption by cyclic control".  

4. Menon, P.K.A.  

5. An.  
"Approximate methods for estimation of cruise range and endurance: aeroplanes with turbojet and turbofan engines".  
6. An. "Lost range, fuel and time due to climb and descent: aircraft with turbojet and turbofan engines". ESDU Data Sheet No. 74018, August 1974.


Appendix: DERIVATION OF THE LOST FUEL DURING CLimb TO CRUISE ALTITUDE

The "lost fuel" during climb is defined as the difference in fuel that is actually burnt during the climb and the fuel consumed during a cruising flight over the same horizontal distance. For jet aircraft:

\[ W_{\text{lost}} = \int_{\text{cl}} C_T T \, dt - \int_{\text{cr}} C_T T \, dt \]  
(A-1)

Introducing the overall efficiency,

\[ n = \frac{TV}{F/g H} = \frac{V}{C_T R_H} \quad (R_H = H/g) \]  
(A-2)

the lost fuel can be rewritten as follows:

\[ W_{\text{lost}} = \int_{\text{cl}} \frac{T \, ds}{n R_H} - \int_{\text{cr}} \frac{T \, ds}{n R_H} \]  
(A-3)

The thrust during the climbing flight is obtained from the equation of motion in the direction of flight:

\[ \frac{W \, dV}{g \, dt} = \frac{W}{g} \frac{dV}{ds} = T - D - W \frac{dh}{ds} \]  
(A-4)

or:

\[ T = D + W \frac{d}{ds} \left( h + \frac{V^2}{2g} \right) = D + W \frac{h + v}{ds} \]  
(A-5)

where the energy height \( h = h + \frac{V^2}{2g} \).

Substitution of (A-5) into (A-3) yields with \( T = D \) during cruising flight:

\[ W_{\text{lost}} = \int_{\text{cl}} \frac{W \, dh}{n R_H} + \int_{\text{cl}} \frac{D \, ds}{n R_H} - \int_{\text{cr}} \frac{D \, ds}{n R_H} \]  
(A-6)

During the climb the energy height is assumed to increase from approximately zero - at sea level - to the value in cruising flight,

\[ (h_e)_{\text{cr}} = h_{\text{cr}} + \frac{V^2}{2g} \]  
(A-7)

The variation in AUW and overall efficiency are assumed to be relatively small, so that mean values can be defined. Furthermore, the cruising flight is assumed to be steady. Hence, (A-6) can be rewritten, using \( D/W = C_D/C_L \),

\[ \frac{W_{\text{lost}}}{W} = \left( \frac{(h_e)_{\text{cr}}}{n cl R_H} + \frac{R_{\text{cl}}}{R_H} \left[ \frac{C_D/C_L}{n} - \frac{C_D/C_L}{n} \right] \right) \]  
(A-8)

The first term is straightforward: it represents the amount of fuel required to bring the aircraft from zero energy at sea level (take-off) to the total energy level at the cruising altitude. The second term represents the fuel lost due to the deterioration of the range parameter during the climb compared with the cruising flight at altitude. Its value depends in particular on the actual flight profile selected for the climb and the variation of the overall efficiency with speed. Furthermore, the climb technique affects the actual
distance travelled during the climbing flight, \( R_{cl} \). The lost fuel is therefore subject to optimization of the flight profile, for which several (economic) criteria can be adopted. Equation A-8 has therefore no unique answer. For example, if the climb takes place at a constant EAS, which is equal to the value in cruising flight, the \( C_D/C_L \) ratio is constant and we obtain:

\[
\frac{W_{f,\text{lost}}}{W} = \frac{(h_e)_{cr}}{\eta_{cr} R_H} + \frac{R_{cl}}{R_H} \left( \frac{C_D}{C_L} \right)_{cr} \left[ \frac{1}{\eta_{cl}} - \frac{1}{\eta_{cr}} \right]
\]

or:

\[
\frac{W_{f,\text{lost}}}{W} = \frac{(h_e)_{cr}}{\eta_{cr} R_H} \left[ \frac{\eta_{cr}}{\eta_{cl}} + \frac{R_{cl}}{(h_e)_{cr}} \left( \frac{C_D}{C_L} \right)_{cr} \left( \frac{\eta_{cr}}{\eta_{cl}} - 1 \right) \right]
\]

In the hypothetical case that \( \eta_{cl} = \eta_{cr} \), the bracketed term is equal to one. This will be a reasonable approximation for low-speed aircraft, cruising at low altitudes. However, for high-subsonic turbofan aircraft such an approximation will result in an underprediction. For example, taking typical values:

- \( R_{cl} = 240 \text{ km} \), \( (h_e)_{cr} = 12 \text{ km} \), \( (C_D/C_L)_{cr} = 0.06 \) and \( \eta_{cr}/\eta_{cl} = 1.20 \), we have:

\[
\frac{W_{f,\text{lost}}}{W} = 1.44 \frac{(h_e)_{cr}}{\eta_{cr} R_H}
\]

A statistical evaluation in Ref. 6 has indicated that an average constant 1.40 can be used for transport aircraft. For low cruise altitudes (e.g. for a diversion) this factor results in a relative overprediction, but the lost fuel is than small in absolute magnitude.
<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>MTOW (lb)</th>
<th>( R_h ) (nm)</th>
<th>( \Delta \text{W/AR} ) (lb/mm)</th>
<th>( \Delta )</th>
<th>(nL/D)_{cr}</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>Airbus A 300-B2</td>
<td>302030</td>
<td>860</td>
<td>25.82</td>
<td>4.93</td>
<td>4.55</td>
<td>LRC. For HSC: nL/D = 3.56</td>
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<td>Airbus A 310-220</td>
<td>291000</td>
<td>1245</td>
<td>21.48</td>
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<td>5.16</td>
<td>For A 320/100: nL/D = 4.60</td>
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<td>158510</td>
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<td>LRC</td>
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<td>Boeing 707-320 3</td>
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<td>4280</td>
<td>19.61</td>
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<td>4.70</td>
<td>4.29</td>
<td></td>
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<tr>
<td>Boeing 737-300</td>
<td>124500</td>
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<td>5.06</td>
<td>4.64</td>
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<td>48.39</td>
<td>7.25</td>
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<td>19.75</td>
<td>7.15</td>
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<td>B. Aerospace 146-100</td>
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<td>3.67</td>
<td>3.26</td>
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<tr>
<td>B. Aerospace 146-200</td>
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<td>1175</td>
<td>11.83</td>
<td>3.31</td>
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<td>99% max. range</td>
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<tr>
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<td>BAC 1-11-500</td>
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<td>880</td>
<td>10.49</td>
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<td>Canadair Challenger</td>
<td>43100</td>
<td>2557</td>
<td>3.45</td>
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<td>4.04</td>
<td>CF-34 engines, LRC, M = 0.74</td>
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<td>65000</td>
<td>440</td>
<td>12.71</td>
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<td>1.96</td>
<td>HSC</td>
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<td>426000</td>
<td>2500</td>
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<td>6.21</td>
<td>5.05</td>
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<td>MDD DC-8-61</td>
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<td>2550</td>
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<td>MDD DC-8-63</td>
<td>356000</td>
<td>3400</td>
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<tr>
<td>MDD DC-9-50</td>
<td>121000</td>
<td>765</td>
<td>11.53</td>
<td>4.42</td>
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<td>MDD DC-9-80</td>
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<td>820</td>
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<tr>
<td>MDD DC-10-10</td>
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<td>2330</td>
<td>30.07</td>
<td>6.17</td>
<td>5.09</td>
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<td>MDD DC-10-30</td>
<td>572000</td>
<td>4167</td>
<td>34.98</td>
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<td>VFW-614</td>
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<td>658</td>
<td>7.81</td>
<td>2.38</td>
<td>2.08</td>
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Table 2: The derived range parameter for propeller transports.

<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>MTOW (lb)</th>
<th>$R_h$ (nm)</th>
<th>$-\Delta W_p / \Delta R$ (lb/nm)</th>
<th>$\Delta$</th>
<th>$(\eta L/D)_{cr}$</th>
<th>Remarks</th>
</tr>
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<tr>
<td>ATR 42-100</td>
<td>32440</td>
<td>776</td>
<td>3.00</td>
<td>4.56</td>
<td>4.22</td>
<td>LRC; for HSC: $\eta L/D = 3.58$</td>
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<tr>
<td>ATR 42-200</td>
<td>34290</td>
<td>910</td>
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<td>4.86</td>
<td>4.46</td>
<td>LRC; for HSC: $\eta L/D = 3.82$</td>
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<tr>
<td>ATR 72</td>
<td>44070</td>
<td>0</td>
<td>4.47</td>
<td>4.15</td>
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<td>LRC</td>
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<tr>
<td>B.A. ATP</td>
<td>49500</td>
<td>560</td>
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<td>LRC</td>
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<td>B.A. Jetstream</td>
<td>15322</td>
<td>600</td>
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<td>3.06</td>
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<td>LRC; for HSC: $\eta L/D = 2.60$</td>
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<td>44000</td>
<td>678</td>
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<td>LRC; for HSC: $\eta L/D = 3.74$</td>
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<td>Dornier Do-228-200</td>
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<td>3.43</td>
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<td>LRC</td>
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<td>Fokker F-27-MK500</td>
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<td>600</td>
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<td>41865</td>
<td>100</td>
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<td>4.43</td>
<td>4.39</td>
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<td>900</td>
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<td>4.53</td>
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<td>HS748</td>
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<td>Piaggio P-180</td>
<td>10510</td>
<td>800</td>
<td>1.03</td>
<td>4.28</td>
<td>3.93</td>
<td>Cruise speed 320 kts.</td>
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<td>SAAB-Fairchild 340</td>
<td>27275</td>
<td>300</td>
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<td>3.43</td>
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<td>Fairchild Metro III</td>
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<td>760</td>
<td>1.79</td>
<td>3.77</td>
<td>3.43</td>
<td></td>
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</table>
Note: $n = M^n$; or $C_T/\sqrt{\theta} = M^{1-n}$

Data obtained from engine brochures for maximum cruise rating $0.6 \leq M \leq 0.9$

Upper limit of data for $H_p = 20,000$ ft
Lower limit of data for $H_p \gg 36,089$ ft

O Indicates data points for $H_p = 30,000$ ft

Fig. 1. The engine efficiency power law exponent according to Ref. 5.
Fig. 2. Typical mission profile for long-range flights.
Fig. 3. Fuel load and payload envelopes vs. range.
Fig. 4. Determination of the range parameter from a payload vs. range diagram.