J.W.I.J. Frénaij  TIME — DEPENDENT SHEAR TRANSFER IN CRACKED REINFORCED CONCRETE
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Cover:
Figuration of subsequent crack displacements (scale 50:1) due to a sustained shear loading at the instants 0.1: 10: 1000 and 100,000 h respectively.
(figure 4.15a, middle curve on page 64)
TIME — DEPENDENT SHEAR TRANSFER
IN CRACKED REINFORCED CONCRETE

PROEFSCHRIFT

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I respectfully dedicate this thesis to my parents.
'Man muss also gewissen Menschen ihr Alleinsein gönnen und nicht so albern sein, wie es häufig geschieht, sie deswegen zu bedauern.'

Friedrich Nietzsche,
'Menschliches, Allzumenschliches', Ein Buch für freie Geister, Band I, Aphorismus N. 625, 1880.
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1. INTRODUCTION

1.1. Scope of the research
The development of cement in the first part of the nineteenth century was to herald a new era for the use of building materials. In the last hundred years concrete has become a structural tool for civil engineers. Applications are found in many different fields, such as housing, plants, road traffic (pavements, tunnels, fly-over crossings, bridges) and onshore facilities (dams, piers, sluices, barriers, loading platforms). The typical mechanical properties of concrete are due to its material structure. Concrete is a multiphase granular material consisting of aggregate particles of various sizes and irregular shape, embedded in hardened cement paste. Air voids, microcracks and interfacial bond microcracks are caused by the manufacture and the physico-chemical processes taking place during the hardening of the cement. As a consequence of this heterogeneous structure concrete displays a non-linear and time-dependent deformation response under sustained loading. Another significant aspect is that the concrete tensile strength is only a fraction (commonly 5-10%) of its uniaxial compressive strength. This is why the structural applications of plain concrete are rather limited. The problem was solved by embedding steel reinforcement in the concrete zones of the structure where tensile stresses are to be expected. These zones may be affected for instance by external loading combinations and/or by imposed deformations in statically indeterminate structures. The cooperation of concrete and reinforcing steel (either passive or active reinforcement, i.e. prestressing steel) which is due to satisfying bond properties, produced good practical opportunities for concrete in the building-industry since the first part of this century.

It should be realized that the progress and prosperity taking shape in Europe and North-America over the last few decades were often due to the special interplay between 'society-pull' and 'technology-push'. There was a general trend to build more cost-effectively and more efficiently. Moreover, the development of material science contributed to improved mechanical properties of both concrete and steel. These facts actuated the erection of higher, taller and more slender structures. Special attention had to be paid to the structural safety of these buildings; on the one hand more severe loading conditions were met, on the other hand complex and slender structures react rather sensitively to applied loads or deformations.
At the same time completely new fields of interest were explored, which would hardly have been feasible without the use of structural concrete. Examples of sophisticated modern applications are:
- nuclear containment vessels;
- high-rise buildings and long-span concrete bridges;
- offshore platforms and storage facilities for liquified gasses.
These types of structures can usually be characterized by their large-scale dimensions and severe loading conditions. Thus, these phenomena do not allow in advance the use of conventional design techniques and criteria or reliance upon engineering experience only. Local damage or even structural failure could spell dire consequences for the community. The risk for human life associated with these structures could have prolonged effects both temporally and spatially. The civil engineer should attempt to master these potential effects. This demands a thorough analysis of reinforced concrete structural behaviour, taking into account its physical and geometrical non-linear responses. For coping with these problems elaborate computational methods have been developed based on ongoing research.

It should be recognized that a tremendous amount of experimental research has been carried out regarding plain concrete under various loading conditions, as well as structural members under service and limit state conditions. As measuring techniques improved, there was a follow-up of more detailed investigations, such as the bond between concrete and reinforcing bars, strain-softening of plain concrete and aggregate interlock, i.e. shear transfer across cracks. The theoretical research in the seventies focused on the development and formulation of suitable and realistic constitutive laws, describing the observed mechanical behaviour of concrete [71,79,96, 141]. The primary purpose of these efforts was the implementation of the numerical models in finite element programs aimed at simulating and computing the behaviour of complicated reinforced concrete structures. The finite element method offers a model of the structure consisting of an assemblage of simple elements. The elements are coupled by nodal points and basically behave according to the constitutive model with respect to the kinematical and static boundary conditions [45]. The application of non-linear finite element methods demanding fast iterative solution techniques has been supported and extended by the evolution in computer technology, leading to a remarkable increase of memory storage and electronic processing speed.
Regarding the above-mentioned examples of sophisticated concrete buildings, structural problems could occur if excessive loading or restraint deformations introduce cracks. For instance, the seismic excitations of an earthquake may induce a leakage of the cooling system of a nuclear plant, leading to an over-pressurization in the nuclear containment vessel. Safe design criteria should take into account the shear loads which are to be resisted by the cracked concrete [56]. These criteria may also contribute to environmental protection against disasters. Similar structural problems may arise in the substructure of an off-shore platform comprising structural shells resting on the sea bed, see figure 1.1.

![Figure 1.1 Substructure of a reinforced concrete offshore platform.](image1)

![Figure 1.2 In-plane shear transfer in the substructure.](image2)

Oil and gas exploration activities take place in water depths which already exceed 200 meters. Under severe loading conditions the shells act as shear walls. Figure 1.2 shows a part of the substructure; the reinforced concrete shear wall resists horizontal forces (wave and wind actions) and concentrated vertical loads mainly due to the dead weight of the structure. In practice the wall is subjected to horizontal and vertical in-plane shear loads. As a result of unequal settlements and temperature gradients - for example hot oil in tanks accommodated in the substructure and subjected to a subsequent temperature drop - additional cracks may develop [211] inducing a redistribution of internal loads. See figures 1.3a-b. The required stiffness perpendicular to the crack plane is provided by restraint through adjacent concrete elements and/or by the fairly high amounts of reinforcement and prestressing applied in offshore structures.
Both examples presented experience with in-plane shear transfer across existing cracks. Interface shear transfer may vitally contribute to the bearing capacity of structures [29, 33, 46, 53, 56, 67, 79]. While in the case of bending, the behaviour of reinforced concrete members has been extensively investigated and the physical model is generally accepted, there is still a lack of knowledge and modelling in the case of shear forces in cracked concrete. The reason is that shear loading leads to complicated physical mechanisms, such as multiaxial stress conditions with inclined crack formation in the web and compression flange of beams or panels, interlocking of cracks, dowel action of the longitudinal reinforcing bars and reduced bond characteristics between the bars and the concrete. That is why the conventional beam theory with plane cross-sections can not simply be applied to the shear design of structural members.

Some 80 years ago Mörsch [1] developed design formulas for the shear reinforcement of concrete beams assuming a truss model with 45° diagonal compression struts. Today, extensive experimental research has led to modern shear design criteria which have either an empirical or a theoretical background. A few recent developments should be mentioned:

- the yield-line theory or theory of plasticity of Johansen [6] which later was extended to the shear design of reinforced concrete slabs and beams by Nielsen and Braestrup et al. [12, 13];

- the truss model with a variable inclination $\theta$ of the concrete compression field-diagonals, as proposed by Thürlimann [62]. The theory of plasticity accounts for the interaction of bending and shear loading. The design shear force $V_d$ (fig. 1.4a) may depend on the web compression failure mechanism. The distribution of forces is represented by truss action (shear

Figure 1.3 (a)-(b) Additional cracking due to restraint deformations.
reinforcement and inclined concrete struts). By virtue of the action of aggregate interlock in the cracks, a redistribution of forces is possible: the direction of the compression struts decreases with the increase of the load, so that more stirrups are activated. As a result, however, the stress in the concrete diagonals increases too. Failure occurs if the crushing strength of the concrete is reached. The 'refined model' of the CEB-model code 1978 [61] is based on this approach. Here, the inclination of the struts is limited to $\theta = 30^\circ$; the shear friction analogy is a valuable method to estimate the ultimate shear force transmitted across a crack plane in a reinforced concrete member. This approach is widely used by structural engineers in North-America. This method was originally applied to design the shear capacity between precast members and cast-in-place concrete [10,11,18] but was later modified for the design of other structures, such as corbels and shear walls [20,27,30,37,39,40,76]. Due to crack dilatation the reinforcement is stressed and clamps both crack faces together; the maximum shear capacity corresponds to the yielding of the bars, see figure 1.4b.

![Figure 1.4](image_url) (a) Cracked concrete shear wall with web reinforcement [62] and (b) shear friction mechanism according to [20].

The above-mentioned computational methods have been checked and calibrated by means of test results and provide reliable and simple design tools. In modern theories, taking account of the capacity of reinforced concrete to redistribute the internal forces, the frictional resistance of the rough cracks plays an important role. Collins and Vecchio [59,98,99] introduced the compression field theory which takes account of a compatibility condition for the strains of the web reinforcement and of the diagonal compres-
sion struts. They derived relationships for the transmission of forces across cracks based on the modelling of the true physical behaviour of the material [112-114]. Reineck [77,78] stated that the web compression failure mechanism - see figure 1.4a - depends on:
- local strength condition of the material;
- actual crack width (and all the parameters influencing it);
- assumed 'full interlocking' with regard to the cracks;
- deviation of the compression struts from the crack inclination, i.e. \( \theta \) (crack) \( \neq \theta \) (strut). This phenomenon refers to a gradual redistribution of forces which is also considered in the draft of the Eurocode for concrete structures [110].

Recently, the present computational methods were verified in experiments carried out by Kupfer, Mang and Kirmair et al. [84,85,105] and Gambarova et al. [68-71,100]. They all adopted the interlocking relation according to Walraven [112-114]. Similar types of shear tests on cracked concrete panels were performed by Perdikaris et al. [88] and Kollegger et al. [106]. Others used empirical relations - for example, the shear retention factor \( \beta \) - to take account of the shear transfer mechanism of cracks.

In many cases the crack pattern of a loaded panel can be simplified as a series of parallel cracks with known distances, see figure 1.4a. Therefore, much research effort has been devoted to the shear transfer mechanism across a single crack, see figure 1.5. The behaviour of cracked reinforced concrete panels can now be satisfactorily predicted for monotonic short-term shear loading conditions. However, nothing is known about the shear transfer mechanism in concrete in the case of a sustained shear loading [210]. An additional problem concerns cracks in high-strength concrete, in view of the applications to offshore conditions [60,78,92,97,125,146].

Figure 1.5 In-plane shear transfer across a crack in concrete.
Research should be carried out enabling a more accurate prediction of the non-linear response of such structures. For this reason it was decided to start a joint project "Betonmechanica" (concrete mechanics) in The Netherlands.

This project includes experiments, theoretical modelling and numerical implementation of the results in order to extend and improve non-linear finite element programs. The project is being conducted by Rijkswaterstaat (Ministry of transport and public works), TNO-IBBC (Institute for applied scientific research on building materials and building structures) and the Universities of Technology at Delft and Eindhoven, respectively. The supervision and a part of the financial support were supplied by the CUR (Netherlands centre for civil engineering research, recommendations and codes). The first phase of the project which ended in 1982 mainly addressed short-term (static) loading conditions [114,138]. The present study is a part of the second phase which will focus attention on the effects of cyclic and sustained loading conditions [118,134].
1.2 Aim of the study
This research should be applied to predict the time-dependent mechanical behaviour of cracked reinforced concrete structures when subjected to sustained shear loading. The results may also be useful to quantify the redistribution of internal forces. For this purpose the present study will consider the in-plane transfer mechanism of sustained shear loading across a single crack in reinforced and plain concrete. For the static loading case the corresponding numerical formulation is an incremental stress-strain relation which can be used in non-linear finite element programs (see figure 1.5):

\[
\begin{bmatrix}
\Delta \sigma \\
\Delta \tau
\end{bmatrix} = \begin{bmatrix} S_{11} & S_{21} \\
S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
\Delta \delta_n \\
\Delta \delta_t
\end{bmatrix}
\]  \(N/mm^2\)  (1.1)

Equation (1.1) was quantified by means of a physical model developed previously as a part of the concrete mechanics project [112,144]. This model should be adapted now to time-dependent circumstances. As no experimental data were available yet, a number of sustained shear tests were carried out. Special attention was paid to high-strength concrete, small initial crack widths (0.01-0.10mm) and fairly high shear stress levels.

1.3 Contents of this study
This study begins with a literature survey reviewing recent experimental research efforts of shear transfer across a crack in concrete; the aggregate interlock and dowel action mechanisms are both treated for the monotonic loading case. Chapter 2 also deals with some theoretical models which describe the observations on an empirical or a phenomenological basis. Chapters 3 and 4 outline the set-up of an experimental program and include the most important test results, especially regarding the time-dependent displacement increases of the opposing crack faces due to an external sustained shear loading.

Chapter 5 is concerned with modelling: the crack response is related to the time-dependent behaviour of cement-based materials (creep) and of the embedded bars across the crack (bond). Chapter 6 addresses the application of this research for a two-dimensional cracked concrete panel subjected to sustained shear loading. Today, these engineering models have only been developed for monotonic loading conditions. The notation and a list of references are given in the last part of this study.
2. LITERATURE SURVEY

2.1 Basic mechanisms of shear transfer
In chapter 1 the in-plane shear transfer in cracked reinforced concrete panels has been formulated as the starting-point of this research. Special attention should be paid to the time-dependent structural behaviour when cracks are subjected to sustained shear loading. However, this specific problem has not yet been addressed in the literature. Hence this chapter is restricted to a brief literature survey of the transfer mechanisms occurring under monotonic shear loading.

Figure 2.1a shows a unit area of the crack plane which is crossed by one embedded reinforcing bar at a random angle $\theta$ ($0^\circ \leq \theta < 90^\circ$). The initial crack displacements are a separation $\delta_{no} > 0$ and a slip $\delta_{to} = 0$. In medium-strength concrete, cracks usually initiate along the relatively weak bond zones between the matrix material and the aggregate particles. The contact areas between the opposing crack faces depend on the mix composition and on the actual crack displacements [112,114]. The application of a shear stress $\tau$ results in an increase of these displacements; meanwhile the contact areas tend to diminish gradually. The displacement values depend on the normal and shear stiffness of the crack plane. The reinforcing bar in figure 2.1b accounts for a restraint of the crack plane. Once the axial bar stiffness is too low, for instance due to lack of bond or yielding of the steel bar, no equilibrium can be found and the displacements increase more and more. This case is defined as shear failure which is characterized by the shear strength $\tau_{u}$.

![Figure 2.1 Unit area of crack plane in (a) unloaded and (b) loaded state.](image-url)
The external shear load is transferred partly by the interlocking of the opposing crack faces and partly by the embedded reinforcing bars. In figures 2.2a-b these two transfer mechanisms can be identified separately:

(a) **Aggregate interlock of the rough crack faces.** Generally, the crack follows an irregular path and shows an uneven surface. Shear transfer is provided by the mechanical locking of the particles protruding from one face and pushing into the matrix of the opposing crack face. The interlock mechanism is stipulated to occur if the crack plane is sufficiently restrained;

(b) **Dowel action of the reinforcing bar.** Dowel action is defined as the load carrying capacity of a bar in the direction perpendicular to the longitudinal bar axis. For $\theta \neq 90^\circ$, the bars will contribute to the stiffness parallel as well as perpendicular to the crack plane.

![Diagram](image)

Figure 2.2 (a)-(b) Basic mechanisms for shear transfer across a crack; $\theta=0^\circ$.

It is important to consider the interaction of both mechanisms which takes place in cracked reinforced concrete. The application of shear stress causes slip and in addition causes a tendency for the crack surface to separate slightly. The reinforcing bar is stressed in tension; the steel tensile strains and the bond slip permit a crack width increase. Moreover, the steel bars restrain the crack plane and hence they influence the dowel mechanism, which initially determines the direction of the crack-opening \[148\]. In assessing the mechanisms of shear transfer it is clear that the primary variables are \[112,114\]:

- concrete grade and bar characteristics (diameter, steel yield strength);
- restraint perpendicular to the crack plane.

Section 2.2 deals with previous experiments which relate to the static shear transfer mechanisms. Emphasis lies on the qualitative influence of variables on the crack response. Section 2.3 reviews a few theoretical models, as well as numerical modelling techniques.
2.2. Experiments on shear transfer
2.2.1 Aggregate interlock mechanism

One of the first investigations was carried out by Colley and Humphrey [23] in 1967 and later by Nowlen [26]. The concrete pavements tested were provided with a transverse centric crack and were supported uniformly by a subsoil (figure 2.3). Alternating repeated shear forces were applied on either side of the crack plane thus simulating a heavy traffic loading. The number of cycles to failure depended on the type and size of the aggregates used and it significantly decreased as a certain crack width was exceeded.

![Figure 2.3 Test set-up of Colley and Humphrey [23] and Nowlen [26].](image)

In the years 1968-1980 several static shear tests were conducted on cracked concrete. Usually, the test set-up concerned two plain concrete blocks separated by a preformed crack which was sufficiently restrained. Shear force-displacement relations were established by, amongst others, Fenwick and Paulay [29], Taylor [32,33], White and Holley [42] and Laible et al. [56]. Either a constant crack width was maintained or the crack width was variable. In general the initial crack widths were rather large, i.e. 0.25-0.75mm. They are related to the field of application; reinforced concrete wall structures with flexural or shear cracks caused by external loads and/or imposed deformations, cracks due to over-pressurization in a nuclear containment vessel, etcetera. 'Free' sliding of the crack faces occurred before they made contact. The test results were significantly affected by the size and type of aggregate, the concrete grade, $\delta_{no}$ and by the restraint stiffness.

Paulay and Loeber [48] carried out displacement-controlled (with respect to $\delta_t$) static shear loading tests, see figure 2.4a. The crack width, slip and restraining stress perpendicular to the crack plane were recorded. Three
types of gravel aggregate (rounded: 9.5mm and 19mm max. size and crushed: 19mm), as well as three different constant crack widths were chosen as experimental variables. The 102mm cube concrete strength ranged between 36 and 40 N/mm². One empirical shear stress-displacement relation was found for the variable crack width tests (constant stress-crack width ratio):

\[ \tau_a = 0.51 + 7.077 \sqrt{\delta} \]  

\[ [\text{N/mm}^2] \]  

(2.1)

This formula resembles well with the predicted curve as derived from the constant crack width tests, see figure 2.4b. Houde and Mirza [50] performed similar push-off tests and found that \( \tau_a \) is almost proportional to \( \sqrt{f_{cy}} \) and \( \delta_{no}^{-1.5} \).

Walraven et al. [112,114] used 32 push-off type specimens similar to those of Mattock [37-40], see figure 2.5a. By means of nuts the external restraint rods were fastened to stiff steel plates fixed on the small sides of the specimens. Dowel action of these bars was negligible. The shear loading was applied in a displacement-controlled manner. The variables of the tests were: initial crack width (0.01; 0.2 and 0.4mm), 150mm cube strength and type of aggregate (Fuller grading curve, gravel: \( f_{cc} = 19.9-56.1 \text{ N/mm}^2 \), Korlin light-weight aggregates: \( f_{cc} = 38.2 \text{ N/mm}^2 \)) and the maximum size (16mm but \( D_{max} = 32 \text{mm} \) for the high-strength type of concrete). The change of the rod diameter enabled variation of the normal stiffness to the crack plane. The test results of six specimens are presented in figure 2.5b; \( \delta_{no} \) had a considerable influence, but there was a rather slight effect.
on $\tau_a - \delta_t$ relations of $D_{\text{max}}$ in the range tested. Empirical bilinear stress-displacement relations were found which accurately fit to the data recorded:

$$\tau_a = \frac{f_{\text{cc}}}{30} + [1.80 \delta_n^{-0.80} + (0.234 \delta_n^{-0.707} - 0.20) f_{\text{cc}}] \cdot \delta_t \quad [\text{N/mm}^2] \quad (2.2a)$$

$$\sigma_a = \frac{f_{\text{cc}}}{20} + [1.35 \delta_n^{-0.63} + (0.191 \delta_n^{-0.552} - 0.15) f_{\text{cc}}] \cdot \delta_t \quad [\text{N/mm}^2] \quad (2.2b)$$

Similar equations were obtained for the light-weight concrete which exhibited a less steep crack-opening curve indicating a relatively smooth crack surface, probably caused by cracks that run mainly through the aggregate particles which are weaker than the matrix material.

**Figure 2.5** Tests of Walraven et al. [112,114]; (a) specimen with external restraint rods and (b) shear stress-displacement relations and measured crack-opening curves for normal-weight concrete.

Reinhardt [129,132] pointed out that the linear fracture mechanics approach can be applied to crack growth phenomena in concrete. Normal and shear stresses on a crack plane correspond to modes I and II respectively. Shear failure is related to the critical stress intensity factor of the material. Thus for crack lengths $c_1 = \lambda c_2$ ($0 < \lambda < 1$) then $\tau_{u,1} = \tau_{u,2}/\sqrt{\lambda}$. This relation shows good agreement with shear test results of beams unreinforced in shear [111,129]. It can be deduced that the ratio $\delta_t/\delta_n$ of eqs. (2.2a-b) increases for small $\lambda$-values assuming that crack width and crack length are geometrically scaled. Today, more sophisticated non-linear fracture mechanics approaches exist.
Recently, extensive research projects focused on the shear transfer of plain concrete with a relatively small initial crack width of 0.05-0.20 mm. Tests were conducted by Millard et al. [64,65], Daschner et al. [80,81], Sture [83], Nissen [101] and Divakar et al. [104].

Tassios and Vintzeleou [93,103] investigated prismatic concrete blocks in which two small parallel cracks ($\delta_{no} < 0.1$ mm) were initiated, see figure 2.6a. The variables investigated were: roughness of the interface (smooth, sand-blasted or rough), concrete cylinder strength ($f_{cv} = 16-40$ N/mm$^2$ using crushed limestone aggregates with $D_{max} = 30$ mm) and the constant compressive stress on the crack plane ($\sigma_c = 0.5-2.0$ N/mm$^2$). Figure 2.6b shows that the friction coefficient of a rough interface depends on the compressive stress. These results qualitatively correspond to those of Daschner et al. [80,81], Walraven [112,114], Leichnitz [89] with respect to rock joints. Interface characteristics of concrete are also related to other granular materials such as rock, soil, etcetera [89,96,141]. Figure 2.6c shows that the crack-opening curves seem to be influenced by the normal compressive stress which is in accordance with results of other researchers [64,101,104,112], who investigated rather high $\delta_{no}$-values. Bilinear relations ($\delta_n = 0.7\delta_t$ and $0.4\delta_t$ for larger slips) are proposed based on quantitative observations of the crack roughness. The general trend of these curves and those of Walraven [112,114] is described by $\delta_n = 0.6\delta_t^{2/3}$ although significant scatter is found, probably due to the ignored influence of the normal stress. Another similarity between both research projects concerns the shear stress displacement relation for constant normal stress.

Figure 2.6 (a) Test set-up of Tassios et al. [93,103]; (b) $\mu$-$\sigma_c$ relations and (c) measured crack-opening paths.
2.2.2 Dowel mechanism

The dowel mechanism of embedded steel bars which cross a crack is related to the shear transfer in reinforced concrete. The following subjects will be outlined in this section:

- a short review of the first studies of the dowel mechanism, including the load-displacement behaviour and the dowel strength;
- recent experimental research on dowel action in two-dimensional elements, such as containment vessels, shell structures, etcetera.

One of the first research projects conducted by Teller and Sutherland [4], concerned dowel action of transverse construction or expansion joints in concrete pavements. At the same time Timoshenko et al. [3] modelled the dowel as a beam of semi-infinite length, placed on a foundation assuming linear-elastic material behaviour. The mathematical solution of the fourth order differential equation was verified later by Friberg [5] for concrete applications. According to the theory it follows that (figure 2.7a):

\[
\delta_y(x = 0) = \frac{V - \beta M_0}{2. \beta^3 E_s I} \quad \text{and} \quad L = \frac{1}{\beta} \arctan(1 - \frac{V}{\beta M_0}) \quad \text{[mm]} \quad (2.3)
\]

for \( \beta = \frac{4k_d b}{(4E_s I)} \) [mm\(^{-1}\)]; \( k \) = modulus of subgrade support [N/mm\(^3\)]; \( E_s I \) = flexural stiffness of the beam [Nm\(^2\)]. If \( M_0 = -V\delta_n/2 \), it yields that:

\[
M_{\text{max}} = \frac{-V e^{-\beta L'}}{2. \beta} \sqrt{1 + (1 + \beta \delta_n)^2} \quad \text{[Nmm]} \quad (2.4a)
\]

\[
L' = \frac{1}{\beta} \arctan\left(\frac{1}{1 + \beta \delta_n}\right) \quad \text{[mm]} \quad (2.4b)
\]

![Figure 2.7 Beam on an elastic foundation: (a) long beam; (b) short beam and (c) reinforcing dowel bar close to a crack plane [4,7].](image-url)
The ratio of consecutive extreme values of the bending moment or the shear loading is $e^{-\pi} = 0.043$ in the x-direction. Thus the behaviour close to the crack hardly changes if the bar ends at point P in figure 2.7b. Assuming $k = E_c/d_b = 3.10^4/d_b$ [N/mm$^3$] so that $\beta = 0.92/d_b$, then it can be found that $L_{\min} = L' (eq.(2.4b)) + 2\pi/\beta = 8d_b$. This result agrees with other investigations [7,8]. Further computations yield $M_{\max} = 0.25V_d$ to $0.42V_d$ and $L' = 0.85d_b$ to $0.76d_b$ for $\delta_n = 0$ and $\delta_n = 0.2d_b$ respectively.

If $k$ is halved then $M_{\max}$ and $L'$ increase only by 15%-19%; $\beta \to \infty$ gives the elastic deflection of a cantilever fixed at the crack plane (figure 2.7c). Experimental values of $k$ differ widely, $k = 500 - 1500$ N/mm$^3$ [9]. A particular reason is the non-linear behaviour of the concrete due to crushing; thus $k$ should vary parallel to the bar axis. Moreover, the dowel behaviour is influenced by the casting direction, the concrete strength, the bar characteristics and the position of the bar.

According to the theory of elasticity the supporting concrete situated directly under the bar is subjected to radial and circumferential stresses, see figure 2.8a. Marcus [8] investigated uniformly loaded embedded reinforcing bars (figure 2.8b). He found that the bearing strength exceeds $f_{\text{cyl}}$. Fig. 2.8c shows that this ratio depends on the bar diameter and the embedment length. In general the bearing stresses in the concrete strongly account also for the shear transfer of the dowel, besides the considerable contribution of the steel bar itself. Broms [15,16] related the dowel mechanism to the lateral resistance of foundation piles in cohesive soils. The proposed model agreed well with measured pile-deflections at the soil sur-
face (about 0.2d_b at ultimate loading). The pile deformation depends on the position of the plastic hinge in the cross-section provided that the beam is long, i.e. \( \beta l > 5 \). The model of a beam on elastic foundation is not appropriate here. A similar behaviour can be expected in the case of a dowel embedded in concrete. Due to a redistribution of the reaction forces the maximum bending moment may move away from the surface. Sorouheshian et al. [108] established that the bearing strength \( f_b \) and the modulus of subgrade support were proportional to \( \sqrt{f_{cyl}} \) and to \( d_b^{1/3} \) which closely agrees with figure 2.8c; \( f_b/f_{cyl} \) ranged from 1.2 to 3.0.

Recent experimental research. Basically three types of investigation were performed (figures 2.9a-c). The direct shear tests are often related to small diameter bars and a thick concrete cover, so that the bearing capacity is governed by steel yielding and concrete crushing under the bar. Divided-beam and beam-end tests were often developed in order to study the concrete splitting failure mechanism and the anchorage length of the bar.

![Figure 2.9 (a)-(c) Different types of dowel test set-up.](image)

Rasmussen [14] carried out ten direct dowel tests on smooth steel bars (figure 2.10a) with \( f_{cyl} = 11-44 \) N/mm\(^2\); \( f_{sy} = 225-439 \) N/mm\(^2\) and \( d_b = 16-26 \) mm. On the basis of a simple model according to figure 2.10b, he found:

\[
V_{du} = c.\left[\sqrt{(cc)^2 + 1 - (cc)}\right] \cdot d_b^2 \cdot \sqrt{f_{cyl}} \cdot f_{sy} [\text{N}] (2.5)
\]

where \( \epsilon = 3.0.\sqrt{f_{cyl}/f_{sy}} / d_b \). From tests \( c = 1.31 \) was found if \( \epsilon \) is neglected. The mean bar deflection was 0.17d_b at 0.92V_{du}. Dowel tests of Friberg [5] and Marcus [8] also show a proportionality of \( V_{du} \) and \( d_b^2 \). Dulacska [44] carried out 16 direct dowel tests, see figure 2.11a. Interlocking was eliminated by means of two lubricated 0.2mm thick brass sheets placed in the shear plane. The major parameters investigated were the
effects of $\theta$ (10-40°); $d_b$ (6.5-14mm) and $f_{cc}$ (10-32 N/mm²). In general, the measured load-deflection relations indicate an elasto-plastic dowel behaviour (figure 2.11b). At a slip $\delta_t = \delta_{no} = 0.4$mm the dowel strength was almost reached; at $0.8V_{du}$ the shear slip was $1/2.8t = 0.145d_b$ for $e = 1/2.8n = 0.036d_b$ on average. Based on a load transfer model presented in figure 2.11c, the following formula was derived:

$$V_{du} = 0.2d_b^2 \rho f_{sy} (\sin \theta) \left[ \sqrt{1 + \frac{f_{cc}}{0.03 f_{sy} \sin^2 \theta}} - 1 \right] \text{[N]} \quad (2.6)$$

where $\rho = 1 - (N/N_{sy})^2$ accounts for the axial bar force. Note that if the actual dowel force eccentricity is considered, then the average ratio of calculated and measured $V_{du}$-values is reduced from 1.14 to 1.05. Additional investigations on dowel action are reported in [29,49,54].

Figure 2.11 (a) Test specimen of Dulacska [44]; (b) typical load-displacement curve and (c) proposed model for dowel action.
Mills [52] assumed a bilinear bearing stress diagram under the dowel. Based on his own tests and on results of Dulacska [44], he found that:

\[ V_{du} = 1.5 \cdot d_b^2 \cdot (13.3 \sqrt{f_{cyl}} - f_{cyl} \cdot \tan \theta) \]  

[N]  

(2.7)

Note, that for \( f_{Sy} = 284 \text{ N/mm}^2 \) (mean value of tests) and \( \theta = 0^0 \), eq. (2.7) gives on average 9% lower \( V_{du} \)-values than Rasmussen’s formula. Results of dowel action experiments of Millard et al. [64] displayed that the measured initial shear stiffness of a dowel can be predicted quite accurately by the model of a beam on an elastic foundation for \( k = 750 \text{ N/mm}^3 \) (\( f_{cc} = 35 \text{ N/mm}^2 \)). \( V_{du} \)-values are also satisfactorily described by eq. (2.5) using \( \rho \) of eq. (2.6) for the tests with \( \sigma_{so} > 0 \). Now both models can be combined using eq. (2.3) with \( \delta_y(x=0) = 1/2. \delta_t, \rho = 1.0 \) and \( M_o = 0 \):

\[ \xi = \frac{(dV_d/d\delta_t) \cdot \delta_t}{V_{du}} \geq \frac{0.167k^{3/4} \cdot E_s^{1/4} \cdot d_b^{7/4} \cdot \delta_t}{1.3d_b^2 \cdot \sqrt{\rho f_{Sy} \cdot f_{cc}}} \approx 3.27d_b^{-1/4} \cdot \delta_t \]  

[-]  

(2.8a)

The shear load-displacement curve could be approximated with:

\[ V_d = V_{du} \cdot [1 - \exp(-\xi)] \]  

[N]  

(2.8b)

Eq. (2.8a) neglects the influence of loading eccentricity \( e \). Jimenez et al. [67] found that the 'initial' shear stiffness was about proportional to \( d_b^{9/4} \). The authors reported that the axial steel stress caused significant damage to the concrete surrounding the bar on both crack halves. Utescher et al. [86] investigated the behaviour of smooth embedded dowels \( (d_b = 14-25 \text{mm with } f_{Sy} = 270 \text{ N/mm}^2; \text{ embedment length } l = 6d_b-10d_b) \). In the case of a thick concrete cover \( (c_\| > \text{approximately } 5d_b) \) a small crater-shaped area was observed close to the bar, indicating high local bearing stresses. Failure was satisfactorily described by Rasmussen's eq. (2.5).

Paschen et al. [87] computed the three-dimensional stress and strain fields in the concrete close to an embedded dowel and found that the principal strains are significantly changed in a spherical volume of \( 3d_b \) diameter close to the concrete surface. Enhanced bond properties occurred under the dowel bar. Mills [52] reported vertical splitting cracks under the dowel. The \( V_{du} \)-values were approximately proportional to \( f_{cc}^{2/3} \) which indicates that the concrete tensile strength is an important parameter for this
phenomenon. In case of steel yielding the plastic moment was situated about $d_b$ away from the concrete surface. Vintzeleou et al. [93,94] found a distance of $0.6d_b - d_b$. They performed dowel action tests. Each specimen had two equal embedded reinforcing bars ($d_b = 8-18$mm). Based on the beam on elastic foundation theory, simple approximations were found for $V_{dcr}$. In the case of steel yielding, the bar is modelled as a pile, see Broms [15,16]. The plastic yielding moment of the bar is expressed as a function of the load eccentricity and the bearing strength which is assumed to be doubled due to local confinement [1], so that:

$$V_{du}^2 + (10f_{cyl,d_b}).V_{du} - 1.7d_b^4.f_{cyl}.f_{sy} = 0 \quad [N^2] \quad (2.9a)$$

and:

$$\delta_{tu} = \delta_{tel} + \delta_{tpl} \approx 0.05d_b \quad [mm] \quad (2.9b)$$

For $e=0$ eq. (2.5) is found. Vintzeleou only qualitatively indicated the reduced dowel action in case of axial stresses in the bar.

With respect to the divided-beam tests (figure 2.9b) several experiments were carried out; information about the test set-ups is summarized in table 2.1. Nearly all the specimens exhibited longitudinal (side) splitting and a residual dowel strength of $0.5-0.8V_{dcr}$ was reported. Other research was conducted by Johnson and Zia [36], Houde et al. [50] and Kemp and Wilhelm, as reported in [79].

<table>
<thead>
<tr>
<th>Reference</th>
<th>no. of tests</th>
<th>$f_{cyl}$</th>
<th>$d_b$</th>
<th>no. of bars</th>
<th>$\phi/d_b$</th>
<th>$\delta_{no}$</th>
<th>stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td>[22]</td>
<td>12</td>
<td>19</td>
<td>22-29</td>
<td>2</td>
<td>1.2-2</td>
<td>0.6</td>
<td>no</td>
</tr>
<tr>
<td>[31,33]</td>
<td>46</td>
<td>13-39</td>
<td>6 or 22</td>
<td>2</td>
<td>1.2</td>
<td>1.5</td>
<td>yes</td>
</tr>
<tr>
<td>[34]</td>
<td>31</td>
<td>12-62*)</td>
<td>16-26</td>
<td>2-8**)</td>
<td>1-2.2</td>
<td>0.1-10</td>
<td>yes</td>
</tr>
<tr>
<td>[95]</td>
<td>14</td>
<td>-</td>
<td>10-22</td>
<td>2</td>
<td>2.2-3.1</td>
<td>2</td>
<td>no</td>
</tr>
</tbody>
</table>

*) $f_{cc}$-values     **) in two layers
2.2.3 Combined mechanism

In all practical situations where shear forces should be transferred across a crack, both the aggregate interlock and the dowel action mechanisms will simultaneously be activated as pointed out in section 2.1. In the preceding sections 2.2.1 and 2.2.2 the individual mechanisms have been experimentally isolated to assess their most important parameters. Attention will now be paid to the observed shear failure and to the shear-friction hypothesis for cracked reinforced concrete. Next, tests are shortly reviewed which more or less refer to the state of serviceability of cracked structural concrete.

Observations near shear failure. Part of the research originally focused on the shear transfer problem of a slab. Investigations are reported by, amongst others, Johansen [6], Nielsen [12], Morley [18], Prince and Kemp [25], Mills [52] and Millard [63]. However, all the tests neither consider effects of confinement of the reinforcement near the crack plane due to concentrated bearing stresses (multiaxial state of stress) nor shear transfer due to aggregate interlock. Many static in-plane push-off tests have been performed, originally intended to determine the shear strength of reinforced connections between precast and cast-in-place concrete. See also Anderson [10], Hanson [11], Fenwick et al. [29] and Mattock et al. [30,37,39,40]. Other research efforts are summarized in table 2.2. From the observations it can be concluded that:

- **Low reinforcement ratios.** Steel yielding and slip in the shear plane occurred. For $\rho f_{sy} < 4 \text{ N/mm}^2$ the concrete strength did not affect $\tau_u$;

- **High reinforcement ratios.** The cracked and uncracked specimens exhibited diagonal tension cracks at an angle of 40-50° with the shear plane. See figure 2.13a. Crack spacings and crack lengths were 50-100mm. Slip took place as a result of rotation of the concrete struts between the cracks. The crack roughness and the high reinforcement ratio provided a 'locking up' so that the shear strength was hardly affected by the presence of the cracked shear plane. This phenomenon did not occur in light-weight concrete, probably due to the minor crack roughness [40];

- for equal values of $\rho f_{sy} + \sigma_N$ the shear strength was hardly influenced by a change of $d_b$, $f_{sy}$ or by the sign of $\sigma_N$ subjected perpendicular to the shear plane. Concrete corbels displayed a similar behaviour [19,27,46];

- Mattock et al. [37] also used reinforcement provided with rubber sleeves 25mm on each side of the crack plane, leading to reduced shear strengths for $d_b = 9.5\text{mm}$. Ultimate slips were found to be six times higher.
Walraven [112] pointed out that the sleeves reduce both dowel action and the bond behaviour of the bars. Paulay et al. [49] reported that the contribution of dowel action is usually not more than 16%, but this ratio may increase if significant slips can occur. Pruijssers [148] also dealt with this subject.

The shear friction hypothesis for cracked reinforced concrete was proposed by Birkeland and Birkeland [20], see figure 2.13b. Equilibrium can be expressed in terms of stress:

$$ \tau_u = \rho \cdot f_{sy} \cdot \tan(\phi) \quad \text{[N/mm}^2\text{]} \quad (2.10a) $$

where $\tan(\phi)$ is the coefficient of internal friction (based on tests [10, 11] $\tan(\phi) = 1.7$ for monolithic concrete; $\tan(55^\circ) \approx 1.4$ for artificially roughened construction joints and 0.8-1 for ordinary construction joints). Taylor and Broms [17] found $\tan(\phi) = 0.62$-0.73 for the interface strength of aggregate connected to mortar; Clarck [91] measured $\tan(\phi) = 0.75$ for smooth joints in plain concrete. However, these values are related to uniaxially loaded specimens. In the case of lateral compression (figure 2.13c) $\sigma_2 \approx 0.1\sigma_1$ is found for $\tan(\phi) = 1.4$. Mast [27] proposed to use $\rho f_{sy} + \sigma_N$ in the above formula. Moreover, a cohesive strength (or: 'dowel strength') was added according to [30, 37, 39, 49, 58, 102]:

Figure 2.12 Push-off specimen used by (a) Anderson [10]; (b) Hanson [11] and (c) test results.
\[ \tau_u = 2.8 + 0.8(\rho \cdot f_{sy} + \sigma_N) \]  
\[ \text{[N/mm}^2\text{]} \] (2.10b)

where \( \tau_u < 0.3f_{cyl} < 10.8 \text{ N/mm}^2 \). Supplementary tests of Mattock et al. [40] revealed cohesive strengths of 1.40-1.75 N/mm\(^2\) in the case of light-weight concrete, with \( \tau_u < 0.2f_{cyl} < 5.6 \text{ N/mm}^2 \).

Figure 2.13 (a) shear transfer in initially uncracked concrete [37]; (b) shear-friction model and (c) \( \alpha \) acc. to Mohr-Coulomb criterion.

Table 2.2 Overview of push-off tests (\( D_{max} = 16-22\text{mm} \)).

<table>
<thead>
<tr>
<th>reference</th>
<th>type of( \ast )</th>
<th>number of( \ast * )</th>
<th>( f_{cyl} ) ( \ast * \ast )</th>
<th>( \rho f_{sy} ) ( \ast \ast * \ast \ast )</th>
<th>( d_b )</th>
<th>( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[30,37]</td>
<td>ps</td>
<td>15/23</td>
<td>17-36</td>
<td>0.4-10.2</td>
<td>9.5-16</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>pl</td>
<td>6/6</td>
<td>35</td>
<td>1.3-5.3</td>
<td>6.4-9.5</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>mp( \ast * * )</td>
<td>6/10</td>
<td>28-45</td>
<td>2.2-6.9</td>
<td>9.5</td>
<td>0-75°</td>
</tr>
<tr>
<td>[39]</td>
<td>cp</td>
<td>0/12</td>
<td>28</td>
<td>2.5-4.1</td>
<td>9.5-13</td>
<td>e &gt; 0</td>
</tr>
<tr>
<td></td>
<td>ps( \ast * * * )</td>
<td>9/6</td>
<td>28</td>
<td>3.6-5.9</td>
<td>9.5</td>
<td>-</td>
</tr>
<tr>
<td>[40]</td>
<td>ps</td>
<td>28/32</td>
<td>18-42</td>
<td>0.9-9.8</td>
<td>9.5</td>
<td>lightw. c.</td>
</tr>
<tr>
<td>[49]</td>
<td>ps</td>
<td>6/30</td>
<td>21-31</td>
<td>0.9-4.3</td>
<td>6.5-13</td>
<td>-</td>
</tr>
<tr>
<td>[112]</td>
<td>fig. 2.5a</td>
<td>0/8</td>
<td>29</td>
<td>2.4</td>
<td>8</td>
<td>45-135°</td>
</tr>
</tbody>
</table>

\( \ast \) cp = corbel push-off ps = push-off pl = pull-off mp = modif. push-off
\( \ast \* \) uncr/cr. \( \ast \* \* \) \( \sigma_N \) compression \( \ast \* \* \* \) \( \sigma_N \) tension \( \ast \* \* \* \* \) \( f_{sy} = 298-465 \text{ N/mm}^2 \)

Observations before failure. The first investigations focused on the shear stress-displacement behaviour of cracked and uncracked push-off specimens.
Displacement-controlled shear tests on cracked reinforced concrete specimens were also conducted by Walraven et al. [112,114], for $\delta_{n0} = 0.01-0.09$mm. See table 2.2 and figures 2.5a and 2.14a-b. The crack-opening curves appeared hardly influenced by the reinforcement characteristics ($d_b = 4-16$mm; $\theta = 0-90^\circ$; $\rho = 0.56-2.24\%$ and $D_{max} = 16$mm). The following empirical relations were found for $f_{cc} = 20-40$ and $56$ N/mm$^2$ resp.:

$$\delta_t = 1.40\delta_n^{1.2}\quad\text{and}\quad\delta_t = 1.87\delta_n^{1.4}$$

(2.11)

Only for $\rho < 1.0\%$ there was an influence of $D_{max}$. The light-weight and the high-strength gravel concrete revealed rather flat crack-opening curves (also observed on four specimens provided with short rubber sleeves near the shear plane). In contrast with an unreinforced crack, the crack-opening curve of a reinforced crack is hardly affected by the axial stiffness. Moreover, the opening of the crack after the shear test revealed a considerable amount of loose particles which enabled a so-called 'second mechanism' of interlock. The corresponding force is directed normal to the crack-opening path. Large forces occur for higher $\rho$. A calculated superposition of the dowel mechanism and the two interlock mechanisms shows good agreement with the measurements. Supplementary tests were carried out by Jimenez et al. [67] and by Vintzeleou [93]. Internal crack inspection by Millard [65] after the push-off of the cracks, displayed uniform crack widths and a small bond slip of the bars ($\delta_{cs} = 0.1$mm). The cone-shaped cracked areas near the bar as observed by Walraven [112] were not reported.

Figure 2.14 Tests of Walraven [112]; (a) $f_{cc} = 31$ and (b) $f_{cc} = 56$ N/mm$^2$. 

![Figure 2.14 Tests of Walraven [112] (a) $f_{cc} = 31$ and (b) $f_{cc} = 56$ N/mm$^2$.](image-url)
2.3 Modelling of shear transfer

2.3.1 Theoretical models

In this section the modelling of the in-plane shear transfer in cracked concrete will be outlined for a monotonically increasing shear loading. The physical observations presented in the sections 2.1-2.2 will be described mathematically, accounting for the mechanically-based conditions of equilibrium, constituency (i.e. the stress-deformation relation of the material) and compatibility. The displacement response of the crack is complex and highly non-linear: analytical expressions approximating the test data can often be formulated by means of statistical methods. However, the final models should be physically based and should permit implementation in finite element programs as will be pointed out in section 2.3.2.

Two extreme crack response curves can be distinguished for the case of a displacement-controlled shear loading [79,101], namely retaining a constant crack width, related to an infinite normal stiffness of the crack plane, or a constant normal stress which can be achieved by a constant external normal force together with a zero normal stiffness. See figures 2.15a-b.

In actual structures the crack plane is often partially constrained by means of reinforcing bars crossing the crack. Apart from a certain normal stiffness \( \frac{d\sigma}{d\delta_n} \), the dowel and aggregate interlock mechanisms provide a shear stiffness \( \frac{d\tau}{d\delta_t} \). Empirical values of the normal and shear stiffness with respect to both mechanisms are reported in [29,50,93,94] and can also be derived from eqs. (2.1), (2.3) and (2.8a) respectively.

![Figure 2.15 Shear stress-displacement behaviour of a crack for (a) constant crack width and (b) constant normal stress.](image-url)
A theoretical model should consider the interaction between the stresses and displacements ($\sigma$, $\tau$, $\delta_n$, $\delta_t$). In general, the crack behaviour may show a slight path-dependency which is also observed in other granular materials [89,141]. A few recently developed theoretical models will be reviewed:

a. Rough-crack model of Bazant and Gambarova [70,71,74,75]

This model gives a mathematical description of the observed crack behaviour. The interface stresses are assumed to depend on the displacement ratio $R = \delta_t/\delta_n$. 'Free sliding' can occur ($\sigma_a = 0$) until both crack faces make contact. The maximum shear stress is stipulated by the crushing of mortar material (figure 2.16). The aggregate particles have a Fuller grading curve. The formulae presented are based on shear tests of Paulay et al. [48], but may also be used for other research [29,80,108,112]:

$$\tau_a = 0.25f_{cyl} \cdot \left(1 - \sqrt{\frac{2\delta_n/\delta_{max}}{R}}\right) \cdot \frac{a_3 + a_4 \cdot |R|^3}{1 + a_4 \cdot R^4} \quad [N/mm^2] \quad (2.12a)$$

$$\sigma_a = -a_1 \cdot a_2 \cdot \tau_a \cdot \frac{\delta_n \cdot R}{(1 + R^2)^{0.25}} \quad [N/mm^2] \quad (2.12b)$$

where $a_1, a_2, a_3$ and $a_4$ are constants related to $f_{cyl}$. Note, that $\tau_a$ has a boundary value of $0.25f_{cyl}$ which is in agreement with other tests [91]. The crack-opening curves are restricted to $\delta_t = c.\delta_n$ ($a > 1$). The model does not describe shear transfer in reinforced cracks due to lack of reliable test data. Later, Gambarova [69] proposed to implement the tension-stiffening effect of embedded bars. Recently, the model was also used to determine the contribution of aggregate interlock to the shear transfer of cracked reinforced concrete beams [100].

Figure 2.16 Calculated response according to eqs. (2.12a-b), see [69-71,74].
Bážant and Gambarova [70,71,75] also developed a so-called micro-plane model which has the advantage of path-dependency. The concrete is considered as a system of randomly orientated weak planes, which represent the thin layers of matrix material between the aggregates. In these planes the cracks are concentrated in a crack-band. The matrix deterioration is expressed in terms of stress-strain relations providing an easy implementation in finite element programs.

b. Two-phase model of Walraven [112,114]

This model suggests that concrete is a two-phase material consisting of stiff aggregate particles embedded in an ideally-plastic cement matrix (figures 2.17a-b). In gravel concrete the low bond strength between the matrix and these particles may usually lead to crack initiation. The particles are idealized as spheres, as suggested - in the qualitative sense - previously by Nowlen [26]. The shear plane consists of a distribution of rigid spheres of a range of sizes embedded to various depths in the matrix material. The model does not consider interaction between spheres from opposite crack faces. An expression is derived to predict the chances of finding a particular sized aggregate particle at a certain embedment depth. Equilibrium is related to frictional sliding and crushing of matrix along the contact areas $a_x$ and $a_y$ (figure 2.17c), which depend on $\delta_t, \delta_n$ and the mix proportions ($D_{\text{max}}$ and the volumetric percentage of aggregate). A Fuller grading curve is used for the particle distribution. The constitutive relations of the crack are unique - i.e. there is path-independency - acc. to:

$$
\sigma_a = \sigma_{pu} \cdot (A_x - \mu A_y) \quad \text{and} \quad \tau_a = \sigma_{pu} \cdot (A_y + \mu A_x) \quad \text{[N/mm}^2\text{]} \quad (2.13a)
$$

where: $\mu = \tau_{pu} \cdot \sigma_{pu} = \text{coefficient of friction} = 0.40 \quad [-] \quad (2.13b)$

$$
\sigma_{pu} = \text{matrix yield strength} = 6.39 f_{cc}^{0.56} \quad \text{[N/mm}^2\text{]} \quad (2.13c)
$$

and: $A_x = \Sigma a_x, A_y = \Sigma a_y = \text{contact areas per unit area of crack plane}.$

The model closely agrees with Walraven's static shear tests described in section 2.2.1 (figure 2.5b) and with the experiments of Paulay et al. [48] for a given normal restraint 'stiffness' of the crack plane. Combining eqs. (2.13a-c) results in the curves presented in figure 2.17d. It can be seen that the 'free slip' at $\sigma_a = 0$ increases as the initial crack-opening is en-
larged. Simple bilinear expressions have been derived according to eqs. (2.2a-b). A further analysis revealed that the path-dependency of the interlock mechanism can almost be neglected if $\delta_t < 2/3, \delta_n$ [115]. The two-phase model will be treated in detail in chapter 5. Note, that Walraven [112, 114, 130] combined the aggregate interlock and the dowel mechanisms in order to simulate the response of cracked reinforced push-off specimens: see section 2.2.3.

![Model of Walraven [112]](image)

**Figure 2.17** Model of Walraven [112]; (a)-(b) assumed matrix deformation; (c) contact areas and (d) stresses for a single crack.

c. Model of Fardis and Buyukozturk [72]

Fardis et al. [72] theoretically modelled the shear transfer across a reinforced crack: see figure 2.18a. The general roughness of the crack retains its shape - which contrasts with observations - and is described by a stochastically based series of parabolic pieces, whereas the local roughness due to small matrix asperities may diminish gradually. The shear slip generates at least two contact points between both crack halves. The corresponding stresses are a result of material friction and local roughness. The model accounts for dowel action and for the normal restraint stiffness of the reinforcing bars crossing the crack. The model reasonably describes various shear experiments on reinforced cracks subjected to a constant restraint stress [42, 56, 66]. However, it does not consider the concrete grade as a major parameter of the interlock mechanism.
d. Model of Divakar et al. [104]

A constitutive model is proposed applicable for mixed mode fracture problems in plain cracked concrete, assuming decoupling of displacements and stresses as used previously by others [74, 89, 112]. Based on his own tests and the tests of Daschner et al. [80], empirical expressions were derived for $\sigma_a$ and $\delta_n/\delta_t$ as functions of the peak shear stress $\sigma_{tp}$ (a function of $\sigma_a$ and $f_{cy}$, compared with [93] where no $\sigma_a$ influence was assumed) and the initial shear stiffness (a function of $\delta_{no}$ and $f_{cy}$, see fig. 2.18b). The computed crack response curves closely agreed with measured data [64, 65, 112, 114].

e. Other models

Millard et al. [64, 65] combined the empirically derived dowel model of Rasmussen [14] with Walraven's two-phase model, taking account of the effects of axial steel stresses (i.e. dowel stiffness reduction and crack widening) and the interaction between the normal restraint stiffness of the bars and the shear stiffness of the crack; see figures 2.15a-b. The measured crack-opening curve or the measured normal restraint stiffness is used as input for the calculations. For $\delta_{no} > 0.25\text{mm}$ the predicted response was too stiff probably due to local bond deterioration which is not incorporated in the combined model.
Nissen [101] modelled the crack roughness by means of triangular concrete teeths of different geometry; see figure 2.18c. Each tooth ABC is characterized by values of $h$, $l$ and $\eta$ each having a statistical distribution function based on roughness measurements. If the opposite crack halves make contact, the teeth may either shear-off (for $h/l > 0.5 - 1.0$) or crush. These two failure mechanisms are in accordance with the Mohr-Coulomb criterion. The model can also take account of path-dependency and shows good agreement with own tests and tests of Walraven [112] for $\delta_{no} > 0.05$mm. Note, that for cracked rock materials the roughness is also modelled as a set of triangular-shaped asperities in [96].

Pruijssers [148] modelled the crack-opening direction of a cracked reinforced push-off specimen. He stated that the initial direction is governed by deformation of the bars. After the development of plastic hinges in the bars, the crack-opening path is dominated by the aggregate interlock mechanism. This approach is illustrated and extended in sections 5.3-5.6.
2.3.2 Numerical approach

Two distinct approaches of the finite element method [45,79,82] have been developed in order to model the cracking of concrete; figures 2.19a-b show examples of the discrete and the smeared (or: distributed) crack method. The crack is assumed to initiate in a direction perpendicular to the principal tensile stress (or: strain).

![Figure 2.19](image)

**Figure 2.19** (a) Discrete and (b) smeared crack approach.

In general the mechanical behaviour of a single crack is described by an asymmetric incremental stiffness matrix of which the coefficients may vary depending on the induced stresses, displacements, $D_{\text{max}}$ and $f_{\text{cyl}}$ [74,107]:

$$
\begin{bmatrix}
\Delta \sigma_{nn} \\
\Delta \sigma_{nt}
\end{bmatrix} =
\begin{bmatrix}
S_{nn} & S_{nt} \\
S_{tn} & S_{tt}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_{n} \\
\Delta \delta_{t}
\end{bmatrix}
$$

(N/mm$^2$) (2.14a)

or in tensor notation:

$$
\Delta \sigma = S^{\text{cr}} \cdot \Delta \delta
$$

(N/mm$^2$) (2.14b)

where:
- $S_{nn} = k_t/(\mu_f \cdot \beta_d)$;
- $S_{nt} = -k_t/\mu_f$;
- $S_{tn} = -k_t(1-\xi)/\beta_d$;
- $S_{tt} = k_t$;
- $k_t = (\partial \sigma_t/\partial \sigma_{nt})^{-1}$;
- $k_n = (\partial \sigma_n/\partial \sigma_{nt})$;
- $\mu_f = (-\partial \sigma_{nn}/\partial \sigma_{nt})^{-1}$;
- $\beta_d = (\partial \sigma_n/\partial \sigma_t)$ and $\xi = \mu_f \cdot \beta_d \cdot k_n/k_t$.

In case $\xi > 0$ complete coupling is adopted, i.e. there is path-dependency with respect to the shear transfer across a crack. Non-linear functions can be derived for $k_t$, $k_n$, $\beta_d$ and $\mu_f$ based on a regression analysis of experimental results [107]. Next, matrix $[S^{\text{cr}}]$ can be related to the uncracked concrete: $[S^{\text{cr}}] = \psi \cdot [S^{\text{co}}]$ where $\psi$ is a one-dimensional matrix accounting for the material damage due to cracking. This model can be simply incorporated in existing finite element programs.
The discrete method allows cracks to propagate usually only along the boundaries of elements by a disconnection of the nodes. The method is a powerful tool for local fracture prediction. The stiffness of embedded steel bars can be represented by sets of orthogonal springs (figure 2.19a) as proposed by Ngo et al. [24]. Nilson [28] modelled the non-linear stress-strain relation of concrete in compression. Later, Houde and Mirza [50] and Grootenboer [127] introduced crack formation through the elements. Constitutive stress-displacement relations of a crack can be easily implemented in discrete finite element computer programs.

The smeared crack approach represents relative crack displacements by crack strains which could be regarded as an infinite number of small parallel equidistant cracks. Thus, relations according to eqs. (2.14a-b) can be used for the finite element program. This method - contrary to the discrete concept - fits closely to continuum mechanics. Kollegger et al. [106] pointed out that the finite element method should consider the specific non-linear aspects of reinforced concrete, such as shear modulus reduction, tension-stiffening and potential reorientation of the direction of crack propagation. Note, that tension-stiffening refers to the macrolevel, whereas on the microlevel detailed bond models consider the slip layer close to the bar surface [136].

The reduced shear transfer due to cracks in plain or reinforced concrete, is usually expressed by the retention factor $\beta$ (figure 2.20a):

$$G_{cr} = \beta G_{co} \quad [\text{N/mm}^2] \quad (2.15a)$$

where $G_{co} = 0.5E_c/(1+\nu_c)$. If flexural deformations prevail then $\beta$-values hardly influence the results of non-linear finite element computations [51]. From figure 2.20a it can be derived that:

$$G_{cr} = (G_{co}^{-1} + (K_1)^{-1})^{-1} \quad [\text{N/mm}^2] \quad (2.15b)$$

where $l_c$ = crack spacing and $K = \partial r/\partial \delta_t$ = shear stiffness of a crack. Note, that $K \to \infty$ leads to $\beta = 1.0$. Perdikaris et al. [88] applied eq. (2.15b) to the observed behaviour of orthogonally cracked reinforced concrete panels subjected to increasing shear as well as constant biaxial tensile loading.
Figure 2.20 (a) Shear deformation of a cracked panel; (b) shear retention factor $\beta$ for plain concrete.

In the case of reinforcing bars crossing the crack, several empirical formulas for $G_{cr}$ were reported by, amongst others, Isenberg et al. [35], Duchon [43], Hand et al. [47], Houde et al. [50], Schimmelpfennig [55], Cedolin et al. [57], Collins [59], Fardis et al. [72,73], Hsu et al. [102] and Kolmar [109]. See also figure 2.20b. Some formulas distinguish separately the mechanisms of aggregate interlock and dowel action, which is usually related to local bond deterioration near the crack plane. Jimenez et al. [67] and Vecchio et al. [98] both found $G_{cr} = 0.05-0.07G_{co}$ for cracked reinforced concrete panels subjected to monotonic shear loading.

Rots et al. [142] stated that the assumption of zero off-diagonal terms in eq. (2.14a) might be too simple. However, there is still a lack of experimental data and of sound numerical solution techniques. Pruijssers [144] used a decomposition of the concrete strains and an expression based on the model of Walraven et al. [114]:

$$G_{cr} = \frac{1}{\alpha \varepsilon_{nn} + 1} G_{co} \quad [N/mm^2] \quad (2.15c)$$

where $\alpha$ is a function of $\gamma/\varepsilon_{nn}$, $f_{cc}$ and $D_{max}$. See figure 2.20b. Vecchio et al. [98] later derived a similar formula. For the numerical modelling of shear transfer across a crack in concrete, two assumptions can be made:

- decomposition of the total strain increments (figure 2.20a):

$$[\Delta \varepsilon] = [\Delta \varepsilon^{co}] + [\Delta \varepsilon^{cr}]$$

[-] \quad (2.16a)
incremental $\sigma$-$\epsilon$ relations for the cracked (cr) and uncracked (co) parts. One combined relation $[\Delta g] = [N].[\Delta \epsilon]$ can be derived for the cracked concrete \cite{141,142} with:

$$[N] = \alpha \begin{bmatrix} 1 & \nu_c & 0 \\ \nu_c & E_c/E_{ct} & 0 \\ 0 & 0 & \gamma \end{bmatrix}$$

(2.16b)

where $\alpha = E_c \cdot E_{ct}/(E_c - \nu_c^2 \cdot E_{ct})$ and $\gamma = (E_c - \nu_c^2 \cdot E_{ct}) \cdot \beta/(2 \cdot E_{ct} + 2 \cdot \nu_c \cdot E_{ct})$. The damage parameters $E_{ct}$ and $\beta$ are pointed out in figures 2.21a-b. The cracked part of the concrete deformation (superscript cr) is related to a certain element size or a measuring length. Note that eq. (2.16b) is based on a linear elastic behaviour of the uncracked concrete and on zero off-diagonal terms in the incremental stiffness matrix of the cracked part, i.e. no interaction of shear and normal stresses in the crack.

![Diagram](image)

**Figure 2.21** Definitions according to Rots et al. \cite{142}; (a) normal retention factor and (b) shear retention factor.
2.4 Conclusions
The aggregate interlock mechanism of cracked concrete has been investigated thoroughly by many researchers. Previous static push-off tests focused mainly on shear stress-slip relations, but recently researchers also reported reliable crack width data and information about the external restraint force acting perpendicular to the crack plane. The measurements provide the crack-opening curves which should relate to the actual in-plane behaviour of cracks in for instance structural concrete panels. In general, either the crack width or the restraint force was kept constant during the tests. Results can be employed to formulate incremental stress-strain relations according to eq. (1.1), as used in sophisticated finite element programs.

The response curves of the aggregate interlock tests are significantly influenced by the concrete grade and the restraint 'stiffness' of the crack plane. Cracks usually develop along the weak interfaces between the cement matrix and the particle surfaces. The contact areas between the matrix material and the protruding particles of the opposing crack face enable transfer of forces parallel and perpendicular to the crack plane. Light-weight aggregates may display less steep $\delta_t - \delta_n$ curves and cause a rather smooth crack surface. The shear strength $\tau_u$ is proportional to $f_{cc}^a$ where $a = 0.33 - 0.67$ as derived from different experiments. This relation indicates that the matrix material dominates the interlock mechanism. For $\delta_t > 0.5\text{mm}$ the crack-opening curves of similar types of test are almost parallel and display a minor influence of the particle distribution and particle size.

With respect to the dowel behaviour of statically loaded embedded bars, premature failure may occur due to splitting of the concrete cover (divided beam and beam-end tests). Another mechanism is the steel yielding and the crushing of the supporting concrete. This failure mode is studied predominantly by means of direct shear tests, which display highly non-linear shear stress-displacement relations. Different authors report that the dowel strength of smooth bars is proportional to $d_b^2 \sqrt{f_{cy}}$. The bar deflection varies between 0.15-0.17$d_b$ at 80-92% of the dowel strength. All these results correspond to zero eccentricity of the dowel force with respect to the concrete edge. They may be significantly affected by the bar position during the concrete cast. The dowel strength decreases considerably for axial steel stresses $\sigma_s > 0.75f_{sy}$. Various empirically based retention factors have been proposed.
An embedment length of less than $8d_p$ may greatly change the dowel force displacement relationship. The behaviour can be satisfactorily predicted by means of rather simple theoretical models (beam on elastic foundation; transversely loaded foundation pile). The maximum bending moment in the steel bar is assumed to be situated at 0.5-1.5$d_p$ away from the crack plane; the distance increases if the concrete locally crushes due to high bearing stresses. These stresses also improve the local bond properties — particularly for small diameter bars — so that only minor slip or even ‘perfect bond’ may occur. This phenomenon may support the preliminary conclusion of recent dowel tests on ribbed bars which also fit to eq. (2.5) of Rasmussen. The sustained dowel behaviour may be affected by creep of concrete and by time-dependent bond slip when subjected to lateral confinement.

The combined mechanism of aggregate interlock and dowel action has been studied by means of cracked reinforced concrete push-off specimens for short-term circumstances. The shear friction hypothesis predicts the ultimate shear stress as a function of $\rho_{fsy}$ and the external normal stress $\sigma_N$. This approach does not yet consider the concrete grade as a parameter. The static crack-opening curves are hardly affected by the reinforcement characteristics. Less steep curves were observed for high-strength gravel concrete, for light-weight concrete and if dowel action was eliminated by means of soft rubber sleeves. Special attention should be paid to the axial reinforcement stiffness in combining both mechanisms; the bond may be deteriorated in the vicinity of the crack caused by pre-cracking of the concrete specimen. It is concluded that the previous research did not investigate the stress-displacement relations for sustained shear loading conditions with crack widths $\delta_n < 0.20\text{mm}$. Experiments are needed to study this time-dependent shear transfer problem, see the chapters 3 and 4.

The theoretical models of Walraven et al. [112,114] and Gambarova et al. [69-71] have been successfully applied to observed interlocking of cracked concrete subjected to monotonic shear loading. With respect to the static loading of cracked reinforced concrete, the two-phase model of Walraven has been combined with Rasmussen’s formula, eq. (2.5). A few modifications are necessary for the combined model, see Walraven et al. [114], Millard et al. [64,65] and Pruijssers [148]. The theoretical predictions fit well to the test results. This approach may now be extended in order to model the results of the sustained shear transfer tests, see chapter 5.
3. EXPERIMENTS

3.1 Introduction
In chapter 2 it was concluded that the shear transfer mechanism of cracked concrete needs to be studied for sustained loading conditions. Accordingly, two types of push-off test should be carried out. The first type concerns 34 tests on reinforced cracked concrete. The second type (12 tests) is based on the shear transfer of plain cracked concrete so that pure aggregate interlock dominates. The effect of various parameters on the time-dependent displacement behaviour of a single crack has been observed.

This chapter deals with the most important aspects of the test set-up and the parameters chosen. Comprehensive overviews are presented in [119-126]. Sections 4.2 and 4.3 describe the main test results as well as the statistically-based analyses. Complementary information and a few additional detailed tests are treated in the final sections of chapter 4.

3.2. Testing equipment
The shape of the push-off specimen chosen is almost identical to the type used by Mattock et al. [40] and Walraven [112] for static experiments. The dimensions of the crack area are 120x300 mm². See figures 3.1a-b. Auxiliary reinforcement reduced secondary cracking which might cause premature fail-

\[
\begin{align*}
\text{Figure 3.1 Details of push-off specimens with a single crack for (a) reinforced concrete and (b) plain concrete restrained by rods.}
\end{align*}
\]
ure. The cantilevers at both top and bottom of the specimen were transversely post-tensioned, to improve the gradual introduction of the external shear force into the crack plane.

For the reinforced crack, 8mm diameter closed stirrups - each overlapped on the short side to ensure effective anchorage - perpendicularly intersected the crack plane. In the other type of specimen (figure 3.1b) four 22mm diameter cylindrical holes - preformed by temporarily placed inflatable tubes - were installed. Steel rods of 10 or 16mm diameter passed through these holes. They were fastened between 20mm thick demountable steel plates fixed on the short sides of the specimen. The plates were installed in the moulds before the concrete was cast, so that they fitted closely to these sides. This procedure ensured an almost constant restraint stiffness perpendicular to the crack plane. Due to an enlarged diameter of the holes close to the crack plane (5.9% reduction of the shear plane) the rods are not expected to transfer any dowel force. The axial steel stress of each rod is measured by means of - bending compensated - strain gauges attached on the bar surface. The initial steel stress was adjusted by bolts.

Prior to the actual test each specimen is pre-cracked in a vertical position by means of a three point bending test (figures 3.2a-b and 3.6a); a steel knife was pushed into a V-shaped groove along the shear plane. Successively, the front and the rear side of the specimen were split. At the top and bottom of the shear plane the crack widths were measured with displacement transducers (Hewlett-Packard type 7-DCDT-100, 0.01mm accuracy at 5mm range) attached to steel reference points stuck to the concrete surface.

Figure 3.2 (a)-(c) Details of the splitting procedure of the specimen.
Figure 3.3 Front view of specimen and loading arrangement.
Next, the cantilevers were post-tensioned and the specimen was placed centrally in a metal frame (figure 3.3). Prior to the long-term test, the desired shear loading level was applied step-wisely by a Freyssinet flat hydraulic jack, range 400 kN ($\tau_{\text{max}} \approx 11.1 \text{ N/mm}^2$) measured by a load cell installed under the specimen. An oil-accumulator was added to the hydraulic system to compensate for small pressure losses.

Special care was taken that the loading frame did not restrain the crack displacements. Also, several measures ensured a centric introduction of the shear force into the crack plane [119,120,135]. Due to loading eccentricity a maximum bending moment of 2.4 kNm was transferred by the crack at ultimate shear loading. The bending moment hardly affected the measuring accuracy of the displacement transducers.

In chronological order the displacements of the crack were recorded in two ways (figures 3.4a-b):
- duration $t \leq 24$ hours: displacement transducers measured the crack width and the shear slip on both faces of the specimen;
- duration $t > 24$ hours: change to another measuring system due to the limited number of transducers permanently available for the complete test series. A hand-held measuring device with a built-in opto-electric displacement transducer (Onno-Sokki type GS-555, 0.003mm accuracy at 5mm range) was used here. See figure 3.5.

![Figure 3.4 (a)-(b) Front view of both measuring systems.](image)
Figure 3.5 Removable hand-held device for measuring displacements.

This transducer was connected to one movable leg. The two other legs were fixed. The device was calibrated on a steel plate provided with fixed reference points. Improved and reliable data were achieved by repeating each recording three times. The recording system displayed good reproductive features; the measuring error was 0.005mm. In order to be able to measure, the specimen needed to be placed horizontally. Therefore, it was turned to a horizontal position approximately 24 hours after load application. See figure 3.6b.

![Image of a removable hand-held device for measuring displacements.]

Figure 3.6 (a) Precracking of a push-off specimen and (b) loading arrangement (horizontal position).
3.3 Testing procedure
The push-off specimens were cast in steel moulds. Synthetic moulds were used for the standard tests on 150mm cubes. Immediately after casting, the specimens were covered with plastic sheets. After two days they were demoulded and stored in a fog room (20 °C, 99% R.H.). Next, at an age of 22 days they were placed in the laboratory (20 °C, 50% R.H.). In this way all the specimens were exposed to the same curing conditions.

The test period can be divided into four phases:
- preparation of the push-off specimen. The concrete surface was locally roughened and steel reference points were fixed using epoxy glue. After pre-cracking and the post-tensioning of the concrete cantilevers (and adjustment of the steel stresses in the restraint rods by means of bolts), the final centring of the specimen in the loading frame took place;
- application of the shear loading in increments of about 25 kN at an average rate of 10-30.10^{-3} N/mm^{2} per second;
- periodical measurements under sustained loading of both the shear force and the crack displacements. Initially, there was a short period between successive measurements, but after 21 days the recordings were repeated weekly. During the tests the maximum loss of the sustained stress was less than 0.11 N/mm^{2} (4.0 kN);
- removal of the sustained loading. Unloading occurred after at least 90 days of load application. The remaining crack displacements were periodically measured over a period of three weeks. Next, the reinforced specimens were pushed-off by a monotonically increasing shear force at a constant loading rate.

Experimental data handling by micro-computer was opted for. It provides reliable data storage. Moreover, corrections and additional calculations can easily be programmed and it ensures a quick presentation (tables, plots) of test results, see figure 3.7. The measured displacements were corrected in view of the direct and the time-dependent (creep, shrinkage) deformations of the concrete between the reference points near the crack (figure 3.8). Simple formulae were found for the corrections [120] based on detailed supplementary investigations and on data from the literature.
Figure 3.7 Experimental data handling in chronological order.

CORRECTION OF MEASURED DISPLACEMENTS

Figure 3.8 Difference between measured and actual crack displacements.
3.4 Experimental parameters

The experimental program comprised five parameters:

- **cube compressive strength** $f_{cc}$. Two different concrete mixes were used both containing glacial river aggregates of 16mm maximum size. See table 3.1 and appendix 11.1. The particle distribution was in accordance with Fuller. The concrete contained 325 kg (wcr=0.50) and 420 kg (wcr=0.375) of Portland cement type B per m$^3$ resp. Average 28-day cube strengths were 51 and 70 N/mm$^2$ respectively. High-strength concrete was chosen in view of the application to offshore structures [125] and for research on the crack roughness. Cracks are expected to run partly through the aggregate particles due to the good properties of the matrix material;

<table>
<thead>
<tr>
<th>Table 3.1 Mix proportions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>mix A</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>cement content [kg/m$^3$]</td>
</tr>
<tr>
<td>wc-ratio</td>
</tr>
<tr>
<td>agg-c-ratio</td>
</tr>
<tr>
<td>superpl. [%]</td>
</tr>
<tr>
<td>water content [%]</td>
</tr>
<tr>
<td>sp. weight [kg/m$^3$]</td>
</tr>
<tr>
<td>$f_{cc}$ [N/mm$^2$]</td>
</tr>
<tr>
<td>$e_{cc}$ [N/mm$^2$]</td>
</tr>
</tbody>
</table>

- **restraint** of the crack plane. Either 8mm diameter embedded reinforcing bars (figure 3.1a) or four restraint steel rods ($d_b = 10$ or 16mm, see figure 3.1b) were used. The reinforcement ratios were between 1.12% (4 stirrups) and 2.24% (8 stirrups). The reinforcement and the rods all crossed the crack plane perpendicularly;

- **initial crack width** $\delta_{no}$. It was hardly possible to accurately adjust the desired initial crack width. During pre-cracking the crack width amounted to about 0.10mm and was reduced to 0.01-0.06mm after removal of the knife-edge;

- **steel grade** $f_{sy}$ or **initial normal compressive stress** $\sigma_{co}$ on the crack area. Ribbed steel bars were used with a yield stress of $f_{sy} = 460$ or 550 N/mm$^2$ (spec. rib areas $f_R = 0.050$ or 0.059 respectively). The ultimate strain of the steel was at least 24.10$^{-3}$. The adjusted stress of the external rods corresponded to $\sigma_{co} = 1.2$ N/mm$^2$ at the onset of the shear loading test;
The shear stress-level $\tau / \tau_u$. The level was based on the static shear strength $\tau_u$ which is a calibration value (figures 3.9a-b). From 67 static tests on cracked reinforced concrete push-off specimens it was found that [116]:

$$\tau_u = \alpha (\rho f_{sy})^\beta$$

provided that: $\tau_u \leq 0.26f_{cc}$ [N/mm$^2$] (3.1a)

where $\alpha = 0.822f_{cc}^{0.406}$ and $\beta = 0.159f_{cc}^{0.303}$. The ultimate shear stress of cracked plain concrete push-off specimens was related to test results of Daschner et al. [80] and Walraven [112]:

$$\tau_u = 1.647\sigma_{co}^{0.427}f_{cc}^{0.321}$$

provided that: $\tau_u \leq 0.26f_{cc}$ [N/mm$^2$] (3.1b)

assuming an initial crack width of 0.1mm.

The shear stress-levels used in the test series are relatively high compared with the serviceability state of structural applications. This was done to obtain crack displacements in a measurable range. In the experimental program (section 3.5) it is shown that for reinforced push-off specimens $\tau = 5.7-11.5$ N/mm$^2$, so that $\tau / \tau_u = 0.45-0.89$. The nominal shear stresses in reinforced concrete beams are usually restricted to 8-10 N/mm$^2$ at the ultimate state. For the plain concrete push-off specimens $\tau = 4.0-6.5$ N/mm$^2$ so that $\tau / \tau_u = 0.49-0.84$.

![Figure 3.9 Static shear strength $\tau_u$ for (a) cracked reinforced concrete and (b) cracked plain concrete push-off specimens.](image-url)
3.5 Experimental program
The sustained shear tests had a load application period of at least 90 days. This duration is chosen for practical reasons in view of the time schedule. Also, this period is expected to provide reliable extrapolation values of the crack displacements during the life-time of structural applications.

The tests were performed in an arbitrary sequence in order to minimize systematic errors. Each cast consisted of a series of two push-off specimens and nine 150mm cubes for the standard tests (see section 3.4). A few sustained tests were similarly repeated to get some idea of the influence of the scatter of the test results. See also [128]. All the specimens were assigned an indentifying code indicating the type of test and consisting of six indications (figures 3.10a-b).

![Identification code for reinforced concrete specimens](a)

![Identification code for plain concrete specimens](b)

**Figure 3.10** Codification of (a) reinforced and (b) plain concrete specimens.

For the reinforced concrete push-off specimens, figures 3.11-3.13 display the distributions of the experimental parameter values chosen in this research. It concerns the cube compressive strengths, reinforcement ratios, initial crack widths and the adjusted sustained shear stresses, respectively. The program comprised 17 series; in one series the specimens remained uncracked and two specimens were loaded at $t_0 = 10$ days. See tables 3.2 and 3.4. Note, that $\delta_{\text{neq}}$ and $\delta_{\text{tel}}$ represent the direct displacements of the crack observed immediately after the application of the desired load level; $\delta_{\text{neq}} > 0.16$mm in less than 20% of the tests.

For the plain concrete push-off specimens, four static tests and eight sustained shear tests were performed. See tables 3.3 and 3.4. The sustained shear stress values are relatively low as the shear load must be completely transferred by aggregate interlock between the opposing rough crack faces.
Figure 3.11 (a)-(b) Distribution of the 28-day cube compressive strengths related to the reinforced and plain concrete push-off specimens.

Figure 3.12 (a)-(c) Distribution of $\delta_{no}$, $\rho$ and $\sigma_{no}$-values over the test program.

Figure 3.13 (a)-(b) Distribution of constant $\tau$-values over the test program.
Table 3.2 Survey of the test program for reinforced concrete specimens.

<table>
<thead>
<tr>
<th>$t_0$</th>
<th>shear plane</th>
<th>mix</th>
<th>$f_{cc}$ [$N/mm^2$]</th>
<th>$\rho$ **)</th>
<th>$f_{sy}$ [$N/mm^2$]</th>
<th>$\delta_{no}$ [mm]</th>
<th>$\tau/\tau_U$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>28 cr.</td>
<td>A</td>
<td>15</td>
<td>48-54</td>
<td>1.12-2.24</td>
<td>460 or 550</td>
<td>0.01-0.06</td>
<td>0.45-0.89</td>
</tr>
<tr>
<td>28 cr.</td>
<td>B</td>
<td>15</td>
<td>67-74</td>
<td>1.12-2.24</td>
<td>460 or 550</td>
<td>0.01-0.06</td>
<td>0.55-0.86</td>
</tr>
<tr>
<td>10 cr.</td>
<td>A</td>
<td>1</td>
<td>44</td>
<td>1.12</td>
<td>550</td>
<td>0.01</td>
<td>0.78</td>
</tr>
<tr>
<td>10 cr.</td>
<td>B</td>
<td>1</td>
<td>58</td>
<td>1.12</td>
<td>550</td>
<td>0.01</td>
<td>0.74</td>
</tr>
<tr>
<td>28 uncr.</td>
<td>A</td>
<td>1</td>
<td>50</td>
<td>1.68</td>
<td>550</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>28 uncr.</td>
<td>B</td>
<td>1</td>
<td>70</td>
<td>1.12</td>
<td>550</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.3 Survey of the test program for plain concrete push-off specimens.

<table>
<thead>
<tr>
<th>$t_0$</th>
<th>loading</th>
<th>mix</th>
<th>$f_{cc}(t_0)$ [$N/mm^2$]</th>
<th>$d_b$ [mm]</th>
<th>$\sigma_{co}$ [N/mm²]</th>
<th>$\delta_{no}$ [mm]</th>
<th>$\tau/\tau_U$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>stat.</td>
<td>A</td>
<td>56</td>
<td>10</td>
<td>2.5</td>
<td>0.02</td>
<td>-</td>
</tr>
<tr>
<td>28-65</td>
<td>stat.</td>
<td>B</td>
<td>65-75</td>
<td>-10 or 16</td>
<td>0.4-1.1</td>
<td>0.01-0.03</td>
<td>-</td>
</tr>
<tr>
<td>28-79</td>
<td>sust.</td>
<td>A</td>
<td>49-58</td>
<td>10 or 16</td>
<td>0.9-2.0</td>
<td>0.01-0.02</td>
<td>0.49-0.80</td>
</tr>
<tr>
<td>28-35</td>
<td>sust.</td>
<td>B</td>
<td>64-73</td>
<td>10 or 16</td>
<td>1.0-2.0</td>
<td>0.02-0.03</td>
<td>0.74-0.84</td>
</tr>
</tbody>
</table>

Table 3.4 Overview of the sustained shear stress levels and of the related instantaneous crack displacements.

<table>
<thead>
<tr>
<th>type of crack</th>
<th>$t_0$ [d]</th>
<th>concrete mix</th>
<th>no. of spec.</th>
<th>$\tau$ [N/mm²]</th>
<th>$\delta_{nel}$ [mm]</th>
<th>$\delta_{tel}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>reinf.</td>
<td>28</td>
<td>A</td>
<td>15</td>
<td>5.7-10.0</td>
<td>0.07-0.22</td>
<td>0.04-0.15</td>
</tr>
<tr>
<td>reinf.</td>
<td>28</td>
<td>B</td>
<td>15</td>
<td>8.5-11.5</td>
<td>0.07-0.18</td>
<td>0.02-0.13</td>
</tr>
<tr>
<td>plain</td>
<td>28-79</td>
<td>A</td>
<td>5</td>
<td>4.0-6.5</td>
<td>0.03-0.08</td>
<td>0.04-0.06</td>
</tr>
<tr>
<td>plain</td>
<td>28-35</td>
<td>B</td>
<td>3</td>
<td>5.0-6.5</td>
<td>0.04-0.06</td>
<td>0.02-0.04</td>
</tr>
</tbody>
</table>

*) number of specimens    **) 1.12% (4 stirrups), 1.68% (6) or 2.24% (8)
4. EXPERIMENTAL RESULTS AND PARAMETER ANALYSIS

4.1. Introduction
The sections 4.2 and 4.3 present the most important results of the tests on cracked push-off specimens. The crack plane is perpendicularly crossed by either embedded reinforcement or free restraining rods. Each section starts with a few characteristic examples of the observed crack movements due to sustained shear loading. Emphasis lies on the development of the instantaneous and the time-dependent displacements parallel and perpendicular to the shear plane. Both sections end with a first analysis of the tests. The observed behaviour has been statistically determined as a function of the experimental variables. The predicted long-term crack displacements are dealt with in section 4.4.

In sections 4.5-4.6 attention is paid to a few supplementary investigations carried out on the reinforced concrete push-off specimens, such as the crack displacements at unloading, the residual static shear strength and detailed research on the dowel action mechanism. For this purpose, pull-out tests on reinforcing bars as well as in-situ measurements of the axial steel stresses in some push-off specimens were carried out. A short critical review of the experiments is given in section 4.7. Comprehensive information is presented in appendices 11.2-11.4 and in [120-126].

4.2. Shear transfer tests on cracked reinforced concrete specimens
4.2.1 Instantaneous displacements
During splitting, the crack initiation of the shear plane of the specimens was visually checked. In some of the more heavily reinforced specimens slight secondary cracking was observed at the top and bottom parts of the crack face. This so-called 'compressive spalling' was caused by stress concentrations.

The crack displacements $\delta_{n1}$ and $\delta_{t1}$ perpendicular and parallel to the crack surface respectively, are related to the instant $t = 0$ hrs for which the desired sustained shear loading has just been applied. The displacements were observed to increase time-dependently with $\delta_{nc}(t)$ and $\delta_{tc}(t)$ as functions of the duration of load. This can be written as (see figures 4.1a-b and table 3.4):

$$ t \geq 0 \text{ hrs: } \delta_n(t) = \delta_{n1} + \delta_{nc}(t) \quad [\text{mm}] \quad (4.1a) $$
$$ t \geq 0 \text{ hrs: } \delta_t(t) = \delta_{t1} + \delta_{tc}(t) \quad [\text{mm}] \quad (4.1b) $$
Figure 4.1 (a)-(b) Definitions of instantaneous and incremental ('creep') displacements as functions of $\tau$ and $t$.

Note that on average, the observed crack widths $\delta_{\text{tel}}$ of 15 specimens (mix A: $f_{\text{ccm}} = 51 \text{ N/mm}^2$) deviated less than 5% from data calculated by Walraven et al. [112,130] for $f_{\text{cc}} = 56 \text{ N/mm}^2$. In general the measured $\delta_{\text{tel}}$ values tended to be somewhat lower than might be expected from eq. (2.11), even if the necessary corrections according to figure 3.8 were considered. Probably, this was caused by the different application methods used (load- or displacement-controlled).

All the plotted displacements of the specimens are average values of observations on the front and rear side of the shear plane. Probably due to the casting direction and the method of compaction, displacements on the front (i.e. the upper side of the mould) are systematically larger (figs. 4.2a-b). The developments of measured displacements after pre-cracking and during the application of the shear loading are illustrated in figures 4.3-4.4 for two specimens. The codification is specified in figure 3.10a.

Figure 4.2 (a)-(b) Distributions of the ratio of measured displacements on front and rear side of the specimen.
4.2.2. Time-dependent displacements

Periodically, the average crack displacements of each specimen have been recorded. The displacement increments of two tests are presented in figures 4.5-4.6. These data refer to an application period of 90 days (\( \approx 2150 \) hours). The regression curves and the associated statistical boundaries of the 90% confidence areas - see section 4.2.3 - are indicated.

Note, that \( 10^5 \) hrs \( \approx 11.4 \) years. On combining the instantaneous and the time-dependent increments according to eqs. (4.1a-b), the crack-opening path is found. Each curve shows the crack response from the very moment of pre-cracking to the last periodical measurement under sustained shear loading. Two examples are given in figures 4.7-4.8. The two initially uncracked specimens were regularly observed. Nearly one month after the load was applied, a longitudinal crack developed between the V-shaped grooves, probably due to the combined action of shrinkage and shear loading.
Figure 4.5 Measured $\delta_{nc}$ and $\delta_{tc}$-values; specimen no. 4.

Figure 4.6 Measured $\delta_{nc}$ and $\delta_{tc}$-values; specimen no. 16.
Figure 4.7 Measured crack-opening path of specimen no. 6. Figure 4.8 Measured crack-opening path of specimen no. 17.
4.2.3 Statistical analysis of the measured displacements

All the curves presented in figures 4.5-4.8 have been calculated by non-linear regression analysis for various reasons:
- the time-dependent behaviour exhibits considerable scatter [196]. For one thing, this is the case with observations on individual specimens because of the heterogeneity of concrete giving rise to complicated time-dependent effects (shrinkage, creep, bond between concrete and steel). Furthermore, measurements performed on two or more 'identical' specimens will differ from one another by amounts that are by no means always negligible. The scatter and reliability of the test data need to be quantified;
- a statistical analysis provides a tool for an objective description;
- extrapolation to a longer sustaining period which should be done with care as the results have to be in agreement with physical reality;
- simple comparison of the responses of the specimens is desired. Moreover, the influence of the variables then can be analysed more easily.

The time-dependent displacement curves have all been expressed by power-functions so that the measuring data should be logarithmically transformed. Generally this type of function can satisfactorily describe the creep deformations of plain concrete for short and long periods of observation [178]:

\[ y = \alpha_1 + \alpha_2 ( x + \alpha_3 )^{\alpha_4} \]  

in which \( x, y \) = two independent groups of measurements and \( \alpha_1-\alpha_4 \) = regression coefficients. For each specimen eq. (4.2) has been applied three times:

\[ \delta_{nc}(t) = \alpha_1 + \alpha_2 ( t + \alpha_3 )^{\alpha_4} \]  

\[ \delta_{tc}(t) = \alpha_5 + \alpha_6 ( t + \alpha_7 )^{\alpha_8} \]  

\[ \delta_t = \alpha_9 + \alpha_{10} ( \delta_n + \alpha_{11} )^{\alpha_{12}} \]

in which \( t \) = duration of load application and \( \alpha_1-\alpha_{12} \) = regression coefficients which have been iteratively determined by means of a computer-program especially developed for this purpose. The least square sum \( \Sigma = \sum_{i=1}^{n} ( y_i \text{meas} - y_i \text{regr} )^2 \) is stepwisely minimized. The mean difference between an individual measurement and its regression value is defined as:

\[ \tilde{d} = \sqrt{\Sigma / n} \]
where \( n \) denotes the number of observations. Theoretically \( \tilde{a} \) should be zero. From figures 4.9a-c it is concluded that \( \tilde{a} \) satisfactorily approximates the 0.005mm error of the measured displacements. The values at the onset of the sustained test (\( t = 0 \) hrs; \( \delta_{nc} = \delta_{tc} = 0 \) mm and \( \delta_t = 0 \) mm for \( \delta_n = \delta_{no} \)) are also accurately described by means of eqs. (4.3a-c). All the regression curves according to eq. (4.3c) are presented in appendix 11.3.

An extrapolation to a longer application period does not lead to a progressive change of the predicted displacements, see figures 4.10a-b. These extrapolated values depend partly on the algebraic expression chosen and on the duration \( t_{\text{max}} \) of the measuring period: if it exceeds 1500 hrs then figures 4.10a-b indicate that the computed crack width increments \( \delta_{nc} \) at \( t = 2000 \) and \( 10^5 \) hrs tend to stabilize. The same is true for the parallel displacement increments of the crack plane. Thus, it is concluded that an observation period of at least three months will be sufficient.

![Figure 4.9](image)

**Figure 4.9** (a)-(c) Distributions of \( \tilde{a} \) acc. to eqs. (4.3a-c) respectively.

![Figure 4.10](image)

**Figure 4.10** Influence of the observation period \( t_{\text{max}} \) on the computed displacements acc. to eq. (4.3a) for (a) \( t = 2000 \) and (b) \( 10^5 \) hrs.
4.2.4 Parameter influence on the displacement response

In section 4.2.3 the displacement behaviour of the separate specimens has been statistically analysed. Before a physical explanation of the observations is formulated in the chapters 5-6, the measured displacements must be represented as functions of the experimental parameters.

This section shows a few results of the mathematical description which provides a so-called sensitivity analysis of the parameters. It also gives a simulation of non-tested conditions. The choice of the magnitudes of the variables which had to be introduced into the calculations, is confined to the experimental range. Thus:

\[
\begin{align*}
20 & \leq f_{cc} \leq 80 \quad [N/mm^2] \\
0.01 & \leq \delta_{no} \leq 0.05 \quad [mm] \\
3 & \leq \rho f_{sy} \leq 12 \quad [N/mm^2] \\
0.30 & \leq \tau/\tau_u \leq 0.90 \quad [-] \text{ provided that } \tau \geq 3 \text{ N/mm}^2 \quad (4.5)
\end{align*}
\]

The complete test series has been mathematically described in two steps:

**Step 1.** The calculation of unique \( \tau-\delta_n \) and \( \tau-\delta_u \) relations for monotonically increasing shear loading. Good agreement with the measurements was found if these relations are a linear combination of a power function and a reciprocal function. The method employed is illustrated by the shear stress-crack width response (see figure 4.11a):

\[
\tau(\delta_n) = \frac{\tau_u}{2} \cdot \left( \frac{\delta_n - \delta_{no}}{\delta_1} \right)^{\beta_1} + \frac{\left( \frac{\delta_n - \delta_{no}}{\delta_1} \right)}{1 + \beta_2 \tau_u} \quad [N/mm^2] \quad (4.6a)
\]

where: \( \delta_1 = \delta_{ntop} - \delta_{no} \) and with \( \tau_1 = \tau(\delta_n = 0.1\text{mm}) \):

\[
\begin{align*}
\beta_1 &= \ln\left( \frac{\tau_1}{\tau_u} \right) / \ln\left( 0.10 - \frac{\delta_{no}}{\delta_{ntop}} \right) \quad [-] \quad (4.6b) \\
\beta_2 &= \left( \tau_1 \delta_1 - \tau_u (0.10 - \delta_{no}) \right) / \left( \tau_u \tau_1 \delta_1 \right) \quad [mm^2/N] \quad (4.6c)
\end{align*}
\]

Eq. (4.6a) has four unknown coefficients, \( \beta_1, \beta_2, \delta_1 \) and \( \tau_u \); four prescribed values are needed to quantify the shear stress-crack width relation...c

- \( \tau=0 \text{ N/mm}^2 \) for \( \delta_n = \delta_{no} \text{ [mm]} \); simple empirical formulae were found for \( \delta_{ntop}, \tau_1 \) and \( \tau_u \), see also eq. (3.1a). These formulae are based on test results of Hofbeck et al. [30], Walraven [112] and Frénaij [120]. For chosen values of \( \delta_{no}, f_{cc} \) and \( \rho f_{sy} \), the four coefficients can now be computed so that eq. (4.6a) is known.

The displacement \( \delta_{nel} \) - and \( \delta_{tel} \) [120] - associated with a constant shear stress, is computed by means of the Newton-Raphson iterative method.
Table 4.1 shows that the prescribed values given by the empirical formulae closely agree with the measurements. It can be concluded that no significant influence of the method of shear load application - either load- or displacement-controlled - was stated.

**Step 2.** The object is to write the time-dependent displacement response as a function of $f_{cc}$, $\rho f_{sy}$, $\delta_{no}$, $\tau/\tau_{u}$. Tests with only 8mm diameter stirrups were considered. It was hard to find reliable functions for the regression coefficients $\alpha_1$-$\alpha_{12}$ of eqs. (4.3a-c). Thus as a certain drawback, an implicit method is required, illustrated now for the crack width increment $\delta_{nc}(t)$ in figure 4.11b. Four prescribed values were proposed according to:

\begin{align}
\delta_{nc}(t = 0 \text{ hrs}) &= \alpha_1 + \alpha_2(\alpha_3) \alpha_4^4 = 0 \quad \text{[mm]} \quad (4.7a) \\
\delta_{nc}(t = 2000 \text{ hrs}) &= \alpha_1 + \alpha_2(\alpha_3 + 2000) \alpha_4^4 = g_1 \quad \text{[mm]} \quad (4.7b) \\
d\delta_{nc}/dt(t = 100 \text{ hrs}) &= \alpha_2\alpha_4(\alpha_3 + 100) \alpha_4^{-1} = g_2 \quad \text{[mm/hrs]} \quad (4.7c) \\
d\delta_{nc}/dt(t = 2000 \text{ hrs}) &= \alpha_2\alpha_4(\alpha_3 + 2000) \alpha_4^{-1} = g_3 \quad \text{[mm/hrs]} \quad (4.7d)
\end{align}

Next, statistically-based functions were determined for $g_1$-$g_3$ which all express a proper formulation of the prescribed values, see also table 4.1. After the prescribed values are known for each parameter combination, the (new) coefficients $\alpha_1$-$\alpha_4$ are iteratively solved from the set of eqs. (4.7a-d). For the derivation of $\delta_{tc}$-$\delta_{n}$ and $\delta_{tc}$-$t$ relations, the procedure is as outlined above for $\delta_{nc}(t)$. 
Table 4.1 Comparative overview of the computed and 'measured' prescribed values.

<table>
<thead>
<tr>
<th>relation</th>
<th>prescribed value</th>
<th>no. of tests</th>
<th>( \bar{x} ) *)</th>
<th>v.c. **)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau - \delta_n ) and ( \tau - \delta_t )</td>
<td>( \tau )</td>
<td>67</td>
<td>0.99</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>( \tau(\delta_n = 0.1\text{mm}) )</td>
<td>31</td>
<td>1.04</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>( \tau(\delta_t = 0.1\text{mm}) )</td>
<td>31</td>
<td>1.06</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>( \delta_{ntop} )</td>
<td>17</td>
<td>0.94</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>( \delta_{ttop} )</td>
<td>17</td>
<td>0.95</td>
<td>0.23</td>
</tr>
<tr>
<td>( \delta_{nc}(t) )</td>
<td>( g_1, g_2 ) and ( g_3 )</td>
<td>24</td>
<td>1.03</td>
<td>0.27</td>
</tr>
<tr>
<td>( \delta_{tc}(t) )</td>
<td>( g_4, g_5 ) and ( g_6 )</td>
<td>24</td>
<td>1.06</td>
<td>0.23</td>
</tr>
<tr>
<td>( \delta_{t}(\delta_n) )</td>
<td>( g_7, g_8 ) and ( g_9 )</td>
<td>24</td>
<td>1.00</td>
<td>0.20</td>
</tr>
</tbody>
</table>

*) \( \bar{x} \) = calc./meas.   **) coefficient of variation = \( s(\bar{x}) / \bar{x} \)

Note, that the somewhat arbitrary choice of \( \bar{x} \) as a measure of comparison does not mean that a relatively large coefficient of variation is always significant. Special attention should be paid to the crack-opening curve which considers the full duration of the test (step 1 and step 2). As an advantage, this curve has a relatively large statistical reliability.

According to figure 4.12 there are two ways of projecting the time axis. For an arbitrary point of time \( t = t_j > 0 \) hrs, the points A and B will in general not coincide, i.e. there is no unique \( \delta_n - \delta_t - t \) relation. The final position is located midway at the most probable point D. The displacement shifts are attributed to scatter which is inherent to (time-dependent) phenomena in concrete \[189,196,200\]. The width of the 90% confidence interval of the crack-opening curve determines the maximum shift (or: the sustaining period \( t_{\text{max}} \)) allowed. On the assumption of a normal statistical distribution of the total shifts, it was found from 32 tests that:

\[
\bar{t}_{\text{max}} = 10.570 \text{ hrs} \pm 57.2 \quad \text{and:} \quad t_{\text{max,10\%}} = 8.3 \quad \text{[years]} \quad (4.8)
\]

All these periods are lower bound values due to the constant width of the 90% confidence interval assumed in the calculations. The computer program listed in [120] summarizes all the information presented in this section.
4.2.5 Results of the parameter analysis

Figures 4.13a-b and 4.14a-b show a few displacement responses which have been obtained by means of the computer program discussed in section 4.2.4. The discontinuity in the time-displacement curves relates to the increments \( \delta(t = 0.1 \text{ hrs}) - \delta_{el} \) which become larger for higher \( \tau/\tau_u \) ratios. Some other conclusions may be drawn:
- the \( \tau-\delta_n \) curves exhibit a displacement increase between equidistant points (equal decimal ratios) of time for higher shear stress values;
- the total displacements are virtually equal when the steel yield stress is increased by 20\% \( (\rho_{sy} = 5.15 \text{ and } 6.16 \text{ or } 7.73 \text{ and } 9.24 \text{ N/mm}^2) \). This conclusion is valid for equal values of \( \tau/\tau_u \), \( t \) and \( f_{cc} \);
- for \( f_{cc} = 51 \) and \( 70 \text{ N/mm}^2 \) the ratio \( \delta_{nc}/\delta_{ne} \) ranges between approximately 0.8-1.1 and 0.4-0.8 respectively if \( \tau/\tau_u = 0.90 \text{ and } t = 10^3 \text{ hrs.} \)

This last conclusion denotes the considerable influence of the concrete compressive strength on the crack displacements. It is known that the values adopted for the experimental parameters are an approximation of the actual values. Differences arise in consequence of a limited measuring accuracy (adjustment of \( \tau \) and \( \delta_n \)), variations of material properties \( (\rho, f_{sy}, \text{inhomogeneity of concrete}) \) and environmental changes \( (T \text{ and } \text{R.H.}) \).
For instance, the sustained loading level is based on the mean compressive strength of the concrete test cubes of the batch. Since \( \tau_u \) depends on \( f_{cc} \) - eq. (3.1a) - the actual level may deviate from the assumed level. This phenomenon is shown in figures 4.15a-b for mix A and B.
Figure 4.13 Computed $\tau-\delta_n$ relations for (a) mix A and (b) mix B.
Figure 4.14 (a)-(b) Identical to figure 4.13 for the displ.-time relations.
Figure 4.15 (a)-(b) Variation of $f_{cc}$: characteristic (5%, 95%) and mean (50%) concrete strengths based on a normal distribution.

Apart from the initial crack-opening direction and the $\delta_{\text{nel}}$-values, the displacement response was hardly influenced by a variation of $\delta_{\text{no}}$ [120]. The two uncracked push-off specimens (table 3.2) displayed small time-dependent deformations (denoted by $\delta_{\text{nc}}$ and $\delta_{\text{tc}}$) in comparison with the simulated test results of similar cracked specimens. At $t = 2000$ hrs the differences were 24% and 31% for mix A and mix B respectively.
4.3 Shear transfer tests on cracked plain concrete specimens

4.3.1 Instantaneous displacements

Most observations agree qualitatively with test results presented in section 4.2. The instantaneous crack displacements given previously in tables 3.3-3.4 and in figures 4.1a-b, were now also systematically larger on the front sides of the specimens. The mean ratios were (n = 12):

\[ \frac{\delta_{ne1(\text{front})}}{\delta_{ne1(\text{rear})}} = 1.20 \text{ [-]} \quad \text{and} \quad 1.03 \text{ for } \delta_{tel} \] (4.9)

Figures 4.16-4.17 show the results of two static push-off tests [121]. The codification is explained in figure 3.10b. The measured normal compressive stress on the concrete shear plane is indicated. The measured \( \tau_u \) -values differ less than 9% from eq. (3.1b).

Figure 4.16 Measured response on application of the shear load.

Figure 4.17 Measured response on application of the shear load.
4.3.2 Time-dependent displacements

The data handling and the results of the eight sustained shear tests are similar to those reported in section 4.2.2, at least in the qualitative sense. Appendix 11.4 shows the crack-opening curves which were based on a statistical treatment of the recorded displacements. Two somewhat divergent curves are probably caused by an initial over-estimation of the adjusted constant loading level (specimen no. 9: $\tau_a = 4.9 \text{ N/mm}^2$ during 22 hours and no. 10: $\tau_a = 5.2 \text{ N/mm}^2$ for 10 hours). Note, that $f_{cc}$ is related to the concrete age at the onset of the shear test; for mix A: $t_o = 28-79$ days and for mix B: $t_o = 28-35$ days.

The initial compressive stress $\sigma_{co}$ on the crack surface corresponds to the initial crack width of the unloaded specimens. On application of the shear loading, the displacements of the crack-faces will increase. In order to maintain equilibrium, the restraint bars must be able to develop a sufficiently large increase of $\sigma_c$. Table 4.2 summarizes a few experimental data; the corresponding time-dependent curves are given in appendix 11.4.

Table 4.2 Measured $\sigma_c$-values of the sustained tests.

<table>
<thead>
<tr>
<th>specimen no.</th>
<th>$\sigma_{co}$ [N/mm²]</th>
<th>$\sigma_{cl}$ *) [N/mm²]</th>
<th>$\sigma_{c2}$ **) [N/mm²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.01</td>
<td>2.19</td>
<td>2.10</td>
</tr>
<tr>
<td>6</td>
<td>1.02</td>
<td>1.21</td>
<td>1.30</td>
</tr>
<tr>
<td>7</td>
<td>1.03</td>
<td>1.11</td>
<td>0.91</td>
</tr>
<tr>
<td>8</td>
<td>1.02</td>
<td>1.14</td>
<td>1.31</td>
</tr>
<tr>
<td>9</td>
<td>0.85</td>
<td>1.01</td>
<td>1.22</td>
</tr>
<tr>
<td>10</td>
<td>1.99</td>
<td>2.05</td>
<td>1.68</td>
</tr>
<tr>
<td>11</td>
<td>1.13</td>
<td>1.31</td>
<td>1.23</td>
</tr>
<tr>
<td>12</td>
<td>1.99</td>
<td>2.14</td>
<td>2.42</td>
</tr>
</tbody>
</table>

*) at $t = 0$ hrs. **) at $t_{max}$

Two final remarks should be made:
- the initial normal restraint stiffness $\alpha$ of the crack plane is approximated on the assumption that the axial bar elongation is equal to the crack width increase. Hence:
\[ \alpha = \frac{d\sigma_c}{d\delta} = \frac{E_s A_s}{1.4C} = \text{constant} \cdot d_b^2 \quad \text{[N/mm}^3\text{]} \quad (4.10) \]

so that \( \alpha = 4.2 \) and 11.7 N/mm\(^3\) for \( d_b = 10 \) and 16mm respectively. The experimentally found mean \( \alpha \)-values are 4.2 and 8.4 N/mm\(^3\). Thus there is reasonable agreement with the test data.

- After \( t = 50-250 \) hrs the normal stress sometimes tended to drop as a function of the crack widths. The drying shrinkage of the concrete surrounding the restraint rods may particularly contribute to this phenomenon. Shrinkage was measured to manifest usually a few days after the wet push-off specimens were placed in the laboratory at 20 °C, 50% R.H. This explanation is confirmed by the results of specimens nos. 7 and 10 which exhibited a rather early start of the \( \sigma_c \)-reduction, probably due to a longer drying period before the actual tests started at \( t_0 = 59 \) and 79 days respectively.

4.3.3 Statistical analysis of the measured displacements

Formulae according to eqs. (4.3a-c) have been statistically calculated for each sustained shear test performed. It was now also substantiated that the mean difference between the measurements and the computed values was smaller than the measuring accuracy.

4.3.4 Parameter influence on the displacement response

In broad outline the mathematical assessment corresponds with the method published previously in [120] and in section 4.2.4. A computer-program has been developed enabling the determination of unique time-dependent displacement relations of the crack plane for each parameter combination chosen within the following ranges:

\[
\begin{align*}
50 & \leq f_{cc} \leq 75 \quad \text{[N/mm}^2\text{]} \\
0.01 & \leq \delta_{no} \leq 0.03 \quad \text{[mm]} \\
0.50 & \leq \sigma_{co} \leq 2.00 \quad \text{[N/mm}^2\text{]} \\
10 & \leq d_b \leq 16 \quad \text{[mm]} \\
0.50 & \leq \tau_a/\tau_u \leq 0.90 \quad \text{[-]} \quad \text{provided that } \tau_a \geq 2.5 \text{ N/mm}^2 \quad (4.11)
\end{align*}
\]

In [121] simple formulae have been derived for \( \delta_{ne} \) and \( \delta_{te} \), which closely fit to the test data. An example is shown in figure 4.18. Note, that the measuring accuracy of the displacements is about 0.01mm.
Figure 4.18 Calculated development of $\delta_{ne1}$ as a function of the shear stress and the concrete grade.

Next, the time-dependent behaviour was simulated by means of an implicit calculation method. It has been endeavoured to accurately describe the experiments. According to table 4.3 the results reasonably approximated the test data, especially for the crack-opening curves. None of the eight sustained shear tests was repeated so that the scatter of the measurements might have a rather important effect on the computational results. It was hard to find selection criteria for the small number of test data and thus the reliability of the calculated responses should be considered with care.

Table 4.3 Comparative overview of the computational results.

<table>
<thead>
<tr>
<th>relation</th>
<th>prescr. value</th>
<th>no. of tests</th>
<th>$\bar{x}$ *)</th>
<th>v.c. **)</th>
</tr>
</thead>
<tbody>
<tr>
<td>static</td>
<td>$\delta_{ne1}$</td>
<td>10</td>
<td>0.88</td>
<td>0.18</td>
</tr>
<tr>
<td>behaviour</td>
<td>$\delta_{te1}$</td>
<td>10</td>
<td>0.80</td>
<td>0.27</td>
</tr>
<tr>
<td>$\delta_{nc}(t)$</td>
<td>$g_1$-$g_3$</td>
<td>8</td>
<td>1.04</td>
<td>0.22</td>
</tr>
<tr>
<td>$\delta_{tc}(t)$</td>
<td>$g_4$-$g_6$</td>
<td>8</td>
<td>0.93</td>
<td>0.15</td>
</tr>
<tr>
<td>$\delta_t(\delta_n)$</td>
<td>$g_7$-$g_9$</td>
<td>8</td>
<td>0.95</td>
<td>0.12</td>
</tr>
</tbody>
</table>

*) $\bar{x} = \text{meas./calc.}$  **) coefficient of variation $= s(\bar{x}) / \bar{x}$
4.3.5 Results of the parameter analysis
The effect of a parameter variation on the displacement response has been systematically studied. The approach used was similar to that presented in section 4.2.5 for the embedded push-off specimens subjected to sustained shear loading. A few computational results are presented in figures 4.19a-b and 4.20. They are related to the compressive strengths based on the average concrete age at the onset of the static and sustained shear tests:

\[
\begin{align*}
\text{mix A:} & \quad \bar{t}_0 = 46 \text{ days} : f_{ccm} = 55.2 \text{ N/mm}^2 \quad (4.12a) \\
\text{mix B:} & \quad \bar{t}_0 = 38 \text{ days} : f_{ccm} = 69.1 \text{ N/mm}^2 \quad (4.12b)
\end{align*}
\]

For a constant shear stress, the lower the total time-dependent displacements, the higher the concrete grade. No significant influence of the initial crack width was found so that the mean value \(\bar{\delta}_{n0} = 0.02\text{mm}\) is applied.

As shown in figures 4.21a-b a change of the concrete compressive strength - and to a lesser extent of \(\sigma_{co}\) - usually strongly affects the crack-opening curves, especially for the 10mm diameter restraint rods. According to eq. (3.1b) the bar diameter has no effect on the shear strength of the cracked concrete, which contrasts with physical reality. The formula is valid for \(\sigma_{co}\) remaining constant during the experiment. For decreasing bar diameters the displacement curves become steeper but displacement differences remain smaller than 0.05mm after \(10^5\) hrs of load application.

The computational results of the parameter analyses presented in sections 4.2.5 and 4.3.5 are based on an implicit calculation method. Next, these displacement responses were easily fitted by fairly simple formulae for the complete test series [123], even though the relations were obtained indirectly. It should be considered that the implicit method is rather complicated but it provides a more accurate description.
Figure 4.19 The $\tau_a - \delta_n$ relations with $d_b = 10\text{mm}$ and $\sigma_{co} = 1.5$ or $2.0 \text{N/mm}^2$ computed for (a) mix A and (b) mix B.
Figure 4.20 Example of a few displ.-time curves for mix A and B.

Figure 4.21 Crack-opening curves; variation of (a) $f_{cc}$ and (b) $\sigma_{co}$. 
4.4 Long-term crack displacements

Table 4.4 presents the extrapolated values of crack width and parallel displacement for a sustained loading period of $t = 10^5$ hrs $\approx 11.4$ years. The predicted displacements are reasonably reliable, as concluded in section 4.2.3, figures 4.10a-b. It can be generally conceded from the results for $\delta_{no} = 0.01-0.05$mm that:
- the instantaneous and time-dependent crack widths and parallel displacements of the reinforced concrete push-off specimens are relatively small compared with the values of the cracked plain concrete specimens;
- for low shear stress-levels the cracks display $\delta_n > \delta_t$. The opposite conclusion relates to higher levels $\tau/\tau_u \approx 0.80$, for which the crack widths seem hardly to be influenced by the concrete grade;
- in the case of a reinforced concrete crack, $\delta_n$ and $\delta_t$ are smaller than 0.20mm for $\tau/\tau_u \leq 0.50$. This crack width limit corresponds to permissible values at the serviceability state of structural concrete in case of a fairly aggressive environment [216].

**Table 4.4** Predicted displacements of a crack in reinforced (r) or plain (p) concrete subjected to a sustained shear loading; $t = 10^5$ hrs.

<table>
<thead>
<tr>
<th>crack</th>
<th>$\tau/\tau_u$ [-]</th>
<th>$\tau$ [N/mm$^2$]</th>
<th>$f_{cc}$ [N/mm$^2$]</th>
<th>$\delta_n$ [mm]</th>
<th>$\delta_t$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>r *)</td>
<td>0.5</td>
<td>3.0-4.6</td>
<td>30</td>
<td>0.13-0.16</td>
<td>0.12-0.15</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>2.5-4.4</td>
<td>51</td>
<td>0.04-0.09</td>
<td>0.04-0.07</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>3.1-5.8</td>
<td>70</td>
<td>0.03-0.08</td>
<td>0.03-0.05</td>
</tr>
<tr>
<td>r</td>
<td>0.8</td>
<td>4.8-7.4</td>
<td>30</td>
<td>0.35-0.45</td>
<td>0.47-0.54</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>6.7-11.7</td>
<td>51</td>
<td>0.37-0.45</td>
<td>0.38-0.44</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>8.3-15.5</td>
<td>70</td>
<td>0.35-0.43</td>
<td>0.28-0.36</td>
</tr>
<tr>
<td>p **)</td>
<td>0.8</td>
<td>4.8-6.4</td>
<td>55</td>
<td>0.24-0.32</td>
<td>0.18-0.35</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>5.3-7.1</td>
<td>70</td>
<td>0.20-0.30</td>
<td>0.16-0.40</td>
</tr>
</tbody>
</table>

*) for $\rho f_{sy} = 4-12$ N/mm$^2$  **) for $\sigma_{co} = 1-2$ N/mm$^2$ and $d_b = 10-16$mm.
In order to adapt the observed time-dependent behaviour of structural concrete to a more common approach used in the engineering design practice, the results have been expressed by means of 'creep' coefficients $\phi_n$ and $\phi_t$ which have been defined according to [123,124]:

$$\delta_n(t) = \delta_{nel} + \delta_{nc}(t) = \delta_{nel}(1 + \phi_n(t)) \quad [\text{mm}] \quad (4.13a)$$

$$\delta_t(t) = \delta_{tel} + \delta_{tc}(t) = \delta_{tel}(1 + \phi_t(t)) \quad [\text{mm}] \quad (4.13b)$$

Figure 4.22 (a)-(b) Development of $\phi_t(t)$ for cracked concrete [123], with $\delta_{no} = 0.01-0.02\text{mm}$ and $\tau_u$ according to eq. (3.1a) or (3.1b).

From the analyses of $\phi$-values and their time-dependent developments a few conclusions can be drawn [123,124]:
- after a sustained loading period of $10^5$ hrs no final values are attained, so far as the adopted formulae can accurately and reliably predict the development of time-dependent displacements;
- the calculated 'creep' coefficients display non-linearity (figures 4.22a-b) with respect to $\tau/\tau_u$ and $f_{cc}$. The coefficients of cracked plain concrete restrained by free rods are relatively high;
- a reduction of $\tau/\tau_u$ from 0.80 to 0.60 leads to a substantial 25-45% increase of both creep coefficients in the case of a reinforced crack. This is mainly caused by the relatively large decrease of the instantaneous displacements $\delta_{\text{inel}}$ and $\delta_{\text{tel}}$;  

- $\phi_t$-values are systematically larger than $\phi_n$-values; 30-55% at $t = 10^5$ hrs and these percentages increase the lower the concrete grade. A first reason of the differences observed is that $\delta_{\text{tel}} < \delta_{\text{inel}}$ for displacements smaller than about 0.20mm. Moreover, in general $\delta_{\text{tc}} > \delta_{\text{nc}}$. Consequently, the crack-opening curves become steeper for a longer application period;  

- creep coefficients $\phi_c(t)$ of plain concrete based on the CEB model-code 1978 display similar types of curves as were found for $\phi_n$ and $\phi_t$, see [120]. However, $\phi_c$ refers to the serviceability limit state of structural concrete, whereas $\phi_n$ and $\phi_t$ correspond more or less to ultimate limit state conditions for which non-linear behaviour is likely to occur.
4.5 Unloading and reloading to failure

After a sustained loading period of 90-273 days the cracked reinforced concrete specimens were unloaded. The displacement behaviour was partly recoverable. In figures 4.23a-b the decreases $\Delta \delta$ of the displacements - occurring immediately after the release of the external shear loading - have been presented as a function of the original stress level $\tau/\tau_u$. The ratios of $\Delta \delta/\delta$ at $t_{\max}$ varied approximately between 15-35% for the crack widths and 20-50% for the parallel displacements. These ratios increased by 5-10% after an unloaded period of at least three weeks.

Next, a total number of 20 specimens were statically loaded to failure at a constant loading rate for concrete ages between 160 and 407 days. A relatively brittle behaviour was observed - similar to the static bond behaviour of embedded bars after a pre-loading [207, 205] - probably caused by the force-controlled application of the load. The observations were similar to those reported in [112].

No short cracks inclined to the main crack plane as observed by Mattock et al. [37-39] were shown in any of the specimens.

Figure 4.23 (a)-(b) Decreases of crack width and shear slip measured at the instant of unloading.
Eq. (3.1a) proved to fit accurately to the shear strength values, at least if the actual concrete strength was implemented. Thus $\tau_u$ displayed no significant influence of the load history [116]. This conclusion is based on a thorough analysis of test data of different researchers [30,114,120,145]. The aggregates should be sufficiently strong even if pre-loading occurs. Therefore, the percentage of fractured particles has been counted of ten pre-loaded push-off specimens of the type shown in figure 3.1b. The mean ratios $A_{\text{agg}}/A_c$ are low, approximately 23.3% (mix A) and 25.5% (mix B) of the concrete volume [121]. For mix B microcracking has been detected on cross-sections perpendicular to the crack-plane. See figures 4.24a-b. Figure 4.24b shows 24 points systematically observed by means of a microscope. These points contain aggregate particles embedded in the matrix material of cement paste and fine sand. There is no clear orientation of the micro-cracks; they may be induced by shrinkage of the concrete rather than by high stress-concentrations.

With respect to the cracked plain concrete specimens, the release ratio $\Delta\delta/\delta$ appeared to be 1.4-1.9 times higher than in the case of the reinforced concrete push-off specimens. The displacement decreases were almost proportional to the previously applied sustained shear stress.
4.6 Additional tests

As stated in section 2.2.3, the shear transfer mechanism across a single crack is characterized by an interaction between the dowel force and the axial steel stresses of the embedded bars. The response of the combined aggregate interlock and dowel mechanisms strongly depends on the restraint stiffness of the reinforcing bars which cross the crack plane. For a deeper understanding two specific problems still remain:
- the small crack widths - usually 0.05-0.50mm in a push-off specimen - differ from dowel tests with loading eccentricities of some millimeters.
- close to the crack plane the concrete and the steel bar cooperate in resisting the bending moment. This mechanism is due to good bond properties especially on the very part of the bar supported by concrete. As a result the neutral axis of the bar is expected to shift within a length of 1-2 times the bar diameter (figure 4.25).
Thus, it was decided to carry out supplementary research. Section 4.6.1 focuses on the results of pull-out tests on 8mm diameter bars. The axial stresses of embedded bars in push-off specimens have been quantified by means of strain gauge measurements and microscopic examination: see sections 4.6.2 and 4.6.3.

![Figure 4.25 Assumed steel yielding of an embedded steel bar [122,148].](image)

4.6.1 Pull-out tests

The experiments concerned the static and sustained loading of 8mm diameter deformed steel bars ($f_{sy} = 550 \text{ N/mm}^2$, $f_R = 0.059$) each centrically embedded in a $\phi$ 200x200mm concrete cylinder over a length of 40mm.
The displacement rate of the 12 static tests was 0.3-0.5 mm/min. The concrete, either mix A or B, was cast opposite to the loading direction of the bars which differs from the method used for the push-off specimens reported in chapter 3. The cylindrical specimens were removed from the fog room, one day prior to the actual test at 20°C and 50% R.H. The testing arrangement - developed by Van der Veen [152] - consisted of a hydraulic jack, range 400 kN and a displacement transducer (HBM, type W173, 0.002mm accuracy at 2mm.
range) at the unloaded bar end [122]. In all cases the failure mode was a combination of steel yielding and shearing-off. See section 5.2.2. For \( t_0 = 28-90 \) days it was found for \( \tau_{CS} < \tau_{CSU} = 27.5 \, \text{N/mm}^2 \) that (fig. 4.26a):

\[
\text{mix A: } \tau_{CS} = 65.2 * \delta_{CS}^{0.511} \quad \text{and} \quad \delta_{CSU} = 0.231 \, \text{mm} \quad [\text{N/mm}^2] \quad (4.14a)
\]

\[
\text{mix B: } \tau_{CS} = 154.7 * \delta_{CS}^{0.680} \quad \text{and} \quad \delta_{CSU} = 0.086 \, \text{mm} \quad [\text{N/mm}^2] \quad (4.14b)
\]

where \( \delta_{CS} \) denotes the bar slip. These relations display good agreement with other tests [176] for \( \delta_{CS} \leq 0.1 \, \text{mm} \), assuming that \( \tau(\text{vertical})/\tau(\text{horiz.}) \approx 2 \).

Six sustained pull-out tests were performed which started at \( t_0 = 30-144 \) days at a constant loading level \( \tau_{CS}/\tau_{CSU} = 0.62-0.76 \) related to axial steel stresses \( \sigma_s = 341-418 \, \text{N/mm}^2 \). The application period ranged from 46 to 173 hours. The results shown in figure 4.26b are based on a statistical analysis of the recordings [122]. The time-dependent slip values are significantly affected by the concrete age and the concrete grade. To some extent these data provide information about the normal restraint stiffness of the crack plane in a push-off specimen. However, attention should also be paid to the local confinement of the bar and/or to the partial loss of bond near the tension zone of the reinforcement. Note, that the instantaneous slip may be taken as approximately inversely proportional to the concrete strength, see section 5.2.2. As the time-dependent development of \( f_{cc} \) is known - see appendix 11.2 - a part of the descending curves in figure 4.26b is explained.

![Figure 4.26](image)

**Figure 4.26** Development of slip for (a) static and (b) sustained loading.
4.6.2 Strain gauge measurements

The tests were conducted on cracked reinforced push-off specimens of the type shown in figure 3.1a. Either four 8mm diameter ($f_{sy} = 550 \text{ N/mm}^2$; $f_{R} = 0.059$) or three 12mm diameter ($f_{sy} = 500 \text{ N/mm}^2$; $f_{R} = 0.066$) stirrups perpendicularly crossed the shear plane. The axial strain of the steel bars was measured locally by means of so-called TML bolt gauges (type BTM-8 of Tokyo Sokki Kenkyujo, working length 8 x 1mm) cemented in the cross-sectional area of the bar and fastened with epoxy resin. These gauges intersected the crack plane although there is no full guarantee. The method chosen prevented any change of the bond characteristics of the reinforcement used. See also figures 4.27a-b.

Two monotone and two sustained shear loading tests were performed [122]. Apart from the steel stress recordings, the tests were especially used to compare the measured static crack-opening path with results from Walraven [112]. The steel stresses are expected to exceed the values needed for the interlock mechanism alone. Results are presented in appendix 11.3. The theoretical analyses are elaborated in chapter 5.

Figure 4.27 (a) Position of the strain gauges in the crack plane and (b) close-up of one bolt gauge.
4.6.3 Microscopic observations

The strain gauge measurements do not provide the experimental evidence necessary for verifying the dowel action mechanism. According to this mechanism, as presented in section 2.2.2 and figure 4.25, there should be localized plastic deformation of the steel crystals close to the crack plane of the push-off specimen. This plastic deformation of the steel has been examined under a microscope (magnification 500 times). After the pushing-off of the specimen, the concrete of one half of the element was carefully removed, so that the bars remained in their original position. The precise position of the crack plane was marked on the bars by means of a small saw-cut. Next, a small piece of the reinforcing bar was sawn out (figure 4.28). This specimen had a length of $3d_b$, which is presumed to be important for the dowel mechanism. The final shape was attained by the removal of one half of the steel material. The surface was then fixed in polyester resin, polished and etched. During the observations the approximate orientation and yielding of the steel crystals were recorded of 14 specimens, part of which were pre-loaded before the actual static push-off test. Nearly 70% of the selected observations indicated localized orientation of the crystals, which supports the assumption of a local shift of the neutral axis [122,148]. The results also showed that a point of contra-flexure occurred in the bar; however, this point did not always coincide with the assumed location of the crack plane.

![Crack plane](image)

**Figure 4.28** Test specimen used for the microscopic examinations.
4.7 Discussion of results
Chapter 4 discusses the recorded time-dependent displacement behaviour of a single crack in concrete subjected to sustained shear loading. The crack widths observed were usually restricted to $\delta_n < 0.25\text{mm}$, which was the original intention of the research. In none of the long-term tests did shear failure occur. The displacement responses were characterized by significant scatter, not only of the single specimens but also with respect to the experimental reproducibility. These phenomena should be attributed to the heterogeneity of the concrete and they may particularly manifest in case of time-dependency, such as creep and shrinkage of concrete [189,196,200].

It was difficult to distinguish the effect of each experimental parameter. Therefore, an implicit calculation method was developed resulting in unique empirical relationships which objectively and satisfactorily describe the specimen's response. The parameter combinations chosen should be confined to the test range applied. The computational method also supplied sensitivity analyses and simulations of non-tested circumstances. Moreover, the reliability and the accuracy of the results were quantified by means of minimum and maximum response values. A variation of the concrete strength $f_{cc}$ was found to strongly influence the shape of the time-dependent crack-opening curve. It should be realized that under laboratory conditions the region indicated in figure 4.29 will have a smaller area than conceivable in actual practice.

![Figure 4.29 Schematic example of scatter of the displacement response.](image)

The results reveal that the displacement response is highly non-linear with respect to the shear stress-level and the concrete grade. No significant effect of the initial crack width was found. The cracked plain concrete restrained by steel rods displayed relatively small instantaneous crack
widths and limited time-dependent crack width increases for high shear stress-levels. However, the 'creep' coefficients of the plain concrete with respect to $\delta_n$ and $\delta_t$ remarkably exceeded those of the reinforced concrete specimens. The latter had a small displacement decrease at the very instant of the sustained loading release. The static shear strength values were hardly affected by the load history of the specimens. An extrapolation of the sustained loading period did not yield final displacement values. These predictions are based on a mathematical treatment of the test data; advanced reliability is achieved when the specific internal structure of the concrete and its time-dependent characteristics are considered. For these reasons a theoretical model established on a physical basis is to be preferred.

The final sections of this chapter provide some detailed investigations on the contribution of dowel action to the complete shear transfer mechanism. This part of the research is restricted to the bond characteristics of embedded bars and to the measured axial steel stresses of the reinforcement located at the shear plane. With respect to the time-dependency of the aggregate interlock mechanism, the development of the compressive strength, stress-strain relationship, shrinkage and loss of weight have been reported in appendix 11.2 for both types of concrete. A clear distinction in the behaviour of the two mixes was observed. This information supports a fundamental approach which is dealt with in chapter 5.
5. THEORETICAL MODELLING OF SHEAR TRANSFER

5.1 Introduction
This chapter deals with a theoretical approximation of the observed shear transfer mechanism across a crack in reinforced concrete, when subjected to a constant shear loading. Sections 4.2-4.4 presented a statistical treatment and a parameter analysis of the time-dependent crack displacements. However, the approach has a phenomenological basis and is not strictly related to the true time-dependent material behaviour. Some additional tests focused on the microcracking of concrete and on the bond characteristics of the embedded bars close to the crack plane, see sections 4.5-4.6.

The shear transfer across a single crack in plain concrete is satisfactorily described for static loading by means of Walraven's two-phase model \([112,114]\) as pointed out in sections 2.2.1, 2.3.1 and figures 2.17a-d. In the case of a sustained shear loading, the time-dependency of the interlock mechanism - i.e. the interaction between the stiff aggregate particles of a crack face and the compressed matrix material of the opposite crack half - should be considered. With regard to a crack in reinforced concrete, dowel action is another transfer mechanism, which so far has only been studied for static shear loading conditions, see sections 2.2.2 and 2.2.3. Two specific points of interest are raised in sections 5.2.1 and 5.2.2 resp.:
- the time-dependent deformation of matrix material, especially for multi-axial loading conditions;
- the sustained bond behaviour of embedded bars. Time-dependent crack-widening is strongly governed by the bond mechanism. A particular problem in structural engineering concerns cracked concrete \((\delta_n = 0.10-0.30\text{mm})\) subsequently applied by a shear loading.

Next, aspects of these two subjects are used in the sections 5.3-5.7 which present an extension of existing theoretical models to the case of long-term shear transfer across a crack in reinforced concrete.

5.2 Time-dependent aspects
5.2.1 Creep and strength of cement-based materials
The application of a constant compressive stress on cement-based multiphase materials, such as hardened cement paste, mortar and concrete, produces a typical time-dependent deformation denoted as creep \(\epsilon_c(t)\). If no shrinkage occurs, then (figures 5.1a-c):
\[ \varepsilon_{\text{tot}}(t) = \varepsilon_{e1} + \varepsilon_{c}(t) \]  
\[ \text{or: } \varepsilon_{\text{tot}}(t) = \varepsilon_{e1}(1 + \phi_{c}(t)) \]

where the instantaneous strain \( \varepsilon_{e1} = \sigma/E_c \) and the creep coefficient is \( \phi_{c} \).

Figure 5.1 (a) Uniaxial creep test set-up and (b)-(c) time-dependent total deformation of concrete for different ages \( t_0 \).

The creep process is closely related to the properties of cement paste consisting of hydrated cement (gel particles), voids, adsorbed and capillary water and unhydrated cement. The internal surface of the gel is about 200 \( \text{m}^2/\text{gram} \) and the gel has an average void diameter of 2nm (1nm = \( 10^{-9} \text{m} \)) [178]. The material structure is even more complex due to the aging or hydration process. As a result, the creep curves of young concrete diverge, whereas convergence occurs later. A tremendous amount of research has been focused on the creep of concrete [128, 158-160, 165-168, 174, 178, 181, 183, 185-187, 214] and mortar (or: matrix material). A condensed overview is given now on four specific subjects of creep: important influencing factors; mechanism and theories; multiaxial non-linear behaviour; numerical modelling.

**Important influencing factors** The field of interest is usually limited to \( \sigma < 0.5f_{cyl} \) for which creep and induced stress are almost linearly proportional. The observed time-dependent deformation may be affected by internal factors (mix proportion, specimen size, material strength, age at loading, time-duration of loading) and by external factors, namely environmental temperature and humidity [128, 131, 178, 183, 185, 193, 212].
Figures 5.2a-b show the influence of the stress level and the R.H. respectively for concrete prisms. Usually the measured \( \varepsilon_c/\sigma \) ratio is between 50-200 \( 10^{-6} \) mm\(^2\)/N (gravel: less than 1% of these values) but for old concrete prisms applied at \( t_0 = 6-9 \) years, \( \varepsilon_c/\sigma = 10-25.10^{-6} \) mm\(^2\)/N was still found after three years of load application. These results refer to \( f_{cyl} = 28-42 \) N/mm\(^2\) and 65% R.H. [159,199]. In general, the creep deformation is inversely proportional to the static strength. The strength depends on the porous system of the matrix which is affected by the water-cement-ratio. The following relation is valid for a constant sustained stress [131,159]:

\[
\varepsilon_c - f_{cyl}^{-1} \sim (wcr)^2
\]

(5.1c)

Other significant parameters on creep are the volumetric percentages of aggregate (g) and of the unhydrated cement particles (u) [164,185]:

\[
\varepsilon_c(\text{concrete}) = \varepsilon_c(\text{cement paste}).(\frac{100-g-u}{100})^\alpha
\]

(5.1d)

where \( \alpha > 0 \) is an empirical constant depending on the elastic deformations of the matrix and type of aggregate used [159]. Other internal factors of interest on the creep deformation are: pore size distribution, cement fineness, casting direction and duration of vibration of the concrete. The creep phenomenon has a stochastic nature; measured values do not necessarily have a normal distribution [128]. Concrete is subjected to drying or swelling when there is no hygral equilibrium with its ambient environment.

Figure 5.2 (a)-(c) Creep test results of L'Hermite and Mamillan [165] on 70x70x280mm concrete prisms.
The moisture gradient depends on the geometry of the specimen (figure 5.2c) and leads to additional deformations called 'drying creep' and 'drying shrinkage' which are closely related [187]. The creep rate depends on the flux of diffusion of water between the capillary pores and the micropores in the cement gel [212]. The corresponding additional stresses may even cause local microcracking; hence creep and shrinkage deformations can not simply be added, see figure 5.3a. Note, that completely wet specimens show relatively large creep deformations, whereas specimens dried at 105°C exhibit hardly any creep [131]. Figure 5.3b shows a few results of creep tests on Ø 150x600mm cylinders subjected to sustained compressive loading (f_{cyl} = 37 N/mm²; t₀ = 28 days; 50% R.H.). Initially, mix 1 shows relatively small deformations for high stress levels, probably due to an advantageous strength development. A factor finally mentioned are admixtures. For example superplasticizers may cause structural changes - such as the size and distribution of pores and the surface energy of capillary water - potentially affecting the creep deformation [186].

Mechanism and theories

Each analysis of the creep phenomenon is closely related to the structural level of observation. The microlevel concerns the physical properties of hydrated cement gel particles with interlayer hydrate water (< 1nm pore diameter), adsorbed water in gel pores (1-4nm dia.) and capillary water with a high surface energy (4-10nm dia.) [131,179].
Four important mechanisms of creep can be distinguished [178,181,186,201]:
- 'viscous flow' of gel particles lubricated by adsorbed water layers, as proposed by Ruetz [164]. However, in contrast with behaviour of metals [155], creep of concrete relates to a volumetric reduction;
- consolidation due to 'seepage' of either adsorbed water (load bearing water) or hydrate water. The motive force is an externally applied pressure (basic creep) or a vapour pressure (drying creep). This hypothesis was formulated by Ali and Kesler [161];
- 'delayed elasticity' due to a gradual stress redistribution from the cement paste to the elastic skeleton of aggregate particles;
- 'permanent deformation' caused by either local fracture or microstructural repair of the gel structure. The hydration process may be accelerated by the presence of a permanent compressive loading [191].

The main disagreement between researchers concerns the role of water in the cement paste: is it a fundamental cause of creep (seepage) or does it have a secondary effect on the mobility of gel particles (viscous flow)? Is there a contraction at the ends of interlayer surfaces leading to seepage (figure 5.4a) or does a so-called disjoining pressure exist due to hindered adsorption (figure 5.4b)? The above-mentioned mechanisms can be roughly related to either short-term creep (reversible components, i.e. seepage and delayed elasticity, both corresponding to movement of capillary water) or long-term creep (irreversible components, i.e. viscous flow and local fracture, both connected with solid movement). There is still a lack of fundamental evidence to support the assumed theories.

The 'Munich model' is another microstructural approach [179]. The wetting of the cement paste is assumed to increase the average gel particle distance due to a surface energy decrease, caused by the hydrate and adsorbed

Figure 5.4 Different creep models [128,131,164,178,179,186] for hardened cement paste (a) seepage effect; (b) effect of disjoining pressure and (c) swelling according to the Munich model.
water. For higher values of humidity, a disjoining water pressure separates the particles more and more so that their mobility increases (figure 5.4c). The Van der Waals forces then lose their importance. The particle's departure of a position of equilibrium depends on the amount of potential energy supplied and is governed by T, R.H. and the external loading. This hypothesized mechanism is quantified by the theory of rate process [178,181,186] connected with observations of creep on the mesolevel (pores, cracks):

$$\varepsilon_c = c \cdot t_0^{-m} \cdot (t-t_0)^n$$

where $c$, $m$ and $n$ depend on the properties of the cement paste and the environmental conditions. This approach has been successfully applied to other cement-based materials (mortar, concrete). The rate of creep is derived to be inversely proportional to the volume of aggregate particles.

On the macrolevel empirical material laws (see table 5.1) are used: the empirical constants $\alpha$, $\beta$ are calculated by means of statistical methods. Additive or multiplicative combinations of the formulae may better predict the time-dependent deformations [214]. It should be questioned as to whether a very accurate description is needed in view of the stochastic nature of creep [128,180]. For usual life-time periods of structures (30-50 years) the formulae should not necessarily reach a final value. In the engineering practice, the calculation methods of creep often refer to solutions of simple differential equations based on the superposition principle. Examples are the rate of creep method, improved Dischinger method and the age adjusted effective modulus method [139,149]. For the final creep coefficients, the coefficients of variation range from 24 to 31% [180]. Brooks [184] found coefficients less than 20% for estimating ten-year values of basic creep and shrinkage from 28-day test data.

Table 5.1 Overview of creep and shrinkage formulae $\varepsilon_c(t)$.

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<td>$\alpha + \beta \cdot \log(t+1)$</td>
<td>$\alpha \cdot (1 - \exp(-\beta t))$</td>
<td>$\alpha \cdot t/(\beta + t)$</td>
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Multiaxial non-linear behaviour This type of material behaviour has been reasonably explored, particularly for short-term loading, see for instance [140,181]. The non-linearity is related to:
- viscous flow and moisture differentials of water;
- non-proportional stress-strain relationships in compression and tension as well as (micro)cracking;
- material softening and stiffening due to sustained loading.

Figures 5.5a-b show the longitudinal and transverse deformation measured on sealed concrete specimens applied by uni- or biaxial compressive loading [67,169]. Wittmann [178] indicated the similar response of different types of loading which is confirmed by research of Kobayashi et al. [175].

In case of biaxial compressive loading the maximum increases of static strength reported in [195] are about 16% (mortar), and 27% (concrete), in [190] 25% (concrete) and in [192] 38% (mortar). Transverse cracking of the concrete is postponed by the confining pressure. Thus the material behaviour is more ductile. The long-term strength of uniaxially and biaxially loaded specimens was found to vary between 0.85-0.88f' [166,190]. The biaxially loaded plates showed similar fracture at failure; the Poisson ratio is hardly influenced by the type (uni- or multiaxial) or duration of the compressive loading, up to the point when the minimum volumetric strain has just been reached [159,168,169,170,174,185,204]. This point is generally believed to be the beginning of severe microcracking and
the onset of material failure. This conclusion is also applicable to reinforced concrete [98,208]. Figure 5.6c shows that compressive confining pressure reduces creep [186]. Bazant et al. [213] reported a 20% reduction of the volume and a remarkable stiffening of triaxially loaded cement paste specimens ø 19x44 mm; the ratio $\varepsilon_c/\sigma \approx 1-2.10^{-6}$ mm²/N.

The results of multiaxial tests on heterogeneous material are strongly influenced by the geometrical and boundary conditions [142,202]. Van Mier [137] proposed a physical three-phase model for the fracture process in concrete (microcracking; crack-joining and localization). In the second phase, stable delayed crack extension may occur due to crack arrest in mortar and concrete. Concrete multiaxial strength ratios are hardly affected by $f_{cyl}$ (19-59 N/mm²) and the maximum size of the aggregate (if > 5mm) [169,202,208]. Linse et al. [190] reported a few preliminary results of short-term triaxial compression tests (figure 5.6b). An extensive cooperative research project focused on the triaxial behaviour of sealed mortar and concrete specimens. For low stress levels a uniform representation of the stress-strain behaviour is reported [192,195,204].

A lower bound of the ultimate principal compressive stress $\sigma_{1\text{max}}$ can be derived from results of triaxial tests on 123 different types of concrete and mortar, carried out between 1928-1972. For $\sigma_2 = \sigma_3 < 2f_{cyl}$ it is found that:

$$\frac{\sigma_{1\text{max}}}{f_{cyl}} = 1.08\left(\frac{\sigma_2}{f_{cyl}} + 1\right)^{1.57}$$

Figure 5.6 (a) Failure envelopes for biaxial tests on 200x200x50mm plates [190]; (b) results of multiaxial compression tests on 100mm cubes [190] and (c) creep for multiaxial compression [170,186].
Different models have been developed describing the observed multiaxial short-term behaviour of mortar and concrete. Examples are the plasticity theory (criteria of Mohr-Coulomb and Drucker-Prager, approximated by various parameter models), plastic-fracturing theory, endochronic theory and non-linear elastic models [140,143,150,181].

Griffith [156] theoretically simulated the unstable crack propagation in a brittle homogeneous and elastic material such as hardened cement paste. For a small crack length in a plate (plane stress condition) subjected to uniaxial tension he found that (figure 5.7a):

$$\sigma_{cr} = \frac{2Ec\gamma}{\pi c} \quad \text{[N/mm}^2\text{]} \quad (5.3a)$$

where $\gamma$ = material surface energy $= 0.4$ N/m. Shah et al. [173] demonstrated that mortar and concrete exhibit a more ductile behaviour due to crack arrest (figure 5.7b). In addition to this fracture mechanics approach, the crack growth in a visco-elastic material with randomly distributed pores was simulated by Wittmann [178,179]. Assuming equal critical crack lengths $S^*$ for short-term and long-term failure (figure 5.7c) it was found for the strength ratio $\eta$:

$$\eta(t, t_0) = \frac{f_{cc}(\text{long-term})}{f_{cc}(\text{short-term})} = m(t, t_0) \cdot \frac{f_{cc}(t, t_0)}{f_{cc}(t_0)} \cdot \frac{E_c(t_0)}{E_c(t, t_0)} \cdot \frac{1}{1 + \phi_c} \quad [-] \quad (5.3b)$$

where $\eta^*$ refers to the long-term strength of the material (figures 5.8a-b).

![Diagram](image)

**Figure 5.7** Crack propagation of cement-based material (a) Griffith criterion [156], (b) crack arrest and (c) simulated development of crack length in a porous material [178,187].
Figure 5.8 (a)-(b) Examples of development of $\eta$ and $\eta^*$ [178,187] for sustained compressive loading on concrete.

Eq. (5.3b) takes account of the two opposing processes in the material: crack growth (creep) and crack arrestment (hydration; $f_c$ and $E_c$ gradually increase). The release of internal stress is expressed by $m > 1.0$ denoting that the short-term strength increases after a period of pre-loading [117, 160,167,178,187,194,201]. Types of $\eta$-curves according to figure 5.8a also describe the sustained biaxial compression deformation [190]. This conclusion agrees with the observed similar fracture processes in cement-based material subjected to various types of loading.

The numerical modelling of the creep deformation of cement-based materials in case of $\sigma < 0.5 f_{\text{cyl}}$ is often related to the Volterra integral equation:

$$\varepsilon_{\text{tot}}(t) = \frac{\sigma}{E_c} + \int_{-\infty}^{t} K(t-t_0) \cdot \sigma(t_0) \cdot dt_0$$

(5.4a)

where $K$ = kernel of the hereditary theory of elasticity [139,163,178]. For eq. (5.4a) small and gradual changes of $\sigma$ are allowed and the principle of superposition is assumed. Usually, single curves according to figure 5.1b can be described satisfactorily by means of simple $K$-functions, for example $K = \alpha (t-t_0)^\beta$ where $\alpha$, $\beta$ are empirical constants. Eq. (5.4a) may lead to extensive computation times in finite element programs using the smeared approach [138], see section 2.3.2. Moreover, the complete history of strains of each integration point should be stored [139,141,149,150,181]. A solution is provided by expressing $K$ in the form of a Dirichlet series:

$$K = \sum_{i=1}^{\eta} \left( \frac{1}{\alpha_i(t_0)} \cdot (1 - \exp(\frac{t - t_0}{\beta_i})) \right)$$

[N/mm$^2$s] (5.4b)
where $\alpha_j, \beta_j$ are empirical constants of the discrete spectrum [143]. The integral can be numerically solved for adequately chosen time steps. The above-mentioned constitutive relation may be physically interpreted as a rheological model based on the linear visco-elastic theory [178]. Parallel Maxwell chains are preferred to model the time-dependent behaviour in cases of material aging and changes of ambient temperature or humidity [139,163]. If cracking occurs, this mathematical model can be combined with a theoretical tension-softening representation [140,149]. The approach allows the engineer to take the material heterogeneity into consideration [179].

5.2.2 Bond in reinforced concrete

Proper bond behaviour plays a vital role in structural concrete, both under service conditions (control of crack widths and deflections) and for limit state design (reliable anchorage of reinforcement, transfer of shear and bending) [22,29]. Recent research focused on the parameters influencing bond and on the constitutive relations [71,154]. In case of sustained shear loading, the embedded bars that cross the crack plane are subjected to axial tensile stresses as well as confining compressive stresses of the supporting concrete close to the crack edge. Four important aspects of bond are briefly presented here: static bond stress-slip relations; theoretical modelling of bond; effects of lateral pressure and sustained load application; relation between bond behaviour and crack width.

Static bond stress-slip relations Tests usually aimed at providing bond stress-slip relations of either smooth or ribbed (i.e. deformed) embedded steel bars. See figures 5.9a-d and [218]. According to Rehm [157] the slip $\delta_{CS}$ is caused by concrete deformation due to: elastic deformation of comb-like concrete parts between the ribs, localized crushing near the rib surface and internal cracking in the so-called slip-layer.

![Figure 5.9 (a)-(d) Some test set-ups regarding bond of embedded bars.](image-url)
Pull-out tests with usual embedment lengths $l = 3-5d_b$ were carried out by, amongst others, Martin [176], Martin and Noakowski [177] (more than 1300 tests with $f_{cc} = 12-45 \text{ N/mm}^2$; $d_b = 8-32\text{mm}$; $l = 3-20d_b$ and $f_R = 0.015-0.120$) and Vos [136]. For $\tau_{cs} \leq 0.75\tau_{csu}$, simple empirical relations were found for ribbed bars if a homogeneous distribution of the bond stress is assumed:

By Martin [176]:
$$\frac{\tau_{cs}}{f_{cc}} = a_0 + b_0 \cdot (\delta_{cs})^{1/\beta}$$

And in [177]:
$$\frac{\tau_{cs}}{f_{cc}} = \alpha \cdot (\delta_{cs})^\beta$$

Later, Svensvik [205] proposed simple bilinear relations. As shown in figure 5.10a the sliding resistance of ribbed bars is provided by adhesion, material friction (increased by radial stresses caused by transverse cracks) and by mechanical interlock of concrete between the steel ribs.

![Figure 5.10](image-url)

**Figure 5.10** (a) Formalistic model of bond stress-slip behaviour; (b) relative rib area according to Rehm [157] and (c) failure modes.

The values presented in figure 5.10a are: $\tau_1 = 0.05f_{cc}$ (adhesion lost); $\tau_2 \approx f_{ct}$ (transverse cone-shaped cracks); $\tau_2 - \tau_3$ (radial cracks) and $\tau_3 - \tau_{csu}$ (sliding near the ribs). The bond stress-slip relation is influenced by:
- the bar profile characterized by the relative rib area $f_R$ defined as the ratio of the circumferential area $A_R$ and the axially projected area $A_M$;
- mechanical properties of the cement matrix which surrounds the bar. For $\delta_{cs} < 0.1\text{mm}$ the bond stress is proportional to $f_{cc}^{1/2}$ (smooth bars) or to $f_{cc}$ (ribbed steel) [157,177]. A decrease of the maximum particle size $D_{max}$ of the concrete leads to improved bond characteristics as more matrix material becomes available;
- the concrete cover, the embedment length of the bar and the position of the bar during the cast of concrete [157,177];
lateral confinement by means of external stresses or spiral reinforcement, which may significantly improve the bond strength and the axial stiffness $\frac{d\sigma_{cs}}{d\delta_{cs}}$ [177,217]. There is a minor influence of the bar diameter ($d_b = 8-32\text{mm}$) and the concrete age at loading for $t_o \geq 28\text{ days}$.

A pull-out specimen will fail as a result of either steel yielding (in the case of a thick concrete cover), sliding of the bar (see figure 5.10c) or splitting of the concrete cover (usually $c < 1.5-2d_b$). Typical crack patterns were found by Goto [172] who observed sawn cylindrical specimens after ink injection during the application of tensile loading ($d_b = 19$ or $32\text{mm}$; $c = 40$ or $45\text{mm}$; $f_{cyl} = 30\text{ N/mm}^2$). Further increase of the loading leads to secondary cracks between the primary cracks, so that a stabilized crack pattern develops. A thin concrete cover may cause cracks parallel to the bar axis extending from the internal microcracks near the ribs. Schmidt-Thrö [218] calculated the multiaxial compressive stress of the concrete near the ribs in case of sliding. He found $\sigma > 4f_{cyl}$.

Theoretical modelling of bond An approximate description of the bond behaviour in a tension-pull specimen with one centric reinforcing bar, is based on solutions of the differential equation [157,176]:

$$\frac{d^2\delta_{cs}}{dx^2} = k_1^2 \tau_{cs}(x)$$

where $k_1^2 = 4(1+(E_sA_s)/(E_cA_c))/(\pi d_b)$. Note that $\tau_{cs}(x)$ is related to a unique relation along the bar and local cracking is not explicitly considered. The differential equation is solved either numerically or analytically. Later, linear bond stress-slip relations were proposed by Koch [205] and expressed as a function of the local steel stress, $d_b$ and $f_{cyl}$.

More advanced computer-based simulations were carried out by Ngo et al. [24], Nilson [28] and Isenberg et al. [35], see figure 2.19a. Improved input data were found by means of tests of Houde and Mirza [50] and Cedolin et al. [57]. Martin [176] divided the area surrounding the steel rib into three zones (each with specific values of $\nu_c$ and $E_c$). Detailed finite element calculations considering local microcracking and the multiaxial matrix strength were carried out by Martin et al. [176,177], De Groot et al. [133], Vos [136] and Dragosavić et al. [151].
Effects of lateral pressure and sustained load application Untrauer and Henry [162] were one of the first to investigate the effect on the bond strength of normal pressure applied on two parallel faces of a 150mm cubic pull-out specimen ($d_b = 19$ or $28$mm; $f_R = 0.071$). The slip $\delta_{CSU}$ at the loaded end of the bar was proportional to the normal pressure. For the two bar diameters one relation was derived (figure 5.11a):

$$\tau_{CSU} = (1.50 + 0.45\sqrt{\sigma_N}) \cdot f_{cyl}^{1/2} \text{ [N/mm}^2]$$ (5.7)

Dörr [203] carried out bond tests on $\phi$ 150x600mm cylinders each provided with a centric reinforcing bar ($d_b = 16$mm; $f_R = 0.065$-0.075). Precracking was initiated by means of a metal sheet placed in the cross-sectional centre of the specimen. Similar push-off tests are reported in [215,218].

Dörr's results (figure 5.11b) were analysed further by Vos [136] who found a shear stress increase $\Delta \tau_{CS} = 0.17$-$0.23\sigma'_N$ for constant slip-values where $\sigma'_N$ denotes the radial pressure at the bar surface ($\sigma'_N = 1.5\sigma_N$). From his numerical calculations he concluded that the local areas (of plastic strain and of microcracking near the ribs) both extend due to radial pressure.

Klein et al. [182] performed displacement-controlled tensile tests on 500x500x50 mm$^3$ tensile prisms each having one embedded ribbed steel bar $d_b = 10$ or 16mm which crossed a preformed centric crack of 0.40mm maximum width. See figure 5.11c. For $\theta < 90^0$ the bar is subjected to dowel loading as
well, causing bond deterioration and hence a reduction of the bond strength and the bond stiffness. The crack widths (calculated from the measured steel strains) are systematically smaller than were observed on the concrete surface, probably caused by unevenness of the crack plane [172]. Dörr [203] proposed a damage parameter \( k \leq 1.0 \) according to:

\[
k = \frac{\tau_{cs}(\theta)}{\tau_{cs}(\theta = 90^0)} = (2.04 - 0.01150)(\theta/90^0)
\]  

(5.8)

The concrete cover may strongly affect the bond mechanism for \( c < 1.5d_b \). Previously, Martin [176] presented \( \tau_{cs} - c \) (cover) relations as a function of \( f_R, f_{sy} \) and \( f_{cc} \). See also [66]. Based on some previous test results, the maximum dowel force is reduced by \( \rho = \sqrt{1 - (N/N_{sy})^2} \) according to Jimenez et al. [66], Millard et al. [64,65] and Vintzeleou et al. [94]. Note, that Dulacska [44] used \( \rho' = \rho^2 \). The parameters \( \rho \) and \( \rho' \) are tentatively assumed for a sufficient concrete cover as reliable experimental evidence is still lacking. Recently, Perdikaris et al. [88] found no significant effect of biaxial tensile loading \((N \leq 0.9N_{sy})\) on the shear stiffness of reinforced cracked concrete panels. They concluded that there is a minor bond deterioration due to tensile loading.

Sustained pull-out tests are reported by Rehm [157], Franke [188], Martin et al. [177] and recently in [151,205-207,215,217]. For a constant bond stress level \( \tau_{cs}/\tau_{csu} \leq 0.5 \) the time-dependent slip increase \( \Delta\delta_{cs}(t) \) is related to the instantaneous slip:

\[
\phi_\delta = \frac{\Delta\delta_{cs}(t)}{\delta_{cs}(t = 0)} = \alpha t^\beta
\]  

(5.9a)

where \( \phi_\delta \) is the slip creep coefficient, \( t \) is the duration of load application in hours and \( \alpha, \beta \) are empirical constants. Franke [188] found that:

\[
\phi_\delta = (1+10.t)^2-1.0
\]  

(5.9b)

where \( a = 0.082 \) for \( \tau_{cs}/\tau_{csu} = 0.35-0.55 \) and \( t \leq 700 \) hours. See figure 5.12a. Rohling [217] found more reliable extrapolations of the slip for \( a = 0.043-0.085 \) based on an elimination of test data for \( t < 100 \) hours. Svensvik [205] combined eqs. (5.5a) and (5.9a) which resulted in time-dependent bond stress-slip relations as shown in figure 5.12b.
Relation between bond behaviour and crack width

Goto [172] observed that the crack width $\delta_n$ reaches a maximum near the concrete surface. Rohling [217] reported that $\delta_n$ (concrete surface) / $\delta_n$ (steel surface) = 1.2-1.5 for $c/d_b = 3.5$. The crack width ratio tends to increase for high steel stresses. Martin [176] simply proposed that $\delta_n = 2 \delta_{cs}$ taking into account the scatter of the tensile strength. Later, Schiessl [216] stated that not only the crack width, but especially the concrete cover, its permeability as well as the structural detailing would determine the durability of reinforced concrete. Noakowski [211] and Krips [209] emphasized that the stochastic nature of the crack width is mainly a consequence of the scatter of both the bond behaviour and the concrete tensile strength. They mathematically described the development of crack width and crack spacing in monotonically loaded reinforced tension-pull elements, using empirical $\tau_{cs}$-$\delta_{cs}$ relations acc. to eq. (5.5b). Walraven and Reinhardt [114] reviewed empirical relations for the crack spacing, number of cracks and loading state, which were combined according to eq. (1.1).

Unstable crack patterns are characterized by a time-dependent crack-widening due to bond creep which corresponds to an increase of the transmission length of the embedded bars, see figure 5.12c. These effects can be quantified by inserting fictitious time-dependent bond stress-slip relations into crack width formulae [153,205,211]. In case of stabilized crack patterns, decreasing tension-stiffening occurs in the crack spacing zone. Rohling [217] assumes that the steel stress 'gap' between two cracks gradually reduces, so that $\Delta \sigma_s(t = \infty) = 0.5 \Delta \sigma_s(t = 0)$. Both above-mentioned types of crack pattern are initiated by imposed deformation or/imposed loading.
5.3 Shear transfer model - monotonic loading

5.3.1 Basic mechanisms

This section addresses the load transfer across a single crack crossed perpendicularly by reinforcing bars. The two transfer mechanisms of the static loading case are shortly discussed: the formulae are also adapted to sustained loading conditions in section 5.4.

A sound and reliable physical model of the aggregate interlock was formulated by Walraven [112-114,130], see section 2.3.1. Eqs. (2.13a-c) determine the stress-displacement relations of a plane crack. As the crack opens and slides the total contact areas $\Sigma a_x$ and $\Sigma a_y$ of the stiff granular particles gradually increase that is provided by plastic deformation of matrix material, see figure 5.13a. Pruijssers [148] stated that a minimum slope $\partial \delta_t / \partial \delta_n$ exists for each combination of displacements, see figure 5.13b. This is analogous to a sufficient restraint of the crack halves. As the shear slip develops, an increasing percentage of the aggregate particles reaches 'full contact' with the matrix material provided that $\delta_t > \delta_n$ and that the embedment depth is sufficiently small ($u < R - \delta_n$). For these individual particles it is found that:

$$\delta_t > \sqrt{R^2 - u^2} - \sqrt{R^2 - (u + \delta_n)^2}$$ \hspace{1cm} \text{[mm]} \hspace{1cm} (5.10)

Figure 5.13 (a) Contact areas and deformed matrix and (b) possible crack-opening directions (shaded area).

Dowel action of reinforcing bars is the second mechanism of shear transfer in a crack. Several empirical formulae were given in section 2.2.2. Eq. (2.5) provides a reliable formula that was found by Rasmussen [14]. The dowel strength is reduced by bending due to eccentricity $e$, see figure 5.14a. Pruijssers [147,148,150] derived:

$$V_{du} = 1.35[\sqrt{1+\epsilon'} - \epsilon'].d_b^2 f_{ccm} f_{sy}$$ \hspace{1cm} \text{[N]} \hspace{1cm} (5.11)
where \( \varepsilon' = e \sqrt{f_{ccm}/f_{sy}} / d_b \). This formula resembles eq. (2.5) but now \( V_{du} \) is increased by 10-12% provided \( f_{cyl} = 0.85f_{ccm} \) and \( e \leq 0.015d_b \) and \( f_{cyl}/f_{sy} = 0.05-0.10 \). Eq. (5.11) fits well with test results of various researchers, for a sufficient cover of the concrete i.e. no premature splitting failure. Rigid bond (no slip at the interface) is adopted on the supported part of the dowel close to the crack plane. Yet, high bond stresses occur as a result of elastic deformation in the interface layer, see [136,172,209,211]. These stresses are enhanced by local multiaxial confinement of the concrete, see for example figs. 2.8c and 5.11a-b. Actually, a 'push-in' mechanism takes place which may postpone local cracking. As for the dowel strength, a better concrete strength not only improves the bond characteristics, but it also provides higher multiaxial resistance inducing an additional rise of the bond. The analysis of previous dowel test results revealed a shift of the neutral axis towards the supported side of the bar. The cooperation of the embedded bar and the surrounding concrete causes a considerable increase of the plastic bending moment of the bar at dowel failure, see figures 5.14a-b:

\[
M_{pl} = V_{du}(e + ax) = 0.22d_b^3 f_{sy} \quad [\text{Nmm}] \quad (5.12)
\]

Note that \( M_{pl, bar} = 0.166d_b^3 f_{sy} \) [31] is exceeded by about 34%. The computations are based on the stress distribution at the plastic hinge of the bar. The precise distribution of bond stresses parallel to the bar axis is not considered in view of the small length of \( x \) in figure 5.14a. Combining eqs. (5.11-12) with \( \varepsilon = 0.45 \) (fig. 2.10b) and \( e = 0 \) leads to:

\[
x = \frac{M_{pl}}{0.45V_{du}} = 0.273d_b \sqrt{f_{sy}/f_{ccm}} = 0.8-1.1d_b
\]

Figure 5.14 (a)-(b) Assumed dowel mechanism according to Pruijssers [148] and (c) assumed formalistic dowel model.
This distance agrees quite well with several research results [14, 44, 87, 93, 94] and with calculations reported on pp. 18 of this report. Figure 5.14c points out that the dowel strength is reached if the crack halves slide parallel over $\delta_t \geq \delta_{no}$, with a proposed minimum of 0.10mm for $\delta_t$. Here $\delta_{no}$ is defined as the crack width at the instant that the shear load is just applied. This simple approximation is in reasonable agreement with test data in [44, 49, 64, 65, 108]. The 'normal restraint' of the crack is delivered by the embedded bars. High axial stresses reduce the shift of the neutral axis, see figure 5.14b. The interaction of axial and shear stresses is formulated empirically by means of a yield criterion [64-66, 94] so that for $\sigma_s \leq 0.45f_{sy}$, $V_{du}$ decreases by less than 21%, see also figure 5.15:

$$V_d = \gamma_d V_{du} = \sqrt{[1 - (\sigma_s/f_{sy})^2]} V_{du} \quad (\gamma_d \leq 1) \quad [N] \quad (5.13)$$

Two final remarks are made on the dowel model adopted in figure 5.14c:
- Another approach to verify the formalistic model is based on an elastic response of the concrete. With $\delta_{no} = 0.005d_b$, $\beta = 0.92/d_b$ and $V_{du}$ according to eq. (5.11) the formula presented in fig. 2.7c yields $\delta_t = 2.5 \times 10^{-4} d_b / f_{sy} f_{ccm} = 5\delta_{no}$ [mm]. Reduced dowel deflection due to local triaxial confinement is neglected. Therefore, the predicted slip overestimates the real value. Rasmussen [14], Broms [15, 16] and Vintzeleou et al. [93, 94] report $\delta_t = 0.10-0.17d_b$ at $V_d = 0.9V_{du}$, but these displacements refer to a rather large eccentricity (or: large $\delta_{no}$) demanding a significant shear slip before the bearing stresses have fully developed;
- Figure 5.14c indicates an initial dowel stiffness $\partial \sigma_d / \partial \delta_t = 22.7 \, N/mm^3$ for eight 8mm diameter steel bars *). Eq. (2.8a) yields 10.7 $N/mm^3$. Before the dowel force reaches its maximum value, the 'shear stiffness' is almost fully governed by interlocking. This is suggested by the high values presented in fig. 2.18b.

*) $f_{ccm} = 35$; $f_{sy} = 400 \, N/mm^2$; $\delta_{no} = 0.01mm$; $\rho = 1.1\%$ and $k = 2500 \, N/mm^3$. 
5.3.2 Model for a reinforced crack

The complete shear stress to be transferred is defined as:

\[ \tau = \frac{V}{A_c} \quad \text{and:} \quad \tau_{du} = \frac{V_{du}}{A_c} \quad [N/mm^2] \quad (5.14a) \]
\[ \rho = 0.25 * \pi d_b^2 / A_c \quad [-] \quad (5.14b) \]

On combining eqs. (2.13a-c), (5.11) and (5.13) it yields:

\[ \tau = \tau_a + \gamma_d \tau_d \quad [N/mm^2] \quad (5.15a) \]
\[ \sigma = \sigma_a \quad [N/mm^2] \quad (5.15b) \]

so that for \( d_b = 8\text{mm} \) and \( A_c = 120 \times 300 \text{ mm}^2 \):

\[ \tau = \sigma_{pu}(A_y + \mu A_x) + 5.40 \gamma_d \rho a \sqrt{(f_{ccm} f_{sy})} / \pi \quad [N/mm^2] \quad (5.15c) \]
\[ \sigma = \sigma_{pu}(A_x - \mu A_y) \quad [N/mm^2] \quad (5.15d) \]

where \( \sigma_{pu} = 6.39 f_{ccm} 0.56 \); \( \mu = 0.40 \) and \( \alpha = (\sqrt{1+(\epsilon')^2} - \epsilon') \). Pruijssers [147,148] proposed to multiply both interlock stresses by \( \gamma_a < 1 \) in case of a low reinforcement ratio. This adaptation is not considered in this report. Eqs. (5.15a-d) denote that the stresses \( \sigma, \tau \) are uniquely related to \( \delta_n, \delta_t \).

If the plastic hinges have fully developed in the bar then the crack-opening is dominated by the rough-crack model of Walraven, because the dowel bars no longer affect the compatibility of the crack. At the onset of the shear test - for small shear slips \( \delta_t \leq 0.10\text{mm} \) - the bearing stresses under the dowel are relatively low. Then the model of a beam on elastic foundation (see section 2.2.2) may be applied [148]:

\[ \tau_d = \tau_{du} \left( \frac{\delta_t}{0.10 + \delta_{t,e}} \right)^{1/3} \leq \tau_{du} \quad [N/mm^2] \quad (5.16a) \]

where: \( \delta_{t,e} = 1.31 \times 10^{-7} d_b 0.60 \sqrt{(f_{ccm} f_{sy})} \) \[mm\] \quad (5.16b)

The crack-opening curve is now given by an empirical relationship:

\[ \delta_t = (\delta_{no} f_{sy}/2f_{ccm})^{1/2} (\delta_n - \delta_{no})^{2/3} \quad [mm] \quad (5.16c) \]

The combined model fits with tests of Mattock [38], Millard et al. [64] and Walraven [112]. The computational procedure is summarized in appendix 11.5.
5.4 Shear transfer model - sustained loading

5.4.1 Matrix strength

The crack halves of the push-off specimens subjected to a sustained shear loading, reveal time-dependent displacement increases $\delta_{nc}(t), \delta_{tc}(t)$, see eqs. (4.1a-b) and figs. 4.1a-b [120,121,123]. The equilibrium state of the crack basically depends on the values of $\sigma_{pu}$ and $\mu$ according to the rough crack model. The matrix material is expected to display creep deformation due to the long-term compressive forces caused by the aggregate particles embedded in the opposing crack half. Realistic formulations for $\sigma_{pu}$ and $\mu$ are needed as functions of the application period. This section highlights some influencing factors.

The matrix consists of hydrated and unhydrated cement particles, air voids, small sand particles ($D \leq 0.25\text{mm}$), water and admixtures. The strength $\sigma_{pu}$ of the matrix corresponds to local crushing of the material. Moreover, air voids may partly be compressed. The macroscopic deformation is assumed to be high, see fig. 2.17a, but it is restricted to a volume close to the surface of an aggregate particle. As shown in table 5.2, the average thickness of the matrix layer embedding a gravel particle is less than $t = 0.20\text{mm}$:

$$t = \frac{\text{matrix volume } V}{\sum (\pi D_i)^2} \quad [\text{mm}] \quad (5.17)$$

where $D_i$ ranges from $0.25\text{mm}$ to $16\text{mm}$. Eq. (5.17) presumes that the volumetric parts are related to an arbitrary plane area in the concrete mix. Consequently, the maximum shear slip amounts to $2t*\sqrt{2} \approx 0.45\text{mm}$ before two opposing particles make contact in the crack plane. It can be deduced [118] that the smallest particles first make contact with larger 'spheres'; for $D \geq 0.50\text{mm}$ the matrix layer permits a maximum shear slip of about $0.80\text{mm}$ for both types of mix used so that sliding is dominated by matrix deformation. The elastic deformation provides a negligible slip of about $t\sigma_{pu}/0.3E_{agg} \approx 4*10^{-3}t$ [mm], see [219]. The matrix stiffness is expected to be higher due to local confinement. Virtually, the calculation considers an unfavourable situation: the interlock model appears applicable for a parallel displacement of more than $1\text{mm}$ so that the real thickness of the matrix layer may be under-estimated.
Table 5.2 Information related to eq. (5.17), see also appendix 11.1.

<table>
<thead>
<tr>
<th>aggregates</th>
<th>$D_1$ [mm]</th>
<th>surface area [m²/m³] *)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mix A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8-16</td>
<td>12</td>
<td>117.7</td>
</tr>
<tr>
<td>4-8</td>
<td>6</td>
<td>166.5</td>
</tr>
<tr>
<td>2-4</td>
<td>3</td>
<td>235.5</td>
</tr>
<tr>
<td>1-2</td>
<td>1.5</td>
<td>333.4</td>
</tr>
<tr>
<td>0.50-1</td>
<td>0.75</td>
<td>471.5</td>
</tr>
<tr>
<td>0.25-0.5</td>
<td>0.375</td>
<td>665.9</td>
</tr>
<tr>
<td>mix B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>117.7</td>
<td>112.5</td>
<td></td>
</tr>
<tr>
<td>166.5</td>
<td>159.2</td>
<td></td>
</tr>
<tr>
<td>235.5</td>
<td>225.1</td>
<td></td>
</tr>
<tr>
<td>333.4</td>
<td>320.0</td>
<td></td>
</tr>
<tr>
<td>471.5</td>
<td>448.6</td>
<td></td>
</tr>
<tr>
<td>665.9</td>
<td>633.9</td>
<td></td>
</tr>
</tbody>
</table>

*) $V$(matrix) = 0.307 and 0.346 m³/m³ and $t$ = 0.15 and 0.18 mm resp.

Creep phenomena of concrete are usually characterized by short and long-term components of deformation, see pp. 87 of this study. The shift or 'change-over' manifests by means of a clear adjustment to a lower creep rate after a rather short sustaining period. Yet, this has not even been noticed for the crack displacements after an observation period of more than nine months, see chapter 4. Besides the normal stresses on the contact areas $\sigma_x$, $\sigma_y$ (see figs. 5.13a-b) the matrix also experiences a shear stress $\tau_{pu} = \mu \sigma_{pu}$. Thus, a multiaxial compression state exists which is even more complex due to time-dependent aspects. Another point is that the assumed rigid-plastic constitutive relation of the matrix for the static loading case can not be simply transformed into a gradual 'weakening' according to figure 5.3b for the sustained loading case.

Based on the preceding remarks, it was decided to use a strength criterion for the description of the time-dependent constitutive relation of the matrix. A major parameter is the 28-day cube concrete strength $f_{ccm}$ which affects both the interlocking of the crack faces and the dowel action of the bars, see eq. (5.15c). It is postulated that a strength reduction may evolve as a function of $f_{ccm}$, the duration of load application and the concrete age at the onset of loading denoted by $t$ and $t_0$, respectively. In the qualitative sense, this strength decrease agrees with the observed macroscopic behaviour of cement-based materials subjected to a constant
normal stress, see figs. 5.8a-b. The sustained stresses cause material 
damage related to the development and propagation of microcracks repre­ 
sented by the measured macroscopic strain and expressed by $\phi_C$ in eq. (5.3b). 
Aggregate particles or inclusions such as pores may arrest the crack exten­ 
sion either within the matrix material or on its boundary: microstructural 
repair may even occur due to ongoing hydration of cement particles. All 
these effects are not restricted to uniaxial loading conditions. Figs. 
5.5a-b suggest that a similar, macroscopic response of the material is 
expected in case of multiaxial actions. The long-term strength $f$ is directly 
associated with the uniaxial compressive strength $f_{ccm}$ of concrete at 
t_0 = 28 days [118,124,126]:

$$f(t) = \lambda_f(t,t_0,f_{ccm}) \times f_{ccm} \quad (\lambda_f \leq 1) \quad [N/mm^2] \quad (5.18a)$$

where $\lambda_f = 1.0$ for t_0 = 28 days at the instant t = 0 h that the long-term 
loading is applied: $\lambda_f$ denotes a damage parameter.

Eq. (5.18a) presumes a multiplicative character of the material deterio­ 
ration process. To solve $\lambda_f$ is essential in arriving at a proper model for 
the time-dependent shear transfer mechanism. Obviously, the actual behav­ 
ior of the matrix is based on processes acting at a lower level demanding 
a deep insight and knowledge of the internal structure of the material. 
However, eq. (5.18a) is to be incorporated in a so-called engineering model 
which does not consider the physical phenomena of the detailed (meso- or) 
microlevel of observation.

The approach chosen is based on an overall description apt for practical 
use. One should realize the scatter of the test results as pointed out in 
chapter 4. Therefore, a strictly accurate formulation is not useful and 
does not improve predictions of the structural response of the crack.

With respect to the short-term behaviour of the concrete interface, the 
interlock stresses $\sigma_a$ and $\tau_a$ are proportional to $f_{ccm}^{0.56}$ and the dowel 
strength $\tau_{du}$ is directly related to $f_{ccm}^{0.50}$, see eq. (5.15c). These two 
exponents lie within the same range so that a change of the concrete 
properties approximately affects both transfer mechanisms similarly. 
This assumption is extended to long-term behaviour.
Special attention should be paid to high-strength concrete in view of the increasing brittleness expressed by the ratio $\sigma_{pu}/f_{cc}$, see figure 5.16 and references [125,131]. Eibl et al. [219] stated that the ratio also depends on $D_{max}$ for a constant water-cement ratio. This influence is not specifically considered here. The shear plane of the crack was extensively observed after the static push-off tests, see section 4.5. However, the high-strength concrete used for the tests - mix B with $f_{ccm} \approx 70$ N/mm$^2$ - revealed no systematic difference in percentage of fractured particles than mix A ($f_{ccm} \approx 51$ N/mm$^2$). These ratios remained low related to the significant number of bond cracks along the interface of the gravel aggregates. Consequently, the observations indicate that the rough crack model is valid for all the experiments carried out. Eq. (2.13c) for $\sigma_{pu}$ may now underestimate the true matrix strength. The matrix strength should at least be as high as $f_{cc, 95\%}$. This yields for mix B, see figure 3.1lb:

$$\sigma_{pu} > f_{cc, 95\%} = 69.3 \times (1 + 0.063 \times 1.64) = 76.5 \text{ N/mm}^2$$

This condition is satisfied for a 15% increase of the exponent 0.56, thus:

due to dowel action: $f_{ccm}^{0.50} \approx 0.56 \times f_{ccm}^{0.64}$ \text{[N}^{1/2}/\text{mm}] (5.18b)

due to interlocking: $\sigma_{pu} \approx 4.76 \times f_{ccm}^{0.64}$ \text{[N/mm}^2\text{]} (5.18c)

Eqs. (5.18b-c) deviate less than 5% from the original formulae, for $f_{ccm}$ ranging from 40 to 75 N/mm$^2$, see figure 5.16.

![Figure 5.16 Basic and modified formulae.](image)
5.4.2 Friction coefficient

Besides the aggregate interlock model offers a second 'degree of freedom', viz. $\mu$ which was established 0.40 for the case of monotonic shear loading. Friction contributes significantly to the transfer of $\tau_a$ across a crack. It also affects the relationship between the restraining stress $\sigma_a$ and the crack displacements. Microscopic undulations cause a frictional mechanism. The actual contact area of two solids does not solely depend on the geometry of these micro-roughnesses, but it is also determined by how these irregularities deform. Generally, friction can be divided into an adhesive and a deformational component. This last term particularly relates to the contact between a rather stiff and a soft material, such as the interaction between gravel and the cement matrix respectively. The literature on tribology of materials does not supply any information about the time-dependency of $\mu$. Two opposing processes may be important:

- gradual smoothing of the contact surface mainly due to the high compressive and shear stresses. They locally compress the matrix more and more, depending on the mix composition as well as on the local distribution and size of the pores. Loose small-sized granulars can form in the macrocrack when it is initiated ('split') or during the sliding of both crack halves parallel to one other. See figure 5.17. These particles may also originally be gathered in air voids and then become compacted later under shear loading. This might affect the (macroscopic) friction [137];

- enhanced roughening of the contact surface. The sustained multiaxial loading may induce local fracture of the matrix layer close to the contact areas, see table 5.2. Another process concerns microstructural changes on the contact surface by a growth of C-S-H crystals during the hydration of cement particles.

Two other phenomena should be mentioned in this respect:

* other material damage on the mesolevel such as effects of residual stresses due to imposed deformation as a result of moisture gradients, see pp. 87 and in [180,181]. Probably small cracks initiate in the matrix layer perpendicular to the contact surface. The material is considered as a set of parallel struts which permit rotations on application of $\tau_{pu}$;

* subsequent repair of the deteriorated structure by re-hydration [201].

It is not clear which of the above mentioned mechanisms prevails as time passes. It is tentatively proposed to incorporate a constant friction coefficient $\mu = 0.40$ for the analysis of the sustained shear tests.
5.4.3 Extended model for a reinforced crack

Attention is now paid to the interaction of aggregate interlock and dowel action, provided that:

\[ \tau(t) = \bar{\tau} = \text{constant} \quad [\text{N/mm}^2] \quad (5.19a) \]

Combining eqs. (5.13), (5.15c-d) and (5.18b-c) yields for the case of monotonic shear loading:

\[ \tau = \sigma_{pu}^* (A_y + \lambda \mu A_x) + 0.204 \alpha \gamma_d \rho f_{sy} 0.50 \quad [\text{N/mm}^2] \quad (5.19b) \]

\[ \sigma = \sigma_{pu}^* (A_x - \lambda \mu A_y) \quad [\text{N/mm}^2] \quad (5.19c) \]

where \( \lambda = 1.0; \mu = 0.40; \alpha = \sqrt{1+(\varepsilon')^2} - \varepsilon' \) see eq.(5.11), and with \( \rho = 0.25 \cdot n \cdot d_b / A_c \). With eqs. (5.18a), (5.19a-c) it is derived for the case of a sustained shear loading, that:

\[ \tau(t) = \bar{\tau} = \tau \beta(t) / \beta = c \lambda_F^* (t,t_0,f_{ccm}) \tau(\text{static}) \quad [\text{N/mm}^2] \quad (5.20a) \]

\[ \sigma(t) = c \lambda_F^* (t,t_0,f_{ccm}) \sigma(\text{static}) \quad [\text{N/mm}^2] \quad (5.20b) \]

where \( c \) provides the state of equilibrium at \( t = 0 \): \( \lambda_F' = \lambda_F 0.64 \) refers to a multiplicative process, see section 5.4.1. Parameter \( \lambda \mu > 1.0 \) takes account of the time-dependent change of the friction coefficient, see section 5.6. Consequently, the long-term behaviour of the crack is treated quasi-statically according to appendix 11.5. There is a comparable effect of \( \lambda_F' \) on both transfer mechanisms. All the formulae refer to a fully developed dowel load at the instant that the constant shear load has just been applied on the reinforced crack. Consequently, the sustained shear stress should conform to a minimum value, see eqs. (5.23a-b).
5.5. Review of computations

This section points out the calculation procedures. A computer program was especially developed for a computational analysis of the theoretical models presented in sections 5.3-5.4. Section 5.6 deals with the instantaneous and the time-dependent displacements of a single crack exerted by a sustained shear loading. They have been compared with test data given in sections 4.2 and 4.3. As for the reinforced crack the initial cracks widths ranged from 0.01-0.06mm: two concrete grades and three reinforcement ratios were investigated. The material properties used in the computer program are:

- concrete:  
  mix A:  \( D_{\text{max}} = 16\text{mm}; \ p_k = 0.70; \ f_{ccm} = 51 \text{N/mm}^2 \)  
  mix B:  \( D_{\text{max}} = 16\text{mm}; \ p_k = 0.67; \ f_{ccm} = 70 \text{N/mm}^2 \)  
- reinforcing steel:  \( f_{\text{sy}} = 460 \text{or} \ 550 \text{N/mm}^2; \ d_b = 8\text{mm}; \ \rho = 1.12-2.24\% \)

Contribution of the interlock mechanism The contact areas  \( A_x \) and  \( A_y \) for a unit crack area have been determined by numerical integration according to Walraven [112,113]. Preliminary, \( \mu = 0.40 \) was taken for the complete observation period of the experiments. Next, the crack-opening curves \( 100\sigma_a/\sigma_{pu} = \alpha (\alpha = 0.5,1,2,\ldots,16) \) and \( 100\sigma_a/\sigma_{pu} = \beta (\beta = 0.5,1,2,\ldots,12) \) have all been individually described by sets of third-order polynomials according to:

\[
\delta_t = \gamma_1 + \gamma_2 \delta_n + \gamma_3 (\delta_n)^2 + \gamma_4 (\delta_n)^3 \quad \text{[mm]} \quad (5.21)
\]

where  \( \gamma_1-\gamma_4 \) are empirical constants. Separate formulas were derived for  \( 0.02 \leq \delta_n \leq 0.20\text{mm} \) and for  \( 0.20 < \delta_n \leq 0.80\text{mm} \) in order to attain more accurate approximations. For given  \( \delta_n,\delta_t \) the interlock stresses are linearly interpolated between the polynomials. Thus a discretization of interlock relationships is used. The computational results lie within the measuring accuracy. Figures 5.18a-b and 5.19a-b refer to displacement paths of the crack for a monotonic shear loading. Steeper paths are attained if  \( D_{\text{max}} \) or  \( \sigma_{pu} \) decreases, for light-weight and for high-strength concrete in view of a more ‘smooth’ shear plane [112-114] (less overriding of the undulations). Another reason is the relatively low value of  \( p_k \) usually applied for high-strength concrete. For  \( \delta_t < 1/3\delta_n \), denoted as ‘free slip’,  \( \sigma_a \) becomes negative (compression) so that path-dependency may be neglected [115].

Contribution of the dowel mechanism Figure 5.20 shows the development of  \( \tau_{du} \) for  \( A_C = 36000 \text{ mm}^2 \) and  \( d_b = 8\text{mm} \), according to eqs. (5.11-5.14a-b).
Figure 5.18 (a)-(b) Static $\delta_t - \delta_n$-curves acc. to the rough crack model.

Figure 5.19 (a)-(b) Identical to figs. 5.18a-b for small crack widths.
5.6 Experimental verification

5.6.1 Instantaneous displacements

This section points out the calculated response of a single crack due to a monotonically increasing shear loading. Three subjects are highlighted:
- comparison between the predicted and recorded stresses $\sigma_s$ and $\tau$ transferred across the shear plane;
- an analogous analysis with respect to the static crack-opening curves;
- remarks on different displacement behaviour observed in previous research.

Measured average shear stress-crack width relations are compared with the calculations in figures 5.21a-b: the test data are given for $\tau \leq 0.9\tau_u$. The computed development of $\tau_d$ (dowel contribution) has been indicated separately. The predicted initial stiffness $\partial \tau / \partial \delta_n$ is relatively high. For widths $< 0.20\text{mm}$ - valid for nearly all the instantaneous displacements of the sustained tests - the shear stresses differ less than $1.4 \text{ N/mm}^2$. The adjusted stress levels of the long-term tests were between 0.45 and 0.89. Therefore, the average ratios $\bar{x}$ of computed and recorded shear stresses indicate a reasonable agreement [116,120], see table 5.3: the coefficients of variation were less than 5%. The different response may particularly be attributed to the sensitivity of the model for a change of the initial crack width according to eq. (5.16c). Improved agreement was found for an exponent of 0.25 instead of 0.50, probably due to the load-controlled application method used [135]. As a consequence, the crack opening direction at a fully developed dowel load is less sensitive to a change of $f_{cc}$ and $f_{sy}$. Another reason for the deviations may be the parameters of the rough crack model, which are based on other crack-opening curves, see pp. 114.
The computed compressive stresses perpendicular to the shear plane are not affected by a variation of \( \rho \). They agree quite well with strain gauge measurements carried out on one specimen of each mix (\( \rho = 1.12\% \)), see section 4.6.1 and appendix 11.3. Actually, \( \sigma_s / \sigma_c \) refers to the centre of the shear plane. The measurements may be affected by the 8mm working length of a strain gauge and the uncertainty about the precise position of the crack plane, see fig. 4.25. The computations revealed a fully developed dowel load at \( \delta_t = 0.11 - 0.13 \text{mm} \), compared with \( \delta_t = \delta_{no} \) and \( \geq 0.10 \text{mm} \) in fig 5.14c. At low axial stresses the expected shift of the neutral axis should be related to localized orientation of the steel crystals near the plastic hinges of the bar. This assumption of the dowel model has been successfully substantiated by microscopic observations, see section 4.6.3.

<table>
<thead>
<tr>
<th>( \tau ) at ( \delta_n = )</th>
<th>( \bar{x} )</th>
<th>mix A</th>
<th>mix B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10mm</td>
<td>1.20</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>0.15mm</td>
<td>1.08</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>0.20mm</td>
<td>1.00</td>
<td>0.94</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.3 Ratios of predicted and measured shear stresses.**

Figure 5.21 Response on a monotonic loading for (a) mix A and (b) mix B.
Figures 5.22a-b show computed crack-opening paths which agree accurately with the previously mentioned empirical relation of Walraven [112]. Steeper curves are attained for higher ratios $f_{sv}/f_{ccm}$ according to eq. (5.16c). A similar conclusion can be derived from eq. (2.11) for $\delta_n < 0.23\text{mm}$. The effect of a variation of $f_{ccm}$ (mix A, figure 5.22b) is similar to curves in figure 4.15a for the time-dependent case, although these are systematically flatter. Another interesting feature concerns specimen no. 5 in fig. 5.22b applied by shear loading at $t_o = 10$ days with $f_{cc} = 43.6 \text{N/mm}^2$. Probably, the effect of material age on the time-dependent response is particularly governed by the actual concrete strength.

It was found that the recorded crack-opening curves until the onset of the sustained loading (see figs. 4.13a-b and 4.14a-b) were flatter than expressed by eq. (2.11) for $f_{cc} = 56 \text{N/mm}^2$. There is a maximum difference of $\Delta\delta_t = 0.05\text{mm}$ for $\rho = 1.12\%$ and $f_{cc} = 70 \text{N/mm}^2$.

Figure 5.22 (a) Calculated static curves for mix A and B; (b) calculated (static) and measured (sustained) curves for mix A.
The small distinction between the static crack-opening curves of Walraven ($f_{ccm} = 56 \text{ N/mm}^2$) and those of this research ($f_{ccm} = 51$ or $70 \text{ N/mm}^2$) is similar to $c \neq 1$ in eqs. (5.20a-b). The differences are attributed to:
- different application methods used, viz. displacement-controlled with respect to the shear slip (Walraven's tests) and load-controlled (this research). Gradual displacement increases of $\delta_n$ and $\delta_t$ were observed in between the stepwise application intervals of the external shear loading, particularly if $\tau \geq 0.5\tau_U$, see figure 5.23;
- displacement range used for the empirical formulae denoted by eq. (5.22a). Walraven based his relationship on the recordings of the complete shear stress-displacement relations including the descending branch for which $\delta_t, \delta_n > 1.0 \text{mm}$. However, the sustained shear tests usually refer to a maximum slip of less than $0.50 \text{mm}$.

![Figure 5.23 Schematic response curves for two types of load application.](image-url)
5.6.2 Time-dependent displacements
This section addresses the time-dependent crack-opening path. Next, section 5.6.3 deals with proposed empirical relations for $\lambda'_f(t)$ and $\mu(t)$ illustrated by a working example.

Figures 5.24a-b show some examples of $\delta_n(t)$ curves up to an application period of $10^5$ hours. They are established from figs. 4.14a-b. There is a slight influence of both the shear stress level and the ultimate normal restraining stress $\rho f_{sy}$. The overall curves display only a significant effect of the concrete grade used, see figure 5.25. As shown, the static curves of the crack which refer to monotonically increasing shear load are systematically steeper. Reliable relations were obtained for each test series according to:

$$\delta_t = \alpha(\delta_n)^\beta$$  \hspace{1cm} (5.22a)

with correlation coefficient $r > 0.99$ in all cases, see also table 5.4. The sustained shear tests refer to $\tau/\tau_0 = 0.50-0.90$; $\rho f_{sy} = 5.15-12.32 \text{ N/mm}^2$; $\delta_{no} = 0.01-0.06 \text{ mm}$ and $0 \leq t \leq 10^5$ hours.

Figure 5.24 Recorded crack-opening curves for (a) mix A and (b) mix B.
Table 5.4 Coefficients $\alpha$, $\beta$ of eq. (5.22a).

<table>
<thead>
<tr>
<th>loading</th>
<th>$f_{ccm}$ [N/mm$^2$]</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>static</td>
<td>20-40</td>
<td>1.400</td>
<td>1.200</td>
</tr>
<tr>
<td>static</td>
<td>56</td>
<td>1.870</td>
<td>1.400</td>
</tr>
<tr>
<td>sustained</td>
<td>51</td>
<td>1.249</td>
<td>1.285</td>
</tr>
<tr>
<td>sustained</td>
<td>70</td>
<td>1.195</td>
<td>1.375</td>
</tr>
</tbody>
</table>

Figure 5.25 Curves according to eq. (5.22a), see also table 5.4.

The instantaneous crack widths $\delta_{nei}$ were accurately described [123] by a type of formula similar to eq. (5.22a):

$$\delta_{nei} = n \left( \tau / \tau_u \right)^m$$ [mm] \hspace{1cm} (5.22b)

where $n = 0.238 (\rho f_{sy})^{0.135}$ and $m = 4.127 (\rho f_{sy})^{-0.221}$, see figure 5.26. The average ratio of observed and calculated values amounts to 1.001 with a variation coefficient of less than 7.5%. Combining eqs. (5.22a-b) yields $\delta_{tei} < \delta_{nei} < 0.26$mm for all the tests conducted. Assuming that the dowel force of the reinforcing bars has fully developed for $\delta_{tei} > 0.10$mm, then the minimum sustained shear stress levels are:
Mix A: \[ \frac{\tau}{\tau_u} \geq 0.815 - 0.01* \rho_{sy}^f \] [N/mm²] (5.23a)
Mix B: \[ \frac{\tau}{\tau_u} \geq 0.860 - 0.01* \rho_{sy}^f \] [N/mm²] (5.23b)

where \( \tau_u \) refers to eq. (3.1a). These empirical formulae are based on computations according to the static model presented in section 5.3.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig5.26}
\caption{Development of \( \delta_{neq} \) according to eq. (5.22b).}
\end{figure}

Section 5.3 pointed out that the interlock mechanism dominates the static crack-opening path for a sufficiently large shear slip, see figs. 5.18-19. In case of a sustained shear loading 'damage' parameters were introduced into the theoretical model, see section 5.4, enabling other crack-opening directions to occur. The distinction between the static and the time-dependent crack-opening curves is illustrated in figure 5.27a by a gradual shift from A to B. It is thought that bond creep of the embedded reinforcement leads to a reduced restraint of the crack. This assumption should now be analysed further. The following points are successively reviewed:

- computation of steel stresses and related crack widths for short-term (\( t = 0 \) h) and long-term (\( t = 10^4 \) h) shear loading exerted to a specimen at a constant shear slip \( \delta_t = 0.55 \)mm;
- estimation of the time-dependent fictitious shift of the crack width assuming a gradual decrease of the axial stiffness of the embedded reinforcement which crosses perpendicularly to the crack, see eqs. (5.24a-d);
- proposed formula eq. (5.25) describing the complete time-dependent crack-opening curve.

As a constant shear loading is maintained on the crack plane, it is thought that the restraint stiffness of the reinforcement perpendicular to the crack plane is gradually reduced; whereas, the shear stiffness of the dowel remains nearly constant apart from the shear retention factor \( \gamma_d \) accounting for the yield criterion in the bar. For a shear slip of 0.55mm, eq. (5.22a) yields for \( f_{ccm} = 51-56 \) N/mm²:
\[ \Delta \delta_n = (0.55/1.249)(1/1.285)(0.55/1.870)(1/1.400) = 0.528 - 0.417 = 0.11 \text{mm}. \]

Next, the models presented in sections 5.3-5.4 have been applied to find the restraining steel stress in the crack. The computations are based on the measured crack-opening curves with \( f_{ccm} = 51 \text{ N/mm}^2 \) and \( f_{sy} = 460 \) or \( 550 \text{ N/mm}^2 \). As can be seen in table 5.5, the average displacements correspond closely to the positions A,B indicated in figure 5.27a. It follows that:

\[ |\Delta \delta_n| = 0.108 \text{mm}; |\Delta \delta_t| = 0.014 \text{mm}; |\Delta \tau| = 0.42 \text{ N/mm}^2; |\Delta \tau_d| = 0.22 \text{ N/mm}^2 \]

where \( \Delta \) refers to the difference between both types of loading for equal \( \rho_{f_{sy}} \) values. The average differences of the shear components appear to be relatively small. Therefore, it is permitted to make further comparisons.

**Table 5.5** Calculated results for an initial crack width \( \delta_{no} = 0.04 \text{mm} \).

<table>
<thead>
<tr>
<th>( \rho_{f_{sy}} ) [N/mm(^2)]</th>
<th>\text{static loading}</th>
<th>\text{sustained loading *)}</th>
<th>\text{sustained loading *)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_n ) [mm]</td>
<td>( \delta_t ) [mm]</td>
<td>( \sigma_{s1} ) [N/mm(^2)]</td>
<td>( \delta_n ) [mm]</td>
</tr>
<tr>
<td>5.15</td>
<td>0.40</td>
<td>0.53</td>
<td>256</td>
</tr>
<tr>
<td>6.16</td>
<td>0.40</td>
<td>0.56</td>
<td>227</td>
</tr>
<tr>
<td>7.73</td>
<td>0.40</td>
<td>0.53</td>
<td>170</td>
</tr>
<tr>
<td>9.24</td>
<td>0.40</td>
<td>0.56</td>
<td>152</td>
</tr>
<tr>
<td>12.32</td>
<td>0.40</td>
<td>0.56</td>
<td>128</td>
</tr>
</tbody>
</table>

*) \( \tau/\tau_u = 0.90; \ t = 10^4 \text{ hrs and } \mu = 0.56, \text{ see section 5.6.3.} \)

**Figure 5.27** (a) Gradual transformation of the displacement path; (b) calculated steel stresses at the centre of the crack plane.
The calculated steel stresses have been displayed in figure 5.27b. For a constant crack width, these stresses are hardly influenced by $p$, see figs. 5.21a-b. The average steel stress during the shift A-B is approximated by:

$$\sigma_{s12} = \left(\sigma_{s1} + \sigma_{s2}\right) / 2 \quad [\text{N/mm}^2] \quad (5.24a)$$

where $\sigma_{s1}$ and $\sigma_{s2}$ are given in table 5.5. Now, suppose that the axial tensile stresses in the reinforcement are responsible for a time-dependent deformation increase of the concrete that surrounds the embedded bars. The slip at the steel-concrete interface is negligible, see fig. 5.14a. Vos [136] estimated the interface layer to have a thickness of one bar diameter. A length of $1.5d_b$ minus a 'bond free zone' of 1.5mm is taken as a reasonable approximation of the activated layer parallel to the bar axis. The precise distribution of bond stresses parallel to the dowel is not known. The average bond stresses in the interface layer - on the supported side of the dowel - are expressed as a function of the steel stress in the crack (figure 5.28):

$$\bar{\tau}_{cs} = 0.5A_s \sigma_s(t) / [0.5pd_b(1.5d_b-1.5)] = \sigma_s(t) / (6-6/d_b) \quad [\text{N/mm}^2] \quad (5.24b)$$

This calculation is based on a linear distribution of $\tau_{cs}$ across the slip layer. For $d_b = 8\text{mm}$ the average bond stress is equal to 19% of $\sigma_s$: the assumption of a parabolic distribution parallel to the bar axis and along the circumference $0.5pd_b$ yields: $\tau_{\max} = \bar{\tau}_{cs} \cdot 2 \cdot (1.5)^2 = 0.86\sigma_s(t)$. These high bond stresses can be transferred as a result of multiaxial confinement of the material under the dowel, situated close to the crack plane. Another point is the favourable effect of a kind of push-in mechanism enabling a surplus of bond stress to be generated.
Now the time-dependent deformation of the interface layer is related to $\Delta \delta_n$:

$$\Delta \delta_n = 2*\text{slip} = 2d_b(\gamma_{12} - \gamma_1) = 0.38d_b(\frac{\sigma_{s12}}{G_{ml2}} - \frac{\sigma_{s1}}{G_{ml1}}) \quad [\text{mm}]$$  \hfill (5.24c)

where $G_{ml1}, G_{ml2}$ denote the shear moduli of the interface material at $t = 0$ and $t \leq 10^4$ hrs (average value), respectively. It is assumed that matrix material dominates in the interface layer. The Poisson's ratio is reported to hardly change by the load duration [168,170,202] or by the multiaxial compressive stresses so that $\nu_m = 0.25$. Furthermore:

- a similar time-dependent development of the modulus of elasticity (concrete) and the shear modulus (matrix material) is adopted. The CEB-FIP model code [61] reveals that $E^{\prime}/E_0 = 0.43$ for $f_{ccm} = 51$ N/mm$^2$;
- there is a negligible effect of the confinement [192] on the modulus of elasticity only at extremely high multiaxial compressive stresses [213];
- $G_{ml1} = 0.4E_0 \approx 0.12E_{agg} = 6000$ N/mm$^2$ and $G_{ml2} = 1.1*0.43*6000 = 2838$ N/mm$^2$ are chosen. Combining eqs. (5.24a-c) and table 5.5, yields:

$$\Delta \delta_n = 5.36*10^{-4}(\sigma_{s2} + 0.054\sigma_{s1}) \quad [\text{mm}]$$  \hfill (5.24d)

where $\Delta \delta_n$ ranges from 0.028-0.061mm. Actually, the observed deviation of the crack-opening path at $t = 0$ hrs (see pp. 114) due to the different application methods used should be added. If the approximations are considered, it can be concluded that the computational results agree quite well with the recorded displacement shift A-B in figure 5.27a. In fact both $\delta_n$ and $\delta_t$ exhibit a shift: for simplicity the slip is kept constant for the calculations presented.

The high-strength concrete (mix B) displayed a larger shift in time. This can be easily verified with the above-mentioned calculation procedure. In [61] the modulus of elasticity is almost proportional to $f_{ccm}^{0.33}$. The steel stress is proportional to $f_{ccm}^{0.64}$ as it is directly related to the interlock stress of the crack. Therefore, it can be expected that a larger shift of the crack width occurs so that a flatter crack-opening curve is obtained unless there is a strong reduction of the time-dependent deformation of the matrix. As for the stiffness of the dowel a gradual decrease is established in the axial direction during the sustaining period.
The shift $\Delta \delta_n$ appeared proportional to $f_{ccm}$. See also figs. 5.25 and 5.27a. Therefore, a simple power-function is proposed for the time-dependent $\delta_n - \delta_t$ relationship, provided that $\rho = 1.1-2.2\%$. Then the coefficients of eq. (5.22a) need be adapted to:

$$\alpha = 2.166 f_{ccm}^{0.140} \text{[mm]} \quad \text{and} \quad \beta = 0.554 f_{ccm}^{0.214} \text{[-]} \quad (5.25)$$

A final remark should be made about the observations of the crack plane after the shear tests were carried out. Local bond deterioration of the embedded bars close to the crack plane may have been caused at the instant of cracking when $\delta_n \approx 0.10-0.15\text{mm}$ was recorded, see [112,120,145]. Bond damage noticed after the actual shear test may also be due to the separation of both crack halves which was needed in order to inspect the shear plane of the concrete. Similar damage is reported by Utescher et al. [86] and by Dulacska [44], see fig. 2.11c, contrary to results of Millard et al. [64,65] for $\delta_{no} > 0.125\text{mm}$. The total 'bond free zone' parallel to the bar axis is estimated to be less than 5mm which is remarkably smaller than reported on reinforced tensile specimens as shown in fig. 5.12c. The short length is related to local confinement of the concrete on the supported side of the steel bar, see fig. 5.14a. Indeed, 'perfect bond' was qualitatively observed after the static pushing-off of the specimen when the cover of the steel bar was carefully removed (section 4.6.3). On the supported side of the bar a small concrete part had virtually been pressed to the steel surface of the 8mm diameter bar over a length of about 20mm near the plastic hinges (fig. 5.14a). Consequently, the approximate crack width increase is smaller than $5f_{sy}/E_s \approx 0.01\text{mm}$ and can be neglected.

5.6.3 Derivation of damage parameters

This section deals with a verification of the extended model presented in section 5.4.3. Emphasis lies on the evaluation of the multiplicative damage parameter $\lambda_f$. The study is restricted to a fully developed dowel mechanism so that the stress level is to satisfy a minimum value, see eqs. (5.23a-b). The formulae (5.20a-b) have been applied to find $c$ at $t = 0h$ (the onset of the long-term experiments) incorporating the recorded displacements of each test as input for the calculations. It followed that [124,126]:

mix A: $\bar{c} (t = 0h) = 1.218 \quad \text{v.c.} = 4.7\% \quad (5.26a)$
mix B: $\bar{c} (t = 0h) = 1.391 \quad \text{v.c.} = 3.7\% \quad (5.26b)$
where v.c. refers to the variation coefficient. The small scatter confirms that the displacement path is dominated by the actual concrete strength as concluded in the previous section. Eqs. (5.26a-b) indicate higher values for an increasing compressive strength [125] resembling flatter curves for the $\delta_n - \delta_t$ relationship, see fig. 5.25. The coefficient $c(t = 0)$ > 1 expresses the imbalance of shear forces so that the external load $V$ exceeds the theoretical contributions of the interlock and dowel mechanisms. The phenomenon is mainly attributed to small distinctions between the static displacement paths of the crack, viz. < 0.05mm. However, expressed as a ratio the deviations amount to 10-20%, see also figs. 5.23, 5.25, 5.27.

The interlock stresses are governed by $\mu$ and $\sigma_{pu}$. They have been empirically formulated for the two-phase model of Walraven covering the whole static shear test range up to the instant of failure of each specimen [112,113]. Yet, the tests of this research refer to $\delta_{tel} < \delta_{nel} < 0.26$mm. It was decided not to change the basic empirical relationships of the monotonic case. Instead, the 'initial value' $c$ has been successfully proposed as a calibration value for the time-dependent behaviour. The instantaneous displacements (eqs. (5.22b) and (5.25)) provide sufficient data to establish $c$.

Next, the development of $\lambda_f(t)$ has been calculated for all the tests conducted. In figure 5.29a the shaded area corresponds to 90% of all the computational results of mix A. In general, the curves displayed a strong dependency of both the constant shear stress level applied and of the ultimate restraining stress $P_{f_{sy}}$. However, these effects are not incorporated in the damage parameter according to the theoretical model. After a thorough analysis it was found that a time-dependent increase of the friction coefficient $\mu$ induces a considerable reduction of the shaded area. Figures 5.29b-c show two examples where final values $\mu(t = 10^5 \text{ hrs})$ have been chosen 0.50 and 0.60, respectively. For $\mu$ a logarithmic expression with respect to the application period of the constant shear stress appeared to satisfy well. A sensitivity analysis based on the results of figure 5.29c revealed that a further reduction of the 'confidence area' is not useful in view of the accuracy and reliability of the test data. Therefore, a simple formula is suitable to fit all tests:

$$\lambda_f(t) = \mu(t)/0.40 = 1.00 + 100 \times 10^{-3} \times \log(t + 1) \quad [-] \quad (5.27)$$
Figure 5.29 Development of $\lambda_f(t)$ for (a) $\mu = 0.40$ and (b)-(c) a gradual increase of the friction coefficient.

where $0 \leq t \leq 10^5$ hrs, see figure 5.30a. There is a remarkable change of $\lambda_\mu$ immediately after the onset of the test. Consequently, the previous assumption in section 5.4.2 of a constant friction coefficient does not agree with the experimental findings. The same conclusion is valid for mix B. The second damage parameter $\lambda_f$ has been successfully approximated by two empirical formulae for each mix, see figure 5.30b:

mix A or B: $\lambda_f(t) = A + B \log(t)$

where $10^{-1} \leq t \leq 10^5$ hrs. B-values of each mix are gathered in table 5.6. The A-values simply follow from $\lambda_f(0.1 \text{ h}) = 1.0$. Both $\lambda_f'$ and $\lambda_\mu$ are written as logarithmic functions which resemble other material damage in case of a long-term application period, such as creep and fatigue [117,128,131,148, 164,178,185,201]. Apparently there is a significant distinction of the deterioration processes in both types of concrete (or: matrix material). As is indicated in figure 5.30b, mix A experiences relatively strong physical changes even for $t > 10^3$ hrs. The different responses of the concrete mixes can not be fully attributed to the different macroscopic material strengths $\sigma_{pu}$ and $f_{ccm}$. Using $\lambda_f'$ and $\lambda_\mu$ as input for the calculations, the predicted displacement response of a single reinforced crack has been compared with the test data. The average ratios of measured and computed values ranged from 0.93 to 1.09 covering the two concrete grades tested. The computational procedure is outlined in figure 5.31 and appendix 11.5.
Working example The time-dependent behaviour of a single crack subjected to sustained shear loading has been predicted by means of the method outlined in figure 5.31. The concrete properties are: mix A; $D_{\text{max}} = 16\, \text{mm}$; $\rho = 0.70$; $f_{\text{ccm}} = 51\, \text{N/mm}^2$. The initial crack width is $0.02\, \text{mm}$. The 120*300 $\text{mm}^2$ shear plane is crossed perpendicularly by twelve 8mm diameter steel bars ($f_{\text{sy}} = 460\, \text{N/mm}^2$; $f_{R} = 0.050$) so that $\rho = 1.68\%$. A stress $\tau = 0.9\tau_{\mu} = 10.65\, \text{N/mm}^2$ has been permanently installed on the shear plane. The calculated instantaneous displacements amount to $\delta_{\text{inel}} = 0.238\, \text{mm}$, $\delta_{\text{tel}} = 0.197\, \text{mm}$ according to eqs. (5.22b) and (5.25) which corresponds to a calibration value $c = 1.291$. The results have been compared with the test data in table 5.7. As shown in figures 5.32a-b there is a satisfactory agreement. Figure 5.32c indicates a minor redistribution between both transfer mechanisms. It is clear that $\lambda_{f}'(t)$ affects them equally.
Table 5.7 Comparison of the test data with the predicted time-dependent response. The damage parameters are indicated ([*] = [N/mm²]).

<table>
<thead>
<tr>
<th>t [h]</th>
<th>(\delta_n^{\text{exp}}) [mm]</th>
<th>(\delta_t^{\text{exp}}) [mm]</th>
<th>(\delta_n^{\text{cal}}) [mm]</th>
<th>(\delta_t^{\text{cal}}) [mm]</th>
<th>(\lambda_{\mu}) [-]</th>
<th>(\lambda_f^i) [-]</th>
<th>(\alpha\gamma_d) [-]</th>
<th>(\sigma_s) [ ]</th>
<th>(\tau_a) [ ]</th>
<th>(\tau_d) [ ]</th>
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<tr>
<td>0</td>
<td>0.250</td>
<td>0.217</td>
<td>0.238</td>
<td>0.197</td>
<td>1.000</td>
<td>1.000</td>
<td>0.982</td>
<td>35</td>
<td>5.18</td>
<td>5.47</td>
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<td>1</td>
<td>0.324</td>
<td>0.304</td>
<td>0.317</td>
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</tr>
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<td>10¹</td>
<td>0.361</td>
<td>0.354</td>
<td>0.364</td>
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<td>1.105</td>
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<td>0.966</td>
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<td>5.26</td>
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<td>10²</td>
<td>0.409</td>
<td>0.414</td>
<td>0.392</td>
<td>0.375</td>
<td>1.200</td>
<td>0.958</td>
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<td>5.47</td>
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<td>5*10²</td>
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<td>0.957</td>
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<td>5.71</td>
<td>4.94</td>
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</tbody>
</table>

Figure 5.31 Review of computations to predict the time-dependent response in case of a crack subjected to a sustained shear loading.
Figure 5.32 (a)-(b) Comparison of displacement responses and (c) predicted redistribution between shear components.
5.6.4 Evaluation of damage parameters

Further research focused on the damage parameters established. The results are summarized within the next three points:
- remarks on the increasing friction coefficient;
- use of the extended model for cracked plain concrete;
- effects of load history.

The friction coefficient is supposed to enlarge gradually. Apparently there is one prevailing physical mechanism of those mentioned in section 5.4.2 which induces an increasing roughness of the contact surface of the matrix material. In figure 5.33 the recorded steel stress increments of the embedded bars crossing the crack of a push-off specimen are presented. See also section 4.6.2, appendix 11.3 and in [122]. The predicted curve no. 1 corresponds to the measured crack-opening path: actually, this path is unrealistic in view of the steeper curves found for the complete test series. A more likely path according to eq. (5.25) provides curve no. 2 in figure 5.33. It closely approximates the recorded steel stresses.

![Figure 5.33 Steel stress increments $\Delta \sigma_s$ of a sustained shear test.](image)

$\Delta \sigma_s [\text{N/mm}^2]$ vs. $\log \text{t}[\text{h}]$

The successive increase in time of the friction coefficient enhances the redistribution of internal shear stresses, see figures 5.34a-d. Note that the dowel mechanism is reduced the smaller the reinforcement ratio is ($\sigma_d$ in eq. (5.20a) and $\gamma_a < 1.0$). These findings are realistic from a structural point of view. Consequently, the gradual 'structural weakening' represented by the time-dependent $\tau$-$\delta$ curves in fig. 4.13a is fully considered by the theoretical model.
Figure 5.34 (a)-(d) Effect of $\mu(t)$ on $\tau_a$, $\tau_d$, $\sigma_s$ and $\lambda_f$.

$\left(f_{ccm} = 51 \text{ N/mm}^2; f_{sy} = 460 \text{ N/mm}^2; \rho = 1.12\% \text{ and } \tau = 8.6 \text{ N/mm}^2\right)$

The sustained shear tests conducted on cracked plain concrete have also been analysed by the extended version of Walraven's model. The experimental results of seven specimens were used. Generally, $c(t = 0h)$ exceeded 1.0, viz. average values amounted to 1.44, v.c. 12.6% and 1.39, v.c. 10.0% for concrete mix A and B respectively. Previously, the rather small instantaneous displacements $\delta_nel$ were expressed by a simple formula [121] accounting for the initial values $\sigma_{co}$, $\delta_{no}$, the compressive strength $f_{cc}$ and the adjusted constant shear stress $\tau$. Thus the values of $c$ are known in advance.

The restraining stress-crack width relations were used as input for the calculations, approximated by $\sigma_c(t) = \sigma_{co} + \alpha \sqrt{(\delta_n - \delta_{no})}$ where $\alpha$ is derived from eq. (4.10). The damage parameters $\lambda_f$ and $\lambda_\mu$ were also incorporated. The computed crack-opening paths agreed reasonably with the recordings given in section 4.3. The calculations revealed a strong influence of the normal restraint stress used. This is confirmed by other research, see figs. 2.5b and 2.6c. For $t = 0-10^5$ hrs the average ratios $\bar{x}$ of measured and computed (constant) shear displacements of the seven tests yielded:
As shown in appendix 11.4 the computations are based on too high a normal stress on the crack for a long loading period. This may explain the low shear slip values predicted. It is concluded that the theoretical model may also be applied to the time-dependent interlocking mechanism separately if the sensitivity of the calculations and the actual measuring accuracy of the displacements are taken into consideration.

The supplementary investigations reveal another plausible conclusion. The damage parameters $\lambda_f$ and $\lambda_\mu$ have originally been derived from the equilibrium of shear forces in the crack plane. This is true for a crack in plain or reinforced concrete. Apparently, these parameters also account for the time-dependent transfer of forces perpendicular to the shear plane.

Another phenomenon concerns the effects of load history on the monotonic stress-displacement response of a reinforced crack. Two points are reviewed:

- At the instant of unloading the displacement decreases $\Delta \delta_n$ and $\Delta \delta_t$ are almost proportional to the previously applied shear stress level, see figs. 4.23a-b. No distinct influence of the load duration is observed. On reloading the 'initial' crack widths are relatively large, viz. 0.060-0.218mm [120]. These recorded values were added to the crack width data obtained on 'virgin' push-off specimens when the ultimate shear stress has just been reached. The predicted values were in good agreement with displacements measured on the pre-loaded specimens. An analogous behaviour is reported on the bond stress-slip relations of pull-out specimens, see in [206,207];

- The static shear strength $\tau_u$ is not influenced [116] unless the actual concrete strength is implemented in the calculations. This was investigated for concrete ages ranging from 10 to 417 days. The particles penetrate a part of the matrix material which has not been damaged before ($\lambda_f = \lambda_\mu = 1$). It is thought that any strength increase due to ongoing hydration of cement particles is compensated by deterioration processes within the material structure. Another effect is the larger crack width at the onset of reloading.
5.7 Long-term strength

This section deals with the phenomena which influence the time-dependent development of \( \lambda_f' \) in eq. (5.28). Virtually, the damage parameter refers to a strength criterion. A simple model is established which predicts a minimum value of \( \lambda_f \) denoted as the long-term strength ratio of a cement-based material. A first attempt is to pay attention to concrete as research on this material is extensively reported in the literature. Moreover, concrete and matrix material resemble considerably [118, 134]:
- both are cement-based, so that crack formation is influenced by the hydration process;
- both materials display crack arrest by aggregate particles or air voids.

The following subjects are successively discussed in this section:
- failure in case of sustained loading;
- development of the short-term and long-term strength;
- damage parameter \( \lambda_f \) and long-term strength or \( \eta(t, t_0) \).

Failure in case of sustained loading A permanent loading involves material deterioration expressed by a gradual decrease of the short-term strength. This phenomenon links up with the coefficient \( \lambda_f \) in eq. (5.18a). As for cement-based materials, Fouré [224] proposed an empirical formula between the stress level \( \sigma_{cr}/f_c(t_0) \) and the time to failure \( t_f \) (in minutes):

\[
\sigma_{cr}/f_c(t_0, t_f) = C - D \ln(t_f)
\]

where \( t_0 \) refers to the concrete age at the instant of load application. As an advantage, different \( t_0 \)-values can be represented by a single curve. Table 5.8 shows values of \( C \) and \( D \) which have been derived from creep experiments on concrete specimens applied by either a constant compressive or a constant tensile loading. The correlation coefficients \( r \) indicate that the curves match satisfactorily to the observed behaviour. One should take into consideration the scatter of the test results which is inherent to the time-dependent response of a heterogeneous material such as concrete, see in [196]. Another point is that an unequal number of tests were usually performed for each stress level adjusted. The 95%-confidence interval of \( D \)-values in table 5.8 ranges between 0.005 and 0.049. If the stress at failure is related to \( f_c(t_0) \) then eq. (5.30a) converts into:

\[
\sigma_{cr}/f_c(t_0) = (\sigma_{cr}/f_c(t_0, t_f)) \times \left( f_c(t_0, t_f)/f_c(t_0) \right)
\]
Table 5.8 Coefficients C and D of eq. (5.30a).

<table>
<thead>
<tr>
<th>literature source</th>
<th>$f_{cm}$ [N/mm²] concrete*</th>
<th>type of loading**</th>
<th>nos. of tests</th>
<th>eq. (5.30a) C</th>
<th>D</th>
<th>r</th>
</tr>
</thead>
<tbody>
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<td>35 G</td>
<td>C</td>
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<td>1.043</td>
<td>0.0388</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>25-35 limest. G</td>
<td>C</td>
<td>22</td>
<td>0.969</td>
<td>0.0229</td>
<td>0.82</td>
</tr>
<tr>
<td>Sell et al.[167]</td>
<td>6 L</td>
<td>C</td>
<td>21</td>
<td>0.888</td>
<td>0.0101</td>
<td>0.92</td>
</tr>
<tr>
<td>Prokopović [178]</td>
<td>- G</td>
<td>T</td>
<td>4</td>
<td>1.019</td>
<td>0.0147</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>- G</td>
<td>T</td>
<td>41</td>
<td>0.870</td>
<td>0.0006</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>- G</td>
<td>T</td>
<td>49</td>
<td>1.080</td>
<td>0.0357</td>
<td>0.93</td>
</tr>
<tr>
<td>Wittmann [178]</td>
<td>26 G</td>
<td>C</td>
<td>13</td>
<td>0.946</td>
<td>0.0346</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>33 L</td>
<td>C</td>
<td>6</td>
<td>0.910</td>
<td>0.0382</td>
<td>0.83</td>
</tr>
<tr>
<td>Luijerink [180]</td>
<td>35 G</td>
<td>C</td>
<td>7</td>
<td>0.750</td>
<td>0.0085</td>
<td>0.99</td>
</tr>
<tr>
<td>Al-Kubaisy [222]</td>
<td>2.5 G</td>
<td>T</td>
<td>12</td>
<td>0.843</td>
<td>0.0390</td>
<td>0.98</td>
</tr>
<tr>
<td>Nishibayashi [223]</td>
<td>3.0 G</td>
<td>T</td>
<td>34</td>
<td>0.914</td>
<td>0.0199</td>
<td>0.99</td>
</tr>
<tr>
<td>Fouré [224]</td>
<td>2.5 L</td>
<td>T</td>
<td>37</td>
<td>0.924</td>
<td>0.0171</td>
<td>0.94</td>
</tr>
<tr>
<td>Reinhardt et al. [225]</td>
<td>39-57 G ****)</td>
<td>T</td>
<td>32</td>
<td>0.922</td>
<td>0.0132</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>44-48 L G ****)</td>
<td>T</td>
<td>54</td>
<td>0.907</td>
<td>0.0167</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>2.6-2.9 G ****)</td>
<td>T</td>
<td>29</td>
<td>0.825</td>
<td>0.0336</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>1.9-2.2 G ****)</td>
<td>T</td>
<td>29</td>
<td>0.820</td>
<td>0.0389</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>2.6-2.9 G</td>
<td>T</td>
<td>35</td>
<td>0.832</td>
<td>0.0432</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>1.9-2.2 G</td>
<td>T</td>
<td>32</td>
<td>0.837</td>
<td>0.0456</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>1.9-2.9 G ****)</td>
<td>T</td>
<td>58</td>
<td>0.799</td>
<td>0.0224</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>1.9-2.9 G</td>
<td>T</td>
<td>72</td>
<td>0.833</td>
<td>0.0405</td>
<td>0.99</td>
</tr>
</tbody>
</table>

*) G = gravel, L = light-weight, **) C = compression, T = tension, *** incl. short-term tests, ****) T = 4 °C

**Development of the short-term strength** It is obvious that the uniaxial strength of cement-based material is determined by various parameters, such as the mix composition, type of cement, compaction method and curing conditions (T, RH). A purely theoretical prediction of the material strength is beyond the scope of this research. Thus, a phenomenological approach has been chosen. The first derivative of $f_c(t_0)$ is described by either a hyperbolic expression or by a combination of a hyperbolic and an exponential expression. This rough approximation of the actual behaviour provides two monotonically increasing formulae for the short-term strength:
\[
f_c(t_0) = \frac{f_{cm} \cdot t_0}{(a + b \cdot t_0)} \quad \text{[N/mm}^2\text{]} \quad (5.31a)
\]

and:
\[
f_c(t_0) = f_{cm} \cdot (a \cdot f_1(t_0, \xi) + b \cdot f_2(t_0)) \quad \text{[N/mm}^2\text{]} \quad (5.31b)
\]

where:
- \( f_{cm} \) = mean 28-day uniaxial strength of the material [N/mm\(^2\)]
- \( a, b, \xi \) = empirical coefficients based on test results
- \( t_0 \) = material age [days]
- \( f_1 = \{ 1 - \exp(-\xi t_0) \} \cdot (1/\xi) \) and \( f_2 = \ln(2) + \ln\left(\frac{t_0 + 1}{t_0 + 2}\right) \)

For \( t_0 = 0 \) both formulae yield \( f_c = 0 \) and for \( t_0 \to \infty \) the maximum strength of the material is: \( f_{cm}/b \) (eq. (5.31a)) or \( f_{cm} \cdot (a/\xi + b \cdot \ln(2)) \) (eq.(5.31b)).

For usual circumstances the ratio \( f_c/f_{cm} \) varies from 1.0 to 2.0. Then the ratio \( \gamma \) of the 14-day and the 28-day material strength is:
- eq. (5.31a): \( \gamma = 0.67 - 1.00 \);
- eq. (5.31b): \( \gamma = 0.36 - 1.00 \) (\( \xi = 1/28 \)).

Consequently, eq. (5.31b) considers a relatively wide spectrum of time-dependent strength curves. Instead, eq. (5.31a) incorporates a limited strength increase for \( t_0 > 28 \) days if a strong initial rise occurs, see figures 5.35a-b.

![Figure 5.35 (a)-(b) Some curves acc. to eq. (5.31a) and its first derative.](image)

Values of \( a \) and \( b \) were calculated from short-term tests performed at different ages of the concrete. They are gathered in table 5.9. The results refer to the same types of concrete which have been used for the creep tests presented in table 5.8. No significant effects on \( a, b \) of the ambient circumstances of the test specimens were established. The ratios \( f_c/f_{cm} \) are relatively high when low quantities of cement were used. For ratios which exceed about 1.20 the agreement between eqs. (5.31a-b) tend to diminish.
Table 5.9 Coefficients \( a, b \) of eqs. (5.31a-b).

<table>
<thead>
<tr>
<th>literature source</th>
<th>type of loading*</th>
<th>type of concrete</th>
<th>( \text{eq. (5.31a)} )</th>
<th>( \text{eq. (5.31b)**} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rüsch et al. [166] ***</td>
<td>C</td>
<td>S1</td>
<td>7.89</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>S4</td>
<td>2.13</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>S6</td>
<td>2.01</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>S9</td>
<td>4.11</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>S3</td>
<td>6.84</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>S8</td>
<td>-0.01</td>
<td>0.97</td>
</tr>
<tr>
<td>Sell [167]</td>
<td>C</td>
<td>1</td>
<td>2.07</td>
<td>0.90</td>
</tr>
<tr>
<td>Wittmann [178]</td>
<td>C</td>
<td>2</td>
<td>3.10</td>
<td>0.90</td>
</tr>
<tr>
<td>Nishibayashi [223]</td>
<td>T</td>
<td>3</td>
<td>2.21</td>
<td>0.92</td>
</tr>
<tr>
<td>Fouré [224]</td>
<td>T</td>
<td>4</td>
<td>3.38</td>
<td>0.88</td>
</tr>
<tr>
<td>Reinhardt [225]</td>
<td>T</td>
<td>5</td>
<td>6.46</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>6</td>
<td>1.59</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>7</td>
<td>6.72</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>8</td>
<td>2.78</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>9</td>
<td>2.21</td>
<td>0.92</td>
</tr>
</tbody>
</table>

*) \( C = \) compression. \( T = \) tension  
**\( \xi = 1/28 \)

On combining eqs. (5.30a-b) and (5.31a) the long-term strength can be found: it corresponds to a minimum of the \( \sigma_{cr}/f_c(t_0) - t_f \) relationship denoted by \( (t_f^*, \sigma_{cr}/f_c(t_0)) \), see also in [224]:

\[
\frac{d}{dt_f} \left[ \frac{\sigma_{cr}}{f_c(t_0)} \right] = 0 \rightarrow \alpha^*(\frac{t_0}{t_f^*})^2 + (2\alpha - \frac{\sigma_{cr}^*}{f_c(t_0)^*})^*(\frac{t_0}{t_f^*}) + \alpha = 0 \quad (5.32)
\]

where \( \alpha = (1+b*t_0/a) \) and provided that \( \sigma_{cr}/f_c(t_0) \geq 4\alpha D \). Formula (5.32) implicitly indicates the location of the minimum. The time to failure is \( t_f^* \leq t_0 \) for real solutions of the second-order equation. This means that a specimen subjected to a constant loading is expected to fail at an age between \( t_0 \) and \( 2t_0 \) according to this simple mathematical model. No failure occurs for stress levels which are lower than \( \sigma_{cr}/f_c(t_0) \). Figure 5.36 shows that for chosen values \( \alpha = 1 + b*t_0/a \) and \( D \), the adjusted constant stress level should satisfy to a minimum.
Development of the long-term strength An expression of the long-term strength ratio has been derived from an extensive Russian research program reported in [178] on pp. 107. The constant compressive stress levels were between 0.60 and 1.00; \( t_0 = 7-500 \) days; \( f_{ccm} = 28-60 \) N/mm\(^2\). During the creep tests the ambient conditions were constant: 20 °C and 70% RH. The (safe) 5% lower-bound values of \( \sigma_{cr}^*/f_c(t_0) \) have been expressed by a simple function of \( t_0 \) and the compressive strength of the concrete (see also figs. 5.8a-b and in [187,194]):

\[
\frac{\sigma_{cr}^*}{f_c(t_0)} = \frac{\log(t_0)}{p + q \log(t_0)}
\]

where

\[
p = -1.1530 + 0.0364 \times f_{ccm} - 3 \times 10^{-4} \times (f_{ccm})^2
\]
\[
q = 2.5314 - 0.0376 \times f_{ccm} + 3 \times 10^{-4} \times (f_{ccm})^2
\]

Figure 5.37 shows that the long-term strength ratio (\( \approx 0.77-0.80 \)) displays a small variation for young concrete of moderate strength. Remarkable differences occur for a concrete age \( t_0 \geq 14 \) days. Virtually, the Russian test results revealed a considerable effect of the water content added to the concrete mix. A similar conclusion follows from the CEB-FIP model code [61] which prescribes the effect of the mix consistency upon the time-dependent deformations (creep, shrinkage) of concrete.
Figure 5.37 Values of $\sigma^*/f_c(t_0)$ according to eq. (5.33).

A few examples of the relation between the long-term strength ratio and the time to failure are shown in figures 5.38a-b. The curves become less steep for an increasing age of the concrete at the instant of load application. Generally, $t_f$ increases if the strength development proceeds longer - this corresponds to a higher ratio of $f_{cm}/f_{cm}$ - so that failure of the material is temporarily postponed. A similar effect occurs for low values of $D$. By means of eq. (5.32) an analytical expression is found for the long-term strength. A similar approach can be adopted for eq. (5.31b). Yet, then an iterative computation is needed to find $t_f^*$.

Figure 5.38 (a)-(b) Examples of the relation between the constant stress level and the duration of load application $t_f^* \leq t_0$. 
The calculation procedure outlined above, is summarized as follows:
- input of concrete age \( t_0 \), the average 28-day strength \( f_{cm} \) of the cement-based material, the coefficients \( a, b \) of eq. (5.31a) and \( D \) of eq. (5.30a);
- use eqs. (5.32-33) to find the long-term strength ratio \( \sigma_{cr}^*/f_c(t_0) \) and the related loading period \( t_f^* \);
- solve \( C \) so that the complete failure envelope curve according to eq. (5.30b) is known: some examples are displayed in figures 5.39a-b.

![Figure 5.39](image-url) (a)-(b) Calculated curves which represent the relationship between the sustained stress level and the time to failure.

The parts of the curves located behind the minima are fictitious, i.e. they have no physical meaning. They show that the time-dependent rise of the short-term strength (that corresponds to continuous hydration of cement) is dominated by the gradual deterioration of material (extension of micro-cracks) which is represented by \( D \) in eq. (5.30a). In figure 5.39b the minima are less pronounced as the long-term strength ratio is almost equal to \( 4aD = 4*(1+b*t_0/a)*D. \) At a concrete age of about 1-3 years, any change of \( f_c(t_0) \) can further be neglected: then the slope of the curves is governed by \( D \). Two final remarks are made on this subject:
- The minimum concrete age for which hydration of cement hardly contributes to a change of the short-term strength, is derived from an arbitrary criterion \( df_c/dt_0 = 0.001 \text{ N/mm}^2 \) per day. Accordingly, the minimum value of \( t_f \) can be traced back to the assumption that the ratio \( f_c(t_0+t_f)/f_c(t_0) \) becomes 1.005 in eq. (5.30b). These two criteria yield \( t_0 \approx 250-1000 \text{ days} \) and \( t_f \approx 170-1400 \text{ days} \). In most cases \( t_f \) appears smaller than \( t_0 \), see table 5.10. The same conclusion is stated independently by eq. (5.32).
The analyses of the test results in table 5.8 revealed an empirical expression for \( C \) in eq. (5.30a): 
\[
C \approx 0.762 + 5.751 \times D
\]
where the average ratio \( C \) (predicted)/\( C \) (table 5.8) is 1.025 with a variation coefficient of 12.5%. The theoretical failure period of a short-term test can now be found with eq. (5.30a):
\[
\sigma_{cr}/f_c(t_0, t_f) = 0.762 + D(5.751 - \ln(t_f))
\]
\( D = 0.027 \) yields an application period \( t_f \approx 0.047 \) minutes \( \approx 3 \) seconds which is a realistic estimation in view of the logarithmic time scale used. Thus, the curves in figs. 5.39a-b properly describe the failure behaviour in case of short-term as well as long-term application periods.

**Table 5.10** Minimum values of \( t_0 \) and \( t_f \) (between parentheses), expressed in days, which refer to a minor time-dependent change of \( f_{cc} \).

<table>
<thead>
<tr>
<th>( f_{cw}/f_{cm} )</th>
<th>( f_{cc} [\text{N/mm}^2] )</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>1.10</td>
<td>245 (194)</td>
<td>348 (577)</td>
<td>427 (1381)</td>
</tr>
<tr>
<td>1.25</td>
<td>411 (174)</td>
<td>585 (424)</td>
<td>718 (762)</td>
</tr>
<tr>
<td>1.40</td>
<td>549 (182)</td>
<td>781 (424)</td>
<td>959 (725)</td>
</tr>
</tbody>
</table>

**Damage parameter \( \lambda_f \) and long-term strength** The failure envelope curves were determined for the two concrete mixes used for the sustained shear tests. The calculation method presented on pp. 136 has been applied:

- mix A: \( f_{Ccm} = 51 \text{ N/mm}^2 \); \( \alpha = 1 + b \times t_0/a = 1 + 0.862 \times 28/3.854 = 7.263 \), see eq. (11.2); \( \sigma^*/f_c(t_0) = 0.746 \) so that \( t_f^* = 0.593 \times 28 = 16.6 \) days;
- mix B: \( f_{Ccm} = 70 \text{ N/mm}^2 \); \( \alpha = 1 + b \times t_0/a = 1 + 0.908 \times 28/2.570 = 10.893 \); \( \sigma^*/f_c(t_0) = 0.768 \) so that \( t_f^* = 0.844 \times 28 = 23.6 \) days;

An analysis of the test data in table 5.8 revealed a tendency of \( D \) to be inversely proportional to \( f_{Ccm} \). Thus \( D \) is chosen 0.0240 and 0.0175 for mix A and B, respectively. A long application period \( t \) yields for the damage parameter in eq. (5.18a):
\[
\lambda_f \approx \frac{\sigma^*}{f_c(t_0)} \quad [-] \quad (5.34)
\]
For \( t = 10^5 \) h the \( \lambda_f \)-values (\( \lambda_f = \lambda_f^{1/0.64} \) are 0.844/b = 0.844/1.16 = 0.728 and 0.918/1.10 = 0.835 for mix A and B respectively, when the maximum ratio \( f_{c,c}/f_{c,m} \) is assumed to be reached *). Consequently, combining eqs. (5.28) and (5.34) leads to longer periods \( t \):

Mix A: \( \log(t) = \frac{(1.0152 - (0.746*1.16)^{0.64})}{23.749*10^{-3}} \rightarrow t = 2.6 \) years
Mix B: \( \log(t) = \frac{(0.9926 - (0.768*1.10)^{0.64})}{10.751*10^{-3}} \rightarrow t \gg \)

The value of mix B is excessive in comparison with the life-time of concrete structures. Moreover, eq. (5.33) considers lower-bound values of \( \sigma_{cr}^* \) so that the average failure times are smaller. Both calculation methods reveal that mix B has a relatively large long-term strength ratio. One should bear in mind that the observations of the matrix material with respect to \( \lambda_f \) indicate an exponent 0.64 for the compressive strength of the concrete. This value was derived from the short-term tests but it is not sure if it can be strictly maintained for the time-dependent case. This aspect should also be considered when comparing \( t \) and \( t_f^* \).

**Damage parameter \( \lambda_f \) and \( \eta(t,t_0) \) of eq. (5.3b)** The \( \eta \)-function was presented in section 5.2 and it accounts for two opposing physical processes which take place on the microstructural level of the material, viz. continuous hydration of cement and release of internal stress versus initiation and extension of microcracks. For a sufficiently large period of load application it follows from eqs. (5.3b) and (5.31a) that:

\[
\eta(t,t_0) = m(t,t_0)^{b^*} \frac{\sqrt{(b)^{-1/3}}}{1 + \phi_c(t,t_0)} \quad [-] \quad (5.35)
\]

where the modulus of elasticity is suggested to be in accordance with the CEB-FIP model code [61]. Coefficient \( m \) accounts for the release of internal stress. Usually \( m \) is taken as the ratio of the short-term strengths with and without a previous loading exerted to the specimen. Eq. (5.35) appears applicable to different types of loading:

- uniaxial compression, see Wittmann [178], Sčerbakov et al. [187], Zaitsev et al. [194], Sousa Continho [191] and Stöckl [220];
- uniaxial tension, see for example Wittmann [178] and Domone [221];
- repeated compression or tension, see Garrett et al. [201], Frénaij [117],

* The time-dependent effect of \( f_{c,c} \) is not considered in eq. (5.28); moreover then \( \gamma_d \) and \( \alpha \) in eqs. (5.19b), (5.20a) display a negligible change.
Generally, m depends on the type of loading, its duration and the constant stress level adjusted. Based on the references stated, m was chosen 1.10. With b = 1.16 (mix A) and 1.10 (mix B) and with long-term strength ratios \( \lambda_f = 0.728 \) (mix A) and 0.835 (mix B), the creep coefficients are known:

\[
\lambda_f = \eta(t,t_0) \rightarrow \phi_c(t,t_0) = 1.93 \text{ (mix A)} \quad \text{and} \quad \phi_c(t,t_0) = 1.04 \text{ (mix B)}
\]

so that \( \phi_c(\text{mix B})/ \phi_c(\text{mix A}) = 1.04/1.93 = 0.54 \). According to eq. (5.1c) the creep deformation is approximately proportional to \((wcr)^2\). The instantaneous deformation of the matrix material is equal to the ratio of the constant stress \( \lambda f \) and the modulus of elasticity \( \lambda_{fc} \). Now the ratio of the creep coefficients is:

\[
\frac{\phi_c(\text{mix B})}{\phi_c(\text{mix A})} = \frac{0.38}{0.50} \cdot \frac{51}{70} \cdot 0.31 = 0.52
\]

This ratio agrees well with the theoretical prediction given above.

The degree of hydration of the cement grains corresponds to a certain macroscopic short-term strength of the cement-based material. For young concrete at \( t_0 = 7 \) days, fig. 5.37 indicates that the curve of the long-term strength ratio \( \sigma_{cr}/f_c(t_0) \) diverges from the other curves. Timusk et al. [226] established a similar phenomenon denoted as 'maturing creep': young concrete exhibits a surplus of creep deformation as a function of the constant stress level applied. Tests revealed that hydration stops at low temperatures (-11 °C) so that the stress-creep deformation curves became linear again. Ghosh [227] proposed four mechanisms to explain 'maturing creep'. It decays continuously with increasing maturity. The time-dependent physical processes are mainly governed by the rate and degree of hydration. Khalil et al. [228] found no other response of mortar material in case of admixtures, such as used for mix B (see table 3.1).

Hydration is quantified by the measured loss of weight of concrete specimens heated to 105 °C and to 1050 °C respectively. These types of measurements were carried out for mix A and B at four different ages of the concrete [120], but the detailed analyses of the investigations are abandoned in this thesis.
6. APPLICATION TO A REINFORCED CONCRETE WALL

The results of the investigations presented in the chapters 4-5 are now illustrated by means of a practical example. It concerns the extension of a multi-storey building, which is to be placed on an existing foundation. Figure 6.1 shows the present building, connected monolithically with a L-shaped reinforced concrete abutment which was previously used as a part of the entrance. The 225mm thick wall has a length of 60 meters. Any deformation parallel to the wall (temperature, shrinkage) is restrained by the ground-floor. No structural or expansion joints were used so that the wall is locally cracked.

The wall contains two layers of welded wire mesh (12mm diameter bars) which have a 140mm grid spacing. Owing to a lack of space in the neighbourhood of the building and due to want of time, it is decided to maintain the wall which rests on piles. These piles are able to carry the enlarged structure and no differential settlements are expected. The transverse stiffness of the piles is negligible as a result of soft soil layers situated under the wall. Therefore, the wall is hardly constrained by the piles. Due to the short distance between the successive piles, in-plane shear loading prevails. The structural stability is sufficient. The wall was originally designed for a three-storey building. The question is whether it can temporarily resist a surplus of another three stories.

Figure 6.1 (a) Overview of the extension of the multi-story building and (b) details of the wall structure.
It is expected that the cracks will widen and slide due to shear loading. The main problem might be the structural strength of the reinforced cracked concrete. Other points of interest are the durability of the concrete and the local deformation of the upper-surface of the wall which is to fit closely to the facade of the new building.

Materials gravel concrete

\[ f_{ccm} = 40 \text{ N/mm}^2, \text{ i.e. strength-class C30 acc. to Eurocode [110]}; \]
\[ f_{ctm} = 2.8 \text{ and } E_{cm} = 32000 \text{ N/mm}^2; D_{max} = 16\text{mm}; \]

high-bond steel bars

\[ f_{sy} = 500 \text{ and } E_s = 2.1 \times 10^5 \text{ N/mm}^2; \rho = A_s/A_c = 905/(225 \times 560) = 0.72\% \]

Loadings A uniformly distributed line-load is adopted on the substructure over its full length inducing the following maximum shear stresses:
- \( \tau = 5.0 \text{ N/mm}^2 \) (service limit state);
- \( \tau = 7.0 \text{ N/mm}^2 \) (ultimate limit state).

The safety factors are taken according to the Eurocode [110].

Additional data
- \( \alpha_T = 12 \times 10^{-6} \text{ °C}^{-1} \) for steel and concrete;
- creep coefficient \( \phi = 1.50 \) and shrinkage \( \epsilon = 230 \times 10^{-6}; \)
- concrete cover 35mm and permitted mean crack width \( \delta_n = 0.30\text{mm}; \)
- temperature gradient 10 °C (solar radiation);
- ambient conditions: 20 °C and 70% RH.

Three subjects are analysed in this chapter:

a. Estimation of the initial crack widths and crack spacings due to restraint (shrinkage, temperature);

b. Determination of the maximum shear stress which acts on a crack in the case of either the service or the ultimate limit state;

c. Prediction of the structural behaviour of the crack immediately after the completion of the building and after a certain application period.
Ad a The expected mean initial crack width due to imposed deformation follows from the steel stress increase at the instant of cracking [90, 211]:

\[
\sigma_{so} = \left(\frac{E_s}{E_{cm}}\right) * 0.62f_{ctm} = 11 \text{ N/mm}^2 \tag{6.1}
\]

\[
\sigma_{s,cr} = \left(\frac{E_s}{E_{cm}} + \frac{1}{\rho}\right) * 0.62f_{ctm} = 253 < 500/1.5 \text{ N/mm}^2 \tag{6.2}
\]

\[
\delta_{no} = 2\left(\frac{\alpha+1}{\beta} \right) d_1 \left(\frac{1}{E_s} \right) \sigma_{s,cr} (\sigma_{s,cr} - \sigma_{so}) \left(\frac{1}{\beta}\right) = 0.114 \text{mm} \tag{6.3}
\]

where \(\alpha = 0.38\) and \(\beta = 0.18\) determine the bond-stress slip relation of the bar. The minimum crack spacing (fully developed crack pattern) is given by:

\[
1_{\text{min}} = \frac{\delta_{no} E_s}{(1-\beta) \sigma_{s,cr}} = 115 \text{mm} \tag{6.4}
\]

and \(1_{\text{max}} = 230\text{mm}\) according to the model of Noakowski [211]. Cracks are initiated by imposed deformations, related to the large length of the wall:

\[
\Delta \varepsilon = \Delta \varepsilon_{cs} + \Delta \varepsilon_T = (230 + 12*10) \times 10^{-6} = 350 \times 10^{-6}
\]

\[
\varepsilon_{\text{max}} = (35 + 2.7 \sigma_{s,cr}) \times 10^{-6} = 653 \times 10^{-6} > \Delta \varepsilon = 350 \times 10^{-6}
\]

The crack pattern is not fully developed so that crack formation is well controllable. Cracks usually initiate near the vertical bars due to stress concentrations. The grid spacing chosen is then favourable as it contributes to a distributed crack pattern.

Ad b Eq. (3.1a) provides a reliable estimation of the ultimate shear stress of a crack in concrete crossed perpendicularly by reinforcing bars: \(\tau_u = 6.85 \text{ N/mm}^2\) for \(\rho f_{Sy} = 3.60 \text{ N/mm}^2\) and \(f_{cmm} = 40 \text{ N/mm}^2\). Additional reinforcement in the tension zone of the wall near the pile connections is not considered to transfer any shear loading across the crack. A critical location in the wall occurs for a crack close to the edge of the pile connection. For the service limit state it follows that \(\tau/\tau_u = 5.00/6.85 = 0.73\). Compression strut failure is not expected as \(\tau < 0.3 f_{cylk} = 9 \text{ N/mm}^2\).

Ad c According to the theoretical model given in the sections 5.3-5.6, a minimum shear stress level should be considered with respect to the time-
dependent behaviour of the crack. Eqs. (5.23a-b) are transformed for the C30 concrete grade so that $\tau/\tau_u \geq 0.72$. Thus, the actual stress level is sufficient. Table 6.1 presents the calculated displacements—eqs. (5.16a-c, 5.19a-b)—of the crack as the shear loading on the wall is successively increased. The dowel strength has just been attained for a shear slip $\delta_t \approx 0.14$ mm. According to the static model (fig. 5.14c) this displacement is about equal to $\delta_{no}$. At the instant $t = 0$ h the building is just completed and the full service loading is installed on the six stories. The time-dependent displacement increases of the crack follow from eqs. (5.20a-b). A basic condition is that $\delta_n, \delta_t$ correspond to the equilibrium of shear forces which demands an iterative solution technique. The equilibrium perpendicular to the crack plane yields $\sigma_s = \sigma_{s,cr} + (\sigma_a/\rho)$. The 'damage' parameters $\lambda_\mu$ and $\lambda_F$ (according to eq. (5.28) for mix A) have both been incorporated into the calculations. After $t = 5000$ h = 7 months of load application, the permissible crack width is attained. After about 11 years, the steel stress is still approximately 25% smaller than the yield stress of the steel. At that time the time-dependent crack width increase amounts to 0.14 mm.

It is concluded that the wall may serve as a temporary solution. Another option is the use of 'external' prestressing cables, placed parallel to the structure. To prevent restraint of the wall, joints are needed then.

Table 6.1 Predicted stress-displacement relations of a single crack **).

<table>
<thead>
<tr>
<th>$t$ [h]</th>
<th>$\delta_n$ [mm]</th>
<th>$\delta_t$ [mm]</th>
<th>$\lambda_\mu$ [-]</th>
<th>$\lambda_F$ [-]</th>
<th>$\sigma_a$ [*]</th>
<th>$\sigma_s$ [*]</th>
<th>$\alpha\gamma_d$ [-]</th>
<th>$\tau_a$ [-]</th>
<th>$\tau_d$ [-]</th>
<th>$\Sigma\tau$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.114</td>
<td>0.0</td>
<td>1.00</td>
<td>1.00</td>
<td>0.0</td>
<td>253</td>
<td>0.86</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.140</td>
<td>0.075</td>
<td>1.00</td>
<td>1.00</td>
<td>0.02</td>
<td>256</td>
<td>0.86</td>
<td>2.04</td>
<td>1.50</td>
<td>3.54</td>
</tr>
<tr>
<td>100</td>
<td>0.160</td>
<td>0.109</td>
<td>1.00</td>
<td>1.00</td>
<td>0.06</td>
<td>261</td>
<td>0.85</td>
<td>3.08</td>
<td>1.49</td>
<td>4.57</td>
</tr>
<tr>
<td>1000</td>
<td>0.180</td>
<td>0.138</td>
<td>1.00</td>
<td>1.00</td>
<td>0.35</td>
<td>302</td>
<td>0.80</td>
<td>3.59</td>
<td>1.39</td>
<td>4.98</td>
</tr>
<tr>
<td>0</td>
<td>0.216</td>
<td>0.184</td>
<td>1.00</td>
<td>1.00</td>
<td>0.67</td>
<td>346</td>
<td>0.72</td>
<td>3.75</td>
<td>1.26</td>
<td>5.01</td>
</tr>
<tr>
<td>10</td>
<td>0.253</td>
<td>0.204</td>
<td>1.11</td>
<td>0.97</td>
<td>0.78</td>
<td>361</td>
<td>0.69</td>
<td>3.80</td>
<td>1.21</td>
<td>5.01</td>
</tr>
<tr>
<td>100</td>
<td>0.282</td>
<td>0.242</td>
<td>1.30</td>
<td>0.94</td>
<td>0.84</td>
<td>369</td>
<td>0.68</td>
<td>3.83</td>
<td>1.18</td>
<td>5.01</td>
</tr>
<tr>
<td>1000</td>
<td>0.354</td>
<td>0.322</td>
<td>1.50</td>
<td>0.90</td>
<td>0.90</td>
<td>378</td>
<td>0.66</td>
<td>3.86</td>
<td>1.15</td>
<td>5.01</td>
</tr>
</tbody>
</table>

*) [N/mm$^2$]  **) $\tau_{du} = 1.75$ N/mm$^2$ for $\alpha\gamma_d = 1.0$. 
7. CONCLUSIONS AND OUTLOOK

The aim of the present study was to model the behaviour of a reinforced crack in concrete subjected to sustained shear loading. The tests and analyses resulted in a simple theoretical model which is a combination of Walraven's two-phase model and the modified empirical formula of Rasmussen. Both approaches originally refer to the short-term loading case. The long-term shear tests on cracks in plain and reinforced concrete were both adequately reflected by the analytical model provided that two empirical 'damage' parameters are incorporated.

As stated before, fairly high shear stress levels were applied in view of the serviceability limit state of ordinary concrete structures. These levels were specifically chosen to attain reliable test data for long periods of load application, since the displacement increments of the crack tend to decrease gradually. Virtually, the measuring error of the crack displacements strongly influenced the shear stress levels applied in this research. The experimental results refer to complex practical applications for which severe and concentrated in-plane shear stresses may dominate the stiffness and/or stability of structures. A reinforced concrete panel subjected to shear leads to a set of equidistant cracks. The cracks between the compression struts transfer shear loading by virtue of the interlock mechanism. A constant external loading - for example in the substructure of a tall building or in a beam-column joint - may cause a gradual rotation of the struts. The cracks enable a redistribution of internal forces. This phenomenon similarly occurs if the external loading is increased. Advanced nonlinear finite element programs should incorporate this research to support the design of modern concrete structures when shear plays an important role.

The research also provides to predict accurately the time-dependent response of a crack in case of low shear stress levels, which correspond to usual structural applications where no excessive shear stresses are to be expected. Supplementary research may focus on the following subjects:

- **Numerical implementation** of the analytical model. As the sustained shear transfer is modelled quasi-statically the approach proposed by Pruijssers [144,148] for the static loading case may be applied. Preferably, the time-dependency is represented by parallel Maxwell chains, see section 5.2.1 and in [125,126];
- **Damage parameters.** It is thought that the change of the friction coefficient is related to the fictitious period of contact between a certain part of the matrix layer and an aggregate particle of the opposing crack half. This period is estimated to be inversely proportional to $\partial \delta / \partial t$. Consequently, $t$ multiplied by 10 provides contact periods which are expected to become 6.3-8.5 times longer. For an application period $t = 10^4 - 10^5$ hrs, the average displacement velocity is about one micron per hour. This corresponds to less than 0.010mm crack displacement per year. The thickness of the matrix layer initially is 15-20 times larger. The mix proportion of the concrete affects the damage parameters. Indeed, both mixes investigated display different formulae for $\lambda_f(t)$. Its time-dependent development might relate to the porous structure and the degree of hydration of the matrix. In the macroscopic sense material deterioration is related to the long-term uniaxial compressive strength of concrete.

- Experimental verification of the extended model for large initial crack widths which exceed 0.10mm. Although the theory is expected to provide good predictions for both transfer mechanisms (interlocking and dowel action), there is a lack of test data to substantiate them. It is expected that larger crack widths induce shear failure within the life-time of a structure, provided that a sufficient concrete cover prevents premature splitting. Failure might occur if the crack displacements reach the descending branch of the static $\tau-\delta^H$ envelope curve. Other related points of interest are the response on long-term shear loading for:
  * cracks crossed by reinforcing bars at inclination $\neq 90^0$, see [38,44];
  * cracks in case of an arbitrary degree of prestressing, viz. a combination of prestressing steel and conventional bars, see [90,153];
  * crack extension in case of mixed-mode loading conditions when tensile stresses are transferred by the uncracked concrete parts, see [132,134];
  * hydraulic fracturing that occurs in mining and off-shore engineering.

- Although the microscopic observations of this study clearly revealed the presence of a plastic hinge close to the shear plane and a local shift of the neutral axis in the steel bar, the actual bond mechanism of the dowel is not known. The rather flat crack-opening curves of the sustained tests are attributed to the time-dependent shear deformation of matrix material in the interface layer of the dowel. However, the explanation is based on a rather rough approximation of the real mechanism.
8. SUMMARY / SAMENVATTING

Chapter 1 The introduction gives recent developments in shear design criteria for reinforced concrete. Usually, structural concrete exhibits cracks in the service limit state. There is a lack of experimental and theoretical research on the behaviour of cracked concrete subjected to in-plane sustained shear loading. This subject is the aim of the present study.

Chapter 2 A short survey of relevant literature is presented. The basic mechanisms of shear transfer across a crack are discussed, viz. interlocking of the opposing crack halves due to protruding irregularities and dowel action by the embedded reinforcing bars that cross the shear plane. The review is mainly concerned with short-term tests as no results of sustained shear tests are available yet. The combined mechanism is significantly affected by the concrete compressive strength, the steel yield stress of the bars and by the restraint of the crack. Next, two modelling techniques are reviewed which attempt to define at realistic constitutive laws for the crack response, viz. analytical expressions and a numerical approach associated with non-linear finite element computer programs.

Chapter 3 The set-up of an experimental program is outlined. Tests were conducted on 46 push-off specimens each provided with a central crack. The initial crack width did not exceed 0.10mm. In view of the application to offshore conditions, high-strength concrete was chosen with 28-day compressive strengths $f_{ccm} = 51$ or $70 \text{ N/mm}^2$ of 150mm cubes. To analyse separately the shear transfer mechanisms, the 120*300 mm$^2$ shear plane was restrained in two ways. Either free steel rods were applied or 8mm diameter reinforcing bars ($\rho =1.1-2.2\%$) perpendicularly crossed the crack plane. Ribbed steel bars were used with a yield stress of $f_{sy} = 460$ or $550 \text{ N/mm}^2$. The adjusted constant stress levels ranged from 45 to 89% of the static shear strength. The movements parallel and perpendicular to the crack plane were regularly recorded on either side of the specimen. These measurements were carried out for at least 90 days. The measuring error was 0.005mm. The displacement data were corrected in view of the direct and the time-dependent (creep, shrinkage) deformations of the concrete between the reference points near the crack plane.

Chapter 4 The instantaneous as well as the time-dependent displacements of the crack are presented. The data of each test have been treated by non-
linear statistical techniques inducing a tool for a reliable and objective description in view of the considerable displacement scatter established. This is particularly owing to the heterogeneity of the concrete. The computations revealed that an observation period of three months is sufficient for extrapolation. The recorded displacements have been mathematically represented as functions of the experimental parameters. An implicit calculation procedure was employed. A sensitivity analysis proved the large influence of $f_{cc}$ on the time-dependent crack-opening curves. Shear stress-crack width relations of the reinforced cracks showed a gradual 'weakening'. Generally, a highly non-linear response occurred with respect to the concrete grade and the shear stress level adjusted. No final displacement values were attained. For $\tau/\tau_u \leq 0.50$ the extrapolated crack widths were less than 0.20mm after eleven years of load application. The shear slip of the crack in a plain concrete push-off specimen was rather large. Upon unloading, the displacement decreases revealed to be almost proportional to the previously applied load level. Supplementary research focused on microscopic observations of the shear plane and on the restraint stiffness of the crack plane, measured by means of strain gauges.

Chapter 5 The observed behaviour is considered theoretically. It concerns an extended version of Walraven's two-phase model accounting for the interlocking of aggregates and matrix material of the opposing crack half. The dowel mechanism is described by a modification of Rasmussen's formula. At the onset of the sustained test, the dowel load is assumed to be fully developed. Therefore, $\tau \geq 0.7-0.8\tau_u$ is needed. These two models have been combined and adapted to the time-dependent behaviour of a crack subjected to sustained shear loading. A damage parameter $\lambda_f \leq 1.0$ is introduced that accounts for a gradual decrease in the short-term concrete strength according to $\lambda_f(t)f_{cc}$. The reduction represents the deterioration of the cement-based matrix material. This strength criterion induces increased (time-dependent) movements of the crack halves so that the total contact area between the crack faces is enlarged. In this way the equilibrium of forces is maintained for the crack. The two transfer mechanisms are equally affected by $\lambda_f(t)$: a bilinear relation is derived for each type of concrete investigated. A second adaptation concerns the static friction coefficient $\lambda_\mu(t)\mu$ for the contact between aggregate particles and matrix material. The parameter $\lambda_\mu \geq 1$ is expressed by one empirical formula. Both parameters $\lambda_f$ and $\lambda_\mu$ appear proportional to $\log(t)$, and they provide reliable predictions of the experimentally obtained time-dependent stress-displacement re-
lations of a single crack in plain or reinforced concrete. The calculated restraining stresses correspond quite well with in-situ strain gauge measurements on the reinforcing bars which perpendicularly cross the crack. The time-dependent constitutive relation of a single crack is outlined by a working example which illustrates the satisfactory agreement with the test data. The results of supplementary investigations are summarized as follows:

- Generally, the time-dependent $\delta_t-\delta_n$ curves were rather flat. A fictitious transformation of the static curves is substantiated by the shear deformation of matrix in the interface layer of the dowel. The reduction in normal restraint stiffness is virtually induced by bond creep;

- The static shear strength of a reinforced crack appeared hardly to be influenced by a previous sustained loading unless the actual concrete strength is implemented in the empirical formula for $\tau_u$;

- The dowel mechanism of the embedded steel bars is delivered by a cooperation between steel and concrete under the dowel. At low axial stresses a shift of the neutral axis is expected in the bar. This was confirmed by microscopic observations of localized orientation of steel crystals near the plastic hinges in the dowel;

- $\lambda_f(t)$ has been compared with data of uniaxial creep tests on cement-based material given in the literature. The relation between sustained stress (compressive or tensile) and time to failure depends upon two opposing processes, viz. continuous hydration of cement and release of internal stress versus extension of microcracks. These phenomena have been described by a simple model which predicts the long-term strength.

Chapter 6 The theory is put into practice. An existing reinforced concrete wall, 60 meters in length and placed on piles, is initially cracked due to imposed deformations. The wall is then subjected to in-plane shear loading by a building. The (time-dependent) behaviour of the crack is predicted.

Chapter 7 The results of this study are reviewed. Fairly high shear stress levels were used compared with values given in the design codes. On the one hand these levels were chosen to attain reliable test data for long periods of load application as the decreasing displacement increments tend to approximate the measuring accuracy. On the other hand, lower stress levels are also taken into account by the theoretical model. In practice, concentrated shear may affect the stability and/or stiffness of structures. Therefore, the results of this study should be used in non-linear finite element programs to support the design of complex concrete applications.
SAMENVATTING: 'TIJDSAFHANKELIJKE SCHUIFSPANNINGS-OVERDRACHT IN GESCHEURD GEWAPEND BETON'

Hoofdstuk 1 In de inleiding worden recente ontwikkelingen van dwarskracht-ontwerpcriteria genoemd voor gewapend beton. In het gebruiksstadium toont constructief beton gewoonlijk scheuren. Er is een tekort aan experimenteel en theoretisch onderzoek naar het gedrag van gescheurd beton, dat onderworpen aan langeduur-schuifbelasting, aangebracht in het vlak van de scheur. Dit onderwerp is doel van de onderhavige studie.

Hoofdstuk 2 Er wordt een overzicht gegeven van relevante literatuur. Besproken worden de basismechanismen van dwarskracht-overdracht over één scheur, nl. het contact tussen aangrenzende scheurhelften door uitstekende onregelmatigheden en deuvelwerking door ingestorte wapeningsstaven, die het schuifvlak doorsnijden. Het overzicht heeft vooral betrekking op korteduurproeven omdat (nog) geen resultaten van tijdsafhankelijke schuifproeven beschikbaar zijn. Het gecombineerde mechanisme wordt significant beïnvloed door de betondruksterkte, de vloei spanning van de staven en de mate van opsluiting van de scheur. Vervolgens worden twee modelleringstechnieken besproken, gericht op realistische, constitutieve relaties voor het scheergedrag, te weten analytische uitdrukkingen en numerieke benaderingen, zoals toegepast in niet-lineaire eindige-elementenprogramma's voor computers.

Hoofdstuk 3 De opzet van een proevenprogramma wordt aangegeven. Er zijn proeven uitgevoerd op 46 afschuifelementen, elk voorzien van één centrale scheur: de initiële wijdte was kleiner dan 0.10mm. Ten behoeve van offshore toepassingen is gekozen voor hogesterkte beton met 28-daagse druksterkten $f_{ccm} = 51$ of 70 N/mm$^2$ van 150mm kuben. Om de overdrachtsmechanismen over de scheur apart te analyseren was het schuifvlak, afm. 120*300 mm$^2$, op twee wijzen opgesloten: of door middel van 'vrije' staven of via wapeningsstaven van 8mm diameter ($\rho = 1.1-2.2\%$), die dat vlak loodrecht doorsneden. Geribd staal is toegepast met een vloei spanning $f_{sy} = 460$ of 550 N/mm$^2$. Het ingestelde constante schuifspanningsniveau bedroeg 45-89% van de statische afschuijsterkte. De verplaatsingen, evenwijdig aan en loodrecht op het scheurvlak, zijn regelmatig geregistreerd aan beide zijden van het proefstuk gedurende tenminste 90 dagen. De meetfout was 0.005mm. De metingen zijn gecorrigeerd in verband met instantane en tijdsafhankelijke (kruip, krimp) vervorming van het beton aan weerszijden van het scheurvlak.

Hoofdstuk 4 De gemeten directe en tijdsafhankelijke scheurverplaatsingen
zijn weergegeven. Door de aanzienlijke spreiding van de verplaatsingen, zijn de gegevens van iedere proef met niet-lineaire statistische technieken behandeld, zodat een betrouwbare en objectieve beschrijving mogelijk wordt. Deze spreiding is vooral veroorzaakt door de heterogeniteit van het beton. De berekeningen gaven aan dat een waarnemingsperiode van drie maanden voldoende is voor extrapolatie. De gemeten verplaatsingen zijn daarna wiskundig verwerkt en geschreven als impliciete functie van de experimentele parameters. Een gevoeligheidsanalyse toonde de grote invloed aan van f<sub>cc</sub> op de tijdsafhankelijke scheuropeningen-curven. De schuifspanning-scheurwijdte relaties van de gewapende scheur vertoonden een geleidelijke 'ontsteving'. In het algemeen was de responsie niet-lineair met betrekking tot de betonsterkte en de ingestelde schuifspanning. Er zijn geen eindwaarden van de verplaatsingen bereikt. Voor τ/τ<sub>u</sub> < 0.50 waren de geëxtrapoleerde scheurwijdten kleiner dan 0.20mm na een belastingduur van elf jaar. De langsvrplaatsingen van de ongewapende afschuifproefstukken waren relatief groot. Bij ontlasten waren de verplaatsingsafnamen ongeveer evenredig met het oorspronkelijke belastingniveau. Aanvullend onderzoek richtte zich op microscopische waarnemingen van het schuifvlak en op de opsluit-stijfheid van het scheurvlak door middel van rekstrookmetingen.

Hoofdstuk 5 De waarnemingen zijn theoretisch geanalyseerd. Voorgesteld is een uitbreiding van het twee-fasen model van Walraven dat de 'interlocking' van twee scheurhelften beschrijft. De deuvelwerking is aangeduid met een modificatie van Rasmussen's formule. Verondersteld is, dat de deuvelkracht volledig is ontwikkeld bij het begin van de langduurschuifproef indien τ/τ<sub>u</sub> ≥ 0.7-0.8. Deze twee modelleringen zijn gecombineerd en aangepast voor het tijdsafhankelijk gedrag van een scheur die is onderworpen aan een constante schuifbelasting. Een beschadigingsparameter λ<sub>f</sub> ≤ 1.0 is voorgesteld die een geleidelijke vermindering van de korteuwer-sterkte van het beton veroorzaakt volgens λ<sub>f(t)</sub>*f<sub>cc</sub>. De reductie vertegenwoordigt de beschadiging van het cementgebonden materiaal tussen de grove toeslagkorrels. Dit sterkte-criterium veroorzaakt toenemende (tijdsafhankelijke) verplaatsingen van de scheurhelften zodat het totale contactoppervlak tussen de scheurzijden zich uitbreidt. Zodoende blijft het krachtenevenwicht voor de scheur gehandhaafd. Beide overdrachtsmechanismen - deuvelwerking en interlocking - worden op gelijke wijze beïnvloed door λ<sub>f(t)</sub>; voor elke onderzochte betonsoort is een bilineaire relatie afgeleid. Een tweede aanpassing betreft de statische wrijvingscoëfficiënt λ<sub>µ(t)</sub>*µ voor het contact tussen toeslagkorrels en matrixmateriaal: λ<sub>µ</sub> ≥ 1 is weergegeven met één empirische formule.
Zowel $\lambda_f$ als $\lambda_\mu$ blijken evenredig met $log(t)$ en zorgen voor een betrouwbare voorspelling van de experimenteel vastgestelde tijdsafhankelijke spanningverplaatsing relaties van één scheur in ongewapend of gewapend beton. De berekende spanningen loodrecht op het scheurvlak komen goed overeen met in-situ rekstrookmetingen, uitgevoerd aan wapeningsstaven die het scheurvlak doorsnijden. De constitutieve relatie van een scheur is toegelicht met een rekenvoorbeeld, waaruit de bevriddigende overeenstemming met de proeven blijkt. De resultaten van aanvullend onderzoek zijn hieronder samengevat:

- Veelal waren de tijdsafhankelijke $\delta_t-\delta_R$-curven tamelijk vlak. Dit is aannemelijk gemaakt door een fictieve transformatie vanuit de statische curven: feitelijk wordt de afnemende normaalstijfheid veroorzaakt door aanhechtkruip van matrixmateriaal onder de deuvel;

- De statische afschuiuifsterkte van een gewapende scheur werd nauwelijks beïnvloed door een constante voorbelasting;

- Deuvelwerking van de ingestorte wapeningsstaven wordt geleverd door samenwerking tussen staal en beton onder de deuvel. Bij lage axiale spanningen leidt dit tot een verschuiving van de neutrale lijn in de staaf. Dit is bevestigd door microscopische waarnemingen die duidden op lokale kristaloriëntatie in het staal nabij de plastische scharnieren in de deuvel;

- $\lambda_f(t)$ is vergeleken met literatuurgegevens van éénassige kruipproeven op cementgebonden materiaal. De relatie tussen de constante spanning (druk of trek) en de breuktijd hangt af van tegengesteld werkende processen, nl. voortgaande hydratatie van cement en inwendige spanningsafbouw versus uitbreiding en initiatie van microscheuren. Deze verschijnselen zijn beschreven met een simpel model dat de langeduur-sterkte voorspelt.

Hoofdstuk 6 De theorie is toegepast voor een bestaande gewapend betonwand van 60 meter lengte, die is gescheurd door opgelegde vervorming. Het (tijdsafh.) gedrag is voorspeld als de wand fungeert als fundering van een gebouw.

9. **NOTATION**

Unless otherwise stated the dimensions are N, mm or N/mm².

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{x}, a_{y} )</td>
<td>projected contact areas ( A_{x}, A_{y} ) for a particle [( \text{mm}^2 )]</td>
</tr>
<tr>
<td>( c )</td>
<td>concrete cover or crack length</td>
</tr>
<tr>
<td>( d_b )</td>
<td>bar diameter</td>
</tr>
<tr>
<td>( e )</td>
<td>eccentricity of load</td>
</tr>
<tr>
<td>( f_b )</td>
<td>bearing strength of concrete under a bar</td>
</tr>
<tr>
<td>( f_{cc} )</td>
<td>cube compressive strength of concrete</td>
</tr>
<tr>
<td>( f_{cspl} )</td>
<td>cube tensile splitting strength of concrete</td>
</tr>
<tr>
<td>( f_{ct} )</td>
<td>cube tensile strength of concrete</td>
</tr>
<tr>
<td>( f_{cyl} )</td>
<td>cylinder compressive strength of concrete</td>
</tr>
<tr>
<td>( f_i )</td>
<td>indicates displacement formulae ((i = 1, 2, \ldots))</td>
</tr>
<tr>
<td>( f_{sy} )</td>
<td>steel yield stress</td>
</tr>
<tr>
<td>( f_R )</td>
<td>specific rib area of reinforcing bar [-]</td>
</tr>
<tr>
<td>( g_i )</td>
<td>boundary condition formulae or values ((i = 1, 2, \ldots))</td>
</tr>
<tr>
<td>( h )</td>
<td>depth of beam</td>
</tr>
<tr>
<td>( k )</td>
<td>modulus of subgrade support ([\text{N/mm}^3])</td>
</tr>
<tr>
<td>( l )</td>
<td>bar length</td>
</tr>
<tr>
<td>( l_c )</td>
<td>crack spacing</td>
</tr>
<tr>
<td>( m )</td>
<td>coefficient of internal stress release [-]</td>
</tr>
<tr>
<td>( n )</td>
<td>ratio ( E_s/E_c ) or number of observations [-]</td>
</tr>
<tr>
<td>( p )</td>
<td>probability [-]</td>
</tr>
<tr>
<td>( p_k )</td>
<td>volume of the particles/total concrete volume [-]</td>
</tr>
<tr>
<td>( q )</td>
<td>distributed load ([\text{N/mm}])</td>
</tr>
<tr>
<td>( r )</td>
<td>radius of particle or coefficient of correlation [-]</td>
</tr>
<tr>
<td>( t )</td>
<td>duration of load applic. or thickness of matrix layer [( \text{d} )]</td>
</tr>
<tr>
<td>( t_0 )</td>
<td>age of concrete at start of test [( \text{d} )]</td>
</tr>
<tr>
<td>( t_f )</td>
<td>duration to failure [( \text{d} )]</td>
</tr>
<tr>
<td>( w_{cr} )</td>
<td>water-cement ratio by w. [-]</td>
</tr>
<tr>
<td>( x, y )</td>
<td>Cartesian coordinate axes</td>
</tr>
<tr>
<td>( A_c )</td>
<td>cross-sectional area of concrete shear plane ([\text{mm}^2])</td>
</tr>
<tr>
<td>( A_s )</td>
<td>cross-sectional area of reinforcement ([\text{mm}^2])</td>
</tr>
<tr>
<td>( A_{x}, A_{y} )</td>
<td>total proj. contact areas between particles and matrix material per unit area ([\text{mm}^2])</td>
</tr>
<tr>
<td>( D_{max} )</td>
<td>maximum particle size</td>
</tr>
<tr>
<td>( E_c )</td>
<td>modulus of elast. of concrete</td>
</tr>
<tr>
<td>( E_s )</td>
<td>mod. of elasticity of steel</td>
</tr>
<tr>
<td>( F )</td>
<td>external force</td>
</tr>
<tr>
<td>( G_c )</td>
<td>shear modulus of concrete</td>
</tr>
<tr>
<td>( I )</td>
<td>moment of inertia of cross-section ([\text{mm}^4])</td>
</tr>
<tr>
<td>( K )</td>
<td>normal or shear stiffness ([\text{N/mm}^3]) or kernel of the Volterra integr. eq. ([\text{mm}^2/\text{Ns}])</td>
</tr>
<tr>
<td>( M_{pl} )</td>
<td>plastic bending moment ([\text{Nmm}])</td>
</tr>
<tr>
<td>( M_u )</td>
<td>ultimate bending moment ([\text{Nmm}])</td>
</tr>
<tr>
<td>( N )</td>
<td>internal axial force</td>
</tr>
<tr>
<td>( S )</td>
<td>total crack length</td>
</tr>
<tr>
<td>( S_{ij} )</td>
<td>stiffness coeff. of the incr. stress-strain rel. ([\text{N/mm}^3])</td>
</tr>
<tr>
<td>( V )</td>
<td>shear load</td>
</tr>
<tr>
<td>( V_d )</td>
<td>dowel force</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>empirical constant</td>
</tr>
</tbody>
</table>
\[ \beta \] empirical constant or shear retention factor [-]
\[ \gamma \] shear deformation [-] or material surface en. [N/mm]
\[ \gamma_{a}, \gamma_{d} \] reduct. factor for normal and/or shear stress [-]
\[ \delta_{cs} \] bond slip of reinforcing bar
\[ \delta_{n} \] separation increase due to sustained loading
\[ \delta_{tel} \] instantaneous separation
\[ \delta_{ntop} \] separation at \( \tau = \tau_{u} \)
\[ \delta_{s} \] slip (or: parallel displ.)
\[ \epsilon_{c} \] creep deform. of concrete [-]
\[ \epsilon_{cs} \] shrinkage deform. [-]
\[ \eta \] strength ratio or normal retention factor [-]
\[ \theta \] angle of inclination [-]
\[ \lambda_{f}, \lambda_{\mu} \] 'damage' parameters [-]
\[ \mu \] coefficient of friction [-]
\[ \nu_{c} \] Poisson's ratio of concrete [-]
\[ \rho \] reinforcement ratio \((A_{s}/A_{c})\) or dowel reduction factor [-]
\[ \sigma_{a} \] normal stress due to aggregate interlock
\[ \sigma_{c} \] concrete stress
\[ \sigma_{pu} \] yield strength of the matrix material
\[ \sigma_{s} \] steel stress
\[ \sigma_{N} \] external normal stress applied on the crack plane
\[ \tau_{a} \] shear stress due to aggregate interlock
\[ \tau_{cs} \] bond stress of a reinforcing bar
\[ \tau_{d} \] shear stress due to dowel action = \( V_{d} / A_{c} \)
\[ \tau_{pu} \] shear strength of the matrix material
\[ \tau_{u} \] ultimate shear stress
\[ \phi \] angle of internal friction [-]
\[ \phi_{c} \] creep coeff. of concrete [-]
\[ \phi_{n} \] creep coefficient of cracked concrete = \( \delta_{nc}(t)/\delta_{tel} \) [-]
\[ \Delta \] increment or measured difference [-]
\[ \Sigma \] summation [-]
[ ] refers to references or to unit used

subscripts
\[ a \] aggregate interlock
\[ c \] creep
\[ co \] uncracked concrete part
\[ cr \] cracked concrete part
\[ cs \] shrinkage or bond
\[ d \] dowel action
\[ el \] elastic or instantaneous
\[ m \] mean value or matrix material
\[ n \] normal direction (perpendicular to the crack plane)
\[ p \] cement paste or permanent load
\[ q \] live load
\[ s \] steel
\[ t \] transverse direction (parallel to the crack plane)
\[ u \] ultimate
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# 11. APPENDICES

## Appendix 11.1 Mix proportions

<table>
<thead>
<tr>
<th>components [kg/m³]</th>
<th>sieve opening [mm]</th>
<th>[kg]</th>
<th>[cum. %]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mix A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sand</td>
<td>877.2</td>
<td>623.7</td>
<td>100.0</td>
</tr>
<tr>
<td>gravel</td>
<td>1065.0</td>
<td>441.3</td>
<td>67.9</td>
</tr>
<tr>
<td>cement-B</td>
<td>325.0</td>
<td>312.1</td>
<td>45.2</td>
</tr>
<tr>
<td>water</td>
<td>162.5</td>
<td>220.9</td>
<td>29.1</td>
</tr>
<tr>
<td></td>
<td>0.50 - 1</td>
<td>156.2</td>
<td>17.7</td>
</tr>
<tr>
<td></td>
<td>0.25 - 0.50</td>
<td>110.3</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>0.10 - 0.25</td>
<td>77.7</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2429.7</td>
<td></td>
</tr>
<tr>
<td>mix B</td>
<td></td>
<td>1942.2</td>
<td></td>
</tr>
<tr>
<td>sand</td>
<td>857.3</td>
<td>596.5</td>
<td>100.0</td>
</tr>
<tr>
<td>gravel</td>
<td>1018.5</td>
<td>421.9</td>
<td>68.2</td>
</tr>
<tr>
<td>cement-B</td>
<td>420.0</td>
<td>298.3</td>
<td>45.7</td>
</tr>
<tr>
<td>water</td>
<td>147.0</td>
<td>212.0</td>
<td>29.8</td>
</tr>
<tr>
<td>superpl.2.5%</td>
<td>10.5</td>
<td>148.6</td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td>0.50 - 1</td>
<td>105.0</td>
<td>10.6</td>
</tr>
<tr>
<td></td>
<td>0.25 - 0.50</td>
<td>93.5</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>0.10 - 0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2453.3</td>
<td></td>
</tr>
</tbody>
</table>

![Diagram](image_url)
Appendix 11.2 Mechanical properties of concrete

Two concrete mixes were used throughout the test series (see appendix 11.1). Each batch consisted of two push-off specimens and at least nine 150 mm cubes. The following mechanical properties of both types of concrete were determined [120,122]:

- 28-day strengths $f_{cc}$ and $f_{csp1}$ of 150mm cubes;
- time-dependent development of $f_{cc}$;
- stress-strain relation recorded for prisms and cylinders;
- shrinkage deformation and weight loss of prisms.

All the tests were conducted in a Toni-Technik testing machine, range 3000 kN. The average load application rates were $0.20 \text{ N/mm}^2 \cdot \text{s}$ (for $f_{cc}$), $0.05 \text{ N/mm}^2 \cdot \text{s}$ (for $f_{csp1}$) and $10^{-6}/\text{sec}$ (for the stress-strain relation).

Static strengths. The 28-day compressive strengths approximately displayed a normal distribution as shown in figures 3.11a-b. Generally, there was no difference between the mean strengths of each cast. Statistical analyses made it plausible to consider all the casts as one sample. These conclusions also refer to $f_{csp1}$. At $t_0 = 28$ days it was found that:

For mix A:

$$f_{csp1m} = 3.4 \text{ N/mm}^2 \quad (24 \text{ cubes; v.c.} = 0.13) \quad (11.1a)$$

For mix B:

$$f_{csp1m} = 4.0 \text{ N/mm}^2 \quad (19 \text{ cubes; v.c.} = 0.12) \quad (11.1b)$$

Next, tests were performed on 150mm cubes at age $t_0 = 7$-149 days. The development of the static strength was described by the following relation:

$$f_{cc}(t_0) = f_{ccm} \cdot t_0 / (\alpha + \beta t_0) \quad [\text{N/mm}^2] \quad (11.2)$$

where $f_{ccm}$ denotes the mean 28-day cube strength and $\alpha, \beta$ = regression coefficients. The results are summarized in table 11.1 and figures 11.1a-b. Values of $\delta\alpha$ and $\delta\beta$ represent half of the 90% confidence intervals of $\alpha$ and $\beta$ respectively. The rather poor fit for mix B is traced back to the brittle behaviour of high-strength concrete, causing significant scatter; it showed an initially high static strength increase, probably due to the considerable amount of cement (affecting the hydration process) and the low water-cement ratio. From eq. (11.2) final strength values can be computed:

For mix A:

$$f_{cc}(t_0 \to \infty) = 1.15 - 1.21f_{ccm} \quad [\text{N/mm}^2] \quad (11.3a)$$

For mix B:

$$f_{cc}(t_0 \to \infty) = 1.10 - 1.14f_{ccm} \quad [\text{N/mm}^2] \quad (11.3b)$$
Table 11.1 Results of regression analyses based on eq. (11.2).

<table>
<thead>
<tr>
<th>mix</th>
<th>no. of observ.</th>
<th>$\alpha \pm d\alpha$ [days]</th>
<th>$\beta \pm d\beta$ [-]</th>
<th>$r$</th>
<th>$f_{ccm}$ [N/mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>34</td>
<td>3.785 ± 0.434</td>
<td>0.847 ± 0.020</td>
<td>0.93</td>
<td>54.1</td>
</tr>
<tr>
<td>B</td>
<td>44</td>
<td>2.533 ± 0.306</td>
<td>0.895 ± 0.016</td>
<td>0.89</td>
<td>69.6</td>
</tr>
</tbody>
</table>

Figure 11.1 Development of $f_{cc}$ for (a) mix A and (b) mix B.
Stress-strain diagram Figure 11.2 shows the stress-strain relations recorded for both types of concrete, conducted on $\phi$ 150x400mm cylinders and 150x150x600mm prisms, load-controlled at a rate of 0.2 N/mm$^2$.s. The moduli of elasticity at $t_0 = 28$ days, are:

- **mix A:** $E_{c0} = 32400$ and $E_c = 31100$ [N/mm$^2$] (11.4a)
- **mix B:** $E_{c0} = 40300$ and $E_c = 37750$ [N/mm$^2$] (11.4b)

where the secant modulus $E_c$ corresponds to a straight line between $\sigma = 0$ and $1/3.f_{cyl}$ in the $\sigma$-$\epsilon$ diagram. Note that the average ratios $E_{c0}(t_0 = 690 \text{ d})/E_{c0}(t_0 = 28 \text{ d})$ are 1.18 and 1.13 for mix A and mix B respectively. These values closely approximate eqs. (11.3a) and (11.3b). The ratios $f_{cyl}/f_{cc} \approx 0.77$ and $f_{cp}/f_{cc} \approx 0.71$ are according to Lewandowski [171].

Figure 11.2 Stress-strain diagrams for cylinders (---) and prisms (----).
Shrinkage tests were carried out in order to correct the measured crack widths of the push-off specimen, see figure 3.8. The longitudinal deformation of 150x150x600mm prisms was measured by means of Fahr-millimess dial-gauges (0.005mm accuracy at 2mm range and 400mm measuring length) at 20°C and 50% R.H. The recordings started at $t_0 = 23-26$ days, one hour after the removal of the prisms from the fog room. It was established that:

$$\varepsilon_{cs}(t) = \frac{t}{\alpha + \beta t}$$

(11.5)

for $10 \leq t \leq 460$ days, where $\alpha = 116.29$, $\beta = 2.54$ (mix A) and $\alpha = 91.70$, $\beta = 2.95$ (mix B), correlation coefficient $r = 0.98$. The results of mix A agree closely with previous measurements [117]. Shrinkage deformation of mix A exceeds the deformation of mix B for $t \geq 60$ days.

Figures 11.3a-b present the observed loss of weight of the prisms as a function of the drying period. These measurements should contribute to finding relations between the degree of hydration and the time-dependent mechanical properties of concrete. The initial moisture content of both concrete types is almost equal. However, after one year of exposure, the loss of weight of mix B is barely half that of mix A. This is probably caused by the high density (low water-cement ratio) and the large amount of cement demanding a moisture surplus for the hydration process. Hobbs [193] found linear relations between shrinkage and loss of weight indicating that ultimate shrinkage values were not yet attained. Non-linear relations of both mixes developed after about 300-350 days of exposure.

Figure 11.3 Loss of weight of 150x150x600mm prisms for (a) mix A and (b) B.
Appendix 11.3  Sustained shear test results: reinforced concrete
Development of the concrete compressive stress perpendicular to the crack plane: \( \sigma_c = (A_s / A_c) \sigma_s \) (embedded bar). See also section 4.6.2.
### Appendix 11.4 Sustained shear test results: plain concrete

<table>
<thead>
<tr>
<th>MIX A</th>
<th>$f'_{c} [N/mm^2]$</th>
<th>$d_{c} [mm]$</th>
<th>$x'_{u}$</th>
<th>$f'_{u} [N/mm^2]$</th>
<th>$d_{u} [mm]$</th>
<th>spec. no.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>58.0</td>
<td>10</td>
<td>0.99</td>
<td>0.09</td>
<td>58.0</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>52.6</td>
<td>16</td>
<td>0.65</td>
<td>0.22</td>
<td>58.0</td>
<td>16</td>
<td>7</td>
</tr>
<tr>
<td>58.0</td>
<td>16</td>
<td>0.13</td>
<td>0.01</td>
<td>58.0</td>
<td>16</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MIX B</th>
<th>$f'_{c} [N/mm^2]$</th>
<th>$d_{c} [mm]$</th>
<th>$x'_{u}$</th>
<th>$f'_{u} [N/mm^2]$</th>
<th>$d_{u} [mm]$</th>
<th>spec. no.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>69.8</td>
<td>10</td>
<td>0.79</td>
<td>0.02</td>
<td>73.2</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>73.2</td>
<td>16</td>
<td>0.04</td>
<td>0.02</td>
<td>73.2</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>73.2</td>
<td>16</td>
<td>0.04</td>
<td>0.02</td>
<td>73.2</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>
Development of $\sigma_c = (A_s / A_c) \cdot \sigma$ (steel rods) where $A_s = \pi (0.25)^2$ or $\pi (0.5)^2$. See section 4.3.2.
Appendix 11.5 Computational procedure

The procedure given below can also be applied to transfer of sustained shear stress $\tau$ (indicated by $\tau$) unless the friction coefficient $\mu(t)$ and a damage parameter $\lambda(t)$ are considered, see sections 5.4-5.6. Note that $\delta_n$, $\delta_t \leq 0.80$mm. The choice of the magnitudes of the variables should relate to the test range. Iterative computations are not indicated specifically.

```
Input: concrete: $D_{\text{max}}$, $p_k$, $f_{\text{ccm}}$, $\delta_{\text{no}}$
       steel : $d_b$, $f_{\text{sy}}$, $\rho$

$\delta_n \rightarrow \delta_t$

Find
$\delta_{\text{nel}}, \delta_{\text{tel}}$

$A_x, A_y$

Find
$[\lambda_f(t), \lambda_\mu(t)]$

$\sigma_a, \tau_a, \tau_{\text{du}}$

$\gamma_d$

Find $\sigma, \tau$

$\tau \leq 0.9\tau_u$

(shear) stress-displacement relationships of crack found
```
Appendix 11.6 Curriculum vitae

naam : Frénaij, Jeroen Willem Ignatius Jozef

geboren : 29 september 1956 te Geertruidenberg (NB)


sinds 1-3-1989 : senior wetenschappelijk onderzoeker bij het IMAG (Instituut voor Mechanisatie, Arbeid en Gebouwen), ressorterend onder de Directie Landbouwkundig Onderzoek (DLO) te Wageningen.