Numerical simulation of the LAGEOS thermal behavior and thermal accelerations

J. I. Andrés,¹ R. Noomen,¹ and S. Vecellio None²

Received 5 July 2005; revised 23 February 2006; accepted 21 April 2006; published 27 September 2006.

[1] The temperature distribution throughout the LAGEOS satellites is simulated numerically with the objective to determine the resulting thermal force. The different elements and materials comprising the spacecraft, with their energy transfer, have been modeled with unprecedented detail. The radiation inputs on the satellites are direct solar (eclipse modulated), Earth albedo, and Earth infrared radiations. For each satellite the lifetime temperature (behavior) of 2133 nodes is computed. On the basis of this distribution, individual forces and the net instantaneous accelerations are obtained. Simulations yield typical temperature variations ranging between 30 and 100 K for different elements and materials, whereas the net instantaneous accelerations are on the order of 70 pm s^{-2} , in good agreement with previous results. Simulations also show the importance of the consideration of a proper orientation of the satellite: LOSSAM yields acceleration differences of up to three times the acceleration obtained with a constant spin axis orientation. The temperature of the four germanium retroreflectors deviates up to 70 and 100 K with respect to their silica counterparts for LAGEOS I and II, respectively. This generates thermal acceleration differences of several pm s^{-2} , up to 25% of the postulated difference in reflectivity between hemispheres. Two factors play a major role: the spin rate and the Sun aspect angle with respect to the spin axis. On the basis of the latter, two characteristic periods can be distinguished: a rapid spin, slow drift period (until 13 and 8 years after launch for LAGEOS I and II, respectively) and a slow spin, rapid wobbling afterward. The acceleration results will be used in a refined orbit computation in a subsequent investigation.

Citation: Andrés, J. I., R. Noomen, and S. Vecellio None (2006), Numerical simulation of the LAGEOS thermal behavior and thermal accelerations, *J. Geophys. Res.*, *111*, B09406, doi:10.1029/2005JB003928.

1. Introduction

[2] The Laser Geodynamics Satellites LAGEOS I and LAGEOS II are orbiting Earth since 4 May 1976 and 22 October 1992, respectively. With 426 corner cube reflectors (CCRs) embedded in a spherical surface and a very low area-to-mass ratio, the LAGEOS satellites are among the best tools for space geodetic research: the large number of retroreflectors increases the possibilities of laser ranging returns (i.e., return signal strength) and the low area-to-mass ratio augments the accuracy of orbital solutions by reducing the so-called surface accelerations. By analyzing satellite laser ranging (SLR) observations, geophysical phenomena such as variations of the Terrestrial Reference Frame (TRF) with respect to the geocenter, global-scale, low-degree gravity field terms, Earth Orientation Parameters (EOPs) and plate tectonic motions can be accurately measured, their accuracy directly dependent on

that of the ground laser instrumentation and the accuracy of the orbit determination. As a matter of fact, the contribution of the LAGEOS pair is indispensable for the assessment of the origin and scale (the latter shared with contributions from very long baseline interferometry, VLBI) of the practical implementations of the terrestrial reference system, such as ITRF2000 [Altamimi *et al.*, 2002], which is at the heart of the far majority of crustal deformation (and other) studies. Orbit quality is crucial here; the current investigation provides results that will be used in a follow-up study aimed at a further improvement of this essential orbit quality.

[3] Intensive orbital analyses yielded a decrease in the semimajor axis of the orbit of LAGEOS I, at a rate of 1.3 mm d^{-1} , shortly after launch [Smith and Dunn, 1980; Rubincam, 1982]. A similar decay has been observed for LAGEOS II. Various physical processes (or a combination of them) have been proposed as possible causes for this acceleration, namely, thermal thrust [e.g., Rubincam, 1987a], ionospheric drag (neutral and charged particles [e.g., Rubincam, 1987a; Afonso *et al.*, 1989; Barlier *et al.*, 1986]), radiation from celestial bodies (Earth and Sun) and reradiation from the satellite itself, together with eclipse dependencies of the (re)radiation [e.g., Anselmo *et al.*, 1983; Afonso *et al.*, 1989; Rubincam, 1990; Slabinski, 1997]. This

¹Delft Institute for Earth Observation and Space Systems, Delft University of Technology, Delft, Netherlands.

²Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Milan, Italy.

Figure 1. Schematic drawing of the 4 germanium CCRs spatial distribution for LAGEOS I and II (left and right, respectively), measured with respect to a satellite-fixed reference frame.

decay can be modeled by an empirical along-track acceleration with a mean value of about -3.4 pm s^{-2} [e.g., *Rubincam*, 1987a; *Afonso et al.*, 1989]. In the course of these investigations, the importance of the so-called thermal forces has been demonstrated. A significant number of publications has been dedicated to this matter, addressing either the relevance of a particular radiation source in the total amount of radiation absorbed by the satellite (e.g., Earth's albedo), or the eclipse passages in thermal reradiation (Yarkovsky-Schach effect), or a certain temperature distribution calculation (purely theoretical or numerical approaches, e.g., *Rubincam* [1987a] or *Slabinski* [1997], respectively). A most important factor in the calculation of the thermal forces is the orientation of the spin axis [e.g., *Rubincam*, 1982; *Scharroo et al.*, 1991; *Slabinski*, 1997; *Andrés et al.*, 2004], especially when the despinning of the satellite has reached a situation where the spin period is comparable to the thermal characteristic time [*Farinella et al.*, 1996; *Andrés et al.*, 2004].

[4] A very interesting postulated effect which has not found a proper physical explanation so far [*Vokrouhlický and Métris*, 2004], is the effective difference in reflectivity δ between the two hemispheres of LAGEOS. This was first suggested by *Rubincam* [1987b] and successfully followed up in the calculations done by *Scharroo et al.* [1991] and *Farinella et al.* [1996] in which values of 1.5% and 2.3% were obtained for LAGEOS I and II, respectively. According to *Lucchesi* [2003, 2004], the maximum amplitude of the (spin-averaged) effect (12 and 18 pm s^{-2} for the aforementioned values of δ) for an angle between Sun and spin axis vectors of 90° , is due to the special position of the four germanium CCRs (forming a tetrahedron, see Figure 1). However, *Vokrouhlický and Métris* [2004] have demonstrated that the calculated amplitude is two thirds smaller, which would leave an unexplained effect still present. On the other hand, *Lucchesi* [2003] rightly refers to the fact that for a general geometry configuration, not all Ge CCRs are illuminated at the same time; thus, typically, a latitudinal asymmetry in the temperature of these CCRs generally exists. Moreover, this asymmetry is reinforced when considering Earth's albedo in the radiation budget received by the germanium retroreflectors. It is the suspicion of the authors that a combination of a proper spin axis modeling together with the inclusion of the germanium CCRs, and a revised charged particle drag contribution,

could give answer to the residuals attributed to the reflectivity difference effect for three reasons: (1) *LOSSAM* [*Andrés et al.*, 2004] has demonstrated to agree significantly better with observations than previous models [see *Lucchesi et al.*, 2004], (2) as mentioned above, the germanium contribution is not negligible, and (3) the charged particle drag has lacked some modeling accuracy. The contribution of the first two is addressed in this study whereas the latter is the subject of a publication in course. The main objective of this study is to arrive at the most reliable and most detailed description of the accelerations of the two spacecraft based on their thermal behavior. In addition, these detailed simulations will allow two other things: (1) to ascertain the importance of individual parameters for relevant physical effects and (2) to reduce uncertainties of thermal acceleration models, in order to benefit studies of other acceleration contributors. Direct observations of the temperatures and/or thermal acceleration as such do not exist (the solution for the empirical accelerations that are typically solved for in orbit computations are influenced by an ensemble of various forces and/or residuals thereof), which by definition implies that no confrontation with reality can be done in this study; instead, a sensitivity analysis of the consequences of assumptions and/or physical properties used will be included. The final objective of this study is to contribute to a complete reassessment of the LAGEOS nongravitational forces which includes both spinning regimes, this study being the first step toward it.

[5] As mentioned above, this publication aims at providing a thorough calculation of the thermal forces by the inclusion of the LAGEOS spin axis evolution, as given by *LOSSAM*, and the development and use of a detailed structural model of the vehicles (including the germanium retroreflector characteristics), so as to diminish the uncertainty of the (modeling of the) net thermal forces, and thus to improve the analysis options of other possible forces at pm s^{-2} scale, e.g., charged particle drag. First, the thermal problem and the modeling considerations are presented. This is followed by results in terms of temperature and net accelerations (including sensitivities and uncertainties) for the entire lifetime of both satellites. Conclusions will be drawn in the last part of this paper, together with some recommendations for further study. Details of the actual numerical method used in these simulations are presented in appendices.

2. Thermal Problem

[6] A proper understanding of the LAGEOS thermal problem requires a complete heat transfer analysis in order to properly describe and quantify the different types of heat exchange between the aforementioned elements. The different elements of the satellite (including fabrication materials, physical and geometrical properties) of each satellite are described in Appendix A. The development of the thermal model is described here first. The importance of the spin axis modeling and rotation behavior for the thermal problem is discussed next (in a qualitative way).

2.1. Model

[7] Quantification of the energy balance for an arbitrary LAGEOS element reveals that only conduction and radia-

Figure 2. Retroreflector assembly geometry.

tion must be considered as heat transfer mechanisms. Provided that (1) most heat transfer occurs via radiation and (2) the ratio between conductive and radiative resistances is small (i.e., the process of transferring heat via conduction is much faster than the typical radiation process), the particular element can be considered as isothermal [Slabinski, 1997]. This ratio Q can be written as

$$Q = \frac{4\epsilon\sigma(\mathcal{V}/\mathcal{A})T_0^3}{k} \quad (1)$$

where ϵ is the emissivity, σ is the Stefan-Boltzmann constant, $\mathcal{V}$ is the volume, $\mathcal{A}$ is the radiating area, T_0 is the mean temperature, and k is the thermal conductivity. In the case of a retroreflector this ratio is less than 0.05 for a mean temperature of about 430 K; the same holds for the aluminium shells up to a temperature of about 1800 K. Hence these elements can be regarded as isothermal. As for the KEL-F rings, only one hypothesis is satisfied: the ratio is small up to a temperature of 327 K, but most heat transfer does not occur via radiation. To overcome this problem and following the example of Slabinski [1997], mounting rings are subdivided into three isothermal elements corresponding to the upper ring, the lower ring and the group of three posts (compare Figure 2).

[8] With the assumption of isothermal elements, the temporal change of total heat content Q of element i (its outer surface consisting of M elements) can be written as [after Slabinski, 1997]

$$\dot{Q}_i = \mathcal{H}_i \frac{\partial T_i}{\partial t} = \sum_{j=1}^M (\mathcal{P}_j - \epsilon_j \sigma \mathcal{A}_j T_i^4) + \frac{J_i - E_{b_i}}{\frac{1-\epsilon_i}{\epsilon_i \mathcal{A}_i}} + \mathcal{R}_{i,al} \sigma (T_{al}^4 - T_i^4) + \sum_{j=1}^L \mathcal{C}_{i,j} (T_j - T_i) \quad (2)$$

where $\mathcal{H}_i$ is the heat capacity, $\mathcal{P}_j$ is the summed absorbed power from all external heat sources (i.e., Earth and Sun),

$\mathcal{A}_j$ is the subarea j of element i radiating toward the external source, J_i is the radiosity of node i , E_{b_i} is the black-body radiation emitted by element i , $\mathcal{A}_i$ is a radiating area within an enclosure (like the CCR cavity), $\mathcal{R}_{i,al}$ is the radiation coupling factor between those surfaces in (almost) contact with the aluminium cavity, T_{al} is the temperature of the relevant shell, and $\mathcal{C}_{i,j}$ is the thermal conductance between elements i and j . Values for the various parameters (areas and respective coefficients) can be obtained upon request from J. I. Andrés.

[9] So the mechanisms affecting $\dot{Q}_i$ are (for each corresponding term in the right-hand side of equation (2)): (1) radiation exchanged with all other elements and/or external sources, (2) the radiating surface resistance due to the difference between radiation from a black-body and the element under consideration, (3) the aforementioned radiation exchange between an element and the shell exclusively, and (4) the conductance between the (L) adjacent elements and element i . Specifically, the thermal model includes 2133 elements: an upper and a lower shell, an inner core, and 426 retroreflectors assemblies with 5 elements each. In doing so, possible azimuthal variations in the temperature distribution are allowed for, contrary to Slabinski [1997]. As in the work by Slabinski [1997], all possible sources of radiation have been considered: solar radiation, infrared radiation from Earth, and Earth's albedo radiation. The Sun has been considered as a point-like source with a solar energy constant at 1 AU of $\Phi_{\odot,1\text{ AU}} = 1370 \text{ W m}^{-2}$, thus the radiation arriving at the satellite can be calculated as $\Phi_{\odot,\text{sat}} = \Phi_{\odot,1\text{ AU}} (1/\tilde{r}_{\odot})^2$, with $\tilde{r}_{\odot}$ the distance to the Sun measured in AU. As for Earth's infrared radiation, this has been considered as being produced by a circular disk with a radius $R_{IR} = 6407.92 \text{ km}$ [Slabinski, 1997], a value slightly larger than the physical Earth's mean radius $R_{\oplus} = 6378.14 \text{ km}$, with an Earth's exitance value of $\Phi_{\oplus} = 232 \text{ W m}^{-2}$ [Jet Propulsion Laboratory, 1995], a bit larger than $\Phi_{\oplus} = 223 \text{ W m}^{-2}$, which was used by Slabinski [1997]. The eclipse function has been evaluated for a perfectly spherical Earth with radius $R_{\oplus}$ and a point-like Sun at infinite distance (i.e., no penumbra has been considered, thus $\nu = 0, 1$). The albedo contribution has been modeled by a constant term for the entire Earth [after Eckert and Drake, 1972]

$$\Phi_{\text{alb}} = \mathcal{C}_{\text{alb}} \Phi_{\odot,\text{sat}} \mathcal{A}_{\% \text{light}} \mathcal{F}_{\oplus-\text{sat}} \quad (3)$$

with the albedo coefficient $\mathcal{C}_{\text{alb}} = 0.31$, and $\mathcal{A}_{\% \text{light}}$ representing the illuminated fraction of the Earth disk as seen by the satellite. The portion of Earth seen by the satellite is analytically calculated for the specific geometry at that step of integration. It can be demonstrated that, provided that the albedo coefficient is constant, the interaction between the spherically approximated Earth and the satellite is equivalent to the interaction between two disks: the first one as the circle over the Earth's surface as seen by the satellite, and the second one as (approximately) the satellite cross section. Therefore the Earth-satellite view factor can be calculated readily, yielding $\mathcal{F}_{\oplus-\text{sat}} = 0.295$, a value close to the one used by Slabinski [1997] ($\mathcal{F}_{\oplus-\text{sat}} = 0.293$), and larger than the one used by Rubincam [1987a] ($\mathcal{F}_{\oplus-\text{sat}} = 0.273$).

[10] Equation (4) gives J_i , expressed as a function of the view factors $\mathcal{F}_{ij}$ between surfaces i and j [Eckert and Drake, 1972]:

$$\frac{E_{b_i} - J_i}{\frac{(1-\epsilon_i)}{\epsilon_i A_i}} = \sum_{j=1}^M \frac{(J_i - J_j)}{\frac{1}{A_i \mathcal{F}_{ij}}} \quad (4)$$

Inspecting equation (2), it is clear that its formulation involves highly nonlinear elements (i.e., fourth power of the temperature). Appendix B gives details on how this aspect is handled.

[11] Supposing a pure Lambertian radiator, the thermal force exerted by the emission of photons exiting the finite surface dA , with the speed of light c , in the direction normal to the surface $\mathbf{n}$, can be written as [Slabinski, 1997]

$$d\mathbf{F} = -\frac{2}{3} \frac{\epsilon \sigma T^4}{c} dA \mathbf{n} \quad (5)$$

Once the numerical model has yielded the temperature of the various elements, equation (5) can be integrated over all outer surface elements to give the net thermal force.

2.2. Spin Axis

[12] As has already been mentioned, one of the most important factors for a correct calculation of the instantaneous thermal thrust is the angular velocity of the satellite: both orientation and magnitude of the angular velocity need to be taken into account. To represent reality as well as possible, LOSSAM (LAGEOS Spin Axis Model [Andrés et al., 2004]) has been used for both satellites. This model is based on the integration of the Euler equation for the evolution of the angular momentum of a solid with a number of external torques as excitation factors. LOSSAM gives the temporal evolution of the spin period and the direction of the spin axis unit vector, with a claimed accuracy of about 1 s and 9° (LAGEOS I) and 0.46 s and 0.6° (LAGEOS II), respectively. This already gives a direction of the computed thermal force closer to reality than previous studies (e.g., Slabinski [1997] spin axis orientation taken after Scharroo et al. [1991]).

[13] A change in the orientation of the axis of rotation causes a change in orientation of the retroreflectors and thus in that of the resulting force. The spin period governs the rotational regime and thus the temperature distribution over the satellite: only latitudinal variations for the rapid rotation regimes, and also azimuthal variations for slow ones.

2.3. Satellite Rotation

[14] The ratio between the spin period τ_s and the other relevant characteristic times, thermal τ_{th} (here understood as an average of the values for the retainer ring and the retroreflectors, the main elements responsible for the thermal forces), computational Δt , and orbital τ_o , conditions the different regimes and thus the assumptions that can be made. In particular, the thermal characteristic time is in the order of one hour [Scharroo et al., 1991], the rotational period of LAGEOS I is about one hour at the end of 2004 [Andrés et al., 2004], and the orbital period is about 225 minutes for both satellites. This causes the different regimes to be governed by two parameters: one physical and one computational.

[15] The first ratio $\chi = \tau_s / \tau_{th}$ governs the rate at which a component of a retroreflector assembly exchanges heat for a typical rotational step. For rapid spin regimes, the assembly rotates fast enough not to suffer an excessive temperature decay due to its passing through the unilluminated part of its rotation. However, as the rotation rate decays, χ increases and so does the temperature decay for a typical rotation angle, therefore creating a so-called diurnal temperature distribution over the satellite surface.

[16] The second ratio $\zeta = \tau_s / \Delta t$ governs the aliasing problem. This can be described as a numerical artefact caused by an apparent (almost) fixed position of a certain retroreflector assembly with respect to the radiation sources for successive temporal steps; this is a sampling problem. The simulation software computes the temperature according to the net radiation that a retroreflector assembly has received during every time step of integration. Therefore, if ζ is such that between successive integration steps, the assembly completes an (almost) integer number of revolutions, the apparent orientation of the assembly does not change between those time steps; in reality, a number of revolutions has been completed, which yields a change in the radiation that the particular assembly has received. If this were not taken into account, an overheating (or underheating, depending on its relative position), and thus an unrealistic temperature, would be produced.

[17] Measures have been taken to make sure that this aliasing problem is properly treated for every rotational regime in the actual simulations. More details on this can be found in Appendix B.

3. Results

[18] Calculations have been done for both satellites, from their respective launch dates until the end of 2004. Results (available upon request from J. I. Andrés) will be given in terms of temperatures of the various elements and instantaneous (net) accelerations, calculated at every step of integration ($\Delta t = 120$ s). In order to show the dependence of the results on the spin axis evolution and the eclipse passages, the Sun aspect angle β (the angle formed by the Sun position vector and the satellite spin axis) and the eclipse function will be depicted together with the results of the simulation. Furthermore, the latter function will be depicted in such a way that it shows time spans (i.e., full orbital periods) in which eclipses occur, rather than their instantaneous value. In order to calculate this function, the position of the satellite has been calculated using the NASA program for orbit computation GEODYN [Pavlis et al., 1998].

[19] First, the nominal temperature behavior of the various elements of the model and the net instantaneous accelerations resulting from this temperature distribution are presented. In addition to this, a sensitivity analysis is performed in order to assess the importance of the spin axis model (uncertainties), and the contribution of the germanium retroreflectors.

3.1. Temperature Behavior

[20] The simulation provides the time history of the temperature of every individual element of the numerical model. As a first illustration, Figure 3 shows the temperature behavior of the main structural elements (aluminium

Figure 3. Temperature behavior of shells, inner core, retainer rings, and CCRs for LAGEOS II, simulated over one particular day (30 November 1992), together with a zoom in for a particular eclipse passage (indicated by a grey band).

shells and inner core) as well as that of the CCR assemblies' components (with their different materials, germanium and silica). Figures 4 and 5 focus on the long-term behavior of the CCR components only.

[21] The temperature history over a period of one day (about 6 orbital revolutions) of a number of representative elements is shown in Figure 3, for LAGEOS II. At this time the Sun was over the upper shell at an angle of 67° with

Figure 4. Temperature behavior of several retainer rings and CCRs for LAGEOS I since launch. Temperatures for retroreflectors 278 and 288 almost overlap for the rapid spin period and become slightly different for the slow regime (end of simulation).

Figure 5. Temperature behavior of several retainer rings and CCRs for LAGEOS II since launch.

respect to the spin axis. One can easily identify how the solar eclipses (present from the third revolution on this day onward) affect the temperature of most components: a sharp drop followed by a (more) gradual increase when in sunlight again. Clearly, the temperature of the upper shell is about 4 K higher than that of the lower shell (a situation which depends on the orientation of the satellite with respect to the radiation sources). Not surprisingly, the temperature of the inner core remains in between those of the two shells at all times. Figure 3 (bottom) also shows the detailed thermal behavior of several components of CCR assemblies, chiefly RR and CCR temperatures, for different retroreflector groups in the upper shell (1 and 89, Si CCR) and in the lower shell (317, Ge CCR). First of all, it turns out that these components (in particular the CCRs) can be warmer or colder than the hemispheres. Also, they show the typical eclipse behavior. Considering that the thermal characteristic time is given by [Scharroo *et al.*, 1991]

$$\tau_{th} = \frac{m C_p}{4\epsilon A \sigma T_0^3} \quad (6)$$

where C_p is the specific heat content, both absolute temperatures and the variation during eclipse are related to (1) the different materials (aluminium, germanium, silica and beryllium-copper), (2) the different average temperatures T_0 , and (3) the different heat capacities $\mathcal{H} = m C_p$: note the amplitude of the temperature decay during eclipse

passages and the increase of this decay as the mean temperature decreases for a particular element, e.g., RR 89. Furthermore, this decay shows the well-known exponential character (a first-order solution of the black-body equation) as shown in previous theoretical studies [e.g., Afonso *et al.*, 1989]. In agreement with this first-order character, the heating up after an eclipse shows the same behavior, modulated by the orientation of the retroreflector (see humps for the hours before the start of the eclipse periods). Similar results (not depicted here) have been obtained for LAGEOS I, albeit that the temperatures for this satellite are typically some 20 K lower, due to the higher value for the emissivity (compare Table A1). The temperature variation of the shells and the inner core, when entering and leaving the umbra, turns out to be small for such a short period; however, the total evolution when considering a longer time span is significant (reaching decays of 20 K), due to the progressive decrease in temperature as the hemispheres receive solar heating over only a part of each orbit during eclipse season.

[22] It is equally, if not more, interesting to take a look at the lifetime behavior of the satellite components: the temperature history over almost 30 years (LAGEOS I) and 13 years (LAGEOS II). Results are depicted in Figures 4 and 5, respectively, for a number of retainer rings and CCRs. Results for the shells and inner core are not included here, for clarity reasons. Their temperature can be described as an almost constant value with troughs exclusively mod-

Figure 6. Net accelerations caused by surface temperature differences and spin axis-Sun geometry angles, for LAGEOS I and II, respectively.

ulated by the eclipse passages, yielding typical temperature values of 315 ± 15 K and 325 ± 10 K for LAGEOS I and II, respectively. The depth of these troughs is directly correlated with the length of the eclipse passages. As for the temporal behavior of a typical (silica) retroreflector and retainer ring, this shows a periodicity which is not clearly associated with the eclipse passages (see Figures 4 and 5). Furthermore, for both satellites, the behavior of the upper shell elements clearly follows an inverted trend with respect to those in the lower one. In fact, a comparison with the Sun aspect angle and its periodicities, depicted in Figure 6, shows a strong correlation between these events. This correlation is clearly evident when analyzing retroreflector L-I reflector 1 (see Figure 4): whenever the CCR is illuminated ($\beta_{\text{Sun-SA}} < 90^\circ$), the high germanium absorptance causes the temperature of the retroreflector to increase significantly, and to decrease proportionally when $\beta_{\text{Sun-SA}} > 90^\circ$. A similar situation happens for retroreflector L-II reflector 317, with a lower amplitude due to the colatitude angle of this retroreflector with respect to the spin axis ($\theta_0 = 121.231^\circ$). Furthermore, the amplitudes of the temperature variations of the elements are dependent on the temporal evolution of the spin axis behavior: compare Figures 4 (bottom) and 5 (bottom) with their respective Sun aspect angle plot, and see how these amplitudes increase with time (e.g., for retainer ring and retroreflector 426, LAGEOS I). It is also obvious that the temperature variations depend on the retroreflector material:

the amplitudes for germanium retroreflectors are twice as large as those of their silica counterparts, and differences may reach up to 100 K. This is clearly due to the different optical properties of germanium with respect to silica (the absorptance is 4 times larger, whereas the emissivity is 9 times smaller, compare Table A1). This also causes the temperature of a typical germanium retroreflector to be much higher, by almost 100 K, for a given hemisphere (e.g., L-I reflector 288 in Figure 4).

[23] In addition to these variations caused by the (gradually changing) geometry, the temperature results are modulated by the eclipse passages, which cool down every element and hence increase the variability of the signal (the amplitude of the high-frequency variations) when entering the eclipse zones. An exception to this is the L-I reflector 1 germanium retroreflector, which again, due to its particular position on top of the satellite and the larger thermal inertia of the construction material, undergoes much larger eclipse modulations (compare Figure 4).

[24] The latitudinal temperature differences for each satellite can be compared in Figures 4 (top) and 5 (top). The average temperatures for the retainer rings in the upper and lower shell show typical differences of about 10 K and 20 K (for rows ± 4 for LAGEOS I and II, respectively). Polar retainer rings show similar differences. Computations done for the case of all retroreflectors being made of silica show average temperature differences of the polar retroreflectors

Table 1. Thermal Characteristic Times at the Beginning of Eclipse Periods for Different Components and/or Materials^a

Parameter	Elements			
	RR	CCR _{SiO₂}	CCR _{Ge}	Shell
<i>Lageos I</i>				
(ϵA) [cm ²]	1.14	11.34	1.026	798.16
T_0 [K]	[280,340]	[260,300]	[200,500]	[315,322]
τ_{th} [min]	[21,38]	[60,94]	[151,237]	[2660,2842]
<i>Lageos II</i>				
(ϵA) [cm ²]	1.14	11.34	1.026	199.54
T_0 [K]	[300,350]	[280,310]	[375,475]	[329,338]
τ_{th} [min]	[19,31]	[54,74]	[176,359]	[9202,9978]

^aMinimum and maximum temperatures, from the entire calculation time span, have been used for the calculations, thus providing a range of time values.

of about 10 K and 25 K for LAGEOS I and II, values more in agreement with the calculations done by *Afonso et al.* [1989] (21 K) than those from *Scharroo et al.* [1991] (55 K), and in reasonable agreement with those computed by *Slabinski* [1997], who reported a value of 36 K. It should be emphasized that the results from previous authors were calculated for a spin axis pointing toward the Sun ($\beta = 0$ degrees), which, under the assumption that LOSSAM represents the true spin axis orientation, is clearly never the case for any of the satellites (see Figure 6). As for the temperature differences between hemispheres, computations yield typical differences of about 4 K (see Figure 3), in clear agreement with the estimates from *Weiffenbach* [1973] (3 K) and the results given by *Slabinski* [1997] (5 K), and a factor of 2 smaller than the estimates from *Bendix* [1974] (8 K).

[25] With the values for mean, maximum, and minimum temperatures when going through the eclipse periods, it is possible to provide values for the thermal characteristic times for every component and/or material (e.g., Figure 3). Table 1 contains the aforementioned values calculated for the most important components, for different (temperature) conditions (compare equation (6)). Depending on the temperature variation, these parameters can show a significant variation in magnitude. The values reported in the literature cover the silica retroreflectors only for estimated temperature values, e.g., $\tau_{th} = 3270$ s (54.5 min) for $T_0 = 316$ K [*Afonso et al.*, 1989], $\tau_{th} = 3121$ s (52.0 min) for $T_0 = 263$ K [*Scharroo et al.*, 1991]. One of the interesting aspects of the current investigation is that we now have realistic estimates for this parameter for all LAGEOS components, based on a numerical evaluation rather than an analytical one.

3.2. Accelerations

[26] So far, this paper has dealt with the temperature behavior of the satellite components. The main objective of the study, however, is to provide a mechanism to better understand and describe the accelerations that the spacecraft exert, in order to improve the quality of the orbit determination (results) and that of derived products such as geocenter and gravity field coefficients. To that aim, the temperature effects of the various surface elements are integrated (compare equation (5)) and the resulting net accelerations will be discussed here.

[27] Figure 6 depicts the values of the three components of the instantaneous net thermal acceleration given in the in-

orbit reference frame for both satellites. As mentioned previously, the Sun aspect angle β is also depicted for both satellites to show the spin axis dependency of the results.

[28] The order of magnitude of the total acceleration is up to 70 pm s^{-2} for LAGEOS I and II, which agrees with Figure 8 of *Slabinski* [1997], with a typical value of $-3 \times 10^{-8} \text{ N}$, or 73 pm s^{-2} . The periodicity reported for the temperatures is reflected in the acceleration pattern, with acceleration values going to zero when $\beta \approx 90^\circ$, due to the equal heating of both hemispheres (rapid spin case). As the spin slows down, the last statement is not fulfilled anymore, e.g., see year 2002 for LAGEOS I and the associated accelerations. The calculated radial and along-track accelerations show an apparent symmetry along the abscissae axis, which is easily understood considering the 200+ min orbital period and the regularly alternating orientation of the in-orbit reference frame (a pattern more clear for LAGEOS II) and an amplitude of the cross-track component half of the others (or equal in the case of LAGEOS II).

[29] The spin axis dependence is also shown in the above mentioned figures: the acceleration patterns change gradually with a change in the spin axis direction (and its associated decrease of the rotational period), which causes a progressive increase of the spin axis wobbling and therefore increasing variations in the Sun aspect angle. These variations cause an increase in the amplitude of the accelerations approximately 13 years after launch for LAGEOS I and 8 years for LAGEOS II. The above mentioned differences can be perfectly seen in Figure 7, which depicts a first-period situation for LAGEOS II and a second-period one for LAGEOS I with significant deviations caused not only by the length of the eclipse passage but also by the spin axis behavior (e.g., wrinkles in the cross-track component for LAGEOS I). Clearly, in all cases, when a satellite enters an eclipse, the temperature differences become smaller and the net acceleration modulus decreases.

3.3. Sensitivity: Spin Axis

[30] To better illustrate the importance of a proper spin axis modeling (or to assess the sensitivity of the acceleration results, as a minimum), the differences of considering the orientation of the satellite as given by LOSSAM as opposed to a fixed orientation for both satellite versions are shown in Figure 8. The orientation of the spin axis becomes a critical factor for the obtained solution, understood as the moment in which the differences have the same amplitude as the calculated accelerations, approximately 13 years after launch for LAGEOS I and 8 years for LAGEOS II, in agreement with the aforementioned intervals. As a matter of fact, the deviation of the LOSSAM solution with respect to the constant spin axis orientation (as included in Figures 4 and 7 of *Andrés et al.* [2004]) is small for the periods mentioned above, which in turn causes small Sun aspect angle differences (not plotted here) and therefore an agreement in terms of temperatures and accelerations.

[31] The theoretized dependence of the along-track component on the Sun aspect angle (due to the temperature dependence of the rotation body on this angle [e.g., *Rubincam*, 1987a; *Afonso et al.*, 1989]), can be seen in Figure 6 for LAGEOS I. As for LAGEOS II, a clear Sun aspect angle correlation is more evident for the cross-track component rather than for the along-track one.

Figure 7. Thermal acceleration simulated over one particular day (1 January 2000 and 30 November 1992 for LAGEOS I and II respectively), together with a zoom in for a particular eclipse passage (indicated by grey band).

[32] Table 2 provides the sensitivity of the accelerations, for a constant spin axis orientation (as taken in previous studies) with respect to LOSSAM. For obvious reasons a distinction has been made between a rapid spin regime and a slow spin

regime. The maximum amplitude of the differences for these periods is $[25,75] \text{ pm s}^{-2}$ for LAGEOS I and $[15,75] \text{ pm s}^{-2}$ for LAGEOS II, with RMS values of $[9.8,27.1] \text{ pm s}^{-2}$ and $[5.5,22.4] \text{ pm s}^{-2}$ for LAGEOS I and II, respectively.

Figure 8. Differences in net thermal accelerations obtained with the LOSSAM spin axis solution versus a constant orientation for LAGEOS I and II, respectively.

Table 2. Differences of the Net Acceleration With Respect to a Nominal Model^a

Case	LAGEOS I				LAGEOS II			
	First Period		Second Period		First Period		Second Period	
	Max	RMS	Max	RMS	Max	RMS	Max	RMS
422 SiO ₂ , 4 Ge constant SA orientation	25	9.828	75	27.086	15	5.462	75	22.361
426 SiO ₂ constant SA orientation	25	9.760	75	26.997	10	6.353	75	22.133
426 SiO ₂ , LOSSAM	0.5	0.246	2.0	0.315	2.0	1.839	4.0	1.651

^aLOSSAM spin axis (SA) evolution and (four) germanium retroreflectors. All values are in $\mu\text{m s}^{-2}$.

3.4. Sensitivity: Germanium CCRs

[33] An interesting bonus of the current simulation is the possibility to quantify the contribution of the germanium CCRs to the acceleration. As has been mentioned in the Introduction, these elements could be held responsible for the occurrence of net forces along the spin axis, or explain (together with a more realistic spin axis behavior) suggested differences in reflectivities between the LAGEOS hemispheres. To illustrate the relevance of the contribution of the germanium retroreflectors, Figure 9 shows the temperature distribution for all surface elements of LAGEOS I and II on 1 January 2002, a noneclipse period, taken as an arbitrary example. Clearly visible are the homogeneous temperature values of both LAGEOS I shells due to $\beta \approx 90^\circ$, whereas a pronounced difference is present for LAGEOS II for which $\beta \approx 45^\circ$, and the relatively hot and cold temperature of other surface elements (CCRs, RRs). In particular, notice the “off” temperatures of the germanium CCRs.

[34] An analysis similar to the one done for the spin axis sensitivity in section 3.3 yields that the germanium CCRs contribute significantly to the total thermal force (compare

Figure 10) and that this contribution exists always, diminishing for $\beta \approx 90^\circ$, however (e.g., compare zeroes in Figures 6 and 10 for the first 5 years for LAGEOS II). This is due to two main asymmetries: (1) the geometry configuration is such that not all germanium CCRs are illuminated at the same time during a time step, confirming the claim by *Lucchesi* [2003], and (2) the germanium CCRs distribution is not symmetric for LAGEOS I. The amplitude of the differences is about $1 \mu\text{m s}^{-2}$ for LAGEOS I and about $2 \mu\text{m s}^{-2}$ for LAGEOS II (using LOSSAM), larger for the second version due to the aforementioned different geometry distribution (two CCRs per hemisphere) and the larger temperatures attained for (almost) constant temperature periods (compare CCRs reflectors 332 and 288 in Figures 4 and 5 for their stable periods). This could be in consonance with the computed value of the (effective) difference in reflectivity, found to be larger for LAGEOS II than for LAGEOS I.

[35] As mentioned in section 3.3, Table 2 provides the sensitivity of the accelerations, understood as the results obtained with the 4 germanium retroreflectors modeled as such, minus the results for the case in which all retrore-

Figure 9. Temperature distribution on 1 January 2002 (in kelvin). The colatitude of the Sun (β) at that epoch was about 90° and 45° , for LAGEOS I and II, left and right, respectively. The high temperature of the germanium retroreflectors is clearly visible for both satellites.

Figure 10. Differences in net thermal accelerations obtained with the LOSSAM spin axis solution, for the case in which Ge CCRs are included versus the case in which all CCRs are made of silica for LAGEOS I and II, respectively.

flectors have been assumed to be made from silica, for a constant spin axis orientation (as taken in previous studies) and for LOSSAM, respectively. The maximum amplitude of the differences for these periods is $[25,75] \text{ pm s}^{-2}$ and $[10,75] \text{ pm s}^{-2}$ for a constant spin axis orientation, and $[0.5,2] \text{ pm s}^{-2}$ and $[2,4] \text{ pm s}^{-2}$ when using LOSSAM, for LAGEOS I and II, respectively. This corresponds to RMS values of $[9.8,27.0] \text{ pm s}^{-2}$ and $[6.4,22.1] \text{ pm s}^{-2}$ for a constant spin axis orientation, and $[0.2,0.3] \text{ pm s}^{-2}$ and $[1.8,1.7] \text{ pm s}^{-2}$ when using LOSSAM, for LAGEOS I and II, respectively. Clearly, the contribution of the germanium retroreflectors is of importance at pm s^{-2} level, but the proper modeling of the spin axis behavior is by far the most important element in these calculations. Moreover, the germanium issue itself does not give a full answer to the difference in reflectivity (its maximum amplitude represents 10% and a 25% of it only, see section 1) in agreement with *Vokrouhlický and Métris* [2004], but the aforementioned results show that the combination of a wrong spin axis model and the mismodeling of germanium creates amplitudes large enough to respond to this effect.

4. Conclusions and Recommendations

[36] As a result of the numerical modeling of the LAGEOS spacecraft, we have been able to model the

lifetime temperature distribution within the satellite using a valid linearized equation for expressing their thermal evolution. The results show (1) temperature variations of up to 200 K because of lighting conditions (Sun aspect angle); (2) differences of about 100 K in average CCR temperatures as a consequence of construction materials; (3) effects of up to 20 K due to eclipse passages; (4) steady temperatures of the shells in daylight conditions; (5) a major contribution from the spin axis modeling; and (6) a net contribution from the germanium CCRs.

[37] From the temperature distribution, the net thermal instantaneous accelerations have been obtained, with values of tens of pm s^{-2} , which agrees very well with solutions previously obtained by other authors [e.g., *Slabinski*, 1997]. The spin axis model proved to be most important in the simulation: differences of the nominal model (i.e., using LOSSAM) with respect to a constant spin axis model yield variations of up to 70 pm s^{-2} (a variation of 100% with respect to the simulated effect), equal to the nominal instantaneous acceleration. Clearly, and related to the longer in-orbit lifetime of the satellite, LAGEOS I is more sensitive on this than LAGEOS II.

[38] Two different periods can be distinguished: a minor spin axis wobbling (thus minor Sun aspect angle variations, 15 and 30 degrees for LAGEOS I and II, respectively)

Figure A1. Structure of the LAGEOS satellites [after Cohen and Smith, 1985].

which approximately lasts 13 and 8 years (1976–1989 and 1992–2000) for LAGEOS I and II, respectively, and an increase of the aforementioned wobbling afterward, with increasing Sun aspect angles for both satellites and, consequently, larger acceleration variations.

[39] As for the physical explanation of the postulated difference in reflectivity, it has been shown that the germanium CCRs contribute with a net effect in the order of several pm s^{-2} , up to 25% of the values given in literature, which agrees with previous studies [Vokrouhlický and Métis, 2004]. In addition to providing answers to a number of questions, the current study also triggers new issues which can be dealt with in follow-up investigations. One of them is the modeling of Earth's radiation: it is expected that some variations arise when considering the most simple latitudinal variation, i.e., the $\sin 2\phi$ dependence or a more detailed modeling [e.g., Martin and Rubincam, 1996] instead of a constant albedo model. A second issue is related to the possible explanation of the difference in reflectivity by means of the combined effect of a more realistic spin axis model and the consideration of the germanium CCRs. In order to assess this, the same modeling effort should be applied to the charged particle drag. A most important recommendation is to take a thorough look at the present-day rotational behavior of LAGEOS I in particular. Although LOSSAM gives the best possible description for this, it is a prediction (the last observations for LAGEOS I date 1997), and its modeling has major consequences for the net acceleration. Every effort should be undertaken to (independently) ascertain the correctness of the current spin axis model. New kHz SLR measurements could help in this matter.

[40] Irrespective of this, the time series of accelerations currently obtained yield material that will be applied in precise orbit determination. It is expected that this will lead to improvements in the latter, in direct orbit solutions and/or in the quality of derived properties. Finally, being able to model the thermal accelerations in such detail now creates new options to readdress questions on other accelerations experienced by the pair of LAGEOS satellites, such as charged particle drag, electromagnetic forces and such. Effectively, the range of uncertainty for such phenomena

has been reduced, possibly opening options for new insights for the latter phenomena. These will be the subject of a follow-up publication.

Appendix A: Structure

[41] The two LAGEOS satellites are basically composed of two aluminium hemispheres, a massive internal cylindrical core, a retaining stud (the latter two fabricated from beryllium copper [Slabinski, 1997]), and 426 retroreflectors (Figure A1). The symmetry axis of the internal elements coincides with the main axis of symmetry of the entire structure as to favor this as the axis with the largest moment of inertia. Hereinafter, core and stud will be considered as one element: the inner core (IC). As for the CCRs, these are “regularly” distributed over its surface in order to reflect light in the direction of its source. From these 426 retroreflectors, 422 are fabricated from fused silica, a variety of vitreous silica with a high degree of transparency (National Scientific Company, Clear fused quartz tubing and rod, available at <http://www.quartz.com>). The remaining 4 have been fabricated from germanium (Ge), and are located in different positions for every satellite; compare Figure 1. Every (fragile) retroreflector is held in its proper position and orientation by means of three tabs which lie within the so-called ring assembly. This consists of five elements (Figure 2): a lower mounting ring (LR), three ring posts (RPs), an upper mounting ring (UR), a retainer ring (RR), and three screws. Both the retainer ring and the screws are made of aluminium, while the mounting rings and ring posts are machined from clear transparent PolyChloroTriFluoro-Ethylene [Slabinski, 1997] (PCTFE), commonly known by its trademark KEL-F. The mounting rings main function is to fix the retroreflector into its nominal position without harming it (hence the use of the plastic material). The

Table A1. Relevant Material Properties Adopted for the Thermal Model

Property ^a	CCR Assembly						Main Structure	
	CCR						Hemisphere	IC
	RR ^b	UR ^b	Ge ^{c,d}	SiO ₂ ^b	RP ^b	LR ^b		
<i>Lageos I</i>								
α_s	0.42		0.52	0.15			0.42 ^d	
ϵ	0.20	0.92	0.09	0.81	0.92	0.92	0.20 ^b	
C_p	896 ^c	900	320	753	900	900	896 ^c	418 ^c
$\mathcal{H}$	1.3	1.96	26.4	25.3	0.8	3.67	96488.0	73542.0
<i>Lageos II</i>								
α_s	0.39		0.52	0.15			0.39 ^d	
ϵ	0.05	0.92	0.09	0.81	0.92	0.92	0.05 ^b	
C_p	896 ^c	900	320	753	900	900	896 ^c	418 ^c
$\mathcal{H}$	1.27	1.99	26.4	25.0	0.8	3.7	96488.0	73542.0

^aProperties are absorptance α_s [dimensionless], emissivity ϵ [dimensionless], specific heat C_p [$\text{J kg}^{-1} \text{K}^{-1}$], and element heat capacity $\mathcal{H}$ [J K^{-1}] for the different elements. The heat capacity values are calculated from the mass values from Slabinski [1997].

^bSlabinski [1997].

^cMatWeb (2005, online material data sheet, available at <http://www.matweb.com>).

^dElectro Optical Industries, Inc. (2005, Material emissivity properties, available at http://www.electro-optical.com/bb_rad/emissivity/matlemisvity.htm).

screws fix the retainer ring to the aluminium shell going through each mounting ring post and into the shell.

[42] To describe the thermal behavior of the individual elements correctly, the optical and geometrical properties of every material need to be known. Table A1 contains an overview of the properties of the different materials. A sensitivity analysis concerning (the effects of) conflicting values found in the literature (an exhaustive description of this issue is presented by *Slabinski* [1997]) has been done, resulting in typical differences of less than 1 pm s^{-2} with respect to nominal results. As for the geometrical properties, values have been taken from *Slabinski* [1997], which contains a thorough description of every element of the spacecraft (not reproduced here).

Appendix B: Thermal Approach

[43] The intricate geometry of the satellites, the number of elements forming it, and the various mechanisms for heat transfer between them, make the numerical approach the only possible way to obtain an accurate description of the LAGEOS thermal behavior. Thus it is required to take into account the different types of retroreflectors (fused silica versus germanium), different elements in every reflector assembly, radiation budgets based on realistic contributions of the celestial bodies at every step of integration, and above all, the orientation and rotation rate of the satellite varying with time.

[44] The present study assumes a linearization of the highly nonlinear heat equation. This is solved by means of an iterative procedure to compute the temperature of each element at each time step. Provided that the ratio between the increment of temperature at integration step N to the temperature at the same integration step is small, i.e., $(\Delta T / T)_{t_N} \ll 1$, it is possible to develop $T(t_N)$, the temperature of node i at temporal step N , in a Taylor's series as a function of $T(t_{N-1})$, the temperature at the previous integration step, and its derivatives (subindex i corresponding to the node considered is not written for ease of reading):

$$T^4(t_N) = T^4(t_{N-1} + dt) = T^4(t_{N-1}) + \left. \frac{\partial T^4}{\partial t} \right|_{(t_{N-1})} dt + \mathcal{O}((dt)^2) \quad (\text{B1})$$

where $\left. \frac{\partial T}{\partial t} \right|_{(t_{N-1})} dt$ can be approximated as ΔT_N , the increment in temperature at temporal step N , or

$$T_N^4 \simeq T_{N-1}^4 + 4T_{N-1}^3 \Delta T_N \quad (\text{B2})$$

As mentioned, the value for ΔT_N is obtained from an iterative process where convergence is verified. The numerical method has been validated by comparing results obtained for different time steps ($\Delta t = 60 \text{ s}$ and $\Delta t = 120 \text{ s}$). The intrinsic initial value problem associated with the heat equation is overcome by taking a temperature of 300 K for the elements for the first epoch, for an initial (idealized) circular orbit and that particular epoch's geometry, until (after a number of orbital revolutions) convergence is achieved. Afterward, computations follow the normal process with these initial values. All numerical values used in the simulation are provided as additional material to the electronic version of this study.

Figure B1. Geometry definition of the aliasing problem for a given retroreflector at time t_{i-1} and colatitude θ .

[45] On the other hand, as mentioned in section 2.3, various aliasing problems may play a role depending on the rotation regime. The averaging over an integration step size (i.e., 120 s) can be done with respect to the temperatures of the outfacing LAGEOS elements or with respect to the instantaneous energy influx Φ . The first approach requires a more complex method, with a detriment of the simulation speed, and it proves to be more sensitive to the relative position of the celestial bodies. The second approach requires a number of considerations: (1) the parabolic character of the heat equation and (2) the rotational regime. The possible integer number of revolutions between successive temporal steps and the amount of radiation received are to be taken into account in a single formulation.

[46] Mathematically, consider a reference system with its Z axis directed along the spin axis, its X axis pointing toward the Sun projection on the XY plane and the Y axis completing a right-handed system (see Figure B1). The normal influx incident on an element having a normal $\mathbf{n}$ with a colatitude θ (here considered as constant between successive integration steps) and a longitude λ can be written as (a similar formulation is valid for Earth radiation, IR + albedo, albeit with different parameters):

$$\Phi_{n_\odot} = \Pi_\odot(\mathbf{s} \cdot \mathbf{n}) = \Pi_\odot(\theta; t)(\sin \theta \cos \lambda(t) \sin \theta_\odot + \cos \theta \cos \theta_\odot) \quad (\text{B3})$$

with $\mathbf{s}$ the unit vector in the direction of the Sun and $\Pi_\odot(\theta; t)$ the solar influx. The longitude angle between successive temporal steps can be approximated by $\lambda_i = \lambda_{i-1} + \omega_{s_{i-1}} \Delta t$. $\Pi_\odot(\theta; t)$ is a temporal function as well, due to the shadowing effect of the satellite rotation, which can be written as

$$\Pi_\odot(\theta; t) = \Phi_\odot[H(t - t_{i-1}) - H(t - t_{in}) + H(t - t_{out}) - H(t - t_i)] \quad (\text{B4})$$

Figure B2. Typical shape of the influx function $\Pi_{\odot}(\theta; t)$ for a given colatitude. The “ 2π ” on top represents one complete revolution of the vehicle around its spin axis.

for less than one revolution. Here, H represents the Heaviside function, $\Phi_{\odot}$ is the (constant) influx coming from the Sun at that temporal step and that particular distance from the Sun. Considering the general case depicted in Figure B1, the times at which the retroreflector assembly under consideration enters and exits the shadow can be calculated from the spherical triangle ABC , yielding

$$t_{in} = \frac{1}{\omega_{s_{i-1}}} \left[-\lambda_{i-1} + \frac{\pi}{2} + \xi_{\odot} \right]$$

and

$$t_{out} = \frac{1}{\omega_{s_{i-1}}} \left[-\lambda_{i-1} + \frac{3\pi}{2} - \xi_{\odot} \right]$$

where the angle $\xi_{\odot}$ can be obtained from the above mentioned spherical triangle yielding $\sin \xi_{\odot} = \cot \theta \cot \theta_{\odot}$. The typical form of the function $\Pi_{\odot}$ is shown in Figure B2. Generalizing for one or more complete revolutions and considering the periodicity of the function $\Pi_{\odot}$, the average influx over the integration interval $\bar{\Phi}_{n_{\odot}}$ can be expressed as

$$\begin{aligned} \bar{\Phi}_{n_{\odot}} = \frac{\Phi_{\odot}}{\Delta t} & \left\{ \int_{t_{i-1}}^{t_{in}} (\sin \theta \cos \lambda \sin \theta_{\odot} + \cos \theta \cos \theta_{\odot}) dt \right. \\ & + \Lambda \int_{t_{out}}^{t_{in}} (\sin \theta \cos \lambda \sin \theta_{\odot} + \cos \theta \cos \theta_{\odot}) dt \\ & \left. + \int_{t_{out}}^{t_i} (\sin \theta \cos \lambda \sin \theta_{\odot} + \cos \theta \cos \theta_{\odot}) dt \right\} \quad (B5) \end{aligned}$$

where Λ is the integer part of the quotient $(\omega_s \Delta t / 2\pi) = (\Delta t / \tau_s) = \zeta^{-1}$. Some other particular geometrical cases are not discussed here, although they have been taken into account in the numerical simulation.

[47] It must be remarked here that the above mentioned formulation is valid for every rotational regime due to the averaging of the incoming influx with respect to time (rotation in particular).

[48] **Acknowledgments.** The authors would like to thank D. Lucchesi and J. R. Sanmartín, for their helpful discussions and continuous encouragement, and the International Laser Ranging Service (ILRS) for providing (part of) the information that was essential for this study. Also the authors want to express their sincere gratitude to the reviewers V. Slabinski and

E. C. Pavlis for their extremely useful and critical comments and suggestions.

References

- Afonso, G., F. Barlier, M. Carpino, P. Farinella, F. Mignard, A. Milani, and A. M. Nobili (1989), Orbital effects of LAGEOS seasons and eclipses, *Ann. Geophys.*, 7(5), 501–514.
- Altamimi, Z., P. Sillard, and C. Boucher (2002), A new release of the International Terrestrial Reference Frame for earth science applications, *J. Geophys. Res.*, 107(B10), 2214, doi:10.1029/2001JB000561.
- Andrés, J. I., G. Bianco, D. Currie, R. Noomen, and T. Otsubo (2004), The spin axis behavior of the LAGEOS satellites, *J. Geophys. Res.*, 109, B06403, doi:10.1029/2003JB002692.
- Anselmo, L., P. Farinella, F. Mignard, A. Milani, and A. M. Nobili (1983), Effects of the Earth reflected sunlight on the orbit of the LAGEOS satellite, *Astron. Astrophys.*, 117, 3–8.
- Barlier, F., M. Carpino, P. Farinella, F. Mignard, A. Milani, and A. Nobili (1986), Non-gravitational perturbations on the semimajor axis of LAGEOS, *Ann. Geophys., Ser. A*, 4, 193–210.
- Bendix, (1974), Laser Geodynamics Satellite thermal/optical/vibrational analyses and testing, final report, vol. II, *Tech. Rep. DR MA-04*, Bendix Aerospace Syst. Div., Ann Arbor, Mich.
- Cohen, S. C., and D. E. Smith (1985), LAGEOS scientific results: Introduction, *J. Geophys. Res.*, 90(B11), 9217–9220.
- Eckert, E. R. G., and R. M. J. Drake (1972), *Analysis of Heat and Mass Transfer*, McGraw-Hill, New York.
- Farinella, P., D. Vokrouhlický, and F. Barlier (1996), The rotation of LAGEOS and its long-term semimajor axis decay, *J. Geophys. Res.*, 101(B8), 17,861–17,892.
- Jet Propulsion Laboratory (1995), Thermal environments, JPL D-8160, Pasadena, Calif.
- Lucchesi, D. M. (2003), The asymmetric reflectivity effect on the LAGEOS satellites and the germanium retroreflectors, *Geophys. Res. Lett.*, 30(18), 1957, doi:10.1029/2003GL017741.
- Lucchesi, D. M. (2004), LAGEOS satellites germanium cube corner reflectors and the asymmetric reflectivity effect, *Celestial Mech. Dyn. Astron.*, 88(3), 269–291.
- Lucchesi, D. M., I. Ciufolini, J. I. Andrés, E. C. Pavlis, R. Peron, R. Noomen, and D. G. Currie (2004), LAGEOS-II perigee rate and eccentricity vector excitations residuals and the Yarkovskiy-Schach effect, *Planet. Space Sci.*, 52(8), 699–710.
- Martin, C. F., and D. P. Rubincam (1996), Effects of Earth albedo on the LAGEOS-I satellite, *J. Geophys. Res.*, 101(B2), 3215–3226.
- Pavlis, D. E., et al. (1998), GEODYN-II systems description, NASA Goddard Space Flight Center, Greenbelt, Md.
- Rubincam, D. P. (1982), On the secular decrease in the semimajor axis of LAGEOS’ orbit, *Celestial Mech.*, 26, 361–382.
- Rubincam, D. P. (1987a), LAGEOS orbit decay due to infrared radiation from Earth, *J. Geophys. Res.*, 92(B2), 1287–1294.
- Rubincam, D. P. (1987b), Earth anisotropic reflection and the orbit of LAGEOS, *J. Geophys. Res.*, 92(B11), 11,662–11,668.
- Rubincam, D. P. (1990), Drag on the LAGEOS satellite, *J. Geophys. Res.*, 95(B4), 4881–4886.
- Scharroo, R., K. F. Wakker, B. A. C. Ambrosius, and R. Noomen (1991), On the along-track acceleration of the LAGEOS satellite, *J. Geophys. Res.*, 96(B1), 729–740.
- Slabinski, V. J. (1997), A numerical solution for LAGEOS thermal thrust: The rapid spin case, *Celestial Mech. Dyn. Astron.*, 66, 131–179.
- Smith, D. E., and P. J. Dunn (1980), Long term evolution of the LAGEOS orbit, *Geophys. Res. Lett.*, 7, 437–440.
- Vokrouhlický, D., and G. Métris (2004), LAGEOS asymmetric reflectivity and corner cube reflectors, *J. Geophys. Res.*, 109, B10401, doi:10.1029/2003JB002921.
- Weiffenbach, G. (1973), Use of a passive satellite for Earth-physics applications, *Rep. 301-170*, Smithsonian Astrophys. Obs., Cambridge, Mass.

J. I. Andrés and R. Noomen, Delft Institute for Earth Observation and Space Systems, Delft University of Technology, Kluyverweg 1, NL-2629 HS Delft, Netherlands. (j.i.andres@tudelft.nl; r.noomen@tudelft.nl)

S. Vecellio None, Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Via La Masa 34, I-20156 Milan, Italy. (svn77@libero.it)