Stellingen behorende bij het proefschrift
“First-order oscillators”
van Chris Verhoeven

1. Van eerste-orde oscillatoren kunnen zowel de frequentie als de periodetijd zeer lineair geregeld worden door een spanning of een stroom.
   (Dit proefschrift, hoofdstuk 8)

2. Het belangrijkste voordeel van eerste-orde oscillatoren is het feit dat ze volledig integreerbaar zijn.

3. Regeneratieve schakelingen zijn niet noodzakelijk voor de constructie van een eerste-orde oscillator.
   (Dit proefschrift, hoofdstuk 2)

4. Een eerste-orde oscillator is geschikter om er een tijdbasisgenerator voor een analoge oscilloscoop mee te maken dan een tweede-orde oscillator.
   (Dit proefschrift, hoofdstuk 3)

5. Het uitsluitend streven naar kleinere afmetingen in de microelektronica is niet zinvol.

6. Kwalitatief inzicht is het belangrijkst bij het kiezen van een ontwerpstrategie.

7. Eerste-orde oscillatoren worden al eeuwen toegepast.

8. Onderwijsactiviteiten moeten op een universitaire instelling hoger gewaardeerd worden dan publicaties van onderzoeksresultaten.

9. Wanneer er met behulp van een computer gemeten wordt, dan kan dit het beste een computer met een “UNIX operating system” zijn.

10. De tijd, besteed aan een afgewezen publikatie, zou ook gewaardeerd moeten worden omdat de afwijzing vaak niet met het niveau van de publikatie te maken heeft.

11. Programma’s zonder bugs bestaan niet. Het nemen van een pincode is daarom onverstandig.

12. Fabrikanten van elektronische onderdelen die zich uitsluitend richten op de halfgeleidertechnologie maken een historische fout.

13. Een moderne buizenversterker heeft dezelfde functie als een Ferrari voor de deur.

14. Wie eenmaal met videotechniek begint kan nooit meer van een opname genieten.

15. Het zoeken naar financiële middelen ten behoeve van onderzoek gaat ten koste van de wetenschappelijke vooruitgang.

16. Wanneer we vlak na een 1jstijl zouden leven, dan zou het broeikaseffect veel erger zijn.

17. Babies hebben een grotere mensenkennis dan volwassenen.

First order oscillators
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1. Introduction

This thesis treats the subject of oscillators which may be referred to as first-order oscillators. These are distinguished by the fact that a single time constant determines the timing. So-called "multivibrators" or regenerative oscillators are by far the most popular oscillators in this category, and will be discussed extensively in the text. Attention is, however, also devoted to less obvious first-order oscillators which cannot be classified as regenerative. The title of this thesis has conscientiously been chosen to include these variants.

An important feature of all first order oscillators is the fact that they can easily be fully integrated. Nowadays integrability has become a very important quality factor of electronic circuits. This is why the overall performance—including the price-performance ratio—of a first order oscillator in an electronic system can be better than that of a theoretically better, but not fully integrable oscillator of another class.

In this thesis the behavior of first order oscillators will be modeled such that a designer can easily manipulate the models. Simplicity is therefore preferred to sophistication, since complicated models often no longer provide insight in the behavior of the circuit. The models in this thesis are comprehension models. They are intended to visualize the physical behavior of the oscillators.

1.1 The structure of this thesis

The thesis has been divided into eight chapters. In chapter 2 the definition of a first order oscillator will be given. The first order oscillator is introduced as a system in which several basic functions have to be performed. Different ways of constructing a first order oscillator from the basic functions will be described. One of the basic functions is integration. An integrator can be used to make a time variant signal out of a constant signal, but it cannot make a periodical signal by itself. For this, extra circuits are necessary for performing other basic functions. Another very important basic function in a first order oscillator is the binary memory function. In chapter 2, two different implementations for this function will be discussed: regenerative and non regenerative memories. Regenerative circuits are frequently used as binary memories, but other circuits, like sample and hold circuits, can also be used.

In chapter 3 a description "language" for behavioral models will be introduced. This language has the purpose to support understanding of the fundamental aspects of the operation of first order oscillators. For this reason it makes use of graphical symbols. With the aid of this description language, the oscillators can be grouped into several classes. Each class has its own special properties. With the use of this classification the designer can determine whether a difference in behavior between two electronic implementations is caused by technology or something more fundamental. In the first case an improvement or change of technology could be induced by the designer, resulting in better oscillators. In the latter case this could be a waste of time and money, since the selection of another fundamentally different type of oscillator could produce a better result without changing technology. For example, a new IC-process with low base resistances could be designed to reduce the timing jitter caused by the noise of the resistors, but redesigning an oscillator to be less sensitive for that noise is generally cheaper and faster.

Chapter 4 treats regenerative memories. The Schmitt-trigger is an electronic circuit that fits in this chapter. The loop gain-excitation model is introduced as a comprehension model for
regenerative circuits. The model is also used to generate numerical results, although it was not primarily developed for this purpose. It produces results similar to those of other models, for example those used by SPICE.

The non regenerative memories are described in chapter 5. It is a very short chapter, because a lot of research still has to be done on these circuits. It is however a very essential chapter, since many designers think the only way to implement a binary memory for a first order oscillator, is by using a regenerative circuit, which is not true.

The influence of noise on first order oscillators will then be discussed in chapter 6. Some definitions of noise measures will be given. Many compromises have been made in this chapter between general validity of the mathematics and simplicity, but the net result is proven to be of practical use.

In chapter 7 first order oscillators are described which make use of a regenerative memory. This chapter covers most of the first order oscillators that are used in practice: the regenerative oscillators.

In chapter 8 first order oscillators using non regenerative memories are dealt with. For these oscillators a lot of research is still necessary, but the expectations are high.

In both chapters describing oscillators (7 and 8) electronic implementations will be given. They are electronic circuits that have been integrated and verified experimentally. Bipolar technology has been used because this was easily available. The design theory itself is independent of technology.

The main purpose of this thesis is to provide the designer with an understanding of the behavior of first order oscillators, in such a way that he knows what to do when he wants to change the oscillator properties. The understanding is mainly based on qualitative aspects. When exact numerical predictions of the oscillator behavior are required, also computer simulators such as SPICE can be used. Designers who want to know the effect of every move they make, will find the modeling presented here most beneficial.
2. Fundamentals of first order oscillators

When the properties of a system can be determined on a high hierarchical level, this will ease the design considerations very much. The models used can have a wide range of application, and will not be limited to one specific domain of interpretation. On top of that, more “abstract” simulation programs like PSI can be used, instead of low level simulators like SPICE. In order to understand the basic operation of a system, high level models are very useful, and general design criteria may result. When a model is derived for a certain system, it may very well also be applicable to other systems that “look like it”. Systems that have specific common properties are said to belong to the same class. Especially at high hierarchical levels useful classifications can be made, on the basis of fundamentally different behavioral aspects.

The class of systems of interest in this thesis is that of first order oscillators. Oscillators are systems that are able to generate a periodical signal out of constants.

Thus at the input ports, only constants are available (e.g. power supply), and at the output a periodical signal is generated. The name constant should not be interpreted too strictly. The constants are signals that contain no timing information that can be used in the oscillator to determine the frequency. At the most they can be parameters that influence the frequency, by, for example modulating it.

In practice a number of fundamentally different ways to generate the periodical signal exist, so different classes of oscillators can be distinguished at this level. The classification is based on the order of the oscillator. One class is the class of first order oscillators. Only this class will be dealt with in this thesis.

The name “first order oscillators” has been given to this class because many aspects of the behavior of these oscillators can be described using a one pole approach. This means that in the oscillator one single pole has a dominant influence on the oscillator properties. In this chapter these properties will be discussed. It will be explained why a classification based on the order of the oscillators is useful. A few properties of two other classes of oscillators, the second order oscillators, as described in [7], and the third order oscillators will be discussed briefly in order to indicate the fundamental differences between the classes. Next a basic first order oscillator will be constructed. Starting from this oscillator, practical oscillators will be constructed. Then two (sub)classes of first order oscillators will be defined, the regenerative oscillators and the two-integrator oscillators. The first order two-integrator oscillator and the linear two-integrator oscillator as described in [5] are not the same. The fact that a first order oscillator can contain two integrators may look strange at first sight, but since the behavior can be described using the one pole approach, the oscillator does belong in this class.

2.1 The generation of a time variant signal

An oscillator generates a periodical signal out of constants, so internally the oscillator has to have some time reference. The nature of this time reference can be characterized by way of a pole pattern.

Before describing some classes by way of their pole patterns, first an important fact has to be discussed.

It is always possible to generate a periodical signal with the aid of a non periodical time variant signal with a known time behavior.
For example a ramp with a known slope can be used to generate pulses at equal time intervals, by detecting equidistant levels of the ramp. The pulse train that is generated is periodical, although the original signal supplying the time reference is not. Extra system components will be necessary to perform the measurements, but it is fundamentally possible. So when a system is able to generate a time variant signal out of constants, it can be used to construct an oscillator.

Now the influence of the pole pattern on the generation of this time variant signal will be discussed with the order as a parameter.

2.1.1 First order systems

A first order system contains only one pole. This pole will be situated on the real axis. Suppose it is in the origin. In that case the system is an ideal integrator. When a constant (unequal to zero) is applied at the input of the integrator, the output signal changes linearly with time. A time variant signal with a well known time behavior is created in this way. When the input signal for the integrator is $\alpha$ which is a constant, and $E_o(t)$ is the output signal of the integrator, the following relation exists:

$$\frac{dE_o(t)}{dt} = \alpha$$

(2.1)

This is a very simple equation, which makes it very easy to generate a periodical signal from $E_o(t)$.

From (2.1) it can be seen that first order systems are able to create time variant signals out of constants, so is must be possible to construct oscillators with them.

A practical realization of this part, which creates the time variant signal, can be the combination of a constant current (the constant) that charges a capacitor (the integrator). Very often the current source is replaced by a resistor. This can be modeled as a first order system having its pole in the left half plane.

When the pole of a first order system is not in the origin, the slope of $E_o(t)$ will decrease when it is in the left half plane and increase exponentially when it is in the right half plane (fig. 2.1). The non-linear relation between the level of $E_o(t)$ and time needs not to be a problem for the creation of the periodical signal, but other properties may degrade. For example it may have influence on the linearity when the oscillator is tuned. When the pole is in the left half plane the integrator can be said to have losses. Due to these losses, first order oscillators, may show unstable output frequencies and amplitudes. The oscillation is damped.

Sometimes the pole is shifted along the real axis by some control loop around the oscillator, in order to control an oscillator property like the amplitude or the frequency. This will be dealt with later, when the amplitude stabilization of switching two-integrator oscillators is discussed. Generally the ideal integrator is the best option, and the distance between the pole and the origin can be seen as a figure of merit. The ideal relation between time and the level is taken linear—which is the case for an ideal integrator—and the position of the pole determines the "straightness" of the actual signal and thus the quality of the slope. Later it will be shown that generally the pole is always in the left half plane in practical oscillators, unless special undamping measures are taken. It will be shown too that the quality of the oscillator tends to improve when the pole of the system without an undamping mechanism, comes closer to the origin.

In this system the sign of the integration constant is initially switched by one of the comparators.
2.1.2 Second order oscillators

Second order oscillators have two dominant (complex) poles. These oscillators are discussed in detail in [7]. In the oscillator the poles are forced onto the imaginary axis. When the poles are exactly on this axis, an output signal with a stable amplitude and frequency can be generated. A very nice property of the oscillators belonging to this class is the fact that a periodical signal is generated from nature, so no additional steps are necessary as they are for first order oscillators. In first order oscillators, the element that generates the time variant signal is an integrator. In this case the element involved is a resonator. A resonator is a second order device which will oscillate on a frequency of preference when it is excited. Extra system components may be needed to undamp the resonator in order to obtain two poles on the imaginary axis. The distance between the poles of the resonator and the imaginary axis can be used as a figure of merit. In second order oscillators the quality of the generated periodical signal is directly related to the quality of the resonator.

Fig. 2.2 The pole pattern signal of the resonator

2.1.3 Third order oscillators

Third order oscillators have a pole pattern containing three poles. Two of these poles will be on the imaginary axis during stable oscillation. The third pole will have to be on the real axis. Usually it is in the left half plane. This means that losses will be present fundamentally, since the pole has a real part unequal to zero.
Fig. 2.3 The pole pattern of a third order oscillator

2.1.4 Summary

The pole patterns of all oscillators determine the quality of the time variant signal that is generated. In first order oscillators this signal is not periodical. However it is always possible to generate a periodical signal out of this time variant signal. The extra steps that are involved to make the periodical signal may cause a degradation of the accuracy. The basic accuracy of the first order oscillator is used indirectly.

Second order oscillators and oscillators of higher order generate periodical signals by nature. The basic accuracy is used directly, and no extra step are needed that may degrade the accuracy. Third order oscillators always contain losses, since one of the poles usually has a real part unequal to zero. This will cause inaccuracy (i.e. due to noise).

2.2 The generation of a periodical signal in a first order oscillator

In this section the basic first order oscillator will be built. Although it is not a practical system, it is a very useful system to demonstrate some special properties of the oscillator. First the time variant signal will be generated out of a constant by way of an ideal integrator. This means that the pole is situated in the origin, and relation (2.1) is exact. The constant $\alpha$ which is integrated will be called the integration constant from now on. Since $\alpha$ is a constant, the following relation can be used:

$$E_0(T_i + t_o) - E_0(t_o) = \alpha T_i$$ (2.2)

So at any point of time a time interval $T_i$ can be measured by way of observing the level of $E_0(t)$. Now extra system components will be used to measure time intervals. The first element used will be a memory. The memory stores the value of $E_0(t)$ at $t = t_o$. The memory takes a sample of $E_0(t)$ infinitely fast, and will then retain it until a new sample is to be taken. An adder adds an offset $E_{off}$ to the output signal of the memory $E_m$. The result is used as the reference level for a comparator. The comparator compares $E_0(t)$ with the output signal of the adder, and will generate a pulse when both signals are equal, so when:

$$E_0(t) = E_m + E_{off}$$ (2.3)

and

$$E_{off} = \alpha T_i$$ (2.4)

the pulse will occur a time interval $T_i$ after the intake of the sample.

The pulse can be used to initiate the intake of a new sample into the memory. When $E_{off}$ is a constant, which is not necessary, all time intervals measured will be of equal length. Then the pulse train $E_p(t)$ coming from the comparator is periodical. In fig.2.4 and fig.2.5 the block diagram of a basic first order oscillator and some signal occurring in it have been depicted.
respectively. It can be seen that the original time variant signal is still non-periodical, but the pulse train generated with the aid of this signal has become periodical. The generated signal is periodical because $E_{off}$ and $\alpha$ are constants. However, the system does not demand them to be constants. It has no frequency of preference and will immediately respond when they are changed.

The fact that a periodical signal is generated may be called pure coincidence, since $E_{off}$ happens to be a constant. When it is not, there may be no periodicity.

2.2.1 Tuning the oscillators

Generally some influence on the oscillator frequency is desirable. This influence can be effected by changing a constant which is a parameter of the time varying signal (like $\alpha$), or by changing
a parameter that alters the pole positions directly. On top of that in first order oscillators the offset signal may be manipulated. All manipulations will result in a change in the frequency of the oscillator. Some changes will be more predictable than others. In first order oscillators generally a linear relation between the modulating signal and the frequency or the period is feasible. Since there is no element in the oscillator which has a frequency of preference, the oscillator will be tunable linearly over large frequency ranges, by just changing a constant. In general the tuning of a resonator over a wide range is less easy.

In practice the modulating signals very often are voltages or currents. Generally the offset signal $E_{off}$ is a voltage and the integration constant $\alpha$ is a current. Both parameters determine the frequency of the output signal, and principally no energy transport is involved when one of these quantities is changed. Because of this the frequency of a first order oscillator can be changed very fast, even within one period, which is very fast compared to a second order oscillator. In practice tuning a resonator will require the tuning of a capacitor or an inductance. The energy contents of a resonator also depends on the value of these elements. So when the amplitude of the oscillation is to remain constant, the energy contents of the resonator has to change. This cannot be done infinitely fast if signals are to remain within bounds, so the tuning of the oscillator will be speed limited. Also the relation between the tuning signal and the frequency will be non-linear.

2.2.2 Summary

In order to generate a periodical signal in a first order oscillator two extra functions are needed, memorization and level comparison. These functions can be performed by separate system components, but they can also be combined in one component. The quality of these functions also affects the accuracy of the output signal of the oscillator. The basic accuracy, which is determined by the quality of the slope, is used indirectly. Since frequency changes do not require transport of energy, they can be made very fast.

There is no element in the first order oscillator that has a preference for a specific frequency, as there is in second order oscillators. This generally implies large tuning ranges for first order oscillators.

2.3 The realization of practical first order oscillators.

In practice the time variant signal of the basic first order oscillator will cause problems. Theoretically this signal, which is a ramp, will grow without bounds. In practice it will finally be limited by physical constraints. (f.i. available supply voltage) When limiting occurs, the time dependency disappears, the signal becomes constant, and the generation of a periodical signal will be impossible.

The only way to prevent the time variant signal from being limited is changing the integration constant. Because of this the integration constant becomes time dependent! A very convenient way to do this, is changing just the sign of the integration constant. This method is nearly always used in practice. In this way the quality of the time variant signal remains unaffected, the absolute straightness remains unchanged. Of course the sign can be changed at arbitrary moments, for example when limiting is bound to happen, but this will complicate the rest of the system very much. The system that measures the level and generates the pulse train has to take the sign changes into account, because they may occur within the time interval to be measured. Then the offset signal $E_{off}$ has to be adapted. This requires extra system components. Especially when more than one sign change occurs within the interval, extra
memories etc. will be needed. On the other hand it is very simple to change the sign of the integration constant every time a pulse occurs at the output of the comparator. Now it is certain no sign changes will occur within the time interval to be measured.

When the output pulse train is periodical, the time variant signal will become periodical too. Now only two reference levels exist with a difference equal to the offset signal \( E_{off} \). Only when \( E_o(t) \) crosses one of these reference levels, the sign of the integration constant \( \alpha \) is changed (fig.2.6).

When at a certain point of time the integrator signal is measured, there are two directions into which it may change, depending on the sign of the integration constant. The sign depends on which reference level was reached last. This information is contained in the memory. Now the memory can be of the binary type. It only has to store the sign of the integration constant. So in this practical situation the memory can be more simple than the memory in the basic first order oscillator. In the basic oscillator the memory must be able to store the level of the integrator signal, which may have any value.

In the next sections two different ways to construct the memory are discussed. With the type of memory as a parameter, two classes of first order oscillators can be distinguished. In practical first order oscillators four essential functions are performed:

1. Integration
2. Comparison
3. Switching the sign of the integration constant
4. Memorization

The oscillator can be made such that the every system component performs only one function, but a component may also perform several functions.

2.4 Regenerative oscillators

In this section first order oscillators will be built using a regenerative circuit to perform the memory function. The other functions will be performed each by a separate system component. The oscillator that is obtained is the basic regenerative oscillator. Later some functions will be combined in one system component. The consequences of this will be discussed.

The memory is one of the extra system components needed to generate the periodical signal out of the time variant signal. In the basic first order oscillator this memory has to be able to store any value \( E_o(t) \) can have. When generating a periodical signal, the memory can be a binary memory, so it does not have to store the actual value of the level of \( E_o(t) \) any more. In the case the time variant signal is made periodical, the memory can be a binary memory. It only needs to store two different values, as will be explained in this section. A memory that does just this is commonly known as the Schmitt trigger. Sometimes it is also given the name flip flop, or comparator with hysteresis. In chapter 3 it will be shown that the circuits denoted by these names are either identical to the Schmitt trigger, or extended systems containing a Schmitt trigger.

Using the Schmitt trigger as a memory, practical first order oscillators can be built, all belonging to the class of regenerative oscillators. The word regenerative indicates the fact that a regenerative circuit, the Schmitt trigger (in whatever form it may appear), performs the memory function. In chapter 4 the special properties of regenerative circuits are discussed
with the aid of the loop gain-excitation model. In chapter 7 the class of regenerative oscillators is discussed in more detail. For now the basic way to construct a regenerative oscillator will be discussed.

2.4.1 The basic regenerative oscillator

In the previous section it has been shown that in order to obtain practical first order oscillators, at least the sign of the integration constant has to be switched before the time variant signal is limited by some physical constraint. Although there is no fundamental need to introduce a correlation between the periodical signal at the output and the switching of the sign of the integration constant, it is very convenient to do so. In that case the extra system that generates the periodical signal out of the time variant signal, will be the simplest. The most simple configurations are obtained when the period of the periodical signal at the output is the same as the period of the signal that switches the sign of the integration constant.

In the regenerative oscillator two different levels that \( E_o(t) \) can have will be labeled as reference levels. The difference between the two levels, called \( E_h \) and \( E_l \) will be equal to \( E_{off} \):

\[
E_h - E_l = E_{off} \tag{2.5}
\]

When \( E_o(t) \) becomes equal to a reference level, the sign of the integration constant will be changed such that \( E_o(t) \) starts changing into the direction of the other reference level. The configuration should be such, that, when the regenerative oscillator is in steady state (is producing a stable periodical signal), \( E_o(t) \) always remains between the two reference levels. In fig.2.6 \( E_o(t) \), which has become a periodical signal, has been depicted.

![Graph showing the periodical output signal of the integrator](https://via.placeholder.com/150)

Fig. 2.6 The periodical output signal of the integrator

The information which has to be stored in the memory, is which reference level was reached last by \( E_o(t) \). It determines the sign of the integration constant, and thus towards which reference level \( E_o(t) \) is changing. So basically only the sign of the integration constant is determined by the memory, and since only two signs are possible, the memory only has to be able to store two different things, it can be a binary memory. The two reference levels have fixed values.\(^1\) So they can be applied to the oscillator as constants. Now two comparators are needed, one for each reference level. An example of such an oscillator has been depicted in fig.2.7, and

\(^1\)They may also show changes when they are used as a means to modulate the oscillator. This makes no real difference for the way of operation.
can be found in [9, 24]. Three constants are applied to this oscillator. First there is $\alpha$, the integration constant, which is multiplied by the sign signal coming from the memory. In this case the memory is a flip flop (which is just a two input Schmitt trigger). The output signal of the memory $Q$ can be either 1 or $-1$. The other two constants are the two reference levels each applied to one of the reference inputs of the comparators. The output signal of the integrator

![Diagram of a regenerative oscillator with two comparators]

**Fig. 2.7** A regenerative oscillator with two comparators

$E_o(t)$ will be like the one in fig.2.6. When $Q = 1$, the constant $\alpha$ at the input is multiplied by 1. Since $E_o(t)$ is a periodical signal it must be true that $E_o(t) \leq E_h$, when the signal at the input of the integrator is positive. So when $\alpha$ is positive, $E_o(t)$ changes towards $E_h$. When $E_h$ is reached, the top most comparator will generate a pulse which resets, the flip flop, causing its output to change to $-1$. Now $\alpha$ is multiplied by $-1$ and $E_o(t)$ starts changing towards $E_l$. When this reference level is reached the other comparator will generate a pulse and set the flip flop again.

In this oscillator each function is performed by a separate building block. Oscillators with "split functions" like this are built in practice [9]. All properties can be optimized easily, since the optimization of one property will have hardly any influence on another. Every building block can be optimized for the one function it has to perform, and the best circuit to do it can be chosen. A problem can be caused by the complexity of this type of oscillator. Amongst other things it may cause speed problems and the bias may be too complicated, or require too much power.

**Further reduction of the influence of the regenerative memory**

In fig.2.8 a system has been depicted in which measures have been taken to decrease the influence of the regenerative memory further. In this system the sign of the integration constant is initially switched by either one of the comparators. This is a negative feedback mechanism which can stop the oscillation. When the sign of the integration constant is switched, the integrator signal starts changing into the other direction, thus removing the cause for the comparator to produce the output signal necessary to keep the sign changed. Then the integration constant would be switched back to the original value again. However, the comparator also excites the regenerative memory which eventually takes over the decision of the comparator. So when the comparator stops generating the right output signal, the memory prevents the
integration constant from being switched back[17].

When the switching levels of the comparator and the memory are chosen correctly, the memory properties will have less influence on the timing of the complete oscillator. The memory does not have to produce a sufficient output signal sooner than the moment at which the output signal of the comparator does not keep the integration constant at the right value any more. This implies that the memory has more time to switch. The memory delay is not added to the delay of the comparator just like that, because it partly switches in parallel with the comparator. Apart from the fact that the memory has to switch before the comparator stops generating an output signal, no further timing requirements are necessary. The timing jitter of the memory does not affect the accuracy of the moment at which the integration constant is switched, because this is determined by the comparator. The memory only has to sustain the change of the integration constant that was initiated by the comparator. The switching function is less influenced by the properties of the regenerative memory, so the separation of the functions is carried out further.

In chapter 7 an electronic implementation will be discussed in full detail (section 7.7.1).

### 2.4.2 Reduced regenerative oscillators

The oscillator system described above can be reduced. This means that some of the functions can be combined in one system component. It will be shown that without the use of separate comparator blocks, still first order oscillators can be built. In the next subsection these oscillators will be discussed. First one comparator will be deleted, and then the other one. The comparators were introduced, in order to compare $E_o(t)$ with two different reference levels. This comparison is essential for the oscillator, so this function will have to be taken over from the comparators when they are deleted. This will cause correlations between the different functions that are performed in the first order oscillator, so it will no longer be possible to optimize them separately. It may no longer be possible to use the most suitable circuit for one function, since such a circuit may not be able to perform all the functions it has to.

**Basic first order oscillator with one comparator**

When one comparator is deleted, the other one will have to compare $E_o(t)$ with both reference levels. This means the signal at the reference input of the comparator cannot be constant. Depending on towards which reference level $E_o(t)$ is changing, the appropriate reference level must be at the reference input. Since this dependence is present in the memory output, this signal will be used to generate the reference signal for the comparator. The only thing that has to be done is amplifying the unity signal by the appropriate factor. When this factor is chosen to be $\frac{1}{2}E_{off}$, the amplitude of $E_o(t)$ will be $E_{off}$ again. In fig.2.9 an oscillator of this type has been depicted.
Fig. 2.9 A basic first order oscillator using one comparator.

**Basic first order oscillator without comparators**

When the last comparator is deleted, the memory itself will have to compare $E_0(t)$ with the reference levels. The Schmitt trigger is a circuit that is able to do this. It has two stable states, and an internal reference signal. This signal can have two different values, the internal reference levels. When the input signal crosses such a reference level, the Schmitt trigger switches to another state. The internal reference levels in the Schmitt trigger can be used as the reference levels of the complete oscillator. Then the Schmitt trigger should be designed such that the two internal reference levels have a difference of $E_{off}$. In that case the system as depicted in fig.2.10 is obtained.

Fig. 2.10 Basic first order oscillator without comparators

Finally the multiplier at the input can be deleted. Then the output signal of the memory has to have the values $\alpha$ and $-\alpha$ in stead of 1 and -1. Now a first order oscillator is obtained in which one building block, the Schmitt trigger, performs all extra functions that are necessary to generate a periodical signal out of a time variant signal. It will not be possible to optimize this block for every function it has to perform. Later it will be shown that the Schmitt trigger, although it is able to perform all these functions, it is not the optimal circuit for all of them. Of course it is a very suitable bistable memory (in fact the only function it should be used for).

In fig.2.11 this oscillator and a transistor implementation have been depicted. The transistor circuit is the well know emitter coupled oscillator. The integrator is a capacitor, and a differential pair is interconnected as Schmitt trigger. The bias currents for the Schmitt trigger
are also used as the integration constant, and the voltage drop the currents cause across the resistors is used as the reference signal. The Schmitt trigger generates the reference levels, switches the sign of the integration constant, compares the integrator signal with the reference levels, and forms the memory. It is one of the simplest first order oscillators that can be built. Also it is the oscillator with the most compromises for the different functions involved.

![Diagram of basic first order oscillator]

Fig. 2.11 The most simple basic first order oscillator, and a practical implementation.

2.4.3 Summary

Regenerative oscillators are named after the type of memory that is used. This memory is a Schmitt trigger, or a closely related circuit, and has a bistable character. Three constants determine the frequency, one integration constant and two reference levels. The comparison of \( E_o(t) \) with these reference levels can be done by special purpose system components, but also by the memory. In the most expanded version every essential function is performed by a separate system component. In this way every component can be optimized. When the number of separate components is reduced, oscillators can still be built, but the optimization will become more complicated.

2.5 Two-integrator oscillators

The output signal of an integrator at a certain point of time depends not only on the value of the input signal at that time, but also on the values the signal has had in the past. Therefore the integrator can be used as a memory.

2.5.1 Two-integrator oscillators with a sample and hold memory

A common example of such a memory is the sample and hold memory. The basic sample and hold memory consists of an integrator and a comparator/gate system. In fig.2.12 such a memory has been depicted. When the gate signal is off, the output signal of the comparator \( E_c \) is equal to zero. Since this signal is the input signal for the integrator, \( E_o(t) \) remains constant. When the gate pulse is on, the value of \( E_c \) will be such that \( E_o(t) \) changes into the direction of \( E_i \), and will become equal to zero when \( E_o(t) = E_i \). When the gain of the comparator is infinite and the value of \( E_c \) is without bounds, \( E_o(t) \) will become equal to \( E_i \) instantaneously. Then the gate signal may be infinitely short. These are the properties of the ideal sample and hold memory. When \( E_c \) is bounded, it will take some time to change \( E_o(t) \) and the gate pulse has to have a finite length. The sample and hold memory could be the memory that is used in the basic first order oscillator, as depicted in fig.2.4, since it is able to store any value \( E_o(t) \) can have.
Fig. 2.12 A sample and hold memory

When $E_o(t)$ is periodical, only a binary memory is needed. This can be made out of the sample and hold memory with the use of a comparator. When the reference level of the comparator is set to zero, the output signal will contain only information about the sign of the signal at the output of the sample and hold memory. Mathematically this action can be represented by a $sgn$-function. For correct operation the $sgn$-block has to be inverting. In fig.2.13 the output signal of the sample and hold memory can be either $\frac{1}{2}E_{off}$ or $-\frac{1}{2}E_{off}$, and the output of the $sgn$-block will be -1 or 1 respectively. When the output signal of the integrator reaches $E_h$,

Fig. 2.13 A first order oscillator with a sample and hold memory.

the top most comparator turns on the gate signal and the sample and hold will take over this value. Now the output signal of the $sgn$-block becomes -1, so $E_o(t)$ will start changing towards $E_l$. Since this implies that $E_o(t)$ becomes smaller than $E_h$ again, the comparator switches off the gate signal. So the sample and hold memory will retain the value $E_h$ at its output until the bottom most comparator switches the gate signal on, because $E_o(t)$ has reached $E_l$. Then $E_o(t)$ will be made to change to $E_h$ again.
Of course reductions can be made like the ones described in 2.4.2. For example when the \textit{sgn}-block is replaced by a (linear) amplifier having a gain $G$ equal to:

$$G = -\frac{2\alpha}{E_{off}}$$  \hspace{1cm} (2.6)

the output signal of the amplifier can be fed directly into the integrator, and the multiplier can be deleted. The reductions will not be discussed in detail here.

The oscillators built in this way contain two integrators, but only one integrator and thus one pole determines the character of the time variant signal. The other integrator is merely a memory. It is the one used in the sample and hold system. So this two-integrator oscillator really is a first order oscillator, although one could be misled by the name.

2.5.2 Symmetrical two-integrator oscillators

In practical first order oscillators measures are taken in order to keep the time variant signal within bounds. This can be accomplished by changing the sign of the integration constant when the time variant signal reaches a predefined reference level. Two of such reference levels are defined and the time variant signal changes periodically between the two levels (fig.2.6). Extra system components are needed to determine when the time variant signal reaches a reference level. A memory has to keep the integration constant of the integrator constant and of the appropriate sign, when the time variant signal is between the reference levels. All this can be done by a second integrator as will be shown in this section. A first order oscillator will be built containing two integrators in which one acts as a memory for the other integrator and vice versa.

When an integrator is given a starting value \(\hat{E}\), and the input signal at \(t = 0\) is changed from zero to a negative value \(-\alpha\), the output signal of the integrator \(E_{o1}(t)\), will start changing towards zero. Since \(\hat{E}\) and \(\alpha\) are known constants, the time the integrator needs to reach zero is known also.

$$E_{o1}(t_x) = 0$$  \hspace{1cm} (2.7)

$$t_x = \frac{\hat{E}}{\alpha}$$  \hspace{1cm} (2.8)

When a second integrator is given a starting value equal to zero, and at \(t = 0\) the input signal is changed from zero to \(\alpha\), then the output signal \(E_{o2}(t)\) will reach the value \(\hat{E}\) at \(t = t_x\). When \(\hat{E}\) is chosen to be one of the reference levels, the zero crossing of the first integrator can be used to switch the sign of the integration constant of the second integrator. Then \(E_{o2}(t)\) starts changing towards zero. When it reaches zero, \(E_{o1}(t)\) has become equal to \(-\hat{E}\). This is chosen to be the other reference level, so the zero crossing of the second integrator can be used to change the sign of the integration constant of the first one.

A special type of first order oscillator can be obtained in this way. It will be shown that it really is a set of two first order oscillators coupled to each other.

The comparators have a reference level equal to zero and their output signal can have a value equal to \(\alpha\) or \(-\alpha\), so they can be represented by \textit{sgn}-block having a gain of \(\alpha\). For proper operation one of these blocks will have to be inverting. In fig.2.14 a two-integrator oscillator has been depicted. The signals appearing in this oscillator have been depicted in fig.2.15.

Now it will be shown that this oscillator is a coupled oscillator. The oscillator is symmetrical, it consists of two identical integrator–\textit{sgn} pairs. The \textit{sgn}-blocks isolate the integrators from each other. Looking at the terminals of one integrator, there is (at this stage) no way to
Fig. 2.14 A symmetrical two-integrator oscillator

Fig. 2.15 The signals in a symmetrical two-integrator oscillator

determine whether a regenerative memory or an integrator memory is connected. In order to obtain a better understanding of the symmetrical two-integrator oscillator first arbitrarily one integrator will be labeled as the time variant signal generator (the $t$-integrator) and the other as memory (the $m$-integrator). Now some very special properties of the two-integrator oscillator will be indicated. The two-integrator oscillator is dealt with in full detail in chapter 8.

The time variant signal generated by the $t$-integrator is still determined by one pole. The reaching of the reference level is not measured directly, as it is in regenerative oscillators and sample and hold oscillators. It is supposed to be related to the zero crossing of the output signal of the $m$-integrator. When the $m$-integrator and the $t$-integrator are identical, the relation is exact. When the pole pattern of the $m$-integrator does not match with the pattern of the $t$-integrator, timing errors will occur in the switching of the integration constant of the $t$-integrator. The quality of the time variant signal (the absolute straightness) remains unaltered. This is characteristic for first order oscillators. Only one pole determines the quality of the time variant signal. The pole of the $m$-integrator only determines the quality of the memory function.

Of course it is not possible to indicate one integrator particularly as $t$-integrator and the other as $m$-integrator. The two-integrator oscillator is symmetrical. But from the point of view of one integrator, the other integrator can be considered to be just a memory that does not interfere with the generation of the time variant signal. So the two-integrator oscillator can be considered to be a very closely interwoven set of two coupled first order oscillators.
When the period of the two-integrator oscillator is calculated it is found to be:

\[ T = \frac{4\dot{E}}{\alpha} \]  

(2.9)

So the period is linear dependent on the starting value \( \dot{E} \). When there are losses in the integrators, the starting value will be eventually lost, so the frequency of the two-integrator oscillator will be unstable. The frequency will increase while the amplitude of the oscillation is exponentially damped. Therefore in practical oscillators, as described in [10], the losses have to be compensated by an amplitude stabilization loop. Then \( \dot{E} \) is preserved and the frequency is stable. The amplitude stabilization circuits are discussed in section 8.8.

2.6 Implementation details

In the first order oscillators that are dealt with until now no noise sources are fundamentally present. Therefore the frequency stability of the first order oscillator can be made arbitrarily accurate. There are no noise sources because no components having a frequency independent transfer with a dimension (f.e. \( \Omega \)) are used. Such components will always add noise to the signal they transfer. This noise is at least equal to the noise that is generated by a resistor with a value equal to the transfer (expressed in \( \Omega \) or \( \Omega^{-1} \)). (It can be made somewhat lower at the expense of extra power consumption chapter 6). In practice such elements cannot be avoided, so noise will fundamentally be present in the practical first order oscillators. This noise will affect the frequency stability. When only the noise sources are considered that are generated by the transfers, the fundamental noise floor of the oscillator can be determined. In the next subsections the building blocks of the oscillator are discussed in this respect.

Active components like transistors will introduce extra noise. Optimization of these noise sources will never lead to a noise floor lower than the fundamental noise floor.

2.6.1 Integrators

The basic building block around which the first order oscillator is constructed is the integrator. In network theory only two integrators are available, the inductance and the capacitance. In electronic circuits the majority of the integrators used are capacitors. One (modern) reason for this is the fact that inductors can hardly be integrated, whereas the demand for completely integrated oscillators is growing.

The integrators will show losses. These losses can be modeled by way of a pole in the left half plane in stead of in the origin. Now a real part is added to the transfer of the integrator. Considering the capacitor this can be modeled as parallel resistance. This "resistance" introduces noise in the circuit. In the basic first order oscillator (section 2.2), this is the only noise source present, and it will set the noise floor of the oscillator. Especially in the case of the capacitor, the noise source can be very small. It is related directly to the quality of the integrator. In practical first order oscillators, in which the sign of the integration constant is switched, another fundamental noise source will be present, which usually dominates in the fundamental noise floor. This will be explained now.

The integration constant for an integrator built with a capacitor has the dimension of a current. The output signal of this integrator has the dimension of a voltage. So the dimension of the transfer of the integrator is not just time [s], but the product of time and resistance [\( \Omega \) s]. This resistance is not to be confused with the resistance that models the quality of the integrator as described earlier. It implies that the circuit around the integrator, which is needed for the
generation of the periodical signal has to have a dimension too \([\Omega^{-1}]\). This admittance is real, so it will generate noise.

Apparently two noise sources set the noise floor of the first order oscillator. Both sources introduce a noise contribution in the integration constant. This noise is integrated by the integrator, so it will cause instability of the time variant signal and consequently frequency instability.

The two noise sources may very well be much smaller than the noise caused by other circuit components, but they set the absolute noise floor of the oscillator. In chapter 6 it will be shown how large this fundamental noise floor is and how it can be minimized. Then also the expression \(\frac{\partial T}{\partial F}\) will be discussed, which is a formula that describes the uncertainty in the capacitor voltage when it is measured.

Since also the inductor integrator needs an additional circuit that fundamentally generates noise, an analogous behavior of first order oscillators using an inductor can be expected.

### 2.6.2 Non-linear circuits

In the first order oscillator non-linear relations between signals are necessary and unavoidable. The output signal of the capacitor is a continuous signal, the input signal is switched, so at least one non-linearity must be present.

In practical first order oscillators several non-linear circuits may be present. For example the sgn-blocks have only two possible values for the output signal, whereas the input signal can have any value. Also the memory very often contains a limiting element. Of course these non-linearities should not be frequency dependent, since in principle in the first order oscillator only the integrator may have a time (and frequency) dependent behavior.

*All electronic components with a frequency independent non-linear transfer introduce noise.* Many of them have a real transfer with a dimension.\(^2\) An example of this is the differential pair. The output current is eventually limited by the bias current, when the input voltage is increased. Transistors are well known noise sources.

Since most non-linear transfers have a dimension and are real, they will introduce noise in the first order oscillator. In the previous subsection it has been shown that at least one real transfer with a dimension is needed to make an electronic first order oscillator. The non-linear element could fulfil this demand. In principle no more transfers with a dimension are needed. This implies that the non-linearity in the first order oscillator is the cause of the second noise source as described in the previous subsection.

### 2.6.3 Summary

In practical first order oscillator circuits, some of the building blocks unavoidably introduce noise. This is because they have a real transfer with a dimension, usually that of an impedance or an admittance. The fact that the transfer of the integrator has a dimension containing more than just time, implies that the rest of the oscillator has to contain at least one block having a real transfer with a dimension. Consequently the character of the integrator implies the presence of noise in the oscillator although it is not generated by the integrator itself.

The integrator itself may also generate noise. This is the case when the pole of the integrator is not exactly in the origin. Due to losses the pole will be in the left half plane for practical

\(^2\)It may be so that the only way to make an electronic non-linearity is using a transfer with a dimension. No proof of this has been found yet.
integrators. The real part of the pole can be related to the noise introduced by the integrator itself.

Non-linearities always introduce noise in the first order oscillator.

Apart from these fundamental noise sources, the active circuits and bias components introduce extra noise. These noise sources, that are very often dominant, will be discussed later.
3. Classification and high level modeling

In this chapter the classification which was partially started in chapter 2 will be expanded. First the first order oscillators will be described by a "high level" model. It is called a high level model because no electronic circuits are involved, it describes the oscillators on a high hierarchical level. Especially in the case of coupled oscillators, the model is very useful when their structure has to be determined. This structure will be shown to be very important for their quality. The first aim of the model is to provide a better understanding of the behavior of the oscillators. This understanding can be used to make clear what actions should be taken to improve the quality.

After this the model will be used to describe basic first order systems. Most of them will be oscillators, but some of them will be of a degenerated type. These systems contain states in which the integration constant is equal to zero, so they will produce a degenerated time variant signal. It will be constant in some states. In these states the generation of a periodical signal will be impossible without external (time variant) signals. Electronic circuits showing this behavior are known as mono-stable multivibrators.

First order oscillators can be coupled to each other. Time variant signals extracted from one oscillator, can be injected into a second oscillator, and vice versa. In this way the periodical signals generated by the two oscillators become related. The performance of a system of coupled first order oscillators can be much better than that of the separate oscillators. It will be shown that there are different ways of both extracting and injecting such a signals, resulting in different properties which will be discussed.

Finally, with the aid of the state-model, the classification started in chapter 2 will be completed.

3.1 Description languages for first order oscillators

In order to describe different classes of first order oscillators, a description language is needed. For human perception the use of pictures mostly provides the clearest descriptions. For computer manipulations such a "language" is usually very inconvenient. Then description languages like GLASS can be used [14]. In this chapter a description with pictures will be used to describe the different first order oscillators. This is because the structure of the oscillators has to be made clear to the human designer. After that the behavior of the first order oscillators will be related to the properties of the different parts of which they are constructed. It can be investigated whether practical steady state conditions are feasible for a given first order oscillator structure, and when it produces a periodical signal, the frequency can be determined. A very important feature is the possibility to investigate coupled first order oscillator systems. For example the influence of the coupling on the noise behavior can be determined.

3.1.1 The symbols used

Symbols for the different states

Different states can be distinguished in practical first order oscillators. In order to avoid limiting of the time variant signal due to physical constraints, the sign of the integration constant is switched before limiting occurs (section 2.3). This implies that the integration
constant can have several (discrete) values. Very often there are only two different values with opposite signs and equal magnitude. The integration constant is used as parameter to distinguish different states. An oscillator with two different values for the integration constant has two different states.

In the state diagrams a state will be represented by a large circle. In this circle the value of the integration constant can be placed. For now only three symbols for the integration constant will be used, a plus-sign, a minus-sign and a zero. In fig.3.1 different states that can be defined in this way have been depicted. The first two, (a) and (b), are states in which the integration constant is positive, in (c) and (d) it is negative and in (e) it is equal to zero. This fifth state is a degenerated state, since no time variant signal is generated when the first order oscillator is in this state. The integrator output signal is constant. The symbols (a), (c) and (e) represent externally excited states and the states (b) and (d) represent auto-excited states. The difference between auto-excited states and externally excited states will be discussed in section 3.2, which deals with the properties of the different states and transitions.

Symbols for the transitions

The transitions between the states are represented by arrows. In fig.3.2 two different options have been depicted. An internal transition is represented by one arrow (f). In some systems external signals may invoke an external transition. Such an external signal is represented by a "lightning shaped" arrow that points towards the transition it invokes (g).

Examples

In fig.3.3 an example of a regenerative oscillator has been depicted. It could be the regenerative oscillator as depicted in fig.2.7. The oscillator has two states. In one state the integration constant is positive, in the other one it is negative. The oscillator changes state without the influence of external signals, so they are interconnected via two internal transitions, and the states are auto-excited.

In fig.3.4 an electronic circuit and its state-model representation have been depicted. A constant current source is connected either to a capacitor or to ground via a switch. In this circuit
the integrator is a capacitor. Only the external signal (S), which is the signal that operates the switch, can invoke a change of state. The two states are interconnected via external transitions. The output signal of this circuit is either a ramp with a positive slope or a constant, depending on the external signal.

3.2 Properties of states and transitions

3.2.1 Externally excited states

When a first order oscillator is in a certain state, the slope of the output signal has a known value. If the integrator has a known starting value when the state is entered, then at any time after this event the value of the integrator signal is known. When the starting value is denoted by \( \dot{E} \) and the integration constant is denoted by \( \alpha \), then time and the integrator signal \( E_o(t) \) are related by a linear relationship:

\[
E_o(t) = \alpha t + \dot{E}
\]  

(3.1)

The change of \( E_o(t) \), denoted by \( \Delta E_o(t) \) is defined as:

\[
\Delta E_o(t) = E_o(t) - \dot{E} = \alpha t
\]  

(3.2)

This relation is one of the properties that can be assigned to a state. In the circuit of fig.3.4 this property can be used to change the output voltage with a predefined amount. At the start the circuit is kept in the state with an integration constant equal to zero. Then it is put in the other state by the external signal just long enough to obtain the desired voltage change. It is not necessary to measure the output voltage. When the timing of the external signal is right, the desired voltage change will be obtained. In section 3.3 systems like this will be used to build two-integrator oscillators.
The external signal has to meet with certain conditions in order to be able to invoke a transition. These conditions are set by the state in which the system is at that time. In the case of first order oscillators, a certain reference level exists to which the external signal is compared. When it crosses this reference level, the state is left. Different states usually have different reference levels. The value of the reference level is another property of a state. The crossing of the reference level is one condition the external signal has to meet with, another one is usually the duration of the period the first condition is met. When for example the reference level is crossed for a short time due to some spike in the external signal, the system may be to slow to leave the state in time. When the spike disappears, the excitation of the system disappears also, and if the state is not already left at that time, it will not be left at all.

3.2.2 Auto-excited states

Sometimes a system has the capability to leave a state without the influence of an external signal. The states described in the previous paragraph could be left only via transitions that were initiated externally. When in a certain state the integrator signal is compared to a reference level, a signal can be generated that forces the system to leave it. This happens when the integrator signal reaches the reference level belonging to this state. Transitions starting from this state are internal transitions and states like this will be called auto-excited states. In the state diagrams these states are indicated with a small circle around the sign of the integration constant (fig.3.1b and d). Of course states with an integration constant equal to zero can never be auto-excited.

Since the reaching of the reference level invokes a transition, the value of the integrator signal is known when the state is left. Then the value of the integrator signal is equal to the reference level $E_{ref}$. When the state is entered with a certain starting value $\hat{E}$ for the integrator signal, the time $T_x$ the system remains in this state is given by:

$$T_x = \frac{E_{ref} - \hat{E}}{\alpha} \quad (3.3)$$

This relation is a property that can be assigned to auto-excited states only. The time $T_x$ the system remains in the state will be called residence-time.

In regenerative oscillators like the one in fig. 3.3, all states are auto-excited. When one of the states is entered, the value of the integrator signal is equal to the reference level of the state that was just left via the internal transition. Therefore the residence-time of this state is completely fixed. In equation (3.3) $E_{ref}$ and $\alpha$ are determined by the state in which the system is at present, and $\hat{E}$ is equal to the reference level of the previous state. When the system leaves the state again, the integrator signal has the value of the reference level of this state. It is the starting value for the integration in the new state. Also in this new state the residence-time is completely fixed.

In auto-excited states very often the sum of an external signal and the integrator signal is compared to the reference level. Therefore it is possible that the transition is initiated by the external signal. Then an external transition is made, and relation (3.3) cannot be used. Also the integrator signal will not be equal to the reference signal ($E_{ref}$) when the state is left. Hence the properties that belong to auto-excited states only, are lost. This also holds for some unfavorable properties that belong to auto-excited states (e.g. jitter), so internal transitions may be avoided intentionally in order to improve the quality. This will be discussed later.
3.2.3 Degenerated states

Basically two different types of degenerated states can be distinguished. The difference has some effect on the properties of the complete system in which the degenerated state is present. There are two ways in which the integration constant can be made equal to zero. It can be switched to zero, which means that its value is set in this way by the memory. In this case there is no real difference between this state and an ordinary externally excited state. It can also become equal to zero due to limiting. An example of this has been given in fig.3.5. An electronic circuit showing a degenerated state, its state model representation and some signals appearing in the circuit have been depicted. The diode is an idealized diode. As long as the voltage across it is lower than $U_d$ it behaves like an open circuit, and when it becomes equal to this voltage, it starts behaving like a voltage source with a value $U_d$. Hence the voltage across the diode will never become larger than $U_d$. A constant current charges the capacitor. As long as the capacitor voltage is lower than $U_d$, it increases linearly with time. When this voltage is reached, all current flows through the diode and the capacitor voltage remains unaltered from now on. The system has reached the degenerated state, but the contents of an eventual memory would not have changed.

This has some implications for the systems in which these states occur. When there is a reference level, indicated as $E_{ref}$ in fig.3.5, this level will never be reached by the integrator signal alone due to the limiting. When this level could be crossed, the system would make a transition to another state. This transition can only be an external transition, and one way to initiate such a transition is adding an external signal to the integrator signal. This sum might very well cross the reference level. In fig.3.6 a system in which this happens has been depicted. When the sum of the external signal and the integrator signal crosses the reference level a transition to state (c) is made. When the system is still in state (a) when the reference level is crossed, so when the residence-time $T_o$ of state (a) has not expired yet, the transition is made from (a) to (c) directly and state (b) is skipped. As a result the auto-excited properties of state (a) are lost, so the integrator signal is not equal to the limiting level, which is the reference level for the internal transition from state (a) to state (b). Consequently the residence-time of state (c) is changed because it depends on the value of the integrator signal when the state is entered.

Fig. 3.5 A first order system with a degenerated state due to limiting

Fig. 3.6 An external transition in a first order system with a degenerated state due to limiting
When the integration constant has become equal to zero due to the memory contents, the reference levels of this state and the preceding state usually are unequal and transitions that cause skipping of the degenerated state are less likely to occur.

### 3.2.4 External influences on the state-parameters

States have two parameters, the integration constant and the reference level. Both parameters can be influenced by external signals. From (3.3) it can be seen that both parameters have influence on the residence-time of an auto-excited state. This influence is well determined and can be used to modulate the signals that are produced by the system. For example in regenerative oscillators a tuning voltage can be used to modulate the integration constants of the states. Since the period is the sum of the residence-times of all states, in this way the frequency of the oscillator can be modulated. Also the reference levels can be influenced by an external signal. An example of this is given in chapter 8.

The sum of the integrator signal and an external signal is compared to the reference level. When it crosses the reference level of an auto-excited state, a transition takes place. Sometimes the external signal is applied to the oscillator intentionally, but always disturbing signals like noise will be present. The external signal may initiate an external transition. Of course there is no real difference between the case in which the external signal is added to the integrator signal and the case in which the external signal modulates the reference level.

Suppose an auto-excited state (with a positive integration constant) is entered with a starting value \( \bar{E} \) equal to zero, and the reference level is modulated with a factor \( 1 + \beta(t) \). Then the residence time is:

\[
T_{z0}(t) = T_{z0} + \beta(t) \frac{E_{ref}}{\alpha}
\]

(3.4)

\( T_{z0} \) is the unmodulated residence time (3.3).

This situation can also be seen as an summation of the integrator signal \( E_o(t) \) and an external signal \( E_{ext}(t) \):

\[
E_{\text{sum}}(t) = E_o(t) + \beta(t)E_{ref} = E_o(t) + E_{ext}(t)
\]

(3.5)

A transition is made when:

\[
E_{\text{sum}}(t) = E_o(t) + E_{ext} \geq E_{ref}
\]

(3.6)

which is the case when:

\[
\beta(t) \geq \frac{E_{ref} - E_o(t)}{E_{ref}}
\]

(3.7)

From (3.7) and (3.6) it can be seen that signals that add to the integrator signal (or the reference level) and signals that modulate the reference level can invoke transitions. The closer the first order oscillator gets to the unmodulated transition moment, the more sensitive it becomes for these signals.

(For states with a negative integration constant the \( \geq \)-signs in (3.7) and (3.6) must be replaced by \( \leq \)-signs.)

When an external signal invokes external transitions the oscillator becomes synchronized with it. Some signals may occasionally synchronize one transition, other signals may synchronize a transition every period. In the first case the effect on the oscillator may be neglectable, in the latter case serious effects on the oscillator may result. Signals that are able to do this usually are periodical signals with a frequency close to the oscillator frequency or odd harmonics of it. (It should be kept in mind that white noise contains such frequency components [27].)

Sometimes an external signal synchronizes every transition. Then the first order oscillator produces a carrier at the frequency of the external signal in stead of a carrier at its "free
running" frequency. This effect is called \textit{pulling}. Pulling may disturb measurements. For example when extra noise is injected into a first order oscillator to ease the measurement of noise behavior, the synchronization effects may dominate the effects that determine the noise behavior at low noise levels. The effect of noise on the properties of a state are calculated in chapter 6.

From (3.6) and (3.7) it can be seen that when the external signal is large enough an external transition can be initiated always. This property is used to synchronize first order oscillators intentionally, which will be discussed in subsection 3.4. Also signals that modulate the integration constant can initiate transitions, but since they are integrated, this effect diminishes rapidly with the frequency.

Generally the influence of external signals on the state-parameters and thus on the oscillator properties are well determined, so when the properties of the states are dominant in the properties of the complete system, the system behavior is well determined also.

\subsection{3.2.5 Transitions}

Two types of transitions exist, \textit{internal} transitions and \textit{external} transitions. Internal transitions occur when they are initiated due to the crossing of a reference level by the integrator signal in an auto-excited state. External transitions occur when an external signal initiates the transition. The origin of such a signal does not lie within the oscillator itself. Usually external transitions are initiated by signals that add to the integrator signal. This is the way in which synchronization inputs are usually made in first order oscillators. They may also be initiated via modulation of the state parameters, but modulation of the integration constant is not a commonly used method to intentionally initiate transitions. Modulation of a reference level and adding a signal to the integrator signal are not really different actions, but when modulation of the reference levels is discussed, the modulating signals are not supposed to invoke external transitions that seriously affect the oscillator behavior. This is the case for the usual modulating signals with frequencies that are less than half the oscillator frequency.

In practice the transitions take time. The exact duration of a transition is uncertain due to noise, and is often determined by obscure parameters.

Usually transitions that are indicated to be external are meant to occur only when they are initiated by the external signal. However, the state in which the system is at that time, may have auto-excited properties. Then after a time given by (3.3), the state will be left via an \textit{internal} transition, if it is not left before that time via an external transition. This phenomenon is usually to be avoided, but sometimes it can be used favorably. This is the case when a system is needed that has to generate a transition within a certain time interval always. Lacking an external signal (due to some error condition) a transition is still guaranteed. Examples of such systems are the time-base of an oscilloscope in the "auto-trigger mode" and synchronized oscillators (subsection 3.3.3). In section 8.8 the phenomenon will be used to stabilize the amplitude of two-integrator oscillators.

\subsection{3.2.6 External influences on the transition parameters}

The duration of a transition usually depends on obscure parameters such as the $f_T$ of transistors, parasitic, non-linear voltage and temperature dependent capacitors and bias conditions. Such parameters are difficult to control and the influence of external signals is hard to determine. When external signals are used to modulate first order systems, the effect of these signals on the transitions is usually not known accurately. Hence transitions will introduce
inaccuracy in the modulation of first order systems. Noise also modulates the transition times. A calculation of the influence of noise on the transition times is performed in chapter 6. Chapter 4 especially deals with the influence of external signals (also noise) on transitions made by regenerative circuits. Then transitions made by Schmitt triggers will be shown to differ very much in properties from those made by limiters.

### 3.3 First order systems described with the state model

In this section different first order systems will be described using the "description language" as described in section 3.1. In the first subsection, systems containing transitions to states with an integration constant equal to zero will be discussed. States like this are called degenerated states and therefore these first order systems will be called degenerated systems. Degenerated states can only be left via an external transition, so all degenerated systems have inputs for external signals. A distinction is made between systems containing degenerated states only, and systems that contain other states also. Among the latter type the mono-stable multivibrators are found.

After that first order oscillators will be discussed. During normal operation a periodical signal is generated, and degenerated states are never reached. At the end synchronized and coupled first order systems will be dealt with. Different ways to synchronize and couple first order systems will be shown and the behavior of coupled systems will be investigated.

Unpractical systems like the one in fig.3.7 will not be considered. In systems like this the

![Diagram](image)

Fig. 3.7 An unpractical system

...sign of the integration constant never changes. At the most it may occasionally become equal to zero. Hence the value of the integrator signal can never cross a reference level twice. In practice this implies it can never reach a state twice, so the generation of a periodical signal is impossible. Only systems will be considered that contain a loop of states and transitions in which occurs at least one sign change.

The basic first order oscillator of section 2.2 and fig.2.4 is described with the state model symbols in fig.3.8. All states have equal integration constants. From the left to the right their

![Diagram](image)

Fig. 3.8 The basic first order oscillator

reference levels increase with steps equal to $E_{off}$. Each time a transition is made, a pulse is generated. The pulse train is a periodical signal. This circuit was shown to be unpractical before. Lacking the desired loop, the integrator signal has to grow without bounds when a (continuous) periodical signal is to be generated (fig.2.5).
3.3.1 Degenerated first order systems

Systems containing only degenerated states

First order systems containing degenerated states only, form an exceptional class of first order oscillators. Since the output signal of the integrator is constant always, the use of a true integrator is rather useless. It can just as well be replaced by some element that produces a constant (e.g. a voltage source). In practice an integrator is never implemented in this case. Of course for the fundamental way of operation, the use of either an integrator or another element, producing a constant, is of no importance. Therefore in the modeling an integration constant equal to zero will still be assigned to the states, even when the integrator is lacking in the practical circuit.

Systems containing just two degenerated states will be discussed now. In fig.3.9 two first order systems have been depicted, with two degenerated states. One of them is known commonly as the Schmitt trigger and the other as flip-flop. The transitions between the states can only be

![Schmitt trigger and flip-flop diagrams](image)

Fig. 3.9 A Schmitt trigger (a) and a flip-flop (b)

external transitions. Since the integrator signal does not change, it will never cross a reference level. Only events like this would initiate an internal transition, so internal transitions will never occur. In fig.3.9 the differences between a Schmitt trigger (a) and a flip-flop (b) can be seen easily. In a Schmitt trigger only one input is available for the external signal. The state in which the Schmitt trigger is determines which transition is initiated by the external signal (S). The Schmitt trigger can only be in one state at a time, so the selection of the transition is unambiguous. The flip-flop has two inputs (S) and (R). Each input can be used to invoke only one of the transitions. The flip-flop is sensitive for external signals at only one input at a time. The selection of an input depends on the state in which the flip-flop is.

The conditions that have to be met by the external signal to invoke a transition are determined by the state in which the system is. There is some reference level that has to be crossed by the external signal. This reference level is a property of the state, and not of the origin of the external signal. The fact that in a Schmitt trigger only one input is available, and there are two in a flip-flop, is not important. The behavior of the system is determined only by the properties of the states and the transitions. In fig.3.10 the system has been reduced to its essential parts, and an electronic equivalent has been given (level shifts have been left out).

Starting from this system both the Schmitt trigger and the flip-flop can be constructed by merely choosing other inputs. In fig.3.9 this was already shown this to be true when the state-model representation is used, now the same will be shown for the electronic equivalent. In fig.3.11 two currents are injected into the circuit. When the left hand transistor is conducting and the other one is off, an extra current from source (R) in the indicated direction will
introduce a voltage drop across the right hand resistor. When this voltage drop is large enough, the left hand transistor will turn off and the right one will turn on. The fact that the voltage drop has to be "large enough" indicates that there is a reference level. A current from source (S) in the indicated direction would have no effect in this situation. Then the left hand transistor remains on and the right one remains off. Starting from the situation in which the right hand transistor is on, the roles of both current sources are exchanged. In stead of current sources transistors can be used. This case has also been depicted in fig.3.11. It is a practical implementation of a flip-flop.

The circuit described above can also be used as a Schmitt trigger. For example the (R)-source could be deleted, whereas the (S)-source is enabled to supply currents in both directions. It is not to difficult to determine the behavior in this case, and since only one input is used, this circuit is a Schmitt trigger. Of course the (S)-source cannot be replaced by just one transistor.

In fig.3.12 another example of a Schmitt trigger implementation has been depicted (again without level shifts).
Only one voltage source (S) is introduced which acts as the input for the external signal. The voltage can be positive or negative. In order to make clear that this is a Schmitt trigger, the circuit diagram is drawn in another way. First node (G) is used as a new ground node. This has no effect on the way of operation of the circuit, since the use of the ground symbol in circuit diagrams is just used as a convenient way to show that some nodes are interconnected. When this interconnection is not disturbed another node in the circuit diagram may be called ground. The fact that the “ground node” of the current source is connected to another point only introduces a level shift. This does not influence the behavior either. In fig.3.12 two equivalent circuit diagrams have been depicted. The right hand circuit is a well known implementation of the Schmitt trigger.

From all this it follows that Schmitt triggers and flip-flops are actually identical circuits, as far as their behavior is concerned. The way the external signal is introduced in the circuit is the only difference, the properties of the states and the transitions are the same. So the behavior of larger systems containing for example flip-flops cannot be changed fundamentally by replacing them by Schmitt triggers. A preference for one of them may exist due to implementation details.

3.3.2 Mono-stable multivibrators

In this subsection systems will be described that contain one degenerated state in the loop of states it passes through. An external signal is needed to make the system leave the degenerated state. It will be shown there are two different ways for the external signal to do this. The first way is that it initiates a transition as described before, by crossing a reference level set by the state. Most practical mono-stable multivibrators work in this way.

The second way is that the external signal modulates a state-parameter. The parameter that is usually influenced is the integration constant. The external signal can make it unequal to zero. If the state is auto-excited, it is eventually left. Apparently in this situation a degenerated state exists that is potentially auto-excited. When the integration constant is modulated such that it becomes unequal to zero, it behaves like an ordinary auto-excited state.

Mono-stable multivibrators are used to generate one pulse with a predefined duration. It must be delivered after a “request” from an external origin. An external signal initiates a sequence that delivers the pulse. The duration of the pulse is not to be determined by the external signal. The very reason for the use of first order systems was to generate time variant signals out of constants, so no timing information about the pulse width is expected from the external signal.

When the system has to deliver a pulse with a predefined duration, auto-excited states are needed. The duration of the stay of the system in these states is fixed by internal parameters of the system only. Therefore in this subsection only systems containing auto-excited states will be considered. Only when systems are coupled, externally excited states become of practical interest. This will be dealt with in section 3.4. In the remainder of this subsection the two different types of mono-stable multivibrators will be discussed. The first part is about mono-stable multivibrators in which the external signal initiates an external transition. The second part is about mono-stable multivibrators in which the external signal modulates the integration constants, eventually causing an internal transition.

Mono-stable multivibrators with an external transition

In fig.3.13 a state-model representation of a mono-stable multivibrator has been depicted. It
Fig. 3.13 A mono-stable multivibrator

contains three states, two auto-excited states and one degenerated state. The degenerated state is left via an external transition that is initiated by the external signal (R). Since the two auto-excited states have integration constants with opposite signs, a loop of states and transitions is possible and a practical implementation should be feasible.

Suppose the system is in the degenerated state (a). The value of the integrator signal is known, since the only way the system could have entered this state is the transition that departs from state (c). This state is an auto-excited state, and the transition is an internal transition so the value of the integrator signal has to be equal to the reference level of state (c). When the external signal brings the system in state (b), all parameters of this state are fixed. In equation (3.3) the starting value $\bar{E}$ is equal to the reference level of state (c), since the integrator signal does not change in the degenerated state. The other two parameters in (3.3), the integration constant $\alpha$ and the reference level $E_{\text{ref}}$, are properties of state (b) itself. State (b) is an auto-excited state, so after a time given by (3.3) an internal transition is made from (b) to (c). Also of this state all parameters are fixed, and after a time given by (3.3) an internal transition is made towards (a). Now the mono-stable multivibrator is ready to receive a next trigger pulse to start the sequence again.

In this mono-stable multivibrator two auto-excited states are involved (and one degenerated state), so two residence-times determine the duration of the pulse. In practice the residence-time of one of the auto-excited states is chosen much larger than the other. Then the pulse-time of the mono-stable multivibrator is taken equal to the residence-time of one state. Apart from the fact that the duration of the transitions themselves are neglected, also the residence-time of the other state is neglected. The integrator signal is a periodical signal, so in both states the change of the integrator signal is the same. In equation (3.3) the reference levels of both states are always present. One plays the role of the starting value $\bar{E}$ and the other one the role of $E_{\text{ref}}$. Depending on the state in which the system is, they trade places. Hence a difference in the residence-times can only be obtained when the integration constants of the states have different magnitudes. In practical mono-stable multivibrators the integration constants usually are currents, and these currents may become very large in some states!

A practical example has been depicted in fig.3.14. The integrator has been implemented in this circuit as a capacitor. The current $I_{\text{charge}}$ that flows through the capacitor and the value $C_t$ of the capacitor form the integration constant.

$$\alpha = \frac{I_{\text{charge}}}{C_t} \quad (3.8)$$

The memory element is a set-reset flip flop. A pulse at the S input will set the output Q to one, thus turning on the transistor, and a pulse at the R input will set Q to zero in which case the transistor is turned off.

Suppose the flip flop is in the state $Q=1$. All current from the current source flows through the transistor. Since no current is flowing through the capacitor the integration constant is equal to zero, so obviously the circuit is in the degenerated state (a). The capacitor voltage
is equal to zero which means that the voltage at the positive input of the comparator is lower than the reference voltage at the negative input. Because of this no signal is present at the output of the comparator that would set the flip flop.

When an external signal at the R-input resets the flip flop, the transistor is turned off. All current from the source is flowing through the capacitor now, causing the capacitor voltage to raise. Now the circuit is in state (b). When the capacitor voltage becomes equal to the reference voltage, the comparator generates a pulse that sets the flip flop. The transistor is turned on and the capacitor is discharged through it. As long as a current from the capacitor flows through the transistor, the circuit is in state (c). When the capacitor voltage has become equal to zero, this current stops and the integration constant is equal to zero again. The circuit finally has reached state (a) again.\footnote{Circuits like this can be built for example with the 555 timer IC.}

As described above two auto-excited states exist, but only the residence-time of one of them, state (b), is well defined. Even when the current source is implemented as a resistor, the time the capacitor voltage needs to raise from zero to the reference voltage can be calculated easily. However, the discharge current that flows in state (c) is determined by the transistor parameters. Excessive currents may flow when the transistor is placed in parallel with the capacitor in this way. This can be avoided by limiting the current for example by way of a resistor in series with the collector. Then the residence-time of state (c) will become longer. As long as the circuit has not left state (c), no external pulses should be applied. If this happens, state (c) is left towards state (b) via an external transition (not indicated in fig.3.13). Then the value of the integrator signal depends on the point in time at which the pulse was delivered. The auto-excited properties of state (c) are lost. State (b) is entered with a starting value that is not equal to zero, so its residence-time becomes shorter. The time the circuit spends in state (c) can be seen as some sort of recovery time. During the recovery the circuit "prepares" itself for a new pulse sequence. It should be short if the circuit is to be used at high repetition rates, and since it is an integrator output signal that is to be changed, a large integration constant will be needed to do this quickly. This may impose extra demands on the components used.

**Mono-stable multivibrators with a modulated parameter**

Mono-stable multivibrators contain a state in which the integration constant is equal to zero. Such a degenerated state can only be left via an external transition. However, it can also be left by an internal transition when the integration constant is modulated. An external signal can be used to add a certain amount to the integration constant of a state. In the case of a degenerated state this implies the integration constant is made unequal to zero, and the state is no longer degenerated. When it has auto-excited properties in that case, a residence-time is
defined, and the state is left after that time. Hence modulating the integration constant is a way to make a system leave its degenerated state.

In this subsection an example will be given of a mono-stable multivibrator, in which the external signal $S$ can have two different values. It can be either equal to zero, or it can have the negative value $-\alpha_{ext}$. In the case of electronic first order oscillators, usually the signal $S$ causes a change of the integration constants of all states. When $S$ has a certain value, it is added to all integration constants in the system.

In fig. 3.15 a mono-stable multivibrator with modulated integration constants has been depicted. Depending on the value of $S$, the multivibrator behaves like the top most system $(a-b)$ or the bottom most system $(a'-b')$. States (a) and (a') are not really different states. In the picture two separate circles have been drawn to indicate the fact that the integration constant is modulated. In (a') the extra amount $-\alpha_{ext}$ is added to the integration constant, and in (a) it is not. The same holds for (b') and (b). Two signs are found in (b'), which indicates that a negative amount, indicated by the ‘−’ sign, is added to the positive integration constant, indicated by the ‘+’ sign. The reference level of both (a) and (a') is equal to $E_a$, and it is lower than $E_b$, which is the reference level of both (b) and (b').

Ideally the system is desired to behave like the (a–b) system. Since there is no internal transition possible from (a) to (b) the system is temporarily made to behave like the (a'–b') system in which the desired transition is possible. The commonly used pulse time of the multivibrator, is equal to the residence-time of state (b) in this state model description. This residence-time should not be influenced by the external signal. First order oscillators are used to generate time references, and timing information is not expected to be present in the external signal. It is true that the method of modulating the integration constants, may cause influence of the timing of the external signal on the residence-time of (b). Before dealing with this first the ideal behavior will be discussed.

Suppose the system has entered state (a) via an internal transition coming from (b). The integrator signal will be equal to $E_b$ which is the reference level of (b). The system has to be brought back into state (b) by way of the external signal $S$. This can only be done by temporarily making the multivibrator behave like the (a'–b') system. State (a) is transformed into state (a'), which is an auto-excited state, by the external signal. The residence-time $T_{a'}$ of this state is equal to:

$$T_{a'} = \frac{E_a - E_b}{-\alpha_{ext}} \tag{3.9}$$

After a time $T_{a'}$ the system switches to state (b'), but when S is switched to zero immediately when the transition occurs, state (b') is transformed to (b) immediately also. Obviously, the
multivibrator needs a time \( T_a \) to “charge”, before it can generate the desired pulse. Since the reference level of state \((a')\) is well determined, the residence-time of \((b)\) is fixed also:

\[
T_b = \frac{E_b - E_a}{\alpha_b}
\]  

(3.10)

It can be seen easily that the immediate switching of \(S\) when the transition from \((a')\) to \((b')\) occurs, is essential for an accurate residence-time of \((b)\). Now the effect of inaccurate timing of \(S\) will be investigated.

When \(S\) is switched to zero before the residence-time of \((a')\) has expired, the system just ends up in the degenerated state again and no pulse is generated. The integrator has been precharged however, so next time \(S\) transforms \((a)\) into \((a')\) the transition from \((a')\) to \((b')\) will come sooner.

When \(S\) is switched to zero a time \(T_{delay}\) after the transition has taken place, the integrator signal \(E_o\) will change with the amount of \(\Delta E_o\):

\[
\Delta E_o = (\alpha_b - \alpha_{ext})T_{delay}
\]  

(3.11)

In which \(\alpha_b\) is the (positive) integration constant of state \((b)\) and a part of the integration constant of \((b')\). As a result of this the residence-time of \((b)\) will become equal to:

\[
T_b' = \frac{E_b - E_a}{\alpha_b} - \frac{\Delta E_o}{\alpha_b} = T_b - \frac{\alpha_b - \alpha_{ext}}{\alpha_b}T_{delay}
\]  

(3.12)

From (3.12) it can be seen that the residence-time of \((b)\) has become dependent on the timing of the external signal! When \((\alpha_b - \alpha_{ext})\) is positive, a positive residence-time \(T_b'\) exists for \((b')\). Then a transition from \((b')\) to \((a')\) occurs if \(T_{delay} > T_{b'}\), and when \(S\) kept unequal to zero indefinitely, the system behaves like a regenerative oscillator instead of a mono-stable multivibrator.

From all this it can be seen that the main problem is caused in state \((b')\). In this state unwanted changes occur in the integrator signal. Since the system can only be kept out of this state by way of an exact timing of the external signal, which is usually not feasible, the damage that is done when the system enters this state should be kept as small as possible. The problem can be solved by degenerating state \((b')\). In degenerated states the value of the integrator signal is unchanged, so even when the system resides in \((b')\) for some time, the integrator signal will remain equal to \(E_a\) and the residence-time of \((b)\) is well determined again. It can be seen from (3.12) that when \(\alpha_b = \alpha_{ext}\), the influence of \(T_{delay}\) is eliminated.

In fig.3.16 a mono-stable multivibrator with a degenerated \((b')\)-state has been depicted. Now the system contains two degenerated states, and consequently the “parasitic” transition from \((b')\) to \((a')\) has disappeared. When the system has to generate a pulse, first state \((a)\) is transformed to \((a')\) by the external signal. Then the multivibrator is “charged”. After the residence-time for \((a')\), the system switches to state \((b')\). The system is ready to deliver an accurately timed pulse as soon as \(S\) changes to zero. The only timing demand imposed on \(S\) is that \(S\) is not set to zero before the residence-time of \((a')\) has expired. When it is set to zero before that time no pulse is generated. These timing requirements for the external signal are less strict, especially when \(T_b\) is made short.

In the electronic circuit of fig.3.16 state \((b')\) is degenerated by way of a transistor. When the capacitor voltage is unequal to zero and the transistor is on, the system is in state \((a')\). The capacitor is discharged very fast, depending on the transistor properties. When the capacitor voltage has become equal to zero, state \((b')\) is entered. All current coming from source \(I_x\) will flow through the transistor, and none will flow through the capacitor, thus making the integration constant equal to zero. When the transistor is switched off, an accurately timed pulse is delivered.
Mono-stable multivibrators with both external transitions and modulated parameters

In fig.3.17 a state model representation and a practical implementation of a mono-stable multivibrator has been depicted in which the two phenomena described above are both present. The external transition from state (a) to state (b) is intended and does not affect the accuracy of the residence-times of states (b) and (c). The signal causes the Schmitt trigger to switch. The signal also modulates the integration constants of states (b) and (c); they are transformed into (b') and (c'). State (c') is a degenerated state. It has already been shown that this does not need to cause problems. The pulse time of mono-stable multivibrators like this is usually determined by the residence-time of state (b). It can be seen that this residence-time is seriously affected due to the modulation by S. So S should be set to zero as soon as the transition from (a) to (b) has taken place. For the mono-stable multivibrator of fig.3.16, errors due to an inaccurate timing of S were avoided by degenerating the state that causes the problems (b'). In this case this is impossible since the pulse time of the multivibrator is determined by the residence-time of this state. In fig.3.18 an extra capacitor has been added in the electronic circuit which has to prevent the modulation of the integration constant. The capacitor will reduce the modulation of the integration constant especially for low frequencies, but still some influence is left. It mainly depends on its value, which should be small compared to the value of the capacitor that acts as the integrator. However, in these circuits a temporary transformation form (b) to (b') cannot be avoided. The exact duration of this transformation depends on the behavior of S, which may be unknown.
3.3.3 Oscillators

Oscillators are able to produce periodical signals without the interference of external signals. They contain no degenerated states. In most first order oscillators all states are auto-excited. The only exceptions are found among the coupled oscillators, that will be described in the next section. During steady state operation the oscillator cycles through a loop of states, in which opposite signs have to be present. Most first order oscillators are two-state oscillators, described by the state-model representation of fig.3.19. The integrator signal appearing in the oscillator has been depicted in fig.3.19 also. The integrator signal changes periodically between

\[ E_a \quad \text{and} \quad E_b \]

which are the reference levels of state (a) and state (b) respectively. In this case the integration constants have equal magnitudes.

Ideally, the sum of the residence-times of the states \( T_a \) and \( T_b \), is equal to the period of the oscillator signal. The transitions are supposed to be infinitely fast. In practice, especially for high frequencies, the transition times have to be taken into account also. The period of the oscillator is:

\[ T = T_a + T_b + T_{a\rightarrow b} + T_{b\rightarrow a} \]  \hspace{1cm} (3.13)

\( T_{a\rightarrow b} \) and \( T_{b\rightarrow a} \) are the transition times.

The application of first order oscillators usually requires the oscillators to be electronically tunable. When the parameters of the states are modulated, voltage controlled oscillators (VCO's) or current controlled oscillators (CCO's) can be made. In that case the oscillators can for example be used in phase lock loops (PLL's). From (3.3) it was already seen that the residence-time of a state can be proportional or inversely proportional to a modulating signal. This implies that either the frequency or the period of a first order oscillator can be linearly modulated by an external signal, when the transition times are negligible. The finite transition times will introduce non-linearity. The most simple case is found when the parameters of the transitions are not influenced by the modulating signal at all.

Suppose the integration constants of a first order oscillator are modulated in order to obtain a linear modulation of the frequency. This implies that (a part of) the integration constant of the states is changed proportionally to the modulating signal. For state (a) and state (b) the
residence-times are respectively:

\[ T_a = \frac{E_a - E_b}{\alpha_a} \]  \hspace{1cm} (3.14) \\
\[ T_b = \frac{E_b - E_a}{\alpha_b} \]  \hspace{1cm} (3.15)

When \( \alpha_a = -\alpha_b = \alpha \), the frequency of the oscillator is:

\[ f = \frac{1}{T} = \frac{1}{\frac{E_a - E_b}{\alpha} + \frac{E_b - E_a}{-\alpha} + T_{a\rightarrow b} + T_{b\rightarrow a}} \]  \hspace{1cm} (3.16)

After some calculations it follows:

\[ f = \frac{\alpha}{2(E_a - E_b)} \left( \frac{1}{1 + \alpha^2 \frac{T_{a\rightarrow b} + T_{b\rightarrow a}}{2(E_a - E_b)}} \right) \]  \hspace{1cm} (3.17)

For small transition times this can be approximated by:

\[ f = \frac{\alpha}{2(E_a - E_b)} - \frac{\alpha^2}{4(E_a - E_b)^2} (T_{a\rightarrow b} + T_{b\rightarrow a}) \]  \hspace{1cm} (3.18)

A quadratic distortion results that is proportional to the transition times. Enlarging the difference between the two reference levels \( E_a \) and \( E_b \) will not reduce the distortion. When oscillators are compared, they must generate signals with the same frequencies. When the difference between the reference levels is changed, the integration constants have to be changed with the same factor, so the distortion is not reduced.

When there is influence of the modulating signal on the transition times, more than just the quadratic distortion of (3.18) may result. Sometimes a modulating signal that is intended to influence the integration constants, also has some influence on the reference levels of the states. This causes distortion also, but very often the (parasitic) modulation of the reference levels can be determined easily, since it is a property of a state. Compensation of this non-linearity may therefore be possible.

### 3.4 Synchronized and coupled oscillators

When an external signal is available with a well defined time behavior, it can be used to influence the timing of a first order oscillator. Then the first order oscillator will take over the timing properties of the external signal. The periodical signal generated by the first order oscillator becomes related to the external signal.

The origin of the external signal can be either a second first order oscillator , or a completely different system. An example of the first case is the master–slave first order oscillator. One first order oscillator, the slave, is synchronized with a second free-running first order oscillator, the master. The phase relation between the two oscillators can be controlled electronically. An example of the second case is the synchronization of a first order oscillator with a crystal time-base. Synchronized first order oscillators will be dealt with in subsection 3.4.2.

It is also possible to synchronize two first order oscillators to each other. A signal derived from the first oscillator is used to synchronize a second, and a signal from the second is used to synchronize the first oscillator. These coupled systems will be dealt with in subsection 3.4.3. Especially these coupled systems are interesting, since they can have very special properties, that will be discussed in subsection 3.4.4. Now first the different ways to extract signals for synchronization from a first order oscillator, and the different ways to inject signals into a first order oscillator to synchronize it, will be introduced.
3.4.1 Generation and use of synchronizing signals

Signals can be derived from a first order oscillator that are related to the time variant signals in the oscillator. The synchronizing signals discussed in this section will all be of the binary type. They can only have two different values. This restriction is made for convenience. In practice usually synchronizing signals for first order oscillators can be seen as binary signals. The signals are used to synchronize first order oscillators by either invoking a transition or by transforming a state, in which case the state parameters are modulated.

When they have to invoke a transition, they may succeed or fail to do that, so they may be capable or incapable for it which is a binary condition. Later (chapter 4) it will be shown that fast changing external signals are the best for invoking transitions (e.g. for optimal noise behavior). Binary signals ideally make infinitely fast transitions, so optimal practical signals for synchronization generally show a binary behavior.

When the transformation of states by modulating the state parameters is used as method to synchronize first order oscillators, PLL-like systems result. Phase lock is obtained by way of tuning the oscillators. Only some very special cases of this synchronization method will be discussed in this thesis. The synchronizing signals will be restricted to binary types, and are supposed to be derived from other oscillators directly and not from phase detectors.

Extra symbols have to be added to represent the extraction of a signal from a first order oscillator. In fig.3.20 the two different options have been depicted. The first method to derive

![Symbols to represent the extraction of signals](image)

Fig. 3.20 Symbols to represent the extraction of signals

a synchronizing signal from a first order oscillator is comparing the integrator signal with a certain reference level by means of a comparator. When the integrator signal crosses the reference level the comparator output signal changes from one limiting level to the other. The output signal of the comparator is a binary signal. In fig.3.20a the representation of this method has been depicted. The synchronizing signal originates from within the state. Ideally, the state parameters themselves are not influenced at all by this method.

The second method is to detect transitions in the first order oscillator. This is done by way of comparing the actual integration constant, or signals related to it, with a reference level by way of a comparator. (In practice most signals appearing in the memory suffice.) A binary signal results. In fig.3.20b the representation of this method has been depicted. The synchronizing signal originates from a transition. Ideally, the state parameters and the transition parameters are not influenced by this method.

Sometimes the time-derivatives of the synchronizing signals are used in stead of the signals themselves.

Examples

Some examples of the different ways of using the synchronizing signals have been depicted in fig.3.21, fig.3.22 and fig.3.23. Only the transitions that are relevant for the examples have been depicted. In fig.3.21 a synchronizing signal is derived from the integrator signal within a state of the left hand oscillator. It is produced by a comparator that compares the integrator signal
with a reference level. The signal is used to initiate a transition in the right hand system. This can be done by adding the comparator output signal to the integrator signal.

![Diagram](image1)

Fig. 3.21 A synchronizing signal derived from a state that initiates a transition

In fig. 3.22 a synchronizing signal is derived from a transition of the left hand oscillator. In this case this signal can be derived from the signals appearing in the memory. When the left hand oscillators changes state, the memory contents is changed. So the memory changes state also and the transitions in the memory can be used to generate the synchronizing signal. The signal is used to initiate a transition in the right hand system, thus synchronizing the transitions.

![Diagram](image2)

Fig. 3.22 A synchronizing signal derived from a transition that initiates a transition

In fig. 3.23 again a synchronizing signal is derived from a transition of the left hand oscillator. Now the signal is used to transform a state of the right hand system. Depending on the value of the signal the value of the integration constant is reduced or enlarged. In this case the signal is derived from the memory of the left hand oscillator. Depending on the state of the oscillator, the memory can contain two different values that represent the different values of the integration constant. This memory contents is used to make the synchronizing signal.

![Diagram](image3)

Fig. 3.23 A synchronizing signal derived from a transition that transforms a state

The synchronizing signal in the first example is derived from the integrator signal, in the latter two it is derived from the integration constant. In all cases it is derived by way of a comparator function. The time-derivatives of the comparator signal can be used also, but in the third example it is not of much practical use. In practice synchronizing signals related to integration constants, are usually derived from signals appearing in the memory. These
signals already have a binary character, so a separate comparator is unnecessary. Only some scaling and level shifting may be needed. When synchronizing signals are derived from the integrator signal a separate comparator function will be needed to generate a binary signal. Of course also a scaled and level shifted version of the integrator signal itself may be used as synchronizing signal, but this situation will not be discussed here.

### 3.4.2 Synchronized oscillators

Synchronized oscillators contain transitions that take place at points of time determined by an external signal. The external signal can establish this by way of initiating an external transition, or by way of modulation of the state parameters. A common example of the latter type is found in PLL systems. For now oscillators will be dealt with in which external transitions occur. The external transitions are initiated by external signals that are added to the integrator signal. All states are auto-excited in the examples discussed in this subsection, but this is not necessary. When for example the external signal applied to the mono stable multivibrator of fig.3.13 is a periodical signal, the output signal of the multivibrator will be a periodical signal with the same frequency and with one transition that is synchronized with the external signal. In fig.3.24 a synchronized first order oscillator and its integrator signal have been depicted. If no external synchronizing signal is present, the integrator signal changes periodically between the two indicated reference levels. When such a level is reached the sign of the integration constant is changed. When the oscillator is in state (a), the external signal can initiate an external transition to state (b). This is what happens at the second and the subsequent pulses of the external signal S. Then the residence time of state (a) is shortened, so the integrator signal does not reach $E_a$ anymore. Because of this the residence time of state (b) becomes shorter also. As a result the period of the oscillator—which is the sum of both residence times—becomes shorter, such that it becomes equal to the period of the external signal. Now the oscillator is synchronized with the external signal, it produces a periodical signal with a frequency higher than the free running frequency. State (a) has lost its auto-excited properties, state (b) is still auto-excited.

According to (3.6) the sum of the external signal and the integrator signal has to be larger than the reference level of the state in which the oscillator is, in order to obtain a transition. For a given amplitude of the external signal there is a value of the integrator signal at which the sum becomes large enough to initiate the transition. In the example of fig.3.24 the external signal was assumed to be large enough by itself always, in fig.3.25 the integrator signal has to have reached at least the value of $E_{sa}$ before the sum of the external signal and the integrator signal can become larger than the reference level. This level has been indicated in the figure. The first time a pulse in S appears after the integrator signal has reached the value $E_{sa}$ a transition
Fig. 3.25 A first order oscillator with one synchronized transition and limited sensitivity

from (a) to (b) is made. This implies that the oscillator will be insensitive for some of the pulses in S. Pulses that initiate a transition are indicated with a large arrow. The transitions from (a) to (b) are synchronized with the external signal, but the oscillator frequency is a sub-harmonic of the external signal. Again state (a) has lost its auto-excited properties.

In the system of Fig. 3.26 both transitions are synchronized by a signal at a frequency higher than the oscillator frequency. This first order oscillator also produces a sub-harmonic of the external signal. Since both transitions are external, the auto-excited properties of both states are lost. Internal transitions of the oscillator, that occur when the oscillator is not synchronized, show phase jitter that may be intolerable in certain applications. When a high frequency external signal is applied to the first order oscillator, that is for example generated with a crystal time-base, the transitions will show the jitter of that time-base. Temperature drift and noise will have no effect on the transitions, until their influence is so large that the transitions synchronize to another pulse of S. A highly stable first order oscillator results that can be tuned in discrete steps. The higher the frequency of the synchronizing external signal, the smaller the steps can be. Oscillators like this are called raster oscillators. They combine the large tuning range of first order oscillators with the stability of for example crystal reference oscillators.

Fig. 3.26 A first order raster oscillator
Master–slave oscillators

In the three synchronized first order oscillators of fig.3.24, fig.3.25 and fig.3.26, the external signal can be generated in a second first order oscillator. This second oscillator is called the master and the synchronized oscillator is called the slave. The synchronizing signal can be derived from a transition, or from the integrator signal of the master. In the first case transitions become synchronized. When the two oscillators produce equal frequencies the output signals are in phase. In the second case, a comparator is used to generate the synchronizing signal. Then the synchronizing pulses can be generated at any point of time in the period of the master oscillator. Since the slave oscillator synchronizes with these pulses, the phase difference between the two oscillator output signals can have any value. It depends on the parameters of the comparator. Examples of both cases have been depicted in fig.3.27.

Fig. 3.27 The state-model representation of two different master–slave oscillators

3.4.3 Coupled oscillators

Coupling oscillators is done very often to obtain an improved noise behavior. When for example two crystal oscillators are coupled [30], their carriers become strongly correlated and most of their noise sources remain uncorrelated. When both output signals are added, the carrier power doubles (3dB), and the noise power is halved (3dB) if all noise sources are uncorrelated. The coupled system will theoretically show a noise behavior that is improved with 6dB compared to a single oscillator. (About 5dB was measured in [30].) Based on this principle, coupling first order oscillators should also improve the noise behavior with a maximum of 6dB. However, for first order oscillators there is more.

When Schmitt triggers are used in first order oscillators, they introduce phase jitter. In chapter 4 it will be shown that the reaction of a Schmitt trigger to noise depends on the properties of the large signal at its input [2]. A non-linear relation exists between the phase jitter at the output of a Schmitt trigger and the noise at its input. Coupling first order oscillators can be used to improve the noise behavior of the Schmitt triggers, and consequently the coupling may result in an improvement of the noise behavior of more than the theoretical 6dB. Later it will be shown that regenerative circuits are only really suitable to be used as memories. Although they can perform other functions also, as described in chapter 2, there usually are other circuits that can do those things better.
Apart from a reduction of the phase noise, also some other properties of coupled first order oscillators make them of interest. For example quadrature signals are easily generated and residence-times can be influenced independently of each other. Now first different ways to construct coupled first order oscillators and their properties will be discussed, and afterwards two examples will be given of practical electronic circuits.

**Coupled oscillators, using mono-stable multivibrators**

Coupled first order oscillators using mono-stable multivibrators are not often recognized as coupled oscillators. The oscillators of which a practical coupled system is composed usually share many components, but always two integrators are present. For both integrators integration constants and thus states can be defined, and in this case some of these states are degenerated. In fig.3.28 a system of two coupled mono-stable multivibrators has been depicted. For the states (a), (c), (d) and (f) the residence-times \( T_a, T_c, T_d, T_f \) can be defined respectively. When:

\[
T_c < T_f
\]  

and:

\[
T_d < T_a
\]

the periodical signal generated by this system has a period \( T_{sys} \) equal to:

\[
T_{sys} = T_a + T_f
\]

The contributions of the transitions to the period are neglected. \( T_c \) and \( T_d \) need not to be accurate as long as (3.19) and (3.20) are valid. The system consists of two separate mono-stable multivibrators that each contribute to the period with one residence-time. Modulation of the parameters of one of the mono-stable multivibrators needs not to have influence on the parameters of the other mono-stable multivibrator. This property can be used advantageously when the two residence-times \( T_a \) and \( T_f \) have to be accurate while their magnitudes have to differ much. For example when a pulse train has to be generated, consisting of short pulses of a well defined duration, that lie far apart in time, \( T_a \) could be made (nearly) equal to the period of the pulse train and \( T_f \) could be made equal to the pulse width. The period of the output signal can then be controlled by way of modulation of the parameters of mono-stable multivibrator M1, which will have no influence at all on the pulse width. When signals like this are made with a single first order oscillator, the difference between the residence-times is obtained by way of large changes in the magnitude of the integration constant when the oscillator changes state. Especially when the difference in magnitude is large, it is difficult to do this accurately and it
is more complicated than just changing the sign of an integration constant. In an electronic version of the coupled system discussed in this section, the integrators would be capacitors, and a large difference between the integration constants can be obtained by using capacitors with different values. There are many applications of systems like this (using two integrators), for this reason. An example of such an application if given in section 3.5.1 in which a time base generator for an oscilloscope is described.

When the conditions of (3.19) and (3.20) are not met, for example when $T_d > T_a$, depending on the synchronizing signal and on the nature\(^2\) of the degenerated state different things might happen.

When the degenerated state is not degenerated due to limiting, but the effect of the memory contents, $M2$ is insensitive for the synchronization signals coming from $M1$ as long as it is in state (d). When the synchronizing signal is a pulse—so it is the time-derivative of a binary signal extracted from $M1$—and $M2$ has not yet reached state (e) when it occurs, $M2$ will never leave state (e) again when it is entered. The pulse that would have made $M2$ leave the state has gone. Finally the systems end up with both mono-stable multivibrators in their degenerated states.

When the degenerated state is degenerated due to limiting, and again a pulse is used as synchronizing signal, a synchronizing pulse delivered when $M2$ is still in state (d) might invoke a transition directly from state (d) to state (f), see section 3.2.3. This will happen only when the pulse is large enough. In that case state (d) looses its auto-excited properties, which means that the value of the integrator signal is no longer equal to the reference level of state (d) when state (f) is entered. This implies that $T_f$ is changed since this residence-time depends on that (starting) value. Now the modulation of the parameters of $M1$ influences $T_f$ also.

Suppose state (d) is entered via an internal transition, so the integrator signal at that time is equal to $E_f$, which is the reference level of state (f). State (d) is left due to a pulse produced when the residence-time $T_a$ expires. Then the value of the integrator signal $E_{o2}$ of $M2$ is:

$$E_{o2} = E_f + \alpha_d T_a$$  \hspace{1cm} (3.22)

This is the starting value $\dot{E}$ in (3.3) for the integrator signal in state (f). The new residence-time of state (f) is:

$$T_f' = \frac{E_f - \dot{E}}{\alpha_f} = -\frac{\alpha_d}{\alpha_f} T_a$$  \hspace{1cm} (3.23)

(Please note that the integration constants $\alpha_d$ and $\alpha_f$ have opposite signs, so $T_f'$ is positive.) When condition (3.20) had been fulfilled, $T_a$ in expression (3.22) would have been replaced by $T_d$, so then $T_f'$ would be determined by properties of $M2$ only. Now $T_f'$ is influenced by properties of $M1$ also. The period of the system is:

$$T_{sys}' = T_a + T_f' = \left(1 - \frac{\alpha_d}{\alpha_f}\right) T_a$$  \hspace{1cm} (3.24)

When the pulse is not large enough to invoke the direct transition from state (d) to state (f), the system will normally enter state (e) after the residence-time of state (d) has expired, and stay there for ever since the pulse that could have invoked the transition to state (f) has gone. Again the system will end up with both mono-stable multivibrators in their degenerated states.

When the synchronizing signal is a step signal, and it occurs when $M2$ is in state (d) it may not be large enough to initiate a transition at once. However, when the integrator signal has grown somewhat until the sum of both signals is large enough, a transition may finally occur. The properties of this transition will be as if it were an internal transition, because the external

---

\(^2\)It might be degenerated due to limiting or due to the memory contents, see section 3.2.3

45
signal does not invoke the transition, it merely created the right conditions for it to happen. When state (e) is not degenerated due to limiting, M2 will immediately make a transition from state (e) to state (f) after it has made the transition from state (d) to state (e). This is because the external signal that can invoke the second transition is already present when state (e) is entered. This is what will happen in the system of fig.3.28 since direct transitions that skip the degenerated states are not present. Obviously in this system the degeneration does not occur due to limiting. When for example in this system condition (3.20) is not met, integrator the expression for the period \( T_a \) is replaced by \( T_d \).

\[
T_{sys} = T_d + T_f
\]  

(3.25)

Now the period depends on the residence-times of only one of the mono-stable multivibrators, and most advantages of the coupled system are lost.

The use of step signals in stead of pulses for synchronization will prevent the system from being stopped with both mono-stable multivibrators in their degenerated states, when the conditions of (3.19) and (3.20) are not met. Depending on the nature of the degenerated states, the residence-times of the states following the degenerated states become affected, or the residence-times of state (c) or state (d) will replace one of the other residence-times in the expression for the period.

**Coupled oscillators, using single first order oscillators**

The synchronization signal that is used in section 3.4.2 can be derived from a second (identical) first order oscillator. When this second oscillator is synchronized with signals derived from the first oscillator a *coupled first order oscillator* is created. In fig.3.29 the basic structure of such a system has been depicted. Both first order oscillators synchronize each other, so no particular

![Fig. 3.29 The basic structure of a coupled first order oscillator](image)

oscillator can be pointed out as “master” or “slave”. Coupled first order oscillators show a several advantageous properties compared to the single first order oscillators that are used to construct them.

When the synchronizing signals are used to initiate transitions, two different types of coupled oscillators can be distinguished if the coupled system is kept symmetrical.

In fig.3.30 the state-model representation of a coupled first order oscillator has been depicted in which the synchronizing signals are derived from transitions. The first order oscillator that is the first to make a transition will also initiate a transition in the other oscillator. Since the coupled system is symmetrical, the only phenomenon that can make one oscillator faster than the other is noise. This implies that in the average both oscillators will have an equal influence on the transitions of the coupled system.

When, for a single first order oscillator, only the starting point in time of a transition depends on white noise, that is added to the integrator signal (or the reference level), a Gaussian probability density can be assumed for the time at which the transition takes place. This is discussed in more detail in chapter 6. Now the probability density of a transition of the coupled system will be calculated, when both first order oscillators show identical Gaussian
probability densities. The probability density of oscillator A and oscillator B are $p_A(t)$ and $p_B(t)$ respectively, and the transition in a noiseless first order oscillator is supposed to take place at $t = 0$. Then it follows:

$$p_A(t) = p_B(t) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \frac{t^2}{\sigma^2}} \tag{3.26}$$

$\sigma$ is the standard deviation.

The two noise processes in both oscillators are supposed to be uncorrelated. The probability that a transition takes place in oscillator A for $t \geq t_1$ is given by the probability distribution $P_A(t)$:

$$1 - P_A(t_1) = 1 - \text{prob}\{t < t_1\} = 1 - \int_{-\infty}^{t_1} p_A(t) dt = \int_{t_1}^{\infty} p_A(t) dt \tag{3.27}$$

A similar expression is found for oscillator B. The probability distribution of the coupled system, $P_S(t)$ is the product of the probabilities of both oscillators. Since when the system transition has to take place later than $t_1$, in both oscillators the transitions have to be later than $t_1$. Then it follows:

$$1 - P_S(t_1) = 1 - \int_{-\infty}^{t_1} p_S(t) dt = \int_{t_1}^{\infty} p_A(t) dt \int_{t_1}^{\infty} p_B(t) dt \tag{3.28}$$

The probability density function $p_S(t)$ is found by differentiating the distribution function. This results in:

$$\frac{dP_S(t_1)}{dt} = p_S(t_1) = P_A(t_1) \int_{t_1}^{\infty} p_B(t) dt + P_B(t_1) \int_{t_1}^{\infty} p_A(t) dt \tag{3.29}$$

This is the sum of two conditional probability densities: the probability oscillator A makes a transition while oscillator B still has not, and vice versa. Since the oscillators are identical, the distribution functions are identical also, which simplifies expression (3.29):

$$p_S(t_1) = 2p(t_1) \int_{t_1}^{\infty} p(t) dt \tag{3.30}$$

With:

$$p_A(t) = p_B(t) = p(t) \tag{3.31}$$

In fig. 3.31 the probability density of the transitions of the coupled system $p_S(t)$ has been plotted as a function of the difference between the actual time at which the transition takes place, and the time at which the transition of the noiseless system would have occurred. For comparison also the probability density of one of the oscillators of which the coupled system consists is plotted in the same figure. It can be seen that on the average the transitions of
Fig. 3.31 The probability density of the transitions of the coupled system (fat) and the probability density of the transitions of one single oscillator (thin)

the coupled system will come sooner than those of a single oscillator, so the frequency of the periodical signal produced by the coupled system will be somewhat higher. Also the width of the density function (the standard deviation) is reduced. This reduction of the uncertainty in the timing of the transitions results in a better frequency stability of the coupled system. Of course, the amount of oscillators present in the coupled system needs not to remain restricted to two. When more oscillators are used, a larger reduction of the uncertainty is obtained. However, the reduction is less than proportional to the number of oscillators, so every time an extra oscillator is added, a smaller improvement is obtained.

The phenomenon that reduces the uncertainty in coupled first order oscillators like this can be compared to that in a laser, in which the output signal is also generated by a large amount of "signal generators" that stimulate each other. Large oscillator arrays will produce very stable signals.

In fig.3.32 the state-model representation of a coupled first order oscillator has been depicted in which the synchronizing signals are derived from the integrator signals. This is done by way of comparing the integrator signals with a certain reference level. One of the examples of electronic implementations in section 3.5, is a coupled oscillator that can be described by fig.3.32. This way of coupling first order oscillators has some very advantageous properties. This will be explained now. When in a single first order oscillator the integrator signal reaches the reference level, the sign of the integration constant is switched. This implies that the integrator signal starts moving away from the reference level again, so the transition that is to follow tends to neutralize its own cause.

This negative feedback mechanism is present in all auto-excited states. It is able to slow down and sometimes even stop the transition. There are several different solutions to this problem. A delicate selection of the values of the different reference levels is a method that can be used to counteract the negative feedback. This method will be discussed in chapter 7. In regenerative oscillators the positive feedback loop in the Schmitt trigger usually has a loop gain that is sufficient to dominate the negative feedback loop. However, the behavior of the Schmitt trigger is still hampered by it.

In the oscillator of fig.3.32, the problem is fundamentally solved. When designed properly, all
transitions are initiated by the synchronizing signals. These signals are derived from integrator signals that are not directly influenced by the transition that takes place. When the integrator signal of one of the oscillators, crosses the reference level, thus invoking a transition in the other oscillator, the integration constant of the first oscillator is not switched. Now the transition in the second oscillator has no direct influence on its cause, so no negative feedback is present. A second advantage of this coupled system is the fact that the timing of the transitions is largely determined by the comparators, and the influence of the Schmitt triggers is reduced. Ideally the influence of the Schmitt triggers can be removed completely. In that case they could even be left out. Then a coupled first order oscillator as depicted in fig.3.33 is obtained. In this system the auto-excited states have been replaced by externally excited states. Ideally

there is no difference between the systems of fig.3.32 and fig.3.33. In practice due to losses in the integrators and delays, an amplitude stabilization circuit is needed. In the system of fig.3.32 the Schmitt triggers will stabilize the amplitude. Lacking Schmitt triggers, the system of fig.3.33 needs an additional stabilization circuit. Both types will be described in more detail in chapter 7 and chapter 8 respectively. Then also some electronic implementations will be given.

A very large advantage of systems like this is the fact that when the system is symmetrical, identical output signals that are exactly in quadrature, are available. This quadrature relation is frequency independent. Very often such signals are generated with the use of a first order oscillator, running on twice the desired frequency, and two dividers. This generally consumes more power, and the two dividers will introduce extra jitter. (Sometimes a first order oscillator on twice the frequency is not even feasible.)

A third way to couple two first order oscillators is shown in fig.3.34. In this case the synchro-
nizing signals are used to modulate the integration constants. Also in this case quadrature

![Diagram of coupled oscillator with coupling via modulation of integration constants](image)

Fig. 3.34 A coupled oscillator with coupling via modulation of the integration constants

output signals are feasible, but the quality of the oscillator is generally not as good as it can be in the oscillators as depicted in fig.3.32 and fig.3.33. Unlike the previous system, in this system all transitions remain internal transitions. The large arrows in fig.3.34 indicate the direction into which the state transformations occur during normal operation. It can be seen that in the left hand oscillator the integration constants are enlarged by the external signal before it makes an internal transition. This increase of the integration constant is usually favorable for many properties such as the noise behavior. In chapter 4 it will be shown that an increase of the integration constant reduces the sensitivity of a Schmitt trigger for disturbances. So the properties of the left hand oscillator are improved by this way of coupling. However, in the right hand oscillator the external signal decreases the integration constant before it makes its internal transition. Hence the properties of this oscillator are degraded by this way of coupling. The accuracy of the period of the coupled system is determined by all four transitions. Looking for example at the left hand oscillator, it can be seen that the periodical signal generated by this oscillator, depends on the two internal transitions and on the timing of the transformation of its states. So the oscillator with the improved internal transitions has become "infected" with the jitter of the degraded oscillator. Therefore the timing of the coupled system cannot be better than that of the worst oscillator it is constructed with. It can be found easily that coupled systems, in which a stable phase relation between the separate oscillators has to be present, will always contain one oscillator with a degraded and one oscillator with an improved behavior. So as far as frequency stability is concerned, this way of coupling is not to be preferred. It is however a valid way to obtain identical output signals with a predefined phase relation [15].

There are more configurations possible of coupled systems than described in this section. Permutations of the different ways of deriving and using the synchronizing signals will result in other systems. However, all consequences of a specific way of working with the synchronizing signals have been paid attention to in this section, and it is not complicated to investigate the properties of other coupled first order oscillators, as soon as their state-model representation is available.

### 3.4.4 Properties of coupled oscillators

There are basically three different ways to couple first order oscillators. The way that seems to result in the best coupled systems is the use of synchronizing signals that initiate external transitions, but that are not derived from transitions themselves. Then the influence the signals
have on the system will not cause their own origin to be counteracted. So the inherent negative feedback that is present in all other first order oscillators, is avoided in this case. Usually coupling oscillators improves the noise behavior. Only in the case of coupled systems in which the integration constants are influenced by the synchronizing signals, the noise behavior may become worse. Due to the coupling, noise will cause oscillators that are coupled in phase to produce a somewhat higher output frequency. Large arrays of oscillators that are coupled in phase can be used to produce very stable carriers. The phenomenon could be compared to LASERS. The effect of noise on the coupled oscillators with quadrature coupling will be discussed in chapter 6.

3.5 Examples

3.5.1 A time base oscillator of an oscilloscope

In fig. 3.35 a simple time-base generator for an oscilloscope has been depicted. Although circuits like this are not used in modern oscilloscopes anymore, it is a good example to demonstrate the use of the state-model description. The tube used is called a thyatron. It is a gas-filled tube which has a low impedance from cathode to anode when the gas is ionized, and a high impedance when it is not. The voltage difference between cathode and anode that is needed to start the ionization $E_{ign}$ is higher than the minimal voltage that is needed to sustain ionization once it has started $E_{sus}$. When the voltage across the tube is between these two voltages, two different states of the tube are possible. It can be ionized or not. This depends on the conditions the tube encountered before. When the tube voltage has been higher than $E_{ign}$ before, the tube will be in the ionized state, and when the tube voltage was lower than $E_{sus}$ before, it is extinct. Hence the tube can be used as a memory. Two reference levels exist, $E_{ign}$ and $E_{sus}$, and the crossing of these reference levels determines the state of the tube. In this respect the tube behavior can be compared to that of a Schmitt trigger. It shows hysteresis.

When a capacitor is used as an integrator, and the tube is in the extinct state, the capacitor is charged via the resistor. The oscillator is in state A. When the oscillator is in steady state, the starting value of the capacitor voltage in this state is equal to the reference level of state B. This state B is left when the tube becomes extinct, so the capacitor voltage at that time is equal to $E_{sus}$. It is not difficult to calculate the time needed for the capacitor voltage to reach $E_{ign}$, since when the tube is extinct, only a RC-network remains. So an accurate residence-time of state A is feasible. When the resistor is chosen to be very large, it behaves like a current source. Then the capacitor voltage will change linearly with time. Although this is not necessary to obtain an accurate residence-time, it is of course a must for a time-base oscillator of an oscilloscope. When $E_{ign}$ is reached, the tube becomes ionized. The impedance of the tube becomes very low compared to the resistor, so the capacitor is discharged quickly.
When the oscillator is in this state, the residence-time is not (and needs not to be) accurate. The oscillator is in the so-called "fly-back" state, in which the spot on the screen is brought into the starting position again. Via a small resistor in series with the cathode of the tube a blanking signal for the spot could be derived. This is an example of a signal derived from the memory directly.

The oscillator has to be synchronizable in order to be of any use in an oscilloscope. This is achieved by way of the grid in the tube. When a positive voltage \( S \) is applied at time \( t_s \) to this grid, the ignition voltage of the tube is lowered. So the residence-time of state A can be made shorter. It is an example of the synchronizing method discussed in this chapter, in which the external synchronizing signal is added to the integrator signal. When the tube is ionized, the grid has no influence. Looking at the state-model representation it can be seen that an oscillator is obtained in which one of the transitions can be externally initiated. The external signal can only reduce the residence-time of state A. Hence the oscillator can be synchronized when its free running frequency is chosen somewhat smaller than the (fundamental) frequency of the signal it has to be synchronized with. In fig.3.36 the capacitor voltage and the capacitor current have been plotted as functions of time. It can be seen that the capacitor voltage alternates between the two reference levels, and that the discharge current when the thyratron is ionized is of larger magnitude than the charging current. When at \( t = t_s \) a synchronizing pulse is applied to the grid the ignition voltage is temporarily lowered (for the duration of the synchronization signal).

![Diagram](image)

**Fig. 3.36** The capacitor voltage and the capacitor current in the time-base oscillator.

When the residence-time of state A becomes shorter, the amplitude of the saw-tooth voltage across the capacitor reduces also. This will result in a shorter time axis on the screen. This is an effect that can be observed in all oscilloscopes that are equipped with time-base oscillators that work like this. When large frequency differences occur, dramatic reductions of the time axis on the screen can be observed. It should be kept in mind that only the length of the trace is shortened, but the scale (e.g. \( \frac{\Delta S}{\Delta t} \)) remains unaltered. This scale is only determined by \( R \) and \( C \), and not by \( E_{sus} \) and \( E_{ign} \). The accuracy of the scale only depends on the basic accuracy of the first order oscillator, which is the slope of the integrator signal. In this respect the basic accuracy of a harmonic oscillator could not be used directly. The slope of the output signal of a harmonic oscillator can be made well-determined only by way of extra circuitry (e.g. amplitude stabilization). Also the slope would be frequency dependent, which is of course very inconvenient for an oscilloscope, so the amplitude stabilization would have to be made frequency dependent too.

This example shows that obviously harmonic oscillators are best suited for generating accurate frequency references, and first order oscillators are best suited for generating accurate time references.

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3.5.2 A coupled first order oscillator

A symmetrical two-integrator oscillator as described in section 2.5.2 can be described with the state-model representation of fig.3.33. An electronic implementation of such an oscillator has been depicted in fig.3.37. The amplitude and thus the frequency of the oscillator is determined

Fig. 3.37 A symmetrical two-integrator oscillator

by the sum of the initial voltages across the capacitors when the oscillator is started. The sum of these two initial voltages is called the starting value of the oscillator. Due to losses and delays this starting value becomes corrupted. So in practice additional circuitry is necessary to preserve the starting value. This can be done by way of an amplitude detector and some a.g.c. action or by way of Schmitt triggers. The latter method has been used in this coupled oscillator. In fig.3.38 a reduced circuit diagram of the oscillator has been depicted. It can be described with the state-model representation of fig.3.32. Ideally the Schmitt triggers have no influence on the circuit at all. Since in that case the circuit itself would completely preserve its starting value, and no correcting action of the Schmitt triggers is needed. In that case the fact that due to the Schmitt triggers all states have become auto-excited is of no importance. The behavior is not changed. In practice the fact that the states have become auto-excited will guarantee a stabilized amplitude. From the state-model representation of fig.3.32 it can

Fig. 3.38 A coupled oscillator (a two-integrator oscillator), with Schmitt triggers that stabilize the amplitude

be seen that the oscillator can also be seen as a set of two coupled first order oscillators, since
any of the two integrator sections could also oscillate independently. This is why oscillators like this are also called coupled first order oscillators. Not all symmetrical two-integrator oscillators are coupled first order oscillators, but every coupled first order oscillator is intended to behave like a symmetrical two-integrator oscillator. In chapter 7 the coupled oscillator will be discussed in more detail, and some measurement results will be presented. Another electronic implementation of a symmetrical two-integrator oscillator, which is not a coupled first order oscillator, will be discussed in chapter 8.

3.6 Classification review

In this chapter as well as the previous chapter it has been shown there are different ways to construct a first order oscillator. In fig.3.39 a classification diagram is depicted. In the large class of oscillators one of the subclasses is that of first order oscillators. In chapter 2 the definition of a first order oscillator has been given.

![Classification diagram for first order oscillators](image)

Fig. 3.39 A classification diagram for first order oscillators

Two main families of first order oscillators can be distinguished at this level. The distinction is made according to the type of memory used. When a regenerative memory is used, e.g. a Schmitt trigger, the oscillator belongs to the class of regenerative oscillators.

The memory function could also be obtained by way of a second integrator. In that case no regenerative circuits are found in the basic oscillator concept. This class is the class of the
two-integrator oscillators. Although in practice most first order oscillators are regenerative oscillators, the existence of the two-integrator oscillator class should not be forgotten. Some very interesting oscillators that will be discussed later belong to this class. Not every first order oscillator is a regenerative (relaxation) oscillator!

On the next level of the classification the oscillator classes are subdivided further. The class of single regenerative oscillators is the largest class among the first order oscillator classes. Nearly all practical first order oscillators belong to this class.

When a first order oscillator system is constructed, in which a number of first order oscillators are coupled to each other, the system, when viewed at the output terminals, can be looked upon as a so-called coupled first order oscillator. Such oscillators belong to the coupled regenerative oscillator class.

The two-integrator oscillator class can also be subdivided into two separate classes. When the second integrator introduced for the memory function, is chosen identical to the first integrator, the oscillator belongs to the symmetrical two-integrator oscillator class.

It is also possible that the second integrator is actually optimized for just the memory function and that the influence on the residence-times is minimized. Such an integrator is usually implemented as a sample and hold circuit. In that case the oscillator belongs to the class of sample and hold oscillators.

It has been shown that there are basically three different ways to couple first order oscillators, so the class of coupled regenerative oscillators can be subdivided into three classes. A special type of oscillator is found in one of these three classes: those that belong to the class of coupled oscillators with quadrature coupling via external transitions. This is because some oscillators that belong to the class of symmetrical two-integrator oscillators, are constructed in such a way, that they are identical to these coupled regenerative oscillators. The oscillators that belong to this class can be looked upon as either two-integrator oscillators or as coupled regenerative oscillators. In fig.3.39 they have been put in the same box. The classification of the oscillator merely depends on the starting point of the design. When the basic circuit consists of two identical integrators—so the oscillator is a symmetrical two-integrator oscillator—to which Schmitt triggers are added for amplitude stabilization, the oscillator would belong to the class of two-integrator oscillators with amplitude stabilization by way of Schmitt triggers.

When two regenerative oscillators are coupled in quadrature via external transitions, the start of the design is made with two separate regenerative first order oscillators. Then the coupled system would belong to the class of coupled regenerative oscillators with quadrature coupling via external transitions. The resulting system however does not differ from the two-integrator oscillator with amplitude stabilization via Schmitt triggers. This can be shown easily with the use of the state-model representation. The regenerative oscillator is represented by fig.3.32, and the basic symmetrical two-integrator oscillator by fig.3.33. The only difference between the two representations is the fact that the former one has auto-excited states and the latter has not. However, due to the amplitude stabilization via the Schmitt triggers, the states of the two-integrator oscillator also become auto-excited. Then there is no difference anymore. Usually coupling regenerative oscillators is intended to minimize the influence of the Schmitt triggers. It is clear that when the coupling is perfect, a coupled regenerative oscillator will behave like an ideal symmetrical two-integrator oscillator.

There are also two-integrator oscillators that contain an amplitude detector and some amplitude control loop. These oscillators are described according to fig.3.33. They contain no auto-excited states and consequently no regenerative circuits at all. Oscillators like this belong to the class of symmetrical two-integrator oscillators with amplitude stabilization via an undamping loop.
4. Regenerative memories

Most first order oscillators are regenerative (relaxation) oscillators. The word regenerative indicates that the memory function in the first order oscillator is performed by a regenerative circuit. Furthermore this circuit may also perform other functions (like current switching) which may well be assigned to other types of circuits.

Any oscillator can be constructed with at least two memories. In a first order oscillator one of these memories is a binary memory, which can contain only two different values. The other memory is an analog memory: the integrator.

In electronics most binary memories are regenerative. Hence this chapter is mainly dedicated to the behavior of regenerative systems. Their properties when used as binary memories will be discussed. There are also other applications for regenerative systems, like flip-flop sensors [16], but they will not explicitly be discussed in this thesis. Of course many properties of regenerative systems used as memories, are also common to the other applications.

Binary memories need not be regenerative. With, for example, sample and hold circuits combined with limiters, non-regenerative binary memories can be constructed also. At the end of this chapter these memories will be discussed and some examples will be given.

In the beginning of this chapter the behavior of regenerative systems will be explained. Starting from simple systems a qualitative description of the behavior will be given. The Loop-Gain Excitation model will be introduced to ease the understanding of the behavior. It is intended to clarify the reaction of regenerative systems external influences. Design rules will follow. Attention will particularly be paid to the qualitative aspects of measures a designer can take to optimize the behavior of a regenerative system. The usefulness of very complicated calculations is doubtful, since they usually only apply to one specific implementation of a regenerative system. Here, on the other hand, most of the introduced design rules are generally valid. Simulators can be used in a specific case to find the "exact" values for a certain application. When these values do not fulfil the demands, the design rules in this chapter will provide a way to obtain (fundamental) improvement. Since the regenerative systems are meant to be used as memories in first order oscillators, also specific design rules for first order oscillators will be discussed.

Memory systems other than the regenerative type can also be used in first order oscillators. Such non-regenerative binary memories will be discussed in the next chapter(s), where also some examples of systems comprising these memories will be given.

A simple simulation program has been written that makes use of the newly introduced model. It is mainly intended as an educational simulator, so it works on a rather abstract level. All parameters that are used in the Loop-Gain Excitation model are available in the simulator, so their influence on the system behavior can be investigated. Results of this dedicated simulator will be used as examples in this chapter. The same results could also be obtained with other simulators such as SPICE, (this has been verified), which, however, is much more time consuming.
4.1 Regenerative systems

Regenerative systems are called regenerative because they contain a part that "generates its own input signals". This is not as special as it seems. The fact that a part of the output signal of a system finds its way back to the input of the system again, is a very common phenomenon in modern electronics. It is known as feedback, a method which is for example of great use in the design of high performance amplifiers [3]. All these amplifiers partly generate their own input signal. In negative feedback systems, the returned output signal ideally annihilates an external signal at the input of the system. In that case the value of the output signal of the system is dominantly controlled by the external signal. Just when positive feed back is used, the output signal of the system that is returned to the input again, tends to become the dominant part of the complete input signal, and thus it dominantly controls the value of the output signal. Then it can be said the system "generates its own input signal", so it is regenerative. This will be discussed in more detail in the next section.

4.2 Linear feedback systems

Basically a regenerative system is just a feed back system. For now it is assumed to contain no non-linearity etc. In this section the behavior of a linear feedback system will be discussed. The influence of the loop gain $A_l$ on the behavior will be focussed on.

The system can be modeled as has been depicted in fig.4.1. A linear amplifier with an amplification factor equal to $A$ is put into a feed back loop. The output signal of the system is called $X$. In this chapter $X$ will be used as general output indication for feed back (memory) systems. It will be called the state variable of a system. All time-variant signals within a system depend on $X$. So any practical signal in the system is a function of $X$. In this system the loop gain is equal to the gain of the amplifier:

$$A_l = A$$  \hspace{1cm} (4.1)

The loop gain of the system is a small signal quantity, the gain of the amplifier is not. At this point this is not of great importance, but later, when non-linearities are introduced the difference becomes crucial.

When the system is in steady state, the time-derivatives of $X$ are equal to zero. The first thing to be done is the investigation of the steady state value of $X$, called $X_{ss}$, as a function of the input signal $E_i$. When the output signal of the amplifier is equal to $X_{ss}$, its input signal $E_{amp}$, is equal to:

$$E_{amp} = \frac{X_{ss}}{A_l}$$  \hspace{1cm} (4.2)

In steady state this value should be equal to the value of the output signal $E_{sum}$ of the adder. This output signal is equal to:

$$E_{sum} = E_i + X_{ss}$$  \hspace{1cm} (4.3)
Equating (4.2) and (4.3) results in the steady state value for $X$ as a function of $E_i$.

$$X_{ss} = \frac{A_l}{1 - A_l} E_i$$  \hspace{1cm} (4.4)

Two interesting values for $A_l$ seem to exist when (4.4) is examined. When $A_l = 0$, $X_{ss}$ is independent of $E_i$. From fig.4.1 it can be seen that this is a rather trivial conclusion, since when $A_l = 0$ no signal path exists from the input to the output. When $A_l = 1$ an infinite gain is found from $E_i$ to $X_{ss}$. Unity loop gain is exactly the separation between regenerative and non-regenerative behavior of a feedback system. When it is larger than one, regenerative behavior can be expected, and when it is smaller than one, the common non-regenerative amplifier behavior can be expected. This will be made more clear in the remainder of this section.

In fig.4.2 $X_{ss}$ has been plotted as a function of $E_i$ for various values of $A_l$. When a start is made with a large negative value for $A_l$, and then it is gradually changed towards a large positive value, the straight line that represents the relation between $X_{ss}$ and $E_i$ rotates counter clockwise. The rotation starts from the dashed line, and when $A_l$ is changed from $-\infty$ to $\infty$, a rotation of $180^\circ$ is made. For $A_l = 0$ a horizontal line is found and for $A_l = 1$ a vertical line. From fig.4.2 it can be seen that three intervals for $A_l$ can be distinguished.

<table>
<thead>
<tr>
<th>gain $A$</th>
<th>$-\infty &lt; A &lt; 0$</th>
<th>$0 &lt; A &lt; 1$</th>
<th>$1 &lt; A &lt; \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sign of loop gain $A_l$</td>
<td>negative</td>
<td>positive</td>
<td>positive</td>
</tr>
<tr>
<td>slope of the line</td>
<td>negative</td>
<td>positive</td>
<td>negative</td>
</tr>
</tbody>
</table>

Table 4.1

When for a certain value of $A_l$ the state of the system is such that it is a steady state—so the
operating point of the system is on the line—the system will not change state. This implies:

\[
\frac{dX}{dt} = 0
\]  

(4.5)

Now for the three intervals indicated in table 4.1 it will be investigated what happens when the steady state condition is disturbed. The time scale on which the events to be discussed take place, will not be considered now. Just the sequence of events is of importance at this moment. Later the timing will be taken into account also. For now it suffices to state that some events cause others to happen, so some events necessarily precede others. Since no slowness has been expressed in the model at this time, everything happens infinitely fast.

4.2.1 The first interval: \(-\infty < A_t < 0\)

As an example the value of \(A\) is chosen to be equal to \(-1\). The steady state curve of this system has been depicted in fig.4.3. When the steady state condition is disturbed, this can only be caused by a change of the input signal. Suppose \(E_i\) is changed with an amount of \(\Delta E_i\). Then the operating point first shifts into the horizontal direction. It is assumed that the system does not react as yet. In fig.4.3 this event has been depicted for the case in which \(\Delta E_i\) is positive. This new operating point is not on the steady state line, so the state of the system has to change.

The output signal of the adder is no longer equal to the steady state value, belonging to the state in which the system is at that time. It can be calculated easily that an “excess” signal equal to \(\Delta E_i\) is present at the output of the adder. This excess signal will be called excitation from now on. The symbol that will be used for the excitation is \(E_{exc}\), with:

\[
E_{exc} = E_i - E_{i_{ss}}
\]  

(4.6)

\[
= E_i - \frac{1 - A_i}{A_i}X
\]  

(4.7)

In this formula \(E_{i_{ss}}\) represents the value \(E_i\) should have if the state \(X\) in which the system actually is, has to be a steady state. Note that in (4.7) the state variable \(X\) has not been given the index \(ss\), since in this formula it is not imperative for \(X\) to be a steady state.

Fig. 4.3 \(X_{ss}\) as a function of \(E_i\) with \(A_t = -1\)
When the right steady state input signal, as given by (4.4), is applied the excitation is found to be equal to zero.

When a change equal to $\Delta E_i$ occurs in the input signal, then:

$$E_{exc} = \Delta E_i$$  \hspace{1cm} (4.8)

In this case the excitation is found as a part of the input signal of an amplifier with a negative amplification factor. Hence a positive excitation results in a negative change of state of the amplifier. So after a shift of the operating point from the steady state line to the right, the state of the system will change such that it goes down. A similar behavior is found when the operating point is moved to the left (negative excitation). Then the change of state will be such that the operating point is shifted upwards. In both cases the operating point moves towards the steady state line, so finally the system will end up in a new steady state again. This has been illustrated in fig.4.3 also. The two large arrows in fig.4.3 indicate in which vertical direction an operating point will move when it is not on the steady state line. This movement is always into the direction of the steady state line. In the figure the horizontal distance between the operating point and the steady state line is equal to the magnitude of the excitation. When the operating point moves, the excitation reduces.

**In negative feed back systems, the reaction of the system to excitation is such that the excitation is nullified.**

4.2.2 The second interval: $0 < A_i < 1$

Systems represented by the diagram of fig.4.1, in which $0 < A_i < 1$, are positive feed back systems. In fig.4.4 the steady state line of such a system (with $A = 0.2$) has been depicted. Again two large arrows indicate into which vertical direction an operating point moves when it is not on the steady state line. It can be seen that again the operating point always moves towards the steady state line. In this case the reaction of the system to positive excitation is a change of state in the positive direction. Compared with negative feed back systems the sign of this change is opposite, but the sign of the slope of the steady state line is opposite also (see table 4.1).

![Diagram](image.png)

**Fig. 4.4 $X_{ss}$ as a function of $E_i$ for a positive feed back system with $A = 0.2$**

60
In positive feedback systems, with a loop gain smaller than unity, the reaction of the system to excitation is such that the excitation is nullified.

4.2.3 The third interval: $1 < A_l < \infty$

In fig.4.5 the steady state line of a positive feedback system with $A_l = 2$ has been depicted. It is a system that belongs to the third interval of table 4.1. All systems with a loop gain larger than one, have a steady state line with a negative slope. When the system is in steady state and a positive change of the input signal occurs, a positive excitation results. Since the loop gain $A_l$ is positive, the system changes state in the positive direction. Again two large arrows in fig.4.5 indicate the vertical direction of the movement of the operating point when it is not on the steady state line. It can be seen that the reaction of the system to a non-zero excitation is such that the excitation is enlarged. Systems that tend to enlarge their excitation are called regenerative.

It can be seen from fig.4.5 that as the operating point moves up, the excitation keeps growing. In the end an "infinite" excitation results, compared to which the original input signal is neglectable. The system generates its own input signal.

In positive feedback systems with a loop gain larger than unity, the reaction of the system to excitation is such that the excitation is enlarged.

4.2.4 A special case: $A_l = 1$

A special case is found when $A_l = 1$. Circuits with a loop gain equal to unity are on the "edge" of being regenerative. In fig.4.6 the steady state line of such a system has been depicted. When the operating point is not on the steady state line, the system changes state in the direction indicated by the large arrows. The positive excitation combined with the positive loop gain $A_l$ results in a positive change of state of the system. However, a change of state does not influence the magnitude of the excitation. The operating point moves in parallel with the steady state line. When $A_l = 1$ is substituted in (4.7), this formula reduces to:

$$E_{exc} = E_i$$  \hspace{1cm} (4.9)
Fig. 4.6 $X_{ss}$ as a function of $E_i$ for a positive feed back system with unity loop gain

The value of the excitation only depends on the external input signal and not on the state of the system. This can be seen in fig.4.6.

_in positive feed back systems with a unity loop gain, the reaction of the system to excitation does not influence the excitation itself._

### 4.2.5 A special case: $A_t = 0$

When $A = 0$ the steady state line cannot be left via changes of $E_i$. Changes of $E_i$ will cause the operating point to move horizontally. Since the steady state line is a horizontal line, the operating point will remain on this line always. The excitation can be seen as the distance between the steady state line and the operating point, so the system will never be excited, and consequently it will never change state. Mathematically this situation is somewhat complicated since expression (4.7) is not valid for $A_t = 0$, but in (4.6) it can be seen that the excitation is equal to zero always, since in this system $E_i = E_{iss}$ always.

### 4.2.6 Summary

A new signal quantity has been introduced which is called excitation. Only when the excitation of a system is unequal to zero, it will change state. The reaction of a system to non-zero excitation depends on the value of the loop gain. When the loop gain is negative, the reaction to non-zero excitation will always be such that the excitation is annihilated. This is true for any negative loop gain.

When the loop gain is positive, but smaller than unity, the reaction to excitation is still such that it is annihilated. Only when the loop gain is larger than unity, the reaction of the system is such that the excitation is enlarged. When this happens the system is called regenerative. The excitation is neither reduced nor enlarged when the loop gain exactly equals unity.

### 4.3 Positive feedback systems containing a limiting amplifier.

When binary memories are to be constructed, regenerative systems can be used. The system of which the steady state line has been depicted in fig.4.5 already shows a memory function. When the input signal of the system is changed such, that it is brought from the steady state, to for example a state in which a positive excitation results, this positive excitation will be enlarged by the system. Since it will become extremely large, after this event the value of the
input signal is of no importance anymore. When the sign of the state variable is considered, it is equal to the sign of the excitation when the steady state line was first left. Apparently the system retains this information. Unfortunately it will never be possible to change this information, so this memory has a "write once" behavior, which makes it, without further measures, useless for first order oscillator design. It is possible to bring a regenerative system back into steady state by way of changes in the input signal. When it is changed with the amount:

$$\Delta E_i = -E_{exc}$$

the system enters steady state. When the change is larger than this value, the operating point can even be brought to the other side of the steady state line.

In the case of the regenerative system of fig.4.5, the excitation becomes infinite. This implies that an "infinite" change of the input signal would be necessary to bring the system into steady state again. Of course this is not a practical option. Some non-linearity is needed to limit the value of the excitation. This means that for a practical value of the state variable the system has to enter a steady state, which can be easily obtained by way of replacing the linear amplifier by a limiter which reduces the loop gain to zero as soon as the system reaches a predefined state. The transfer function of such a limiter has been depicted in fig.4.7. A long

![Diagram](image)

Fig. 4.7 The transfer function of a limiting amplifier (limiter)

as the limiter is in its linear region, \(|X| \leq E_l\), it has a gain equal to \(A\), so the loop gain is equal to \(A\) also. The value of the loop gain \(A_l\) and thus the value of \(A\) should be larger than one in this linear region in order to obtain regenerative operation. The limiter starts limiting when:

$$E_i > \frac{E_l}{A}$$

or when:

$$E_i < -\frac{E_l}{A}$$

When \(E_i\) is in either one of the two intervals, the output signal of the limiter becomes constant and equal to \(E_l\) or \(-E_l\) respectively. Then the effective large signal gain of the limiter, \(A_{eff}\), is equal to:

$$A_{eff} = \frac{E_l}{E_i}$$

More important is the value of the loop gain at this point. Small changes in \(E_i\) have no influence on the output signal \(X\) any more. The small signal amplification factor of the limiter has become equal to zero, and hence the loop gain has become equal to zero. Apparently in regenerative systems like fig.4.1.1 that contain a limiter, the loop gain becomes equal to zero as soon as limiting occurs. In that case a horizontal steady state line can be expected. In fig.4.8 the steady state lines of such a system have been depicted. Three lines can be seen in this
Fig. 4.8 The steady state lines of a regenerative system containing a limiter

The two horizontal lines are found when the loop gain reduces to zero which is the case when limiting occurs. Since limiting occurs for \( X = E_l \) and \( X = -E_l \), the horizontal lines are found at these levels. The lines are interconnected via a line with a negative slope. This is a steady state line belonging to a positive feedback system with a loop gain larger than unity. This situation applies when the limiter is in its linear mode.

The loop gain has become state dependent; \( A_l \Rightarrow A_l(X) \).

Two large arrows indicate the direction of the change of state of the system when the operating point is not on the steady state line. It can be seen that when it is not, it will always move to either one of the two horizontal lines. Apparently two states exist with \( X = E_l \) and \( X = -E_l \), in which the system is likely to be when it is in steady state. It may also be on the slanted steady state line that interconnects the two horizontal lines, but any change in the input signal will cause the system to leave this line, thus causing excitation that will drive the operating point of the system towards one of the horizontal lines. The event of such an unstable operating point will be discussed later on.

When fig.4.8 is compared to fig.4.5, it can be seen that as long as \(-E_l < X < E_l\), the system is regenerative. So when the system has a non-zero excitation, it changes state such that the excitation is enlarged. The operating point of the system containing the limiter will finally meet one of the two horizontal lines. Then the excitation reduces to zero and the change of state stops.

There are two values of \( E_l \) for which the horizontal steady state lines coincide with the slanted line. These two values are:

\[
E_{ih} = -E_l \left( \frac{1 - A_l}{A_l} \right)
\]  

and:

\[
E_{il} = E_l \left( \frac{1 - A_l}{A_l} \right)
\]

\( E_{ih} \) applies for the bottom most horizontal line and \( E_{il} \) for the top most line. Both values have been indicated in fig.4.8. When the input signal \( E_l \) is within the interval:

\[
[E_{il}; E_{ih}],
\]

two stable operating points on the horizontal steady state lines are possible, and one unstable operating point on the slanted steady state line. A stable operating point on a steady state line occurs when the excitation drives an operating point towards the line when it is close to that line. An unstable operating point occurs when the excitation drives an operating point away from the line when is close to the line. In this case the unstable operating point will
not be considered, because when the regenerative system is used as a binary memory in a first order oscillator, it usually does not encounter such operating points (but it does for instance in flip-flop sensors).

When the input signal leaves the interval given by (4.16), only one operating point on one of the two horizontal lines is possible for steady state. When it is not on such a line the system cannot be in steady state so it changes state. The excitation drives the operating point into the direction of the horizontal steady state line until it is reached. Then the excitation reduces to zero. When after this the value of the input signal is brought within the interval again, the operating point remains on this line. The fact that the input signal previously was outside the interval caused the operating point to move towards one of the two horizontal steady state lines, and as long as the input signal remains within the interval this information is retained. Depending on the state in which the system is, it can be determined if the last excursion of the input signal has been across the upper or the lower boundary of the interval.

Since there are two stable steady states possible when \( E_i \) is within the interval given by (4.16), the memory is capable of retaining two different events. Therefore it can be used as a binary memory. This is exactly the type of memory needed to construct first order oscillators (chapter 2).

The interval is bounded by two values, given by (4.14) and (4.15). They are the threshold levels at which the system switches from one state to the other. Due to the limiting, these threshold levels have practical values. Also the two steady state values \( X = E_i \) and \( X = -E_i \) are determined by the limiter and thus confined to practical values. In the modeling presented in chapter 3, each one of these steady states is represented by a circle (fig.3.1).

### 4.3.1 The unstable operating point

Very often a first order oscillator shows an unstable operating point on the steady state line during start-up. The state in which the first order oscillator finds itself at that time can be modeled as an extra state in the state-model representation. This has been depicted in fig.4.9. As usual this state is characterized by a certain value for the integration constant. During

![State model representation of a regenerative oscillator containing the startup state](image)

Fig. 4.9 The state model representation of a regenerative oscillator containing the startup state

start-up it is this value that determines the behavior of the integrator output signal. This signal is, sometimes via an amplifying circuit, used as the input signal for the memory. Hence the integrator output signal determines the external part of the excitation of the memory. Since the operating point of the memory is on the steady state line, a change of state can only be expected as a result of a change of the integrator signal, which would force the operating point off the steady state line.

Start-up problems can be expected when the extra state is a degenerated state. Then the integrator signal does not change, so no change of state of the memory is initiated either. Then the first order oscillator will never start. However, apart from the intended integrator
signal, also noise is present at the input of the memory in a practical circuit. This noise is also a part of the excitation, and it will cause the operating point to leave the steady state line. Due to this excitation—which is usually very small at the start—the operating point moves to one of the horizontal steady state lines.

_The oscillator is started by noise._

In simulators "numeric" noise, due to the limited accuracy of the computer, provides the same effect. This may however take a considerable amount of time in accurate computers. Then the insertion of a start-up pulse can be used to bring the memory in a desired state. When only white noise is present, the system has no preference for one of the two states. Some deterministic signals may introduce such a preference. This phenomenon is made use of in flip-flop sensors[16].

### 4.3.2 Example

In fig.4.10 the input signal and the state of a binary memory have been depicted. At the start the system is in state \( X = -E_1 \). The starting value of the input signal \( E_i \) is equal to \( E_1 \), which is a value that lies within the interval given by (4.16). The input signal is slowly changed to \( E_2 \) and then back to \( E_1 \) again, thereby crossing the reference level \( E_{ih} \). It can be seen that at the moment the reference level is crossed, the system switches to state \( X = E_1 \). In fig.4.11

![Diagram](image)

Fig. 4.10 The input signal \( E_i \) and the state \( X \) of a regenerative system

the steady state lines of the system have been depicted. Some characteristic points have been labeled. These labels correspond to the labels given to some points in fig.4.10. The movement of the operating point can be easily tracked in this way. When the operating point arrives at point 2, and moves further to the right, it comes off the steady state line and is consequently driven towards the other horizontal steady state line. Since at this moment no slowness has been modeled, the operating point moves infinitely fast from point 2 to point 3. After this the operating point will remain on the top most steady state line, even when \( E_i \) drops below its starting value. Only when \( E_i < E_{ii} \), the system will switch back to the bottom most steady state line.
4.4 The transient behavior of a regenerative system

4.4.1 The effect of slowness

Until now, the reaction of a regenerative system to excitation happened infinitely fast. In practice this is not possible, because there will always be some slowness in the system. The system takes time to react on excitation; it cannot change state at an infinite speed. In order to take this effect into account, an extra block containing a time constant $\tau$ is inserted in the system of fig.4.1. Actually this time constant is caused by the low-pass character of some element within the limiter. In an electronic implementation of a first order oscillator all components introduce some low-pass filtering action that limits the speed. The filtering action can be modeled with time constants. Some of them have a more dominant influence on the behavior of the first order oscillator than others. Very often it suffices to describe the behavior of a first order oscillator with only one (the most dominant) time constant. Therefore only one time constant is introduced in the model.

The new system containing the time constant has been depicted in fig.4.12. The amplifier part is a limiter with a transfer function as depicted in fig.4.7. The gain $A$ of the limiter when it is in the linear mode, is chosen to be larger than unity, because only in that case the system is of practical use as a binary memory.

![Diagram](image)

Fig. 4.12 A regenerative system containing a limiter and slowness

The differential equation describing the dynamic behavior of this system is:

$$\frac{dX}{dt} = \frac{A_l}{\tau} E_i - \frac{1 - A_l}{\tau} X$$  \hspace{1cm} (4.17)
\[ \frac{dX}{dt} = \frac{A_i}{\tau} E_{exc} \]  

(4.19)

For convenience the state variable \( X \) will be normalized from now on, so it will be confined to the interval \([-1;1]\). For the limiter this implies:

\[
\begin{align*}
E_l &= 1 \\
E_i &= -1
\end{align*}
\]

(4.20) (4.21)

and the threshold levels are:

\[
\begin{align*}
E_{ih} &= \frac{A_i - 1}{A_i} \\
E_{il} &= \frac{1 - A_i}{A_i}
\end{align*}
\]

(4.22) (4.23)

These two values are always smaller than one. The hysteresis \( E_{hys} \) is defined as the difference between the two threshold levels:

\[ E_{hys} = E_{ih} - E_{il} = \frac{A_i - 1}{A_i} - \frac{1 - A_i}{A_i} = 2\frac{A_i - 1}{A_i} \]

(4.24)

When \( A_i \) becomes smaller, the threshold levels move towards each other, and finally they coincide when the loop gain equals unity. Then the hysteresis equals zero.

With decreasing loop gain, the hysteresis decreases also.

The hysteresis is the memory phenomenon that is made use of in first order oscillators. A hysteresis equal to zero makes the memory unusable.

### 4.4.2 The loop gain-excitation model

Three quantities determine the transient behavior of a regenerative system. They are:

1. loop gain \( (A_i) \)
2. excitation \( (E_{exc}) \)
3. slowness \( (\tau) \)

In the loop gain-excitation model, these three quantities are used to describe the transient behavior of regenerative systems. Usually all three quantities depend on the state variable \( X \). For now the state dependency of the time constant will be neglected.

The excitation of a regenerative system can be split up into two components. They are called the internal excitation \( (E_{exc}^i) \) and the external excitation \( (E_{exc}^e) \). The internal excitation is the part of the excitation that is generated due to changes of state of the regenerative system. The external excitation is the part of the excitation that is generated by the input signal. When during a transition the excitation changes, it can have two causes. Either the input signal is changed, or it is caused by a change of state. Before the start of a transition the excitation is equal to zero, so then the excitation will not be changed by way of a change of state. At that time the excitation can be made non zero only by the external signal.
Fig. 4.13 The graphical representation of the internal and the external excitation

In fig. 4.13 the graphical interpretation of the internal and the external excitation has been given. The internal excitation is the distance between the steady state line and an imaginary vertical line that contains the steady state operating point the system was in just before the transition started.

The external excitation is the horizontal distance between the actual operating point and the imaginary vertical line. The excitation has a sign. When the operating point is on the left side of the steady state line or the imaginary vertical line, the internal and the external excitation are negative respectively. When they are on the right side of their reference they are positive.

The external excitation usually is much smaller than $E_{hp}$. Especially in the case of first order oscillators in which the integrator output signal is used as input signal for the memory directly. The integrator signal is a ramp, which usually changes slowly compared to the switching speed of the memory. Very often the change is so slow that the ramp can be considered constant during the transition. As long as the ramp has not reached the threshold level yet, the system does not switch, and the excitation is equal to zero. When the threshold level is crossed with just a very small amount, the system starts changing state. It starts increasing the excitation, and very soon the internal excitation dominates the external excitation. Then the system completes its transition at a speed that is nearly independent of the input signal.

At some state, labeled as $X_{eq}$, the internal and the external excitation are of equal magnitude:

$$E_{exc}^e = E_{exc}^i$$  \hfill (4.25)

When the system is in this state, the internal and the external excitation amount equally to the total excitation. Usually $X_{eq}$ has a value close to the starting value of $X$. For the remainder of the transition the internal excitation dominates. This implies that external influences (like noise) on the transition are mostly found at the start of the transition. During this first stage the total excitation is very small, since the (intended) external excitation is very small. This has two consequences:

1. The system changes state very slowly at the beginning of a transition. Therefore this part of the transition may contribute considerably to the total transition time, whereas the state does not change very much. Usually all this takes place before the state has changed with 10%. Then this part of the transition time is not taken into account in the 10–90% rise time, which may lead to an estimate of the maximum operating frequency that is too optimistic.
2. Noise may have a considerable influence since the signal to noise ratio of the external excitation usually is very poor. Because of this the duration of the first part of the transition becomes modulated deeply by noise. At this part of the transition most of the timing jitter is generated.

Next the reaction of a regenerative system to the external excitation will be determined. First the reaction on a step will be dealt with and then the reaction on a ramp. Then also the effect of noise will be taken into account.

4.4.3 The reaction on a step signal at the input

The reaction of the regenerative system will be determined when the excitation is stepped from zero to the value \( \Delta E_{exc} \). The step size of \( E_i \) to obtain this is of no importance, since the reaction of the system is based on the excitation. At the start the system is supposed to be in state \( X = -1 \). The “active” threshold level—which is the level to be crossed to obtain a change of state—is \( E_{ih} \). The steady state lines of the system have been depicted in fig.4.14. When \( E_i \) is stepped beyond the threshold level, the excitation is equal to:

![Fig. 4.14 The steady state lines of a regenerative memory](image)

\[
\Delta E_{exc} = E_i - E_{ih} \quad (4.26)
\]

This excitation drives the operating point of the system towards the top most steady state line. During this change of state the excitation increases proportionally to the state variable \( X \), since the external excitation caused by \( E_i \) is constant after the initial step. The complete expression for the excitation as a function of \( X \) and time is found to be:

\[
E_{exc}(X,t) = E_{exc}^e + E_{exc}^i \quad (4.27)
\]

\[
= \Delta E_{exc} \varepsilon(t) + E_{as}(X_{start}) - E_{as}(X) \quad (4.28)
\]

\[
= \Delta E_{exc} \varepsilon(t) + \frac{1}{2} E_{hys} - \left(-\frac{1}{2} X E_{hys}\right) \quad (4.29)
\]

\[
= \Delta E_{exc} \varepsilon(t) + \frac{1}{2} (1 + X) E_{hys}. \quad (4.30)
\]

in which \( \varepsilon(t) \) represents the unit step function. \( E_{as}(X) \) is the value the input signal should have in order to make \( X \) a steady state. The magnitude is equal to the horizontal distance of
the operating point to the steady state line. With (4.24) substituted in (4.30), it follows:

\[ E_{\text{exc}}(X, t) = \Delta E_{\text{exc}}(t) + (1 + X) \frac{A_l - 1}{A_l} \]  

(4.31)

When this expression is substituted in (4.19), a linear differential equation is found that can be used to determine the transition properties of the regenerative memory:

\[ \frac{dX}{dt} = \frac{A_l}{\tau} \Delta E_{\text{exc}}(t) + (1 + X) \frac{A_l - 1}{\tau} \]  

(4.32)

The Laplace transformation technique can be used to solve this equation. This calculation can be found in appendix A.1. The state as a function of time is found to be:

\[ X(t) = \frac{A_l}{A_l - 1} \Delta E_{\text{exc}} \left( e^{(A_l-1)\frac{t}{\tau}} - 1 \right) - 1 \]  

(4.33)

From (4.33) it can be seen that for values of the loop gain \( A_l \) close to unity, the state of the system \( X \) is very sensitive to changes of the excitation. However, the reaction to excitation will be very slow, since the effective time constant,

\[ \tau_{\text{eff}} = \frac{\tau}{A_l - 1} \]  

(4.34)

is large for values of the loop gain close to unity.

With (4.33) the switching speed of the regenerative memory can be calculated, when the external excitation is a step function. The situation is considered in which a transition is made from \( X = -1 \) to \( X = 1 \). After some calculations, from (4.33) an expression for the transition time of the memory can be derived as a function of \( X \):

\[ T_{\text{tr, st}}(X) = \frac{\tau}{A_l - 1} \ln \left( (1 + X) \frac{A_l - 1}{A_l \Delta E_{\text{exc}}} + 1 \right) \]  

(4.35)

When \( X = 1 \) is substituted, the time needed for the complete transition from \( X = -1 \) to \( X = 1 \) is found. The time constant of the memory \( \tau \) has a linear relation to the transition time. An increase of the loop gain makes the memory faster. This is not unexpected, since in negative feedback amplifiers an increase of the loop gain usually increases the bandwidth too. Also an increase of the excitation increases the switching speed. This phenomenon will be used later indeed to increase the speed of regenerative circuits.

In fig.4.15 the change of state of a regenerative system with \( A_l = 5 \), has been depicted for various values of \( \Delta E_{\text{exc}} \). The external excitation is stepped from zero to \( \Delta E_{\text{exc}} \) to start the transition. It can be seen that all transitions start slowly, and that very soon they continue at equal speed. Apparently the changes in \( \Delta E_{\text{exc}} \) only have influence on the speed of the transition at the start.

### 4.4.4 The reaction on a ramp signal at the input

A more realistic input signal for the regenerative system is a ramp. In most first order oscillators, the integrator output signal is used as input signal for the memory directly. Ideally this signal is a ramp.

\[ E_o(t) = \alpha t + E_{ih} \]  

(4.36)

\( E_o(t) \) is the integrator output signal, and \( \alpha \) is the integration constant as described in chapter 2. The integration constant \( \alpha \) determines the slope of the ramp. When the system is in state \( X = -1 \), the integration constant is positive.
Fig. 4.15 The state as a function of time for $\Delta E_{exc} = 0.001, 0.006, 0.011, 0.016$ and $0.021$ from the right to the left

The excitation is now equal to:

$$E_{exc}(X, t) = at + \left( 1 + X \right) \frac{A_l - 1}{A_l}$$  \hspace{1cm} (4.37)

When this excitation is substituted in (4.19), after some calculation the state as a function of time is found:

$$X(t) = \frac{A_l}{A_l - 1} \frac{\alpha \tau}{A_l - 1} \left( e^{(A_l-1)\frac{\tau}{A_l}} - 1 \right) - \frac{A_l}{A_l - 1} at - 1$$  \hspace{1cm} (4.38)

The calculation of $X(t)$ can be found in appendix A.2.

In many circuits $\tau$ and the time scale on which the level of the ramp changes differ in orders of magnitude. Since the transition time is of the same order of magnitude as $\tau$, this implies that $t$ in (4.38) remains restricted to a few $\tau$. This makes the term $\frac{A_l}{A_l - 1} at$ of (4.38) neglectable compared to the $-1$. When this term is deleted, the new equation only differs from (4.33) by the factor $\frac{\alpha \tau}{A_l - 1}$ which replaces $\Delta E_{exc}$.

$$X(t) = \frac{A_l}{A_l - 1} \frac{\alpha \tau}{A_l - 1} \left( e^{(A_l-1)\frac{\tau}{A_l}} - 1 \right) - 1$$  \hspace{1cm} (4.39)

With (4.39), the transition time as a function of $X$ can be derived:

$$T_{tr, rs}(X) = \frac{\tau}{A_l - 1} \ln \left( (1 + X) \frac{A_l - 1}{A_l} \frac{A_l - 1}{\alpha \tau} + 1 \right)$$  \hspace{1cm} (4.40)

When $X = 1$ is substituted, the time needed for the complete transition from $X = -1$ to $X = 1$ is found.

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4.4.5 A comparison between step and ramp input

The time the system externally excited with a ramp needs to complete a transition is found by substituting \( X = 1 \) in (4.40). At the start of the transition, the external excitation is equal to zero, and at the end it is equal to:

\[
\alpha T_{tr,ra}(1) = \frac{\alpha \tau}{A_l - 1} \ln \left( 2 \frac{A_l - 1}{A_l - 1} \right) + 1 \quad (4.41)
\]

When the responses to a step and a ramp used as external excitation are compared, it is not unnatural to give the step \( \Delta E_{exc} \) this value. So the step immediately reaches the level the ramp reaches at the end of the transition. For convenience a factor \( \lambda \) will be introduced now in order to reduce the complexity of the expressions:

\[
\lambda = \ln \left( 2 \frac{A_l - 1}{A_l - 1} \right) + 1 \quad (4.42)
\]

Then it follows:

\[
\Delta E_{exc} = \lambda \frac{\alpha \tau}{A_l - 1} \quad (4.43)
\]

For a system with \( \tau = 3.3 \text{ns}, A_l = 10 \) and \( \alpha = 10^7 \text{s}^{-1} \), it follows \( \lambda \approx 6.2 \).

The time the system needs to change state with an amount equal to \( \Delta X \) as a function of \( X \) will be calculated now. The starting state of the system is chosen to be \( X = -1 \) again. For the step input it follows:

\[
T_{st}(\Delta X, X) = \frac{\tau}{A_l - 1} \ln \left( 1 + \frac{\Delta X}{\lambda \frac{A_l}{A_l - 1} \cdot \frac{\alpha \tau}{A_l - 1} + (1 + X)} \right) \quad (4.44)
\]

And for the ramp input it follows:

\[
T_{ra}(\Delta X, X) = \frac{\tau}{A_l - 1} \ln \left( 1 + \frac{\Delta X}{\frac{A_l}{A_l - 1} \cdot \frac{\alpha \tau}{A_l - 1} + (1 + X)} \right) \quad (4.45)
\]

The calculation of (4.44) and (4.45) can be found in appendix A.3.

Usually \( \alpha \tau \) is small \((<< 1)\), so with increasing \( X \), very soon the contribution of terms containing this factor can be neglected with respect to the contribution of \((1 + X)\). When this happens there is no difference between (4.44) and (4.45) any more. Also, for increasing \( X \), the speed at which the system changes state increases. This is caused by the increasing internal excitation.

For both situations the state \( X_{eq} \) can be calculated for which the internal and the external excitation amount equally to the transition time. For the step input and the ramp input, the value are respectively:

\[
X_{eq, st} = -1 + \lambda \frac{A_l}{A_l - 1} \frac{\alpha \tau}{A_l - 1} \quad (4.46)
\]

\[
X_{eq, ra} = -1 + \frac{A_l}{A_l - 1} \frac{\alpha \tau}{A_l - 1} \quad (4.47)
\]

When the system changes state beyond this value, the influence of the external excitation rapidly decreases, so the speed in the two cases described does not differ any more. Since \( \alpha \tau \) usually is small, in both cases \( X_{eq} \) is close to the starting value \( X = -1 \). However, the fact that the state does not change much in this area does not imply that it does not take much time!
When (4.46) is substituted in (4.35), and (4.47) in (4.40), the duration of the first part of the transition is found for both situations:

\[ T_{tr, st}(X_{eq, st}) = T_{tr, ra}(X_{eq, ra}) = \frac{\tau}{A_l - 1} \ln(2) \]  

(4.48)

For the practical values of \( \tau, A_l \) and \( \alpha \) given before, this amounts to about 10% of the total transition time for the system externally excited with a ramp. The change of state that occurs during this time is hardly noticeable.

The time the system excited with a ramp needs to reach the same state \( X_{eq, st} \), as the system excited with the step, is found by substituting (4.46) in (4.40):

\[ T_{tr, ra}(X_{eq, st}) = \frac{\tau}{A_l - 1} \ln (1 + \lambda) \]  

(4.49)

The delay between the arrival of the two systems at \( X_{eq, st} \) is equal to:

\[ T_{delay}(X_{eq, st}) = T_{tr, ra}(X_{eq, st}) - T_{tr, st}(X_{eq, st}) = \frac{\tau}{A_l - 1} \ln \left( \frac{1}{2} (\lambda + 1) \right) \]  

(4.50)

When the delay is calculated between the systems reaching \( X = 0 \), which is the zero crossing frequently used in the application of first order oscillators, it is found:

\[ T_{delay}(0) = T_{tr, ra}(0) - T_{tr, st}(0) = \frac{\tau}{A_l - 1} \ln \left( \frac{1}{2 + \frac{1}{A_l - 1}} (\lambda + 1 + \frac{1}{A_l - 1}) \right) \]  

(4.51)

The calculation of this delay time is found in appendix A.4.

When the loop gain is large, (4.50) and (4.51) become equal. When \( A_l = 10 \) and \( \lambda = 6 \), the difference is about 4%. This means that most of the delay between the zero crossings of a system excited with a step and a system excited with a ramp, is generated before the systems reaches state \( X_{eq} \).

Although hardly any change of state occurs during the first part of the transition, a considerable influence on the timing of the transition can be expected there.

For the remainder of the transition the internal excitation has become so large that the external excitation has a negligible influence.

### 4.5 Design considerations

The first part of the transition \((-1 < X < X_{eq})\) plays an important role in the timing of the complete transition. In this part the excitation is still very small and largely determined by the external excitation. Because the excitation is small, the system changes state very slowly, although the actual change of state is hardly noticeable. It takes a considerable amount of the total transition time for the system to complete this part of the transition.

Small variations of the external excitation have a considerable influence on the speed of the system during this first part. It may cause a considerable shift of the zero crossing of the transition. For most of the “visible” part of transition (when most of the change of state occurs \( X > X_{eq} \)), the external excitation has a negligible influence, so all transitions are completed at equal speed.

Since the excitation is small at the start of the transition, noise can have a considerable influence. The signal to noise ratio of the excitation is usually very bad at the start. Especially when the external excitation is a ramp—which it usually is in first order oscillators—the first part of the transition takes much time and is very susceptible for noise.
From (4.40) it can be seen that the duration of the first part of the transition can be made shorter by increasing $\alpha$. Then there is less time for noise to influence the transition. The best external excitation would be a step, which improves the signal to noise ratio of the external excitation dramatically. Since in practice the input signal always has a finite rise time, $\tau$ should not be too small. Then the external excitation has time to “build up” before the memory switches. Apart from this reason to make $\tau$ not too small, it will also limit the noise conversion band, which will be discussed in section 4.11.

From all this it can be concluded that it is favorable for the noise performance of a first order oscillator to make $\alpha$ as large as possible and to make $\tau$ as large as can be allowed by other design criteria. The increased external excitation at the start of the transition will bring the memory quickly beyond the state $X = X_{eq}$, where the influence of noise rapidly decreases. Also the transition time decreases in this way.

An increase of $\alpha$ implies that the slope of the input ramp of the memory is increased. In first order oscillators this can be achieved by way of making the signal amplitude at the output of the integrator as large as possible for a given frequency, or by way of using limiters, which will be discussed in section 4.6 of this chapter. In practical first order oscillators this comes down to the use of a large current to charge the capacitor used as integrator. As usual power consumption is traded for noise performance. It will be shown later that on other grounds it is not advisable for the noise performance the reduce the value of the capacitor to obtain a larger $\alpha$.

The $f_T$ of the transistors should not be chosen larger than necessary in order to make $\tau$ not too small.

### 4.6 Improving the performance with limiters

In some first order oscillators, $\alpha$ as discussed in the sections above actually is the integration constant. It should be made as large as possible, but due to other constraints it cannot be increased without bounds. Limiting factors in practice are, for example, the maximum allowable value for the capacitor used as integrator, or the available supply voltage. Still when due to these constraints the slope of the integrator signal cannot be increased any further, the effective $\alpha$ for the memory can be enlarged. The slope of the signal at the input of the memory can be enlarged by way of amplification. This can be obtained by inserting an amplifier between the integrator output and the memory input. When the output signal of the integrator is amplified by a factor $A_{lim}$, the slope increases with the same factor. Hence the effective $\alpha$ for the memory is increased with a factor $A_{lim}$. This amplification cannot be realized with an ordinary linear amplifier. Since in an optimal design the amplitude of the integrator signal is already chosen as large as possible, a further increase of the amplitude with a factor $A_{lim}$ cannot be allowed. However, the amplification is only needed within a restricted range of the integrator signal. A large slope of the memory input signal is only required when the memory is about to switch. The threshold level has to be crossed with a large slope. For the rest of the time the behavior of the memory input signal is of no importance. When the output signal of the amplifier is limited outside the interval of interest, this makes absolutely no difference for the memory. In fig.4.16 this has been made more clear. The window in which the memory is sensitive for the input signal has been indicated with dashed lines. Within this window the two signals show no differences. Limiters typically show this type of behavior. Within a restricted range of the input signal, they behave like a linear amplifier, and for the remainder of the input signal the output signal remains within bounds (fig.4.7). The limiter acts as some kind of “magnifying glass” for the memory.

Since there are two threshold levels, and the magnifying action of the limiter is restricted
to a small interval around a threshold level, special measures have to be taken to obtain a magnifying action around both threshold levels. There are several ways to accomplish this. In chapter 2 first order oscillators containing limiters are discussed. Two limiters can be used (fig.2.7), or the reference level of the limiter can be shifted to the appropriate level by using the output signal of the memory (fig.2.9). The value of the reference level of the limiter determines the signal level around which the magnifying action occurs.

The noise influence of the memory, and the noise sensitivity of the memory are both reduced when limiters are used. Of course, the noise performance of the limiters should be optimized for the input level at which the memory is about to switch. Also the accuracy of the integrator signal level at which the memory switches can be determined more accurate. Very often the reference levels of the regenerative memory itself are temperature dependent, and depend on the exact value of the loop gain. This exact value is not always easy to determine or to design. When the slope of the memory input signal is increased by a limiter, variations of the threshold level of the memory will have less influence. In the ideal case, when the limiter has an infinite gain, variations of the threshold level of the memory have no effect at all. Limiters with an infinite gain behave like comparators. (Comparators produce a binary output signal, that is an indication whether the input signal of the comparator is above or below its reference level. Usually the comparator function is implemented as a high gain limiter.)

The accuracy of the threshold level for the integrator signal is now determined by the accuracy of the reference level of the limiter. Usually it is less difficult to design this reference level accurately.

_For both the noise performance and the accuracy the use of limiters in front of the regenerative memory is favorable._

The limiters introduce an extra delay in the first order oscillator. Still the maximum operating frequency of the oscillator does not necessarily have to deteriorate. The memory is excited with a signal that changes faster than the original ramp coming from the integrator directly. In the previous section it has been shown that this increases the switching speed of the memory. Sometimes this effect compensates for the extra delay introduced by the limiters.

### 4.6.1 Summary

At the start of a transition the excitation is almost completely determined by the external excitation. It usually is small, because it is a ramp in most first order oscillators, which causes noise to have a considerable influence.

Due to the small excitation at the start of the transition, the state only changes slowly, and
noise has a large influence on the exact duration.

The remainder of the transition is nearly independent of the external excitation. During this part of the transition the most of the change of state takes place.

In order to reduce the sensitivity for disturbances as much as possible, the slope of the external excitation should be made as large as possible. This is achieved by making $\alpha$ as large as possible.

The time constant of the memory should not be chosen smaller than necessary. Then the external excitation gets the time to build up.

Limiters can be used to increase the effective $\alpha$ for the memory beyond the value that can be maximally achieved in the integrator. This will improve the noise behavior and the accuracy, whereas it does not necessarily have to decrease the maximum operating frequency of the first order oscillator.

4.6.2 Example: calculation the maximum operating frequency of a simple first order oscillator

When a regenerative circuit is operating at its maximum frequency, it is just able to make two transitions in one period. Using (4.35) an expression for $f_{max}$ can be found:

$$f_{max} = \frac{1}{2T_{tr}} = \frac{A_I - 1}{2\tau \ln \left( \frac{2(A_I-1)}{A_I \Delta E_{exc}} + 1 \right)}$$ \hspace{1cm} (4.52)

A simple electronic memory circuit constructed with two bipolar transistors, can be satisfactory described with one time constant equal to $\tau = \frac{1}{2\pi f_T}$. An example of such a circuit is a Schmitt trigger as depicted in fig.3.12. A realistic value for the loop gain could be $A_I = 10$. When the input signal of the Schmitt trigger changes very slowly (a low frequency first order oscillator with relatively high frequency transistors), the initial excitation, $\Delta E_{exc}$ mainly originates from noise. For now the noise is assumed to generate a step voltage at the start of the transition. This way of modeling can be used if only a rough estimate of the maximum operating frequency is requested. An equivalent effective input noise voltage of approximately $2.5V_{\sqrt{Hz}}$ is not unusual when the hysteresis voltage is about 1V.

From these values an estimate of $\Delta E_{exc}$ of the normalized system can be made. It follows $\Delta E_{exc} = 4.10^{-9}$. When all these data are substituted in (4.52), it is found:

$$f_{max} = 0.26 f_T$$ \hspace{1cm} (4.53)

This value is valid for a system with a loop gain that abruptly changes from zero to a constant value and back to zero again during a transition. This makes this system somewhat faster than practical systems for which the loop gain is a continuous function of $X$. For practical systems values around $0.2 f_T$ are common.

4.7 The transient behavior when the loop gain is state dependent

In this section the transient behavior of a regenerative system with a state dependent loop gain will be discussed. The loop gain of the systems described before abruptly changed from zero to a constant value and then abruptly back to zero again during the transition. In practice this is not possible. A practical loop gain function is always continuous. Many different loop gain functions are possible. In this section the effects of a state dependent loop gain will be discussed with the use of a very common first order oscillator as an example. This first order
oscillator has been depicted in fig.4.17. It is the well known emitter coupled oscillator. This simple first order oscillator consists of an integrator (C) and a Schmitt trigger. The Schmitt trigger performs the memory function of this oscillator. In this section the transient behavior of the Schmitt trigger will be discussed. Therefore the integrator can be replaced by a voltage source \( U_{in} \).

The differential pair consisting of Q1 and Q2 acts as the limiter. The state variable \( X \) is linearly

\[
\Delta U_{be} = (1+X)I - (1-X)I
\]

related to the transistor currents. Now the relation between \( \Delta U_{be} \) and \( X \) will be calculated for a steady state situation. The fact that a steady state is described will be indicated with the index \( ss \) (\( X_{ss} \)).

The relation between \( X_{ss} \) and the input voltage \( \Delta U_{be} \) is found with the use of the exponential relation between the base–emitter voltage and the collector current of a bipolar transistor:

\[
I_c = I_s \left( e^{\frac{\Delta U_{be}}{V_T}} - 1 \right),
\]

with \( V_T = \frac{kT}{q} \).

Using this relation it follows:

\[
\Delta U_{be} = U_{be1} - U_{be2}
\]

\[
= V_T \ln \left( \frac{1 + X_{ss} + \frac{I}{R}}{1 - X_{ss} + \frac{I}{R}} \right) \approx V_T \ln \left( \frac{1 + X_{ss}}{1 - X_{ss}} \right)
\]

The base currents have been neglected. When \( X_{ss} \) is not too close to its extreme values (\( X_{ss} = -1 \) and \( X_{ss} = 1 \)), the latter approximation of (4.56) is allowed. From (4.56) the state as a function of \( \Delta U_{be} \) can be derived:

\[
X_{ss} = \frac{e^{\frac{\Delta U_{be}}{V_T}} - 1}{e^{\frac{\Delta U_{be}}{V_T}} + 1}
\]

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Using this limiter, a regenerative memory can be constructed. To accomplish this a positive feedback loop has to be made around the limiter. Since the transfer of the limiter has the dimension of an admittance, the feedback network has to have the dimension of a impedance. In the most simple case it can be composed of linear resistors. Later the effect of non-linear feedback networks will be discussed.

In fig.4.19 the Schmitt trigger has been depicted in a more “abstract” way. The limiter block contains the two transistors. In fig.4.19 a common mode bias current source \((2I)\) has been depicted also. This is because at the output of the limiter the contribution of this current is of importance also. The division of this current between the two outputs, indicated by \(X\), determines the small-signal parameters of the limiter.

Now the loop gain of this system as a function of \(X\) will be calculated. The loop gain that is used in the loop gain-excitation model is a small-signal quantity. It is calculated for a constant \(X\), which does not necessarily have to represent a steady state. When \(X\) is changed, the division of \(2I\) between the two transistors is altered, and consequently the loop gain changes. The small-signal transfer of the limiter is equal to:

\[
H_{lim} = \frac{g_{e1}g_{e2}}{g_{e1} + g_{e2}} 
\]  
(4.58)

\(g_{e1}\) and \(g_{e2}\) are the transconductances of \(Q1\) and \(Q2\) respectively.

The transfer of the feedback network is:

\[
H_{fb} = 2R  
\]  
(4.59)

The loop gain is the product of these two transfers:

\[
A_l = H_{lim}H_{fb} = 2R \left( \frac{g_{e1}g_{e2}}{g_{e1} + g_{e2}} \right)  
\]  
(4.60)
The transconductances are dependent on \( X \):

\[
ge_{e1} = \frac{I(1 + X)}{V_T} \tag{4.61}
\]

\[
ge_{e2} = \frac{I(1 - X)}{V_T} \tag{4.62}
\]

After some calculations the expression for the loop gain as a function of \( X \) is found:

\[
A_l(X) = A_{\text{max}} \left( 1 - X^2 \right) \tag{4.63}
\]

With:

\[
A_{\text{max}} = \frac{RI}{V_T} \tag{4.64}
\]

From (4.63) already some requirements for the circuit to ensure proper regenerative operation can be extracted. The maximum of the loop gain is found for \( X = 0 \), and it is equal to \( A_{\text{max}} \). The maximum has to be larger than unity. If it is not, the circuit will never become regenerative. It is possible also, to determine the values for \( X \), at which the loop gain equals unity:

\[
X_{\text{unity}} = \pm \sqrt{1 - \frac{1}{A_{\text{max}}}} = \pm X_{th} \tag{4.65}
\]

When the input signal of the Schmitt trigger is approaching the threshold level, the loop gain is still smaller than unity. In that case the Schmitt trigger behaves like any normal negative feedback amplifier; it reduces its excitation. When the slowness of the Schmitt trigger is neglectable on the time scale of the changes of the input signal (static approach), the reduction of the excitation happens so fast, that the Schmitt trigger can be dealt with as if it were in steady state. Then (4.56) can be used to determine \( \Delta U_{be} \) for a given \( X \). The Schmitt trigger “switches” when loop gain equals unity.

The input signal \( U_{in} \) of the system in steady state is equal to:

\[
U_{in} = \Delta U_{be} - 2X_{ss}RI = V_T \ln \left( \frac{1 + X_{ss}}{1 - X_{ss}} \right) - 2X_{ss}RI \tag{4.66}
\]

All signals in the Schmitt trigger can be normalized with respect to \( 2RI \). In that case the output signal of the Schmitt trigger becomes equal to \( X \) itself. Then it follows:

\[
E_i = \frac{U_{in}}{RI} = \frac{1}{2A_{\text{max}}} \ln \left( \frac{1 + X_{ss}}{1 - X_{ss}} \right) - X_{ss} \tag{4.67}
\]

When the two possible values for \( X_{\text{unity}} \) (\( X_{th} \) and \( -X_{th} \)) are substituted, the static threshold levels are obtained.

The hysteresis is the difference between the two threshold levels:

\[
E_{\text{hys}} = E_{th} - E_{il} = \frac{U_{th}}{2RI} - \frac{U_{il}}{2RI} \tag{4.68}
\]

With:

\[
E_{th} = E_i(-X_{th}) = \frac{1}{2A_{\text{max}}} \ln \left( \frac{1 - X_{th}}{1 + X_{th}} \right) + X_{th} \tag{4.69}
\]

\[
E_{il} = E_i(X_{th}) = \frac{1}{2A_{\text{max}}} \ln \left( \frac{1 + X_{th}}{1 - X_{th}} \right) - X_{th} \tag{4.70}
\]

it follows:

\[
E_{\text{hys}} = \frac{1}{A_{\text{max}}} \ln \left( \frac{1 - X_{th}}{1 + X_{th}} \right) + 2X_{th} \tag{4.71}
\]
In a first order approximation the logarithmic term is usually neglected. Since $0 < X_{th} < 1$, the logarithmic term is negative always, so it reduces the hysteresis. When used in a first order oscillator, this causes the output frequency to be higher than given by the first order approximation. In high loop gain systems, the logarithmic term vanishes and the approximation becomes more accurate.

4.7.1 The steady state curve of the Schmitt trigger

Expression (4.67) can be used to generate the steady state curve of the Schmitt trigger. In fig.4.21 $X_{ss}$ as a function of $E_i$ has been plotted for $A_{max} = 1, 2, 5$ and 10. When this figure is compared with fig.4.2, a large similarity can be seen for values of $X_{ss}$ around zero. For $A_{max} = 1$ a vertical line is found. For larger values of $A_{max}$ lines with a negative slope are found. In those cases the same “Z-like” shape occurs as it does in fig.4.8. Only now the sharp edges have been rounded.

The derivative of the steady state curve can be expressed in small-signal quantities:

$$\frac{dX_{ss}}{dE_i} = \frac{A_{max}(1 - X_{ss}^2)}{1 - A_{max}(1 - X_{ss}^2)}$$

(4.72)

When the value of $X_{ss}$ is varied, the tangent line of the curves rotates. At $X_{ss} = 0$, the tangent line starts with a negative slope. When $X_{ss}$ is changes towards one of its bounds, the tangent line rotates clockwise. It is a vertical line when the loop gain equals unity. After that the slope of the tangent line becomes positive. In section 4.2 it has already been stated that regenerative operation can only be expected when the tangent line of the steady state curve has a negative slope. The regenerative area is bounded by the states at which the tangent line is vertical.

The movement of the operating point can be easily predicted with the theory discussed in the previous sections. When it is not on the steady state curve, it moves up when it is on the right side of the curve, and it moves down when it is on the left side of the curve. Again the horizontal distance from the operating point to the steady state curve is equal to the magnitude
of the excitation. When the loop gain is smaller than unity, the operating point moves such that the excitation is reduced, and when the loop gain is larger than unity, it moves such that the excitation is enlarged.

### 4.7.2 The reaction to a step signal at the input

The excitation is the horizontal difference between the operating point and the steady state curve. First the system is assumed to be in a steady state with unity loop gain. Then the operating point is on the steady state curve at a point where the tangent line is vertical. In that case the excitation is equal to the difference between the input signal and the threshold level (see (4.26)). The threshold level is equal to:

$$E_{\text{ith}} = \frac{U_{\text{th}}}{2RI} = \frac{1}{2A_{\text{max}}} \ln \left( \frac{1 + X_{\text{unity}}}{1 - X_{\text{unity}}} \right) - X_{\text{unity}}$$  \hspace{1cm} (4.73)

When the operating point moves along the vertical tangent line, the expression for the excitation as a function of $X$ is:

$$E_{\text{exc}}^i(X) = E_{\text{ith}} - E_i = \frac{1}{2A_{\text{max}}} \ln \left( \frac{(1 + X_{\text{unity}})(1 - X)}{(1 - X_{\text{unity}})(1 + X)} \right) + (X - X_{\text{unity}})$$  \hspace{1cm} (4.74)

This expression equals zero when $X = X_{\text{unity}}$.

Small variations of the excitation as a result of small variations of $X$, can be expressed in terms of small-signal quantities. First the steady state curve is approximated by its tangent line. Suppose the Schmitt trigger is in state $X = X'$. When the operating point moves, a linear change of the excitation can be expected, for small changes, given by the slope of the tangent line. With (4.72) it follows:

$$\frac{dE_{\text{exc}}}{dX} = \frac{A_i(X') - 1}{A_i(X')}$$  \hspace{1cm} (4.75)

During the small change of state $X'$ is considered to be a constant, so the loop gain is a constant. The small-signal expression for the internal excitation for a given value of $X'$ is:

$$e_{\text{exc}}^i(X) = E_{\text{exc}}^i(X')\varepsilon(t) + \frac{dE_{\text{exc}}}{dX} dX$$  \hspace{1cm} (4.76)

$$= E_{\text{exc}}^i(X')\varepsilon(t) + \frac{A_i(X') - 1}{A_i(X')} (X - X')$$  \hspace{1cm} (4.77)

For the small changes discussed, $E_{\text{exc}}^i(X')$ is a constant. When an external step signal equal to $2RI\Delta E_{\text{exc}}$—in which $\Delta E_{\text{exc}}$ is the normalized step of the excitation—is applied at the input of the Schmitt trigger, the complete small-signal expression for the excitation is:

$$e_{\text{exc}}(X) = \Delta E_{\text{exc}} \varepsilon(t) + E_{\text{exc}}^i(X')\varepsilon(t) + \frac{A_i(X') - 1}{A_i(X')} (X - X')$$  \hspace{1cm} (4.78)

This expression can be used in the differential equation that describes the reaction of a regenerative system on excitation:

$$\frac{dX}{dt} = \frac{A_i(X')}{r} e_{\text{exc}}(X)$$  \hspace{1cm} (4.79)

This equation is very similar to (4.19), but in this case it can only be used to calculate small changes of state. For the calculation of a complete transition, the value of $X'$ has to be adapted repeatedly. This quasi stationary approach can be easily implemented in a computer program.
to calculate transitions of regenerative systems with a state dependent loop gain. This has been done in the LOEX simulator, which is a dedicated simulator that makes use of (4.78) and (4.79) to calculate a transition.

Using the Taylor expansion of the logarithm and the approximation:

\[
\frac{A_l(X) - 1}{A_l(X)} \approx \frac{A_{max} - 1}{A_{max}},
\]

(4.80)

after some calculations it is found:

\[
e_{exc}(X) = \Delta E_{exc}(t) + \frac{A_{max} - 1}{A_{max}}(X - X_{unity})
\]

(4.81)

The expression has become independent of \(X\). As long as both approximations are valid, the system behaves like a system with a constant loop gain. The excitation of such systems has been given by (4.31). When \(X_{unity} = -1\) is substituted in (4.81), both equations are identical. Unfortunately, for the most interesting part of the transition the approximations, especially the one for the loop gain, are not valid. When \(X\) is around the threshold value, the loop gain is near unity. This implies that the start of the transition, is largely affected by the state dependency of the loop gain. During the fast part of the transition, in which \(X\) is around zero, the approximations can be useful, since they allow an analytical solution for the differential equation.

In fig.4.22 the internal excitation according to (4.74) has been plotted for two different values of \(A_{max}\). When \(A_{max} = 3\) the threshold state is \(X_{unity} \approx \pm 0.8\) and when \(A_{max} = 10\) it is \(X_{unity} \approx \pm 0.95\). The logarithmic terms of the internal excitation have been plotted separately in the figure as dotted lines. Especially for larger values of \(A_{max}\), neglecting the logarithmic term does not introduce significant errors. It can be modeled as a reduction of \(A_{max}\). The approximation of the excitation function by a straight line does not much error. The logarithmic term mainly disturbs the linearity of the actual excitation function at the edges somewhat. The main problem is caused by the state dependency of the loop gain. During important parts of the transition the approximation of the loop gain by a constant is not allowed.

Fig. 4.22 The internal excitation as a function of \(X\) for \(A_{max} = 3\) and \(A_{max} = 10\) respectively

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4.8 The reaction of a regenerative system with a state dependent loop gain on excitation

Now the differential equation will be solved for small changes of $X$ around a constant $X'$. When (4.78) is substituted in (4.79) it follows:

$$\frac{dX}{dt} = \frac{A_i(X')}{\tau} \left\{ \left( \Delta E_{exc} + E_{exc}'(X') \right) \varepsilon(t) + \frac{A_i(X') - 1}{A_i(X')} (X - X') \right\}$$  (4.82)

For a constant $X'$ this equation can be solved. After some calculations, similar to those found in appendix A.1, it follows:

$$X(t) = \frac{A_i(X')}{A_i(X') - 1} \left( \Delta E_{exc} + E_{exc}'(X') \right) \left( t^{(A_i(X') - 1/2)} - 1 \right) + X'$$  (4.83)

This expression is very similar to (4.33), but in this case it is valid for small changes of state only. The reaction speed of the system to excitation is determined by the effective time constant:

$$\tau_{eff} = \frac{\tau}{A_i(X') - 1}$$  (4.84)

The larger the loop gain, the faster the system reacts to excitation.

In a first order oscillator a very important part of the behavior is determined when it is in a state close to the threshold states. Suppose the integrator signal in the oscillator slowly approaches the threshold level. Then the loop gain of the memory is still below unity, so it behaves like a negative feedback amplifier. This implies that the operating point moves towards the steady state curve when it is not on it. The memory tries to reduce its excitation. Therefore, when the threshold level is reached, the excitation is very small. The speed of this reduction is determined by the effective time constant, and when the loop gain equals unity it becomes infinite. Then the memory behaves like an integrator, so the reaction to excitation will be very slow. Many unfavorable properties are present around the threshold level:

1. The excitation is very small since just before the reaching of the threshold level, the memory was still reducing it, as if it were a negative feedback amplifier.

2. Regenerative systems with unity loop gain do change state, but this change has no effect on the excitation itself. It is neither reduced nor increased (fig.4.6). Hence the excitation remains small.

3. In any regenerative system a small excitation causes a slow change of state. The small value of the loop gain and the large effective time constant around the threshold states cause the reaction to become even slower.

In fig.4.23 a typical transition of a regenerative memory with a state dependent loop gain has been depicted. At the origin of the plot ($t = 0$) the system has reached the unity loop gain state, and a small step in the excitation is generated externally. This has been repeated for different values of the external step. It can be seen that the fast part of the transition is preceded by a much longer slow part, in which only a minor change of state can be observed. The duration of this first part of the transition largely (and non-linearly) depends on the value of the external step. All transitions are completed in a similar way, since during the fast part of the transition the internal excitation usually dominates.

The duration of a transition can have any value depending on the input signal. From the regenerative circuit itself it cannot be derived how much delay there will be between the zero crossing of the output signal and the crossing of a threshold level.
Fig. 4.23 The state of a regenerative memory with a state dependent loop gain plotted against time, for various values of the external step. From the left to the right $\Delta E_{exc}=0.20$, 0.15, 0.10 and 0.05.

When slowly changing input signals are used for the memory, it will be able to reduce the excitation to zero until the threshold level is reached. Especially in that case unwanted signals (like noise) can have a large influence on the first part of the transition. The exact place in time of the fast part of the transition, including the zero crossing, will be determined by the unwanted signals. This component of the phase jitter of a first order oscillator is caused in the first part of the transition.

In some applications of regenerative memories, the unwanted signals may manage to postpone the transition such, that the intended input signal is no longer across the threshold level of the memory, when the disturbance disappears. Then it will not switch any more and the memory contents is wrong! This will not happen in first order oscillators, because the memory contents controls the direction of the change of the input signal. In that case the input signal will always cross the threshold level eventually. In other (logic) applications however, errors like this are possible indeed. For those applications the reliability of regenerative memory cells will never be 100%.

An improvement of the behavior in this respect can be obtained in several ways, that have already been mentioned for regenerative systems with a constant loop gain.

1. The excitation should be increased at the start of a transition. This implies that the slope of the external input signal should be as large as possible. Again the use of limiters may be favorable.

2. A noise optimization of the memory should focus on the situation in the threshold states.

3. The time the memory spends around the threshold level also depends on the value of the loop gain. An increase of the slope of the loop gain function around the threshold level
will decrease this time. The loop gain inevitably is equal to unity at the threshold level, but after that it should increase as fast as possible.

In order to optimize the noise behavior and the switching speed apparently the derivatives of both the loop gain and the excitation should be increased. The loop gain is a function of $X$, so $\frac{dA_t(X)}{dX}$ has to be considered. The excitation is as well a function of time as of the state. However, during the first part of the transition, the external excitation has a dominant influence on the behavior. The external excitation only depends on time, so $\frac{dE_{ext}(t)}{dt}$ has to be considered at the time the threshold level is reached. The product of both slopes at $X = X_{unity}$ can be used as a figure of merit for a first order oscillator. It could be labeled as the loop gain-excitation product (LOEX-product).

### 4.8.1 Departure from a state with zero loop gain

When $X = 1$ or $X = -1$, the loop gain equals zero. According to (4.79), the state will not change for zero loop gain, since then $\frac{dX}{dt} = 0$ always. Apparently the model is not valid for these extreme values, since in practice regenerative circuits are able to depart from a state with zero loop gain. Evidently (4.79) is not the correct expression to describe the behavior of the system at zero loop gain. The small-signal approach that was used to obtain the expressions for the loop gain-excitation model cannot be used. The large signal transfer function of the limiter has to be used, which still defines a relation between $X$ and the input signal at zero loop gain.

The limiter has two input terminals. At one of them the input signal is connected, at the other the signal coming from the feedback network. When the loop gain equals zero, the limiter is completely "saturated". Its input signal has no influence on the output signal. Therefore the signal coming from the feedback network can be seen as an independent constant signal. The sum of this signal and the input signal is found at the input of the limiter. This sum is such that the limiter becomes saturated. The input signal can be changed such that the sum signal does not keep the limiter saturated any longer. As soon as this happens, the output signal of the limiter, and thus $X$, changes according to the changes of the input signal. Due to this change the loop gain becomes unequal to zero. Only a very small change of $X$ is necessary to close the loop. Until that moment the direct transfer of the limiter from the input to $X$ without a loop determines the behavior.

Although this effect is necessary for completeness, it is not of much importance in the description of the behavior of first order oscillators for designers. Therefore no more attention will be paid to this aspect of the transient behavior.

### 4.9 A solution of the non-linear differential equation

In order to find a solution to the non-linear differential equation, some simplifications will be made. The solution applies to the specific case discussed in this chapter, in which a regenerative memory is used that contains a bipolar differential pair as limiter and a resistive feedback network. According to (4.63), the loop gain is equal to:

$$A_t(X) = A_{max}(1 - X^2)$$  \hspace{1cm} (4.85)

The logarithmic term in (4.74) is linearized:

$$\frac{1}{2A_{max}} \ln \left\{ \frac{(1 + X_{unity})(1 - X)}{(1 - X_{unity})(1 + X)} \right\} \approx -\frac{1}{A_{max}}(X - X_{unity})$$  \hspace{1cm} (4.86)
Then the internal excitation is equal to:

\[ E_{\text{exc}}^i = \frac{A_{\text{max}} - 1}{A_{\text{max}}} (X - X_{\text{unity}}) \]  \hspace{1cm} (4.87)

The input signal is chosen to be a ramp since that is a good approximation for practical input signals.

\[ E_{\text{exc}}^e = \alpha t \]  \hspace{1cm} (4.88)

When the expression for the complete excitation is substituted in the differential equation, it follows:

\[ \frac{dX}{dt} = \frac{A_{\text{max}} (1 - X^2)}{\tau} \alpha t + \frac{A_{\text{max}} - 1}{\tau} (1 - X^2) (X - X_{\text{unity}}) \]  \hspace{1cm} (4.89)

From here two different differential equations will be considered. For the slow part of the transition (4.89) is approximated by:

\[ \frac{dX}{dt} = \frac{A_{\text{max}} (1 - X^2)}{\tau} \alpha t \]  \hspace{1cm} (4.90)

The change of state during the slow part of the transition is small, so \( X - X_{\text{unity}} \approx 0 \). For the fast part of the transition it will be approximated by:

\[ \frac{dX}{dt} = \frac{A_{\text{max}} - 1}{\tau} (1 - X^2) (X - X_{\text{unity}}) \]  \hspace{1cm} (4.91)

In this case \( \alpha t \) is assumed to be small compared to \( X - X_{\text{unity}} \).

4.9.1 The solution for the approximation of the slow part

In (4.90) the terms can be rearranged:

\[ \frac{dX}{1 - X^2} = \alpha A_{\text{max}} \frac{t}{\tau} dt \]  \hspace{1cm} (4.92)

Integrating both sides of the equation results in:

\[ \ln \left( \frac{1 + X}{1 - X} \right) = \alpha A_{\text{max}} \frac{t^2}{\tau} + C_1 \]  \hspace{1cm} (4.93)

\( C_1 \) is a constant that is determined by the starting value of \( X \). After some calculations it follows:

\[ X = \frac{e^{\alpha A_{\text{max}} \frac{t^2}{\tau}} - 1}{e^{\alpha A_{\text{max}} \frac{t^2}{\tau}} + 1} + X_{\text{unity}} \]  \hspace{1cm} (4.94)

4.9.2 The solution for the approximation of the fast part

In (4.91) the terms are rearranged:

\[ \frac{dX}{(1 - X^2)(X - X_{\text{unity}})} = \frac{A_{\text{max}} - 1}{\tau} dt \]  \hspace{1cm} (4.95)

Integrating both sides of the equation results in:

\[ \frac{2 \ln(X - X_{\text{unity}}) + X_{\text{unity}} \ln \left( \frac{X - 1}{X + 1} \right) + \ln(1 - X^2)}{2(1 - X_{\text{unity}}^2)} = \frac{A_{\text{max}} - 1}{\tau} t + C_2 \]  \hspace{1cm} (4.96)
$C_r$ is a constant that is determined by the starting value of $X$. For the values of $X$ concerned, the following approximations can be made:

\[
\ln \left( \frac{X - 1}{X + 1} \right) \approx 0 \quad (4.97)
\]

\[
\ln(1 - X^2) \approx 0 \quad (4.98)
\]

After some calculations it follows:

\[
X = e^{(A_{\text{max}} - 1)(1 - X_{\text{unity}}^2)^{1/2}} + X_{\text{unity}} \quad (4.99)
\]

Since $X_{\text{unity}}^2 = 1 - \frac{1}{A_{\text{max}}}$ the expression can be written as:

\[
X = e^{(A_{\text{max}} - 1)^{1/2}} + X_{\text{unity}} \quad (4.100)
\]

The final part of the transition where the loop gain approaches unity again is not covered in this calculation. However, with the expressions derived, the events in the two most important parts of the transition can be calculated. With expression (4.94) the delay can be calculated between the reaching of the threshold level and the start of the fast part of the slope, and with (4.100) the rise time can be calculated. Equation (4.101) can be used to determine which one of the two solutions should be used.

\[
\frac{A_{\text{max}}(1 - X^2)}{\tau} \alpha t = \frac{A_{\text{max}} - 1}{\tau} (1 - X^2)(X - X_{\text{unity}}) \quad (4.101)
\]

This expression equates both right hand terms of (4.89). It can be simplified to:

\[
t = \frac{1}{\alpha} \left( \frac{A_{\text{max}} - 1}{A_{\text{max}}} \right) (X - X_{\text{unity}}) \quad (4.102)
\]

Using this equation and (4.94) the delay time can be found. Numerically this is no problem. From (4.102) it can be seen that an increase of $\alpha$, as expected, decreases the delay time. In fig.4.24 the delay time is determined graphically. Expression (4.102) is represented by a straight line. At the point at which it intersects with the curve given by (4.94) the delay time is found. When $\alpha$ is increased, the slope of the straight line decreases, so the intersection with the curve shifts to the left.

From (4.100) very quickly the rise time can be derived. For this purpose $X = 1$ is substituted.

\[
t_{\text{rise}} = \frac{A_{\text{max}} - 1}{A_{\text{max}}} \tau \ln(1 - X_{\text{unity}}) \quad (4.103)
\]

The rise time does not depend on the slope of the ramp, it is merely determined by parameters of the regenerative memory itself. Unfortunately the complete transition time also contains the delay time that can be given any value via the external input signal. Although once sufficiently excited, regenerative systems can be very fast, it cannot be predicted from the parameters of the system itself how much time passes between the reaching of the threshold level and the zero crossing. When an external input pulse is available longer than the rise time of the regenerative system one would expect the system will always succeed to complete a full transition. However, when the level of the pulse is too low, due to the delay the regenerative system might still not be able to switch in time. When the pulse disappears again, the regenerative system has not switched and consequently the memory contents is wrong.
Fig. 4.24 The delay time determined graphically. \( t \) is plotted versus \( X - X_{\text{unity}} \)

4.9.3 Discussion

In the section above some calculations have been performed that apply to a specific type of regenerative memory. They were intended as example. Usually simulators can provide better numeric results than the analytical solutions that were obtained by allowing a lot of approximations. These approximations limit the validity of the solutions. However, they allow the designer to determine the effect of the variation of system parameters in a more intelligent way than the "trial and error" computer investigations the sole use of simulators would lead to.

4.10 The effects of non-linear feedback networks

Until now linear resistors have been used in the feed back network. Non-linear feed back networks are frequently used when all functions that are necessary to build a first order oscillator, are carried out by the memory circuit. So apart from the memory function, it also determines the amplitude of the integrator output signal and generates the integration constant. In order to make such simple first order oscillators electronically tunable, a non-linear feed back network is necessary. When the functions in the first order oscillator are separated (i.e. each function is performed by a separate circuit), the use of non-linear feed back networks can be avoided. In this section it will be shown that the use of a non-linear feed back network usually deteriorates the properties of the regenerative memory, and that conflicting design rules may occur. In chapter 7 this separation of functions will be discussed, and electronic implementations of such circuits will be given.

Together with the transfer function of the limiter, the feed back network determines the shape of the loop gain function and the excitation function. In this section the consequences of some non-linear loads will be briefly discussed. Again the bipolar differential pair will be used as a limiter. A non-linear feed back network can consist of diodes. In fig.4.25 four regenerative memories with different feedback networks have been depicted. The loop gain functions of circuits (a) and (b) have been depicted in fig.4.26. In the linear case a parabola is found according to \( A_l(X) = A_{\text{max}}(1 - X^2) \). In fig.4.27 the loop gain functions for the circuits (c) and
Fig. 4.25 Regenerative memories with linear (a) and non-linear feed back networks (b), (c) and (d)

Fig. 4.26 The loop gain functions for the linear feed back network (a) and the non-linear feed back network (b)

(d) have been depicted. All loop gain functions have been calculated with SPICE. The circuits were simulated with comparable parameters. In all figures unity loop gain has been indicated with a dotted line. Below the different circuits containing a non-linear feed back network will be compared to the circuit with the resistive feed back network.

4.10.1 Circuit (a) compared to circuit (b)

For a certain range of values given by:

\[ X \in [-1; -X_r] \cup [X_r; 1], \]

(4.104)

with \( X_r > 0 \), the currents through the diodes are such that their impedance can be neglected compared to the impedance of the resistors. One of the diodes conducts so little current, that the impedance across the parallel network of the diode and the resistor, is equal to the
Fig. 4.27 The loop gain functions for the non-linear feedback networks (c) and (d)

impedance of the resistor. The other diode conducts so much current that its impedance is much smaller than the impedance of the resistors. It can be considered as a short circuit in the feedback network. Now the feedback network can be modeled as a single resistor, with a resistance equal to that of one of the resistors in the original feedback network. Then the circuit behaves just like the circuit of fig. 4.25a. The loop gain function tracks the parabola.

For some value of $X$ given by:

$$X \in [-X_r; X_r],$$  \hspace{1cm} (4.105)

the diode impedances both start influencing the impedance of the feedback network. When at this stage the diode voltages are assumed to be equal to $U_{\text{diode}}$, and constant enough to assume that a constant current flows through the resistors, it follows:

$$I_r = \frac{U_{\text{diode}}}{R}$$  \hspace{1cm} (4.106)

The diode impedances are:

$$r_{d1} = \frac{V_T}{(1 + X)I - I_r}$$  \hspace{1cm} (4.107)

$$r_{d2} = \frac{V_T}{(1 - X)I - I_r}$$  \hspace{1cm} (4.108)

In the same way as used in (4.60) the loop gain function can be determined:

$$A_l(X) = (r_{d1} + r_{d2}) \frac{g_{e1}g_{e2}}{g_{e1} + g_{e2}}$$  \hspace{1cm} (4.109)

$g_{e1}$ and $g_{e2}$ are the transconductances of the two transistors given by (4.61) and (4.62) respectively. After some calculations and with the use of (4.106) it follows for $X = 0$:

$$A_l(0) = 1 + \frac{U_{\text{diode}}}{RI}$$  \hspace{1cm} (4.110)
For large values of $R$ it is close to unity. When a circuit has exactly unity loop gain—which it has when the resistors are removed—it is just not regenerative. In that case the diode impedances are equal to the small-signal base-to-emitter resistances of the corresponding transistors always.

(In practice, due to some non-idealities such as the presence of finite base currents, a slight difference occurs. Then a loop gain slightly larger than unity is found and regenerative operation is obtained.)

Due to the fact that the resistors reduce the diode currents somewhat, the diode impedances increase. The non-linear behavior of the diodes causes the impedance of the parallel network of a diode and a resistor to be higher than that of a diode without a resistor.

In fig.4.26b, the reduction of the loop gain at the center can be observed. Relatively small resistors have been used, so the effect is moderate. For a large range of $X$ the parabola is still tracked. Small resistors cause $X_r$ to be small also. The reduction of the loop gain at the center slows down the transition. Especially in the case of large resistances, the loop gain nearly equals unity, so a large effective time constant can be expected. A non-linear feedback network like this can be used in a simple first order oscillator, like an emitter coupled oscillator, in order to make the oscillator electronically tunable. The diode are then used to make the hysteresis independent of the current $I$. Then this current can be used for frequency modulation of the oscillator. A slow transition causes non-linearity in a current or voltage controlled oscillator (CCO or VCO), so in order to avoid this cause for non-linearity, small resistances should be used. However, due to the resistors the hysteresis becomes dependent on $I$ again. This causes another type of non-linearity of the controlled oscillator. Apparently two conflicting design rules exist for the value of the resistances, when this feedback network is used in a simple electronically tunable first order oscillator.

4.10.2 Circuit (a) compared to circuit (c)

In fig.4.25c another two-diode configuration for the feedback network has been depicted. The loop gain function, depicted in fig.4.26c, shows no similarity with the parabola at all. For $X = 0$ a very high loop gain is obtained. This may mislead the designer, since this high loop gain is available in a very small range of $X$ only. For the rest it is very small. Then it is even below the parabola shaped loop gain function that is valid for circuits with a resistive feedback network. Also at unity loop gain, the slope of the loop gain function is smaller than it would be in the resistive case. More delay and a larger noise sensitivity can be expected at the start of the transition.

It is possible to manipulate the circuit such that the unity gain line moves upwards. Then a large slope of the loop gain function at unity loop gain can be obtained (see fig.4.28). However, this method has some major drawbacks:

1. When this feedback network is used in a simple first order oscillator like an emitter coupled first order oscillator, at unity loop gain the capacitor current is nearly equal to zero. A characteristic formula for the capacitor current is:

   \[ I_C = X I \]  \hspace{1cm} (4.111)

   When $X$ is close to zero at unity loop gain, the capacitor voltage hardly changes. The result of this is a very small excitation at the start of the regenerative transition. This effect may very well annililate the advantage of the increased slope of the loop gain function. The small excitation causes a larger noise sensitivity also.

2. The hysteresis is reduced very much.

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3. For a considerable part of the transition the circuit is not regenerative. It behaves like a normal amplifier, so its output signal changes at a rate that is determined by the input signal. In this case this signal is a slowly changing capacitor voltage. So the output signal of the memory will contain very slow slopes with a small somewhat faster regenerative part in the middle. Apart from the fact that this effect causes non-linearity of the CCO or VCO, it may also be an inconvenient input signal for circuits that use the oscillator output signal as input. Slow slopes will for instance deteriorate the noise performance of a mixer[18].

Both effects can be reduced when the network of fig.4.25d is used. It can be seen as a combination of the other two diode networks, and consequently has properties that are similar to those of both other networks. Compared to network (c) the hysteresis is increased, and the reduction of the excitation at the start of the transition due to partial switching of the capacitor current, is avoided. For $X$ close to zero, the two anti parallel diodes have a neglectable influence on the impedance of the network. Then the properties are similar to network (b). Also in this case not all the currents that flow through the transistors are flowing through the diode. The two current sources $(1-a)I$ reduce the diode currents. Therefore a loop gain larger than unity is found. At the center of the loop gain function, for $X = 0$, the loop gain is:

$$A_l(0) = 1 + (1 - a) \quad (4.112)$$

This is a local minimum of the loop gain function. For the small values of $a$ that result in a large hysteresis, the loop gain is usually close to 2. The transition will not be slowed down that much. An interesting property of the network is found when the unity gain line is moved upwards. Then a situation can occur in which the loop gain drops below unity at the center.

![Fig. 4.28 The loop gain functions of circuit (c) and (d) when extra emitter resistors are inserted](image)

Three stable steady states are found in stead of two. Then, for example, a trinary Schmitt trigger could be built.

A method to move the unity gain line upwards is the insertion of small resistors in the emitter leads of the transistors. This introduces a reduction of the loop gain. The effect on the loop
gain functions of the circuits of fig.4.25c and fig.4.25d have been depicted in fig.4.28. The total value of the inserted emitter resistances is about 10% of the resistance of the resistive feedback network of circuit (a). These values can be rather low. In the spice simulation that was done to obtain fig.4.28, the extra emitter resistance was 20Ω. From this it can be concluded that care has to be taken with parasitic emitter resistances, because they may seriously harm the loop gain function, and consequently degrade the overall performance of the circuit. In this case circuit (d) even obtained a third stable state for \( X = 0 \). In fig.4.29 and fig.4.30 two

![Graph showing input and output signal of a Schmitt trigger with a non-linear feedback network of type (d), simulated without emitter resistances. A binary behavior can be observed.](image)

Fig. 4.29 The input and output signal of a Schmitt trigger with a non-linear feedback network of type (d), simulated without emitter resistances. A binary behavior can be observed.

SPICE simulations show the binary and trinary behavior of circuit (d) respectively. The binary behavior occurs when no emitter resistors are present and the trinary behavior occurs when the resistors are inserted. Then the loop gain drops below one in the center of the loop gain function. As a result the Schmitt trigger behaves like a normal negative feedback amplifier, so the output signal tracks the input signal. This can be seen in both fig.4.29 and fig.4.30, where at the start of each regenerative transition, the slope of the output signal is proportional to the slope of the input signal.

4.10.3 Discussion

Non-linear feedback networks are necessary when a regenerative circuit has to function as a simple current controlled oscillator (CCO). An example of such an oscillator is the emitter coupled oscillator with a non-linear feedback network. The properties of the oscillators equipped with a non-linear feedback network always tend to be worse than that of a comparable oscillator with resistive feedback. However, a simple regenerative first order oscillator with a resistive feedback network has no usable CCO properties. The separation of functions as described in chapter 2 is a method to use a regenerative memory with a resistive feedback network in a regenerative CCO. This separation of functions is also discussed in chapter 7 in
which also some electronic implementations of first order oscillators with separated functions will be given.

4.11 The noise conversion band of the regenerative system

In this section the system will be externally excited with a sine wave superimposed on a ramp. The difference between the reaction of this system to this excitation and to just a ramp will be calculated as a function of the frequency of the sine wave. Due to its slowness the system will not be able to react on excitation at relatively high frequencies; it has a low pass character. The bandwidth of this low pass filtering action is called the noise conversion band. In chapter 6 it will be discussed in detail. For now only an estimate of the noise conversion band will be made.

The external excitation is equal to:

\[ E_{exc}^e(t) = \alpha t + \beta \sin \omega t \]  

(4.113)

The expression for \( X(t) \) that is found after some calculations is very similar to (4.38). Two extra terms are found, one containing sines and cosines, the other a signal growing exponentially with time. The latter term is equal to:

\[ \delta X = \frac{A_l}{A_l - 1} \frac{\tau}{A_l - 1} \left( \beta \frac{\omega}{(\frac{\omega \tau}{A_l - 1})^2 + 1} \right) e^{(A_l - 1) \frac{t}{\tau}} \]  

(4.114)

Adding this term to (4.39) results in:

\[ X(t) = \frac{A_l}{A_l - 1} \frac{\tau}{A_l - 1} \left( \alpha + \beta \frac{\omega}{(\frac{\omega \tau}{A_l - 1})^2 + 1} \right) e^{(A_l - 1) \frac{t}{\tau}} \]  

(4.115)
The factor \( \beta \omega \) gives the slope of the sine wave around its zero crossings. For low frequencies the sine wave can be approximated by a ramp with this slope. This can be done when the duration of the sensitive part of the transition \((-1 < X < X_{eq})\), is small compared to the period of the sine wave (4.48).

For a certain value of \( \omega \), the slope of the ramp equals the slope of the sine wave. This happens when:

\[
\beta \omega = \alpha
\]  
(4.116)

(For these relatively low frequencies the term \( \left( \frac{\omega \tau}{A_{t} \beta} \right) \) can be neglected.) Now both signals contribute equally to the transition time. Note that when \( \beta \) is negative, there will be no transition, because then the excitation is exactly equal to zero. Before this the ramp dominates because it changes faster than the sine wave. In fig.4.23 it was already made clear that small changes in the external excitation may cause large changes in the duration of the first part of the transition. So when the regenerative system is excited with a ramp with a very low \( \alpha \), still a considerable influence can be expected from disturbing signals with a low frequency. In section 7.3.2 an example of such a minimally excited system will be discussed.

The maximum influence, with \( \beta \) kept constant, is found for \( \omega = \frac{A_{t}}{\tau} \). For higher frequencies the influence of the sine wave starts decreasing. Of course the ratio between \( \alpha \) and \( \beta \) still determines the sensitivity.

Two frequencies can be given that give an indication of the size of the noise conversion bandwidth:

\[
\omega_{\text{max}} = \frac{A_{t} - 1}{\tau}
\]  
(4.117)

\[
\omega_{\text{min}} = \frac{\alpha}{\beta}
\]  
(4.118)

A large \( \tau \) reduces the noise conversion band at the upper side and a large \( \alpha \) gives a reduction of the noise conversion band at the lower side. A large \( \alpha \) with respect to \( \beta \) also reduces the noise sensitivity for all frequencies. Again a large \( \alpha \) and a large \( \tau \) are good for the noise performance. The loop gain also appears in the expressions. Generally, due to other design constraints, it is less convenient to use it to optimize the noise performance. It should not be made higher than necessary as far as the noise conversion bandwidth is concerned.

In this section the influence of a sine wave on the duration of the transition has been discussed. This is not the only phenomenon that causes frequency instability in first order oscillators! It has been found that for low frequencies, the transition time needs not to be much affected. This does not mean that the complete first order oscillator is not influenced by signals at these frequencies. The point in time at which the input signal reaches the threshold level of the memory can still be shifted in time by the sine wave. Only for low frequencies, the transition time is not affected. This effect will be profoundly discussed in chapter 6. Only for higher frequencies \( (\omega > \frac{\beta}{\alpha}) \), the extra effect of modulation of the transition time occurs. So the noise performance is made worse due to this characteristic behavior of regenerative systems.

The calculations in this section are actually somewhat simple. The phase of the sine wave at the start of the transition has not been taken into account. This phase would have to be a random variable[27] and statistics would have to be used to obtain expressions that in the end are likely to produce similar results.
5. Non regenerative memories

In most first order oscillators, the binary memory is a regenerative memory. Also when a transfer that contains hysteresis is requested from an electronic circuit, a regenerative solution is usually chosen. In this chapter it will be shown that there are other non regenerative types of memories that are able to perform the binary memory function just as well.

5.1 The sample and hold system as memory

A sample and hold system can be used to construct a binary memory. It contains a storage part and a sampling part. The sampling part is a circuit that can be in two different modes; it can be activated or deactivated. An external signal determines the mode in which the sampling part is. It is labeled as the sampling pulse. The sampling part, when activated, generates a copy of its input signal in the storage part. When the sampling part is deactivated, this copy is retained in the storage part until the sampling part becomes active again. Ideally the generation of the copy happens infinitely fast, and the data can be retained in the storage part infinitely long.

In practical electronic sample and hold circuits, the storage part usually is a capacitor, although other elements, like inductors, could be used as well. When a capacitor is used, the sampled signal will be a voltage. The sampling circuit can be a switch or a gain-cell. A gain-cell is an amplifier of which the amplification factor can be controlled electronically. It can be deactivated by way of making the gain equal to zero. The amplifier has to establish a well defined relation between its output voltage and some input quantity.

When the sampling part is activated, it has to change the capacitor voltage as fast as possible to the desired value. The capacitor voltage depends on the charge that is stored in it, so a current has to flow into or out of the capacitor. This current has to be supplied by the sampling part. When a switch is used (for example a MOS-transistor), this current has to be supplied by the source of the sample and hold circuit. When this cannot be allowed the sample and hold circuit has to be preceded by a buffer amplifier. This buffer has to be designed such that it can supply enough current, to generate the largest voltage change at the capacitor that can be expected within the time the switch is activated (sampling time). The same holds for the maximum current that can be supplied by the gain-cell. For now the source of the sample and hold circuit is supposed to be able to supply the requested current. Then the sample and hold circuit can be connected to the source $U_s$ directly. In fig.5.1 a sample and hold circuit containing a switch has been depicted.

![Sample pulse](image)

**Fig. 5.1** A sample and hold circuit containing a switch
5.1.1 Hysteresis with a sample and hold circuit

In order to construct a circuit that shows hysteresis some extra elements are needed. Two comparators are used to set the threshold levels of the hysteresis curve. Several voltage sources are needed to set some reference voltage levels. In fig.5.2 the circuit has been depicted. The sample and hold circuit consists of the usual hold capacitor and two switches. Each switch is operated by a separate comparator. In this case the comparators are voltage comparators, but that is not essential. Comparator C1 is activated when the input voltage is above $U_{rh}$ and comparator C2 is activated when the input voltage is below $U_{rl}$. The reference voltages $U_{rh}$ and $U_{rl}$ are chosen such that $U_{rh} > U_{rl}$. In that case a range of input voltages exists in which none of the comparators is activated.

Suppose the input signal of the complete circuit is such that comparator C1 is activated. Then switch S1 is closed, and the capacitor voltage becomes equal to $U_h$. When the input signal changes again and causes C1 to switch off, S1 opens. Then at the hold capacitor the voltage $U_h$ is retained, until again one of the comparators is activated. When this happens to be C2, the capacitor voltage will be changed to $U_l$. Hence at the output of the sample and hold circuit, it can be determined which comparator was active the last, so which threshold level was crossed last. The circuit behaves like a binary memory as needed in first order oscillators.

The circuit can be reduced. When, for example, $U_{rh}$ and $U_h$ are equal, the two voltage sources can be replaced by one. A more drastic reduction can be made when one of the switches is deleted. In fig.5.3 a circuit containing only one switch has been depicted. Again two comparators are used to detect the crossing of the two threshold levels. Now their output signals are combined, so the same switch is closed when either one of the comparators is activated. Of course a distinction has to be made between the two events that cause the switch to close. This difference is not expressed in the closing of a different switch any more, since only one switch is present. Therefore the input signal for the comparators is also used as input signal for the sample and hold circuit itself. There are two different levels of the input signal for which the switch closes. Hence two different values at the output of the sample and hold circuit can be expected. Suppose comparator C1 is activated. This means that the input signal is above $U_{rh}$. At the output of the sample and hold circuit a copy of the input signal is found that is updated continuously as long as the switch is closed. As soon as the input signal drops below $U_h$, the switch opens and the value of the input signal at the time of the opening is retained. At the time of the opening the value of the input signal was exactly equal to $U_h$, so this is the value that will be retained. An analog description of the behavior can be given when comparator C2 is activated. Then the value $U_l$ is retained when the switch is opened.
Fig. 5.3 A sample and hold memory with one switch

again. The fact that the output signal of the sample and hold system tracks any change of the input signal when the switch is closed usually does not introduce problems. The binary character of the memory when the switch is open suffices for the use in first order oscillators. If necessary a third comparator (C3), which has a reference level $U_{ref}$ with $U_1 < U_{ref} < U_h$, can be used as a buffer for the signal at the hold capacitor. At the output of this limiter real binary information if available. From this it can be concluded that some droop of the capacitor voltage can be allowed. This droop will not disturb the binary information contents as long as it is not too large. However, it does affect the timing. When the voltage at the hold capacitor is changed with a constant current, due to the droop, less time is needed to change the binary information. The voltage at the hold capacitor has already come somewhat closer to the value it is about to be forced to by the sampling part. In section 8.1 the effect of the droop on the behavior of a first order oscillator is discussed.

When the voltage at the storing capacitor, is not an exact copy of the input signal, due to a limited charging time, offset voltages and so on, this does not harm the basic operation. As long as the two different events can be distinguished at the output, operation as a binary memory is feasible, but again the timing may be affected.

Usually the source of the memory circuit does not allow the sample and hold part to be directly connected to it. When the switch closes, the currents necessary for fast voltage changes at the hold capacitor cannot be supplied. In that case a buffer amplifier has to precede the memory. When the gain of this buffer can be controlled, the switch of the sample and hold can be omitted. The amplifier itself acts as “switch”. This situation has been depicted in fig.5.4. The combined output signal of the comparators now controls the gain of the amplifier. The amplifier has been implemented as a negative feedback amplifier, with unity feedback. When the active part has nullor properties when activated, the circuit does not load the source at all, and is also able to supply enough current to change the capacitor voltage in time.

5.2 The integrator as memory

In chapter 2 the integrator has been introduced as memory. Actually the sample and hold systems also belong to the class of integrator memories, but they are dealt with separately. This is because the systems usually referred to as integrator, are supplied with well defined input signals, whereas this is not the case in sample and hold systems. Then the input signals for the integrators in the sample and hold circuits are expressed in terms of “as large as possible” and so on. For example, when the input quantity of a capacitor is chosen to be a current, then it is clearly an integrator, because a capacitor integrates currents. In sample
Fig. 5.4 A sample and hold memory with a gain-cell

...and hold circuits containing a capacitor, all relevant information quantities are expressed in voltages. A capacitor does not integrate voltages, so the fact that the capacitor operates as an integrator seem to be less obvious. Still to obtain a voltage change at the capacitor, a current has to be supplied. Hence the sampling part has to have a transfer with the dimension of an admittance. Especially in the case of switches, this transfer is not well determined, and only minimal values are given. Then the current to be integrated is expressed in the terms mentioned previously.

When an integrator, supplied with a well determined input signal, is used as memory, it can still be used for the memory function in a first order oscillator. Only now the finite switching time is not considered to be a parasitic effect that has to be minimized. It is now a well determined part of the total period of the oscillator. All this was explained in section 2.5.2. In chapter 8 two-integrator oscillators will be dealt with in full detail.

5.3 Summary

In this section it has been shown that sample and hold systems can be used to construct systems that show hysteresis. This property can be used to make the binary memories, which are needed in first order oscillators. There is no fundamental difference between memories like this and regenerative binary memories. They are able to perform the same basic function. In any application of regenerative memories, the sample and hold memory could be used also. Of course in some applications the use of these memories is less obvious than it is in others. In digital practice both types of memories are used. The regenerative memories are known as static RAM cells, and the sample and hold memories as dynamic RAM cells.

It has been indicated that sample and hold systems actually are based upon the integration of signals. Therefore the sample and hold system can be classified among the integrators (chapter 2). Consequently sample and hold first order oscillators are two-integrator oscillators.
6. Noise in first order oscillators

In this chapter the effect of noise on first order oscillators will be discussed. The noise behavior will be dealt with on the system level, so no special transistor implementations will be discussed. All basic functions are assumed to be separated. The memory is assumed to have no influence on the noise behavior. Especially in the case of regenerative memories, this would complicate the calculations. Moreover, when designed correctly, a first order oscillator does not have to suffer from the noise behavior of the memory. The noise behavior of regenerative memories has been discussed in chapter 4.

Noise influences the oscillator in two ways. This can be seen easily from the general formula for the output frequency:

\[ \omega_0 = 2\pi \frac{\alpha}{2E_{hys}} \]  
\[(6.1)\]

Both the integration constant \( \alpha \) and the hysteresis \( 4E_{hys} \) can contain an unpredictable (noise) part.

For the noise sources the Bennet model will be used. According to Bennett[27], white noise can be described as the sum of an infinite number of sinusoidal voltages (or currents) having equal amplitudes, different frequencies and a random phase. The phase \( \phi \) is a random variable with a uniform probability density function in the interval \([\pi; \pi]\). Using this model, the effect of noise on a circuit can be calculated in a simple way. The effect of only one frequency component has to be calculated and then the effect of the complete noise spectrum can be found by way of superposition. Of course the effects concerned have to be linear to allow the use of this noise model.

For the noise sources discussed in this chapter, this condition is assumed to be met. For electronic first order oscillators that make use of a capacitor to implement the integration function, a somewhat more specific expression for the output frequency can be given:

\[ \omega_0 = 2\pi \frac{I}{2U_{hys}C_i} \]  
\[(6.2)\]

\( U_{hys} \) is the hysteresis voltage of the oscillator and \( C_i \) is the value of the timing capacitor that implements the integrator function. \( I \) is the amplitude of the current that charges the capacitor. The current can contain a noise component \( i_n \) and the hysteresis voltage a noise component \( u_n \). For relatively low frequency noise, the expression for the output frequency becomes:

\[ \omega_0 = 2\pi \frac{I + i_n}{2C_i(U_{hys} + u_n)} \]  
\[(6.3)\]

The amplitudes of both \( i_n \) and \( u_n \) are relatively small.

The first order oscillator is a time variant system. Therefore the noise sources can also be time variant. Four different types of noise sources can be distinguished when this aspect is taken into account. The Bennet components of the noise sources can be described as:

\[ u_{n1}(\omega_m, t) = sgn(\alpha)\hat{u}_{n1} \sin(\omega_m t + \phi_{u1}) \]  
\[(6.4)\]

\[ i_{n1}(\omega_m, t) = sgn(\alpha)\hat{i}_{n1} \sin(\omega_m t + \phi_{i1}) \]  
\[(6.5)\]

\[ u_{n2}(\omega_m, t) = \hat{u}_{n2} \sin(\omega_m t + \phi_{u2}) \]  
\[(6.6)\]

\[ i_{n2}(\omega_m, t) = \hat{i}_{n2} \sin(\omega_m t + \phi_{i2}) \]  
\[(6.7)\]
Noise sources $u_{n1}$ and $i_{n1}$ are correlated to the switching action in the oscillator. This is expressed by the factor $\text{sgn}(\alpha)$. In fig.6.1 the four noise sources have been inserted in a first order oscillator. All basic functions except the integrator function are performed by the active circuit. The transfer of the active part has the dimension of an admittance. The input quantity is a voltage and the output quantity is a current. These conditions are required by the integrator which has a transfer with the dimension of an impedance.

The noise sources are "equivalent noise sources". They do not need to be present in the actual circuit. Their value is such that when the active part is considered noiseless, the equivalent noise sources cause the same noise effects in the oscillator as the actual noise sources in the active part would have done[3]. The use of equivalent noise sources normally eases the noise calculations. The calculation of the values of the noise sources may be complicated by the fact that some of the transfers in the active part are time variant. However, there are methods to overcome this[28].

6.1 The origin of correlated and uncorrelated sources

In this section one of the ways noise sources can become correlated to the switching action will be discussed. The noise currents will be taken as an example. The largest contribution to the noise currents $i_{n1}$ and $i_{n2}$ is generally made by the output signal of the current switch. When implemented as shown in fig.6.2, a correlated noise current is found at the output of the current switch. When implemented as shown in fig.6.3, an uncorrelated noise current is found

![Fig. 6.2 The generation of a noise source correlated to the switching action](image)

at the output.

6.1.1 The minimal noise current of the switch

In electronic first order oscillators a general statement can be made about the minimal noise current that is introduced by the current switch. Somewhere in the oscillator system an independent noise source exists from which the noise current originates. Such a source is basically implemented as shown in fig.6.4. The smallest noise current that can be obtained
Fig. 6.3 The generation of a noise source that is not correlated to the switching action

Fig. 6.4 The basic noise source

from this source is determined by the value of the resistor $R_i$. [25] Its spectral density is given by:

$$S(i_n) = \frac{4kT}{R_i} \quad (6.8)$$

The value of $R_i$ is bounded by the available supply voltage. The current $I$ has to be as large as possible for a large signal-to-noise ratio of the integration constant. A large value of $R_i$ is also favorable for this, hence a large voltage across $R_i$ is desired. The power that is dissipated in $R_i$ is equal to:

$$P_{R_i} = R_i I^2 \quad (6.9)$$

Apparently this power dissipation has to be maximized for an optimal noise behavior. When for a specific design the maximum power consumption has been specified, immediately the maximum attainable signal-to-noise ratio of the integration constant can be calculated. From this value also predictions can be made about the noise behavior of the complete oscillator.

6.2 Noise measures

Before going into detail about the noise sources and their effects, first some noise measures will be discussed.

The carrier-to-noise ratio (CNR) is the most widely used figure of merit for oscillators. It is defined as:

$$\mathcal{L}(\omega_m) = \frac{P(\Delta \omega_m)}{P(\omega_c)} \quad (6.10)$$

$P(\Delta \omega_m)$ is the signal power in a 1Hz sized frequency band at a distance $\Delta \omega_m$ from the carrier $\omega_c$. $P(\omega_c)$ is in principle the power in the complete output spectrum, but in practice the power of the carrier is taken. Usually this introduces a neglectable error. For a signal that causes FM-modulation, it can be derived that the ratio of the magnitude of the carrier and the magnitude of a sideband is equal to half the modulation index. The square of this ratio is also the CNR:

$$\mathcal{L}(\omega_m) = \left( \frac{\Delta \omega_c}{2 \omega_m} \right)^2 \quad (6.11)$$
In this case \( \Delta \omega_c \) is the peak deviation of the carrier frequency caused by a noise component of the Bennet model and \( \omega_m \) is its frequency. It can be seen that in that case the shape of a CNR graph has a \( \frac{1}{\omega_m} \) character (20dB/dec). Both quantities can be measured directly, which makes the CNR very easy to use. For applications of oscillators in communication systems, the CNR at one specific frequency can be used as a figure of merit. Generally the CNR at a distance from the carrier equal to the distance between two neighboring channels of the communication system is used. Noise power in a neighboring channel can degrade the performance of the communication system.[18] For other applications the CNR is less convenient, since then there is no specific distance from the carrier at which the CNR is of special importance. On top of that the operating frequencies of oscillators to be compared can differ very much. Since the value of the CNR depends on the carrier frequency it is not easy to use in that case. Another noise measure is the mean square fractional frequency fluctuation density, labeled as \( S_y(\omega_m) \). It is defined as:

\[
S_y(\omega_m) = \frac{2\omega_m^2 C(\omega_m)}{\omega_0^2}
\]  

(6.12)

When a noise source causes frequency modulation of the oscillator, it is independent of the carrier frequency of the oscillator and can therefore be used to compare oscillators that operate on different frequencies. For FM modulation, the \( S_y(\omega_m) \) is also independent of \( \omega_m \). As a result the graph of \( S_y(\omega_m) \) is horizontal if the noise causes FM modulation. It is more convenient to compare the levels of the horizontal parts of the \( S_y(\omega_m) \) graphs of two oscillators than the CNR's at an arbitrary distance from the carriers.

For variations of the carrier frequency, the \( S_y(\omega_m) \) should be constant. There are however phenomena that can cause a dependency of \( S_y(\omega_m) \) on the carrier frequency. An important phenomenon is the sampling of noise by the oscillator. Due to this effect noise at high frequency bands is folded around the carrier. This increases the noise level around the carrier. When the oscillator is tuned to a lower frequency, it folds back more noise bands to the same place around the carrier. Then the noise power around the carrier is increased further. This effect can be noticed from the \( S_y(\omega_m) \), because it increases when the oscillator is tuned to a lower frequency. It can be used to investigate the presence of noise folding.

In practice, for relatively low noise frequencies, the \( S_y(\omega_m) \) is not horizontal, but slanted with a negative slope. This is usually caused by \( \frac{1}{\omega} \)-noise. At large distances from the carrier the \( S_y(\omega_m) \) tends to start increasing with the distance to the carrier frequency. When this happens usually the white noise floor of the oscillator is reached.

6.3 The influence of correlated noise sources

The noise sources that are correlated to the switching action \((i_{n1}, u_{n1})\) have the same influence on the oscillator as signals that are intentionally applied to the oscillator for FM-modulation. The only difference is the fact that normal modulating signals do not contain frequency components above half the carrier frequency, whereas the noise sources generally do.

6.3.1 The influence of a correlated noise current source

Suppose a noise component \( i_{n1} \) is present in the charging current \( I \). It is one of the components from the Bennet noise model of a noise current source. Then the two different values the integration constant can have both contain a noise component. When the two states in which the memory of the oscillator can be are labeled \( x \) and \( y \), their respective integration constants
\[ \alpha_x = \alpha + \Delta \alpha \sin(\omega_m t + \phi) \quad (6.13) \]
\[ \alpha_y = -\alpha - \Delta \alpha \sin(\omega_m t + \phi) \quad (6.14) \]

The oscillator behavior is described by two integral equations:
\[ \int_0^{\frac{1}{2}T_o} \alpha_x dt = U_{hys} \quad (6.15) \]
\[ \int_{\frac{1}{2}T_o}^{T_o} \alpha_y dt = -U_{hys} \quad (6.16) \]

\( T'_o \) is the actual period, that is found due to the noise. From these equations, after some calculation, the following integral equation can be derived:
\[ \int_0^{T'_o} [\alpha + \Delta \alpha \sin(\omega_m t + \phi)] dt = 2U_{hys} \quad (6.17) \]

After some calculation it is found:
\[ T'_o = T_o \left( \frac{1}{1 + \frac{\Delta \alpha}{\omega_m} \frac{1}{T_o} \left[ \cos(\phi) - \cos(\omega_m T'_o + \phi) \right]} \right) \quad (6.18) \]

With:
\[ T_o = \frac{2U_{hys}}{\alpha} \quad (6.19) \]

\( T_o \) is the period of a noiseless oscillator.

A closed form expression for \( T'_o \) has to be found. In the denominator of the fraction in (6.18), still \( T'_o \) is present. At these places the complete expression (6.18) can be substituted for \( T'_o \). When this is done, repeatedly, convergence to a solution may be obtained. At some stage, it will be sufficient for the accuracy to substitute \( T_o \) for \( T'_o \) in stead of (6.18). Here only one iteration step will be made, so \( T_o \) is substituted directly for \( T'_o \).

Using:
\[ T_o = \frac{2\pi}{\omega_o} \quad (6.20) \]

the following closed form expression is found:
\[ T'_o = T_o \left( \frac{1}{1 + \frac{\Delta \alpha}{\omega_o} \frac{\omega_o}{\omega_m} \left( \cos(\phi) - \cos(2\pi \frac{\omega_m}{\omega_o} + \phi) \right)} \right) \quad (6.21) \]

The expression for the frequency is then:
\[ \omega'_o(\omega_m, \phi) = \omega_o \left( 1 + \frac{\Delta \alpha}{2\pi} \frac{\omega_o}{\omega_m} \left( \cos(\phi) - \cos(2\pi \frac{\omega_m}{\omega_o} + \phi) \right) \right) \quad (6.22) \]

In this expression the random variable \( \phi \) is still present. It can be canceled when the first and the second moment of \( \omega'_o(\omega_m, \phi) \) are calculated. The first moment, also known as the mean is defined as:
\[ \bar{\omega}_o(\omega_m) = \int_{-\infty}^{\infty} \omega'_o(\omega_m, \phi) P(\phi) d\phi \quad (6.23) \]

The function \( P(\phi) \) is the probability density function of \( \phi \). In this case it has a constant value \( \frac{1}{2\pi} \) in the interval \([-\pi; \pi]\) and it is zero outside the interval. With \( P(\phi) \) defined as such, the mean can be calculated as:
\[ \frac{1}{2\pi} \int_{-\pi}^{\pi} \omega'_o(\omega_m, \phi) d\phi \quad (6.24) \]
When (6.22) is substituted in (6.24), after some calculations the mean is found:

\[ \bar{\omega}_o(\omega_m) = \omega_o \]  

(6.25)

Apparently the mean of the frequency is not affected by the disturbing signal (which has a mean equal to zero).

The second \textit{centralized} moment, also known as the \textit{standard deviation} is the next quantity to be calculated. The standard deviation gives the effective value of the fluctuations of the frequency. Using the same probability density function as before, the standard deviation is given by:

\[ \Delta \omega_o(\omega_m) = \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} (\omega_o(\omega_m, \phi) - \bar{\omega}_o(\omega_m))^2 \, d\phi} \]  

(6.26)

A substitution of (6.22) and (6.25) in (6.26) results in:

\[ \Delta \omega_o(\omega_m) = \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} \left( \omega_o \frac{\Delta \alpha}{\omega_m} \frac{\omega_o}{\alpha} \frac{\omega_o}{\omega_m} (\cos(\phi) - \cos(2\pi \frac{\omega_m}{\omega_o} + \phi)) \right)^2 \, d\phi} \]  

(6.27)

After some calculations it follows:

\[ \Delta \omega_o(\omega_m) = \omega_o \left| \frac{1}{2} \sqrt{2} \frac{\Delta \alpha}{\alpha} \sin \left( \frac{\pi \omega_m}{\omega_o} \right) \right| \]  

(6.28)

From (6.28) it can be seen that for \( \omega_m = \omega_o \) and \( \omega_m = 2\omega_o \) the frequency is not modulated.

The CNR can be expressed in terms of \( \Delta \omega_o \):

\[ \mathcal{L}(\omega_m) = \left( \frac{\Delta \omega_o}{2\omega_m} \right)^2 = \left( \frac{\sqrt{2} \Delta \omega_o(\omega_m)}{2\omega_m} \right)^2 \]  

(6.29)

A substitution of (6.28) in (6.29) results in:

\[ \mathcal{L}(\omega_m) = \frac{1}{4} \left( \frac{\omega_o}{\omega_m} \right)^2 \left( \frac{\Delta \alpha}{\alpha} \right)^2 \left( \sin \left( \frac{\pi \omega_m}{\omega_o} \right) \right)^2 \]  

(6.30)

The factor \( \frac{\Delta \alpha}{\alpha} \) is the signal-to-noise ratio of the charging current. With:

\[ \Delta \alpha = \frac{\dot{i}_{n1}}{C_t} \]  

(6.31)

\[ \alpha = \frac{I}{C_t} \]  

(6.32)

it follows:

\[ \frac{\Delta \alpha}{\alpha} = \frac{\dot{i}_{n1}}{I} \]  

(6.33)

\( \dot{i}_{n1} \) is equal to the amplitude of the noise current. When the noise comes from a current source from which the spectral power density is given by (6.8), the amplitude is:

\[ \dot{i}_{n1} = \sqrt{2} \sqrt{\frac{4kT}{R_i}} \]  

(6.34)

For relatively low noise frequencies, the third factor between brackets of (6.30) yields to unity. Then the CNR is:

\[ \mathcal{L}(\omega_m) = \left( \frac{\omega_o}{\omega_m} \right)^2 \frac{2kT}{R_iI^2} \]  

(6.35)
This equation has been verified by measurement (section 7.7.1,17).

The method described above is not the simplest way. In section 6.4 a simpler way is suggested. Only in that case the third factor of (6.30) that introduces the dependency on \( \omega_m \) is not found. It is assumed constant and equal to unity. This restricts the range for which the simplified equation can be used.

6.3.2 The effect of a correlated noise voltage source

Similar calculations as described above can be performed in order to find the CNR caused by a noise voltage source that is correlated to the switching action. Suppose the hysteresis voltage is affected with a noise voltage:

\[
\nu_{n1} = \hat{\nu}_{n1} \sin(\omega_m t + \phi)
\]  

(6.36)

From the integral equations (6.15) and (6.16) the following equations can be derived:

\[
\int_{0}^{T_x} a dt = U_{hys} + \frac{1}{2} \hat{\nu}_{n1}(\sin(\omega_m T_x + \phi) + \sin(\phi))
\]  

(6.37)

\[
\int_{0}^{T_y} -a dt = -U_{hys} - \frac{1}{2} \hat{\nu}_{n1}(\sin(\omega_m T_x + \omega_m T_y + \phi) + \sin(\omega_m T_x + \phi))
\]  

(6.38)

\( T_x \) is the actual duration of the first half period of the oscillator. At the start and at end of the interval the integrator signal is compared with the fluctuating hysteresis voltage. The value of the noise voltage at those two moments is important. The difference between the two values is a measure for the influence of a noise component at a certain frequency on the hysteresis. The value of the noise at the start of the interval is taken before the oscillator switches, because at that moment the value of the hysteresis is important. The same holds for the value of the noise at the end of the interval. The oscillator switches one time between the determination of the two values. Hence they have opposite signs, because their sign is correlated to the switching action. Therefore a sum of the two values is found in (6.37). The factor \( \frac{1}{2} \) is necessary to obtain the correct amplitude of the noise. This can be easily seen when \( \omega_m = 0 \) is substituted. Then the effective value of the sum of the two sine-functions is \( \sqrt{2} \hat{\nu}_n \). The effective value of the source added to the hysteresis should however be \( \frac{1}{2}\sqrt{2} \hat{\nu}_n \).

\( T_y \) is the actual duration of the next half of the period. It is determined in a similar way as described above. Because this half period directly follows the half period described by \( T_x \), a phase shift \( \omega_m T_x \) is found in the arguments of the sine-functions. This phase shift expresses the correlation between the noise voltages in the two half periods.

After some calculations, similar to those for the noise current, the mean and the standard deviation of the frequency can be calculated. With (6.1) it follows:

\[
\alpha = 2U_{hys} \left( \frac{\omega_o}{2\pi} \right)
\]  

(6.39)

Using this expression, the mean and the standard deviation can be expressed in terms of the signal-to-noise ratio of the hysteresis.

The mean of the frequency is:

\[
\bar{\omega}_o(\omega_m) = \omega_o
\]  

(6.40)

The standard deviation is:

\[
\Delta \omega_o(\omega_m) = \frac{1}{2} \omega_o \left( \frac{\hat{\nu}_{n1}}{U_{hys}} \right) \frac{1}{2}\sqrt{2} \left( \cos(\pi \frac{\omega_m}{\omega_o}) + 1 \right)
\]  

(6.41)
For \( \omega_m = \omega_0 \) there is no FM-modulation.

For relatively low frequencies the last factor between bracket in (6.41) yields to 2. The dependency on \( \omega_m \) is lost. Taking into account that \( \Delta \omega \) is an effective value, the CNR can be calculated:

\[
L(\omega_m) = \left( \frac{\sqrt{2} \Delta \omega}{2 \omega_m} \right)^2 = \frac{1}{4} \left( \frac{\omega_0}{\omega_m} \right)^2 \left( \frac{u_{n1}}{U_{hyd}} \right)^2
\]  \hspace{1cm} (6.42)

When the noise is caused by resistors, like base resistances of transistors, the square amplitude of the noise is:

\[
u_{n1}^2 = 8kT R_u
\]  \hspace{1cm} (6.43)

Then it follows:

\[
L(\omega_m) = \left( \frac{\omega_0}{\omega_m} \right)^2 \frac{2kT R_u}{E_{hyd}^2}
\]  \hspace{1cm} (6.44)

This equation has been verified by measurement.

### 6.4 Simplified calculation of the influence of noise sources

Starting from (6.2) very quickly the CNR caused by the correlated noise sources can be calculated. It all comes down to finding the peak deviation of the carrier frequency caused by the noise. When this is known, it can be substituted in the expression for the CNR (6.10). For a noise current \( i_{n1} \) it follows:

\[
\omega_0 + \Delta \omega = 2\pi I + i_{n1} = \omega_0 + 2\pi \frac{i_{n1}}{2U_{hyd} C_t}
\]  \hspace{1cm} (6.45)

\[
\Delta \omega = 2\pi \frac{i_{n1}}{2U_{hyd} C_t} = \omega_0 \frac{i_{n1}}{I}
\]  \hspace{1cm} (6.46)

For a noise voltage \( u_{n1} \) it follows:

\[
\omega_0 + \Delta \omega = 2\pi \frac{I}{2(U_{hyd} + u_{n1}) C_t} = \frac{\omega_0}{1 + \frac{u_{n1}}{U_{hyd}}} \approx \omega_0 \left( 1 - \frac{u_{n1}}{U_{hyd}} \right)
\]  \hspace{1cm} (6.47)

\[
\Delta \omega = -\omega_0 \frac{u_{n1}}{U_{hyd}}
\]  \hspace{1cm} (6.48)

These expressions have proved to be very useful. Only the range of \( \omega_m \) for which they are valid has to be kept in mind.

### 6.5 The spectrum of a duty cycle modulated signal.

Before the effect of noise sources that are not correlated to the switching action are dealt with, first the spectrum of a duty cycle modulated signal will be investigated. This is because the uncorrelated noise sources basically modulate the duty cycle of the first order oscillator.

#### 6.5.1 The Fourier series describing a duty cycle modulated signal

The effect on the spectrum of one sine wave component at a frequency \( \omega_m \) will be investigated. It can be seen as one of the components of the Bennet noise model. A duty cycle modulated
signal has been depicted in figure 6.5. The period of this signal is $T$, and the pulse width is $d_c T$. Then the duty cycle is:

$$\frac{d_c T}{T} = d_c$$  \hspace{1cm} (6.49)

The Fourier series describing this signal is:

$$f(t) = \sum_n f_n(t) = \sum_n c_n e^{in\omega_0 t}$$  \hspace{1cm} (6.50)

$$c_n = \frac{1}{n\pi} \sin(n\pi d_c)$$  \hspace{1cm} (6.51)

$$\omega_0 = \frac{2\pi}{T_0}$$  \hspace{1cm} (6.52)

The modulation of the duty cycle can be modeled as a time dependency of the duty cycle:

$$d_c(t) = d_{co} + \dot{d}_c \sin(\omega_m t)$$  \hspace{1cm} (6.53)

The average duty cycle is described with a constant $d_{co}$. The time dependent part is described by a modulation frequency $\omega_m$ and an amplitude $\dot{d}_c$. Now the coefficients $c_n$ become time dependent:

$$f_n(t) = c_n(t) e^{in\omega_0 t}$$  \hspace{1cm} (6.54)

AM signals are described in a similar way, and also in this case AM-like sidebands are generated. In order to determine the amplitude of these sidebands $c_n(t)$ will be looked at first. Substitution of (6.53) in (6.51) results in:

$$c_n(t) = \frac{1}{n\pi} \sin \left( n\pi \left[ d_{co} + \dot{d}_c \sin(\omega_m t) \right] \right)$$  \hspace{1cm} (6.55)

$$= \frac{1}{n\pi} \left\{ \sin(n\pi d_{co}) \cos \left( n\pi \dot{d}_c \sin(\omega_m t) \right) \right\}$$

$$+ \frac{1}{n\pi} \left\{ \cos(n\pi d_{co}) \sin \left( n\pi \dot{d}_c \sin(\omega_m t) \right) \right\}$$  \hspace{1cm} (6.56)

Expression (6.56) consists of products of sines and cosines. Now every factor of these products will be reduced in order to simplify the interpretation of (6.56). Also the factor $\Delta d_c$ is introduced, which represents the static deviation from a duty cycle equal to a half. So $\Delta d_c$ is a constant.

$$d_{co} = \frac{1}{2} + \Delta d_c$$  \hspace{1cm} (6.57)

$$|\Delta d_c| < \frac{1}{2}$$

With (6.57) and by assuming:

$$n\pi \dot{d}_c \ll 1$$  \hspace{1cm} (6.58)

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the following reductions can be made:

\[
\cos \left( n\pi \hat{d}_c \sin(\omega_m t) \right) \approx 1 - \frac{1}{4} \left( n\pi \hat{d}_c \sin(\omega_m t) \right)^2 \\
= 1 - \frac{1}{4} \left( n\pi \hat{d}_c \right)^2 \left( \frac{1}{2} - \frac{1}{2} \cos(2\omega_m t) \right) \\
= 1 - \left( \frac{1}{2} n\pi \hat{d}_c \right)^2 + \left( \frac{1}{2} n\pi \hat{d}_c \right)^2 \cos(2\omega_m t) \tag{6.59}
\]

\[
\sin \left( n\pi \hat{d}_c \sin(\omega_m t) \right) \approx n\pi \hat{d}_c \sin(\omega_m t) \tag{6.60}
\]

For the simplification of the other two factors, odd and even \( n \) are separated.

**Even \( n \), except \( n = 0 \):**

\[
\sin(n\pi d_{co}) = \sin \left( \frac{1}{2} n\pi + n\pi \Delta \hat{d}_c \right) = \xi_n \sin(n\pi \Delta \hat{d}_c) \tag{6.61}
\]

\[
\cos(n\pi d_{co}) = \cos \left( \frac{1}{2} n\pi + n\pi \Delta \hat{d}_c \right) = \xi_n \cos(n\pi \Delta \hat{d}_c) \tag{6.62}
\]

**Odd \( n \):**

\[
\sin(n\pi d_{co}) = \xi_n \cos(n\pi \Delta \hat{d}_c) \tag{6.63}
\]

\[
\cos(n\pi d_{co}) = -\xi_n \sin(n\pi \Delta \hat{d}_c) \tag{6.64}
\]

With:

\[
\xi_n = \cos \left( \frac{1}{2} n\pi \right) + \sin \left( \frac{1}{2} n\pi \right) \tag{6.65}
\]

Note: \(|\xi_n| = 1\).

Now (6.56) can be reduced for odd and even \( n \):

**Even \( n \), use (6.61) and (6.62).**

\[
c_{ne}(t) = \frac{\xi_n}{n\pi} \left\{ \sin \left( n\pi \Delta \hat{d}_c \right) \left[ 1 - \left( \frac{1}{2} n\pi \hat{d}_c \right)^2 + \left( \frac{1}{2} n\pi \hat{d}_c \right)^2 \cos(2\omega_m t) \right] \right\} \\
+ \frac{\xi_n}{n\pi} \left\{ \cos \left( n\pi \Delta \hat{d}_c \right) n\pi \hat{d}_c \sin(\omega_m t) \right\} + \frac{1}{2} \delta(n) \tag{6.66}
\]

With \( \delta(0) = 1 \) and \( \delta(n) = 0 \) for \( n \neq 0 \).

A special case is found for \( n = 0 \). Then a division by zero takes place in (6.56). When the limit \( n \to 0 \) is calculated, it follows the value \( c_0(t) = d_{co} + \hat{d}_c \sin(\omega_m t) \) for \( n = 0 \). When the same limit is calculated for (6.66) the result has to be the same. Calculations show that due to the simplifications a term \( \frac{1}{2} \) would be missing for \( n = 0 \), if the term \( \frac{1}{2} \delta(n) \) were omitted.

In stead of \( d_{co} + \hat{d}_c \sin(\omega_m t) \), it would be found \( \Delta \hat{d}_c + \hat{d}_c \sin(\omega_m t) \). For this reason the term \( \frac{1}{2} \delta(n) \) has been introduced in (6.66). In that case (6.56) and (6.66) produce the same result for \( n = 0 \).

**Odd \( n \), use (6.63) and (6.64).**

\[
c_{no}(t) = \frac{\xi_n}{n\pi} \left\{ \cos \left( n\pi \Delta \hat{d}_c \right) \left[ 1 - \left( \frac{1}{2} n\pi \hat{d}_c \right)^2 + \left( \frac{1}{2} n\pi \hat{d}_c \right)^2 \cos(2\omega_m t) \right] \right\} \\
- \frac{\xi_n}{n\pi} \left\{ \sin \left( n\pi \Delta \hat{d}_c \right) n\pi \hat{d}_c \sin(\omega_m t) \right\} \tag{6.67}
\]

An inspection of (6.66) and (6.67) shows that apart from frequency components at a distance \( \omega_m \) from the carrier, frequency components at a distance \( 2\omega_m \) can be expected also.\(^1\)

\(^1\)Sidebands may also be generated at \( 4\omega_m \) etc. when the Taylor series expansion of the cosine in (6.59) would be continued. For now these components are considered to be negligible.
6.5.2 Sidebands in the duty cycle modulation spectrum

The sidebands that exist around $\omega = 0$, $\omega = \omega_o$ and $\omega = 2\omega_o$ are the most interesting, and will be concentrated upon. So $c_n(t)$ will be calculated for values of $n$ that contribute to these sidebands. Apart from $n = 0$, $n = 1$ and $n = 2$, also $n = -1$ and $n = -2$ should be taken into account. It can be seen from (6.51) that the sign of $n$ has no influence, so the results are identical.

$n = 0$ is substituted in (6.66):

$$c_0(t) = \frac{1}{2} + \Delta d_c + \hat{d}_c \sin(\omega_m t) \quad (6.68)$$

A term at $\omega_m$ and a DC-term are generated,

$n = 1$ is substituted in (6.67):

$$c_1(t) = \frac{1}{\pi} \cos(\pi \Delta d_c) \left[ 1 - \left( \frac{1}{2} \pi \hat{d}_c \right)^2 + \left( \frac{1}{2} \pi \hat{d}_c \right)^2 \cos(2\omega_m t) \right]$$

$$- \sin(\Delta d_c) \pi \hat{d}_c \sin(\omega_m t) \quad (6.69)$$

A component is generated at $\omega_o$ itself and sidebands are generated at distances $\omega_m$ and $2\omega_m$ from $\omega_o$.

$n = 2$ is substituted in (6.66):

$$c_2(t) = \frac{-1}{2\pi} \sin(2\pi \Delta d_c) \left[ 1 - \left( \pi \hat{d}_c \right)^2 + \left( \pi \hat{d}_c \right)^2 \cos(2\omega_m t) \right]$$

$$- \cos(2\pi \Delta d_c) \hat{d}_c \sin(\omega_m t) \quad (6.70)$$

A component is generated at $2\omega_o$ itself and sidebands are generated at distances $\omega_m$ and $2\omega_m$ from $2\omega_o$.

The same results as above are found for $n = -1$ and $n = -2$. Theoretically the contribution of these coefficients will be around $-\omega_o$ and $-2\omega_o$. In practice these sidebands are added to those occurring around the corresponding positive frequencies, which are therefore doubled. The sum of the amplitudes of the left and the right sideband is given by (6.69) or (6.70). So each separate sideband has an amplitude equal to one half of this value. The contributions of the negative $n$ values double this value again.

Because of this the values found with (6.69) and (6.70) can be used to determine the amplitude of one single sideband. Of course $n$ should be taken positive only now.

In the table 6.1 the amplitude of the separate spectral components have been listed.

6.5.3 Duty cycle 50%

When the duty cycle is 50%, $\Delta d_c = 0$. Then a number of sidebands will disappear.

In first order oscillators very often the duty cycle is made 50%, so this special condition needs extra attention. In table 6.2 the remaining sidebands are listed.

As expected for a symmetrical square wave the spectrum does not contain even harmonics, so no energy is present at $2\omega_o$ itself. However, the sidebands belonging to DC and $2\omega_o$ are the largest sidebands in the spectrum, and they dominate the spectrum very often.

The carrier itself is influenced also. A part of the energy of the carrier at $\omega_o$ is transferred to sidebands at a distance of $2\omega_m$. The amplitude has a quadratic relation to $\hat{d}_c$. Due to these
<table>
<thead>
<tr>
<th>frequency of the harmonic</th>
<th>Size of the spectral components at a distance:</th>
<th>( \omega_m )</th>
<th>( 2\omega_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \frac{1}{2} + \Delta d_c )</td>
<td>( \ddot{d}_c )</td>
<td>0</td>
</tr>
<tr>
<td>( \omega_o )</td>
<td>( \frac{1}{2} \cos (\pi \Delta d_c) \left[ 1 - (\frac{1}{2} \pi \ddot{d}_c)^2 \right] )</td>
<td>( \sin (\pi \Delta d_c) \ddot{d}_c )</td>
<td>( \frac{1}{2} \cos (\pi \Delta d_c) \left( \frac{1}{2} \pi \ddot{d}_c \right)^2 )</td>
</tr>
<tr>
<td>( 2\omega_o )</td>
<td>( -\frac{1}{2\pi} \sin (2\pi \Delta d_c) \left[ 1 - (\pi \ddot{d}_c)^2 \right] )</td>
<td>( -\cos (2\pi \Delta d_c) \ddot{d}_c )</td>
<td>( -\frac{1}{2\pi} \sin (2\pi \Delta d_c) \left( \pi \ddot{d}_c \right)^2 )</td>
</tr>
</tbody>
</table>

Table 6.1 Spectral components around DC and the two first harmonics.

<table>
<thead>
<tr>
<th>frequency of the harmonic</th>
<th>Size of the spectral components at a distance:</th>
<th>( \omega_m )</th>
<th>( 2\omega_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \ddot{d}_c )</td>
<td>0</td>
</tr>
<tr>
<td>( \omega_o )</td>
<td>( \frac{1}{\pi} \left[ 1 - (\frac{1}{2} \pi \ddot{d}_c)^2 \right] )</td>
<td>0</td>
<td>( \frac{1}{\pi} \left( \frac{1}{2} \pi \ddot{d}_c \right)^2 )</td>
</tr>
<tr>
<td>( 2\omega_o )</td>
<td>0</td>
<td>( -\ddot{d}_c )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.2 Spectral components when \( d_{co} = \frac{1}{2} \).

two properties this effect may not always be neglectable compared to the sidebands that are generated around \( 2\omega_o \). Especially when the modulation is large—as it may be the case when extra noise is injected in an oscillator for measurement purposes—these sidebands may have a significant influence.

6.5.4 Duty cycle \( \neq 50\% \)

When \( d_{co} \neq \frac{1}{2} \) sidebands around the carrier \( \omega_o \) appear.\( \text{(table 6.1)} \) This can be very inconvenient, since especially this part of the spectrum may very well be in the frequency range of interest of the system that makes use of this signal. For example low frequency (\( \frac{1}{2} \)) noise that normally would have effect only around \( 2\omega_o \) will now also have its effect around \( \omega_o \), within the pass band of the system.
This is a good reason to keep the duty cycle of an oscillator equal to 50\% as much as possible.
6.6 The effect of uncorrelated noise sources

In regenerative oscillators both the duty cycle and—as a second order effect—the frequency are modulated by noise sources that are not switched by the oscillator. At the start of this chapter some of these sources have been indicated.

6.6.1 The effect of uncorrelated current sources

Suppose the disturbing signal (noise) is described by:

\[ f_m(t) = \Delta \alpha \sin(\omega_m t + \phi) \]  \hspace{1cm} (6.71)

This is one of the components of the Bennet model for the noise. The phase \( \phi \) is a random variable with a uniform probability density in the interval \([-\pi; \pi]\). The duty cycle of a regenerative oscillator is modulated by adding this signal to the integration constant. The period \( T \) can be split into two parts \( T_x \) and \( T_y \), in which the integration constant has the value of \( \alpha_x(t) \) and \( \alpha_y(t) \) respectively.

\[
\alpha_x(t) = \alpha_{x0} + \Delta \alpha \sin(\omega_m t + \phi) \]  \hspace{1cm} (6.72)
\[
\alpha_y(t) = \alpha_{y0} + \Delta \alpha \sin(\omega_m t + \phi) \]  \hspace{1cm} (6.73)

\( \alpha_{x0} \) and \( \alpha_{y0} \) are constants. Without noise the duty cycle still does not have to be equal to a half. When the static duty cycle is equal \( d_{co} \), then with (6.57) it follows:

\[
\alpha_{x0} = \alpha(1 - 2\Delta d_c) \]  \hspace{1cm} (6.74)
\[
\alpha_{y0} = -\alpha(1 + 2\Delta d_c) \]  \hspace{1cm} (6.75)

These equations give the relation between the static duty cycle and the static difference between the integration constants. When there are large differences, they are usually intentionally introduced by the designer. For example, a pulsed output signal may be required in the system in which the oscillator is used. Another example is a temperature sensor in which the static duty cycle contains the temperature information.[21]

The output signal of the oscillator has been depicted in fig.6.6.

The following equations describe the oscillator:

\[ \int_0^{T_x} \alpha_x(t) dt = E_{hys} \]  \hspace{1cm} (6.76)
\[ \int_{T_x}^{T_x+T_y} \alpha_y(t) dt = -E_{hys} \]  \hspace{1cm} (6.77)

Substituting (6.72) and (6.73) in (6.76) and (6.77) results in:

\[
T_x + \frac{\Delta \alpha}{\alpha \omega_m} \frac{1}{1 - 2\Delta d_c} \left[ \cos(\phi) - \cos(\omega_m T_x + \phi) \right] = T_{x0} \]  \hspace{1cm} (6.78)
\[
T_y - \frac{\Delta \alpha}{\alpha \omega_m} \frac{1}{1 + 2\Delta d_c} \left[ \cos(\omega_m T_x + \phi) - \cos(\omega_m T_x + \phi + \omega_m T_y) \right] = T_{y0} \]  \hspace{1cm} (6.79)

With:

\[
T_{x0} = \frac{E_{hys}}{\alpha(1 - 2\Delta d_c)} \]  \hspace{1cm} (6.80)
\[
T_{y0} = \frac{E_{hys}}{\alpha(1 + 2\Delta d_c)} \]  \hspace{1cm} (6.81)
Fig. 6.6 An oscillator output signal.

$T_{x0}$ and $T_{y0}$ are the unmodulated values of $T_x$ and $T_y$ ($\Delta \alpha = 0$). Since time interval $T_y$ starts immediately after the end of $T_x$, the phase of the modulating signal has shifted with the amount of $\omega_m T_x$. This phase shift must be taken into account. It represents the correlation between the effects that occur in the two time intervals.

Closed form expressions for $T_x$ and $T_y$ have to be found. First the terms of (6.78) and (6.79) are rearranged:

\[
T_x = T_{x0} \frac{1}{1 + \frac{\Delta \alpha}{\omega_m} \frac{1}{T_x} \frac{1}{1 - 2 \Delta d_c} \left[ \cos(\phi) - \cos(\omega_m T_x + \phi) \right]} \tag{6.82}
\]

\[
T_y = T_{y0} \frac{1}{1 - \frac{\Delta \alpha}{\omega_m} \frac{1}{T_y} \frac{1}{1 + 2 \Delta d_c} \left[ \cos(\omega_m T_x + \phi) - \cos(\omega_m T_x + \phi + \omega_m T_y) \right]} \tag{6.83}
\]

It can be seen that in the right side of both (6.82) and (6.83) $T_x$ and $T_y$ are still present. A way to solve this problem is to substitute (6.82) and (6.83) for these terms. When this is done repeatedly convergence to a solution may be obtained. Then the error that is introduced when finally $T_{x0}$ is substituted for $T_x$ and $T_{y0}$ for $T_y$ can be made small enough. In this case this substitution can be made in the first step since the disturbing signals are supposed to be very small (noise), so $\Delta \alpha$ is very small. Substituting $T_{x0}$ and $T_{y0}$ in (6.82) and (6.83) at the appropriate places results in:

\[
T_x = T_{x0} \frac{1}{1 + \frac{\Delta \alpha}{\omega_m} \frac{1}{T_{x0}} \frac{1}{1 - 2 \Delta d_c} \left[ \cos(\phi) - \cos(\omega_m T_{x0} + \phi) \right]} \tag{6.84}
\]

\[
T_y = T_{y0} \frac{1}{1 - \frac{\Delta \alpha}{\omega_m} \frac{1}{T_{y0}} \frac{1}{1 + 2 \Delta d_c} \left[ \cos(\omega_m T_{x0} + \phi) - \cos(\omega_m (T_{x0} + T_{y0}) + \phi) \right]} \tag{6.85}
\]

\[
T_o = \frac{2E_{\text{hyd}}}{\alpha} \tag{6.87}
\]

In these expressions the period of the oscillator $T_o$ will be introduced. The unmodulated period when $d_c = \frac{1}{2}$ ($\Delta d_c = 0, \Delta \alpha = 0$) is:

\[
T_o = \frac{2E_{\text{hyd}}}{\alpha} \tag{6.87}
\]

Using this equation, $T_{x0}, T_{y0}$ and their sum can be expressed in terms of $T_o$:

\[
T_{x0} = \frac{1}{2} T_o \left( \frac{1}{1 - 2 \Delta d_c} \right) = \frac{\pi}{\omega_o} \left( \frac{1}{1 - 2 \Delta d_c} \right) \tag{6.88}
\]
\[ T_{\nu 0} = \frac{1}{2} T_o \left( \frac{1}{1 + 2\Delta d_c} \right) = \frac{\pi}{\omega_o} \left( \frac{1}{1 + 2\Delta d_c} \right) \]  
(6.89)

\[ T_{\nu 0} + T_{\nu 0} = T_o \left( \frac{1}{1 - 4\Delta d_c^2} \right) = \frac{2\pi}{\omega_o} \left( \frac{1}{1 - 4\Delta d_c^2} \right) \]  
(6.90)

With:

\[ T_o = \frac{2\pi}{\omega_o} \]  
(6.91)

\( \omega_o \) is the output frequency of the oscillator, also referred to as the carrier frequency. Now the following closed form expressions for \( T_x \) and \( T_y \) are found:

\[ T_x = \frac{1}{2} T_o \frac{1 - 2\Delta d_c}{1 + \frac{1}{\pi} \frac{\Delta \alpha}{\alpha} \frac{\omega_o}{\omega_m} \left[ \cos(\phi) - \cos(\pi \left( \frac{1}{1 - 2\Delta d_c} \right) \frac{\omega_m}{\omega_o} + \phi) \right]} \]  
(6.92)

\[ T_y = \frac{1}{2} T_o \frac{1 + 2\Delta d_c}{1 - \frac{1}{\pi} \frac{\Delta \alpha}{\alpha} \frac{\omega_o}{\omega_m} \left[ \cos(\pi(\frac{1}{1 - 2\Delta d_c} \frac{\omega_m}{\omega_o} + \phi) - \cos(2\pi \left( \frac{1}{1 - 4\Delta d_c} \right) \frac{\omega_m}{\omega_o} + \phi) \right]} \]  
(6.93)

Two functions are defined that can be used to simplify (6.92) and (6.93).

\[ f_x(\omega_m, \phi) = \frac{\cos(\phi) - \cos(\pi(\frac{1}{1 - 2\Delta d_c} \frac{\omega_m}{\omega_o} + \phi)}{\pi(\frac{1}{1 - 2\Delta d_c} \frac{\omega_m}{\omega_o})} \]  
(6.94)

\[ f_y(\omega_m, \phi) = \frac{\cos(\pi(\frac{1}{1 - 2\Delta d_c} \frac{\omega_m}{\omega_o} + \phi) - \cos(2\pi(\frac{1}{1 - 4\Delta d_c^2} \frac{\omega_m}{\omega_o} + \phi)}{\pi(\frac{1}{1 + 2\Delta d_c^2} \frac{\omega_m}{\omega_o})} \]  
(6.95)

When these functions are introduced, \( T_x \) and \( T_y \) can be written as:

\[ T_x = \frac{1}{2} T_o \frac{1}{1 - 2\Delta d_c + \Delta \alpha \alpha f_x(\omega_m, \phi)} \]  
(6.96)

\[ T_y = \frac{1}{2} T_o \frac{1}{1 + 2\Delta d_c - \Delta \alpha \alpha f_y(\omega_m, \phi)} \]  
(6.97)

**DC-disturbances**

Now first the expressions will be verified for a simple (degenerated) case. The signal in (6.71) is transformed into a DC-disturbance by substituting \( \omega_m = 0 \) and \( \phi = \frac{1}{2} \pi \). A DC-signal having the value \( \Delta \alpha \) is left. Since:

\[ f_x(0, \frac{1}{2} \pi) = \sin(\frac{1}{2} \pi) = 1 \]  
(6.98)

\[ f_y(0, \frac{1}{2} \pi) = \sin(\frac{1}{2} \pi) = 1 \]  
(6.99)

both the expressions for the duty cycle \( d_c \) and the frequency \( \omega \), containing the influence of the disturbance, can be calculated easily. With:

\[ T_x = \frac{1}{2} T_o \frac{1}{1 - 2\Delta d_c + \frac{\Delta \alpha}{\alpha}} \]  
(6.100)

\[ T_y = \frac{1}{2} T_o \frac{1}{1 + 2\Delta d_c - \frac{\Delta \alpha}{\alpha}} \]  
(6.101)

\[ d_c = \frac{T_x}{T_x + T_y} \]  
(6.102)

\[ \omega = \frac{2\pi}{T_x + T_y} \]  
(6.103)
it follows:

\[ d_c = \frac{1}{\alpha} \left[ 1 - \left( \frac{\Delta \alpha}{\alpha} - 2 \Delta d_c \right) \right] \quad (6.104) \]

\[ \omega = \omega_0 \left[ 1 - \left( \frac{\Delta \alpha}{\alpha} - 2 \Delta d_c \right)^2 \right] \quad (6.105) \]

From (6.105) it can be seen that duty cycle modulation also affects the frequency. The transfer is quadratic, so nonlinear distortion of the output spectrum occurs.

*The maximum of \( \omega \) is found for a duty cycle equal to a half. Any deviation from this duty cycle results in a lower output frequency.*

### 6.6.2 Cancellation of \( \phi \) when \( \Delta d_c = 0 \)

Again the random variable \( \phi \) introduced in (6.71), is an unwanted factor in (6.94) and (6.95). Therefore the first and second moment of both expressions will be calculated. Because the calculations tend to become very complicated when \( \Delta d_c \neq 0 \) in this section in the formulas for the standard deviation, \( \Delta d_c \) will be set to zero. This is not unrealistic, since in many oscillators the duty cycle is made equal to a half, and in section 6.5 it has also been shown it is wise to do so.

For the expressions of the duty cycle and the frequency, the mean and the standard deviation will be calculated now.

#### The mean of the duty cycle

When (6.96) and (6.97) are substituted in (6.102), it follows:

\[ d_c(\omega_m, \phi) = \frac{1}{\alpha} \left[ 1 + \frac{2 \Delta d_c - \Delta \alpha}{\alpha} f_y \right] \quad (6.106) \]

The mean of the duty cycle \( \bar{d}_c(\omega_m) \) is:

\[ \bar{d}_c(\omega_m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d_c(\omega_m, \phi) d\phi \quad (6.107) \]

\[ \approx \frac{1}{4\pi} \int_{-\pi}^{\pi} \left( 1 + 2 \Delta d_c - \frac{\Delta \alpha}{\alpha} f_y \right) \left( 1 - \frac{1}{2} \frac{\Delta \alpha}{\alpha} (f_x - f_y) \right) d\phi \quad (6.108) \]

The averages of \( f_x, f_y \) and \( f_x - f_y \) are equal to zero. When these terms are deleted, the following equation remains:

\[ \bar{d}_c(\omega_m) = \frac{1}{4\pi} \int_{-\pi}^{\pi} \left[ 1 + 2 \Delta d_c + \left( \frac{\Delta \alpha}{\alpha} \right)^2 f_y (f_x - f_y) \right] d\phi \quad (6.110) \]

\[ \approx \frac{1}{\alpha} (1 + 2 \Delta d_c) \quad (6.111) \]

When \( \Delta \alpha \) is small enough, (6.111) is usable. For noise this is supposed to be true.

\[ ^2 \text{In order to keep the expressions readable, the functions } f_x(\omega_m, \phi) \text{ and } f_y(\omega_m, \phi) \text{ will be written as: } f_x \text{ and } f_y. \text{ For all other functions, the dependencies will still be indicated.} \]
The standard deviation of the duty cycle

The standard deviation of the duty cycle $\sigma_d(\omega_m)$ is:

$$\sigma_d(\omega_m) = \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} \left( d_c(\omega_m, \phi) - \bar{d}_c(\omega_m) \right)^2 d\phi}$$

(6.112)

$$= \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} \left( -\frac{\Delta\alpha}{\alpha} \frac{f_x + f_y}{1 + \frac{\Delta\alpha}{\alpha} (f_x - f_y)} \right)^2 d\phi}$$

(6.113)

After some calculations it follows:

$$\sigma_d(\omega_m) = \frac{\Delta\alpha}{\alpha} \sin \left( \frac{\pi \omega_m}{\omega_o} \right)$$

(6.114)

It can be seen that for $\omega_m = \omega_o$ and $\omega_m = 2\omega_o$ the duty cycle is not modulated. The expression gives the sensitivity of the oscillator for a certain frequency component at $\omega_m$. The place in the output spectrum where the sideband appears can be found in table 6.2.

In section 6.5 the modulation of the duty cycle was given by (6.53). In this expression the amplitude of the variation in the duty cycle is a constant. When the integration constants are modulated with a signal as defined in (6.71) a modulation of the duty cycle is the effect. Note that a positive excursion of the disturbing signal in (6.53) results in an increase of the duty cycle, whereas a positive excursion of the disturbing signal given by (6.71) results in a decrease of the duty cycle. Hence the effects have opposite signs. This has to be taken into account.

In this section it has been shown that this way of duty cycle modulation is frequency dependent. This can be modeled as a frequency dependence of $d_c$, so (6.53) should then be written as:

$$d_c(t) = d_{co} + \tilde{d}_c(\omega_m) \sin(\omega_m t)$$

(6.115)

The effective value of the modulation is given by (6.114). The relation between the amplitude $E$ and the effective value $E_{eff}$ of a sine-wave is:

$$E = \sqrt{2} E_{eff}$$

(6.116)

So the amplitude is found by using (6.116) and (6.114), and by taking into account the sign of the changes as described above:

$$\tilde{d}_c(\omega_m) = -\frac{1}{2} \left| \frac{\Delta\alpha}{\alpha} \sin \left( \frac{\pi \omega_m}{\omega_o} \right) \right|$$

(6.117)

Check for a DC component

Again the expression will be verified for a simple case. For a DC component ($\omega_m = 0$) the duty cycle will be calculated. Starting from (6.53) a DC-duty cycle is described by:

$$d_c(t) = d_{co} + \hat{d}_c,$$

(6.118)

in which $\hat{d}_c$ represents the amplitude of the DC component. A substitution of $\omega_m = 0$ in (6.117) results in:

$$\hat{d}_c(0) = -\frac{1}{2} \frac{\Delta\alpha}{\alpha}$$

(6.119)

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When a modulating DC signal is added to the integration constants, according to (6.104) with $\Delta d_c = 0$ the duty cycle is:

$$d_c = \frac{1}{2} - \frac{1}{2} \frac{\Delta \alpha}{\alpha}$$

Equating (6.118) and (6.120) confirms (6.119).

The CNR as a result of uncorrelated noise current sources

The most important sidebands that are produced by a noise current that produces duty-cycle modulation are according to table 6.2 found at a distance $\omega_m$ from the frequency $2\omega_o$ and at $\omega_m$ itself. The latter sideband and the sideband to the left of $2\omega_m$ can also be seen as two sidebands belonging to $\omega_o$. Their distance to the carrier at $\omega_o$ is then $\omega_o - \omega_m$. The two sidebands have a 180 degree phase difference, so one could label them as FM-sidebands of $\omega_o$. This is the reason why a limiter does not remove these sidebands although they have been calculated before as AM-sidebands of the frequency $2\omega_o$. A limiter would remove AM-sidebands related to the carrier signal that causes it to switch, and this is not a carrier at $2\omega_o$, but at $\omega_o$.

The magnitudes of the sidebands are given by (6.117) and table 6.2 shows the magnitude of the carrier. With these values, the CNR can be calculated:

$$L(\omega_o - \omega_m) = \pi^2 \left( \frac{\Delta \alpha}{\alpha} \frac{\sin(\pi \frac{\omega_m}{\omega_o})}{\sinh(\frac{\omega_m}{\omega_o})} \right)^2$$

The argument of the CNR is the distance between sideband and the carrier. When a noise component with a frequency $\omega_m$ modulates the duty cycle, the resulting sideband is found at a distance $|\omega_o - \omega_m|$ from the carrier. Therefore the argument of the CNR is $|\omega_o - \omega_m|$ instead of $\omega_m$. It can be seen from the expression that especially far from the carrier, noise is introduced. This noise is caused by the low frequency components of the noise current. From this it can be concluded that since the effects of low frequency components are found far from the carrier, low frequency noise sources, like $f$-sources, generally do not have to have much influence on the system performance of the oscillator. A condition that has to be met to obtain this is the fact that the duty-cycle of the oscillator has to be equal to a half. If not, it can be seen from table 6.1, that sidebands are found around $\omega_o$ itself. In that case low frequency noise sources, like the $f$-source, introduce noise close to the carrier. It is therefore advisable for the noise performance of the oscillator to keep the duty-cycle of the oscillator as close to a half as possible.

6.6.3 The mean and the standard deviation of the frequency

It has been stated before that duty-cycle modulation also affects the frequency of the oscillator. The expression for the frequency as a function of $\omega_m$ and $\phi$ is found by way of a substitution of (6.96) and (6.97) in (6.103). For ease of calculation the duty-cycle is chosen to be equal to a half ($\Delta d_c = 0$). Similar calculations as performed before result in:

$$\bar{\omega}(\omega_m) = \omega_o \left( 1 - \frac{\Delta \alpha}{\alpha} \left( \frac{\cos(\pi \frac{\omega_m}{\omega_o}) - 1}{\sinh(\frac{\omega_m}{\omega_o})} \right)^2 \right)$$

From this expression it can be seen that, when the duty-cycle is equal to a half, noise sources that are not correlated to the switching action always decrease the carrier frequency. The oscillator is the most sensitive for noise components close to the carrier frequency.

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The standard deviation is:

\[ \Delta \omega(\omega_m) = \omega_o \frac{1}{2} \sqrt{2} \frac{\Delta \alpha \cos(\pi \frac{\omega_m}{\omega_o}) - 1}{\frac{\omega_m}{\omega_o}} \]  

(6.123)

From this expression the CNR can be calculated again.

\[ L(|\omega_o - \omega_m|) = \left( \frac{\Delta \omega_{\text{peak}}}{2(\omega_o - \omega_m)} \right)^2 = \frac{1}{4} \left( \frac{\omega_o}{\omega_o - \omega_m} \right)^2 \left( \frac{\Delta \alpha}{\alpha} \right)^2 \left( \frac{\cos(\pi \frac{\omega_m}{\omega_o}) - 1}{\frac{\omega_m}{\omega_o}} \right)^2 \]  

(6.124)

Also according to the expression, the oscillator is the most sensitive for frequencies close to the carrier frequency. The resulting sidebands are close to the carrier, so the noise components of an uncorrelated noise source that are close to carrier frequency are of importance for the noise performance of an oscillator.

The influence of uncorrelated noise voltage sources

Also for the noise voltage sources that are not correlated to the switching action the mean and the standard deviation of the frequency can be calculated in a similar way as it was done before in this chapter. Again only the value of the noise source at the time the oscillator switches is of importance. (The same as with the calculation of the effect of the correlated noise voltage source, also in this case special care should be taken with the signs of the noise voltages in the integral equations.)

The results are:

\[ \bar{d}_c = \frac{1}{2} \]  

(6.125)

\[ \hat{d}_c = \frac{1}{2} \frac{\bar{n}_2}{U_{\text{hyd}}} \frac{1}{2} \sqrt{2} \sin(\pi \frac{\omega_m}{\omega_o}) \]  

(6.126)

As with the current sources, the sidebands that are the result of the noise voltage are mainly far from the carrier again, provided that the duty-cycle is close to a half. In contrary with the current sources, a DC voltage component has no effect on the oscillator. A DC voltage in series with the capacitor is compensated with an opposite voltage on the capacitor. The triangular waveform at the capacitor terminals only shows a DC shift, but it does not change shape.

Frequency modulation due to a noise voltage source

Via the modulation of the duty-cycle, the noise voltage source also modulates the frequency. The effect on the frequency can also be expressed in terms of a mean and a standard deviation. The results are:

\[ \bar{\omega}_o(\omega_m) = \omega_o \]  

(6.127)

\[ \Delta \omega_o(\omega_m) = \omega_0 \frac{\bar{u}_n}{U_{\text{hyd}}} \frac{1}{2} \sqrt{2} \left( \cos(\pi \frac{\omega_m}{\omega_o}) - 1 \right) \]  

(6.128)

The oscillator is the most sensitive for noise voltage components that are close to the carrier frequency. The CNR is equal to:

\[ L(|\omega_o - \omega_m|) = \frac{1}{4} \left( \frac{\bar{u}_n}{U_{\text{hyd}}} \right)^2 \left( \frac{\omega_o}{\omega_o - \omega_m} \right)^2 \left( \cos(\pi \frac{\omega_m}{\omega_o}) - 1 \right)^2 \]  

(6.129)

Fig.6.7 shows a graph of this expression plotted on a logarithmic scale. This graph has also been measured[26].

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Fig. 6.7 The CNR of a first order oscillator as the result a noise voltage source as a function of $\frac{\omega_m}{\omega_0}$ (the normalized distance to the carrier)

6.7 The oscillator as a sampling system

Within one period two decisive moments occur. At these moments the integrator signal reaches one of the threshold levels. Only at these moments noise can have its influence.

The noise currents are integrated during a half period. At the end of this half period the result is "measured". It depends on the result of the integration at what exact time the integrator signal reaches the threshold level. It is obvious that for high frequencies most of the noise is averaged out by the integrator. The presence of a factor $\frac{1}{\omega_m}$, which can be seen as the transfer of an integrator, in the formulas expresses the effect of this averaging.

The value of the noise voltage sources is only of importance at the time the threshold level is reached. This can be seen also from the integral equations that are used to calculate the influence of the noise sources. Only their values at the decisive moments appear in those equations. Apparently only three points can be pointed out in a period where the value of the sources is important. They are the switching moment at the start of the period, the switching moment after a half period and finally the switching moment at the end of the period. There are many sinusoidal waveforms that fit through these three points.

The oscillator cannot distinguish them.

In general it can be stated that all sinusoidal waveforms $f(t)$ that are described as:

$$f(t) = \sin((\omega_m + 2n\omega_0)t)$$  \hspace{1cm} (6.130)
$$n \in N$$  \hspace{1cm} (6.131)

have the same effect in the oscillator. This can also be seen in the expression for the noise effects derived earlier in this chapter. They all contain a periodical component like:

$$\sin(\pi \frac{\omega_m}{\omega_0})$$  \hspace{1cm} (6.132)
When \( \omega_m \) is the variable, the expression is periodical with \( \frac{1}{2\omega_c} \).

Since all these components have the same effect on the oscillator, they also produce a sideband at the same place. So there are many noise components of the Bennet model that produce a sideband at the same place in the output spectrum of the oscillator. Generally this effect is named "the folding of noise".

The noise conversion bandwidth

When the expressions derived above are considered, and the sampling effect is taken into account, it can be seen that especially in the case of noise voltage sources the folding of noise works for frequencies up to infinity. This would result in infinite noise power around the carrier, which is of course not the reality. Until now the switching was assumed to be infinitely fast. In that case only at a point in time the value of the noise voltages is of importance. Then folding up to infinite frequencies would occur indeed. In practice the speed of the circuits that perform the basic functions in the oscillator is limited. This implies that the oscillator is not able to react on signals that have a fundamental frequency that is beyond the capability of the oscillator. There is not a point in time at which the value of the noise is of importance, but a small time interval. The noise is "integrated" within this interval due to the limited small-signal bandwidth of the oscillator circuits. In this way the number of noise components that is folded becomes limited. The larger the small-signal bandwidth of the oscillator circuits is, the more noise components are folded. In practice it has been measured that, out of two oscillators with different small-signal bandwidths, that are producing the same output frequency, the fastest oscillator is the noisiest (see fig. 7.16).

The effect of the limited noise conversion bandwidth (NCB) can be introduced in the expressions for the CNR as an extra multiplication factor \( N_{cb}^2(\omega_m) \), with:

\[
N_{cb}(\omega_m) = \left| \frac{H(\omega_m)}{H(0)} \right| \tag{6.133}
\]

\( H(\omega_m) \) is the small-signal transfer function of the circuit that limits the NCB. An example of an equation that contains this factor can be found in [26]. The NCB has been calculated by Boon[26]. It is defined as:

\[
B_{ncb} = \int_{0}^{\infty} \frac{|H(\omega)|^2 d\omega}{|H(0)|^2} \tag{6.134}
\]

The effective number of noise bands that is folded due to the sampling is easily calculated with the NCB. The noise bands have a width equal to \( 2\omega_c \), so the number of band that is folded is equal to:

\[
n_{fold} = \frac{B_{ncb}}{2\omega_c} \tag{6.135}
\]

Example

The effect of a noise voltage source, that is not correlated to the switching action, on the CNR will be calculated now as an example. The noise source is supposed to be a white noise source. At a distance \( \omega_m < \omega_b \) from the carrier the sideband of a noise component with a frequency \( (\omega_c - \omega_m) \) is found. The standard deviation caused by this component is given by (6.128). Also the noise component at \( (\omega_c + \omega_m) \) produces a sideband at a distance \( \omega_m \) from the carrier. The magnitude is equal to that of the first component.

There is also a noise contribution at a distance \( \omega_m \) from the carrier caused by noise at higher frequencies due to the sampling of the oscillator. The following values for the frequency \( \omega_{a,b} \)
of a noise component all produce sidebands at a distance \( \omega_m \) from the carrier.

\[
\begin{align*}
\omega_{a,n} &= (\omega_o - \omega_m) + 2n\omega_o \quad (6.136) \\
\omega_{b,n} &= (\omega_o + \omega_m) + 2n\omega_o \quad (6.137) \\
n &\in N \quad (6.138)
\end{align*}
\]

Then it follows:

\[
L(\omega_m) = \sum_{n=0}^{\infty} \frac{1}{4} \left( \frac{\omega_o}{\omega_m} \right)^2 \left( \frac{\tilde{u}_{n2}}{U_{hys}} \right)^2 N_{cb}^2(\omega_{a,n}) \left( \cos\left(\frac{\omega_{a,n}}{\omega_o}\right) - 1 \right)^2
+ \sum_{n=0}^{\infty} \frac{1}{4} \left( \frac{\omega_o}{\omega_m} \right)^2 \left( \frac{\tilde{u}_{n2}}{U_{hys}} \right)^2 N_{cb}^2(\omega_{b,n}) \left( \cos\left(\frac{\omega_{b,n}}{\omega_o}\right) - 1 \right)^2
\]  

(6.139)

The cosine terms have the same value for \( \omega_{a,n} \) and \( \omega_{b,n} \) for every \( n \). When it also is assumed that:

\[ N_{cb}(\omega_{a,n}) \approx N_{cb}(\omega_{b,n}) \approx N_{cb}(2n\omega_o), \quad (6.140) \]

then it follows that:

\[
L(\omega_m) = \sum_{n=0}^{\infty} \frac{1}{4} \left( \frac{\omega_o}{\omega_m} \right)^2 \left( \frac{\tilde{u}_{n2}}{U_{hys}} \right)^2 N_{cb}^2(2n\omega_o) \left( \cos\left(\frac{\omega_m}{\omega_o}\right) + 1 \right)^2
\]  

(6.141)

The CNR consists of the sum of the effects of all components given by (6.136) and (6.137). When \( N_{cb}^2(2n\omega_o) \) becomes small enough, the summation can be stopped. The number of noise bands that is folded is given by (6.135). The use of this expression results in:

\[
L(\omega_m) = \frac{1}{2} n_{fold} \left( \frac{\omega_o}{\omega_m} \right)^2 \left( \frac{\tilde{u}_{n2}}{U_{hys}} \right)^2 \left( \cos\left(\frac{\omega_m}{\omega_o}\right) + 1 \right)^2
\]  

(6.142)

### 6.8 Conclusions

In this chapter several noise calculations have been performed in order to find the CNR of first order oscillators. Two types of noise sources have been distinguished. Noise sources of the first type are voltage or current sources that are correlated to the switching action of the oscillator. Noise sources of the second type are voltage or current sources that are not correlated to the switching. Whether or not noise sources are correlated to the switching action, influences the place where their effects are perceived in the output spectrum of the oscillator. In the case of white noise, this may be not so spectacular since all noise frequencies are equally present and a calculation of the output spectrum remains correct even if the frequencies are interchanged. For noise which is not white, for example, \( \frac{1}{f} \)-noise, it is however important to know if the source is correlated or not.

Several design rules can be given as a result of the investigations made in this chapter:

1. The duty-cycle of an oscillator should be equal to a half, since in that case \( \frac{1}{f} \)-noise sources that are not correlated to the switching action, have no effect close to the carrier.

2. The signal-to-noise ratio of the integration constant \( \Delta t_o \) should be maximized. This implies that the signal-to-noise ratio of the charging current should be maximized.

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3. The signal-to-noise ratio of the hysteresis should be maximized. This implies that the amplitude of the integrator signal should be as large as possible.

4. The signal-to-noise ratio of the integration constant is limited by the available supply power, so all power available should be used to realize the integration constant.

5. The amplitude of the integrator signal is limited by the supply voltage, so level shifts in series with the capacitor should be avoided since they limit the voltage swing.

6. The small-signal bandwidth of the circuits in the oscillator should not be chosen larger than necessary in order to avoid excess folding of high frequency noise.
7. Regenerative oscillators

This chapter deals with regenerative oscillators. Regenerative oscillators have one common property. They contain a regenerative memory as described in chapter 4. The basics of these oscillators have been described in chapter 2. Regenerative oscillators are the most commonly used class of first order oscillators, though not the only class. The class of non regenerative oscillators, will be dealt with in the next chapter. A first order oscillator actually is a system. It combines various basic functions. Therefore the design also involves a consideration of the first order oscillator on a system level. The first distinctions that will be made in this chapter between various regenerative oscillators will be on this system level. After the set-up of the system has been decided, the basic circuits that perform the various basic functions are designed. When a basic function cannot be implemented such that the design requirements of the complete oscillator are met, a different system set-up may become necessary. In this chapter various system set-ups will be discussed. Electronic implementations of these set-ups will be given. Some of these implementations have been integrated and verified by measurements.

Apart from single regenerative oscillators, also systems of coupled regenerative oscillators have been investigated. At the end of this chapter, these coupled oscillator systems will be discussed. An electronic implementation will also be given of this oscillator. Measurements verifying the theory have been performed on an integrated version of a coupled oscillator system, and experimental results will be presented.

7.1 Design considerations

First order oscillators can be characterized by several parameters. The most important parameters are the amplitude \( \hat{E}_o \) of the integrator output signal, the value of the integration constant \( \alpha \) and the period \( T_o \). Whenever parameters are changed in this chapter to improve the quality, the period is kept constant. So when the effects of different parameter settings are compared, it is done at the same output frequencies.

Nearly all regenerative first order oscillators have two different states in which they can be (chapter 3). This means that the integration constant can have two different values, one for each state. These values will be labeled \( \alpha_{\text{min}} \) and \( \alpha_{\text{max}} \). When practical oscillators are made, \( \alpha_{\text{min}} \) and \( \alpha_{\text{max}} \) must have opposite signs.

In this thesis a capacitor is used to implement the integration function. This implies that the amplitude \( \hat{E}_o \) is a voltage and that the integration constant is of the form \( \frac{\text{current}}{\text{capacitance}} \). For these first order oscillators, the following relations can be given:

\[
T_o = \frac{2\hat{E}_o}{|\alpha_{\text{min}}|} + \frac{2\hat{E}_o}{|\alpha_{\text{max}}|} \tag{7.1}
\]

\[
\alpha_{\text{min}} = \frac{I_{\text{min}}}{C_t} \tag{7.2}
\]

\[
\alpha_{\text{max}} = \frac{I_{\text{max}}}{C_t} \tag{7.3}
\]

\( C_t \) is the value of the integration capacitor.

Some general design rules can be given for the oscillator parameters.

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Design considerations for $\dot{E}_o$

$\dot{E}_o$ is the amplitude of the integrator signal. This signal contains a certain amount of power in the frequency spectrum at the carrier frequency $f_c$. Ideally only at this frequency power should be present. Practically, also in the rest of the frequency spectrum signal power is found. In order to obtain an optimal performance of the oscillator, as much power as possible should be put in the carrier. In that way, the noise floor has the least influence. Circuits that use the integrator signal as input signal also produce noise. Also the influence of this noise can be reduced to a minimum if the carrier is made as large as possible.

*It is always favorable to make the amplitude $\dot{E}_o$ of the integrator output signal as large as possible.* (unless the measures to be taken severely damage the oscillator performance)

Design considerations for $\alpha$

An important property of the integrator output signal is its slope. The signal is used to obtain timing information by way of level comparison. In order to be able to determine the crossing of a reference level as accurately as possible, the integrator signal should cross the reference level as fast as possible. In chapter 4 it has already been shown that the noise sensitivity of a regenerative memory decreases when the slope of the input signal is increased.

Additive noise is always present in the integrator output signal. It may originate from within the integrator, but is can also come from an equivalent input noise source of the circuit that makes use of the integrator signal. Additive noise can manifest itself as amplitude noise or as timing jitter. Timing jitter is the most serious form of the noise, since once the timing of a system is disturbed, it can never be corrected. Zero crossings shifted in time due to noise, cannot be distinguished from unaffected zero crossings, in a first order oscillator. The measure in which the additive noise manifests itself as timing jitter depends on the slope of the integrator signal. This has been illustrated in fig.7.1. A signal $f_a(t)$ with finite slopes that

![Figure 7.1](image)

Fig. 7.1 The influence of additive noise on the zero crossings of a signal with a finite (a) and an infinite (b) slope

is contaminated with additive noise shows a shift of the zero crossings in time (jitter). Signal $f_b(t)$ has infinite slopes and shows no shift of the zero crossings at all. In this case additive noise only manifests itself as amplitude noise. Obviously the faster the slope, the less jitter additive noise can cause.

*A large slope is achieved with a large integration constant and it is favorable for the oscillator performance to do so.*

When (7.2) and (7.3) are considered it can be seen there are apparently two different ways to increase the value of the integration constant. It can be achieved by either an increase of $I$ or a decrease of $C_i$. As far as the effect of noise added to the integrator signal by the circuit that
uses it as input signal, there is no difference. Then only the slope is of importance. However, for the signal to noise ratio of the integrator signal itself there is a difference indeed.

Inevitably some losses will be present in a practical integrator. The capacitor, that is the electronic implementation of the integrator, is connected to various circuits that load the capacitor. The larger the value of \( C_t \), the smaller the effect of the loads will be. Also noise currents that are injected by the circuits connected to the capacitor have a smaller effect on the output signal if \( C_t \) is enlarged.

The accuracy of \( C_t \) can also improve when its value is enlarged, since then the contribution of (inaccurate) parasitic capacitances to \( C_t \) becomes smaller.

**A capacitor value \( C_t \) as large as possible is favorable for the oscillator performance.**

This statement conflicts with the demand for a large integration constant. Apparently an increase of \( \alpha \) should be established with an increase of \( I \) and not with a decrease of \( C_t \). In practice, the current \( I \) will be contaminated with noise. An increase of \( I \) generally improves the signal to noise ratio of \( I \), so also in this respect a value for \( I \) as large as possible is good.

**A current as large as possible is favorable for the oscillator performance.**

**Summary**

With respect to the oscillator parameters \( \dot{E}_o \) and \( \alpha \), the following design criteria can be formulated:

1. \( \dot{E}_o \) should be chosen as large as possible.
2. \( \alpha \) should be chosen as large as possible.

With respect to the two components of which \( \alpha \) exists, the following design criteria can be formulated:

1. \( C_t \) should be as large as possible.
2. \( I \) should be as large as possible.

Two **conflicting** design rules apply to \( C_t \). It should be large for the over-all performance of the oscillator, but small to obtain an optimal integration constant. This implies that there is an optimum for \( C_t \) and \( I \), which depends on the implementation or application. Additional criteria can be the power consumption, the availability of capacitors in a IC-process and so on.

The parameters were allowed to be changed under the condition that the period remained constant. An increase of \( \alpha \) should therefore be accompanied by an increase of the amplitude of the integrator signal. This produces no conflicting demands on the parameters. A first aim should be to design the oscillator such that a maximum amplitude of the integrator signal is achieved. Theoretically an amplitude equal to the supply voltage can be reached. After that the integration constant has to be designed such that the period is correct, while the maximum current that can be allowed is used. Finally a capacitor of the correct value has to be selected. The oscillator should be designed such that as much of the available supply current as possible is used in the integration constant.

In IC-processes very often the value of \( C_t \) will be a limiting factor in the design optimization.

In simple regenerative oscillators, in which several basic functions are combined into the regenerative memory, the parameters of the memory may impose extra limitations on the oscillator parameters. This will be discussed in the next section.
7.2 The parameters of the memory and their influence of the oscillator properties

In the same way several parameters can be used to characterize the complete oscillator, there are also some parameters that characterize the properties of the memory. The most important parameters are the hysteresis \( E_{hys} \), the threshold levels \( E_{ll} \) and \( E_{ih} \), and the extreme values of the output signal \( E_{min} \) and \( E_{max} \). Depending on the specific implementation of the regenerative oscillator, these parameters have more or less influence on the parameters of the complete oscillator. For example, in some implementations the amplitude of the integrator signal is determined by the hysteresis of the regenerative memory and in others it is not.

The output signal of the memory has not been characterized with an amplitude, but with two extreme values. This is because the output signal of the memory by itself is not necessarily periodical. The periodicity of the memory output signal is caused by the system set-up and not by the memory itself. Also, the amplitude only gives a measure for the difference between the two extreme values, whereas for the determination of the output frequency, their absolute value is of importance. In degenerated first order oscillators, like mono-stable multivibrators, the necessity of this distinction becomes more evident. Then one of the extreme values is essentially equal to zero. This would not be indicated by the value of an amplitude.

Generally the parameters of the memory are not completely independent of each other. A dependency that is found very often, is a linear relation between the extreme values of the output signal and the hysteresis. The interdependencies generally hamper the design optimization. It has already been mentioned in chapter 2, that a separation of the basic functions that have to be performed in a first order oscillator, gives the most freedom for the designer. The basic functions are:

1. Integration
2. Comparison
3. Switching the sign of the integration constant
4. Memorization

In principle, the only function that has to be performed by the memory circuit is memorization. For comparison and switching, other basic circuits can be used. (The memory function can also be performed by a non regenerative circuit, but this will be discussed in the next chapter.) Apart from separating the basic functions in a first order oscillator, also an attempt can be made to reduce the interdependency of the memory parameters.

Obviously two different methods can be used to ease the design optimization. The first method (and probably the best) is on the system level. By way of changing the system set-up, the basic functions can be distributed over several basic circuits, that each can be optimized for one single task.

The second method is on the level of the basic circuits. When several basic functions are performed by one circuit, the parameters that characterize these functions are made independent of each other. This is done by redesigning the circuit that performs these functions. In the next section the behavior of regenerative oscillators in which the interdependencies of the parameters of the memory are reduced will be discussed. After that oscillators with really separated function will be paid attention to.
7.3 Oscillators with reduced interdependencies of the memory parameters

In some applications it is very convenient that regenerative memories are able to perform also other functions than just the memory function in a first order oscillator. The resulting oscillators usually have a very simple topology. The circuits are small and therefore some design aims, like a high operating frequency, are easily reached. This is the main reason why the emitter-coupled oscillators, like the one depicted in fig.7.3, are used so often. Evidently all parameters of the regenerative memory are very dominating in the behavior of the complete oscillator. Hence physical limitations for the parameters of the regenerative memory are also limitations for the parameters of the complete oscillator. A very common phenomenon is the fact that in this type of regenerative oscillators, the amplitude \( \hat{E}_o \) of the integrator output signal is equal to the hysteresis \( E_{hys} \) of the memory.

Another problem is caused by the fact that the parameters of a Schmitt trigger are not independent of each other without extra measures. A general diagram of an electronic first order oscillator with all functions—except integration—combined in the regenerative memory, has been depicted in fig.7.2. The memory is implemented as a Schmitt trigger.

![Fig. 7.2 A basic regenerative oscillator with all functions except integration combined in a Schmitt trigger](image)

The expressions for the period \( T_o \), the amplitude \( \hat{E}_o \) of the integrator signal and the integration constants are:

\[
T_o = \frac{E_{hys}}{|\alpha_{min}|} + \frac{E_{hys}}{|\alpha_{max}|} \quad (7.4)
\]

\[
\alpha_{min} = \frac{I_{min}}{C_t} \quad (7.5)
\]

\[
\alpha_{max} = \frac{I_{max}}{C_t} \quad (7.6)
\]

\[
\hat{E}_o = \frac{1}{2} E_{hys} \quad (7.7)
\]

\( \alpha_{min} \) and \( \alpha_{max} \) are the integration constants, each belonging to one of the two states in which the oscillator can be (chapter 3). Very often \(-\alpha_{min} = \alpha_{max}\), and consequently \(-I_{min} = I_{max} = I\). This property will be assumed for the rest of this section.

When (7.4) is compared with (7.1), it can be seen that now the period has become dependent on a memory parameter.

With the use of the simple, but widely used, emitter-coupled oscillator as an example, the dependencies between the various parameters of the Schmitt trigger, and their influence on the oscillator properties, will be investigated. Also methods to reduce the dependencies between the parameters will be discussed. A reduction of the influence of the Schmitt trigger parameters on the oscillator properties can be achieved by changing the system set-up, but this is not to be discussed in this section.
7.3.1 The relation between the hysteresis and the amplitude of the integrator output signal

An electronic implementation of the emitter-coupled oscillator has been depicted in fig. 7.3. The capacitor \( C_t \) performs the integrator function, and all other functions are performed by the Schmitt trigger, formed by the differential pair and a resistive feed back network. The hysteresis of the Schmitt trigger is equal to \( E_{hys} = 4RI \). In order to prevent saturation of the transistors, it has to be limited to a value in the order of \( 2V_{be} \). Therefore the amplitude of the integrator signal cannot be given a value in the order of two times the supply voltage, which is possible in some other first order oscillators. The physical limitation of the hysteresis conflicts with the requirements for an optimal noise behavior. The hysteresis can be enlarged if components that introduce a level shift are inserted. A larger hysteresis results in a larger amplitude of the integrator signal, which generally improves the oscillator performance. In the circuit of fig. 7.4 two emitter followers have been inserted as level shifters. Another means to accomplish an increase of the hysteresis is the use of complementary elements. An example of such a first order oscillator has been depicted in fig. 7.5. With this circuit an amplitude of the integrator signal close to the supply voltage can be obtained.

Both measures to increase the hysteresis have some drawbacks. The insertion of emitter followers does not result in a very large increase of the hysteresis. Extra bias sources \( (I_b) \) are necessary. Also emitter followers can have an output impedance with an inductive character. Together with parasitic capacitances this output impedance can form a frequency selective network. This network can cause "ringing". Especially in high frequency oscillators, this ringing can introduce a considerable distortion in the waveforms of the signals in the oscillator. The frequency selectivity also interferes with the mechanism that normally determines the output frequency of the oscillator. The tuning properties of the oscillator become affected.
Fig. 7.5 An emitter-coupled oscillator that makes use of complementary elements to increase the hysteresis

may cause hysteresis in the tuning curve, and disturb its monotony. Close to the frequency of preference the oscillator is pulled to that frequency. It can be expected that this pulling effect sometimes is able to improve the frequency stability. This has been observed in measurements. However, the predictability and the designability of the phenomenon is poor as yet.

The use of complementary devices requires the availability of these components in an IC-process. Very often the large differences in performance of the devices prevent the implementation of first order oscillators with complementary elements (in the signal path).

Tuning the oscillator

When the oscillator is to be used as a current controlled oscillator (CCO), the dependencies between the memory parameters also play an important role. In principle a first order oscillator can be modulated when the integration constant is modulated. In this circuit the integration constant is determined by the capacitor value and a current that is sourced by the Schmitt trigger. The absolute value of the integration constant is given by:

$$|\alpha| = \frac{I}{C_i}$$ (7.8)

A modulation of the current can be a means to modulate the oscillator, since this modulates the integration constant. However, in this case the interdependency of the parameters of the Schmitt trigger causes the output frequency to remain constant. The same current that is found in the expression for the integration constant, also determines the hysteresis of the Schmitt trigger. This is because the Schmitt trigger also performs the switching function. With respect to the integrator, the current sourced by the Schmitt trigger, is the output signal of the Schmitt trigger. The extreme values of the output signal are \(-I\) and \(I\) respectively. The value of the integration constant is determined by these extreme values. In first order approximation the hysteresis and the value of the integration constant change proportionally when the current is modulated. The output frequency of the oscillator is:

$$f_o = \frac{|\alpha|}{2E_{\text{hys}}} = \frac{I}{2C_i(4RI)} = \frac{1}{8RC_i}$$ (7.9)

It can be seen that the output frequency is independent of \(I\), due to the interdependency of the parameters of the memory circuit.

When the behavior of the oscillator is examined in more detail, some dependency of the frequency on \(I\) is found due various effects, such as the influence of the current on the loop gain of the Schmitt trigger. An expression for the hysteresis of a Schmitt trigger as used in the emitter-coupled oscillator of this section is given by (4.71). Substitution of (4.64) and (4.65) in
this expression and a multiplication of the result by 2RI, because the expressions in chapter 4 have been normalized, results in:

$$E_{hyd} = 2V_T \ln \left( \frac{1 - \sqrt{1 - \frac{V_T}{RI}}}{1 + \sqrt{1 - \frac{V_T}{RI}}} \right) + 4RI \sqrt{1 - \frac{V_T}{RI}}$$  \hspace{1cm} (7.10)

When this expression is used to calculate the output frequency, it follows:

$$f_o = \frac{1}{8RC_t \sqrt{1 - \frac{V_T}{RI}} + 4V_T C_t \ln \left( \frac{1 - \sqrt{1 - \frac{V_T}{RI}}}{1 + \sqrt{1 - \frac{V_T}{RI}}} \right)}$$  \hspace{1cm} (7.11)

This relation has been verified by measurement. A temperature dependency of the frequency is introduced via $V_T$. Usually it is not too difficult to compensate it, because $V_T$ only appears in combination with $I$ in the expression. Then $I$ has to be made proportional to the ambient temperature (PTAT). However, prevention is always better than compensation. A separation of the basic functions can make the amplitude of the integrator signal independent of the temperature dependent hysteresis of the memory. This is a very good alternative, because more than once, the lack of good transistor parameters makes compensation circuits undesiriable.

The dependency of the hysteresis and the integration constant on the same current prevented the use of the oscillator as CCO. There are two basic ways to solve this tunability problem. The first method is the separation of basic functions, which will be discussed in section 7.5. The second method, which is used very often, is reducing the dependency of the hysteresis on the current. This can be done with the use of a non-linear feed back network in the memory circuit. Regenerative memories with a non-linear feed back network have been described in section 4.10.

Again the emitter-coupled CCO will be used as an example to show the advantages and disadvantages of this method. The CCO has been depicted in fig.7.6. The shape of the loop gain function of this circuit has been depicted in fig.4.26b. Bulk resistances may cause the center of this function to drop below unity, in which case a trinary behavior of the memory is the result. The diodes limit the hysteresis when the current $I$ is large enough. For small currents the voltage across the resistors is too small to obtain any effect from the diodes. In that case the tuning curve is described by (7.11), so the circuit is certainly not a linear CCO for small currents. For currents that are large enough, the diodes more or less fix the hysteresis, because of the exponential relation between the voltage across the diodes and the current that flows through them. When the Schmitt trigger is close to a threshold state, one diode will be limiting, because nearly all current flows through its branch. The small current through the

Fig. 7.6 An emitter-coupled oscillator CCO
other branch causes a voltage drop across the corresponding resistor. The diode in parallel with this resistor generally has a neglectable influence, since the voltage drop is small. Therefore around the threshold states the Schmitt trigger can be modeled as if it were a Schmitt trigger with a resistive feed back network. The resistance is equal to $R$ and the conducting diode is modeled as a short circuit. A similar expression as (4.71), describing the dependency of the threshold level on the loop gain and hence on the current can be derived. Together with the non ideal limiting of the diodes this effect causes non-linearity of the tuning curve of the CCO. For larger currents also the bulk resistances of the diodes will affect the linearity. Again most effects can be compensated, but the designability of compensating circuits may be poor.

In a lot of cases monotony of the tuning curve is sufficient. When the linearity is not of great importance, in this respect a configuration with a non-linear feed back network may suffice. Still other problems caused by the non-linear network remain. It generally limits the hysteresis to a relatively small value. This conflicts with the design rule that the amplitude of the integrator output signal should be as large as possible, because via the hysteresis the network also limits this amplitude. Hence the non-linear feed back network tends to hamper the optimization of many other quality aspects like the frequency stability. When the oscillator performance has to be really optimized, the use of a system set-up with separated functions becomes almost inevitable.

### 7.3.2 The relation between the integration constant and the memory output signal

In simple regenerative oscillators, like the emitter-coupled oscillator, the output signal of the memory is proportional to the integration constant. This output signal $I_o$ is a current, which depends on the state variable $X$ of the memory. In the case of the emitter-coupled oscillator the relations are:

\[
I_o = \dot{I}_o X \quad (7.12)
\]

\[
\alpha = \frac{\dot{I}_o}{C_i} X \quad (7.13)
\]

in which $\dot{I}_o$ represents the amplitude of the charging current.

In situations where the threshold states are close to each other, this relation will cause problems. An example of a loop gain function with the threshold states close to each other can be seen in fig.4.27c. Before a threshold state is reached, the regenerative memory behaves like a normal amplifier, so it changes state at the rate of its input signal. In the situation where the threshold states are close to each other, the state has to change much before regenerative operation is achieved. This implies that $I_o$ and consequently the integration constant shows a considerable decrease before the memory switches regeneratively. As a result the slope of the integrator signal decreases when approaching the threshold level. For $X = 0$ the slope would have become equal to zero, but before that happens the regenerative memory should have switched. This effect has two major consequences:

1. The period becomes inaccurate.

2. The excitation of the memory is decreased so the noise sensitivity is enlarged, and the transition time is enlarged (mainly in the first part of the transition).

Not only by breaking the proportional relation between the state of the memory and the integration constant by way of a separation of basic functions can solve the problem, but alternatively a change of the loop gain function can be used. Keeping in mind that the
slope of the loop gain function at the threshold states should be large, a resistive feed back network appears to be the best solution to this problem. Unfortunately the use of this network introduces the relation between the hysteresis and the integration constant, which causes other problems. Non-linear feed back networks (for example with diodes) have to used. Generally relatively simple oscillators result with this method. When simplicity is a design requirement, and the performance is acceptable for the application, this solution can be used. In all other cases a separation of basic functions usually provides a better performance.

7.4 The influence of non idealities of the integrator

The ideal integrator has only one pole in the origin of the p-plane. The transfer contains no real part. In practice this ideal situation can only be approximated. To start with, the integrator itself has losses. Generally it is implemented as a capacitor. The losses of a capacitor can be very low. A quality factor can be defined for a capacitor, and when this is compared to the quality factor of an LC-resonator, it is usually much better. The non idealities are therefore usually not caused by the bad quality of the integrator itself, but by the circuits that perform the other basic functions in a first order oscillator. Unfortunately, the quality of the capacitor is only indirectly related to the essential output quantity of a first order oscillator: the frequency. The extra circuits that are necessary to make a periodical signal, are not able to retain the good quality of the basic accuracy in the first order oscillator (chapter 2). The circuits that perform the other basic functions load the capacitor. These loads can be modeled as integrator losses. Therefore both effects have been modeled in fig. 7.7 with one resistor \( R_p \) in parallel with the capacitor. A second resistor \( R_s \) models the bulk and other parasitic resistances in series with the capacitor.

The impedance of the integrator composed of the capacitor and the two resistors, for \( R_s \ll R_p \), is:

\[
Z_{int} = R_p \frac{1 + R_s C_t p}{1 + R_p C_t p}
\]  

(7.14)

Due to \( R_p \), the pole of the impedance is no longer in the origin and on top of that \( R_s \) introduces a zero. Both the shift of the pole and the presence of a zero can seriously affect the oscillator performance. Both effects will be dealt with separately below.

The influence of \( R_p \)

The resistor in parallel with the capacitor \( R_p \), distorts the output wave form of the integrator. Instead of a ramp, an exponential function is found. A quarter period of the integrator output signal can be described with:

\[
E_o(t) = R_p I_0 \left(1 - e^{-\frac{t}{\tau_p}}\right)
\]  

(7.15)

\[
\tau_p = R_p C_t
\]  

(7.16)
The distortion causes non-linearity when the oscillator is used as a CCO. Also the amplitude $\dot{E}_o$ of $E_o(t)$ has become bounded by a maximum:

$$\dot{E}_{omax} = R_p I$$  \hspace{1cm} (7.17)

When $\dot{E}_{omax} < \frac{1}{2} E_{hp}$, the oscillator does not produce a periodical output signal, because the integrator signal never reaches a threshold level. But even before the oscillation really stops, the oscillator performance becomes affected. The slope of the integrator signal decreases when the integrator signal approaches a threshold level. It has already been stated that this deteriorates the oscillator performance. Especially when $\dot{E}_{omax}$ comes close to $\frac{1}{2} E_{hp}$, the frequency stability becomes very bad. The very low excitation of the regenerative memory causes it to be very noise sensitive. Also the linearity when used as a CCO becomes very poor.

For an optimal performance of the oscillator, $\tau_p$ should be made as large as possible. A value for $C_i$ that is as large as possible has already been advised in section 7.1. Another way to increase $\tau_p$ is the optimization of $R_p$. This can be done preferably by making it as large as possible, or—if the application allows some deterioration of the frequency stability—making it accurate so that some effects can be compensated. A separation of functions may very well ease the optimization of this aspect of the oscillator performance.

The influence of $R_s$

The complete oscillator system contains a negative feed back loop. When the integrator output signal reaches a threshold level, the events that start taking place are such that the sign of the integration constant is changed. Due to this mechanism, the integrator output signal starts moving away from the reference level again. Apparently, by approaching a threshold level, the integrator output signal initiates a mechanism that counteracts this approach. This is negative feed back. If present solely, this feed back loop can stabilize the integrator output signal at a threshold level. There are several reasons why the loop is not able to do this. In two-integrator oscillators, the loop is never closed because of the presence of comparators that break it and by virtue of a delicate selection of threshold levels and switching sequences.

In regenerative oscillators, the regenerative positive feed back loop of the memory dominates the negative feed back loop. The switching of the memory happens relatively fast, and since the negative feed back loop contains an integrator, the positive feed back loop can easily dominate it. The negative feed back loop simply has no time to react. This situation changes when the resistor $R_s$ introduces a zero. Then an instantaneous reaction of the negative feed back loop can be expected. It depends on the respective values of the loop gains which loop will dominate the behavior. Also in regenerative oscillators it is possible to break the negative feed back loop by way of careful selection of reference levels and switching sequences. This usually requires the separation of basic functions. In simple regenerative oscillators, this is not possible. Calculations to determine the two loop gains are common place and will not be discussed. In this section, with the aid of the loop gain-excitation model, the effect of $R_s$ will be investigated.

The presence of the resistor $R_s$ introduces an extra term in the expression for the external excitation. When $I_o$ is the current that flows through the capacitor $C_i$, the expression yields to:

$$E_{exc}^e = V_{ct} + R_s I_o$$  \hspace{1cm} (7.18)

In a simple regenerative oscillator, like the emitter-coupled oscillator, usually $I_o$ is proportional to the state variable $X$:

$$I_o = \dot{I}_x X$$  \hspace{1cm} (7.19)
This implies that the external excitation has become state dependent. The capacitor voltage $V_{ct}$ is assumed to be constant during the transition. In section 7.3.2 the influence of a state dependent $I_o$ on the transient behavior has already been discussed. This change was particularly significant outside the regenerative area of the transition. The presence of $R_s$ gives an additional reduction of the slope similar to that discussed in section 7.3.2. In this section the influence of $R_s$ during the regenerative part of the transition is discussed. For this part of the transition $V_{ct}$ can be assumed to be constant.

The state dependency of the external excitation has its influence on the trajectory of the operating point in the steady-state diagram of the regenerative memory. Fig.7.7 illustrates the effect. The slope of the trajectory if the capacitor voltage is constant is given by:

$$\frac{dX}{dE_i} = -\frac{1}{I_o R_s}$$  \hspace{1cm} (7.20)

The excitation of the memory is equal to the horizontal distance between the operating point and the steady state line. Obviously the presence of $R_s$ reduces the excitation. In the case of trajectory t2, the reduction is so large that the operating point ends up on the slanted steady state line. This is a stable operating point! Regenerative operation stops. The memory does not stop changing state as yet, because unless $X = 0$, the capacitor is still charged. Due to this effect $V_{ct}$ slowly increases, thus moving the operating point somewhat to the right. Then the operating point moves in the direction of t2 towards the slanted steady state line again. As a result the operating point moves slowly, at the rate of change of the capacitor voltage along the slanted steady state line until the state $X = 0$ is reached. There the operating point stops because the capacitor voltage does not change any more. Apparently a “critical value” exists for $R_s$. When $R_s$ is larger than this value, the regenerative memory ends hung up in the state $X = 0$. The slope of the slanted steady state line is given by the normalized equation (4.72) for the emitter-coupled memory by denormalizing it with the normalization factor $2R_I I_o$. The it follows:

$$\frac{dX_{ss}}{dE_i} = \left( \frac{1}{2RI_o} \right) \frac{A_{max}(1 - X^2_{ss})}{1 - A_{max}(1 - X^2_{ss})}$$  \hspace{1cm} (7.21)

The linearized steady state curve of fig.7.8 can then be described by:

$$\frac{dX}{dE_i} = \left( \frac{1}{2RI_o} \right) \frac{A_{max}}{1 - A_{max}}$$  \hspace{1cm} (7.22)
With this expression the critical value for $R_s$ can be calculated:

$$R_{crit} = -2R \left( \frac{1 - A_{max}}{A_{max}} \right) \tag{7.23}$$

**Example**

Although this value seems very large, the effect has been found to be responsible for a degradation of several practical oscillators. When a first order oscillator is completely integrated the value of capacitors is usually limited. Several techniques exist to make the best use of the available capacitors. When a junction capacitor is used, its value can be made the largest when the voltage across it is as small as possible. Suppose an oscillator implementation is chosen with a grounded capacitor. Then the "ground" terminal of the capacitor can be connected to a voltage source with a value just below the lowest value the other terminal can have, instead of being connected to ground directly (fig.7.9) In that case the capacitance is the largest. This

![Diagram](image)

Fig. 7.9 The ground terminal of a timing (junction)capacitor connected to a voltage source instead to increase the capacitance

voltage source could be implemented as an emitter follower, to prevent the injection of signal current to a common reference source. When, in order to save power, the current through the emitter follower is chosen very low, the output impedance of the emitter follower may cause severe damage to the oscillator properties. A bias current of 10μA result in an output impedance of about 2500Ω. In first order oscillators that operate on these current levels, the resistance of the feedback network can be in the order of 25kΩ to obtain a maximum of the loop gain of about 10. Then $R_s$ is already at 10% of its critical value.

**7.5 Oscillators with separated basic functions**

Each basic function that has to be performed in a first order oscillator can be implemented electronically in several ways. When such a circuit is used to implement just one of the basic functions, it can usually be optimized easily. This is why a distribution of the basic functions among several basic circuits, is a convenient way to obtain so called "high-performance" first order oscillators. In the previous sections it has been shown that circuit properties that are necessary for the implementation of one basic function, very often hamper the quality of another basic function that has to be performed by the same circuit. Especially when this problem occurs, the conflicting functions should be separated. In this section, the implementation of basic functions in dedicated circuits will be discussed. The memory function has already been discussed in chapter 4 and chapter 5 and will not be discussed in this section any more.
7.5.1 Level comparison

The level comparator is the electronic implementation of the \textit{sgn}-function. The output signal of the level comparator can only have three different states:

1. The input signal is \textit{above} the reference level.
2. The input signal is \textit{at} the reference level.
3. The input signal is \textit{below} the reference level.

The output signal of the level comparator is used as input signal for the memory. Different ways to interconnect the circuits have been discussed in chapter 2. Basically two level comparators are required, since there are two threshold levels in a first order oscillator, but implementations of first order oscillators are possible in which only one level comparator is required. Then the reference level of the level comparator is switched to the appropriate value by the memory. Examples of circuits using two comparators can be found in [9] and in section 7.7.1. In fig.7.10 a regenerative oscillator equipped with one comparator has been depicted. The output quantity

![Diagram of a regenerative oscillator with one comparator](image)

Fig. 7.10 A regenerative oscillator with one comparator

of the Schmitt trigger is a voltage, which is converted to a current by a linear transadmittance amplifier (G). The comparator is a voltage comparator with a voltage output. Some signals appearing in the circuit are shown in fig.7.11. The memory output signal is the output voltage

![Diagram showing signals in a regenerative oscillator with one comparator](image)

Fig. 7.11 Some signals in a regenerative oscillator with one comparator

of the Schmitt trigger, and hence also the reference voltage for the comparator. The current
that charges the capacitor has the same waveform as the memory output signal. Suppose the oscillator is in a state in which the signal values are as depicted at the most left side of the time axis of fig.7.11. When the integrator signal (capacitor voltage) reaches the reference level of the comparator, the comparator changes its output very fast. As a reaction to this, the Schmitt trigger starts changing state. Because of the large excitation delivered by the comparator, the transition time of the Schmitt trigger is well defined. Then at the zero crossing of the Schmitt trigger, the sign of the current changes, and the integrator signal starts changing into the other direction. The speed at which the memory output signal changes is much larger than that of the integrator signal. Therefore the reference level of the comparator remains below the integrator signal. The new reference level of the comparator is reached long before the integrator signal reaches this value. The negative feedback loop described in section 7.4 is broken in this circuit. This is not achieved due to the fact that the Schmitt trigger takes time to switch, but due to the fact that the sign of the current switches after the Schmitt trigger has made a zero crossing. In that case it is guaranteed that the integrator signal will not catch up with the memory output signal before it is at the new reference level. The loop is not broken due to delay, but due to a delicate selection of switching levels and sequences. Even when the comparator and the Schmitt trigger are infinitely fast, the oscillator will work properly.

The electronic implementation of the level comparator is usually a high-gain limiter. In a very small area of the input signal, such a circuit behaves like a linear amplifier. The Schmitt trigger is only sensitive for the limiter output signal when it is around its threshold level. This threshold level is in the linear area of the limiter output signal. Therefore as far as the Schmitt trigger is concerned there is no difference between a linear amplifier and the limiter. This has already been discussed in section 4.6. The limiter can therefore be optimized for an optimal noise behavior at the operating point that produces the threshold level of the Schmitt trigger at its output. Standard amplifier theory can be used to do so.[3]

7.5.2 Switching of the integration constant

In many first order oscillators, the value of the integration constant is proportional to the state of the memory (7.19). This causes distortion of the integrator output signal, which was described earlier in this chapter. In fig.7.11 this distortion can be observed. Also, very often an interdependency exists between the output signal of the memory and other memory parameters, like the hysteresis. When a separate circuit is used to switch the integration constant, these problems can be avoided. A current switch can be used that switches an independent current.

\[ \text{sgn}(E_{\text{mem}}) \]

\[ I_{\text{ct}} \]

\[ c_L \]

\[ E_{\text{mem}} \]

Fig. 7.12 A first order oscillator in which the integration constant is switched by a separate circuit

The relation between the state of the memory and the integration constant becomes strongly non-linear. In the ideal case it is the electronic implementation of the \text{sgn}-function with a
scaling factor:

\[ \alpha = \frac{\dot{I}}{C_t} \text{sgn}(E_{mem}) \]  

(7.24)

\( \dot{I} \) is an independent current, and \( E_{mem} \) is the output signal of the Schmitt trigger. The \( \text{sgn} \)-function can be implemented as a high gain limiter. In practice, circuits like an OTA can be suitable. In the next chapter there is an example of an OTA that is used as a current switch (section 8.1.2). When the current switch acts according to (7.24), the distortion of the integrator output signal disappears. The fact that \( \dot{I} \) is an independent source, makes it a very convenient means to modulate the oscillator. Other parameters, like the amplitude of the integrator signal, are not affected. This increases the accuracy of the CCO transfer from modulation input to output frequency. In fig.7.13 some signals appearing in the circuit of fig.7.12 have been depicted. When this figure is compared to fig.7.11, it can be seen that the non-linearity of the integrator signal has disappeared.

The negative feedback loop has been broken again by way of a careful selection of reference levels and switching sequences.

The feedback network of the Schmitt trigger can be of the resistive type, which generally results in the best performance of the Schmitt trigger. Now also the hysteresis can be controlled separately from the integration constant. This is achieved by modulation of the bias current of the Schmitt trigger. Then the period of the oscillator changes proportionally to the modulating quantity. The dependency of the loop gain function on the current can cause non-linearity in this case, but several methods are known to reduce it. One of them is, of course, the separation of functions. Fig.7.14 shows a simple circuit in which this has been done.

**7.5.3 Bypassing the memory.**

When both the level comparison function and the switching function are performed by separate circuits, it is possible to reduce the influence of the regenerative memory on the timing. The memory can be bypassed. Fig.7.15 shows an oscillator in which this has been done. Both the comparators and the memory operate the switch. The switch is initially operated by either
one of the comparators. This is a negative feedback mechanism which can stop the oscillation. When the sign of the integration constant is switched, the integrator signal starts changing into the other direction, thus removing the cause for the comparator to produce the output signal that changed the position of the switch. Then the integration constant would be switched back to the original value again. However, the comparator also excites the regenerative memory which eventually takes over the “decision” of the comparator. So when the comparator stops generating the necessary output signal, the memory prevents the switch from switching back into its original position[17].

When the switching levels of the comparator and the memory are chosen correctly, the memory properties will have less influence on the timing of the complete oscillator. The memory does not have to produce a sufficient output signal sooner than the moment at which the output signal of the comparator does not keep the switch in the right position anymore. This implies that the memory has more time to switch. The memory delay is not added to the delay of the comparator just like that, because it partly switches in parallel with the comparator. Apart from the fact that the memory has to switch before the comparator stops generating the right output signal, no further timing requirements are necessary. The timing jitter of the memory does not affect the accuracy of the moment at which the integration constant is switched. This is determined by the comparator. The memory only has to sustain the change of the integration constant that was initiated by the comparator. The switching function is less influenced by the properties of the regenerative memory, so the separation of the functions is carried out further. In section 7.7.1 an electronic implementation will be discussed.

7.5.4 High frequency behavior

The circuits that perform the basic functions in a first order oscillator need time for this. For example, a limiter that is used to implement the level comparison function, does not switch its
output signal immediately when its input signal reaches the reference level, but after a time \( \Delta T \). This delay causes an error in the timing of the oscillator. Since the integration constant is not switched immediately, the integrator signal shows an overshoot. The period becomes longer. Two times every period a threshold level is reached, which requires a reaction from a level comparator. Hence two times the delay of the comparator has its effect on the period. The integrator integrates a time \( \Delta T \) longer, and needs another time \( \Delta T \) with the integration constant having the opposite sign to compensate for this extra signal. Hence a delay equal to \( \Delta T \) results in a contribution of \( 2\Delta T \) to the period:

\[
T_o = T_{\text{ideal}} + 4\Delta T
\]  
(7.25)

\( T_{\text{ideal}} \) is the period if no delays are present.

Fig. 7.13 shows an example, in which the memory has a delay. It can be seen that the integrator signal shows an overshoot for the duration of two times the delay.

The influence of delays on CCOs

When the oscillator is used as a CCO, delays introduce non-linearity in the CCO transfer. Especially for relatively high frequencies the influence can become very large. The maximum operating frequency of a first order oscillator is limited by the sum of all delays in the system.

\[
T_{\text{omax}} < 4\Delta T
\]  
(7.26)

The period can never become shorter than the sum of the delays, but this is not always considered the maximum operating frequency. When for example the linearity of a CCO has to be very good, the delays have to be small compared to the period. Then the maximum frequency at which the oscillator can be used is much lower. Modulation of the integration constant only results in a modulation of the ideal period \( T_{\text{ideal}} \). The delays are not influenced, or they are influenced in an inaccurate way. For now the delay is assumed to remain constant.

The static non-linearity of a transfer can be expressed in terms of the differential error \( \varepsilon \) [3]. It is defined as:

\[
\varepsilon = \frac{TF - TF(Q)}{TF(Q)}
\]  
(7.27)

\[
TF = \frac{df_o}{dE_i}
\]  
(7.28)

\( TF(Q) \) is the transfer in the quiescent point, \( TF \) is the transfer in another operating point and \( E_i \) is the input signal for the CCO. When the tuning constant of the CCO is \( k \), the ideal output frequency is without delays is:

\[
f_{\text{ideal}} = kE_i
\]  
(7.29)

Due to the delay, the actual output frequency is:

\[
f_o = \frac{f_{\text{ideal}}}{1 + 4\Delta Tf_{\text{ideal}}}
\]  
(7.30)

When a maximum frequency deviation of \( \Delta f \) from the carrier frequency \( f_{\text{center}} \) is assumed, the maximum differential error can be calculated:

\[
|\varepsilon_{\text{max}}| = 8\Delta T\Delta f \left( \frac{1 + 2\Delta T f_{\text{center}} + \Delta f}{1 + 4\Delta T(f_{\text{center}} + \Delta f)^2} \right)
\]  
(7.31)
This expression is derived in appendix B. For small delays, this expression yields to:

\[ |\epsilon_{\text{max}}| = 8\Delta T \Delta f \]  
(7.32)

The maximum delay allowed for a certain differential error is:

\[ \Delta T_{\text{max}} = \frac{|\epsilon_{\text{max}}|}{8\Delta f} \]  
(7.33)

When for example a 10MHz CCO is tuned with a maximum frequency deviation of 1MHz, and a differential error of 1% is requested, the maximum delay allowed in the oscillator is 1.25ns. According to (7.26) the maximum operating frequency of such an oscillator is 200MHz!

**The effect of delays on the noise behavior**

First order oscillators are systems that contain time variant transfers. In such systems noise of one frequency band can be converted to another frequency band. In first order oscillators this means that noise in frequency bands around multiples of the carrier frequency can be folded back into the band around the carrier. In chapter 6 this has been discussed in more detail. It depends on the bandwidth of the various basic functions how many noise bands are folded back. When an oscillator is operated on a frequency that is relatively low compared to its maximum operating frequency, many noise bands will be folded back. As a result this oscillator shows more phase jitter than an oscillator that has a lower maximum operating frequency. In fig.7.16 the measured output spectra of two oscillators have been depicted. The first one was obtained from an oscillator that was build with germanium transistors with an \( f_T \) of 2.5MHz. In the other case the \( f_T \) was 1.4GHz. The operating frequency was the same. It can be seen that the high frequency oscillator shows considerably more phase jitter. With respect to the noise performance, the maximum operating frequency should not be chosen higher than necessary. Unfortunately for linearity it should be as high as possible, so two conflicting design rules result. A solution of this problem could be the use of a *synchronous* frequency divider. Then the oscillator can be operated closer to its maximum frequency, so it does not fold back much noise. The divider does not influence the position of the zero crossings, so the jitter of the zero crossings at the input of the divider is equal to the jitter at the output. Only at the output the period is larger, so the relative contribution of the jitter to the period is reduced. When the oscillator itself was operated at the output frequency of the divider, the jitter would have been larger because then the oscillator would have folded back noise.

**The influence of the system set-up on the high frequency behavior**

The origin of the delays depend on the implementations of the basic functions. They have to be calculated for each specific circuit. A selection of the devices that are used to implement the basic functions has to be made on those ground. Apart from this, the system set-up can be adapted for the high frequency behavior. This is the only solution when one or more of the implementations of a basic function do not meet with the requirements. Very often the influence of parasitic capacitances can be successfully reduced by choosing the right system set-up. When the oscillators have to be completely integrated—which is especially important for high frequency oscillators—the available capacitors usually have a considerable parasitic capacitance to the substrate (ground) at one of their terminals. Then a system set-up with a grounded capacitor seems the obvious way to implement an oscillator. In that way the parasite can be shorted and has no effect, or if its accuracy in considered sufficient, it can be used in parallel with the normal capacitor. Other specifications for the system, like temperature
Fig. 7.16 The output spectra of oscillator equipped with transistors with an \( f_T \) of 2.5MHz (lower trace) and an \( f_T \) of 1.4GHz (upper trace) respectively

stability or the prevention from signal currents being injected in the power supply rails, may require a balanced system set-up. In that case the capacitor has to be floating. Then more subtle solutions for the problem caused by the parasite have to be found. In fig.7.17, a solution has been depicted. Since the only solution is the connection of one of the capacitor terminals to

![Diagram of a balanced first order oscillator with grounded capacitors](image)

Fig. 7.17 A balanced first order oscillator with grounded capacitors

ground, this has been done here also. However, now two capacitors are needed. The parasitic capacitors \( C_p \) are made use of also. The fact that the oscillator has been connected to ground at this place, only has no effect if no other point in the oscillator is referenced to ground.
This implies that the supply of the oscillator has to be arranged by current sources solely, because current sources do not fix the voltage across them. Only current sources will not introduce an extra ground reference. A closer inspection if fig.7.17 shows that in this circuit current sources are placed in series. If their sum is not exactly equal to zero, which is likely to happen in practice, charge is stored in the circuit. Then its potential with respect to ground linearly increases or decreases with time, until a current source saturates. A bias control loop is necessary. This loop has been added to the circuit in fig.7.18. At the output of the adder,

![Bias Control Loop Diagram](image)

Fig. 7.18 A balanced first order oscillator with grounded capacitors and a bias loop

the common mode voltage of the emitters is available. The output signal is compared with a reference voltage. The difference is used to control the current source that injects current into a common mode point of the oscillator. In this way the common mode voltage at the emitters is kept equal to $V_{ref}$.

The bias loop only works for common mode signals. The adder can be seen as a filter that is able to separate common mode and differential mode signals. The property that makes this possible is the fact that these signals are orthogonal. The high frequency signals are all differential mode signals, so the oscillation is not affected by the bias loop. Another example of such a loop will be discussed in section 7.7.1.

### 7.5.5 Temperature stability

Many parameters of the devices that are used to implement a first order oscillator depend on temperature. In bipolar devices, like transistors, $V_T$ is an example of a temperature dependent parameter:

$$V_T = \frac{kT}{q} \tag{7.34}$$

$T$ is the absolute temperature, $k$ is Boltzmanns constant and $q$ the charge of one electron. Via $V_T$ the loop gain of the bipolar first order oscillator discussed in section 4.7 becomes temperature dependent. Hence also the hysteresis becomes temperature dependent. When this hysteresis also determines the amplitude of the integrator signal, the output frequency of the oscillator becomes temperature dependent too. Two different ways can be followed to solve this particular problem. The temperature dependency can be compensated, or avoided by way of a separation of functions. When the Schmitt-trigger is biased with a PTAT current, the temperature dependency is compensated[20]. A zero temperature coefficient is obtained for one temperature and around this temperature it is reduced. When the bias current is given
a bit more complicated temperature behavior, a complete compensation can be obtained[22]. A detailed discussion of PTAT sources is beyond the scope of this thesis.

When the functions are separated, the level comparison can be performed by a circuit in which temperature effects are for example balanced out.

In the simple two-transistor Schmitt-trigger of fig.4.17, there is no balancing although the circuit diagram looks symmetrical. This is because in the states where the temperature dependency is important, close to unity loop gain, the current distribution among the two transistors is very asymmetrical.

There are also examples of an intentional implementation of a certain temperature behavior. Usually the integration constant is given a temperature dependency. An application in a temperature transducer is described in [21]. In this case other than the desired temperature dependencies must be avoided. A separation of functions is again a way to do this.

7.6 Coupled first order oscillators

In section 3.4 already a wide variety of methods to couple first order oscillators has been presented. In this section only one type of coupling will be discussed. It is the type of coupling with two oscillators coupled in quadrature via external transitions. The external transitions of one oscillator are initiated by a signal that is derived from the zero crossings of the integrator signal of the other oscillator. Fig.7.19 shows a state model representation of this type of coupled oscillator. This type of coupled oscillators has some properties that makes

![Diagram of coupled first order oscillators](image)

Fig. 7.19 The state model representation of a coupled oscillator with two regenerative first order oscillators coupled in quadrature

them preferable above the other types described in section 3.4. In these coupled oscillators, the negative feed back loop that tends to be present in single first order oscillators, is absent. The signal that initiates the transition is not influenced by this transition directly. After a zero crossing of the integrator signal of one oscillator, the slope of this integrator signal does not change directly. This happens after a quarter period. Furthermore the comparators that have to be used to detect the zero crossings of the integrator signals also provide amplification of these signals before they are used as exciting signals. Hence the slope of these exciting signals is increased which is favorable for the accuracy and frequency stability of the oscillator.

The temperature has less influence also. The temperature dependent threshold levels of the regenerative memories have no influence on the period anymore. In the comparators the temperature effects can be balanced out to some degree around the zero crossings. Therefore the comparators set less temperature dependent threshold levels.
7.6.1 The synchronization of two first order oscillators

In practice the two first order oscillators to be coupled usually do not produce output frequencies that are exactly equal. In the coupled system, the output frequencies are made equal, so some tuning mechanism has to be present. The fastest oscillator speeds up the slowest oscillator, and the slowest slows down the fastest oscillator. The system frequency will be the average of both free-running frequencies.

Fig. 7.20 The steady-state lines of a first order oscillator with external transitions

In fig. 7.20 the steady state lines of a first order oscillator have been depicted. The transitions are initiated by an external signal. The dashed lines indicate the vertical trajectory of the operating point if no external synchronizing signals are applied. The operating point moves in a rectangle at different speeds for different parts of the rectangle. On the horizontal fat parts it moves at the same speed as the integrator output signal. A change of the memory input signal equal to $\Delta E_i$ takes a time $\Delta T$ with:

$$\Delta T = \frac{\Delta E_i}{\alpha}$$  \hspace{1cm} (7.35)

On the thin parts of the rectangle, the operating point moves faster. In the horizontal direction the change of $E_i$ is caused by the synchronizing signals coming from the other oscillator. These signals are derived from the zero crossings of the integrator signal of the other oscillator by way of a comparator. When this comparator has a small-signal gain of $A$, the slope of its input signal is found increased with a factor $A$ at its output. The time $\Delta T_s$ the operating point needs for a horizontal movement equal to $\Delta E_i$ due to the synchronizing signal is equal to:

$$\Delta T_s = \frac{\Delta E_i}{A\alpha}$$  \hspace{1cm} (7.36)

In this case $\alpha$ is the integration constant of the oscillator from which the synchronizing signals are derived.

The speed of the vertical change is determined by the switching speed of the memory. A complete vertical trajectory is crossed by the operating point within the transition time of the memory. Fig. 7.20 is a very idealized diagram, since the synchronizing signals are assumed to have infinite slopes. In practice the synchronizing signals have a finite slope. Once a threshold level is crossed, the memory starts changing state. So while the memory is changing state, the synchronizing signals are still "building up". When, for example, the synchronizing signal and the memory both change at the same speed, a state diagram as shown in fig. 7.21 is found. When the memory switches very fast compared to the synchronizing signals, first a
fast transition of the operating point (nearly) along the dashed lines is found, followed by a movement to the starting point of the corresponding fat horizontal line. The memory is nearly minimally excited by this signal, so the noise behavior of the memory will not improve much. If the gain $A$ of the comparator is equal to unity, the slope of the integrator signal from which the synchronizing signal is derived is also found for the output signal of the comparator. Since both oscillators are designed to be equal, this is the same slope as that of the integrator signal of the oscillator that is to be synchronized. The input signal for the memory $E_i$ is the sum of the two signals, so only twice the slope of the integrator signal is obtained. This does not increase the external excitation with much effect. The memory has switched already before a significant extra excitation has been built up by the synchronizing signal. In high-frequency oscillators, the bandwidth of the comparator itself can be the cause of a low extra excitation. Then it is difficult to obtain a signal from the comparator that makes a significantly faster transition than the memory does.

In all cases, independent of what happens during the fast movements of the operating point, the synchronizing signals can establish synchronization between the oscillators. The two fat horizontal parts of the state diagram, that dominantly determine the period of the oscillator concerned, are of equal length in all cases. In first order approximation the period is equal to the time the operating point spends moving along these fat parts.

From the diagram shown in fig.7.20 it can be seen that the period of the oscillator is enlarged. The horizontal fat parts are longer than the distance between the two dashed lines. The places where the fast parts of the diagram, due to the synchronizing signals appear, are determined by the phase relation between the two oscillators.

Suppose there are two oscillators that are not in quadrature yet. The places where the fast parts of the diagram appear, determine the length of the period of the oscillator. Fig.7.22 shows a state diagram of one of the oscillators. In this situation the period of the oscillator is the shortest. The other oscillator has a larger period, so fast parts p1 and p2 are not at equal distances from the start of the corresponding fat parts. Fast part p2 appears later. Each subsequent period, the two fast parts move further clockwise in the state diagram. Finally one of the fast parts (in this case p2) is able to initiate a transition. This situation has been depicted in fig.7.23. Then an extra time $\Delta T_{p2} = \Delta E_o$ is added to the period of the oscillator. Hence the oscillator is slowed down. When the other oscillator is still faster, the fast parts p1 and p2 keep on moving clockwise. First $\Delta T_{p2}$ increases to a maximum, then also p1 will be
Fig. 7.22 The steady-state lines of a first order oscillator with synchronizing signals that do not initiate transitions

Fig. 7.23 The steady-state lines of a first order oscillator with synchronizing signals of which one initiates a transition

able to initiate a transition. Then an extra time $\Delta T_{p1}$ will be added to the period too. The oscillator will be slowed down more and more as the fast parts $p1$ and $p2$ move clockwise. When the periods of the two oscillators have become equal, the movement of $p1$ and $p2$ stops. Then the oscillators are in lock.

If the maximal increase of the period that $p1$ and $p2$ can establish is not sufficient, lock is impossible. Then the oscillators can be in "quasi-lock" for a several periods, while $p1$ and $p2$ keep on moving clockwise very slowly. Then the lock is lost for some time and $p1$ and $p2$ move clockwise faster until the quasi-lock is obtained again. A coupled oscillator system that is in this mode shows a very "wild" output spectrum. In practice this is observed when the amplitudes of the synchronizing signals are increased from zero to the values for which real locking occurs.

Until now for only one of the two oscillators the effect of the synchronizing signals has been discussed. In the other oscillator, the opposite effect is present. The fast parts of the state diagram caused by the synchronizing signals move counter-clockwise. Via this movement the oscillator is speeded up when there is no lock. This will not be discussed in detail here, because the analysis is very similar to the one described above.
Apparently a control mechanism exists within a coupled oscillator system that is able to equate the period of both oscillators. This mechanism can only succeed if the synchronizing signals are able to establish a sufficient change of the periods of the two oscillators. When the free-running frequencies of the oscillators are equal an exact quadrature coupling is obtained. Then the state diagrams of both oscillators are identical. When the free-running frequencies are not equal the state diagrams differ. When the difference is not too large, all four fast parts remain able to initiate a transition. Then effectively quadrature is obtained. The transitions of one oscillator still coincide with the zero crossings of the integrator signal of the other oscillator.

When the oscillators are tuned, measurements showed that lock is not lost. Hence the coupled oscillator system, referred to in short as coupled oscillator, can be used as a CCO with output signals that have an accurate quadrature relation. This phase relation is independent of the operating frequency of the coupled oscillator. Both quadrature signals have identical properties. Advantageous properties of normal first order oscillators, like linearity and a large tuning range, are also present in the coupled oscillator. Certainly a lot of theory can (and should) be developed about the dynamics of the locking mechanism. A detailed discussion of the control loop that can be defined in the coupled oscillator system is, however, beyond the scope of this thesis.

When the difference in free-running frequencies is very large, using the graphical method described above, it can be shown that synchronization between the two oscillators is still possible. Then it follows:

\[ f_{\text{osc1}} = n f_{\text{osc2}} \]  \hspace{1cm} (7.37)
\[ n \in N^+ \]  \hspace{1cm} (7.38)

Apart from the fact that more than two fast parts caused by synchronizing signals occur in the steady state diagram of the slowest oscillator, a similar analysis as described above can be applied.

7.7 Electronic implementations

7.7.1 A low-noise 100-MHz balanced regenerative oscillator

In this section a fully balanced architecture for high-frequency, low-noise regenerative oscillators will be discussed. Differential operation is achieved with the use of two grounded capacitors. The oscillator circuit has been realized in a high-frequency \( f_T = 3 \)GHz} bipolar process. The tuning range extends to 150MHz. At an oscillation frequency of 115MHz, measured phase noise was -118dBc/Hz at 1MHz distance from the carrier. The design has been published in [17].

A problem in traditional regenerative oscillators, based on the emitter-coupled oscillator, is the presence of parasitic capacitors \( C_p \) at each side of the floating timing capacitor. These parasitic capacitors are not used as a part of the timing capacitor \( C_t \). The timing capacitor has to be large compared to the parasitic capacitors in order reduce their influence on the timing in the oscillator. In the design of high-frequency oscillators with small on-chip timing capacitors, this it not possible. Integrated capacitors have relatively large parasitic capacitors to the substrate. For high-frequency oscillators a topology that uses the parasitic capacitors as a part of the timing capacitors is therefore preferable [20, 23].

It has been stated before that the emitter-coupled oscillator is not really a balanced regenerative oscillator. In section 7.5.4 a method to obtain full balancing has been suggested. The absence of a floating capacitor and the possibility to use much smaller timing capacitors are important.
Fig. 7.24 An emitter-coupled oscillator with parasites

advantages of such an oscillator topology. Another result is a reduction of the sensitivity for power supply variations.

The system set-up

The system set-up has been depicted in fig.7.25. The level comparison function is performed by separate circuits. The memory function is performed by a set-reset flip-flop. The flip-flop is bypassed by the outputs of the comparators. The comparators directly operate the current switch. The only demand made on the speed of the flip-flop is that it takes over the control over the switch before the comparator output signal that initiated the switching disappears.

Fig. 7.25 The system set-up of the oscillator
The removal of the flip-flop from the signal path is favorable for both high-speed operation (parallel processing) and for low-noise operation due to an increased slope of the external excitation, and the exclusion of the timing of the flip-flop from the timing of the complete oscillator. For very high-frequency oscillators the slope of the external excitation of the flip-flop can hardly be increased anymore by the comparators. Then bypassing the memory is the only way to further decrease the influence of the flip-flop noise.

In order to reduce the sensitivity to power supply variations a fully differential architecture is desirable. Fig. 7.25 shows a diagram of the balanced version of the basic circuit of fig. 7.24. When \( C_1 = C_2 = C \), the output frequency of the balanced oscillator is:

\[
fo = \frac{I}{2U_{hys}C}
\]  

(7.39)

Threshold levels in the balanced architecture are floating. The voltages at \( C_1 \) and \( C_2 \) are balanced with respect to \( U_{ref} \). A difference between \( U_{ref} \) and the common-mode signal \( (U_1 + U_2)/2 \) results in an adjustment of the discharging current source such that the difference is reduced. The common-mode feedback loop establishes equality of the sum of the charging currents \( I \) sources and the discharging source \( 2I \).

When the voltage difference \( U_1 - U_2 \) is equal to \( U_{hys} \), one of the voltage comparators switches. Its output signal excites the set-reset flip-flop and switches the current sources. As a result opposite triangular waveforms are found at \( C_1 \) and \( C_2 \).

An essential feature of this architecture compared to many other differential regenerative oscillators [24] are the bypassed memory function and the common-mode feedback loop. The use of this common-mode feedback is the reason why grounded capacitors can still be used in a differential architecture.

The electronic circuit

The circuit diagram of the oscillator based on the system set-up described above has been depicted in fig. 7.26. It was intended to be used in a phase-lock loop in a frequency range of 100MHz to 130MHz. In order to make the PLL designable, some demands on the linearity of the oscillator were made.

The voltages at the timing capacitors are level-shifted by \( Q2a \)–\( Q3a \) and \( Q2b \)–\( Q3b \). The threshold \( U_{hys} \) voltage is set by the diodes \( Q4a \) and \( Q4b \), so \( U_{hys} \) is about 0.7V. The differential stages \( Q5a \)–\( Q5b \) and \( Q6a \)–\( Q6b \) are used as voltage comparators. The output currents of these stages are transformed into pulse voltages by means of \( R2a \) and \( R2b \) respectively. These pulse voltages switch a next differential stage (\( Q1a \)–\( Q1b \)), which is the implementation of the current switch. They also set and reset the set-reset flip-flop (\( Q7a \)–\( Q7b \)). The waveform of the differential voltage \( V_s \) across the current switch has been depicted in fig. 7.27. It is the result of a SPICE simulation, but it can also be measured. It can be seen that \( V_s \) quickly rises after one of the voltage comparators switches on. When the comparator switches off again, \( V_s \) decreases to a level determined by the flip-flop. This level has been chosen large enough to keep the current switch \( Q1a \)–\( Q1b \) in the right position. The voltage \( V_s \) across the current switch is also used to drive an output buffer.

The common-mode loop

The adder that is needed to obtain the common-mode voltage of the two timing capacitors is implemented with two resistors (\( R1a \) and \( R1b \)). The common-mode signal is level-shifted with
Fig. 7.26 The circuit diagram of a balanced regenerative oscillator

$Q_{12}, Q_{13}, Q_{14}$. Then it is transformed into a current by means of a series stage ($Q_{16}$ and $R3$). Neglecting high-frequency poles, the open loop common-mode transfer function $G(p)$ is:

$$H(p) = \frac{1}{2pR_3C} \quad (7.40)$$

If $C = 2.5$ pF (inclusive of the parasitic capacitors) and $R3 = 400\Omega$ the common-mode bandwidth is about 80MHz. Simulations show that other poles in the common-mode loop seem to be non-dominant, so the stability of the common-mode loop is assured.

Instead of fixing the common-mode voltage to a fixed reference voltage $U_{ref}$, the sum of the voltages of $Q_{12}, Q_{13}, Q_{14}, Q_{16}$ and the voltage across $R3$ determines the common-mode voltage. Since the voltage across $R3$ and the tuning current $I$ change proportionally, the common-mode voltage also depends on the tuning current. This construction of the common-mode loop saves an extra voltage reference and avoids the use of lateral PNP-transistors in the loop. Low-frequency poles due to these PNP-transistors in the common-mode loop could have causes stability problems.

The voltage $U_r$ has to be approximately equal to the emitter voltage of $Q_{12}$ in order to avoid
Fig. 7.27 The voltage $V_s$ as a function of time

saturation of either the voltage comparator or the current switch. This can be established easily by means of the insertion of a transistor between the base of $Q12$ and $U_r$. In a later version of this oscillator this method has been successfully applied.

**Experimental results**

The phase noise spectrum was measured, using special phase noise measurement equipment, at carrier frequency of 115MHz. The absolute value of the phase noise is $-118\, \text{dBc}$ at 1MHz distance from the carrier. Because low-noise current sources were used, there is a dominating influence of the sampled high-frequency voltage noise. Analysis of the oscillator circuit reveals that there are effectively seven transistors in series with the switching threshold. The total equivalent noise resistance $R_n$ is 2300Ω, mainly due to the base noise resistance ($r_b = 3000\Omega$) of the transistors. The noise conversion bandwidth is about 400MHz. Because of the switching time delay the effective value of $U_{hp}$ at $f_0 = 115$MHz is 1V, which is somewhat larger than the 0.7V for low-frequency operation. Using (6.135) and (6.142) a value of $-120\, \text{dBc}$ is found for $f_m = 1$MHz and $f_0 = 115$MHz. This value differs only 2dB from the measured phase noise.

### 7.7.2 A coupled oscillator

An electronic version of a coupled oscillator system has been integrated in a bipolar IC-process ($f_T=3$GHz). It consists of two emitter-coupled oscillators. Two extra differential pairs are used to implement the comparators that detect the zero crossings of the integrator signal. The circuit diagram has been depicted in fig.7.28. For simplicity the current sources have been represented by the symbol for ideal current sources. In reality they were implemented as transistor-resistor combinations. The emitter-coupled oscillators have a resistive load, so in principle they do not operate as CCO's. The bias currents of the comparators also flow...
through these resistors. Apart from the fact that in this way they establish the coupling, they also influence the hysteresis. By modulating the bias currents of the comparators, the hysteresis and consequently the period of the coupled oscillator can be modulated. The circuit should be seen as a first test of the feasibility of this type of coupled oscillator, but already some very interesting results were obtained. The maximum operating frequency of the oscillator was found to be about 380MHz. At this frequency the period of the oscillator could still be modulated. The optimal operating frequency at which the frequency stability was measured to be the best was about 200MHz. At that frequency the carrier-to-noise ratio measured at a distance of 1MHz from the carrier was about $119_{\text{dBc}}^{1}$Hz. At this frequency not too much high-frequency noise is folded back around the carrier, and the relative influence of jitter of the transitions is not too large compared to the length of the period. The oscillator is relatively simple compared to other oscillators that are able to supply quadrature signals. The oscillator can also be classified as a two integrator oscillator (chapter 3), and is probably one of the most simple implementations of these. Quadrature can also be obtained with a single oscillator, using frequency dividers. This option generally consumes more power than the coupled oscillator, which generates high quality quadrature on the actual operating frequency. A coupled oscillator has already been implemented in an experimental radio system for this purpose[29]. A patent is pending for this type of oscillator.
8. The first order two-integrator oscillator

In chapter 2 two-integrator oscillators have been introduced. Two different types have been distinguished. Both types will be discussed in this chapter. The first type is the oscillator with a sample and hold circuit as binary memory. Only the principle of this oscillator type will be discussed along with but a few of its properties. The second type to be discussed is the symmetrical two-integrator oscillator, the properties of which will be discussed in more detail. It is true, especially when a gain-cell is used in the sample and hold circuit, that the distinction between sample and hold oscillators and symmetrical two-integrator oscillators is rather vague. It merely depends on the way the integration constants for the two integrators are generated. In the case of the symmetrical two-integrator oscillator they are both well determined, whereas in the case of the sample and hold oscillator one of the integration constants is controlled by the properties of the sampling part of the sample and hold circuit. This integration constant is usually not well known.

In symmetrical two-integrator oscillators the amplitude is dependent on the starting values of the integrators. Due to all kinds of non-idealities, the starting values are eventually lost. An amplitude stabilization circuit has to be added to the two-integrator oscillator in order to obtain stable oscillation. Different amplitude stabilization circuits will be discussed in this chapter. An important feature of symmetrical two-integrator oscillators is the fact that they generate two identical output signals with an accurate quadrature relation. The generation of quadrature signals with a two-integrator oscillator can be much more profitable than the usual way of incorporating an oscillator on the double frequency and frequency dividers, when power consumption and $f_T$ requirements of the components are concerned.

8.1 A first order oscillator with a sample and hold memory

Using the binary memories described chapter 5, first order oscillators can be constructed. An example of such a first order oscillator has been depicted in fig.8.1 It makes use of the sample and hold memory containing a gain-cell, as shown in fig.5.4. Some signals appearing in the oscillator are depicted in fig.8.2.

The first order oscillator containing a sample and hold memory belongs to the two-integrator
oscillator class. A closer inspection of the sample and hold circuit reveals that it is actually an integrator. The output signal of an integrator depends on its past. The actual input signal only determines the time derivative of the output signal, but not its absolute value. In the sample and hold circuit the input signal for the integrator is repeatedly changed from zero (storage mode) to a certain non-zero value (sample mode) which is usually not a constant. The sampling circuit in front of the hold capacitor is designed to adapt its output signal—which is the integrator input signal—such that the voltage at the hold capacitor becomes a copy of the input signal of the sampling circuit. The voltage at a capacitor can be changed to a certain value by supplying a current to it, with the right value for the right duration. The capacitor integrates this current. Mostly the design aim is a sampling time that is as short as possible. This requires a current that is as high as possible. Usually a sample and hold circuit is not looked upon as an integrator, but in this case it is essential. When the charging current is limited, the sampling interval will have a finite length in which the voltage at the hold capacitor linearly changes with time. The effect of this will be discussed with the use of the first-order oscillator of fig. 8.1.

When the voltage at the timing capacitor reaches the threshold level of one of the comparators, the gain cell becomes activated. When it has a large gain, but can supply only a current \( I_{\text{max}} \) at its output at maximum, it will act as a current source that starts charging the hold capacitor. Suppose the previous threshold level that was crossed was equal to \( U_{\text{ref}} \). Then the voltage at the hold capacitor \( C_s \) is equal to \( U_{\text{ref}} \). Comparator C3 will switch when the voltage at the hold capacitor becomes equal to \( U_{\text{ref}} \). The time this voltage needs to reach the value \( U_{\text{ref}} \) is equal to:

\[
\Delta T = \frac{(U_{\text{ref}} - U_{\text{ref}})C_s}{I_{\text{max}}} \tag{8.1}
\]

During this time interval the voltage at the timing capacitor keeps increasing. Instead of changing direction as soon as the voltage reaches the threshold level, some overshoot occurs. When finally C3 switches after a time \( \Delta T \), the direction of the voltage at the timing capacitor changes. This voltage will need another time again equal to \( \Delta T \) to get back to the threshold level. For a time equal to \( 2\Delta T \) an overshoot occurs, and all this time the gain cell remains activated. The value the voltage at the hold capacitor reaches within this time, determines
the length of the overshoot interval at the next crossing of a threshold level. During steady state oscillation the overshoot at both threshold levels will stabilize to a constant value. When \( U_{\text{ref}} \) is exactly between the two threshold levels, the overshoots at both threshold levels will be equal.

The overshoot deteriorates the designability of the output frequency of the first order oscillator, since generally the parameters that determine the overshoot are not well known. Then the system parameters have to be designed such that the overshoot becomes minimized.

If all parameters in (8.1) are well known, it is possible to generate an accurate output frequency, even when \( \Delta T \) is not small compared to the period. Especially in the case of a gain cell, the parameters of (8.1) can be well determined. When the oscillator is used as a VCO, the length of \( \Delta T \) can be modulated also, to prevent a non-linear tuning curve due to the delay. The peak to peak voltage across the timing capacitor \( C_t \) is:

\[
U_{\text{pp}} = (U_{rh} - U_{rl}) + 2 \frac{I_t}{C_t} \Delta T
\]

(8.2)

\( I_t \) is the magnitude of the current that charges and discharges the timing capacitor. The period \( T_o \) of the oscillator is equal to:

\[
T_o = 2 \frac{U_{\text{pp}} C_t}{I_t} = 2 \frac{(U_{rh} - U_{rl}) C_t}{I_t} + 4 \Delta T
\]

(8.3)

\[
= 2 \frac{(U_{rh} - U_{rl}) C_t}{I_t} + 4 \frac{(U_{\text{ref}} - U_{rl}) C_s}{I_{\text{max}}}
\]

(8.5)

When the oscillator is to be frequency modulated with a signal \( m(t) \), this can be done by modulating both currents:

\[
I_t = I_{t0} (1 + m(t))
\]

(8.6)

\[
I_{\text{max}} = I_{\text{max}0} (1 + m(t))
\]

(8.7)

In that case the effective period \( T_{\text{eff}} \) is equal to:

\[
T_{\text{eff}} = T_o + \frac{(U_{rh} - U_{rl}) C_t}{m(t) I_t} + 2 \frac{(U_{\text{ref}} - U_{rl}) C_s}{m(t + \Delta t_1) I_{\text{max}}} + \frac{(U_{rh} - U_{rl}) C_t}{m(t + \Delta t_2) I_t} + 2 \frac{(U_{\text{ref}} - U_{rl}) C_s}{m(t + \Delta t_3) I_{\text{max}}}
\]

(8.8)

The period is composed of four smaller time intervals. The duration of these time intervals is not influenced by \( m(t) \) at the same time. This has been expressed with the delay times \( \Delta t_{1 \ldots 3} \) in the arguments of the various instances of \( m(t) \) in (8.8). The delay times give the time that has passed between the start of the period and the start of the interval concerned.

When \( m(t) \) does not change significantly within a period of the oscillator signal, \( \Delta t_{1 \ldots 3} \) can be assumed equal to zero. Then the associativity theorem can be applied on the factor \( m(t) \) that has become a common factor now. In that case the frequency is equal to:

\[
f = \frac{(1 + m(t))}{2 \frac{(U_{rh} - U_{rl}) C_t}{I_t} + 4 \frac{(U_{\text{ref}} - U_{rl}) C_s}{I_{\text{max}}}}
\]

(8.9)

Then the delay does not cause non-linearity. It is not difficult to generate two currents with the properties of (8.6) and (8.7). When a gain cell is used in the sample and hold circuit is not difficult to charge the hold capacitor with the desired current also. From (8.9) it can be seen that inaccuracy of \( C_s \) affects the accuracy if the output frequency, but not the linearity of the VCO.

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In a regenerative memory it is not that easy to control the delay, as it has been shown in chapter 4. In applications that make use of a minimally excited first order oscillator, that consequently shows a large delay, it may be better to use a sample and hold oscillator.

The assumption that $\Delta t_{1.3} = 0$ is only really correct when $m(t)$ is a constant. Only for a DC input signal the linearity of the CCO is not affected by $\Delta T$. When $m(t)$ is time variant, the distortion increases with increasing frequency. The static distortion due to $\Delta T$ can be eliminate completely, but still some dynamic distortion remains. It is however smaller, than the static distortion that $\Delta T$ causes when it is not modulated with $m(t)$.

It has been stated before that actually the sample and hold oscillator is a two-integrator oscillator. The integration constant of one of the integrators (the hold capacitor) is manipulated by an additional circuit (the sampling part). Due to this manipulation a stabilization of the amplitude is obtained. Hence the sample and hold oscillator can be seen as a two-integrator oscillator with a rather exotic type of amplitude stabilization.

### 8.1.1 The effect of losses in the sample and hold circuit

Due to leakage effects the voltage at the hold capacitor is not constant any more. The droop causes the time the sampling part needs to generate a certain voltage at the hold capacitor to change. As a result the value of $\Delta T$ is changed. In (8.1) $U_{el}$ is replaced by another voltage, that changes with time due to the droop. The starting value of this voltage is $U_{el}$. When the period of the oscillator is large, the droop is large, so the influence on $\Delta T$ will be large. Still, losses do not fundamentally prevent the use of the non regenerative memories, they only impose some restrictions on the timing.

### 8.1.2 A practical implementation of a sample and hold oscillator

In fig.8.3 a complete first order oscillator circuit diagram has been depicted, build with standard integrated circuits\(^1\). The CA3080 is an OTA, of which the gain can be controlled via the bias input (pin 5). In this oscillator, it is used as the input for the sample pulse. The LM3900 is a voltage comparator with an open collector output, so the summing of the output signals is achieved easily. The two PNP transistors form a current mirror that makes the output signal of the comparators suitable for the bias input of the OTA. At the output of the sample and

---

\(^1\)At the moment this section was written a dedicated integrated circuit was not available yet.
hold circuit a buffer is used. It is a limiter, so only the sign of the signal on the hold capacitor is found at its output, but that is sufficient. The output signal of this limiter is used as input signal for a second OTA. Because of the limiting, it acts as a current switch. At the output of this OTA a current is available that is equal in magnitude to the current flowing into the bias input. The direction of the output current depends on the input voltage. For proper operation of the OTA, the output signal of the limiter is attenuated somewhat before it is applied to the input of the OTA. The timing capacitor of the first order oscillator is charged and discharged by the output current of the OTA. The voltage across the timing capacitor is used as input for the gain cell and the two comparators again. The threshold levels can be derived from the supply voltages with voltage dividers. A practical implementation was designed to produce an output frequency of about 5kHz. The measurement results were as predicted by the theory. This implementation should be seen as the start of a larger investigation of oscillators of this type. Later fully integrated high frequency oscillators are expected to be tested.

8.2 The symmetrical two-integrator oscillator

In this section the symmetrical two-integrator oscillator will be discussed. The basic operation has already been discussed in section 2.5.2. Only at the start of this section some attention will be paid to the situation in which the two-integrator oscillator is not symmetrical.

8.2.1 The output frequency and the amplitudes of the integrator signals

In a two-integrator oscillator the value of the output signal of one of the integrators determines the sign of the integration constant of the other and vice versa. When the output signal reaches a reference level, the sign of the corresponding integration constant has to be changed. So by some means the reaching of the reference level has to be detected. It can be done by two comparators. In the oscillator depicted in fig. 8.4 the comparators $comp_1$ and $comp_2$, are used to detect the reaching of the reference levels, of the two integrators, $int_1$ and $int_2$. In this case

![Diagram of a two-integrator oscillator](image)

Fig. 8.4 A two-integrator oscillator

the comparators detect the zero crossings of the integrator output signals. Suppose $int_2$ starts with an output signal equal to zero, and $int_1$ starts with an accurately known starting value ($\bar{E}$). The sign of the integration constant of $int_1$ is assumed to be such that its output signal changes towards zero.

When the zero level is reached, this is detected by $comp_2$, that changes the sign of the integration constant of $int_2$. Since both the starting value and the integration constant are known accurately, the time $int_1$ needs to change its output signal from the starting value to zero is accurately known also.

During this time $int_2$ has integrated from zero to a value that depends on its integration con-
stant, so this value is accurately known also. From now the behavior of the system continues in a similar way as described above, only now int1 starts at zero and int2 starts with a certain non-zero starting value.

### 8.2.2 The period of the two-integrator oscillator

The system of figure 8.4 is described by:

\[
\begin{align*}
E_{o1}(t) &= \text{sgn}(E_{o2}(t))\alpha_1 t + \dot{E}_1 \\
E_{o2}(t) &= -\text{sgn}(E_{o1}(t))\alpha_2 t + \dot{E}_2
\end{align*}
\]  

(8.10)  (8.11)

With:

\[
\text{sgn}(x) = \begin{cases} 
1 & \text{for } x > 0 \\
0 & \text{for } x = 0 \\
-1 & \text{for } x < 0 
\end{cases}
\]

and:

\[
\begin{align*}
\alpha_1 &> 0 \\
\alpha_2 &> 0 \\
\dot{E}_1 &= \dot{E} > 0 \\
\dot{E}_2 &= 0
\end{align*}
\]  

(8.12)  (8.13)  (8.14)  (8.15)

\(\dot{E}_1\) and \(\dot{E}_2\) are the starting values of the integrators and \(\alpha_1\) and \(\alpha_2\) are the integration constants. For a symmetrical two-integrator oscillator \(\alpha_1\) and \(\alpha_2\) are equal. This system behaves like an oscillator. The initial conditions at \(t = 0\) are:

\[
\begin{align*}
E_{o1}(0) &= \dot{E} \\
E_{o2}(0) &= 0 \\
\frac{dE_{o2}(0)}{dt} &= -\alpha_2
\end{align*}
\]  

(8.16)  (8.17)  (8.18)

And when \(t > 0\) then:

\[
\frac{dE_{o1}(0)}{dt} = -\alpha_1
\]  

(8.19)

Since \(\dot{E} > 0\) \(E_{o1}\) changes towards zero. Suppose the time needed to reach zero is called \(T_A\). Then:

\[
T_A = \frac{\dot{E}}{\alpha_1}
\]  

(8.20)

And the output signal of int2 at \(t = T_A\) is:

\[
E_{o2}(T_A) = -\frac{\alpha_2}{\alpha_1}\dot{E}
\]  

(8.21)

Now time interval \(T_B\) is supposed to start. During this interval the time derivatives of the output signals are:

\[
\begin{align*}
\frac{dE_{o1}(t)}{dt} &= -\alpha_1 \\
\frac{dE_{o2}(t)}{dt} &= \alpha_2
\end{align*}
\]  

(8.22)  (8.23)
Within $T_B$ $E_{o2}$ changes towards zero. So:

$$E_{o2}(T_A + T_B) = E_{o2}(T_A) + E_{o2}(T_B) = 0 \quad (8.24)$$

With (8.21) it follows:

$$T_B = \frac{\dot{E}}{\alpha_1} \quad (8.25)$$

From (8.11) and (8.25) it follows:

$$E_{o1}(T_A + T_B) = -\dot{E} \quad (8.26)$$

Apart from the sign of $E_{o1}(T_A + T_B)$, this state of the system is equal to that at $t = 0$. So for the time interval $T_C$ that starts now, a behavior as found for $T_A$ will be found, and for $T_D$ following $T_C$ a behavior as found for $T_B$ can be expected:

$$E_{o1}(T_A + T_B + T_C) = 0 \quad (8.27)$$

$$E_{o2}(T_A + T_B + T_C) = \frac{\alpha_2}{\alpha_1} \dot{E} \quad (8.28)$$

$$E_{o1}(T_A + T_B + T_C + T_D) = \dot{E} \quad (8.29)$$

$$E_{o2}(T_A + T_B + T_C + T_D) = 0 \quad (8.30)$$

At this point of time ($t = T_A + T_B + T_C + T_D$) the situation is identical to that of $t = 0$. Apparently the signals $E_{o1}$ and $E_{o2}$ are periodical with a period:

$$T = T_A + T_B + T_C + T_D = 4 \frac{\dot{E}}{\alpha_1} \quad (8.31)$$

The amplitudes of the two signals are equal to:

$$\dot{E}_{A1} = \dot{E} \quad (8.32)$$

$$\dot{E}_{A2} = \frac{\alpha_2}{\alpha_1} \dot{E} \quad (8.33)$$

The amplitudes and the period depend on the starting value of int1.

In order to produce an accurate period (frequency) it is necessary that the starting value is preserved during oscillation.

The two integrators can be seen as memories between which the starting value is exchanged periodically.

### 8.2.3 Two starting values unequal to zero

When both integrators have a starting value unequal to zero, with:

$$\dot{E}_1 > 0 \quad (8.34)$$

$$\dot{E}_2 > 0 \quad (8.35)$$

the situation at $t = 0$ is:

$$E_{o1}(0) = \dot{E}_1 \quad (8.36)$$

$$E_{o2}(0) = \dot{E}_2 \quad (8.37)$$

$$\frac{dE_{o1}(0)}{dt} = \alpha_1 \quad (8.38)$$

$$\frac{dE_{o2}(0)}{dt} = -\alpha_2 \quad (8.39)$$
Hence \( E_{o2} \) will change towards zero. After a time \( T_S \), \( E_{o2}(T_S) \) becomes equal to zero. Then it follows:

\[
T_S = \frac{\hat{E}_2}{\alpha_2} \tag{8.40}
\]

\[
E_{o1} = \frac{\alpha_1}{\alpha_2} \hat{E}_2 + \hat{E}_1 \tag{8.41}
\]

When a new constant \( \hat{E} \) is defined with:

\[
\hat{E} = \frac{\alpha_1}{\alpha_2} \hat{E}_2 + \hat{E}_1, \tag{8.42}
\]

and a new time is defined with:

\[
t_{new} = t_{old} - T_S, \tag{8.43}
\]

then starting from \( t_{new} = 0 \), the same situation exists as described in the previous paragraph. (\( int1 \) starting at \( \hat{E} \) and \( int2 \) starting at zero)

So with (8.42) substituted in (8.31), (8.32), and (8.33), the expressions for the amplitudes and the period is found:

\[
T = 4 \left( \frac{\hat{E}_1}{\alpha_1} + \frac{\hat{E}_2}{\alpha_2} \right) \tag{8.44}
\]

\[
\dot{E}_{A1} = \frac{\alpha_1}{\alpha_2} \hat{E}_2 + \hat{E}_1 \tag{8.45}
\]

\[
\dot{E}_{A2} = \frac{\alpha_2}{\alpha_1} \hat{E}_1 + \hat{E}_2 \tag{8.46}
\]

Again, the accuracy of the generated periodical signal depends on the quality of the preservation of the starting value \( \hat{E} \).

In a symmetrical two-integrator oscillator, both integrators, both comparators and both integration constants are equal:

\[
\alpha_1 = \alpha_2 = \alpha \tag{8.47}
\]

From now on only symmetrical two-integrator oscillators will be discussed.

### 8.3 Design considerations

The only real difference between two-integrator oscillators and regenerative oscillators, is the type of memory that is used. The principle of operation as a first order oscillator is identical for both types of oscillators. Therefore the general design considerations for regenerative oscillators, discussed in section 7.1 also apply to two-integrator oscillators.

The differences between the various memory types, do not result in different design rules. Only the measure of influence of a certain parameter can differ. This is why generally a decision for either a regenerative first order oscillator or a two-integrator oscillator is made on the system level. Various properties of two-integrator oscillators tend to be more desirable than their equivalents in regenerative oscillators. An example of this is the difference between the switching speeds of a regenerative memory and a sample and hold memory. The first tends to be very fast, but somewhat unpredictable and uncontrollable, whereas the latter is not that fast, but rather accurately to predict, and to control. A compensation for the effects of the sample and hold memory is therefore much more feasible than it is for a regenerative memory. An example of this compensation has been given in section 8.1.
8.4 The phase diagram

The output signals $E_{o1}(t)$ and $E_{o2}(t)$ of the integrators are in quadrature. In a phase diagram as depicted in fig. 8.5 this phase relation can be observed. In the diagram $E_{o1}(t)$ is plotted along the X-axis and $E_{o2}(t)$ is plotted along the Y-axis. For a symmetrical two-integrator

Fig. 8.5 A phase diagram

oscillator the phase diagram is a rectangle. The sum of the absolute values of $E_{o1}(t)$ and $E_{o2}(t)$ is constant, which can be seen in the figure. So:

$$|E_{o1}(t)| + |E_{o2}(t)| = \hat{E}_A$$

(8.48)

$\hat{E}_A$ is the sum of the absolute values of the starting values of the two integrators. It is also equal to the amplitudes of the output signal of either integrator. From (8.48) it can be seen that the sum of the two time varying signals is constant. This property will be used later to make an amplitude detector.

The operating point of the oscillator can be defined as a point on the rectangular phase diagram. This operating point moves along the diagram with a speed that is determined by the integration constants. The direction of the change has been indicated with arrows. The speed is not determined by the starting values. In fig. 8.6 two phase diagrams have been depicted. Since in both cases the operating point moves at equal speed, the time it takes to

Fig. 8.6 Two phase diagrams for different starting values

complete one round is longer for the right hand situation than it is for the left hand one. Hence the period is larger in the right hand situation.

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8.5 The effects of offsets

The comparators may show offsets equal to \(dE_1\) and \(dE_2\) at their respective inputs. In that case they will not switch at the zero crossing of their input signals but when the input signals are at levels \(-dE_1\) and \(-dE_2\) respectively. Since the integration constants are not changed, the slopes of the sides of the rectangle in the phase diagram will not change either. Therefore the complete phase diagram is just shifted with an amount equal to the offsets. In fig.8.7 this has been made more clear. The offsets do not disturb the operation of the two-integrator oscillator. Only now the sum of the absolute values of the two integrator output signals given by (8.48), is no longer constant. This causes problems when the sum is used for amplitude detection. Then a ripple is found at the output of the detector. Also the output frequency is not solely determined by the starting values of the integrators anymore. The offsets have to be taken into account to find the exact output frequency.

8.6 The two-integrator oscillator with non ideal integrators

In the oscillator system the value of a constant (8.42) has to be stored 'for ever'. When the integrators have a real part in their transfer function, they loose a part of the information that is stored. First this effect will be shown for one single integrator.

8.6.1 An integrator with losses

In figure 8.8 an integrator with losses has been depicted, consisting of and ideal integrator having an integration constant \(c\) and a real transfer \(G\), in a feedback loop. When the integrator has a starting value unequal to zero, this can be simulated in the Laplace domain by way of an extra signal \(E(0^-)\) inserted in the system. When this is done the system is at rest for \(t < 0\), thus allowing the application of the usual Laplace operations [6]. When the starting value is equal to \(\dot{E}\), it follows:

\[
E(0^-) = \dot{E}z(t)
\]  

(8.49)
Fig. 8.8 Integrator with losses and a starting value

e(t) is the step function.
In the Laplace domain the expression for the output signal of the integrator is:

$$E_o(p) = \frac{\alpha}{p} \left( E_i(p) - GE_o(p) \right) + E(0^-)$$  \hspace{1cm} (8.50)

When $E_i(p)$ is a step function, and with (8.49) it follows:

$$E_o(p) = \frac{\alpha}{p} \left( \frac{1}{p} - GE_o(p) \right) + \frac{\dot{E}}{p}$$  \hspace{1cm} (8.51)

When a time constant is defined with:

$$\tau_i = \frac{1}{\alpha G}$$  \hspace{1cm} (8.52)

The expression for the output signal of the integrator becomes:

$$E_o(p) = \frac{\alpha}{p} \left( \frac{1}{p + \tau_i^{-1}} \right) + \frac{\dot{E}}{p + \tau_i^{-1}}$$  \hspace{1cm} (8.53)

The corresponding time function is:

$$E_o(t) = \alpha \tau_i \left( 1 - e^{-\frac{t}{\tau_i}} \right) + \dot{E} e^{-\frac{t}{\tau_i}}$$  \hspace{1cm} (8.54)

It can be seen that the contribution of the starting value $\dot{E}$ to $E_o(t)$ decreases exponentially with a time constant $\tau_i$, independent of the input signal.

When the exchange of the information introduces no losses in the oscillator, and the two integrators have equal integration constants and time constants, the memory function for the starting value is identical to that of one single integrator. In that case the starting value is lost exponentially, with a time constant $\tau_i$.

Now with (8.31), (8.32), (8.33) and $\alpha_1 = \alpha_2 = \alpha$, the expressions for the amplitude and the period of the lossy two-integrator oscillator become equal to:

$$T = \frac{4\dot{E}}{\alpha} e^{-\frac{t}{\tau_i}}$$  \hspace{1cm} (8.55)

$$A = \dot{E} e^{-\frac{t}{\tau_i}}$$  \hspace{1cm} (8.56)

When losses are present both the amplitude and the period are affected. The phase diagram is not closed anymore. The trajectory of the operating point is bent towards the center of the diagram. In fig.8.9 the phase diagram of a two-integrator oscillator with losses has been depicted. For comparison two dotted straight lines have been inserted in the figure to indicated the trajectory of the operating point when no losses would be present. The further the operating point is away from the center, the more the abbreviation from the straight line will be.
Fig. 8.9 The phase diagram for a two-integrator oscillator with losses

8.7 The effect of delay in the comparators

When the comparators have a delay, they establish a change of sign of the integration constant somewhat later than the event of the zero crossing of their input signal. In fig.8.10 the effect of the delayed switching of the integration constant can be seen. In stead of changing direction

Fig. 8.10 The phase diagram for a two-integrator oscillator with comparators that have delay

on the axis, the phase curve always changes direction somewhat beyond the axis. As a result the phase diagram is not closed any more. The amplitudes and the period of the output signals of the integrators increase. The sides of the curve remain straight.

Usually both delay and losses are present in a two-integrator oscillator, so when no further amplitude stabilization is present, due to these effects still eventually a stable oscillation will be reached. This is because the effect of the losses increases with the amplitude, whereas the increasing effect of the delay is independent of the amplitudes. For small amplitudes the delay dominates the behavior, so the amplitudes increase. Then the losses become larger also.
8.7.1 The stationary amplitude caused by the comparator delay and the integrator losses

According to (8.52), the losses of the integrators can be described with a time constant $\tau_i$. The integration constants are both equal to $\alpha$, and the comparators are supposed to have a delay equal to $t_{\text{delay}}$. When a stable amplitude is reached, the phase diagram is closed. In that case the losses introduce a phase shift that exactly compensates for the delay causes by the comparators. Due to the losses the zero crossing of an integrator output signal will come sooner than it would without losses. From (8.54) the time the output signal of a lossy integrator needs to reach zero form a starting value $\hat{E}_A$ can be calculated:

$$t_{\text{zero}} = \tau_i \ln \left( \frac{\hat{E}_A - \alpha \tau_i}{\alpha \tau_i} \right)$$  \hspace{1cm} (8.57)

Now the events in one half of the period of the oscillator will be examined. At the start of the time interval ($t = 0$) one integrator, arbitrarily labeled as $\text{int1}$, is supposed to have the maximal output signal equal to $\hat{E}_A$.

$$E_{\text{o1}}(0) = \hat{E}_A$$  \hspace{1cm} (8.58)

The other integrator, labeled as $\text{int2}$ had a zero crossing a time $t_{\text{delay}}$ before.

$$E_{\text{o2}}(-t_{\text{delay}}) = 0$$  \hspace{1cm} (8.59)

At $t = t_{\text{zero}}$ the output signal of $\text{int1}$ reaches zero. A time $t_{\text{delay}}$ after the zero crossing of $\text{int1}$, $\text{comp2}$ changes its output signal, so then the output signal of $\text{int2}$ is at its maximal or minimal output level.

$$|E_{\text{o2}}(t_{\text{zero}} + t_{\text{delay}})| = \hat{E}_A$$  \hspace{1cm} (8.60)

After a time $t_{\text{zero}}$, the output signal of $\text{int2}$ will reach zero again.

$$E_{\text{o2}}(2t_{\text{zero}} + t_{\text{delay}}) = 0$$  \hspace{1cm} (8.61)

This zero crossing results in a change of sign of the integration constant of $\text{int1}$ after a delay equal to $t_{\text{delay}}$. Then the output signals of $\text{int1}$ is at its minimal value:

$$E_{\text{o1}}(2t_{\text{zero}} + 2t_{\text{delay}}) = -\hat{E}_A$$  \hspace{1cm} (8.62)

Apart from the sign, this is a situation similar to that at $t = 0$. Apparently a half period is equal to:

$$\frac{1}{2}T = 2t_{\text{zero}} + 2t_{\text{delay}}$$  \hspace{1cm} (8.63)

When (8.57) and (8.62) are substituted in (8.54), the value for the stationary amplitude can be calculated:

$$\hat{E}_A = \alpha \tau_i \sqrt{1 - e^{-2t_{\text{delay}}/\tau_i}}$$  \hspace{1cm} (8.64)

When the losses and the delays are well known, the two-integrator oscillator can be designed such that it produces a predictable stable output frequency.

When later an additional amplitude stabilization circuit is added, the effects mentioned above will introduce an amplitude dependent error in the amplitude that is to be established by the stabilization circuit. In a similar way as it is the case in normal negative feedback amplifiers, the amplitude error can be minimized by increasing the loop gain of the amplitude stabilization loop. Then variations of the minimized error are of less importance. The accuracy of the amplitudes is of great importance, since they are linearly related to the period of the output signal of the oscillator. So the amplitude stability also determines the frequency stability.
8.8 Two-integrator oscillators with stabilized amplitudes

When a well determined frequency has to be generated with a two-integrator oscillator with losses and delays, usually the amplitude has to be stabilized with an extra circuit. This is because the losses and the delays, which also set a stable amplitude, are generally not accurate or even designable. In principal, this extra circuit is a memory which has to store the starting value, since the lossy integrators are not suitable for this function. Also a circuit is needed that compensates for the losses and the delays according to this memory value. When the losses are manipulated, also the effects of the delays can be compensated, so from now on only the compensation of the integrator losses will be dealt with explicitly.

Two different ways to compensate the losses will be discussed.

First a method will be discussed in which the losses of the integrators are directly compensated by way of 'undamping' the integrators. This method needs an amplitude detector, to obtain a measure that can be used to control the compensating circuits. In this chapter only one special type of amplitude detector, the sum-moduli detector will be discussed, which has the property that basically no filtering is required at its output [11][12][10]. Other amplitude detectors are not discussed since they belong to the standard electronic circuits.

Secondly, a method will be discussed in which the timing of the change of the signs of the integration constants is influenced by an amplitude stabilization circuit. This method, which incorporates the use of Schmitt triggers, is not discussed in detail in this chapter, because they have already been dealt with in chapter 7. In the classification tree in fig.3.39 it can be seen that this type of oscillator is considered to be a member of both the regenerative oscillator class as the two-integrator oscillator class.

8.9 Undamping the integrators

In this chapter the losses of an integrator have been modeled as a real transfer G in a feedback loop around the integrator (figure 8.8). An extra feedback loop around the integrator with a transfer \( H_c = -G \) would exactly compensate the other loop that causes the losses. Unfortunately the value of \( G \) is not known accurately so it is not possible to solve the problems just by inserting a loop having a constant transfer \(-G\). Also, when an oscillator is started, usually the starting values of both integrators are equal to zero. So these oscillators would never start, since the amplitude would be kept constant at zero.

To reach a desired amplitude the transfer of the compensating loop should have a value that is somewhat more than the transfer of the loop that causes the losses. Effectively a negative loss exists, which causes the amplitude to grow exponentially. When the desired amplitude is reached the transfer has to be reduced until it exactly compensates the losses. Apparently the transfer \( H_c \) of the compensating loop is a function of the amplitude:

\[
H_c = -G[1 + F(\hat{E}_A(t))],
\]

in which the amplitudes are given by:

\[
\hat{E}_A(t) = |E_{o1}(t)| + |E_{o2}(t)|
\]

During steady state oscillation \( \hat{E}_A(t) \) is constant. This expression is comparable with (8.48), which is the expression for the sum of the amplitudes for integrators without losses. In that case the starting value was preserved so the sum was inherently constant. In this case the starting values are lost, so now the sum can be used to find a measure for the losses that can be used to compensate. When the sum is kept constant, the amplitudes and output frequency are constant. To do this, the sum has to be compared with a reference, which can be seen as a memory for the starting values, which replaces the memory function of the integrators.

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When the output wave forms of the integrators are ideal triangular wave forms, the output signal of an amplitude detector that determines $\dot{E}_A(t)$ according to (8.66) is a DC-signal. It is available always and needs no further filtering. This detector will be referred to as sum-moduli detector. When the output signals of the integrators are not purely triangular, a ripple will appear at the output of the amplitude detector. Then extra filtering is necessary and it will take at least one period to measure the amplitude. Very often, due to other design considerations, it takes more time to measure the amplitude.

Ripple due to limited gain of the comparators

When the input signal of a comparator with limited gain approaches the reference level, it starts behaving like a linear amplifier. This happens when the comparator function is implemented as a (high gain) limiter. A limiter always has a transfer as depicted in fig.4.7, so a linear area—which can be made smaller by way of increasing the small-signal gain of the limiter—around the reference level is always present. It is good practice to make the gain of a limiter not too large to prevent noise from being folded back.[7] Therefore around the peaks of the triangular wave forms, the input signal of the integrator concerned is not a constant, but the linearly amplified output ramp of the other integrator. The integrator integrates this ramp so at its output a quadratic waveform is found. It has the form of a parabola, with its axis of symmetry at the zero crossing of the input signal of the integrator. Hence the peaks of the triangular waveforms are rounded off due to the limited gain of the comparators. Then the sum given by (8.66) is not a constant any more. This can be seen also in fig.8.11. Every time the output signal of either one of the integrators reaches a peak value, the sum is reduced somewhat. Every period of the two-integrator oscillator four times a peak value is reached (twice by each integrator), so four times every period a dip in the output signal of the amplitude detector can be expected. Hence a ripple with a fundamental frequency equal to four times the oscillator frequency occurs in the output signal of the detector.

8.10 The undamping network

In order to be able to control the losses in an integrator, an undamping network must be constructed around it. In this section two different undamping networks will be dealt with. First a linear undamping network will be discussed. In this network a linear multiplier is used to obtain an undamping network that can be externally controlled.
Secondly a switching undamping network will be discussed. In this network a switching multiplier is used to obtain an externally controllable undamping network.

At the end of this chapter, an electronic implementation of a two-integrator oscillator, with an amplitude stabilization loop that makes use of undamping circuits, will be presented.

### 8.10.1 The linear undamping network

In the lossy oscillator of figure 8.12 two undamping loops have been constructed around the integrators. In principle it suffices to introduce just one undamping loop around only one of the integrators. The loss of the other integrator will also be compensated by this loop. This can be expected from the fact that only one starting value has to be remembered and corrected, so only one memory is needed. For reasons of symmetry both integrators have been equipped with an undamping loop.

The transfer function of the linear feed back network in this compensating loop is labeled $H_c$. The magnitude of the transfer of this feed back network can be controlled via a control input. When the control signal has a value $E_{control}$, the transfer of the feed back network is:

$$H_c = k_c E_{control} \quad (8.67)$$

In this case it is a real transfer.

The amplitude $\hat{E}_A(t)$, given by (8.66), is time variant due to the losses. The time variant behavior can be described with a differential equation:

$$\frac{d\hat{E}_A(t)}{dt} = -\frac{1}{\tau_i} \hat{E}_A(t) \quad (8.68)$$

During steady state oscillation, which occurs when the losses are exactly compensated, it follows:

$$H_c = -\frac{1}{\tau_i} = [\hat{E}_A(t) - \hat{E}_{ref}]k_c \quad (8.69)$$

The parameter $k_c$ determines the loop gain of the undamping loop. The larger $k_c$, the smaller the amplitude error will be. Substituting (8.69) in (8.68) results in:

$$\frac{d\hat{E}_A(t)}{dt} = k_c[\hat{E}_A(t) - \hat{E}_{ref}]\hat{E}_A(t) \quad (8.70)$$

This differential equation can be used to determine the reaction of the oscillator to changes of $\hat{E}_{ref}$. These changes can be caused by unwanted signals like noise, but they can also be generated intentionally in order to modulate the period of the oscillator.
When the amplitude of the oscillator has stabilized, the amplitude error will be small when $k_c$ is sufficiently large. Then the following approximation can be made:

$$\hat{E}_A(t) \approx \hat{E}_{ref}$$

(8.71)

Using the approximation, (8.70) yields to:

$$\frac{d\hat{E}_A(t)}{dt} = -\frac{1}{\tau_{loop}} [\hat{E}_A(t) - \hat{E}_{ref}]$$

(8.72)

with:

$$\tau_{loop} = -\frac{1}{k_c \hat{E}_{ref}}$$

(8.73)

The solution to this differential equation is:

$$\hat{E}_A(t) = \hat{E}_{ref} \left(1 - e^{-\frac{t}{\tau_{loop}}}\right)$$

(8.74)

During steady state oscillation, the amplitude reacts to a variation of $\hat{E}_{ref}$ according to a time constant $\tau_{loop}$.

During start-up different approximation can be made. Then $\hat{E}_A(t)$ is very small compared to $\hat{E}_{ref}$. Using this property, (8.70) yields to:

$$\frac{d\hat{E}_A(t)}{dt} = -\frac{1}{\tau_{loop}} \hat{E}_A(t)$$

(8.75)

The solution to this differential equation is:

$$\hat{E}_A(t) = \hat{E}_A(0)e^{\frac{t}{\tau_{loop}}}$$

(8.76)

When the oscillator is started, the amplitudes of the integrators are equal to zero, so $\hat{E}_A(0) = 0$. Then from (8.76) it follows that the amplitude remains equal to zero for ever. Apparently some start-up pulse is necessary. Then an exponentially growing amplitude can be expected. In practice noise usually causes the oscillator to start.

### 8.10.2 The switching undamping network

Instead of using the actual output signal of the integrator as input signal for the compensating network with a transfer $H_c$, it is also possible to use just the sign of this output signal. This implies that a comparator is put in front of the undamping network. In fig.8.13 this situation has been depicted. In this case it follows:

$$\frac{dE_{o1}(t)}{dt} = \alpha \text{sgn}(E_{o2}(t)) + H_c \text{sgn}(E_{o1}(t))$$

(8.77)

$$\frac{dE_{o2}(t)}{dt} = -\alpha \text{sgn}(E_{o1}(t)) + H_c \text{sgn}(E_{o2}(t))$$

(8.78)

From (8.66) the time derivative of $\hat{E}_A(t)$ can be derived:

$$\frac{d|E_{o1}(t)|}{dt} + \frac{d|E_{o2}(t)|}{dt} = \frac{d\hat{E}_A(t)}{dt}$$

(8.79)

When (8.77) and (8.78) are substituted in (8.79), it follows:

$$\frac{d\hat{E}_A(t)}{dt} = 2H_c$$

(8.80)
Fig. 8.13 Lossy two-integrator oscillator with a switching compensating loop

The solution to this differential equation is:

$$\dot{E}_A(t) = 2H_c t + \dot{E}_A(0)$$  \hspace{1cm} (8.81)

It looks like the oscillator does not need a start-up pulse when (8.81) is considered. However, this is not true, since at the start both \(E_{o1}(0)\) and \(E_{o2}(0)\) are equal to zero. According to the definition of the \(\text{sgn}\) function, it becomes equal to zero when its argument equals zero. Then from (8.77) and (8.78) it follows that the time derivatives of the amplitudes of the integrator output signals equal zero. These amplitudes started at zero, so they will remain zero. Again a start-up pulse is required.

The value of \(H_c\) is given by:

$$H_c = k_c [\dot{E}_A(t) - \dot{E}_{ref}]$$  \hspace{1cm} (8.82)

When this expression is substituted in (8.80) it follows:

$$\frac{d\dot{E}_A(t)}{dt} = 2k_c [\dot{E}_A(t) - \dot{E}_{ref}]$$  \hspace{1cm} (8.83)

The solution to this differential equation is:

$$\dot{E}_A(t) = (\dot{E}_A(t) - \dot{E}_{ref})e^{-\frac{t}{\tau_{loop}}} + \dot{E}_{ref}$$ \hspace{1cm} (8.84)

$$= \dot{E}_{ref} \left(1 - e^{-\frac{t}{\tau_{loop}}}\right)$$ \hspace{1cm} (8.85)

with:

$$\dot{E}_A(0) = 0$$ \hspace{1cm} (8.86)

$$\tau_{loop} = \frac{1}{2k_c}$$ \hspace{1cm} (8.87)

Also in this case the oscillator reacts to a disturbance according to a time constant \(\tau_{loop}\). Due to the extra \(\text{sgn}\)-block, the time constant has become independent of the amplitude \(\dot{E}_A(t)\). Expression (8.85) can be used to describe both the start-up behavior and the reaction to small disturbances during steady state operation. In the latter case the expression is similar to (8.74), so during steady state operation oscillators with linear and switching undamping circuits show a similar dynamic behavior.
A comparison between the linear and the switching undamping network

In the linear amplitude stabilization loop the time constant of the loop depends on the amplitude. When the period of the oscillator is modulated, the bandwidth of the loop varies with the period. This results in a signal dependent modulation bandwidth of the oscillator. At the worst, the oscillator may be tuned such that the loop becomes unstable. This is not the case for the switching amplitude stabilization loop. However, due to the switching action of the \textit{sgn}-block, noise may be folded back into the frequency range of interest\cite{10}. On the other hand, the linear multiplier in the linear loop may introduce more noise itself than the switching multiplier would do. At this moment it is difficult to determine which one of the loops is the best to use. Also the feasibility of an electronic implementation will play a role.

8.10.3 Tuning the oscillator

When the oscillator is to be tuned by an external circuit, for instance when it is used in a phase locked loop, according to (8.31) two different ways of tuning are possible. Either the amplitudes ($\hat{E}_A(t)$) or the integration constants ($\alpha$) can be changed. When the amplitudes are controlled, the period of the output signal is proportional to the tuning signal. The output frequency is proportional to the tuning signal if the integration constants are used to tune the oscillator. For relatively small frequency deviations, and in applications that only require monotony of the tuning characteristic, both tuning methods can be used.

In many practical situations the first method is the most linear way to tune the oscillator, but in the previous section it has been shown that when $\hat{E}_\text{ref}$ is changed, the frequency cannot change instantaneously. This is caused by the low pass filtering in the compensation loop which has a time constant $\tau_{\text{loop}}$. Even when $\tau_{\text{loop}}$ is small, extra filtering can be present in the loop that is needed to reduce the ripple caused by the distortion of the triangular waveforms. The time constant needed to do this, is at least larger than a quarter of the period of the oscillator output signal. Sometimes the time constant is chosen much larger, due to noise consideration or to ensure stability of the compensation loop. Then the modulation bandwidth can become too small. Since in practice the loop tends to be at least a second order loop, overshoot and other stability problems may occur. Overshoot problems can be avoided when the loop is not excited. In that case this modulation method cannot be used as tuning parameter, since $\hat{E}_{\text{ref}}$ must be kept constant.

It has been shown before that a large output signal of the integrators is favorable for the noise behavior of the oscillator. When the amplitude is changed for period modulation, this demand cannot be fulfilled. For example, the center frequency cannot be chosen such that the amplitude is at maximum.

When the zero crossings are affected with timing jitter, a decrease of the period will increase the effect of the jitter in the frequency stability. This is because then the jitter delivers a relatively larger contribution to the total period of the oscillator. Hence when the jitter is constant, an increase of the output frequency of the oscillator results in a worse frequency stability anyway. However, in this case the jitter is not constant. When the amplitudes are decreased, the jitter becomes larger. The amplitudes are made smaller when the period is made smaller, so also this effect decreases the frequency stability when the output frequency is increased. Apparently two effects that deteriorate the noise performance of the oscillator are present when period modulation is applied to a two-integrator oscillator.

In the second method the integration constants are used as a means to modulate the oscillator. Via the integration constants an instantaneous change of the output frequency can be achieved. This can be seen in fig.8.14. For this method of modulation, amplitudes can be chosen at 173
maximum, which is good for the noise performance of the oscillator. The output frequency of the oscillator can be increased by way of increasing the integration constants. Large integration constants are good for the noise performance. This effect may compensate for the deterioration of the noise performance, caused by the larger effect jitter gets when the period is made smaller. Ideally the amplitudes are independent of the integration constants. However, in practice the parameters that are able to change the integration constants also affect other properties, like the amplitudes, of the circuit. Depending on the implementation, the influence on the various parameters may cause non-linearity of the tuning characteristics or even instability of the amplitude stabilization loop.

Discussion

Two different modulation methods exist for a two-integrator oscillator. The period can be modulated, by way of changing the amplitudes of the integrator output signals, or the frequency can be modulated, by way of changing the integration constants. Both methods have their advantages and disadvantages. The frequency stability may be better when the integration constants are used for modulation, and the linearity of the tuning curve may be better when the amplitudes are used as means of modulation. The particular application or implementation has to be decisive.

8.11 Affecting the sign changes of the integration constant

In the lossless oscillator the sign of the integration constants are changed at the exact moment the corresponding integrator output signal has reached the desired output value. In the case of the lossy oscillator, the integrator has not yet reached this value. An extra circuit can be used to postpone the change of sign until the integrator has reached the desired value. This extra circuit consists of a memory in order to store the starting value, and circuitry that is able to influence the timing of the sign changes of the integration constant. In figure 8.15 the sign of the integration constant of one of the two integrators is controlled by such a circuit. The extra circuit is the basic discrete memory circuit, the Schmitt trigger. It will only change its output signal when both the output signal of the integrator has a predefined value and at the other input the zero crossing of the other integrator is indicated.

Actually, the latter condition is not really true. When the output signal of the integrator raises too much above the desired value, the Schmitt trigger will switch anyway, independent of the status of the other integrator. So this circuit is able to oscillate on it own also. Oscillators
of this type are the most common type of regenerative oscillators. Apparently a "parasitic" regenerative oscillator exists in this two-integrator oscillator.

Usually, for reasons of symmetry, both integrators are equipped with an amplitude stabilizing Schmitt trigger. Then the oscillator can be looked upon as a coupled oscillator system, in which the two parasitic regenerative oscillators are coupled, thus producing two strongly correlated output signals. This coupled oscillator belongs to a general class of coupled oscillators. In this class an arbitrary number of oscillators is coupled to each other, all running on frequencies that are integer multiples of a basic frequency. The two-integrator oscillator as described here is one special case of this class; two oscillators, generating the same frequency. When the coupling is perfect, this oscillator system behaves like the ideal lossless two-integrator oscillator.

In chapter 7 coupled regenerative oscillators and their electronic implementation have been discussed in detail. Therefore in this section no implementations and further discussion will be found.

Tuning the oscillator

Since the amplitude stabilization works instantaneously, the modulation bandwidth is not limited by any filtering action of, for example, an amplitude detector. This may not always be an advantage since a modulation bandwidth that is too large will probably result in more phase jitter than necessary.

8.12 An electronic implementation

A two-integrator oscillator with an amplitude stabilization loop has been integrated on a semi-custom chip. The amplitude detector used, is a sum-moduli detector as described in this chapter.

8.12.1 The basic oscillator

In fig.8.16 the basic oscillator has been depicted. The amplitude stabilization circuit has not been inserted yet. Two differential pairs perform the comparator function. The integrator function is performed by capacitors. For each integrator two capacitors have been placed in series. This is because "on chip" capacitors are used, which have a considerable parasitic capacitance to the substrate at one terminal. In order to reduce the influence of these parasites, the corresponding terminals have been connected to ground. Ideally no signal currents will flow to the substrate via the parasites, since because of the symmetry of the circuit, no AC-signals
are present at the center node between two capacitors. The common mode voltage at the collectors of the differential pairs is controlled by transimpedance amplifiers consisting of two resistors and four PNP-transistors. The integration constants of the integrators are proportional to the current $I_a$, that is supplied to the differential pair via a current mirror. This circuit is able to produce a stable output frequency due to delays and losses, as given by (8.64).

8.12.2 The amplitude detector

In fig.8.17 and implementation of the sum-moduli detector has been depicted. Two differential pairs equipped with extra emitter resistors act as transadmittance amplifiers (T1, T2). They convert the integrator output voltages into currents. These currents are used as input signals for the four current switches (S1...S4). These switches are also controlled by the integrator output voltages, so they work as "rectifiers" for the integrator signals. The output currents of the current switches are a measure for the absolute value of the integrator output signals. Via a current mirror all currents are added. The fact that the current mirror is an inverting current follower is used to add all currents with the right sign. The resulting output current of the detector is the sum of the currents that are a measure for the absolute values of the integrator output signals, so this circuit determines a measure for the amplitude as given by (8.66). This current, labeled as $I_{amp}$ can be used to control the undamping circuit.

8.12.3 The undamping circuit

In fig.8.18 an undamping circuit has been depicted. Across the output terminals of this circuit a negative impedance is established that is proportional to the bias current of the circuit. This bias current is equal to the difference of $I_{amp}$ and $I_{ref}$. The reference value for the amplitude is labeled as $I_{ref}$ because in this implementation it has the dimension of a current. Two of these undamping circuits have been inserted in the two-integrator oscillator of fig.8.16, one for each integrator.

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2In the expressions at the start of the chapter it appears in the expressions as $E_{ref}$.
Fig. 8.17 An amplitude detector of the sum-moduli type

Fig. 8.18 An undamping circuit

8.12.4 The complete oscillator

The basic oscillator circuit with the additional undamping network has been depicted in fig. 8.19. The complete oscillator consists of this circuit combined with the amplitude detector of fig. 8.17.

Spice simulations showed a behavior of the oscillator as expected from the theory. It has also been integrated in a 2GHz bipolar process. The carrier frequency was chosen to be 75MHz. The carrier to noise ratio has been measured at a distance of 1MHz from the carrier. It was about 110dB, which is a common value for regenerative oscillators in this frequency range. Both amplitude and frequency modulation were possible over a wide range. This oscillator should be seen as a first step in the investigation of this type of oscillators. More two-integrator oscillators of this type are intended to be build for a more detailed study. This design was mainly intended to investigate the feasibility of a high frequency two-integrator oscillator with an amplitude stabilization loop. It is expected that, when more effort is put into the design,
Fig. 8.19 A two-integrator oscillator with undamping circuits

the performance will improve.
A. Solutions to the differential equations of the state variable

A.1 The state as a function of time with a step at the input

The system discussed in this section has a constant loop gain, equal to \( A_l \) when the limiter is in the linear mode. It has been described in section 4.4.3. The expression for the excitation is:

\[
E_{exc}(X) = \Delta E_{exc} \epsilon(t) + (1 + X) \frac{A_l - 1}{A_l} \tag{A.1}
\]

in which \( \epsilon(t) \) is the unit step function. Substituted in (4.19), it follows:

\[
\frac{dX}{dt} = \frac{A_l}{\tau} \Delta E_{exc} \epsilon(t) + (1 + X(t)) \frac{A_l - 1}{\tau} \tag{A.2}
\]

When the Laplace transformation is applied, it follows:

\[
x(p) - x_{t=0} = \frac{A_l \Delta E_{exc}}{p} + \left( \frac{1}{p} + x(p) \right) \frac{A_l - 1}{\tau} \tag{A.3}
\]

After some calculations and with \( x_{t=0} = -1 \), an expression for \( x(p) \) is found:

\[
x(p) = \frac{1}{p - A_l^{-1}} \left\{ -1 + \frac{1}{p} \left( \frac{A_l \Delta E_{exc} + A_l - 1}{\tau} \right) \right\} \tag{A.4}
\]

With the aid of the inverse Laplace transform, \( x \) as a function of time is found:

\[
x(t) = -e^{(A_l^{-1}) \frac{t}{\tau}} + \left( \frac{A_l \Delta E_{exc} + A_l - 1}{\tau} \right) \int_0^t e^{(A_l^{-1}) \frac{t'}{\tau}} dt' \tag{A.5}
\]

\[
= \frac{A_l}{A_l - 1} \Delta E_{exc} \left( e^{(A_l^{-1}) \frac{t}{\tau}} - 1 \right) - 1 \tag{A.6}
\]

A.2 The state as a function of time with a ramp at the input

The system discussed in this section has a constant loop gain, equal to \( A_l \) when the limiter is in the linear mode. It has been described in section 4.4.4. The expression for the excitation is:

\[
E_{exc}(X, t) = \alpha t + (1 + X) \frac{A_l - 1}{A_l} \tag{A.7}
\]

Substituted in (4.19), it follows:

\[
\frac{dX}{dt} = \frac{A_l}{\tau} \alpha t + (1 + X(t)) \frac{A_l - 1}{\tau} \tag{A.8}
\]

Using the Laplace transformation it follows:

\[
x(p) - x_{t=0} = \frac{A_l \alpha}{\tau p^2} + \left( \frac{1}{p} + x(p) \right) \frac{A_l - 1}{\tau} \tag{A.9}
\]
After some calculations and with \( X_{[t=0]} = -1 \), the equation is solved for \( X(p) \):

\[
X(p) = \frac{1}{p - \frac{A_l}{\tau}} \left\{ \frac{1}{p^2} \frac{A_l}{\tau} + \frac{1}{p} \frac{A_l - 1}{\tau} - 1 \right\}
\]  

(A.10)

With the aid of the inverse Laplace transformation \( X \) as a function of time is found:

\[
X(t) = \frac{A_l}{\tau} \int_0^t \int_0^t e^{(A_l-1)\frac{\alpha \tau}{\tau}} dt''dt' + \frac{A_l - 1}{\tau} \int_0^t e^{(A_l-1)\frac{\alpha \tau}{\tau}} dt' - e^{(A_l-1)\frac{\alpha \tau}{\tau}}
\]

(A.11)

\[
= \frac{A_l}{A_l - 1} \frac{\alpha \tau}{A_l - 1} \left( e^{(A_l-1)\frac{\alpha \tau}{\tau}} - 1 \right) - \frac{A_l}{A_l - 1} \alpha \tau - 1
\]

(A.12)

### A.3 The calculation of \( T(\Delta X, X) \) for a system with a constant loop gain

With (4.35) and (4.40) the time the system needs to change state with an amount equal to \( \Delta X \) can be calculated for a step and a ramp input respectively. The calculation will be carried out for the ramp input. First expression (4.40) for the \( T_{sr}(X) \) is repeated:

\[
T_{sr,ra}(X) = \frac{\tau}{A_l - 1} \ln \left( \frac{1 + X}{A_l - 1} + \frac{A_l - 1}{A_l} \right)
\]

(A.13)

The time interval to be calculated is defined as:

\[
T_{sr}(\Delta X, X) = T_{sr,ra}(X + \Delta X) - T_{sr,ra}(X)
\]

(A.14)

Substituting (A.13) results in:

\[
T_{sr}(\Delta X, X) = \frac{\tau}{A_l - 1} \ln \left( \frac{1 + X + \Delta X}{A_l - 1} \right)
\]

(A.15)

\[
= \frac{\tau}{A_l - 1} \ln \left( \frac{1 + \Delta X}{\frac{A_l - 1}{A_l}} + 1 \right)
\]

(A.16)

\[
= \frac{\tau}{A_l - 1} \ln \left( 1 + \frac{\Delta X}{\frac{A_l - 1}{A_l}} + 1 \right)
\]

(A.17)

A similar expression is found for a step input. Expression (4.40) can be transformed into (4.35) by way of inserting the factor \( \lambda \) as defined in (4.42) in front of the term \( \frac{\alpha \tau}{A_l - 1} \). When this is done in (A.17) also, the expression for \( T_{sr}(\Delta X, X) \) is found:

\[
T_{sr}(\Delta X, X) = \frac{\tau}{A_l - 1} \ln \left( 1 + \frac{\Delta X}{\lambda \frac{A_l - 1}{A_l}} + 1 \right)
\]

(A.18)

### A.4 The calculation of \( T_{delay}(0) \) for a system with a constant loop gain

Starting from:

\[
T_{delay}(0) = T_{sr,ra}(0) - T_{sr,rd}(0)
\]

(A.19)

the expression is found by substituting \( X = 0 \) in (4.35) and (4.40):

\[
T_{delay}(0) = \frac{\tau}{A_l - 1} \ln \left( \frac{A_l - 1}{A_l} \frac{\alpha \tau}{A_l} + 1 \right)
\]

(A.20)

\[
= \frac{\tau}{A_l - 1} \ln \left( \frac{\Delta E_{sec} \frac{A_l - 1}{A_l} + \frac{A_l}{A_l - 1}}{1 + \frac{A_l}{A_l - 1}} \right)
\]

(A.21)

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With (4.42) it follows:

\[ T_{delay}(0) = \frac{r}{A_l - 1} \ln \left( \frac{\lambda + \frac{A_l}{A_l - 1}}{1 + \frac{A_l}{A_l - 1}} \right) \]  
(A.22)

\[ = \frac{r}{A_l - 1} \ln \left( \frac{1}{2 + \frac{1}{A_l - 1}} \left( \lambda + 1 + \frac{1}{A_l - 1} \right) \right) \]  
(A.23)
B. The differential error of a CCO transfer due to delays

According to the definition given in [3], the differential error is:

$$\varepsilon = \frac{TF - TF(Q)}{TF(Q)}$$  \hspace{1cm} (B.1)

$$TF = \frac{df_o}{dE_i}$$ \hspace{1cm} (B.2)

$TF(Q)$ is the transfer in the quiescent point, $TF$ is the transfer in another operating point and $E_i$ is the input signal for the CCO. The tuning constant of the CCO is $k$. The ideal output frequency is without delays is:

$$f_{ideal} = kE_i$$  \hspace{1cm} (B.3)

The actual output frequency is:

$$f_o = \frac{f_{ideal}}{1 + 4\Delta T f_{ideal}}$$ \hspace{1cm} (B.4)

The maximum frequency deviation is $\Delta f$, and the center frequency is $f_{\text{center}}$.

$$f_{\text{center}} = \frac{kE_i(0)}{1 + 4\Delta T k E_{i0}}$$ \hspace{1cm} (B.5)

$E_{i0}$ represents the input signal for a zero frequency deviation. Using (B.1) and (B.2) the expression for the differential error can be calculated:

$$\varepsilon = \frac{d(f_{\text{center}} + \Delta f)}{dE_i} + \frac{df_{\text{center}}}{dE_i}$$  \hspace{1cm} (B.6)

$$= \frac{k}{(1 + 4\Delta T(f_{\text{center}} - \Delta f))^2} - \frac{k}{(1 + 4\Delta T f_{\text{center}})^2}$$ \hspace{1cm} (B.7)

$$= -8\Delta T \Delta f \frac{1 + 2\Delta T(2f_{\text{center}} + \Delta f)}{(1 + 4\Delta T(f_{\text{center}} + \Delta f))^2}$$ \hspace{1cm} (B.8)
C. References


D. Summary

After a short introduction in chapter 1, the fundamentals of first order oscillators are discussed in chapter 2. First the definition of a first order oscillator is given as an oscillator in which the timing is controlled by only one time constant. This time constant originates from an integrator which is used to make a time variant signal out of a constant. However, an integrator cannot produce a periodical signal by itself, and additional circuits must be included. These extra circuits perform the basic functions that are necessary for generating a periodical signal from the time variant signal of the integrator. Several system concepts for first order oscillators are given, and various ways of implementing the basic functions are suggested.

The integrator is usually implemented as a capacitor which is charged by a constant current. When capacitors are available in an IC-process, first order oscillators are fully integratable. This is a very important feature of first order oscillators.

One of the basic functions that must be performed in a first order oscillator is a binary memory function. A binary memory can be built with a regenerative circuit, like a Schmitt trigger. Oscillators that are equipped with this type of binary memory are labeled regenerative oscillators or multivibrators. There are other types of binary memories that are not regenerative, and the concepts of oscillators making use of these memories are also given.

In chapter 3 a description language for behavioral models is introduced. It is designed to facilitate the understanding of the fundamental aspects of the oscillator operation. For this reason it makes use of graphical symbols. With the aid of the description language, the oscillators can be grouped into several classes. Each class has its own special properties. Emerging from this classification is the feasibility of other first order oscillator configurations than just the commonly used regenerative oscillators. Some interesting oscillator concepts are devised with the aid of the description language.

In chapter 4 the regenerative binary memories are dealt with, the Schmitt trigger being one example. The loop gain-excitation model is introduced to model the switching behavior of regenerative circuits. It is used to investigate the noise behavior of regenerative circuits and the influence of non idealities on their switching speed. Different implementations are discussed.

In chapter 5 non regenerative memories are discussed. Sample and hold circuits and other circuits containing an integrator can also be used to build a binary memory. Several implementation are discussed.

Chapter 6 deals with the noise behavior of first order oscillators. The noise behavior is dealt with on the system level, so no calculations are performed for a specific electronic implementation. The fact that a first order oscillator is a time variant system implies a rather complicated noise behavior. The noise is sampled by the oscillator, which causes the transfer of noise power from high frequency bands to a frequency band around the carrier frequency. Expressions describing this effect are derived, and have been used to predict the noise behavior of the electronic implementations discussed in the next two chapters. The predictions appeared to be in very good agreement with the measurements.

Chapter 7 and chapter 8 both deal with the implementation of first order oscillators. In the former chapter regenerative oscillators are discussed and in the latter the non regenerative oscillator. In both chapters the properties of various implementations are discussed and electronic implementations are given. These have furthermore been integrated in a bipolar IC-process and verified experimentally.
E. Samenvatting

Na een korte introductie in hoofdstuk 1 worden in het tweede hoofdstuk de basis principes behandeld die ten grondslag liggen aan de werking van een eerste-orde oscillator. Als eerste wordt de definitie van de eerste-orde oscillator gegeven. Een eerste-orde oscillator is een oscillator waarin de periodetijd wordt bepaald met behulp van één enkele tijdconstante. De tijdconstante is afkomstig van een integrator. De integrator wordt gebruikt om een tijdvariërend signaal te genereren vanuit een constant signaal. Een integrator alleen kan echter geen periodiek signaal opwekken. Om dat te bereiken zijn er extra circuits nodig. Deze circuits voeren de basisfuncties uit in het oscillator systeem die nodig zijn voor het opwekken van een periodiek signaal vanuit het tijdsvariërend integrator signaal. Enkele systeemconcepten voor eerste-orde oscillatoren worden behandeld en verschillende manieren om de basic functies te implementeren worden voorgesteld.

De integrator wordt meestal uitgevoerd met een condensator, die met een constante stroom geladen en ontladen wordt. Wanneer in een IC-proces geïntegreerde condensatoren beschikbaar zijn, dan is een eerste-orde oscillator volledig integreerbaar. Dit is een zeer voordelige eigenschap van dit type oscillatoren. Een van de basic functies die in een eerste-orde oscillator moet worden uitgevoerd is de binaire geheugenfunctie. Een binaire geheugen kan met een regeneratief circuit, zoals een Schmitt trigger gemaakt worden. Oscillatoren die met een regeneratief binair geheugen zijn uitgevoerd worden ook wel regeneratieve oscillatoren of multivibratoren genoemd. Er zijn echter ook binaire geheugens die niet-regeneratief zijn. Ook voor oscillatoren uitgevoerd met dit soort geheugens worden systeemconcepten gegeven.

In hoofdstuk 3 wordt een beschrijvingstaal geïntroduceerd voor eerste-orde oscillatoren. De taal maakt gebruik van grafische symbolen om het gedrag te beschrijven. Met behulp van de beschrijvingstaal kunnen de oscillatoren onderscheiden worden in verschillende klassen. Elke klasse heeft specifieke eigenschappen. De klassificatie toont aan dat er meer implementaties van eerste-orde oscillatoren mogelijk zijn dan alleen de veel toegepaste regeneratieve oscillatoren. Enkele zeer interessante implementaties worden met behulp van de beschrijvingstaal afgeleid.


In hoofdstuk 5 worden de niet-regeneratieve geheugens behandeld. Sample and hold circuits en andere circuits die een integrator bevatten kunnen ook als binair geheugen worden ingezet. Ook hier worden verschillende implementaties behandeld.

Hoofdstuk 6 behandelde het ruisdagrad van eerste-orde oscillatoren. Het ruisdagrad wordt op systeenniveau behandeld. Er worden geen berekeningen uitgevoerd aan één specifieke elektronische implementatie. Het feit dat een eerste-orde oscillator een tijdvariant systeem is, maakt het ruisdagrad nogal gecompliceerd. De ruis wordt bemerkt door de oscillator met als gevolg dat ruis uit hogere frequentiebanden wordt teruggevoerd naar een frequentieband rond de grondharmonische van het oscillatorsignaal. Er worden uitdrukkingen afgeleid die dit effect beschrijven. De resultaten van de ruisberekeningen zijn gebruikt om het ruisdagrad van enkele elektronische implementaties te voorspellen. Metingen hebben aangetoond dat de
uitdrukkingen zeer bruikbaar zijn.

De hoofdstukken 7 en 8 behandelen de eigenlijke implementaties van eerste-orde oscillatoren. In het eerstgenoemde hoofdstuk worden de regenerative oscillatoren behandeld en in het laatstgenoemde de niet-regeneratieve. In beide hoofdstukken worden diverse electronische implementaties gegeven. Sommige daarvan zijn geïntegreerd in een bipolair IC-proces en door middel van metingen geverifieerd.
F. Acknowledgements

The work on which this thesis is based would have been impossible without the support of many people at the Delft University of technology. The accurate nature of Klaas van Zalinge stimulated me to be very precise in defining the models. He also contributed very much to the verification of the design theory. As an experienced designer he was able to build and verify every oscillator concept very expediently. Jan-Wim Eikenbroek has always been a great help when the mathematical problems tended to become too large to tackle. I would also like to thank Jack Sneep and Jaap van der Plas. Their implementations of first order oscillators have been of great use for the verification of the theory. David van Maaren and Ernst Nordholt have, with their remarks, inspired many of my more useful ideas. Hans Stoffels forced me think about many fundamental questions concerning the development of a design theory, the result of which is the approach found in this thesis.

Further, I would like to thank Jan Nusteling and Rob Janse for their efforts to keep all necessary programs running on the various computer systems.

The ORION computer solved many mathematical problems for me with the REDUCE program. It considerably reduced the time I needed to derive the expressions that describe the noise behavior of the oscillators.

At this point, I would also like to express my gratitude to my graduate students who have made significant contributions via their of implementation of the system concepts of the oscillators. Jaap Noordeloos built the first coupled oscillator and Dick van de Broeke proved the feasibility of the high frequency switching two integrator oscillator.

Apart from acquainting me with IC-technology, Lis Nanver also helped me to write this thesis in an English that can be understood by more than just the Dutch.

The project that resulted in this thesis has been made possible by the Delft University of Technology (beleidsruimte).
G. Biography

Chris Verhoeven was born in The Hague on the 25th February, 1959. When he was six he received his first electronic construction kit as a present. From that time on he became more and more interested in electronics. In high-school he used most of his spare time constructing of all kinds of electronic circuits. After a visit to the Delft University of technology he decided that being an electronics engineer would be even more interesting than being a air plane pilot. In 1978 he began his studies in Electrical Engineering at the Delft University of technology. After his graduation in 1985, he continued the research already pursued as a graduate student for the purpose of preparing a thesis.