Estimating atmospheric stability from observations and correcting wind shear models accordingly

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Abstract. Atmospheric stability strongly influences wind shear and thus has to be considered when performing load calculations for wind turbine design. Numerous methods exist however for obtaining stability in terms of the Obukhov length $L$ as well as for correcting the logarithmic wind profile. It is therefore questioned to what extend the choice of adopted methods influences results when performing load analyses. Four methods found in literature for obtaining $L$, and five methods to correct the logarithmic wind profile for stability are included in the analyses (two for unstable, three for stable conditions). The four methods used to estimate stability from observations result in different PDF’s of $L$, which in turn results in differences in estimated lifetime fatigue loads up to 81%. For unstable conditions hardly any differences are found when using either of the proposed stability correction functions, neither in wind shear nor in fatigue loads. For stable conditions however the proposed stability correction functions differ significantly, and the standard correction for stable conditions might strongly overestimate fatigue loads caused by wind shear (up to 15% differences). Due to the large differences found, it is recommended to carefully choose how to obtain stability and correct wind shear models accordingly.

1. Introduction
Wind shear is a major cause of cyclic loads of wind turbines, and should therefore be described as accurately as possible when designing new wind turbines. The IEC standard [1] prescribes the use of either a power law or logarithmic wind profile (log-profile) as a shear model when performing load calculations. These profiles are independent of atmospheric stability, while it is well known that stability has a major impact on wind shear. As such, recently studies have been carried out to assess the impact of atmospheric stability on resource assessment [2], wind turbine performance [3] and fatigue loads [4].

The impact of stability on wind shear, and subsequently on wind turbine design, can be studied based on observation data. One has to choose however i) how to determine or classify stability from observations and ii) how to correct wind profiles accordingly. For both of these choices various methods are used in literature. This raises the question if choosing a specific methodology to determine stability and correct shear profiles accordingly has a significant impact on wind turbine design. In this research the impact of both choices on estimated lifetime fatigue loads of wind turbines is analysed. The impact of using specific stability correction functions on wind shear is analysed for five stability correction functions (two for unstable and three for stable conditions). The dependency of the frequency of occurrence of stability on a chosen method to
estimate stability is studied for four specific methods. As a final analyses the impact of choosing specific methods (both for obtaining and correcting wind shear for stability) on wind turbine design is studied based on load simulations of a reference wind turbine.

2. Theory

2.1. Wind shear and Atmospheric Stability

Wind shear in the atmospheric boundary layer is typically well described with a logarithmic shear profile. Neglecting stability effects, the neutral logarithmic wind profile (neutral log-law) is given by

\[ u(z) = u_* \frac{\ln \left( \frac{z}{z_0} \right)}{\kappa} \]  

(1)

Here \( u(z) \) is the wind speed at height \( z \), \( u_* \) is the friction velocity, \( \kappa \) is the von Karman constant and \( z_0 \) is the roughness length. Following Monin-Obukhov similarity theory, the neutral log-law can be rewritten to include stability effects

\[ u(z) = u_* \left[ \ln \left( \frac{z}{z_0} \right) - \Psi \left( \frac{z}{L} \right) + \Psi \left( \frac{z_0}{L} \right) \right] \]  

(2)

Here \( \Psi \) is a stability correction function that depends on the Obukhov length \( L \). The last term in equation 2 is generally neglected since \( \Psi \left( \frac{z}{L} \right) \gg \Psi \left( \frac{z_0}{L} \right) \). The Obukhov length is defined as

\[ L = \frac{-\overline{\theta_v u_s^3}}{\kappa g (\overline{w' \theta'_v})_s} = \frac{\overline{\theta_v u_s^2}}{\kappa g \theta_*} \]  

(3)

Here \( \overline{\theta_v} \) is the mean virtual potential temperature, \( g \) is the acceleration due to gravity, \( (\overline{w' \theta'_v})_s \) is the surface virtual potential heat flux and \( \theta_* \) is the surface layer temperature scale. If \( L \) is negative the atmosphere is unstable, while for positive values the atmosphere is stable.

The definition of the stability correction function in equation 2 varies in literature. Generally the standard corrections proposed by Businger and Dyer (BD-functions [5, 6]) are used, which are defined as

\[ \Psi(L \leq 0) = 2 \ln \left( \frac{1+x}{2} \right) + \ln \left( \frac{1+x^2}{2} \right) - 2 \arctan (x) + \frac{\pi}{2} \]

\[ x = (1 - \gamma \frac{z}{L})^{1/4} \]

(4)

\[ \Psi(L \geq 0) = -\beta \frac{z}{L} \]

Where the parameters \( \beta \) and \( \gamma \) where first determined as 4.7 and 15 based on the Kansas experiments [5]. In literature various other values of \( \beta \) and \( \gamma \) are found, and here the correction proposed by Högström [7] is adopted (\( \beta = 6 \) and \( \gamma = 19.3 \)). The validity of the BD-functions is questionable, either based on dimensional analyses (for unstable conditions) or based on observational data (for stable conditions). For extreme unstable conditions stresses are no longer significant, and one can show that the BD-functions are incorrect [8]. For such conditions \( u_* \) is no longer a scaling parameter, and based on dimensional analyses one obtains a stability correction function that should hold in the free convection limit

\[ \Psi(L \leq 0) = \frac{3}{2} \ln \left( \frac{y^2 + y + 1}{4} \right) - \sqrt{3} \arctan \left( \frac{2y + 1}{\sqrt{3}} \right) + \frac{\pi}{\sqrt{3}} \]

\[ y = (1 - \gamma \frac{z}{L})^{1/3} \]  

(5)

Here the parameter \( \gamma \) is set to 10 [8]. A similar expressions was already proposed in the seventies [9], still generally the BD-functions are applied. For stable conditions observations show that the BD-functions overestimate wind shear, specifically for very strong stable stratifications. As
such, it is proposed independently by Brutsaert and Holtslag to (empirically) alter the stability correction function for stable conditions [10, 11]. The formulation of Brutsaert is given by

\[
\Psi(L \geq 0) = \begin{cases} 
-\beta \frac{z}{L} - \beta \ln \frac{z}{L} & \text{if } 0 < \frac{z}{L} \leq 1 \\
-\beta \ln \frac{z}{L} - \beta & \text{if } \frac{z}{L} > 1 
\end{cases}
\] (6)

Whereas the formulation of Holtslag is given by

\[
\Psi(L \geq 0) = -\frac{z}{L} - \frac{2}{3} \left( \frac{z}{L} - \frac{5}{0.35} \right) \exp \left( -0.35 \frac{z}{L} \right) - \frac{10}{1.05}
\] (7)

In equation 7 the proposed parameters of Beljaars and Holtslag are used [12]. One can show with equation 2 that the ratio of the wind speed at two heights becomes a function of height, stability and the roughness length only if one assumes \( u_* \) is constant with height (which is approximately true in the atmospheric surface layer, the lowest 10% of the boundary layer).

2.2. Obtaining \( L \) from observations

One can use various methods to estimate \( L \) from regular observations [13]. If high temporal resolution observation data is available, one can calculate \( L \) directly based on the eddy-covariance method and the observed turbulent fluxes of momentum and heat. In absence of this data however, one must rely on empirical methods to estimate \( L \). In general these empirical methods depend either on the Richardson number (RI-methods), or one estimates \( L \) iteratively from wind and temperature profiles (Profile-methods).

The Richardson number is given by

\[
RI = \frac{g \Delta \overline{v} \Delta z}{\overline{u}^2 \Delta \theta^2}
\] (8)

One can come up with a gradient Richardson number by considering wind speed and temperature measurements at two heights in the atmosphere (thus considering the gradient of wind speed and temperature within the atmosphere). If one considers the surface conditions (with \( u(z_0) = 0 \text{ m s}^{-1} \)) in combination with wind speed and temperature observations of the atmosphere at one height, one comes up with a bulk Richardson number (thus considering the atmosphere as one bulk layer). Subsequently we define a gradient-Richardson method (RI-Grad) and a bulk-Richardson method (RI-Bulk) that can both be used to estimate \( L \). The general form of both methods is similar, but parameter values differ. In this study the RI-Grad method is defined as [2]

\[
\frac{z}{L} = \begin{cases} 
\frac{RI}{1 - 5RI} & \text{if } RI \geq 0 \\
\frac{RI}{1 - 5RI} & \text{if } RI \leq 0 
\end{cases}
\] (9)

and the RI-Bulk method is defined as [14]

\[
\frac{z}{L} = \begin{cases} 
\frac{10RI}{1 - 5RI} & \text{if } RI \geq 0 \\
10RI & \text{if } RI \leq 0 
\end{cases}
\] (10)

For the profile methods it is assumed that the stability corrected log-profiles are valid (thus observations must be carried out in the atmospheric surface layer). The following set of equations can iteratively be solved in combination with equation 3

\[
\begin{align*}
\frac{u_*}{u} &= \frac{\Delta u}{\ln (z_2/z_1) - \Psi(z_2/L) + \Psi(z_1/L)} \\
\frac{\theta_*}{\theta} &= \frac{\Delta \theta}{\ln (z_2/z_1) - \Psi(z_2/L) + \Psi(z_1/L)}
\end{align*}
\] (11)
Where $z_2 > z_1$ and $z_1$ equals $z_0$ if the sea surface is considered as lowest observation height. The $\Psi$-functions for the temperature profile are not equal to those used for the wind profile. One can find the $\Psi$-functions for temperature in literature [15, 16]. For the calculation of $u_*$ we consider the BD-correction functions, which as discussed might be inappropriate for very stable/unstable conditions. Just as for the Richardson-methods, we define two profile-methods: one considering observations of surface conditions (Profile-Sea method) and one considering observations of wind speed and temperature at two heights in the atmosphere (Profile-Air method).

Both the Richardson and Profile methods depend on Monin-Obukhov similarity theory which is strictly speaking only valid for stationary conditions. We adopt a similar filtering procedure as [13], and only consider situations where the wind speed, wind direction, sea temperature and air temperature do not change significantly in time.

3. Methodology

3.1. Observation data analyses

Both the accuracy of the various $\Psi$-functions and the differences found when using the various methods to estimate $L$ are analysed based on observation data of the OWEZ wind farm. At the OWEZ wind-farm a meteorological observation mast is present where wind and temperature observations are carried out at 3 heights (21, 70 and 116m), as well as observation of water temperature, waves and currents. Due to the presence of the wind farm North-East of the meteorological mast, only a limited amount of undisturbed observation data is available. In this study 10 months of data (from July-2005 until May-2006) is included in the analyses. For a detailed description of the meteorological mast and the sensors used, the reader is referred to the website of the OWEZ wind farm (www.noordzeewind.nl).

All temperature sensors have an accuracy of 0.1 °C, and wind sensors have an accuracy of at least 95%. The methodologies assessed to determine stability are most sensitive to measurement errors when $\Delta u$ or $\Delta \theta_v$ is small. Since both wind speed and temperature gradients between 21m and 70m height are far smaller than those between 0 and 21m height, the RI-Grad and Profile-Air methods are most sensitive to measurement errors. This is especially true for near neutral conditions ($\Delta \theta_v \approx 0$) or very unstable conditions ($\Delta u \approx 0$). Besides, both profile methods assume validity of the logarithmic wind speed and temperature profiles, either up to 21m height or up to 70m height. These logarithmic profiles are valid in the lowest 10% of the boundary layer, and for very stable conditions the observation heights (especially at 70m height) are likely no longer located within the surface layer. As such the accuracy of both profile methods decreases for increasing atmospheric stability. Since the bulk-Richardson method is least sensitive to measurement errors, and does not depend on the assumption of validity of the logarithmic wind and temperature profiles, we consider the Richardson-bulk method as reference methodology.

It is noted in [13] that the sea temperature observations have a -0.82 °C offset compared to ECMWF Re-analysis data, hence we correct all sea temperature observations by subtracting 0.82 °C.

3.2. Calculating wind turbine fatigue loads

The impact of choosing specific stability correction functions and choosing a method to estimate stability from observations on the design of wind turbines is analysed by looking at lifetime fatigue loads of a wind turbine. For these analyses the software package Bladed is used, and the 5MW NREL wind turbine is used as a reference wind turbine since it is widely used in literature for similar studies. Notice that the 5MW NREL wind turbine has a hub height of 90 m while the OWEZ metmast does not have observations at this height. It is therefore decided to use an iterative method similar to the profile methods to estimate the 90m wind speed based on the observed wind speed (and temperature) at 70m and 116m height, and subsequently fit a Weibull distribution to the calculated wind speed data.
Table 1. Boundaries of stability classes in terms of $L$

<table>
<thead>
<tr>
<th>Class name</th>
<th>Class boundaries</th>
<th>Value for load calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Unstable (VU)</td>
<td>$-200 \leq L &lt; 0$</td>
<td>$L = -100$</td>
</tr>
<tr>
<td>Unstable (U)</td>
<td>$-500 \leq L &lt; -200$</td>
<td>$L = -350$</td>
</tr>
<tr>
<td>Neutral (N)</td>
<td>$</td>
<td>L</td>
</tr>
<tr>
<td>Stable (S)</td>
<td>$200 &lt; L \leq 500$</td>
<td>$L = 350$</td>
</tr>
<tr>
<td>Very Stable (VS)</td>
<td>$0 &lt; L \leq 200$</td>
<td>$L = 100$</td>
</tr>
</tbody>
</table>

Since only wind shear is considered in this study, a steady state situation is imposed in Bladed, thereby neglecting fatigue loads caused by turbulence. Despite being less realistic, this enables us to focus on the impact that stability has on the loads caused by wind shear only. The fatigue load analysis are carried out for the blade root, since here maximum bending moments due to cyclic loadings occur. The bending moments calculated with Bladed for a given wind profile and hub height wind speed are converted to stresses according to

$$\sigma(u, L) = \frac{M(u, L)y}{I}$$  \hspace{1cm} (12)

Here $\sigma$ is the stress at the blade root, $M$ is the bending moment calculated with Bladed, $y$ is the distance at which the moment acts and $I$ is the area moment of inertia. The blade root is simulated as a thin cylinder with an inner radius of 3.42 m and an outer radius of 3.5 m (hence $y = 3.5$ m and $I = 0.65$ m$^4$). These stresses are plotted against time, and from the (nearly) sinusoidal pattern the stress amplitude $S$ is calculated. One generally converts load cycles to actual fatigue damage with a SN-curve, but since there is no data available for this test case the Damage Equivalent Load ($D_{EQ}$) is introduced [17].

$$D_{EQ} = \left[ \sum S(u, L)^m N(u) \right]^{1/m}$$  \hspace{1cm} (13)

For the analyses it is assumed that $m = 12$ ($m$ is the Wöhler exponent), $N_{EQ} = 10^7$ and $N(u)$ is the lifetime number of cycles for a given wind speed. We assume a wind turbine lifetime of 20 years, and $N(u)$ is calculated with the known RPM of the turbine

$$N(u) = RPM(u) \times (20 \times 365 \times 24 \times 60)$$  \hspace{1cm} (14)

By defining several stability classes (here ranging from very unstable to very stable) and assuming the cut-in and cut-out wind speed is respectively 4 and 25 m s$^{-1}$, the cumulative lifetime fatigue load $D_{EQC}$ is calculated following

$$D_{EQC} = \sum_{u=4}^{25} \sum_{L=VU}^{VS} D_{EQ}P(L|u)P(u)$$  \hspace{1cm} (15)

Here $P(L|u)$ equals the chance that for a given hub height wind speed $u$ the given stability class $L$ occurs, and $P(u)$ is the chance that a given hub height wind speed occurs. The boundaries of the stability classes are found in table 1, where also the specific Obukhov length for each class is given that is used to create wind shear profiles. This classification is based on [18], though here only five classes are included for computational efficiency. Furthermore, extremely stable and unstable situations are also included here that were neglected in the original classification.
4. Results

4.1. Stability and wind shear

The sensitivity of the occurrence of stability classes to the specified methods to determine $L$ is analysed first. The frequency of occurrence of stability as a function of wind speed is plotted in figure 1. It is clear that the frequency of occurrence of the five stability classes differs when using various calculation methods. In general it is found that for weak wind speeds (below $10 \text{ m s}^{-1}$) very unstable conditions prevail, while for strong wind speeds (above $15 \text{ m s}^{-1}$) neutral conditions start to occur more frequently. For moderate wind speeds however there is a significant difference in the occurrence of stability classes for the various methodologies considered here. Those methodologies considering surface temperature find prevailing (very) unstable conditions for wind speeds between $10$ and $15 \text{ m s}^{-1}$, while both other methods find far more (very) stable conditions. There is a striking similarity of the general stability distribution if one either considers surface observations or only atmospheric observations. Especially in terms of (very) stable conditions, those methods considering surface observations find a gradual increase in stable conditions for increasing wind speeds. In contrary, those methods considering only atmospheric observations find a majority of stable conditions for wind speeds between $10$ and $20 \text{ m s}^{-1}$. It is generally thought that for strong wind speeds the atmosphere becomes neutrally stratified. Although we do find an increase in neutral conditions for very strong wind speeds, notice that there is still a significant amount of non-neutral observations for wind speeds above $20 \text{ m s}^{-1}$. Notice also that when using the RI-Bulk and Profile-Sea methods, one finds no stability classification for wind speeds above $22 \text{ m s}^{-1}$ in contrary to the RI-Grad and Profile-Air methods. Reason is that for the few events where such strong wind speeds were observed, sea
Figure 2. Observed and theoretical wind shear between 21 and 70 m height as a function of stability (here calculated with the RI-Bulk method).

temperature observations were not available and stability could not be calculated.

The impact of using specific stability correction functions for the stability corrected log-law can be seen in figure 2. We consider here stability as calculated by the RI-Bulk method on the x-axes. The assumption frequently used in wind energy that wind shear is independent of stability, assuming a power law (here with a power of 0.2) or neutral logarithmic shear profile, is clearly invalid. For unstable conditions up to \(100/L = -1\) the stability corrected log-profile performs well, no matter which correction function is used. Differences between both correction functions are small, and the neutral log law and power law both perform worse and overestimate shear. For neutral conditions a lot of scatter is found in the observations and the logarithmic shear profile tends to underestimate wind shear, while the power law overestimates shear. A potential cause of the increase in wind shear compared to the theoretical profiles might be the occurrence of internal boundary layers, as discussed in [2]. For stable conditions scatter increases, though the stability corrected logarithmic wind profile tends to perform reasonably well up to \(100/L = 2\) if one considers the stability correction of Holtslag. Clearly the BD-functions cause an overestimation of wind shear for very stable situations, which has also been found in other studies. Both other stability correction functions perform better, and the formulation of Holtslag perform slightly better than the formulation of Brutsaert. The power law typically overestimates wind shear for \(100/L < 1\), and underestimates shear for \(100/L > 1\).

4.2. Equivalent load analyses

Multiple simulations with Bladed are performed with various shear models and stability distributions, and calculated equivalent loads are visualised in figure 3. As a reference we choose the stability corrected log law with BD-functions and the stability distribution obtained with the RI-Bulk method (third bar in figure 3). The remaining bars show the equivalent loads calculated when changing either the shear profile (bar 1 and 2), the \(\Psi\)-functions (bar 4, 5 and 6) or stability distribution (bar 7, 8 and 9) from the reference case. One can see here that using the power-law or neutral logarithmic wind profile results in significant differences compared to using the stability corrected logarithmic wind profile. When using the power law, we find lifetime fatigue...
loads nearly twice as high compared to considering atmospheric stability, while using the neutral log law results in an underestimation of the fatigue loads by 17%. When changing the stability correction function for unstable conditions little differences are found (1%), hence changing this correction is not significant when calculating lifetime equivalent loads. In contrary, applying the different correction functions for stable conditions leads to a reduction in calculated lifetime equivalent loads of 8% (Brutsaert) or 15% (Holtslag) compared to using the original Businger-Dyer correction functions for stable conditions. Although these differences are significant, the impact of using specific methodologies to estimate stability is even more profound. Using the stability distributions obtained from the RI-Grad and Profile-Air methods result in a significant increase in the expected lifetime fatigue loads (respectively 30% and 70% increase). This is primarily caused by the fact that these methods estimate more frequently stable conditions, which in turn results in high shear and high fatigue loads. Using the stability distribution obtained from the profile-sea method results in a slight decrease in fatigue loads since there are slightly less (very) stable conditions for moderate wind speeds.

As a final step, the equivalent loads as a function of wind speed are analysed when using specific shear models, stability correction functions and stability distributions (figure 4). Here again the equivalent loads are normalised similar as was done in figure 3. Results indicate that the simulated equivalent loads are most sensitive to the shear model used and the methodology used to estimate stability. One can also see here that for wind speeds above 11 m s\(^{-1}\) the calculated equivalent loads decrease for all simulations due to pitching of the blades.

Since the power law typically overestimates shear, it is not surprising that equivalent loads are also highest for the far majority of wind speeds considered here. The neutral log law underestimates shear, but the resulting fatigue loads do not differ much compared to using the stability corrected logarithmic wind profile, at least for wind speeds up to 13 m s\(^{-1}\). This can be explained by the fact that for these wind speeds the atmosphere is primarily (very) unstable or neutrally stratified, and wind shear decreases slightly when the atmosphere changes from neutral to unstable conditions. For higher wind speeds the atmosphere becomes more often

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**Figure 3.** Relative lifetime equivalent loads for various methods (see x-axes) normalised with the equivalent loads calculated when using the stability corrected log law with BD-functions and the stability distribution obtained with the RI-Bulk method (third bar).
The simulated fatigue loads are very sensitive to the methodology used to obtain a stability distribution. One can clearly see here that fatigue loads calculated when using stability distribution obtained with the RI-Bulk and Profile-Sea methods correlate quite well, though the loads calculated with the RI-Bulk method are typically slightly higher. Despite the small differences shown here, this accounts for a 10% difference on the lifetime fatigue loads as shown in figure 3. Using the RI-Grad or profile-air methods results in a significant increase in fatigue loads due to the increase in stable conditions, primarily found for wind speeds between 10 and 16 m s$^{-1}$.

When looking at the impact of using various stability corrections functions, little differences are found for unstable conditions. For stable conditions, the fatigue loads decrease similarly as wind shear decreases when applying either the Brutsaert (little decrease in wind shear) or Holtslag (large decrease in wind shear) stability correction functions. Difference are relatively small however compared to the impact of using various shear models, or various methodologies to estimate stability as was shown in figure 3 as well.

5. Conclusions
In this study we assessed the accuracy of shear profiles, and the sensitivity of fatigue load analyses to specific shear models, stability corrections and stability distributions. It is found that wind shear depends strongly on atmospheric stability, especially for stable conditions. The shear...
profiles that do not consider stability (power law and neutral log law) deviate strongly from the stability dependence observed. For unstable conditions the exact formulation of the stability correction is not significant. For stable conditions however results do differ and the stability correction of Holtslag tends to perform best. The distribution of stability is very sensitive to the method used to determine stability. Especially the choice of incorporating surface observations has a significant impact on the stability distribution.

The meteorological results found have a distinct impact on the calculated fatigue loads for the simple reason that if shear increases fatigue loads increase as well. The equivalent loads are most sensitive to the shear profile used (differences up to 88% in lifetime fatigue loads) and the methodology used to obtain the PDF of stability (differences up to 71% in lifetime fatigue loads. The sensitivity to the exact stability correction is less pronounced, but for stable conditions we find that the Businger-Dyer functions that are typically used overestimate fatigue loads by 15%.

Based on the results it is proposed to consider the stability corrected logarithmic wind profile as a shear model in combination with the "Free Convection" and "Holtslag" stability correction functions. Besides it is suggested to consider the Bulk-Richardson method to estimate stability from regular observations since this method is least sensitive to observation errors.

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References