Memorandum M-633

AN ENERGY BASED MESH REFINEMENT PROGRAM
FOR THE STAGS-A COMPUTER CODE

H. A. J. Knops

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Delft University of Technology
Faculty of Aerospace Engineering
Delft, The Netherlands
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Abstract

To get in touch with mesh refinement techniques a program is developed to perform such a mesh refinement for the STAGS-A computer code. After consulting some articles on this subject I decided to use the so-called strain energy density method. To illustrate the working of the program a few examples are dealt with. These examples prove the mesh refinement program to yield excellent results for values of $N (=\text{rows} \times \text{columns})$ small compared to the ones found in the traditional way of calculating. Finally in appendix 1 the input and output files for this program are described and explained.
1. Introduction

For rather simple engineering problems it is obvious that the ananalytical approach will yield an exact solution. Most problems though encountered in applied engineering science are of such a complexity that it is impossible to find an exact analytical solution. For these problems various numerical methods have to be applied, yielding approximate values of selected unknown variables at discrete points throughout the continuum. These solutions will not be exact, because the use of such a numerical method implies that the continuous mathematical model has to be replaced by a discrete model. Generally there is no guarantee that this discrete model is close enough to the mathematical model to find a good approximation of the exact solution. One way of approaching such a complex model is to perform a number of calculations on successive models using various mesh refinements. In this way the solution will converge towards a value that is at least near the exact solution. The number of elements, needed to find such a solution, can be enormous and through that the cpu-time involved becomes unreasonably long.

This encouraged scientists to search for the optimal grid layout, i.e. a grid layout such that for a minimum number of elements the approximation of the exact solution reaches its optimum. Some of the principal investigations on this subject have been made by Turcke [1], Turcke and McNeice [2] and Melosh and Marcal [3]. They made use of the fact that in the optimized grid layout the nodes had aligned themselves generally along contour lines of some response function [4]. Consequently they plotted various contours and trajectories and upon examination the following provided very definite characteristics similar to the optimized grid

- isostatics, principal stress trajectories
- isoenergetics, contours of constant strain energy density

Further McNeice and Marcal [5], Turcke and McNeice [2] and Turcke [6] concluded that the problem of finding the optimal grid layout is one of minimizing the potential energy or maximizing the strain energy. The
consistent response function with respect to the potential energy formulation would be the strain energy density function and thus the isoenergetics, in general, should be used to indicate the proper shape along which the edge of an element can be aligned. These isoenergetics can be attained from an initial or coarse grid analysis. Turcke and McNeice [2], McNeice and Marcal [5] and Melosh and Marcal [3] analyzed a few problems and proved this approach to be an excellent way of finding an optimal grid layout and through that a close approximation of the exact solution.

This encouraged me to apply this way of grid optimization on STAGS-A [7], a computer code developed for the static and dynamic analysis of arbitrary shells. Actually this is not a finite element but a finite difference program. The STAGS-A analysis is based on an energy formulation. This finite difference method was chosen because of two reasons.

In the choice of the grid layout there is a restriction to rows and columns, i.e. finite difference approximations in two geometrically defined directions e.g. for a cylinder the axial and circumferential direction. Because of this restriction only an approximation of the optimal grid layout will be found. The advantage of this restriction though is that the mesh refinement program won’t be too complex. This will keep me from getting entangled in a net of problems and difficulties with respect to the programming part of the work and not getting in touch with the real difficulties concerning the search for the optimal grid layout e.g. high strain gradients and stress singularities.

The second reason for choosing STAGS-A is that in my sphere of activity (vakgroep C2, Faculty of Aerospace Engineering, TU Delft) the stability of shells is one of the main subjects and consequently STAGS-A is a commonly employed computer code. This means that there are a few persons in my vicinity who are acquainted with this computer code and therefore are capable of giving constructive criticism concerning the new mesh refinement program.
2. Criteria for mesh refinement

Before starting with the development of a mesh generator the criteria on which this mesh refinement is based will be explained. The first and most obvious one has to do with the strain energy density gradient. The second one deals with the buckling stability of structures and its consequences for the grid layout. The explanation of these two criteria will be given on the basis of a finite element method but also holds for a finite difference method.

2.1 The strain energy density criterion

To find a numerical solution that is close enough to the exact solution a grid layout has to be chosen such that a close approximation of the strain energy density at any point of the structure can be achieved. The most important criterion to find this optimal grid layout is the strain energy density gradient. At places where this gradient is small the strain energy will vary little. At these places as far as the grid layout is concerned a limited number of elements will do. At places though where the gradient starts to grow, e.g. near stress concentrations, the number of elements has to be raised in order to get a close approximation of the rapidly varying strain energy density. The best way to explain this is on the basis of an example. Suppose we have a square plate (fig. 1) subjected to concentrated corner loads [2]. In figure 2 the strain energy density along edge AB is plotted. First the strain energy density will be approximated by three similar linear elements (fig. 3). At places where the strain energy gradient is small the approximation is rather good in contrast with places where this gradient starts to increase. Here the number of elements has to be raised. In figure 4 one can see that in this way a close approximation of the total strain energy density curve for low and high gradients will be found.
2.2 The buckling criterion

The second criterion has to do with the buckling stability of thin walled structures, one of the most important fields of research in my 'vakgroep'. Buckling of thin walled structures occurs with a buckling mode consisting of displacement waves. If one wants to determine the buckling load in a numerical way one has to be aware of the fact that the model must have enough degrees of freedom to describe the buckling mode. Suppose you want to calculate the buckling load of an axially compressed simply supported cylinder (fig. 5). The strain energy density along line AB is plotted in figure 6. If the first criterion would be applied now the only places where the mesh will be refined are the areas near the boundaries (fig 7). Suppose the critical buckling mode consists of three half waves in axial direction (fig. 8). A close approximation of the exact critical buckling load cannot be made now with the grid layout of figure 7. To describe this buckling mode better and with that to find a more reliable buckling load the number of elements has to be raised. One has to define a minimum number of elements for each half wave e.g. 6 elements per half wave.

On the basis of these two criteria a numerical method can be developed in order to determine the optimal grid layout.
3. The numerical method for mesh refinement

In this chapter the numerical approach of the mesh refinement problem will be discussed. After the explanation of the two criteria for an optimal grid layout in chapter 1 they have to be made available in a program.

3.1 Application of the strain energy density criterion

The numerical approach of the strain energy density criterion is based on the following principle:

Make a division of the strain energy density of the total structure into levels of growing magnitude and see to it that for each grid element at most 1 strain energy density level is passed. If more levels are passed a local mesh refinement procedure has to be applied.

This principle needs some explanation. The first thing to do is to define a starting mesh to perform a numerical calculation. After that the strain energy density at the discrete points throughout the continuum must be calculated. Next a subdivision into levels can be made, where the lowest level is somewhere near the minimum and the upper level is somewhere near the maximum strain energy density. Assuming, for instance, 10 subdivisions one can see how many SED-levels (strain energy density levels) are being passed for each element. If this number is 0 or 1 the size of the element can stay unchanged. If it is greater than 1 the number of elements must be divided into a number of subelements equal to the number of SED-levels that are being passed, e.g. if for an element 3 SED-levels are being passed this element must be divided into 3 subelements. After this mesh refinement a new numerical calculation can be performed. If the answer is still too far away from the exact solution the mesh refinement program may be entered again.

The subdivision of the total strain energy density into levels can be done in two ways, linear and quadratic subdivision. For a linear subdivision the distance between two succeeding SED-levels is constant
for growing strain energy density (fig. 9). For a quadratic subdivision the distance between the square roots of two succeeding levels is constant. This implies that the distance between two SED-levels increases linearly with respect to a growing strain energy density (fig. 10). This last one can be used for cases where one has to deal with huge stress concentrations, e.g. crack tip analysis (fig. 11). The reason for choosing a quadratic subdivision here is the following. The formula for the strain energy density at mesh station $i$ can be written as

$$\Delta U^i = \frac{1}{2} S^i D^i S^i$$

where

$D^i = \text{stiffness matrix}$

$S^i = \text{vector of stress resultants}$

For cases where one has to deal with high stress concentrations a constant difference between the principal stresses corresponding to the SED-levels can be aimed at. In the formula these stresses are available in a quadratic form. This implies that one has to make a quadratic subdivision of the total strain energy density.

Finally there is an opportunity to enter a mesh station for which the SED-levels reach a maximum. This may help if a numerical analysis has to be performed for a case where one has to deal with stress singularities. The example in figure 1 is such a case. In the upper right corner of the plate the stress resultants will go towards infinity. By establishing a mesh station a little bit away from the corner where the SED-levels must reach their maximum one prevents the stress singularity to define the maximum of the SED-levels and through that to cause an excessive concentration of SED-levels in the corner of the plate.

3.2 Application of the buckling criterion

After performing an initial buckling calculation, the number of half waves the buckling mode consists of can be determined. Now the number of elements one wants to define for each half wave must be entered. If this
number is multiplied by the number of half waves in a certain direction the maximum size of the elements in that direction is found and the grid layout can be determined again.

It is obvious that thin walled structures will buckle at places where the stress resultants reach their maximum values. This implies that the mesh refinement procedure defined in the previous paragraph only has to be applied at places where the strain energy density is greater than e.g. 0.75 times the maximum strain energy density. In this way the program is kept from producing a refined mesh at places where buckling won’t occur.

In the following chapter to explain these different possibilities of mesh refinement a few examples will be presented.
4. Examples

4.1 Cylinder with cutouts

The first example is a cylinder with two cutouts subjected to an axial compression load (fig. 12). Because of symmetry only 1/8 of the cylinder has to be analyzed. Looking at figure 13 one may expect to find a rather high stress concentration at corner A and the mesh refinement could be restricted to this region. Both the linear and quadratic subdivision of the total strain energy density will be used starting with the linear one.

4.1.1 Linear subdivision

First a calculation of the buckling load is made keeping the grid layout homogeneous for the whole structure. For increasing mesh refinement the buckling load will converge towards the final numerical solution as expected (fig. 14).

Next a calculation is performed starting with an initial grid layout of 11 rows and 13 columns and using the mesh refinement program with linear subdivision of the total strain energy. Looking at figure 14 again one can see that the speed of convergence has increased but the final value of the buckling load is a little above the numerical solution found with the homogeneous grid layout (see curve I). This is caused by the fact that the initial mesh of 11 rows and 13 columns is too coarse. To prove this another calculation is performed starting with an initial mesh of 21 rows and 25 columns. This time the final value of the buckling load is very close to the one found with the homogeneous mesh (see curve II).

Finally an initial mesh is chosen taking into account that one knows where to expect a stress concentration. The initial mesh is already refined a little at corner A. Now again the final buckling load is very close to the one found with the homogeneous mesh (see curve III). In figure 15-17 one can see how the mesh refinement works for this
final case.

4.1.2 Quadratic subdivision

The method of quadratic subdivision of the total strain energy density has been developed for cases where one has to deal with high stress concentrations. In our example we have to deal with a stress concentration and therefore one can try to find the critical buckling load using this way of mesh refinement. To make it possible to compare the two methods the same initial meshes as for linear subdivision are used.

Looking at figure 18 now one can see that for the initial mesh I and III the numerical solution is not very close to the one found with the homogeneous mesh. With initial mesh II though this numerical solution is approximated very closely. It is rather striking that the final solution for initial mesh III, for which one took into account the knowledge of expecting a stress concentration, is even higher than the solution for the very coarse initial mesh I.

The explanation for this is that, for a buckling calculation, mesh I and III are too coarse to start with. This can be proved as follows. Suppose one has found the critical buckling load using the mesh refinement program. The sensitivity of the final mesh to slight changes must be low now, i.e. if a few rows or columns are added to the final mesh the solution may not change dramatically. First let us take the final mesh III plotted in figure 19 and add two rows. One can see that the difference between the two solutions is rather large. Now the final mesh II, plotted in figure 20, is refined by adding 6 rows. Here the difference between the two solutions is small. Consequently the sensitivity of the final mesh to slight changes is rather low compared to the final mesh II. This proves that mesh III is highly unstable for slight changes due to the fact that the initial mesh is too coarse. From this one can learn that the choice of the initial mesh is very important, because it is the base of the whole calculation. If this initial mesh is too coarse the convergence of the final solution cannot
be guaranteed.

4.2 Shear loaded cylinder

The second example is a cylinder subjected to a transverse shear load (fig. 21). Because of symmetry only 1/2 of the cylinder has to be analyzed. The stress concentrations, one expects to find, are not of that size that the quadratic subdivision of the total strain energy density should be applied. Therefore linear subdivision of the strain energy density is used. Two different kinds of prebuckling boundary conditions were applied:

- case 1 : \( w_2 = 0 \)
- case 2 : \( w_2 \) free

These two cases will be dealt with separately.

4.2.1 Case 1 : \( w_2 = 0 \)

First the whole calculation is performed again with a homogeneous grid layout. This causes the critical buckling load to converge towards the final numerical approximation of the exact buckling load (see curve A, fig. 22). Next the critical buckling load is calculated starting with a mesh of 16 rows and 16 columns and only taking into account the strain energy density criterion. The buckling load found in this way is completely different from the one found with the homogeneous grid layout (see curve B). Finally the buckling criterion is activated too and one can see that now the buckling load, found with the homogeneous grid layout, is approximated closely (see curve C).

To see what happens two plots have been made of the final mesh obtained with and without the buckling criterion (fig. 23 and 24). The strain energy density criterion will only cause considerable mesh refinement in axial direction at boundary 1 and in circumferential direction only a few columns are added to the initial mesh. Although we are left now with an area of high strain energy density (boundary 1) where buckling will occur the number of columns in this area is small.
This will cause the underestimation of the critical buckling load found with the homogeneous grid layout. Activating the buckling criterion will result in a final mesh plotted in figure 24. The number of columns in the high strain energy density area has been raised compared to figure 23 and the final buckling load is very close to the one found with the homogeneous grid layout (fig. 22).

4.2.2 Case 2 : $w_2$ free

This case is even more extreme with respect to the importance of the buckling criterion. In figure 26 one can see that using the strain energy density criterion will not provide us with a final mesh that is sufficiently refined in axial and circumferential direction. Therefore the accuracy of the final approximation totally depends on the buckling criterion for this case. After including the buckling criterion one will end up with a final mesh as in figure 27. The buckling load now is very close to the one found with the homogeneous grid layout (fig. 25).

From these two cases one can learn that, if we have to deal with buckling stability of thin walled structures, the buckling criterion is a very important tool to select the appropriate mesh in order to get a close approximation of the exact buckling load.

4.3 Corner loaded plate

It is obvious that for a corner loaded plate (fig. 1) a stress singularity will occur in the upper right corner. The strain energy density in this corner will be extremely high and this will cause a concentration of SED-lines in the upper right corner. Therefore there is an option to enter a mesh station where the SED-levels reach their maximum. This was already mentioned in chapter 3. To illustrate this the SED-levels are plotted in four ways

1 linear subdivision (fig. 28)

2 linear subdivision, maximum SED-level for $x=47.5$ inch, $y=47.5$ inch (fig. 29)
3 quadratic subdivision (fig. 30)

4 quadratic subdivision, maximum SED-level for x=47.5 inch, y=47.5 inch (fig. 31)

One can see that for the cases where no limit for the maximum SED-levels is entered (fig. 26 and fig. 28) a concentration of SED-lines will occur in the upper right corner. The stress concentration in this corner is caused by the fact that the real model had to be replaced by a discrete model. The stresses here are not an accurate reproduction of the real stresses. Therefore in practice one is not interested in the numerical values of the stresses in the upper right corner. A little bit away from the corner the real situation can be closely approximated again and the mesh refinement program can be applied yielding useful results. To restrict the mesh refinement to this area a mesh station can be entered for which the SED-levels reach their maximum (x = 47.5 in; y = 47.5 in). The results are shown in figure 29 and figure 31. It should be noted that the grid layout beyond the entered maximum SED-level may not be too coarse in order to get a stress distribution close to reality. Therefore the grid layout beyond the maximum SED-level is automatically adapted to the refined mesh.
5. Conclusion

All in all the mesh refinement program turns out to give good results if the initial mesh is not too coarse. This is caused by the fact that the program does not produce a completely new grid layout but only refines the initial mesh. These problems could be avoided if the new grid layout is independent of the initial one.

In table 1 a comparison is made of the cpu time needed to calculate the critical buckling load of the axially compressed cylindrical shell with two cutouts (fig. 14) using two different kinds of meshes:

1. the homogeneous grid layout
2. the grid layout obtained with the adaptive mesh generator starting with initial mesh number III

Indeed one can see that it is cheapest to use the adaptive optimum mesh generator.

Further the same difficulties were encountered as by previous researchers. They also had problems how to deal with e.g. stress singularities. This note also confirms the applicability of the strain energy density approach. Looking at figure 14 for instance one can see that good results can be obtained for a value of N (= rows x columns) that is small compared to the homogeneous grid layout. N is a measure for the size of the stability matrix and through that for the cpu-time needed for the buckling analysis. Further it is also clear that, because one also wants to perform buckling calculations, an accurate approximation of the internal stress resultants is not enough. The model must be provided with enough degrees of freedom to buckle freely. This possibility is also built in in the mesh refinement program.

Finally one may conclude that, besides getting in touch with the real difficulties of mesh refinement techniques, a useful program has been developed to obtain an optimum grid layout for thin walled cylindrical or elliptical structures and flat plates and this can be a firm base for developing an adaptive optimum mesh generator.
Literature


<table>
<thead>
<tr>
<th>mesh</th>
<th>rows x columns</th>
<th>CPU time (sec)</th>
<th>buckling load (lbf/in)</th>
<th>curve number III</th>
<th>rows x columns</th>
<th>CPU time (sec)</th>
<th>buckling load (lbf/in)</th>
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<td>57803</td>
<td>338,9</td>
<td></td>
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</table>

table 1: CPU time needed to calculate the critical buckling load of the axially compressed cylindrical shell with two cutouts (fig. 14) using:
1. a homogeneous grid layout
2. meshes obtained with the adaptive optimum mesh generator starting with initial mesh number III
The calculation are performed on a SUN-workstation with a DP Linpach processor, 10 MIPS, 1.5 MFLOPS
\[ p = 25 \times 10^6 \text{ lbs} \]

Poisson's ratio: \( \nu = 0 \)

Young's modulus: \( E = 10 \times 10^6 \text{ psi} \)

Thickness: \( t = 1.0 \text{ in} \)

Length: \( L = 100 \text{ in} \)

**Figure 1:** Corner loaded square plate

**Strain energy density**

\( (x10^4 \text{ lbf/in}^2) \)

**Figure 2:** Strain energy density along edge AB of the corner loaded square plate (fig. 1)
strain energy density
($10^4$ lbf/in$^2$)

figure 3: Approximation of the strain energy density curve by three similar straight elements
strain energy density
($x10^4$ lbf/in$^2$)

figure 4: Approximation of the strain energy density curve while the size of the elements depends on the strain energy density gradient
figure 5: Axially compressed circular cylindrical shell

strain energy density

distance along line AB

figure 6: A qualitative representation of the strain energy density along line AB of the axially compressed circular cylindrical shell (fig. 5)
strain energy density

figure 7: Approximation of the strain energy density with straight line elements according to the strain energy density criterion only.

$w$ displacement normal to the cylinder surface

figure 8: An approximate buckling mode of the axially compressed circular cylinder in along line AB.

20
minimum strain energy density

1  2  3  4  5  6  7  8  9

→ level number

maximum strain energy density

figure 9: Linear subdivision of the total strain energy density

minimum strain energy density

1  2  3  4  5  6  7  8  9

→ level number

maximum strain energy density

(1)(4)(9)(16) (25) (36) (49) (64) (81)

figure 10: Quadratic subdivision of the total strain energy density

strain energy density

distance along line AB

figure 11: Qualitative representation of the strain energy density of a notched plate
Young's modulus: $E = 10 \times 10^6$ psi
Poisson's ratio: $\nu = 0.3$
thickness: 0.1 in

**figure 12**: Circular cylindrical shell with two cutouts subjected to an axial compressive load

\[ u \text{ free} \quad \quad \quad u = 0 \]
\[ v = 0 \quad \quad \quad v \text{ free} \]
\[ w = 0 \quad \quad \quad w \text{ free} \]
\[ w_x' \text{ free} \quad \quad \quad w_x' = 0 \]

**figure 13**: Part of the cylindrical shell needed for the buckling analysis
Figure 14: The critical buckling load for an axially compressed cylindrical shell with two cutouts (linear subdivision)
buckling load: $N_a = 375.9 \text{ lbf/in}$
strain energy density levels 1-10
linear subdivision

figure 15: Distribution of the rows and columns for initial mesh number III (step 1)
buckling load : $Na = 349.2$ lbf/in
strain energy density levels 1-10
linear subdivision

figure 16: Distribution of the rows and columns for initial mesh number III (step 2)
buckling load: $Na = 339.1 \text{lbf/in}$
strain energy density levels 1-10
linear subdivision

Figure 17: Distribution of the rows and columns for initial mesh number III (step 3)
figure 18: The critical buckling load for an axially compressed cylindrical shell with two cutouts (quadratic subdivision)
figure 19: The sensitivity of final mesh number III to additional rows
( quadratic subdivision )
additional rows
buckling load without additional rows : Na = 338.9 lbf/in
buckling load with additional rows : Na = 334.7 lbf/in

figure 20 : The sensitivity of final mesh number II to additional rows (quadratic subdivision)
Young's modulus : $E = 5.83 \times 10^6$ psi
Poisson's ratio : $\nu = 0.3$
thickness : $t = 0.0267$ in

figure 21: A shear loaded cylindrical shell
critical buckling load Nbv (lbf/in)

\[ Nbv = \frac{VL^2}{\pi R} \]

\[ x=4 \]
\[ x=5 \]

A: homogeneous grid layout
B: only SED criterion
C: SED and buckling criterion
x: number of elements per half wave

initial grid

16

rows x columns (log)

100

1000

10000

figure 22: Critical buckling load for a shear loaded cylindrical shell case 1: \( w_2 = 0 \) (linear subdivision)
Figure 23: Final mesh for the shear loaded cylindrical shell taking into account the SED criterion only (case 1: $w_2 = 0$)
Figure 24: Final mesh for the shear loaded cylindrical shell taking into account the SED and the buckling criterion with 4 elements per half wave (case 1: $w_2 = 0$)
critical buckling load $N_{bv}$ (lbf/in)

$$N_{bv} = \frac{VL^2}{RR}$$

A: homogeneous grid layout
B: only SED criterion
C: SED and buckling criterion
$x$: number of elements per half wave

initial grid

figure 25: Critical buckling load for a shear loaded cylindrical shell case 2: $w_2$ free (linear subdivision)
Figure 26: Final mesh for the shear loaded cylindrical shell taking into account the SED criterion only (case 2: \( w_2 \) free)
figure 27: Final mesh for the shear loaded cylindrical shell taking into account the SED and buckling criterion with 4 elements per half wave (case 2: $w_2$ free)
figure 28: SED-levels for a corner loaded plate (see fig. 1)
- linear subdivision
figure 29: SED-levels for a corner loaded plate (see fig. 1)

- linear subdivision
- maximum SED-level for $x = 47.5$ inch
  $y = 47.5$ inch
figure 30 : SED-levels for a corner loaded plate ( see fig. 1 )
- quadratic subdivision
figure 31: SED-levels for a corner loaded plate (see fig. 1)
- quadratic subdivision
- maximum SED-level for $x = 47.5$ inch
  $y = 47.5$ inch
Appendix 1

Mesh refinement program for the STAGS-A computer code

Explanation of the input and output file

The mesh refinement program is written in such a way that the output file of the STAGS-A computer code can be used as input file for this program, after omitting a few lines. It only may contain:

1. a shell type definition line (G-1 card)
2. a line with the surface properties of the shell (G-2 card)
3. a line with the number of rows and columns and the row and column numbers of eventual cutouts (D-1 card)
4. if present - a line with the number of segments in X-direction (D-2 card)
   - lines with the length of the segments in X-direction (D-3 card)
   - lines with the number of mesh spaces within each segment in X-direction (D-4 card)
   - a line with the number of segments in Y-direction (D-5 card)
   - lines with the length of the segments in Y-direction (D-6 card)
   - lines with the number of mesh spaces within each segment in Y-direction (D-7 card)
5. lines where the stiffness matrix is printed out (attention: this must be 6 lines)
6. lines with row numbers, column numbers and the corresponding internal stress resultants

An example of such a file is given in figure 32. To run the mesh refinement program now this file must be named "fort.99". After starting the program you will get some questions about the input file and the way of producing a new grid layout. These questions will concern:

- the format of the stiffness matrix

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- the format of the stress resultants
- the number of SED-levels
- the choice for linear or quadratic subdivision of the SED-levels
- the choice for a maximum SED mesh station
- in case of a buckling calculation
  - the number of half waves in x- and y-direction
  - the number of elements per half wave

The output file of the mesh refinement program is "fort.98". It will contain the maximum and minimum SED, the SED for each mesh station and the corresponding SED-level, the coordinates of the SED-curves and a new grid layout. This new grid layout is also printed in file "fort.97" in the format of the STAGS-A computer code. In this way it can be copied into the STAGS-A input file very easy.

Programs and subroutines used

<table>
<thead>
<tr>
<th>Program</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>STAGSMR</td>
<td>main program for calculation of the SED-levels</td>
</tr>
<tr>
<td>INVERS</td>
<td>subroutine to invert a matrix</td>
</tr>
<tr>
<td>EQSOV</td>
<td>subroutine to solve a matrix equation</td>
</tr>
<tr>
<td>SEDPL</td>
<td>subroutine to make a plot of the SED-curves</td>
</tr>
<tr>
<td>PRCONT</td>
<td>subroutine to keep the plot on the screen as long as you wish</td>
</tr>
<tr>
<td>NEWMESH</td>
<td>subroutine to produce a new grid layout</td>
</tr>
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Figure 32: Input file for the mesh refinement program