

## Indication of the Ferromagnetic Instability in a Dilute Two-Dimensional Electron System

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The magnetic field  $B_c$ , in which the electrons become fully spin polarized, is found to be proportional to the deviation of the electron density from the zero-field metal-insulator transition in a two-dimensional electron system in silicon. The tendency of  $B_c$  to vanish at a *finite* electron density suggests a ferromagnetic instability in this strongly correlated electron system.

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At sufficiently low electron densities, an ideal two-dimensional (2D) electron system becomes strongly correlated, because the kinetic energy is overpowered by energy of electron-electron interactions (exchange and correlation energy). The interaction strength is normally described by the Wigner-Seitz radius,  $r_s = 1/(\pi n_s)^{1/2} a_B$  (where  $n_s$  is the electron density and  $a_B$  is the effective Bohr radius in semiconductor), which is equal in the single-valley case to the ratio of the Coulomb and the Fermi energies as calculated for 2D band electrons. There are several possible candidates for the ground state of the system, for example, (i) a Wigner crystal characterized by spatial and spin ordering [1], (ii) a ferromagnetic Fermi liquid with spontaneous spin ordering [2], and (iii) a paramagnetic Fermi liquid [3]. The Wigner crystal is expected to form in a very dilute limit, at  $r_s \gtrsim 35$  [4]. The spin ordering may survive at higher electron densities (lower  $r_s$ ) up to the threshold determined by ferromagnetic (Stoner) instability [2] as caused by the competition between the exchange energy and the Pauli principle.

In the strongly interacting limit ( $r_s \gg 1$ ), all results given by theoretical approaches are very approximate, not to mention that in real 2D electron systems, the influence of disorder needs to be taken into account, which complicates the problem drastically. For an ideal 2D electron system, a direct transition from the Wigner crystal to the paramagnetic Fermi liquid was predicted by quantum Monte Carlo calculation [4], although near the transition the energies of all three states are very close to each other. On the other hand, a tendency to spontaneous spin polarization has been found recently in numerical studies of a disordered and interacting electron gas [5]. In strongly disordered 2D systems, the influence of the disorder can dominate the interaction effects leading to a disorder-driven localization, whereas in the least disordered 2D systems, the metal-insulator transition may be caused by interaction effects

[6]. The problem of the ground state of strongly interacting 2D systems is therefore far from being solved.

In magnetic fields parallel to the 2D electron plane, the spin effects should dominate the properties of a 2D electron system once the orbital quantization is quenched. Indeed, the resistance of a 2D electron gas in silicon metal-oxide-semiconductor field-effect transistors (MOSFETs) was found to be isotropic with respect to in-plane magnetic field,  $B$ , and rise steeply with the field saturating to a constant value above a critical magnetic field  $B_c$  which depends on electron density [7]. Moreover, an analysis of Shubnikov-de Haas oscillations in tilted magnetic fields has established recently that the field  $B_c$  corresponds to the onset of full spin polarization of the electron system [8,9].

In this Letter, we study low-temperature parallel-field magnetotransport in a 2D electron system in silicon in a wide range of electron densities. We find that the saturation (or polarization) magnetic field,  $B_c$ , is a strictly linear function of  $n_s$ :  $B_c \propto (n_s - n_c)$  where  $n_c$  is the critical electron density for the  $B = 0$  metal-insulator transition. Since above  $n_c$ , the 2D band tail of localized electrons is negligibly small, as inferred from the low-field/low-temperature Hall effect measurements, vanishing  $B_c$  at a finite electron density gives evidence in favor of the existence of a ferromagnetic transition (at  $n_s$  close to  $n_c$ ) in this strongly correlated 2D electron system.

Measurements were made in a rotator-equipped Oxford dilution refrigerator with a base temperature of  $\approx 30$  mK on high-mobility (100)-silicon samples similar to those previously used in Ref. [10]. The resistance was measured by a standard four-terminal low-frequency technique. Excitation current was kept low enough to ensure that measurements were taken in the linear regime of response. Contact resistances in our samples were minimized by using a split-gate technique that allows one to maintain a high electron density in the vicinity of the contacts regardless

of its value in the main part of the sample. In this paper we show results obtained on a sample with a peak mobility close to  $3 \text{ m}^2/\text{Vs}$  at 0.1 K.

Typical curves of the low-temperature magnetoresistance  $\rho(B)$  in a parallel magnetic field are displayed in Fig. 1. The resistivity increases approximately quadratically with field until it saturates at a constant value above a certain density-dependent magnetic field. In the vicinity of the metal-insulator transition, the magnetoresistance strongly depends on temperature  $T$ , as was reported, e.g., in Ref. [11]. As one moves away from the transition, however, this temperature dependence weakens and eventually disappears at very low temperatures where the metallic resistivity at  $B = 0$  saturates and becomes independent of temperature. Therefore, as  $n_s$  is lowered, the low-temperature limit is realized for progressively narrower initial intervals on the curve  $\rho(B)$ . The data discussed in this paper are obtained in this low-temperature limit where the magnetoresistance becomes temperature independent.

In Fig. 2, we show how the normalized magnetoresistance, measured at different electron densities, collapses onto a single curve when plotted as a function of  $B/B_c$ . The scaling parameter,  $B_c$ , has been normalized to correspond to the magnetic field at which the magnetoresistance saturates (within the accuracy with which the latter can be determined) and hence full spin polarization of the electrons is reached [8,9]. The observed scaling is remarkably good for  $B/B_c \leq 0.7$  in the electron density range  $1 \times 10^{16} \text{ m}^{-2}$ , although with increasing  $n_s$  the scaled experimental data occupy progressively shorter intervals on the resulting curve. Both at  $B/B_c > 0.7$  and outside the indicated range of electron densities, the scaled data start to noticeably deviate from the universal curve. In particular, the scaling breaks down when one approaches ( $n_s < 1.3n_c$ ) the metal-insulator transition which in this sample occurs at zero magnetic field at  $n_c = 8 \times 10^{14} \text{ m}^{-2}$  [12]. This is not surprising as the magnetoresistance near  $n_c$  de-

pends strongly on temperature; see above. We note that the observed scaling dependence is described reasonably well by the theoretical dependence of  $\rho/\rho(0)$  on the degree of spin polarization  $\xi = gm\mu_B B/\pi\hbar^2 n_s = B/B_c$  (where both the  $g$  factor and the effective mass  $m$  may be enhanced due to interactions; see below) as predicted by the recent theory [13] based on the spin-polarization-dependent screening of a random potential deep within the metallic regime; see Fig. 2.

In Fig. 3, we show the first important and unexpected result of this paper: with high precision,  $B_c$  is proportional to the deviation of the electron density from its critical value, i.e., to  $(n_s - n_c)$ , over a wide range of electron densities. In other words, the field, at which the magnetoresistance saturates, tends to vanish at  $n_c$  (see also Refs. [14,15]). We emphasize that our procedure provides high accuracy for determining the behavior of the field of saturation with electron density, i.e., the functional form of  $B_c(n_s)$ , even though the absolute value of  $B_c$  is determined not so accurately. Note that at  $n_s$  above  $2.4 \times 10^{15} \text{ m}^{-2}$ , the saturation of the resistance is not reached in our magnetic field range; still, the high precision of the collapse of the high-density experimental curves onto the same scaling curve as the low-density data allows us to draw conclusions about the validity of the obtained law  $B_c(n_s)$  over a much wider range of electron densities.

The fact that the saturation (or polarization) field tends to vanish at  $n_c$  would be trivial if the density of the delocalized electrons approached zero at the metal-insulator transition. This crucial point can be verified with the help of the Hall effect measurements in the limit of weak magnetic fields and low temperatures [16]. As seen in the inset of Fig. 4, above  $n_c$ , the electron density obtained from the Hall effect measurements coincides within experimental uncertainty with the one determined from Shubnikov-de Haas oscillations, which shows that in the metallic phase, the 2D band tail of localized electron states is negligible; i.e., all the electrons are delocalized [17].

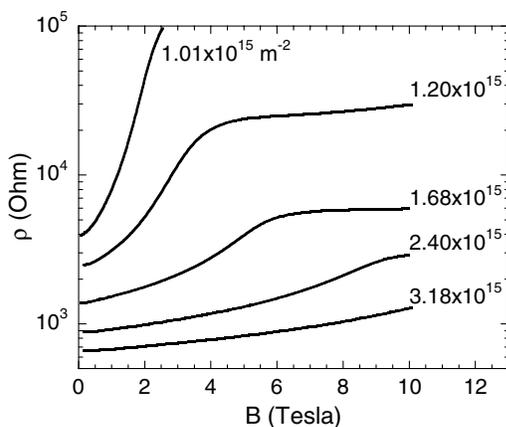


FIG. 1. Low-temperature magnetoresistance in parallel magnetic fields at different electron densities above the critical density for the  $B = 0$  metal-insulator transition. The lowest density curve is outside the scaling region as described in the text.

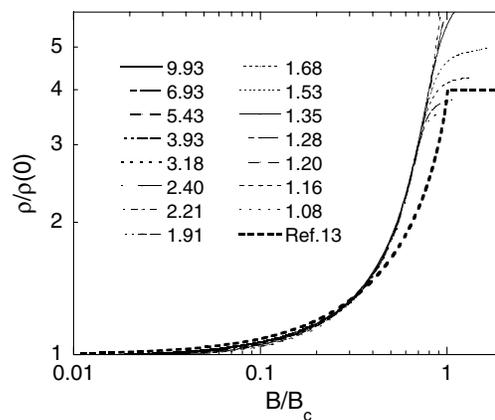


FIG. 2. Scaled curves of the normalized magnetoresistance at different  $n_s$  vs  $B/B_c$ . The electron densities are indicated in units of  $10^{15} \text{ m}^{-2}$ . Also shown by a dashed line is the normalized magnetoresistance calculated in Ref. [13].

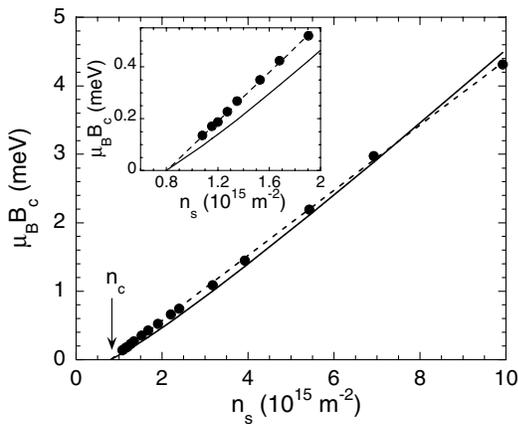


FIG. 3. Dependence of the field  $B_c$  on electron density. The dashed line is a linear fit which extrapolates to the critical electron density for the  $B = 0$  metal-insulator transition. The fit using Eq. (1) with  $\gamma = 0.095$  is shown by a solid line. A close-up view of the region near  $n_c$  is displayed in the inset.

Therefore, the observed tendency for  $B_c$  to vanish at a finite electron density gives an indication of the long-awaited ferromagnetic instability in strongly correlated 2D electron systems [18].

Based on the data in Fig. 3 and the above form of the spin polarization parameter  $\xi$ , we have determined the product,  $gm$ , of the effective  $g$  factor and the effective mass as a function of electron density; see Fig. 4. The dependence  $gm(n_s)$ , including the drastic enhancement of  $gm$  at  $n_s \rightarrow n_c$ , is consistent with the one obtained from the analysis of Shubnikov–de Haas oscillations in dilute silicon MOSFETs [8,15,19–21]. This gives additional support to our procedure and conclusions.

Subject to the occurrence of a ferromagnetic (Stoner) instability [2] at  $n_s \approx n_c$ , it is easy to calculate the behavior of  $B_c$  vs  $n_s$  based on the functional form of the

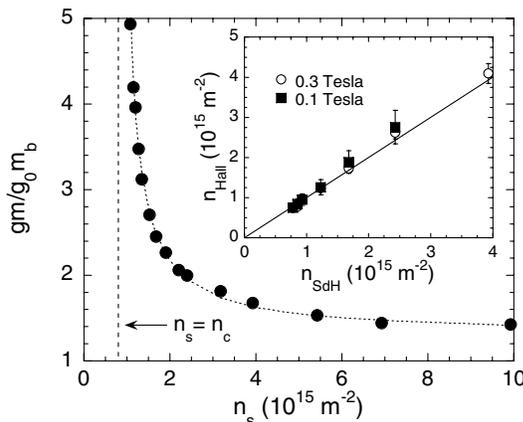


FIG. 4. The product  $gm/g_0m_b$  vs  $n_s$  obtained from the data in Fig. 3, based on the above form of the spin polarization parameter  $\xi$ . The dashed line is a guide to the eye. The inset compares the electron densities determined from Shubnikov–de Haas oscillation and weak-field low-temperature Hall measurements. For the latter, averages were taken for opposite magnetic field directions. The line in the inset corresponds to  $n_{\text{Hall}} = n_{\text{SdH}}$ .

exchange and correlation energy which is valid in the strong-interaction limit. Apparently, at the onset of a spontaneous spin polarization, the chemical potentials of a spin polarized and unpolarized 2D electron system (with interaction energy taken into account) must be equal. In the opposite limiting case, when the Zeeman energy,  $g_0\mu_B B$ , strongly exceeds the energy of electron-electron interactions, the chemical potentials of spin polarized and unpolarized 2D electron system at the boundary of a full spin polarization must be equal as well. Assuming that this equality holds also for intermediate magnetic fields, and writing the exchange and correlation energy in their explicit form, we get for the chemical potential difference at the onset of full spin polarization and at  $B = 0$

$$\frac{\pi \hbar^2 n_s}{2m_b} - \gamma \frac{e^2}{\kappa} n_s^{1/2} - \frac{1}{2} g_0 \mu_B B_c = 0. \quad (1)$$

Here  $\hbar$  is the Planck constant, the band mass  $m_b = 0.19m_0$  ( $m_0$  is the free electron mass),  $e$  is the electron charge,  $\kappa$  is the dielectric constant, the Landé  $g$  factor in bulk silicon is equal to  $g_0 = 2$ , and  $\mu_B$  is the Bohr magneton. The first term in Eq. (1) is given by the difference of the bare Fermi energies of spin-polarized and spin-unpolarized systems which differ by a factor of 2 originating from the spin degeneracy (valley degeneracy being taken into account). The interaction (second) term contains the unknown numerical factor  $\gamma$  which is positive since the exchange and correlation energy is always negative and its absolute value enhances with spin polarization. Note that the interactions are expected to lead to corrections to both  $m_b$  and  $g_0$ ; the problem of the interaction-enhanced effective mass  $m$  and  $g$  factor lacks a definite answer so far.

As seen from Fig. 3, the resulting dependence  $B_c(n_s)$  with the factor  $\gamma = 0.095$  determined from the condition  $B_c(n_c) = 0$  describes the experimental finding well enough. Formally, the best description of the experimental data might be obtained with a bit smaller  $\gamma = 0.090$ , which corresponds to the expected ferromagnetic transition point just below  $n_c$ , at  $n_s = 7 \times 10^{14} \text{ m}^{-2}$ . Although the accuracy of the  $B_c$  normalization hardly allows one to distinguish, the conclusions of the paper would still hold for that case. We emphasize, however, that the  $B_c(n_s)$  dependence given by Eq. (1) is weakly superlinear, whereas our experiments yield strictly linear dependence. Clearly, theoretical efforts are needed to explore a possible relationship between the Stoner instability and the metal-insulator transition in 2D.

In summary, we have studied the low-temperature magnetoresistance in parallel magnetic fields in a wide range of electron densities above the critical electron density for the zero-field metal-insulator transition,  $n_c$ . The normalized magnetoresistance is found to scale with  $n_s$  defining the scaling parameter,  $B_c$ , that corresponds to the magnetic field in which the full spin polarization is achieved. Over a wide range of electron densities, this scaling parameter changes in precise accordance with the relation  $B_c \propto (n_s - n_c)$ . Although in the metallic regime, the band

tail of the localized electrons has been found negligibly small,  $B_c$  tends to vanish at a finite electron density. This gives evidence in favor of the existence of a ferromagnetic transition in this strongly correlated 2D electron system.

Recently, a conclusion about possible spontaneous spin polarization in a dilute 2D electron system has been reached by Vitkalov *et al.* [22].

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