A New Eulerian Model for Turbulent Evaporating Sprays in Recirculating Flows

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Summary

A new Eulerian model for the computation of turbulent evaporating sprays in recirculating flows is derived. It comprises droplet heating and evaporation processes by solving separate transport equations for the droplet’s temperature and diameter. Full coupling of the droplet and the gaseous phase is achieved by the exchange of source terms due to momentum, heat and mass transfer. The partial differential equations describing the droplet's transport and evaporation in the new method can be solved using the same numerical procedure as for the gas phase equations. The validity of the model is established by comparison with a well known Lagrangian approach and with experimental data. For this purpose calculations of a recirculating droplet charged air flow within a model combustor are presented.

Nomenclature

$A, B, C$ - coefficients of the Cox-Antoine vapour pressure equation
$C_1, C_2, C_3$ - constants of the $k$, $\epsilon$-turbulence model
$c_D$ - drag coefficient
$c_{p,p}$ - $J/(kg \ K)$
$c_{p,v}$ - $J/(mol \ K)$
$c_{rel}$ - $m/s$
$c_o$ - $kg/kg$
$D$ - diameter
$D_x$ - characteristic diameter in the Rosin-Rammler drop size distribution
$\vec{F}$ - $N/m^3$
$F_r$ - Frössling number
$h$ - $J/kg$
specific enthalpy
$\Delta h_v$ - $J/kg$
specific enthalpy of evaporation
$k$ - $m^2/s^2$
kineetic energy of turbulence
$L_x$ - $m$
length scale of the $k$, $\epsilon$-turbulence model
$Le$ - Lewis number
$M$ - $kg/mol$
molecular weight
$N$ - parameter in the Rosin-Rammler drop size distribution
$Nu$ - Nusselt number
$n$ - $mol/m^3$
molar concentration
$nd$ - number of discrete drop-size classes
$Pr$ - Prandtl number
$p$ - $N/m^2$
pressure
$Re$ - Reynolds number
$S$ - source term (equation dependent)
$Sc$ - Schmidt number
$T$ - temperature
$t$ - time
$u, v, w(u_i)$ - $m/s$
velocity components
$V$ - $m^3/s$
volume flow rate
$x, y, z(x_i)$ - $m$
cartesian coordinates
$v_c$ - volume fraction of the dispersed phase
$X$ - mol fraction

Greek symbols

$\epsilon$ - $m^2/s^3$
turbulence energy dissipation rate
$\Gamma$ - $m^2/s$
diffusion coefficient
$\lambda$ - $W/(mK)$
thermal conductivity
$\mu$ - $Ns/m^2$
dynamic viscosity
$\nu$ - $m^2/s$
kineamtic viscosity
$\omega$ - $s^{-1}$
particle response frequency
$\rho$ - $kg/m^3$
density
$\sigma_k, \sigma_\epsilon$ - constants of the $k$, $\epsilon$-turbulence model
$\tau$ - $s$
particle response (relaxation) time

Introduction

Liquid fuel combustion processes in gas turbine combustors so far are not well understood and, therefore, object of numerous theoretical and experimental investigations. In particular, the accurate prediction of these processes is extremely difficult due to the combination of complex physical and chemical phenomena. Dispersed phase/turbulence interactions and turbulence effects on chemical reactions are only two examples for the complexity. In addition nonreacting two-phase flows include a variety of unsolved problems, e.g. turbulent droplet dispersion, turbulence effects on heat and mass transfer between the phases, vapour/air mixing etc.. Besides, nonreacting turbulent two phase flows are important by itself. Premixed-prevaporized gas turbine combustors, Diesel engine sprays and rocket plumes are some examples of this type of flow (Mostafa and Elghobashi (1984)).

Approaches for the prediction of droplet transport and evaporation in combustion systems can be classified by two fundamentally different methods. In the Lagrangian approach, the spray is represented by discrete droplets. Each computed droplet represents a number of physical droplets and is observed on its trajectory until it leaves the calculational domain or it evaporates completely. The equations describing the droplet behaviour can be simplified to ordinary differential equations. In turbulent flows, droplet motion and evaporation is simulated by a stochastic or Monte Carlo approach (Gosman and Ioanides (1983), Wittig et al. (1987), Kneer et al. (1990)). In the Eulerian approach, the evaporating spray is treated as an interacting and interpenetrating continuum. The resulting equations are similar to the
equations describing the turbulent gas phase. The application of the Eulerian approach for sprays requires the consideration of the continuum assumption (Batchelor (1974)). This assumption is valid, when each computational element contains a large number of droplets in the way that statistically averaged properties can be assigned to the droplets. Crowe (1982) showed that most practical systems satisfy the continuum assumption.

This paper is addressed to the further improvement of a recently developed Eulerian method for the numerical simulation of turbulent evaporating sprays (Hallmann et al. (1993)). Results of the new model are compared with both, experimental data and computations using a Lagrangian approach.

Eulerian methods for the prediction of turbulent two-phase flows have been used by various groups of researchers (e.g. Melville and Bray (1979), Elghobashi et al. (1984), Chen and Wood (1986), Krämer (1988), Abou-Arab and Rocco (1990), Simonin (1990)). They all deal with questions concerning the dispersion of particles in parabolic flows neglecting heat and mass transfer between the two phases. Mostafa and Elghobashi (1985) as well as Mostafa and Mongia (1987) report on Eulerian methods for the computation of turbulent jets with droplet vaporization. However in these papers rather simplified assumptions were used. The authors assumed isothermal flow conditions and a constant droplet (saturation) temperature. In addition, a simplified mass transfer model for the calculation of the droplet diameter was applied. In contrast to these investigations, the present study introduces a new model for the computation of fuel spray characteristics including droplet heating and evaporation by solving separate transport equations for the droplet’s temperature and diameter. This model is applicable not only to parabolic flows but also to turbulent recirculating non-isothermal evaporation sprays, which are of major interest within the design process of real gas turbine combustors.

Governing equations

Gas phase equations

For the steady mean flow under consideration, the time averaged continuity equation and the stationary Reynolds equations

$$\frac{\partial}{\partial z_i} (\rho u_i) = S_{p,p}$$

(1)
\[
\frac{\partial}{\partial x_i} (\rho u_i u_j) = \frac{\partial}{\partial x_i} (\mu_{eff} \frac{\partial u_i}{\partial x_j} + \frac{2}{3} \rho \frac{\partial u_k}{\partial x_i} \delta_{ij} ) - \frac{\partial}{\partial x_j} (p + 2 \rho \alpha) + F_j + S_{u_j,\rho}
\]

in conjunction with the standard \( k, \epsilon \)-turbulence model

\[
\frac{\partial}{\partial x_i} (\rho u_i k) = \frac{\partial}{\partial x_i} (\mu_{eff} \frac{\partial k}{\partial x_i}) + \nabla \cdot (\rho \nabla k) + G_k - \rho \epsilon + S_{k,\rho}
\]

\[
\frac{\partial}{\partial x_i} (\rho u_i \epsilon) = \frac{\partial}{\partial x_i} (\mu_{eff} \frac{\partial \epsilon}{\partial x_i}) + \frac{\epsilon}{k} (C_1 G_k - C_2 \rho \epsilon) + S_{\epsilon,\rho}
\]

are solved numerically by a Finite Volume discretization method. In the above equations

\[
\mu_{eff} = \mu + \mu_t
\]

is the frequently used effective viscosity with the laminar viscosity \( \mu \) and the eddy viscosity

\[
\mu_t = \rho C_p k^2 \frac{\nabla k}{\epsilon}
\]

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\]

External forces acting on the fluid, e.g. gravity are denoted by \( F_j \) in the momentum equations (2) and are assumed to be negligible in the present investigations. Turbulent transport of enthalpy and vapour concentration is calculated via additional transport equations:

\[
\frac{\partial}{\partial x_i} (\rho u_i h) = \frac{\partial}{\partial x_i} (\mu_{eff} \frac{\partial h}{\partial x_i}) + S_{h,\rho}
\]

\[
\frac{\partial}{\partial x_i} (\rho u_i c_a) = \frac{\partial}{\partial x_i} (\mu_{eff} \frac{\partial c_a}{\partial x_i}) + S_{c_a,\rho}
\]

with

\[
\frac{\mu_{eff}}{Pr_{eff}} = \frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} + \frac{\mu_{eff}}{Sc_{eff}} = \frac{\mu}{Sc} + \frac{\mu_t}{Sc_t}
\]

The constants of the standard \( k, \epsilon \)-turbulence model used and the turbulent Prandtl and Schmidt numbers are given in Table 1. Source terms \( (S_{\rho,\rho}) \) due to gas/droplet interactions are described in combination with the new droplet model. The wall function method is used to eliminate the large number of grid points needed to resolve the laminar sublayer ([Launer and Spalding (1974)])

\[
\frac{d D_p}{dt} = -F \frac{d M_{e,\rho} n_{ref} T_{ref} \ln (\frac{1-X_{e,\infty}}{1-X_{e,p}})}{p D_p}
\]

with

\[
F = 1 + 0.276 \sqrt{Re_p Sc^*}
\]

is given by Faeth (1983) and Wittig et al. (1988) assuming a constant liquid density. The calculation of the reference values and the mol fraction of the vapour is shown in Appendix A.

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_{\mu} )</th>
<th>( \sigma_k )</th>
<th>( \sigma_\epsilon )</th>
<th>( Pr_t )</th>
<th>( Sc_t )</th>
</tr>
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<tbody>
<tr>
<td>1.44</td>
<td>1.92</td>
<td>0.09</td>
<td>0.90</td>
<td>1.30</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 1: Constants of the \( k, \epsilon \)-turbulence model

**Droplet equations**

The equations describing continuity and momentum exchange of the liquid phase for laminar flow conditions can be derived from a mass- and momentum balance at an infinitesimal small fluid volume:

\[
\frac{\partial}{\partial x_i} (v_c \rho_p u_i,\rho) = S_{c_e}
\]

\[
\frac{\partial}{\partial x_i} (v_c \rho_p u_i,\rho, u_j,\rho) = 3 \frac{1}{4} v_c \rho_c \frac{C_D}{D_p} \epsilon_{ret}(u_j - u_j,\rho) + F_j + S_{c_e,\rho}
\]

with

\[
c_{ret} = \| \bar{u} - u_p \|
\]

\[
Re_p = \frac{D_p \epsilon_{ret}}{\nu}
\]

\[
c_D = 0.36 + 5.48 R e_p^{-0.573} + \frac{24}{Re_p}
\]

The mass source term due to droplet evaporation is obtained by

\[
S_{c_e} = v_c \rho_p \frac{3}{D_p} \frac{d D_p}{dt}
\]

The term on the l.h.s. of Eq. (13) represents the inertia force per unit volume due to droplet acceleration. The first term on the r.h.s. represents the drag force due to the slip between the two phases, the second contains external forces like gravity and the third takes momentum loss due to droplet evaporation into account. Pressure gradient terms for the dispersed phase are neglected. An expression for the decreasing diameter of a single droplet in Lagrangian coordinates

\[
F = 1 + 0.276 \sqrt{Re_p Sc^*}
\]

is given by Faeth (1983) and Wittig et al. (1988) assuming a constant liquid density. The calculation of the reference values and the mol fraction of the vapour is shown in Appendix A.
In order to derive a transport equation for the droplet diameter we first write:

$$\frac{\partial}{\partial t} (v_c \rho_p u_{i,p} D_p) = D_p \frac{\partial}{\partial x_i} (v_c \rho_p u_{i,p}) + v_c \rho_p u_{i,p} \frac{\partial D_p}{\partial x_i}$$

Including Eqs. (12) and (17) yields:

$$\frac{\partial}{\partial x_i} (v_c \rho_p u_{i,p} D_p) = \frac{4}{3} S_{\nu,D} D_p \tag{20}$$

A transport equation for the droplet temperature can be derived via an energy balance in the same manner as the momentum equation:

$$\frac{\partial}{\partial x_i} (v_c \rho_p u_{i,p} T_p) = v_c \frac{6 \nu F \lambda_{ref}}{c_{p,p} D_p^2} (T_{\infty} - T_p) + S_{\nu, T} \tag{21}$$

with

$$N u = \frac{2 L e \ln \left( \frac{1 - X_{p,\infty}}{1 - X_{p,p}} \right)}{\frac{1 - X_{p,\infty}}{1 - X_{p,p}} - 1} \tag{22}$$

$$L e = \frac{\Gamma_{ref} c_{p,v,ref}}{\lambda_{ref}} \tag{23}$$

$$F r = 1 + 0.276 \sqrt{R e_p} P r^{1/4} \tag{24}$$

The model implies a uniform temperature distribution in the droplet assuming an infinite thermal conductivity. In Eq. (21) the first term of the r.h.s. represents droplet heating due to the temperature gradient between the two phases, the second droplet cooling due to droplet evaporation. The specific heat of the liquid $c_{p,p}$ is assumed to be constant. An exact derivation of the expressions describing droplet heating and cooling in the source term of Eq. (21) is given by Faeth (1983) and Wittig et al. (1988).

**Turbulence modelling**

Turbulence effects are evaluated by introducing fluctuation quantities for the volume fraction and the droplet's velocity, temperature and diameter:

$$\Phi_p = \Phi_p + \Phi_p' \tag{25}$$

$$\Phi_p = \frac{1}{\Delta t} \int_{t}^{t + \Delta t} \Phi_p dt \tag{26}$$

Inserting Eq. (25) in the l.h.s. of the Eqs. (12), (13), (20) and (21), time averaging and using the following gradient hypothesis for the second order correlations representing the turbulent fluxes of momentum, mass and enthalpy

$$- \rho_p v'_c u'_j, p = \mu_{*p} \frac{\partial v'_c}{\partial x_i} \tag{27}$$

$$- \rho_p v'_c u'_j, p = \mu_{*p} \frac{\partial u'_j}{\partial x_i} \tag{28}$$

$$- \rho_p \frac{\partial D'_p}{\partial x_i} = \mu_{*p} \frac{\partial T'_p}{\partial x_i} \tag{29}$$

leads to a set of equations for turbulent evaporating sprays which can easily be casted in a form identical to commonly used gas phase equations:

$$\frac{\partial}{\partial x_i} (v_c \rho_p u_{i,p} u_{j,p}) = \frac{\partial}{\partial x_i} \left( \frac{\mu_{*p}}{S_{ci,p}} \frac{\partial v_c}{\partial x_i} \right) + S_{\nu, u_{i,p}} \tag{31}$$

$$\frac{\partial}{\partial x_i} (v_c \rho_p u_{i,p} u_{j,p}) = \frac{\partial}{\partial x_i} \left( \frac{\mu_{*p}}{S_{ci,p}} \frac{\partial u_{i,p}}{\partial x_i} + \frac{\partial u_{j,p}}{\partial x_j} \right) + 3 \frac{v_c \rho c_D c_{v,ci,j}(u_j - u_{i,p})}{D_p} + F_{j} + S_{\nu, u_{i,p}} \tag{32}$$

$$\frac{\partial}{\partial x_i} (v_c \rho_p u_{i,p} D_p) = \frac{\partial}{\partial x_i} \left( \frac{\mu_{*p}}{S_{ci,p}} \frac{\partial D_p}{\partial x_i} \right) + 4 \frac{1}{3} S_{\nu, D_p} \tag{33}$$

$$\frac{\partial}{\partial x_i} (v_c \rho_p u_{i,p} T_p) = \frac{\partial}{\partial x_i} \left( \frac{\mu_{*p}}{S_{ci,p}} \frac{\partial T_p}{\partial x_i} \right) + \frac{6 \nu F \lambda_{ref}}{c_{p,p} D_p^2} (T - T_p) + S_{\nu, T} \tag{34}$$

Correlations involving fluctuations in the liquid phase density ($\rho_p$, $v_c$) are taken into account only in the continuity equation.

The closure hypothesis (27) and (28) for the turbulent mass flux and momentum transfer have been tested successfully by a lot of researchers (Chen and Wood (1984), Elghobashi et al. (1984), Melville and Bray (1979)) and have been extended by Hallmann et al. (1993) for turbulent heat and mass transfer in evaporating sprays (Eqs. (29), (30)). The turbulent viscosity $\mu_{*p}$ of the dispersed phase is modeled using the approach of Melville and Bray (1979):

$$\mu_{*p} = \mu_t \frac{\rho_p E_p}{\rho k} \tag{35}$$
with

\[ k_p = \frac{1}{2}(u_p'^2 + v_p'^2 + w_p'^2) \]  

(36)

The ratio of the turbulent kinetic energies of the dispersed and the gas phase is calculated following the approach of Krämer (1988):

\[ \frac{k_p}{k} = \frac{1}{1 + \omega^2 \tau^2} \]  

(37)

Since in general the droplets do not follow the motion of the surrounding fluid from one point to another it is expected that the ratio \( k_p/k \) is different from unity and varies with the particle relaxation time \( \tau \) and local turbulence quantities (Elghobashi et al. (1984)). Krämer recommends the following equations for the frequency of the particle response

\[ \omega = \frac{1}{\tau} \left( \frac{3\sqrt{2} k}{L_x} \right)^{0.25} \]  

(38)

\[ \tau = \frac{1}{18} \frac{\rho_p D^2}{\nu} \frac{1}{1 + 0.133 R_e^{0.687}} \]  

(39)

with a characteristic macroscopic length scale of turbulence

\[ L_x = C_\mu^{0.75} \frac{k^{1.5}}{\epsilon} \]  

(40)

For the turbulent Schmidt number \( Sct,p \) of the dispersed phase Krämer suggests a value of 0.3. In the course of our work we found this value to be particle size dependent. Because more detailed information is not available at the moment we recommend a median value of 2.5 for problems treated in this paper.

**Coupling with the gas phase equations**

The gaseous phase is affected by the dispersed phase due to momentum, heat and mass exchange. The easy way of coupling the two phases is an evident advantage of the new model. Due to an identical mathematical formulation the same terms appear for both phases, simply with different sign. A drawing up of the coupling terms for the gas phase equations is given in Table 2.

The expressions are added up for all nd discrete drop-size classes. In addition phase coupling is guaranteed by the use of temperature, pressure and vapour concentration dependent properties in the calculations of the gaseous phase.

Source terms \( (S_{k,p}, S_{\epsilon,p}) \) for the equations of the \( k, \epsilon \)-turbulence model due to the presence of the dispersed phase were modeled following the approach of Chen and Wood (1984, 1986). Because the calculations presented in this paper do not show any significant influence of these terms they are not taken into consideration here.

### Table 2: Coupling terms for the gas phase equations due to droplet transport and evaporation

<table>
<thead>
<tr>
<th>( \Phi )</th>
<th>( S_{\Phi,p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>(- \sum_{k=1}^{nd} (S_{\rho,c}) )</td>
</tr>
<tr>
<td>( c_\alpha )</td>
<td>(- \sum_{k=1}^{nd} (S_{\alpha,c}) )</td>
</tr>
<tr>
<td>( u_j )</td>
<td>(- \sum_{k=1}^{nd} \left( \frac{3}{2} \rho_p \frac{\partial G}{\partial x_j} \epsilon_{rel} (u_j - u_{j,p}) + S_{\epsilon_c} u_{j,p} \right) )</td>
</tr>
<tr>
<td>( h )</td>
<td>(- \sum_{k=1}^{nd} \left( c_{\rho,p} \left( \frac{v_e}{\epsilon_{rel}} \left( c_{\epsilon,p} \right) \frac{D^2}{\nu} (T - T_p) + S_{\epsilon_c} \left( \frac{\delta S_{\epsilon,p}}{T_p} \right) \right) \right) )</td>
</tr>
</tbody>
</table>

**Solution steps**

As mentioned above, the equations of both phases can be solved numerically using the same finite volume discretization method (Noll and Wittig (1991), Noll (1992)). After calculating the droplet field with the preceding values of the gas field properties, the gas field is recalculated with the coupling terms due to droplet transport and evaporation. This procedure is repeated until the coupling terms converge, i.e. both phases have statistically constant values.

**Results and discussion**

For the verification of the new model, measurements of a recirculating droplet charged air flow (Himmelebach (1987), Wittig et al. (1987), Wittig et al. (1988)) within a model combustor are compared with. The test section used in our laboratory is shown in Fig. 1. It consists of a rectangular flow channel with a cross sectional area of 100*300 mm². A prefilming two-dimensional airblast nozzle is incorporated into the test section. The airflow enters the channel through four slots with 60 m/s mean air velocity. Two of these slots are charged with a liquid film. The experimental studies were performed with ethanol at inlet gas temperatures of 320 K and 520 K.

The symmetric flow field in the model combustor is characterized by a recirculation zone induced by the centerbody of the nozzle (Fig. 2). As reported earlier (Wittig et al. (1987)) it can be predicted with
sufficient accuracy. In the calculations a 68•36 computational grid has been used with the MLU-scheme (Noll (1992)) for the discretization of the convective terms. The convective terms of the droplet equations are discretized using the well known UPWIND-scheme.

Droplet motion and evaporation were calculated for ten discrete drop-size classes. In Figs. 3 and 4 calculated volume fractions of the liquid phase and characteristic droplet diameters of the spray are plotted against experimental data given by Wittig et al. (1987) for an inlet gas temperature of 320 K. The initial conditions for the droplets were determined from elaborate studies of air-blast atomizers as a function of the operating parameters of the nozzle and the properties of the liquid (Aigner (1986), Sattelmayer (1989)). They can be described with the distribution parameters \( D_{63,2} = 63 \, \mu m \) and \( N = 2 \) in the Rosin Rammel distribution (see Appendix B). Both, measurements and predictions show that in the recirculation zone mainly small droplets are found due to turbulent dispersion, while the forward flowing regions are dominated by large droplets. Highest droplet concentrations occur close to the atomizer's edge decreasing rapidly towards the recirculation zone. Turbulent particle dispersion is slightly underpredicted. Nevertheless measurements and predictions are in good agreement.

Fig. 5 shows the predicted spatial distribution of the decreasing droplet diameter in the upper half of the combustor for an inlet gas temperature of 520 K. For comparison results of a Lagrangian approach are shown. Both computational approaches are based on the same evaporation model, the well known 'Uniform Temperature' law (Faeth (1983), Wittig et al. (1988), Hallmann et al. (1993)). The excellent agreement of the two methods in almost all details is proof that the new Eulerian model yields adequate results for the diameter decrease of the evaporating droplets. It should be noted that the shape of the Eulerian solution in Fig. 5 can be influenced by variations of the turbulent Schmidt number \( Sc_{t,p} \). As mentioned before, \( Sc_{t,p} \) is not a constant and further investigations in the area of turbulent droplet dispersion are necessary to guarantee a general application of the new model. Fluctuations, which can be seen in the Lagrangian results, are due to the statistical nature of the Monte Carlo sampling method. They could be damped by increasing the number of particles used to evaluate mean droplet diameters.

Figs. 6 and 7 show calculated and measured volume fractions (Himmelsbach (1987)) and characteristic droplet diameters for \( T = 520 \, K \) (Wittig et al. (1987, 1988)). The larger diameters compared to the cold flow conditions are caused by larger initial diameters \( D_{63,2} = 78 \, \mu m, \, N = 2 \) but also by the faster evaporation of small droplets. The computational results are of sufficient agreement with the measurements for both, volume fractions and characteristic diameters of the dispersed phase.

Effects of phase coupling can be seen in Fig. 8 and Fig. 9 for hot flow conditions. The vapour concentration of the gas phase shows a strong increase along the way of the evaporating droplets combined with a decrease of the gas temperature. The recirculating flow transports parts of the cold vapour/air mixture back to the centerbody of the nozzle resulting in very high vapour concentration and temperature gradients near the atomizing edge. At the channels outlet the vapour concentration reaches approximately 10 % resulting in a temperature decrease of more than 100 degrees.

Conclusions

A new Eulerian model for the computation of turbulent evaporating sprays has been developed. In contrast to former Eulerian approaches it comprises transport equations for droplet heating and evaporation and is applicable for recirculating flows. The coupling of the gaseous and the droplet phase is guaranteed by the exchange of source terms due to momentum, heat and mass transfer and by the calculation of temperature, pressure and vapour concentration dependent properties of the gaseous phase. Comparison of the new model with Lagrangian calculations and experiments with respect to diameter distributions and concentrations reveal good agreement.

However, the physical understanding of the turbulent particle dispersion processes need further improvements for a more general application. Nevertheless, the similar structure of the transport equations obtained by the Eulerian approach with the commonly used gas phase equations offers the opportunity for an easy incorporation of our new model in standard CFD-codes. This is a great advantage in contrast to Lagrangian methods which require different numerical procedures for the two phases. In addition, the implementation of Lagrangian methods in codes for boundary fitted non-orthogonal coordinates or unstructured grids is very complicated, but is easily accomplished with the present formulation.

Acknowledgements

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References


Appendix A

The reference values in the models presented are determined according to the 1/3-rule of Sparrow and Gregg (1958)

\[ T_{ref} = \frac{2}{3} T_p + \frac{1}{3} T_\infty \]  

\[ X_{v,ref} = \frac{2}{3} X_{v,p} + \frac{1}{3} X_{v,\infty} \]  

\[ n_{ref} = n(T_{ref}) \]  

\[ \Gamma_{ref} = \Gamma(T_{ref}) \]  

\[ c_{p,v,ref} = c_{p,v}(T_{ref}) \]  

\[ \lambda_{ref} = \lambda(T_{ref}) \]  

The mol fraction of vapour at the droplet's surface is given by an exponential law following Cox-Antoine

\[ X_{v,p}(p_s) = \frac{p_s}{p} \]  

\[ p_s(T_p) = \exp \left( A - \frac{B}{T_p + C} \right) \]  

where A, B and C are specific values for the droplet liquid under consideration.

The relation between the vapour concentration \( c_\alpha \) and the mol fraction of vapour \( X_v \) is

\[ c_\alpha = \frac{X_v M_v}{X_v M_v + (1 - X_v) M_{air}} \]

Appendix B

In the Rosin Rammler drop-size distribution

\[ D_z = D_{63.2} \ln \left( \frac{100}{100 - z} \right)^{\frac{1}{b}} \]

\( z \) indicates the volume percentage of all droplets with smaller diameters than \( D_z \). Two characteristic diameters are needed to completely define the distribution. In presenting the results the volumetric mean diameter \( D_{60} \) and a characteristic diameter for small droplets \( D_{10} \) are used.

Figures

Figure 1: Test section - two dimensional combustor model (Wittig et al. (1987))
Figure 2: Calculated flow field within the model combustor

Figure 3: Volume fractions for cold flow conditions; △ measurements, — calculations

Figure 4: Characteristic diameters for cold flow conditions; measurements: △ \( D_{10} \), ○ \( D_{50} \), — calculations
Figure 5: Spatial distribution of the droplet diameter for a starting diameter of 17.67 μm; upper diagram: Eulerian approach, lower diagram: Lagrangian approach

Figure 6: Volume fractions for hot flow conditions; △ measurements, — calculations

Figure 7: Characteristic diameters for hot flow conditions; measurements: Δ $D_{10}$, ○ $D_{50}$, — calculations
Figure 8: Calculated fuel vapour distribution for hot flow conditions

Figure 9: Calculated gas temperature distribution for hot flow conditions

Discussion

Question 1. Dr. L. Ianovski
Why do you not take account of the thermophoresis force in your model for droplets?

Author's Reply
In the flow considered, these forces are of negligible influence.

Question 2. A. Kleitz
In your experimental device, you use a Malvern which doesn't give a "true" local size measurement. What is the size of the measuring volume?

Author's Reply
Knowing that the Malvern Particle Sizer integrates along the laser beam, we have chosen two-dimensional experimental conditions which allows us locally resolved measurements.

Question 3. C. Hassa
Have you compared calculations with experimental investigations giving a liquid flux distribution: for instance, Snyder and Lumley, JFM 1977?

Author's Reply
We compared our predictions with measurements from Hishida et al, which were presented at the "5th Workshop on Two Phase Flow Modelling" in Erlangen. These measurements include particle velocities and volume fluxes for different flow configurations, and these measurements are good.