Stellingen behorende bij het proefschrift van Piotr Ptasinski


2. In de relatie tussen de frictiefactor en het Reynoldsgetal bij hoge weerstandsvermindering is het van essentieel belang om rekening te houden met het feit dat de betreffende polymeeroplossingen sterk afschuifverdunnend gedrag vertonen. Voor een correcte bepaling van Virk's asymptoot dient men het Reynolds getal te baseren op de viscositeit aan de wand (dit proefschrift).

3. Het is te verwachten dat de uiteindelijke verklaring voor weerstandsreductie eerder uit de reologie dan uit de stromingsleer komt, omdat de reologische vergelijkingen zelfs onder eenvoudige condities tot nu toe minder goed verifieerbaar zijn gebleken dan de stromingsvergelijkingen.

4. De juiste bepaling van reologische parameters in een eenvoudig karakterisatie-experiment biedt geen garantie dat deze parameters toereikend zijn in een complexe stromingsberekening.

5. De werkzaamheden tijdens een promotieonderzoek zijn veel meer die van een baan dan van een studie. De Engelse benaming *Ph.D. student* is dan ook misleidend.

6. De beslissing om universitaire onderzoeksgebieden af te stoten zou genomen moeten worden op grond van wetenschappelijke en inhoudelijke argumenten in plaats van toevallige factoren zoals bijvoorbeeld vertrek of pensioen van de hoogleraar.

7. De introductie van de mobiele telefoon heeft geleid tot een grotere bereikbaarheid maar een kleinere vrijheid.

8. Van wetenschappelijke publicaties zouden in de 'submitted version' de auteurs moeten worden weggelaten en in de 'final version' de reviewers met naam moeten worden genoemd.

9. Resultaten van numerieke simulaties kunnen alleen als waar worden aangenomen indien zij experimenteel verifieerbaar zijn.

10. Managers functioneren goed als je hun aanwezigheid niet merkt.

11. Een promotieonderzoek is vaak zeer specialistisch, zodat de promovendus steeds meer kennis opdoet over een steeds kleiner vakgebied. Het is dan ook van groot belang te zorgen voor voldoende verbreding op een ruim vakgebied, zodat men aan het eind van het promotietraject meer weet dan absoluut alles van absoluut niets.
Turbulent flow of polymer solutions near maximum drag reduction

Experiments, simulations and mechanisms
Turbulent flow of polymer solutions near maximum drag reduction

Experiments, simulations and mechanisms

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Summary

Turbulent flow of polymer solutions near maximum drag reduction: experiments, simulations and mechanisms

Piotr K. Ptasinski

Dissolving a small amount of long-chain polymers in the turbulent flow of a Newtonian solvent has a significant effect on the character of turbulence. The result is for instance a large reduction of frictional drag for turbulent pipe or channel flows. This phenomenon was discovered in 1949 and has received a lot of attention afterwards, partly because of its practical use in various applications such as for instance in oil pipe lines, heating and sewer systems. In terms of turbulence theory the most interesting fact is that the polymers are primarily active on the smallest length scales, but they are nevertheless able to influence the macroscopic scales of the flow and the drag. Such behaviour contrasts with the commonly accepted picture of turbulence. Therefore, understanding the phenomenon of drag reduction is challenging both from an applied and a fundamental point of view.

This thesis deals with turbulent flows of polymer solutions in particular near maximum drag reduction. Maximum drag reduction means that the drag reduction approaches an empirically found asymptote, called the maximum drag reduction or Virk asymptote. This happens when the polymer concentration is high enough and when strongly extensible polymers are used. The objective of this thesis is to study the behaviour of turbulent flows near this asymptote and to get insight in the effect of polymers on the dynamics of turbulence and in the mechanism of drag reduction. This study is carried out by means of experimental and numerical methods. In all experiments and simulations with polymers, the flow remains fully turbulent, but its statistics are significantly modified with respect to Newtonian flow.

The experiments are performed in a recirculatory pipe flow facility with a length of 34m and a diameter of 40mm. Laser Doppler Velocimetry (LDV) is used to measure the mean velocity and turbulence statistics. The polymer solutions consist of a small amount (in the order of 0.01% mass fraction) of linear high-molecular weight polymers dissolved in water. Only one polymer type (polyacrylamide called Superfloc A110) is used, because of its relative insensitivity to mechanical degradation. A variation in the amount of drag reduction is made by using three different polymer concentrations. Values
near the maximum drag reduction asymptote are reached for the highest concentrations. The polymer solutions show significant shear-thinning behaviour. This effect has been taken into account through the definition of the Reynolds number, which is based on the local viscosity at the pipe wall. It turns out that by using this definition of the Reynolds number a good agreement is obtained with the maximum drag reduction asymptote, while this is not the case when the Reynolds number is based on the viscosity of the solvent.

In the second part of this thesis direct numerical simulations of turbulent polymer flow in a rectangular channel are considered. The computations are performed in a so-called minimal flow unit (MFU), which is the smallest flow geometry in which the essential turbulence characteristics are present. The polymers are modeled as elastic dumbbells using the FENE-P model. This model leads to a closed-form constitutive equation for the polymer stress and this keeps the computational effort relatively economic. In the computations the equations of the polymer model are two-way coupled with the flow equations, i.e. the polymers are deformed by the flow and on their turn they influence the flow by exerting a polymer stress. A variation of polymer parameters, such as the maximum extension, the elasticity and the concentration, is considered to obtain different amounts of drag reduction. For the case of highly extensible polymers the results of our simulations are very close to the maximum drag reduction or Virk asymptote. For the discretization of the equations pseudospectral and finite difference methods are used. For the time integration a mixed explicit-implicit algorithm is used in order to prevent the dumbbells exceeding their maximum length.

In general, the agreement between the results of the experiments and the simulations, both qualitatively and quantitatively, is very good. The main changes for the drag reducing flows with respect to Newtonian flow can be summarized as follows. The mean velocity profile shows for small drag reduction a parallel translation upwards with respect to the Newtonian profile in the logarithmic region. For maximum drag reduction the slope is significantly higher and consistent with the theoretical profile of the asymptote. The turbulence intensities show initially an increase of the peak for the streamwise direction, but a decrease at maximum drag reduction. A continuous decrease as function of polymer concentration is found for the spanwise and wall-normal directions. The Reynolds stress is very strongly suppressed, but remains definitely non-zero. This decrease of Reynolds stress is compensated by a large polymer stress, which can be almost 60% of the total mean shear stress. The edge of the buffer layer and the maxima of the turbulence intensities are shifted away from the wall leading to a thickening of the buffer layer. This is consistent with Lumley's proposal for the mechanism of drag reduction. The polymers are strongly extended, especially near the wall. Furthermore it is found that the largest part of the kinetic energy present in the flow is transferred directly into elastic energy of the polymers where it is dissipated by polymer relaxation. This contrasts with the standard way of turbulent dissipation of energy. The redistribution of the turbulent kinetic energy over the three coordinate directions is also strongly reduced and this provides an explanation for the changes of the turbulence intensities, especially the increase of the streamwise fluctuations.
These simulation results are explained here by two mechanisms. Firstly, as suggested by Lumley (1969) the polymers damp the cross stream or wall-normal velocity fluctuations and suppress the bursting in the buffer layer. Secondly, the 'shear sheltering' mechanism acts to amplify the streamwise fluctuations in the thickened buffer layer, while reducing and decoupling the motions below and above this layer. The expression for the substantial reduction in the wall drag, derived by considering the time scales of the non-linear fluctuations of this damped shear layer, are shown to be consistent with the experimental data of Virk.
Samenvatting

Turbulente stroming van polymeeroplossingen bij maximale weerstandsvermindering: experimenten, simulaties en mechanismen

Piotr K. Ptasinski

Het oplossen van een kleine hoeveelheid van lange polymeerketens in een turbulente vloeistofstroming heeft een significant effect op de eigenschappen van turbulentie van deze stroming. Het resultaat is bijvoorbeeld een sterke vermindering van de wrijvingsweerstand in turbulente buis- of kanaalstromingen. Dit verschijnsel is ontdekt in 1949 en heeft sindsdien veel aandacht gekregen, deels vanwege de toepassingen in bijvoorbeeld in oliestromingen, verwarmings- en afvoersystemen en deels vanwege de fysische achtergrond van dit verschijnsel. Wat betreft de theorie van turbulentie is het meest interessante feit dat polymeren primair actief zijn op de kleinste lengteschalen, maar toch in staat zijn om de macroschalen en de weerstand te beïnvloeden. Dit is in tegenspraak met de klassieke theorie van turbulentie. Derhalve is het begrijpen van dit verschijnsel van weerstandsreductie uitdagend zowel vanuit een toegepast als ook vanuit een fundamenteel perspectief.

Dit proefschrift behandelt turbulente stromingen van polymeeroplossingen, in het bijzonder nabij maximale weerstandsvermindering. Bij maximale weerstandsvermindering nadert de weerstand tot een asymptotische waarde, genaamd de maximale weerstandsvermindering (maximum drag reduction, MDR) of Virk asymptoot. Deze asymptoot wordt gevonden bij gebruik van voldoende hoge polymeerconcentratie en sterk strekbare polymeren. Het doel van dit proefschrift is het bestuderen van turbulente stromingen nabij deze asymptoot en inzicht te verkrijgen in het effect van polymeren op de dynamica van turbulentie en in het mechanisme van weerstandsvermindering. Het onderzoek is uitgevoerd door middel van experimentele en numerieke methoden. Tijdens alle experimenten en simulaties met polymeren blijft de stroming turbulent, maar veranderen de statistieken van de stroming significant ten opzichte van een Newtonse stroming.

De experimenten zijn uitgevoerd in een circulaire buis met een lengte van 34m en een diameter van 40mm. De techniek van Laser Doppler Velocimetry (LDV) is gebruikt voor het meten van de gemiddelde snelheid en van de statistieken van turbulentie. De polymeeroplossingen bestaan uit een kleine hoeveelheid (in de orde van 0.01% massafractie)
van lineaire polymeren met een hoog molecuulgewicht, die opgelost zijn in water. Er is één polymerte type (polycrylamide genaamd Superfloc A110) gebruikt, omdat deze weinig mechanische degradatie ondervindt. Door het gebruik van drie verschillende polymerconcéntraties kan een variatie in de hoeveelheid weerstandsvermindering worden bereikt. Het gebied dichtbij de maximale weerstandsvermindering wordt bereikt voor de hoogste concentraties. De polymeeroplossingen vertonen sterk afschuwverdunnend gedrag. Hiermee moet rekening worden gehouden in de definitie van het Reynolds getal, welke wordt gedefinieerd met behulp van de lokale viscositeit aan de wand. Het blijkt dat met deze definitie een goede overeenkomst wordt bereikt met de in de literatuur bekende maximale weerstandsverminderings asymptoot, hetgeen niet het geval is wanneer het Reynolds getal wordt bepaald met de viscositeit van de solvent.

In het tweede gedeelte van dit proefschrift worden directe numerieke simulaties van turbulent polymerstromingen in een rechthoekig kanaal behandeld. De berekeningen zijn uitgevoerd in een zogenaamde ‘minimal flow unit’ (MFU), die de kleinste geometrie is waarin de essentiële turbulentie karakteristieken bestaan. De polymeren worden gemodelleerd als elastische halters met behulp van het FENE-P model. Dit model leidt tot een gesloten constitutieve vergelijking voor de polymerspanning en zorgt ervoor dat de berekeningen relatief efficiënt uitgevoerd kunnen worden. In de berekeningen zijn de vergelijkingen van het polymere model tweezijdig gekoppeld met de stromingsvergelijkingen, hetgeen wil zeggen dat de polymeren enerzijds worden gedefinieerd door de stroming en anderszijds dat zij de stroming beïnvloeden door het uitoefenen van een polymeerspanning. Het variëren van de diverse parameters van de polymeren, zoals de maximale lengte, de elasticiteit en de concentratie, leidt tot verschillende waarden van de weerstandsverminderings. Voor het geval van sterk strekbaar polymeren liggen de resultaten van de simulaties vlakbij de maximale weerstandsverminderings asymptoot van Viskr asymptoot. Voor de discretisatie van de vergelijkingen zijn pseudospectrale en eindige differentie methoden gebruikt. Voor de tijdsintegratie is gebruikt gemaakt van een gemengd expliciet/impliciet algoritme, om te vermijden dat de polymeren hun maximale lengte overschrijden.

In het algemeen is de overeenkomst tussen experimenten en simulaties, zowel kwalitatief als kwantitatief, erg goed. De belangrijkste veranderingen ten opzichte van een Newtonse stroming zijn de volgende. Het profiel van de gemiddelde snelheid vertoont voor geringe weerstandsverminderings een parallele verschuiving omhoog ten opzichte van een Newtonse stroming in het logaritmische gebied. Voor maximale weerstandsverminderings is de helling groter en overeenkomend met het theoretische profiel van de asymptoot. Het maximum van de turbulentie intensiteit in de stromingsrichting vertoont aanvankelijk een toename, maar neemt weer af voor maximale weerstandsverminderings. Voor de dwarsrichting en de richting loodrecht op de wand neemt de turbulentie intensiteit af als functie van de polymeerspanning. De Reynoldsverminderings neemt zeer sterk af, maar blijft ongelooflijk aan nul. De afname van de Reynoldsverminderings wordt gecompenseerd door een hoge polymeerspanning, welke kan oplopen tot 60% van de totale gemiddelde schuifspanning. De bufferlaag en de maxima van de turbulentie intensiteiten verschuiven van de wand af, het-
geen leidt tot een dikkere bufferlaag. Dit komt overeen met Lumley's mechanisme voor weerstandsvermindering. De polymeren worden sterk gestrekt, in het bijzonder vlakbij de wand. Het grootste gedeelte van de kinetische energie van de stroming wordt direct omgezet naar elastische energie van de polymeren, waar het gedissepeerd wordt door polymeerrelaxatie. Dit is anders dan de Newtonse dissipatie van turbulente energie. De herverdeling van de turbulente kinetische energie over de drie richtingen wordt sterk onderdrukt, hetgeen de veranderingen in de turbulentie intensiteiten verklaart, met name de toename van de intensiteit in de stromingsrichting.

De resultaten van de simulaties worden verklaard door twee mechanismen. Ten eerste, zoals voorgesteld door Lumley, onderdrukken de polymeren de snelheidsfluctuaties loodrecht op de wand en het proces van 'bursting' in de bufferlaag. Ten tweede, zorgt het mechanisme van 'shear sheltering' voor een toename van de snelheidsfluctuaties in de stromingsrichting in de dikkere bufferlaag, terwijl de interactie van de stroming boven en onder deze laag wordt onderdrukt. De relatie voor de sterk verminderde weerstand wordt afgeleid uit de tijdschalen van de niet-lineaire fluctuaties in de buffer laag en komt goed overeen met de experimenten van Virk.
Chapter 1

Introduction

1.1 The phenomenon of drag reduction

The character of turbulence in dilute polymer solutions differs substantially from that in Newtonian fluids. One of the most notable differences is that addition of a small amount of polymers to water leads to a drastic reduction of the frictional drag of turbulent pipe or channel flow. This phenomenon was first discovered by Toms (1949) and is nowadays known as 'turbulent drag reduction'. Toms found that addition of a very minute amount of polymers, e.g. a few weight parts per million (wppm), reduces the frictional drag by several tens of percents. Since this discovery the subject has received a lot of attention, not the least because of its practical use in various applications.

One of the first large-scale applications of polymeric drag reduction was in the Trans Alaska Pipeline in 1979 (Burger et al. 1980). In this case oil-soluble polymers are continuously injected in the system, which results in a large increase of the flow rate at a constant pressure drop and hence an increased production per unit of time. The process has turned out to be so beneficial that two pump stations did not have to be built. Since this time much progress has been made regarding the effectiveness of the polymers and in 1995 one achieved an increase of the flow rate by 33%, when using an amount in the order of 1 wppm of polymers. This improvement was based on the observation that a small amount of high molecular weight polymers obtained by means of a special polymerization technique is more effective and less expensive than a large amount of low molecular weight polymers. In these drag reducing flows the heat exchange is also reduced. This leads to a higher temperature of the crude oil and therefore a lower viscosity and thus a higher flow rate at a constant pressure drop.

Another application is in sewer systems (see Sellin & Ollis 1980). In case of overload of the sewer system after heavy rainfall, one can add polymers to the system and thus increase the flow rate and the capacity of the system. This is a quick and economic solution opposite to constructing and building of additional pumping capacity.

In the future some biological and medical applications might become interesting. For
instance, some fish or plant produce a slime, which may work as a drag reducer. Such slime could become beneficial in biomedical products, because it can have a higher acceptability to living bodies than artificial additives. Addition of appropriate additives to blood can reduce its pressure and enhance the circulation. Finally, there exist applications in irrigation systems, regional and city heating systems, ship flows and fire fighting.

Although drag reduction is the most important consequence of polymer addition for practical applications, it is also known that the polymers have a more fundamental effect on the turbulence. In particular the character and structure of the flow with additives differs substantially from that without. This different character of non-Newtonian turbulent flows and the phenomenon of drag reduction have now been studied for more than 50 years. Nevertheless a full and convincing explanation for the physical mechanism is still lacking. Therefore, also from a fundamental point of view the subject remains challenging. It is probable that the different flow character cannot be attributed to one effect only, but that there are several effects acting simultaneously by which polymers are able to influence turbulence on all length scales. Clearly the macroscopic turbulent flow behaviour has in some way an interaction with the microscopic behaviour of the polymers. Turbulence upto now is a not fully understood phenomenon and by studying its interaction with polymers and the way polymers modify structures one can perhaps get more insight in the dynamics of turbulence itself. On the other hand, it is known that the polymers are strongly deformed in turbulent flows (e.g. linear polymers in straining flow become strongly extended) and that the polymer configuration depends on a complex time-dependent deformation history. Therefore, insight in the behaviour of polymers in turbulence can lead to improvement of polymer models in complex time-dependent flows. In the following two sections a short introduction of the fields of turbulence and polymers will be presented.

1.2 Turbulence

In general one can distinguish two types of flow, which have a completely different nature: laminar and turbulent. While laminar flows are layered, smooth and determined, turbulent flows are characterized by irregularity, fluctuations and chaos. A flow is considered to be turbulent, when non-linear inertial forces are dominant over viscous forces. In other words, turbulence exists when the Reynolds number, which is a dimensionless ratio of inertial and viscous forces, exceeds a certain value. One of the characteristics of turbulence is that it efficiently transports momentum and therefore the frictional drag of turbulent flows is much larger than of laminar flows.

As the Reynolds number for most flows in nature and in engineering is large, these flows are usually turbulent. Examples are flows in oceans, rivers and the atmosphere, but also flows occurring in chemical, oil and aeronautical industries. Turbulence changes various flow processes, such as e.g. mixing, particle and droplet deposition, heat transfer, chemical reactions and combustion. Despite the fact that the equations of motion are
fully known (the Navier-Stokes equations), the non-linear effects make their full solution non-trivial. Individual realizations of a turbulent flow with almost the same boundary and initial conditions have complete different solutions and are in some sense unpredictable. Therefore it is common to use statistical techniques, which will be often applied in this thesis.

For a general review of turbulence theory the reader is referred to several textbooks (e.g. Tennekes & Lumley 1973; Hinze 1975; Pope 2000). In general, turbulence can be characterized as a multi-scale, chaotic, intermittent and time-dependent state of motion, in which three-dimensional vortical regions and regions of strong deformation occur. A turbulent flow consists of a broad range of spatial scales. On one side of the spectrum there is the macrostructure, which consists of large flow scales dependent on the geometry of the flow and which determines the macroscopic transport processes which are responsible for the drag. For example, in a pipe flow a characteristic length scale $L$ is related to the pipe diameter and a characteristic velocity scale $U$ is related to the friction velocity. The dynamics of the macrostructure are mainly driven by non-linear inertia effects and because of the high Reynolds number the macrostructure does not depend on viscous effects. Therefore, it is said that turbulence is a property of the flow and not of the fluid. On the other side of the spectrum there is the microstructure, which consists of the smallest scales in the flow. Its dynamics is mainly determined by viscous effects, which take care for the dissipation of turbulent kinetic energy, which is present in the macrostructure. The microstructure is characterized by the kinematic viscosity of the fluid $\nu$ and by the dissipation rate $\epsilon$, which is the rate of energy transported from the macrostructure to the microstructure. The microscopic scales (also called the Kolmogorov scales) are defined by:

$$
\eta = \left( \frac{\nu^3}{\epsilon} \right)^{\frac{1}{4}} \quad \nu = (\nu \epsilon)^{\frac{1}{4}} \quad \tau = \left( \frac{\nu}{\epsilon} \right)^{\frac{1}{4}}
$$

(1.1)

where $\eta$, $\nu$ and $\tau$ are the microscopic length, velocity and time scales, respectively.

The energy transport from large-scales to small-scales is known as the cascade process and continues until the Kolmogorov scales where dissipation of kinetic energy takes place. This energy transport along the cascade can be related to the macroscopic parameters $L$ and $U$ by assuming that the rate at which large eddies pass energy to the small eddies is proportional with the time scale of the large eddies. This time scale is actually the large eddy turnover time and is proportional to $L/U$. The rate at which large eddies supply their energy to small eddies is proportional to the rate of dissipation. Taking the energy per unit mass of the large eddies being proportional to $U^2$ gives as estimation for the rate of dissipation $\epsilon$:

$$
\epsilon \sim \frac{U^3}{L}\quad (1.2)
$$

From equation (1.1) it follows that the ratio of the Kolmogorov length scale and the macroscopic length scale is equal to:

$$
\frac{\eta}{L} \sim Re^{-3/4}\quad (1.3)
$$
where \( Re = \frac{UL}{\nu} \) is the macroscopic Reynolds number. The different flow scales will be discussed further in chapter 5 when presenting a possible mechanism of drag reduction.

Concluding this section we want to mention a few measuring techniques used in turbulent flows. In early times one used visualization techniques, for instance by adding some smoke or dye into the flow. Though this clearly indicates that turbulence is of chaotic (but not random) nature, these observations remain rather qualitative. In later times quantitative information became available by using measurement instruments like pitot tubes, hot wire anemometry and Laser Doppler Velocimetry (LDV). In the experimental part of this thesis the latter technique has been used in order to measure the velocity. Laser Doppler Velocimetry (see Drain 1986; Durst et al. 1976) enables one to determine more than one velocity component simultaneously at a single point in space and to reach a high accuracy. An important aspect of LDV is that the flow is not disturbed by the measuring instrument.

In addition to the above mentioned experimental techniques, numerical simulations have lately become very popular in studying turbulent flows as a result of increasing computer capabilities. One of these simulation techniques is a direct numerical simulation (DNS), which is essentially a three-dimensional time-dependent solution method of the Navier-Stokes equations without any modeling assumptions such as a turbulence closure model. DNS is now frequently used to study turbulence properties and is a helpful tool in the setup and verification of turbulence models. A general overview of direct numerical simulations is provided by Rogallo & Moin (1984) and more recently by Moin & Mahesh (1998). An important achievement was made by Kim et al. (1987), who performed the first direct numerical simulation of a fully developed turbulent channel flow. The advantage of DNS is that it provides information about variables in the flow, which are difficult to measure (like pressure, polymer stress and cross correlations or covariances). A drawback is that in a DNS one has to resolve all significant flow scales. On a three-dimensional grid this means that the number of gridpoints should be proportional with the third power of the length ratio of macro- and microscales, so that from equation (1.3) follows that the number of gridpoints is proportional with \( Re^{9/4} \). DNS is therefore limited to relative low Reynolds numbers and simple flow geometries. In our case it turns out to be an excellent tool in the simulations of polymeric turbulent flows.

1.3 Polymers

The branch of fluid mechanics that describes the behaviour of non-Newtonian materials, among which polymer solutions and melts, is called rheology. Research on polymers originated with the development of the chemical industry in the beginning of the 20th century. Polymer solutions and melts are now widely used in the production of plastics, coatings, paints and synthetic materials. Polymer materials have various applications, e.g. in the oil, food, cosmetics, aeronautical and chemical industry. In all these areas it is important to model the behaviour of the polymers in flow.
INTRODUCTION

The polymers considered in this thesis are molecules that consist of a large sequence of identical monomer units. They are long-chain linear molecules, typically consisting of $10^5$ monomers, and have a molecular weight in the order of $10^6 - 10^7$ g/mol. The polymers are dissolved in a low molecular weight solvent like water, which is for small concentrations denoted as a dilute polymer solution.

Dilute polymer solutions show some unpredictable behaviour. An example is the rod-climbing effect: when a rod is rotated in a beaker, the fluid level around can climb the rod, which is opposite to the effect found in a Newtonian fluid. Another phenomenon is the die swell effect: a fluid that exits from a capillary will increase in diameter instead of the decrease in diameter seen for a Newtonian fluid. These effects appear because the non-Newtonian polymer fluid possesses besides viscous properties also elastic properties. Therefore they are often referred as viscoelastic fluids.

In order to describe the influence of the polymers on the flow behaviour, additional equations are required besides the conservation equations of mass, momentum and energy, which describe a Newtonian fluid flow. These additional equations express the relation between stress and deformation history and they are called constitutive equations. Newtonian fluids like water consist of simple molecules (say at most a few atoms per molecule) so that interactions between the molecules are relatively simple and independent of the details of the molecules. The constitutive equation for a Newtonian fluid is very simple: $\tau = 2\eta \mathbf{D}$, with $\tau$ the stress, $\eta$ the constant shear viscosity and $\mathbf{D} = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$ is the rate-of-deformation tensor (with $\mathbf{u}$ the velocity vector). The material properties are fully specified by the (constant) viscosity. On the other hand, non-Newtonian fluids contain complex molecules (consisting of many atoms), so that more complex interactions between the molecules and complex internal configurations can play a role. Therefore for non-Newtonian fluids the constitutive equations are usually much more complex and more material functions (describing for instance normal stress and extensional viscosity) can play a role. The development of models that describe the relations between stress and flow field (constitutive equations) and their verification in flows is an important topic in rheology.

Initially, constitutive models were often based on a continuum mechanics approach. In this case the stress in the viscoelastic fluid is related to some kinematic properties like the deformation gradient. Sometimes it is possible to write the stress-strain relations in the form of a differential constitutive equation. However, this approach is somewhat phenomenological: the constitutive equations are postulated in such way that they contain properties like elasticity or shear thinning behaviour, but the unknown material parameters have to be fitted to the results of simple rheological experiments like purely shear or elongational flow. Examples of these models are the Maxwell model and the Giesekus model. The advantage of this approach is that such constitutive equations can easily be implemented in numerical flow simulations. Because of this simplicity they can be used in computations of flows in complex geometries and they remain computationally economic. The disadvantage is that the parameters in the constitutive equations have been fitted to simple flows (e.g. shear flow), so that they can be only considered accurate to predict
the behaviour for this particular kind of flow. The effect is that for new and complex flow geometries these equations are not necessarily predicting with the same accuracy. This disadvantage is the consequence of not considering the details and interactions of the polymers at a molecular level.

Since the 1970s the development of models which are based on a statistical description of the microstructure of the fluid have been developed. The goal of these models is to predict macroscopic flow quantities based on the dynamics and interactions of many polymer molecules. Very suitable in this approach are mechanical models which are based on kinetic theory. The polymers are modeled as relatively simple mechanical objects, for instance as elastic dumbbells, bead-springs or bead-rods systems. Obviously one should always take in mind that macromolecules have a complex internal structure with many degrees of freedom and that these models are therefore strong simplifications. In the elastic dumbbell model a polymer is represented as two beads, which are connected by an elastic spring. These dumbbells are supposed to be convected through the flow and are subject to deformations by the flow and by thermal fluctuations. The latter are modeled by a Brownian force. It is possible to derive from the equation of motion of the dumbbells an evolution equation for the probability density function of the dumbbell configurations. This equation has the form of a Fokker-Planck equation. Now two approaches are possible to find the relation between the polymer stress and the flow field.

Firstly, with help of Brownian dynamics it is possible to derive from the Fokker-Planck equation a set of stochastic differential equations, which are more suitable than the Fokker-Planck equation for computer simulations of the flow. This idea was introduced by Laso & Öttinger (1993) and this approach was much improved with the introduction of the Brownian configuration fields method by Hulsen et al. (1997). In this microscopic approach the polymers are modeled at a more fundamental level by capturing the essential polymer dynamics. This is very satisfying from a physical point of view and enables one to use models for which no closed-form constitutive equation is available. The drawback is that the method is computationally very demanding. Macroscopic flow simulations using this method can be performed: the essential difference with the use of a constitutive equation is that the polymer stress is computed from a large ensemble of polymer configurations.

The second approach is to apply a closure for the unknown terms in the Fokker-Planck equation. This avoids solving this difficult equation and leads to a closed-form constitutive equation from which the configuration of the dumbbells and the polymer stress can be computed directly. This approach is only possible for some polymer models of which the Oldroyd-B model and the FENE-P model are examples. This approach is computationally much more economic than the first approach. In this thesis the FENE-P model has been selected for the modeling of the polymer molecules and it will be used in the direct numerical simulations of the turbulent flows of dilute polymer solutions. This will be discussed further in chapter 4.
1.4 Polymers in turbulent flows

Based on the review of rheology and turbulence as described in the previous two sections (especially in relation to the concept of various scales in turbulence), it would seem at least unlikely that the main large-scale turbulence characteristics could be modified due to an addition of a minute amount of polymers to the fluid. This is because the polymers, even in their extended configuration, have a size that is much smaller than the smallest length scale of the turbulence. One can imagine that the effect of the polymers will lead to some increase of the shear viscosity of the fluid, as this is a well-known effect, which can be measured in a shear viscometer. Therefore one would expect that the polymers at most can affect the microstructure of the turbulence, but it seems improbable they would change the macrostructure, which is connected with the main characteristics of the flow and is responsible for transport of momentum which results in frictional drag.

However, the numerous results and applications mentioned in section 1.1 indicate that the story is completely different. Though the polymers are primarily active on the smallest length scales, they are nevertheless able to influence strongly the macroscopic scales of the flow by which the drag is determined. This unexpected behaviour of polymers in turbulent flows makes a study of the interaction of polymers and turbulence very challenging. From a fundamental point of view its solution could perhaps shed more light on the turbulence problem itself.

1.5 Overview of previous research on turbulent drag reduction

During the last 50 years numerous papers have been written, which discuss several experimental, numerical and theoretical aspects of drag reduction of turbulent flows by polymer additives. For a general review of drag reduction by additives the reader is referred to Lumley (1969) and to the more recent work of Gyr & Bewersdorff (1995). The additives causing drag reduction can be divided in three groups: polymers, surfactants and fibres. A short introduction on the rheology of polymers was already given in section 1.3. A key point is that only strongly elongated polymers, which can reach a high end-to-end distance, result in drag reduction. This aspect is related to a high molecular weight and a high number of monomer units of the polymers. Surfactants are molecules that consist of a hydrophilic (water soluble) and hydrophobic (water insoluble) part. Beyond a critical concentration these molecules start to form aggregates, which are called micelles. The micelles can have different shapes and if they are rod-like drag reduction can occur. Fibres are long cylinder-like objects with high aspect (length to width) ratio. They can orient themselves along the main direction of the flow and in this case drag reduction of the flow can occur. This thesis is restricted to long linear polymer molecules dissolved in water.

The first studies on the interaction between turbulence and polymers have dealt with the onset of drag reduction. This is connected to the so-called onset Reynolds number,
which is the point where the drag of the polymer solutions starts to deviate from the drag of Newtonian fluids. Two onset hypotheses based on a dimensional analysis have been put forward. Virk et al. (1967) proposed a length scale argument, indicating that the onset criterion follows from the exceeding of a characteristic length scale of the polymer (e.g., the radius of gyration) over a characteristic length scale of the turbulence (e.g., $\nu/u_*$, with $\nu$ the kinematic viscosity and $u_*$ the friction velocity). On the other hand, Lumley (1969) proposed a similar argument, but based on the time scales of the polymer (e.g., the relaxation time) and the turbulence (e.g., $\nu/u_*^2$). Berman & George (1974) performed a quantitative study of these two suggestions and concluded that the time scale hypothesis is much more consistent with experimental observations and that the experimental data even predict trends opposite to the length scale hypothesis.

In the following years the theories focused on the interaction between turbulence and polymers. Regarding the mechanism of drag reduction, two principal theoretical concepts have been put forward as explanation. The first theory can be attributed to Lumley (1969, 1973), who has proposed a mechanism based on the extension of the polymers. He postulated that stretching of randomly coiled polymers, primarily in regions with strong deformations, such as the buffer layer, will increase the effective (extensional) viscosity. The result is damping of small eddies, a thickening of the viscous sublayer and consequently drag reduction.

The second theory is attributed to De Gennes (1990) who argues that drag reduction is caused by the elastic rather than the viscous properties of polymers. The background behind this suggestion is the fact that drag reduction also occurs in experiments where the polymers have been injected in the centre of a pipe (McComb & Rabie 1979, 1982) so that wall effects can be neglected. De Gennes’ explanation is that the shear waves, which are caused by the elasticity of the polymers, prevent production of turbulent velocity fluctuations at the small scales. Both theories, however, are somewhat qualitative and they give no explanation of the dynamics of wall turbulence quantitatively.

A major result in polymer drag reduction was found by Virk et al. (1967, 1970) and Virk (1975). The authors observed that the amount of drag reduction is limited by an empirical asymptote, called the mean drag reduction asymptote (MDR) or Virk asymptote. This means that for each value of the Reynolds number the friction is limited by a lower bound (which is still larger than the friction of a laminar flow). The asymptote was originally proposed for very dilute polymer solutions for which the shear viscosity can be taken constant and equal to that of the solvent. By a small modification of the Reynolds number however, it can be generalized to more concentrated solutions of which the viscosity of the solution is no longer constant (Gyr & Bewersdorff 1995, p. 105).

Regarding experiments, most studies present results of various statistical quantities in drag reducing flows. Worth mentioning here are the LDV measurements in channel flow (Harder & Tiederman 1991; Wei & Willmarch 1992; Willmarth et al. 1987) and pipe flow (Pinho & Whitelaw 1990; Den Toonder et al. 1997; Den Toonder 1995). In all these experiments similar trends are observed regarding mean velocity, turbulence intensities and Reynolds stresses. Summarizing, in all experiments a small upward shift of the mean
velocity profile is found. Measurements of the turbulence intensities show an increase of the root mean square in the streamwise direction and a decrease of the root mean square in the spanwise and wall-normal directions. Also, a decrease of the Reynolds stress is observed. However, these experiments have all been carried out with dilute polymer solutions for which the amount of drag reduction is far away from the maximum drag reduction asymptote. Experimental data in the high drag reduction regime have been reported by Gampert & Yong (1989) for duct flow and by Warholic et al. (1999a,b, 2001) for channel flow. These authors suggest that for maximum drag reduction the Reynolds stress almost vanishes. This has the implication that polymers should produce rather than dissipate turbulent kinetic energy.

Other experimental work worth mentioning is that of Tiederman (1989) who focus on the flow behaviour in the buffer layer and on turbulent structures. Finally, other experiments (Bonn et al. 1993; Cadot et al. 1995; Douady et al. 1991) show that polymer additives suppress the appearance of strong vortices and reduce the formation of large eddies.

Besides the experimental studies it has become possible to perform direct numerical simulations (DNS) of turbulent drag reducing flows. This approach became possible as result of the increasing computer capabilities and the development of advanced viscoelastic models. The advantage of this approach is that one can more easily isolate and study the effect of various polymer properties (like elasticity, stretching and concentration) on the flow.

Generally, it is assumed that polymers in turbulent flow must be highly stretched before they can effectively change the turbulence (Hinch 1977). Based on this assumption, Orlandi (1995) performed a direct numerical simulation by coupling the extensional viscosity to the local deformation of the flow and by proposing an anisotropic stress model. In this work it is assumed that a highly stretched polymer orients itself into the flow direction while exerting in this direction a viscous anisotropic stress. A similar approach was used by Den Toonder et al. (1995b, 1997). Their simulations confirm that such a viscous anisotropic stress indeed leads to drag reduction and the results agree at least qualitatively with the experimental observations. However, the types of polymer models used are still quite simple and do for instance not take relaxational effects of the polymers into account. The importance of such effects follows from the studies of Massah et al. (1993) and Massah & Hanratty (1997), who studied the configurational changes of single polymer molecules in simple rheological shear and elongational flows and in turbulent flows by representing the polymers as FENE bead-spring chains. Their results reconfirm the unraveling of the polymers by strong straining deformations. In addition they show that the polymer configuration mainly changes to an extended state in the buffer layer where the polymers introduce additional stresses associated with their stretching. However, in these simulations only the influence of the turbulence on the polymers is considered and not the effect of the polymers back on the flow. The next step was taken by Suresh Kumar et al. (1997) and Dimitropoulos et al. (1998) who have performed a DNS in combination with a more realistic polymer model, the so-called FENE-P model, which
will be discussed in detail in chapter 4. These simulations, which have been carried out for cases representative of very dilute polymer concentrations or small drag reduction, predict a change in turbulence statistics comparable to what is found in the experiments. They also provide some criteria for the onset of drag reduction, which seems to agree with the hypothesis of Lumley as discussed above.

1.6 Objectives and outline of this thesis

The main objective of this thesis is to study turbulence in relation to polymer dynamics in order to find a mechanism or explanation for the phenomenon of turbulent drag reduction. In particular the focus is on the regime near maximum drag reduction, because the channel flow experiments of Warholic et al. (1999a,b) suggest that the Reynolds stress at maximum drag reduction is close to zero. This brings them to a hypothesis that the criterion for maximum drag reduction is a zero Reynolds stress. This hypothesis has large implications for the turbulence and its production in polymer flows. During our study this hypothesis will be validated by means of experiments and simulations of turbulent flow with polymer additives at maximal drag reduction, i.e. at conditions which are close to the maximum drag reduction (MDR) or Virk (1975) asymptote.

The first part of this thesis deals with experiments in turbulent pipe flow at high drag reduction. To interpret the effects of the polymers on turbulence at these high concentrations, the results will be compared with data for Newtonian flow (water) and with results obtained with very dilute solutions (Den Toonder et al. 1997, Den Toonder 1995). The objective is to investigate the effect of polymer additives on the various flow statistics like turbulence intensities and stresses. Furthermore, polymer stresses and their contribution to the total shear stress will be discussed.

The second part of this thesis deals with numerical simulations. Upto now no numerical simulations have been performed in the high drag reduction regime, i.e. in the neighbourhood of the maximum drag reduction asymptote. Our objective is to perform such simulations. It is possible to reach this regime by use of a rather high Reynolds number, together with relatively high concentrations of strongly extensible polymers. The results of these simulations are compared with those obtained for Newtonian flow and for small drag reduction. Furthermore, the polymer contribution to the total stress and the influence of polymers with respect to kinetic energy budgets and the elastic energy of the polymers are discussed. Finally, a mechanism for drag reduction based on the notion of shear sheltering (Hunt et al. 1989) is suggested.

The outline of this thesis is as follows. In the following two chapters the experiments are considered. Chapter 2 is concerned with the main features of turbulent drag reducing flows and the definitions of the parameters that will be measured. The experimental setup and equipment are also described in this chapter. In chapter 3 the results of the LDV experiments are presented in terms of various statistics as function of the polymer concentration. Chapters 4 and 5 deal with numerical simulations. The governing equations
for the flow, the constitutive equations for the polymer model (the FENE-P model) are given in chapter 4. In this chapter also a description of the numerical procedures and parameters of the flow is given, together with the characteristics of the polymers and the flow geometry. In chapter 5 the results of the simulations are presented, focusing on the flow statistics, the kinetic energy and its connection with elastic energy of the polymers and the pressure. In particular a detailed comparison with the experimental results discussed in the first part of this thesis is presented. Also in this chapter various length scales in the flow and a possible mechanism for drag reduction are discussed. Finally the main conclusions of this thesis are drawn in chapter 6.
Chapter 2

Turbulent pipe flow characteristics and experimental setup

Abstract
In this chapter an overview is given of the experimentally observed relations between friction and Reynolds number for Newtonian and non-Newtonian pipe flow. In particular the regime near maximum drag reduction is considered and special attention is paid to the definition of the Reynolds number in the flow of shear-thinning fluids. It appears that in the definition of Virk asymptote the Reynolds number has to be based on the viscosity at the pipe wall. The governing equations and main characteristics of turbulent pipe flow are presented, extended with an analysis for non-Newtonian behaviour. Furthermore, the experimental flow facility, the experimental equipment and the measurement procedures are described. Finally, an overview is given of the rheological parameters of the polymer solutions.

2.1 Introduction
The following two chapters deal with the laboratory experiments in turbulent drag reduction. All experiments are performed in a recirculatory pipe flow facility at the Laboratory for Aero and Hydrodynamics of Delft University of Technology. In the past most experiments have been carried out in channel flow (see section 1.5 for a literature overview). A pipe flow has been selected for the following reasons. Firstly, as was stated in the first chapter, measurements for pipe flow for high drag reduction are lacking. Another reason is the fact that pipe flow is fully symmetric around the centreline, while in channel flow one always has to deal with corner effects. The symmetry is beneficial for determining variables, which have to be integrated over the cross section (as is the case in the present study for kinetic energy budgets). A practical reason is that a very good pipe flow facility is already present in our laboratory. The main difficulty of pipe flow opposite to channel flow is the curved geometry, which provides more difficult experimental conditions.
This can especially be problematic for LDV measurements near the wall, which lead to problems with refraction of the laser beams by the curved geometry.

2.2 Gross flow observations

2.2.1 Relations between friction factor and Reynolds number

Consider a fully developed incompressible and isothermal flow through a straight pipe with smooth walls. The flow is driven by a constant pressure gradient. From the equations of motion can be derived that the mean shear stress $\tau$ has a linear profile as function of the radial location (Schlichting 1987) for Newtonian and non-Newtonian fluids and for all flow regimes (laminar and turbulent). This shear stress has the value zero in the centre of the pipe and the value $\tau_w$ at the pipe wall, which is directly related to the pressure gradient by:

$$\tau_w = \frac{D}{4} \frac{\Delta P}{\Delta x}$$  \hspace{1cm} (2.1)

where $\Delta P/\Delta x$ is the constant pressure gradient and $D$ is the diameter of the pipe. Instead of relations between pressure gradient (or wall shear stress) and flow rate, it is conventional to use relations between the Fanning* friction factor $f$ and the Reynolds number, which is defined by:

$$Re = \frac{\rho U_b D}{\eta}$$  \hspace{1cm} (2.2)

with $U_b$ the mean or bulk velocity in the pipe, $\rho$ the density and $\eta$ the constant viscosity of the fluid. The wall shear stress is expressed in terms of the friction factor $f$ by:

$$f = \frac{\tau_w}{\frac{1}{2} \rho U_b^2} = 2 \left( \frac{u_*}{U_b} \right)^2$$  \hspace{1cm} (2.3)

where $\tau_w = \rho u_*^2$, with $u_*$ the friction velocity. The relations between $f$ and $Re$ are called laws of friction or laws of resistance and are usually expressed in so-called Prandtl-Karman coordinates, i.e. $f^{-1/2}$ vs. $Re f^{1/2}$. In physical terms $f^{-1/2} = U_b/\sqrt{2u_*}$ is the ratio of the mean and friction velocities and $Re f^{1/2} = \sqrt{2}D/(\eta/u_*\rho)$ can be interpreted as the ratio of the pipe diameter and the turbulent length scale.

For a Newtonian fluid the expression reads according to Schlichting (1987) for laminar flow:

$$f = \frac{16}{Re} \Leftrightarrow f^{-1/2} = \frac{Re f^{1/2}}{16}$$  \hspace{1cm} (2.4)

and for fully turbulent pipe flow:

$$f^{-1/2} = 4.0 \log(Re f^{1/2}) - 0.4$$  \hspace{1cm} (2.5)

*Alternatively the Moody friction factor $f_M$ is used instead of the Fanning friction factor $f$. The relation between these is: $f_M = 4f$. 
Figure 2.1: Relations between the friction factor and the Reynolds number in Prandtl-Karman coordinates for different types of polymer, polymer concentrations and pipe dimensions. Figure taken from Virk (1975).

These relations are known as the Hagen-Poiseuille law and the Prandtl-Karman law, respectively. Often, the latter is approximated by the following Blasius relation, which is an empirical relation valid for smooth pipes:

\[ f = 0.0791 R_e^{-1/4} \]  

(2.6)

After addition of a small amount of polymers, the friction factor initially follows relations (2.4) and (2.5). At a certain Reynolds number, which is called the onset Reynolds number, the friction factor for this dilute solution will deviate from equation (2.5) and this is the point where drag reduction sets it. The onset Reynolds number depends on the polymer concentration and on the type of additives used. The amount of drag reduction increases with increasing Reynolds number. Dependent on several variables (like the type of polymer, the type of solvent, the polymer concentration and the pipe diameter), the amount of drag reduction ultimately attains a maximum value. This means that the friction factor for a given Reynolds number is limited by a lower bound, which is called the maximum drag reduction (MDR) asymptote. The asymptote is an empirical relation and was first proposed by Virk (1975) and is also known as the Virk asymptote. The case
described above is also known as type A drag reduction.

However, it is also possible that the curve immediately follows the asymptote after the flow becomes turbulent. This is the case for polymers, which are already somewhat extended before transition to turbulence occurs and is called type B drag reduction (Virk & Wagger 1989). The $f-\text{Re}$ relation for the maximum drag reduction asymptote reads for both types (A and B) of drag reduction:

$$f^{-1/2} = 19.0 \log(\text{Re} f^{1/2}) - 32.4$$  \hspace{1cm} (2.7)

This relation was originally proposed for fluids with a constant viscosity. In the next section an extension will be made for cases where the viscosity is shear-rate dependent.

In figure 2.1 some examples are shown for the friction-Reynolds number relations obtained from several experiments (figure taken from Virk 1975).

### 2.2.2 Modifications of the Reynolds number

The theory presented in the previous section is derived for very dilute solutions. In this case the viscosity is assumed to be constant and the definition of the Reynolds number gives no problems (equation 2.2). For more concentrated polymer solutions the viscosity is in general shear-rate dependent and thus depends on the location in the flow. As a result the definition of the Reynolds number based on a constant viscosity cannot be used. Simply using the viscosity of the solvent will not collapse the data on the curves of equations (2.4) and (2.7). Metzner & Reed (1955) proposed an alternative by using the power-law model (Bird et al. 1987a), which describes the viscosity $\eta$ as function of shear rate by:

$$\eta = K \dot{\gamma}^{n-1}$$  \hspace{1cm} (2.8)

in which $K$ is the consistency index, $n$ is a dimensionless power-law index and $\dot{\gamma}$ is the shear rate. This approach leads to an analytical solution for the laminar flow in a pipe for which equation (2.4) is valid, provided the so-called modified Reynolds number $Re_n$ is used:

$$Re_n = \frac{\rho U_b^{2-n} D^n 2^{3-n}(3 + \frac{1}{n})^{-n}}{K}$$  \hspace{1cm} (2.9)

However, the power-law model is not very realistic, because it does not describe the viscosity for $\dot{\gamma} \to 0$ and it gives $\eta = 0$ for $\dot{\gamma} \to \infty$, while the viscosity of a polymer solution approaches the viscosity of the solvent for high shear rates. For more complex models analytical solutions are much more difficult or not available.

Therefore, in this thesis another modification for the Reynolds number will be applied. In this approach the Reynolds number is based on the viscosity at the pipe wall, $\eta_w \equiv \eta(\dot{\gamma}_w)$ which by definition is equal to

$$\eta_w = \frac{\tau_w}{\dot{\gamma}_w}$$  \hspace{1cm} (2.10)
with $\dot{\gamma}_w$ the local shear rate at the wall. A similar approach was used by Pinho & Whitelaw (1990) and Draad et al. (1998). The wall Reynolds number is then defined by:

$$Re_w = \frac{\rho U_b D}{\eta_w} \quad (2.11)$$

The wall Reynolds number can be used together with various rheological models. For the special case of the power-law model the relation between the wall and the modified Reynolds numbers depends only on $n$ and is equal to:

$$Re_w = Re_n \frac{3n + 1}{4n} \quad (2.12)$$

The polymer solutions used in our experiments always show a shear-thinning behaviour ($n < 1$), i.e. a decreasing shear viscosity with increasing shear rate. For most solutions $n \approx 0.7 - 0.8$, which gives for laminar flow a difference of $6 - 10\%$ between $Re_n$ and $Re_w$ for the power-law model. For each of these polymer solutions the relation between the viscosity and the shear rate has been measured in a viscometer. To these observations the Carreau model (see Bird et al. 1987a) has been fitted. The Carreau model is widely used as a fitting model to rheometric measurement data. The viscosity as function of the shear-rate is given by:

$$\eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty) \left[1 + (\lambda \dot{\gamma})^2\right]\left(n-1\right)/2 \quad (2.13)$$

in which $\eta_0$ is the zero-shear-rate viscosity, $\eta_\infty$ is the infinite-shear-rate viscosity, $\lambda$ is a time constant and $n$ is the dimensionless power law index ($n < 1$, comparable to the power-law model). For very high shear rates, where the viscosity should reach its \$\eta_\infty$-plateau, the measurements in the viscometer show some instabilities. Hence, for the infinite-shear-rate viscosity the viscosity of the solvent (water) is taken. The Carreau parameters of our polymer solutions are given in section 2.5.3.

The value of the wall viscosity (and the wall Reynolds number) is determined as follows. For a given value of $\tau_w$ (which is known from pressure drop measurements by equation 2.1) the corresponding $\eta_w$ and $\dot{\gamma}_w$ are determined by solving equations (2.10) and (2.13) numerically. Using equation (2.11) the wall Reynolds number is computed.

As a check on this approach the shear rate at the wall is also determined by an alternative method. This is done by estimating the derivative of the streamwise velocity ($U_x$ is measured with LDV which will be discussed in section 2.4.2) where $\dot{\gamma}_w = -dU_x/dr$ for $r = D/2$ (the pipe wall). It turns out that the two values of $\dot{\gamma}_w$ (the numerical procedure with the Carreau model and the derivative of the velocity using LDV) do not differ more than by 3% so that our procedure appears to be well justified.

The maximum drag reduction asymptote (equation 2.7) was originally proposed for fluids with a constant viscosity, which is equal to the viscosity of the solvent. For the case of shear-thinning fluids Gyr & Bewersdorff (1995) (page 105) show that plotting the friction factor vs. the wall Reynolds number collapses the data on a single curve which is
the Virk asymptote. This does certainly not happen when the Reynolds number based on the solvent viscosity is used. In our opinion this justifies the use of the wall Reynolds number $Re_w$ in equation (2.7) by the procedure described above.

### 2.2.3 Definition of the amount of drag reduction

The definition of the amount of drag reduction is not trivial, as one has to compare the flow of a Newtonian fluid with that of a non-Newtonian fluid. As long as the polymer solution has a constant viscosity (or at least not too much shear-thinning behaviour) this difference in fluids does not pose major problems. Two definitions are widely used. In the first definition the relative change of pressure drop due to polymer addition at a constant flow rate $Q$ (or constant $Re$) is determined. Alternatively, one can imagine drag reduction as an increase of flow rate while the pressure drop is kept constant. The corresponding definitions are:

\[
DR\% = \frac{\Delta P_N - \Delta P_P}{\Delta P_N} \cdot 100\% \quad \text{at constant } Q \tag{2.14}
\]

\[
DR\% = \frac{Q_P - Q_N}{Q_P} \cdot 100\% \quad \text{at constant } \Delta P \tag{2.15}
\]

The suffices ‘N’ and ‘P’ stand for the Newtonian (non drag-reducing) and the polymer (drag-reducing) fluids respectively. The first definition can for a constant viscosity be directly expressed in terms of $Re$ and $f$.

In order to remain consistent with the use of the $f - Re_w$ relations, the percentual amount of drag reduction for shear-thinning fluids is in this thesis defined by:

\[
DR\% = \frac{\Delta P_N - \Delta P_P}{\Delta P_N} \cdot 100\% = \frac{f_N - f_P}{f_N} \cdot 100\% \tag{2.16}
\]

at constant $Re_w$.

### 2.3 Description of turbulent pipe flow

#### 2.3.1 Basic equations

The governing equations for fluid flow consist of conservation laws for mass, momentum and energy. For a derivation and background of these equations the reader is referred to standard textbooks, such as Batchelor (1967). For isothermal and incompressible flows in absence of external forces only the first two conservation laws have to be taken into account and they are known as the continuity and the Navier-Stokes equations. These equations read:

\[
\nabla \cdot \mathbf{u} = 0 \tag{2.17}
\]
and
\[ \frac{D\mathbf{u}}{Dt} = \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot \mathbf{\tau} \] (2.18)

In these equations \( \mathbf{u} \) is the velocity vector, \( t \) is the time, \( p \) is the pressure and \( \mathbf{\tau} \) is the (total) stress tensor. For a fluid in which a small amount of polymers is dissolved in a Newtonian solvent it is possible to decompose the total stress in a solvent (Newtonian) part \( \mathbf{\tau}^{(s)} \) and a polymeric (non-Newtonian) part \( \mathbf{\tau}^{(p)} \) according to:
\[ \mathbf{\tau} = \mathbf{\tau}^{(s)} + \mathbf{\tau}^{(p)} \] (2.19)

The contribution of the solvent is described by the constitutive equation of a Newtonian fluid:
\[ \mathbf{\tau}^{(s)} = 2\eta_s \mathbf{D} \] (2.20)

where \( \mathbf{D} = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \) is the rate-of-deformation tensor and \( \eta_s \) is the solvent viscosity. The polymer stress should follow from a constitutive equation that relates the stress and the flow deformation history. However, in this experimental part the polymer stress will remain unspecified, but a constitutive model for the polymers will be used in the numerical simulations, which are described in chapters 4 and 5.

As turbulence is a chaotic and fluctuating state of motion it is common to deal with a statistical approach to describe the flow. Here, the well-known Reynolds decomposition is used, which means that the velocity vector \( \mathbf{u} \), the pressure \( p \) and the polymer stress tensor \( \mathbf{\tau}^{(p)} \) consist of a mean flow part (denoted by an upper case) and a fluctuating turbulent part (denoted by a prime). Alternatively, a mean value is sometimes indicated by an overbar (\( \bar{\ldots} \)). The decomposition can be written as:
\[ \mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}', \quad p = \bar{p} + p', \quad \mathbf{\tau}^{(p)} = \mathbf{\tau}^{(p)} + \mathbf{\tau}'^{(p)} \] (2.21)

with \( \mathbf{U} = \bar{\mathbf{u}}, \quad P = \bar{p} \) and \( \mathbf{T}^{(p)} = \bar{\mathbf{T}}^{(p)} \).

For a pipe flow it is convenient to write the equations (2.17) and (2.18) in a cylindrical coordinate system \((x, r, \theta)\) for the axial, radial and tangential coordinate respectively. Applying the Reynolds decomposition (2.21) and the stress decomposition (2.19) to these equations gives for the equations of continuity and axial momentum of the mean flow quantities:
\[ \frac{\partial U_x}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r U_r) + \frac{1}{r} \frac{\partial U_\theta}{\partial \theta} = 0 \] (2.22)
\[ \rho \left( \frac{\partial U_x}{\partial t} + U_x \frac{\partial U_x}{\partial x} + U_r \frac{\partial U_x}{\partial r} + U_\theta \frac{\partial U_x}{\partial \theta} \right) = \frac{\partial P}{\partial x} - \rho \left( \frac{\partial}{\partial x} u_x^2 + \frac{1}{r} \frac{\partial}{\partial r} (ru_x u_r) + \frac{1}{r} \frac{\partial}{\partial \theta} u_x u_\theta \right) + \eta_s \left( \frac{\partial^2 U_x}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial U_x}{\partial r}) + \frac{1}{r^2} \frac{\partial^2 U_x}{\partial \theta^2} \right) + \frac{\partial \mathbf{T}_{xx}^{(p)}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r \mathbf{T}_{xx}^{(p)}) + \frac{1}{r} \frac{\partial \mathbf{T}_{\theta x}^{(p)}}{\partial \theta} \] (2.23)
In our experiments the pipe flow is assumed to be stationary and fully developed. Furthermore the flow is axisymmetric around the centreline of the pipe. For the case of a fully developed pipe flow only the mean axial velocity component is non-zero and all statistics are only a function of $r$. Therefore $U_r = U_\theta = 0$, all quantities are independent of time and all mean flow quantities are independent of $x$ and $\theta$. This reduces equation (2.23) for our case to:

$$0 = -\frac{\partial P}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( -\rho u'_x u'_r + \eta_s \frac{\partial U_x}{\partial r} + T^{(p)}_{rx} \right) \right)$$  \hspace{1cm} (2.24)

From the momentum equation for the radial direction it can be derived that the pressure gradient in the streamwise direction is not a function of $r$. Multiplying equation (2.24) with $r$ and integrating from 0 to $r$ gives the following result:

$$\frac{r}{2} \frac{dP}{dx} = T_{rx} = -\rho u'_x u'_r + \eta_s \frac{\partial U_x}{\partial r} + T^{(p)}_{rx}$$  \hspace{1cm} (2.25)

Due to homogeneity in the $x-$direction the pressure gradient is constant and this means that the pressure is a linear function of $x$ only. The right-hand-side of equation (2.25) denotes the total mean shear stress $T_{rx}$ and consequently this is a linear function of the radial location. For $r = D/2$ (the absolute value of) the total stress is equal to the wall shear stress defined in equation (2.1). The total stress consists of three contributions: the turbulent or Reynolds stress, the viscous stress of the solvent and the polymer stress, respectively. This equation will be used in the discussion of the results of the individual stress contributions in the next chapter.

### 2.3.2 Energy budgets

In this section the index notation is used for the velocity and the polymer stress. The summation convention is applied, which means that a repeated index denotes a summation over all coordinate directions of the index.

#### Kinetic energy of the mean flow

The kinetic energy of the mean flow is equal to $\frac{1}{2} \rho U_t^2$. Its balance equation can be derived by multiplying equation (2.24) with $U_x$ and reads:

$$0 = P_u + T^{(r)}_u + T^{(s)}_u + T^{(p)}_u + D_u - \epsilon_u - W_u$$  \hspace{1cm} (2.26)
with

\[ P_u = -U_z \frac{\partial P}{\partial x} \]  \hspace{1cm} (2.27)

\[ T_u^{(r)} + T_u^{(s)} + T_u^{(p)} = \frac{1}{r} \frac{\partial}{\partial r} \left( r U_z \left( -\rho u'_z u'_r + \eta_s \frac{\partial U_z}{\partial r} + T_{rx}^{(p)} \right) \right) \]  \hspace{1cm} (2.28)

\[ D_u = \rho u'_z u'_r \frac{\partial U_z}{\partial r} \]  \hspace{1cm} (2.29)

\[ \epsilon_u = \eta_s \left( \frac{\partial U_z}{\partial r} \right)^2 \]  \hspace{1cm} (2.30)

\[ W_u = T_{rx}^{(p)} \frac{\partial U_z}{\partial r} \]  \hspace{1cm} (2.31)

\( P_u \) is the production of mean flow energy by the imposed pressure gradient. The three terms \( T_u^{(r)} + T_u^{(s)} + T_u^{(p)} \) describe the transport of mean flow energy by respectively Reynolds stresses, mean viscous stresses and mean polymer stresses. \( D_u \) is called deformation work and this term will return as a production term in the equation for the turbulent kinetic energy, to be discussed below. \( \epsilon_u \) is the viscous dissipation by the mean flow and \( W_u \) is the dissipation by mean polymer stresses. The production \( P_u \) is positive and the deformation work \( D_u \) is negative. The two dissipative terms (\( \epsilon_u \) and \( W_u \)) are both positive (but have of course a negative contribution to the energy balance because of the minus signs in equation 2.26).

Kinetic energy of the turbulence

By a similar procedure as used to obtain equation (2.23) one can derive the momentum equation for the fluctuation \( u'_z \). Multiplying this equation with \( u'_z \) gives the equation for the \( x \)-component of the turbulent kinetic energy. Repeating this procedure for all coordinate directions gives an equation for the turbulent kinetic energy, \( \frac{1}{2} \rho u'^2 \). Here only the final result is given for the flow conditions in our case (stationarity, no mean flow in radial and tangential directions, no mean derivatives in axial and tangential directions). The balance equation for the kinetic energy of the turbulence, \( \frac{1}{2} \rho u'^2 \), reads:

\[ 0 = P_k + T_k^{(r)} + T_k^{(s)} + T_k^{(p)} + T_{kx}^{(p)} - \epsilon_k - W_k \]  \hspace{1cm} (2.32)
with

\[
P_k = -\rho \overline{u'_x u'_r} \frac{\partial U_x}{\partial r} \quad (2.33)
\]

\[
T_k^{(r)} + T_k^{(\sigma)} + T_k^{(a)} + T_k^{(p)} =
\frac{1}{r} \frac{\partial}{\partial r} \left( r \left( -\frac{1}{2} \rho \overline{u'_r u'_r} - \overline{p' u'_r} + \frac{1}{2} \eta_s \frac{\partial}{\partial r} \overline{u'_r u'_r} + \overline{u'_r \tau_{rr}^{(p)}} \right) \right) \quad (2.34)
\]

\[
\epsilon_k = \eta_s \frac{\partial \overline{u'_r}}{\partial x_j} \frac{\partial \overline{u'_r}}{\partial x_j} \quad (2.35)
\]

\[
W_k = \tau_{ij}^{(p)} \frac{\partial \overline{u'_i}}{\partial x_j} \quad (2.36)
\]

\( P_k \) is the production of turbulent kinetic energy, which is the opposite of the deformation work \( D_u \) in the equation for the mean flow energy (2.26). Thus the production is always positive. The four terms \( T_k^{(r)} + T_k^{(\sigma)} + T_k^{(a)} + T_k^{(p)} \) are again transport terms and they result in redistribution of turbulent kinetic energy. They represent the transport due to velocity fluctuations, due to pressure fluctuations, due to fluctuating viscous stresses and due to fluctuating polymer stresses, respectively. The last two terms are the viscous (solvent) dissipation \( \epsilon_k \) and the polymer stress work \( W_k \). The viscous (solvent) dissipation is always positive, whereas the sign of the polymer stress work is unknown.

These budgets of the kinetic energy will be discussed further in the discussion of the experimental results in the next chapter.

### 2.4 Experimental setup

A very detailed overview of the experimental facility that has been used in the experiments, together with its design, components and construction, can be found in Draad et al. (1998) and Draad (1996). Here only the most important characteristics will be mentioned.

#### 2.4.1 Flow facility

For the experiments with both Newtonian and non-Newtonian flow a recirculatory pipe flow facility has been used, which is shown schematically in figure 2.2. The two most important parts of this facility for this study are the pipe itself, in which a fully developed turbulent flow is established, and the test section in which the actual measurements are performed. The measurement pipe consists of a smooth straight tube with a length of 34 m and an inner diameter of 40.37 mm. The pipe has a circular cross-section and is made of transparent Plexiglas. The pipe consists of several sections, each with a length of 2 m, which are attached to each other by flanges. The pipe segments are centred on the inner diameter in order to avoid discontinuities of the pipe wall.
The fluid is pumped from an open free-surface reservoir with a volume of 0.96 m³. For flows of polymers solutions in a recirculating facility it is important to use a pump that minimizes the amount of mechanical degradation of the polymers. Mechanical degradation is defined as breakup of long-chain polymers (e.g. by a pump or by intense shearing in the flow). As such breakup changes the rheological properties of the polymers and its drag reducing capability, mechanical degradation should be minimized (and if possible avoided). Degradation will be discussed further in section 2.5.1. The pump should provide a constant flow rate and any variations or pulsations should be avoided. Following a previous study (Den Toonder et al. 1995a) a disc pump, manufactured by Begemann, has been used. A disk pump works similar as a centrifugal pump, though with the difference that its rotor is made of a number of parallel disks. The fluid enters the pump through a suction opening in the centre of the disks and leaves the pump by centrifugal forces at the outlet.

After the pump the flow passes through a flow straightener and a settling chamber. The settling chamber was designed to generate a smooth and uniform inflow in the pipe and this is mainly relevant for transition experiments (Draad 1996). As for this study a fully turbulent flow is needed, the flow is forced to turbulence by means of a (disturbance or trip) ring just after the entrance of the pipe. This gives a sudden narrowing of the pipe and causes a large disturbance and direct transition to turbulence. In the measurement pipe, after the flow is considered to be fully developed, the pressure drop and the flow rate are measured. A part of the pipe is replaced by a specially designed test section (to be discussed below), in which the LDV experiments are performed. At the exit of the pipe the flow passes through a discharge chamber (a Plexiglas box with volume 0.25 m³) and a returns via a return pipe to the reservoir. The return pipe has a diameter of 150 mm so that the velocity in the return pipe is much lower than in the measurement.
pipe, so that degradation is minimized. Under normal operating conditions the total fluid volume in the entire facility is 1.5 m³. The use of a large fluid volume is advantageous, because this minimizes the number of times the fluid has to circulate and this minimizes the amount of degradation. The whole measurement pipe is insulated in order to avoid temperature variations, which may cause secondary flow. LDV measurements show that such secondary flows do not occur in turbulent flow conditions. Furthermore, contact of the fluid with metals is avoided everywhere in the facility, because such contact might lead to a chemical reaction with the polymers.

![Diagram of test section](image)

Figure 2.3: Side view of the test section. In the perspex box (the test section) the pipe wall is replaced by a thin sheet of Teflon FEP, which has approximately the same refraction index as water.

The use of LDV in a pipe flow is complicated due to optical refraction by the curved surface of the pipe. This is especially problematic for measurements near the pipe wall and for measurements in which two velocity components have to be measured simultaneously. The refraction problems are caused because of the difference in refraction indices of air ($n = 1.00$), water ($n = 1.33$) and Plexiglas ($n = 1.49$). To minimize this problem, a small part of the pipe is replaced by a special test section, which consists of a rectangular Plexiglas box which is placed around the pipe and which is filled with water at approximately the same pressure as occurs in the pipe itself. The pipe wall inside the optical box is replaced by a thin cylindrical sheet of Teflon FEP, manufactured by Du Pont, with a thickness of 190 μm. This is illustrated in figure 2.3. The refraction index of the sheet is $n = 1.344$ which is close to that of water ($n = 1.33$). This refraction index together with the small thickness of the sheet minimizes the refraction of the laser beams. The inner diameter of the sheet in the test section is equal to the diameter in the rest of the pipe. The location of the test section is 26 m = 650D after the entrance of the pipe and the flow at this location can be considered as fully developed (Draad 1996).
2.4.2 Measuring equipment

Laser Doppler Velocimetry (LDV)

For measurements of the velocity statistics a two-component LDV system is used. The method of LDV (Laser Doppler Velocimetry) is one of the most widely used experimental techniques in fluid dynamics. The key advantage of LDV above many other techniques is the fact that the flow is not disturbed by the measuring instrument and that high accuracies can be achieved. For general theory of LDV the reader is referred to Drain (1986) and Durst et al. (1976). The principle of LDV is based on diffraction or scattering of light by a particle in the flow. The scattered light has frequency shifted with respect to the incoming light. The frequency shift, which is proportional to the velocity of the particle, is known as the Doppler effect. As the particles do not influence the flow, LDV makes it possible to determine the velocity of the flow by means of the particles.

![LDV Diagram](image)

**Figure 2.4:** Schematic overview of the LDV system.

A schematic overview of the LDV measuring equipment is shown in figure 2.4. The light source of the system is a 5 W Argon-ion laser (Spectra Physics, model 2020). From the laser the light enters a transmitter box in which the laser beam is split in two different wavelengths (colours) (green light with wavelength 514.5 nm and blue light with wave-
length 488.0 nm). Each colour is split into two separate beams, of which one is shifted in frequency by means of a Bragg cell. This pre-shifting is necessary in order to determine the direction of the fluid velocity. Via optical fibres the set of four laser beams (two green and two blue, of each colour one shifted and one unshifted beam) is transmitted to the measurement probe. In the probe the four laser beams are focused by a front lens (focal length 80 mm), with one pair of beams located in the horizontal plane and the other pair located in the vertical plane. The beams are aligned such that the focal point coincides in one location, which is called the measurement volume. The dimensions of the measurement volume in water are approximately 20 μm, 20 μm and 100 μm in the streamwise (axial), normal (radial) and spanwise (tangential) direction, respectively.

The LDV measurement probe is placed on a 3D traversing mechanism manufactured by Dantec, which is operated by a PC. The minimum step for the traverse is 0.05 mm. In the LDV experiments vertical traverses have been performed along the symmetry line of the pipe. Each pair of beams (the green pair and the blue pair) is used to measure one velocity component so that when all four beams coincide in one point two velocity components can be measured simultaneously. The green pair (514.5 nm) is used for the streamwise (axial) velocity component and the blue pair (488.0 nm) for the normal (radial) velocity component. The signal of scattered light from a particle is only accepted when it is detected by both beams pairs. This is called coincidence. The exact location of the measurement volume with respect to the pipe wall is determined by traversing the probe vertically to a position where the measurement volume just hits the wall. This becomes apparent in the signal on an oscilloscope, which at the wall does not show single bursts like in the flow, but a high noise level because of scattering from the wall. A detailed description of procedures for the alignment of the probe and for the determination of the exact position of the measurement volume is given by Tabitu (1994).

The LDV equipment operates in a so-called backscatter mode, which means that the diffracted (scattered) light is processed by the same measurement probe as used for transmission and focusing. The diffracted light is transmitted via an optical fibre to a colour separator. The two separated colours go into two photo multipliers in which the signal is amplified. The signal from the photo multipliers is analyzed by two Burst Spectrum Analysers (BSAs) (Dantec, types Enhanced 57N20 and Enhanced slave 57N35) and is further analyzed by a PC using the program Burstware (Dantec).

A minute amount of seeding consisting of pigment particles (TiO$_2$) has been added to the flow in order to increase the data rate of the LDV signal. The amount is small, so that it does not affect the rheological properties of the fluid. The data rate of the LDV signal used for the calculation of the flow statistics is approximately 30 Hz. The actual data rate is higher but in order to avoid velocity bias the measurements are performed in so-called dead-time mode. This means that only one burst is accepted during the period of the dead time. The data presented in the next chapter are obtained with a measuring time of 240 s for each radial position. The estimated (relative) statistical errors are: approximately 2% for the first moment (mean velocity) and 4% for the second moment (root mean squares and stresses).
Flow rate

The flow rate through the pipe is measured with a magnetic inductive flow meter (Krohne-Altometer, type M950/6) just after the test section. The accuracy of this flow meter according to the calibration certificate is 0.4% with a minimum velocity of 1 mm/s. This corresponds with a minimal error in the flow rate of 4.5 l/hr which is larger than the discrete step of the flow meter of 3 l/hr. Furthermore, the flow rate was checked by integrating the velocity profile (both for laminar and turbulent flow) and the agreement between these is also within the specified accuracy.

Pressure drop

The pressure drop over a segment of the pipe, as shown in figure 2.2, is measured with a membrane differential pressure transducer (Validyne Engineering Corp., type DP15-20). The transducer measures the deflection of a stainless membrane as result of the pressure difference over this membrane and translates the deflection into a voltage. Via a calibration of the transducer using a Betz manometer this can be transferred into a pressure drop. The transducer is connected to pressure taps with a diameter of 1 mm in the pipe. The full scale of the pressure transducer corresponds with a pressure of an 88 mm water column. The relative measurement error of the pressure drop measurements is 0.5%. The pressure difference is measured over a segment of 8 m, which is chosen as large as possible in order to minimize the relative measuring error. The begin of the segment is located 16 m (400 pipe diameters) after the entrance of the pipe. The flow is fully developed so that the pressure gradient is constant in the entire segment. Draad (1996) has shown that for non-Newtonian flow this is the case after 280 pipe diameters. As the signal from the pressure transducer shows some fluctuations it is integrated in time over a period of 30 s. The integrated signal is stable and is used in the determination of the pressure drop and the friction factor.

Temperature

The temperature of the fluid is measured with two PT100 elements located in the free surface reservoir and in the discharge chamber. Also the air temperature is measured by means of a mercury thermometer. The scale of all instruments is 0.1 °C. It is important to keep the temperature as constant as possible during an experiment, as temperature variations lead to large changes in the fluid viscosity. During measurements of pressure drop and the velocity profiles using LDV the temperature remains constant within 0.2 °C.

2.5 Polymer solutions

The viscoelastic fluids used in the experiments are so-called dilute polymer solutions, which consist of a Newtonian solvent (water) in which polymers are dissolved. The poly-
mer used is Superfloc A110 (Cytec Industries), which consists of partially hydrolyzed polyacrylamide (PAMH). Superfloc polymers have applications in the oil, food and chemical industry. The chemical structure of this polymer is shown in figure 2.5. The molecular weight of this polymer is relatively high (approximately $6 \cdot 8 \cdot 10^6$ g/mol according to the manufacturer). A single monomer A110 has a molecular weight of approximately 70 g/mol; therefore the macromolecule consists of about $10^5$ monomers.

\[
\begin{array}{c}
\text{CH}_2 \quad \text{CH} \quad \text{CH}_2 \quad \text{CH} \\
\mid \text{C = O} \mid \text{C = O} \\
\mid \text{OH} \mid \text{NH}_2
\end{array}
\]

Figure 2.5: The chemical structure of Superfloc A110, which is the polymer used in the experiments.

### 2.5.1 Mechanical degradation

Mechanical degradation was already shortly addressed in section 2.4.1 during the discussion of the characteristics of the pump. Mechanical degradation is the process of rupture of the polymers into smaller molecules or break-up of aggregates due to mechanical forces. This can happen due to shear forces caused by the flow through the pipe and especially through the pump. The result of mechanical degradation is reduction of the molecular weight and this is known to lead to a strong decrease of the drag reducing capability of the polymer solution. Compared to other available drag reducing polymers (e.g. Separan AP-273, a polyacrylamide by Dow Chemical, or Polyox WSR-301, a polyethylene oxide by Union Carbide) Superfloc A110 has been found to be much better resistant to mechanical degradation (Den Toonder et al. 1995a). This is the main reason to use this polymer, especially for measurements in the neighbourhood of Virk asymptote where a high amount of drag reduction has to be achieved.

However, mechanical degradation cannot be fully avoided. In order to make sure that the flow conditions remain near the maximum drag reduction asymptote during the whole experiment, higher polymer concentrations are used than would be strictly necessary to reach the asymptote. Inerthal & Wilski (1985) and Kenis (1971) state that such an excess of polymers leads to a decrease of the effects of degradation. In our measurements the pressure drop remains constant during an LDV experiment. Furthermore the shear viscosity of the solutions has been measured at the beginning and at the end of an experiment and this has been found not to change, while mechanical degradation would certainly lead
to a decrease of the plateau value of the zero-shear-rate viscosity. Summarizing, the use of a disk pump, a large fluid volume and Superfloc A110 as polymer type lead together to a minimal amount of mechanical degradation.

Another type of degradation that can occur in drag reducing flows is chemical or oxidative degradation, which is caused by reactions of the polymer with metals or with chlorines (see Choi et al. 1992). In our system there is no contact of the polymers with metals or other degrading substances and therefore this type does not play a role in our system.

### 2.5.2 Preparation procedure

Before the actual experiments the solutions are prepared by first creating a solution with a relatively high fixed concentration (e.g. 3000-5000 weight parts per million) of Superfloc A110 in water. This is done by slowly dissolving the polymers in water\(^1\) in a stirred vessel by a continuous rotation. After stirring for about 10 hours, the mixture is left alone for about one day in order to let small air bubbles escape from the vessel. Afterwards this solution is pumped into the system and is simultaneously diluted with fresh water until the prescribed concentration is obtained. Slowly mixing by a mixer in the free surface reservoir and pumping of the solution at a low flow rate through the entire setup is continued until the solution becomes homogeneous. After this the solution is ready for the measurements. Note that there is a difference between our experiments, where the polymer solution is first made homogeneous and then circulated during the experiments and the work of Warholic et al. (1999a), in which the polymer is injected in the setup during the experiments.

### 2.5.3 Rheological characterization

In the experiments, which will be discussed in the next chapter, three different polymer solutions have been used, consisting of 103, 175 and 435 wppm (weight parts per million) Superfloc A110. For each of these polymer solutions a rheological characterization has been performed, which means that some fluid parameters of these solutions are determined by means of rheometric measurements. One of the goals of this characterization is to determine the viscosity of the fluid at the pipe wall and hence the wall Reynolds number as described in section 2.2.2. To this end the steady shear rate viscosity \(\eta\) is measured as function of the shear rate \(\dot{\gamma}\) in a viscometer. This is done at the same temperature at which the LDV measurements are carried out. A necessary condition the viscometer has to meet is that it is able to measure stresses in the order of \(10^{-2}\) mPa as this is the level where our aqueous solutions reach their plateau value for the zero-shear-rate viscosity. Such stress level can be measured with the Contraves LS-40, which has a Couette geometry (with rotating cup, cupdiameter 13 mm, bobdiameter 12 mm, boblength 18 mm).

\(^1\)For very high concentrations one can eventually dissolve the polymers first in a small amount of ethanol.
Table 2.1: Carreau parameters of the three polymer solutions used in the LDV experiments as defined in equation (2.13).

<table>
<thead>
<tr>
<th>solution</th>
<th>103 wppm</th>
<th>175 wppm</th>
<th>435 wppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>temp. (°C)</td>
<td>18.0</td>
<td>18.4</td>
<td>17.4</td>
</tr>
<tr>
<td>( \eta_0 ) (mPas)</td>
<td>3.005</td>
<td>4.446</td>
<td>10.60</td>
</tr>
<tr>
<td>( \eta_\infty ) (mPas)</td>
<td>1.053</td>
<td>1.043</td>
<td>1.090</td>
</tr>
<tr>
<td>( n )</td>
<td>0.7906</td>
<td>0.7605</td>
<td>0.7185</td>
</tr>
<tr>
<td>( \lambda ) (s)</td>
<td>0.4359</td>
<td>0.4543</td>
<td>0.5635</td>
</tr>
</tbody>
</table>

The measured viscosity curves are fitted to the Carreau model as described by equation (2.13). In Table 2.1 the Carreau parameters for the three polymer solutions are given and the viscosity curves are presented in Figure 2.6. As it is not possible to reach very high shear rates in the viscometer, it is assumed in the fitting procedure that \( \eta_\infty = \eta_{\text{solvent}} = \eta_{\text{water}} \). It is very clear from this figure that our three polymer solutions show significant shear thinning behaviour and that the effect becomes stronger with increasing concentration. The results of the experiments with these three polymer solutions will be compared with those of a solution consisting of 20 wppm Superfloc A110 (Den Toonder et al. 1997; Den Toonder 1995). This solution does not show shear thinning behaviour and the (constant) viscosity of this solution is \( \eta = 1.10 \) mPas.

![Figure 2.6: Viscosity as a function of shear rate for the three polymer solutions used in the LDV experiments. Symbols: measurement; lines: fit.](image-url)
Chapter 3

Experimental results

Abstract
In this chapter the experimental results of a fully developed turbulent pipe flow with polymer additives are presented. The experiments consist of pressure drop and LDV measurements. The Reynolds number (based on the mean velocity, the pipe diameter and the local viscosity at the wall) is for all experiments approximately 10000. Polymer solutions consisting of three different concentrations have been used, which have been chosen such that maximum drag reduction occurs. The amount of drag reduction found is 60-70 %. Our experimental results are compared with results obtained with water and with a very dilute solution, which exhibits only a small amount of drag reduction. The objective is to investigate the effect of polymers on various elements of the flow statistics like turbulence intensities and stresses. Also the various contributions to the total mean shear stress, which consist of a turbulent, a solvent (viscous) and a polymer part, have been measured. The polymers are found to contribute significantly to the total stress. The mean velocity profile shows that the buffer layer is thickened and that the slope of the logarithmic profile is increased. For the streamwise velocity fluctuations an increase of the root mean square value is found at low polymer concentration, but a return to values comparable to those for water at higher concentrations. The root mean square of the wall-normal velocity fluctuations shows a strong decrease. Also the Reynolds (turbulent) shear stress and the correlation coefficient between the streamwise and the normal components are drastically reduced at each location in the pipe. In all cases the Reynolds stress stays definitely non-zero at maximum drag reduction. The consequence of the drop of the Reynolds stress is a large polymer stress, which can be 60 % of the total stress. The kinetic-energy balance of the mean flow shows a large transfer of energy directly to the polymers instead of the route by turbulence. The kinetic energy of the turbulence suggests a possibly negative polymeric dissipation of turbulent energy.
3.1 Introduction

In the experiments polymer solutions have been used of one type of a linear high-molecular weight polymer (Superfloc A110), which properties are described in section 2.5. The three solutions differ in the polymer concentration, which are 103, 175 and 435 wppm (weight parts per million), respectively. The focus of the experiments is on the high drag reduction regime near Virk asymptote. Our experimental results will be compared with results obtained with a Newtonian fluid (water) and with a very dilute solution (20 wppm), which exhibits only a small amount of drag reduction (Den Toonder 1995 and Den Toonder et al. 1997). In the first part of this chapter pressure drop measurements are presented in a friction-Reynolds number diagram. In section 3.3 the results of the LDV measurements are presented focusing on the observations of turbulence statistics and on the various contributions to the total shear stress. Finally kinetic energy budgets, both for mean flow and for the turbulence, are discussed.

3.2 Pressure drop measurements

For three different fluids (water and the polymer solutions containing 103 wppm and 435 wppm of polymers) pressure drop measurements have been performed for a range of different flow rates in the pipe flow facility. The results of these measurements are shown in figure 3.1 in terms of relations between the Fanning friction factor and the (wall) Reynolds number in Prandtl-Karman coordinates. For definitions of these quantities, see section 2.2.1.

For water the agreement between experiment and theory (equation 2.5), is very good for the turbulent flow regime. For laminar flow there is a small discrepancy with equation (2.4), which can be caused because for laminar flow the low range of the pressure drop transducer is used and the signal from the transducer is somewhat fluctuating. For the 103 wppm solution the experimental data follow the same profile as water for laminar flow, but deviate as soon as the flow becomes turbulent. Directly after transition a high amount of drag reduction occurs and for high values of the wall Reynolds number the relation approaches Virk asymptote (equation 2.7). For the most concentrated solution (435 wppm) a similar behaviour is observed. The only difference is that transition to turbulence is delayed until a higher Reynolds number and that the data are immediately close to the asymptote. The data seem to slightly exceed the asymptote, which is probably caused by the strong shear-thinning behaviour of this solution (see figure 2.6), which might lead to an inaccuracy in the determination of the wall viscosity and hence the wall Reynolds number.

In figure 3.1 also the values of the friction factor for the cases for which detailed LDV measurements have been performed are shown (denoted with +, and the polymer concentration in brackets). The results of these measurements will be discussed in detail in section 3.3. From this figure follows that the 20 wppm solution exhibits only minor drag
Experimental Results

![Graph of friction factor vs. log(Re_w \sqrt{f})](image)

Figure 3.1: The relation between the friction factor and the (wall) Reynolds number in Prandtl-Karman coordinates. The LDV measurements, which will be discussed in section 3.3, are denoted with +, together with the concentration in wppm in brackets. The data for water and for the 103 and 435 wppm solutions are presented at a number of (wall) Reynolds numbers as indicated by the symbols. The three lines refer to the theoretical relations for laminar flow (equation 2.4), Newtonian turbulent flow (the Prandtl-Karman law, equation 2.5) and the maximum drag reduction or Virk asymptote (equation 2.7).

Table 3.1: Results of the friction factor measurements for the cases for which LDV experiments have been carried out. Q: flow rate; Re_w: wall Reynolds number; \dot{\gamma}_w: local shear rate at the wall; \tau_w: shear stress at the wall; f: friction factor determined with pressure drop measurement; DR%: percentual drag reduction as defined by equation (2.16).

<table>
<thead>
<tr>
<th>solution</th>
<th>water</th>
<th>20 wppm</th>
<th>103 wppm</th>
<th>175 wppm</th>
<th>435 wppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q (L/h)</td>
<td>1089</td>
<td>1461</td>
<td>2466</td>
<td>2565</td>
<td>4185</td>
</tr>
<tr>
<td>Re_w</td>
<td>9573</td>
<td>11636</td>
<td>11952</td>
<td>9764</td>
<td>10805</td>
</tr>
<tr>
<td>\dot{\gamma}_w (s^{-1})</td>
<td>226</td>
<td>263</td>
<td>218</td>
<td>184</td>
<td>275</td>
</tr>
<tr>
<td>\tau_w (Pa)</td>
<td>0.225</td>
<td>0.285</td>
<td>0.396</td>
<td>0.422</td>
<td>0.933</td>
</tr>
<tr>
<td>f (\times 10^{-3})</td>
<td>8.00</td>
<td>5.75</td>
<td>2.75</td>
<td>2.73</td>
<td>2.27</td>
</tr>
<tr>
<td>DR%</td>
<td>0</td>
<td>23</td>
<td>63</td>
<td>65</td>
<td>70</td>
</tr>
</tbody>
</table>

Reduction. The 103 wppm solution shows slightly less drag reduction than Virk asymptote, the 175 wppm solution shows approximately the same drag reduction as the asymptote and the 435 wppm solution even slightly exceeds the asymptote. It will be shown in the next section that the 175 and the 435 wppm solutions have only minor differences in the LDV experiments. This means that indeed a limit for drag reduction is reached. In table 3.1 some relevant parameters applicable to these experiments are presented. The wall
Reynolds number $Re_w$ and the shear rate at the wall $\dot{\gamma}_w$ are determined following the procedure described in section 2.2.2. The wall shear stress $\tau_w$ and the friction factor $f$ are obtained directly from the measurement of the pressure drop, using equations (2.1) and (2.3).

### 3.3 LDV experiments

In this section the results obtained from the LDV experiments are described. The focus is on the effect of the polymers on the turbulence statistics for the three different concentrations in the high drag reduction regime (103, 175, 435 wppm). The results of these experiments are compared with LDV measurements for water and for a very dilute polymer solution of 20 wppm of the same polymer (Den Toonder et al. 1997; Den Toonder 1995). All results are obtained at approximately $Re_w \approx 10000$. An overview of the experimental conditions is given in table 3.2.

<table>
<thead>
<tr>
<th>solution</th>
<th>water</th>
<th>20 wppm</th>
<th>103 wppm</th>
<th>175 wppm</th>
<th>435 wppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>symbol</td>
<td></td>
<td>□</td>
<td>▼</td>
<td></td>
<td>▲</td>
</tr>
<tr>
<td>$Q$ (l/h)</td>
<td>1089</td>
<td>1461</td>
<td>2466</td>
<td>2565</td>
<td>4185</td>
</tr>
<tr>
<td>$Re$ or $Re_w$</td>
<td>9573</td>
<td>11636</td>
<td>11952</td>
<td>9764</td>
<td>10805</td>
</tr>
<tr>
<td>$Re_*$</td>
<td>608</td>
<td>624</td>
<td>443</td>
<td>361</td>
<td>364</td>
</tr>
<tr>
<td>$U_p$ (mm/s)</td>
<td>236</td>
<td>317</td>
<td>535</td>
<td>557</td>
<td>908</td>
</tr>
<tr>
<td>$u_*$ (mm/s)</td>
<td>15.0</td>
<td>17.0</td>
<td>19.9</td>
<td>20.6</td>
<td>30.6</td>
</tr>
<tr>
<td>$U_c/u_*$</td>
<td>20.2</td>
<td>23.7</td>
<td>39.1</td>
<td>42.7</td>
<td>47.6</td>
</tr>
<tr>
<td>$f \times 10^{-3}$</td>
<td>8.00</td>
<td>5.75</td>
<td>2.75</td>
<td>2.73</td>
<td>2.27</td>
</tr>
<tr>
<td>$\eta_w/\rho u_*$ (mm)</td>
<td>0.066</td>
<td>0.065</td>
<td>0.091</td>
<td>0.112</td>
<td>0.111</td>
</tr>
</tbody>
</table>

In turbulence it is common to display the flow variables in dimensionless units, the so-called wall units. This means that velocities are made non-dimensional with the friction velocity $u_* = \sqrt{\tau_w/\rho}$ and lengths with $\eta_w/(u_*\rho)$. The non-dimensional quantities are denoted with a plus superscript. Consequently stresses are scaled with the wall shear stress $\tau_w = \rho u_*^2$. Note that in these definitions the viscosity at the wall is used (see section 2.2.2). In addition to the Reynolds number based on the bulk velocity in the pipe, one can introduce the Reynolds number based on the friction velocity $u_*$, thus $Re_* = \rho u_* D/\eta_w$. The distance from the wall is denoted by $y^+ = y u_* \rho/\eta_w$, with $y$ the actual distance from the pipe wall. Using these definitions $y^+ = 0$ denotes the pipe wall and $y^+ = D^+/2 = Re_*/2$ denotes the centre of the pipe.
3.3.1 Velocity statistics

Mean velocity

The profile of the mean axial velocity, $U_x^+ = U_x/u_*$, is shown as function of the radial coordinate $r/D$ in figure 3.2 for different polymer concentrations. For reasons of symmetry only half of the pipe is shown. The effect of drag reduction due to the addition of polymers is evident: for approximately similar Reynolds numbers the dimensionless mean velocity (and thus the flow rate) increases monotonically with increasing polymer concentration.

![Graph showing mean axial velocity $U_x^+$ as a function of $r/D$.]

Figure 3.2: LDV measurements of the dimensionless mean axial velocity $U_x^+$ as function of the radial coordinate $r/D$ for water and for the polymer solutions. $r/D = 0$ denotes the centre of the pipe, $r/D = 0.5$ is the pipe wall.

In figure 3.3 the same quantity $U_x^+$ is plotted in terms of wall units. From theory (e.g. Tennekes & Lumley 1973) it is known that for a fully developed turbulent pipe flow of a Newtonian fluid the theoretical mean velocity profiles in the viscous sublayer and in the logarithmic region are given by respectively:

$$U_x^+ = y^+ \quad \text{for } y^+ < 5 \quad \text{(3.1)}$$

$$U_x^+ = 2.5 \ln(y^+) + 5.5 \quad \text{for } y^+ > 30 \quad \text{(3.2)}$$

For drag reducing fluids the same profile is found in the viscous sublayer. For maximum drag reduction the logarithmic profile shows an increase of the slope and the profile is according to Virk (1975) given by:

$$U_x^+ = 11.7 \ln(y^+) - 17.0 \quad \text{for } y^+ > 30 \quad \text{(3.3)}$$
Figure 3.3: LDV measurements of the mean axial velocity $U_x^+$ as function of the distance from the wall $y^+$ for water and for the polymer solutions. The lines are the theoretical profiles of the viscous sublayer (equation 3.1), the Newtonian logarithmic profile (equation 3.2) and the Virk asymptote (equation 3.3). Flow conditions are given in table 3.2.

For water the agreement between relations (3.1) and (3.2) and the experimental data is very good. The profile of the 20 wppm solution shows the same profile as water in the viscous sublayer ($y^+ < 5$), but in the logarithmic region it is shifted upwards almost parallel to the Newtonian profile. The logarithmic region starts further away from the wall. This means that the viscous sublayer and buffer layer, where the latter is the part between the viscous sublayer and the logarithmic region, are thickened. Similar observations are found in other low drag reduction experiments in pipe (Pinho & Whitelaw 1990) and channel flow (Harder & Tiederman 1991, Wei & Willmarth 1992 and Warholic et al. 1999a). There is some argument in literature whether the slope of the logarithmic profile is fully parallel to the profile in the Newtonian case. An extensive study by Virk (1975) suggests a parallel profile corresponding with the elastic sublayer model of Virk. The experiments of Den Toonder (1995) and Den Toonder et al. (1997) (the 20 wppm solution) indicate that the slope in the logarithmic region is very slightly increased and is thus not fully parallel to the Newtonian profile. As the focus of this study is on our high drag reduction experiments, this difference will not be further discussed. For the polymer solutions with high concentrations there is also no difference in the viscous sublayer with Newtonian flow and with equation (3.1). It is very clear however, that there is a significant increase in the slope of the logarithmic profile. The slope and thus the velocity gradient become larger when the polymer solutions become more concentrated. Also the start of the logarithmic region in further delayed, which corresponds with thickening of the buffer layer. These effects become even more clear if physical units are taken into account.
From the lowest row of table 3.2 it follows that the physical size of the viscous units is larger for the polymer solutions than for water. Therefore the same value of \( y^+ \) in figure 3.3 corresponds with a shift from the wall towards the centre of the pipe for the highly concentrated solutions. The Virk asymptote is slightly exceeded for the 175 wppm and particularly for the 435 wppm solution. This was also observed in figure 3.1. Such effect has been also noted by Warholic et al. (1999b) and can again probably be attributed to the strong shear thinning behaviour of our solutions, which may result in some inaccuracies in the determination of the wall viscosity. Note that the difference in velocity between the two highest concentrations is small and only present near the centre of the pipe (velocities at the centreline differ by about 10% while the concentration is a factor 2.5 higher). Therefore it seems that a limit of the profile is reached for the highest concentrations.

**Turbulence intensities**

The dimensionless root mean square (r.m.s.) of the velocity fluctuations is defined by:

\[
\text{rms}(u^+_i) = \frac{1}{u_*} \sqrt{\langle (u_i - U_i)^2 \rangle}
\]  

(3.4)

where \( \langle \cdots \rangle \) denotes an averaging over all velocity samples taken during the measurement time. The quantity is determined for the axial and radial velocity components as a function of the radial position in the pipe.

In figure 3.4 the root mean square of the axial velocity fluctuations, \( \text{rms}(u^+_x) \), is presented. For the lowest concentration (20 wppm) the height of the peak increases with respect to the Newtonian case (water) and the maximum value of the peak is shifted away from the wall. However, when the concentration is increased further, the peak of the root mean square first remains high but later decreases to approximately the same value as for the solvent. This means that the maximum of the peak is connected to an intermediate concentration. As a function of the polymer concentration there is a shift of the peak away from the wall (to higher wall units), which again can be interpreted as a thickening of the buffer layer for high drag reducing flows.

Figure 3.4 also shows that addition of polymers results in a decrease of the r.m.s. of the axial velocity fluctuations near the wall, while the value is increased in the logarithmic layer with respect to Newtonian flow. This corresponds with a larger radial gradient of the r.m.s. value. A possible explanation for this is that the polymers produce a layer at the top of the buffer layer, with high turbulence above it and low turbulence below it, as result of the shear sheltering mechanism (Hunt & Durbin 1999). In other words, shear sheltering decouples these two regions. This will be further discussed when a possible mechanism for drag reduction is proposed in chapter 5. In the centre of the pipe the results approach to a similar value for all concentrations.

The root mean square of the radial velocity fluctuations, \( \text{rms}(u^+_r) \), is plotted in figure 3.5. It appears that the radial fluctuations are suppressed as a result of polymer addition for all locations in the pipe except in the viscous sublayer. The decrease is strongest for
Figure 3.4: LDV measurements of the root mean square of the axial velocity fluctuations, \( \text{rms}(u_z^+) \), as function of the distance from the wall \( y^+ \) for water and for the polymer solutions. Flow conditions are given in table 3.2.

Figure 3.5: LDV measurements of the root mean square of the radial velocity fluctuations, \( \text{rms}(u_r^+) \), as function of the distance from the wall \( y^+ \) for water and for the polymer solutions. Flow conditions are given in table 3.2.
the high polymer concentrations for which maximum drag reduction is found. For small
drag reduction (20 wppm solution) the profile shows only a small decrease and its value
approaches the value of the Newtonian case near the centre of the pipe, whereas the high
drag reducing solutions have a significantly lower value in the centre. For all polymer
solutions the location of the peak is shifted away from the wall though the shift for the
radial velocity component is smaller than for the axial component.

Similar effects as found here for both the axial and wall-normal r.m.s. values are
reported in the experiments of Harder & Tiederman (1991), Pinho & Whitelaw (1990), Wei
& Willmarth (1992) and Warholic et al. (1999a) and in the direct numerical simulations
of Den Toonder et al. (1997) and Sureshkumar et al. (1997). It should be noted however,
that except the channel flow experiments of Warholic et al. (1999a) all other studies have
been carried out for small drag reduction. The main conclusion of the velocity fluctuation
measurements in our study is that drag reduction creates a strong anisotropy (as one
component is increased and the other is decreased) of the turbulent velocity field and
modification of turbulent energy particularly in the buffer layer. Both for the axial and
for the radial r.m.s. the difference between the two highest concentrations is very small
which indicates that a limit is obtained.

Higher order statistics

Accurate data for higher-order statistics, like third (skewness) and fourth (flatness) mo-
mments, are more difficult to obtain experimentally. This is caused by high scatter of the
fluctuating velocity signals and therefore for an accurate result a long measurement time
is necessary. This problem is larger in the vicinity of the wall. To our knowledge only Wei
& Willmarth (1992), Den Toonder (1995) and Den Toonder et al. (1997) have measured
these statistics and all these experiments are restricted to the low drag reduction regime.

Here only the results of the axial skewness and flatness are presented, because the data
for the radial component have very much scatter. Measurements of Den Toonder (1995)
indicate that for the radial component the skewness and flatness are more scattering and
that the difference for drag reducing flows is only minor with respect to Newtonian flows.
The definitions of the axial skewness $S_x$ and flatness $F_x$ are:

$$S_x = \frac{\langle (u_x - U_x)^3 \rangle}{[\text{rms}(u_x)]^3}$$  \hspace{1cm} (3.5)

$$F_x = \frac{\langle (u_x - U_x)^4 \rangle}{[\text{rms}(u_x)]^4}$$  \hspace{1cm} (3.6)

The results of the LDV measurements for the axial skewness are shown in figure 3.6.
For a Gaussian distribution the skewness is equal to zero. It is clear that the axial velocity
distribution is not Gaussian. Very close to the wall ($y^+ < 5$) there is still some scatter
in the data, but further away from the wall the axial skewness seems higher for drag
reducing flows than for the Newtonian case. This difference increases and is present over
Figure 3.6: LDV measurements of the axial skewness, $S_x$, as function of the distance from the wall $y^+$ for water and for the polymer solutions. Flow conditions are given in table 3.2.

Figure 3.7: LDV measurements of the axial flatness, $F_x$, as function of the distance from the wall $y^+$ for water and for the polymer solutions. Flow conditions are given in table 3.2.
a larger part of the pipe for the higher polymer concentrations. Close to the centre of the pipe the value is for all solutions similar to the Newtonian case.

The axial flatness profile is depicted in figure 3.7. For the lowest concentration the flatness is not changed very much. For the high drag reducing flows the axial flatness is significantly higher than for Newtonian flow in the buffer region ($5 < y^+ < 30$). Close to the centre of the pipe the differences are quite small and close to the value for a Gaussian distribution which is equal to 3.

### 3.3.2 Shear stresses

The total mean shear stress in a fully developed pipe flow is a linear function of the radial coordinate varying between the zero value at the centreline and the value $\tau_\infty$ given by equation (2.1) at the wall (see section 2.3.1). The mean shear stress $T$ can be written as the sum of the Reynolds stress $T^{(r)}$, the solvent (viscous) stress $T^{(s)}$, and the polymer stress $T^{(p)}$, as is shown in equations (2.24) and (2.25). Note that according to our notation the total stress and all its components are negative, because the pressure gradient is negative. Though for reasons of simplicity and consistency with the usual way of presenting of these results, the stress contributions will be displayed as their absolute values. For the dimensionless values of these stresses (scaled with $\tau_\infty$) then follows:

\begin{align}
T^+ &= \frac{2r}{D} \\
T^{+(r)} &= \frac{1}{u_\infty^2} \langle u'_x u'_r \rangle \\
T^{+(s)} &= -\frac{1}{\rho u_\infty^2 \eta_s} \frac{dU_x}{dr} \\
T^{+(p)} &= T^+ - T^{+(r)} - T^{+(s)}
\end{align}

(3.7) \hspace{1cm} (3.8) \hspace{1cm} (3.9) \hspace{1cm} (3.10)

The Reynolds stress is calculated directly from the experimental data, taking into account that in the averaging procedure both velocity components should be measured in coincidence mode. The solvent stress is computed from a derivative of the mean velocity profile, which is determined by differentiation of a spline interpolation of the measurements. Note that the solvent stress is based on the viscosity of the solvent (water) only. In this experimental part the polymer stress is not specified by a constitutive equation. Therefore its contribution cannot be computed directly but it is by definition the difference of the total stress and the sum of the other two contributions.

**Reynolds shear stress**

The turbulent shear stress (Reynolds stress) is shown in figure 3.8. For the lowest concentration (20 wppm) a small decrease of the Reynolds stress is observed, except near the centre of the pipe, where the value is similar to the Newtonian case. For the high drag
Figure 3.8: LDV measurements of the Reynolds stress, $T_+^{(r)}$, as function of the distance from the wall $y^+$ for water and for the polymer solutions. Flow conditions are given in table 3.2.

reduction experiments the decrease of the Reynolds stress is much stronger (up to 75% of the value found for the Newtonian case). This decrease becomes stronger when the concentration increases and at the two most concentrated solutions (175 and 435 wppm) a lower limit is reached. The decrease of the Reynolds stress for low polymer concentration has been also reported in other experimental data (Harder & Tiederman 1991, Wei & Willmarth 1992, and Den Toonder 1995) and in direct numerical simulations (Sureshkumar et al. 1997). However, the effects are weaker because in these cases lower polymer concentrations are used. In similar experimental conditions as used here, Warholic et al. (1999a,b) have reported an almost zero Reynolds stress for surfactant and polymer solutions near maximum drag reduction. In our experiments the Reynolds stress at maximum drag reduction is significantly reduced but it remains definitely non-zero.

Though the reduction of the Reynolds stress is very clear, one can argue whether this is due to a decrease of the correlation of the two velocity fluctuations or due to a decrease in magnitude of one of them. Therefore in figure 3.9 the correlation coefficient $\rho_{xr}$ of the axial and radial velocity components is shown, which is defined by:

$$\rho_{xr} = \frac{\langle u'_x u'_r \rangle}{\sqrt{\langle u'_x^2 \rangle \langle u'_r^2 \rangle}}$$

The results indicate that the correlation decreases at all radial positions. The suppression is a monotonic function of the polymer concentration. However, the relative decrease is less than for the turbulent stress, so the decrease of the Reynolds stress can partly be attributed to the decrease of the velocity fluctuations themselves.
EXPERIMENTAL RESULTS

**Figure 3.9:** LDV measurements of the correlation coefficient of the axial and radial velocity components for water and for the polymer solutions. Flow conditions are given in table 3.2.

### Contributions to the total shear stress

The individual contributions to the mean shear stress are for Newtonian flow (water) shown in figure 3.10. The two contributions are the Reynolds stress $\tau^{(+)}$ and the solvent stress $\tau^{(+)}$, which are both measured with LDV. Also the sum of these two contributions is shown and compared to the theoretical mean shear stress $\tau^{+}$, which is obtained from pressure drop measurements. The total stress is a linear function across the radial location. The agreement of the measured data with the linear function is excellent except for the region near the wall. This can be attributed to small inaccuracies in the determination of the solvent stress, which is obtained from an interpolation of a steep velocity gradient.

The contributions of the Reynolds stress, the solvent stress and the polymer stress for the polymer solutions are plotted in figures 3.11-3.14. For all these viscoelastic flows the sum of the solvent and turbulent stresses no longer equals the total mean stress. This is called the Reynolds stress deficit and it should be compensated by a polymer stress. For the 20 wppm solution the polymer stress contribution is quite small and only non-negligible close to the wall. The results for the more concentrated solutions show that the polymer stress takes over the role from the Reynolds stress as main contributor to the total stress. Especially the 175 wppm and the 435 wppm solutions have a polymer stress contribution which is about 60% of the total stress. In particular, the polymer stress becomes very large with respect to the turbulent stress in the neighbourhood of the wall. For the strong drag-reducing solutions the polymer stress is almost a linear function of the radial location. This means that the relative contribution of the polymer stress to the total stress is approximately the same at each radial location.
Figure 3.10: The various contributions to the mean shear stress as function of the radial location $r/D$ for water. $T^{+}(r)$: Reynolds stress; $T^{+}(s)$: solvent stress; $T^{+}$: (theoretical) mean shear stress. $Re \approx 9600$.

Figure 3.11: The various contributions to the mean shear stress as function of the radial location $r/D$ for the 20 wppm solution (Den Toonder 1995). $T^{+}(r)$: Reynolds stress; $T^{+}(s)$: solvent stress; $T^{+}(p)$: polymer stress; $T^{+}$: (theoretical) mean shear stress. $Re_{w} \approx 11600$. 
Figure 3.12: The various contributions to the mean shear stress as function of the radial location $r/D$ for the 103 wppm solution. $T^+_r$: Reynolds stress; $T^+_s$: solvent stress; $T^+_p$: polymer stress; $T^+$: (theoretical) mean shear stress. $Re_w \approx 12000$.

Figure 3.13: The various contributions to the mean shear stress as function of the radial location $r/D$ for the 175 wppm solution. $T^+_r$: Reynolds stress; $T^+_s$: solvent stress; $T^+_p$: polymer stress; $T^+$: (theoretical) mean shear stress. $Re_w \approx 9800$. 

3.4 Kinetic energy budgets

In this section the kinetic energy balance equations (2.26) and (2.32) are applied to our experimental data. In particular a comparison is made between Newtonian flow and the drag reducing polymer solutions. Considered are the area-integrated budgets, which means that all terms in equations (2.26) and (2.32) are integrated over the pipe cross section (denoted with ‘S’). Because of the divergence theorem, the contribution of the transport terms (equations 2.28 and 2.34) to the area-integrated budget becomes zero.

3.4.1 Mean flow kinetic energy

For the kinetic energy of the mean flow (equation 2.26) the integration over the pipe cross section means that the production of mean flow energy due to the imposed pressure gradient (‘gain’) should be in balance with the sum of (the opposite of) the deformation work, the viscous dissipation by the mean flow and the dissipation by mean polymer stresses (‘loss’):

$$\int_S P_u = \int_S -D_u + \int_S \epsilon_u + \int_S W_u$$  \hspace{1cm} (3.12)

The terms in equation (3.12) are given in table 3.3 for water and for the polymer solutions. In this table also the sum of all terms on the right-hand-side is given, which should be equal to the production term.
EXPERIMENTAL RESULTS

Table 3.3: The various mean flow energy budget terms for water and for the polymer solutions. All quantities are made non-dimensional with $\eta_w u^2$.

<table>
<thead>
<tr>
<th>solution</th>
<th>water</th>
<th>20 wppm</th>
<th>103 wppm</th>
<th>175 wppm</th>
<th>435 wppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_\mu$ ($\times 10^4$)</td>
<td>2.94</td>
<td>3.66</td>
<td>3.71</td>
<td>3.08</td>
<td>3.28</td>
</tr>
<tr>
<td>$-D_\mu$ ($\times 10^4$)</td>
<td>1.16</td>
<td>1.41</td>
<td>0.76</td>
<td>0.48</td>
<td>0.38</td>
</tr>
<tr>
<td>$\epsilon_\mu$ ($\times 10^4$)</td>
<td>1.85</td>
<td>2.02</td>
<td>1.33</td>
<td>0.96</td>
<td>0.88</td>
</tr>
<tr>
<td>$W_\mu$ ($\times 10^4$)</td>
<td>0</td>
<td>0.21</td>
<td>1.59</td>
<td>1.63</td>
<td>2.01</td>
</tr>
<tr>
<td>sum r.h.s. ($\times 10^4$)</td>
<td>3.01</td>
<td>3.65</td>
<td>3.68</td>
<td>3.07</td>
<td>3.27</td>
</tr>
</tbody>
</table>

Figure 3.15: The various mean flow energy budget terms for water. $Re \approx 9600$.

For the case of water all terms have been measured independently (obviously the polymer dissipation $W_\mu$ is zero). This means that the terms on the right-hand-side are measured with LDV and the mean flow production is determined from the measured pressure gradient. Within an error of about 2% the sum of both sides of equation (3.12) is the same and this is consistent with the agreement found for the stress profiles of water shown in figure 3.10. This gives us confidence that our data are sufficiently accurate. As additional information, the distribution of the terms in equation (3.12) for water across the radial location is shown in figure 3.15.

The most important result for the mean energy budget of the polymer solutions is that the polymer dissipation term $W_\mu$ becomes more important as the polymer concentration increases. The contribution of the polymers to the kinetic energy budget is computed using the polymer stress, which is computed using equation (3.10). For the 20 wppm solution the contribution by the polymers is still rather small, but for the other solutions
it becomes the largest loss term on the right-hand side of the budget. This implies that most of the energy is transferred directly to the polymers and not by the route of turbulence. This is also shown in figure 3.16 where the various budget terms are plotted for the 175 wppm solution. Most dissipation of mean flow kinetic energy by polymers occurs in the region near the pipe wall.

3.4.2 Turbulent kinetic energy

In the kinetic energy balance of the turbulence, equation (2.32), the production $P_k$ is equal to the opposite of the deformation work $D_u$ in the mean flow balance. After integration of equation (2.32) over the pipe cross section there should be a balance between the production of turbulent energy on one side and the viscous dissipation and the polymer stress work on the other side. This reduces this equation to:

$$\int_S P_k = \int_S \varepsilon_k + \int_S W_k$$  \hspace{1cm} (3.13)

In this budget only the production $P_k$ can be measured directly. This is an important term, since it is responsible for the generation of turbulence by transferring energy from the mean flow to the turbulence. This production term is plotted as a function of the distance from the wall in figure 3.17. For the polymer solutions the peak of the production is lower and it is shifted away from the wall. This is consistent with the shift observed in the r.m.s. profiles. The effect becomes stronger with increasing polymer concentration. However, the turbulent kinetic energy of the axial component is for the polymer cases larger or equal than for the Newtonian case (see figure 3.4). Therefore the smaller production suggests
that most energy remains in the axial component and is not transferred to the radial and tangential directions. This is confirmed by the results the radial turbulence r.m.s. values which decrease continuously with increasing polymer concentration (see figure 3.5). The production term acts as an input for the axial component only, whereas for the other components energy is generated by the pressure-strain correlation. Therefore the pressure-strain correlation should also be reduced. In chapter 5 will be shown by means of direct numerical simulations that the pressure-strain correlation is indeed strongly reduced as result of the addition of polymers.

![Figure 3.17: LDV measurements of the turbulent kinetic energy production, $P_k^+$, as function of the distance from the wall $y^+$ for water and for the polymer solutions. Flow conditions are given in table 3.2.](image)

In the budget for the Newtonian flow the only terms in equation (3.13) are the viscous dissipation $\epsilon_k$ and the production $P_k$ and thus they should by definition be equal. It is not possible to measure the viscous dissipation since this requires the determination of spatial derivatives of the fluctuating velocity, which cannot be measured with LDV. Therefore an approximation of the viscous dissipation has been made, by using a similar approach as proposed in section 1.2 (see also Tennekes & Lumley 1973). This means that the dissipation is taken to be proportional to the energy of the large eddies divided by their turnover time (equation 1.2) and that $\epsilon_k$ is approximated by:

$$\epsilon_k \approx c_D \rho \frac{\epsilon^{3/2}}{D}$$  \hspace{1cm} (3.14)

where $c_D$ is a dimensionless constant and $\epsilon = \frac{1}{2} \overline{u'^2}$ is the turbulent kinetic energy per unit mass, which is the sum of the axial, radial and tangential contributions. Since the tangential component $\overline{u_\theta^2}$ has not been measured, an estimate by discussing the order
of magnitude is made. Direct numerical simulations (e.g. Den Toonder et al. 1997 and Sureshkumar et al. 1997 and our own results that will be presented in chapter 5) show that \( \overline{u'^2} \) is of the same order as \( \overline{u''^2} \) for both non drag-reducing and drag-reducing flow. Therefore the turbulent kinetic energy is approximated by \( e = \frac{1}{2}(\overline{u'^2} + 2\overline{u''^2}) \). This estimation gives for our Newtonian flow a value of \( c_D = 7.21 \).

The value of the constant \( c_D \) depends on the pipe geometry, e.g. through the Reynolds number, and on the dynamics of turbulence by which energy is transferred through a cascade process into heat at the microscales. In order to say something about the polymer cases, it will be assumed that \( c_D \) remains the same for the turbulent flow of the polymer solutions. This makes possible to compute \( \epsilon_k \) for the polymer solutions. The polymer stress work \( W_k \) can then be computed from the difference of \( P_k \) and \( \epsilon_k \). The results are given in table 3.4.

Table 3.4: The various turbulent kinetic energy budget terms for water and for the 175 wppm solution. All quantities are made non-dimensional with \( \eta_w u''^2 \).

<table>
<thead>
<tr>
<th>solution</th>
<th>water</th>
<th>20 wppm</th>
<th>103 wppm</th>
<th>175 wppm</th>
<th>435 wppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \int_S P_k \times 10^4 )</td>
<td>1.16</td>
<td>1.41</td>
<td>0.76</td>
<td>0.48</td>
<td>0.38</td>
</tr>
<tr>
<td>( \int_S \epsilon_k \times 10^4 )</td>
<td>1.16</td>
<td>1.65</td>
<td>1.49</td>
<td>0.81</td>
<td>0.85</td>
</tr>
<tr>
<td>( \int_S W_k \times 10^4 )</td>
<td>0</td>
<td>-0.21</td>
<td>-0.73</td>
<td>-0.33</td>
<td>-0.47</td>
</tr>
</tbody>
</table>

Based on the above approximation the polymer stress work \( W_k \) becomes negative for all drag reducing solutions. In other words, the fluctuating polymer stresses produce rather than dissipate kinetic energy. For the 20 wppm solution the polymer stress work is rather small, but it is much higher for the other concentrations. However, it must be stressed that this result depends strongly on our assumption that the constant \( c_D \) remains the same which requires an independent verification. Therefore the conclusion is that although energy production by fluctuating polymer stresses cannot be excluded, there is certainly no conclusive evidence for this.

The process that energy is produced by fluctuating polymer stresses has also been suggested by Warholic et al. (1999a,b) though the difference compared to our results is that in their experiments the production of kinetic energy \( P_k \) is zero because of a zero Reynolds stress. Therefore the viscous dissipation \( \epsilon_k \), which is always positive, must be the opposite of the polymer stress work \( W_k \) and the latter has to be negative.

### 3.5 Conclusions and discussion

In this chapter the results of pressure drop and LDV measurements in a turbulent pipe flow with polymer additives have been presented. The polymer used is Superfloc A110, which is partially hydrolyzed polyacrylamide. The conditions of the flow are at maximum
drag reduction, i.e. near the Virk asymptote (Virk 1975). It is important to notice that in all cases that have been studied, the flow is turbulent.

The amount of drag reduction is measured in terms of a change in the Fanning friction factor as function the (wall) Reynolds number. The use of the wall Reynolds number follows from the fact that for non-Newtonian fluids, like the shear thinning polymer solutions used in our case, the viscosity does not have a constant value. Therefore the usual Reynolds number definition cannot be used, because it is based on a constant viscosity everywhere in the flow. Instead the wall Reynolds number is used, which is based on the wall viscosity $\eta_w$ which is related to the wall shear stress.

All our LDV experiments with polymer solutions are performed at $Re_w \approx 10000$ and show a high amount of drag reduction. The difference in the measured velocity statistics between the two highest concentrations is very small, becoming independent of the concentration, so that an asymptotic value for drag reduction is reached. The results of these measurements are compared with our measurements with a Newtonian fluid (water) and with measurements with a less concentrated polymer solutions (Den Toonder et al. 1997, Den Toonder 1995). Most effects are much stronger for the strong drag-reducing solutions compared to the results of Den Toonder.

The main results of the profile measurements are the following: as a result of the addition of polymers the flow rate strongly increases, which is consistent with drag reduction at constant pressure drop. In the wall region the buffer layer thickens and the velocity profile in the logarithmic region has a larger slope. The measured profiles of the mean velocity in the logarithmic region are consistent with the profile proposed for maximum drag reduction (equation 3.3).

The peak of the root mean square of the axial velocity fluctuations is increased for small polymer concentrations and shifts away from the wall. However, for the highest concentrations the height of the peak returns to the value found for water. So a maximum value is reached at some intermediate concentration. The root mean square of the radial velocity fluctuations shows a strong decrease and a small shift of the peak towards the centre of the pipe. Thus most turbulent kinetic energy remains in the streamwise (axial) component at the cost of the normal (radial) component.

The Reynolds stress strongly decreases at the high polymer concentrations, but without doubt remains non-zero. This is in contrast with the observations of Warholic et al. (1999a,b), who concluded from their measurements that the Reynolds stress is very close to zero for surfactant and for polymer solutions at maximum drag reduction. Because of the greatly reduced Reynolds stress, the sum of the solvent and Reynolds stress no longer equals the total stress. This is called the Reynolds stress deficit and the effect is more dominant for strong drag reducing solutions. This means that the polymers contribute significantly to the total stress, compensating the reduced role of the turbulence.

The comparison of the various terms in the mean energy budget shows that in polymer flows the largest part of the mean flow kinetic energy is transferred directly to the polymers and not to the turbulence. Also the turbulent kinetic energy is strongly suppressed.
together with a probable reduction of the pressure-strain correlation. Depending on the assumption made for the viscous dissipation it is possible that the polymer stress work is negative, which is equivalent to a production of turbulence by the fluctuating polymer stresses.
Chapter 4

Viscoelastic modeling and simulation techniques

Abstract
In this chapter the equations for the fluid flow and for the modeling of the polymers are presented. The polymers are modeled in a microstructural way by means of the elastic dumbbell theory. From this theory the constitutive equation for the FENE-P model is derived. The advantage of this model is that it provides a closed-form constitutive equation that can directly be used to compute the polymer stress. The constitutive equation is solved simultaneously with the equations for the flow. Finally, the numerical techniques, parameters and procedures used in the simulations of polymer flows are discussed.

4.1 Introduction

In the previous two chapters the results of our laboratory experiments have been discussed and a comparison has been made with some other experiments. In this chapter the numerical simulations of turbulent flows of dilute polymer solutions are introduced. The technique of direct numerical simulations (DNS) is used, which means that the conservation equations of mass and momentum for the turbulent flow field are solved in space and time without using any turbulence model. In addition to this, the equations for the polymers, which relate the polymer stress to the flow field, are solved. The latter equations are denoted as constitutive equations.

Both laboratory experiments and numerical simulations play an important role in present day fluid dynamics. While experiments make it possible to obtain flow information in complex geometries and at very high Reynolds numbers, the advantage of simulations is that they provide a full three-dimensional flow field in space and time from which information can be obtained which is impossible to measure. For instance, information on quantities like polymer stresses and pressure in the flow is extremely difficult to obtain from measurements, but can directly be obtained from simulations. An other advantage is
that in simulations one can isolate different polymer properties (like elasticity, maximum stretching and concentration) and study their separate effect on the flow. Therefore simulations contribute to the insight in the physical mechanism of drag reduction.

A difference between our experiments and simulations is the flow geometry. While the experiments are performed in pipe flow, which is more suitable for the reasons mentioned in section 2.1, the simulations are carried out in channel flow geometry. This is partly due to computational reasons, since computations of pipe flow are more complex especially near the centre of the pipe. The inclusion of the polymer model makes the whole computer code much more expensive than a standard Newtonian fluid code and therefore most computational resources should be reserved for the polymer part. Despite this difference in flow geometry, from a physical point of view there is a similarity between pipe and channel flows, because both types belong to the class of wall-bounded turbulent flows and their flow statistics (especially near the wall) are very similar. Therefore, the results of the simulations will be compared with with the experimental results.

A review of the simulations in drag reducing turbulent flows is given in section 1.5. Though, some of these simulations (Orlandi 1995; Den Toonder 1995; Den Toonder et al. 1997) have the disadvantage that the polymer models used are not ‘frame-indifferent’ or ‘objective’. Frame-indifference means that the constitutive equations should be independent of the chosen frame. For instance they should be invariant for an arbitrary rotation of the system. Furthermore, in these simulations rather simple polymer models are used, which do not take all physical properties of the polymers into account and are thus not very satisfactory from a physical point of view. Only very recently some simulations (Sureshkumar et al. 1997) have been reported, which do not have the disadvantages mentioned above. Upto now no simulations have been performed for a high amount of drag reduction. Our experiments show that the effects of polymers are most pronounced in this regime and therefore it is expected that simulations will provide a better insight in the mechanism of drag reduction by polymers. Therefore, similarly as in the experiments, the simulations have been performed in the regime near the maximum drag reduction asymptote (MDR) or Virk (1975) asymptote. It is possible to perform direct numerical simulations in this regime by using a relatively high Reynolds number together with high polymer concentrations and by using strongly extensible polymers with a high elasticity.

### 4.2 Governing equations for the flow field

The general equations for the isothermal flow of an incompressible fluid are given by the continuity equation and the equation for the conservation of momentum, which read:

\[ \nabla \cdot u = 0 \]  \hspace{1cm} (4.1)

and

\[ \frac{D\rho u}{Dt} \equiv \rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u = -\nabla p + \nabla \cdot \tau \]  \hspace{1cm} (4.2)
In these equations $t$ is the time, $\rho$ is the density, $\mathbf{u}$ is the velocity vector and $\mathbf{\tau}$ is the total extra stress tensor. $D/Dt$ denotes the material derivative. The fluid considered is a polymer solution, which consists of a Newtonian solvent in which long-chain linear polymer molecules are dissolved. The solution is assumed to be sufficiently dilute, so that the polymers do not interact with each other. The total stress can be written as the sum of a Newtonian part due to the solvent $(\mathbf{\tau}^{(s)})$ and a non-Newtonian part due to the polymers $(\mathbf{\tau}^{(p)})$ according to:

$$\mathbf{\tau} = \mathbf{\tau}^{(s)} + \mathbf{\tau}^{(p)} = \beta \eta_0 (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \mathbf{\tau}^{(p)}$$  \hspace{1cm} (4.3)

where $\beta = \eta_s/\eta_0$, the ratio of the solvent viscosity $\eta_s$ and the total zero-shear-rate viscosity of the solution $\eta_0$. Equation (4.2) then becomes:

$$\rho \frac{D\mathbf{u}}{Dt} = \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \beta \eta_0 \nabla^2 \mathbf{u} + \nabla \cdot \mathbf{\tau}^{(p)}$$ \hspace{1cm} (4.4)

The polymer stress $\mathbf{\tau}^{(p)}$ must be related to the flow field and to the polymer configuration. For this a polymer model is needed which is able to describe the polymer dynamics in a turbulent flow realistically.

### 4.3 Polymer modeling

#### 4.3.1 Elastic dumbbell theory

In this section an outline of the elastic dumbbell theory is given. For details the reader is referred to Bird et al. (1987b).

**Modeling of the polymers by an elastic dumbbell**

Polymers have a great diversity in structure and molecular weight distribution and therefore it is complicated to describe their configuration in detail. Since the 1960s it has become popular to use simplified mechanical models for the description of polymer chains. An example is the elastic dumbbell model, which has its roots in kinetic theory (see Bird et al. 1987b). In this model the polymers are represented by small mechanical objects (the dumbbells) and using kinetic theory it is possible to derive constitutive equations and to calculate rheological properties.

The elastic dumbbell consists of two identical spherical beads with mass $m$, which location is specified by the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. The beads are connected by a non-bendable elastic spring, specified by the connector vector $\mathbf{Q} = \mathbf{r}_1 - \mathbf{r}_2$. The dumbbell model is schematically shown in figure 4.1. The connector vector describes the orientation and the internal configuration of the dumbbell. The entire molecule is fully specified by its length and its direction, represented by $\mathbf{Q}$, and by the position of the centre of mass $\mathbf{r}_c = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$. Obviously this is a substantial simplification because the molecular
architecture and internal configurations are neglected. Nevertheless, the model contains two essential characteristics of linear polymers namely orientability and stretchability. These properties are found to be essential to describe the rheological properties of the polymer solution.

![Figure 4.1: Schematic overview of the elastic dumbbell with the configuration given by the vector \( Q \).](image)

It is assumed that the polymers are dissolved in a Newtonian solvent. The solution is sufficiently dilute so that the polymers do not interact with each other. The number of polymers per unit volume is \( n \). Furthermore, the flow field is homogeneous on the length scale of the dumbbells and is incompressible. The distribution of the ensemble of the dumbbells is described by a statistical distribution function in the position-velocity space. This function can be written as the product of a configuration-space distribution function and a velocity-space distribution function. It is assumed that the configuration-space distribution function is independent of the location of the centre of mass of the dumbbell and that the velocity-space distribution function is given by a Maxwellian distribution around the velocity of the centre of mass.

**Equation of motion for the elastic dumbbell**

Using the statistical distribution functions it is possible to derive the evolution equation in the form of a diffusion or Fokker-Planck equation for the probability density function of the dumbbells. This is done by formulating a force balance for each of the beads separately and by subtracting these equations one obtains the equation of motion for the dumbbell vector \( Q \). It is assumed that inertia is neglected due to the small mass of the beads and that the beads slowly move through the solution, so that their drag can be described by the Stokes’ law. The equation of motion for the dumbbell connector vector reads:

\[
[\dot{Q}] = \kappa \cdot Q - \frac{2kT}{\zeta} \frac{\partial}{\partial Q} \ln \psi - \frac{2}{\zeta} \mathbf{F}^{(c)}
\]  

(4.5)
where $\kappa$ is the (constant) velocity-gradient tensor, $\zeta$ is the isotropic friction coefficient of the beads, $k$ is the Boltzmann constant, $T$ is the absolute temperature, $\psi(Q,t)$ is the normalized configuration-space distribution function and $F^{(c)}(Q)$ is the connector force in the spring between the beads. The operator $[\cdots]$ denotes averaging over the velocity-space distribution function. The three terms of the right-hand-side come from the three forces acting on the beads: the hydrodynamic drag force, the Brownian force and the intramolecular force. The hydrodynamic drag force is a Stokes resistance force that a bead experiences during the movement through the solution; the force is proportional to the difference of the velocity of the bead and the velocity of the solution at the location of the bead. The Brownian force is caused by thermal fluctuations of the solution and leads to a rapid and irregular motion of the beads. The average of this fluctuating force is usually expressed using the configuration-space distribution function $\psi$. The intramolecular or connector force is the force in the spring, which is the gradient of the spring potential energy. In general, also external forces (e.g. gravitational and electrical) can be acting on the dumbbells, but their contribution is zero if the same force acts on each bead, which is the case in our system.

**Continuity and diffusion equations for the configuration-space distribution function $\psi$**

The continuity equation of the normalized configuration-space distribution function $\psi(Q,t)$ states that the function $\psi$ satisfies the normalization condition

$$\int_Q \psi(Q,t) dQ = 1$$

when integrating over all possible dumbbell configurations $Q$. The continuity equation has the form of a conservation equation and reads:

$$\frac{\partial \psi}{\partial t} = - \frac{\partial}{\partial Q} \cdot [\dot{Q}] \psi \quad (4.6)$$

The diffusion equation for $\psi(Q,t)$ can be derived by substituting the equation of motion (4.5) for $[\dot{Q}]$ in the continuity equation. It is a second-order partial differential equation and describes the change of the distribution of configurations in an instationary flow field $\kappa(t)$ and is sometimes called the Fokker-Planck equation. It reads:

$$\frac{\partial \psi}{\partial t} = - \frac{\partial}{\partial Q} \left( \kappa \cdot Q \psi - \frac{2kT}{\zeta} \frac{\partial}{\partial Q} \psi - \frac{2}{\zeta} F^{(c)} \psi \right) \quad (4.7)$$

Multiplying this equation with the second-order tensor $QQ$ and integrating over the entire configuration space (denoted with the operator $\langle \cdots \rangle$) gives the following equation
of change for $\langle QQ \rangle$:

$$\frac{D\langle QQ \rangle}{Dt} = \frac{\partial \langle QQ \rangle}{\partial t} + u \cdot \nabla \langle QQ \rangle =$$

$$\nabla u \cdot \langle QQ \rangle + \langle QQ \rangle \cdot (\nabla u)^{\tau} + \frac{4kT}{\zeta} I - \frac{A}{\zeta} \langle QF^{(c)} \rangle$$

(4.8)

This equation describes the change in dumbbell configurations due to the convection of the dumbbells through the flow and their deformation by various forces. Convection is related to the motion of the centre of mass of the dumbbells while the deformation changes the dumbbell configuration, i.e. the orientation and the separation of the beads. On the right-hand side the various contributions of the forces acting on the dumbbell can be recognized: the first two terms represent deformation by hydrodynamic forces, the third term refers to Brownian motion and the fourth term is the elastic retraction of the spring. For a system in equilibrium (that is without flow and all time derivatives set to zero) equation (4.8) becomes:

$$\langle QF^{(c)} \rangle_{eq} = kTI$$

(4.9)

which will be used further on.

### 4.3.2 Stress tensor

The total polymer stress tensor $\sigma^{(p)}$ consists of a contribution from the intramolecular or connector force ($\sigma^{(p)}_{c}$) and a contribution from the motion of the beads ($\sigma^{(p)}_{b}$). Using kinetic theory (see Bird et al. 1987b for details) the following expressions for these contributions can be derived:

$$\sigma^{(p)}_{c} = n \langle QF^{(c)} \rangle$$

(4.10)

$$\sigma^{(p)}_{b} = -2nkTI$$

(4.11)

The total polymer stress is the sum of these two contributions and is thus equal to:

$$\sigma^{(p)} = \sigma^{(p)}_{c} + \sigma^{(p)}_{b} = n \langle QF^{(c)} \rangle - 2nkTI$$

(4.12)

The polymer stress in equilibrium is equal to (using equation 4.9):

$$\sigma^{(p)}_{eq} = \sigma^{(p)}_{c,eq} + \sigma^{(p)}_{b,eq} = n \langle QF^{(c)} \rangle_{eq} - 2nkTI = -nkTI$$

(4.13)

The total polymer stress tensor is the sum of an isotropic contribution (present in equilibrium) and a contribution due to the flow. The polymer stress tensor, as used in equation (4.4), is the contribution due to the flow only. Therefore, the equilibrium contribution has to be subtracted from the total polymer stress and the extra polymer stress becomes:

$$\tau^{(p)} = -nkTI + n \langle QF^{(c)} \rangle$$

(4.14)

This expression is called the Kramers form of the stress tensor.
4.4 Constitutive equations

In the previous section the elastic dumbbell theory has been discussed without any specification for the connector force. By specifying this force, constitutive equations can be derived, which are the equations to be used in the numerical simulations. A very simple example for a connector force is a linear spring, i.e. \( F^{(c)} = HQ \) with \( H \) the spring constant. This is called a Hookean dumbbell and its substitution in the equations (4.8) and (4.14) leads to the Oldroyd-B model. However, this model has some drawbacks, for instance a constant shear viscosity (while the experiments show that our solutions are shear-thinning) and a possible infinite separation of the beads, which leads to an infinite value of the elongational viscosity at finite elongation rates.

Therefore, it is more reasonable to use a model with a finite maximum separation of the beads. One of the most common and realistic models nowadays is the FENE (finitely extensible non-linear elastic) model, which was introduced by Warner (1972). The corresponding force is specified by:

\[
F^{(c)} = \frac{HQ}{1 - (Q/Q_0)^2}
\]  
(4.15)

where \( Q = |\mathbf{Q}| = \sqrt{Q \cdot Q} \) is the length of the spring and \( Q_0 \) is the maximum value for its length. The FENE spring is linear for a small separation of the beads, but will become stiffer for higher extensions. It is clear that as \( Q \to Q_0 \) the spring force will approach infinity and therefore \( Q \) cannot exceed \( Q_0 \). The disadvantage of this model is that it does not lead to a closed-form of the constitutive equation. In other words this means that after substitution of equation (4.15) in the equation of change (4.8) for \( \langle QQ \rangle \) and the equation (4.14) for the polymer stress \( \tau^{(p)} \) there are still terms which cannot be obtained from \( \langle QQ \rangle \) alone, so that macroscopic flow simulations cannot be performed easily.

To overcome this problem, a modification of the FENE model has been proposed by Bird et al. (1980). This is called the FENE-P model, for which the connector force \( F^{(c)} \) reads:

\[
F^{(c)} = \frac{HQ}{1 - \langle Q^2 \rangle/Q_0^2}
\]  
(4.16)

where \( \langle Q^2 \rangle = \text{tr} \langle QQ \rangle \) is the ensemble averaged squared length of the polymer. This pre-averaging in the denominator of equation (4.16) is known as the Peterlin approximation and distinguishes the FENE-P from the FENE model. Substitution of this connector force in the equations (4.8) and (4.14) now provides closed form expressions (i.e. expressions in \( \langle QQ \rangle \) only).

In order to simplify the equations, it is convenient to scale the dumbbell vector with the value \( \sqrt{kT/H} \), so that \( \mathbf{Q} = \mathbf{Q}/\sqrt{kT/H} \) and to introduce a dimensionless length parameter \( b = HQ_0^2/kT \). Furthermore, it is useful to introduce the relaxation time of the dumbbell \( \lambda = \zeta/4H \) and the dimensionless conformation tensor \( c = \langle \mathbf{Q} \mathbf{Q} \rangle \). The equations (4.14) and (4.8) combined with the dumbbell force (4.16) then lead to the
following equations:

$$\tau^{(p)} = nkT (-I + fc)$$  \hspace{1cm} (4.17)

and

$$\frac{\partial c}{\partial t} = -u \cdot \nabla c + \nabla u \cdot c + c \cdot (\nabla u)^T + \frac{1}{\lambda} (I - fc)$$  \hspace{1cm} (4.18)

with

$$f = \left(1 - \frac{1}{b} \tr c\right)^{-1}$$  \hspace{1cm} (4.19)

The polymer stress is characterized by three parameters: the relaxation time $\lambda$, the length parameter $b$ and the parameter $nkT$ which is related to the viscosity ratio $\beta$. The relation between $\beta$ and the parameter $nkT$ (determining the polymer concentration) is used in order to compute the polymer stress in equation (4.17) and reads according to Wedgewood \\& Bird (1988):

$$\eta_p \equiv (1 - \beta)\eta_0 = nkT\lambda \frac{b}{b+3}$$  \hspace{1cm} (4.20)

where $\eta_p$ is called the polymer viscosity. The conservation equations (4.1-4.3) together with the equations (4.17-4.19) form a closed set of equations and their numerical solution will be discussed in the next section.

### 4.5 Numerical simulations

#### 4.5.1 Simulation techniques

For the numerical solution the equations (4.1), (4.2) and (4.18) are discretized on a three-dimensional grid and integrated in time. The flow geometry is a channel with height $H$ and the flow is considered to be fully turbulent. The equations are described in an orthogonal Cartesian coordinate system $(x, y, z)$ for the streamwise, spanwise and wall-normal direction respectively. The corresponding components of the velocity vector are $u = (u, v, w)$. The total wall shear stress is denoted by $\tau_w$, which defines the friction velocity $u_* = \sqrt{\tau_w/\rho}$ with $\rho$ the fluid density. It is convenient to introduce dimensionless variables, $\tilde{x} = x/H$, $\tilde{u} = u/u_*$ and $\tilde{t} = tu_*/H$. The pressure and the polymer stress tensor are made dimensionless with $\rho u_*^2$. This gives the following equation for the conservation of momentum:

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} = -\tilde{u} \cdot \tilde{\nabla} \tilde{u} - \tilde{\nabla} \tilde{p} + \beta \frac{1}{Re_*} \tilde{\nabla}^2 \tilde{u} + \tilde{\nabla} \cdot \tau^{(p)}$$  \hspace{1cm} (4.21)

where $Re_* = \rho u_* H/\eta_0$ is the Reynolds number. The evolution equation for the dimensionless conformation tensor becomes:

$$\frac{\partial c}{\partial \tilde{t}} = -\tilde{u} \cdot \tilde{\nabla} c + \tilde{\nabla} \tilde{u} \cdot c + c \cdot (\tilde{\nabla} \tilde{u})^T + \frac{1}{\lambda} I - \frac{1}{\lambda} f c$$  \hspace{1cm} (4.22)
where $\tilde{\lambda} = \lambda u_* / H$ is the dimensionless relaxation time. Note that the tensor $c = \langle \tilde{Q} \tilde{Q} \rangle$ is already dimensionless. From now on the tilde symbol will be omitted and these equations will be considered in their dimensionless form only.

### Spatial discretization

The various variables, like velocity, pressure and polymer stress are discretized on a three-dimensional grid. A number of methods are available in literature for the numerical solution of the governing equations for fluid flow. Some well-known examples are (pseudo)spectral methods, finite element methods and finite volume/finite difference methods (see Ferziger & Perić (2002) for a general overview).

In a spectral method the physical variables are expanded on a finite number of orthogonal basis functions. For instance trigonometric functions are often used with periodic boundary conditions. The advantage of spectral methods is that they offer the highest possible numerical accuracy, but a drawback is that the choice of the flow geometry is limited. For a general review on spectral methods the reader is referred to Canuto et al. (1988). In a full spectral method the Navier-Stokes equations are transformed into spectral space, e.g. by expanding the variables in terms of three-dimensional finite Fourier series. For instance the velocity vector $u$ is expanded by:

$$u(r, t) = \sum_k \hat{u}_k(t) e^{i k \cdot r} \quad (4.23)$$

where $r = (x, y, z)$ is the position vector in physical space and $k = (k_x, k_y, k_z)$ is the wave number vector in spectral space. The summation for $k_\alpha (\alpha = x, y, z)$ is from $-N_\alpha / 2$ to $N_\alpha / 2$, where $N_\alpha$ is the number of gridpoints in the $\alpha$-direction. The entire simulation is carried out in spectral space and a transform back to physical space is done only when this is necessary, for example for data processing. A disadvantage of this approach is the large number of operations required for the non-linear advective terms, because a multiplication in physical space corresponds with a convolution in spectral space, which is computationally very expensive. Therefore in this study a slight modification is used, which is called a pseudospectral method. This means that the determination of derivatives in the $x$- and $y$-directions is performed by only transforming the various terms (like the advective, diffusive or polymer terms) into the spectral space using one-dimensional Fourier series. Now the first or second order derivatives in the $\alpha$-direction can be determined by multiplying the transformed terms with $(i k_\alpha)$ or $(-k_\alpha^2)$ respectively. Finally these terms are transformed back to the physical space. Because our channel geometry is periodic in the streamwise ($x$) and spanwise ($y$) directions, the set of equations (4.21) and (4.22) is in these directions discretized using this so-called pseudospectral method.

In the wall-normal ($z$) direction a finite difference method is used, because this direction is not periodic and Fourier expansions cannot be used. Another possibility would be using a spectral method based on Chebyshev polynomials, but this method is quite expensive. The finite difference method is based on Taylor expansions to determine the
derivatives. This is a standard method in fluid dynamics (see Hirsch 1988 for details), especially on regular uniform meshes. A staggered grid is used in the $z-$direction, so that the pressure and the $x-$ and $y-$velocity components are determined in the centres of the gridcells, i.e. at $(n - \frac{1}{2})\Delta z$, while the $z-$velocity component is determined at the faces of the cells, i.e. at $n\Delta z$, with $n$ an integer number. For the conformation tensor $c$ and the polymer stress tensor $\tau^{(p)}$ the $xz-$ and $yz-$components are defined at the cell faces and the other four components in the cell centres. In the case that a variable is needed at a location where it is not defined, the value is determined using linear interpolation. The reason for the grid-staggering in this way is that all gradients in the wall-normal direction are determined over the smallest possible grid spacing and thus provide an accurate discretization of the gradients in this direction.

Periodic boundary conditions are applied in the two homogeneous directions and no-slip boundary conditions on the two channel walls.

A problem which requires some attention is that the integration in time of the discretized equations does not necessarily produce a positive definite value for the conformation tensor $c$. Positive definiteness of $c$ for instance implies that $\text{tr } c > 0$ and is essential (see Dupret & Marchal 1986), because $\text{tr } c$ is related to the (positive) length of the polymers. The violation of this constraint may occur in the numerical solution of time-dependent viscoelastic flow problems and becomes worse as elasticity effects become more important. To avoid this problem a small artificial diffusive term is added to the equation for the conformation tensor in order to improve the stability of the computations (see Sureshkumar et al. 1997). More precisely, an extra term $\kappa \nabla^2 c$ is added to term $G(u, c)$ of equation (4.22), with $\kappa = \kappa/(u, H)$. The effect of this diffusive term should approach to zero when the grid size and time step decrease. In our case the artificial diffusivity is equal to $\bar{\kappa} = 1.2 \cdot 10^{-2}$. In appendix 5.A the effect of varying this diffusivity is studied for selected results and it is shown that a variation does not significantly modify our results. A detailed analysis on the addition of the stress diffusivity has been performed by Sureshkumar & Beris (1995) and Sureshkumar et al. (1997), who used a value of $\bar{\kappa} = O(10^{-2})$.

Temporal discretization

The velocity at a discrete time level $n\Delta t$ (with $\Delta t$ the time step) is denoted $u^n$. The time integration method is called explicit if the velocity at the new time level $u^{n+1}$ is determined using only variables at previous ($\leq n$) time levels. Here, the momentum equations (4.21) are integrated in time using a second-order Adams-Bashforth method for the advective, diffusive and polymer terms. This is a second-order accurate explicit method. Mass conservation at the new time level is enforced with the standard pressure projection method. The following procedure is applied for these equations: first the equations are integrated forward in time without the pressure gradient terms (predictor
step) giving vector $\mathbf{u}^*$:

$$\mathbf{u}^* = \mathbf{u}^n + \Delta t \left( \frac{3}{2} F^n - \frac{1}{2} F^{n-1} \right)$$  \hspace{1cm} (4.24)$$

where $F(\mathbf{u}, \tau^{(p)}) = -\mathbf{u} \cdot \nabla \mathbf{u} + \frac{\beta}{Re} \nabla^2 \mathbf{u} + \nabla \cdot \tau^{(p)}$. However, this intermediate velocity $\mathbf{u}^*$ does not necessarily satisfy the continuity equation (4.1). Therefore this intermediate velocity $\mathbf{u}^*$ is corrected to obtain the velocity at the new time level (corrector step) with:

$$\mathbf{u}^{n+1} = \mathbf{u}^* - \Delta t \nabla p^{n+1}$$  \hspace{1cm} (4.25)$$

where the pressure at the new time level is determined by taking the divergence of this equation and applying continuity at the new time level:

$$\nabla \cdot \mathbf{u}^{n+1} = 0 = \nabla \cdot \mathbf{u}^* - \Delta t \nabla^2 p^{n+1}$$  \hspace{1cm} (4.26)$$

This leads to a Poisson equation for the pressure, which is solved in a way consistent with the momentum equations, i.e. spatial derivatives in the $x$- and $y$-directions are carried out in Fourier space and in the $z$-direction the finite difference technique is used. This leads to the application of FFTs in the homogeneous directions and tridiagonal matrices in the wall-normal direction. Finally, equation (4.25) gives the velocity at the new time level $\mathbf{u}^{n+1}$. The time step is determined using the following heuristic criterion:

$$\Delta t = \frac{C}{\left| \frac{u}{\Delta x} \right| + \left| \frac{v}{\Delta y} \right| + \left| \frac{w}{\Delta z} \right| + \frac{1}{Re} \left( \frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2} \right)}$$  \hspace{1cm} (4.27)$$

where the constant $C = 0.3$ is a safety factor to avoid numerical instabilities.

For the integration of the discretized equations for the polymer conformation tensor (4.22), which are solved simultaneously with the continuity and the momentum equations, a second-order Adams-Bashforth scheme is used for the advection and the deformation due to the flow and a second-order Adams-Moulton scheme is used for the FENE-P force. The latter is an implicit scheme and is used in order to prevent the dumbbells from exceeding their maximum length, so that always $tr \mathbf{c} < b$. This leads to the following discrete equation:

$$\mathbf{c}^{n+1} = \mathbf{c}^n + \Delta t \left( \frac{3}{2} G^n - \frac{1}{2} G^{n-1} + \frac{1}{2} H^n + \frac{1}{2} H^{n+1} \right)$$  \hspace{1cm} (4.28)$$

where $G(\mathbf{u}, \mathbf{c})$ (including the artificial diffusion term) and $H(\mathbf{c})$ are given by equation (4.22). The resulting expression for $\mathbf{c}$ at the new time level then becomes:

$$\mathbf{c}^{n+1} \left( 1 + \frac{\Delta t}{2\lambda} \frac{1}{1 - \frac{1}{b} tr \mathbf{c}^{n+1}} \right) = \mathbf{c}^n + \Delta t \left( \frac{3}{2} G^n - \frac{1}{2} G^{n-1} + \frac{1}{2} H^n \right)$$  \hspace{1cm} (4.29)$$

Taking the trace of this equation and rearranging the terms results in a quadratic equation for $tr \mathbf{c}^{n+1}$ which can be solved directly. With this value of $tr \mathbf{c}^{n+1}$ all components of $\mathbf{c}$ at the new time level are calculated using equation (4.29). Finally by using equation (4.17) the polymer stress at the new time level is obtained.
4.5.2 Flow parameters

All physical quantities to be obtained from our simulations will be expressed in dimensionless wall units, denoted with the superscript ‘+’, similarly as done in the experiments. This means that all velocities are scaled with the friction velocity \( u_\tau \) and stresses with the wall shear stress \( \tau_w = \rho u_\tau^2 \). The wall shear stress \( \tau_w \) is via the streamwise momentum balance connected to the pressure gradient according to:

\[
\tau_w = \frac{H}{2} \frac{\Delta P}{\Delta x} \tag{4.30}
\]

All lengths are scaled with \( \eta_w/\rho u_\tau \), with \( \eta_w \) the local viscosity at the wall, similarly as in the experiments. The viscosity has to be specified at a fixed position in the flow because of the shear-thinning behaviour of the FENE-P model. The viscosity is not constant, but is decreasing with increasing shear rate. The wall viscosity is determined by:

\[
\eta_w = \frac{\tau_w}{(\partial U_z/\partial z)_w} \tag{4.31}
\]

where the wall shear stress and the velocity gradient follow directly from the simulation. The simulations are performed in a so-called minimal flow unit (MFU) with dimensions \( 1.5H \times H \times H \) in the streamwise \((x)\), spanwise \((y)\) and wall-normal \((z)\) direction respectively. Turbulent flow in such a minimal flow unit has been studied in detail by Jimenez & Moin (1991). The concept of the MFU is to create a minimal size flow geometry in which the essential turbulent characteristics, dynamics and morphology resemble the turbulence properties of a full-scale turbulent channel flow. The idea of using an MFU in our case is that this minimizes the amount of grid points needed for computation of the flow field, so that the remaining computational resources can be used for the computation of the polymer stress, which is computationally very expensive. Jimenez & Moin (1991) found that the requirements for the MFU are a spanwise extent of at least 100 wall units and a streamwise extent of 250 – 350 wall units. Because of the use of polymer fluids, for which turbulent structures like streaks usually become larger than for Newtonian flow, the computational domain is chosen bigger than these minimal requirements. Based on the Reynolds number \( Re_\tau = \rho u_\tau H/\eta = 360 \) for which the Newtonian flow simulation is carried out, the dimensions of our channel are chosen as \( 540x^+, 360y^+ \) and \( 360z^+ \) for the streamwise \((x)\), spanwise \((y)\) and wall-normal \((z)\) directions respectively. It is relevant to point out that our flow is fully turbulent in the whole channel at all times.

According to Dean’s correlation (Dean 1978) for the relationship between the friction factor and the Reynolds number, the Reynolds number for Newtonian flow based on \( U_b \) and \( H \) is equal to \( Re = 5500 \). This value also follows from our simulation for Newtonian flow. For the viscoelastic cases the (wall) Reynolds number is defined by:

\[
Re_w = \frac{\rho U_b H}{\eta_w} \tag{4.32}
\]

The computations are carried out on a grid with \( 48 \times 32 \times 100 \) gridpoints in the \( x-, y- \) and \( z- \) directions respectively. The dimensionless time step is determined using equation
(4.27) and is set equal to $\Delta t = 2 \cdot 10^{-4}$ for the Newtonian case. For the viscoelastic cases the time step is set to halve this value, which was needed in order to keep the simulations stable. The entire algorithm for the flow including the FENE-P model needs about 6 times more CPU time per time step than for the case of a Newtonian flow only. Therefore the viscoelastic simulations require about 12 times more CPU resources compared to the Newtonian simulation. Our results for Newtonian flow agree very well with the results of Kim et al. (1987), which is considered as the standard reference for direct numerical simulations of channel flow at moderate Reynolds numbers.

As was discussed in section 4.4, three parameters can be freely chosen in the FENE-P model: the extensibility parameter $b$, the relaxation time $\lambda$ and the ratio of solvent to the total viscosity $\beta$. An objective of our study is to investigate how these parameters affect the flow and the drag reduction. Therefore, here an explanation of these parameters in a flowing system will be given.

From equations (4.18) and (4.19) can be derived that for a system in equilibrium (with $\nabla \cdot u = 0$ and $\partial c/\partial t = 0$):

$$\text{tr} \ c_{eq} = \frac{3b}{b + 3} \quad (4.33)$$

while for the maximum value $\text{tr} \ c_{\text{max}} = b$, so that:

$$\sqrt{\frac{\text{tr} \ c_{\text{max}}}{\text{tr} \ c_{eq}}} = \sqrt{\frac{b + 3}{3}} \quad (4.34)$$

The extensibility parameter $b$ thus determines the ratio of the maximum and the equilibrium polymer length.

The relaxation time $\lambda$ is usually written as a dimensionless Weissenberg number which gives the ratio of the relaxation time of the polymers to the characteristic time scale of the flow:

$$We_* = \frac{\lambda}{\eta_0/\rho u_t^2} = \frac{\rho \lambda u_t^2}{\eta_0} \quad (4.35)$$

In the definition of the Weissenberg number the zero-shear-rate viscosity $\eta_0$ is used.

The viscosity ratio $\beta$ is directly related to the polymer concentration via equation (4.20).

One simulation for Newtonian flow and five simulations for the polymer solutions have been performed. In all simulations the Reynolds number is equal to $Re_* = 360$, but a variation is made in the three parameters of the FENE-P model. An overview of the numerical parameters for which computations have been carried out, is given in table 4.1, where the letter ‘N’ denotes the Newtonian flow simulation and the letters ‘A’, ‘B’, ‘C’, ‘D’ and ‘E’ refer to the viscoelastic cases. Runs A and B differ in the extensibility parameter, i.e. the maximum polymer length, with run B being representative for a longer polymer than run A; runs B and C differ with respect to the relaxation time, i.e. the Weissenberg number, with run C being representative for a stiffer polymer than run B; the difference between runs B, D and E is the parameter $\beta$, i.e. the polymer concentration $n$, with
Table 4.1: Parameters of the FENE-P model used in the direct numerical simulations. All simulations are
carried out with $Re_\infty = 360$. ‘N’ indicates the Newtonian flow simulation, the other letters the FENE-P
simulations. $\beta$: ratio of solvent and zero-shear rate viscosity; $We_\ast$: Weissenberg number; $b$: extensibility
parameter; $Re_w$: wall Reynolds number; $\eta_w/\eta_0$: ratio of viscosity at the wall and zero-shear rate viscosity;
$\%DR$: percentual amount of drag reduction (see equation 2.16 for the definition).

<table>
<thead>
<tr>
<th>run</th>
<th>$\beta$</th>
<th>$We_\ast$</th>
<th>$b$</th>
<th>$Re_w$</th>
<th>$\eta_w/\eta_0$</th>
<th>$%DR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>5486</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>A</td>
<td>0.6</td>
<td>54</td>
<td>100</td>
<td>8609</td>
<td>0.792</td>
<td>26</td>
</tr>
<tr>
<td>A</td>
<td>0.6</td>
<td>54</td>
<td>1000</td>
<td>10246</td>
<td>0.938</td>
<td>61</td>
</tr>
<tr>
<td>C</td>
<td>0.6</td>
<td>72</td>
<td>1000</td>
<td>10574</td>
<td>0.967</td>
<td>66</td>
</tr>
<tr>
<td>D</td>
<td>0.8</td>
<td>54</td>
<td>1000</td>
<td>7020</td>
<td>0.977</td>
<td>40</td>
</tr>
<tr>
<td>E</td>
<td>0.4</td>
<td>54</td>
<td>1000</td>
<td>12616</td>
<td>0.810</td>
<td>64</td>
</tr>
</tbody>
</table>

increasing polymer concentrations going from runs D, B to E. The value of $\beta$, especially
for run E, might seem to be low (i.e. a high polymer concentration) compared to other
studies like Sureshkumar et al. (1997). However, the polymer solutions in the experiments
for high drag reduction (Warholic et al. 1999a; Ptasinski et al. 2001 and our experiments
described in chapters 2 and 3) show very significant shear-thinning behaviour (see figure
2.6). This implies that these values of $\beta$ should be considered as realistic.

4.5.3 Computational procedures

To compute the various flows mentioned in table 4.1 the following procedure is applied. First a computation of the Newtonian flow is carried out by running the code without the
FENE-P model. The starting point is an initial field, in which the velocity profile is fitted
to experimental observations in order to begin with some realistic initial conditions. To
this mean velocity profile a small disturbance is added, generated by a random number
generator in order to force the flow to turbulence. The simulation is started and continued
until it becomes independent from the initial conditions and until it reaches a statistically
steady state. This takes approximately $20T$, where the timescale $T$ is defined as $H/u_\ast$.

The fully developed velocity field of the Newtonian turbulent channel flow is used as the
initial condition for the polymer computations. The initial condition for the conformation
tensor $c$ is computed by substituting the mean velocity profile $U(z)$ for Newtonian flow,
which is for this initialization assumed to be a time-independent unidirectional shear flow,
in equation (4.22). This is done to develop a not too unrealistic field for $c$ as starting
condition. Starting with this initial condition of $c$ and the fully developed velocity field
$u$, the equations (4.21) and (4.22) are integrated in time without coupling the fields of
$u$ and $c$ (i.e. $\beta = 1$). This integration is continued until a statistically steady state is
reached for $c$.

At this point a coupled simulation is started, by setting $\beta$ to the value given in table
4.1: $\beta \neq 1$. The coupled equations (4.21), (4.22) and (4.17) are then integrated until again a steady state is reached, which takes approximately $10T$. This is the point where the coupled simulation is independent from all initializations.

The computations are then continued for another $10 - 12T$, during which data fields are collected with a separation of $0.1T$. This time separation is large compared to the integral time scale of the turbulence, so that these data fields can be considered as statistically independent. During post-processing these data fields are used to compute various statistics, which are obtained by spatial averaging over the streamwise and spanwise directions and temporal (ensemble) averaging over the data fields. The average values obtained in this way are denoted with an overbar ($\overline{\cdots}$).
Chapter 5

Results of the direct numerical simulations

Abstract
In this chapter the results of the direct numerical simulations of drag reducing flows will be presented. Turbulent flows of a Newtonian fluid and of polymer solutions using the FENE-P model have been investigated. A variation of polymer parameters, such as the maximum extension, the elasticity and the concentration, is made. For the case of highly extensible polymers the results of our simulations are very close to the maximum drag reduction or Virk (1975) asymptote. The simulation results show that at approximately maximum drag reduction the slope of the mean velocity profile is increased with respect to the standard logarithmic profile in turbulent wall flows. For the root mean square of the streamwise velocity fluctuations initially an increase of the peak is found, which near maximum drag reduction changes to a decrease in magnitude. For the velocity fluctuations in the spanwise and wall-normal directions a continuous decrease with increasing drag reduction is found. The Reynolds shear stress is strongly reduced, especially near the wall, and this is compensated by a polymer stress, which at maximum drag reduction is about 40% of the total stress. These results have been compared with LDV experiments (chapter 3 and Ptasinski et al. 2001) and the agreement, both qualitatively and quantitatively, is in most cases very good. In addition an analysis of the turbulent kinetic energy budgets has been performed. The main result is a reduction of energy transfer from the streamwise direction, where the production of turbulent kinetic energy takes place, to the other directions. A substantial part of the energy of the mean flow is transferred directly into elastic energy of the polymers. The turbulent velocity fluctuations also contribute energy to the polymers. The elastic energy of the polymers is subsequently dissipated by polymer relaxation. Furthermore, the various contributions to the pressure fluctuations have been computed. Finally, some cross covariances and various length scales are presented. Based on these simulation results a mechanism is proposed which appears to play a significant role in the drag reduction by polymers. It is based on the effect of shear sheltering caused by the strong velocity gradient at maximum drag reduction. This shear sheltering leads
to decoupling of the flow below and above the shear layer. The analysis also provides a scaling of the friction factor, which is close to the experimentally found behaviour.

5.1 Introduction

In this chapter the results of the direct numerical simulations (DNS) of turbulent channel flow with and without polymers are presented. In the viscoelastic flows the polymers are modeled using the constitutive equation for the FENE-P model as discussed in the previous chapter. The dilute polymer flow simulations contain two-way coupling between the flow and the polymers. This means that the deformations of the flow are used to compute the polymer conformation tensor and the polymer stress tensor, while simultaneously the polymer stress tensor is used in the equations of the flow field. In all simulations the flow is fully turbulent.

As mentioned earlier, no numerical simulations have been performed for the high drag reduction regime, while the results of Warholic et al. (1999a, 2001) and our experiments presented in chapter 3 and in Ptasinski et al. (2001) indicate that the polymer behaviour near the maximum drag reduction asymptote has major consequences for turbulence and its production. Therefore, the focus is on simulations in this regime. It is possible to reach this regime by use of a rather high Reynolds number, together with high polymer concentrations and strongly extensible polymers.

Varying the three parameters of the FENE-P model (\(\beta\), \(\text{We}^\star\), and \(b\)), which determine the concentration, elasticity and maximum extension of the polymers respectively, simulations with five different parameter sets (see table 4.1, denoted with different letters) have been performed, of which three (B, C and E) are near the maximum drag reduction asymptote. The objective is to compute various turbulence statistics and to interpret the effects of polymers on the turbulence. Furthermore, the influence of the polymers with respect to kinetic energy budgets of the mean flow, the turbulence and the elastic energy stored in the polymers is discussed. Finally, cross covariances and various length scales in the flow are presented and a hypothesis is setup for a buffer layer mechanism to explain the behaviour of the friction factor near the maximum drag reduction asymptote.

Besides the discussion of the simulation results, a comparison with experimental data is made, in particular with our experimental results, which are presented in chapter 3 and in Ptasinski et al. (2001). An overview of the relevant parameters of these experiments is presented in the tables 3.1 and 3.2. From these experiments four cases have been selected, which in view of their amount of drag reduction are close to the simulations. They are denoted with the abbreviation ‘exp’ and the same letter as the corresponding simulation. For completeness, an overview of the corresponding simulations and experiments is presented in table 5.1.

The simulation data are usually presented in their dimensionless form (denoted by a '+') superscript. This means that velocities are scaled with the friction velocity \(u_\star = \sqrt{\tau_w/\rho}\) and lengths with \(\eta_w/(u_\star \rho)\). Stresses are scaled with the wall shear stress \(\tau_w\). For reasons
of symmetry only half of the domain is shown. The distance from the wall is denoted by \( z^+ = zu_*\rho/\eta_w \), with \( z \) the actual distance from the wall. Some data are presented in physical units \( z/H \). In this case \( z/H = 0 \) denotes the channel wall and \( z/H = 0.5 \) denotes the centre of the channel.

Table 5.1: Overview of the corresponding simulations (see table 4.1) and experiments (see tables 3.1 and 3.2) which are used in the comparison of the results.

<table>
<thead>
<tr>
<th>simulations</th>
<th>run</th>
<th>% DR</th>
<th>experiments</th>
<th>run</th>
<th>concentration</th>
<th>%DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>-</td>
<td>exp N</td>
<td>none (water)</td>
<td>exp A</td>
<td>20 wppm</td>
<td>23</td>
</tr>
<tr>
<td>A</td>
<td>26</td>
<td>exp A</td>
<td></td>
<td>exp B</td>
<td>103 wppm</td>
<td>63</td>
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<tr>
<td>B</td>
<td>61</td>
<td>exp B</td>
<td></td>
<td>exp C</td>
<td>435 wppm</td>
<td>70</td>
</tr>
</tbody>
</table>

5.2 Relation between friction factor and Reynolds number

The percentual amount of drag reduction \( DR\% \) and the friction factor \( f \) are defined similarly as in the experiments (see section 2.2 and equations 2.3 and 2.16). For completeness, the definitions are:

\[
DR\% = \frac{f_N - f_P}{f_N} \cdot 100\% \tag{5.1}
\]

and

\[
f = \frac{\tau_w}{\frac{1}{2}\rho U_b^2} \tag{5.2}
\]

at equal (wall) Reynolds number for the polymer and Newtonian fluid. The suffices 'N' and 'P' stand for the Newtonian and polymer case, respectively, \( \tau_w \) (see equation 4.30 for its relation to the pressure gradient in channel flow) is the wall shear stress and \( U_b \) the bulk or mean velocity.

According to Dean (1978) the relations between the friction factor and the Reynolds number are for a Newtonian fluid for laminar and turbulent flows respectively:

\[
f = \frac{12}{Re} \tag{5.3}
\]

and

\[
f = 0.073 Re^{-1/4} \tag{5.4}
\]

where the Reynolds number is defined in terms of the channel height \( H \) and the bulk velocity \( U_b \). Note that due to the general similarity between pipe and channel flow,
equation (5.4) is very similar to Blasius’ relation for pipe flow (equation (2.6)), with the only difference being that the value of the constant is 0.073 for channel flow instead of 0.073 for pipe flow. When polymers are added the drag in turbulent flow will be lower (at least if the onset Reynolds number is exceeded). At high polymer concentrations the friction factor will approach an empirical asymptote, called the maximum drag reduction asymptote (MDR) or Virk (1975) asymptote, which reads:

\[ f^{-1/2} = 19.0 \log(Re_w f^{1/2}) - 32.4 \]  

(5.5)

\( Re_w \) is the wall Reynolds number defined in equation (4.32). This relation was originally proposed for pipe flow. Comparison of the Newtonian cases of pipe and channel flow shows that the velocity profiles (cf. equations 3.2 and 5.7) and the Blasius relations (cf. equations 2.6 and 5.4) are almost identical. At maximum drag reduction the velocity profiles are also assumed to be identical for pipe and channel flow (see Warholic et al. 1999a,b). The \( f - Re \) relations are directly connected to the integrated velocity profiles (see Schlichting 1987). Given the full similarity between pipes and channels for Newtonian flow and the similarity for maximum drag reduction regarding the velocity profile, it is assumed that equation (5.5) is also valid for channel flow.

The relation between the friction factor \( f \) and the (wall) Reynolds number \( Re_w \) is shown in figure 5.1. The results of the numerical simulations are indicated by +-symbols.
and labeled with letters. The Newtonian simulation is in good agreement with equation (5.4). For a relatively small value of the extensibility parameter \((b = 100, \text{run A})\) only a small amount of drag reduction is found. A higher value of this parameter \((b = 1000, \text{run B})\) increases the drag reduction to a value close to the maximum drag reduction asymptote. When increasing the elasticity to \(We_*= 72, \text{run C}\) or the polymer concentration \((\beta = 0.4, \text{run E})\) the results remain near the asymptote. A smaller value of the concentration \((\beta = 0.8, \text{run D})\) leads to a smaller amount of drag reduction.

Also in figure 5.1 the data obtained from pressure drop experiments in turbulent pipe flow are plotted (with symbols and letters, see table 5.1 for notations). For details regarding these experiments the reader is referred to chapters 2 and 3 and to Den Toonder et al. (1997) and Ptasinski et al. (2001).

### 5.3 Flow statistics

#### 5.3.1 Velocity statistics

**Mean velocity**

The mean streamwise velocity profile for fully developed turbulence at large Reynolds numbers obeys the following relations in the viscous sublayer and in the logarithmic layer, respectively (see Kim et al. 1987):

\[
U_x^+ = z^+ \quad \text{for } z^+ < 5 \quad \text{(5.6)}
\]

\[
U_x^+ = 2.5 \ln(z^+) + 5.0 \quad \text{for } z^+ > 30 \quad \text{(5.7)}
\]

These equations are similar to those for pipe flow, see equations (3.1) and (3.2). For the maximum drag reduction asymptote Virk et al. (1970) and Virk (1975) have proposed the following velocity profile (see also equation 3.3):

\[
U_x^+ = 11.7 \ln(z^+) - 17.0 \quad \text{for } z^+ > 30 \quad \text{(5.8)}
\]

Originally this profile was proposed for pipe flow, but Warholic et al. (1999a,b, 2001) established in their experimental (LDV and PIV) work that this profile is also valid for channel flow.

In figure 5.2 the results of the dimensionless mean streamwise velocity profile are shown as function of the distance from the wall \(z^+\) for the Newtonian and the viscoelastic flow simulations together with the expressions given above. In the viscous sublayer all profiles collapse to the profile of equation (5.6) irrespective of the presence of polymers. Further away from the wall the velocity for the polymeric flows increases with respect to Newtonian case, which is consistent with drag reduction. The profile for the Newtonian flow is in excellent agreement with equation (5.6) in the viscous sublayer and with equation (5.7) in the logarithmic layer, both for the simulations and for the experiments. For a small value of the extensibility parameter \((b = 100, \text{case A})\) the profile in the logarithmic
Figure 5.2: Mean streamwise velocity profile as function of the distance from the wall $z^+$. (a): DNS of Newtonian flow and the runs A, B and C. (b): Experiments (compare figure 3.3) for cases comparable with the DNS cases N, A, B and C. (c): DNS of Newtonian flow and the runs B, D and E. Also shown are the theoretical profiles in the viscous sublayer (5.6), the logarithmic layer for Newtonian flow (5.7) and the Virk asymptote (5.8).
layer is shifted upwards parallel to the logarithmic profile of the Newtonian simulation (figure 5.2(a)). This implies that the additive constant in equation (5.7) is increased from 5.0 to 8.4. The same behaviour is seen in the experimental results for low drag reduction (case 'exp A') shown in figure 5.2(b). In the channel flow experiments of Warholic et al. (1999a) and in the DNS of Sureshkumar et al. (1997) the same behaviour is found for the mean velocity profile at low drag reduction. Furthermore, the upward shift of the logarithmic profile can be interpreted as a thickening of the buffer layer. This has been used by Lumley (1969, 1973) to explain the phenomenon of drag reduction.

For the simulations with higher drag reduction, which are primarily connected to an increase of the elongational parameter $b$ (runs B, C, D and E) the slope of the logarithmic profile deviates from the value given by equation (5.7). For the cases with the highest drag reduction (C and E) the slope is close to the value given by the maximum drag reduction asymptote (equation 5.8). To our knowledge this is the first simulation in which the maximum drag reduction asymptote is confirmed. Other simulations (e.g. Dimitropoulos et al. 1998) had either a too low value of the Reynolds number or a too high value of $\beta$, i.e. a too low polymer concentration. The corresponding experimental results (cases 'exp B' and 'exp C', figure 5.2(b)) show that the data are very close to the maximum drag reduction asymptote. The maximum drag reduction profile is even slightly exceeded for case 'exp C', which can be attributed to very strong shear-thinning behaviour of the fluid for this case and therefore some inaccuracies in determination of the viscosity at the wall.

The effect of the viscosity ratio $\beta$ (and thus of the polymer concentration) is illustrated in figure 5.2(c). For the lowest values of $\beta$ (runs B and E) the slope of the velocity profile is almost the same and is close to the maximum drag reduction asymptote, while for a lower viscosity ratio (run D) only a minor increase of the slope with respect to the Newtonian profile is found.

For the cases with smaller values of $b$ (not shown here) only a minor effect on the velocity profile with respect to Newtonian flow was found. Summarizing, a high extensibility parameter $b$ in combination with high elasticity given by large values of $W_e$, and large values of the polymer concentration or alternatively low values of $\beta$ are necessary to reach the maximum drag reduction regime.

**Turbulence statistics**

The well-known Reynolds decomposition is used for fluctuating variables by splitting the variables in a mean part (denoted by a upper case) and a fluctuation (denoted by a lower case with a prime). Alternatively the mean part is indicated by an overbar ($\overline{\ldots}$). The dimensionless root mean square of the velocity fluctuations in the $\alpha$-direction ($\alpha = (x, y, z)$) is defined by:

$$\text{rms}(u_\alpha^+)=\frac{1}{u_*}\sqrt{\overline{u'_\alpha^2}} \quad (5.9)$$

The root mean square of the streamwise velocity fluctuations, $\text{rms}(u^+)$, is shown in figure 5.3 for both simulations and experiments. The results of figure 5.3(a) indicate that
Figure 5.3: Profiles of the root mean square of the streamwise velocity fluctuations as function of the distance from the wall. (a): DNS of Newtonian flow and the runs A, B and C. (b): Experiments (compare figure 3.4) with cases comparable with the DNS cases N, A, B and C. (c): DNS of Newtonian flow and the runs B, D and E.
the peak value for the polymer solutions is higher than that for the Newtonian case. The peak value increases with increasing extensibility parameter $b$ (compare for instance runs A and B). However, for the case of the highest Weissenberg number (run C, for which the largest drag reduction is observed) the value is decreased with respect to case B. This behaviour of the peak value (including the decrease at the highest drag reduction) is confirmed by the experiments shown in figure 5.3(b). However, it appears that the height of the peak found in the simulations (especially in the high drag reduction runs) is higher than the peak value observed in the experiments. This fact might be due to shortcomings in the FENE-P model. Brownian dynamics simulations of FENE (without pre-averaging) and FENE-P (with pre-averaging) dumbbells (Herrchen & Öttinger 1997; Keunings 1997) and chains (Van den Bruij 1993) in instationary elongational flows show for instance that the Peterlin approximation can cause some changes in the rheological behaviour (like overprediction of the polymer stresses). The largest difference between simulations and experiments is found in the buffer layer, which is the region where the highest elongation occurs. Close to the wall (for $z^+ < 10$) and in the centre of the channel (for $z^+ > 100$) the value of $\text{rms}(u^+)$ for the polymeric flows does not differ much from the Newtonian case. Figure 5.3 also shows that for the polymer cases the location of the peak is shifted to the centre of the channel, both for experiments and simulations.

In figure 5.3(c) the behaviour of $\text{rms}(u^+)$ for the cases with varying viscosity ratio $\beta$ is shown. The maximum value increases monotonically with decreasing $\beta$. Also the location of the peak moves continuously away from the wall with decreasing $\beta$.

In figure 5.4 the root mean square of the spanwise velocity fluctuations, $\text{rms}(v^+)$, is
Figure 5.5: Profiles of the root mean square of the wall-normal velocity fluctuations as function of the distance from the wall. (a): DNS of Newtonian flow and the runs A, B and C. (b): Experiments (compare figure 3.5) with cases comparable with the DNS cases N, A, B and C. (c): DNS of Newtonian flow and the runs B, D and E.
shown as function of the distance from the wall. For $b = 100$ (case A) a small decrease
with respect to the Newtonian case and again a shift of the peak to the centre of the
channel are found (figure 5.4(a)). The values in the centre of the channel are comparable
to those for the Newtonian flow. For the larger value of $b$ (run B) a strong decrease across
the whole channel is observed. Also the shift of the peak value toward the centre is larger.
For high elasticity (run C) the values do not alter significantly with respect to those of
run B except near the centre of the channel. The effect of varying the concentration is
shown in figure 5.4(c) and indicates a general decrease of $\text{rms}(v^+)$ for increasing polymer
concentration, i.e. decreasing $\beta$, although only a minor difference is found between the
two highest concentrations. The overall behaviour found for $\text{rms}(v^+)$ is in reasonable
agreement with experiments of Pinho & Whitelaw (1990), who are one of the few who
measured spanwise velocity fluctuations.

The root mean square of the wall-normal velocity fluctuations, $\text{rms}(w^+)$, is shown in
figure 5.5 for both the simulations and the experiments. The results of the simulations
show an almost continuous decrease across the channel for the runs A, B and C or in
other words for increasing drag reduction. Similarly as found for the spanwise r.m.s.
the values decrease with increasing polymer extensibility and decrease only slightly with
increasing elasticity (figure 5.5(a)). The experiments shown in figure 5.5(b) confirm the
behaviour of the $\text{rms}(w^+)$ as function of the drag reduction. The peak value of $\text{rms}(w^+)$
is shifted towards the centre of the channel, similarly as for the other directions. The
agreement between simulation and experimental results is very good, both qualitatively
and qualitatively. The effect of polymer concentration (figure 5.5(c)) shows an overall
decrease as a function of increasing concentration (decreasing $\beta$) except for the two highest
concentrations where the change is minimal.

### 5.3.2 Mean polymer extension

In some explanations of drag reduction (Lumley 1969, 1973) the influence of the polymers
on the turbulent flow is connected to the extension of the polymers by the flow deformation.
This would for instance result in a high extensional viscosity, which can possibly affect the turbulent structures. Therefore the mean extension of the polymers has been
investigated, which is the actual polymer length compared to the equilibrium length. The
dimensionless length of the polymers is by definition equal to $\sqrt{tr \ c} = |\langle Q \bar{Q} \rangle|$. The
maximum length is equal to $\sqrt{tr \ c_{\text{max}}} = \sqrt{\beta}$ and the ratio between the maximum and the
equilibrium length is given by equation (4.34). This latter equation implies for instance
that for $b = 1000$ (cases B, C, D and E) the polymers can be stretched to about 18 times
their equilibrium length.

In figure 5.6 the mean polymer extension is shown as function of the distance to the
wall. For the smallest value of the extensibility parameter $b$ ($b = 100$, case A) the average
extension is small and this case corresponds with a small amount of drag reduction.
Nevertheless, the polymers are close to fully extended near the wall. For the value of
$b = 1000$ (cases B, C, D and E) much larger stretching of the polymers with respect to
the equilibrium is found and again most stretching occurs in the region near the wall. Therefore it can be concluded that a sufficiently high value of the extensibility parameter is essential for large drag reduction. Figure 5.6 also shows that the highest value of the Weissenberg number (case C) results in the largest extension. This case also corresponds with the highest drag reduction. The simulations for $b = 1000$ show that the polymers do not reach a fully extended state.

5.3.3 Shear stresses

For a stationary fully developed flow one can rewrite equation (4.21) to a balance equation between the pressure gradient and the total stress. Together with the stress decomposition in a solvent and a polymer part and with the Reynolds decomposition this equation for the $xz$-component of the shear stress becomes:

$$0 = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial z} \left( \frac{-u_x' u_z' + \frac{\beta}{Re_*} \partial U_x}{\tau^+} + T_x^{(p)} \right)$$

(5.10)

with $T^+ = T^+^{(r)} + T^+^{(s)} + T^+^{(p)}$ indicating the turbulent or Reynolds stress, the viscous stress of the solvent and the polymer stress, respectively. The sum of these three contributions is the total shear stress which, because of the constant pressure gradient, follows
Figure 5.7: Profiles of the turbulent (Reynolds) shear stress as function of the distance from the wall. (a): DNS of Newtonian flow and the runs A, B and C. (b): Experiments (compare figure 3.8) with cases comparable with the DNS cases N, A, B and C. (c): DNS of Newtonian flow and the runs B, D and E.
Figure 5.8: Shear stress contributions as function of the distance from the wall computed with DNS. $T^{+(r)}$: Reynolds stress, $T^{+(u)}$: solvent stress, $T^{+(p)}$: polymer stress, $T^+$: total stress. (a): Newtonian run; (b): run A; (c): run B; (d): run C; (e): run D; (f): run E.
Figure 5.9: Shear stress contributions as function of the distance from the wall from experiments (compare figures 3.10, 3.11, 3.12 and 3.14) with cases comparable with the DNS cases N, A, B and C. $T^{+(r)}$: Reynolds stress, $T^{+(s)}$: solvent stress, $T^{+(p)}$: polymer stress, $T^{+}$: total stress. (a): exp N; (b): exp A; (c): exp B; (d): exp C.
a linear profile over the distance from the wall with the value zero at the centre of the channel and the value \( \pm \tau_w \) given by equation (4.30) at the walls. The (dimensionless) total shear stress is equal to \( T^* = 1 - 2z/H \).

The results for the turbulent stress obtained from simulations and from experiments are presented in figure 5.7. All viscoelastic simulations indicate a shift of the peak away from the wall and a decrease of the magnitude compared to the Newtonian simulation. The decrease becomes larger for increasing polymer extensibility, elasticity and concentration. The general behaviour is confirmed by the experiments shown in figure 5.7(b) and also by other experiments (Harder & Tiederman 1991; Wei & Willmarth 1992). The strongest reduction in the stress found in the simulations is about 50\% for run C. This is less than observed in the experimental work (see figure 5.7(b)), which results in reduction of dimensionless Reynolds stress by about 75\%. Nevertheless the trend found in our simulations is confirmed by the experiments. Warholic et al. (1999a,b) suggest that at maximum drag reduction the Reynolds stress should nearly vanish. Both our simulation and experimental results of figure 5.7 do not confirm this suggestion.

The various contributions to the total shear stress (equation 5.10) obtained from the simulations are shown in figure 5.8. All values are presented in dimensionless form (scaled with \( \rho u_r^2 \)). For reasons of symmetry only half of the domain is shown and all stresses are depicted as positive quantities. For Newtonian flow (figure 5.8(a)) only the Reynolds stress and the viscous stress are non-zero and they add up to the total stress. For the viscoelastic simulations (figures 5.8(b)-(f)) all three contributions are plotted (\( T^{+\text{(r)}} \), \( T^{+\text{(b)}} \), \( T^{+\text{(p)}} \)) and these are all directly computed from simulations. The most obvious fact from these latter figures is that for the viscoelastic simulations the Reynolds and viscous stresses do not longer add up to the total stress. This is called the ‘Reynolds stress deficit’, which was also found in the experiments (section 3.3.2). The Reynolds stress deficit implies that there must be a contribution of the polymers to the stress. For the low drag reduction simulations (figures 5.8(b) and 5.8(e)) the polymer stress contribution is relatively small and it is present mainly close to the wall. For the drag reduction close to maximum (figures 5.8(c), 5.8(d) and 5.8(f)) this contribution is large and can become 40 – 50\% of the total stress. Moreover, the polymer stress is in this case important across the whole channel.

The various stress contributions obtained from the experiments are shown in figure 5.9. In contrast to the simulations the polymer stress is in this case computed indirectly as the difference between the total stress and the sum of the Reynolds and solvent stress. It is clear that the polymer stress increases monotonically with increasing drag reduction and for the highest case (exp C, figure 5.9(d)) it contributes to 60\% of the total stress. This is even more that in found in the DNS and is mainly due to the very strong Reynolds stress reduction in the experiments.
5.4 Energy budgets

In this section the various budget terms in the balance equation of the kinetic energy are discussed. Also a balance equation for the elastic energy of the polymers is derived. The index notation and the well-known summation convention are used (which means that a repeated index indicates a summation over all coordinate directions). Starting point in the derivation of the kinetic energy equations is the (dimensionless) momentum conservation equation (4.21). Application of the Reynolds decomposition to this equation and averaging gives an equation for the mean velocity $U_i$: 

$$
\frac{\partial U_i}{\partial t} = -U_j \frac{\partial U_i}{\partial x_j} - \frac{\partial P}{\partial x_i} + \frac{\beta}{Re_*} \frac{\partial^2 U_i}{\partial x_j^2} + \frac{\partial T_{ij}^{(p)}}{\partial x_j} \tag{5.11}
$$

Subtracting this equation from the original equation (4.21) gives an equation for the fluctuating velocity $u'_i$: 

$$
\frac{\partial u'_i}{\partial t} = -U_j \frac{\partial u'_i}{\partial x_j} - u'_i \frac{\partial U_i}{\partial x_j} - \frac{\partial}{\partial x_j} (u'_i u'_j) + \frac{\partial}{\partial x_j} (u'_i' u'_j') - \frac{\partial P'}{\partial x_i} + \frac{\beta}{Re_*} \frac{\partial^2 u'_i}{\partial x_j^2} + \frac{\partial T_{ij}^{(p)}}{\partial x_j} \tag{5.12}
$$

5.4.1 Kinetic energy of the mean flow

The kinetic energy of the mean flow is defined by $\frac{1}{2} U_i U_i$. An equation for this quantity is found by multiplying equation (5.11) with $U_i$. The result reads (cf. equation (2.26) for pipe flow): 

$$
\frac{D}{Dt} \left( \frac{1}{2} U_i U_i \right) = P_u + T_{ii}^{(r)} + T_{ii}^{(s)} + T_{ii}^{(p)} + D_u - \epsilon_u - W_u \tag{5.13}
$$

with 

$$
P_u = -U_i \frac{\partial P}{\partial x_i} \tag{5.14}
$$

$$
T_{ii}^{(r)} + T_{ii}^{(s)} + T_{ii}^{(p)} = \frac{\partial}{\partial x_j} \left( -U_i u'_i u'_j + \frac{\beta}{Re_*} U_i \frac{\partial U_i}{\partial x_j} + U_i T_{ij}^{(p)} \right) \tag{5.15}
$$

$$
D_u = u'_i \frac{\partial U_i}{\partial x_j} \tag{5.16}
$$

$$
\epsilon_u = \frac{\beta}{Re_*} \left( \frac{\partial U_i}{\partial x_j} \right)^2 \tag{5.17}
$$

$$
W_u = T_{ij}^{(p)} \frac{\partial U_i}{\partial x_j} \tag{5.18}
$$

In these equations, $P_u$ is the production of mean flow kinetic energy, i.e. the work performed by the mean pressure gradient. The terms $T_{ii}^{(r)} + T_{ii}^{(s)} + T_{ii}^{(p)}$ denote the transport of mean kinetic energy by turbulent velocity fluctuations, by viscous stress and by polymer
stress, respectively. \( D_u \) is called the deformation work. Finally, \( \epsilon_u \) and \( W_u \) stand for dissipation of energy by the Newtonian shear stress and by mean polymer stress, respectively. The production of energy is positive; the deformation work and the two dissipation terms have a negative contribution to the budget (because the minus signs in equation (5.13) \( D_u < 0, \epsilon_u > 0 \) and \( W_u > 0 \)).

### 5.4.2 Kinetic energy of the turbulence

The equation for the turbulent kinetic energy, \( \frac{1}{2} u'_i u'_i \), follows from multiplying equation (5.12) with \( u'_i \) and averaging. The result reads (cf. equation (2.32) for pipe flow):

\[
\frac{D}{Dt} \left( \frac{1}{2} u'_i u'_i \right) = P_{ii} + T^{(r)}_{ii} + T^{(\pi)}_{ii} + T^{(s)}_{ii} + T^{(p)}_{ii} - \epsilon_{ii} - W_{ii}
\]  

(5.19)

with

\[
P_{ii} = -u'_i u'_j \frac{\partial U_i}{\partial x_j}
\]

(5.20)

\[
T^{(r)}_{ii} = \frac{\partial}{\partial x_i} \left( -\frac{1}{2} u'_i u'_j \right)
\]

(5.21)

\[
T^{(\pi)}_{ii} = \frac{\partial}{\partial x_i} \left( -u'_i p \delta_{ij} \right)
\]

(5.22)

\[
T^{(s)}_{ii} = \frac{\partial}{\partial x_j} \left( \frac{1}{2 \beta} \frac{\partial}{\partial x_j} u'_i u'_i \right)
\]

(5.23)

\[
T^{(p)}_{ii} = \frac{\partial}{\partial x_j} \left( u'_i \tau_{ij}^{(p)} \right)
\]

(5.24)

\[
\epsilon_{ii} = \frac{\beta}{Re_s} \left( \frac{\partial u'_i}{\partial x_j} \right)^2
\]

(5.25)

\[
W_{ii} = \tau_{ij}^{(p)} \frac{\partial u'_i}{\partial x_j}
\]

(5.26)

Here, \( P_{ii} \) is the production of turbulent kinetic energy and \( P_{ii} = -D_u \) so that this term will be always positive. \( T^{(r)}_{ii}, T^{(\pi)}_{ii}, T^{(s)}_{ii} \) and \( T^{(p)}_{ii} \) are transport terms, defining energy transport by the fluctuating Reynolds stress, by pressure fluctuations, by the fluctuating viscous stress and by the fluctuating polymer stress, respectively. The viscous dissipation \( \epsilon_{ii} \) is always positive and the polymer stress work \( W_{ii} \) can in principal both be positive or negative. All energy budget terms are made dimensionless consistent with the scaling of the velocity statistics in section 5.3.1.

The energy budgets of three different simulations will be discussed here; Newtonian flow and for two polymer flows: runs A \((b = 100)\) and B \((b = 1000)\), both with \( \beta = 0.6 \) and \( We_\ast = 54 \). In figures 5.10 and 5.11 the various terms in the turbulent kinetic energy
Figure 5.10: Turbulent kinetic energy budgets \( \langle \frac{1}{2} u_i' u_i' \rangle \) as function of the distance from the wall \( z^+ \) for Newtonian flow (thick lines) and for the FENE-P model with \( \beta = 0.6, \; \mathrm{We}_* = 54 \) and \( b = 100 \) (run A, closed symbols) and \( b = 1000 \) (run B, open symbols). Plotted are the lines for production \( (P_a) \), viscous dissipation \( (\epsilon_u) \) and polymer stress work \( (W_u) \).

Figure 5.11: Turbulent kinetic energy budgets \( \langle \frac{1}{2} u_i' u_i' \rangle \) as function of the distance from the wall \( z^+ \) for Newtonian flow (thick lines) and for the FENE-P model with \( \beta = 0.6, \; \mathrm{We}_* = 54 \) and \( b = 100 \) (run A, closed symbols) and \( b = 1000 \) (run B, open symbols). Plotted are the lines for transport by turbulent stress \( (T_{iu}^{(T)}) \), viscous (solvent) stress \( (T_{iu}^{(v)}) \) and polymer stress \( (T_{iu}^{(p)}) \). The transport by pressure fluctuations \( (T_{iu}^{(f)}) \) is very small and is not shown in this graph. Only \( z^+ < 100 \) is shown as all transport terms are almost zero in the centre of the channel.
budget terms are plotted as a function of the distance from the wall. The results for Newtonian flow are in very good agreement with data from Mansour et al. (1988). For viscoelastic flow the production is decreased with increasing extensibility parameter. For the highest value of $b$ a significant shift of the peak away from the wall is observed, which is consistent the thickening of the buffer layer. The viscous dissipation for the polymer flows is much smaller than for the Newtonian flow. Finally, the polymer stress work is has a negative contribution to the balance everywhere in the channel and is thus acting as a dissipative term. The polymer stress work becomes more important with increasing $b$. The contributions of the transport terms illustrated in figure 5.11 show a similar behavior: all budget terms (for turbulent, viscous and polymer transport) become smaller in magnitude with increasing $b$ and show a significant shift away from the wall.

5.4.3 Turbulent kinetic energy per direction

The turbulent kinetic energy of an individual coordinate direction is equal to $\frac{1}{2} u'_a u'_a$ (no summation over $\alpha$). The balance equation for the $\alpha$-component reads:

$$\frac{D}{Dt} \left( \frac{1}{2} u'_\alpha u'_\alpha \right) = P_{\alpha\alpha} + T^{(e)}_{\alpha\alpha} + T^{(e)}_{\alpha\alpha} + T^{(e)}_{\alpha\alpha} + \Pi_{\alpha\alpha} - \epsilon_{\alpha\alpha} - W_{\alpha\alpha}$$

(5.27)

with all budget terms defined as in the equations (5.20-5.26). However, one extra term ($\Pi_{\alpha\alpha}$) appears in equation (5.27), which is called the pressure strain and which is equal to:

$$\Pi_{\alpha\alpha} = \frac{\partial u'_\alpha}{\partial x_\alpha}$$

(5.28)

This term is responsible for redistribution of energy over the three coordinate directions and obviously the sum of the three pressure strain components must be zero.

In figure 5.12 the terms in the kinetic energy budget for the streamwise direction, which is the largest one in magnitude, are shown. The production term of this budget is equal to the production of the total kinetic energy. The viscous dissipation of the streamwise direction is also almost equal to the dissipation in the total energy budget. The most important result of these graphs is the very strong decrease of the pressure strain for the polymer cases in comparison the Newtonian case. This means that the energy transfer from the streamwise component to the other components is much smaller than for a Newtonian flow and thus that most kinetic energy remains in the streamwise direction. This explains that the r.m.s. of the streamwise velocity fluctuations increases when polymers are added (figure 5.3(a)) and that the r.m.s. of the spanwise and wall-normal fluctuations decrease (figures 5.4(a) and 5.5(a)). The polymer stress work is dissipative everywhere in the channel. Contrary to the viscous dissipation, the largest

*Sometimes the pressure strain is written as $\Pi_{\alpha\alpha} = -u'_\alpha \frac{\partial p'}{\partial x_\alpha} = \mu \frac{\partial^2 u'_\alpha}{\partial x_\alpha^2} - u'_\alpha p'$, which is the sum of the pressure strain and pressure transport in our case. However, here the definition of equation (5.28) is used, because in this case $\sum_{\alpha} \Pi_{\alpha\alpha} = 0$. 
Figure 5.12: The turbulent kinetic energy budgets of the streamwise direction ($\frac{1}{2}u'^2$) as function of the distance from the wall $z^+$ for Newtonian flow (thick lines) and for the FENE-P model with $\beta = 0.6$, $W_{\delta} = 54$ and $b = 100$ (run A, closed symbols) and $b = 1000$ (run B, open symbols). (a): production ($P_{zx}$) and viscous dissipation ($-\epsilon_{zz}$); (b): pressure strain ($\Pi_{zz}$) and polymer stress work ($-W_{xx}$); (c): transport by turbulent stress ($T_{xx}^{(t)}$), viscous (solvent) stress ($T_{xx}^{(s)}$) and polymer stress ($T_{xx}^{(p)}$). The transport by pressure fluctuations ($T_{xx}^{(p)}$) is negligibly small and is not shown here. Only $z^+ < 100$ is shown as all transport terms are almost zero in the centre of the channel.
(absolute) value is not found at the wall but in the buffer layer. This confirms the hypothesis that the buffer layer is the principal region where polymers contribute the most to the changes in turbulence. The transport contributions to the energy budget of the streamwise component do not significantly differ from the budget of the total kinetic energy.

The terms of the budget for the spanwise direction are shown in figure 5.13. Again a decrease in magnitude for the pressure-strain term with increasing \( b \) is found, which is the only positive contribution to this budget (apart from small transport contributions). The decrease of this term is consistent with the smaller root mean square values of the spanwise velocity fluctuations (see figure 5.4(a)). Furthermore, for the highest value of \( b \) a strong decrease of the viscous dissipation is found, which implies that most dissipation is due to the polymers. Note also that the transport terms have almost vanished for the highest drag reduction case.

The results for the wall-normal energy budget, presented in figure 5.14 show a similar behaviour as for the spanwise component. Again the pressure strain becomes smaller with increasing polymer extensibility (and drag reduction), corresponding with a smaller r.m.s. (figure 5.5(a)). The main dissipation is again due to the polymers.

![Figure 5.13: The turbulent kinetic energy budgets of the spanwise direction \((\frac{1}{2}v'^2)\) as function of the distance from the wall \(z^+\) for Newtonian flow (thick lines) and for the FENE-P model with \( \beta = 0.6 \), \( \bar{W}_{\text{ex}} = 54 \) and \( b = 100 \) (run A, closed symbols) and \( b = 1000 \) (run B, open symbols). (a): pressure strain \((\Omega_{\text{ex}})\), viscous dissipation \((-\epsilon_{\text{ex}})\) and polymer stress work \((-W_{\text{ex}})\); (b): transport by turbulent stress \((T_{\text{ex}}^{u'v'})\) and viscous (solvent) stress \((T_{\text{ex}}^{\sigma})\). The transport by pressure fluctuations \((T_{\text{ex}}^{p'})\) and polymer stress \((T_{\text{ex}}^{p'p'})\) is negligibly small and is not shown here. Only \(z^+ < 100\) is shown as all transport terms are almost zero in the centre of the channel.](image)
Figure 5.14: The turbulent kinetic energy budgets of the wall-normal direction ($\frac{1}{2}w'^2$) as function of the distance from the wall $z^+$ for Newtonian flow (thick lines) and for the FENE-P model with $\beta = 0.6$, $\text{We}_s = 54$ and $b = 100$ (run A, closed symbols) and $b = 1000$ (run B, open symbols). (a): pressure strain ($\Pi_{ww}$), viscous dissipation ($-\varepsilon_{ww}$) and polymer stress work ($-W_{ww}$); (b): transport by turbulent stress ($T_{ww}^{(s)}$) and pressure fluctuations ($T_{ww}^{(p)}$). The transport by viscous (solvent) stress ($T_{ww}^{(s)}$) and polymer stress ($T_{ww}^{(p)}$) is negligibly small and is not shown here. Only $z^+ < 100$ is shown as all transport terms are almost zero in the centre of the channel.

5.4.4 Elastic energy of the polymers

Another form of energy is the potential energy stored in the elastic polymers. For the elastic energy a balance equation can be derived from the connector force in the spring. The connector force is the gradient of the intramolecular potential in the elastic spring (see section 4.3.1). In the elastic dumbbell model the connector force $F^{(e)}$ in general reads $F^{(e)} = F^{(e)}(Q)$ with $Q$ the vector of the spring and therefore the elastic energy $\phi$ is defined by:

$$F^{(e)} = \frac{\partial}{\partial Q} \phi$$

(5.29)

where $\phi$ is the elastic (potential) energy stored in the spring. The elastic energy is then equal to:

$$\phi = \int F^{(e)} \cdot dQ$$

(5.30)

This is the energy of one dumbbell and it has to be multiplied with $n$ (the number of polymers per unit volume) to obtain the elastic energy per unit volume. For the FENE-P model where the force $F^{(e)}$ is specified by equation (4.16), the polymer energy becomes:

$$\phi = n \int F^{(e)} \cdot dQ = -\frac{nHQ_0^2}{2} \ln \left(1 - \frac{\langle Q^2 \rangle}{Q_0^2}\right) = \frac{1}{2}nkTb \ln f$$

(5.31)
with \( f = f(\text{tr} \, c) \) specified in (4.19). Taking the trace of the equations for the polymer stress (4.17) and the evolution of the polymer conformation tensor (4.18) gives (in index notation):

\[
\tau_{ii}^{(p)} = n k T (-3 + fc_{ii})
\]

and

\[
\frac{Dc_{ii}}{Dt} = \frac{\partial c_{ii}}{\partial t} + u_j \frac{\partial c_{ii}}{\partial x_j} = 2 \frac{\partial u_i}{\partial x_j} c_{ij} + \frac{1}{\lambda} (3 - fc_{ii})
\]

The time derivative of the polymer energy follows from equation (5.31):

\[
\frac{\partial \phi}{\partial t} = \frac{1}{2} n k T b \frac{\partial f}{\partial c_{ii}} \frac{\partial c_{ii}}{\partial t} = \frac{1}{2} n k T f \frac{\partial c_{ii}}{\partial t}
\]

and a similar expression can be obtained for \( \partial \phi / \partial x_j \). Combination of equations (4.17), (5.32), (5.33) and (5.34) gives for the material derivative of the polymer elastic energy:

\[
\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + u_j \frac{\partial \phi}{\partial x_j} = \frac{1}{2} n k T f \left( \frac{\partial c_{ii}}{\partial t} + u_j \frac{\partial c_{ii}}{\partial x_j} \right)
\]

\[
= n k T f \left( \frac{\partial u_i}{\partial x_j} c_{ij} + \frac{1}{2\lambda} (3 - fc_{ii}) \right)
\]

\[
= (\tau_{ij} + n k T f \delta_{ij}) \frac{\partial u_i}{\partial x_j} - \frac{1}{2\lambda} f \tau_{ii}
\]

\[
= \tau_{ij} \frac{\partial u_i}{\partial x_j} - \frac{1}{2\lambda} f \tau_{ii}^{(p)}
\]

Applying the Reynolds decomposition to the polymer energy \( (\Phi = \bar{\phi} + \phi') \) and averaging of equation (5.35) leads to:

\[
\frac{D\Phi}{Dt} = \frac{\partial \Phi}{\partial t} + U_j \frac{\partial \Phi}{\partial x_j} = - \frac{\partial}{\partial x_j} (u'_j \phi') + T_{ij}^{(p)} \frac{\partial U_i}{\partial x_j} + \tau_{ij}^{(p)} \frac{\partial u'_i}{\partial x_j} - \frac{1}{2\lambda} f \tau_{ii}^{(p)}
\]

The first term on the right-hand-side denotes the transport of elastic energy by velocity fluctuations. The second and third terms denote the interaction of the polymers with the mean and fluctuating velocity field, respectively, and the last term is a dissipative term. The two interaction terms are exactly equal to the polymeric dissipation of mean flow kinetic energy \( W_u \) in equation (5.13) and the polymer stress work \( W_{ii} \) in equation (5.19). In the balance of the elastic energy they result in production by the mean flow and by turbulence respectively. The dissipative term is the transfer of energy from polymers into heat by relaxation of the polymers from an extended state to their equilibrium state.
Figure 5.15: The various terms of the energy budgets for mean flow kinetic energy, turbulent fluctuations kinetic energy, elastic energy and their interactions.

Figure 5.16: Polymer energy budgets as function of the distance from the wall $z/H$ for the FENE-P model. (a): run A ($\beta = 0.6$, $W_0 = 54$ and $b = 100$); (b): run B ($\beta = 0.6$, $W_0 = 54$ and $b = 1000$). The transport term is negligible and is therefore not shown here.
Note that in this term the relaxation time appears as a timescale in which elastic energy is dissipated.

The interaction between the various processes in the balances of the kinetic and elastic energy is schematically illustrated in figure 5.15.

In figure 5.16 the various contributions of the elastic energy balance (except the transport term, which is negligibly small) are shown for two different polymer simulations. From this figure it follows that close to the wall the balance reduces to a difference between two large terms, i.e. mean flow production and dissipative relaxation. Further away from the wall, i.e. in the buffer layer, the production by the fluctuating velocity field becomes larger than the mean flow term. However, the largest contribution in total comes from the mean flow, which is consistent with the importance of the polymer stresses in the mean momentum balance at large drag reduction.

5.5 Pressure fluctuations

Pressure fluctuations in turbulent flows play for instance an important role in turbulence modeling. For a review on the role of pressure in turbulence the reader is referred to Willmarth (1975) and Eckelmann (1988). In the previous section was found that an important effect of the polymers is the reduction of the pressure strain in the turbulent kinetic energy budget. Therefore, here pressure fluctuations in the simulations and their role in the energy balances will be investigated in more detail.

![Figure 5.17: Root mean square of the pressure fluctuations as function of the distance from the wall z/H for Newtonian flow and for all simulations with the FENE-P model.](image)
In figure 5.17 the root mean square of the pressure fluctuations is shown as a function of distance from the wall. Our results for Newtonian flow are in good agreement with the results of Kim (1989). This figure shows that the pressure fluctuations are damped for viscoelastic flows over the entire channel height. The effect is stronger as drag reduction increases. Also the maximum of the r.m.s. is located further away from the wall, which indicates again the thickening of the buffer region.

To study the pressure in detail, the Poisson equation for the pressure fluctuations (which is the divergence of equation (5.12)) can be written in the following way:

\[
\frac{\partial^2 p'}{\partial x_i^2} = \frac{\partial^2}{\partial x_i \partial x_j} \left( - u'_i U_j - U_i u'_j \right) + \frac{\partial^2}{\partial x_i \partial x_j} \left( - u'_i u'_j + \overline{u'_i u'_j} \right) + \frac{\partial^2}{\partial x_i \partial x_j} \tau''(p) \tag{5.37}
\]

From the right-hand-side three of this equation it can be seen that the pressure fluctuations consist of three parts. Therefore the pressure fluctuations can be decomposed in these three contributions by writing \( p' = p'_{\text{rapid}} + p'_{\text{slow}} + p'_{\text{polymer}} \), which stand for the ‘rapid’, ‘slow’ and ‘polymer’ pressure and these are defined by:

\[
\frac{\partial^2 p'_{\text{rapid}}}{\partial x_i^2} = \frac{\partial^2}{\partial x_i \partial x_j} \left( - u'_i U_j - U_i u'_j \right) \tag{5.38}
\]

\[
\frac{\partial^2 p'_{\text{slow}}}{\partial x_i^2} = \frac{\partial^2}{\partial x_i \partial x_j} \left( - u'_i u'_j + \overline{u'_i u'_j} \right) \tag{5.39}
\]

\[
\frac{\partial^2 p'_{\text{polymer}}}{\partial x_i^2} = \frac{\partial^2}{\partial x_i \partial x_j} \tau''(p) \tag{5.40}
\]

The background for the splitting of the pressure is given by Wilcox (1993) and Kim (1989). In short, it can be said that the rapid part is due to the deformation by the mean flow field and the non-linear slow part is caused by the interaction of turbulence. The polymer part depends on the fluctuating polymer stress. By solving these three Poisson equations the individual pressure contributions can be computed.

In figure 5.18 the r.m.s. of the individual contributions to the pressure fluctuations is shown. The results for Newtonian flow are shown together with two viscoelastic simulations. For the viscoelastic simulations the rapid part is reduced with respect to Newtonian flow near the wall, but it is slightly increased in the centre of the channel (in particular for case B). As this contribution is due to the mean flow, this effect is the direct result of the change in the mean velocity profile. The slow part, that deals with the non-linear turbulence effects, is for the viscoelastic flows everywhere smaller than for Newtonian flow and this effect becomes stronger with increasing drag reduction. Furthermore, for the Newtonian flow the slow part is almost everywhere in the channel larger than the rapid part, while for case B they become both of the same order of magnitude. As we saw that drag reduction generally leads to damping of turbulence (e.g. the Reynolds stress reduction shown figure 5.7), this manifests itself in this case primarily as reduction of the non-linear (slow) part. The influence of the polymers results in an increase of polymer part of the pressure fluctuations with increasing drag reduction, which is opposite to the
Figure 5.18: Root mean square of the rapid, slow and polymer pressure contributions as function of the distance from the wall \( z/H \). (a): Comparison of Newtonian flow (lines) and the FENE-P model with \( \beta = 0.6, \ We_\ast = 54 \) and \( b = 100 \) (case A, symbols). (b): Comparison of Newtonian flow (lines) and the FENE-P model with \( \beta = 0.6, \ We_\ast = 54 \) and \( b = 1000 \) (case B, symbols).

Figure 5.19: The contributions to the streamwise pressure strain by the three individual pressure fluctuations as function of the distance from the wall \( z^+ \). (a): Comparison of Newtonian flow (lines) and the FENE-P model with \( \beta = 0.6, \ We_\ast = 54 \) and \( b = 100 \) (case A, symbols). (b): Comparison of Newtonian flow (lines) and the FENE-P model with \( \beta = 0.6, \ We_\ast = 54 \) and \( b = 1000 \) (case B, symbols).

general reduction of the other contributions. For case B the r.m.s. polymer contribution is about 20% of the total r.m.s.

The individual contributions by the three pressure fluctuations to the pressure-strain term in the streamwise direction \( \Pi_{xx} \) are shown in figure 5.19. In figure 5.12(b) was shown that the total pressure strain is strongly reduced with increasing drag reduction and the maximum values becomes located further away from the wall. Regarding the
three individual contributions to the pressure strain, the results indicate that each of these contributions is also reduced but the relative magnitude with respect to each other stays about the same. The polymer part of the total pressure strain is negligible and this means that fluctuating polymer stresses do not contribute directly to the exchange of kinetic energy between the three velocity components.

5.6 Shear sheltering in turbulent drag reduction

In this section some ideas on a mechanism for turbulent drag reduction by polymers will be presented. To this end some cross covariances in the buffer layer are computed for Newtonian and polymer flows. First a shear-free turbulent boundary layer over a surface (i.e. the mean velocity and the dissipation do not vary with the height) is considered. The wall-normal velocities of a large turbulent eddy (centred at a height \( z_1 \) above the surface, velocity \( w_l \)) and a small eddy (centred at a height \( z \), velocity \( w_s \)) are only very weakly correlated, thus \( \overline{w_l w_s} \simeq 0 \) (Hunt et al. 1989). The velocity at height \( z \) above the surface will, however, be slightly modified by the induced velocity of the large eddy given by:

\[
w(z) \simeq w_s + \frac{z}{z_1} w_l
\]  (5.41)

Multiplying this result with the velocity of the large eddy \( w_l = w(z_1) \) and ensemble averaging gives the following relation of the velocities at heights \( z \) and \( z_1 \):

\[
\overline{w(z)w(z_1)} \simeq \frac{z}{z_1} \overline{w(z_1)w(z_1)}
\]  (5.42)

The ‘top-down’ normalized covariance has the form:

\[
\hat{R}_{ww}(z, z_1) = \frac{\overline{w(z)w(z_1)}}{w^2(z_1)} \simeq \frac{z}{z_1} \quad \text{for } z < z_1
\]  (5.43)

In turbulent layers with shear this covariance can be generalized according to Hunt et al. (1989) and becomes:

\[
\hat{R}_{ww}(z, z_1) \simeq f\left(\frac{z}{z_1}\right) \quad \text{for } z < z_1
\]  (5.44)

This self-similar form has been confirmed in many types of boundary layers (Hunt et al. 1989 and references therein). For \( Re \to \infty \) the effect of shear is less, so that relation (5.44) approaches (5.43). In figure 5.20(a) this covariance is shown for the direct numerical simulations of Newtonian flow. Here, \( z_1 \) varies over the channel height and the data collapse on a single curve for an extensive range of \( z_1 \) proving the self-similarity of (5.44). Note that the usually defined correlation \( R_{ww} \), normalized in the conventional way, i.e. with
Figure 5.20: The cross covariance $\hat{R}_{uuw}$ of the wall-normal velocities (see equation 5.43) computed by DNS. (a): Newtonian flow (case N); (b): FENE-P model with $\beta = 0.6$, $We_* = 54$ and $b = 100$ (case A); (c): FENE-P model with $\beta = 0.6$, $We_* = 54$ and $b = 1000$ (case B).
RESULTS OF THE DIRECT NUMERICAL SIMULATIONS

Figure 5.21: Sketch of the shear sheltering mechanism. The maximum of the velocity gradient is located at the top of the shear sheltering layer, where in connection with the large velocity gradient disturbances are growing by the Kelvin-Helmholtz mechanism and finally breakdown. The momentum transport resulting from these disturbances is then transmitted through the shear sheltering layer by viscous effects. Indicated are a velocity scale $U$ (the bulk velocity of the friction velocity), a macroscopic length scale $L$ (e.g. the size of large eddies or a length scale depending on the channel dimensions) and a microscopic length scale $l$ (the thickness of the shear sheltering layer).

\[
\sqrt{w^2(z) w^2(z_1)},
\]

is not self-similar.

In figure 5.20(b) and (c) the same covariance $\hat{R}_{ww}(z, z_1)$ is shown for two viscoelastic simulations (cases A and B). For a small value of the extensibility parameter (case A, figure 5.20(b)), the self-similar behaviour is still present and the covariance for this viscoelastic simulation does not differ much from the Newtonian one. However, as the extensibility parameter and drag reduction increase, figure 5.20(c), a significant decrease of the covariance is observed except for the smallest value of $z_1$. Also the self-similar form of the curves is no longer found.

Our explanation for this behaviour begins with a consideration of how large eddies in a turbulent flow approach a rigid boundary. If there is no mean shear the wall-normal velocity component of these eddies is damped and their integral length scale is greatly reduced. In the presence of strong shear in the buffer layer near the wall, the length scale is further reduced. Thus, although the shear production of small scale eddies increases, there is a blocking or 'shear sheltering' (Hunt & Durbin 1999) mechanism at some distance from the wall. As a consequence of this damping of the larger scale eddies, they are blocked and are prevented from penetrating into the near-wall region. As a result the interactions between the structures below and above the layer are then significantly reduced.

Our hypothesis is that the addition of polymers implies a strengthening of the shear sheltering mechanism. A sketch of the process is shown in figure 5.21. At the top of the buffer layer, since here the rate of straining is maximum in a Newtonian flow, the polymer stretching rate is large and damps the wall-normal fluctuations (Lumley 1969). This leads to blocking of the large eddies coming from the centre of the channel. These
Figure 5.22: The wall-normal velocity $u(x,z)$ along the streamwise direction normalized similarly as the covariance in equation 5.43 for Newtonian flow and for the FENE-P model with $\beta = 0.6$, $We_* = 54$ and $b = 1000$ (case B). $z^+ = 28.8$, $z^+_i = 72$.

have a velocity $U$ and a length scale $L$ and cause amplification of streamwise velocity fluctuations over the lifetime $L/U$ of the eddies at the top of the buffer layer. Also the natural Kelvin-Helmholtz instability of this shear layer is suppressed by action of the polymers, so that there is an increase in the time taken for the breakdown of this local shear layer as seen by the signal of the wall-normal velocity (normalized similarly as the covariance $\hat{R}_{umu}$) shown in figure 5.22. From this figure follows that the interval between two events is larger for polymer flows than for Newtonian flow and that the flow has a more wavy character. A similar shear sheltering effect is seen when damped wall-normal free stream turbulence drives instabilities at the top of a laminar boundary layer (Wu et al. 1999). Furthermore, both the intense shear and the stability of the layer reduce the wall-normal velocity fluctuations below this layer. This is in agreement with the computed flow statistics, which indicate that most turbulent activity is moved away from the wall and that the finite thickness of the buffer layer increases.

Although there is a strong decrease of interactions of the flow between the regions above and below the shear sheltering layer, the upper region nevertheless affects the one below, where the wall shear stress is generated. It is likely that the intermittent breakdown of the shear sheltering layer provides the mechanism for the interaction with the outer flow and the determination of the wall shear stress. This is the basis for our order of
magnitude model described below. The non-linear wavelike fluctuation of the buffer layer with polymers shown in figure 5.22 is consistent with De Gennes (1990) 'elastic' concept. This is also supported by the data of McComb & Rabie (1979, 1982), Sreenivasan & White (2000) and Min et al. (2002).

In the order of magnitude discussion firstly the the inertial and viscous terms in the shear sheltering layer are balanced. The inertial terms can be estimated as $U^2/\mathcal{L}$, the viscous terms as $\nu U/\ell^2$, where $U$ is a characteristic macroscopic velocity scale, whose order of magnitude lies between the mean velocity $U_b$ and the friction velocity $u_\ast$. The macroscopic length scale $\mathcal{L}$ is of the order of the channel height and the microscopic length scale $\ell$ is of the order of the thickness of the shear-sheltering layer. The dynamics of the shear sheltering layer with strong polymer actions is quite different from that of the wall layer in a Newtonian flow, because its considerable stability means that the time scale is determined by the outer flow. Therefore:

$$\frac{U^2}{\mathcal{L}} \sim \frac{\nu U}{\ell^2}$$

(5.45)

so that the ratio of $\ell$ to $\mathcal{L}$ is given by:

$$\frac{\ell}{\mathcal{L}} \sim \left(\frac{\nu}{U \mathcal{L}}\right)^{1/2} = \text{Re}^{-1/2} \quad \text{if } U \sim U_b$$

(5.46)

and

$$\frac{\ell}{\mathcal{L}} \sim \left(\frac{\nu}{U \mathcal{L}}\right)^{1/2} = \text{Re}^{-1/2} \left(\frac{U_b}{u_\ast}\right)^{1/2} = \text{Re}^{-1/2} f^{-1/4} \quad \text{if } U \sim u_\ast$$

(5.47)

The Reynolds number $Re$ is always based on $U_b$ and $\mathcal{L}$. This shows why with polymers the wall layer thickness $\ell$ is much greater than in a Newtonian flow where:

$$\frac{\ell}{\mathcal{L}} \sim \text{Re}^{-1} f^{-1/2}$$

(5.48)

Scaling the mean velocity gradient in the shear sheltering layer as $U_b/\ell$ gives for the wall shear stress:

$$\tau_w \sim \frac{\rho \nu U_b}{\ell}$$

(5.49)

which gives for the estimation of the friction factor $f$ (equation 5.2) the following relation:

$$f \sim \frac{\tau_w}{\rho U_b^2} \sim \frac{\nu}{U_b \ell} = \text{Re}^{-1} \frac{\mathcal{L}}{\ell}$$

(5.50)

Depending on the estimate of $U$, for the friction factor $f$ (equation 5.2) the following two possibilities can be derived using equations (5.46) and (5.47):

$$f \sim \text{Re}^{-1/2} \quad \text{for } U \sim U_b$$

(5.51)

$$f \sim \text{Re}^{-2/3} \quad \text{for } U \sim u_\ast$$

(5.52)
These results can be compared with the maximum drag reduction asymptote (2.7), which can be approximated by a power-law expression of the form $f = ARe^{-n}$ with $n \approx 0.55$ (Virk et al. 1967) or $n \approx 0.58$ (Virk 1975) and which is valid for middle-range Reynolds numbers $4000 < Re < 40000$. This estimate based on the shear sheltering effect is broadly consistent with the power-law estimate for the Virk asymptote.

Note that for $U \sim u_*$ a macroscopic time scale of the flow can be defined by $\tau_f = \mathcal{L}/u_*$. A comparison of this time scale with the relaxation time of the polymer $\lambda$ gives:

$$\frac{\lambda}{\tau_f} \sim \frac{\lambda u_*}{\mathcal{L}} \quad (5.53)$$

which is a Weissenberg number based on macroscopic variables. Typically the macroscopic length scale $\mathcal{L} \sim 0.1H$, which gives for this Weissenberg number $We > 1$. Thus the proposed mechanism is consistent with the polymer physics and the usual criterion of the onset of drag reduction.

### 5.7 Conclusions and discussion

Direct numerical simulations for a turbulent flow of a Newtonian fluid with dissolved polymers have been performed. The polymers are modeled by means of a realistic constitutive equation using the FENE-P model. The coupling between the polymers and the flow field is two-way, i.e. the polymers are deformed by the velocity field and the resulting polymer stress is returned into the momentum equations of the flow. The flow is fully turbulent all the time. Regarding the polymer model a variation of three parameters has been made: the extensibility parameter $b$, which is proportional to the square of the maximum polymer length, the ratio $\beta$ of the solvent to the total zero-shear-rate viscosity, which determines the polymer concentration and the Weissenberg number $We_*$, which is the ratio of the relaxation time $\lambda$ of the polymers to the turbulence time scale. Necessary criteria to obtain the maximum drag reduction asymptote are, besides a sufficiently high Reynolds number, highly extensible polymers and a high polymer concentration. Flow conditions and polymer characteristics have been selected such that our simulations are very close to the maximum drag reduction or Virk (1975) asymptote.

As the pressure drop is kept constant in the simulations drag reduction manifests itself as an increase of the mean or bulk velocity. With respect to the mean velocity profile both in low drag reduction experiments and simulations only an offset in the additive constant of the logarithmic profile for the Newtonian flow is found. At high drag reduction also a large increase of the slope of the profile is observed. Especially in the latter cases the buffer layer is significantly thickened, as the logarithmic profile starts further away from the wall. No changes are found in the viscous sublayer. The changes of the mean velocity profile are qualitatively and quantitatively in excellent agreement with the experiments of Den Toonder et al. (1997) and our own experiments (chapter 3 and Ptasinski et al. 2001).

The peak value of the r.m.s. of the streamwise velocity fluctuations increases with increasing parameter $b$ compared to Newtonian flow. At the same time the maximum of
the r.m.s. of the streamwise velocity is located further away from the wall, reconfirming the thickened buffer layer. For the simulation of the highest drag reduction case, which has also the largest Weissenberg number, the peak value is reduced to an intermediate value. Such behaviour, i.e. increasing and then decreasing peak values of the streamwise velocity fluctuations when drag reduction increases to its maximum value, has also been found in the experiments. A difference between the simulations and the experiments is that the changes in the simulations are larger than in the experiments. This might be related to shortcomings of the FENE-P model. Especially in time-dependent elongational flows (see Herrchen & Öttinger 1997; Keunings 1997) the FENE-P model fails to fully represent the rheological behaviour. A possible solution would be to perform Brownian dynamics simulations, e.g. by using the Brownian configuration fields method (Hulsen et al. 1997). This method makes it possible to use the FENE model (without the Peterlin approximation), but as it does not provide a single closed-form constitutive equation, the computational requirements are much more demanding.

The r.m.s. of the spanwise and wall-normal velocity fluctuations show a decrease and a shift away from the wall monotonically with increasing drag reduction. This agrees very well with the experiments. The Reynolds stress is strongly decreased, though less than in the experiments. The resulting Reynolds stress deficit is compensated by a polymer shear stress which for the highest drag-reduction cases is 40 - 50% of the total stress. Contrary with observations of Warholic et al. (1999a), but in agreement with our experiments (chapter 3 and Ptasinski et al. 2001), the Reynolds stress does not vanish at maximum drag reduction. Computational results for the polymer length show a high extension of the polymers compared to their length in equilibrium, especially in the neighbourhood of the wall. It follows that ability of polymers to stretch is an essential ingredient for high polymer drag reduction.

The analysis of the kinetic energy budgets shows that the pressure-strain term, which is responsible for energy transfer from the streamwise direction to the other directions, is the most dominantly reduced term in the budget and this explains the enhanced anisotropy of the flow. Furthermore, the polymer stress work is found to be positive for all cases, which means that polymers dissipate (and not produce) energy. The highest dissipation by polymers is in the buffer layer, in contrast to the viscous dissipation, which is maximal at the wall. The polymer dissipation of the mean flow and the turbulent kinetic energy are transferred into elastic energy of the polymers, which in its turn is then dissipated by polymer relaxation.

Finally, from an analysis of cross covariances it can be concluded that drag reduction produces a strong shear sheltering layer in the buffer layer. The result is a decoupling of the structures above and below this layer. From an analysis of various length scales a power-law behaviour for the friction factor as a function of the Reynolds number is found, which resembles the behaviour of the maximum drag reduction asymptote. So the changes of the flow statistics and structures in the buffer layer in combination with the shear sheltering mechanism play a key role in the high drag reduction regime.
5.A Effect of variation of the artificial diffusivity

In section 4.5.1 and more precisely in the integration procedure for the polymer conformation tensor, equation (4.28), an artificial diffusivity term $\tilde{\kappa} \nabla^2 c$, with $\tilde{\kappa} = \kappa/(u_\ast H)$, was introduced. This term has to be added to equation (4.22) for numerical reasons in order to avoid instabilities. However, this term should obviously not have too much influence on the macroscopic flow parameters like velocities and stresses. A detailed study on the effect of the artificial stress diffusivity on the stability of viscoelastic flow calculations is given by Sureshkumar & Beris (1995). In this appendix the reason for the introduction of this term will be briefly summarized. For selected simulation results it will be shown that the artificial diffusivity indeed does not significantly alter the results.

Viscoelastic calculations do not only introduce extra variables like the polymer conformation and stress, but more importantly they can lead to a change in type of the total system of equations for the momentum and for the polymer conformation tensor (Joseph 1990). This is in particular the case for high Weissenberg number problems. A detailed analysis of the type of the equations and its consequences on numerical methods is provided by Keunings (1989). The change of type is associated to the loss of evolution, which leads to an instability in which short waves will grow in amplitude (Dupret & Marchal 1986; Joseph & Saut 1986; Marchal & Crochet 1987). In our problem this will lead to a violation of the positive definiteness of the polymer conformation tensor $c$, which is an essential property of this tensor. In principle, for most differential models the conformation tensor will remain positive definite as long as it is positive definite initially (Hulsen 1990). However, in a computation this property can be lost due to numerical errors during time-integration.

Positive definiteness and thus stability can be conserved by using so-called "streamwise upwinding", as is often done in finite element techniques (e.g. Marchal & Crochet 1987). In our case this will lead to less accuracy. Another possibility is the use of diffusivity which is proposed by Sureshkumar & Beris (1995) and which is also applied in this thesis.

This will increase stability, but should not alter the main results significantly. To show this, the results of one of our viscoelastic turbulent simulations (run A, with $\beta = 0.6$, $We_\ast = 54$ and $b = 100$ and $\tilde{\kappa} = \kappa/(u_\ast H) = 0.012$) are compared with a simulation with the same parameters in the FENE-P model but with $\tilde{\kappa} = \kappa/(u_\ast H) = 0.008$. This comparison is shown for some selected statistics in figures 5.23-5.26. These figures clearly show that the difference is very small providing us with evidence that the addition of a small amount of artificial diffusivity does not significantly modify the simulation results.
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Figure 5.23: Mean streamwise velocity profile as function of the distance from the wall $z^+$ with the FENE-P model (parameters $\beta = 0.6$, $We_\ast = 54$ and $b = 100$) for two different values of the artificial diffusivity $\kappa$. $\bar{U}_x^+$: streamwise component; $\bar{U}_y^+$: spanwise component; $\bar{U}_z^+$: wall-normal component.

Figure 5.24: Root mean square profiles of the velocity fluctuations as function of the distance from the wall $z^+$ with the FENE-P model (parameters $\beta = 0.6$, $We_\ast = 54$ and $b = 100$) for two different values of the artificial diffusivity $\kappa$. $u'^+$: streamwise component; $v'^+$: spanwise component; $w'^+$: wall-normal component.

Figure 5.25: Various shear stress contributions as function of the distance from the wall $z/H$ with the FENE-P model (parameters $\beta = 0.6$, $We_\ast = 54$ and $b = 100$) for two different values of the artificial diffusivity $\kappa$. R: Reynolds stress; S: solvent (viscous) stress; P: polymer stress; T: total stress.

Figure 5.26: Mean polymer extension as function of the distance from the wall $z/H$ with the FENE-P model (parameters $\beta = 0.6$, $We_\ast = 54$ and $b = 100$) for two different values of the artificial diffusivity $\kappa$. $\bar{\varepsilon}^+$: mean polymer extension; $\bar{\varepsilon}_f^+$: mean fluid extension.
Chapter 6

Conclusions

The aim of this research was to study turbulent flows of polymer solutions at maximum drag reduction and to get more insight in the phenomenon of drag reduction. This has been done by means of experimental and numerical methods. The polymer solutions used in the experiments and computations consist of long-chain linear polymers dissolved in a Newtonian solvent. In the following two sections the results of the experiments and the simulations will be presented separately.

6.1 Conclusions of the experiments

The experiments consist of LDV measurements and are performed in a recirculatory pipe flow facility. The maximum drag reduction asymptote is reached by using relatively high polymer concentrations. These concentrations are slightly higher than the critical concentration to reach the MDR in order to minimize the effect of mechanical degradation. The main conclusions of the experiments are the following.

- The profile of the mean streamwise velocity shows for a small amount of drag reduction (low polymer concentration) an almost parallel upward shift in the logarithmic region. Close to maximum drag reduction the slope in the logarithmic region is significantly increased and is consistent with the empirical profile for the Virk asymptote.

- The peak of the root mean square of the streamwise (axial) velocity fluctuations is for small concentrations increased with respect to Newtonian flow, but for the highest concentrations it is decreased. This means that a maximum value for the magnitude of the peak is found for some intermediate concentration. The root mean square of the wall-normal (radial) velocity fluctuations is significantly reduced with respect to Newtonian flow.

- The Reynolds stress is very strongly suppressed with respect to Newtonian flow, but is definitely non-zero. This is different from the results of Warholic et al. (1999a,b),
who reported an almost zero Reynolds stress for both surfactant and polymer solutions.

- The suppression of Reynolds stress causes a ‘Reynolds stress deficit’ and this is compensated by a large polymer stress, which can be almost 60% of the total shear stress.

- The start of the logarithmic region of the mean velocity profile and the maximum values of the r.m.s. statistics are located further away from the wall. This shift corresponds with a thickening of the buffer layer and agrees with Lumley’s theory for drag reduction.

- The kinetic energy budgets for the high drag reducing cases show that most energy is transferred directly to polymers instead of to turbulence. The consequence is a smaller production of turbulent kinetic energy. The suggestion that the polymer stress work acts as production term of kinetic energy is considered with help of the assumption that the scaling of the viscous dissipation is independent of the presence of polymers. This assumption is somewhat crude and does certainly not provide a definite evidence for production of turbulence by fluctuating polymer stresses.

### 6.2 Conclusions of the simulations

The simulations are performed in a minimal channel flow geometry. The polymers are modeled using the FENE-P model. The regime near maximum drag reduction is reached by a combination of a high Reynolds number and high concentrations of strongly extensible polymers. The main conclusions of the direct numerical simulations are the following.

- Because all simulations are performed at the same pressure gradient, drag reduction manifests itself by an increased flow rate. The mean velocity profile is increased for all non-Newtonian simulations. For a small amount of drag reduction only a parallel shift of the velocity profile in the logarithmic region is observed, while near maximum drag reduction also the slope is increased approaching the maximum drag reduction asymptote. This is in full agreement with the experimental results.

- The root mean square of the streamwise velocity fluctuations is in general increased with increasing drag reduction, with the exception of the highest amount where a decrease is observed. This behaviour agrees with the experiments. However, the magnitude of the value in the simulations is much higher than in the experiments, which can be caused by shortcomings of the FENE-P model. Simulations with more advanced rheological models, for instance Brownian dynamics simulations, could possibly clarify this difference. For the spanwise and wall-normal velocity fluctuations the r.m.s. values are always decreased.
CONCLUSIONS

- The Reynolds stress is decreased with respect to Newtonian flow, but remains non-zero, again in agreement with the experiments. Its suppression is compensated by the polymer stress.

- All statistics show a shift away from the wall confirming the thickening of the buffer layer, which has also been found in the experimental data.

- The polymers are strongly extended in the flow, especially in the region near the channel wall.

- The kinetic energy budgets show that the pressure-strain term, which is responsible for the redistribution of kinetic energy over the three directions, is significantly reduced. This explains the behaviour of the r.m.s. profiles. The polymer stress work is everywhere a positive quantity, which means that polymers always dissipate energy. This result contradicts the hypothesis that polymers may produce turbulent kinetic energy. The maximum value of the polymer stress work is found in the buffer layer, contrary to the viscous dissipation, which is maximal at the wall.

- Most energy from the mean flow is transferred directly into elastic energy of the polymers and is dissipated by polymer relaxation.

- A possible mechanism for drag reduction by polymers is the enhanced role of a shear sheltering layer at the top of the buffer layer, which leads to decoupling of the regions below and above this layer. An analysis based on the shear sheltering effect and the various scales in the flow shows that the scaling of the friction factor for high drag reduction is close to the experimentally found value.
References


REFERENCES


REFERENCES


# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>extensibility parameter</td>
<td>–</td>
</tr>
<tr>
<td>( c )</td>
<td>conformation tensor</td>
<td>–</td>
</tr>
<tr>
<td>( D )</td>
<td>pipe diameter</td>
<td>m</td>
</tr>
<tr>
<td>( D_u, D_{ii} )</td>
<td>deformation work (in energy budgets)</td>
<td>m²s⁻³</td>
</tr>
<tr>
<td>( D )</td>
<td>rate-of-deformation tensor</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>( e )</td>
<td>turbulent energy per unit mass</td>
<td>m²s⁻²</td>
</tr>
<tr>
<td>( f )</td>
<td>friction factor</td>
<td>–</td>
</tr>
<tr>
<td>( F )</td>
<td>flatness</td>
<td>–</td>
</tr>
<tr>
<td>( F^{(c)} )</td>
<td>connector force</td>
<td>N</td>
</tr>
<tr>
<td>( H )</td>
<td>channel height</td>
<td>m</td>
</tr>
<tr>
<td>( H )</td>
<td>spring constant</td>
<td>Nm⁻¹</td>
</tr>
<tr>
<td>( k )</td>
<td>Boltzmann constant</td>
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</tr>
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<td>( K )</td>
<td>consistency index (in Carreau model)</td>
<td>Pas</td>
</tr>
<tr>
<td>( \ell )</td>
<td>microscopic length scale</td>
<td>m</td>
</tr>
<tr>
<td>( L )</td>
<td>characteristic length scale</td>
<td>m</td>
</tr>
<tr>
<td>( n )</td>
<td>number of dumbbells per unit volume</td>
<td>m⁻³</td>
</tr>
<tr>
<td>( n )</td>
<td>power-law index (in Carreau model)</td>
<td>–</td>
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<tr>
<td>( p )</td>
<td>pressure</td>
<td>Pa</td>
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<tr>
<td>( \Delta P/\Delta x )</td>
<td>mean pressure gradient</td>
<td>Pam⁻¹</td>
</tr>
<tr>
<td>( P_u, P_k, P_{ii} )</td>
<td>production (in energy budgets)</td>
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</tr>
<tr>
<td>( Q )</td>
<td>flow rate</td>
<td>m³s⁻¹</td>
</tr>
<tr>
<td>( Q )</td>
<td>dumbbell connector vector</td>
<td>m</td>
</tr>
<tr>
<td>( Q )</td>
<td>length of the connector vector</td>
<td>m</td>
</tr>
<tr>
<td>( Q_0 )</td>
<td>maximum length of the connector vector</td>
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</tr>
<tr>
<td>( R )</td>
<td>pipe radius</td>
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</tr>
<tr>
<td>( \hat{R} )</td>
<td>covariance function</td>
<td>–</td>
</tr>
<tr>
<td>( S )</td>
<td>skewness</td>
<td>–</td>
</tr>
<tr>
<td>( t )</td>
<td>time</td>
<td>s</td>
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<tr>
<td>( T )</td>
<td>absolute temperature</td>
<td>K</td>
</tr>
<tr>
<td>( T_u, T_k, T_{ii} )</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>--------</td>
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<td>$T$</td>
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<tr>
<td>$u_*$</td>
<td>friction velocity</td>
<td>ms$^{-1}$</td>
</tr>
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<td>$U_b$</td>
<td>bulk (mean) velocity</td>
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</tr>
<tr>
<td>$U_c$</td>
<td>centerline velocity</td>
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<tr>
<td>$U$</td>
<td>characteristic velocity scale</td>
<td>ms$^{-1}$</td>
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<tr>
<td>$W_u, W_k, W_i$</td>
<td>polymer stress work (in energy budgets)</td>
<td>m$^2$s$^{-3}$</td>
</tr>
<tr>
<td>$x,y,z$</td>
<td>Cartesian coordinates (for streamwise, spanwise and wall-normal directions respectively)</td>
<td></td>
</tr>
<tr>
<td>$x,r,\theta$</td>
<td>cylindrical coordinates (for streamwise, (axial), wall-normal (radial) and spanwise (tangential) directions respectively</td>
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</table>

**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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</thead>
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<tr>
<td>$\beta$</td>
<td>ratio of solvent viscosity and total viscosity</td>
<td>–</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>shear rate</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$\gamma_w$</td>
<td>shear rate at the wall</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time step</td>
<td>s</td>
</tr>
<tr>
<td>$\Delta x, \Delta y, \Delta z$</td>
<td>grid size</td>
<td>m</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>rate of dissipation</td>
<td>m$^2$s$^{-3}$</td>
</tr>
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<td>$\epsilon_u, \epsilon_k, \epsilon_i$</td>
<td>viscous dissipation (in energy budgets)</td>
<td>m$^2$s$^{-3}$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>friction coefficient</td>
<td>kgs$^{-1}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Kolmogorov length scale</td>
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</tr>
<tr>
<td>$\eta_s$</td>
<td>viscosity</td>
<td>Pas</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>solvent viscosity</td>
<td>Pas</td>
</tr>
<tr>
<td>$\eta_w$</td>
<td>polymer viscosity</td>
<td>Pas</td>
</tr>
<tr>
<td>$\eta_{\tau}$</td>
<td>viscosity at the wall</td>
<td>Pas</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>zero-shear rate viscosity</td>
<td>Pas</td>
</tr>
<tr>
<td>$\eta_\infty$</td>
<td>infinite-shear rate viscosity</td>
<td>Pas</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>velocity-gradient tensor</td>
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<tr>
<td>$\lambda$</td>
<td>time constant (in Carreau model)</td>
<td>s</td>
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<tr>
<td>$\lambda$</td>
<td>relaxation time</td>
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<tr>
<td>$\nu$</td>
<td>kinematic viscosity</td>
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<td>$\Pi_{\alpha\alpha}$</td>
<td>pressure strain (in energy budgets)</td>
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<td>$\rho$</td>
<td>density</td>
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<td>$\rho$</td>
<td>correlation coefficient</td>
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<td>$\sigma$</td>
<td>total stress tensor</td>
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<tr>
<td>$\tau$</td>
<td>Kolmogorov time scale</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_{\tau}$</td>
<td>stress tensor</td>
<td>Pa</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>shear stress at the wall</td>
<td>Pa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kolmogorov velocity scale</td>
<td>ms$^{-1}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>elastic energy per unit mass</td>
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</tr>
<tr>
<td>$\psi$</td>
<td>configuration-space distribution function</td>
<td>–</td>
</tr>
</tbody>
</table>
Subscripts and mathematical symbols

\( (\ldots)' \) fluctuation quantity

\( (\ldots) \) non-dimensionized with \( u_\star \) and \( H \)

\( (\ldots)^+ \) non-dimensionized with wall variables \( (u_\star, \eta_w) \)

\( \langle (\ldots) \rangle \) ensemble average

\( (\ldots) \) time and space average

\( (\ldots)_{eq} \) equilibrium

\( (\ldots)_{\text{max}} \) maximum

\( (\ldots)(p) \) polymer

\( (\ldots)(r) \) Reynolds/turbulent

\( (\ldots)(a) \) solvent

\( (\ldots)(\pi) \) pressure

\( (\ldots)_{\text{v}} \) turbulent budget term (without summation over all directions)

\( (\ldots)_{\text{v1}, (\ldots)_{\text{v2}}} \) turbulent budget term (including summation over all directions)

\( (\ldots)_{\text{u}} \) mean flow budget term

\( (\ldots)_{\text{w}} \) at the (pipe or channel) wall

\( D(\ldots)/Dt \) material time derivative

\( I \) unit tensor

\( \text{tr} \) trace of tensor

\( \int_S \) integration over the pipe cross section

Dimensionless numbers

\( Re \) Reynolds number (based on bulk velocity)

\( Re_n \) modified Reynolds number (see eq. 2.9)

\( Re_w \) wall Reynolds number (based on viscosity at the wall)

\( Re_\epsilon \) Reynolds number (based on friction velocity)

\( We_\epsilon \) Weissenberg number (based on friction velocity)

Abbreviations

BSA Burst Spectrum Analyser

DNS Direct Numerical Simulation

DR\% percentual drag reduction

FENE finitely extensible non-linear elastic

FFT fast Fourier transform

LDV Laser Doppler Velocimetry

MDR maximum drag reduction

MFU minimal flow unit

PAMH hydrolyzed polyacrylamide

PIV Particle Image Velocimetry

rms root mean square

wppm weight parts per million (0.0001 \%)
Curriculum vitae

Piotr Ptasinski was born on July 3, 1973 in Warsaw. In 1992 he graduated from pre-university education (VWO, gymnasium B) at the ‘Lorentz-Lyceum’ in Eindhoven. From 1992 to 1997 he studied Applied Physics at Delft University of Technology. The graduation project was performed in the group of Fluid Dynamics under supervision of prof. dr. ir. B.H.A.A. van den Brule and was on experiments and simulations of a viscoelastic fluid flow in a complex geometry.

In 1998 he started working on his Ph.D. project at the Laboratory of Aero- and Hydrodynamics of Delft University of Technology under supervision of prof. dr. ir. B.H.A.A. van den Brule and prof. dr. ir. F.T.M. Nieuwstadt. The title of this project is ‘Polymer liquids in complex time-dependent flows’. The goal of this research project was to get more insight in the behaviour of polymers in turbulent flows and consisted of experiments as well as numerical simulations. The research presented in this thesis has resulted in two journal publications and was presented at international conferences and meetings in Aix-les-Bains (France), Prague (Czech Republic), Madison (WI, USA), Bethesda (MD, USA), Cambridge (United Kingdom), Herning (Denmark) and Southampton (United Kingdom). Besides the Ph.D. work he was treasurer and president of the board of the ‘Promovendi Overleg Delft’ (the Delft Ph.D. researchers organization).

Journal Publications

Based on chapters 2 and 3:

Based on chapters 4 and 5:
Conference contributions


Acknowledgements

This Ph.D. thesis is based on four years of research performed at the Laboratory for Aero- and Hydrodynamics of Delft University of Technology. It is on turbulent flows of polymer solutions, which lead to the phenomenon of drag reduction. This phenomenon has been studied for more than half a century, but is still not fully understood and therefore it remains challenging. This thesis combines two fascinating areas of fluid mechanics, turbulence and rheology, and I have enjoyed a lot working on these two subjects as well as performing experiments and numerical simulations. I would like to express my gratitude to all people who have contributed to the realisation of this thesis.

First of all, I would like to thank my promotors, Frans Nieuwstadt and Ben van den Brule. They gave me the opportunity and the perfect conditions to perform this research. I appreciate the freedom they gave me to develop my own ideas, which has certainly contributed to this final result. Their guidance, critical view and the frequent discussions have always been a great stimulation.

Many thanks are addressed to Bendiks Jan Boersma, who stands at the basis of the code used in the numerical simulations. His tremendous efforts and continuous help with the numerical and computer problems are greatly appreciated. I am grateful to Julian Hunt for the motivating discussions we had on turbulence theory and on shear sheltering during the final year. I thank Martien Hulsen for our discussions on rheology and on numerical methods. These have been very stimulating and have largely contributed to my knowledge.

I would like to thank all my colleagues for their interest and support. It has been a great experience and a big pleasure to work at the Laboratory for Aero- and Hydrodynamics. In particular I thank Frank Peters for the discussions we had on rheology and polymer modeling and the many talks outside the lab (and for the idea of the cover of this thesis), Mathieu Pourquié for his frequent help with many numerical and computer problems and numerous discussions and René Delfos for the help with the cover of this thesis.

During my Ph.D. time I have greatly enjoyed attending scientific conferences and working meetings. I thank the ‘Stichting voor Fundamenteel Onderzoek der Materie (FOM)’ for the financial support and Antony Beris, Radhakrishna Sureshkumar and Jay Schieber for the discussions at various conferences.

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Finally and last but not least, I thank my parents for their interest in my work and their neverending support and confidence.

Piotr Ptasinski
Delft, August 2002

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