**Mass transport in gravity waves on a
sloping bottom**

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<u>CONTENTS</u>	<u>page</u>
SUMMARY	
1. INTRODUCTION	1
2. REVIEW OF PREVIOUS WORK	2
3. THEORETICAL CONSIDERATIONS	4
4. EXPERIMENTS	13
4.1 Experimental arrangement	13
4.2 Preliminary experiments	16
4.3 Waves	17
4.4 Mass transport measurements	21
5. DISCUSSION OF RESULTS	23
5.1 Horizontal bottom	23
5.2 Sloping bottom	23
5.2.1 Smooth bottom	23
5.2.2 Rough bottom	30
5.2.3 Artificial ripples	31
6. CONCLUSIONS	32
LIST OF SYMBOLS	33
REFERENCES	34
Appendix I - Wave analysis (figs. I-1 to I-30)	36
Appendix II - Measured mass transport velocity profiles (figs. II-1 to II-31)	67
Appendix III - Mass transport velocity profiles on the various slopes (figs.III-1 to III-26)	99

SUMMARY

In the present investigation the influence of bottom slope on mass transport by progressive waves was investigated, both theoretically and experimentally. Theoretical considerations, based on linear wave theory, show the greatest influence of the slope on the bottom drift velocities for relatively long waves and steep slopes. The numerical values, however, remain rather small (influence less than 20%). In addition, the experiments show that, in the cases examined, the bottom drift velocities are more determined by the local parameters than by the magnitude of the bottom slope.

Considering the net bottom velocities, the discrepancy between the horizontal bottom theory (Longuet - Higgins) and experimental results is considerable. Taking into account the first harmonic of the local wave form and the small slope effect in horizontal bottom theory does show, however, the same tendency as the experimental results, for relatively small depths.

1 INTRODUCTION

The complexity of the mechanism of sediment movement by wave action makes the choice of the proper model bed material in coastal movable bed models often difficult. Therefore a better knowledge of one of the details of the mechanism, namely the net mass transport in waves, would assist in determining the correct sediment scale or at least in recognizing scale effects, if they are inevitable. Though the role of the mass transport is not fully understood, experiments from past investigations suggest that it influences the transport of sediment near the bottom and in suspension. In models especially the influence of the beach slope on the behaviour of this phenomenon is then important, because of the frequently observed distortion in these models.

This paper reports results of experiments to determine mass transport velocities on three sloping bottoms. In particular attention was paid to the resultant bottom drift velocities, since those are most important for sediment studies. The considerations are limited to the offshore region.

Section 2 gives a brief survey of previous work. Section 3 aims at predicting the net bottom velocities as they occur on a gently sloping bottom; section 4 describes the various experiments carried out; section 5 and 6 compare the experimental and theoretical results and give the conclusions. In appendices I, II and III the results of wave analysis and mass transport measurements, respectively, are represented.

2 REVIEW OF PREVIOUS WORK

Theoretical or experimental results for mass transport velocities on gently sloping bottoms are very scarce. Most investigations have been carried out with horizontal bottoms. In this connection Longuet-Higgins (1) has made a most important theoretical contribution. He gives solutions for very low waves (laminar boundary layer conduction solution). For these waves good agreement between theory and experiment was obtained for values of kh (k = wave number, h = water depth) between 0.9 and 1.5. Experiments for higher values of kh generally yielded a mass transport velocity profile resembling better the Stokes profile. This is reported in (2).

Despite of the fact that Russel and Osorio (3) carried out their experiments with higher waves, the tendencies predicted by Longuet-Higgins were confirmed. Though in most of their cases the boundary layer was turbulent, Longuet-Higgins showed in the appendix of their paper that, in particular, the velocity just outside the boundary layer at the bottom does not depend on the value of the (eddy) viscosity, provided it is constant. In general, however, the results of the experiments for the low waves show better agreement with theory than for the high waves.

Brebner, Askew and Law (4) examined the influence of roughness on the mass transport. In general they found a smaller velocity at the bottom than predicted by Longuet-Higgins theory. They suggest that this is caused by the turbulent boundary layer.

Noda (5) - using in his computations an eddy viscosity, being a function of distance from the wall (see also (6)) - attempted to explain this phenomenon, but he only found a significant effect for the case of standing waves.

Sleath (7) calculated the mass transport velocities taking into account convective acceleration terms. His corrections, (both positive and negative) amount to an order of 10% for the cases considered. Another investigation of Sleath (8) is based upon the introduction of damping waves in the longitudinal direction. His mathematical formulation yields three possible solutions for the Longuet-Higgins case (very low waves). Unfortunately, it is not clear under which circumstances the various solutions hold. Moreover the experimental support in the report is weak.

For a gently sloping bottom Russel and Osorio (3) observed, for a

single case considered, no significant differences from the results on a horizontal bottom. Lau and Travis (9) also considered a sloping bottom, but they were primarily interested in the consequences of partial reflection on mass transport velocities.

Concluding, one may remark that net velocity profiles in physical models, although already very schematized, are not satisfactorily predicted by existing theories for cases with relatively high waves. In order to indicate tendencies, at best Longuet-Higgins theory can be used.

3 THEORETICAL CONSIDERATIONS

In this section it is attempted to predict the behaviour of the net bottom velocities in progressive waves propagating on a gently sloping bottom. As a consequence of the latter limitation, partial reflection of the progressive waves will be disregarded.

A conventional x, z - coordinate system is used as shown in fig 1.

Figure 1: definition sketch.

The particle velocity in progressive gravity waves can be approximated by the following expression (see fig. 2):

$$U = u + \frac{\partial u}{\partial x} \Delta x + \frac{\partial u}{\partial z} \Delta z \quad (1)$$

with

$$\Delta x = \int_0^t U dt \quad \text{and} \quad \Delta z = \int_0^t W dt \quad (2)$$

It should be noticed that U and W give the time-history of the velocity of a certain fluid particle (Lagrangian velocity), while u and w express the velocity at a certain location in space as a function of time (Eulerian velocity)

Figure 2: Position of a fluid particle.

The approximation according eq.(1) is reasonable if the displacements are small. In that case the particle velocities in eq.(2) can be replaced by the Eulerian velocities. The expression (1) then becomes

$$U = u + \frac{\partial u}{\partial x} \int^t u \, dt + \frac{\partial u}{\partial z} \int^t w \, dt \quad (3)$$

Averaging over a wave period results in the so-called mass transport velocity

$$\bar{U} = \bar{u} + \overline{\frac{\partial u}{\partial x} \int^t u \, dt} + \overline{\frac{\partial u}{\partial z} \int^t w \, dt} \quad (4)$$

It should be noticed that the second and third term on the right hand side in general are not equal to zero. The mass transport velocity thus is not identical to the time-average of the velocity, measured on a fixed location in space. Usefull discussions on this point can be found in ref. 10, 11 and 12.

From expression (4) it can be seen that if the expressions from linear wave theory are substituted in the quadratic terms, the mean Eulerian velocity must be calculated to the second order. This will be carried out for the bottom boundary layer.

For small displacements the following equation describes the first order two-dimensional motion in the boundary layer at the bottom (see also ref.(13) and (14));

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = \nu \frac{\partial^2 u}{\partial z^2} \quad (5)$$

The corresponding equation of motion just outside the boundary layer, has the form:

$$\frac{\partial u_{\infty}}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \quad (6)$$

Assuming that the pressure gradient in the thin boundary layer equals the gradient just outside, the pressure term in eq. (6) can be substituted in eq.(5) resulting in (see ref. 13):

$$\frac{\partial u}{\partial t} - \frac{\partial u_{\infty}}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} \quad (7)$$

In the case under consideration (simple-harmonic progressive wave motion) the horizontal first order fluid motion is given by (see ref. (13) and (14)) the following solution of eq.(7):

$$u = A(x) \left[\cos(\omega t - \phi(x)) - e^{-\frac{h+z}{\delta}} \cos(\omega t - \phi(x) - \frac{h+z}{\delta}) \right] \quad (8)$$

in which

$$\delta = \left(\frac{2\nu}{\omega} \right)^{\frac{1}{2}}$$

$$u_{\infty} = A(x) \cos(\omega t - \phi(x))$$

and

$$k(x) = \frac{\partial}{\partial x} \phi(x)$$

The expression for u can be conceived of as the first order result for the Eulerian velocity in the boundary layer. The time average of this result yields a zero net velocity. To determine the mean second order horizontal Eulerian velocity, the equations of motion have to be applied again.

From the equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (9)$$

the vertical velocity can be derived, after substituting (8) and integrating. This yields:

$$w = + A\delta k \left[\left\{ \frac{1}{2} e^{-\mu} (\sin \mu + \cos \mu) - \frac{1}{2} \right\} \cos \psi + \right. \\ \left. + \left\{ -\mu + \frac{1}{2} e^{-\mu} (\sin \mu - \cos \mu) + \frac{1}{2} \right\} \sin \psi \right] +$$

$$\begin{aligned}
 & - \delta \frac{dA}{dx} \left[\left\{ \mu - \frac{1}{2} e^{-\mu} (\sin \mu - \cos \mu) - \frac{1}{2} \right\} \cos \psi + \right. \\
 & \quad \left. + \left\{ \frac{1}{2} e^{-\mu} (\sin \mu + \cos \mu) - \frac{1}{2} \right\} \sin \psi \right] + \\
 & - A \frac{dh}{dx} \left[\left\{ -e^{-\mu} \cos \mu + 1 \right\} \cos \psi + \left\{ e^{-\mu} \sin \mu \right\} \sin \psi \right]
 \end{aligned} \tag{10}$$

in which $\mu = \frac{h+z}{\delta}$

and $\psi = \omega t - \phi(x)$

The complexity of this expression is due to the fact that derivatives of $A(x)$ and $h(x)$ must be taken into account.

The first order velocity components being known, the equation of motion, including convective terms, must be applied to get the mean second-order Eulerian velocity. This will be carried out by means of a momentum balance.

Figure 3: Spatial fixed volume in boundary layer.

Application of the law of conservation of momentum to a spatial fixed volume in the boundary layer (PQRS in fig. 3) results in the equation:

$$\frac{\partial}{\partial t} \iiint_V \rho u dV = (\rho u w)_{dx} - (\rho u w)_{\infty} dx + \tau dx - \int_{z=-h}^{-h+D} \frac{\partial}{\partial x} (p + \rho u^2) dx dz \tag{11}$$

Taking the time average yields for a laminar boundary layer:

$$v \frac{\partial \bar{u}}{\partial z} = \overline{(uw)} - \overline{(uw)}_{\infty} - \frac{-h+D}{z} \overline{\left(\frac{p}{\rho} + u^2 \right)} dz \quad (12)$$

since the left member of eq.(11) is then equal to zero, due to the periodicity of the motion.

The pressure term in eq.(12) will again be derived from the interior of the fluid. Taking the time average of the governing equation of motion, in that region, yields:

$$\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} = -u \frac{\partial \bar{u}}{\partial x} - w \frac{\partial \bar{u}}{\partial z} \quad (13)$$

At the bottom, just outside the boundary layer, the third term being small compared to the second, this equation becomes:

$$\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} = -\frac{1}{2} \frac{\partial}{\partial x} \overline{u_{\infty}^2} \quad (14)$$

Physically the gradient of the time mean pressure is due to a variation of the mean water level (see ref. 11).

To the order of magnitude under consideration this mean pressure gradient may be substituted in the boundary layer equation (12) (see also ref. 13). This equation then becomes:

$$\bar{u} = \frac{1}{v} \int_{-h}^z \overline{(uw)} - \overline{(uw)}_{\infty} dz - \frac{1}{v} \int_{-h}^z \int_z^{-h+D} \frac{\partial}{\partial x} \left(-\frac{1}{2} \overline{u_{\infty}^2} + \overline{u^2} \right) dz dz \quad (15)$$

It should be noted that the last term in eq.(15) does not occur on a horizontal bottom when the wave field is horizontally homogeneous. After substituting eqs.(8) and (10), the integration in eq.(15) can be carried out, resulting in

$$\bar{u} = \frac{A^2 k}{\omega} \left[-\frac{1}{2} \mu e^{-\mu} (\sin \mu + \cos \mu) + \frac{1}{2} e^{-\mu} \sin \mu - e^{-\mu} \cos \mu + \frac{1}{4} e^{-2\mu} + \frac{3}{4} \right] +$$

$$+ \frac{A}{\omega} \frac{dA}{dx} \left[\frac{1}{2} \mu e^{-\mu} (\sin \mu - \cos \mu) + 2 e^{-\mu} \sin \mu + \frac{1}{2} e^{-\mu} \cos \mu + \frac{1}{4} e^{-2\mu} - \frac{3}{4} \right] \quad (16)$$

The result obtained is the second-order Eulerian mean velocity. The mean particle velocity or mass transport velocity can be derived after calculating the second and third term of eq.(4).

Substituting eqs.(8)and(10)in these expressions results in

$$\begin{aligned} \overline{\frac{\partial u}{\partial x} \int_0^t u dt} + \overline{\frac{\partial u}{\partial z} \int_0^t w dt} &= \\ &= \frac{A^2 k}{\omega} \left[\frac{1}{2} \mu e^{-\mu} (\sin \mu + \cos \mu) - \frac{1}{2} e^{-\mu} \sin \mu - e^{-\mu} \cos \mu + \frac{1}{2} e^{-2\mu} + \frac{1}{2} \right] + \\ &+ \frac{A}{\omega} \frac{dA}{dx} \left[-\frac{1}{2} \mu e^{-\mu} (\sin \mu - \cos \mu) - \frac{1}{2} e^{-\mu} \cos \mu + \frac{1}{2} e^{-2\mu} \right] \quad (17) \end{aligned}$$

Eqs.(16)and(17)substituted in eq.(4)give the final result:

$$\overline{U} = \frac{A^2 k}{\omega} \underbrace{\left[-2e^{-\mu} \cos \mu + \frac{3}{4} e^{-2\mu} + \frac{5}{4} \right]}_{f(\mu)} + \frac{A}{\omega} \frac{dA}{dx} \underbrace{\left[2e^{-\mu} \sin \mu + \frac{3}{4} e^{-2\mu} - \frac{3}{4} \right]}_{g(\mu)} \quad (18)$$

At the edge of the boundary layer this expression becomes:

$$\overline{U}_{\infty} = \frac{5}{4} \frac{A^2 k}{\omega} - \frac{3}{4} \frac{A}{\omega} \frac{dA}{dx} \quad (19)$$

It should be emphasized now that the concept of a gently sloping bottom is of importance only as far as the resulting wave field varies gradually in horizontal direction. In the foregoing it was shown how the mean Eulerian velocity arises from the non-uniformity of the motion in horizontal direction. The first term in eq.(18) expresses the influence of phase change in horizontal direction, caused by the progressive character of the waves. Vertical velocity components are induced, being in phase with the horizontal velocity. These in-phase components produce a mean flux of horizontal momentum towards the boundary layer. For purely progressive waves this flux is balanced by a mean shear stress at the bottom. This mechanism does not occur in standing waves, since the phase change in horizontal direction is equal to zero. In this latter case, however, the mean shear stress is balanced by a horizontal gradient of horizontal momentum flux, re-

sulting from the varying velocity-amplitude. In the present study this latter influence is expressed by the second term of eq.(18).

Longuet-Higgins, elaborating his fundamental theory on mass transport for progressive and standing waves separately, determined already the respective dimensionless functions $f(\mu)$ and $g(\mu)$ in eq.(18) of the present study. The fact that both influences are present together in eq.(18) for the case of progressive waves with gradually varying amplitude is explained satisfactorily by the foregoing.

Mei, Carter and Liu (2) and also Lau and Travis (9) derived expressions containing both influences for the case of partially reflected progressive waves on a beach. Their expressions, however, only contain amplitude variation by reflection and not by shoaling.

Consequently the corresponding term vanishes when there is no reflection.

Figure 4: Graphs of $f(\mu)$ and $g(\mu)$.

In fig. 4 the functions $f(\mu)$ and $g(\mu)$ are represented. In cases where the first term, with $f(\mu)$, dominates (for instance for purely progressive waves) the flow in the boundary layer is in the direction of wave propagation. If velocity-amplitude variation dominates (standing waves for instance) the flow in the lower part of the boundary layer is in the direction in which the velocity-amplitude increases (towards the nodes in standing waves). In the upper part of the boundary layer the particles move in the opposite direction (see graph of $g(\mu)$ in fig. 4).

In the following the specific case of gravity waves propagating on a sloping bottom will be considered. A quantitative estimate of the influence of the gradual variation of the wave field, as caused by decreasing depth, will be given, using linear wave theory. Only the velocity at the edge of the boundary layer, according to eq.(19), will be considered. As a result of the assumption of gradually decreasing depth, it will be assumed that the wave properties are functions of initial wave parameters and local depth, and not of the magnitude of the bottom slope.

Since A is the amplitude of the horizontal velocity at the bottom in the interior of the fluid (at the edge of the boundary layer) it can be expected to increase continuously in a progressive wave, propagating on the slope before breaking. Also in the beginning, when waves arrive from deep water and wave amplitude decreases (up to a point where kh has reached a value of about 1.2) the direct influence of decreasing depth on bottom velocity will dominate. Therefore, the first term of the right member of eq.(19) indicates a continuous increase of the net bottom velocity in a progressive wave, propagating over a bottom with constant slope. Consequently, the derivative of A with respect to x is positive in that case, so that the second term of the right member of eq.(19) diminishes the net bottom velocity, as predicted for progressive waves on a horizontal bottom.

An assessment of the influence of the latter term is possible by considering the transformation of a progressive wave on a beach using the energy concept (linear theory). Then the amplitude of the oscillating bottom velocity can be expressed in the deep water wave amplitude and the local value of kh :

$$A = \frac{a_0 \omega}{\sinh kh} \left[\frac{2 \cosh^2 kh}{2 kh + \sinh 2 kh} \right]^{\frac{1}{2}} \quad (20)$$

As this expression does not contain the horizontal coordinate or the bottom slope, the gradient of the velocity amplitude will be proportional to the bottom slope:

$$\frac{dA}{dx} = \frac{dA}{dh} \frac{dh}{dx} \quad (21)$$

Substituting eq.(21) by eq.(19) and normalizing the second term by the first one yields:

$$\bar{U}_{\infty} = \frac{5}{4} \frac{A^2 k}{\omega} \left[1 - \frac{3}{5} \frac{1}{Ak} \frac{dA}{dh} \frac{dh}{dx} \right] \quad (22)$$

For a constant slope the dimensionless term between brackets is a function of $k_0 h$ (or kh) only. The varying part is graphically represented in fig. 5.

Figure 5: Slope effect according to eq. 22.

The figure shows that in general the influence of the correction term is small. Its influence is greatest for the relatively long waves; or, in other words, near the breaker zone. Under practically all conditions its value will remain below 20%, for slopes up to 1 : 10. It is very doubtful, however, whether the linear wave theory for this phenomenon can be applied in the region near to the breaker zone. Due to shoaling, for instance, the waves obtain a very pronounced crest and flat trough. The corresponding velocity at the bottom will behave the same, and in fact higher harmonics must be taken into account then. If these harmonics have the same celerity, ref. (3) indicates how to take them into account:

$$\bar{U}_{\infty} = \frac{5k}{4\omega} \left[A_1^2 + A_2^2 + \dots \right] \quad (23)$$

in which $A_1, A_2, \dots$ are the amplitudes of the bottom velocities due to the various harmonics.

4 EXPERIMENTS

4.1 Experimental arrangement

Experiments were carried out in a wave flume, 30 m long, 0.80 m wide and 0.60 m deep. The side-walls of the observation section of the flume consisted of glass plates (fig. 6 and 7). Waves were generated by a wave-paddle, oscillating with different amplitudes at the bottom and still water level.

Three different beach slopes were used, viz. 1 : 10, 1 : 25 and 1 : 40. The slope surface was rigid, whereas its roughness was varied, viz.

Figure 6: Wave flume with sandrough slope surface.

Figure 7: Wave flume.

painted smooth concrete, glued sand grains and artificial ripples. The diameter of the sand grains was between 1.6 and 2.0 mm; length and height of the symmetrical ripples were 80 mm and 18 mm, respectively. The waterdepth in front of the beaches (h_i) was constant for all experiments (0.45 m).

Figure 8: Sandrough bottom.

Figure 9: Artificial ripples.

4.2. Preliminary experiments

Experiments with a horizontal bottom showed the necessity to take special measures against disturbing influences. It turned out that both the wave generator and the wave absorber produced their own specific drift currents, having nothing to do with the wave-induced mass transport. A solution for this problem was found by hanging plastic curtains in front of the wave generator and wave absorber (see fig. 10). The curtains, constituting better boundary conditions, did not disturb the waves.

Figure 10: Plastic curtain.

Applying this solution another disturbing influence arose. Without plastic curtain the mass transport caused the dust-layer on the water surface to move towards the wave absorber, where this layer was destroyed by the breaking waves. As a result this dustlayer did not grow to considerable thickness. The plastic curtain in front of the wave absorber, however, prevented the transport of dust. Now the mass transport velocities near the surface were influenced considerably by the dust-layer. An example is shown in fig. 11.

Figure 11: Mass transport velocity influenced by dust-layer.

Addition of a small amount of surface tension reducing agent solved this problem. The behaviour of the dust-layer, described before, could be observed visually during the experiments.

4.3 Waves

The characteristics of the waves, applied in the various experiments, are shown in table I. Both deep water values and the values, pertaining to the actual depth of the horizontal bottom in front of the beaches, are given.

In all experiments the wave profiles were recorded at small longitudinal intervals along the flume. For wave no.1 (table I) the interval was 0.10 m and for the other waves 0.25 m. To determine the character of the waves and their behaviour on the beaches (higher harmonics, position and type of breaking, etc.) the data were analysed by means of Fourieranalysis. Furthermore, disturbances to the mass transport such as caused by reflection, seiches and free higher harmonics, could be distinguished in this way. The results of the Fourier-analysis, applied to the waverecords, are represented in appendix I. In these figures total wave height (crest-trough distance), amplitudes of first and second harmonic component and mean water level are represented as a function of location along the flume. For the waves under consideration reflection and a free second harmonic component were the most important origins of spatial variation of wave height.

wave no	T in sec.	H in m	H_o/L_o	H_i/L_i	$k_i h_i$	slope	smooth bottom	sand- rough bottom	rippled bottom
1	1.0	0.094	0.058	0.060	1.88	1:40	B - 1	—	—
						1:25	C - 1	—	—
						1:10	D - 1	—	—
3	1.5	0.043	0.010	0.015	1.04	1:40	B - 3	B-3a	B-3b
						1:25	C - 3	C-3a	C-3b
						1:10	D - 3	D-3a	D-3b
4	1.5	0.095	0.026	0.033	1.04	1:40	B - 4	—	B-4b
						1:25	C - 4	C-4a	—
						1:10	D - 4	D-4a	—
5	1.5	0.181	0.046	0.059	1.04	1:40	B - 5	B-5a	B-5b
						1:25	C - 5	C-5a	C-5b
						1:10	D - 5	D-5a	D-5b
7	2.0	0.095	0.014	0.023	0.72	1:40	B - 7	—	—
						1:25	C - 7	—	—
						1:10	D - 7	—	—

table I : Review and initial wave properties.

The first harmonic component of the Fourier series consist of an incoming and a reflected wave. Both have the same angular speed but the celerities are opposite to each other. Therefore the phase difference is only a function of location and not of time. On locations where the two waves are in phase a maximum for the amplitude of the first harmonic occurs, being the arithmetical sum of both amplitudes. The arithmetical difference occurs where the phases are just opposite, giving a minimum amplitude. The relative phase velocity between incoming and reflected wave is twice the original one. Therefore the distance between the maxima is equal to half the wavelength of the components. The amplitudes of the incoming and reflected waves then can be calculated from:

$$a_1 = \frac{a_{1 \max} + a_{1 \min}}{2} \quad (24)$$

$$a_{1r} = \frac{a_{1 \max} - a_{1 \min}}{2}$$

The reflection was small in these experiments. Therefore only the reflection coefficient for the first harmonic component was determined. Disregarding the reflection of the second harmonic, the maxima and minima of the amplitude of this harmonic in principle arise in the same way. This second harmonic consists mainly of a component attached to the incoming first harmonic wave, and a free harmonic component. The former has a frequency equal to twice of that of the first harmonic and equal phase velocity; the latter also has this double frequency, but an independent velocity of propagation, according to that frequency. Therefore the attached component propagates faster than the free one. The distance between locations of equal phase can be approximated by the product of the wavelength of the free component and the phase velocity of the attached component, divided by the difference in phase velocities of the attached and free components.

In a way analogous to that concerning the first harmonic, the amplitudes of the free and attached components of the second harmonic can be calculated from:

$$a_2 = \frac{a_{2 \max} + a_{2 \min}}{2} \quad (25)$$

$$a_{2fr} = \frac{a_{2 \max} - a_{2 \min}}{2}$$

More information about this free harmonic component can be found in ref. 15.

Calculated results regarding reflected and free components can be found in table II. This table also gives a comparison between the measured and theoretical (according to Stokes) values of the ratio between the amplitudes of first and second harmonic components.

wave no	a_1 in m	a_1/a_2	a_1/a_2 theor.	a_{1r}/a_1	a_{2fr}/a_1
1	0.043	8.3	8.8	3.6%	1.2%
3	0.021	16.3	17.0	4.0%	2.0%
4	0.043	8.0	7.5	5.2%	2.3%
5	0.078	3.9	4.2	4.6%	5.9%
7	0.045	5.1	5.1	6.0%	11.8%

table II: Measured properties of the waves on the horizontal bottom in front of the beaches.

From the data in table II it can be concluded that reflection did not exceed 6%, and that the amplitude of the free second harmonic component was always less than 12% of the amplitude of the first harmonic.

It is also shown that the ratios between the measured amplitudes of the first and second harmonics, before reaching the beach, agree rather well with the theoretical values according to Stokes.

The ratio between breaker height and breaker-depth and the type of breaking of the applied waves, are given in table III:

wave no		slope 1:40 (B)			slope 1:25 (C)			slope 1:10 (B)		
		smooth	sandr.	ripp.	smooth	sandr.	ripp.	smooth	sandr.	ripp.
1	H_b/h_b type	0.68 sp	- -	- -	0.76 sp	- -	- -	0.80 pl/sp	- -	- -
3	H_b/h_b type	0.69 sp	0.70 sp/pl	0.64 sp	0.69 sp/pl	0.70 sp/pl	0.75 sp	0.83 pl	0.83 pl	0.74 pl
4	H_b/h_b type	0.71 sp	- -	0.68 sp	0.86 sp/pl	0.82 sp/pl	- -	0.91 pl	0.91 pl	- -
5	H_b/h_b type	0.69 sp	0.65 sp	0.63 sp	0.77 sp/pl	0.75 sp	0.73 sp	0.83 pl	0.83 pl	0.78 pl
7	H_b/h_b type	0.72 sp/pl	- -	- -	0.76 sp/pl	- -	- -	0.90 pl	- -	- -
sp = spilling breaker; pl = plunging breaker.										

Table III: Breaking characteristics of the waves.

Breaker type classification and determination of breaker location were carried out according to the definitions given by Iversen (16) and Galvin (17).

4.4 Mass transport measurements

The mass transport velocities were determined by filming small solid particles, the density of which was close to that of water, viz. 995-1005 kg/m³.

An apparatus was designed and constructed to determine the average particle velocities from the films. A recorded film was projected on a screen, frame by frame. Two pairs of thin threads, each pair consisting of a horizontal and a vertical one, could be moved across the screen. The apparatus plotted the horizontal distance between the two vertical threads as a function of the mean vertical position of the two horizontal threads. By placing the first pair of threads on the initial position of a particle and the second pair on the final position (after an integer number of wave periods) the procedure resulted in the mean particle velocity as a function of depth. The scales were determined by gauging.

In this way the velocity profiles in appendix II were obtained. As can be seen from these plots, scatter is rather large in some cases. This is true particularly close to the breaker. These latter profiles are marked with an asterisk in the figures of appendix II.

In most cases, however, a definite velocity profile can be distinguished from the plotted points. In these cases, an average velocity profile was determined from the mean values at various depths. In appendix III the average profiles for one slope are collected. A certain wave on various slopes, as well as a certain wave on a certain slope, but with varying roughness, are compared.

The mass transport velocities were assumed to equal the average velocities of the corresponding solid particles. This seemed to be justified, since the latter velocities did not depend on the diameter (3 to 8 mm) of the solid particles used, as turned out from experiments concerning this question. However, because of the finite dimensions of the solid particles, the measured velocities represent averages over some height. This averaging effect is greatest near the bottom and the water surface, where the vertical gradients of the mass transport velocity are usually largest. The particles were obviously too large to detect the velocity gradients in the boundary layers. Outside these boundary layers, the deviations amount to only a few percents.

To investigate the stability of the velocity profiles during a longer

range of time, some experiments in this sense were carried out. An example of the results can be seen in fig.12, which shows that the profiles did not change significantly in time.

Figure 12: Mass transport velocity profiles for a certain wave after various time intervals.

Also the velocity profile just after the starting of the wave generator (a time interval of about 15 minutes) hardly differed from the profiles recorded after about fifteen hours. However, to make sure that a steady state was reached, in all experiments the wave generator was started the evening before the measuring day, so that all measurements were carried out at least about fifteen hours after the starting of the wave generator.

The influence of the side-walls of the flume on the drift velocities was thoroughly investigated by Mei et al (2). Applying their results, the conclusion for the present experiments can be that the results for the upper part of the vertical in the deeper regions of the flume must be treated with some caution.

5 DISCUSSION OF RESULTS

5.1 Horizontal bottom

The principal aim of the measurements on the horizontal bottom was to test the measuring method. Therefore, only a limited number of these experiments was carried out.

From a comparison between the results of the present experiments and the theory developed by Longuet-Higgins (1), it follows that:

- the distribution over the depth was rather well predicted for the waves 3, 4 and 5 ($k_i h_i = 1.04$), whereas the agreement became less for the waves 1 and 7 ($k_i h_i = 1.88$ and 0.72 , respectively);
- the predicted values are too large as compared with the measurements. This deviation becomes stronger with increasing wave amplitude;
- the deviations, mentioned before, are stronger for the surface velocities than for those at the bottom.

These conclusions are in agreement with the results of the experiments of Russell and Osorio (3). Therefore, it was decided to pass on to the sloping bottoms.

5.2 Sloping bottom

5.2.1 Smooth bottom

Generally speaking the tendencies shown by the experiments confirm the horizontal bottom theory of Longuet-Higgins. The latter predicts for waves with $kh \approx 1$ a mass transport velocity profile that is more or less symmetrical around mid-depth, with forward (in the direction of wave propagation) flow at surface and bottom, and backward flow in the center of the body of water. When kh becomes larger the surface velocity will increase; the bottom drift will decrease and becomes equal to zero for greater depth. The contrary occurs when kh decreases. When a wave approaches the beach k increases (because of decreasing wave length) and h decreases by definition. Because the decrease in depth predominates, the product kh decreases. As a result, according to horizontal bottom theory the drift profile will finally change into a profile with strong forward bottom drift, backward flow at mid-depth and small forward or even backward flow at the surface.

In the present investigation the latter only occurs when initial steepness of the wave and relative water depth are small. Significant backward flow at the surface only occurred for wave 7 ($k_1 h_1 = 0.72$) on the steepest slope (1 : 10). When propagating on the beach all waves applied reach such values of kh , that, according to horizontal bottom theory, the forward surface drift should change into backward drift. This phenomenon was also observed by Russel and Osorio on a horizontal bottom.

When the water depth becomes very small, the surface velocities increase considerably. This is in agreement with the results of Mei et al. (2), which, however, hold good for waves with very small amplitude on a horizontal bottom. When considering surface velocities the remark in 4.4., concerning the influence of the side-walls, must be kept in mind.

The increase of forward surface drift velocities for small depths, as mentioned before, cannot be analysed unambiguously from the present experiments, because of the quite different behaviour of the mass transport in the region close to the breaker.

In this breaker region the drift profile has the same shape for all waves, namely rather strong forward velocities at the bottom and the surface, with a back flow in between. At the breaker the velocities can attain greater values than predicted by horizontal bottom theory. Within the breaker zone, the net bottom velocities were always directed towards deeper water. Differences in these velocities, which could be caused by breakertype, were not found.

Outside the breaker zone, the values according to horizontal bottom theory (based on local wave height and length) considerably exceeded the experimental values for the bottom velocities. This is shown in figures 13, 14, 15 and 16 in more detail. In figures 13 and 14 the measured values of the bottom drift velocities are given as functions of depth. The experimental bottom velocities, which were normalized by the theoretical values, using measured local wave parameters, are given in figures 15 and 16. This might seem an inconsistent way of normalizing, but the wave heights at the various slopes did not show great differences, provided the initial wave was the same. As a consequence, the values obtained for the various slopes can be compared to each other.

Figure 13: Measured bottom drift velocities and theoretical values in waves with different periods.

Figure 14: Measured bottom drift velocities and theoretical values in waves with different heights.

Figure 15: Bottom drift velocities normalized by theoretical values in waves with different periods.

Figure 16: Bottom drift velocities normalized by theoretical values in waves with different heights.

Figs. 13 and 15 compare three waves with equal wave heights but different periods, each on three different slopes. In figures 14 and 16 the results are given for waves with equal period, but with different heights, also each propagating on three different slopes. The scatter of the experimental data shown in the parts a and b of figures 15 and 16 is smaller than the normalized values might indicate, as can be seen in figures 13 and 14. This greater scatter is caused by the method of normalizing applied to small velocities. The solid curves in figures 15 and 16 for the three slopes were calculated by smoothing the measured values, given in figures 13 and 14. In figures 15 and 16 the correction according to eq. 22 (see also fig. 5) for the 1 : 10 slope is plotted using dotted curves. The correction based on eq. 23, in which only the experimental first harmonic is considered (practically equal for the various slopes provided the initial waves are the same), is plotted in the same way. For a better comparison, in fact, the higher harmonics should be taken into account as well. Since the contributions of the various harmonics in eq. 23 occur in quadratic terms they are neglected here.

In fact the influences according to eqs. 22 and 23 have to be superimposed.

It can be concluded from the figures 15 and 16 that the correction to the horizontal bottom theory using the first harmonic rather than the wave height is generally greater than the correction caused by the slope. In the region where the corrections are most significant (small kh -values), the former can be three times greater than the latter. Though both corrections diminish the theoretical values, these values are always greater than the measured ones. Nevertheless, the tendencies predicted by the corrections are confirmed by the experiments. Considering fig. 15, for instance, the corrections for the longest waves are greatest, and this is rather well confirmed by the experiments. In fig. 16 a similar behaviour of predicted and measured values can be observed.

Noda (5) and Johns (6) numerically calculated the mass transport for progressive waves and standing waves, assuming a turbulent boundary layer and a coefficient of eddy viscosity. Johns concluded that for purely progressive waves the mass transport velocities at the outer edge of the boundary layer do not differ much for the laminar and turbulent cases, as was also shown by Longuet-Higgins in the supplement to the report of Russell and Osorio (3). This conclusion also applies

to the first term in eq. (19) of the present study, in which laminar flow is assumed. However, the tendency of the results for standing waves found by Johns (and also by Noda) applied to the present study means that in the turbulent case, the positive values of the function $g(\mu)$ in eq. 18 (see fig. 4) will be greater and the absolute value of the negative values will be smaller than in the laminar case. This indicates that in the turbulent case the mass transport, represented by the second term in eq. 18, increases in the lower part of the boundary layer and decreases in the upper part, when compared with the laminar case. As a result the total mass transport in the upper part of the boundary layer will increase if the boundary layer is turbulent. In view of the ratios of the diameters of the solid particles and boundary layer thickness, (which) are not small, the possibility of a turbulent boundary layer cannot explain the difference between theoretical and experimental values.

The steepness of the slope does not seem to have a very significant influence on the bottom velocities. The 1 : 40 slope may show somewhat higher values; in general, the differences with the results for the other slopes are small.

Concluding, one can say that the bottom velocities are primarily determined by the local wave parameters, such as depth, height and shape of the wave. The gradient of the bottom has no significant influence. In the cases examined, distortion of a model has only minor consequences for the mass transport velocities at the bottom. Although the distortions used in these experiments were not perfect (the initial water depth remained constant while the slope varied), there is enough experimental support for this conclusion (see also (3)). It is also suggested that distortion of prototype situations with less steep slopes is permitted as far as bottom drift velocities are concerned. The explicit proof, however, has not been delivered.

5.2.2 Rough bottom

The experiments with the slope surface with sand-roughness show the same tendencies as mentioned before. The only difference compared to the results on the relatively smooth slopes is that all velocities at comparable places are somewhat smaller, especially those at the bottom. The thickness of the layer, where the latter velocities occur, decreases

es slightly at corresponding places.

5.2.3 Artificial ripples

The artificial ripples prove to have an important influence on the drift profile. The initially forward velocity at the bottom in the smooth case, is reduced to about zero, whereas close to the bottom a consistent flow is induced. The direction of this current seems to depend at least on the wave steepness. Wave 3 ($H_1/L_1 = 0.015$) shows a forward velocity in that region, whereas waves 4 and 5 ($H_1/L_1 = 0.033$, resp. 0.059) show a strong backward current at the same level (see fig. 17)

Figure 17: Influence of artificial ripples on mass transport velocity profile.

The consistency of the forward current (small wave steepness) seems to depend on the bottom slope. For the artificial ripples applied this phenomenon indicates the existence of a critical wave steepness for the reversal of this current between 0.033 and 0.015.

At this moment the amount of experimental data on rough bottoms like these is not sufficient to draw general conclusions. The profiles observed, however, are so different from the smooth cases, that further investigations in this field are justified.

6 CONCLUSIONS

As far as it is permitted to draw general conclusions for the area outside the breaker zone from this restricted number of experiments, it can be noted that:

- the mass transport velocities at the bottom on the slopes seem to be determined by the local depth rather than by the slope-angle;
- the mass transport velocities at the bottom, predicted by the horizontal bottom theory, are too large for the sloping bottom;
- the discrepancy between theory and experiment increases with decreasing depth and increasing relative wave length and wave height;
- the inclusion of the change in wave form and slope effect appears to explain the behaviour of the bottom velocities qualitatively;
- the drift velocities change slightly for increasing bottom roughness and considerably when a ripple-like roughness is present.

LIST OF SYMBOLS

a = amplitude of surface elevation

A = maximum orbital velocity at the bottom outside the boundary layer

c = wave phase celerity

D = boundary layer thickness

$$\delta = \left(\frac{2\nu}{\omega}\right)^{\frac{1}{2}}$$

h = local still water depth

H = wave height

$$k = \frac{\partial}{\partial x} \phi(x) = \text{local wave number}$$

L = wave length

p = normal pressure

t = time

T = wave period

τ = shear stress

u = horizontal Eulerian velocity component

U = horizontal particle (Lagrangian) velocity component

w = vertical Eulerian velocity component

$$\omega = \frac{2\pi}{T}$$

x = horizontal coordinate

z = vertical coordinate

$$\mu = (h+z)/\delta$$

ν = kinematic viscosity of water

ρ = specific density of water

ϕ = phase, depending on horizontal coordinate x

$$\psi = \omega t - \phi(x)$$

subscript "o" refers to deep water value

subscript "1" or "2" refers to first or second harmonic component, respectively

subscript "i" refers to value on horizontal bottom

subscript " ∞ " refers to value just outside the boundary layer

subscript "b" refers to value at the breaker

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Appendix I

WAVE ANALYSISContents

fig. nr.	wave nr.	slope bottom roughness	fig. nr.	wave nr.	slope	bottom- roughness
I - 1	1	1 : 40 smooth	I - 16	3	1 : 40	sandrough
I - 2	3	1 : 40 smooth	I - 17	5	1 : 40	sandrough
I - 3	4	1 : 40 smooth	I - 18	3	1 : 40	rippled
I - 4	5	1 : 40 smooth	I - 19	4	1 : 40	rippled
I - 5	7	1 : 40 smooth	I - 20	5	1 : 40	rippled
I - 6	1	1 : 25 smooth	I - 21	3	1 : 25	sandrough
I - 7	3	1 : 25 smooth	I - 22	4	1 : 25	sandrough
I - 8	4	1 : 25 smooth	I - 23	5	1 : 25	sandrough
I - 9	5	1 : 25 smooth	I - 24	3	1 : 25	rippled
I - 10	7	1 : 25 smooth	I - 25	5	1 : 25	rippled
I - 11	1	1 : 10 smooth	I - 26	3	1 : 10	sandrough
I - 12	3	1 : 10 smooth	I - 27	4	1 : 10	sandrough
I - 13	4	1 : 10 smooth	I - 28	5	1 : 10	sandrough
I - 14	5	1 : 10 smooth	I - 29	3	1 : 10	rippled
I - 15	7	1 : 10 smooth	I - 30	5	1 : 10	rippled

initial wave nr.1: $T=1.0\text{ s}$, $L \approx 1.50\text{ m}$

slope 1:40

smooth bottom

WAVE ANALYSIS

Exp. B_1

Fig. I_1

MASS TRANSPORT IN GRAVITY WAVES ON A SLOPING BOTTOM SK-12

LABORATORY OF FLUID MECHANICS-DELFT UNIVERSITY OF TECHNOLOGY

initial wave nr. 4: $T=1.5$ s. $L \approx 2.75$ m

slope 1:40

smooth bottom

WAVE ANALYSIS

Exp. B_4

Fig. I_3

MASS TRANSPORT IN GRAVITY WAVES ON A SLOPING BOTTOM SK-12

LABORATORY OF FLUID MECHANICS-DELFT UNIVERSITY OF TECHNOLOGY

initial wave nr.7: $T=2.0s$, $L \approx 3.95m$

slope 1:40

smooth bottom

WAVE ANALYSIS

Exp. B_7

Fig. I_5

MASS TRANSPORT IN GRAVITY WAVES ON A SLOPING BOTTOM SK-12

LABORATORY OF FLUID MECHANICS-DELFT UNIVERSITY OF TECHNOLOGY

initial wave nr.1: $T=1.0s$, $L \approx 1.50m$
 slope 1:10 smooth bottom

WAVE ANALYSIS

Exp. D_1

Fig I_11

MASS TRANSPORT IN GRAVITY WAVES ON A SLOPING BOTTOM SK-12
 LABORATORY OF FLUID MECHANICS-DELFT UNIVERSITY OF TECHNOLOGY

initial wave nr.7: $T=2.0\text{ s}$, $L\approx 3.95\text{ m}$

slope 1:10

smooth bottom

WAVE ANALYSIS

Exp. D_7

Fig I_15

MASS TRANSPORT IN GRAVITY WAVES ON A SLOPING BOTTOM SK-12

LABORATORY OF FLUID MECHANICS-DELFT UNIVERSITY OF TECHNOLOGY

initial wave nr.5: $T=1.5$ s. $L \approx 2.75$ m

slope 1:40

rippled bottom

WAVE ANALYSIS

Exp. B_5b

Fig. I_20

MASS TRANSPORT IN GRAVITY WAVES ON A SLOPING BOTTOM SK-12

LABORATORY OF FLUID MECHANICS-DELFT UNIVERSITY OF TECHNOLOGY

Appendix II

MEASURED MASS TRANSPORT VELOCITY PROFILESContents

fig. nr.	wave nr.	slope	bottom- roughness	fig. nr.	wave nr.	slope	bottom- roughness
II - 2	1	1 : 40	smooth	II - 17	3	1 : 40	sandrrough
II - 3	3	1 : 40	smooth	II - 18	5	1 : 40	sandrrough
II - 4	4	1 : 40	smooth	II - 19	3	1 : 40	rippled
II - 5	5	1 : 40	smooth	II - 20	4	1 : 40	rippled
II - 6	7	1 : 40	smooth	II - 21	5	1 : 40	rippled
II - 7	1	1 : 25	smooth	II - 22	3	1 : 25	sandrrough
II - 8	3	1 : 25	smooth	II - 23	4	1 : 25	sandrrough
II - 9	4	1 : 25	smooth	II - 24	5	1 : 25	sandrrough
II - 10	5	1 : 25	smooth	II - 25	3	1 : 25	rippled
II - 11	7	1 : 25	smooth	II - 26	5	1 : 25	rippled
II - 12	1	1 : 10	smooth	II - 27	3	1 : 10	sandrrough
II - 13	3	1 : 10	smooth	II - 28	4	1 : 10	sandrrough
II - 14	4	1 : 10	smooth	II - 29	5	1 : 10	sandrrough
II - 15	5	1 : 10	smooth	II - 30	3	1 : 10	rippled
II - 16	7	1 : 10	smooth	II - 31	5	1 : 10	rippled
II - 1	location of profiles						

remark: profiles marked with * are located within the breaker-zone are very close to the breaking point.

fig. II-1

Appendix III

MASS TRANSPORT VELOCITY PROFILES ON THE VARIOUS SLOPESContents

III - 1 Location of profiles				
fig. nrs.	exp. nrs.			remarks
The same initial wave on beaches with different slopes (smooth bottom)				
III - 2	B - 1,	C - 1,	D - 1	
III - 3	B - 3,	C - 3,	D - 3	
III - 4	B - 4,	C - 4,	D - 4	
III - 5	B - 5,	C - 5,	D - 5	
III - 6	B - 7,	C - 7,	D - 7	
Waves with different period on the same beach (smooth bottom)				
III - 7	B - 1,	B - 4,	B - 7	slope 1 : 40
III - 8	C - 1,	C - 4,	C - 7	slope 1 : 25
III - 9	D - 1,	D - 4,	D - 7	slope 1 : 10
Waves with different initial wave height on the same beach (smooth bottom)				
III - 10	B - 3,	B - 4,	B - 5	slope 1 : 40
III - 11	C - 3,	C - 4,	C - 5	slope 1 : 25
III - 12	D - 3,	D - 4,	D - 5	slope 1 : 10
The same initial wave on beaches with different bottom roughness				
III - 13	C - 3,	C - 3a,	C - 3b	slope 1 : 25
III - 14	D - 3,	D - 3a,	D - 3b	slope 1 : 10
III - 15	B - 4,		B - 4b	slope 1 : 40
III - 16	C - 4,	C - 4a,		slope 1 : 25
III - 17	D - 4,	D - 4a,		slope 1 : 10
III - 18	B - 5,	B - 5a,	B - 5b	slope 1 : 40
III - 19	C - 5,	C - 5a,	C - 5b	slope 1 : 25
III - 20	D - 5,	D - 5a,	D - 5b	slope 1 : 10
Same wave on beaches with different slopes and the same bottom roughness				
III - 21	B - 5a,	C - 5a,	D - 5a	sandrrough bottom
III - 22	B - 3b,	C - 3b,	D - 3b	rippled bottom
III - 23	B - 5b,	C - 5b,	D - 5b	rippled bottom
Waves with different initial wave height on beaches with the same slope (artificially rippled bottom)				
III - 24	B - 3b,	B - 4b,	B - 5b	slope 1 : 40
III - 25	C - 3b,		C - 5b	slope 1 : 25
III - 26	D - 3b,		D - 5b	slope 1 : 10

I-III

100

fig. III-1

fig. III - 2

fig. III—3

fig. III - 4

fig. III-5

fig. III-6

fig. III-7

fig. III-8

fig. III-9

fig. III-10

fig. III — 11

fig. III-12

fig. III-13

fig. 14

III-15

fig. III-15

114

initial wave : nr. 4

$T = 15 \text{ s.}$

$L_1 = 275 \text{ m}$ $H_1 = 0.09 \text{ m}$

$k_1 h_1 = 104$ $H_0/L_0 = 0.026$

slope 1:25

fig. III - 16

fig. III-17

fig. III-18

fig. III-19

fig. III - 20

fig. III-21

fig. III-22

fig. III-23

fig. III-24

fig. III - 25

fig. III - 26