Manipulator control by energy dissipation

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Manipulator control by energy dissipation

Master of Science Thesis

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Abstract

In this research we investigate a class of systems which can only be controlled by dissipating energy. As an example, we investigate the control of a robotic manipulator without motors, but merely dissipative elements such as brakes and dampers. The energy source of the manipulator is the potential (gravitational) energy of the package which it is handling. This package is supplied to the manipulator at position higher than where it needs to be placed. After dropping the package, the manipulator moves back towards the starting position by using stored energy, for example using springs. The dissipative elements constrain the possible control signal to dissipating energy. We call this robot the passively controlled manipulator.

The challenge is that classical feedback solutions cannot be used to control this class of passive systems. Standard methods rely on the possibility to inject and extract energy into the system using motors, but the passively controlled manipulator has merely dissipative elements. Optimization methods could be used, but these have the downside of being computationally expensive.

In this thesis we analyse this challenge and present a novel control methodology. The developed solution tries to follow a trajectory along which no control is needed in order to reach a target position. If the end effector does not start on this trajectory, it is controlled towards it using a passified velocity field tracking controller.

The developed controller has been successfully implemented on both a simulation and a physical setup, and is capable of controlling the passively controlled manipulator towards multiple target positions. The controller has been compared to a second controller, which is the passified standard impedance controller. It outperforms the passified standard impedance controller on the ability to realize a desired workspace velocity.
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“In theory, there is no difference between theory and practice.
In practice, there is.”

— Lawrence Peter “Yogi” Berra
Chapter 1

Introduction

At the present time, it is impossible to imagine large factories without robots. Robotic arms are ubiquitous in many branches of the manufacturing industry and perform repetitive tasks in a fast and accurate manner. However, in Small and Medium sized Enterprises (SMEs) humans still perform these dreadful repetitive tasks. To encourage the process of moving the robots out of the large factories, and into the SMEs, the Delft Biorobotics Lab (DBL) has initiated a research project with the following goal:

To create robot arms that have the design and skills to perform repetitive tasks in a natural dynamic manner. This will make them more lightweight, more efficient, and safer than current industrial robot arms, allowing application outside the traditional factory environment into the domains of food handling and safe human-robot collaboration.

This thesis focuses on two issues with current robotic arms. The first is safety; the motors on the arm can generate rapid and unexpected link movements, which can form a safety risk. The second is cost; when considering a robotic arm, the most expensive components are the motor, gearbox and control hardware. Research has been performed on reducing the costs and improving the safety, by equipping the robot with smaller motors [1]. Why not take this to an extreme case and remove the motors entirely?

This research investigates the feasibility of a robotic manipulator without any motors, but merely with dissipative actuators such as brakes and dampers. We call this the passively controlled manipulator.

1-1 Passively controlled manipulator

The passively controlled manipulator, shown in Figure 1-1(a), is driven by the energy provided by the potential (gravitational) energy of the package it is handling. The package is supplied to the manipulator at the starting position, which is higher than the target position. After dropping the package, the manipulator moves back towards the starting position using stored energy. In this research it was chosen to store this energy by elongating a spring when moving...
from the start to the target position. Since the pick-and-place manipulator should be able to reach different target positions, control of the manipulator is required. As the manipulator has no motors, it cannot use these motors to control its trajectory. However, the manipulator can be controlled using energy dissipation devices such as brakes or dampers.

(a) Illustration of the passively controlled manipulator. The package is delivered to the manipulator using a conveyor belt and has to be placed on one of the target positions, which are indicated using stars. The spring pulls the arm back towards the starting position. This illustration shows a two link serial arm with revolute joints, other mechanisms can be used. This is discussed in Chapter 4.

(b) Illustration of a ski run, which has similarities with the passively controlled manipulator. The skier starts on the top and wants to move down to the bar or restaurant in the valley. The skier can go back up the mountain using the ski lift.

Figure 1-1: Illustrations of two similar ‘scenarios’. Both use gravity to move downward and can only steer towards the goal by dissipating energy.

At the beginning of this chapter, two downsides of a generic robotic manipulator were stated. These downsides were eliminated by introducing a passively controlled manipulator. This manipulator has the following advantages versus an actively controlled system:

- Removing motors from the manipulator creates a safer device. Without motors, the manipulator is unable to generate sudden fast movements which may injure a nearby human. The only source of energy in the device is gravitational energy. If no package is supplied to the device, the manipulator is not able to move. However, passively controlled manipulation requires an energy storage, such as a spring, which may cause unsafe situations. The unsafe situations occur when the energy release is not properly controlled.

- The absence of motors will decrease the power demand of the manipulator. Theoretically, a passively controlled manipulator does not need to be connected to the power grid. In practice, low power is needed to utilize the dissipative elements needed for steering. This results in a lower energy consumption, which is an important topic in modern companies.

- The absence of motors will decrease the initial and operation costs of the manipulator. The motor, gearbox and electronics form a large part of the costs of a robotic manipulator. The manipulator does not rely on electric power to move the robotic arm, which reduces the operational costs.
1-2 Passively controlled systems background

One of the best known examples of passive systems are the passive dynamic walkers [2, 3], which have been an inspiration for this research. Passive dynamic walkers are able to walk down a slope by using their potential energy. They do not rely on using motor control. The concept of passive dynamic walking was first introduced in 1990 by McGeer [2]. Over the past two decades extensive research has been conducted towards adding actuation [4] and 3D walking [5]. A similarity between the passive dynamic walker and our desired passively controlled manipulator is that in both devices, the energy needed to move is obtained from gravity. Unlike the passive dynamic walker, the passively controlled manipulator allows controlled energy dissipation using brakes or dampers.

Passively controlled haptic [6, 7, 8] and rehabilitation devices [9, 10] show similarities with the passively controlled manipulator. These devices are controlled using friction brakes or dampers, making them similar to the desired passively controlled manipulator. The advantage of using a passive device for haptic and rehabilitation situations is the intrinsic safety of the device. The lack of motors makes it impossible for the device to generate sudden erratic movements. A difference between the passively controlled manipulator and the haptic and rehabilitation devices is the energy source that provides the motion. The haptic and rehabilitation devices are human powered, whereas for the passively controlled manipulator, the only source of energy is gravitational energy. More details on these haptic and rehabilitation devices will be provided in this thesis.

1-3 Related example: ski run

A related example to the passively controlled manipulator is skiing, which is shown in Figure 1-1(b). We will briefly discuss this example here and it will be used in this thesis to illustrate ideas and the reasoning behind the developed control notion of the passively controlled manipulator. Imagine yourself on the top of a ski run, and you want to get to the valley. There are two goal positions in the valley. For lunch you want to ski towards the restaurant on the left and at the end of the afternoon you want to go to the bar, located on the right of the valley. The goal is to get to the restaurant and the bar, starting from the top of the ski run, without having to walk. Due to gravity, you will start to move down the ski slope, what you have to do is steer in the correct direction and make sure you do not go too fast. Both of these are realized by dissipating energy. There is a ski lift in the center which can be used to move back up. This example is very similar to the passively controlled manipulator in that the potential energy is the only source of energy and the goal is to reach the target positions. The spring in the passively controlled manipulator is represented by the ski lift. However, the ski lift in this example is an energy injecting component, whereas the spring is not. The question in this example is: how can you reach your target position, by merely dissipating energy, without having to walk? This is similar to the control problem in the passively controlled manipulator.

The major challenge of such a passively controlled manipulator is its control. This research focuses on the control of such a system, and briefly discusses the mechanics. This leads to the problem definition of this thesis, which is formulated in the next section.
1-4 Problem definition

Standard feedback motion control solutions rely on the possibility to both add and extract energy from the device; whereas dissipative actuators can merely extract energy. Therefore these standard methods cannot be applied to the class of passive systems we address here. Currently no memoryless feedback control solution exists that is both easily implemented on a passively controlled system and provides the desired performance.

1-5 Research goal

The goal of this research is to design a controller for a passively controlled manipulator which can control the end effector of the manipulator towards a target position.

In realizing this goal three subgoals have been specified:

1. Identify the challenges in controlling a passively versus an actively controlled manipulator.
2. Develop a controller to control this passively controlled manipulator.
3. Assess the performance of the designed controller by measuring the ability to realize the desired final position and velocity in both simulation and experiments.

The position and velocity are the key performance indicators, because the passive manipulator performs a pick-and-place task, during which it is the goal to place a package at a desired position. Upon reaching this position, the velocity should not exceed a prespecified value, to prevent damage to the package due to the impact. This measure is used to evaluate the performance of different controllers.

1-6 Thesis outline

A theoretical introduction to passivity and passive control is presented in Chapter 2. The goal of this chapter is to introduce the mathematical interpretation of a passively controlled system, and to illustrate the difficulties in controlling such a system. Chapter 3 follows up on the general analysis of passivity by focusing on passive control of manipulators. This chapter provides a background on related passive devices and how they are controlled, together with important aspects in controlling a passively controlled manipulator. In Chapter 4 a notion of controlling a passively controlled system is described, which is based on the natural dynamics of the system. Since the natural dynamics of a system depend on the mechanism, this is also discussed in this chapter. Thereupon Chapter 5 describes the development of our novel controller. Simulation and experimental results are presented in Chapter 6 and 7. Finally, based on the previous chapters, conclusions and recommendations will be given in Chapter 8. Throughout this thesis references are made towards Appendix C for relevant mathematical derivations.
In figure 1-2 we present a diagram showing the outline of this thesis. This thesis can be read in two ways. The short path starts directly with the controller design and continues with the results. The full path first introduces the challenges in passively controlled systems, for both general systems and manipulator, and subsequently continues on the controller design and the results.

Figure 1-2: A diagram representing the outline of this thesis. The short path starts directly with the controller design and continues with the results. The full path first introduces the challenges in passively controlled systems, for both general systems and manipulator, and subsequently continues on the controller design and the results.
Chapter 2

Passivity

The passively controlled manipulator has to be controlled solely by dissipating energy. Since no energy is added in controlling the manipulator, it is a passively controlled system. This chapter starts with a theoretical introduction of the concept of passivity, which is a constraint on the supply rate of energy to a system. After this, the difficulty of controlling a passively controlled system is illustrated by discussing a simple one dimensional example.

2-1 Definitions

Consider a general time-invariant nonlinear state space system \( \Sigma \):

\[
\Sigma : \begin{cases} 
\dot{x} = f(x, u) \\
y = h(x, u)
\end{cases}, 
\tag{2-1}
\]

with the state \( x \in \mathcal{X} \subseteq \mathbb{R}^l \), output \( y \in \mathcal{Y} \subseteq \mathbb{R}^k \) and input \( u \in \mathcal{U} \subseteq \mathbb{R}^m \). When \( u = 0 \) the system is uncontrolled and reduces to \( \dot{x} = f(x) \) and \( y = h(x) \). In this thesis the focus is on fully actuated systems, resulting in \( m = l \). For clarity the superscript \( m \) will still be used.

Before introducing a passively controlled system, the dissipative system is presented. The definition of a dissipative system is more general than a passive system and is valid for all mechanical systems.

**Definition 1** (Dissipative system [11, 12, 13]). Consider the general nonlinear system as presented in Eq. (2-1). This system is called dissipative with respect to supply rate \( s \) if there exists a function \( S : \mathcal{X} \rightarrow \mathbb{R}^+ \), called the storage function, such that for all \( x(t_0) \in \mathcal{X} \), all time \( t_1 \geq t_0 \), and all input functions \( u \in \mathcal{U} \):

\[
S(x(t_1)) - S(x(t_0)) = \int_{t_0}^{t_1} s(u(t), y(t)) \, dt - \frac{d(t)}{dt}, 
\tag{2-2}
\]

with \( d(t) \geq 0 \) the dissipation effects of the system.

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Which means that a system is dissipative if the energy externally supplied to the system is equal to the difference in stored energy in the system, minus the dissipation.

**Definition 2.** [Passive system [11, 13]] A system is passive if it is dissipative with respect to supply rate:

\[ s(u, y) = u^T y. \]  

(2-3)

Which means that a system is passive if the externally added energy is equal to the difference in stored energy (minus the dissipation), and the supply rate can be written as (2-3).

A general control technique which uses these definitions in designing a controller Passivity Based Control (PBC). It is a general control technique which renders the closed-loop system passive with respect to a desired storage function [12]. This technique is however not suited for our system. More details are provided in Appendix C-6.

### 2-2 Passivity for mechanical systems

The previous definitions considered general systems. In this section the definitions of dissipative and a passive system are used to derive the supply rate for a multibody mechanical system [11]. This section contains a short version of the full derivation which is included in Appendix C-2. The derivation of the supply rate for a mechanical system starts with the Lagrangian \( L(q, \dot{q}) \in \mathbb{R} \), as a function of the generalized position and velocity, respectively \( q, \dot{q} \in \mathbb{R}^n \). Combining this with the Euler-Lagrange (EL) equations:

\[
\frac{d}{dt} \left( \frac{\partial L(q, \dot{q})}{\partial \dot{q}} \right) - \frac{\partial L(q, \dot{q})}{\partial q} = Q,
\]  

(2-4)

with \( Q \in \mathbb{R}^n \) the generalized forces, equal to \( u - F_{diss} \) in a fully actuated system without external applied forces. \( F_{diss} \) represent the dissipation forces, satisfying: \( F_{diss}^T \dot{q} \geq 0 \). The Hamiltonian \( \mathcal{H}(q, \dot{q}) \in \mathbb{R} \), which can be derived from the Lagrangian using the Legendre transformation [14], represents the total energy of the system. The Hamiltonian is defined as:

\[
\mathcal{H}(q, \dot{q}) := \left( \frac{\partial L(q, \dot{q})}{\partial \dot{q}} \right)^T - L(q, \dot{q}),
\]  

(2-5)

Combining (2-4) and (2-5) yields the following equation:

\[
\mathcal{H}(q(t_1), \dot{q}(t_1)) - \mathcal{H}(q(t_0), \dot{q}(t_0)) = \int_{t_0}^{t_1} u^T \dot{q} \, dt - \int_{t_0}^{t_1} F_{diss}^T \dot{q} \, dt,
\]  

(2-6)

which is similar to (2-2), but is now specifically for mechanical systems. The supply rate satisfies the passivity definition of (2-3) and is given by:

\[ s(u, \dot{q}) = u^T \dot{q}. \]  

(2-7)

Using Definition 2 it can be concluded that for a mechanical system to be passive, this supply rate should be less than or equal to zero:

\[ u^T \dot{q} \leq 0. \]  

(2-8)
2-3 Simple example

The effect of the constraint on the supply rate is demonstrated in this section for a simple one dimensional example.

2-3-1 System description

The system under consideration is a falling mass $m$ under the influence of gravity. The mass, of which the position is indicated by $x$, starts at an initial height $x(0)$ above the ground, located at $x = 0$. To control the vertical position an (active) force $u$ can be exerted on the mass.

Using a force analysis or using the EL equation (see Appendix B), the Equations Of Motion (EOM) of the linear system can be derived. This results in:

$$m\ddot{x} + mg = u.$$  \hfill (2-9)

For this standard case, the control action is unconstrained. Now what happens to the system if the control signal is constrained to passive actions? The control action then has to satisfy the passivity constraint derived in (2-8). This means that the mass can only be slowed down and not speed up. The control action $u$ thus has to satisfy:

$$\dot{x}u \leq 0.$$  \hfill (2-10)

The easiest way to satisfy this constraint, without the trivial case, is by setting:

$$u = -\dot{x}\bar{u}^2,$$  \hfill (2-11)

with some control signal $\bar{u} \in \mathcal{U} \subset \mathbb{R}$. By inserting this into the passivity constraint of (2-8), it can be shown that it is always verified (for real valued inputs):

$$-\dot{x}^2\bar{u}^2 \leq 0.$$  \hfill (2-12)

The EOM of the nonlinear system in which the control action is passive is:

$$m\ddot{x} + mg = -\dot{x}\bar{u}^2.$$  \hfill (2-13)

2-3-2 Controllability

The effect of the passivity constraint has a large effect on the controllability of the system. A system is controllable \cite{11, 15} if the system can be controlled towards any desired state. Intuitively, this is not the case for the passively controlled system. The controllability for a linear and a nonlinear system is computed in different manner, which will now be described.
Controllability for linear systems

Consider the following linear (state space) system:

\[ \Sigma_L : \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \]  

with state \( x \in \mathcal{X} \subset \mathbb{R}^l \), output \( y \in \mathcal{Y} \subset \mathbb{R}^k \) and input \( u \in \mathcal{U} \subset \mathbb{R}^m \).

**Definition 3** (Controllability for linear systems [11]). The system \( \Sigma_L \) is said to be controllable if the matrix:

\[ M_c = \begin{bmatrix} B | AB | A^2 B | \cdots | A^{n-1} B \end{bmatrix}, \]  

is full rank.

Controllability of nonlinear systems

The notion of controllability is more involved for nonlinear systems. Consider the following nonlinear (input affine) system:

\[ \Sigma_{NL} : \begin{cases} \dot{x} = f(x) + \sum_{i=1}^m g_i(x) u_i \\ y = h(x) \end{cases} \]  

with state \( x \in \mathcal{X} \subset \mathbb{R}^l \), output \( y \in \mathcal{Y} \subset \mathbb{R}^k \) and input \( u \in \mathcal{U} \subset \mathbb{R}^m \). To explain the controllability for nonlinear systems, first a brief description of reachable sets [11] is needed.

**Definition 4** (Reachable set [11]). Consider the following definitions:

- The set \( \mathcal{R}_\Sigma(x_0, T) \) is a reachable set for system \( \Sigma \) at time \( T > 0 \). It is the set of points that can be reached by starting at point \( x_0 \) after time \( T \).

- The set \( \mathcal{R}_\Sigma(x_0, \leq T) \) is the reachable set within time \( T \): \( \mathcal{R}_\Sigma(x_0) = \bigcup_{t \in [0, T]} \mathcal{R}_\Sigma(x_0, t) \), in which \( \bigcup \) is the union set operator.

- The set \( \mathcal{R}_\Sigma(x_0) \) is the reachable set for all positive time \( t \): \( \mathcal{R}_\Sigma(x_0) = \bigcup_{t \geq 0} \mathcal{R}_\Sigma(x_0, t) \).

The definition of a reachable set can be used for the definition of controllability for nonlinear systems [11, 15]. In increasing importance:

**Definition 5** (Controllability for nonlinear systems [11, 16, 15]). Consider the following definitions:

1. \( \Sigma \) is accessible from \( x_0 \) if \( \text{int}(\mathcal{R}_\Sigma(x_0)) \neq \emptyset \).
2. \( \Sigma \) is locally controllable from \( x_0 \in \text{int}(\mathcal{R}_\Sigma(x_0)) \).
3. \( \Sigma \) is small-time locally controllable from \( x_0 \) if there exists \( T > 0 \) such that \( x_0 \in \text{int}(\mathcal{R}_\Sigma(x_0, \leq T)) \) for each \( t \in [0, T] \).
4. \( \Sigma \) is globally controllable from \( x_0 \) if \( \mathcal{R}_\Sigma(x_0) = \mathcal{X} \).

The set operator \( \text{int}(\mathcal{R}) \) is the interior of set \( \mathcal{R} \), which are all the points in \( \mathcal{R} \) that do not belong the the boundary of \( \mathcal{R} \). A nonlinear is accessible if the controllability matrix of the system is full rank. This matrix is generated using Lie Brackets, the full procedure is shown in Appendix C-3.

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Phase plots

Before going further into the controllability of both the active and passively controlled systems, we will discuss the phase plots of both systems. The phase plots provide a physical interpretation to the aforementioned theory. The phase plot of the active ($\Sigma_A$) and passively controlled system ($\Sigma_P$), respectively presented in (2-9) and (2-13), are shown in Figure 2-1. These figures show along what trajectory the mass can move. The ground is positioned at $x = 0$. The mass cannot go below the ground, therefore only the positive position is shown. The dot represents the starting position $x_0$ and the thick line is the uncontrolled motion due to gravity. From these figures the difference in the reachable set and thus in controllability between the active and passively controlled system is clearly shown. From the figures, the following can be concluded:

- The actively controlled system can generate a control action in any direction, which can speed up or slow down the mass. This creates the possibility to accelerate faster than gravity or to move upwards. Therefore the reachable set for the actively controlled system, indicated by $R_{\Sigma_A}$, is the entire set $\mathcal{X}$.

- Since the passively controlled system can merely be slowed down, and not speed up, the reachable set of this system, indicated by $R_{\Sigma_P}$, is smaller. The reachable set of the passively controlled system is limited to the area confined by the uncontrolled motion due to gravity and the horizontal (zero velocity) axis, which indicates that the system is not globally controllable. The reachable set of the passively controlled system is decreased with respect to the reachable set of the actively controlled system.

Controllability of falling mass

The controllability of the linear system can be easily computed by writing the equations in the state space representations of (2-14), using $x_1 = x$ and $x_2 = \dot{x}$:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g + \frac{u}{m}.$$  \hspace{1cm} (2-17, 2-18)

The controllability of the system can be checked by verifying that the controllability matrix of the linear system $M_{ca}$, constructed using (2-15), is full rank:

$$M_{ca} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{1}{m},$$  \hspace{1cm} (2-19)

which is always true for a nonzero mass.

The controllability of the passively controlled system can be assessed by verifying that the controllability matrix for the passively controlled system in (2-16) is full rank. Applying this procedure on the nonlinear system, which is shown in Appendix C-4, results in the controllability matrix $M_{cp}$ of the nonlinear system:

$$M_{cp} = \begin{bmatrix} 0 & 0 & -2\ddot{u}\dot{x}\tau^{-1} \\ 0 & 2\ddot{u}\dot{x}\tau^{-1} & -2\ddot{u}g\tau^{-1} \\ \tau^{-1} & \tau^{-2} & \tau^{-3} \end{bmatrix},$$  \hspace{1cm} (2-20)
Figure 2-1: The phase plots of the active and passive falling mass example with unitary mass. The reachable set is indicated by the shaded area $\mathcal{R}_A(x_0)$. The subscripts $A$ and $B$ indicate the active or passively controlled system. The thick blue line represents the uncontrolled movement due to gravity. The active controller can move up (positive $x$ direction) and down (negative $x$ direction), whereas the passive controller can only move down. These trajectories are illustrated using the green lines.
with scalar $\tau$, used to rewrite the standard nonlinear system into an input affine nonlinear system. The system is controllable when the determinant of this matrix is not equal to zero. The determinant is:

$$\text{det} (M_{\chi\eta}) = 4\bar{u}^2 \dot{x}^2 \tau^{-3}.$$  \hspace{1cm} (2-21)

This shows that the system is accessible. By using Definition 5, it can be shown that the system is not locally controllable, because the starting position $x_0$ is not in the interior of the reachable set $\mathcal{R}_\Sigma(x_0)$. The starting point is in fact on the boundary of this set. Due to the lack of local controllability it is also not small-time locally controllable or globally controllable. Furthermore, it can be seen that when $\tau$ goes to zero, the determinant goes to infinity. The scalar $\tau$ was introduced to rewrite the standard nonlinear system into a input affine nonlinear system (see Appendix C-4). A smaller scalar $\tau$ corresponds to a smaller difference between the new input affine control signal $v$ and the real control signal $\bar{u}$. If the scalar $\tau$ is zero, the input affine model $v$ represents the actual input $\bar{u}$ perfectly, but this results in a infinite determinant. In physical terms, this means that the control input is multiplied by an infinite gain, when applied on the system.

Constraining the control signal to be decreases the controllability of the system. In the next section we analyze the control of a general passive mechanical system and in the last section we discuss this possible control solutions to this topic.

## 2-4 General control of a passively controlled system

Instead of a simple model, this section contains the analysis of a general system which can only be controlled in a passive manner. Due to the complex structure of the EOM of a general mechanical system, it is hard to make conclusions on the controllability of a general passive mechanical system. However, it is possible to investigate the stability of a general passively controlled system, controlled using a passive controller. The system is therefore investigated by constructing a Lyapunov function [17] and applying LaSalle’s theorem [18] (see Appendix C-1 for details and definitions).

### 2-4-1 Passive actuation

Consider the standard EOM for rigid body systems, of which the derivation is shown in Appendix B:

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = u,$$  \hspace{1cm} (2-22)

with $q \in \mathbb{R}^n$, $M(q) \in \mathbb{R}^{n \times n}$ the configuration space inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ the matrix representing the centrifugal and Coriolis terms and $G(q) \in \mathbb{R}^n$ the vector containing potential forces such as gravity.

This system is controlled using a general control law:

$$u = f(q, \dot{q}),$$  \hspace{1cm} (2-23)

with $f(q, \dot{q}) \in \mathbb{R}^n$ some function of the generalized position and velocity. This controller has to satisfy the passivity criteria (2-7):

$$f^T(q, \dot{q}) \dot{q} \leq 0.$$  \hspace{1cm} (2-24)
Lyapunov

As an attempt to prove stability, the following Lyapunov function \[17\] (see Appendix C-1) is used:

\[ V(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q}. \] (2-25)

Its time derivative is:

\[ \dot{V} = \frac{1}{2} \dot{q}^T \dot{M}(q) \dot{q} + \dot{q}^T M(q) \ddot{q} \]
\[ = \frac{1}{2} \dot{q}^T \dot{M}(q) \dot{q} + \dot{q}^T M(q) M^{-1}(q) \left[-C(q, \dot{q}) q - G(q) - f(q, \dot{q})\right] \]
\[ = -\dot{q}^T \left[G(q) - f(q, \dot{q})\right], \] (2-27)

which shows the time derivative of the Lyapunov function for a general control law. In this derivation the skew-symmetric property of \( \dot{M}(q) - 2C(q, \dot{q}) \) was used. The potential term \( G(q) \) is computed by (see Appendix B):

\[ G(q) = \frac{\partial V(q)}{\partial q}, \] (2-29)

with \( V(q) \) the potential energy. It can be shown that this is equal to (using (B-7) and (A-3)):

\[ G(q) = gJ^T(q) \bar{M} \Gamma, \] (2-30)

with \( g \in \mathbb{R} \) gravity, \( \bar{M} \in \mathbb{R}^{l \times l} \) the workspace mass matrix, \( \Gamma \in \mathbb{R}^l \) the vector used to eliminate the components of \( x \) on which gravity does not act and \( J(q) \in \mathbb{R}^{l \times n} \) the Jacobian, defined in (A-3). By substituting (2-30) into (2-28) and by defining the controller in the workspace, and converting it to the configuration space using the transpose Jacobian:

\[ f(q, \dot{q}) = J^T(q) \bar{f}(q, \dot{q}), \] (2-31)

the following relation is obtained:

\[ \dot{V}(q, \dot{q}) = -\dot{q}^T J^T(q) \left[ gM \Gamma - \bar{f}(q, \dot{q}) \right]. \] (2-32)

For a system to be Lyapunov stable, this equation, which represents the addition of energy over time, has to be smaller than or equal to zero. In this equation, the relation between the controller and the mechanism is illustrated. The added energy is based on the combination of these components. Unfortunately, using this Lyapunov based analysis, nothing more can be concluded.

LaSalle’s theorem

Applying LaSalle’s theorem \[18\] (see Appendix C-1) on the general EOM with the general controller (2-23) provides:

\[ G(q) = f(q, 0), \] (2-33)

which shows that the controller should be able to generate a non-zero value when the velocity is zero, which compensates the gravitational forces.
2-5 Standard control solutions

Standard control solutions can be divided in two classes of control approaches: Model Based Feedback Control (MBFC) and optimization methods. MBFC methods, such as feedback linearization and backstepping, use modeled system information and model manipulation to construct a feedback control law. Since the passively controlled system is not controllable, the standard MBFC methods cannot be used. Optimization methods, which can be used in a feedforward way (e.g. path-planning algorithms), or in feedback (e.g. Model Predictive Control (MPC), reinforcement learning) optimizes the control signal according to some criteria. Optimization methods can be used since they can search the entire input and state space for feasible solutions. This, however, results in computationally expensive procedures. For example, path-planning techniques may not be suitable due to their feedforward control implementation which cannot handle disturbances very well; reinforcement learning requires many trials to learn the optimized control input; and MPC may be required to run nonlinear optimizations at very high rate with increases the complexity of the controller.

We aim to make a compromise between these two classes of control approaches, in order to synthesize the controller for the class of passively controlled systems we address here. The controller uses system information, which is MBFC, yet it also uses precomputed trajectories. The developed control solution, together with the reasoning behind it is explained in the subsequent chapters.

2-6 Conclusion

This chapter has provided an introduction to passivity. From this chapter we concluded that a mechanical system is passive if it satisfies:

$$u^T \dot{q} \leq 0,$$

with input $u \in \mathbb{R}^m$ and configuration space velocity $\dot{q} \in \mathbb{R}^n$. This states that the supply rate of energy to a system has to be smaller then or equal to zero. We showed using a simple system, that standard control solutions cannot be applied to the class of passively controlled systems we address here. Furthermore, the stability of the passive mechanical system is analyzed. It is shown that when designing a passive controller in the workspace, this creates the possibility to separate the mechanism and the (passive) controller in the time derivative of the Lyapunov function.
This chapter discusses related topics to the desired passively controlled manipulator. First, we provide a background on related mechanisms and algorithms from haptic and rehabilitation devices. Second, we show that there are two ways of defining passive control: per joint or for the system in total. Third, we provide two methods for generating a passive controller, based on the two ways of defining passive control.

3-1 Background

This section provides a brief background on research related to the passively controlled manipulator, with the goal to use this information in the analysis and design of the mechanism and controller. We will not provide a full summary, but instead focus on pros and cons and interesting features. First we discuss the implementations found in haptic and rehabilitation devices and second we discuss related control methods.

A difference between the desired passively controlled manipulator and the found haptic and rehabilitation devices lies in the source of energy. Whereas the previously discussed devices got their energy from humans which operate it, the passively controlled manipulator will be powered solely by gravity.

3-1-1 Haptic and rehabilitation devices

Physical implementations of passively controlled systems have been found in haptic and rehabilitation devices. Several examples are shown in Figure 3-1. The implementations are classified into three categories, being dissipation type, mechanism and workspace dimension. An overview of articles, that present implementations of passive actuation on mechanical devices, is shown in Table 3-1, classified into the different categories. First we discuss the dissipative actuators and second the mechanism.
Figure 3-1: Implementation of passive control on haptic devices. A human provides the input force to move the manipulator and the motion is guided using friction brakes in Figure 3-1(a) and Figure 3-1(d) and MR brakes in Figure 3-1(c) and Figure 3-1(b).

Table 3-1: Overview of implementations of passive actuation subdivided by dissipation type, mechanism and workspace dimension.

<table>
<thead>
<tr>
<th>Dissipative type</th>
<th>MR damper</th>
<th>Friction brake</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism</td>
<td>Links</td>
<td>Wires</td>
</tr>
<tr>
<td>2D [9, 22], 3D [20, 10]</td>
<td>2D [23, 19, 24, 25, 26, 27, 28, 29, 30]</td>
<td>2D [21], 3D [31]</td>
</tr>
</tbody>
</table>
Dissipative actuator

MR dampers and friction brakes are the dissipative actuators found in related passively controlled haptic and rehabilitation devices. Electromagnetic (EM) force brakes have not returned any implementations, but since these could be used these are also briefly touched upon.

**MR dampers** Magneto-Rheological (MR) dampers are dampers with a controllable damping coefficient. The damping coefficient can be altered by applying a magnetic field which changes the viscosity of the damper fluid due to tiny iron particles.

**Friction brakes** Dissipative actuation using friction brakes is implemented in two methods, which are proportional friction braking [21] and on/off switching [19]. Both methods use a brake connected between a link and the base to counteract the motion. When a torque is applied on a brake, there is a maximum amount of torque which it can resist until it starts to slip. A brake which is situated between two Degrees of Freedom (DoFs) is called a clutch, in which it couples the motion of both Degree of Freedom (DoF)s.

**Electromagnetic force brakes** Electromagnetic (EM) brakes use EM forces to generate a force opposite to the direction of motion. Two types of EM brakes exist, being Eddy current brakes and regenerative brakes. Eddy current brakes use a generator to convert kinetic energy into heat without a frictional contact. It uses the generation of electromagnetic forces to counteract the movement. Eddy currents are generated due to a movement of a conductor in a magnetic field. Eddy current brakes should not be confused with electromagnetic brakes which are friction brakes actuated by an electromagnetic field. Regenerative brakes use a generator to convert the kinetic into electrical energy. Often a standard electromotor is used in an inverse setting. Implementations of regenerative brakes exist, but mainly consist of non passive devices [32, 33, 34].

There are applications in which electromagnetic force brakes have been used to improve the impedance performance of haptic interfaces [35, 34, 33]. However, these implementations merely cover simple one DoF systems and techniques to prevent oscillation [36, 37, 38, 39]). No implementations on devices similar to the desired passively controlled manipulator have been found.

Mechanisms

The mechanisms of the haptic and rehabilitation devices consist out of links or wires (see Figure 3-1). We will briefly discuss an interesting example of both types.

**Mechanism consisting of links** An example of a passively controlled system using links is the P-TER [24], shown in Figure 3-1(a). The planar device consists of several links, which are connected to the base and to each other using friction brakes and clutches. Whereas the brakes purely dissipate the kinetic energy, the clutches transfer the energy between DoFs. This has the benefit that energy is not dissipated, but redirected. A downside of these dampers is
that the controller which uses these clutches is more complex than without the use of these clutches [19, 25, 26].

**Mechanism consisting of wires** An interesting passively controlled haptic device [21] consisting wires and friction brakes is shown in Figure 3-1(c). The device consists of an end effector which is connected to friction brakes using wires. Since wires can only withstand tension, a minimum of \(l + 1\) wires are needed to control the position of the end effector in \(R^l\) [40]. An advantage of this device is the low inertia of the moving parts, since this consists mainly out of the end effector and wires. A downside is that the wires can only pull the end effector, something which has to be incorporated in the controller. Moreover, the wires need to span the entire workspace, which is not desirable. These downsides make a mechanism consisting out of wires less favourable. Should a large manipulator inertia have a negative influence on the performance of the manipulator, then a mechanism consisting of wires could be further investigated.

### 3-1-2 Related control methods

Related control methods that are of interest for this research are discussed in this section. The control methods are separated into five groups:

- \( u(q, \dot{q}) = -D(q, \dot{q}) \dot{q} \), in which the control force is the multiplication of the damping matrix \( R \in \mathbb{R}^{n \times n} \) and the damper velocity \( \dot{q} \in \mathbb{R}^n \). The damping matrix is diagonal and positive definite, but not always full rank. This is a simple method, but a downside is that it cannot generate a control force when the system is not moving.

- \( u(q) = J^T(q) F_d \), in which the control force is computed using the Jacobian \( J(q) \in \mathbb{R}^{n \times m} \) and the desired dissipative forces \( F_d \in \mathbb{R}^m \). These forces are obtained using vector based force analyses. A downside is that this is not an intrinsically passive method, this means that the configurations space force will need to be clipped to satisfy the criteria of a passive signal.

- \( u(q, \dot{q}) = -\text{sgn}(\dot{q}) \mu F_n(q, \dot{q}) \) with \( \mu \) the friction coefficient, \( F_n \in \mathbb{R}^m \) the normal force which pushes the plates of the friction brakes together and \( \text{sgn}(\cdot) \) the sign function defined by \( \text{sgn}(\cdot) = \frac{\cdot}{|\cdot|} \). The normal force can be computed using different methods such as a realizable force or velocity analyses. This methods allows the usage of clutches which can redirect a part of the kinetic energy instead of fully dissipating it.

- \( u(q, \dot{q}) = \min(f_o) \) representing a minimization of cost function \( f_o \) which depends on the desired and current velocity of the end effector and the decrement in kinetic energy. A positive aspect of this controller is that it includes the use of clutches. A downside is that the optimization is a computationally expensive procedure.

- \( u(q, \dot{q}) = K_T(T_d - T) \) is a controller which is solely used for mechanisms consisting of wires. \( T_d \) are the desired wire tensions, \( T \) the current wire tension and \( K_T \) a gain. This controller is designed specifically for mechanisms consisting of wires, because it has to ensure that the wire does not go slack and it has to incorporate that wires can merely pull the end effector. Therefore this controller cannot be applied on mechanisms consisting of links.
Conclusion

The simple control laws each have their benefits and downsides. The controller which maps a workspace desired force to the configuration space torque presents small downsides, except that the signal will need to be passified in the configuration space. This mapping from workspace to configuration space is therefore used in the designed controller in Chapter 5. It was shown that several implementations of passive control on mechanical systems can be found in literature in the field of haptic and rehabilitation devices. Yet no general theory exists which can be applied for controlling general passive systems.

The design goal of the control algorithms of the haptic and rehabilitation devices is not to move the end effector towards one target position, but to follow a trajectory. In following this trajectory, it does not matter if the controller dissipates all energy (fully brakes/locks the motion) in order not deviate from this trajectory. However, in the control of the desired passively controlled manipulator, it is the goal to reach a target position, and not to remain on a specified trajectory. This contributes to necessity of designing a new controller, which can be used in the control of the passively controlled manipulator.

3-2 Constructing a controller for a passively controlled system

Now we know the state of the art is in passively controlled systems, the next step is to investigate the control of such a passively controlled system in more detail. We first show that the passivity of a mechanical system can be interpreted in two manners, which is joint or system passivity. After this a method is introduced which makes an active controller satisfy both manners, followed by an example. Subsequently a type of controllers is introduced which does not need this passification, since they are intrinsically passive.

3-2-1 Joint or system passive

In Chapter 2 it was concluded that the supply rate of the mechanical system has to be less than, or equal to, zero. This passivity constraint can be interpreted and satisfied in two manners:

Joint passive

The first method treats every joint separately, which means that the supply rate of energy of each joint in the system has to be negative-semidefinite. This means that every individual joint has to be passive and reduces the passivity constraint in (2-8) to:

\[ u_1 \dot{q}_1 \leq 0, \quad u_2 \dot{q}_2 \leq 0, \quad \ldots \quad u_n \dot{q}_n \leq 0. \]  

System passive

The second method is a holistic approach, which means that the total supply rate of energy to the system has to be negative-semidefinite, resulting in:

\[ u_1 \dot{q}_1 + u_2 \dot{q}_2 + \ldots + u_n \dot{q}_n \leq 0. \]
This means that for a system to be passive, not every individual joint has to be passive. This provides the possibility for energy transfer between the joints. The energy transfers can be realized using a connection between DoFs as was previously discussed in Section 3-1-1.

3-2-2 Active to passive control

An active control input \( u_a \in \mathbb{R}^m \) can be modified into a passive input \( u_p \in \mathbb{R}^m \) using some function \( \Phi (u_a, \dot{q}) \), which depends on the control input and the configuration space velocity. Since there are two ways to interpret the passivity constraint, there are at least two of these functions. The most intuitive of these functions are discussed independently for the joint and system passive case, which are discussed next.

**Joint passive**

Joint passivity can be realized using:

\[
\Phi_{pj} (u, \dot{q}) = \begin{cases} 
  u_i & \text{if } u_i \dot{q}_i \leq 0 \\
  0 & \text{if } u_i \dot{q}_i > 0 
\end{cases},
\]

with the subscript \( i \) indicating a joint wise operation, and index \( pj \) indicating joint passivity.

**System passive**

The steps which can be followed to make a system passive are: Check if the total system is passive, by verifying if (3-2) is satisfied. If this is false, make the system passive on joint level according to (3-3). In formulae, this is represented by:

\[
\Phi_{ps} (u, \dot{q}) = \begin{cases} 
  u & \text{if } u^T \dot{q} \leq 0 \\
  \text{apply (3-3)} & \text{if } u^T \dot{q} > 0 
\end{cases},
\]

the subscript \( ps \) indicates system passivity. This is a rather conservative method of making an active signal system passive. We have presented the two most intuitive methods of making an active control signal passive, another method is presented in Appendix C-7.

3-2-3 Example

This subsection provides an example of making an active control law joint passive. This control law will be used later in this thesis as a (simple) comparative controller. Joint passivity is used because this is the easiest to implement on a physical system, since the transfer of energy between DoFs is not needed. In this example we passify a workspace impedance controller. This controller has been chosen because it is a simple but widely used controller for controlling a system, when the exact trajectory is not of great importance.

The impedance is defined as a measure of opposition to motion, when the system is subjected to a force. Impedance control \([11, 41]\) can be implemented by enforcing that the system behaves as a spring damper system.

\[
\ddot{R}_d (\dot{x}_d - \dot{x}) + K_d (x_d - x) = F_{IC}
\]
with $x_d$ and $x \in \mathcal{X} \subset \mathbb{R}^l$ respectively the desired and the current workspace position, $\bar{R}_d \in \mathbb{R}^{l \times l}$ the desired workspace damping and $K_d \in \mathbb{R}^{l \times l}$ the desired workspace stiffness matrix. $F_{IC} \in \mathbb{R}^l$ represents the impedance control forces which have to be generated on the end effector to match the left hand side (LHS) of (3-5). Using the Jacobian $J \in \mathbb{R}^{l \times n}$, the forces in workspace can be mapped to the configuration space:

$$u_{IC} = J^T(q) F_{IC},$$  \hspace{1cm} (3-6)

$$= J^T(q) \bar{R}_d (\dot{x}_d - \dot{x}) + J^T(q) K_d (x_d - x),$$  \hspace{1cm} (3-7)

with $u_{IC} \in \mathbb{R}^n$ the joint torques. This control law is visualized using a block scheme in Figure 3-2. The passive control signal can be obtained by inserting (3-7) into (3-3). Note that for checking the passivity of the total system can also be checked by checking whether the system is passive in the workspace:

$$u^T \ddot{q} = \left( J^T(q) F_{IC} \right)^T J^{-1}(q) \ddot{x}$$  \hspace{1cm} (3-8)

$$= F_{IC}^T J(q) J^{-1}(q) \ddot{x}$$  \hspace{1cm} (3-9)

$$= F_{IC}^T \ddot{x}.$$  \hspace{1cm} (3-10)

Note that this only holds for the total system, and not for each joint.

![Figure 3-2: Block scheme of the passive impedance controller, which is connected to the plant. The inputs to the controller are the current $x$ and target $x_d$ workspace position. These are used to compute the passive impedance control force $F_{PIC}$ using (3-7) in combination with (3-3). This is the input to the plant, of which state $x$ is measured. Figure adopted from [42].](image)

The reciprocal of impedance is admittance (sometimes called mobility), which is the amount of motion which occurs, when the system is subjected to a force. This type of control is less favourable because the input to an admittance controller is force and not position. The Admittance control relies on an active Proportional-Derivative (PD)-controller. To implement admittance control, the plant is typically position controlled, using a simple PD-controller, to enforce a closed loop admittance behaviour.

### 3-2-4 Intrinsically passive control

The previously mentioned type of synthesizing a control law does does not change the fact that the system to be controlled is passive and that the controller is based on the assumption that it is active. To incorporate these limitations, intrinsically passive control could be used.
Intrinsically passive control takes the limitation of only be able to apply a passive into account. These controllers can be based on a mechanical mechanical. The two mechanical devices which are discussed are a damper and a brake.

**Damper**

A damper can be modeled using a linear viscous friction model [43]:

\[ u_D(q) = -R\dot{q}, \]  

(3-11)

with \( u \in \mathbb{R}^m \) and \( \dot{q} \in \mathbb{R}^n \) respectively the configuration space forces and velocities. \( R \in \mathbb{R}^{m \times n} \) is the configuration space damping matrix which is diagonal and positive definite. This matrix can be considered as the control input. Because of the multiplication with the velocity, the damper model cannot generate forces when there is no movement.

**Brake**

A brake can generate a force when there is no movement, which makes it possible to remain at a certain position when the system is at a standstill. A brake can be modeled using numerous different models; an overview is presented in [43, 44]. In this thesis the static model is used, which means that the friction is time independent. It is an often used model consisting of containing stiction, viscous friction and Coulomb friction.

**Stiction** Stiction is short for static friction and describes the friction force when there is no movement. As opposed to viscous and Coulomb friction, stiction counteracts external forces instead of motion. The stiction part of the brake model, denoted by \( u_{Bs} \), can be modeled by:

\[
\begin{align*}
    u_{Bs}(\tau_e) = & \begin{cases} 
    -\tau_e & \text{if } \dot{q}_i = 0 \text{ and } |\tau_e| < u_s \\
    -u_s \text{sgn}(\tau_e) & \text{if } \dot{q}_i = 0 \text{ and } |\tau_e| \geq u_s
    \end{cases}
\end{align*}
\]  

(3-12)

with \( \tau_e \in \mathbb{R} \) the applied configuration space forces and \( u_s \in \mathbb{R}^+ \) the maximum static configuration space force which the brake can generate, which is the control input when using this model. The sgn (·)-operator represents the sign of the variable, which can be computed by dividing the scalar by its absolute value. The stiction forces can vary between \(-u_s\) and \(u_s\) and its magnitude is always smaller then or equal to the applied force.

**Viscous friction** The viscous friction \( u_{Bv}(\dot{q}) \), is identical to damper model in (3-11).

**Coulomb friction** The Coulomb friction depends on the sign of the velocity and can be modeled by:

\[ u_{Bc}(\dot{q}) = -u_c \text{sgn}(\dot{q}), \]  

(3-13)

with \( u_c \) the configuration space Coulomb friction force which can be exerted by the brake, which is the control input when using this model.
The total model, created by combining the three friction models is defined by:

$$u_B(q, \tau_e) = \begin{cases} 
-R\dot{q} - \tau_c \text{sgn}(\dot{q}) & \text{if } \dot{q} \neq 0 \\
-\tau_c & \text{if } \dot{q} = 0 \text{ and } |\tau_e| < u_s \\
-u_s \text{sgn}(\tau_e) & \text{if } \dot{q} = 0 \text{ and } |\tau_e| \geq u_s
\end{cases}$$

(3-14)

From this equation it is clearly visible that the viscous and Coulomb friction models contribute to the friction force when there is a nonzero velocity and the stiction creates the friction force when there is no movement. In general there is no force saturation implemented for the brake models.

The use of an intrinsically passive controller is preferred over the use of an active controller which is made passive. However, this does still require an appropriate choice of matrix $R$ and vector $\tau_c$.

### 3-2-5 Brake model

In the previous chapter, we discussed a general mechanical system which is passively controlled. We can now extend the application of LaSalle’s theorem on the Equations Of Motion (EOM), by incorporating the brake model. Removing the terms which are zero for zero velocity, yields:

$$G(q) = gJ^T(q)\dot{\Gamma} = \begin{cases} 
-\tau_e,i & \text{if } |\tau_e,i| < u_{s,i} \\
-\tau_s,i \text{sgn}(\tau_e,i) & \text{if } |\tau_e,i| \geq u_{s,i}
\end{cases},$$

(3-15)

in which $\tau_e$ is the applied load on the brake, $\tau_s$ is the maximum static friction force which the brake can generate and $|\cdot|$ represents the absolute value of a variable. The subscript $i$ indicates an element wise operation.

Since the system is at standstill, the external torque $\tau_e$ in (3-15) is equal to gravity related terms on the LHS. Substituting $\tau_e$ with $G(q)$ yields:

$$G(q) = \begin{cases} 
-G(q)_i & \text{if } |G(q)_i| < \tau_{s,i} \\
-\tau_{s,i} \text{sgn}(G(q)_i) & \text{if } |G(q)_i| \geq \tau_{s,i}
\end{cases},$$

(3-16)

This provides a lower bound on the static friction which the brake should be able to deliver, depending on the configuration of the mechanism. This shows that the position in which the manipulator can remain at a standstill is influenced by the force which can be generated at standstill (the stiction force).

### 3-3 Conclusion

This chapter has provided more details on passivity and passive control. First, related control algorithms have been presented, from which the controller which mapped workspace to configuration space forces provided the least downsides. The majority of the methods has been developed for one specific mechanism and dissipation type. No general theory exists which...
can be applied on different mechanisms and for the passively controlled manipulator. This shows the reason for a new type of controller.

Second, it was shown that a mechanical system can be passive for each separate joint or for the system in total. Third, two general types of generating a passive control law are presented for the joint and system passivity. The first is illustrated using a standard workspace impedance controller and the second can be realized by modeling the controller as a brake or damper.

Fourth and last, it was shown that LaSalle’s theorem provides a lower bound on the static force which the controller should be able to apply.
This chapter introduces a control notion which decreases the negative effect of being able to apply only passive forces. This control notion is based on the natural dynamics of the system, which is discussed first. The mechanism used of the passively controlled manipulator dictates the natural dynamics of a system, therefore this is discussed second. Subsequently, the required energy of the system is investigated when using this control idea. The controller which uses the natural dynamics is presented in Chapter 5.

4-1 Natural dynamics

The developed control idea for the manipulator is based on the natural dynamics of a system, and is explained using the ski run example which was introduced in the introduction. The natural dynamics are illustrated by discussing Single Degree of Freedom (SDoF) and Multiple Degree of Freedom (MDoF) natural dynamics.

4-1-1 Example

Consider again the ski run example, which was presented in the introduction. The question in this example is: how can you reach your target position, without having to walk?

The solution in which you have the largest chance of reaching your target position is to move directly above the bar or restaurant, and let yourself slide down the hill in a straight line from there. This trajectory will naturally guide you towards the target position. However, the ski run is not perfectly smooth; it contains hills and dips which cause this trajectory to not be perfectly straight, as is shown in Figure 4-1(a). This trajectory is computed, by starting from the target position and assessing how you get there without steering. By repeating this procedure from this new position, this will result in a trajectory uphill, back in time, which is in fact the trajectory which naturally brings you towards the target position.
28 Natural dynamics

Illustration of a ski run, which has large similarities with the passively controlled manipulator. The skier starts on the top and wants to move towards the bar or the restaurant in the valley. The skier can go back up the ski run using the ski lift. The solution in which the skier has the largest chance of reaching the target position is by moving directly above the bar or restaurant and let yourself slide down the hill from there. However, since the ski run is not perfectly smooth, this trajectory which leads towards the target is not perfectly straight.

Illustration of the passively controlled manipulator. The package is delivered to the manipulator using a conveyor belt and has to be placed on the target positions. The spring pulls the arm back towards the start position. This illustration shows a two link serial arm with revolute joints, other mechanisms can be used. The safest way of reaching the target position is by moving towards the trajectory which naturally brings the package towards the target position.

Figure 4-1: Illustrations of two similar scenarios. Both use gravity to move downward and can only steer towards the goal by dissipating energy.

4-1-2 Manipulator

The same procedure can be used to control the passively controlled manipulator. This is illustrated in Figure 4-1(b). There is some trajectory which ends in the target position, indicated by a star, along which there is no control required. The advantage of controlling the system by moving along this trajectory, is that moving along it requires no active control. Since it is merely possible to apply passive control action, the possible control actions are limited. By using no active control, this decreases the impact of this constraint. Before moving further in how a system can be controlled using the previously introduced idea, we provide three definitions.

Definition 6 (Natural dynamics of a system). We define the natural dynamics of a system as the dynamics of the system when it is not time or feedback controlled.

The natural dynamics do not only consist of the uncontrolled motion of a system, but it can be modified by adding stiction, viscous and Coulomb friction to the system, which were addressed in Section 3-2-4. This provides the possibility to manipulate the natural dynamics. These forces are always passive and can be considered as intrinsical system properties. In this thesis we will only use the viscous friction model, because this is a continuous model. The natural dynamics of a system $f(q, u)$ can be described by:

$$\dot{q}^* (t) = f (q^* (t), u^*),$$

with $q^* (t), \dot{q}^* (t) \in \mathbb{R}^n$ respectively the configuration space position and velocity. $u^* \in \mathbb{R}^m$ is the passive control action which suffices:

$$u^* (\dot{q}^* (t)) = -R(q) \dot{q}^* (t),$$

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with \( R(q) \in \mathbb{R}^{n \times n} \) the constant configuration space damping matrix, which can be equal to zero in the case of no added damping along the motion. More details are provided in Section 4-1-4.

**Definition 7** (Natural trajectory of a system). The trajectory which a system will follow based on the natural dynamics, presented in (4-1), is called the natural trajectory \( q^*(t) \).

Examples of natural trajectories are shown in Figure 4-2(a). Corresponding to each natural trajectory, there is a natural velocity which is needed to remain on this trajectory. The circles represent the starting positions, the line represents the natural trajectory and the star represents the target position. It shows that there is one trajectory which reaches the target position, indicated by the thick red line.

**Definition 8** (Target natural trajectory of a system). The natural trajectory of a system which passes trough the target position is defined as the target natural trajectory.

![Illustration of the natural trajectories of a system](image1.png)

![Illustration of the natural trajectories of a system for three starting points shown in the left figure](image2.png)

**Figure 4-2:** Illustrations of the trajectories for a system and how this can be used in controlling a system.

When a starting position is not on the target natural trajectory, but it is possible to reach this trajectory and achieve the velocity along this trajectory, the target position can be reached. This is called natural dynamics based control and is indicated in Figure 4-2(b). Details on the implementation of the natural dynamics in the control of the manipulator are presented in Chapter 5.

The natural trajectory in the configuration space can be mapped to the workspace using (A-1) to obtain \( x^*(t) \). Reaching a desired natural trajectory can be written as:
\[ \exists u \in \mathbb{R}^m, \delta > 0 : |x^*(0) - x(0)| < \delta, \quad (4-3) \]

such that \[ |x^*(t) - x(t)| \to 0, \text{ when } t \to \infty, \quad (4-4) \]

with some parameter \( \delta \) limiting the distance from the starting point of some trajectory to the target natural trajectory. This is depicted in Figure 4-3 for several positions. The set \( S \) is the back reachability set of \( x_d \) and contains all points from which the target position can be reached. \( \bar{S} \) which contains all starting positions which have a nonzero velocity beneficial to reaching the target trajectory. Assessing whether the velocity at the starting position is beneficial to reaching the target position is important, but difficult to determine. Therefore the analysis will be confined to starting positions with a zero velocity.

![Figure 4-3: Trajectory of the target natural trajectory \( x^*(t) \) leading towards a target position \( x_d \). Around this trajectory there is set \( S \), which is the back reachability set of \( x_d \) and contains all points from which the target position can be reached. \( S \) is a subset of \( \bar{S} \) which contains all starting positions which have a nonzero velocity beneficial to reaching the target position.](image)

In Figure 4-4 the passive control problem is considered in a target position centered view. There is one starting position, which can reach a set of target positions. This set is indicated by \( \mathcal{T} \), which is the forward reachability set of \( x(0) \). This set contains all points which can be reached when starting from \( x(0) \). Again, this set contains all positions with a zero starting velocity. The set \( \mathcal{T} \) contains all starting positions which have a velocity which enlarges the set of target positions.

The general notion of the natural trajectory of a system has been introduced. The natural trajectory depends on the Equations Of Motion (EOM), which in turn is dictated by the mechanism. Therefore, the influence of the mechanism is described next.

### 4-1-3 Single degree of freedom natural trajectories

The natural trajectory of a system requires the solving of a differential equation. This can be avoided by constraining the motion to a subset of the total configuration space by locking all but one of the joints. By restraining the motion of the system to one Degree of Freedom (DoF), the natural trajectory of the system is simplified to this one DoF.
Figure 4-4: Starting position $x(0)$ and its natural trajectory. $\mathcal{T}$, which is the forward reachability set of $x(0)$. This set contains all points which can be reached when starting from $x(0)$. Set $\mathcal{T}$ is a subset of set $\bar{T}$ which contains all possible target positions starting from $x(0)$ with an initial nonzero velocity beneficial to reaching the target.

In Figure 4-5 the permissible motion with one free moving joint is shown for several devices. For a revolute joint, this creates a circular natural trajectory and for a slider joint this creates an instantaneous straight natural trajectory. Depending on the mechanism, the total natural trajectory for a slider joint is also a straight line. This is illustrated in Figure 4-5(e) and Figure 4-5(f).

The SDoF trajectories can be used in controlling a system based on its natural dynamics. A control method, based on these SDoF natural trajectories is presented in Chapter 5.

4-1-4 Multiple degrees of freedom natural trajectories

Multiple Degree of Freedom (MDoF) natural trajectories permit motion on multiple joints at the same time. Compared to the SDoF trajectories, the MDoF trajectories are more complex and less intuitive. The natural trajectory of a system depends on several factors:

- The most important factor is the mechanism. Similar as to the SDoF-case this sets the EOM of the system.

- The shape of the natural trajectory can be influenced by added viscous friction (4-2). Increasing this damping causes the target natural trajectories to become steeper and longer, this is shown in Figure 4-6(b). Note that this is only important for MDoF and not for SDoF natural trajectories.

- The velocity upon reaching the target also influences the natural trajectory. In Figure 4-6(b) it is shown that increasing this velocity causes the target natural trajectories to become steeper and longer. In the case of the passively controlled manipulator, this parameter can be chosen to have a certain velocity upon impact.
Figure 4-5: The SDoF natural trajectory for three mechanisms. The illustrations in the top row allow motion on the first joint, and the illustrations in the bottom row allow motion on the second joint. Different mechanisms clearly result in different natural trajectories.
(a) The effect of damping on the natural trajectory of the serial two link mechanism. Increasing the damping causes the target natural trajectories to become steeper and longer.

(b) The effect of velocity upon reaching the target on the natural trajectory of the serial two link mechanism. Increasing this velocity causes the target natural trajectories to become steeper and longer.

Figure 4-6: The effect of parameter changes on the target natural trajectory for the serial two link mechanism and the chosen target natural trajectories.
When these three components have been specified, the natural trajectories $x^*(q)$ can be obtained using a numerical integration. This results in a time dependent trajectory $x^*(t)$ and velocity $\dot{x}^*(t)$. However, the natural trajectory and velocity along it as a function of time, are not of interest. Therefore, these functions are written independent of time, as a function of the (vertical) position $x_2$:

$$\dot{x}^*(x_2) = 0.$$  \hfill (4-5)

It was noted that the mechanism is important for the shape of the natural trajectory of a system, in both the single and multiple DoF case. Therefore, mechanisms which can be used for a passively controlled manipulator are discussed in more detail in the next section.

### 4-2 Mechanism analysis

In this chapter three planar mechanisms are investigated, which are shown in Table 4-1.

- The first mechanism is a two link serial mechanism with revolute joints. The top link is connected to the fixed world and the bottom link is connected in series to the first link.
- The second is a two link parallel mechanism with revolute joints. The sliders are both connected to the world and to each other, which creates a triangular configuration.
- The last mechanism is a two link serial mechanism with slider joints. The first link is connected to the world and the second link is connected to the first link under an angle of $\pi/2$ degrees.

The only source of energy for the passive manipulator is the potential energy. Movement of each DoF should result from gravity, either directly or indirectly. Indirectly means that due to the movement of one DoF, another DoF starts to move. This can be uncontrolled, by Coriolis forces or controlled, by using a clutch. Systems with DoF which merely have uncontrolled indirect gravity effects are harder to control and a clutch may not be available. Therefore we investigate if gravity has an effect on all DoF of the mechanism.

The energy resulting from gravity is expressed as the potential energy $V(x)$. The potential energy is defined in the workspace and is a function of the joint angles $q \in \mathbb{R}^n$, see (B-7). Using the Lagrangian $L$ [11, 45], which is a function of the generalized position and velocity, respectively $q, \dot{q} \in \mathbb{R}^n$, the influence of the potential energy can be related to the motion of a mechanical system, described by its EOM. For a general multibody system, the EOM are:

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = u,$$  \hfill (4-6)

of which all elements have been introduced at (2-22), and are shown in Appendix B. The derivation of (4-6) is shown in Appendix B. The potential force vector $G(q)$ is the only component of the EOM which is affected by gravity.

Examples of the potential force vector for are shown in Table 4-1 for the three mechanisms. It can be seen that the potential force vectors of the first two mechanisms are in general nonzero, while the last mechanism has a zero component. This zero component indicates
**Table 4-1:** Standard mechanisms covered in this thesis with their potential force vector \( G(q) \). A zero in this vector shows a zero influence of gravity on this DoF. The potential energy for these mechanisms is shown in Appendix C-8.

\[
G(q) = \begin{bmatrix}
g l_1 m_1 \sin q_1 + g l_1 m_2 \sin q_2 \\
g l_2 m_2 \sin q_2
\end{bmatrix}
\]

\[
G(q) = \begin{bmatrix}
(m_c_1 + m_c_2) f_2(q_1, q_2) \\
(m_c_1 + m_c_2) f_3(q_1, q_2)
\end{bmatrix}
\]

with: 
\[
f_2(q_1, q_2) = -\frac{g}{4} \frac{q_1^2 - f_1(q_1, q_2)}{\sqrt{\xi_0^2 - f_1(q_1, q_2)^2}}
\]

and: 
\[
f_1(q_1, q_2) = \frac{x_0 + q_1^2 - q_2^2}{4 \xi_0}
\]

\[
G(q) = \begin{bmatrix}
-\frac{1}{2} g m_1 - g m_2 \\
0
\end{bmatrix}
\]

that there is no influence of gravity on this DoF, meaning that this mechanism is not suited for a passively controlled manipulator.

This result can be generalized by investigating the definition of the potential force vector:

\[
G(q) := \frac{\partial V(q)}{\partial q} = g \Gamma \tilde{M} J(q),
\]

(4-7)

of which all elements have been introduced at (2-30), and are shown in Appendix B. Each element of the potential forces vector shows the influence of gravity on the corresponding DoF. Therefore, \( G(q) \) should contain only nonzero elements in order to have an influence of gravity on each DoF. Since all elements in (4-7) other than the Jacobian are constants and do not differ in structure between mechanisms, the influence of gravity on a DoF merely depends on the structure of the Jacobian.
4-3 Energy analysis

For reaching a target position using a passively controlled manipulator, it is important to analyze the energy of the total system. The Hamiltonian $\mathcal{H}$ is total energy in the system, which is the sum of the kinetic $\mathcal{T}$ and potential energy $\mathcal{V}$. The Hamiltonian of a (ideal) system without internal dissipation and to which no control is applied is constant, see Appendix C for more details on the Hamiltonian. The Hamiltonian of the system is defined by $\mathcal{H}$ and the Hamiltonian of the system along the natural trajectory is $\mathcal{H}^\star$. In order to reach a target position, the following criteria need to be satisfied:

- The total energy $\mathcal{H}$ before reaching the natural trajectory has to be larger than the amount of energy on the natural trajectory $\mathcal{H}^\star$. Since the system is passive, the total energy can only decrease or remain constant, therefore this has to be true. This condition always needs to be satisfied, and could be used as a check during the movement. If it is not possible to reach the target natural trajectory, the target position cannot be reached.

- A similar statement can be made on the starting and target position. The total energy in the system at time $t = 0$ has to be larger than the total energy in the system upon reaching the target position:

$$\mathcal{H}(x(0)) \geq \mathcal{H}(x_d). \quad (4-8)$$

- If the velocity at the start and target position is very small, the kinetic energy at these positions is negligible. This means that the previous criterion changes to:

$$\mathcal{V}(x(0)) \geq \mathcal{V}(x_d). \quad (4-9)$$

From this energy analysis we can conclude that the potential energy at the start should be at least equal to the potential energy at the target position, if both the velocity at the starting and target position are small. The maximum size of these velocities depends on the mechanism and the amount of dissipation used in control.

4-4 Conclusion

This chapter has presented the notion of controlling a system based on the natural dynamics of the system. The natural dynamics were used, because moving along the natural trajectory of a system requires no or little control, this decreases the impact of this constraint. The natural trajectory depends on three components, the mechanism, the added viscous friction and the velocity upon reaching the target position. For the mechanisms, it was shown that to be able to be used as a passively controlled manipulator, the potential vector $G(q)$ should not contain any zero-elements. The final section presented an analysis of the required energy in the system. This analysis showed that the potential energy at the start should be at least equal to the potential energy at the target position, if both the velocity at the starting and target position are small.
This chapter explains the developed control algorithm. The idea of controlling a passively controlled system based on its natural dynamics was explained in Chapter 4. This chapter explains how the controller is constructed and why it is constructed in this way. This chapter ends with a step-by-step algorithm which is used to synthesize the controller for a given system. This step-by-step algorithm is used to generate the simulation and experimental results in respectively Chapter 6 and 7.

5-1 Single or multiple degree of freedom control

In Chapter 4 the Single Degree of Freedom (SDoF) and Multiple Degree of Freedom (MDoF) natural trajectories were explained. This section discusses how these trajectories can be used in controlling a system and a choice between the two is made.

Controlling a system based on the SDoF natural trajectory has been implemented on the Passive Trajectory Enhancing Robot (P-TER) [26] as SDoF control. However, this was without discussing natural dynamics based control. This type of natural dynamics control simplifies the generation of the natural trajectories. Reaching a target position is simplified to finding intersections of lines and circles, representing the natural trajectories. However, there are several downsides to this type of control:

- This type of control results in abrupt switching behaviour, if there is a transfer between two Degrees of Freedom (DoFs).
- Energy may be spilled when switching from one SDoF trajectory to another. Dissipating this energy can limit the workspace area that can be reached.
- The order of which joint should be free to move is not clear. This requires tuning for each mechanism and target position.
The velocity upon reaching the target position is confined to the motion of the free moving joint. Since the movement of the system is limited to a subset of the total workspace, it may not be possible to realize the desired velocity.

The SDoF controller only allows movement of one Degree of Freedom (DoF) at the time, which constrains the system to joint passive, instead of system passive. This controller lacks the ability to transfer energy between the DoFs.

Controlling a system based on its SDoF natural trajectories introduces the downsides mentioned above. Controlling all joints using one controller fixes all of these difficulties, but requires the solving of a differential equation to obtain the natural trajectories. These trajectories can be computed using a back in time numerical integration, starting at the target position.

5-2 Choices for the controller

This section discusses the choices which have been made in designing the controller. The choices are grouped in four subsections, being: the natural trajectory, the target positions, parametrization of trajectory and finally the controller itself.

5-2-1 Natural trajectory

In the previous chapter, it was explained that the target natural trajectory of a system depends on three components. These components are the mechanism, the velocity upon reaching the target and the added damping. We will discuss these in the aforementioned order:

- The simulations and also the physical experiment have been conducted on the serial two link mechanism with revolute joints, which is shown in Appendix D, together with all the physical parameters. This mechanism was chosen because gravity influences each DoF. Furthermore, a physical setup of this system was easy to realize.

- The workspace velocity upon reaching the target position has been chosen to be $[0.0, -0.5]$ m/s. This was chosen, by keeping the desired application in mind. The package should approach the target position from above vertically, without a horizontal velocity, because this could result in an impact to other packages. The magnitude of the vertical velocity was chosen to be small. The effect of this velocity is assessed in a sensitivity analysis in Section 6-2-2.

- The added damping was chosen in an iterative procedure. The trajectory back in time is computed for the specified mechanism and workspace velocity upon reaching the target position. The backward computation is performed using a numerical integration with different values of the added damping. A small damping results in trajectories which move up from the target position, but then quickly go down and clash into the ground. This is because a small damping results in a small increase in velocity when integrated back in time. The smaller increase in velocity results in a smaller kinetic energy in the system. Since the kinetic energy is the result of the potential energy, the
potential energy will also remain small, and the end effector will not move upwards. The damping is chosen such that all target natural trajectories move upwards, with the smallest possible damping coefficient. A smaller damping coefficient is desirable since this leads to lower velocities, when computed back in time.

5-2-2 Target positions

The goal is to move towards the selected target positions, which are spaced along a horizontal range and located on the floor located 0.7 m below the shoulder joint of the two link mechanism. These target positions have been chosen to span a horizontal range, which is near the edge of the reachable workspace of the mechanism chosen before. The target positions, indicated in meters below the shoulder joint, are located at:

\[
x_{d,1} = \begin{bmatrix} -0.4 \\ -0.7 \end{bmatrix}, \quad x_{d,2} = \begin{bmatrix} -0.2 \\ -0.7 \end{bmatrix}, \quad x_{d,3} = \begin{bmatrix} 0.2 \\ -0.7 \end{bmatrix}, \quad x_{d,4} = \begin{bmatrix} 0.4 \\ -0.7 \end{bmatrix}
\]

(5-1)

5-2-3 Parametrization of the trajectory and velocity

The target natural trajectories \( x^*(t) \) are obtained by numerically integrating the Equations Of Motion (EOM) of the mechanism with the chosen parameters. In order to implement the vertical position dependent natural trajectory \( x^*(x_2) \) and its velocity magnitude \( ||\dot{x}^*(x_2)|| \) without a look up table, a polynomials \( x^* = p_x(x_2) \) and \( ||\dot{x}^*(x_2)|| = p_{\dot{x}}(x_2) \) are fitted on the trajectory using a least squares approach on the horizontal position and velocity error. This parametrization is needed to compute partial derivatives used in the controller. Computing these derivatives on the \( x^*(x_2) \) requires interpolation, which makes this procedure more complex. Table 5-1 shows the cumulative horizontal position error between polynomial and the target natural trajectory. From this table we can see that the cumulative error decreases the most between the first and second order polynomial. The decrement between the other polynomials is less substantial. A lower order polynomial results in a less complex controller, since the controller is based on this natural trajectory. The chosen second order polynomial, is shown together with the actual trajectory and the target positions, marked by stars, in Figure 5-1. The same procedure has been used for the velocity magnitude parametrization. The chosen third order polynomial, together with the actual velocity curves are shown in Figure 5-2.

\begin{table}[h]
\centering
\caption{This table shows the polynomial order versus the cumulative horizontal position error between the polynomial and the trajectory. From this table we can see that cumulative error decreases over 50 percent between the first and second order, but after the second order this decrement is smaller. Therefore the second order polynomial is chosen.}
\begin{tabular}{|c|c|c|c|}
\hline
Polynomial order & 1 & 2 & 3 & 4 \\
\hline
Cumulative error\(^1\) [m] & 8.2 & 2.8 & 1.5 & 1.0 \\
\hline
\end{tabular}
\end{table}

\(^1\)Cumulative horizontal position error between polynomial and target natural trajectory.
Figure 5-1: This figure shows the natural trajectories for the chosen parameter configuration (solid lines) and their 2nd polynomial parametrization (dashed lines). The target positions are indicated by a star.

Figure 5-2: Velocity magnitude along the target natural trajectories as a function of the vertical position. The actual velocity magnitude (solid line) and the third order polynomial (dashed line) which is used to parametrize it. From these figures we can see that the first and last target position require larger velocity magnitudes at the same workspace position than the second and third target position.
5-2-4 Controller

This subsection discusses the choice for the controller. First the requirements are provided, after which several alternatives are briefly discussed and a choice is made.

There are three important requirements for the controller:

- The end effector must be controlled towards the target natural trajectory (see Definition 8) when it is not on this trajectory.

- The end effector must be guided along the target natural trajectory when it is on this trajectory.

- The velocity of the end effector must be directly controlled. This is necessary because in order to follow a natural trajectory, the end effector must have the correct velocity.

Standard methods such as impedance and admittance control which are passified do not satisfy these criteria. Impedance control computes a desired control force based on the state of the system. This is a state dependent controller, but it has no direct control over the velocity. Admittance control specifies a desired velocity behaviour of the system as a function of the input force. Since the system should be controlled based on the state of the system, and not based on the force, this method also does not qualify.

The combination of velocity control and guiding the end effector as a function of the workspace position can be realized using tracking velocity vector fields controller [11]. Since this is not an intrinsically passive controller, it will have to be passified according to the methods presented in Section 3-2-2. An additional benefit of this controller is that it can be proved that this controller stabilizes the system on a reference vector field. However, this has not yet been realized for the passified version, so this remains for future work.

The passified impedance controller, introduced as an example in Section 3-2-3, showed to be an interesting second choice, due to the state dependent control action. Therefore this controller will be implemented as a comparative controller, in order to compare the results.

As described in Section 3-2-4, an intrinsically passive controller has the preference over an active controller which has been made passive. This topic however remains for future work.

5-3 Tracking velocity vector fields

The obtained natural trajectories are used in combination with the tracking velocity vector fields controller. How this is implemented is described in this section.

The tracking velocity vector fields controller specifies a desired velocity as a function of the position of the end effector. The desired velocity is stored in a velocity vector field.

5-3-1 Standard implementation

The standard implementation of tracking reference vector fields is described in this subsection. The construction of the vector field $f_d(q)$ is described in the subsequent subsection.
desired reference dynamics are described by the velocity vector field $f_d(q) \in \mathbb{R}^n$:

$$\dot{q}_d = f_d(q_d).$$  \hspace{1cm} (5-2)

$F(q, \dot{q})$ is defined as the configuration space velocity error:

$$F(q, \dot{q}) := \dot{q} - f_d(q).$$  \hspace{1cm} (5-3)

Which can be implemented into the control law:

$$u = -KF(q, \dot{q}) - \nabla \mu(q) + M(q) \nabla f_d(q) \dot{q} + C(q, \dot{q}) f_d(q),$$  \hspace{1cm} (5-4)

with $K \in \mathbb{R}^n$ a positive definite gain matrix, $\nabla$ the partial derivative with respect to $q$, $M(q)$ and $C(q, \dot{q})$ respectively the inertia and Coriolis matrices, $\mu(q)$ a Lyapunov function (see Appendix C-1) for the velocity field and $V(q)$ a Lyapunov for the total system, which satisfies:

$$\dot{V}_f(q) = \nabla \mu^T \dot{q} = \nabla \mu^T f_d(q) \leq 0.$$  \hspace{1cm} (5-5)

Using this Lyapunov function, it can be shown that the active version of this controller stabilizes the system for a given vector field. This is shown in Appendix C-9.

### 5-3-2 Total velocity vector field generation

This section describes the construction of the velocity vector field $f_d(q)$. The total vector field is generated by combining two fields. The first field guides the end effector towards the target natural trajectory, and the second field guides the end effector along it. These fields, which are named $\bar{f}_{d,1}(x)$ and $\bar{f}_{d,2}(x)$, are combined using a weighting function. These components are explained in this subsection.

#### Partial velocity vector field generation

The target natural trajectory, parametrized using polynomial $p_x(x_2)$, is the basis for the velocity vector fields. Both partial vector fields are based upon this function. The field which guides the end effector towards the target natural trajectory is constructed using the horizontal error function. This function $G(x)$ is defined by:

$$G(x) = x_1 - p_x(x_2).$$  \hspace{1cm} (5-6)

The function $\bar{f}_{d,1}(x)$ generates a vector field towards the trajectory. This field is generated by:

$$\bar{f}_{d,1}(x) := \nabla \left[ (G(x))^2 \right].$$  \hspace{1cm} (5-7)

This function squares the error, which creates a valley around the target natural trajectory. The partial derivative $\nabla$ is the slope, which points towards the valley.

The function $\bar{f}_{d,2}(x)$ generates a vector field along the trajectory. This field can be generated by computing the tangent vector for a general point along the natural trajectory. This vector, and therefore this field is generated by:

$$\bar{f}_{d,2}(x) := \begin{bmatrix} \frac{\partial p_x(x_2)}{\partial x_2} \\ 1 \end{bmatrix}.$$  \hspace{1cm} (5-8)
This results in a vector field along the natural trajectory, which is independent of the vertical position. Since this partial field is only used when the end effector is close to the natural trajectory, this is not a problem. Finally, the fields $\vec{f}_{d,1}$ and $\vec{f}_{d,2}$ are normalized. This is needed for combining both fields with equal weight.

**Combining the velocity vector fields**

The combined workspace vector field is defined by:

$$\vec{f}_d(x) := ||\dot{x}_d|| \left[ (1 - n(x)) \vec{f}_{d,1}(x) + n(x) \vec{f}_{d,2}(x) \right], \quad (5-9)$$

in which $||\dot{x}_d|| \in \mathbb{R}$ is the desired workspace velocity magnitude, which is discussed in more detail in the next subsection. $n(x) \in (0, 1]$ is a continuous weighting function which is at its maximum on the trajectory and decreases when moving away from it. This weighting function is defined by:

$$n(x) := \left( \frac{1}{1 + (G(x))^2} \right)^\alpha, \quad (5-10)$$

with $\alpha \in \mathbb{R}$ a scalar which manipulates the rate of decline of $n(x)$ around the trajectory $G(x)$; a larger value of $\alpha$ results in a more narrow band around $G(x)$ and vice versa.

The configuration space field generated by mapping (5-9) to the configuration space using (A-2) and (A-1). This results in:

$$f_d(q) = J^{-1}(q) \vec{f}_d(x)$$

$$= J^{-1}(q) \vec{f}_d(h(q)), \quad (5-11)$$

with $h(q)$ the mapping from configuration space to workspace, and $J(q)$ the Jacobian as defined in (A-3) and assuming it is invertible. The Lyapunov function used is:

$$\mu(x) = (G(x))^2, \quad (5-13)$$

which can be transformed to the workspace using methods in Appendix B.

**Velocity magnitude**

The velocity magnitude, shown in (5-9), scales the magnitude of the velocity vectors. Without scaling the velocity vector is of unit length. Two fields have been implemented, the first has a constant velocity magnitude, whereas the second has a velocity magnitude equal to the velocity along the natural trajectory. Both implementations are briefly discussed.

**Constant velocity magnitude** The first implementation is using a vector field with constant velocity magnitude, which is shown in Figure 5-3. The direction is based on the procedure described above, but the velocity magnitude does not match the velocity on the target natural trajectory. The velocity magnitude is constant and has the magnitude of the velocity at reaching the target position of $||\dot{x}_d|| = -0.5 \, m/s$. The controller may be able to track the natural trajectory because the velocity is maximized to the velocity upon reaching the target position. Whether tracking the trajectory is possible, depends on the dynamics of the specific mechanism. An advantage of this method is that the manipulator moves slower, causing smaller accelerations and requiring a slower control loop.
Figure 5-3: Velocity vector fields with constant velocity magnitude based on the target natural trajectories for the serial two link manipulator. The target Position is indicated with a star, and the floor is at the vertical position of $x_2 = -0.7$. 

(a) Target position 1.  
(b) Target position 2.  
(c) Target position 3.  
(d) Target position 4.
**Trajectory dependent velocity magnitude**  The second implementation uses the same vector fields as above, but scales the magnitude depending on the vertical position of the end effector. The scaling factor is defined by the velocity on the target natural trajectory. The velocity magnitude has been parametrized, as was discussed in Section 5-2-3. The workspace velocity magnitude is computed using:

\[
||\dot{x}_d|| = p_k(x),
\]

(5-14)

in which \(p_k(x) \in \mathbb{R}\) is the polynomial of the velocity magnitude as a function of the vertical position. The resulting velocity vector fields are shown in Figure 5-4.

### 5-3-3 Total controller

The total controller consists of the control law provided in (5-4), passified for each joint. All components which are used in the control law have been discussed. The control law is made passive according to the procedure described in (3-3).
Vertical position $x_2$ [m]
Horizontal position $x_1$ [m]

(a) Target position 1.
(b) Target position 2.
(c) Target position 3.
(d) Target position 4.

**Figure 5-4:** Velocity vector fields with velocity magnitude based on the target natural trajectories for the serial two link manipulator. The target position is indicated with a star, and the floor is at the vertical position of $x_2 = -0.7$. 
5-3-4 Algorithm

All steps described in the previous subsections are used to generate the field dependent components of (5-4). The full procedure is summarized in Algorithm 1.

Algorithm 1: Generation of all field dependent components of control law (5-4).

Data: Mapping from configuration to workspace \( x = h(q) \) and workspace mass matrix \( \bar{M} \).

Result: Generation of all field dependent components of control law in (5-4).

1 Specify:
   - \( s \) target positions.
   - Mechanism, characterized by \( x = h(q) \) and \( \bar{M} \).
   - Desired velocity upon reaching target position.
   - Proximity variable \( \alpha \).

2 while Target natural trajectories not satisfactory do

3 Tune added damping.

4 Generate target natural trajectories:
   - Generate Equations Of Motion (EOM) using \( x = h(q) \) and \( \bar{M} \).
   - Perform back in time numerical integration from target position.
   - Select target natural trajectory.

end

5 for \( x_{d,i} = x_{d,1} \) to \( x_{d,s} \) do

6 Parametrize trajectories and velocities:
   - Parametrize trajectories
   - Parametrize velocity magnitude as function of vertical position.

7 Generate fields:
   - Generate field towards target natural trajectory.
   - Generate field tangent to target natural trajectory.
   - Combine fields using weight function.

8 Generate partial derivatives:
   - Compute partial derivative of the velocity vector field.
   - Compute partial derivative of the Lyapunov function.

end

5-4 Conclusion

This chapter has presented the passive velocity vector field tracking controller, which is based on the natural trajectory of a system. This controller has been chosen because we want to use the natural dynamics in controlling the passively controlled system. Since the natural trajectory depends on the velocity, a velocity tracking controller is chosen.
Chapter 6

Simulations

The controller presented in the previous chapter is tested in this chapter using a simulation study in MATLAB. First, we briefly introduce the simulation environment and the implementation. Second, we present the simulation results of the developed natural dynamics based controller and discuss results on the workspace position tracking, supply rate and workspace velocity. Third, we compare the performance of the novel controller to a comparative passive workspace impedance controller. Fourth, a sensitivity analysis is performed to assess the effect of parameter disturbances on reaching the target position.

6-1 Implementation

The simulation is performed using MATLAB. The mechanism under consideration is the two link serial mechanism with revolute joints, which is shown in Figure 6-1. The workspace position is specified by $x = [x_1, x_2]^T$, representing the horizontal and vertical position. The configuration space position is specified by $q = [q_1, q_2]^T$. The origin of the workspace is located at the top joint. The links are modeled using a mass and an inertia. $x_e$ is the workspace position of the end effector. Gravity, friction effects in the joints, and torque limits are included in the simulation. All relevant data is provided in Appendix D. Using this data, the Equations Of Motion (EOM) of the mechanism are derived based on the procedure described in Appendix B. These EOM are numerically integrated using a Runge-Kutta 4 (RK4) algorithm [45].

6-1-1 Parameters

The arm starts in the top position in which the end effector is located at $x_e(0) = [0.045, -0.38]^T$, with respect to the workspace origin, located at the root joint. The starting position is based on the joint limits of the physical system presented in Chapter 7. The goal is to move towards
the selected target positions which are located at:

\[
x_{d,1} = \begin{bmatrix} -0.4 \\ -0.7 \end{bmatrix}, \quad x_{d,2} = \begin{bmatrix} -0.2 \\ -0.7 \end{bmatrix}, \quad x_{d,3} = \begin{bmatrix} 0.2 \\ -0.7 \end{bmatrix}, \quad x_{d,4} = \begin{bmatrix} 0.4 \\ -0.7 \end{bmatrix}
\] (6-1)

The vertical position of \(-0.7\) m represents the floor. All positions are expressed in meters. All used parameters are shown in Table 6-1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation number</th>
<th>[unit]</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedback gain</td>
<td>(5-4)</td>
<td>–</td>
<td>(K)</td>
<td>100</td>
</tr>
<tr>
<td>Gain of weight function</td>
<td>(5-10)</td>
<td>–</td>
<td>(\alpha)</td>
<td>100</td>
</tr>
<tr>
<td>Horizontal starting position</td>
<td>(m)</td>
<td>(x_{e_1}(0))</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>Vertical starting position</td>
<td>(m)</td>
<td>(x_{e_2}(0))</td>
<td>-0.38</td>
<td></td>
</tr>
<tr>
<td>Horizontal starting velocity</td>
<td>(m/s)</td>
<td>(\dot{x}_{e_1}(0))</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Vertical starting velocity</td>
<td>(m/s)</td>
<td>(\dot{x}_{e_2}(0))</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Package mass</td>
<td>(kg)</td>
<td>(m_e)</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Step size of numerical integration</td>
<td>(s)</td>
<td>(T_s)</td>
<td>0.001</td>
<td></td>
</tr>
</tbody>
</table>

## 6.2 Simulation Results

The resulting workspace positions for the end effector are visualized in Figure 6-2(a) for the constant velocity magnitude vector field and in Figure 6-2(b) for the trajectory dependent velocity magnitude vector field. The start position is indicated using a circle and the four target positions using stars. Several observations can be made:

- The controller using the trajectory dependent velocity magnitude vector field reaches the target position with a close to equal deviation from the target position, as the controller using the constant velocity magnitude field. The mean position error for the first is 0.0323 m, whereas for the second this is 0.0345 m.
6-2 Simulation Results

- The end effector moves towards the target position, but does not exactly reach it for all target positions.

- The distance to target position $x_{d,2}$ when reaching the floor (located at $y = -0.7 \, \text{m}$) is larger than the other target positions. This can be the effect of the parametrization of the natural trajectory. In Figure 5-1 the natural trajectories together with their polynomials was shown. The polynomial of the second natural trajectory curves towards the right of the target position when reaching the target vertical position. This natural trajectory is also different from the other natural trajectories in that it is less steep.

Supply rate

In order to verify the passivity of the device, the supply rate is checked. The supply rate, together with the configuration space velocities and torques are shown in Figure 6-3. The figures for the first target position, for both the constant (left) and trajectory dependent velocity magnitude (right) are shown here, the figures for the other target positions are shown in Appendix F-1-1. Several observations can be made:

- The supply rate for both velocity field magnitudes are clearly negative, which shows that the control action is passive.

- From the configuration space velocities it can be seen that the first joint is kept stationary, while the second joint moves. This is needed in order to reach the target position on the far left.

- The torque signal shows a chattering behaviour. A discussion on this is provided in Section 6-3.

- Whereas the controller with the constant velocity field magnitude slows down the second link, the controller with the trajectory dependent field does not do this. Therefore the velocity of the second link increases using the trajectory dependent field.

- As soon as the second joint has reached a certain angle, the first joint is slightly released. The position at which this happens depends on the velocity field, and therefore on the target natural trajectory.

- The configuration space velocities with the controller using the trajectory dependent velocity vector field magnitude are larger then the controller using the constant velocity vector field magnitude. The larger velocity leads to a shorter movement time and and lower velocity leads to a longer movement time.

- The chatter in the supply rate is caused by chatter in the torque signal, which is in turn caused by friction in the system.
(a) Natural dynamics velocity field controller, with a constant velocity field magnitude.

(b) Natural dynamics velocity field controller, with a trajectory dependent velocity field magnitude.

(c) Passive impedance controller, with $K_p = \text{diag}(150, 150)$ and $K_d = \text{diag}(20, 20)$.

(d) Passive impedance controller, with $K_p = \text{diag}(250, 100)$ and $K_d = \text{diag}(20, 20)$.

**Figure 6-2:** Workspace end effector trajectories for the developed controller and the comparative passive impedance controller in simulation. The target position has been indicated by a star and the starting position by a circle. The orientation of the manipulator (blue stripes) is shown during the first movement at four steps evenly spread over the movement time.
6-2 Simulation Results

(a) Supply Rate with a constant velocity field magnitude.

(b) Supply Rate with a trajectory dependent velocity field magnitude.

(c) Torque with a constant velocity field magnitude. Details on the chattering are provided in Section 6-3.

(d) Torque with a trajectory dependent velocity field magnitude. Details on the chattering are provided in Section 6-3.

(e) Configuration space velocity with a constant velocity field magnitude.

(f) Configuration space velocity with a trajectory dependent velocity field magnitude.

\textbf{Figure 6-3:} Supply rate of energy to the system for the natural dynamics velocity field controller for the first target position in simulation. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row). The images on the left use a constant velocity field magnitude and on the right a trajectory dependent velocity field magnitude.
Workspace velocity

In order to provide more insights in the performance of the controller, the workspace velocities are investigated. The workspace velocity over time for reaching the first target position are shown in Figure 6-4(a) and Figure 6-4(b), the figures for the other target positions are again shown in Appendix F-1-2. Both the horizontal ($x$) and vertical ($y$) components are visualized, together with the absolute magnitude. Again, several observations are made:

- The velocity magnitude upon reaching the target position is (almost) equal to the desired $0.5 \text{ m/s}$. This velocity magnitude is largely caused by the vertical velocity. However, there is also a nonzero velocity in the horizontal direction, which may be undesirable when using the manipulator as a pick-and-place device. Note that this holds for all target positions.

- The resulting workspace velocities with passified natural dynamics controller with the trajectory dependent velocity vector field magnitude are larger than when using the constant velocity vector field magnitude, which is to be expected because of the larger desired velocities. This velocity may however be too large, causing the end effector to overshoot the target position on the far left and right, as is shown in Figure 6-2(b).

- The velocities of the controller using the constant magnitude velocity vector field all show no velocities above $0.55 \text{ m/s}$ (see Figure F-7).

6-2-1 Comparative controller

A passified workspace standard impedance controller has been implemented to compare the performance of the passified natural dynamics controller. This comparative controller was chosen because it is a standard control method, that can provide state dependent control (unlike admittance control). The passified impedance controller has been introduced in Section 3-2-3. The used parameters are shown in Table 6-2. The ability of the impedance controller of reaching the target position relies heavily on tuning its gains, whereas this is of less importance in the natural trajectory velocity field tracking controller. The influence of the gains is illustrated using Figure 6-2(c) and 6-2(d).

Table 6-2: The parameters used in the simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Simulation 1</th>
<th>Simulation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness matrix</td>
<td>$K$</td>
<td>150 0 0 150</td>
<td>250 0 0 100</td>
</tr>
<tr>
<td>Damping matrix</td>
<td>$\tilde{R}$</td>
<td>20 0 0 20</td>
<td>20 0 0 20</td>
</tr>
<tr>
<td>Step size of integration</td>
<td>$T_s$</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Based on these trajectories, and by comparing them with the trajectories of passified natural dynamics controller, the following observations can be made:

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Figure 6-4: The workspace velocities for passified natural dynamics controller, together with the comparative passive workspace impedance controller in simulation.
• It can be seen that reaching the target position is possible by using the passive workspace impedance controller. However, this requires considerate tuning of the proportional and derivative gains of the passive workspace impedance controller (can be interpreted as workspace stiffness and damping).

• There is no direct control over the workspace velocity since these can only be tuned indirectly using the derivative gain. When the workspace velocity upon reaching the target position is higher than the desired velocity, the end effector clashes into the ground, which damages the package.

• The mean horizontal error when reaching the target height is smaller compared to using passified natural dynamics controller, but all horizontal positions show a small error. The mean error is .

• The trajectories are more smooth, compared to the trajectories of passified natural dynamics controller.

In the remaining of this comparative controller analysis, only the passive workspace impedance controller that reaches the target positions is discussed.

Supply rate

The supply rate of the passive workspace impedance controller is shown in Figure 6-5, the supply rates for the second until fourth target positions are shown in Appendix F-1-1. Several observations, based on these figures are:

• The supply rate of energy is negative, illustrative for a passively controlled system.

• Likewise to passified natural dynamics controller, the impedance controller permits movement on the second joint and locks the first joint. The control force applied to keep the first joint at a constant value is smaller, compared to passified natural dynamics controller. The velocity of second joint is not reduced by the controller.

• The configuration space velocity curve, shown in Figure 6-5(c) shows large similarities with the configuration space velocity curve of the passified natural dynamics controller-with trajectory dependent velocity magnitude.

Workspace velocity

The workspace velocities of the passive workspace impedance controller are shown in Figure 6-4(c) for the first target position and in Appendix F-1-2 for the other target positions. Comparing these workspace velocities with the velocities of passified natural dynamics controller shows:

• As a result of the similar configuration space velocity, as mentioned above, the workspace velocities are also similar to workspace velocity of passified natural dynamics controller with a trajectory dependent velocity field magnitude.
Figure 6-5: Supply rate of energy to the system for the passive workspace impedance controller for the first target position. The supply rate is the multiplication of the torque (second row) and configuration space velocities (third row).
• A large difference in the workspace velocities of the passive impedance controller is that the velocity upon reaching the target position are larger. This is not the case for the first and fourth target positions, but is for the second and third, as can be seen in Figure F-8. The derivative gains of the impedance controller could be tuned to counteract this, but this requires tuning, and may negatively influence the ability to reach the first and fourth target positions.

6-2-2 Sensitivity

This subsection investigates the effect of several important internal and external parameters of the passified natural dynamics controller. Internal parameters are gain $K$ in (5-4), factor $\alpha$ in (5-10). External parameters are the starting position and velocity, the package mass and the amount of friction. All investigated parameters and their variations are shown in Table 6-3. All parameter variations are independently simulated, while all other parameters are at their standard value. The package mass is an extra mass at the end effector position, in the previous results this was 0.65 kg. The package mass also influences the inertia of the second link. The friction depends on three parameters per joint, which are all scaled by the same friction multiplication factor, in order to assess its influence.

The effect of the parameter variations is shown in Figure 6-6. Several observations are:

• The effects of the parameter variations for both velocity vector fields does not show large differences; the effect is similar.

• A smaller feedback gain $K$, leads to a larger final horizontal position error, however, this effect is only caused at values which are a factor 20 lower than the standard value. The feedback gain $K$ is a proportional feedback gain on the position and velocity. A lower value of the feedback gain results in smaller penalty on an error in the configuration space velocity error.

• A horizontal starting position that is more towards the left than the standard starting position, results in a final horizontal position which is also more towards the left than the standard final position.

• An initial higher vertical starting position causes a larger deviation in the final horizontal end effector position. It was the expectation of the author that this would not have a significant effect.

• A initial horizontal or vertical velocity does not have a large effect on the final horizontal end effector position for the controller using the vector field with constant magnitude. However, the controller with the trajectory dependent velocity magnitude does show an effect. An increasing horizontal velocity increases the position error for all but the fourth target position.

• A smaller package mass causes a larger final end effector position error. Due to the lower package mass, there is not enough potential energy that can be dissipated in order to steer the manipulator. A larger package mass does not cause a larger error, since this extra potential energy can be dissipated up to the limits of the actuators.
- The effect of larger friction parameters does not have a large effect on the final end effector positions. Since the velocities are kept relatively low, it is expected that friction does not play a large role.

- Not all parameter variations that have been examined, have a final horizontal position indicated in the Figure 6-6. This is because the vertical component of the target position has not been reached within two seconds. In practice, this means that the mechanism has stopped moving at a vertical position above the target.

Table 6-3: The parameters variations under investigation. The last row contains the symbols which indicate the final end effector position in Figure 6-6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>[unit]</th>
<th>Symbol</th>
<th>- -</th>
<th>-</th>
<th>Standard</th>
<th>+</th>
<th>++</th>
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<tr>
<td>Feedback gain</td>
<td>-</td>
<td>$K$</td>
<td>5</td>
<td>50</td>
<td>100</td>
<td>500</td>
<td>1000</td>
</tr>
<tr>
<td>Weight function gain</td>
<td>-</td>
<td>$\alpha$</td>
<td>5</td>
<td>50</td>
<td>100</td>
<td>500</td>
<td>1000</td>
</tr>
<tr>
<td>Horizontal starting position</td>
<td>m</td>
<td>$x_{e1}(0)$</td>
<td>-0.1</td>
<td>0.0</td>
<td>0.045</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Vertical starting position</td>
<td>m</td>
<td>$x_{e2}(0)$</td>
<td>-0.12^1</td>
<td>-0.25</td>
<td>-0.38</td>
<td>-0.45</td>
<td>-0.50</td>
</tr>
<tr>
<td>Horizontal starting velocity</td>
<td>m/s</td>
<td>$\dot{x}_{e1}(0)$</td>
<td>-1.0</td>
<td>-0.5</td>
<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Vertical starting velocity</td>
<td>m/s</td>
<td>$\dot{x}_{e2}(0)$</td>
<td>-1.0</td>
<td>-0.5</td>
<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Package mass</td>
<td>kg</td>
<td>$m_e$</td>
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<td>0.235</td>
<td>0.65</td>
<td>2.0</td>
<td>5.0</td>
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<td>Friction multiplication</td>
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<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Symbol in Figure 6-6</td>
<td>□</td>
<td>□</td>
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<td>□</td>
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<td></td>
</tr>
</tbody>
</table>

^1This value is chosen such that $q_1 = -\pi/2$ and $x_e(0) = 0$. 

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Figure 6-6: The effect of parameter variations shown in Table 6-3 versus the final end effector position. The target position is indicated by the vertical striped line, the symbols are explained in Table 6-3 and the horizontal dotted line connects the symbol to the target position.
6-3 Chattering discussion

In the previously discussed simulation results it was observed that the developed controller generated a chattering torque signal, this section discusses this undesirable chattering in more detail.

The torque profile shows a chattering behaviour when the controller tries to maintain the first link at a constant angle. In doing this, the configuration space velocity moves between a negative small value and a positive small value. Depending on the combination of the sign of this velocity and the sign of the desired torque, the desired torque signal is passified. This results in the chattering torque behaviour.

The chattering is the result of making the actuation signal passive. The control signal is clipped to allow merely passive signals. For very small velocities, the torque results in a change of sign of the velocity. The resulting velocity results in the actuator injecting a small amount of energy. To eliminate the possibility of adding energy, the control signal could be applied using a damper model, as presented in Section 3-2-4. This would result in:

\[ u^D (\dot{q}) = -R \dot{q}, \]

with \( u \in \mathbb{R}^m \) and \( \dot{q} \in \mathbb{R}^n \) respectively the configuration space forces and velocities. \( R \in \mathbb{R}^{m \times n} \) is the configuration space damping matrix which is diagonal and positive definite. This damping matrix can be based on the desired control signal:

\[ R_{\alpha i} = \min \left( \frac{|u_{d,i}|}{\epsilon + |\dot{q}_i|}, R_{\max,i} \right), \]  

with \( u_{d,i} \) the torque computed using the passified natural dynamics controller, \( R_{\max,i} \) the maximum damping coefficient and \( \epsilon \) a small value to generate a finite output for a zero velocity. The subscript \( i \) indicates a component wise operation. This however remains for future research.

6-4 Conclusion

This chapter has presented the simulation results of the developed natural trajectory based controller together with a comparative passive workspace impedance controller. Both controllers are capable of reaching the target positions with a relatively small position error. However, using the passified natural dynamics controller it is easier to realize the desired workspace velocity at the target position. Realizing the desired workspace velocity is easier, since this does not require considerate tuning, as opposed to the passified impedance controller. Using the passified impedance controller requires considerate tuning because the velocity and position gains both influence the followed trajectory, and with that the final target position.
In the previous chapters a controller was introduced which is able to control the passively controlled manipulator in simulation. This chapter will demonstrate the performance of the algorithm on a physical system. First we introduce the experiment design. Second, we discuss the results for the developed natural dynamics based controller. Third, we present the results for the comparative passive workspace impedance controller.

7-1 Experiment design

This section describes the physical setup, used to conduct the experiments. Subsequently the approach of the actual experiments is provided.

7-1-1 System description

It was chosen not to design a robotic arm equipped with brakes or dampers, but instead use an actively controlled system in which the control action is limited to passive actions. This choice was made because it is not the goal of this research to design one new device, but rather to analyze a passively controlled manipulator and design a controller which can control it.

For demonstrating the controller, a physical setup identical to the simulation of the serial two Degree of Freedom (DoF) manipulator, described in the previous chapter, was used. The setup, originally designed by M. Plooij for demonstrating a novel spring mechanism [46] is shown in Figure 7-1. For this experiment the spring mechanism is removed. The setup consists of two links created by an 18x1.5mm stainless steel tube, connected in series by two revolute joints. The two motors are both Maxon 60W RE30 with a gearbox ratio of 66:1. The motors are connected to the joints using AT3-gen III 16mm timing belts which provide an additional transfer ratio of 3:1. The controller is implemented in Simulink, which is shown in Figure E-1. The Simulink model is compiled to C-code which runs realtime on a pc using xpc-target.
7-1-2 Approach

The experiment consists of three parts, which will be explained subsequently. The first part demonstrates how a motor can be used as a passive brake, the second part covers the novel controller and the third and final part discusses the comparative controller.

Passive motor

The motor simulates a passive actuator by constraining the reference torque signal to a passive signal. Since there is no controlled coupling of the movement of the Degrees of Freedom (DoFs), both joints have to satisfy (3-1). There is a small difference between the reference torque and the actually applied torque on the joint. Since the passivity constraint is on the reference signal and not the actual torque, this may cause a small positive supply rate. For demonstrating the use of an active motor in a passive setting, a Proportional-Derivative (PD)-controller has been implemented which tries to control the arm towards a configuration space position of $[0, 0]^T$, using a proportional gain of 8 and a derivative gain of 0. The zero-value derivative gain will illustrate the implementation of using the motor as a passive actuator. The passive motor experiment is a side experiment, and not part of the main goal of this thesis. Therefore, the results are presented and discussed in Appendix F-2-1.

Novel controller

The controller is constructed according to the algorithm presented in Section 5-3-4. The Simulink model used is shown in Figure E-1. The controller has been split into four segments; the first three segments contain the gradient of the Lyapunov function, the gradient
of the configuration space velocity field and the workspace velocity field. The fourth section combines these segments using the control law of (5-4).

The experiment starts with the arm in the starting position, at which the end effector is at \(x(0) = [0.05, -0.4]^T\). Until the experiment starts, the arm is kept at this position using a configuration space PD-controller. After this, the passive controller steers the end effector to a target position chosen beforehand. When the end effector moves below the vertical component of the target position, the controller stops. The arm is manually moved back towards the starting position. This is performed manually because the motors cannot generate enough torque to realize this using non-swinging motion. This procedure is repeated four times. All parameters are identical to those used in the simulation described in the previous chapter.

**Comparative controller**

As a comparative controller, a passive impedance controller is implemented, similar to the simulation of Section 6-2-1. This is implemented using the same Simulink model as discussed before, without using the separate controller specific functions.

### 7-2 Experiment Results

#### 7-2-1 Novel controller

The novel passified natural dynamics controller has been implemented using the natural trajectories shown in Figure 5-1 for both the fields with the constant velocity magnitude (Figure 5-3) and the natural dynamics velocity magnitude (Figure 5-4). The workspace position for the end effector is shown in Figure 7-2(a) for the constant velocity magnitude field and in Figure 7-2(b) for the velocity field with natural dynamics based velocity. Four repetitions per target destination have been performed. The starting and target positions are again indicated by respectively circles and stars. Several observations on these trajectories:

- The end effector of the passified natural dynamics controller using both velocity fields reaches the goal. The final end effector position has the largest error for the second and the smallest for the third target position.

- The four repetitions result in a consistent trajectory. Most trajectories overlap, and only small differences are visible. The trajectories obtained using the controller with the trajectory dependent velocity magnitude show small deviations between the trajectories, this is probably due to the larger velocities with respect to the constant velocity magnitude field.

- There is a small difference in the starting position, which causes the multiple circles representing the starting positions.

- Comparing the trajectories of the experiment with the simulation, shown in Figure 6-2 shows no large differences. Small differences are a smaller error in the experiment and a smaller overshoot in the trajectory dependent velocity field magnitude.
(a) Natural dynamics velocity field controller, with a constant velocity field magnitude.

(b) Natural dynamics velocity field controller, with a trajectory dependent velocity field magnitude.

(c) Passive impedance controller, with $K_p = \text{diag}(150, 150)$ and $K_d = \text{diag}(20, 20)$.

(d) Passive impedance controller, with $K_p = \text{diag}(250, 100)$ and $K_d = \text{diag}(20, 20)$.

**Figure 7-2:** Workspace end effector trajectories for the novel and comparative controller in experiment with four repetitions per target destination. The target position has been indicated by a star and the starting position by a circle. The orientation of the manipulator (blue stripes) is shown during the first movement at four steps evenly spread over the movement time.
Supply rate

The supply rate, together with the torque and configuration space velocities are shown in Figure 7-3 for the movement towards the first target position. The figures for the constant velocity field magnitude are again shown on the left and for the trajectory dependent velocity field magnitude on the right. The figures for the other target positions are shown in Appendix F-2-2. Several observations are:

- The supply rate is, similar to the simulations, clearly passive.
- The torque on the first joint shows a chattering behaviour with a large amplitude. This is caused by the near-zero velocity of this joint. The controller tries to keep this joint at a constant angle, while applying a passive control action. Since the velocity chatters between a small positive and negative value, this causes the control action to chatter. Different to the simulation results, this does cause a visible supply rate. A discussion on using an active motor as a passive device is presented in Section 7-3.
- The controller using the trajectory dependent velocity field magnitude does not slow down the second joint, whereas this was the case in the simulation. In the discussion of the workspace velocity, it is shown that the workspace velocity at the target position is below the desired value of $-0.5 \, m/s$, which means that the workspace velocity has decreased due to other effects such as friction.
- The resulting curves of the experiment resemble those of the simulation, when one does not consider the chattering.

Workspace velocity

The workspace velocities for passified natural dynamics controller using the constant velocity field magnitude are shown in Figure 7-4(a) and for the trajectory dependent velocity field magnitude in Figure 7-4(b). These figures correspond to the first target position, the figures for the other target positions are shown in Appendix F-2-2. Several observations are:

- As a result of the chattering configuration space velocities, the workspace velocities also chatter.
- The workspace velocities of the controller with both velocity fields is below the desired value of $-0.5 \, m/s$. The simulation (see Figure 7-4) and experiment again show similar behaviour, but with added noise/chattering.
- The controller with the constant velocity field moves in the positive direction in the end of the movement. This is caused by overshothing the target on the left.
Figure 7-3: Supply rate of energy to the system for the natural dynamics velocity field controller for the first target position in experiment. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row). The images on the left use a constant velocity field magnitude and on the right a trajectory dependent velocity field magnitude.
Figure 7-4: The workspace velocities for passified natural dynamics controller, together with the comparative passive workspace impedance controller in simulation.
7-2-2 Comparative controller

Similar to the simulation, a passified impedance controller was implemented for comparing the target reaching performance. The workspace position for the end effector is shown in Figure 7-2(c) for $K_p = \text{diag}(150, 150)$ and $K_d = \text{diag}(20, 20)$ and in Figure 7-2(d) for $K_p = \text{diag}(250, 100)$ and $K_d = \text{diag}(20, 20)$. These are the same gains as have been used in the simulation, several observations on the results:

- The results are similar to those of the simulations. However, all trajectories are slightly more towards the right. This holds for both workspace stiffness values and may be the result of discrepancies between the modeled and actual inertia distribution. A larger value causes the system to move towards the right, when the elbow joint is on the left. Note that the controller using the workspace stiffness of 150 N/m reaches the third target position.

- An interesting result is the curl in the trajectory towards the fourth target using stiffnesses of 250 and 100 N/m. This is the result of the end effector moving too far towards the right, which results in a fully stretched arm and an elbow angle of $\pi$ rad. Since the arm is fully stretched, a small configuration space velocity results in a large workspace velocity. Furthermore, in the fully stretched situation, the Jacobian (see (B-18)) loses rank and it becomes singular.

Supply rate

The supply rate, together with the configuration space forces and velocities are shown in Figure 7-5. These figures correspond to the first target position, the figures for the other target positions are again shown in Appendix F-2-2. Several observations are:

- The controller first maintains a zero angle for the first joint, while letting the second joint move freely. Subsequently joint one is released in order to reach the target position.

- The torque on the first joint again shows a chattering behaviour with an amplitude of maximally 10 N/m, which is caused by the near-zero velocity of this joint. This is the same as with the passified natural dynamics controller. A discussion on using an active motor as a passive device is presented in Section 7-3.

- There is a large similarity in configuration space velocities resulting from the passified impedance controller and the passified natural dynamics controller with a trajectory dependent velocity magnitude.

Workspace velocity

The workspace velocities for the passified impedance controller are shown in Figure 7-4(c). This figures correspond to the first target position, the figures for the other target positions are shown in Appendix F-2-2. Several observations are:
7-2 Experiment Results

(a) Supply Rate for target 1.

(b) Torque for target 1.

(c) Configuration space velocity for target 1.

Figure 7-5: Supply rate of energy to the system for the passive workspace impedance controller for the first target position in experiment. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row).
• The amplitude of the chattering is smaller when using this controller than when using the passified natural dynamics controller.

• The final workspace velocity is just above the desired value of $-0.5 \, m/s$. As can be seen in Figure F-17, for the second and third target position, the workspace velocity upon reaching the target position is more than twice the desired magnitude.

• The workspace velocity curves resulting from the passified impedance controller resemble the curves of the passified natural dynamics controller using a trajectory dependent velocity field.

7-3 Chattering discussion

In the previously discussed experiments it was observed that the developed controllers resulted in a chattering torque signal, this section discusses this undesirable chattering in more detail. The chattering is a combination of several factors. These factors are divided in two groups, the first group contains factors resulting from the controller and second group resulting from the measurement.

7-3-1 Chattering due to the controller

The inputs to the controller are the position and velocity of the end effector. The position is measured using a position encoder on the motor, which is numerically differentiated to obtain the velocity.

• A velocity which is close to zero causes chattering by switching between clipping and not clipping the active control signal. This was also observed in the simulation results (see for example Figure 6-3). Since the velocity chatters between a small positive and negative value, this causes the control action to chatter. Different to the simulation results, this does cause a visible supply rate. This is because the simulation is constructed using a model of the system, which does not contain effects which are present in the experimental setup such as a dynamics friction model or actuator dynamics.

• The maximum torque which the motor can supply is constrained by the maximum current to $16.19 \, Nm$. In Figure 7-3 it can be seen that this torque is alternately applied in both positive and negative direction. This maximum torque results from the velocity field feedback component of the controller. Lowering this gain will lower the amplitude of the torque signal, which in turn lowers the chattering of the velocity. However, lowering this gain will not guide the end effector towards its target position anymore.

• The passified natural dynamics controller with the constant velocity magnitude shows more chattering than passified natural dynamics controller with trajectory dependent velocity (see Figure 7-4). This is because the velocity error of the passified natural dynamics controller with constant velocity magnitude varies between slightly positive and slightly negative. Since the controller applies a control signal based on this error, this causes chattering. The passified natural dynamics controller with a trajectory
dependent velocity magnitude has a larger error which does not chatter around zero, and therefore this causes less chattering. Whereas the passified natural dynamics controller controls the end effector towards a velocity setpoint, the passive impedance controller controls the end effector towards the target position based on the error in position and velocity. Since the error in position and velocity does not vary between positive and negative, this results in the smallest amount of chattering.

7-3-2 Chattering due to the measurement

The second group of causes of the chattering are the input signals to the controller.

- In physical experiments, noise on the measurement signal is often present. The configuration space velocity is obtained by numerically differentiating the joint angle, which introduces extra errors. An error in the velocity measurement increases the effect of the earlier mentioned controller related causes.

- For practical reasons, the encoders are mounted on the motors. A small gearbox backlash can therefore result in a measured velocity, while the outgoing gearbox axle does not rotate. This results in a discrepancy between the measurements and the actual system behaviour. This discrepancy causes a control action which is not the desired control action for the current state of the system.

7-4 Conclusion

This chapter has presented the results of using the developed natural trajectory based controller together with a comparative passive workspace impedance controller in a physical experiment. Similar to the simulation, both controllers are capable of reaching the target positions with a relatively small position error, but using the passive workspace impedance controller it is harder to realize the desired workspace velocity at the target position. The control signal of the passified impedance controller resulted in a smoother control behaviour. However, this is the result of the structure of this controller. Whereas the passified natural dynamics controller controls towards a reference velocity, the passified impedance controller controls the end effector directly towards the target position.
Conclusions and recommendations

This chapter summarizes the conclusions made in this thesis. Furthermore open issues are explained and based on these issues recommendations for further work are provided.

8-1  Summary and conclusions

The goal of this research is to design a controller for a passively controlled manipulator which can control the end effector of a manipulator towards a target position.

In realizing this goal three subgoals have been specified:

1. Identify the challenges in controlling a passively versus an actively controlled manipulator.

2. Develop a controller to control this passively controlled manipulator.

3. Assess the performance of the designed controller by measuring the ability to realize the desired final position and velocity in both simulation and experiments.

A brief conclusion to these goals will be provided to each subgoal independently:

1. There are two main challenges in passively controlling a system. First, the supply rate of energy to the system is restricted to negative values. This restriction is called the passivity constraint. The negative supply rate causes the passively controlled system to be locally uncontrollable and therefore standard control solutions cannot be applied. Second, in order to move in all directions in the workspace, the passively controlled manipulator requires influence of gravity on each Degree of Freedom (DoF), which can be checked by verifying that there are no zero elements in the gravity vector $G(q)$ of the Equations Of Motion (EOM).
2. Two controllers have been developed. The first controller, named the passified natural dynamics controller, controls the device by constructing a desired velocity vector field which aims at the natural trajectory of the device. This trajectory is the motion of the manipulator along which no active control is needed, therefore reducing the effect of the passivity constraint. The second controller is a passified workspace impedance controller. It is a standard workspace impedance controller of which the control action is also clipped using the passivity constraint. Unfortunately, the author of this thesis was not able to develop a controller which is based on the intrinsic limitation to perform only passive control actions. The developed controllers are both active controllers in which the control signal has been clipped to satisfy the passivity constraint.

3. Both controllers have been successfully implemented on both a simulation and a physical setup, and are capable of controlling the passively controlled manipulator towards multiple target positions. The endpoint position error of both controllers is similar. There are however differences in the endpoint workspace velocity. The velocity of the passive impedance controller needs considerable tuning and the workspace velocity at the target position is hard to control and therefore not realized. The passified natural dynamics controller requires only little parameter tuning, and is robust against uncertainties such as package mass. We can conclude that the novel passified natural dynamics controller results in the smallest position and velocity error, and is therefore best suited for this control challenge.

8-2 Open issues and recommendations

This research resulted in several new research directions, which will be subdivided into three sections: the controller, the physical mechanism, and the total system.

Related to the controller:

- The developed controller is not an intrinsically passive controller, but an active controller of which only the passive part is applied to the system. Since the active control algorithm does not include the limitations of a passive system, it limits the performance of the device. Intrinsically passive controllers, based on physical devices such as brakes or dampers offer several advantages. This is because these controllers are not based on the assumption that energy can be dissipated and injected into the system, and therefore a better controller performance is expected. However, no successful implementations have been realized. It should be investigated further how the limitations of a passively controlled system could be incorporated into the controller.

- This research has shown the possibilities of passive control, however, no proof of convergence has been obtained. Further research should be focused on strengthening the theoretical basis.

- Investigate the possibility of using an intrinsically passive damping control in combination with the passified natural dynamics controller in order to decrease the chattering. In Section 6-3 the chattering of the torque signal during the simulation was discussed, which is caused by injecting energy when the controller tries to maintain a constant
joint angle. A possible solution to this problem is to use a damper model, as presented in Section 3-2-4.
Future research related to the *physical mechanism*:

- Investigate the possibility of a physical implementation of a passively controlled manipulator, which is not passive for each individual joint, but passive in total. The implementation of the passively controlled manipulator in this report is passive for each individual joint. A system that is passive in total can redirect kinetic energy from one DoF to another, which increases the reachable workspace. Transferring energy between DoFs requires special clutch mechanisms which provide the ability to couple the Degrees of Freedom (DoFs). In the Passive Trajectory Enhancing Robot (P-TER) [19] this has been realized using two clutches that can couple the velocity in a one-to-one or an inverting ratio. The largest benefit of the system passivity is achieved by using a variable transmission between the joints.

- The energy which is dissipated in steering the end effector can be stored and later inserted back into the system. The energy can be converted to electrical energy using a motor/generator. If this energy is stored and fed back to the motor/generator when needed, then this energy can be used in reaching a target position. This harvested energy could also actively speed up the manipulator, which in turn could decrease the time needed to move between the pick and place positions.

- This research focuses on controlling the manipulator based on the natural trajectory, without investigating how the mechanism can aid in this control. In this report it was described that the natural trajectory depends among other things on the mechanism of the system. It should be investigated how the natural trajectory can be modified, by modifying the mechanism.

Finally, there is one recommendation related to the *total system*:

- This research has shown that the developed passive controllers work in simulation and practice. The total pick-and-place cycle, in which the manipulator moves back using a spring, has been shown to work in simulation. The passively controlled motion towards the target position has been shown to work in practice. However, the total pick-and-place cycle has not been assessed in physical setup. The next step is to demonstrate the workability of the total system in practice.

### 8-3 Final words

It has been shown that the manipulator, comprising a simple two degree of freedom serial manipulator with revolute joints, can realize a pick-and-place motion without adding energy using motors. A controller has been implemented for steering towards the target position and moving back up gradually. This is realized by applying a constant damping factor, such that the arm moves towards the position of minimal potential energy. At this point the package is again supplied and the cycle can repeat itself. The passively controlled manipulator works!
This appendix provides definitions of the configuration and workspace and lists the transformations between them for the position, velocity and acceleration. This appendix is provided as background information, the equations are used throughout this thesis.

A-1 Definitions

The configuration of a mechanical system, such as a robotic manipulator, is defined in the configuration space.

Definition 9 (Configuration space [11]). The configuration space is the set of all robot configurations.

The parameters which define the configuration in the configuration space (in literature sometimes referred to as joint space) are the generalized coordinates, being the position or angle $q$ and its derivatives.

Definition 10 (Workspace [11]). The workspace is the volume of space which the robot can reach using its end effector.

The end effector position or angle in the workspace is indicated by $x$. 
A-2 Configuration to workspace

The mapping from configuration to workspace is defined by:

\[ x = h(q), \quad (A-1) \]

with \( q \in \mathbb{R}^n \) the configuration space position, \( x \in \mathbb{R}^l \) the workspace position and \( h : \mathbb{R}^n \to \mathbb{R}^l \) the mapping from configuration to workspace with \( n \neq l \) or \( n = l \). The time derivative of (A-1) provides the workspace velocity:

\[ \dot{x} = J(q) \dot{q}, \quad (A-2) \]

with \( \dot{q} \) the configuration space velocity and \( J(q) \in \mathbb{R}^{l \times n} \) the Jacobian (of \( h \)), defined by:

\[ J(q) := \frac{\partial h(q)}{\partial q}. \quad (A-3) \]

The workspace accelerations are obtained by taking the time derivative of (A-2):

\[ \ddot{x} = J\dot{(}q) \ddot{q} + J(q) \dot{\ddot{q}}, \quad (A-4) \]

with \( \ddot{q} \) the configuration space accelerations and \( H(q) \in \mathbb{R}^{l \times n \times n} \) the Hessian (of \( h \)), defined by:

\[ H(q) := \frac{\partial J(q)}{\partial q}. \quad (A-6) \]

A-3 Workspace to configuration space

The mapping from workspace to configuration space is the inverse of (A-1):

\[ q = h^{-1}(x) , \quad (A-7) \]

with \( h^{-1} : \mathbb{R}^l \to \mathbb{R}^n \). The configuration space velocities are obtained by rewriting (A-2):

\[ \dot{q} = J^\dagger(q) \dot{x}, \quad (A-8) \]

with \( J^\dagger(q) \in \mathbb{R}^{n \times l} \) the (left) pseudoinverse of \( J(q) \), needed because \( n \neq l \), defined by:

\[ J^\dagger(q) := \begin{cases} J^T(q) J(q)^{-1} J^T(q), & \text{if full rank} \end{cases} \]

which satisfies \( J^\dagger(q) J(q) = I \) and assuming \( \begin{bmatrix} J^T(q) J(q) \end{bmatrix} \) is full rank. The configuration space accelerations can be determined in two ways:

- Solving (A-4) for \( \ddot{q} \) and rewriting using (A-8) to replace \( \dot{q} \):
  \[ \ddot{q} = J^\dagger(q) \ddot{x} - J^\dagger(q) \dot{J}(q) \dot{q}, \quad (A-10) \]
  \[ = J^\dagger(q) \ddot{x} - J^\dagger(q) \dot{J}(q) J^\dagger(q) \dot{x}, \quad (A-11) \]
  \[ = J^\dagger(q) \ddot{x} - J^\dagger(q) \begin{bmatrix} H(q) \dot{q} \end{bmatrix} J^\dagger(q) \dot{x}, \quad (A-12) \]
  \[ = J^\dagger(q) \ddot{x} - J^\dagger(q) \begin{bmatrix} H(q) J^\dagger(q) \ddot{x} \end{bmatrix} J^\dagger(q) \dot{x}. \quad (A-13) \]

- Taking the time derivative of (A-8):
  \[ \ddot{q} = J^\dagger(q) \ddot{x} + J^\dagger(q) \dot{x}. \quad (A-14) \]
Appendix B

Obtaining equations of motion for multibody systems

This appendix provides the procedure to obtain the Equations Of Motion (EOM) for a general multibody system using the Euler-Lagrange (EL) equation. The equations of motion can be used to analyse the system and to can be numerically integrated to obtain the motion of the system over time.

B-1 Kinetic and potential energy

The start is the Lagrangian $\mathcal{L} [11, 45]$, which is a function of the generalized position and velocity, respectively $q, \dot{q} \in \mathbb{R}^n$:

$$\mathcal{L} (q, \dot{q}) := \mathcal{T} (q, \dot{q}) - \mathcal{V} (q),$$  \hspace{1cm} (B-1)

with $\mathcal{T}$ and $\mathcal{V}$ the kinetic and potential energy. For a general mechanical system, the kinetic energy can be computed by:

$$\mathcal{T} (\dot{x}) = \frac{1}{2} \dot{x}^T \bar{M} \dot{x},$$  \hspace{1cm} (B-2)

with $\dot{x} \in \mathbb{R}^l$ the workspace velocity and $\bar{M} \in \mathbb{R}^{l \times l}$ the workspace inertia matrix. Using (A-2), this can be transformed to the configuration space:

$$\mathcal{T} (q, \dot{q}) = \frac{1}{2} \dot{q}^T J^T (q) \bar{M} J (q) \dot{q},$$  \hspace{1cm} (B-3)

$$\mathcal{T} (q, \dot{q}) = \frac{1}{2} \dot{q}^T M (q) \dot{q},$$  \hspace{1cm} (B-4)

with $J (q) \in \mathbb{R}^{l \times n}$ the Jacobian, defined in (A-3), and $M (q) \in \mathbb{R}^{n \times n}$ the generalized inertia matrix defined by:

$$M (q) = J^T (q) \bar{M} J (q).$$  \hspace{1cm} (B-5)
For a system without springs, the potential energy $\mathcal{V}$ depends only on gravity and can be computed by:

$$\mathcal{V}(x) = g \Gamma^T \bar{M} x,$$

(B-6)

with $g \in \mathbb{R}$ gravity and $\Gamma \in \mathbb{R}^l$ the vector used to eliminate the components of $x$ on which gravity does not act. Using (A-1) this can be rewritten to:

$$\mathcal{V}(q) = g \Gamma^T \bar{M} h(q),$$

(B-7)

with $x = h(q)$ the mapping from configuration to workspace.

### B-2 Euler-Lagrange equations

The EL equations state that:

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}(q, \dot{q})}{\partial \dot{q}} \right) - \frac{\partial \mathcal{L}(q, \dot{q})}{\partial q} = Q,$$

(B-8)

with $Q \in \mathbb{R}^n$ the generalized forces, equal to the input $u \in \mathbb{R}^m$ in a system without internal dissipation and external applied forces. For simplicity it is assumed that there is no underactuation, e.g. $m = n$. Inserting (B-1) into (B-8) results in:

$$\frac{d}{dt} \left( \frac{\partial T(q, \dot{q})}{\partial \dot{q}} \right) - \frac{\partial T(q, \dot{q})}{\partial q} + \frac{\partial V(q)}{\partial q} = u.$$

(B-9)

The time derivative in the first terms can be written as:

$$\frac{d}{dt} \left( \frac{\partial T(q, \dot{q})}{\partial \dot{q}} \right) = \frac{\partial}{\partial \dot{q}} \left( \frac{\partial T(q, \dot{q})}{\partial \dot{q}} \right) \dot{q} + \frac{\partial}{\partial \dot{q}} \left( \frac{\partial T(q, \dot{q})}{\partial \dot{q}} \right) \ddot{q},$$

(B-10)

since there is no explicit time dependency in the kinetic energy, the first right hand side (RHS)-term drops out. Substituting (B-10) in (B-9) yields:

$$\frac{\partial}{\partial \dot{q}} \left( \frac{\partial T(q, \dot{q})}{\partial \dot{q}} \right) \ddot{q} + \frac{\partial}{\partial \dot{q}} \left( \frac{\partial T(q, \dot{q})}{\partial \dot{q}} \right) \dot{q} - \frac{\partial T(q, \dot{q})}{\partial q} + \frac{\partial V(q)}{\partial q} = u.$$

(B-11)

Resulting in the following standard EOM notation for rigid body systems:

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = u,$$

(B-12)

with $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ the matrix representing the centrifugal and Coriolis terms and $G(q) \in \mathbb{R}^n$ the vector containing potential forces such as gravity. An explicit solution for $C(q, \dot{q})$ as a function of the kinetic energy exists, but this is not the point of interest. The accelerations can be obtained from (B-12) by pre-multiplying with the inverse of $M(q)$:

$$\ddot{q} = M^{-1}(q) \left[ -C(q, \dot{q}) - G(q) + u \right],$$

(B-13)

assuming $M(q)$ is full rank. Note that this matrix, being the generalized inertia matrix, can be obtained by computing the shown partial derivatives or by using (B-5).
\section*{B-3 Mechanism specific matrices}

In the procedure described above, several application specific matrices have to be specified. These matrices are provided for the applications covered in this thesis.

\subsection*{Two link mechanism}

For the two link mechanism used in Section 6-1 and shown in Figure D-1, the state vector is chosen to be:

\[ x = [x_{1c1}, x_{2c1}, q_1, x_{1c2}, x_{2c2}, q_2]^T, \]  
\hspace{1cm} \text{(B-14)}

with the subscript \(c\) indicating the position of the Center of Mass (CoM). The state vector lists the positions of the CoM and the angle for both links. The mapping from configuration to workspace \(h(q)\) is:

\[ x = h(q) = \begin{bmatrix} \frac{l_1}{2} \sin(q_1) \\ h_0 - \frac{l_1}{2} \cos(q_1) \\ l_1 \sin(q_1) + \frac{l_2}{2} \sin(q_2) \\ h_0 - l_1 \cos(q_1) - \frac{l_2}{2} \cos(q_2) \\ q_1 \\ q_2 \end{bmatrix}, \]  
\hspace{1cm} \text{(B-15)}

with \(l_{ci}\) the distance along link \(i\) of its CoM, measured from the previous joint, \(l_i\) is the length of link \(i\) and \(h_0\) is the vertical position at which the first link is connected to the world, see Figure D-1.

The position of the end effector \(x_e\) is specified by:

\[ x_e = \begin{bmatrix} x_{e1} \\ y_{e2} \end{bmatrix}. \]  
\hspace{1cm} \text{(B-16)}

The mapping from work to configuration space for \(x_e\) is given by \(h_e(q)\):

\[ x_e = h_e(q) = \begin{bmatrix} l_1 \sin(q_1) + l_2 \sin(q_2) \\ h_0 - l_1 \cos(q_1) - l_2 \cos(q_2) \end{bmatrix}. \]  
\hspace{1cm} \text{(B-17)}

Its Jacobian \(J_e(q)\) (see Appendix A) is:

\[ J_e(q) = \begin{bmatrix} l_1 \cos(q_1) & l_2 \cos(q_2) \\ l_1 \sin(q_1) & l_2 \sin(q_2) \end{bmatrix}. \]  
\hspace{1cm} \text{(B-18)}

The workspace inertia matrix is:

\[ \bar{M} = \text{diag}([m_1, m_1, I_1, m_2, m_2, I_2]), \]  
\hspace{1cm} \text{(B-19)}

with \(m_i\) and \(I_i\) the mass and inertia of link \(i\). The vector \(\Gamma\) corresponding to the state vector is:

\[ \Gamma = [0, 1, 0, 0, 1, 0]. \]  
\hspace{1cm} \text{(B-20)}
Appendix C

Mathematical background

This appendix contains a mathematical background on the information provided in this thesis.

C-1 Stability

Consider a general time-invariant nonlinear state space system $\Sigma$:

$$\Sigma : \begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases}$$ (C-1)

with the state $x \in \mathcal{X} \subseteq \mathbb{R}^l$, output $y \in \mathcal{Y} \subseteq \mathbb{R}^k$ and input $u \in \mathcal{U} \subseteq \mathbb{R}^m$. When $u = 0$ the system is uncontrolled and reduces to $\dot{x} = f(x)$ and $y = h(x)$.

For a general system represented by (C-1), the stability of a point can be defined [11, 17]:

Definition 11 (Stability). The equilibrium point $x_{eq} = 0$ of $\dot{x} = f(x, u)$ is:

- stable if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon) > 0$ such that:
  $$\|x(0)\| < \delta \Rightarrow \|x(t)\| < \varepsilon, \forall t \geq 0$$ (C-2)

- asymptotically stable if it is stable and a $\delta$ can be chosen such that
  $$\|x(t)\| < \delta \Rightarrow \lim_{t \to \infty} x(t) = 0$$ (C-3)

- unstable if not stable

This definition states that an equilibrium is stable when the state of the system remains within a certain distance of this equilibrium point. If the state of the system converges to its equilibrium, then it is asymptotically stable and if none of the above hold then it is unstable.

In order to explain methods which can be used to assess the stability of a system, we need the notion of an invariant set.


C-1-1 Invariant set

**Definition 12** (Invariant set [47]). A set \( X \subset \mathbb{R}^n \) is said to be invariant for the system as presented in Eq. (C-1) if for all \( x(0) \in X \), \( x(t) \in X \) for all \( t \). A system is said to be positive invariant if it is invariant for all \( t > 0 \).

C-1-2 Lyapunov’s method for stability

The stability for a system can be assessed using the Lyapunov stability criterion [11, 17].

**Theorem 1** (Lyapunov stability). Let \( x_{eq} = 0 \) be an equilibrium point for \( \dot{x} = f(x, u) \) and \( X \subset \mathbb{R}^n \) be a domain containing \( x_{eq} \). Let \( V: X \to \mathbb{R} \) be a continuously differentiable function, such that

\[
V(0) = 0 \quad \text{and} \quad V(q) > 0 \quad \text{in} \quad X - \{0\} \quad (C-4)
\]

\[
\dot{V}(q) \leq 0 \quad \text{in} \quad X \quad (C-5)
\]

Then, \( x_{eq} \) is stable. Moreover if

\[
\dot{V}(x) < 0 \quad \text{in} \quad X - \{0\} \quad (C-6)
\]

then \( x_{eq} \) is asymptotically stable.

An advantage of this stability analysis is that no differential equations of the system need to be solved in order to conclude on system stability. ¹

C-1-3 LaSalle’s theorem

LaSalle’s principle, sometimes also called Krasovskii-LaSalle’s principle, provides a criterion for asymptotic stability if the time derivative of the Lyapunov function is negative semidefinite [18, 11].

**Theorem 2** (LaSalle’s theorem). Let \( \Omega \subset X \) be a set that is positively invariant (see Definition 12) with respect to \( \dot{x} = f(x, u) \). Let \( V: X \to \mathbb{R} \) be a continuously differentiable function such that \( \dot{V}(x) \leq 0 \) in \( \Omega \). Let \( \mathcal{E} \) be the set of all points in \( \Omega \) where \( \dot{V} = 0 \). Let \( \mathcal{M} \) be the largest invariant set in \( \mathcal{E} \). Then every solution starting in \( \Omega \) approaches \( \mathcal{M} \) as \( t \to \infty \).

C-2 Supply rate for mechanical systems

The start is the Lagrangian \( L(q, \dot{q}) \in \mathbb{R} \), as a function of the generalized position and velocity, respectively \( q, \dot{q} \in \mathbb{R}^n \). Its time derivative is given by:

\[
\frac{d}{dt}(L(q, \dot{q})) = \left( \frac{\partial L(q, \dot{q})}{\partial q} \right)^T \dot{q} + \left( \frac{\partial L(q, \dot{q})}{\partial \dot{q}} \right)^T \ddot{q}. \quad (C-7)
\]

¹Please note that this criterion does not state anything about the transient response or the performance of the system [48].
Let $\Omega \subset \mathbb{Q}$ be a compact set that is positively invariant with respect to $\dot{q} = h(q)$. The largest invariant set in $\Omega \rightarrow \mathbb{R}$. Let $M$ be the set of all points in $\Omega$. As $t \rightarrow \infty$ $M$ approaches $M$ and represents the total energy of the system. Inserting (C-29) in (C-11) provides:

This equation can be rewritten using:

$$\frac{d}{dt}(\mathcal{L}(q, \dot{q})) = \frac{d}{dt}\left(\frac{\partial \mathcal{L}(q, \dot{q})}{\partial \dot{q}}\right) + \mathcal{L}(q, \dot{q}) \dot{q} - \mathcal{L}(q, \dot{q}) = (u - F_{\text{diss}}) \dot{q}. \quad (C-11)$$

The Hamiltonian $\mathcal{H}(q, \dot{q}) \in \mathbb{R}$, which can be derived from the Lagrangian using the Legendre transformation [14], is defined as:

$$\mathcal{H}(q, \dot{q}) := \left(\frac{\partial \mathcal{L}(q, \dot{q})}{\partial \dot{q}}\right)^T - \mathcal{L}(q, \dot{q}), \quad (C-12)$$

and represents the total energy of the system. Inserting (C-29) in (C-11) provides:

$$\frac{d}{dt}(\mathcal{H}(q, \dot{q})) = u^T \dot{q} - F_{\text{diss}}^T \dot{q}. \quad (C-13)$$

Integrating (C-13) over the time interval $[t_0, t_1]$ gives the following equation:

$$\mathcal{H}(q(t_1), \dot{q}(t_1)) - \mathcal{H}(q(t_0), \dot{q}(t_0)) = \int_{t_0}^{t_1} u^T \dot{q} dt - \int_{t_0}^{t_1} F_{\text{diss}}^T \dot{q} dt. \quad (C-14)$$
Computing controllability matrix for nonlinear systems

The controllability matrix for a nonlinear input affine system, as shown in (2-16), is:

\[
M_c = \begin{bmatrix}
g(x) | \text{ad}_{g_1}^0 (g_j (x)) \cdots \text{ad}_{g_1}^n (g_j (x)) | \text{ad}_{g_2}^0 (g_j (x)) \cdots \text{ad}_{g_2}^n (g_j (x)) | \cdots \\
\end{bmatrix},
\]

(C-15)
in which the ad-operator is defined as:

\[
\text{ad}_f^0 (g) := g, \\
\text{ad}_f^1 (g) := [f, g], \\
\text{ad}_f^2 (g) := [f, [f, g]], \\
\vdots \\
\text{ad}_f^k (g) := [f, \text{ad}_f^{k-1}],
\]

(C-16) - (C-19)

with \([f (x), g (x)]\) the Lie bracket, defined by:

\[
[f (x), g (x)] := \frac{\partial g (x)}{\partial x} f (x) - \frac{\partial f (x)}{\partial x} g (x)
\]

(C-20)

However, this procedure only works for input affine systems. The controllability of general nonlinear systems of the shape \(x = f (x, u)\) can be assessed by constructing a new state \(x = [x, u]^T\) to obtain the input affine form with new control input, defined by:

\[
v := \tau \dot{u} + u,
\]

(C-21)

This creates the new system, with state \(z\) and \(\bar{f} (z)\) and \(\bar{g} (z)\):

\[
\dot{z} = \begin{bmatrix}
\dot{x} \\
\dot{u}
\end{bmatrix} = \begin{bmatrix} f (x, u) \\
\tau^{-1} u
\end{bmatrix} + \begin{bmatrix} 0 \\
\tau^{-1}
\end{bmatrix} v.
\]

(C-22)

Controllability for falling mass example

This section contains the computation of the controllability matrix of the falling mass example, presented in Section 2-3 under passive control. The Equations Of Motion (EOM) of the nonlinear system in which the control action is passive is:

\[
\ddot{x} + g = -\dot{x} u^2,
\]

(C-23)
in which a unitary mass has been used. This system can be written as:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\dot{x}}
\end{bmatrix} = \begin{bmatrix} \dot{x} \\
g \end{bmatrix} + \begin{bmatrix} 0 \\
-\dot{x} u^2
\end{bmatrix}
\]

(C-24)

We construct a new state \(x = [x, u]^T\) and input affine output defined by:

\[
v := \tau \dot{u} + u.
\]

(C-25)
This results in the following system:

\[
\dot{z} = \begin{bmatrix}
\dot{x} \\
\dot{x} \\
\dot{\bar{u}}
\end{bmatrix}
= \begin{bmatrix}
\dot{x} \\
-g - \dot{x}\bar{u}^2 \\
-\tau^{-1}\bar{u}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
-\tau^{-1}
\end{bmatrix} v.
\] (C-26)

Computing the controllability matrix \( M_c \), now comes down to computing:

\[
M_c = \begin{bmatrix}
\bar{g}(z) & \bar{f}(z) \bar{g}(z) & \bar{g}(z)
\end{bmatrix}.
\] (C-27)

The resulting matrix is presented in (2-20).

**C-5 Hamiltonian mechanics**

The Hamiltonian, presented in (C-29) represents the total energy of the system, making it the sum of the kinetic and potential energy:

\[
\mathcal{H}(q, \dot{q}) = T(q, \dot{q}) + V(q)
\] (C-28)

The Hamiltonian is written as a system of \( 2n \) first order differential equations, as opposed to the \( n \) second order equations in Eq. (B-12). The \( 2n \) equations are a function of the generalized coordinates \( q \) and the generalized momenta \( p \in \mathbb{R}^n \):

\[
\Sigma : \begin{cases}
\dot{q} = \frac{\partial \mathcal{H}(q, p)}{\partial p} \\
\dot{p} = -\frac{\partial \mathcal{H}(q, p)}{\partial q} + B^T(q) u
\end{cases}
\] (C-29)

with \( p(q) = M(q) \dot{q}, B(q) \) the input matrix and

\[
\mathcal{H}(q, p) = \frac{1}{2} p^T M^{-1}(q) p + V(q)
\] (C-30)

If the dimension of the input space is different from the configuration space one can define:

\[
y = B^T(q) \frac{\partial \mathcal{H}(q, p)}{\partial p}
\] (C-31)

One can easily show that using this representation, the energy balance is the vector multiplication of the input and the output of the system:

\[
\frac{d\mathcal{H}(q(t), p(t))}{dt} = y^T(t) u(t)
\] (C-32)
C-5-1 Port-Hamiltonian systems

A more compact and general form is the Port-Hamiltonian (PH) representation. A PH system with dissipation is defined as [49]

\[\Sigma: \begin{cases} \dot{x} = (J(x) - R(x)) \frac{\partial H(x)}{\partial x} + g^T(x) u \\ y = g^T(x) \frac{\partial H(x)}{\partial x} \end{cases}\] (C-33)

and \(g(x) \in \mathbb{R}^s\) the input matrix, \(J(x)\) and \(R(x)\) the \(\mathbb{R}^{n \times n}\) interconnection and damping matrix, satisfying \(J(x) = -J^T(x)\) (skew-symmetric) and \(R(x) = R^T(x) \geq 0\) (symmetric and positive definite).

Note that with the following values of \(J(x), R(x)\) and \(g(x)\), the PH system of Eq. (C-33) results in the Hamiltonian system of (C-29) describing a Euler-Lagrange equations:

\[R(x) = 0, \quad J(x) = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \quad \text{and} \quad g(x) = \begin{bmatrix} 0 \\ B(q) \end{bmatrix}\] (C-34)

A PH system captures various physical system domains, such as mechanics, electrical, chemical or thermal systems. This is possible because it uses port-based modeling to model power-connections between the different elements. Power is the product of the effort \(e\) and flow \(f\), which are respectively the current and voltage in an electrical system and the velocity and force/torque in a mechanical system. For a mechanical system, \(x\) satisfies:

\[x = \begin{bmatrix} q \\ p \end{bmatrix} \in \mathbb{R}^s,\] (C-35)

with \(q\) and \(p\) as defined earlier. Note that the size of the state vector is \(s = 2n\). For more details on PH systems, the interested reader could investigate [49, 50].

C-6 Standard passivity based control types

The notion of passivity presented in Section 2-2 is related to, but more strict then the more general Passivity Based Control (PBC). PBC is a general control technique which renders the closed-loop system passive with respect to a desired storage function [12] and was first introduced in [51]. This section provides a brief high level introduction into the large area of PBC. The standard formulation of PBC is to select a control function

\[u = \beta(x) + v\] (C-36)

so that the dynamics of the system satisfies the criteria of a passive system as stated in (2-3), being

\[\mathcal{H}(x(t_1)) - \mathcal{H}(x(t_0)) = \int_{t_0}^{t_1} v^T(t) z(t) ds - d_d(t)\] (C-37)

with \(\mathcal{H}(x) : \mathbb{R}^s \to \mathbb{R}\) the desired Hamiltonian, which has a strict minimum at \(x_{eq}\), \(z(t) \in \mathbb{R}^m\) the new passive output (which may be equal to the normal output \(y\)) and \(d_d(t) : \mathbb{R}^s \to \mathbb{R}^+\)
the desired dissipation of the system. \( x = [q, p] \in \mathbb{R}^s \) is the state of the system with \( q \) and \( p \) respectively the generalized coordinates and momenta. Note that \( s = 2n \), with \( n \) the dimension of \( q \) and \( p \). The desired dissipation is often an addition which is merely added to enforce asymptotic stability.

This control technique controls a system by designing a desired energy function, which is related and therefore of interest for this research. However, this technique uses an active control law to realize this result. This type of controller is therefore not suited.

The three most used types of PBC are Classical Passivity Based Control (C-PBC), Interconnection and Damping Assignment (IDA) and Control by Interconnection (CbI).

### C-6-1 Classical PBC

The goal of the classical or standard PBC is to only shape the desired energy function \( \mathcal{H}_d \), such that the PH system presented in (C-33) results in the following closed-loop dynamics

\[
\Sigma_{cl} : \begin{cases}
\dot{x} = (J(x) - R(x)) \frac{\partial \mathcal{H}_d(x)}{\partial x} + g^T(x) v \\
z = g^T(x) \frac{\partial \mathcal{H}_d(x)}{\partial x}
\end{cases}
\]  

(C-38)

An often used method to obtain this desired energy function is Energy Shaping (ES) and Damping Injection (DI) [13, 11]. The control input is decomposed into two terms:

\[
u(x) = u_{es}(x) + u_{di}(x)
\]

(C-39)

\[
u = \frac{\partial}{\partial x} (V(x) - V_d(x)) + \frac{\partial \mathcal{F}_d(\dot{x})}{\partial \dot{x}}
\]

(C-40)

With \( \mathcal{F}_d \) the desired dissipation function [52], which can be equal to the Rayleigh dissipation function and \( V_d \) the desired potential energy. Writing out the damping injection term results in

\[
u_{di}(x) = -K_{di}(x) y
\]

(C-41)

with \( K_{di}(x) > 0, \in \mathbb{R}^m \) the damping injection matrix on the output \( y \).

### C-6-2 IDA PBC

The objective of IDA PBC is to shape the desired interconnection \( J_d \) and damping matrix \( R_d \) of the PH system, next to the desired energy function \( \mathcal{H}_d \). In the case of physical systems, the interconnection and the damping matrices determine the energy exchange and the dissipation of the system [53]. The following closed loop dynamics are obtained

\[
\Sigma_{cl} : \begin{cases}
\dot{x} = (J_d(x) - R_d(x)) \frac{\partial \mathcal{H}_d(x)}{\partial x} + g^T(x) v \\
z = g^T(x) \frac{\partial \mathcal{H}_d(x)}{\partial x}
\end{cases}
\]

(C-42)

There are three ways of computing the desired control input [53, 54]: Non-Parameterized IDA, Algebraic IDA and Parameterized IDA, but the implementation is outside the scope of this research.
C-7 Least squares method for active to passive control

A least squares method, which minimizes the error between the active and passive torques. The results of this method are less intuitive then those other methods due to its quadratic minimization. This is illustrated using a two Degree of Freedom (DoF) example, a generalization for the \(n\)-DoF case is provided in the end. For a two DoF-system, the squared error \(e_2 \in \mathbb{R}^n\) between the active and passive torques is:

\[
e_2 = \sum_{i=1}^{n} (u_{\text{act},i} - u_{\text{pass},i})^2 \tag{C-43}
\]

\[
e_2 = (u_{\text{act},1} - u_{\text{pass},1})^2 + (u_{\text{act},2} - u_{\text{pass},2})^2. \tag{C-44}
\]

The maximum permissible relation between the inputs is obtained by setting (3-2) equal to zero:

\[
u_{\text{pass},1} = -\frac{\dot{q}_2}{q_1} u_{\text{pass},2} = -ru_{\text{pass},2}, \tag{C-45}
\]

with \(r\) defined by \(r = \frac{\dot{q}_2}{q_1}\) for ease of notation. This is substituted in (C-44):

\[
e_2 = (u_{\text{act},1} + ru_{\text{pass},2})^2 + (u_{\text{act},2} - u_{\text{pass},2})^2, \tag{C-46}
\]

yielding an equation with only one variable. The minimum of (C-46) is obtained by computing the partial derivative with respect to \(u_{\text{pass},2}\):

\[
\frac{\partial e_2}{\partial u_{\text{pass},2}} = 2r (u_{\text{act},1} + ru_{\text{pass},2}) - 2 (u_{\text{act},2} - u_{\text{pass},2}) \tag{C-47}
\]

\[
= u_{\text{pass},2} (r^2 + 1) = u_{\text{act},2} - ru_{\text{act},1}, \tag{C-48}
\]

when set to zero, this provides the minimum value of \(e_2\):

\[
u_{\text{pass},2} = \frac{u_{\text{act},2} - ru_{\text{act},1}}{r^2 + 1}, \tag{C-49}
\]

combining this result with (C-45) gives the two DoF minimized squared error. A similar approach can be used for \(n\)-DoF system, in which \(u_{\text{pass},1}\) can be removed from the equation by computing \(r_2, r_3, \ldots, r_n\) and computing \(n-1\) partial derivatives.

C-8 Potential energy for several mechanisms

Illustrations together with brief descriptions will be added.

Serial two link

\[
V(q) = \begin{bmatrix} g \, m_1 \, (h_0 - l_{c_1} \cos q_1) + g \, m_2 \, (h_0 - l_1 \cos q_1 - l_{c_2} \cos q_2) \end{bmatrix} \tag{C-50}
\]

\[
G(q) = \begin{bmatrix} g \, l_1 \, m_1 \sin q_1 + g \, l_1 \, m_2 \sin (q_1) \\ g \, l_2 \, m_2 \sin q_2 \end{bmatrix} \tag{C-51}
\]
Parallel two link

\[ V(q) = \begin{bmatrix} g m_{c_1} \left( h_0 - \sqrt{q_1^2 - \left( x_0 + \frac{q_1^2 - q_2^2}{4x_0} \right)^2} \right) \\ g m_{c_2} \left( h_0 - \frac{\sqrt{q_1^2 - \left( x_0 + \frac{q_1^2 - q_2^2}{4x_0} \right)^2}}{2} \right) \end{bmatrix} + \begin{bmatrix} \end{bmatrix} \]

\[ G(q) = \begin{bmatrix} g m_{c_1} \left( 2q_1 - \frac{q_1 \left( x_0 + \frac{q_1^2 - q_2^2}{4x_0} \right)}{x_0} \right) \\ g m_{c_2} \left( 2q_1 - \frac{q_1 \left( x_0 + \frac{q_1^2 - q_2^2}{4x_0} \right)}{x_0} \right) \\ -4 \sqrt{q_1^2 - \left( x_0 + \frac{q_1^2 - q_2^2}{4x_0} \right)^2} \\ -g m_{c_2} q_2 \left( x_0 + \frac{q_1^2 - q_2^2}{4x_0} \right) \right) \\
-4 \sqrt{q_1^2 - \left( x_0 + \frac{q_1^2 - q_2^2}{4x_0} \right)^2} \\ -4 \sqrt{q_1^2 - \left( x_0 + \frac{q_1^2 - q_2^2}{4x_0} \right)^2} \end{bmatrix} \]

Overhead crane

\[ V(q) = \begin{bmatrix} g m_2 \left( h_0 - q_1 \right) + g m_1 \left( h_0 - \frac{q_1}{2} \right) \end{bmatrix} \]

\[ G(q) = \begin{bmatrix} -\frac{1}{2} g m_1 - g m_2 \\ 0 \end{bmatrix} \]

General

\[ V(q) = g\Gamma M H(q) \]

\[ G(q) = gJ(q) \bar{M}\Gamma \]

All variables are explained in Appendix B.

**C-9 Stability of tracking reference vector fields controller**

This section proves the stability of the tracking reference vector fields controller [11], introduced in Section 5-3.

Using the following Lyapunov function (see Appendix C-1) we can show that the control law of the tracking reference velocity vector fields, presented in (5-4), stabilizes the system:

\[ V(q, \dot{q}) = \mu(q) + \frac{1}{2} F^T(q, \dot{q}) M(q) F(q, \dot{q}), \]

\[ \text{(C-58)} \]
with $V \in \mathbb{R}$ the positive definite Lyapunov function for the system, $\mu$ the positive definite Lyapunov function for the vector field $f_d$, $M(q)$ the configuration space mass matrix and $F(q, \dot{q})$ the configuration space velocity error:

$$F(q, \dot{q}) := \dot{q} - f_d(q). \quad (C-59)$$

Computing the time derivative of the Lyapunov function of the system shows:

$$\dot{V}(q, \dot{q}) = \nabla \mu^T \dot{q} + \frac{1}{2} F^T \dot{M} F + F^T M (\ddot{q} - \nabla f_d \dot{q}) \quad (C-60)$$

$$= \nabla \mu^T \dot{q} + \frac{1}{2} F^T \dot{M} F - F^T M \nabla f_d \dot{q} + F^T (-C \dot{q} - K F - \nabla \mu + M \nabla f_d \dot{q} + C f_d) \quad (C-61)$$

$$= \nabla \mu^T \dot{q} + \frac{1}{2} F^T \left( \dot{M} - 2C \right) F - F^T K F - F^T \nabla \mu \quad (C-62)$$

$$= -F^T K F + F^T \nabla \mu^T f_d \leq 0 \quad (C-63)$$

In which LaSalle’s (see Appendix C-1) can be used to complete the proof.
This appendix contains all physical parameters of the used mechanisms, together with other constants such as motor friction and gear ratios. These parameters are used in simulation and physical experiments.

<table>
<thead>
<tr>
<th>Table D-1: Physical parameter of the two link serial manipulator. The subscript indicates link $j$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Mass [kg]</td>
</tr>
<tr>
<td>$m_j$</td>
</tr>
<tr>
<td>Inertia $^1$ [kgm$^2$]</td>
</tr>
<tr>
<td>$I_j$</td>
</tr>
<tr>
<td>Length [m]</td>
</tr>
<tr>
<td>$l_j$</td>
</tr>
<tr>
<td>Distance to Center of Mass (CoM)$^2$ [m]</td>
</tr>
<tr>
<td>$L_{c_j}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table D-2: Friction coefficients of two link serial manipulator. The subscript indicates joint $i$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Viscous friction [Nm s/rad]</td>
</tr>
<tr>
<td>Coulomb friction [Nm]</td>
</tr>
<tr>
<td>Torque dependent friction [Nm]</td>
</tr>
</tbody>
</table>

The motor is connected to the joint via a gearbox and timing belts with corresponding pulley ratio. The maximum torque can be computed using the parameters shown in Table D-3:

$$\tau_{\text{max}} = I_{\text{max}} k_m r_G r_P = 16.69 \text{Nm}.$$  \hspace{1cm} (D-1)

$^1$Measured around the CoM. Includes axle around which it rotates, the pulley on this axle and the motor inertia corrected for the gear and pulley ratio.

$^2$Distance along the link, measured from its parent joint.

$^3$Ratio is defined by (outgoing : incoming).
Table D-3: Motor torque (related) parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max current $[A]$</td>
<td>$I_{\text{max}}$</td>
</tr>
<tr>
<td>Motor torque constant $[Nm/A]$</td>
<td>$k_m$</td>
</tr>
<tr>
<td>Gear ratio $^3$</td>
<td>$r_G$</td>
</tr>
<tr>
<td>Pulley ratio</td>
<td>$r_P$</td>
</tr>
</tbody>
</table>

Figure D-1: Two link mechanism used in simulation and physical experiments with symbol definitions.
This appendix provides the Simulink model used in the physical experiment described in Chapter 7.

Figure E-1: Simulink model used in the physical experiment. The orange blocks contain the input and output to the motors and encoders. The yellow blocks contain kinematic computations and EOM matrices. The green blocks contain the control algorithm. The blue blocks contain data loading and logging components.
This appendix contains the resulting data of the simulations and experiments.

**Table F-1:** Page numbers of the simulation data.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Velocity field</th>
<th>Target number</th>
<th>Supply rate</th>
<th>Workspace velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passified Natural Dynamics</td>
<td>Constant</td>
<td>1</td>
<td>Page 53</td>
<td>Page 107</td>
</tr>
<tr>
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<td>Constant</td>
<td>2</td>
<td>Page 101</td>
<td>Page 107</td>
</tr>
<tr>
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<td>Constant</td>
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<td>Page 102</td>
<td>Page 107</td>
</tr>
<tr>
<td>Passified Natural Dynamics</td>
<td>Constant</td>
<td>4</td>
<td>Page 103</td>
<td>Page 107</td>
</tr>
<tr>
<td>Passified Natural Dynamics</td>
<td>Constant</td>
<td>1</td>
<td>Page 53</td>
<td>Page 108</td>
</tr>
<tr>
<td>Passified Natural Dynamics</td>
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<td>2</td>
<td>Page 101</td>
<td>Page 108</td>
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<tr>
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<tr>
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<td>Traj. Dep.</td>
<td>4</td>
<td>Page 103</td>
<td>Page 108</td>
</tr>
<tr>
<td>Passified impedance</td>
<td>-</td>
<td>1, 2</td>
<td>Page 104</td>
<td>Page 109</td>
</tr>
<tr>
<td>Passified impedance</td>
<td>-</td>
<td>3, 4</td>
<td>Page 105</td>
<td>Page 109</td>
</tr>
</tbody>
</table>

**Table F-2:** Page numbers of the experiment data.

<table>
<thead>
<tr>
<th>Controller</th>
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<th>Target number</th>
<th>Supply rate</th>
<th>Workspace velocities</th>
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<td>Page 119</td>
</tr>
<tr>
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<tr>
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<tr>
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<td>Constant</td>
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<td>Page 115</td>
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</tr>
<tr>
<td>Passified Natural Dynamics</td>
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<td>Page 120</td>
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<tr>
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<td>Page 113</td>
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<td>Traj. Dep.</td>
<td>3</td>
<td>Page 114</td>
<td>Page 120</td>
</tr>
<tr>
<td>Passified Natural Dynamics</td>
<td>Traj. Dep.</td>
<td>4</td>
<td>Page 115</td>
<td>Page 120</td>
</tr>
<tr>
<td>Passified impedance</td>
<td>-</td>
<td>1, 2</td>
<td>Page 116</td>
<td>Page 121</td>
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<tr>
<td>Passified impedance</td>
<td>-</td>
<td>3, 4</td>
<td>Page 117</td>
<td>Page 121</td>
</tr>
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</table>
F-1  Simulation

F-1-1  Supply rate
Figure F-1: Supply rate of energy to the system for the natural dynamics velocity field controller for the second target position in simulation. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row). The images on the left use a constant velocity field magnitude and on the right a trajectory dependent velocity field magnitude.
Figure F-2: Supply rate of energy to the system for the natural dynamics velocity field controller for the third target position in simulation. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row). The images on the left use a constant velocity field magnitude and on the right a trajectory dependent velocity field magnitude.
(a) Supply Rate with a constant velocity field magnitude.

(b) Supply Rate with a trajectory dependent velocity field magnitude.

(c) Torque with a constant velocity field magnitude.

(d) Torque with a trajectory dependent velocity field magnitude.

(e) Configuration space velocity with a constant velocity field magnitude.

(f) Configuration space velocity with a trajectory dependent velocity field magnitude.

Figure F-3: Supply rate of energy to the system for the natural dynamics velocity field controller for the fourth target position in simulation. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row). The images on the left use a constant velocity field magnitude and on the right a trajectory dependent velocity field magnitude.
Figure F-4: Supply rate of energy to the system for the passive workspace impedance controller for the first and second target position in simulation. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row).
Figure F-5: Supply rate of energy to the system for the passive workspace impedance controller for the third and fourth target position in simulation. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row).
F-1-2  Workspace velocities
Figure F-6: Workspace velocities for the natural dynamics velocity field controller with constant velocity magnitude in simulation. The velocities for the four target positions are shown.
Figure F-7: Workspace velocities for the natural dynamics velocity field controller with trajectory dependent velocity magnitude in simulation. The velocities for the four target positions are shown.
Figure F-8: The workspace velocities for the passive workspace impedance controller in simulation. The velocities for the four target positions are shown.
F-2  Experiment

F-2-1  Passive motor

In Figure F-9 the position, supply rate, velocity and torque for a the Proportional-Derivative (PD)-control are shown for when the motor is used in an active and passive mode. Initially a disturbance is provided offset is provided on both links.

- Since the controller is lacking a derivative gain, the links start to oscillate around the desired angle with increasing for the active case. When restricting the control torque to be passive, the system is stabilized towards the desired angle.

- The supply rate of the active and passive control modes are shown in the second row. The supply rate of the passively controlled system is negative, except for several small spikes, which have been caused by measurement noise, actuator noise, backlash in the gearbox or the implementation of the passive controller.

- The torque profile of the passive controller shows some chattering, this is caused by the same factors. Especially when the velocity is close to zero, a small disturbance on this signal can cause the system to be active, requiring the torque to be clipped to zero.
Figure F-9: The position, supply rate, velocity and torque for a motor used in an active and passive mode. Initially a small offset is provided, causing a deviation for both joint angles. All variables are expressed in the configuration space.
F-2-2  Supply rate
Figure F-10: Supply rate of energy to the system for the natural dynamics velocity field controller for the second target position in experiment. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row). The images on the left use a constant velocity field magnitude and on the right a trajectory dependent velocity field magnitude.
(a) Supply Rate with a constant velocity field magnitude.

(b) Supply Rate with a trajectory dependent velocity field magnitude.

(c) Torque with a constant velocity field magnitude.

(d) Torque with a trajectory dependent velocity field magnitude.

(e) Configuration space velocity with a constant velocity field magnitude.

(f) Configuration space velocity with a trajectory dependent velocity field magnitude.

**Figure F-11:** Supply rate of energy to the system for the natural dynamics velocity field controller for the third target position in experiment. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row). The images on the left use a constant velocity field magnitude and on the right a trajectory dependent velocity field magnitude.
(a) Supply Rate with a constant velocity field magnitude.

(b) Supply Rate with a trajectory dependent velocity field magnitude.

(c) Torque with a constant velocity field magnitude.

(d) Torque with a trajectory dependent velocity field magnitude.

(e) Configuration space velocity with a constant velocity field magnitude.

(f) Configuration space velocity with a trajectory dependent velocity field magnitude.

Figure F-12: Supply rate of energy to the system for the natural dynamics velocity field controller for the fourth target position in experiment. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row). The images on the left use a constant velocity field magnitude and on the right a trajectory dependent velocity field magnitude.
Simulation and experiment data

Figure F-13: Supply rate of energy to the system for the passive workspace impedance controller for the first and second target position in experiment. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row).
(a) Supply Rate for target 3.

(b) Supply Rate for target 4.

(c) Torque for target 3.

(d) Torque for target 4.

(e) Configuration space velocity for target 3.

(f) Configuration space velocity for target 4.

Figure F-14: Supply rate of energy to the system for the passive workspace impedance controller for the third and fourth target position in experiment. The supply rate is the multiplication of the torque (second row) and workspace velocities (third row).
F-2-3  Workspace velocities
Figure F-15: Workspace velocities for the natural dynamics velocity field controller with constant velocity magnitude in experiment. The velocities for the four target positions are shown.
Figure F-16: Workspace velocities for the natural dynamics velocity field controller with trajectory dependent velocity magnitude in experiment. The velocities for the four target positions are shown.
Figure F-17: The workspace velocities for the passive workspace impedance controller in experiment. The velocities for the four target positions are shown.


## Glossary

### List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMD</td>
<td>BioMechanical Design</td>
</tr>
<tr>
<td>Cbl</td>
<td>Control by Interconnection</td>
</tr>
<tr>
<td>CoM</td>
<td>Center of Mass</td>
</tr>
<tr>
<td>C-PBC</td>
<td>Classical Passivity Based Control</td>
</tr>
<tr>
<td>DBL</td>
<td>Delft Biorobotics Lab</td>
</tr>
<tr>
<td>DI</td>
<td>Damping Injection</td>
</tr>
<tr>
<td>DoF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>DoFs</td>
<td>Degrees of Freedom</td>
</tr>
<tr>
<td>EL</td>
<td>Euler-Lagrange</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>EOM</td>
<td>Equations Of Motion</td>
</tr>
<tr>
<td>ES</td>
<td>Energy Shaping</td>
</tr>
<tr>
<td>IDA</td>
<td>Interconnection and Damping Assignment</td>
</tr>
<tr>
<td>lhs</td>
<td>left hand side</td>
</tr>
<tr>
<td>MBFC</td>
<td>Model Based Feedback Control</td>
</tr>
<tr>
<td>MDoF</td>
<td>Multiple Degree of Freedom</td>
</tr>
<tr>
<td>MPC</td>
<td>Model Predictive Control</td>
</tr>
<tr>
<td>MR</td>
<td>Magneto-Rheological</td>
</tr>
<tr>
<td>PD</td>
<td>Proportional-Derivative</td>
</tr>
</tbody>
</table>
PBC  Passivity Based Control

PH  Port-Hamiltonian

P-TER  Passive Trajectory Enhancing Robot

rhs  right hand side

RK4  Runge-Kutta 4

SDoF  Single Degree of Freedom

SMEs  Small and Medium sized Enterprises

List of symbols

\( J(x) \)  Interconnection matrix (PH)
\( p \)  Generalized momenta
\( \mathbb{R} \)  Set of real numbers
\( \mathbb{R}^n \)  Real vector of size \( n \)
\( X \)  Set
\( \Sigma \)  State space system
\( x \)  State variable
\( x_{eq} \)  Equilibrium point of \( x \)
\( B(q) \)  Input matrix (Hamiltonian)
\( g(x) \)  Input matrix (PH)
\( u \)  Input variable
\( V \)  Lyapunov function
\( y \)  Output variable
\( |\cdot| \)  Absolute value of variable
\( ||\cdot|| \)  Norm of variable
\( \text{sgn}(\cdot) \)  Sign of variable, by \( \text{sgn}(\cdot) = \frac{\cdot}{|\cdot|} \)
\( \hat{\cdot} \)  Unit vector, i.e. \( ||\hat{\cdot}|| = 1 \)
\( t \)  Continuous time instant
\( \alpha \)  Gain of weight function
\( C(q, \dot{q}) \)  Matrix representing centrifugal and Coriolis terms
\( G(q) \)  Potential force vector
\( \mathcal{L} \)  Lagrangian
\( M(q) \)  Configuration space mass matrix
\( \mu \)  Lyapunov function for velocity field
\( M \)  Workspace mass matrix
\( \nabla \)  Partial derivative
\[ q \] Configuration space position
\[ \dot{q} \] Configuration space velocity
\[ \ddot{q} \] Configuration space acceleration
\[ R \] Configuration space damping matrix
\[ \bar{R} \] Workspace damping matrix
\[ \Sigma \] System
\[ J \] Kinetic energy
\[ \tau_e \] External torques
\[ \mathcal{V} \] Potential energy
\[ x \] Workspace position
\[ \dot{x} \] Workspace velocity
\[ \ddot{x} \] Workspace acceleration
\[ F \] Velocity error
\[ f_d \] Desired vector field velocity
\[ F_e \] External forces
\[ H(q) \] Hessian (second order partial derivative w.r.t. \( q \))
\[ h(q) \] Mapping from configuration to workspace
\[ h^{-1}(q) \] Mapping workspace to configuration space
\[ J(q) \] Jacobian (partial derivative w.r.t. \( q \))
\[ J^\dagger(q) \] Pseudoinverse of the Jacobian
\[ n(x) \] Weighting function
\[ p_x \] Polynomial of \( x \)
\[ p_{\dot{x}} \] Polynomial of \( \dot{x} \)
\[ x_{d,i} \] Target position \( i \)
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