Cross-shore mean flow in the surf zone

M.J.F. STIVE and H.G. WIND

Delft Hydraulics Laboratory, Laboratory de Voorst, P.O. Box 152, 8300 AD Emmeloord, The Netherlands

(Received January 28, 1986; revised and accepted May 2, 1986)

ABSTRACT

Stive, M.J.F. and Wind, H.G., 1986. Cross-shore mean flow in the surf zone. *Coastal Eng.*, 10: 325–340.

The seaward mean returnflow or undertow in the surf zone, which compensates for the shoreward mass flux above the wave trough level, is found to be driven by the force imbalance between the wave momentum flux on the one hand and the set-up on the other hand, in qualitative agreement with the model of Dyhr-Nielsen and Sørensen (1970). A model, which quantifies this imbalance, is shown to yield theoretical results in good accordance with experiments when the proper boundary conditions are accounted for.

1. INTRODUCTION

It is a long-known observation that on more or less two-dimensional beaches there exists a relatively strong mean offshore flow below the wave trough level which compensates for the shoreward mass transport in the breaking wave crests. However, it is only recently that this cross-shore current phenomenon has received theoretical attention.

Dyhr-Nielsen and Sørensen (1970) were the first to give a qualitative, theoretical analysis of the phenomenon. They hypothesized that the local imbalance between the depth varying momentum flux and the depth uniform set-up force becomes significant in the surf zone and thus provides a driving force for a vertical circulation. The present experimental analysis of detailed velocity measurements in periodic, breaking waves confirms that their physical ideas in essence provide the explanation for the existence of the undertow. This conclusion is based on a quantification of the force terms in the fluid interior driving the mean flow.

Quantitative evaluations of Dyhr-Nielsen and Sørensen's ideas leading to a model for the flow field were made by Dally (1980), Börecki (1982), Svendsen (1984) and most recently by Buhr-Hansen and Svendsen (1984). Their models

FIG. 1a EXPERIMENTAL ARRANGEMENT

FIG. 1b DETAIL EXPERIMENTAL ARRANGEMENT

Fig. 1. Experimental set-up.

primarily differ in the description of the breaking wave properties. The model presented in this study differs from those mentioned in the treatment of the boundary conditions. This latter aspect appears to be an important element in the quantitative evaluation of the undertow, specifically with respect to its variation close to the bottom.

This paper is built up as follows. Firstly, the experiments are described. Secondly, after an evaluation and experimental quantification of the momentum balance terms in the fluid interior a model for the undertow is constructed and checked experimentally. Finally, the conclusions are summarized.

2. EXPERIMENTS

The measurements from which the present results were derived are the same as those used by the authors in previously published studies (e.g. Stive, 1980; Stive and Wind, 1982). To prevent duplication of the description of details just a brief account of the experimental arrangement, instruments and procedures is given below.

Experimental arrangement

The experiments were conducted in a wave flume of the Delft Hydraulics Laboratory, 55 m long, 1 m wide and 1 m high. Periodic waves with minimal free second-harmonic components were generated in a water depth of 0.85 m. The waves broke on a plane concrete slope 1:40 (see Fig. 1a).

Instrumentation

Surface elevations in the surfzone were measured by conductivity-type wave gauges positioned 1 m apart. Although aeration influences the response of the gauges, the air content in the breaking waves was estimated low enough to cause only negligible deviations.

Velocities were measured by means of a laser doppler velocity meter (LDV). The device was mounted on a carriage such that any desired horizontal or vertical position along the flume could be reached. At different levels the horizontal and vertical component of velocity were measured simultaneously with the surface elevation in the cross-sections indicated in Fig. 1b. In the crests of the breaking waves LDV measurements are prevented by the presence of air bubbles.

Data-analysis

The velocity data were processed with the aid of an ensemble averaging technique in which each wave cycle in the time series was considered to be one realization. The technique results in a description of the horizontal and vertical velocity components as the sum of an 'organized', periodic component (denoted by a tilde) and an 'unorganized', residual component (denoted by a prime), i.e. $u = \tilde{u} + u'$ and $w = \tilde{w} + w'$. The periodic component consists typically of a periodic fluctuation around a constant mean, i.e. a time-average over several wave cycles. In the breaking region the residual component is typically formed by the turbulence due to breaking. A characteristic order of magnitude of its rms-value is $0.1 c$, where c is the wave propagation speed. In the shoaling region the residual component shows rms-values below $0.01 c$. The relatively low values in the shoaling region confirm the consistency of the ensemble averaging technique.

Wave conditions

The experiments described here concern one extensively investigated test. The conditions of this test (see Table 1) represent a type of breaking usually found on gently sloping beaches. The initial breaking behaviour falls in the category spilling breaking. The rapid transitions of wave shape in the region right after breaking develop soon, i.e. after a horizontal distance of several times the breaker depth, into a relatively well-organised, quasi-steady breaking motion, which is virtually independent of the initial breaking behaviour. At that stage of their breaking motion, breakers on a beach may be described as spilling breakers or bores.

TABLE 1

Wave conditions (wave height H , period T , wave length L , where the subscripts o, h and b denote deep water, horizontal section and breakpoint)

H_o (m)	H_h (m)	H_b (m)	T (s)	H_o/L_o (-)
0.159	0.145	0.178	1.79	0.032

3. DERIVATION OF A MODEL FOR THE CROSS-SHORE MEAN FLOW

The qualitative suggestions of Dyhr-Nielsen and Sørensen (1970) with respect to the role of the internal forces in driving a mean cross-shore circulation in the surf zone are based on an analysis of the magnitude of the terms in a local, time-mean momentum balance. Integration of this momentum balance using appropriate boundary conditions should reveal the mean circulation in a two-dimensional surf zone. As described in the introduction a practical evaluation for surf zone waves has been made by several investigators. Their evaluations are all based on the physical considerations of Dyhr-Nielsen and Sørensen (1970), while they primarily differ in the description of the breaking wave properties. Furthermore, less basic differences appear in the boundary conditions applied. Because of the unknown magnitude of some force terms, it seems somewhat uncertain which boundary conditions should be applied. The choice, however, is rather critical for the description of the undertow, specifically with respect to its depth variation close to the bottom. The treatment of the boundary conditions here presented differs from that applied in the above mentioned studies. It is based on the observation that the momentum flux decay in breaking waves above wave trough level is of such a strength that it imposes an important shear stress condition at this level. The different aspects of the problem are described in the following.

The mean internal momentum balance

In the case of a steady, two-dimensional motion in the (x, z) -plane of a fluid of constant density, ρ , the horizontal momentum balance for an elementary volume time-averaged over several wave cycles reads:

$$\frac{\partial}{\partial x}(\bar{p} + \rho \bar{u}^2) + \frac{\partial}{\partial z} \rho(\bar{uw}) = 0 \quad (3.1)$$

where p represents the pressure and u and w represent the velocity in the horizontal x - and vertical z -direction respectively, and the overbar denotes time-averaging. Following the conclusions of Stive and Wind (1982) we adopt the

following approximation for the mean dynamic pressure $\bar{P}$, defined as the difference between the mean pressure and the mean hydrostatic pressure at a level z :

$$\bar{P} \equiv \bar{p} - \rho g(\bar{\zeta} - z) \cong -\rho \bar{w}^2 \quad (3.2)$$

Before reducing the horizontal momentum equation (3.1) with the aid of the result (3.2) an additional approximation is introduced, i.e. if the velocity properties are separated into the sum of an organized and a turbulent contribution, $\bar{u}^2 = \bar{u}^2 + \overline{u'^2}$ and $\bar{w}^2 = \bar{w}^2 + \overline{w'^2}$, the turbulence is found to be practically isotropic (Stive and Wind, 1982). With these approximations the horizontal momentum equation (3.1) reduces to:

$$\frac{\partial}{\partial x} \rho(\bar{u}^2 - \bar{w}^2) + \frac{\partial}{\partial x} \rho g \bar{\zeta} + \frac{\partial}{\partial z} \rho(\bar{u}w) = 0 \quad (3.3)$$

In the case of an imbalance between the first, radiation stress induced, and the second, set-up induced, term of eqn. (3.3) a vertical gradient in the Reynolds stress term must result. Gradients of mean Eulerian velocities would be contained in the Reynolds stress term. The hypothesis of Dyhr-Nielsen and Sørensen (1970) is that the imbalance becomes significant in the surf zone and thus becomes a driving force for a mean flow circulation. Based on the present measurements the imbalance may be quantified. The results are given in Fig. 2a for the shoaling region and in Fig. 2b for the breaking region (note: the difference in horizontal scales). The observations show firstly that compared to the shoaling region the imbalance between the local radiation stress term and the set-up force is indeed significant inside the surf zone, thus supporting Dyhr-Nielsen and Sørensen's hypothesis. Secondly, it appears that in the region of measurement, i.e. below the wave through level the imbalance is practically constant over the depth. These measurement results are used in the following where a model for the mean Eulerian horizontal velocity, $\bar{u}_e(z)$, is derived and verified.

A momentum equation for $\bar{u}_e(z)$

In order to derive a momentum equation for the mean horizontal Eulerian velocity $\bar{u}_e(z)$ the following assumptions are now made: (1) the contribution of the organized wave motion in the surf zone to the Reynolds stress is small relative to that of the turbulent contribution ($\rho \bar{u}\bar{w} \ll \rho \overline{u'w'}$); (2) the Reynolds stress is modelled using the eddy viscosity concept; and (3) variations of $\bar{u}_e$ in propagation direction are negligible compared to its variation over depth. Under these assumptions $\bar{u}_e(z)$ below the wave trough level satisfies the mean horizontal momentum balance as follows:

$$\frac{\partial}{\partial z} \rho \nu_t \frac{\partial \bar{u}_e}{\partial z} = \frac{\partial}{\partial x} \rho(\bar{u}^2 - \bar{w}^2) + \frac{\partial}{\partial x} \rho g \bar{\zeta} \quad (3.4)$$

Fig. 2a. Spatial variation of the local radiation stresses and set-up in the shoaling region.

Fig. 2b. Spatial variation of the local radiation stresses and set-up in the breaking region.

A further reduction of eqn. (3.4) is obtained by adopting the observation, described in the foregoing, that the local imbalance between the momentum flux and the set-up force is virtually constant over the depth, yielding:

$$\frac{d}{dz} \rho \nu_t \frac{d\bar{u}_e}{dz} = \frac{dR}{dx} \quad (3.5)$$

$$R \equiv \rho g \bar{\zeta} + \rho (\bar{u}^2 - \bar{w}^2)$$

where for simplicity a separate symbol, R , for the local force term is introduced. The mean momentum balance (3.5) may be solved directly for $\bar{u}_e$ by integrating twice with the result:

$$\bar{u}_e(z) = \frac{z^2}{2\rho\nu_t} \frac{d}{dx} R + \frac{C_1}{\rho\nu_t} z + C_2 \quad (3.6)$$

where C_1 and C_2 are integration constants. In eqns. (3.5) and (3.6) it is assumed that ν_t beside time-independent is depth-independent as well. Also without the latter assumption the integration of eqn. (3.5) is straightforward. However, as shown by Svendsen (1984) inclusion of a conceptually realistic depth-variation of ν_t has effects of secondary importance when compared to the effects of incorporating alternative boundary conditions at the bottom.

The mean mass balance

An important feature in the present problem is the mass flux or momentum, M , in the region between the wave crests and troughs. This aspect will be treated here.

When we adapt an Eulerian description in a reference frame in which in one point below the trough level the time-mean fluid motion is zero, all the wave momentum, M_{irr} , in an irrotational wave field is contained in the region above the level of the wave troughs. Its magnitude is established by the following relation which is exact if the waves are purely progressive and of constant shape (Phillips, 1977, p. 40):

$$M_{irr} = E/c \quad (3.7)$$

in which E is the energy density and c the wave propagation velocity. Since for a closed wave flume or a two-dimensional, impermeable beach the Eulerian mass balance requires:

$$\int_{-h}^{\zeta} u_e dz = 0 \quad (3.8)$$

it is necessary to introduce a depth-mean Eulerian return flow, $\bar{u}_r$, as a constant contribution to the mean flow in the interior of magnitude:

$$\bar{u}_{r,irr} = -E/(\rho ch) \quad (3.9)$$

where h is the mean water depth including set-up ($h = d + \bar{\zeta}$).

In a wave field breaking on a beach there is a contribution in addition to the above mentioned effect. This is the contribution due to the surface roller, which simply implies that the amount of water contained in the roller is carried shoreward with the wave propagation speed. A quantification of these effects separately and of the surface roller effect specifically has been made by Svendsen (1984). Both effects are automatically included in the present empirical quantification of the depth-mean return flow velocity based on the dimensionless

flow field of quasi-steady, depth-similar breakers presented in Stive and Wind (1982) and made such that the net mass flux below trough level balances that above trough level:

$$\bar{u}_r d_t \equiv - \int_{\zeta_t}^{\zeta_c} \bar{u}_e dz \simeq -\frac{1}{10} (g/h)^{\frac{1}{2}} H d_t \quad (3.10)$$

where d_t is the depth up to the trough level. This result corresponds closely to the net mass flux result (3.7) if we introduce the approximations $c \simeq (gh)^{\frac{1}{2}}$ and $E \simeq \frac{1}{8} \rho g H^2$:

$$M_{irr} \simeq \frac{1}{8} \rho (g/h)^{\frac{1}{2}} H^2 \quad (3.11)$$

The increase of the mass flux according to eqn. (3.10) compared to eqn. (3.11) (on account of $0.8 d_t > H$ generally) is here ascribed to the surface roller contribution. The latter contribution is probably stronger than suggested by the difference between eqns. (3.10) and (3.11) because of the reducing effects of a nonsinusoidal wave form on the estimates following from eqn. (3.11).

Boundary conditions and solution of the momentum balance

For the determination of the Eulerian mean flow $\bar{u}_e(z)$ from eqn. (3.6) boundary conditions are needed to solve the integration constants C_1 and C_2 . One of these seems to be generally accepted in all studies mentioned as a natural choice, i.e. the total mean mass flux below the wave trough level should balance that above this level, implying that:

$$\int_{-h}^{\zeta_t} \bar{u}_e(z) dz = \bar{u}_r d_t \quad (3.12)$$

where ζ_t is the level of the wave troughs. Also in this study we have used this constraint, with $\bar{u}_r$ evaluated quantitatively according to eqn. (3.10).

As the second condition all previous studies rely on a bottom boundary condition specifying the mean flow velocity, i.e. generally the steady stream result at the upper edge of the boundary layer for a uniform, progressive wave field on a horizontal bottom as derived by Longuet-Higgins (1953). It appears however that in the surf zone it is not so much the bottom boundary layer and its induced stresses that determine the internal mean flow distribution but more so the wave momentum loss above the wave trough level. If the loss of this momentum in spatially decaying waves is not replenished by surface forces it must support a shear stress at the trough level, analogous to the situation in temporally decaying waves (see Phillips, 1977, p. 57). The spatial decay rate and resulting shear stress at trough level may be found by deriving a mean

horizontal momentum balance integrated from wave trough level to the wave surface, as done by Svendsen (1985):

$$\frac{\partial S_{xx,s}}{\partial x} + \rho g (\bar{\zeta} - \zeta_t) \frac{\partial \bar{\zeta}}{\partial x} + \bar{\tau}(\zeta_t) = 0 \quad (3.13a)$$

where the radiation stress above the wave trough level (after including the earlier made pressure approximations) is given by:

$$S_{xx,s} = \int_{\zeta_t}^{\zeta_e} \rho (\overline{u^2} - \overline{w^2}) dz + \frac{1}{2} \rho g (\bar{\zeta} - \zeta_t)^2 \quad (3.13b)$$

This result is identical to that for the mean horizontal momentum balance integrated over the total depth (see e.g. equations 18 and 19 of Stive and Wind, 1982) if we realize that it is only the bottom level, $z = -d$, which is replaced by the wave trough level, $z = \zeta_t$, as lower boundary.

In principle, direct theoretical evaluations of eqn. (3.13) lead to a result for $\bar{\tau}(\zeta_t)$. For instance, in the approximation of linear waves on a locally horizontal bottom eqn. (3.13) yields:

$$\bar{\tau}(\zeta_t) = -\frac{\partial}{\partial x} \frac{1}{2} \rho g (\bar{\zeta} - \zeta_t)^2 = -\frac{1}{2} \frac{\partial E}{\partial x} \approx -\frac{1}{16} \rho g \frac{\partial H^2}{\partial x} \quad (3.14)$$

However, it may well be expected that linear theory or for that matter any non-breaking wave theory comes short in predicting the momentum flux properties in the region above wave trough level in breaking waves, even when it only concerns an integrated property as in this case. An important improvement to eqn. (3.14) is found by accounting for the roller contribution as suggested by Svendsen (pers. commun., 1984), yielding:

$$\bar{\tau}(\zeta_t) \approx -\left(\frac{1}{16} + \frac{A_r}{H^2} \frac{h}{L} \right) \rho g \frac{\partial H^2}{\partial x} \quad (3.15)$$

where the second term represents the roller contribution with A_r the cross-sectional area of the surface roller, empirically found to be $A_r \approx 0.9 H^2$. The approximations (3.14) and (3.15) may be checked on the basis of the measurements as follows. Adopting the turbulent shear stress definition $\tau = -\rho \overline{u'w'}$, the assumption $\rho \overline{u\tilde{w}} < \rho \overline{u'w'}$ and the observations of the local force term R , straight forward integration of eqn. (3.3) leads to a result for the shear stress at trough level:

$$\bar{\tau}(\zeta_t) = d_t \frac{dR}{dx} + \bar{\tau}_b \quad (3.16)$$

The experimental observations of Stive and Wind (1982) indicate that the mean bottom shear stress $\bar{\tau}_b$ in a two-dimensional surf zone is negligibly small compared to the set-up force $\rho g h (d\bar{\zeta}/dx)$. Thus, measurement values of the

TABLE 2

Measured and theoretical shear stress at trough level in present experiments

$x(\text{m})$	$\bar{\tau}(\zeta_t) \text{ (Nm}^{-2}\text{)}$		
	measured	linear theory	linear theory with roller contribution
36.5	5.03	2.84	5.96
37.5	4.68	2.17	4.43
38.5	4.51	1.16	2.28
39.5	2.00	0.78	1.48
40.5	1.14	0.61	1.09
41.5	0.73	0.43	0.74

shear stress are based on a quantification of the first term. These results are compared to the theoretical approximations (3.14) and (3.15) using the measured spatial variation of the wave heights (see Table 2). It appears that the discrepancy between linear theory and the measurements is substantially reduced when we account for the roller contribution. In any case, adopting the shear stress at trough level as a boundary condition to (3.6) defines the integration constant C_1 with the result:

$$C_1 = \bar{\tau}(\zeta_t) - \zeta_t \frac{dR}{dx} \quad (3.17)$$

Finally, the condition of zero mass flux when integrated over depth yields the magnitude of the integration constant C_2 in eqn. (3.6). The resulting equation describing the Eulerian mean flow velocity between the top of the bottom boundary layer (of which the thickness is neglected) and the wave trough level reads:

$$\bar{u}_e(z) = \frac{1}{2} [(\eta - 1)^2 - \frac{1}{3}] \frac{d_t^2}{\rho\nu_t} \frac{dR}{dx} + (\eta - \frac{1}{2}) \frac{d_t \bar{\tau}}{\rho\nu_t} + \bar{u}_r \quad (3.18)$$

where $\eta = (z + h)/d_t$.

Before we undertake an experimental verification of this result it should be pointed out that the depth derivative of eqn. (3.18), i.e.:

$$\frac{d\bar{u}_e}{dz} = \frac{(z - \zeta_t)}{\rho\nu_t} \frac{dR}{dx} + \frac{\bar{\tau}(\zeta_t)}{\rho\nu_t} \quad (3.19)$$

indicates that the gradient of $\bar{u}_e(z)$ varies linearly from $\bar{\tau}_b/\rho\nu_t$ at the bottom to $\bar{\tau}(\zeta_t)/\rho\nu_t$ at the trough level. From the experimental observation that in a two-dimensional surf zone $\bar{\tau}_b$ is negligibly small, it follows that the point where $d\bar{u}_e/dz = 0$ is at, or at least very close to, the bottom.

TABLE 3

Numerical values of parameters in present experiments

x (m)	H (m)	h (m)	d_t (m)	ν_t (m ² s ⁻¹)	$\bar{u}_r$ (ms ⁻¹)	$\frac{d_t^2}{\rho\nu_t} \frac{dR}{dx}$ (ms ⁻¹)	$\frac{d_t}{\rho\nu_t} \bar{\tau}(\zeta_t)$ (ms ⁻¹)
36.5	0.119	0.185	0.154	2.6×10^{-3}	-0.086	0.298	0.353
37.5	0.090	0.164	0.131	2.1×10^{-3}	-0.070	0.292	0.276
38.5	0.070	0.144	0.110	1.6×10^{-3}	-0.059	0.310	0.157
39.5	0.061	0.123	0.090	1.2×10^{-3}	-0.057	0.150	0.111
40.5	0.046	0.100	0.073	0.8×10^{-3}	-0.049	0.104	0.100
41.5	0.035	0.077	0.049	0.5×10^{-3}	-0.050	0.072	0.073

Experimental verification

The following experimental verification of the theoretical result (eqn. 3.18) firstly concerns the present measurements and secondly recent measurements of Buhr-Hansen and Svendsen (1984). Based on the above described observations the shear stress at trough level is estimated from the theoretical approximation (eqn. 3.15). Furthermore, estimates are needed for the spatial gradient of the force term R , the depth mean return flow $\bar{u}_r$, and the eddy viscosity coefficient ν_t . The local magnitude of R is determined from the measurements, while $\bar{u}_r$ is evaluated via eqn. (3.10). An estimate for the viscosity coefficient ν_t from the local wave parameters is given in the Appendix, based on the similarity between the flow in wakes and that in quasi-steady breaking waves. The numerical values of the relevant parameters are given in Table 3. Based on these values, theoretical results for $\bar{u}_e(z)$ have been determined according to eqn. (3.18). They are compared with the measurements in Fig. 3. There is an order-of-magnitude agreement, but the scatter in the results and the lack of measurements close to the bottom do not allow a firm conclusion.

In order to check the present theoretical formulation more thoroughly an additional comparison is made with the recently published experimental results of Buhr-Hansen and Svendsen (1984). Contrary to the present experiments these experiments were specifically aimed at the measurement of the undertow and the consistent results indicate that they were carefully conducted. The parameters necessary for the theoretical evaluation of eqn. (3.18) were obtained from the same formulations as described above. The only difference lies in the quantification of the gradient in the force term R which was given by Buhr-Hansen and Svendsen (1984) as a (measured) surf zone average, i.e. $dR/dx \approx 0.14 \rho gh/dx$. The comparison between these experimental results and the theoretical result for $\bar{u}_e(z)$ is presented in Fig. 4. The numerical values of the relevant parameters are given in Table 4.

Fig. 3. Comparison between present theoretical result for $\bar{u}_e(z)$ (solid lines) and the present measurement results (solid points).

In addition the theoretical results derived by Buhr-Hansen and Svendsen (1984) are shown, i.e. their results for $A \equiv [d_t^2/\rho\nu_t c] dR/dx = 0.4$, where they note that the measured values for A are 0.4–0.8 and the best agreement is obtained for A is 0.2–0.4. Their first solution with the net shoreward flow at the bottom is based on adopting the mean flow result derived by Longuet-

Fig. 4. Comparison between the present theoretical results (solid lines) for $u_e(z)$ and the theoretical results (---first solution, -.-second solution) and measurements (solid points) of Buhr-Hansen and Svendsen (1984).

Higgins (1953) at the top of the boundary layer as a boundary condition. This is combined with the condition of zero total mass flux and an exponentially decaying turbulent viscosity coefficient; it is the approach also described in Svendsen (1983). In their second solution the sign of Longuet-Higgins' mean

TABLE 4

Numerical values of parameters in Buhr-Hansen and Svendsen's (1984) experiments

x (m)	H (m)	h (m)	d_i (m)	ν_t (m^2s^{-1})	$\bar{u}_r$ (ms^{-1})	$\frac{d_t^2 dR}{\rho \nu_t dx}$ (ms^{-1})	$\frac{d_t \bar{\tau}(\zeta_t)}{\rho \nu_t}$ (ms^{-1})
22.00	0.116	0.145	0.117	1.7×10^{-3}	-0.095	0.290	0.263
23.00	0.087	0.120	0.097	1.3×10^{-3}	-0.078	0.266	0.219
23.87	0.072	0.101	0.078	1.0×10^{-3}	-0.071	0.221	0.150
24.50	0.058	0.086	0.069	0.8×10^{-3}	-0.062	0.221	0.136

bottom flow result is reversed based on considerations, which are quantitatively only weakly supported.

From the comparison in Fig. 4 it may be observed that the best agreement with the experiments is obtained with the present theory. This applies not only to the depth-averaged value of the mean flow, $\bar{u}_r$, but also and more importantly to the variations of the mean flow close to the bottom. Of Buhr-Hansen and Svendsen's (1984) solutions their second solution yields the better result. Their argumentation however for the sign reversal of the mean flow velocity at the bottom, which parallels that of Longuet-Higgins (1983), is weakly founded and as shown by the present derivations only part of the full argumentation. These findings confirm that the mean flow in the boundary layer is dominated by the forces exerted by the spatially decaying wave motion, as expressed in the present model by an imposed shear stress at the trough level in addition to the set-up force. However, further investigations of the role of the bottom shear stress would clearly be desirable if we wish to generalize the above results to more arbitrary situations with appreciable mean currents, such as in a three-dimensional surf zone.

5. CONCLUSIONS

The cross-shore mean flow in a two-dimensional surf zone is investigated both theoretically and experimentally, leading to a model for the undertow.

The model is in essence based on the physical considerations of Dyhr-Nielsen and Sørensen (1970) related to the internal force balance of a breaking wave field. Their qualitative ideas, quantitatively confirmed by the present experiments, give an explanation for the strong seaward mean flow or undertow below the level of the wave troughs.

In addition to the considerations on the internal force balance the boundary conditions are discussed which are needed to find a theoretical formulation for the mean Eulerian flow below the wave trough level. It is argued that contrary to the assumptions in earlier studies it is not so much the oscillatory boundary layer that imposes a condition on the mean flow in the fluid interior. Instead it appears that the conditions imposed by the strong, spatial decay of the wave motion, expressed by a shear stress at the trough level dominates the flow, such that a mean seaward flow also near the bottom arises.

APPENDIX. Estimation of the eddy viscosity

The eddy viscosity in breaking waves is estimated using the similarity between the flow field in breaking waves and wake flow. This similarity was pointed out by Peregrine and Svendsen (1978) and experimentally confirmed by Stive and Wind (1982).

The eddy viscosity in a wake flow may be estimated as (see Tennekes and Lumley, 1972):

$$\nu_t = U_s l / R_T \quad (\text{A.1})$$

where U_s is the defect velocity, l is a length scale and R_T is the turbulent Reynolds number. For wake flow this number is approximately constant and equal to 12.5.

The defect velocity U_s may be obtained as follows from the measured dimensionless velocity profile given in Stive and Wind (1982). Regard the defect velocity as the difference between the maximum and minimum velocity in each cross-section. Away from the surface roller this leads to defect velocities in the order of 0.2 c to 0.4 c . The length scale l follows from the cross stream defect velocity distribution:

p

$$\frac{U_0 - U}{U_s} = e^{-\frac{1}{2} \alpha \xi^2} \quad (\text{A.2})$$

where $U_0 - U$ is the difference between the undisturbed and actual flow velocity and $\xi = z/l$. Since $\alpha \approx 1$, l is approximately equal to the distance from the free surface to that point of the velocity profile, where $U = U_0 - 0.6 U_s$. This leads to length scales of $\frac{1}{4} h$ to $\frac{1}{2} h$. The resulting eddy viscosity coefficient is then with eqn. (A.1):

$$\nu_t = 10^{-2} c h \quad (\text{A.3})$$

The available information on the velocity profiles does not allow an estimate of the vertical distribution of ν_t .

ACKNOWLEDGEMENT

This work was done as a part of the TOW Coastal Research Programme of the Ministry of Transport and Public Works (Rijkswaterstaat) of the Netherlands. The continuous guidance and valuable comments on successive drafts by Prof. J.A. Battjes are gratefully acknowledged. Prof. I.A. Svendsen and Dr. H.J. de Vriend are thanked for their stimulating discussions in the later stages of the study.

REFERENCES

- Börecki, O.S., 1982. Distribution of wave-induced momentum fluxes over depth and application within the surf-zone. Ph.D. Thesis, Dept. Civil Eng., Univ. of Delaware.
 Buhr-Hansen, J. and Svendsen, I.A., 1984. A theoretical and experimental study of undertow. Proc. 19th Int. Conf. on Coastal Eng., Houston.

- Dally, W.R., 1980. A numerical model for beach profile evolution. M.Sc. Thesis, Univ. of Delaware.
- Dyhr-Nielsen, M. and Sørensen, T., 1970. Some sand transport phenomena on coasts with bars. Proc. 12th Int. Conf. on Coastal Eng., Washington.
- Longuet-Higgins, M.S., 1953. Mass transport in water waves. Philos. Trans. R. Soc. London, Ser. A, 245: 535-581.
- Longuet-Higgins, M.S., 1983. Wave set-up, percolation and undertow in the surf zone. Proc. R. Soc., London, Ser. A, 390: 283-291.
- Peregrine, D.H. and Svendsen, I.A., 1978. Spilling breakers, bores and hydraulic jumps. Proc. 16th Int. Conf. on Coastal Eng., Hamburg.
- Phillips, O.M., 1977. The Dynamics of the Upper Ocean. Cambridge University Press, Cambridge.
- Stive, M.J.F., 1980. Velocity and pressure field of spilling breakers. Proc. 17th Int. Conf. on Coastal Eng., Sydney.
- Stive, M.J.F. and Wind, H.G., 1982. A study of radiation stress and set-up in the nearshore region. Coastal Eng., 6: 1-25.
- Svendsen, I.A., 1984. Mass flux and undertow in a surf zone. Coastal Eng., 8: 347-365.
- Svendsen, I.A., 1985. On the formulation of the cross-shore wave-current problem. Proc. European Workshop on Coastal Zones, Greece.
- Tennekes, H. and Lumley, J.L., 1972. A first course in turbulence. MIT Press, Cambridge, Mass.