MSD in fuselage lap joints

Requirements for inspection intervals for typical fuselage lap joint panels with Multiple Site Damage

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Master thesis
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Preface

This report describes the requirements for inspection intervals for fuselage lap joints. I fulfilled this project as my master thesis at the faculty of Aerospace engineering of the Technical University of Delft, during the period of August 1991 until July 1992.

Although research can bear some inherent frustrations, I experienced this time as an instructive and joyful period. Therefore I would like to thank all people and fellow students of the section B2. Special thanks to J. Snijders and C. Paalvast who helped to realize the tests. Also special thanks to Ir. R. Fredell for giving time saving advice and adequate criticism. The researchers at the NLR and Fokker deserve specific acknowledgment for showing interest in the project; Especially A.U. de Koning and C. Lof from the NLR who performed the finite element calculations and gave useful advice and suggestions for this project; The people from Fokker for producing the specimens, answering all my questions and supplying practical information.

Finally, I would like to thank Professor J. Schijve, supervisor of this project, who supplied constructive criticism, but moreover for giving motivating support and showing great enthusiasm.

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student at the faculty L&R, TU Delft
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Summary

Inspection of failed aircraft structures and fractography on fatigue specimens have shown that after a high number of flights, cracks can appear simultaneously in riveted lap joints. When these crack tips approach each other, a mutual interaction causes an unexpected fast crack growth and next coalescence of cracks (unzipping effect).

The term "Multiple site damage" (MSD) is used when the mutual interaction of two or more damages (cracks) is noticeable.

This report provides a model to predict the fatigue life of a riveted lap joint and the minimum necessary inspection interval for safe aircraft operation. A program (MSDSIM) has been developed to simulate the fatigue process and aircraft inspection. Input for this analysis are scatter data for initiation, the stress distribution between frames and the geometry of the structure.

The fatigue life of a typical lap joint can be divided into 3 stages:

1. Crack initiation
2. Crack growth
3. Link-up/residual fracture

The first stage (crack initiation process) is too complex to model solely by calculation techniques. Many (unknown) factors have a major influence on the initiation mechanism and therefore on the initiation time. Some of these are listed in the next table.
Factors that influence initiation time

<table>
<thead>
<tr>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riveting procedure</td>
</tr>
<tr>
<td>Riveting quality (human factor)</td>
</tr>
<tr>
<td>Rivet and lap joint geometry</td>
</tr>
<tr>
<td>Stress level</td>
</tr>
</tbody>
</table>

Nominal changes of these factors within the same production batch can be held responsible for scatter in initiation time for that specific batch. Operation experience and full scale tests, pointed out that scatter in initiation time is small and significant MSD may occur. For this reason it was considered necessary to do tests on realistic riveted lap joints to quantify the initiation time and corresponding scatter. The tested configuration however did not confirm the previous experience and a single dominant crack caused failure in 7 of 8 tested specimens (S-series specimens). These dominant cracks systematically occurred at the two outer rivets (edge effect) and the results are therefore not representative for a realistic fuselage panel in operation. Additional tests (T-series) supplied some more information on crack initiation mechanisms and the dependence on riveting intensity.

Calculations concerning initiation have been made to predict reinitiation after crack arrest. This has been done by correcting empirically found initiation times for local stress concentration (section 3.3).

An important parameter in calculating the second stage (crack growth) is the stress intensity factor. Stress intensity factors had to be calculated for an arbitrary cracked situation. The method that has been used, is compounding of known solutions. The calculations account for secondary bending, friction between the two sheets and stress redistribution due to local net section yielding.
The third stage (link-up/residual strength) has been modelled and verified by test results.

**The test program brought up the next conclusions:**

- There is a major influence of the riveting intensity on the initiation threshold; Production tolerances for the diameter of the driven head \((1.4 < \text{Driven head/Drivet Shank} < 1.6)\) can result in a difference in fatigue performance corresponding to a factor 3 in lifetime.
- This influence can be explained by a reduction of the stress concentration factor at the rivet sides, which causes a change in initiation mechanism.
- The ratio between the number of cycles until initiation and the number of cycles until failure is approximately 0.85 and more or less constant for all tested specimens (also for different riveting intensities).
- After the first link-up, the remaining life is less than 1.3 % of the total life \((\pm 1500 \text{ cycles for } \sigma=105 \text{ MPa})\). This means that cracks must be found before link-up, if inspection intervals of more than 1500 flights are being used.

**The simulation program has led to the following conclusions**

- The inspection interval can be increased with approximately 75%, if the fatigue life of an entire lap joint panel is simulated instead of the (over) conservative presumption of simultaneously appearance of cracks at all rivet sides.
- The major factor of influence on the interval for periodical inspections is the hoop stress. The hoop stress reduction due to frames and tear straps must therefore be taken into account.
- If the analyzed lap joint is inspected properly, cracks can be found using an inspection interval in the order of 4000 flights for the eddy current inspection technique and less than 2000 flights for visual inspections.
**List of symbols**

- **a** - half crack length
- **april** - crack length at proof test
- **acm** - critical crack length
- **A** - parameter distinguishing the slope of the Weibull distribution plotted on a "Weibull scale"
- **α** - Weibull scale parameter
- **B** - parameter distinguishing the location of the Weibull distribution plotted on a "Weibull scale"
- **β** - Weibull shape parameter
- **C_p** - factor in Paris equation
- **C_{pA}** - scale parameter in Weibull equation for scatter in crack growth rate
- **C_{pO}** - shape parameter in Weibull equation for scatter in crack growth rate
- **C_{pmin}** - location parameter in Weibull equation for scatter in crack growth rate
- **C_{SIF}** - correction factor (for the stress intensity factor)
- **D** - diameter
- **d** - diameter
- **ΔK** - range over which the stress intensity factor fluctuates
- **e** - eccentricity of crack
- **I** - inspection period
- **K** - stress intensity factor
- **K_I** - mode one stress intensity factor
- **K_c** - critical stress intensity factor
- **K_t** - stress concentration factor
- **l** - crack length (from hole perimeter to tip)
- **l_0** - initiation crack length
Symbols

- $m$ - total number of tested specimens or initiation sides.
- $N$ - number of cycles
- $n$ - number of linked-up holes
- $N_0$ - number of cycles until first possible initiation
- $P$ - rivet load
  - probability of detection
- $P_{\text{cum.}}$ - cumulative probability of detection
- $R$ - radius
  - stress ratio ($= \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}$)
- $\sigma$ - remote stress
- $\sigma_{\text{n.s.}}$ - net section stress
- $\sigma_u$ - ultimate stress
- $\sigma_{0.2}$ - yield stress
- $s$ - standard deviation
- $S$ - remote stress
- $S_{\text{bypass}}$ - bypass stress
- $St$ - space between rivet holes
- $t$ - sheet thickness
- $W$ - space between rivet holes
  - finite width in cracked specimen
- $x$ - coordinate
- $x_0$ - Weibull location parameter
1. Introduction

In the past several catastrophic accidents occurred due to explosive decompression of aircraft pressure cabins. In some of these cases (Aloha accident 1988) investigation of the wreckage revealed that fatigue cracks had started to grow from several rivet holes at the same time. If these cracks emanate from rivet holes in the same row, mutual interaction between cracks can cause an unexpected fast crack growth. Because these cracks can suddenly coalesce and form one large crack, the time that is necessary for a crack to grow from "detectable" to "critical" can be significantly shortened (unzipping effect). Consequently, the critical crack length can be (and has been) reached before detection.

If mutual interaction between two or more different damages (cracks) is noticeable, the term "multiple site damage" (MSD) is used.

In order to cope with this problem two different philosophies can be applied:

1. "Safe life": Design the aircraft in such a way that MSD will not occur within the operational life of the aircraft. This means that the aircraft should be retired (for economical reasons), before fatigue cracks start to develop. Consequently, special inspection requirements for MSD are not necessary.

2. "Damage tolerant": Allow the occurrence of MSD within the operational life of the aircraft, but find the damage before a critical situation arises. This means that special and compulsory inspections have to be performed to find MSD before failure.

Chapter 2 gives a brief review of possible methods that can be used to avoid catastrophic accidents due to MSD in view of these philosophies.
For both cases (safe life and damage tolerant), calculations and tests should be carried out to ensure integrity of the structure during operation. This project is focused on the damage tolerant philosophy (crack monitoring). A computer program (MSDSIM) has been developed to simulate the fatigue life of a typical fuselage lap joint and quantify "safe" inspection intervals (chapter 3 and 4).

A test program to support the analysis is described in chapter 5. Nine 500 mm x 1000 mm and three 125 mm x 250 mm riveted lap joint specimens have been fatigue tested and continuously monitored.
2. Review of possibilities to deal with MSD

The most important factor of influence on the structural integrity of an aircraft from the point of view of MSD, is the possibility of sudden link-up of collinear cracks and next strongly increased crack growth (unzipping effect). Moreover, "flapping" might not occur at places where it was expected, because the original resistance along the row of holes (due to frames or tear straps) is strongly reduced by cracks ahead of the critical crack. As opposed to a single crack situation, the cracks can still be small just before a critical situation arises. This means that the damage is difficult to find by inspection.

Another disadvantageous effect of MSD is the fast crack growth; mutual interaction between approaching crack tips causes higher stress intensity factors and therefore higher crack growth rates. This interaction effect should be taken into account if accurate crack growth predictions are needed [23]. This increase in crack growth rate makes that the critical situation can arise significantly sooner than in the case where there is no interaction.

To maintain the same level of safety, one or more of the following measures should be taken:

- Retire the aircraft when its original crack free design life has been reached
- Reduce the inspection interval
- Improve the inspection method
- Find an alternative way to detect the crack before a critical situation arises

*Flapping* is the change of direction of the crack when it approaches a frame or tear strap. Flapping can in some cases cause safe decompression. Some aircraft manufacturers used to argue that this phenomenon provides a sufficient back-up for safety.
In the next 3 paragraphs, 3 possible methods to control this problem are described, that are currently being analyzed and assessed by researchers, manufacturers, airlines and airworthiness authorities.

Section 2.4 supplies an alternative method that might especially be interesting for "hidden" cracks on the inner (less inspectable) side of the aircraft.
2.1 Retire the aircraft: safe life

A measure which seems obvious and reasonable is the retirement of the aircraft, at the moment that its original (crack free) design life is reached (the exploitation of the aircraft has originally been planned according to this goal design life). On the other hand it seems unreasonable to retire an aircraft in good condition. Furthermore, if retirement of the aircraft after a fixed number of flights becomes mandatory because of safety arguments (non-detectability of MSD), this seems to support the safe life philosophy. At this moment, airworthiness authorities only allow this philosophy for a few structural elements like the gear and in some cases engine mounts.

At this moment however, insufficient arguments are available to impose such rigorous airworthiness regulations on the airlines. On the other hand should more catastrophic accidents occur before any effective measures are taken?

2.2 Proof testing: damage tolerant

In order to avoid frequent (and expensive) inspections, the concept of "proof testing" regained interest as a possible alternative for NDI (non-destructive inspection). As opposed to crack monitoring, proof testing is a destructive inspection method.

Proof testing for a fuselage structure means applying the limit load times a "proof factor" (e.g. 1.33 or 1.5); If the structure fails, the critical crack length (or combination of crack lengths in the case of MSD) was present and repairs should be made (if still possible). If the test did not "detect" any critical defects, the aircraft can be used for a number of flights before a new proof test should
be performed. This proof test interval is in the order of 250 to 1500 flights for commercial aircraft (depending on the proof factor and the type of construction considered). The opinions on the reliability of this method are diverse.

The major advantage of this method from the safety point of view is that all crack configurations are covered (although the test interval depends on the crack configuration).

Proof testing brings another important advantage. Because the periodic overloads cause compressive residual stresses at the crack tip, crack growth is strongly retarded. The next two figures summarize the principle of the method.

**Figure 2.1:** residual strength of structure

**Figure 2.2:** crack growth curve

where,

- $\sigma_{op}$ = operational stress
- $a_{pr}$ = half crack length at proof test
- $i_{max}$ = maximum inspection interval
- $\sigma_{pr}$ = proof test stress
- $a_{cr}$ = critical crack length

For a more detailed treatise of this subject the reader is referred to [1].
2.3 Crack monitoring: damage tolerant

The method that is currently being used for longitudinal fuselage lap joints, is visual inspection, or in some cases eddy current inspection of critical places. Some times "critical places" can be major parts of fuselage lap splices. Although good inspection procedures (visual or NDI) might be available, the large size of the fuselage and the human factor are disturbing aspects. Research on NDI procedures to eliminate the human factor are considered to be most worthwhile. Apart from improved eddy current techniques, other NDI methods may offer solutions (refined leakage tests, X-ray under fuselage pressure loads).

For all techniques, the prime question remains,

What would be the appropriate inspection interval, based on which criteria?

An other problem for airworthiness authorities will be the supervision over airlines that the inspections are actually being carried out and proper maintenance is performed. If MSD is being handled by more intensive inspections and retirement becomes an economical decision, sale of old aircraft should be handled with great care. Because of increasing maintenance costs, an airline can decide to sell an old aircraft and purchase a new one. But an obvious question then is, which airline would buy a second-hand aircraft which is apparently more expensive to operate than a new one? Obviously, airlines are tempted to cut down on maintenance costs and this has in some cases led to bad maintenance. Examples of bad maintenance have been found at several places. Schiphol airport lately refused to release an aircraft because of its bad state of maintenance. This is disastrous for the integrity of an aging fleet and should definitely be prevented.
2.4 Alternative: automatic crack detection by crack sensors

This method is based on the idea to place "crack sensors" at critical places where cracks are expected. This sensor should be able to "detect" a crack and send this information to an aircraft operator. Figure 2.2 gives the general look of how such a sensor could be implemented. The sensor consists of an electrical or optical transmitter of information (conductive material or glass tube), that breaks when a crack originates from a rivet hole.

![Diagram of crack detection sensors](image)

Figure 2.2: automatic crack detection by crack sensors

This principle has been used in laboratories to perform automatic measurements of crack lengths. A necessary condition for proper operation is that the sensor should be effectively glued to the structure and that it does not come off.

A condition to make it economically feasible is that the sensor can be easily
implemented and weighs little. It could be interesting to perform a feasibility and reliability study on this method. Especially for less detectable and critical places like the inner critical row of a longitudinal lap splice (see figure 2.2), the method could be a relatively cheap alternative.
3. Theory for quantifying "safe" inspection intervals by simulation

As has been mentioned in chapter 2 of this report one of the main problems for operators, manufacturers and airworthiness authorities is to propose and quantify measures to ensure structural integrity of aging aircraft. Reduction of inspection intervals is a measure that has often been suggested. In this section an approach for quantifying "safe" inspection intervals is proposed. A computer program (MSDSIM) has been developed to accomplish this (a manual for this program is included in appendix A). "MSDSIM" has been developed to simulate the fatigue life (constant amplitude) of a part of a fuselage skin structure (figure 3.1). By simulating inspections, an estimate of the inspection period can be found for a minimum required level of detectability (e.g. 98% cumulative probability of detection: see section 3.9). Figure 3.2 shows the basic flow chart for this program.

Section 3.1 summarizes the most important assumptions that have been made to perform the analysis and compares the proposed model to a previously developed and similar model by D. Broek [2].

Sections 3.3 to 3.9 discuss all separate stages of the analysis that are used to simulate the fatigue life and inspection of a typical fuselage lap joint panel.

Finally sections 3.2 and 3.10 describe the required in- and output of the analysis.
skin thickness \( t = 0.04 \text{ inch} \); tear strap \( = 0.04 \text{ inch} \); pressure \( p = 8.5 \text{ psi} \)

Figure 3.1: typical fuselage stiffened panel for analysis (figure from [2])
Figure 3.2: Flow chart for computer simulation program "MSDSIM"
3.1 Summary of most important assumptions (comparison with model by D. Broek)

To simplify the assessment of the validity of this analysis the most important assumptions have been summarized in table 3.1. A comparison with a previously developed and similar model by D. Broek [2] is included as well.

<table>
<thead>
<tr>
<th>Table 3.1: Most important assumptions (comparison with model by D. Broek)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Subject</strong></td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td><strong>Initiation</strong></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
| | 6. Damage accumulation is accounted for by the Miner rule: \( \varepsilon_v/N = 1 \) (The crack configuration changes continuously). | \[
\frac{N}{N_{\text{from Fig. 3.5}}} = n \times N_s \times 0.5 \times N
\]
| | | \( N \) is now determined from figure 3.5 for all rivers and depends on the local remote stress. If \( n/\text{N} \) of one of the river sides is larger than the assigned \( \varepsilon_v/N \) value, this river is initiated as well. |
| | **Crack growth** | | 3. The influence of other damage on the initiation period is accounted for by raising the remote stress in figure 3.5 according to the mean net section stress rise over three river spacings (e.g., if the mean net section stress is doubled = remote stress (5) is doubled). |
| | 1. Simplification of the geometry: use of \( W_{\text{effective}} \). See figure 3.13 | | Idem except that. |
| | 2. Crack is a through crack. | | * Friction is not accounted for |
| | 3. Friction between the two plates is accounted for by reducing the river bearing loads with a percentage. | | * A different Weibull distribution is used (based on the same test data) |
| | 4. Secondary bending is accounted for by an empirical formula. | | |
| | 5. Scatter on \( C_\text{p} \) values is assumed in the equation, \( da/dN = C_\text{p} D_\text{N}^p \), using previous test data (figure 3.20) | | |

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### Table 3.1: Theory for quantifying "safe" inspection intervals by simulation

<table>
<thead>
<tr>
<th>Subject</th>
<th>Proposed model</th>
<th>Model by D. Broek</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link-up and residual</td>
<td>1. Link-up occurs if one of the next conditions is satisfied,</td>
<td>1. The link-up criterion was taken as the exceedance of net section yield, with some accounting for stress redistribution.</td>
</tr>
<tr>
<td>strength</td>
<td>- Kθ &gt; Kic</td>
<td>- The latter was effected by calculating net membrane section stress at the fastener stations averaged over three fastener spacings.</td>
</tr>
<tr>
<td></td>
<td>- The ligament is smaller than 1 mm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Residual fracture occurs if one of the next conditions is satisfied,</td>
<td>2. Residual fracture occurs if the number of link-ups is larger than 10 (2a = 250 mm).</td>
</tr>
<tr>
<td></td>
<td>- Total net section stress exceeds ( \sigma_{0.2} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- One of the cracks becomes larger than a pre-defined critical crack length (default: 2a = 500 mm)</td>
<td></td>
</tr>
<tr>
<td>Stress distribution</td>
<td>1. The stress distribution is taken from FEM calculations from [21].</td>
<td>1 Idem</td>
</tr>
<tr>
<td>between frames</td>
<td>2. Stress redistribution due to local net section yielding is accounted for (see section 3.6)</td>
<td>2. Stress redistribution is not accounted for.</td>
</tr>
<tr>
<td>Inspection simulation</td>
<td>1. A Weibull distribution is assumed for the probability of detection versus crack length. This curve is based on experimental data of Lewis et al (see figure 3.29). The curves are based on 96 inspectors.</td>
<td>1 Idem</td>
</tr>
<tr>
<td></td>
<td>2. The crack length on the x-axis of figure 3.29 is assumed to be the actual crack length at one of the rivet sides. Both sides are regarded separately.</td>
<td>2. The crack length on the x-axis of figure 3.29 is assumed to be tip-to-tip length minus the hole diameter.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. A refinement has been made to account for the human factor. Every time a crack is detected, the curve is shifted towards better crack detection. After a non-detect, the curve is shifted towards worse crack detection. The changes are limited to a maximum deviation from the default settings.</td>
</tr>
</tbody>
</table>

As can be seen from table 3.1, the major differences concern the initiation simulation. These changes in initiation procedure have been made because of the next three considerations,

1. In the model of D. Broek, \( \Sigma n/N = 0.5 \) is always assigned to a rivet with a higher than average stress level which is not a random assignment.
2. Figure 3.5 is based on specimens which do not have the same geometry as the FEM model. Moreover, figure 3.5 is a cycles to failure curve and not a cycles to initiation curve.
3. The solid line in figure 3.5 differs a factor 3 (in cycles) from the mean line through the test results. This arbitrary factor is used to compensate for laboratory influences (frequency, \( R = 0.1 \), lab. air). Using an other factor (shifting the line up or down) has a major influence on the outcome of the
program (inspection interval). Therefore, an arbitrary choice is of major influence on the results.

An other improvement of the developed computer program over the program by D. Broek is that test- and calculation results obtained by the user, can easily be incorporated in the program (e.g. stress distribution and initiation scatter curves). For a more detailed description of the possibilities of the program, the reader is referred to appendix A.

Figure 3.3: initiation scatter curves
Chapter 3: Theory for quantifying "safe" inspection intervals by simulation

Figure 3.4: Weibull distribution of $\Sigma n/N$ (figure from [21])

Figure 3.5: S-N curve used by D. Broek to model initiation scatter (figure from [2])
3.2 Required input data

The most important input for the analysis (program) consists of the next 4 items,

1 Construction dimensions: rivet pitch, rivet dimensions, frame spacing, distance between rivet rows etc.
2 The differential pressure and stress distribution in the structure: $\sigma_{\text{membrane}}$, $\sigma_{\text{bending}}$ and rivet loads
3 Material properties and baseline data
4 Scatter curves for initiation time (P-S-N curve) and crack growth properties

These items should be available from the manufacturer (1, 2 and 3), or obtained by fatigue tests (3 and 4). In order to find the stress distributions, finite element calculations can be made. In this analysis the finite element results of Foster-Miller Inc. [21] have been used as input data for the computer program.

3.3 Crack initiation

Predicting initiation times in riveted joints is difficult and the results are not very accurate [7]. Because of this, initiation times and initiation scatter curves for the undamaged structure should be found by tests* and used as input for the analysis.

---

* Crack initiation in riveted lap joints is induced by fretting. This means that the initiation period is strongly affected by detail rivet and lap joint geometry, the riveting procedure and the use of anti fretting sealing. In order to get realistic initiation data, the specimen configuration used to obtain initiation data, should be as close to (preferably identical to) the real structure as possible.
The central question in this section, is how to handle continuing damage. That is, when the crack arrests at a fastener hole, how does the analysis deal with the crack reinitiation at the other side of the hole. A method to deal with this problem for an arbitrary damaged configuration, is proposed and discussed in this section. Before, the definition of the initiation period and some general aspects influencing initiation time are given.

**Initiation period definition**

The crack initiation period is defined as the time that is necessary for the crack to grow from length 0 to \( l_0 \) (see figure 3.6).

![Figure 3.6: definition of initiation period](image)

The reason to choose this value is simply because for smaller crack lengths the crack is extremely hard to find and therefore the history of the crack is of no meaning (in this analysis the crack is not supposed to be detectable until \( l_0 \) is reached which is correct for the current inspection procedures). Another reason is that crack growth calculations based on fracture mechanics are inaccurate for cracks smaller than \( l_0 \).

**Factors of influence on initiation time: P-S-N curve**

The time for crack growth to a length \( l_0 \) strongly depends on the next 7 factors:

1. Gross stress
2. Lap joint dimensions
3. Type of rivet
4. Use of anti-fretting sealing

20
5 Riveting procedure/quality
6 Scatter in material fatigue properties
7 Influence of other damage

Ad 1
The first factor is obvious and the dependence of the initiation period on the gross stress can be expressed by generating an S-N curve (see figure 3.7), where N is the number of cycles until initiation. The specimens used to generate this curve should be of the same detail design as used in the calculations.

![Figure 3.7: F-S-N curve for initiation](image)

Ad 2, 3, 4
Factors 2, 3, and 4 are known parameters for each analysis. In principle initiation tests should be performed for every new configuration. Whether initiation data or scatter curves from tests on other (similar) configurations can be used as an approximation for the actual configuration is a relevant question. In the author's opinion tests should be performed in all cases to estimate the threshold values for crack initiation at several stress levels (S-N₀ curve for initiation). This curve
corresponds to the line of \( N_{\text{min}} (= N_0) \) in figure 3.7. To validate the corresponding scatter curves at all of these stress levels would require an extensive test program. The increase in accuracy of the analysis (compared to estimated scatter curves) is probably not large enough to justify such an expensive test program. Moreover, initiation scatter can be estimated conservatively (in the worst case there is no initiation scatter and only a dependence of \( N_0 \) on the stress level).

**Ad 5, 6**

Factor 5 and 6 are accounted for by assuming scatter in test results as is indicated in figure 3.3 and 3.7; Before the analysis starts, the program assigns a fixed cumulative probability* to each hole side at random. Since the riveting procedure and quality can cause enormous differences in initiation time [10], only nominal differences in riveting quality or procedures are covered by these scatter curves (e.g. due to tolerances in the driven head diameter or rivet and riveting tool dimension). Scatter due to quality or procedure differences between several production series (i.e. different aircraft) can only be accounted for by a proper safety factor (initiation threshold) and conservative initiation scatter curves (low scatter; worst case).

The shape of the "scatter curve" is quite arbitrary, but can be validated by test results. The used scatter curve (Weibull distribution) and the required number of tests that is necessary to validate the scatter curve are discussed in appendix B.

**Ad 7: influence of other damage (problem of (re)initiation)**

Finally, the influence of other cracks is accounted for by using linear-elastic stress concentration factors. This directly implies that, as soon as the crack is initiated ahead of the rivet (very good clamping) the theoretical stress con-

*This value is independent of the stress level. This assumption is equivalent with the statement that "a hole side initiated as no. i of n hole sides (isn) at a certain stress level, initiates as no. i at any stress level".
centration factor at the two critical places indicated in figure 3.8 is of no direct meaning to the initiation time and the analysis looses its validity. However, this "strange" initiation behaviour does rarely occur for countersink rivets which are mostly representative for fuselage lap joints.

![Figure 3.8: critical places for initiation](image)

Because the initiation tests have only been related to the gross stress, an equivalent gross stress has to be found, representing an equivalent undamaged situation \( (S_{\text{gross, equivalent}}) \) for which the initiation scatter curve is known. This value of \( S \) is used in the P-S-N curve to find the number of cycles until initiation. Its value is determined by the next formula:

\[
S_{\text{gross, equivalent}} = S_{\text{gross}} \times \frac{K_t}{K_t^*} \tag{3.1}
\]

where, \( K_t^* \) is the stress concentration factor for the undamaged riveted lap joint (tested configuration) and \( K_t \) is the actual stress concentration factor of the cracked configuration. In this way a function \( F(K_t/K_t^*) \) is created, which is 1 for the undamaged situation and increases as soon as a crack is present at the opposite side of the hole or at one of the neighbouring rivets.
This linear elastic "scaling" is not only allowed as long as the peak stress is less than $\sigma_{0.2}$, but also for situations with small scale yielding. The arguments (by Brussat) that support this statement, are taken from [7] and repeated here.

**Some observations on the notch root neighbourhood**

In this section the elastic stress in the neighbourhood of an internal notch in a plate is considered. Figure 3.9 shows an isolated elliptical hole of major diameter $2A_0$ and minimum radius of curvature $R_0$ in an infinite plate subjected to a uniform tension field $S$ normal to the direction of the major diameter.

For this classic case, the peak elastic stress at the edge of the notch is given by

$$\sigma_{22}(0) = (1 + 2\sqrt{A_0/R_0}) S = K_c S \quad (3.2)$$

The expansion

$$\sigma_{ij}(x_1) = \sigma_{ij}(0) + x_1 \sigma'_{ij}(0) + \ldots \quad (3.3)$$

is a valid representation of the elastic stresses for small values of $x_1/R_0$, where $x_1$ is measured from the notch root and prime denotes differentiation with respect to $x_1$. The solution, expressed in the form of equation 3.3, is given by

$$\frac{\sigma_{22}(x_1)}{\sigma_{22}(0)} = 1 - \left(1 + \frac{1}{2K_c}\right) \frac{2x_1}{R_0} + \ldots \quad (3.4)$$

$$\frac{\sigma_{33}(x_1)}{\sigma_{22}(0)} = \frac{2x_1}{R_0} + \ldots \quad (3.5)$$

$$\frac{\sigma_{12}(x_1)}{\sigma_{22}(0)} = 0 \quad (3.6)$$

Note that the term $1 + 1/(2K_t)$ is approximately constant and equal to unity when $K_t \geq 3.0$. There is no other dependence on $K_t$ in equations 3.4 to 3.6.
Thus, in a manner similar to the way the crack tip stress intensity factor governs
the elastic stress field at a crack tip, the peak elastic notch stress \( \sigma_{22}(0) \) governs
the elastic stress field for all \( K_t \geq 3.0 \) notches of equal notch root radius \( R_0 \).

Stated another way, the elastic stress distribution near any long elliptical hole of
minimum radius \( R_0 \) and peak elastic notch stress \( \sigma_{22}(0) \) is approximately identi-
cal to the elastic stress distribution near a circular hole having the same
minimum notch radius and peak elastic stress. This local equivalence of notch
root stresses occurs despite the fact that the \( K_t \) values for the circular and
elliptical holes may be vastly different. The elastic notch-root stress field for a
long slot formed by collinear circular holes connected by a crack, is likewise
expected to be equivalent to that of one circular hole, provided notch radius and
peak stress match. This is expected because of the geometric similarity between
such a slot and a long ellipse, although exact stress gradient expressions for a
long slot are not available.

The equivalence among notch stresses for equal-radius internal notches with
different geometries and \( K_t \) values has important implications for analysis
methods for fatigue crack nucleation and the fatigue propagation of tiny cracks
at notch roots.

Referring to figure 3.9, let \( \rho \) denote the region in the notch-root neighbourhood
of an arbitrary elliptic notch within which the exact elastic stress field \( \sigma_{ij} \) is
essentially equal to \( \sigma_{ij}^{(App)} \), the notch stress field for a circular hole of the same
radius. Suppose that some inelastic behaviour, such as plasticity, fatigue damage,
or microcracking occurs within the subregion \( \delta \), embedded within \( \rho \). If \( \delta \) is small
enough, the inelastic behaviour will have no effect outside the region \( \rho \).
Under this condition of small-scale inelasticity, the mechanical response of the material (i.e., true plastic strain, rate of fatigue damage, or propagation rate and stress intensity factor of a microcrack) near the root of an arbitrary elliptic notch will totally be governed by $\sigma_{ij}^{(App)}$; in other words, it will be the same as that for a circular notch of the same material and notch radius, similarly fabricated and subjected to the same environment and peak elastic notch stress history.

This statement and its supporting argument are analogous to the classical Irwin-Rice arguments for why the elastic stress intensity factor history governs the propagation behaviour of a crack for cases of small scale yielding.

![Diagram of notch root neighborhood, long elliptical notch (figure from [7])](image)

**Stress concentration factors $K_t^*$ and $K_t$**

The assumptions and calculations needed to find $K_t^*$ and $K_t$ as defined in equation 3.1, are explained in the next two sections.
Stress concentration for the undamaged situation: \( K_i^* \)

\( K_i^* \) is the stress concentration factor for the next problem (simplified model for riveted lap joint).

![Simple physical model for a riveted lap joint](image)

**Figure 3.10:** simplified physical model for a riveted lap joint

In figure 3.11 a finite width plate is used for \( W \), but the procedure is also valid for a continuing row of holes. The simplified, finite width problem can be split up into 2 basic problems for which solutions are available.

![Two sub-problems for undamaged configuration](image)

**Figure 3.11:** two sub-problems for undamaged configuration (solutions by Schulz)
The results for the complete problem (a), are plotted in figure 3.12 as a function of \( d/W \) for values of \( S_b/S \) of:

1 (no load transfer by rivets)
\( \frac{1}{2} \) (2 rows of rivets; equal load transfer per row)
\( \frac{2}{3} \) (3 rows of rivets; equal load transfer per row).

For the tested configuration (3 rivet rows), \( d/W_{eff} = 0.16 \) and \( S_b/S \) is assumed to be 0.63 (<\( \frac{2}{3} \) because the middle row transfers less load). From figure 3.12, \( K_t^* \) for \( S_b/S = \frac{2}{3} = 0.63 \) becomes ± 4.3.

Figure 3.12: \( K_t^* \) as a function of \( d/W \) and \( S_b/S \) (solution by Schulz)
Stress concentration for an arbitrary cracked situation: $K_t$

Consider the next arbitrary situation:

![Diagram](image)

Figure 3.13: stress concentration factor for cracked situation

This is a typical problem for which the stress concentration factor should be determined. $W$ is again defined to account for neighbouring holes and possible cracks (as in figure 3.13). Using the principle of superposition, this problem is split up into 2 basic problems.

![Diagram](image)

Figure 3.14: two sub-problems for cracked configuration
Problem (b) can be split-up further as follows.

Figure 3.15: further division of problems 3.14 (b) into 2 basic problems: $K_{t,3}$ and $K_{t,4}$

From figure 3.15 follows that problem (b) of figure 3.14 can be split in the basic problems ③ and ④ ($K_{t,3}$ and $K_{t,4}$). This will turn out to be convenient later. The principle of superposition as used in figure 3.15 to split problem (b) of figure 3.14, is only valid for all points along the hole array. Since the maximum stress is supposed to occur on this line, $K_{t,1} = K_{t,2}$.
**Problem 1:** solution by Tada et al [5] and "similar ratio concept"

Tada et al gives a solution to the problem of an infinite sheet with 2 connected cracks (figure 3.16 (b)). An equation for this configuration fitted to the numerical results, given in [5], is

\[
K_c^{(3.16b)} = \left(1 + 2\sqrt{a/R}\right) \left\{ 1 + \frac{1}{6} \left[ \frac{2}{\pi} \arcsin\left(\frac{a-R}{a}\right) \right]^3 \right\} \tag{3.7}
\]

Next Brussat [8] uses a method he calls the "similarity ratio method" to account for the finite width and eccentricity. In a similar way, this method is used here,

---

**Figure 3.16:** similar ratio method for estimating the influence of the finite width and eccentricity

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The similar ratio method presumes that the ratio between the \( K \)-solution of 3.16c and 3.16d is equal to the ratio between the \( K_t \)-solutions of problem 3.16a and 3.16b.

\[
K_t^{(3.16a)} = K_t^{(3.16b)} \frac{K^{(3.16c)}}{K^{(3.16d)}} \tag{3.8}
\]

The \( K \)-solution to problem 3.16d is well known and equal to \( a/\pi a \)

Isida provides the results for \( K^{(3.16c)} \) (ref. [22]). These results can be divided into a correction for the finite width and a correction for the eccentricity.

\[
K^{(3.16c)} = K^{(3.16d)} \times F_{ecc.} \times F_{f.w.} \tag{3.9}
\]

Only the correction for the eccentricity has been taken from Isida. The correction for the finite width is replaced by the solution of an infinite sheet, containing a periodic array of collinear cracks (Irwin [5]):

\[
F_{f.w.} = \sqrt{\frac{W}{\pi a}} \tan \left( \frac{\pi a}{W} \right) \tag{3.10}
\]

This has been done because the problem in figure 3.13 is more similar to a periodically damaged structure. Therefore, the Irwin solution is likely to be more accurate. (The correction of equation 3.10 is less severe than the finite width correction by Isida, because an infinite sheet restricts lateral contraction; This reduces the principle stresses along the hole array)

Using equation 3.7 and the two corrections, the final solution becomes,

\[
k_t^{(3.16a)} = (1 + 2\sqrt{a/R}) \left\{ 1 + \frac{1}{6} \left[ \frac{2}{\pi} \arcsin \left( \frac{a - R}{a} \right) \right]^3 \right\} \times
\]

\[
\sqrt{\frac{W}{\pi a}} \tan \left( \frac{\pi a}{W} \right) \times F_{ecc.} \tag{3.11}
\]

N.B. In appendix D the accuracy of the similar ratio concept is verified.
Problem 2: solution by "similar ratio concept"

This problem (figure 3.15 ④) is not known analytically; Neither for finite nor for infinite sheets. Therefore, the similar ratio concept is used,

![Diagram of Problem 2](image)

Figure 3.17: similar ratio method for estimating $K_i$ of a strip with a growing crack emanating from a pin loaded hole

similar ratio concept:

$$K_e^{(3.17a)} = K_e^{(3.17b)} \frac{K_e^{(3.17c)}}{K_f^{(3.17d)}}$$  \hspace{1cm} (3.12)

$K/K(l=0)$ is solved by integrating known solutions of Tada [5] for a locally loaded crack. The calculation is further described in appendix D.
The correction for the finite width is made by again using the similar ratio method as follows.

![Diagrams showing different configurations of rivet loads](image)

Figure 3.18: similar ratio method for estimating the finite width correction

The dotted arrows in figure 3.18 are the original rivet loads. Because the solution of problem 3.18c is only solved for the configuration with the solid arrows (Irwin [5]: periodic array of collinear cracks), the rivet loads are moved as indicated in the figure.

The influence of the eccentricity can not be estimated in a reasonable way and is therefore not brought into account. The final solution becomes,

\[
K_t^{(3.18a)} = K_t^{(3.18b)} \frac{K_t^{(3.18c)}}{K_t^{(3.18d)}}
\]

(3.13)
Damage accumulation

Because of changing geometries (crack growth), the value of \( S \) used in the S-N curve changes all the time. Miner's Cumulative Damage Rule is used to account for the accumulated damage after each increment of cycles.

\[
\sum \frac{n}{N} = \sum \frac{n}{N} + \Delta \sum \frac{n}{N}
\]  

(3.14)

where \( \Delta \sum \frac{n}{N} = \frac{cycle\ increment}{N} \)  

(3.15)

\( N \) is determined using \( S_{gross,\ equivalent} \) (equation 3.1) and figure 3.3.

If \( \Sigma n/N = 1 \), the crack is initiated and the crack growth calculation starts.

For constant amplitude loading, the damage rate \((1/N)\) will change only very gradually, since the increase in \( K_c \) with the length of the crack is gradual. Thus the anticipated error associated with the application of Miner's Cumulative Damage Rule is small, since the local loading at the notch is very close to constant amplitude.

N.B. (1) It is not possible to apply scatter on the Miner rule, because available scatter data indirectly include scatter on initiation times and this has already been taken into account.

N.B. (2) Note that scatter data on the Miner rule might be completely due to scatter in initiation and crack growth properties. Consequently, there is no experimental proof (for this application!) that the Miner rule is not exact.
3.4 Crack growth

Once a crack is initiated, further crack growth is calculated by using fracture mechanics. The crack growth rate $\frac{da}{dn}$ is found by calculating the stress intensity factor for the present crack configuration. This stress intensity factor is next converted into a crack growth rate making use of baseline data. To account for scatter in these baseline data, crack growth properties are assigned to each crack before the analysis starts. The crack growth rate $\frac{da}{dn}$ is assumed to follow the Paris equation for stress ratio $R = 0$ as

$$\frac{da}{dn} = (C_p) (\Delta K)^3$$  \hspace{1cm} (3.16)

Analysis of crack growth data of older 2024-T3 alclad aluminium shows that an exponent of 3 in the equation is a good fit for thin sheets ($t=1$ mm).

A Weibull distribution for $C_p$ was taken as (figure 3.19)

$$P = 1 - e^{-\left(\frac{(C_p - C_{p_min})}{(C_{p_0} - C_{p_min})}\right)^{C_p_0}}$$  \hspace{1cm} (3.17)

or

$$C_p = C_{p_{min}} + (C_p^0 - C_{p_{min}}) \times \left[-\ln(1 - P)\right]^{(1/C_p_0)}$$  \hspace{1cm} (3.18)
Stress intensity factors
A relatively easy way is available to make estimations of the stress intensity factors for an arbitrary "MSD configuration" in a lap joint. This method is simple and was used for similar calculations by D. Broek, [2]. The influence of approaching cracks or holes is accounted for by defining a finite width W, similar as used for initiation calculations and illustrated in figure 3.20. This simplification is necessary because all kinds of inhomogeneous MSD configurations can arise (link-up; unequal crack growth; unequal initiation times).
Chapter 3: Theory for quantifying "safe" inspection intervals by simulation

Figure 3.20: definition of W

The crack length $2a = D + L_L + L_R$ is used to calculate stress intensity factors from the next 2 problems.

Figure 3.21: two sub-problems for stress intensity factors
Problem 1

Isida provides the results for this problem (ref. [22]). These results are built up of the basic solution for an infinite sheet \( K = \sigma / \pi a \) and two corrections:
- Finite width correction \( F_{L,W.} \)
- Correction for eccentricity \( F_{ecc.} \)

\[
K^{(3.21a)} = \sigma \sqrt{\pi a} \times F_{occ.} \times F_{L,W.} \tag{3.19}
\]

Only the correction for the eccentricity has been taken from Isida. The correction for the finite width is replaced by the solution of an infinite sheet, containing a periodic array of collinear cracks (Irwin [5]):

\[
F_{L,W.} = \sqrt{\frac{W}{\pi a} \tan \left( \frac{\pi a}{W} \right)} \tag{3.20}
\]

This has been done because the problem in figure 3.20 is more similar to a periodically damaged structure. Therefore, the Irwin solution is likely to be more accurate. (The correction of equation 3.20 is less severe than the finite width correction by Isida, because an infinite sheet restricts lateral contraction; This reduces the principle stresses along the hole array)

Problem 2

This problem is solved in two steps. First the solution is generated for an infinite sheet (fig. 3.22), by integrating the solution of Tada [5] as done in appendix D.
Next the correction for the finite width is found by using the similar ratio concept as was done earlier in this chapter for stress concentration factors.

![Diagram showing stress intensity factors](image)

The influence of the eccentricity can not be estimated in a reasonable way and is therefore not brought into account.

The final solution becomes,

\[
K_I^{(3.23a)} = K_I^{(3.23b)} \frac{K_I^{(3.21c)}}{K_I^{(3.23d)}} = K_I^{(3.22b)} \sqrt{\frac{\pi a}{W_{eff}}} \frac{\tan\left(\frac{\pi a}{W_{eff}}\right)}{\sin\left(\frac{\pi a}{W_{eff}}\right)} \frac{cosh(0)}{W_{eff}} \tag{3.21}
\]
Secondary bending

A correction for secondary bending is made in the same way as done by D. Broek [2],

\[
K = K_{\text{bending}} + K_{\text{basic}}
\]

\[
K_{\text{bending}} = \beta_{\text{bending}} \sigma_{\text{bending}} / \pi a
\]

where \( \beta_{\text{bending}} \) is the geometry factor. From [3] this factor becomes,

\[
\beta_{\text{bending}} = 0.39 \left( 1.0 + 0.16 (a/H)^2 \right), \quad \text{for } v = 0.3
\]  \(3.22\)

3.5 Link-up

Link-up of approaching crack tips can be caused by two phenomena.

1. \( K \) (crack tip) \( \geq K_c \)
2. \( \sigma_{\text{n.s.}} \geq \sigma_{\text{cr.}} \) where \( \sigma_{0.2} \leq \sigma_{\text{cr.}} \leq \sigma_u \)

Although the second criterion has shown to be applicable to residual strength calculations of complete specimens (see section 3.8 and appendix C), an evaluation of test specimens, using fractography revealed that this is not the case for link-up problems (appendix C). Stress redistribution due to local net section yielding is the reason that this criterion is not applicable. For this reason, link-up is supposed to occur if one of the next two conditions is satisfied,

1. \( K_I > K_c \)
2. The ligament between two cracks is smaller than 1 mm
3.6 Stress redistribution

Consider the next cracked situation in the critical rivet row. ((i) indicates that the quantities are local)

![Diagram showing stress redistribution](image)

*Figure 3.24: net section stress in the ligament*

If the net section stress ($\sigma_{n.s.}$) is constant* over the ligament $l(i)$, the next formula gives its value necessary to equilibrate the local remote tension $S(i)$.

$$\sigma_{n.s.}(i) = S(i) \frac{Pitch}{l(i)} \quad (3.23)$$

As $l$ decreases, $\sigma_{n.s.}$ increases until it reaches $\sigma_{0.2}$. Before this happens, the crack growth will cause some remote stress redistribution. The present analysis however, does not account for any stress redistribution up to the point where $\sigma_{n.s.}$ reaches $\sigma_{0.2}$. At this point, stress redistribution must be accounted for.

* Since 2024-T3 is a very ductile material, the stress distribution in the net section spreads out as soon as net section yield occurs and becomes more constant as $\sigma_{n.s.}$ increases.
because of a significant drop in local stiffness. In figure 3.25 the stiffness is assumed to be 0 as soon as $\sigma_{0.2}$ is reached.

Figure 3.25: Schematic representation of the stress-strain curve for 2024-T3 Alclad

According to figure 3.25, $\sigma_{n.s.}$ will not increase as soon as the ligament $l$ decreases below the point at which $\sigma_{n.s.} = \sigma_{0.2}$ (stiffness=0).

Consequently, the local remote stress will drop according to equation 3.23 ($\sigma_{n.s.}(i) = \text{constant} = \sigma_{0.2}$ and $l(i)$ decreases due to crack growth). The difference in remote stress ($\Delta S$) has to be transferred by other ligaments in the structure. To be able to quantify the stress redistribution, the next 3 assumptions were made,

1. As soon as $\sigma_{n.s.}$ reaches $\sigma_{0.2}$, the net section stress is constant over the ligament (discontinuous drop in stiffness as indicated in figure 3.25).
2. The difference in remote stress ($\Delta S$) is equally divided over the adjacent ligaments.
3. The maximum part of the original remote stress that can be redistributed is limited to a fixed percentage of the original remote stress.

In formulas,

\[ \sigma_{n.s.}(i) = S(i) \frac{Pitch}{l(i)} \quad (for \ \sigma_{n.s.}(i) < \sigma_{0.2}) \]
\[ \sigma_{n.s.}(i) = \sigma_{0.2} \quad (for \ \sigma_{0.2}(i) = \sigma_{0.2}) \quad (3.24) \]

If net section yielding occurs:

\[ dS(i) = S(i)_1 - S(i)_2 = \frac{\sigma_{0.2}}{Pitch} (l(i)_1 - l(i)_2) \]
\[ = \frac{\sigma_{0.2}}{Pitch} \quad dl(i) \quad \Rightarrow \quad \frac{dS(i)}{dl(i)} = \frac{\sigma_{0.2}}{Pitch} \quad (3.25) \]

Where, the indices 1 and 2 respectively denote the configuration at \( N_1 \) cycles and at \( N_2 \) cycles (\( N_2 = N_1 + dN \)).

Since the crack growth rate can be expressed by the Paris equation, the stress redistribution rate can also be calculated.

\[ Paris: \quad \frac{da_{(i,s)}}{dN} = C_{p(i,s)} \times \Delta K_{(i,s)}^p \quad (3.26) \]

Where:  
- \( i \) stands for the \( i^{th} \) rivet  
- \( s \) (side) stands for either the left (l) or right (r) crack tip.

\[ Figure \ 3.24: \quad dl(i) = da_{(i-1,r)} + da_{(i,l)} \quad (3.27) \]

Combining equation 3.25, 3.26 and 3.27 yields,

\[ \frac{dS(i)}{dN} = (C_{p(i-1,r)} \times \Delta K_{(i-1,r)}^p + C_{p(i,l)} \times \Delta K_{(i,l)}^p) \times \frac{\sigma_{0.2}}{Pitch} \quad (3.28) \]
Finally, the stress redistribution is limited by the next equation,

\[ S \geq \text{limit} \times S_{original} \quad (\text{limit} < 1) \quad (3.29) \]

### 3.7 Friction

Friction between the facing surfaces of the two sheets can contribute significantly to the load transfer. Although the fraction transferred by friction is unknown, it can be a variable parameter in the analysis. In the program, this parameter is defined as the load transferred by friction divided by the total load transfer by that specific row. The influence of the friction on initiation time and crack growth is modelled by decreasing the rivet loads according to the friction parameter. The decreased rivet loads decrease the stress concentration factor (initiation) and the stress intensity factor (crack growth).

As a crack starts to grow, the rivet is loosened and the friction decreases. The default setting in the program is a linear decline of 50% for the uncracked situation until 0% for a tip-to-tip crack length equal to the rivet pitch.
3.8 Residual strength

Final fracture occurs if one of the next conditions is satisfied,

1. $\sigma_{\text{n.s.}} \geq \sigma_{0.2}$ (for all ligaments)
2. Crack length $> \text{critical crack length (default = 500 mm)}$

![Diagram of residual strength](image)

Figure 3.26: net section yield criterion in lap joint panel

$\sigma_{\text{n.s.}} \cdot A_{\text{n.s.}} = t \int S(x) \, dx$

If $\sigma_{\text{n.s.}} \geq \sigma_{0.2}$, total failure occurs

3.9 Inspection simulation

After each simulated inspection, a simulated probability of detection should be known for all independent cracks. Afterwards, the cumulative probability of detection can be calculated for all cracks by the next formula.

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\[ P_{\text{cum}} = 1 - \prod_{i} (1 - p_i) \] (3.30)

where \( p_i \) is the probability of detection for the \( i \)th simulated inspection.

The variable \( p_i \) can be found making use of already existing data from Lewis et al [24]. The proposed probability of detection curve is taken equal to the one used in [2] and has the next shape.

![Graph showing probability of detection versus crack length]

Figure 3.27: probability of detection versus crack length

The Weibull distributions defined by the next equation,

\[ p = 1 - e^{-[(a - a_0)/(\lambda - a_0)]^\gamma} \] (3.31)
According to [2], these values are representative for easy access and low specificity which is typical for fuselage lap joints.

3.10 Required output

The output should consist of the next items:

- Curves of the cumulative probability of detection versus the inspection interval for different cracks (largest crack, average of all cracks etc.) for visual and eddy current inspections.
- Crack distribution between the frames at any stage in the fatigue life
- Crack growth curves of each independent crack
- Stress distribution between the frames for any stage in the fatigue life
4. Results of simulation program MSDSIM

To evaluate the program, some test results are included in section 4.1. These results correspond to a run with the default input settings. A print of the default settings is included in appendix E.

The outcome of the program evidently depends on the values of the input parameters. Because some of the input had to be estimated, a sensitivity analysis for changes of indefinite input data is performed in section 4.2.

**NOTE:** The results in the "$P_{\text{cum.}}$ versus $I$" graphs represent the results for only one crack. This is the largest crack, because that crack caused final failure of the lap joint.
4.1 Typical results

The output of the program consists of four major parts, that can be displayed and printed using the various menu options,

1. Cumulative probability of detection versus inspection interval
2. Crack distribution between frames
3. Crack growth curves
4. Stress redistribution caused by local net section yielding

The results for all four items are successively discussed below.

**Cumulative probability of detection versus inspection interval**

By selecting the option [1. View cum. Prob. of det. versus I.] of the "VIEW MENU", several graphs of this type can be displayed and saved on file for later use. The first option in the "P_{cum} menu" displays the results for the largest crack. This is the most important information since this crack caused final failure of the aircraft. Figure 4.1 (a) and (b) show the results for the default input settings for one simulated lap joint.
Figure 4.1 (a): cumulative probability of detection versus inspection interval for largest crack; default settings

Figure 4.1 (b): enlargement of figure 4.1 (a)
Two characteristics for this kind of output are clearly notable in this figure:

1. The eddy current inspections yield significant better results than the visual inspections.
2. Certain drop-rise patterns can be recognized as indicated by the solid (e.c. insp.) and dotted (visual insp.) lines.

The differences between visual and eddy current results are directly due to the assumed probability of detection curves for both inspection methods (section 3.9; figure 3.27). The choice of the inspection method now is (and should be) an economical decision; Visual inspections might be cheaper to perform, but need to be done more frequently than eddy current inspections to guarantee the same level of safety.

The sudden drops in cumulative probability of detection, are caused by the following reason; If the lap joint is inspected just before failure, the crack is large and likely to be found; For a slightly larger inspection interval, the same inspection will be just too late to perform (the lap joint is inspected one time less), and the cumulative probability of detection is not increased since the last inspection. The steady rise is caused by the fact that if the same number of inspections is performed, the corresponding inspection for a larger inspection interval is performed later in the fatigue life (consequently, the cracks are larger and easier to detect).

The location of the sudden drops depends on the moment of first inspection. Since the time until first crack initiation can not be estimated accurately, inspection intervals that are longer than the smallest inspection interval that causes an unacceptable cumulative probability of detection are of no relevance. The drop-rise patterns disappear if many runs are simulated and plotted in one graph (see figure 4.2).
The criterion for a "safe" inspection interval could be a cumulative probability of detection of at least 98%. Using figure 4.1, this would lead to an inspection interval of 7000 flights for eddy current inspections and 3000 flights for visual inspections. If the more stringent criterion of 100% cumulative probability of detection is imposed, the inspection intervals drop to 4000 cycles for eddy current inspections and to less than 2000 cycles for visual inspections.

These numbers are rough estimates, because only one lap joint has been simulated. Since other lap joints will have different fatigue lives (because of the random generator that has been built in in the program), a lot of these runs should be performed to obtain a complete set of data. This has been done for the default settings (10 runs) in figure 4.2.

Figure 4.2: cumulative probability versus inspection interval for 10 simulated runs; Default settings
To avoid a chaotic graph, only the eddy current inspection procedure will be considered from now. The "cloud" of points in this figure has a lower boundary (solid line) that should be used for the determination of the inspection interval. Using a 98% cumulative probability of detection, the inspection interval becomes approximately 5100 flights. For a 100% cumulative probability of detection, the inspection interval becomes 3400 flights. Note that these intervals are significantly less than for the previously estimated inspection intervals based on only one simulated lap joint (I=7000 flights for \( P_{\text{cum.}} = 98\% \) and I=4000 flights for \( P_{\text{cum.}} = 100\% \)).

As mentioned before, the time at which the first periodical inspection should be performed, is difficult to estimate. Nevertheless, a reasonable prediction can be made using S-N curves for initiation. These curves should be found by tests on specimens that are representative for the considered lap joint and must be verified by a full scale test program. An appropriate scatter factor (3-5) must be used to cover contingent changes in the production process and material properties.

**Some economical considerations for determining the inspection interval**

If the same level of safety (= cumulative probability of detection) is the criterion for estimating inspection intervals, two choices based on economical considerations or technical restraints can still be made,

- Inspection method
- Inspection procedure

The choice of the inspection method can be imposed by technical restraints: not enough trained people or necessary equipment may be available for a certain inspection method. If this is not the case, economical considerations are decisive. To understand the aspects that are of interest to the operating airline, it is
necessary to know how (scheduled) maintenance of an aircraft is performed. For a Boeing 747, the next four categories of scheduled maintenance exist (the B-check has been eliminated),

1. **A-check**: once a month (± 100 flights); very little structural work; 28 hrs. down time
2. **B-check**: eliminated
3. **C-check**: once a year (± 1100 flights); external inspection of structure, little or no removal of structure or interior for inspection or maintenance of structure; no inspection using NDT; 1 week of down time
4. **D-check**: once every 5 years (± 5500 flights): complete removal of aircraft interior and insulation; chemical stripping of external paint and primer; visual inspection of all structure; high frequency eddy current inspection of known problem areas (in critical rows only); down time 4 to 5 weeks (2 shifts 5 days a week); costs: 3-4 million dollars

Since these checks are being performed because of many other considerations than MSD-problems (mainly corrosion issues and maintenance of aircraft systems), it would be profitable to fit the MSD-inspections into these checks without increasing the down time or affecting the check-interval.

If the inspection interval for eddy current inspections is 6000 flights and the inspection interval for visual inspections is 3000 flights, the next options are available:

- Eddy current inspection at every D-check (5500 cycles)
- Visual inspection every second C-check (2200 flights)
In this case the eddy current inspection can be $5500/2200 = 2.5$ times more expensive than the visual inspection and still result in the same total costs. Not only the determined inspection period is decisive, but also the "scheduling efficiency" is of major importance.

\[
scheduling \text{ efficiency } = \frac{\text{scheduled inspection interval}}{\text{max. allowable inspection interval}}
\]

The scheduling efficiency of the first option is: $5500/6000 = 0.92$ and for the second option: $2200/3000 = 0.73$

Using similar arguments, the inspection procedure can also be chosen. For instance, it can be decided to inspect half of the number of rivets every C-check instead of all rivets every second C-check to prevent extra down time due to one long inspection.

Apart from these considerations, the choice for an inspection method and procedure depends on the direct costs (man-hours; equipment) per inspection for that specific method. These costs are affected by a number of things. An important aspect might be the difference between man-hour costs for visual inspections and those for eddy current inspections (eddy current inspections need higher trained people and are therefore (in this respect) more expensive than visual inspections).

**Crack distribution between frames**

The crack distribution between the frames is mainly determined by the inhomogeneous remote stress distribution caused by frames and tear straps. The default stress distribution is taken from FEM calculations [21] and is plotted in figure 4.6. The crack distribution of all simulated rivets is plotted in figure 4.3.
Figure 4.3: example of crack distributions between frames according to the simulation program: default settings

$N = 43,400$ flights

Figure 4.4: example of crack distributions between frames found in practice
The reason for the consistent cracking between the reinforcements (frames and tear straps) is the strong influence of the local stress level on the number of cycles until crack initiation.

**Crack growth curves**

Crack growth curves for all independent rivets can be viewed using option [4. View crack Growth curve of rivet ...] of the view menu. An example is given in figure 4.5. Comparisons with experimentally found crack growth curves have been made in section 5.3 of chapter 5.

![Crack growth curve](image)

*Figure 4.5: example of crack growth curve according to the simulation program*

**Stress redistribution due to local net section yielding**

The stress redistribution due to cracking, is calculated by the program, and graphs like figure 4.6 and 4.7 display the stress distribution between the frames and tear straps. Figure 4.6 represents a situation with no stress redistribution and figure 4.7 is the (redistributed) stress distribution corresponding to figure 4.3.
Figure 4.6: stress distribution between frames: no stress redistribution

Figure 4.7: example of redistributed stress according to the simulation program: situation corresponding to figure 4.3
4.2 Sensitivity analysis

To evaluate the influence of some of the estimated input, the next parameters have been changed from their default settings,

1. Part of load transferred by friction between the facing sheets
2. Stress distribution between frames
3. Initiation scatter curves

Part of load transferred by friction between the facing sheets
This variable has been changed from 50% (default setting) to 0%. The effect on the inspection interval is a change of +200 flights (I=5300) for eddy current inspections (see figure 4.8; criterion is $P_{cum} = 0.98$). This difference is opposite to the expected change; A decrease in the inspection interval is expected, because of an increased crack growth rate and thus less time (flights) to inspect. Therefore, it can be concluded that,
- The accuracy is in the order of 200 flights (for the relatively small nr. of 10 simulated lap joints)
- Friction has no significant influence on the inspection interval

Note that friction may have a significant influence on the number of flights until the first inspection.
Stress distribution between frames

Every lap joint design has its own stress distribution between the fuselage frames. Floating frames as used in Fokker construction designs for example, cause less hoop-stress reduction than the typical Boeing skin frame attachments. Therefore, a flat (more homogeneous) stress distribution for a panel without tear straps (typical Fokker construction), is analyzed with the developed program.

The assumed stress distribution is given in figure 4.9. Note that the maximum remote stress is equal to the maximum remote stress of the default stress distribution. (Consequently, the average stress in figure 4.9 is higher than for the default settings)
Figure 4.9: assumed (flat) stress distribution between frames

Figure 4.10 gives an example of the crack distribution between the frames for these settings. As can be expected, cracks occur more simultaneously.

Figure 4.10: example of crack distribution between frames: "flat" stress distribution
The influence on the inspection interval is not negligible: from 5000 to 4000 flights for eddy current inspections (figure 4.11). (A reduction of the inspection interval should be expected because of a more homogeneous crack distribution and higher average remote stress)

![Graph showing cumulative probability of detection versus inspection interval for largest crack; flat stress distribution.](image)

* Figure 4.11: Cumulative probability of detection versus inspection interval for largest crack; flat stress distribution

**Initiation scatter curves: P-S-N curve**

The initiation scatter curves have been estimated, but they are not sufficiently validated by test results. Therefore, the most critical situation with respect to these input data has been analyzed; The most critical situation that can arise is the simultaneous appearance of cracks at all rivet sides. This means that there is no scatter in initiation and the stress distribution between the frames is homogeneous ($S_{remote}$ is constant for all rivets). This stress has been made equal to
the maximum remote stress for the default settings. The results have been plotted in figure 4.12.

![Graph showing cumulative probability of detection versus inspection interval for the largest crack, "worst case".](image)

**Figure 4.12:** cumulative probability of detection versus inspection interval for largest crack, "worst case"

For this most critical situation, the eddy current inspection interval becomes 2900 flights ($P_{\text{cum.}} = 98\%$). An interesting conclusion that can be drawn is that the maximum profit that can be gained by assuming scatter and a stress reduction by frames and tear straps is a factor of $5100/2900 = 1.76$. This is a considerable factor and it is worthwhile to achieve less conservative results by further validation of P-S-N initiation curves.
5. Test program

This chapter discusses the test program for the validation of different assumptions. In section 5.1 the objective and scope are discussed. Section 5.2 describes the set-up and execution of the tests and section 5.3 evaluates the actual test results.
5.1 Objective and scope of test program

Any presumed scatter curve for initiation time as well as the assumptions that have to be made to quantify crack growth, link-up behaviour and residual strength, have to be validated by tests. The objective of the test program consists of four parts.

1. Validation/estimation of initiation scatter curves
2. Validation of crack growth
3. Validation of link-up and residual strength criteria
4. General observations of initiation and crack growth behaviour in riveted lap joints

Objective 1, 2 and 3 will be discussed separately in the next three sections.

1. Validation/estimation of initiation scatter curves
It is evident that the more test results are available, the more accurate scatter curves can be estimated. In appendix B (section B-2) a procedure is described to estimate the number of test results that is necessary for a reasonable estimation of a scatter curve. Starting points were,

1. Log(N) is Weibull distributed
2. Four stress levels should be tested
3. There are 20 rivets per specimen, so $2 \times 20 = 40$ possible initiation sides.

This led to 4 specimens per stress level, so $4 \times 4 = 16$ specimens in total. Four extra specimens were made for unforeseen problems or static/residual strength tests.
Due to problems during testing, not enough valid data points could be gathered to attain this part of the objective. The most predominant problem experienced during testing was the early and systematic occurrence of cracks at the side rivets nr. 1 and 20 (see figure 5.1). These cracks linked up to the free side and caused total failure before other holes were initiated. In 7 of 8 tested specimens, a single (side) crack dominated the fracture behaviour.

![Figure 5.1: cracks growing from side rivets](image)

It has been tried to restrain this side effect by taking one or more of the next measures,

- After initiation at one of these side rivets, delay the crack growth by either stop-drill the crack tips or by squeezing the crack tips, producing residual compressive stresses at the crack tip.

- Before starting the tests, squeeze the 4 critical rivets (rivet 1 and 20 at the top and bottom row) 5% more compared to the other rivets. This produces better clamping and therefore better fatigue resistance.

- Replace the 4 critical countersink rivets with protruding head rivets of a larger diameter (D=4.8 mm and D=6.4 mm have been used). By using a larger diameter, part of (D=4.8 mm), or the complete (D=6.4 mm) countersink disappears. Moreover, these rivets produce much better clamping and
therefore have better fatigue properties. An unwanted effect of this measure is that the increased stiffness of the rivets attracts more load.

None of these measures seemed to be sufficiently effective to cause MSD or even an independent crack in the middle 18 rivets. Once the crack at rivet 1 or 20 linked up to one of the free sides, the residual life was in all cases less than 1.3% of the total fatigue life (300 to 5000 cycles). Because no effective measures could be taken to prevent this side effect on short notice, the test program for the S-series was terminated after 9 specimens.

The reason for the systematical occurrence of cracks at rivets 1 and 20 was thought to be a non-uniform stress distribution along the specimen. To examine this presumption, 5 strain gauges have been glued on either side of specimen S-1-2. These measurements however, did not reveal significant differences in local remote stresses. According to figure 5.2, the 3 middle strain gauges indicate even slightly higher stresses.

![Figure 5.2: strain gauge results](image-url)
Two other potential reasons for early initiation could also be
1. Rivet 1 and 20 do not have the "clamping support" from neighbouring rivets at both sides. This might be solved by simulating clamping through squeezing the sheets together with mechanical clamps.
2. The theoretical stress concentration factor for a combined pin remote loaded hole in a finite width plate is higher than for a continuing row of holes; Since the side rivets have a free edge on one side, the stress concentration factor will be slightly higher.

The second unexpected problem was the occasional occurrence of cracks in the critical row at the side of the driven heads. This is surprising because in almost all cases known by the author, the countersink side revealed to be the most critical. This can also be expected because of the extra stress concentration due to the decreased bearing surface. In the extreme case, the bearing surface is so small that a "knife edge" situation causes a vast rise of the stress concentration factor (see figure 5.3). Although for this test program the knife edge situation has been carefully avoided, the remaining bearing surface was expected to be small enough to make the countersink side critical.

![Diagram](image)

*Figure 5.3: knife edge effect for a countersink rivet (figure from [29])*
If however, the load transfer is mainly accomplished by friction between the facing surfaces of the sheets, stress concentration is no longer a dominating factor for initiation and cracks might occur in the sheet with the driven heads. In the case of an intensively riveted lap joint (better hole filling/ expansion and clamping by a larger deformation of the driven head), cracks can even initiate away from the hole perimeter. This can be explained by the fact that fretting between two sheets is the largest at the transition from complete friction (no relative displacements between the sheets) to incomplete/no friction (see also appendix C).

To decrease the friction between the two sheets, 5 specimens have been given an overload (1.33 and 2 times $\sigma_{\text{max}}$) before testing, to cause "unpopping" of the rivets. The term "unpopping" means that the rivets become loosely fitted in the holes. This measure seemed to be effective, since these specimens all failed in the countersink sheet. (In some cases, cracks occurred simultaneously in both sheets).

The expected reason for the occurrence of cracks in the driven head sheet, is the riveting intensity. If the rivets are squeezed a few percent more than standard, the friction between the sheets increases and the stress concentration factor at the countersink sheet is no more a dominant factor for initiation. A dented surface is even visible by looking at one of the specimens (S-series; figure 5.4). These dents at the rivet locations are plastic deformations of the sheet material caused by the riveting procedure.
The influence of the riveting intensity has been examined by testing three supplemental specimens (T-series) and the results show extremely better fatigue performance for more intensively riveted specimens (figure 5.5). The influence of this production parameter on initiation scatter (= MSD sensitivity) however, has never been examined or demonstrated. This information would however, be of great value in view of production standards for newly developed aircraft.
Two provisional, but interesting conclusions can be drawn from this figure,

1. If the tolerance of the driven head diameter ($D_1$) in a production process is defined by: $1.45 < D_1/D_2 < 1.6$, the fatigue life can vary from $\pm 100000$ to $\pm 300000$ cycles. One might be tempted to conclude that the standard of $D_1/D_2=1.5$ should be increased until the lap joint is guaranteed to be crack free during the operational life. This, however, could lead to strains in the deformed rivet head that are larger than the maximum. For the tested lap joints, $D_1/D_2=1.6$ did not lead to rivet head failure and a higher standard for $D_1/D_2$ should seriously be considered. In the authors' opinion the lap joint as tested for this project, can be guaranteed a fatigue life of at least 250000 cycles if the production process is optimized and properly controlled.
2 Scatter in initiation can be expected to be larger for the range $1.4 < D_1/D_2 < 1.6$ than for more intensively riveted lap joints, because the slope, $d(D_1/D_2)/d(N)$ is larger at lower values of $D_1/D_2$; The same scatter in $D_1/D_2$ (tolerances) results in a wider range of possible initiation times $(N)$, thus larger initiation scatter.

2. Validation of link-up and residual strength criteria

For these tests the number of results is less important, because there are no statistics involved. It was presumed that for the link-up criteria all of the link-ups that occur in the initiation tests provide sufficient information. For the residual strength, some of the specimens from the initiation tests can be overloaded when no more useful information can be drawn from them. It was therefore decided that no extra specimens had to be made for residual strength or link-up tests.

Since MSD did not occur in the S-series specimens, only a few link-ups have occurred. For the same reason, residual strength tests have not been performed on these specimens. General observations of link-up behaviour however, showed that it is not a static phenomenon. Up to the point that the ligament is in the order of 1 mm, the cracks continue to grow with increased rate. Only when the ligament is in the order of 1 mm, statical fracture of the remaining net section occurs. This is due to the high ductility of the 2024 alloy.

Specimen T-3 suffered severe MSD and was used to check the net section yield criterion for a MSD configuration.
5.2 Set-up and execution of tests

In this section all aspects concerning the set-up and execution are discussed. The fatigue tests have been performed on a 100 tons MTS fatigue machine of the Faculty of Aerospace Engineering of the Technical University of Delft. A sinusoidal signal at a frequency of 5 Hz has been used.

The lap joint configuration, load spectrum and crack observation are successively discussed in section 5.2.1, 5.2.2 and 5.2.3.

5.2.1 Configuration and fabrication of test specimens

**Specimen dimensions**

The configuration of the S-series specimens are defined in figure 5.6.

![Figure 5.6: test specimen configuration (all dimensions in mm)](image_url)

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Per row, 20 rivets have been used, in order to get enough data from 1 specimen and to be able to quantify the scatter in initiation data for each independent specimen. The length is 2 times the width and the space between the rows have been made equal to the rivet pitch. This configuration is almost identical to the configuration of the finite element model and representative of narrow body fuselage lap joints.

The T-series specimens are of exactly the same detail design, but have smaller outer dimensions: 125 mm x 250 mm (5 rivets per row).

All specimens have been made of material from 1.2 mm sheet (2024 T-3 Alclad) from the same production batch to prevent unwanted scatter due to differences among different batches. (it would be useful to investigate the scatter due to the sheet production- or riveting process, i.e. different aircraft)

Fabrication of test specimens
To exclude scatter due to differences in riveting procedure, it is essential that all specimens are riveted identically and by the same person at the same time. It was therefore decided to rivet the specimens at Fokker using an automatic riveting machine. An effort of Fokker for which I am very grateful.

5.2.2 Fatigue load

All tests have been performed under constant amplitude loading (R=0.05). Tests on the S-series specimens have been performed mainly at a maximum stress level of 105 MPa (representative of fuselage lap joint stress levels). Two specimens were tested at 120 and 125 MPa to see whether the stress level had any influence on scatter. The three additional tests (T-series) have also been tested
at 105 MPa at a stress ration of 0.05. Table 5.1 gives the identification of the specimens and the stress level at which they have been tested.

5.2.3 Observation of crack initiation and crack growth

Crack initiation has been monitored using eddy current equipment (see figure 5.7). Although cracks have been detected at several places with this equipment before they became visible, the initiation time has been taken as the first moment of positive visual detection. This has been done because in several cases, cracks were found by eddy current detection relatively early in the fatigue life (50-70% of the total fatigue life), but stopped growing and never became visible.

Typical inspection intervals have been 1000-5000 cycles, depending on whether cracks had already initiated or not and on the crack growth rates. Crack growth has been monitored during these inspections, using millimetre paper, a magnifying glass and in some cases a microscope (accuracy is \( \pm 0.2 \) mm\(^*\)).

For the 3 T-series specimens (different riveting intensities: \( D_1/D_2 \)), the cracks had a clear tendency to grow further away from the rivet head for more intensively riveted specimens (increasing \( D_1/D_2 \)). This is caused by the increased friction between the facing surfaces of the connected sheets. A larger reduction of the stress concentrations at the expected critical places (A) causes the initiation site to move from places A to B (see figure 5.8). (Table 5.1 supplies more information about these specimens)

\* This concerns measuring accuracy and not absolute accuracy. Since in some cases the crack was a part through-crack (especially in the case of very small cracks), these two qualifications are not identical.
Figure 5.7: Eddy current equipment used for crack detection

(a) Specimen T-1 (D₁/D₂ = 1.45)

(b) Specimen T-2 (D₁/D₂ = 1.5)

(c) Specimen T-3 (D₁/D₂ = 1.6)

Figure 5.8: Change of initiation site from places A to B
5.3 Test results

A summary of the main test results are given in table 5.1 and 5.2.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Maximum stress (MPa)</th>
<th>Overload (MPa)</th>
<th>Crack growth retardation</th>
<th>Maximum nr. of cracks</th>
<th>Failed sheet*</th>
<th>Cycles until first visible crack detection</th>
<th>Cycles until failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1-1</td>
<td>105</td>
<td>none</td>
<td>none</td>
<td>4</td>
<td>1</td>
<td>230000</td>
<td>276000</td>
</tr>
<tr>
<td>S-1-2(1)</td>
<td>105</td>
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<td>none</td>
<td>21</td>
<td>2</td>
<td>275000</td>
<td>351000</td>
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<tr>
<td>S-1-3</td>
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<td>139.7</td>
<td>stop-drill</td>
<td>2</td>
<td>1</td>
<td>220000</td>
<td>270000</td>
</tr>
<tr>
<td>S-1-4</td>
<td>105</td>
<td>210</td>
<td>squeeze</td>
<td>3</td>
<td>1</td>
<td>400000</td>
<td>431500</td>
</tr>
<tr>
<td>S-2-1(2)</td>
<td>105</td>
<td>none</td>
<td>squeeze</td>
<td>5</td>
<td>1(2)</td>
<td>325000</td>
<td>395000</td>
</tr>
<tr>
<td>S-2-2(4)</td>
<td>105</td>
<td>139.7</td>
<td>stop-drill</td>
<td>4</td>
<td>1</td>
<td>310000</td>
<td>361000</td>
</tr>
<tr>
<td>S-2-3(6)</td>
<td>120</td>
<td>139.7</td>
<td>stop-drill</td>
<td>8</td>
<td>1</td>
<td>195000</td>
<td>243000</td>
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<tr>
<td>S-3-1(5)</td>
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<td>139.7</td>
<td>stop-drill</td>
<td>5</td>
<td>1</td>
<td>157000</td>
<td>188000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$D_1/D_2$</th>
<th>Maximum stress (MPa)</th>
<th>Overload (MPa)</th>
<th>Maximum nr. of cracks</th>
<th>Failed sheet*</th>
<th>Cycles until first visible crack detection</th>
<th>Cycles until failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-1</td>
<td>1.45</td>
<td>105</td>
<td>none</td>
<td>?</td>
<td>1</td>
<td>?</td>
<td>110000</td>
</tr>
<tr>
<td>T-2</td>
<td>1.5</td>
<td>105</td>
<td>none</td>
<td>4</td>
<td>2(2)</td>
<td>161000</td>
<td>189000</td>
</tr>
<tr>
<td>T-3</td>
<td>1.6</td>
<td>105</td>
<td>none</td>
<td>10</td>
<td>2</td>
<td>272000</td>
<td>320000</td>
</tr>
</tbody>
</table>

* 1: countersink side  
2: driven head side  
(1) specimen with strain gauges  
(2) cracks occurred in both sheets  
(3) outer rivets have been squeezed 5% more  
(4) outer rivets have been replaced by protruding head rivets ($\phi$ 4.8 mm)  
(5) outer rivets have been replaced by protruding head rivets ($\phi$ 6.4 mm)  
(6) Rivets failed statically due to defect in fatigue machine
Initiation scatter curves (P-S-N diagram)

Since only one specimen (S-1-2) had a significant number of cracks, only one P-N curve (S=105 MPa) could be made. The results from this specimen have been analyzed by the program WEIBULL.EXE and a Weibull curve-fit has been made (figure 5.9). A Weibull distribution is defined as in equation 5.1. Figure 5.9 (a) is a "Weibull" plot: y-scale is chosen such that a Weibull distribution is represented by a straight line. Figure 5.9 (b) has a linear y-scale.

\[ P_{\text{cum.}}(N) = 1 - e^{-\left[ \frac{\log(N) - \log(N_0)}{\alpha} \right]^\beta} \]

where \( \alpha = 0.1477, \beta = 0.9334 \) and \( N_0 = 291 \) kcycles

![Fitted Weibull distribution](image)

Figure 5.9 (a) : scatter curve for initiation ("Weibull" plot): specimen S-1-2

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It should be noted that the curve is just an indication for initiation scatter in fuselage lap joints and can not be considered fully representative. Especially if one is interested in scatter on the countersink side of the lap joint, supplemental tests should be performed (specimen S-1-2 cracked in the driven head sheet).

The initiation scatter curves, necessary for the computer program had therefore to be estimated.
Crack growth

Crack growth has been monitored and some results are plotted in figure 5.10 to 5.17. Most of the crack growth curves are incomplete because cracks have been artificially stopped in many cases. This retardation has been achieved by stop-drilling, or in some cases squeezing the crack tips, which causes residual compressive stresses at the crack tip.

Results of the crack growth analysis, as described in section 3.4 have also been plotted in figure 5.10 to 5.17; Because scatter in crack growth properties has been assumed in the analysis, two curves have been generated: 1 extreme slow growth; 2 extreme fast growth. The "slow growth curve" starts with only a crack on one side of the hole, while the "fast growth curve" starts with both sides cracked. The analysis represents two possible curves for a single crack at one of the two side rivets (a side rivet will grow faster compared to middle rivets because of the free edge). This has been done because most empirical curves represent that configuration.

For comparison, the empirically found crack growth curves have been shifted until the first visible crack corresponds to the "slow crack growth curve" of the analysis. The empirical curves have somewhat irregular shapes because of inaccuracies in crack length measurement and the part through character in the first part of the curve.

The two curves that represent the analysis show that scatter in crack growth properties has a significant influence. In most cases, the average da/dn of the test results fit to the predicted range of da/dn generated by the analysis. In one case (figure 5.17), a crack did not grow further after initiation. This phenomenon has frequently been observed for cracks that had been detected by the eddy current equipment, but never for visible cracks.
Figure 5.10: crack growth curve for specimen S-1-1

Figure 5.11: crack growth curve for specimen S-1-2
Figure 5.12: Crack growth curve for specimen S-1-3

Figure 5.13: Crack growth curve for specimen S-1-4
Figure 5.14: crack growth curve for specimen S-2-1

Figure 5.15: crack growth curve for specimen S-2-2

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Figure 5.16: crack growth curve for specimen S-2-4

Figure 5.17: crack growth curve for specimen S-3-1
Link-up- and residual strength criteria

The link-up phenomenon can be described best as a continuous growth of the cracks, until a very small ligament (in the order of 1 mm) fails statically and causes link-up. The reason for the late statical failure (very small ligament) is the high ductility of the 2024 alloy and stress redistribution due to local net section yielding.

Residual strength has only been tested in one case (specimen T-3). This specimen was loaded quasi-statically until failure. The crack configuration at the moment of the residual strength test is drawn in figure 5.18.

![Figure 5.18: crack configuration at the moment of residual strength test](image)

The specimen failed at 222.7 MPa, which corresponds to a net section stress of 360 MPa. Since this is only a few percent higher than $\sigma_{0.2}$, the net section yield
criterion for residual strength of 2024 Alclad seems to be accurate for small width specimens. A complete panel or part of an aircraft however, can fail long before this criterion is satisfied due to unstable crack extension of a crack that has reached a certain critical crack length. This critical crack length can vary depending on whether (small) cracks are ahead of this crack.

Finally an overview of the ratio between initiation time and cycles to failure, is given in figure 5.19, which shows that the initiation time is 80-90% of the total fatigue life.

![Figure 5.19: ratio between initiation time and cycles to failure](image)

"Hard data" stands for the results as in table 5.1 (S-series) and table 5.2 (T-series). Because cracks have been retarded, the ratio between the number of cycles until initiation and moment of crack retardation has also been plotted in figure 5.19. These data are indicated by "Possible maximum".
6. Conclusions and recommendations for further research

This project has led to several conclusions. They are divided into two categories:

1. Conclusions concerning the test program
2. Conclusions concerning the simulation program

Some of the conclusions are preliminary, but in the author's opinion still helpful to define future research. An overview of research that is worthwhile to perform is also included.
Conclusions concerning the test program:

- There is a major influence of the riveting intensity on the initiation threshold; Production tolerances for the diameter of the driven head \(1.4 < D_{\text{driven head}}/D_{\text{rivet shank}} < 1.6\) can result in a difference in fatigue performance corresponding to a factor 3 in lifetime.
- This influence can be explained by a reduction of the stress concentration factor at the rivet sides, which causes a change in initiation mechanism.
- The ratio between the number of cycles until initiation and the number of cycles until failure is approximately 0.85 and more or less constant for all tested specimens (also for different riveting intensities).
- After the first link-up, the remaining life is less than 1.3 % of the total life (± 1500 cycles for \(\sigma = 105 \text{ MPa}\)). This means that cracks must be found before link-up, if inspection intervals of more than 1500 flights are being used.

Conclusions concerning the simulation program:

- The inspection interval can be increased with approximately 75%, if the fatigue life of an entire lap joint panel is simulated instead of the (over) conservative presumption of simultaneous appearance of cracks at all rivet sides.
- The major factor of influence on the interval for periodical inspections is the hoop stress. The hoop stress reduction due to frames and tear straps must therefore be taken into account.
- If the analyzed lap joint is inspected properly, cracks can be found using an inspection interval in the order of 4000 flights for the eddy current inspection technique and less than 2000 flights for visual inspections.
Recommendations for further research

It is thought to be most worthwhile to conduct further research in the next 5 directions:

- Further validation of presumed scatter curves for initiation
- Empirical verification of the crack growth analysis (using empirically found crack growth curves)
- Empirical validation of the probability of detection curves for visual and eddy current inspections (probability of detection versus crack length).
- A systematical test program to quantify the influence of the riveting intensity on the initiation threshold and initiation scatter (this might lead to a change of production standards)
- Alternative (automatic) measures to detect (large) cracks in fuselage lap joints may be worthwhile to investigate.
Appendix A: manual for program MSDSIM

MSDSIM is a menu controlled set of programs which simulates the fatigue life and inspection of a riveted lap joint under constant amplitude loading (R = 0). The set consists of the main program (MSDSIM.EXE) and 5 sub-programs that can also be called independently.

1. INSTALL.EXE  Installation program
2. SIMUL.EXE    Actual simulation program
3. VIEW.EXE      Program to view the output of simul.exe:
                  - cumulative prob. of detection v. Inspection interval.
                  - a versus N
                  - crack distribution between frames
                  - stress distribution between frames
4. WEIBULL.EXE   Program for statistical analysis. The user is assisted in finding a "Weibull distribution" that is best applicable to a set of test data.
5. PRINT.COM     Dos print program, used to print the parameter settings of a selected run.

The programs will be discussed separately in the next 6 sections.
**MSDSIM.EXE**

This program is the interface between the user and the sub-programs. It displays the menu on the screen (figure A-1) with which the user can call one of the sub-programs (except for INSTALL.EXE). To call the program type `<MSDSIM>` at the DOS prompt.

![Figure A-1: main menu](image)

**INSTALL.EXE**

This program installs the files in a directory named by the user and makes the sub directory `\OUT` for the output files of the program SIMUL.EXE. To call this program, type `<INSTALL>` at the DOS prompt. INSTALL needs only to be run once and can not be called from MSDSIM.EXE.

**3. SIMUL.EXE**

This is the actual simulation program. It is called by selecting option [1. run simulation program] of the main menu or by typing `<SIMUL>` at the DOS prompt (see figure A-1).

All parameters that influence the final results are set at default values, but can easily be changed by the user. This is done through answering questions displayed on the screen. All parameters, their meaning and the default values are listed in table A-1.
<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Unit</th>
<th>Default value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>mm</td>
<td>1.20</td>
<td>Sheet thickness</td>
</tr>
<tr>
<td>R riv. sh.</td>
<td>mm</td>
<td>2.00</td>
<td>Radius of the rivet shank</td>
</tr>
<tr>
<td>R riv h.</td>
<td>mm</td>
<td>3.10</td>
<td>Radius of the rivet head</td>
</tr>
<tr>
<td>Riv. pitch</td>
<td>mm</td>
<td>25.00</td>
<td>Rivet pitch</td>
</tr>
<tr>
<td>R fuselage</td>
<td>mm</td>
<td>2300</td>
<td>Fuselage radius</td>
</tr>
<tr>
<td>Δp fus.</td>
<td>N/cm²</td>
<td>5.86</td>
<td>Difference in inner (cabin) and outer (atmospheric) pressure</td>
</tr>
<tr>
<td>σ₀₂</td>
<td>MPa</td>
<td>310</td>
<td>Yield stress</td>
</tr>
<tr>
<td>Kt</td>
<td>MPa/mm</td>
<td>1265</td>
<td>Critical (mode I) stress intensity factor</td>
</tr>
<tr>
<td>Crit. crack</td>
<td>mm</td>
<td>500</td>
<td>Critical crack length</td>
</tr>
<tr>
<td>load min.</td>
<td>%</td>
<td>75</td>
<td>Minimum remote stress (fraction of the original remote stress; see section 3.6: stress redistribution)</td>
</tr>
<tr>
<td>Fr. par. T.</td>
<td>%</td>
<td>50</td>
<td>Part of load that is transmitted by friction for the Tested configuration</td>
</tr>
<tr>
<td>Fr. par. R.</td>
<td>%</td>
<td>50</td>
<td>Part of load that is transmitted by friction for the Real (simulated) configuration</td>
</tr>
<tr>
<td>A-no Fr.</td>
<td>mm</td>
<td>21.00</td>
<td>Crack length (tip-to-tip) for which the friction between the facing surfaces does not contribute to the load transmission anymore; Fr. par. R.₀ decreases linearly from the setting value (uncracked situation) to 0 (2a = &quot;A-no Fr&quot;)</td>
</tr>
<tr>
<td>Cycle increment</td>
<td></td>
<td>2000</td>
<td>Increment of cycles used to calculate the damage accumulation. This parameter is adjusted (decreased) as the cracks grow larger</td>
</tr>
<tr>
<td>L₀</td>
<td>mm</td>
<td>1.10</td>
<td>Definition of the initiation crack length</td>
</tr>
<tr>
<td>DK exp.</td>
<td></td>
<td>3.00</td>
<td>Exponent for the Paris equation: (da/dn = C_0 \cdot \Delta K^{n})</td>
</tr>
<tr>
<td>C₀</td>
<td>mm/(MPa/√mm)</td>
<td>0.78</td>
<td>Minimum C₀ value for the assumed Weibull scatter distribution in crack growth properties (see section 3.4)</td>
</tr>
<tr>
<td>C₀-lambda</td>
<td>mm/(MPa/√mm)</td>
<td>2.17</td>
<td>&quot;Scale parameter&quot; for the assumed Weibull scatter distribution in crack growth properties (see section 3.4)</td>
</tr>
<tr>
<td>C₀-alpha</td>
<td></td>
<td>2.00</td>
<td>&quot;Shape parameter&quot; for the assumed Weibull scatter distribution in crack growth properties (see section 3.4)</td>
</tr>
<tr>
<td>Nr. of rivets</td>
<td></td>
<td>20</td>
<td>Number of rivets for which the calculations are being made</td>
</tr>
<tr>
<td>Print Nr.</td>
<td></td>
<td>1000</td>
<td>After each number of cycles equal to &quot;Print nr.&quot;, all necessary information is written to disk.</td>
</tr>
<tr>
<td>Stat. link L.</td>
<td></td>
<td>FALSE</td>
<td>This parameter makes it possible to equalize the statistical properties of the left and right side of all rivet holes (concerning initiation):</td>
</tr>
<tr>
<td>Stat link Gr.</td>
<td></td>
<td>FALSE</td>
<td>This parameter makes it possible to equalize the statistical properties of the left and right side of all rivet holes (concerning crack growth):</td>
</tr>
</tbody>
</table>

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SIMUL.EXE is capable of simulating the next 5 fatigue problems,

1. Crack initiation for an arbitrary crack configuration.
2. Crack growth of an arbitrary crack configuration
3. Link-up
4. Residual fracture
5. Stress redistribution due to local net section yield
6. Inspection (eddy current and visual)

During the execution of SIMUL.EXE, the lap joint is graphically displayed on the screen together with the main input variables. This enables the user to monitor the cracks during cycling.

**WARNING:** One run of the program can take a considerable amount of time, depending on input parameters like, the number of simulated rivets, the cycle increment used stresses and the hardware configuration.

**N.B.**

1. Initiation and crack growth properties are assigned randomly, which can yield slightly different results for different runs with the same settings.

2. The inspections are simulated for different inspection intervals. This is done during one fatigue life simulation.
**VIEW.EXE**

This program is called by the option [2. View output] of the main menu or by typing `<VIEW>` at the Dos prompt (see figure A-1). The program uses all the data files with the extension ".DA1" which are created by the program SIMUL.EXE and stored in the subdirectory "\OUT". To view data files of previous runs, answer the question "View results of last run?" with <N> and change the file extension into ".DA?", where ? should be a number from 1 to 5 to view the corresponding run. If you renamed a file manually, you should type the new file name.

All output files of previous runs will be renamed as follows*:

* .DA1 → *.DA2
* .DA2 → *.DA3 etc.

If a graph is displayed, the option is given to save the screen to a specified file (*.pic) for later use. To do this press <S> when the graph is displayed (as indicated on the graph screen).
Option 1 of View menu [1. View cum. Prob. of det. versus I.]

This option assists the user in viewing the cumulative probability of detection for the last run or any previous run of SIMUL.EXE (provided the output has not been suppressed). If this option is selected the screen of figure A-3 is displayed. After selecting any of the options 1 to 4, the next question will be asked:

<Do you want to view the results of the last run? (Y)>

If you press return or answer <Y>, all the files with extension ".DA1" in the sub directory "\OUT" will be used. If you answer <N>, you will be asked to give the filename that contains the data of the run you want to view.

All output files of previous runs will be renamed as follows*:
* .DA1 = *.DA2
* .DA2 = *.DA3 etc.

Option [3. View Pcum. of cracks larger than ..] provides the alternative for the user to exclude cracks smaller than any desired minimum length. This can be helpful if you are interested in the average probability of detection of all linked-up cracks.

*WARNING: The files *.DA5 will be deleted each time SIMULEXE is executed to prevent the creation of too many files. To prevent this, rename the files or move them from the sub directory */OUT".
The graph borders are set automatically, but can be changed manually (you will be asked to press <M> for manual settings). If you select this option, the minima and maxima for both axes must be entered.

**Option 2 of View menu [2. View crack configuration]**

This option assists the user in viewing the crack configuration at failure or at any required number of cycles for the last run or any previous run (provided the output has not been suppressed). The graph is displayed in "bar format". The first 20 rivets are displayed on the screen. To see the Next or Previous 20 rivets press respectively <N> or <P> (as indicated on the graph screen).

Option [2. View crack configuration at failure (lap joint)] will display the lap joint at failure as it was displayed during the execution of SIMUL.EXE (provided the output has not been suppressed). The default name of the file to view is CRAKFL.PIC.

Option [3. View crack configuration at .. cycles (bar graph)] can only display the crack configuration at the number of cycles for which the data have been saved. If any other number is asked, the closest (higher) number for which the data have been saved, is used. To get smaller "save increments", decrease the value of variable "Print Nr." (see table A-1).
Option 3 of View menu [3. View stress distribution]

This option assists the user in viewing the remote stress distribution at failure or at any required number of cycles for the last run or any previous run (provided the output has not been suppressed). The graph is displayed in "bar format". The first 20 rivets are displayed on the screen. To see the Next or Previous 20 rivets press respectively <N> or <P> (as indicated on the graph screen).

Option [2. View stress distribution at .. cycles] can only display the stress distribution at the number of cycles for which the data have been saved. If any other number is asked, the closest (higher) number for which the data have been saved, is used. To get smaller "save increments", decrease the value of variable "Print Nr.".

Option 4 of View menu [4. View crack growth curve of rivet nr. ..]

If this option is selected, you will be asked to give the rivet number for which you want to view the crack growth curve (1=left rivet; i=i\textsuperscript{th} rivet from the left).

To view the crack growth curve of the Next or Previous rivet, press respectively <N> or <P> as indicated on the graph screen.

The graph borders are set automatically, but can be changed manually (you will be asked to press <M> for manual settings). If you select this option, the minima and maxima for both axes must be entered.
PRINT.COM
This is the standard Dos print program and is called by the option [3. Print
parameter settings] of the main menu (see figure A-1).

If this option is selected, the next question is displayed on the screen:

"Do you want to print the data file of the last run? (Y) "

If you answer <Y>, the file RUNPAR.DA1 is printed.
If you answer <N>, you are asked to specify the file you want to print.

All data files of previous runs will be renamed as follows*:
RUNPAR.DA1 → RUNPAR.DA2
RUNPAR.DA2 → RUNPAR.DA3 etc.

It is recommended to print this file every new run because the name of the run and
all parameter settings are saved to this file. This can help to identify the parameter
settings of data files of previous runs. An example of such print (default settings)
is included in appendix E.

*WARNING: The file RUNPAR.DA5 will be deleted each time simul.exe is executed to prevent the creation of too
many files.
WEIBULL.EXE
This program is called by the option [4. Curve-fit a Weibull distribution] or by typing <WEIBULL> at the DOS prompt. The program can be used to "curve fit" a Weibull distribution to a set of test data. The calculated Weibull distribution(s) can be used as initiation input data to the program SIMUL.EXE. The program WEIBULL.EXE contains a (brief) on line explanation of the capabilities. Appendix B (section B-2) explains the two methods that can be used for curve fitting. For a more detailed treatise of the criteria used by these methods, the reader is referred to this section.

N.B. The test data should be stored in a file and should contain the number of cycles until initiation (or failure) in ascending order.
Appendix B: statistical aspects of fatigue test results

This appendix is divided into 2 sections. First some general aspects of statistics in fatigue are discussed (B-1); This section is a free translation of reference [27] with some minor adaptions. Section B-2 gives two procedures used in the test program for this specific project,

1. procedure for finding the required number of specimens for scatter curve validation;
2. procedures for curve fitting a Weibull distribution.
B-1 General aspects of statistics for fatigue

It is common knowledge that fatigue test results can show considerable scatter. Fatigue life estimates for real life structures are usually based on laboratory fatigue tests. Unfortunately, scatter can occur for many reasons. Furthermore, these reasons may be different for the laboratory environment and for the practical usage of the structure. A general classification is,

<table>
<thead>
<tr>
<th>Sources of scatter</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>laboratory tests</td>
<td>structure in practice</td>
</tr>
<tr>
<td>fabrication of specimens</td>
<td>material properties</td>
</tr>
<tr>
<td>test set up</td>
<td>production procedure</td>
</tr>
<tr>
<td>execution of tests</td>
<td>usage in practice</td>
</tr>
</tbody>
</table>

**Laboratory aspects**

During laboratory testing, it is often tried to minimize the sources of scatter. The fatigue process itself will be enough reason for scatter to occur. Furthermore, if the test results are of a comparative nature, any statement will be more difficult if a lot of scatter is present. If for instance, surface treatment A is compared to surface treatment B, the specimens should preferably be made from one homogeneous material quality. If the fatigue threshold is the issue, the specimens should, if possible, be made in one constant production quality. For what's concerning the test set-up-and execution, all sources of scatter should be minimized as much as possible. These factors are considered improper sources, that is sources that are of no relevance in practice. This means that the fatigue machines should be reliable and the test procedure should be accurate.
If all possible measures have been taken to minimize scatter, the test results will yield,
- a mean value trend
- scatter around this mean value

The main issue now is: are the mean value trend and the scatter representative for the problem being studied?

Aspects of the structure in practice
Although a production company can try to deliver a constant good quality, several reasons exist why this is not always possible. The quality of the materials that are purchased is often covered by material specifications which have to meet minimum requirements. This, more or less, guarantees a constant quality. The quality of the production itself can also be subject to fluctuations. Human factors can contribute to this.

A completely different source of scatter is introduced by a different usage of the structure. This source can only be qualified as irrelevant, because it is independent of the fatigue properties of the structure.

Interpretation of the results
Considering the arguments mentioned above, one should realize that two problems must be considered,
1. The interpretation of fatigue test results and the scatter found
2. The possible scatter in fatigue properties that will appear if the structure is widely used in practice

For the first problem, the scatter sources can mostly be tracked, but the second problem is more difficult to analyze. Furthermore, the second problem includes doing predictions. This makes it necessary to use safety factors. The second
problem is more complicated and demands more thought. A classic question is whether safety factors should be applied on fatigue lives or applied stresses (consider the fatigue threshold for instance). A second question is how large should these safety factors be?

**Scatter for identical fatigue tests; the scatter function**

If a series of identical fatigue tests is performed, the obtained lives are different. The table in figure B-1 gives results of 18 identical fatigue tests on unnotched specimens of an Al-alloy. The smallest life is 636 kcycles and the maximum is 2368 kcycles. The ratio of 3.7 is an indication for the scatter. This number however will increase as the number of tests increases. Therefore a more frequently used parameter is the standard deviation.

\[
S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{m-1}} \quad (B-1)
\]

with the mean:

\[
\bar{x} = \frac{\sum x_i}{m} \quad (B-2)
\]

\( \bar{x} \) and \( s \) are estimates of the true mean \( \mu \) and the true standard deviation \( \sigma \) of the population. \( N \) can be set as the variable \( (x_i = N_i) \), but in general this leads to leaning distributions. This is why it is often assumed that \( \log(N) \) has a normal distribution,

\[
p(\log(N)) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\log(N) - \log(\bar{N})}{\sigma} \right)^2} \quad (B-3)
\]

If not a two parameter (\( \sigma \) and \( \log(\bar{N}) \)) normal distribution is assumed, but a three parameter Weibull distribution, the equation becomes,
\[ p(\log(M)) = \frac{a}{\alpha} \left[ \frac{(x \log(N) - \log(N_0))}{\alpha} \right]^{(\delta - 1)} e^{-\left[ \frac{(x \log(N) - \log(N_0))}{\alpha} \right]^\delta} \]  

(B-4)

In the program WEIBULL.EXE, a curve fit is made through a number of data points, assuming that \( \log(N) \) is Weibull distributed (see also section B-2). The Weibull distribution is one of the so called "extreme value distributions". This means that a lower limit exists (denoted \( N_0 \) in formula B-4) as opposed to the normal distribution.

The cumulative distribution is defined as,

\[ P_{\text{cum}}(x) = \int_{-\infty}^{x} p(x) \, dx \]  

(B-5)

\( P_{\text{cum}}(x_1) \) is the chance that an event has taken place for values \( x \) smaller than \( x_1 \). In the case of fatigue life, the chance that the structure has failed at \( N \) cycles (\( P_{\text{cum}}(N) \) or \( P_{\text{cum}}(\log(N)) \)).

For the Weibull distribution, this leads to,

\[ P_{\text{cum}}(x) = 1 - e^{-\left[ \frac{(x - x_0)}{\alpha} \right]^{\beta}} \]  

(B-6)

where, \( x_0 \) is the location parameter: the value of \( x \) for which \( P_{\text{cum}} = 0 \)

\( \alpha \) is the scale parameter: for \( x = \alpha + x_0 \), \( P_{\text{cum}} = 0.63 \)

\( \beta \) is the shape parameter: a measure for the slope of the curve

Figure B-1 gives an example of the cumulative Weibull and normal distributions on probability paper (fatigue analysis). For probability paper, the ordinate is such that the normal distribution becomes a straight line. In the same figure, test results are plotted as well. In order to do this, each test result must be assigned a cumulative probability of failure. This is done through ranking the results in

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ascending order \((N_{i+1} > N_i)\). The estimate of the probability of failure is now given by the formula,

\[ P_{\text{cum}} = \frac{i - \frac{1}{2}}{m} \quad (B-7) \]

where \(m\) is the total number of specimens.

Formula B-8 becomes more accurate as \(m\) becomes larger.

The results in figure B-1 \((m = 18)\) indicate that \(\log(N)\) could very well be normal or Weibull distributed. Tests to support this are available in the literature, but will not be discussed here. The results of figure B-1 don’t give any guarantee that the "tails" of the distribution still satisfy the normal distribution.

In the next section two procedures are described for calculating the three unknown parameters of the Weibull distribution that would be the best "fit" for any set of test results. Furthermore, a qualitative procedure is explained for finding the minimum number of test results necessary for validation of an assumed Weibull distribution function.
Figure B-1: fatigue life for a series of identical tests plotted on probability paper
B-2 procedures for statistical problems of crack initiation

This section gives two types of procedures used in the test program,
1. procedure for finding the required number of specimens for scatter curve validation;
2. procedures (2) for curve fitting a Weibull distribution.

Before these procedures are described, an essential problem inherent to initiation scatter is discussed.

Problem of incomplete initiation data
To find the distribution for initiation times of a riveted lap joint, induces an inherent complication: the set of test data will always be incomplete. The next two reasons can be pointed out,

1. Not all of the hole perimeters will be cracked before total failure.
2. Local stress rise due to other damage may exclude some data points; if a crack is present on 1 side of the hole, the stress at the other side will rapidly increase as this crack propagates (see section 3.2). This will cause sooner crack initiation and therefore it will not be a valid data point. Damage at other holes might as well influence the stress level of the considered (undamaged) hole and so exclude it from the data set.

It is of course not correct to consider all found and valid data points as the complete population. This would exclude the upper part of the population (the part that apparently initiates relatively late) and give a totally wrong picture of the true population.
Some of these problems could be avoided if specimens are used with only one "column" of rivets (see figure B-2). However, the problem of the influence of a crack on one side on the local stress level on the other side still remains. Furthermore, factors like the theoretical stress concentration factor, clamping and secondary bending are different for this specimen compared to a real lap joint. The latest argument was decisive to use wide specimens.

![Figure B-2: specimens with a single "column" of rivets](image)

The criteria used to disqualify data points are,

1. If the crack on the opposite site emerges more than 1 mm from under the rivet head, the data point is no longer valid.

2. If the total crack length exceeds 1/4 of the original net section, no more data points can be found from that specimen (average stress rise in the net section = 33 %).

3. If a crack from an adjacent hole is larger than 1/4 of the original ligament between these holes, the considered hole can not supply data points any more because of local stress rise (local stress rise = 33 %).

4. If link-up has occurred, the two adjacent (uncracked) holes can not supply data points any more (the rivet loads for these rivets will increase considerably due to stress redistribution).
The next example illustrates the effect of invalid data points.

**Example**

Suppose that a specimen with 10 rivets (~ 20 rivet sides) is fatigue tested and the criteria to disqualify data points led to the next table:

<table>
<thead>
<tr>
<th>i</th>
<th>$N_i$ (kycles)</th>
<th>$P = (i-k)/m$</th>
<th>$\Delta i$</th>
<th>i</th>
<th>$N_i$ (kycles)</th>
<th>$P = (i-k)/m$</th>
<th>$\Delta i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>195</td>
<td>0.03</td>
<td>0</td>
<td>11</td>
<td>283</td>
<td>0.53</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>198</td>
<td>0.08</td>
<td>0</td>
<td>12</td>
<td>289</td>
<td>0.58</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>205</td>
<td>0.13</td>
<td>0</td>
<td>13</td>
<td>310 X(2)</td>
<td>0.63</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>218 X(1)</td>
<td>0.18</td>
<td>0</td>
<td>14</td>
<td>324 X(2)</td>
<td>0.68</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>220</td>
<td>0.23</td>
<td>1</td>
<td>15</td>
<td>?????</td>
<td>0.73</td>
<td>—</td>
</tr>
<tr>
<td>6</td>
<td>230</td>
<td>0.28</td>
<td>1</td>
<td>16</td>
<td>?????</td>
<td>0.78</td>
<td>—</td>
</tr>
<tr>
<td>7</td>
<td>245 X(1)</td>
<td>0.33</td>
<td>1</td>
<td>17</td>
<td>?????</td>
<td>0.83</td>
<td>—</td>
</tr>
<tr>
<td>8</td>
<td>249</td>
<td>0.38</td>
<td>2</td>
<td>18</td>
<td>?????</td>
<td>0.88</td>
<td>—</td>
</tr>
<tr>
<td>9</td>
<td>261 X(1)</td>
<td>0.43</td>
<td>2</td>
<td>19</td>
<td>?????</td>
<td>0.93</td>
<td>—</td>
</tr>
<tr>
<td>10</td>
<td>278</td>
<td>0.48</td>
<td>3</td>
<td>20</td>
<td>?????</td>
<td>0.98</td>
<td>—</td>
</tr>
</tbody>
</table>

????: holes not cracked at total failure. The nr. of cycles until initiation is therefore not known.

X(1): invalid data point because of too much influence of neighbouring damage.

X(2): invalid data point because of too much total net section reduction (net section is less than 3/4 of original net section).

$\Delta i$: nr. of places that the considered data point could shift down

The missing data points in row 15 to 20 make that the graph will be incomplete from $N = 324$ kcycles. This means that from there the graph has to be estimated.
The $X(1)$ and $X(2)$ data points are not valid and therefore do not supply any information, except that their i-number can vary from $i-\Delta i$ to $m$. This makes that all following (valid) data points have a range of $\Delta P = \Delta i/m$, where $\Delta i$ is the range of $i$ due to the possible shift of $X(1)$ data points. These effects are plotted in figure B-3, based on table B-1.

**Required number of test specimens**

Starting points for the estimation of the required number of specimens for the test program were,

1. Log(N) is Weibull distributed
2. There are 40 initiation sides per specimen
3. Four stress levels need to be tested

ad 1 In principle any other distribution curve that fits would be equally appropriate. The Weibull distribution has been chosen, because test results indicate that it is applicable and because it has 2 other important advantages:

- There are only 3 parameters and the shape of the curve can still be formed in all required ways that suits this purpose.
The 3 parameters in equation B-5 have clear physical meanings:

\(N_0\): the minimum possible value of \(N\) until initiation

\(\alpha\): \(\alpha + \log(N_0)\) is the value of \(\log(N)\) at which the cumulative probability is 0.63

\(\beta\): a measure for the slope of the curve

Especially the second argument is convenient in this case, because several of these curves have to be estimated (4 different stress levels); The curves should continuously merge for changing values of \(S\). It is suggested that the 3 parameters vary linearly between 2 estimated scatter curves.

If \(N\) is the number of cycles until crack initiation, the next equation represents the (unknown) Weibull distribution:

\[
P = 1 - e^{-\left[\frac{(\log(N) - \log(N_0))}{\alpha}\right]^\beta}
\]  \hspace{1cm} (B-8)

where the parameters \(N_0\), \(\alpha\) and \(\beta\) must be estimated. Two procedures to do this are described later.

The number of test results that is necessary to estimate the 3 parameters of the Weibull distribution is determined with the help of statistics. In a situation where the exact Weibull distribution is known, a sample of tests will not always give the same percentage of initiated rivet sides as the theory would predict (at the same number of cycles). The extent to which the test results correspond to the theoretical distribution curve, depends amongst others on the sample size. This is illustrated with figure B-4 in which the exact Weibull distribution for \(N_0 = 50000\), \(\alpha = 0.12\) and \(\beta = 2\) is plotted. The test results for a binomial distribution can be approximated by a standard distribution. This means that a range in which with a
90% chance, the number of initiated cracks consists of the theoretical value plus or minus the standard deviation times a constant,

$$P_{90\%}(N) = P(N) \pm constant \sqrt{\frac{P(N)(1 - P(N))}{n_{test}}}$$  \hspace{1cm} (B-9)$$

In the case that a 90% level of confidence is required, this constant is 1.64.

In figure B-4, the range in which the test results will lie, with a 90% level of confidence, are plotted for $n_{test} = 10, 20$ and 60. From this figure, it can be concluded that for the 60 test results, a sufficiently narrow band is formed. This means that there is little room for plotting a wrong Weibull distribution between the boundaries of these bands.

![Figure B-4: Weibull distribution with 90% bands](image)

Because a Weibull distribution is determined by three variable parameters, at least three points are necessary to estimate the unknown parameters. To
estimate each of these three points with a sufficient level of confidence, 60 test results per point seem to be required. Of course the "90% band" that is just acceptable is a somewhat arbitrary choice. Also because of the limited time available, more than 60 test results per point would be unattractive. However, real test results can not be obtained at three discrete places (number of cycles until initiation). The results will form a scatter band around the fitted Weibull distribution. This is no problem, because the same level of confidence is achieved in the case that three points are determined with each 60 results as for the case that the parameters are determined by a scatter band, consisting of 180 points (see figure B-5).

![Figure B-5: Illustration of 3 discrete points and scatter band](image)

The number of specimens per stress level becomes, $3 \times 60/40 = 4.5$ specimens (one specimen supplies 40 data points). Because of the already large test program, this number is rounded to 4. The total number of specimens (4 stress levels) becomes, $4 \times 4 = 16$. Four extra specimens have been fabricated for static tests and unforseen problems.
**Procedures for curve fitting a Weibull distribution**

If test data have been generated, they have to be evaluated and interpreted. If one is interested in initiation scatter, this means: find the initiation threshold and a reliable scatter curve. If enough data points have been gathered, both unknowns can be found by curve-fitting a three parameter Weibull scatter curve.

Three parameter Weibull scatter curve:

\[ P_{\text{cum}}(x) = 1 - e^{-\left(\frac{x - x_0}{\alpha}\right)^{\beta}} \]  \hspace{1cm} (B-10)

where,  
- \( x_0 \) is the location parameter: the value of \( x \) for which \( P_{\text{cum.}} = 0 \)
- \( \alpha \) is the scale parameter: for \( x = \alpha + x_0 \), \( P_{\text{cum.}} = 0.63 \)
- \( \beta \) is the shape parameter: a measure for the slope of the curve

This can be done in different ways. Two methods that have been used in the computer program WEIBULL.EXE are discussed here.

1. **Most likelihood parameters**

This method is taken from [28] and is actually for estimating the scale-and shape parameter for a two parameter Weibull distribution. In a two parameter Weibull distribution \( x_0 \) in equation B-10 is equal to 0. By visually estimating a value for \( x_0 \) (using a graph), the method can still be used to calculate the maximum likelihood estimates of the scale-and shape parameter for a three parameter Weibull distribution. The estimates are obtained by solving the pair of likelihood equations given below,

\[ \alpha \beta n - \frac{\beta}{\alpha^{\beta-1}} \sum_{i=1}^{n} X_i^\beta = 0 \]  \hspace{1cm} (B-11)
\[ \frac{n}{\beta} - n \ln(\alpha) + \sum_{i=1}^{n} \ln(X_i) - \sum_{i=1}^{n} \left[ \frac{X_i^\beta}{\alpha} \right] \ln(X_i) - \ln(\alpha) = 0 \]  \hspace{1cm} (B-12)

Equation B-11 can be rewritten as,

\[ \alpha = \left[ \frac{\sum_{i=1}^{n} X_i^\beta}{n} \right]^{1/\beta} \hspace{1cm} (B-13) \]

By substituting equation B-13 into equation B-12, the following equation is obtained.

\[ \frac{n}{\beta} + \sum_{i=1}^{n} \ln(X_i) - \frac{n}{\sum_{i=1}^{n} X_i^\beta} \sum_{i=1}^{n} X_i^\beta \ln(X_i) = 0 \]  \hspace{1cm} (B-14)

Since equation B-14 depends only on the data \( X_1, X_2, ..., X_n \), it can be solved numerically and the solution of \( \beta \) that is obtained, is substituted into equation B-13 to obtain \( \alpha \).

The main disadvantage of this method is that it assumes a complete set of data and can not analyze incomplete sets of data. A method that does not have this limitation is the "minimum correlation coefficient method".

2. Minimum correlation coefficient method

This method is pre-eminently suitable for estimating initiation scatter curves, because it can handle incomplete sets of data.

The method is based on linear regression of the data. In order to do this, the distribution function is rewritten as,
\[ Y = AX + B \]  \hspace{1cm} (B-15)

with,
\[ Y = \ln(-\ln(1-P_f)), \quad X = \ln(x - x_0) \]  \hspace{1cm} (B-16)

where,
\[ x = \log(N_i) \quad \text{and} \quad x_0 = \log(N_0) \]  \hspace{1cm} (B-17)

A and B are determined with the least square method for assumed \( N_0 \) values. By iteration, the \( N_0 \) value is determined for which the correlation coefficient has a maximum. The Weibull constants are,
\[ \alpha = e^{-\frac{B}{A}} \]  \hspace{1cm} (B-18)
\[ \beta = A \]

Figure B-6: maximum correlation coefficient method for estimating \( N_0, \alpha, \) and \( \beta \)
Appendix C: fractography of riveted specimens

In this appendix some of results of the fractographic analysis are presented. Drawings (51) and photographs (7) of some illustrative fracture surfaces are included to demonstrate the differences in, and the complexity of initiation mechanisms in riveted joints. At the end of the appendix, a simple fractographic analysis is performed to demonstrate that cracks can cause significant stress redistribution in the specimen.
Basically 2 different initiation mechanisms exist for riveted joints,
1. Initiation dominated by the stress concentration of the combined pin-remote loaded hole
2. Initiation dominated by fretting between the facing sheet surfaces

Between these two extremes a transition range exists for which the initiation is significantly affected by fretting as well as by the stress concentration factor.

Whether mechanism one or two applies, depends on many aspects. Some of these are listed below,
- Ratio between bypass and remote stress (nr. of rivet rows)
- Clamping of the connected sheets (riveting intensity/quality)
- Hole expansion (riveting intensity)
- Bearing area by which the rivet load is transferred (countersink rivet/protruding head rivet)
- Rivet flexibility
- Surface quality of the sheet material (clad layer; anodized)

All of these factors except the last, owe their influence by more or less reducing the stress concentration at the "expected" critical places (see figure C-1).

If the stress concentration is sufficiently reduced, the initiation site can change from points A to point B in figure C-1.
An interesting example* for which cracks occurred at places A (critical) as well as at place B is shown in figure C-2. The extensive fretting at the interfacing surface is shown in figure C-2 (b) (opposite side of C-2 (a)). Several initiation cracks are clearly visible.

* This specific specimen cracked in both sheets at the (critical) third row.
To summarize the effects of several design/produce parameters on the initiation mechanism, table C-1 has been made. Table C-1 also indicates that the fatigue life generally increases as the initiation mechanism changes from 1 (stress concentration factor dominated) to 2 (fretting dominated).

<table>
<thead>
<tr>
<th>design/produce parameters</th>
<th>Mechanism 1 (SCF dominated)</th>
<th>transition range</th>
<th>Mechanism 2 (fretting dominated)</th>
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<td>rivet type</td>
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<td>increasing number of rivet rows</td>
<td>protruding head rivet</td>
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<td>riveting intensity</td>
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</table>

It might not be surprising that both mechanisms induce different fracture surfaces and result in different fatigue lives. Figure C-3 shows two typical initiation cracks for mechanism 1 (a) and 2 (b). Photograph C-3 (a) has been taken from an ineffectively riveted specimen \(D_1/D_2=1.45\) with a total fatigue life of 110000 cycles. The specimen of figure C-3 (b) has been riveted more intensively \(D_1/D_2=1.6\) and had a total fatigue life of more than 290000 cycles.

* The standard value of \(D_1/D_2\) is 1.5
The semi elliptical crack in figure C-3 (b) can grow to a considerable size before it becomes a through crack and thereby visible from the outside. The crack as shown in figure C-3 (a) on the contrary, becomes visible practically as soon as it emerges from under the rivet head.

**Initiation scatter**
Although significant MSD occurred in only 1 of 9 tested specimens (S-series), fracture surfaces indicate that crack nuclei existed at almost all initiation sites. This confirms that initiation scatter is small and MSD is likely to occur. The reason that MSD did not occur in most of the tested specimens is due to a side effect which caused dominant cracks at the outer rivets (see also section 5.1). Two examples of typical fracture surfaces for the S-series specimens are shown in figure C-4. Figure C-5 shows a fracture surface of specimen T-3 which suffered extensive MSD.
Figure C-4: typical fracture surfaces with many crack nuclei

Figure C-5: MSD in specimen T-3
Finally a overview is given of fracture surfaces at final failure of riveted specimens tested in Delft [26] (figure C-7 to C-12). If it is assumed that no stress redistribution occurred in these cases, figure C-6 gives the average net section stresses for the smallest ligaments that are necessary to equilibrate the (homogeneous) remote stresses. These net section stresses are far beyond $\sigma_{0.2}$ and even $\sigma_u$. Since this is not possible, the remote stress is not homogeneous and significant stress redistribution has appeared.

Figure C-6: net section stresses in case of homogeneous remote stress
Figure C.7: fracture surfaces of riveted specimens reference [26]
Figure C-8: fracture surfaces of riveted specimens reference [26]
Figure C.9: fracture surfaces of riveted specimens reference [26]
Figure C-10: Fracture surfaces of riveted specimens reference [26]
Figure C-11: Fracture surfaces of riveted specimens reference [26]
Appendix C: Fractography of riveted specimens

Figure C-12: Fracture surfaces of riveted specimens reference [26]
Appendix D: verification of the similar ratio method

In this appendix two verification methods are described that have been used to check the "similar ratio method " (SRM). This method has extensively been used in the analysis of chapter 3.

1. **Direct comparison**
   In this case a comparison is made between the SRM results of the unknown problem described in section 3.3 page 16 and finite element calculations. The problem involves the stress concentration factor for a pin loaded hole with a crack on the opposite side (see also figure 3.14 (b) p. 30). The FEM calculations were performed by A.U. de Koning and C. Lof of the NLR in close correspondence with the author.

2. **Indirect comparison**
   The known solution for the stress concentration factor of a hole with a crack on one side in remote tension [5], is compared to SRM results. The differences between both results is an indication for the accuracy of the method.

Both cases will be discussed in the next two paragraphs.
1. **Direct comparison: stress concentration factor for a pin-loaded hole with a crack on one side.**

This problem can be modelled by the next figure.

![Diagram](image)

Figure D-1: finite element model for unknown stress concentration factor

\[ K_t = \frac{\sigma_{max}}{\sigma_{bearing}} \quad (\sigma_{bearing} = \frac{P}{D}) \]

**Finite element calculations**

Figure D-1 is the model used for the FEM calculations. The one way restraints on the left and right side simulate symmetry around these lines. The rivet has been simulated by using gap-elements.

The accuracy of the FEM results is expected to be within 1-3% of the exact solution.
To cover all load situations that are needed for the calculations in section 3.3, the next two cases have been studied.

![Diagram](image)

**Case 1** + **Case 2**

Figure D-2: two cases to be solved

Using the principle of superposition, all linked-up situations can be calculated from the solutions to these two cases. Case 1 involves only one variable, l/R and the second case two, x/l and l/R. The concentrated load on the crack surface in case 2 simulates the contribution of the linked-up hole to $K_t$ on the other side.

**Problem 1: Load on the rivet centre for several values of l/R.**

In this case, the rivet load is inflicted in the centre of the rivet. From this, the resulting load distribution on the hole perimeter is calculated. This load distribution is a function of the variable $l/R$, $l/W$, and the applied load. Figure D-3 shows the load distribution for two cases: $l=0$ and $l=22.35$. Although a difference exists, it is small and not near the point for which $K_t$ is calculated. The load distribution will also change if a load is applied on the crack (simulation of linked-up hole). Strictly, this means that the final results can not be superimposed. This interaction effect is supposed to be small and has little influence on the stress con-
centration factor. $K_t$ has been calculated for 6 values of $l/R$ and has been plotted in figure D-5 together with the SRM results.

![Graph showing stress distribution](image)

**Figure D-3: load distribution on hole perimeter**

**Problem 2:** Load on the crack surface for different values of $x/l$.
Because of the comparative nature of these calculations, only one value of $l/R$ (19.82) has been studied for 7 values of $x/l$. The results are plotted in figure D-8 together with SRM results.

**SRM procedure**
In section 3.2 is explained that two correction factors need to be calculated; One to correct for the growing crack and one to correct for the finite width (see p. 33 and 34). These correction factors are not known for the stress concentration factor but they are known for the "matching" stress intensity factors. The similar ratio method presumes that these correction factors are of equal magnitude. Therefore, if the correction factor for the matching SIF is denoted $C_{SIF}$, the stress concentration factor becomes,
\[ K_t = C_{SIF} K_t^* \]  \hspace{1cm} (D-1) \]

where \( K_t^* \) is the known stress concentration factor for a pin loaded hole in an infinite sheet \( (K_t^* = \sigma_{max}/\sigma_{bearing} = 0.8106; \text{Schulz [12]}) \); 

\( C_{SIF} \) is calculated as follows.

\[ P_n = P_0 \cos(\phi) \]

\[ P_D, i \]

\[ x_1^\infty \]

\[ \phi \]

\[ R \]

\[ x_1, \text{begin} \]

\[ x_1, \text{end} \]

\[ i = 3 \]

\[ i = 2 \]

\[ i = 1 \]

\[ 2a \]

\[ 2R \]

\[ St \]

\[ x \]

\[ x^* \]

Figure D-4: SIF for growing crack; n holes

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The load transfer by the rivet is assumed to be without friction and is therefore perpendicular to the surface.

\[ x_{i, \text{begin}}^* = (n - i) St + l \]  \hspace{1cm} (n is the number of linked-up holes) (D-2)

\[ x_{i, \text{end}}^* = x_{i, \text{begin}}^* + 2R \]

\[ x = x^* - a \] (D-3)

(The sub-script "begin" and "end" denote the left and right side of the holes as indicated in figure D-4)

\[ P_{x,i} = P_{0,i} \cos^2 \phi = P_{0,i} \left[ 1 - \left( \frac{x_{i}^{**}}{R} \right)^2 \right] \]  \hspace{1cm} (D-4)

\[ = P_{0,i} \left[ 1 - \left( \frac{x - R - x_{i, \text{begin}}^*}{R} \right)^2 \right] \]

\[ P_{\text{res},i} = P_{0,i} R \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 \phi \, d\phi = P_{0,i} \frac{\pi R}{2} \Rightarrow \]  \hspace{1cm} (D-5)

\[ P_{0,i} = \frac{2}{\pi R} P_{\text{res},i} \]

For a concentrated force \( P \), the solution by Tada [5] is,

\[ Tada: \quad K = \frac{P}{2 \sqrt{\pi a}} \sqrt{\frac{a + x}{a - x}} \]  \hspace{1cm} (D-6)

\[ \Rightarrow \quad dK = \frac{P_{x,i} \, dx}{2 \sqrt{\pi a}} \sqrt{\frac{a + x}{a - x}} \]  \hspace{1cm} (D-7)

* Note that the resulting rivet load can still be different for all \( i \).
\[ a = \frac{(n - 1)}{2} \text{St} + R + \frac{1}{2} \quad (n = 3 \text{ in the above case}) \quad (D-8) \]

Combining formula D-2, D-3, D-4, D-7 and D-8 yields,

\[ K = \sum_{i=1}^{n} \frac{P_{0,i}}{2\sqrt{\pi a}} \int_{x_{i,\text{begin}}^{x_{i,end}}} \left[ 1 - \left( \frac{x - R - x_{i,\text{begin}}}{R} \right)^2 \right] \sqrt{\frac{a + x}{a - x}} \, dx \quad (D-9) \]

Define:

\[ y = \frac{x}{R}, \quad b_i = \frac{x_{i,\text{begin}}}{R} + 1, \quad c = \frac{a}{R} \]

\[ K = \frac{R}{2\sqrt{\pi a}} \sum_{i=1}^{n} \frac{P_{0,i}}{x_{i,\text{begin}}^{x_{i,end}}} \int_{b_i}^{\frac{x_{i,end}}{R}} \left[ 1 - (y - b_i)^2 \right] \sqrt{\frac{c + y}{c - y}} \, dy \quad (D-10) \]

Finally,

\[ C_{SIF}(a) = \frac{K(a)}{K(a=R)} \quad (D-11) \]

where,

\[ K(a=R) = \frac{R}{2\sqrt{\pi R}} \int_{0}^{1} (1 - y^2) \sqrt{\frac{1+y}{1-y}} \, dy = \frac{P_{0,1}}{4} \sqrt{\pi R} \quad (D-12) \]
For \( n = 1 \) (no link-ups) the integral in equation D-10 can be solved analytically and the result is,

\[
C_{SIF} = \frac{2}{\pi \sqrt{1 + s}} \left\{ \left( \frac{1 - s}{2} \right)^2 \left( \frac{\pi}{2} + \arcsin \left( \frac{1 - s}{1 + s} \right) \right) + 2(1 - s)\sqrt{s} \right\} + \frac{8}{3} s^{\frac{3}{2}}
\]

(D-13)

where, \( s = \frac{1}{2R} \)

This result is graphically shown in figure D-5 (solid line). In order to make a fair comparison with the FEM calculations, the SRM results should be corrected for the finite width and the remote stress. The dotted line in figure D-5 represents the corrected SRM results. This correction has been done in the same way as described in section 3.2. For a more detailed treatise of the correction, the reader is referred to this section.

The same correction can of course be made the other way around for the FEM results. The \( \triangle \) elements in figure D-5 represent the corrected FEM results. A comparison between the solid line and the corrected FEM results shows that the correction for finite width and remote stress is too much; This can be concluded, because it is physically not likely that the correction factor, \( C_{SIF} \), decreases as \( 1/R \) increases (as indicated by the \( \triangle \) elements in figure D-5). The differences between the FEM- and SRM results are therefore mainly due to this correction.
For values of \( n \) not equal to 1 (1 or more link-ups), the integral in equation D-10 must be numerically integrated.

In figure D-6 a comparison is made for \( C_{SIF} \) between SRM results for a cosine load distribution (as assumed in figure D-5) and a concentrated force on the hole perimeter. The solution for the concentrated force is,

\[
C_{SIF} = \sqrt{\frac{1 + \frac{1}{R}}{1 + \frac{1}{2R}}} \tag{D-14}
\]

The limit for \( l/R \to \infty \) for this case is \( \pi/2 \). From figure D-6, it appears that the limit for the cosine distribution is \( \pm 10\% \) less and more accurate when compared to the FEM results in figure D-5.
Finally the relative differences between the FEM results and the SRM results are plotted in figure D-7 for the case of a cosine load distribution and a concentrated force.

Figure D-7: relative difference between FEM- and SRM results
If the FEM results for case 2 (loaded crack: figure D-2) are compared to the SRM results, a larger difference is found (see figure D-8).

![Graph showing comparison between FEM calculations and SRM methods.](image)

**Figure D-8: correction for loaded crack**

This difference is built up of the same three components as for case 1,

1. error in the SRM correction for linked up hole
2. error in finite width correction
3. error in remote stress correction

In contrary to the previous case, it is not possible to reason which contribution has the largest impact.
2. **Indirect comparison: stress concentration factor for a hole with a crack on one side in remote tension (infinite sheet).**

This problem is defined by the next figure.

![Diagram](image)

Figure D.9: stress concentration factor for a hole with a crack on the opposite side in remote tension (infinite sheet)

Tada et al [5] gives a solution to this problem (this solution has been used in the calculations in chapter 3). An equation for this configuration fitted to the numerical results, given in [5], is

\[
C_{SIF} = \left( \frac{1}{3} + \frac{2}{3} \sqrt{a/R} \right) \left\{ 1 + \frac{1}{6} \left[ \frac{2}{\pi} \arcsin \left( \frac{a - R}{a} \right) \right]^3 \right\}
\]  \hspace{1cm} (D-15)

The problem can also be solved with the similar ratio method.

\[
K(l) = S \sqrt{\pi (R+1/2)} \quad \Rightarrow \quad K(l=0) = S \sqrt{\pi + R}
\]  \hspace{1cm} (D-16)

\[
\frac{K(l)}{K(l=0)} = \sqrt{1 + \frac{1}{2R}}
\]  \hspace{1cm} (D-17)
Both solutions are compared in figure D-10. The relative difference (percentage of the SRM results) is plotted in figure D-11 and varies from 0 to ±19 %.

Figure D-10: comparison between SRM results and the solution by Tada et al

Figure D-11: relative difference between SRM results and the solution by Tada et al
Conclusions
From both comparisons, it seems reasonable to conclude that the expected error made by the similar ratio method can vary from 0 to ±20%.

For both examined cases, the similar ratio method gives conservative results. In particular the finite width correction seems to be over-conservative.

The problem for which the similar ratio method is actually used in chapter 3 is estimated to have a deviation varying from 0% to 10% in the extreme case.
Appendix E: default settings for simulation program

This appendix contains the default settings of all run-parameters that are automatically stored in the file "RUNPAR.DA1" every time the simulation program is run. This file can be found in the sub-directory "\OUT" and can be printed by selecting option [3. Print data file] in the main menu.
OUTPUT FILE (RUNPAR.DA1) OF PROGRAM

MM MM SS SS DDD SS SS I MM MM
M M M S D D S I M M M
M M SS SS D D SS S I M M M
M M SS SS D D S S I M M M
M M SS SS DDD SS SS I M M M

THIS FILE CONTAINS THE SETTINGS OF ALL PARAMETERS USED FOR RUN

**** "Default" ****

The names of the parameters correspond to the names in report,

MULTIPLE SITE DAMAGE IN FUSELAGE LAP JOINTS
Requirements for inspection intervals for typical fuselage lap joint panels with Multiple Site Damage

For further explanation of these parameters, the reader is referred to appendix A of this report

FATIGUE SIMULATION PARAMETERS

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<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>L0 (mm)</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>DK exp.</td>
<td>3.00</td>
<td>3.00</td>
</tr>
<tr>
<td>Cp-O (*)</td>
<td>0.78</td>
<td>0.78</td>
</tr>
<tr>
<td>Cp-lambda (*)</td>
<td>2.17</td>
<td>2.17</td>
</tr>
<tr>
<td>Cp-alpha</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Nr. of rivets</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Print Nr.</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Stat. link I.</td>
<td>FALSE</td>
<td>FALSE</td>
</tr>
<tr>
<td>Stat. link Gr.</td>
<td>FALSE</td>
<td>FALSE</td>
</tr>
</tbody>
</table>

(*) Unit is: mm/(MPa·mm)^DK exp.
Weibull constants for initiation scatter (P-S-N curve)

<table>
<thead>
<tr>
<th>Stress level</th>
<th>NO</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
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<tr>
<td>70.00 MPa</td>
<td>500000</td>
<td>0.18000</td>
<td>2.70000</td>
</tr>
<tr>
<td>85.00 MPa</td>
<td>340000</td>
<td>0.12000</td>
<td>2.00000</td>
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<tr>
<td>105.00 MPa</td>
<td>210000</td>
<td>0.08000</td>
<td>1.50000</td>
</tr>
<tr>
<td>120.00 MPa</td>
<td>150000</td>
<td>0.06000</td>
<td>1.20000</td>
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<tr>
<td>150.00 MPa</td>
<td>80000</td>
<td>0.04000</td>
<td>1.00000</td>
</tr>
<tr>
<td>200.00 MPa</td>
<td>30000</td>
<td>0.02000</td>
<td>1.00000</td>
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<tr>
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<td>0.01000</td>
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<tr>
<td>500.00 MPa</td>
<td>500</td>
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<td>1.00000</td>
</tr>
</tbody>
</table>

$P(x) = 1 - \exp \left[ - \left( \frac{\log(N) - \log(NO)}{\alpha} \right)^\beta \right]$

**INSPECTION SIMULATION PARAMETERS**

Weibull constants for detectability of cracks:

Visual: $A_0 = 6.35$ mm  \hspace{1cm} Eddy Current: $A_0 = 3.05$ mm
$\alpha = 12.70$ mm \hspace{1cm} $\alpha = 6.35$ mm
$\beta = 0.50$ \hspace{1cm} $\beta = 0.50$

$P(x) = 1 - \exp \left[ - \left( \frac{A - A_0}{(\alpha - A_0)} \right)^\beta \right]$

Threshold inspection: 210000 cycles

Inspections have been simulated for the next intervals,

2000 cycles 5600 cycles
2200 cycles 5800 cycles
2400 cycles 6000 cycles
2600 cycles 6200 cycles
2800 cycles 6400 cycles
3000 cycles 6600 cycles
3200 cycles 6800 cycles
3400 cycles 7000 cycles
3600 cycles 7200 cycles
3800 cycles 7400 cycles
4000 cycles 7600 cycles
4200 cycles 7800 cycles
4400 cycles 8000 cycles
4600 cycles 8200 cycles
4800 cycles 8400 cycles
5000 cycles 8600 cycles
5200 cycles 8800 cycles
5400 cycles 9000 cycles
## STRESS DISTRIBUTION

<table>
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<tr>
<th>rivet nr.</th>
<th>s (N/mm^2)</th>
<th>sbend (N/mm^2)</th>
<th>p (N/mm)</th>
<th>rivet nr.</th>
<th>s (N/mm^2)</th>
<th>sbend (N/mm^2)</th>
<th>p (N/mm)</th>
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<td>11</td>
<td>77.1</td>
<td>22.6</td>
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<td>88.2</td>
<td>29.5</td>
<td>911.8</td>
<td>12</td>
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<td>942.7</td>
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<td>3</td>
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<td>38.8</td>
<td>911.8</td>
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<td>95.8</td>
<td>38.8</td>
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<td>4</td>
<td>100.7</td>
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<td>929.4</td>
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<td>102.8</td>
<td>42.8</td>
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<tr>
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<td>911.8</td>
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References

Structural integrity of aging airplaines
ISBN 3-540-53461-X (Springer-Verlag Berlin Heidelberg New York)

[2] Broek, D.
The inspection interval for multi-site damage in fuselage lap-joints
FractuResearch inc., Report TR-9002, Galena, Ohio 43021, april 1990

Compendium of stress intensity factors
Her Majesty’s Stationary Office, London, 1976

[4] Rooke, D.P.
Compounding stress intensity factors
Royal Aircraft Establishment, Farnborough, England, 1986

The stress analysis of cracks handbook.
St. Louis, Missouri 63105, 1973, 1985

Stress intensity factors for a crack at the edge of a pressurized hole.

Flaw growth in complex structure. Vol. 1, technical discussion
Lockheed-California Company, Burbank, California december 1977

[8] Brussat, T.R.
Estimating initiation times of secondary fatigue cracks in damage toleran-
ce analysis
A Laboratory Study of Multiple Site Damage in Fuselage Lap Splices
Arthur D. Little, Inc.
Acorn Park, Cambridge, Massachusetts
Reference 63053

[10] Schijve, J.
Preprint: Multiple Site Damage; Fatigue of riveted joints
International Workshop on Structural Integrity of Aging Airplaines
Atlanta, 31 March - 2 April, 1992
Faculty of Aerospace Engineering, Delft University of Technology

On the calculation of stresses in pin-loaded anisotropic plates
Delft University of Technology, Delft 1987

[12] Schulz, K.J.
Over den spanningstoestand in doorboorde platen (on the state of stress in perforated plates).
Delft University of Technology, Delft 1941

[13] Soest, J. van
Elementaire Statistiek (Elementary Statistics)
Delftse uitgevers maatschappij, Delft 1988

[14] Weibull, W.
Fatigue testing and analysis of results
Pergamon press, Sweden 1961

[15] Centre d'Essais Aéronautique de Toulouse p. 1.1/1 - 1.1/87 (Swift, T.)
ICAF - Doc No. 1336: proceedings of the twelfth ICAF symposium:
industrial applications of damage tolerance concepts. Structures of high
fatigue performances.
Toulouse, May 25-27, 1983
References

[16] Swift, T.
Damage tolerance in pressurized fuselages (Douglas paper 7768), 11th Planterma Memorial Lecture (p 32-??)
Presented to 14th symposium ICAF
New materials and fatigue resistant aircraft design
Ottawa, Canada, June 10-12, 1987

[17] Swift, T.
The influence of slow growth and net section yielding on the residual strength of stiffened structure
From: Salvetti, A. and Cavallini, G.
Durability and damage tolerance in aircraft design
The proceedings of the 13th symposium of the ICAF
22-24 May, 1985, Pisa

[18] Asada, H.
Reliability assessment of pressurized fuselages with multiple-site fatigue cracks
5th international conference on structural safety and reliability
ICOSSAR, 1989

[19] Schijve, J.
Vliegtuigmateriale I (Aircraft materials I)
Delft University of Technology, Delft

[20] Schijve, J.
Report LR-360: lecture notes on fatigue, static tensile strength and stress corrosion of aircraft materials and structures
Delft University of Technology, Delft, 1982

[21] Patil, S.
Finite element analysis of lap joint
Foster-Miller Inc. (1990)
[22] Isida, M.
Transactions of American Society of Mechanical Engineers
Journal of Applied Mechanics, Vol. 33, No.3
Sept. 1966, pp. 674-675

[23] Wit, G.P.
Stress intensity factors for multiple site damage in riveted lap joints
Delft University of Technology, Delft, June 1991

[24] Lewis, W.H.
Reliability of non-destructive inspection
San Antonio Air Logistics Center
Report SA-ALC/MME 76-6-38-1 (1978)

[25] Peterson, R.E.
Stress Concentration Factors
John Wiley, 1974

[26] Brandts, M.
Elaminatie gloeinagel (Master thesis)
Delft University of Technology, Delft 1992

[27] Schijve, J.
Vermoeïing van constructies (fatigue of structures)
Post academic course
Delft University of Technology, Delft

[28] Mil.-HDBK-17B
29 February 1988

[29] Cun-Yung Niu, M.
Airframe structural design
Lockheed Aeronautical Systems Company, Burbank, California
Commilit Press LTD.