Master Thesis
Arctic stick-slip effects on a simplified moored vessel in head-on ice conditions

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MASTER THESIS

ARCTIC STICK-SLIP EFFECTS ON A SIMPLIFIED MOORED VESSEL IN HEAD-ON ICE CONDITIONS

by

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ABSTRACT

The stick-slip effect is a quasi-static effect that can occur with every mass-spring system that is subjected to significant amount of friction forces. It is based on the difference between the static and kinetic friction coefficient, where the static friction coefficient normally is much bigger than the kinetic friction coefficient. If the relative velocity between two objects is close to zero, the sliding object will encounter a larger friction force (static) than when the relative velocity is bigger than zero (kinetic). Under certain conditions this friction force can alternate between static and kinetic, which can cause jerky motions between the two sliding objects and finally high spring load peaks.

Practical occurrences of the stick-slip effect are previously studied in several different applications; for example with brake-systems in the automotive industry, earthquakes and glacier movements, but also in the oil and gas industry for drilling or weather-vaning FPSO turret systems. In this master thesis the stick-slip effects on a moored vessel in head-on ice conditions are studied.

A vessel in ice conditions is highly dependent on the ice-hull friction coefficient and therefore it is found that the stick-slip effect can also occur during the station keeping of a vessel in an Arctic environment. A vessel with a single point mooring is able to "ice-vane" towards a head-on ice condition and this ice load is consequently causing surge motions of the vessel. If the surge velocity reaches the same velocity as the ice drift velocity the relative velocity will be close to zero. Consequently the static friction coefficient applies and the vessel will "stick" with the ice-sheet. The vessel will be released again when the restoring mooring forces are big enough to overcome the static friction forces.

Although the stick-slip effect will only occur under specific circumstances, it will normally result in large mooring load peaks, which can ultimately have an effect on the mooring system design. During this thesis the specific stick-slip circumstances and the stick-slip consequences are researched with the help of a created numerical model. The work performed in this master thesis research is divided in three parts:

1. Creation of a numerical model that can predict the response of a simplified moored vessel in head-on ice conditions, including stick-slip effects.

2. Validation of the numerical model with the help of previously performed experiments.

3. With the help of this numerical model further insight is found in the consequences and the probability of stick-slip effects in Arctic mooring systems.

This master thesis research is made possible with the support of the R&D department of SOFEC. The data to validate the numerical model was provided by Vegard Aksnes and the HYDRALAB project consortium.
# Contents

List of Figures \hfill vii
List of Tables \hfill ix

## 1 Introduction
1.1 Offshore mooring systems \hfill 1
1.2 Stick-slip effect \hfill 1
1.3 The Aksnes model experiments \hfill 2
1.4 Design codes and standards \hfill 3
1.5 Goal of the thesis \hfill 4
1.6 Research questions \hfill 4

## 2 Research Method
2.1 SDOF model \hfill 5
2.2 Input parameters \hfill 8
\hspace{1em} 2.2.1 Vessel & mooring system \hfill 8
\hspace{1em} 2.2.2 Ice sheet \hfill 8
\hspace{1em} 2.2.3 Friction coefficient \hfill 9
2.3 External ice load calculations \hfill 10
\hspace{1em} 2.3.1 Ice break load \hfill 10
\hspace{1em} 2.3.2 Ice friction load \hfill 12
\hspace{1em} 2.3.3 Ice rotation load \hfill 13
\hspace{1em} 2.3.4 Ice velocity effects \hfill 14
2.4 Stick-events \hfill 14
\hspace{1em} 2.4.1 Stick-event criteria \hfill 14
\hspace{1em} 2.4.2 Stick event response \hfill 15
2.5 Random variability of the ice sheet \hfill 16
2.6 Numerical implementation \hfill 17
2.7 Assumptions \hfill 18

## 3 Numerical model validation
3.1 Current Arctic operations \hfill 19
3.2 Input: Base-case parameters \hfill 19
3.3 Output: Aksnes experiments \hfill 21
3.4 Output: numerical model \hfill 22
3.5 Stick-event validation \hfill 25
3.6 Global validation \hfill 26

## 4 Result analysis
4.1 Consequences of the stick-slip effect \hfill 30
4.2 Probability of the stick-slip effect \hfill 30
\hspace{1em} 4.2.1 Sensitivity analysis \hfill 31
\hspace{1em} 4.2.2 Environmental sensitivity \hfill 32
\hspace{1em} 4.2.3 Vessel sensitivity \hfill 35

## 5 Conclusions
5.1 Conclusions \hfill 37
5.2 Discussion \hfill 38
5.3 Recommendations \hfill 38
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Internal Disconnectable Turret Mooring - FPSO Pyrenees Venture. Photo courtesy of SOFEC.</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Illustration of the Stick-slip effect with a moored FPSO.</td>
<td>2</td>
</tr>
<tr>
<td>1.3</td>
<td>Illustration of the Aksnes numerical model. Figure courtesy of Aksnes (2010)</td>
<td>2</td>
</tr>
<tr>
<td>1.4</td>
<td>Illustration of the Aksnes model experiments. Figure courtesy of Aksnes (2010)</td>
<td>3</td>
</tr>
<tr>
<td>1.5</td>
<td>Results of the Aksnes experiments. Figure courtesy of Aksnes (2010)</td>
<td>3</td>
</tr>
<tr>
<td>1.6</td>
<td>ISO19906: Ice failure modes. Figure courtesy of ISO19906 [13] (2010)</td>
<td>4</td>
</tr>
<tr>
<td>2.1</td>
<td>Illustration of the single degree of freedom model.</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>Illustration of 1 ice breaking cycle.</td>
<td>7</td>
</tr>
<tr>
<td>2.7</td>
<td>Ice-steel friction coefficient vs. relative velocity by Saeki and Ono (1984) and ISO19906 [13].</td>
<td>9</td>
</tr>
<tr>
<td>2.8</td>
<td>Composition of the external ice force $F_{ext}$ with a homogeneous ice sheet.</td>
<td>10</td>
</tr>
<tr>
<td>2.9</td>
<td>An illustration of the semi-infinite beam with an elastic foundation model.</td>
<td>11</td>
</tr>
<tr>
<td>2.10</td>
<td>Illustration of the friction forces and the bow buoyancy force.</td>
<td>13</td>
</tr>
<tr>
<td>2.11</td>
<td>Composition of the stick force of both the break-stick and the slip-stick events.</td>
<td>15</td>
</tr>
<tr>
<td>2.12</td>
<td>Zoomed in force distribution in the ice sheet when in a stick-event.</td>
<td>15</td>
</tr>
<tr>
<td>2.13</td>
<td>Newly generated ice sheet illustration.</td>
<td>16</td>
</tr>
<tr>
<td>2.14</td>
<td>Composition of the external ice force $F_{ext}$.</td>
<td>17</td>
</tr>
<tr>
<td>2.15</td>
<td>Numerical implementation.</td>
<td>17</td>
</tr>
<tr>
<td>3.1</td>
<td>Current (semi-) Arctic designed systems.</td>
<td>19</td>
</tr>
<tr>
<td>3.2</td>
<td>Filtered and non-filtered time domain mooring forces.</td>
<td>21</td>
</tr>
<tr>
<td>3.3</td>
<td>Numerical time domain output of the normal slip-phase.</td>
<td>22</td>
</tr>
<tr>
<td>3.4</td>
<td>Numerical time domain output of several slip-stick events.</td>
<td>22</td>
</tr>
<tr>
<td>3.5</td>
<td>Numerical time domain output of a break-stick event.</td>
<td>23</td>
</tr>
<tr>
<td>3.6</td>
<td>Orcaflex® preliminary numerical verification.</td>
<td>23</td>
</tr>
<tr>
<td>3.7</td>
<td>Zoomed in break-impulse.</td>
<td>24</td>
</tr>
<tr>
<td>3.8</td>
<td>Stick-event validation: stick-event length.</td>
<td>25</td>
</tr>
<tr>
<td>3.9</td>
<td>Stick-event validation: stick-event mooring load amplitude.</td>
<td>26</td>
</tr>
<tr>
<td>3.10</td>
<td>Total number of stick-events validation. Numerical model vs. Aksnes2100 experiment.</td>
<td>27</td>
</tr>
<tr>
<td>3.11</td>
<td>Number of slip-stick and break-stick events validation. Numerical vs. Aksnes2100 experiment.</td>
<td>28</td>
</tr>
<tr>
<td>4.1</td>
<td>Consequences of a stick-event with a variable ice thickness.</td>
<td>30</td>
</tr>
<tr>
<td>4.2</td>
<td>The break-impulses on a system.</td>
<td>31</td>
</tr>
<tr>
<td>4.3</td>
<td>OFAT analysis results in tornado charts.</td>
<td>32</td>
</tr>
<tr>
<td>4.4</td>
<td>3D graphs of the environmental sensitivity.</td>
<td>33</td>
</tr>
<tr>
<td>4.5</td>
<td>Environmental sensitivity; ice drift velocity vs. ice thickness vs. mooring stiffness.</td>
<td>34</td>
</tr>
<tr>
<td>4.6</td>
<td>Soft or stiff mooring system; 250 or 1250 kN/m.</td>
<td>34</td>
</tr>
<tr>
<td>4.7</td>
<td>Vessel sensitivity; scale factor vs. ice thickness vs. mooring stiffness.</td>
<td>35</td>
</tr>
<tr>
<td>A.1</td>
<td>Photo 1. Photo courtesy of Aksnes (2010).</td>
<td>41</td>
</tr>
<tr>
<td>A.2</td>
<td>Photo 2: ice coverage bow. Photo courtesy of Aksnes (2010).</td>
<td>42</td>
</tr>
<tr>
<td>C.1</td>
<td>Exact numbering of every individual stick-event in the Aksnes2100 experiment.</td>
<td>46</td>
</tr>
<tr>
<td>C.2</td>
<td>Correlation between the stick-length and the stick-amplitude in the Aksnes2100 experiments.</td>
<td>48</td>
</tr>
<tr>
<td>D.1</td>
<td>Monte Carlo histogram fitting.</td>
<td>50</td>
</tr>
<tr>
<td>E.1</td>
<td>Case study: Beaufort Sea environmental conditions in February 1994 to 2001.</td>
<td>51</td>
</tr>
<tr>
<td>E.2</td>
<td>Case study: Beaufort Sea environmental probability.</td>
<td>52</td>
</tr>
</tbody>
</table>
List of Tables

2.1 Input parameters in the numerical model ................................................. 8
2.2 Randomly varying input parameters depicted in red. .............................. 16

3.1 Test matrix Aksnes experiments (2010)]. Data courtesy of Aksnes (2010) and HYDRALAB. . . . 20
3.2 Input parameters Aksnes2100 & Aksnes4100. Data courtesy of Aksnes (2010) & HYDRALAB. . . . 20
3.3 Equilibrium-check of the specific break-impulse of figure 3.7. ............................. 25
3.4 Summary of the global validation of all Aksnes experiments (mean values). .................. 27
3.5 Results validation numerical model vs. Aksnes2100 experiment. ...................... 28

4.1 Summary of the base-case scenario parameters. See table 3.2 for the complete table. .......... 29
4.2 Description of the scale factor. ................................................................. 31

B.1 Input parameters Aksnes experiments. Data courtesy of Aksnes (2010) and HYDRALAB. ........ 43

C.1 Individually analyzed stick-events in the Aksnes2100 experiment. (part 1) ............... 47
C.2 Individually analyzed stick-events in the Aksnes2100 experiment. (part 2) .................... 48

D.1 Monte Carlo parameters .............................................................................. 49
INTRODUCTION

1.1. OFFSHORE MOORING SYSTEMS
Since the late 1940s the oil & gas industry is producing oil from offshore locations. It started with production-platforms situated on large steel jacket structures on the seabed. When oil was found in deeper waters it was a logical step to create production facilities on top of floating structures, such as large oil-tankers. These oil tankers sometimes need to be moored on a single location for over 20 years, this means that its mooring system need to be designed with a high level of integrity. The first Floating Production Storage and Offloading (FPSO) system was deployed in 1977 and since that year ±200 more FPSO systems were created worldwide. Due to the discovery of large oil & gas reserves in the Arctic a lot of research is performed on Arctic FPSO systems. During this thesis the mooring system of a vessel in Arctic conditions is studied with the focus on a specific effect; the Stick-slip effect. This vessel will not be an exact representation of a FPSO due to the simplified barge shaped hull, but the overall goal is to gain further insight on the stick-slip effect itself.

![Internal Disconnectable Turret Mooring - FPSO Pyrenees Venture. Photo courtesy of SOFEC.](image)

1.2. STICK-SLIP EFFECT
The stick-slip effect is a dynamic/quasi-static effect that can occur with every mass-spring system that is subjected to significant amount of friction forces. It is based on the difference between the static and kinetic friction coefficient, where the static friction coefficient normally is much bigger than the kinetic friction coefficient. If the relative velocity between two objects is close to zero, the sliding object will encounter a larger friction force (static) than when the relative velocity is bigger than zero (kinetic). Under certain conditions this friction force can alternate between static and kinetic, which can cause jerky motions between the two sliding objects and finally cause high spring load peaks. Practical occurrences of the stick-slip effect are previously studied in several different applications, for example the brake-systems in the automotive industry [19], earthquakes [5], but also in the oil and gas industry like drilling [14] or in weathervaning FPSO turret systems[6]. The occurrence of stick-slip with ice is also studied in glacier movements in Greenland [21].
In this thesis the stick-slip effects on moored vessels in head-on ice conditions are studied; these effects are caused by the significant ice-hull friction forces, which are present during ice breaking operations. A vessel with a single point mooring is able to "ice-vane" towards a head-on ice condition and this ice load is consequently causing surge motions of the vessel. If the surge velocity reaches the same velocity as the ice drift velocity the relative velocity is zero and thus the static friction coefficient applies and the vessel will "stick" with the ice-sheet. Although the stick-slip effect may probably only occur under specific circumstances, it will normally result in large mooring loads peaks, which can ultimately have an effect on the systems design. A so-called stick-event on an Arctic mooring system can be explained in two phases:

1. If the relative velocity between the vessel and the ice-sheet is zero and the vessel's momentum and/or the mooring restoring force is not big enough to break the ice the vessel will "stick" with the ice sheet. Consequently the vessel will experience an increasing offset and thus an increasing restoring mooring force. The stick-event will last until the restoring mooring forces are in equilibrium with the stick forces.

2. Right after this equilibrium the mooring forces will be bigger than the stick forces, so the vessel will "slip" through the ice again. This results in a relative velocity bigger than zero, and thus a kinetic friction coefficient with decreased ice forces. The vessel will be released towards its mean offset position.

Figure 1.2: Illustration of the Stick-slip effect with a moored FPSO.

1.3. THE AKSNES MODEL EXPERIMENTS

V. Aksnes[1][2][3] performed a PhD research study at the NTNU in Norway into the relation between mooring stiffness and ice forces. The goal of the PhD study was to create & validate a one dimensional numerical model that simulated the interaction between a moored ship and drifting level ice. The numerical model[1] was created with the help of several existing ice break principles, but no stick-slip was expected nor implemented.

Figure 1.3: Illustration of the Aksnes numerical model. Figure courtesy of Aksnes (2010)

To validate the numerical model Aksnes (2010) performed several ice model experiments[2]. During these experiments a very-soft, normal stiff and a completely fixed mooring system were subjected to ice loading in the model experiment setup illustrated in figure 1.4. A summary of the results of the Aksnes experiments is given in figure 1.5. The stick-slip effect was encountered during experiment #2100, where a soft mooring stiffness and a low ice drift velocity were simulated. These unexpected stick-slip effects resulted in a drastic increase of the mooring force amplitude and 3 individual stick-events are indicated with the red circles in figure 1.5.
Currently when a mooring system is designed the design load is based on the highest expected mooring load amplitude. As can be seen in figure 1.5 the mooring load amplitudes of the very soft system (Run #2100) are the highest due to the stick-slip effects, followed by the mooring load amplitudes of the completely fixed system (Run #3100) and finally the mooring load amplitudes of the normal stiff system (Run #4100). Note that these mooring load amplitudes are not arranged in order of mooring stiffness, therefore the goal of this study will be to investigate the relationship between mooring stiffness and the number of stick effects. Under certain configurations the stick-slip effect is more present than others; if these configurations are known it will be possible for a mooring engineer to avoid the stick-slip effects, with its increased mooring load amplitudes. Probably because of the high costs of an ice tank experiment only 6 model tests were performed during this study, with only three different stiffnesses and two different ice drift velocities. These Aksnes experiments are further explained and elaborated in chapter 4. Note that the simplified barge shape of the vessel is causing the broken pieces of ice sheet to “flow” underneath the vessel all the way to the aft of the hull. It is expected that this flow is causing significant friction forces on the bottom of the vessel, which can theoretically cause a stick-slip response.

1.4. Design codes and standards

The main standard for Arctic engineering is documented in ISO19906[13] and for station keeping and mooring design the API-RP-2SK[4] design code is widely used by the industry. Unfortunately the Arctic moorings chapter in ISO19906[13] is relatively small and no information can be found about Arctic engineering in API-RP-2SK[4]. To expand some of these design codes with more Arctic mooring specific information a research is performed in the Arctic Mooring JIP[16], the results are expected to be presented in the end of 2015.

An important parameter in ice-breaking is the way the ice fails against a structure. In ISO19906[13] two different ice failure mechanisms are stated; ice crushing and ice bending failure. Ice crushing occurs with vertical structures and ice bending with inclined/conical structures. Because most ship bows are inclined it is therefore assumed that bending failure is the main ice failure mechanism during ship operations in ice conditions. Both mechanisms are illustrated in figure 1.6
1.5. **GOAL OF THE THESIS**

It is found in ISO19906[13] that a completely fixed mooring system generally experiences larger mean mooring loads than a mooring system that has certain stiffness. But at the same time it is found in the experiments of Aksnes (2010)[2] that at certain low mooring stiffnesses stick-slip effects will occur, which are causing large mooring load peaks.

*The goal of this thesis is to create and validate a numerical model that can predict the response of a simplified moored barge-shaped vessel in head-on ice conditions, including the stick-slip effects. With the help of this numerical model further insight should be found in the consequences and probability of stick-slip effects in Arctic mooring systems.*

The numerical model will be created as a simplified single degree of freedom (SDOF) system, where the vessel will be modeled as a simplistic barge-shaped hull with a relatively small inclined bow, as per the Aksnes (2010)[2] original model in figure 1.4. The ice load will be calculated by using several existing ice load principles and the friction load will be calculated with an existing ice-steel friction model. The numerical model will be completed by adding specific stick-slip conditions to the "normal" ice breaking model. Finally the complete numerical model will be verified with existing experiment test data of Aksnes (2010)[2] and it will be used for a more detailed result analysis to identify the consequences and probabilities of the stick-slip effect.

1.6. **RESEARCH QUESTIONS**

The questions which are driving this study about the stick-slip effects are stated below:

- Can the stick-slip effect be simulated with a simplified SDOF model?
  - How to correctly predict ice breaking loads on the vessel?
  - What is a good estimation of the hydrodynamic, mooring and ice damping coefficient of a moored barge-shaped vessel?
  - What will be the best friction-velocity profile to model the stick-slip effect on a vessel? Is the ISO19906 assumption realistic?
- What will be the consequences of the stick-slip effect?
- What will be the probability of the stick-slip effect?
  - What are the biggest sensitivities of the stick-slip effects?
  - How can you avoid stick-slip effects?
  - What is the effect of changing your vessel properties?
  - Are the environmental conditions where stick-slip effects occur within a realistic range?
In this thesis a numerical model is created based on a single degree of freedom (SDOF) system in surge direction. This model is validated with existing ice model experiments [2] and it will finally be used for an extensive result analysis. In this chapter the creation of the SDOF model is elaborated: after the SDOF model is introduced the input parameters are studied, which will then be used to create a time varying external force profile $F_{ext}(t)$. This external ice force profile is used as an input force for the SDOF model with the surge-displacement $x(t)$ and surge-velocity $\dot{x}(t)$ as the output. In the end the stick event criteria are implemented to get the final stick-slip response of the vessel. For explanatory reasons an homogeneous ice sheet is assumed in the beginning of this chapter, which will be expanded with ice sheet variabilities in section 2.5.

Figure 2.1: Illustration of the single degree of freedom model

### 2.1. SDOF Model

Before any stick-slip criteria can be implemented a "normal" ice break model is designed, which calculates the vessel’s response without any stick-slip criteria. It can be seen as a SDOF model where the vessel will have enough momentum and restoring forces to continuously break the ice; the system will receive periodic impulses while it is continuously breaking off pieces of the intact ice sheet. Aksnes (2010)[2] and several others like Croasdale (1980)[8] and Nevel (1992)[18] previously modeled this slip phase with the help the semi-infinite beam on an elastic foundation theory. A combination of these approaches will be the base of the numerical model created in this thesis. The equation of motion of the vessel can be written as:

$$m \ddot{x}(t) = F_{hd}(t) + F_{moor}(t) + F_{ext}(t)$$ (2.1)

With:

$$F_{hd}(t) = -A_{11}\ddot{x}(t) - c_{hd}\dot{x}(t)$$ (2.2)

$$F_{moor}(t) = -k_{moor}x(t)$$ (2.3)

$m$, $A_{11}$, $c_{hd}$ and $k_{moor}$ are the mass, added mass, hydrodynamic damping and the linear mooring stiffness respectively. The ice forces $F_{ext}(t)$ will be elaborated in section 2.3.
The equation of motion of eq. (2.1) can be rewritten to:

$$\ddot{x}(t) + 2\zeta_{hd}\omega_n\dot{x}(t) + \omega_n^2x(t) = \frac{1}{m + A_{11}}F_{ext}(t)$$ (2.4)

Where:

$$\zeta_{hd} = \frac{c_{hd}}{2(m + A_{11})\omega_n}$$ (2.5)

And:

$$\omega_n = \sqrt{\frac{k_{moor}}{(m + A_{11})}}$$ (2.6)

The external ice force $F_{ext}(t)$ will vary over time with a different combination of loads. It will consist of a combination of breaking-, rotational-, friction- and velocity depending loads. The exact determination of these different load combinations will be explained in section 2.3. The external ice forces $F_{ext}(t)$ will be translated to the surge-direction of the vessel with simple geometric formulas. A step-by-step illustration of a single breaking cycle is given in figure 2.2. For explanation purposes the illustration will consist of a homogeneous ice sheet and the variability of the ice sheet is will be explained in a later stage. Also remember that the stick-slip criteria are not applied yet in this part of the model.

If a periodic break-impulse occurs the system experiences a decrease in momentum, which can be seen in the vessel’s response as a change in velocity. The time between two break impulses is depending on the breaking length of the broken ice block. The width of a break-impulse $\Delta t_{\text{break}}$ is assumed to be 1 second, so that the area underneath the break-impulse will be equal to the calculated break force $F_{\text{break}}$. This assumption is approved by Newton’s second law, which indicates the relation between the momentum and the force acting on the system. This law is stated in eq. (2.7); the left hand side of the equation indicates the change in momentum of the vessel and the right hand side indicates the area underneath a break impulse.

$$m\Delta v = F\Delta t$$ (2.7)

The normal slip-response of the vessel will be calculated with the Duhamel integral, this method is verified in section 3.4. The numerical solution of the Duhamel integral is depicted in eq. (2.8) below.

$$x_{\text{slip}}(t) = \frac{1}{m\omega_d} \int_0^t F(\tau)e^{-\zeta_\omega_d(t-\tau)} \sin(\omega_d(t-\tau)) d\tau$$ (2.8)
Figure 2.2: Illustration of 1 ice breaking cycle.

(a) Illustration of ice break cycle.

(b) External ice force and response in time domain.
2.2. INPUT PARAMETERS
Due to the discreteness of the numerical model and the complex calculation of ice loading a number of different parameters are needed. These input parameters will be divided in 4 different input parameter groups; hull-, mooring-, ice sheet- and friction parameters. It is very difficult, if not impossible to normalize parameters due to the discrete properties of the model. Therefore all parameters in the numerical model will remain individual.

Table 2.1: Input parameters in the numerical model

<table>
<thead>
<tr>
<th>Vessel:</th>
<th>Mooring system:</th>
<th>Ice sheet:</th>
<th>Friction coefficient:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bow angle</td>
<td>Restoring Stiffness</td>
<td>Ice drift velocity</td>
<td>Static friction coefficient</td>
</tr>
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<td>Ice thickness</td>
<td>Kinetic friction coefficient</td>
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<tr>
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<td>Ice break length</td>
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</tr>
<tr>
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<td>Ice density</td>
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<tr>
<td>Hull shape + coverage</td>
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<td></td>
<td></td>
</tr>
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2.2.1. VESSEL & MOORING SYSTEM
There are several hull parameters that are of importance for the numerical analysis. Besides the **dimensions** of the hull, another important property is the vessel’s **bow-angle**, which will be fundamental for its ice breaking capabilities. A downward conical shaped bow, where the ice can break by bending failure (see figure 1.6b), is proven to be the most effective; "Conditions that induce ice failure by flexure generally result in smaller ice actions than for crushing." (ISO19906, 2010[13]).

Another important hull parameter proved to be of high importance is the **mass** and the **added mass** of the vessel. The mass is related to the momentum of the vessel, which is used to break the ice during normal moored circumstances. If the vessel has a large momentum it will less likely reach a relative velocity close to zero ($v_{rel} = 0$) and therefore will less likely run into a so-called stick-event, this will be explained later in the report. The added mass is assumed to be around 2% of the total mass, this is determined with the help of literature[1][2][3].

The stick-slip effect can be seen with systems where large friction forces are present; friction forces are expected to arise from ice transport underneath the vessel, where the buoyancy of the ice sheet is acting as the normal force on the vessel. Therefore the biggest chance to experience stick-slip effects is with vessels that have large **ice concentrations** underneath the bow/hull. Because this thesis is focusing on stick-slip effects the hull shape will be chosen as simple as possible, this includes a 100% ice coverage underneath the bow. This assumption is plausible due to the barge-shaped hull with a shallow bow angle.

The damping in the model comes from two different parts; the **hydraulic damping** and the **ice damping**. The hydrodynamic damping is being calculated regularly with the hydrodynamic damping ratio and the calculation of the ice damping is rather sensitive; it is yet to be agreed by the industry how velocity effects or ice-damping are present for moored vessels. Comfort et al. (1999)[7] found a clear increase of the mean mooring force with respect to the ice drift speed in model test data for the Kulluk. But on the other hand, full-scale measurements of the Kulluk, researched by Wright (1999)[24] don’t show any speed dependence. In the numerical model the ice damping partly is calculated the same as the viscous damping. The exact calculation of the ice damping will be beyond the scope of this thesis.

One of the goals of this thesis is to investigate the influence of the **mooring stiffness** on the stick-slip effects. Concluding the results of the Aksnes (2010)[2] experiments it is expected that changing the mooring stiffness can reduce or avoid the occurrence of stick-slip effects. The mooring system in this thesis will be modeled as a linear spring; in theory the restoring mooring stiffness of such system will be non-linear.

2.2.2. ICE SHEET
There are numerous parameters related to the ice sheet properties. Multiple researches are currently performed to get more understanding of the ice mechanics and how to model them. To simplify the numerical model as much as possible only the following parameters are used to identify an ice sheet; ice drift velocity, ice
2.2. INPUT PARAMETERS

thickness, ice breaking length, ice density, flexural strength and the elastic modulus. Because the inclination
of the bow is relatively small and the vessel will be modeled as a simplified hull shape, it is assumed that there
will be no accumulated ice rubble underneath the vessel and the ice sheet will flow continuously underneath
the hull. Therefore no ice rubble parameters will be taken into account and the ice break calculations will
therefore differ from the ISO19906[13] ice break calculations.

The ice drift velocity and the ice thickness of an ice sheet are relatively easy to measure and they are two
of the key parameters to identify during ice sheet parameterization. The ice drift velocity is defined as the
velocity of the ice sheet that is "hitting" the vessel. This will be the velocity with respect to the ground; the
so-called speed-over-ground of the ice sheet. During the creation of the model it is assumed that the ice sheet
does not react on the vessel; it can be seen that the ice sheet will be infinite large. The ice drift velocity and
ice thickness turned out to play a key role during the stick-slip result analysis.

The ice drift direction in the numerical model is assumed to be head-on, this is a valid assumption be-
cause many operated FPSO systems make use of single point moorings and are capable of weather/ice-vane
towards the most optimal (head-on) heading. The mooring loads will probably be much higher if the ice
sheet is coming from a beam-direction, but modeling these effects will be beyond the scope of this thesis.

The ice break length is defined as the length of which the pieces of ice break of the intact ice sheet.
ISO19906[13] indicates the following rule of thumb: "The ice break length varies in the range of 3 to 10 times
the ice thickness.". This ice break length is an important parameter in the numerical analysis; together with
the relative velocity (ice drift velocity minus the vessel's velocity) it defines the distance between two ice break
impulse peaks. During this research the ice break length is determined from measured distributions of exist-
ing model experiments[3].

The ice density, flexural strength and elastic modulus are parameters of the ice sheet which are more
related to the mechanical properties of the ice. These are difficult to measure in-situ and during this research
they are also determined with measured distributions of existing model experiments. The sensitivities of the
input parameters are elaborated in section 4.2.1.

2.2.3. FRICTION COEFFICIENT

The stick-slip effect is based on the difference between the kinetic and static friction coefficient between ice
and the hull. Therefore it is very important to give a realistic estimation of these kinetic and static friction
coefficients and the transition between those two. The hull normally consists of steel; so a ice-steel friction
coefficient has to be identified. Saeki and Ono (1984) [20] researched the friction coefficient between ice and
steel with the help of model experiments. The blue dots in figure 2.7 indicate the individual results of the
Saeki and Ono model experiments and the blue line indicates the power-trend line of these individual data
points.

Figure 2.7: Ice-steel friction coefficient vs. relative velocity by Saeki and Ono (1984)[20] and ISO19906[13].
There are different ways to simplify this friction coefficient function $\mu(v_{rel})$ and the calculations in this thesis are based on the ISO19906[13] friction model; the friction coefficient will be static ($\mu_s$) if the relative velocity is smaller than 0.01 m/s and kinetic ($\mu_k$) if the relative velocity is bigger than 0.01 m/s. This velocity of 0.01 m/s is called the transition velocity. This ice friction model can be expressed with eq. (2.9) and the red line in figure 2.7.

$$\mu = \begin{cases} 
\mu_s & \text{if } v_{rel} < 0.01 \text{ m/s} \\
\mu_k & \text{if } v_{rel} > 0.01 \text{ m/s}
\end{cases} \quad (2.9)$$

Saeki and Ono (1984)[20] also found that the friction coefficients are mainly affected by the relative velocity, sea ice temperature and surface roughness of the materials. Important laws that describe the friction mechanism are the Amontons’ Laws, they describe the interaction between two ideal materials:

- **First law**: Frictional force is independent of the apparent contact area.
- **Second law**: The coefficients of friction are independent of the vertical stress.

### 2.3. EXTERNAL ICE LOAD CALCULATIONS

As can be seen in the equation of motion (eq. (2.4)) an expression of $F_{ext}(t)$ is needed to calculate the system’s response. In the normal slip-phase this force is varying over time because of the continuous occurrences of break-impulses, this is illustrated in figure 2.2. The varying external ice load is a combination of different individual loads, which are stated below and are illustrated in figure 2.8. Kotras et al. (1983)[15], Frederking and Timco (1985)[11], Valanto (1992, 2001)[22][23] and Aksnes (2010)[1] performed a similar decomposition of the ice forces.

- The **ice break load** (blue); this is the load to break off a single piece of the “semi-infinite” ice sheet.
- The **ice friction loads** (red); these are to all the friction forces on the vessel caused by the ice buoyancy.
- The **ice rotation load** (green); this is to calculate the load due to rotation of the broken ice sheet.
- The **ice velocity depending loads** (yellow); this is to include the ice velocity effects (ice-damping).

**Figure 2.8: Composition of the external ice force $F_{ext}$, with a homogeneous ice sheet.**

#### 2.3.1. ICE BREAK LOAD

The **ice break load** is defined as the load to break a single piece of the intact ice sheet, this load is occurring if the vessel receives a break-impulse. It can be calculated by modeling the ice sheet as a semi-infinite beam on an elastic foundation, this is based on the method of Hetenyi (1949)[12] and is illustrated in figure 2.9. The equation of displacement of this semi infinite beam on an elastic foundation can be written as:

$$EI \frac{d^4 u(x)}{dx^4} + N \frac{d^2 u(0)}{dx^2} + \rho u g B u(x) = 0 \quad (2.10)$$

To solve eq. (2.10) several boundary conditions are needed; one side of the beam is modeled as fixed and the other side only experiences a vertical load $P$. These boundary conditions can be written as:

$$\lim_{x \to \infty} u(x) = 0 \quad \lim_{x \to \infty} \frac{du(x)}{dx} = 0 \quad \frac{d^2 u(0)}{dx^2} = 0 \quad -EI \frac{d^3 u(0)}{dx^3} = -P \quad (2.11)$$
2.3. **EXTERNAL ICE LOAD CALCULATIONS**

Figure 2.9: An illustration of the semi-infinite beam with an elastic foundation model

With the help of the boundary conditions in eq. (2.11) the solution of eq. (2.10) will be:

\[
 u(x) = e^{-\beta x} \left( C_1 \cos(\alpha x) + C_2 \sin(\alpha x) \right) \tag{2.12}
\]

Where:

\[
 C_1 = \frac{2P}{EI} \frac{a\beta}{2a^3\beta^2 + a\beta^4 + \alpha^4} , \quad C_2 = \frac{P}{EI} \frac{\beta^2 - \alpha^2}{2a^3\beta^2 + a\beta^4 + \alpha^4} \tag{2.13}
\]

and:

\[
 \alpha = \sqrt{\gamma^2 + \frac{N}{4EI}}, \quad \beta = \sqrt{\gamma^2 - \frac{N}{4EI}}, \quad \gamma = \sqrt{\frac{\rho w g B}{4EI}} \tag{2.14}
\]

The relation between the horizontal force \( N \) and vertical force \( P \) is divided as:

\[
 N = P\xi \tag{2.15}
\]

With:

\[
 \xi = \frac{\sin(\phi) + \mu \cos(\phi)}{\cos(\phi) - \mu \sin(\phi)} \tag{2.16}
\]

Where \( \mu \) is the friction coefficient between the ice and steel and \( \phi \) is the angle of the bow. The resulting moment on the beam can be expressed as:

\[
 M(x) = -EI \frac{d^2 u(x)}{dx^2} \tag{2.17}
\]

To calculate the maximum failure bending moment of the semi-infinite beam the following steps need to be taken; the maximum flexural strength of a semi-infinite beam can be given as:

\[
 \sigma_{f, max} = \frac{M_0}{I} \frac{y}{2} = \frac{M_0 h}{I/2} \tag{2.18}
\]

With:

\[
 I = \frac{bh^3}{12} \tag{2.19}
\]

Where \( b \) is the width of the beam; in this case the width of the vessel. So the flexural strength \( \sigma_{f, max} \) becomes:

\[
 \sigma_{f, max} = \frac{6M_0}{bh^2} \tag{2.20}
\]

Which can finally be rewritten to:

\[
 M_0 = \frac{1}{6} \sigma_f bh^2 \tag{2.21}
\]
One way to find the horizontal breaking force $F_{\text{break}}$ is to calculate it by iteration; this is done by increasing the horizontal force $N$ until the beam moment $M(x)$ is bigger than the maximum beam moment $M_0$, so $M(x) > M_0$. Another (more direct) way to calculate the horizontal breaking force is with the equation of Hetenyi (1946)[12]. Hetenyi discovered a relation for the maximum bending moment $M_0$ as:

$$M_0 = \frac{V}{\beta \exp\left(\frac{\pi}{4}\right)} \sin\left(\frac{\pi}{4}\right)$$  \hspace{1cm} (2.22)

With $1/\beta$ is a characteristic length:

$$\beta = \left(\frac{K}{4EI}\right)^{\frac{1}{4}}$$  \hspace{1cm} (2.23)

Where $K = \rho_w gb$ is a foundation constant. So the equation will be:

$$M_0 = \frac{V}{\beta \exp\left(\frac{\pi}{4}\right)} \sin\left(\frac{\pi}{4}\right)$$  \hspace{1cm} (2.24)

Equation (2.21) and (2.24) are combined to create the formula for $H$, which is the horizontal (mooring) load needed to break the ice. This can also be seen as the 2D-breaking load, which will be:

$$H = 0.68\sigma_{f,\text{max}} b \left(\frac{\rho_w gh^5}{E}\right)^{\frac{1}{4}}$$  \hspace{1cm} (2.25)

Equation (2.25) is only valid for an ice sheet with exactly the same width as the vessel that is breaking it. It is assumed that in reality the ice sheet will have a bigger width than the vessel and the ice-sheet is therefore expected to break in a circle around the vessel. This is the reason why an extra term needs to be included, which is obtained from ISO19906[13]. For this calculation the characteristic length of the breaking ice sheet $L_C$ is needed, note that this is a different parameter than the breaking length $L_{\text{break}}$ of the ice sheet:

$$L_C = \left[\frac{E h^3}{12 \rho_w g (1 - v^2)}\right]^{\frac{1}{4}}$$  \hspace{1cm} (2.26)

The total equation can then be found by expanding the 2D-breaking load equation with the extra terms:

$$H_{\text{2D}} = 0.68\sigma_{f,\text{max}} b \left(\frac{\rho_w gh^5}{E}\right)^{\frac{1}{4}}$$  \hspace{1cm} (2.27)

$$H_{\text{extra}} = 0.68\sigma_{f,\text{max}} b \left(\frac{\rho_w gh^5}{E}\right)^{\frac{1}{4}} \left(\pi^2 L_C\right)$$  \hspace{1cm} (2.28)

$$H_{\text{total}} = 0.68\sigma_{f,\text{max}} b \left(\frac{\rho_w gh^5}{E}\right)^{\frac{1}{4}} \left[b + \left(\pi^2 L_C\right)\right]$$  \hspace{1cm} (2.29)

This final breaking load formula is valid for both static and kinetic calculations, so for both the stick- and the slip phase. The only parameter which will be varying between static and kinetic calculations is $\zeta$, in which the friction coefficient is present. For the slip-phase the kinetic friction coefficient will be used and for the stick-phase the static friction coefficient.

### 2.3.2. ICE FRICTION LOAD

The friction load is defined as the load caused by two materials that rub against each other with a certain normal force. It is calculated with the basic friction formula shown in eq. (2.30).

$$F_{\text{friction}} = N\mu$$  \hspace{1cm} (2.30)

Where $N$ is the normal load perpendicular to the area of friction. The total ice friction load on the vessel is a combination of different friction loads; the friction caused by the ice underneath the bottom of the vessel, the friction caused by the ice underneath the bow and the buoyancy of the ice sheet on the bow. This is illustrated in figure 2.10 and calculated with eq (2.31).

$$F_{\text{friction}} = F_{fr,bottom} + F_{fr,bow} + B_{bow} = (N_{bottom} + N_{bow})\mu + B_{bow}$$  \hspace{1cm} (2.31)
2.3. **EXTERNAL ICE LOAD CALCULATIONS**

Figure 2.10: Illustration of the friction forces and the bow buoyancy force.

Where $N_{\text{bottom}}$ and $N_{\text{bow}}$ are the the normal force on the bottom and the bow respectively, $B_{\text{bow}}$ is the buoyancy force needed to push the ice underneath the bow and $\mu$ is the static or kinetic friction coefficient depending on which phase the vessel is in; in stick- or slip-phase. All forces will be translated to the surge-direction of the vessel with simple geometric formulas. The normal force of one broken ice block on the bottom of the vessel ($N_{\text{bottom}}$) can be calculated with:

$$N_{\text{bottom}} = C_b h_i L_b (\rho_w - \rho_i) g$$  \hspace{1cm} (2.32)

The normal force of one broken ice block on the bow of the vessel ($N_{\text{bow}}$) can be calculated with:

$$N_{\text{bow}} = C_b h_i L_b (\rho_w - \rho_i) g \cos(\phi)$$  \hspace{1cm} (2.33)

Where Another term is added to express the load to push down a broken ice block under the bow. This force is expressed as the buoyancy of the ice sheet ($B_{\text{bow}}$):

$$B_{\text{bow}} = C_b h_i L_b (\rho_w - \rho_i) g \sin(\phi)$$  \hspace{1cm} (2.34)

Where $C_b$ is the coverage of the ice sheet underneath the hull, $b$, $h_i$ and $L_b$ are the width, ice thickness and breaking length respectively, $\rho_w$ and $\rho_i$ are the water and ice densities, $g$ is the acceleration due to gravity and $\phi$ is the bow angle. The total horizontal friction load will finally become:

$$F_{\text{friction}} = C_b h_i L_b (\rho_w - \rho_i) g ((1 + \cos(\phi)) \mu + \sin(\phi))$$  \hspace{1cm} (2.35)

An important parameter in these calculations is the ice coverage factor underneath the hull $C_b$, which is very sensitive to the ice transportation underneath the hull. With a normal ice-breaking bow the broken pieces of ice will be deflected to the sides. But due to the simplified barge-shaped bow of the studied vessel in this thesis the ice coverage is assumed to be 100%. This assumption is valid because the 100% coverage is seen in the ice model experiments by Aksnes (2010)[2] (see picture A.2). This assumption does not mean that the stick-slip effect will not occur with normal ice-breaking vessels; the ice-friction loads with normal vessels will still be very significant and it can even be increased by the forming of rubble under the hull.

2.3.3. **ICE ROTATION LOAD**

The rotation load is the load needed to turn the ice block after breaking off the intact ice sheet. It is calculated with an energy approach; the difference in potential energy between a floating ice block and an ice block parallel to the bow is calculated with the formula:

$$E_{\text{pot}} = \frac{1}{2} (\rho_w - \rho_i) g L_{br}^2 b h_i \sin(\phi)$$  \hspace{1cm} (2.36)

Where $\rho_w$ and $\rho_i$ are the water and ice density respectively and $L_{br}$, $b$, $h_i$ and $\phi$ the breaking length, width of the vessel, ice thickness and bow angle. Because the ice breaking is only assumed as an small impulse the energy to rotate the ice block will be divided over $L_{\text{break}}$. So the average vertical force to submerge the ice block is:

$$F_{\text{rot}} = \frac{E_{\text{pot}}}{L_{br}}$$  \hspace{1cm} (2.37)

Where $\zeta$ is the relation between the horizontal force and the vertical force eq. (2.16). This horizontal rotation load is relatively small with respect to the friction loads on the vessel and this load will not be present during the breaking of the ice sheet (see the artist impression in figure 2.8).
2.3.4. **ICE VELOCITY EFFECTS**

There are a number of uncertainties with respect to ice-velocity effects. It is yet to be agreed by the industry how velocity effects or ice-damping effects are present with moored vessels: Comfort et al. (1999)[7] found a clear increase of the mean mooring force with respect to the ice drift speed in model test data for the Kulluk. But on the other hand, full-scale measurements of the Kulluk, researched by Wright (1999)[24] don’t show any speed dependence. Therefore it is assumed that the ice damping in this model is acting the same as the hydrodynamic damping with an added term to include the ice drift velocity (right hand side of eq (2.38)). The more detailed equation of motion of eq (2.4) can be written as:

\[
\ddot{x}(t) + 2\omega_n(\xi_{hd} + \xi_{ice})\dot{x}(t) + \omega_n^2 x(t) = \frac{1}{m + A_{11}} F_{ext}(t) + 2\omega_n\xi_{ice} v_d
\]

(2.38)

Where \( \xi_{ice} \) is the ice damping ratio, which is assumed as:

\[
\xi_{ice} = \frac{c_{ice}}{2(m + A_{11})\omega_n}
\]

(2.39)

With \( c_{ice} \) is the ice damping coefficient, which will be determined with the help of experimental data of Aksnes (2010)[2]. Also this ice velocity depending load turned out to be relatively small with respect to the friction and break forces, this is probably due to the relatively low ice drift velocities.

2.4. **STICK-EVENTS**

2.4.1. **STICK-EVENT CRITERIA**

The “normal” ice break model, which is explained in section 2.3 will now be expanded with several stick-event criteria and the respective stick-event response is calculated in section 2.4.2. In case the relative velocity between the vessel and the ice sheet is close to zero and the momentum of the vessel is too small to break the ice the vessel can get into a so-called stick-event. A stick event is described as an event where the vessel "sticks" to the ice sheet and consequently encounters an increasing offset. As a result of this increasing offset the vessel will also encounter increasing restoring mooring forces, which are calculated with eq. (2.3). The vessel will remain into such stick-event until the restoring mooring forces overcome the static stick-forces. If the restoring mooring forces overcome the static stick-forces the vessel will get "catapulted" towards its equilibrium offset position and it will encounter a kinetic friction force again. The momentum of the vessel is determined by the linear momentum equation of eq. (2.40). Newton’s second law indicates the relation between the momentum and the force acting on the system, this is indicated with eq. (2.41). If the momentum of the vessel is bigger than the friction & breaking forces, it will not encounter a stick event and it will just break the ice continuously in its "slip-phase".

\[
P = m\Delta v
\]

(2.40)

\[
m\Delta v = F\Delta t
\]

(2.41)

Where \( P, m, \Delta v, F \) and \( \Delta t \) are the momentum, mass, change in velocity, force and subjected time respectively. So finally there are two criteria which are causing a stick-event to occur; if the relative velocity is close to zero (< 0.01 m/s) or if the vessel’s momentum is too small to break the ice continuously. Summarized the system has to meet one of the two following criteria to encounter a stick-event.

\[
\text{Stick-event if } \begin{cases} 
\nu_{\text{drift}} - \dot{x}(t) \leq 0.01 \\
\text{or} \\
m(\nu_{\text{drift}} - \dot{x}(t)) \leq F_{\text{break}}\Delta t_{\text{break}} 
\end{cases}
\]

(2.42)

Normally it is possible to calculate the subjected time \( \Delta t_{\text{break}} \) exactly, but for simplifying purposes it is assumed to be set on 1 second. This assumption is valid because \( \Delta t_{\text{break}} \) changes linearly with \( F_{\text{break}} \) and only the combination of those two will be of importance.
2.4.2. STICK EVENT RESPONSE

In this section the stick-event response is elaborated. Due to the occurrences of break-impulses in the numerical model there will be two different kind of stick-events:

- **Break-stick event:** A break-stick event is occurring during the breaking of the ice sheet; so during a break impulse. The stick force $F_{stick}$ is composed of both the static friction load and the static breaking load. A break stick event occurs if the relative velocity between the vessel and the ice is close to zero, or if the vessel does not have enough momentum and restoring forces to overcome the break-impulse.

- **Slip-stick event:** A slip-stick event is occurring during the rotation of the ice block. Consequently this will be without any static ice break loads; the stick force $F_{stick}$ is only composed of the static friction load. A slip-stick event can only occur if the vessel's relative velocity is close to zero.

The stick force in a break-stick event $F_{breakstick}$ will be much higher than the stick force in a slip-stick event $F_{slipstick}$, as the composition of the stick forces are different. This is illustrated in figure 2.11.

Figure 2.11: Composition of the stick force of both the break-stick and the slip-stick events.

Due to the absence of a relative velocity, the static friction coefficient applies to the system, which will cause much higher loads. Of the five different load components depicted in section 2.4.1 (break, rotation, buoyancy, friction and velocity depending loads), only the break, buoyancy and friction loads are used in case of a stick event, this is because of the absence of a relative velocity. The calculation of the different ice loads in a stick event is done in the same way as in the slip-phase (see section 2.3), the only difference is the value of the friction coefficient and the combination of different load components.

The static friction load and the static break load can be added to each other during a break-stick event due to the fact that the system first has to overcome the friction & buoyancy forces before it can start breaking the ice. This can be illustrated with figure 2.12; the green arrow indicates the resulting force that is needed to break the ice, where only the vertical part of this force will be of concern (see section 2.3). This vertical part will be much smaller if there is a big upward friction+buoyancy force acting on the intact ice sheet. Therefore this upward friction+buoyancy force first needs to be overcome before the vertical part of the resulting force can be increased. Note that ice buckling and ice compressibility is not being incorporated in the ice-break model; only a vertical load limit of the ice sheet is assumed.

Figure 2.12: Zoomed in force distribution in the ice sheet when in a stick-event.
Both the break-stick and the slip-stick events are modeled the same way: during such stick-event the relative velocity between the vessel and the ice-sheet is close to zero. Therefore the response of the vessel will be modeled as a system with a constant velocity; the ice drift velocity. During a stick-event the offset \( x \) of the vessel is getting bigger over time, as well are the corresponding mooring forces \( F_{\text{moor}} = k_m x(t) \). This process is assumed to occur in a linearly fashion. The stick-event will end after the mooring forces overcome the stick forces \( (F_{\text{moor}} > F_{\text{stick}}) \); then the "normal" slip-phase will continue again. The equilibrium position of the vessel will be used as a starting position in the following slip-phase. The discrete mathematical response equation of the system in a stick-event can be written as:

\[
x_{\text{stick}}(t) = x(t - 1) + v_d \Delta t
\]  

(2.43)

Where \( x(t - 1) \) is the previous position of the vessel. The width of the stick-event in figure 2.11 will depend on when the \( F_{\text{moor}} \) is overcoming the \( F_{\text{stick}} \) and the height of these loads will be elaborated in section 2.3.

2.5. RANDOM VARIABILITY OF THE ICE SHEET

In the previous sections a homogeneous ice sheet was assumed for explanatory reasons. In reality an ice-sheet is never completely homogeneous, not even in ice tank experiments; there will always be a certain variation in ice thickness, strength, breaking length and other properties of the ice sheet. This variation is also incorporated in the numerical model by randomly generating a new ice sheet after every broken piece of ice, i.e. after each breaking cycle. This is illustrated in figure 2.13. The parameters that are varying with every broken ice block are depicted in table 2.2.

Figure 2.13: Newly generated ice sheet illustration.

The parameters used in the result analysis of chapter 4 are varying with certain distributions, which are difficult to obtain in real life. The numerical model in this thesis is made to simulate the Aksnes (2010) model experiments[2]. A lot of parameters were measured during these experiments and the associated distributions are given in one of the papers [3]. A lot of these parameters are correlated to each other in reality, but due to the complexity of the ice and the discreteness of the numerical model it is expected that it is not possible to implement every correlation between the parameters. Nonetheless some correlations are partly implemented by distributing two parameters in the same distribution, for example the ice break length and ice thickness. The exact correlation of the different parameters are beyond the scope of this thesis.

Table 2.2: Randomly varying input parameters depicted in red.
The direct result of this variation of the ice sheet can be shown in the external ice force time domain in figure 2.14; instead of having identical ice break cycles the system will have ice break cycles with varying loads and varying distances between the break-impulses. The consequence of this variation is that there will be even more irregular stick-event occurrences. Also the stick-events will have different stick-event properties (Stick-length, stick-amplitude etc.) and will therefore also vary in width and amplitude. The exact distributions of the variable parameters are elaborated later in section 3.2 and table 3.2.

Figure 2.14: Composition of the external ice force $F_{ext}$ …

(a) … with a homogeneous ice sheet (same as figure 2.8). (b) … with a variable ice sheet.

2.6. NUMERICAL IMPLEMENTATION

To study the consequences and the probability of the stick-events, a discrete time-domain simulation model is created in MATLAB® that can predict the response of a moored vessel. The input of the simulation will be the hull-, mooring- and ice-properties and the output will be the force that is acting on the vessel and the consequent dynamic response of the vessel in surge direction, including stick-slip effects. The “normal” ice break model can be simplified by the flow diagram in figure 2.15a and the complete numerical, with the implemented stick-event criteria, is illustrated in the flow diagram in figure 2.15b. Note that after every “normal slip-mode” a new ice sheet is generated with different properties; this will cause the stick-events to happen more often and irregularly. The complete MATLAB-code of the numerical model is given in appendix F.

Figure 2.15: Numerical implementation

(a) Flow diagram of the normal slip-phase without any stick-event criteria.

(b) Complete flow diagram with the stick-event criteria.
2.7. Assumptions

A number of assumptions have been made to model the complex stick-slip effect with a simplified SDOF system:

1. The numerical model is using a simplified SDOF system. Only the surge motion of the vessel is investigated in this numerical model.

2. Wave, wind and current loads in a level ice condition are neglected as they are minor compared to ice and friction loads.

3. The preliminary model takes a linear restoring curve (linear spring) into account, this assumption is valid for small offset values (for example: Shtokman FPU offset <20m & Terra Nova FPSO offset <10m).

4. The friction vs. relative velocity is modeled as a step function like the red line in figure 2.7, this assumption is based on the ISO19906[13] assumptions.

5. The system goes into a stick-event again when the relative velocity crosses zero. This assumption is valid for most of the cases because then the static friction coefficient applies and it will experience a bigger load (stick-force). In theory it would also be possible to have a negative relative velocity (vessel moving faster than the ice sheet), but this is not implemented in the model.

6. The Duhamel Integral is used for the calculation of the response of the vessel.

7. No ice rubble is present in the numerical model; the ice flow is modeled as a continuous flow of broken ice blocks underneath the vessel. The broken ice blocks can freely flow underneath the ship with a 100% ice coverage. This assumption is valid due to the shallow bow angle; this causes the ice to act as a flow of ice underneath the hull. This was also seen during the Aksnes (2010) experiments (see picture A.2 in appendix A).

8. The ice sheet velocity is not influenced by the interaction with the vessel.

9. The ice damping is implemented the same as the hydrodynamic damping, due to uncertainties in the current method of approach by the industry.

10. Dynamic effects for the beam and the foundation are neglected, due to the low relative velocities.

11. The modeled ice sheet (beam) will break if the force limit is overcome.
3

NUMERICAL MODEL VALIDATION

3.1. CURRENT ARCTIC OPERATIONS

Permanent mooring systems are used in various applications in the oil & gas industry. FPSO systems are sometimes designed to remain stationed on a single location for over 20 years. It speaks for itself that these mooring systems need to be designed and maintained with a high level of integrity, without being over-designed. There is yet no FPSO facility that is subjected to consistent ice loadings at the moment of writing this thesis. This is due to several technical and political complexities of feasibly operating an Arctic FPSO system and therefore it is very difficult to get real time data for ice-load analysis. An extensive real-time data research is performed with conical drill barge Kulluk (figure 3.1a). Furthermore SOFEC was involved in a large study for the Shtokman project (figure 3.1b), which included several ice model experiments in the ice tank of HSVA Hamburg. Ice experiments were also performed during the design of the semi-Arctic Terra Nova FPSO (figure 3.1c), where SOFEC also designed the disconnectable mooring system.

Figure 3.1: Current (semi-) Arctic designed systems

(a) Kulluk drillship  (b) Shtokman FPU  (c) Terra Nova FPSO

3.2. INPUT: BASE-CASE PARAMETERS

Aksnes (2010) [2] performed several ice model test experiments to research the response of a moored system in head-on ice conditions. During these experiments a barge-shaped hull is subjected to a head-on level ice sheet with a relatively small ice drift velocity. The input-parameters of these experiments are now used to scientifically validate the numerical model and they will also be used as a base-case for the result analysis in chapter 4. Six different experiments were performed during the Aksnes (2010) study, where the stiffness was varying between a relatively soft mooring stiffness (250 kN/m) and a completely stiff mooring stiffness and the ice drift velocity was varying between 0.05 m/s and 0.25 m/s. The rest of the parameters were chosen to be modeled as constant. The complete test-matrix of the Aksnes experiments is summarized in table 3.1. As can be seen only a significant amount of stick-events were found during the Aksnes2100 run and therefore will this be the main source of stick-event data for the validation of the numerical model. All the data of these experiments is available for use in agreement with the owner of the data [10]. A more detailed description of the research is given in section 1.3.

19
Table 3.1: Test matrix Aksnes experiments (2010). Data courtesy of Aksnes (2010) and HYDRALAB.

<table>
<thead>
<tr>
<th>Mooring stiffness [kN/m]:</th>
<th>Ice drift velocity [m/s]:</th>
<th>Ice sheet #:</th>
<th>Stick-events:</th>
<th>Length of time plot [h]:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aksnes2100 250</td>
<td>0.05</td>
<td>1</td>
<td>60x</td>
<td>2.64</td>
</tr>
<tr>
<td>Aksnes2200 250</td>
<td>0.25</td>
<td>1</td>
<td>-</td>
<td>0.83</td>
</tr>
<tr>
<td>Aksnes3100 Fixed</td>
<td>0.05</td>
<td>2</td>
<td>-</td>
<td>2.05</td>
</tr>
<tr>
<td>Aksnes3200 Fixed</td>
<td>0.25</td>
<td>2</td>
<td>-</td>
<td>0.87</td>
</tr>
<tr>
<td>Aksnes4100 1125</td>
<td>0.05</td>
<td>3</td>
<td>1x</td>
<td>2.13</td>
</tr>
<tr>
<td>Aksnes4200 1125</td>
<td>0.25</td>
<td>3</td>
<td>-</td>
<td>0.81</td>
</tr>
</tbody>
</table>

The input parameters and parameter distributions of the Aksnes (2010) experiments are specified in detail in several papers about the model experiments [1][2][3]. Both Aksnes2100 and Aksnes4100 experiments are under slow ice drift conditions and therefore are sensitive for stick-events to occur, in addition they are also minimally affected by ice velocity effects. The other Aksnes experiments are done under higher ice drift velocities (0.25 m/s) or fixed mooring stiffnesses and therefore are not very useful in the validation process. The Aksnes2100- and Aksnes4100 parameters will be summarized in table 3.2, where also the distributions are shown in which the respective parameter will be randomly varying. The input parameters of the other Aksnes experiments can be found in appendix B.

Table 3.2: Input parameters Aksnes2100 & Aksnes4100. Data courtesy of Aksnes (2010) & HYDRALAB.

<table>
<thead>
<tr>
<th>Vessel</th>
<th>Length (WL) [m]</th>
<th>Width (WL) [m]</th>
<th>Draft [m]</th>
<th>Bow angle [deg]</th>
<th>Ice coverage [%]</th>
<th>Mass [mt]</th>
<th>Added mass [% of mass]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aksnes2100</td>
<td>106</td>
<td>33</td>
<td>8.8</td>
<td>25</td>
<td>100</td>
<td>28800</td>
<td>2</td>
</tr>
<tr>
<td>Aksnes4100</td>
<td>106</td>
<td>33</td>
<td>8.8</td>
<td>25</td>
<td>100</td>
<td>28800</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mooring</th>
<th>Stiffness [kN/m]</th>
<th>HD damping ratio [-]</th>
<th>Ice damping ratio [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aksnes2100</td>
<td>250</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>Aksnes4100</td>
<td>1125</td>
<td>0.04</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ice sheet</th>
<th>Ice drift velocity [m/s]</th>
<th>Length of the ice sheet [m]</th>
<th>Mean ice thickness [m]</th>
<th>Dev. ice thickness [m]</th>
<th>Mean ice break length/thickness [m]</th>
<th>Gamma [-]</th>
<th>5.7</th>
<th>6.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aksnes2100</td>
<td>0.05</td>
<td>575</td>
<td>0.8</td>
<td>0.0016</td>
<td>0.0012</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aksnes4100</td>
<td>0.05</td>
<td>525</td>
<td>0.7</td>
<td>0.0012</td>
<td>0.0012</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ice properties</th>
<th>Poisson ratio [-]</th>
<th>Mean ice density [kg/m³]</th>
<th>Dev. ice density [kg/m³]</th>
<th>Mean flexural strength [kPa]</th>
<th>Dev. flexural strength [kPa]</th>
<th>Mean elastic modulus [Pa]</th>
<th>Dev. elastic modulus [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aksnes2100</td>
<td>0.3</td>
<td>870</td>
<td>20</td>
<td>850</td>
<td>20</td>
<td>1.30E+09</td>
<td>2.00E+08</td>
</tr>
<tr>
<td>Aksnes4100</td>
<td>0.3</td>
<td>929</td>
<td>7</td>
<td>610</td>
<td>15</td>
<td>1.30E+09</td>
<td>2.00E+08</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Friction</th>
<th>Friction transition velocity [m/s]</th>
<th>Mean static friction coefficient [-]</th>
<th>Dev. static friction coefficient [-]</th>
<th>Mean kinetic friction coefficient [-]</th>
<th>Dev. kinetic friction coefficient [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aksnes2100</td>
<td>0.01</td>
<td>0.1</td>
<td>0.02</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>Aksnes4100</td>
<td>0.01</td>
<td>0.1</td>
<td>0.02</td>
<td>0.05</td>
<td>0.01</td>
</tr>
</tbody>
</table>
3.3. OUTPUT: Aksnes Experiments

The output data of the Aksnes (2010) experiments consists of several different time domain plots. Among others the force graphs are given, which are recorded by several load cells on the hull. The forces recorded by the load cells encounter a lot of high frequency noise, which is probably caused by model test experiment side effects and the summation of these different individual force panels. Therefore the mooring force will be calculated by multiplying the displacement data with the mooring stiffness, this way the inertia of the vessel will act as a filter for all high frequency noise. The comparison between those two can be seen in figure 3.2 below. For more information about the used load cells please see the Aksnes (2010) experiment paper.

Figure 3.2: Filtered and non-filtered time domain mooring forces.

Stick events are mainly happening in the Aksnes2100 experiment, therefore this will be the main source of data for the validation of the numerical model. The stick events are clearly visible in the Aksnes2100 displacement time domain plot (figure 3.2a) due to the vessel’s equal velocity as the ice drift velocity. This means that the relative velocity between the ice sheet and the vessel is zero and thus that the vessel will be subjected to a stick event. All individual stick-events are identified in the time-domain plots of the experiments. In total there is one Aksnes2100 time domain plot available of 2.64 hours of ice load data and in this 2.64 hours there are 60 stick-events identified. The time domain plot with the identified stick-events is shown in appendix C.

During the Aksnes experiments no external ice load profile is recorded, therefore it is impossible to validate the numerical model 1-on-1 with the Aksnes experiments. A statistical Monte Carlo approach is used for validation; first the specific individual stick event properties are statistically validated in the *stick-event validation* and then the number of stick-events per hour of all Aksnes experiments are validated in the *global validation*. The Monte Carlo properties and the fitting of the distributions are elaborated in Appendix D.

<table>
<thead>
<tr>
<th>Property</th>
<th>Main validation data</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stick-event validation</strong></td>
<td>Stick-event length [s]</td>
<td>Aksnes2100: 60 events</td>
</tr>
<tr>
<td></td>
<td>Stick-event amplitude [N]</td>
<td></td>
</tr>
<tr>
<td><strong>Global validation</strong></td>
<td>Number of stick-events per hour [1/h]</td>
<td>Aksnes2100: 2.6 hours</td>
</tr>
</tbody>
</table>

The investigated properties are defined as:

- **The stick-event length** - is defined by the time that the system is in a stick-event.
- **The stick-event amplitude** - is defined by the highest mooring load of the respective stick-event.
- **The number of stick-events per hour** - is defined by the total number of stick-events in a time plot divided by the length of that respective time plot.
3.4. OUTPUT: NUMERICAL MODEL

The numerical model is capable of calculating and plotting of the displacement, vessel velocity, relative velocity and mooring forces and several others. The different stick-events, which are occurring throughout the time domain plots are all recorded individually, together with the individual stick-event properties. In figure 3.3-3.5 the normal slip-phase, slip-stick event and a break-stick event are illustrated. The blue and the red line indicate the mooring load and the external ice load respectively. The mooring load is determined by multiplying the surge-displacement of the vessel with the linear mooring stiffness. The parameters of the Aksnes2100 experiment are used as the input parameters in the numerical model.

Figure 3.3: Numerical time domain output of the normal slip-phase.

Figure 3.4: Numerical time domain output of several slip-stick events.
A preliminary numerical verification is performed in Orcaflex® to verify the numerical calculation method, i.e. the Duhamel integral method. The external input force is being calculated in advance with the method explained in chapter 2. This external input force time trace is then used as an external input force in an Orcaflex® model, which is simulating the base-case scenario. The output of the Orcaflex® model is then compared with the output of the numerical model. The used parameters are stated in table 3.2 and the result of this preliminary verification is shown in figure 3.6.

It can be seen that the red and the green line in figure 3.6 are fairly similar. The small difference between the two can be explained by the difference in calculating the response; it is expected that some loads are calculated differently in Orcaflex® than in the numerical model. This preliminary verification is only to show that the numerical method of calculating the vessel's response with the Duhamel integral is expected to be implemented correctly. Note that this is not validating the complete stick-slip model, but only the numerical response method (i.e. the Duhamel integral) of the "normal" ice break model (figure 3.3).
In the next part of this section a single break-impulse cycle is studied in more detail to give a better understanding of the numerical calculation method. Figure 3.7 is a detail view of a single break-impulse; the external ice load profile (input), surge-displacement, surge-velocity and surge-acceleration are shown for a time range of 20 s (from 4360 s to 4380 s). The three points that define this specific break-impulse are located on 4369 s, 4370 s and 4371 s, the exact location of these points are also stated in each graph.

Figure 3.7: Zoomed in break-impulse.

It can be seen that a break-impulse is causing the surge-displacement, surge-velocity and surge-acceleration to change rapidly. This abrupt change always needs to comply with the equations of motion of the SDOF system at all times. The equation of motion can be written as eq. \((3.1)\), where \(m\), \(A_{11}\), \(c_{\text{total}}\) and \(k_{\text{mooring}}\) are the base-case scenario parameters from table 2.1, they represent the mass, added mass, total damping coefficient and the linear stiffness constant respectively. More information about the equation of motion can be found in section 2.1.

\[
(m + A_{11})\ddot{x}(t) + c_{\text{total}}\dot{x}(t) + k_{\text{mooring}}x(t) = F_{\text{ext}}(t)
\]  

\(3.1\)

The most sensitive area for calculation errors is expected to be during a break-impulse. Therefore the equilibria of the three points indicated in figure 3.7 are checked in more detail, this is done in table 3.3. The first step is to determine the values of the acceleration, velocity and displacement of the three points. These values are then added manually in the equation of motion of eq. \((3.1)\) together with the general parameters of the base-case scenario. The external ice load is also determined from the graph, which will finally be compared to the total force of the response (the left hand side of eq \((3.1)\)). As can be seen in table 3.3 all three external ice forces are in equilibrium with the respective total forces. Therefore it is expected that no calculation errors are implemented in numerical model with respect to the Duhamel integral calculations.
3.5. STICK-EVENT VALIDATION

The numerical calculation method of the "normal" ice break model is preliminary verified, the individual stick-events will be validated. During the Aksnes2100 experiments 60x individual stick-events were recorded. The main properties of an individual stick-event are the stick-length and the stick-amplitude, explained in section 3.3. These properties are individually being identified in appendix C. Because 60 samples are available it is possible to create a histogram of the Aksnes2100 experiment stick-event data. These histograms are then compared with the Monte Carlo results of the numerical model. The results of this comparison is shown in figure 3.8 and 3.9; the gray and black lines are the fitted slip-stick and break-stick distribution in the numerical model respectively. The Monte Carlo parameters and the distributions fitting are elaborated in appendix D.

Figure 3.8: Stick-event validation: stick-event length.

<table>
<thead>
<tr>
<th>Time</th>
<th>Acceleration</th>
<th>Velocity</th>
<th>Displacement</th>
<th>External ice force</th>
<th>Data numerical model</th>
<th>Symbol</th>
<th>Handcalculation</th>
<th>Total force</th>
</tr>
</thead>
<tbody>
<tr>
<td>4369</td>
<td>X2</td>
<td>X1</td>
<td>X</td>
<td>F_ext</td>
<td>-0.000580 m/s^2</td>
<td>0.017091 m/s</td>
<td>2.2601 m</td>
<td>542431 N</td>
</tr>
<tr>
<td></td>
<td>X2</td>
<td>X1</td>
<td>X</td>
<td>F_ext</td>
<td>X2*(m+A)= -17026 N</td>
<td>X1*c_total= -5558 N</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total force: 542431 N</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4370</td>
<td>X2</td>
<td>X1</td>
<td>X</td>
<td>F_ext</td>
<td>0.018027 m/s^2</td>
<td>0.017594 m/s</td>
<td>2.2427 m</td>
<td>1084526 N</td>
</tr>
<tr>
<td></td>
<td>X2</td>
<td>X1</td>
<td>X</td>
<td>F_ext</td>
<td>X2*(m+A)= 529571 N</td>
<td>X1*c_total= -5722 N</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total force: 1084526 N</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4371</td>
<td>X2</td>
<td>X1</td>
<td>X</td>
<td>F_ext</td>
<td>-0.002362 m/s^2</td>
<td>0.000383 m/s</td>
<td>2.2341 m</td>
<td>489253 N</td>
</tr>
<tr>
<td></td>
<td>X2</td>
<td>X1</td>
<td>X</td>
<td>F_ext</td>
<td>X2*(m+A)= -69399 N</td>
<td>X1*c_total= 125 N</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total force: 489253 N</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: Equilibrium-check of the specific break-impulse of figure 3.7.
Figure 3.9: Stick-event validation: stick-event mooring load amplitude.

The shape and the width of the Aksnes2100 histogram of both figure 3.8 and 3.9 are in good comparison with the shape and the width of the fitted distributions of the numerical model. In figure 3.9 it can be seen that the Aksnes2100 stick-amplitude histogram is shifted to the right with ±250 kN with respect to the numerical model, indicating a higher mean value for the stick-events in the Aksnes2100 experiments. This increase is probably caused by certain loads that are not modeled correctly in the numerical model, for example the ice velocity effects. This shift in the mean mooring amplitude is expected to have a minimal effect on the stick-event occurrences.

It can be seen that there are two different peaks in the Aksnes2100 histograms. This is indicating a multi-modal distribution; i.e. multiple effects are recorded in the same histogram. As explained in 2.4 there are two different kind of stick-events; a slip-stick event and a break-stick event. The difference between both events are difficult to identify from the time domain plots of the Aksnes experiments, but are easily recorded in the numerical model. Based on the two identified histogram peaks in the Aksnes2100 stick-events data, two different criteria are being implemented; if an individual stick-event has a stick-length larger than 60 seconds and a stick-amplitude larger than 1750 kN it is called a break-stick event, else it is called a slip-stick event:

\[
\text{Aksnes2100 stick-event} = \begin{cases} 
\text{Stick-length} > 60 \text{s} & \text{Stick-ampl} > 1750 \text{kN} \rightarrow \text{Break-stick event} \\
\text{else} & \rightarrow \text{Slip-stick event}
\end{cases}
\] (3.2)

In tabel C.1 and C.2 in appendix C all the individual stick-events are elaborated and after analysis it is expected that of the total 60x stick-events of Aksnes2100 only 5x are break-stick events and 55x are slip-stick events. Note that it is very difficult to identify and separate a break-stick event from a normal stick-event in model-experiments and therefore this theory is validated statistically in the global analysis in the next section.

3.6. Global validation
Aksnes (2010) performed 6 different model experiments with different input parameters; the test matrix of these experiments is depicted in table 3.1. Stick events were only found in 2 of the 6 model experiments; in Aksnes2100 and Aksnes4100. First the numerical model is validated with every Aksnes experiment to test if the numerical model and the experiments are corresponding in a good agreement. Afterwards the Aksnes2100 experiment is validated in more detail. The input parameters of the respective Aksnes experiment (see appendix B) are inserted into the numerical model and with the help of Monte Carlo simulations the mean number of stick events are found. The results of the overall global validation are stated in table 3.4.
### 3.6. Global validation

Table 3.4: Summary of the global validation of all Aksnes experiments (mean values).

<table>
<thead>
<tr>
<th>Num. model:</th>
<th>Break-stick events [1/h]</th>
<th>Slip-stick events [1/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiments:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2100</td>
<td>23.4</td>
<td>21.5</td>
</tr>
<tr>
<td>#2200</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>#3100</td>
<td>1 * 10^{-2}</td>
<td>4 * 10^{-3}</td>
</tr>
<tr>
<td>#3200</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>#4100</td>
<td>1 * 10^{-4}</td>
<td>8 * 10^{-4}</td>
</tr>
<tr>
<td>#4200</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

There is one Aksnes experiment available that can be used to validate the numerical model on a global level in more detail; the Aksnes2100 experiment with 60x stick-events in 2.6 hour (=23.1 stick-events per hour). First the total amount of stick-events per hour are validated and afterwards the number of slip-stick- and break-stick events are validated individually with the help of the criteria in eq. (3.2). The input parameters of table 3.2 are used as input in the numerical model.

With the help of Monte Carlo simulations the histogram of the number of stick-events is obtained and fitted with a suitable distribution. It will not be possible to create a complete histogram of the Aksnes2100 experiment, because only a single run is performed; therefore only one single point of 23.1 stick-events/hour could be indicated in the fitted distribution, this is done with a red vertical line. The results of the validation of the total number of stick-events per hour can be seen in figure 3.10. More information about the Monte Carlo parameters and the fitting of the distributions can be found appendix D. If more data is obtained a better global validation can be performed; multiple samples are needed to create a complete histogram.

Based upon the criteria created in section 3.5 the individual break- and slip-stick events are studied in more detail. The validation of the individual number of break-stick events and slip-stick events per hour can be seen in figure 3.11. The distributions of the numerical model are indicated in black and gray and the red and orange lines indicate the expected break-stick and slip-stick events in the Aksnes2100 experiment respectively. Note that this is determined on the basis of the criteria of eq. (3.2).
A summary of the more detailed validation of the Aksnes2100 experiment is given in table 3.5. The results of both the individual stick-event validation and the global validation are given.

Table 3.5: Results validation numerical model vs. Aksnes2100 experiment.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Numerical</td>
<td>Experimental</td>
</tr>
<tr>
<td>Stick-length [s]</td>
<td>33.5</td>
<td>35.5</td>
</tr>
<tr>
<td>Stick-amplitude [kN]</td>
<td>845</td>
<td>1248</td>
</tr>
<tr>
<td>Total stick-events [1/h]</td>
<td>23.4</td>
<td>23.1</td>
</tr>
<tr>
<td>Break-stick events [1/h]</td>
<td>2</td>
<td>1.9</td>
</tr>
<tr>
<td>Slip-stick events [1/h]</td>
<td>21.5</td>
<td>21.2</td>
</tr>
</tbody>
</table>

As can be seen in table 3.5 the mean and the deviation of most recorded outputs are in a good agreement. Only the stick-amplitude mean is significantly shifted, this is probably due to an unforeseen load, which is not implemented in the model; expected to be a wrong implemented velocity dependent load. A shift in mean stick-amplitude will not affect the stick-event occurrences in the model (i.e the number of events per hour) and therefore it can be assumed that this shift will not affect the outcome of this validation.

Finally with the results of the global validation of table 3.5 and the specific Aksnes2100 validation of table 3.4 it is assumed valid to conclude that the numerical model simulates the Aksnes experiments with a good agreement. It is therefore assumed valid to use the numerical model in the next chapters for an extensive result analysis.
In the previous chapters a numerical model was created and validated with the help of the Aksnes experiments. It is concluded that the numerical model simulates the Aksnes2100 experiments with a good agreement. In this chapter the numerical model will be used to analyze the results and find the most sensitive parameters. This result-analysis is divided in two different parts; the consequence analysis and the probability analysis of the stick-slip effect. The consequence of a stick event can be seen as the impact it has on the mooring system; this is measured with the maximum mooring load amplitude of such stick event. The second part is to indicate the probability that a stick-event occurs, this will also give more insight in the specific stick-slip circumstances. This second part includes an extensive sensitivity analysis. Both the consequence and the probability will indicate the risk of the stick-slip effect.

The Aksnes2100 experiment is used as the base-case scenario for this analysis. The exact base-case parameters are shown in table 3.2 and a summary is shown in the table 4.1 below. In appendix E a case-study is performed to indicate the stick-slip response of the Aksnes vessel in Beaufort Sea ice conditions. In this case-study the ice drift velocity and ice thickness probability density functions of the Beaufort Sea are combined with the vessel-specific break-stick response.

Table 4.1: Summary of the base-case scenario parameters. See table 3.2 for the complete table.

<table>
<thead>
<tr>
<th>Vessel</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>106</td>
<td>[m]</td>
</tr>
<tr>
<td>Width</td>
<td>33</td>
<td>[m]</td>
</tr>
<tr>
<td>Bow angle</td>
<td>25</td>
<td>[deg]</td>
</tr>
<tr>
<td>Ice coverage</td>
<td>100</td>
<td>[%]</td>
</tr>
<tr>
<td>Mass</td>
<td>28800</td>
<td>[mt]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mooring</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear stiffness</td>
<td>250</td>
<td>[kN/m]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ice sheet</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice drift velocity</td>
<td>0.05</td>
<td>[m/s]</td>
</tr>
<tr>
<td>Ice thickness</td>
<td>0.8</td>
<td>[m]</td>
</tr>
</tbody>
</table>

(a) Summary of the input parameters (b) Illustration of modeled vessel. Courtesy of Aksnes (2010).
4.1. **Consequences of the Stick-Slip Effect**

The stick-slip effect only occurs under certain circumstances, but what are the consequences if a stick-event occurs during a moored vessel operation? The consequence of a stick-event will be defined by the mooring load amplitude of the respective stick-event. The mooring load amplitude is the amplitude of a single mooring load oscillation, so within every time domain graph there will be multiple mooring load amplitudes. The maximum of this mooring load amplitude is normally used as a design load in the design process of a mooring system and is therefore an important parameter in the analysis.

During the analysis of the consequences the base-case parameters of table 4.1 are used and the ice thickness is taken as a variable. This is to see the effects on the mooring load amplitude if the ice thickness is increasing. Four different load amplitudes are analyzed; the ISO19906\[13\] design load; the load amplitudes of the normal slip phase oscillations without any stick-events (figure 3.3); the load amplitudes of the slip-stick events (figure 3.4) and the load amplitudes of the break-stick events (figure 3.5). The consequence analysis is statistically performed with Monte Carlo simulations and the results are shown in figure 4.1. The solid line is the mean mooring load amplitude, while the colored bandwidth indicates the 95% confidence interval. Around 535,000 normal slip-phase samples, 94,000 slip-stick samples and 22,000 break-stick samples per ice thickness were collected for this Monte Carlo analysis, more details can be found in appendix D.

![Figure 4.1: Consequences of a stick-event with a variable ice thickness.](image)

As can be seen in figure 4.1 the slip-stick amplitudes are relatively close to the normal slip phase mooring amplitudes and they are well below the ISO19906\[13\] calculation, which is normally used for mooring load calculations. Therefore it is assumed safe to design a system with the ISO19906\[13\] code if only slip-stick events are present. But on the other hand, if break-stick events are present during an operation, the system can be subjected to 20% higher mooring load amplitudes than initially calculated with ISO19906\[13\]. This is caused by the use of the static friction coefficient during stick-events and the kinetic friction coefficient during ISO19906\[13\] calculations.

Of course it will be an option to design on the safe side by precautionary using the static friction coefficient and a safety factor during the ISO19906\[13\] calculations to cope with optional stick-slip effects. But then there is a chance that the system will be over-designed if no stick-slip effects occur under the respective circumstances. Therefore these consequences have to be analyzed together with the probability of a stick-event to indicate the true risk of the stick-slip effect.

4.2. **Probability of the Stick-Slip Effect**

In the previous section the consequences of all stick-events were analyzed and it was concluded that only a break-stick event will cause an important increase in mooring load amplitude. Therefore the probability of a break-stick event will be the main output in any further analysis. For this analysis a normalized output parameter is needed that is indicating the number of the break-stick event occurrences. This normalized
4.2. **Probability of the Stick-slip Effect**

An output parameter will be called the *break-stick percentage*: it is defined by the number of break-impulses that remain in a stick-event. This is calculated with eq. (4.1) and illustrated in figure 4.2.

\[
\text{Break-stick percentage} = \frac{\text{Total number of break-stick events}}{\text{Total number of break-impulses}} \times 100\% \quad (4.1)
\]

Figure 4.2: The break-impulses on a system.

### 4.2.1. Sensitivity Analysis

Due to the large amount of input parameters in the model, it will not be possible to analyze the sensitivity of all the parameters involved in detail. Therefore, the three most sensitive parameters will be indicated with the help of a one-factor-at-a-time (OFAT) analysis. Together with the mooring stiffness, these three most sensitive parameters will be analyzed in more detail.

An one-factor-at-a-time (OFAT) analysis is generally done by increasing and decreasing every input individual parameter with ±10% to see what the individual effects on the system are. In this case, the total number of stick-events and the break-stick percentage are seen as the main output parameters. The results of this OFAT analysis can be illustrated in a tornado chart; where the variation of the output parameters with respect to the base-case scenario is shown. The base-case scenario resulted in a mean of 23.4 total stick-events per hour and a mean break-stick percentage of 5.1%. The tornado charts of the total stick events per hour and the break-stick percentage are shown in figure 4.3a and 4.3b respectively.

The most sensitive parameters of this OFAT analysis will show up in the upper part of the graph with the widest variation and thus the biggest effect on the system. It can be discussed if a percentage variation or an estimated variation in a realistic range needs to be used for this OFAT analysis. During this study, the 10% was well within every realistic input parameter range, so it was assumed as a valid method.

In both tornado charts, it is clearly visible that the three input parameters which are most sensitive to the 10% change are the scale factor, ice thickness, and the ice drift velocity. In addition to these three input parameters, the mooring stiffness will be taken as a big sensitivity, this is due to the scope of this thesis. The scale factor is defined as a factor to scale the length, width, draft, and the mass of the vessel. The scale factor is explained in figure/table 4.2.

**Table 4.2: Description of the scale factor.**

<table>
<thead>
<tr>
<th>Input parameter:</th>
<th>Scale factor:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length vessel [m]</td>
<td>( A )</td>
</tr>
<tr>
<td>Width vessel [m]</td>
<td>( A )</td>
</tr>
<tr>
<td>Draft vessel [m]</td>
<td>( A )</td>
</tr>
<tr>
<td>Mass vessel [kg]</td>
<td>( A^3 )</td>
</tr>
</tbody>
</table>

(a) Illustration of the scale factor  
(b) Scale factors
4.2.2. ENVIRONMENTAL SENSITIVITY

The ice thickness and ice drift velocity can be seen as environmental parameters, where the scale factor and mooring stiffness are more vessel dependent parameters. Therefore an environmental sensitivity and a vessel dependent sensitivity is performed with the help of a multiple-factor-at-a-time (MFAT) analysis.

To indicate the environmental conditions where stick-slip effects can occur a multiple-factor-at-a-time (MFAT) analysis is performed. This MFAT analysis indicates the sensitivity and correlation by changing multiple parameters at the same time. In this section the sensitivity and correlation between the ice thickness, ice drift velocity and mooring stiffness is analyzed. The same output parameters are used as in the OFAT analysis; the total stick-events per hour and the normalized break-stick percentage. The results can be illustrated in the 3D graphs of figure 4.4, where figure 4.4a indicates the total stick-events per hour and figure 4.4b the break-stick percentage. Note that these results are not verified and it is unknown if the model’s assumed linearity with parameters holds true for values far exceeding the model tests.

In general it can be seen that the number of stick-events is increasing if the ice thickness is increasing. This is caused by the increased break-impulse load (stick load $F_{\text{stick}}$), which is more difficult to continuously overcome with the existing vessel’s momentum ($m\Delta v_{rel} = F_{\text{stick}}\Delta t$). In this case the vessel will not have enough momentum to break the ice continuously and it will remain in a stick-event more often. But on the other hand it can also be seen in figure 4.4a that the total number of stick-events per hour is decreasing if the ice thickness increases above a certain value. This is caused by the increasing stick-length of each stick-event, which consequently causes the number of stick-event per hour to decrease. The break-stick percentage does not have this numerical artifact, therefore this will be seen as the main output parameter. In figure 4.4b can be seen that an increase in ice thickness will also result in an increase in the break-stick percentage.
4.2. Probability of the Stick-Slip Effect

Figure 4.4: 3D graphs of the environmental sensitivity.

(a) The total stick-events per hour as output parameter.

(b) The break-stick percentage as output parameter.
An increase in ice drift velocity will generally decrease the probability of stick-events. This is caused by the increase of the vessel’s momentum during an increased relative velocity ($M = m\Delta v_{rel}$). Therefore it will be more easy for the vessel to overcome the ice break impulses. In addition the chance that a vessel will get the same velocity as the ice sheet will be smaller if the ice drift velocities are high.

In figure 4.5 the contour graphs of the break-stick percentages of 250 kN/m and 1250 kN/m are illustrated, this is just a simplified 2D representation of the 3D graphs of figure 4.4b. It can be seen that the border between stick-events and no stick-events is shifting to the right with higher mooring stiffnesses.

In figure 4.4 and 4.5 it is shown that for most circumstance an increase in mooring stiffness will cause an increase in the break-stick events. On the other hand it was expected in the Aksnes experiments that an increase in mooring systems will lead to a lower chance on stick-events (see figure 1.5). This contradiction can be seen in the crossing of the different mooring stiffness graphs in figure 4.4 and the shifting to the right in figure 4.5. This indicates that it depends on the circumstances if the stiff mooring system or the soft mooring system is governing. These two different mooring stiffness zones are indicated in figure 4.6; the green zone indicates that the soft system has more break-stick events and the red zone indicates that the stiff system has more break-stick events. Note that this graph is only valid for this specific base-case scenario.

**Figure 4.5: Environmental sensitivity; ice drift velocity vs. ice thickness vs. mooring stiffness**

(a) Mooring stiffness of 250 kN/m. (b) Mooring stiffness of 1250 kN/m.

**Figure 4.6: Soft or stiff mooring system; 250 or 1250 kN/m**

*Green = soft system (250 kN/m) governing, red = stiff system (1250 kN/m) governing*
4.2.3. Vessel Sensitivity

The same MFAT analysis as the environmental sensitivity is performed for the vessel sensitivities, i.e. the sensitivity and correlation of the scale factor, ice thickness and mooring stiffness are analyzed. The simplified 2D contour graphs are shown in figure 4.7. The normalized break-stick percentage will be used as the main output of this analysis.

Figure 4.7: Vessel sensitivity; scale factor vs. ice thickness vs. mooring stiffness

Concluding from the graphs in figure 4.7 the break-stick percentage is decreased if the scale-factor is increased; increasing the scale factor results in an increased vessel’s mass, consequently this increased mass will increase the vessel’s momentum \((M = mΔv_{rel})\). Therefore it will be easier for the vessel to continuously break the ice and overcome the break-impulses. Therefore it is expected that the stick-slip effect will only occur with relatively light vessels. This complies with a preliminary market analysis performed during this thesis, where it was shown that the vessel of the Aksnes experiments had a relatively small mass compared to currently operational FPSOs.

An increase in ice-thickness will still lead to an increase in break-stick percentage, but this time there probably is a certain scale factor where an increase in ice thickness will not lead to any stick-events anymore. This can be seen in the “decline” in figure 4.7a. Therefore it can be concluded that the stick-event can be avoided by increasing the mass, and thus the momentum of the vessel.

If the mooring stiffness is increased the break-stick percentage is increasing for most circumstances. On the other hand, like in the environmental sensitivity analysis it is also shown that there are certain circumstances where it will be more beneficial to use a more stiff system.

With the help of this research it can be concluded that the stick-events in the Aksnes2100 experiment were only occurring due to the special circumstances. Normally it is expected to be less beneficial to design a high stiffness mooring system, due to the higher probability of stick-events. This result is in contrary with the preliminary expectations made, which were based on the Aksnes (2010) experiments[2].
5.1. **CONCLUSIONS**
In this thesis the stick-slip effect on a simplified moored vessel in head-on ice conditions is researched. A numerical model is created and validated with the help of several existing ice model experiments. With the help of this numerical model an extensive result analysis is performed. This analysis was focusing on the consequences and the probabilities of stick-events that can occur. The conclusion is divided into three parts; the numerical model validation, the consequences and the probability of a stick-event.

**Numerical model:**
- A statistical Monte Carlo validation is performed on a local level; the 60x individual stick-events of the Aksnes2100 experiment were validated individually. The results found in this analysis were in line with the expectations.
- A statistical Monte Carlo validation is performed on a global level; the numerical model was tested for all Aksnes experiments, with a detailed analysis for the Aksnes2100 experiment. The results found in this analysis were in line with the expectations.
- Therefore it can be concluded that the numerical model created during this study gives a good prediction of the stick-slip effect in the Aksnes experiments.
- Two different stick-events are expected; a slip-stick event and a break-stick event. These two different stick-events distinguish themselves with a different composition of the stick force.

**Consequences of the stick-events:**
- It is expected that a slip-stick event will not cause a significant increase in mooring load amplitude with respect to the normal slip-phase mooring load amplitude. Therefore the mooring loads in a slip-stick event are expected to be well below the ISO19906\[13\] design calculations. This is probably depending on the specific vessel characteristics; the Aksnes2100 experiment was chosen to be the base-case scenario during this study.
- A break-stick event can cause 20% higher mooring loads than current the ISO19906 design calculations. This is because ISO19906 estimates the design load by calculating the break-impulse amplitude with the help of the kinetic friction coefficient, while the stick-event uses the static friction coefficient.
- A preventive use of the static friction coefficient in the ISO19906 calculations, to precautionary cope with the possible stick-events will not always be necessary. First a probability analysis has to be performed to identify the chance of stick-events during the moored vessel operation.

**Probability of the stick-events:**
- The occurrences of stick-slip effects are mostly dependent on the vessel size, the ice drift velocity, ice thickness and mooring stiffness. These parameters are determined by a one-factor-at-a-time (OFAT) analysis and are further analyzed in a multiple-factors-at-a-time (MFAT) analysis.
An increase in ice thickness will generally increase the probability of stick-events. This is caused by the increased break-impulse amplitude, which will be more difficult to continuously overcome with the existing vessel’s momentum \((m\Delta v_{rel} = F_{stick}\Delta t)\).

An increase in ice drift velocity will generally decrease the probability of stick-events. This is caused by the increase of the vessel’s momentum during an increased relative velocity \((M = m\Delta v_{rel})\).

An increase in the size and mass of a vessel will generally decrease the probability of stick-events. This is caused by the increase of the vessel’s momentum and thus the increase in it’s capability to continuously break the ice \((M = m\Delta v_{rel})\).

A decrease of the mooring stiffness will not always increase the probability of stick-events. This is in contrary to the preliminary expectations based on the Aksnes experiments. It is expected that there is a gray zone where it is sometimes more beneficial to have a stiff mooring system, but under most circumstances a soft mooring stiffness will lead to less break-stick events.

With the help of this research it can be concluded that the stick-events in the Aksnes2100 experiment were only occurring due to the special circumstances. Normally it is expected to be less beneficial to design a high stiffness mooring system, due to the higher probability of stick-events. This result is in contrary with the preliminary expectations made based on the Aksnes (2010) experiments[2].

### 5.2. DISCUSSION

The stick-slip effect is a relatively unknown phenomenon in the application of Arctic moored vessels. Therefore a lot of assumptions needed to be made with respect to the numerical model. After the study has been done a lot of questions came up; how realistic will this model be for modeling existing FPSO systems? How big are the chances of stick-slip in real ice conditions? What can be done to improve the model?

It is assumed that the numerical model consists of a barge-shaped hull with a relatively shallow bow shape. This way no rubble is formed underneath the bow and the ice breaking process only consists of a selected amount of ice loads. In the validation part of this thesis it was shown that the numerical model modeled the ice model experiments of Aksnes (2010)[2] with a fairly good agreement. Therefore it is expected that the numerical model gives a good representation of a moored barge in certain level-ice conditions. In reality a FPSO will not have a simplified barge-shaped hull because of its large frontal area that can cause large environmental loads; the hydrodynamic properties of a barge-shaped hull are not ideal as they are sensitive for wave- and slamming loads. In reality an ice-break FPSO will probably have better hydrodynamic properties with an ice deflecting bow shape to decrease the ice coverage underneath the bow. Also the presence of a turret underneath the bow is not included in this study; it will probably cause the ice sheet to break up and deflect underneath the hull. Therefore an ice coverage of 100% will therefore not be completely realistic. After performing a small preliminary market analysis of existing FPSO systems it was also noticed that the mass (and thus the momentum) of the vessel used in the Aksnes (2010) experiments was relatively low. Consequently existing FPSOs will have a higher momentum and therefore they will experience less effort to break the ice continuously.

Even though this study will not give a 1-on-1 representation of a real FPSO system, it still indicates the importance of the stick-slip effect. It is expected that the stick-slip effect can occur at systems where large (ice-)friction forces are present and that they can cause high unexpected mooring loads. In addition, it is expected that possible rubble forces will only increase the amount of ice underneath the hull, which will increase the buoyancy forces and consequently also increase the ice friction forces. A more detailed study about stick-slip effects in rubble conditions is suggested. Finally it is expected that the stick-slip effect will still be very important for relatively light moored vessels with high ice concentrations underneath the hull.

### 5.3. RECOMMENDATIONS

A lot of assumptions were made during this study. This is mainly because the stick-slip is relatively unknown phenomenon for this kind of application. To improve the results in this study the following measurements are proposed to be taken:

- Only one model experiment time series was available for analysis that showed stick-slip effects. To verify the model it would be useful to create more data points by doing more model tests under stick-slip circumstances (slow, head-on level ice with soft mooring system configurations).
• More research is to be done with respect to the friction coefficient profile. The stick slip effect is very sensitive to changes in this profile and therefore it would be interesting to research and implement a more accurate friction coefficient profile.

• The current numerical model consists of a simplified 2D hull shape; before a more realistic 3D hull shape can be examined, an under-the-bow ice transport research is to be performed to examine the friction forces.

• A study into the grouping of the stick-event can be done to examine the correlation between the individual stick-events.

• Design a method of doing ice tank experiments with the help of a alternative material like paraffin.

• More insight can be found in the variability of ice-drift direction. In this study the vessel was assumed to receive a head-on ice loading, but what happens if the ice-drift direction rapidly changes?
Pictures Aksnes (2010) experiments:

Figure A.1: Photo 1. Photo courtesy of Aksnes (2010)
Figure A.2: Photo 2: Ice coverage bow. Photo courtesy of Aksnes (2010)
### Detailed Input Parameters

Input parameters of other experiments and given distributions on courtesy of Aksnes (2010) and HYDRALAB[10].

Table B.1: Input parameters Aksnes experiments. Data courtesy of Aksnes (2010) and HYDRALAB.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>#2100</th>
<th>#2200</th>
<th>#3100</th>
<th>#3200</th>
<th>#4100</th>
<th>#4200</th>
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<tbody>
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<td>106</td>
<td>106</td>
<td>106</td>
<td>106</td>
<td>106</td>
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<td>33</td>
<td>33</td>
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<td>106</td>
<td>106</td>
<td>106</td>
<td>106</td>
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</tr>
<tr>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
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<td>28800</td>
<td>28800</td>
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</tr>
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<td>Ice drift velocity [m/s]</td>
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<td>Mean ice thick [m]</td>
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<td>Mean $L_{br}/th$ [-]</td>
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<td>3.3</td>
<td>3.3</td>
<td>3.3</td>
<td>3.7</td>
</tr>
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<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
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<td>870</td>
<td>870</td>
<td>870</td>
<td>929</td>
<td>929</td>
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<tr>
<td>Dev. ice dens [kg/m³]</td>
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<td>20</td>
<td>20</td>
<td>20</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Mean flex strength [kPa]</td>
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<td>850</td>
<td>665</td>
<td>665</td>
<td>610</td>
<td>610</td>
</tr>
<tr>
<td>Dev. flex strength [kPa]</td>
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<td>20</td>
<td>10</td>
<td>10</td>
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<tr>
<td>Mean elast mod [Pa]</td>
<td>1.30E+09</td>
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<td>1.70E+09</td>
<td>1.30E+09</td>
<td>1.30E+09</td>
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<td>0.01</td>
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</tr>
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<td>0.1</td>
</tr>
<tr>
<td>Dev. stat fric coef [-]</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Mean kin fric coef [-]</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Dev. kin fric coef [-]</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>
In this appendix the individual stick-events in the Aksnes2100 experiment are identified and the stick-event properties (stick-event length and amplitude) are elaborated. These individual stick-events are identified by checking the velocity of the vessel; if the velocity of the vessel is the same as the ice drift velocity it is assumed that the respective peak is a stick-event. The displacement plot is shown in figure C.1 and the individual stick-event properties are elaborated in table C.1 and C.2.
Figure C.1: Exact numbering of every individual stick-event in the Aksnes2100 experiment.
In Table C.1 and C.2 the 60x individual stick-events of the Aksnes2100 experiment are further elaborated. The stick-amplitude is determined by multiplying the displacement with the mooring stiffness \(=250\, kN/m\) and the last column is assumed with the break-stick criteria of section 3.5 and eq (3.2).

Table C.1: Individually analyzed stick-events in the Aksnes2100 experiment. (part 1)

<table>
<thead>
<tr>
<th>Peak number</th>
<th>Stick-length [s]</th>
<th>Stick-amplitude [kN]</th>
<th>Break-stick event?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>921</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>39</td>
<td>1096</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>736</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>22</td>
<td>973</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>1156</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>892</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>43</td>
<td>1277</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>17</td>
<td>894</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>62</td>
<td>1590</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>45</td>
<td>955</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>29</td>
<td>1133</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
<td>910</td>
<td>-</td>
</tr>
<tr>
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<td>25</td>
<td>1245</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>59</td>
<td>1345</td>
<td>-</td>
</tr>
<tr>
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<td>57</td>
<td>1382</td>
<td>-</td>
</tr>
<tr>
<td>16</td>
<td>13</td>
<td>1044</td>
<td>-</td>
</tr>
<tr>
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<td>24</td>
<td>996</td>
<td>-</td>
</tr>
<tr>
<td>18</td>
<td>17</td>
<td>943</td>
<td>-</td>
</tr>
<tr>
<td>19</td>
<td>40</td>
<td>1408</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
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<td>-</td>
</tr>
<tr>
<td>21</td>
<td>16</td>
<td>746</td>
<td>-</td>
</tr>
<tr>
<td>22</td>
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<td>1470</td>
<td>-</td>
</tr>
<tr>
<td>23</td>
<td>27</td>
<td>954</td>
<td>-</td>
</tr>
<tr>
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<td>11</td>
<td>892</td>
<td>-</td>
</tr>
<tr>
<td>25</td>
<td>38</td>
<td>1209</td>
<td>-</td>
</tr>
<tr>
<td>26</td>
<td>56</td>
<td>1133</td>
<td>-</td>
</tr>
<tr>
<td>27</td>
<td>11</td>
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<td>-</td>
</tr>
<tr>
<td>28</td>
<td>25</td>
<td>1100</td>
<td>-</td>
</tr>
<tr>
<td>29</td>
<td>35</td>
<td>1283</td>
<td>-</td>
</tr>
<tr>
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<td>-</td>
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<tr>
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<td>24</td>
<td>1275</td>
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</tr>
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<td>82</td>
<td>1816</td>
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</tr>
<tr>
<td>33</td>
<td>38</td>
<td>790</td>
<td>-</td>
</tr>
<tr>
<td>34</td>
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<tr>
<td>35</td>
<td>23</td>
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<td>36</td>
<td>35</td>
<td>1179</td>
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<td>37</td>
<td>34</td>
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<tr>
<td>38</td>
<td>74</td>
<td>1947</td>
<td>yes</td>
</tr>
</tbody>
</table>
Table C.2: Individually analyzed stick-events in the Aksnes2100 experiment. (part 2)

<table>
<thead>
<tr>
<th>Peak number</th>
<th>Stick-length [s]</th>
<th>Stick-amplitude [m]</th>
<th>Break-stick event?</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>58</td>
<td>1111</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>25</td>
<td>1522</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>63</td>
<td>2015</td>
<td>yes</td>
</tr>
<tr>
<td>44</td>
<td>60</td>
<td>2119</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>60</td>
<td>892</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>20</td>
<td>1299</td>
<td></td>
</tr>
<tr>
<td>47</td>
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<td>1308</td>
<td></td>
</tr>
<tr>
<td>48</td>
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<td>1304</td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>21</td>
<td>1243</td>
<td></td>
</tr>
<tr>
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<td>55</td>
<td>1900</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>25</td>
<td>1785</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>30</td>
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<td></td>
</tr>
<tr>
<td>53</td>
<td>61</td>
<td>1739</td>
<td>yes</td>
</tr>
<tr>
<td>54</td>
<td>30</td>
<td>1151</td>
<td></td>
</tr>
<tr>
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<td>70</td>
<td>1952</td>
<td>yes</td>
</tr>
<tr>
<td>56</td>
<td>60</td>
<td>883</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>20</td>
<td>1191</td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>15</td>
<td>941</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>15</td>
<td>1126</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>45</td>
<td>1244</td>
<td></td>
</tr>
</tbody>
</table>

Note that peak number 39 and 40 are missing in table C.2, this is due to the fact that those peaks are consisting of multiple peaks on top of each other. This is not been modeled in the numerical model and therefore these two peaks are deleted from the data-set.

The correlation between the stick-length and the stick-amplitude is shown in figure C.2. It can be seen that the two parameters have a low positive correlation.

Figure C.2: Correlation between the stick-length and the stick-amplitude in the Aksnes2100 experiments.
A statistical Monte Carlo analysis is performed for the validation of the model and for the result analysis. The Monte Carlo method is described as "A broad class of computational algorithms that rely on repeated random sampling to obtain numerical results." In this study the numerical model has been run a large number of times to indicate the statistical distributions of the outcome. In table D.1 the Monte Carlo parameters are shown. The parameters are divided in the number of simulations and the number of break-cycles (see figure 4.2), which represent the number of runs performed with the same parameters and the length of the time domain plot respectively.

Table D.1: Monte Carlo parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Number of simulations</th>
<th>Number of break-cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stick-event verification Aksnes experiments in section 3.5</td>
<td>10,000</td>
<td>320</td>
</tr>
<tr>
<td>Global verification Aksnes experiments in section 3.6</td>
<td>1</td>
<td>500,000</td>
</tr>
<tr>
<td>Global verification Aksnes2100 in section 3.6</td>
<td>10,000</td>
<td>320</td>
</tr>
<tr>
<td>Consequence study in section 4.1</td>
<td>1</td>
<td>1,000,000</td>
</tr>
<tr>
<td>Tornado charts in section 4.2.1</td>
<td>1</td>
<td>1,000,000</td>
</tr>
<tr>
<td>Environmental sensitivity study in section 4.2.2</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Vessel sensitivity study in section 4.2.3</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

On the next page the fitting of the histogram distributions is elaborated.
The fitted histogram distributions of the numerical model validation of chapter 3.

Figure D.1: Monte Carlo histogram fitting.

(a) Stick-length
(b) Stick-peak mooring load
(c) Total stick-events per hour
(d) Individual stick-events per hour

Note that the stick-length histogram of the slip-stick events (gray histogram in figure D.1a) is composed of two different peaks. The right peak is caused by the occurring of a slip-stick event after each break-stick event. This kind of slip-stick event is almost as big as a break-stick event due to its 1-on-1 correlation (see lower figure ??).
In this appendix a method is elaborated how to use the previous sensitivity analysis in a design procedure of a certain mooring system. In this simple case-study a vessel and a mooring system like the base-case scenario are subjected to the environmental conditions of the Beaufort Sea.

First the environmental conditions need to be determined on the basis of given data or site specific historic distributions. In this case study the distributions of a location in the Beaufort Sea in February are gathered from historic measurement data, which is available to the public [26]. Seven measured distributions are available for the ice thickness and ice drift velocity in February 1994 to 2001. These distributions shown in figure E.1 and are fitted with a gamma distribution. In reality an ice sheet can always change direction and therefore it will always be possible that an ice sheet has a small ice drift velocity. This can also be seen in the ice drift velocity distribution of the Beaufort Sea.

Figure E.1: Case study: Beaufort Sea environmental conditions in February 1994 to 2001.

These two different distributions can combined into one joint probability distribution, which can be illustrated by a contour graph. This joint probability distribution can then be used as a layer into the previously created environmental sensitivity graphs. If the joint probability distribution has a large overlay with the stick-event area in the sensitivity graph there will be a high chance of stick-events to occur.

In this specific case there is a large overlay between the joint probability distribution and the break-stick percentage area. Therefore it is expected that this base-case scenario in the Beaufort Sea will experience break-stick events and a more detailed stick-slip analysis needs to be performed. Note that this will only give a preliminary estimation for the probability of stick-events in the base-case scenario.
Figure E.2: Case study: Beaufort Sea environmental probability.

(a) with a mooring stiffness of 250 kN/m

(b) with a mooring stiffness of 1250 kN/m
The numerical model created in this thesis was modeled in MATLAB®. In this appendix the MATLAB-file is given where the numerical model is written in. The MATLAB-file can also be found in the CD-rom attached to this thesis.

**CONTENTS**

- PARAMETERS
- PRE-ALLOCATION OF NUMERICAL VARIABLES
- NUMERICAL MODEL CALCULATIONS
- STATISTICS & OUTPUT ANALYSIS
- PLOTTING OF FIGURES

```matlab
clc;clear all;close all;

% PARAMETERS
numsim=1;  % Number of simulations (for Monte Carlo purposes)
cycles=500; % Length of the simulation (number of break-cycles)

% General parameters
rho_w=1006; % [kg/m³] density water
g=9.81;    % [m/s²] gravity

% Vessel parameters
length_v=106;  % [m] vessel length
width_v=33;    % [m] vessel width at waterline
draft_v=8.8;   % [m] vessel draft
phi_v=(25/180)*pi; % [rad] vessel bow angle
cov_bot=1;     % [-] ice coverage bottom vessel (see pictures experiment)
cov_length=(length_v-draft_v/tan((25/180)*pi))+(draft_v/sin((25/180)*pi));
mass_v=28800000; % [kg] vessel weight
A11=0.02*mass_v; % [kg] added mass

% Mooring & Damping Ratio
k_m=2500000;  % [N/m] Mooring stiffness coefficient
omega_n=sqrt(k_m/(mass_v+A11)); % Natural frequency
zeta_hd=0.04;  % [-] Hydrodynamic damping ratio
zeta_ice=0.02; % [-] ice damping ratio
zeta=zeta_hd+zeta_ice; % [-] Damping ratio (c_m/c_c)
C_ice=zeta_ice*2*sqrt((mass_v+A11)*k_m); % [kg/s??] Ice damping coefficient
omega_d=sqrt(1-zeta^2)*omega_n;
```
% Ice Sheet parameters
vd=0.05; %[m/s] ice drift velocity
h_i_mean=0.8; % [m] ice thickness mean
h_i_dev=0.05^2*h_i_mean^2; % [m] ice thickness variation
LBhi_mean=5.7; % [m] breaking length/hi mean
LBhi_dev=3.3; % [m] breaking length/hi variation
pressure_i=0; %[Pa] ice pressure sides of ship (not implemented)

% Ice mechanics
nu_i=0.3; %[-] poisson ratio
rho_i_mean=870;
rho_i_dev=20;
sigmaf_mean=850000;
sigmaf_dev=200000;
Ei_mean=1.3*10^9;
Ei_dev=0.2*10^9;

% Friction parameters
vd_ss=0.01; %[m/s] Transition velocity step function static to kinetic (ISO19906)
mu_s_mean=0.1; %[-] Mean static friction coefficient (ISO19906)
mu_s_dev=0.02; %[-] Dev static friction coefficient (ISO19906)
mu_k_mean=0.05; %[-] Mean kinetic friction coefficient (ISO19906)
mu_k_dev=0.01; %[-] Dev kinetic friction coefficient (ISO19906)

% Simulation parameters
simtime=round((1.05*cycles*LBhi_mean*h_i_mean)/vd);
out_simtime_check=simtime;
steadystatet ime=round(cov_length/vd);

PRE-ALLOCATION OF NUMERICAL VARIABLES
b_simtime_end=zeros(1,numsim);
b_after_sticks=zeros(1,numsim);
b_break_sticks=zeros(1,numsim);
b_momentum_sticks=zeros(1,numsim);
b_slip_sticks1=zeros(1,numsim);
b_slip_sticks2=zeros(1,numsim);
b_slip_sticks3=zeros(1,numsim);
b_slip_sticks4=zeros(1,numsim);
b_slipbreak_sticks=zeros(1,numsim);
b_total_TOTAL=zeros(1,numsim);
b_total_slipstick=zeros(1,numsim);
b_total_breakstick=zeros(1,numsim);

c_breaktstart=zeros(cycles,numsim);
c_breaktend=zeros(cycles,numsim);
c_breakxstart=zeros(cycles,numsim);
c_breakxend=zeros(cycles,numsim);
c_slip1tstart=zeros(cycles,numsim);
c_slip1tend=zeros(cycles,numsim);
c_slip1xstart=zeros(cycles,numsim);
c_slip1xend=zeros(cycles,numsim);
c_slip2tstart=zeros(cycles,numsim);
c_slip2tend=zeros(cycles,numsim);
c_slip2xstart=zeros(cycles,numsim);
c_slip2xend=zeros(cycles,numsim);
c_slip3tstart=zeros(cycles,numsim);
c_slip3tend=zeros(cycles,numsim);
c_slip3xstart=zeros(cycles,numsim);
c_slip3xend=zeros(cycles,numsim);
c_slip4tstart=zeros(cycles,numsim);
c_slip4tend=zeros(cycles,numsim);
c_slip4xstart=zeros(cycles,numsim);
c_slip4xend=zeros(cycles,numsim);

for sim=1:numsim % Number of simulations
    x=zeros(1,simtime);
    xd=zeros(1,simtime);
    x1=zeros(1,simtime);
    x2=zeros(1,simtime);
    xrel=zeros(1,simtime);
    P=zeros(1,simtime);
    F_vel=zeros(1,simtime);
    dt=zeros(1,(cycles-1));
    F=zeros(1,cycles);
    time=zeros(1,cycles);
    F_BR_S=zeros(1,cycles);
    F_BR_K=zeros(1,cycles);
    F_B_bot=zeros(1,((round((draft_v/sin(phi_v))/(LBhi_mean*h_i_mean))+1) + round((length_v-draft_v/tan(phi_v))/(LBhi_mean*h_i_mean))));
    F_B_bow=zeros(1,((round((draft_v/sin(phi_v))/(LBhi_mean*h_i_mean))+1) + round((length_v-draft_v/tan(phi_v))/(LBhi_mean*h_i_mean))));
    FBUOYBOW=zeros(1,cycles);
    FBUOYBOT=zeros(1,cycles);
    F_FRIC_S=zeros(1,cycles);
    F_FRIC_K=zeros(1,cycles);
    F_BREAK_S=zeros(1,cycles);
    F_BREAK_K=zeros(1,cycles);
    F_SLIP_S=zeros(1,cycles);
    F_SLIP_K=zeros(1,cycles);
    F_BUOY=zeros(1,cycles);
    F_ROT=zeros(1,cycles);
    F_VELO=zeros(1,cycles);
    MOMENTUM=zeros(1,cycles);
    dt(1)=0;
    F(1)=0;
    time(1)=0;
    xs=0;
    x1s=0;
    x2s=0;
    ts=1;
    xds=0;
    tds=0;
    SLIPSTICK=0;
    BREAKSTICK=0;
    a_break_sticks=0;
    a_slipbreak_sticks=0;
    a_momentum_sticks=0;
    a_after_sticks=0;
    a_slip_sticks=0;
    a_slip_sticks2=0;
    a_slip_sticks3=0;
a_slip_sticks4 = 0;

**NUMERICAL MODEL CALCULATIONS**

% Creating new ice sheet:
\[
\begin{align*}
    h_{i,M} &= \lognrnd(\log((h_{i,\text{mean}}^2)/\sqrt{h_{i,\text{dev}}+h_{i,\text{mean}}^2}),
                         \sqrt{\log(h_{i,\text{dev}}/(h_{i,\text{mean}}^2)+1)},[\text{cycles,1}]); \\
    L_{\text{break,M}} &= \text{gamrnd}(\text{LBhi,mean}^2/\text{LBhi,dev}^2),(	ext{LBhi,dev}^2/\text{LBhi,mean}),[\text{cycles,1}])\times h_{i,\text{mean}}; \\
    \mu_{s,M} &= \text{gamrnd}((\mu_{s,\text{mean}}^2/\mu_{s,\text{dev}}^2),((\mu_{s,\text{dev}}^2/\mu_{s,\text{mean}}),[\text{cycles,1}]); \\
    \mu_{k,M} &= \text{gamrnd}((\mu_{k,\text{mean}}^2/\mu_{k,\text{dev}}^2),((\mu_{k,\text{dev}}^2/\mu_{k,\text{mean}}),[\text{cycles,1}]); \\
    \sigma_{f,i,M} &= \text{normrnd}(\sigma_{f,\text{mean}},\sigma_{f,\text{dev}},[\text{cycles,1}]); \\
    \rho_{i,M} &= \text{normrnd}(\rho_{i,\text{mean}},\rho_{i,\text{dev}},[\text{cycles,1}]); \\
    E_{i,M} &= \text{normrnd}(E_{i,\text{mean}},E_{i,\text{dev}},[\text{cycles,1}]);
\end{align*}
\]

% Break cycle loop
for j=100:cycles % number of cycles
    \[
    h_i = h_{i,M}(j); \\
    L_{\text{break}} = L_{\text{break,M}}(j); \\
    \mu_s = \mu_{s,M}(j); \\
    \mu_k = \mu_{k,M}(j); \\
    \sigma_f_i = \sigma_{f,i,M}(j); \\
    \rho_i = \rho_{i,M}(j); \\
    E_i = E_{i,M}(j);
    \]

%calculating external forces:
\[
\begin{align*}
    zeta_{br,K} &= (\sin(\phi_v) + \mu_k \cos(\phi_v))/((\cos(\phi_v) - \mu_k \sin(\phi_v)); \\
    zeta_{br,S} &= (\sin(\phi_v) + \mu_s \cos(\phi_v))/((\cos(\phi_v) - \mu_s \sin(\phi_v)); \\
    L_C &= ((E_i h_i^{-3})/(12*\rho_w g(1-\nu_i^2)))^{0.25}; \\
    F_{BR,K}(j) &= 0.68 \times zeta_{br,K} \times \sigma_f_i \times ((\rho_w g h_i^5)/E_i)^{0.25} \times (width_v+0.25\pi^2 L_C); \\
    F_{BR,S}(j) &= 0.68 \times zeta_{br,S} \times \sigma_f_i \times ((\rho_w g h_i^5)/E_i)^{0.25} \times (width_v+0.25\pi^2 L_C); \\
    F_{BUOY}(j) &= (\mu_w - \rho_i) \times (L_{\text{break}} h_i width_v cov_bot) \times g; \\
    F_{ROT}(j) &= (0.5 \times (\rho_w - \rho_i) \times g \times (L_{\text{break}} - 2) \times width_v \times cov_bot \times h_i \times \\
                \sin(\phi_v))/L_{\text{break}}) \times zeta_{br,K};
\end{align*}
\]

for k=1:round((draft_v/sin(\phi_v))/(\text{LBhi,mean} \times h_{i,\text{mean}}))
    F_B_bow(k) = F_{BUOY}(j-k);
end
FBUOYBOW(j) = sum(F_B_bow);

for k=(round((draft_v/sin(\phi_v))/(\text{LBhi,mean} \times h_{i,\text{mean}})) + 1):round((\text{length_v-draft_v} \\
                                         /\tan(\phi_v))/(\text{LBhi,mean} \times h_{i,\text{mean}}))
    F_B_bot(k) = F_{BUOY}(j-k);
end
FBUOYBOT(j) = sum(F_B_bot);

FBsheet = sin(\phi_v) \times FBUOYBOW(j);
Fship2 = cos(\phi_v) \times FBUOYBOW(j);
Fship3 = FBUOYBOT(j);
Fsheet2_s = (mu_s \times Fship3)/cos(\phi_v);
Fsheet2_k = (mu_k \times Fship3)/cos(\phi_v);
Fsheet1_s = Fsheet2_s + FBsheet + Fship2 \times mu_s;
Fsheet1_k = Fsheet2_k + FBsheet + Fship2 \times mu_k;
Fsides = 2 \times \text{pressure}_i \times h_i \times length_v;
F_FRIC_S(j)=F_{sheet1_s}/\cos(\phi_v)+F_{sides}*\mu_s;
F_FRIC_K(j)=F_{sheet1_k}/\cos(\phi_v)+F_{sides}*\mu_k;
F_VELO(j)=C_{ice}*v_d;
F_BREAK_S(j)=F_{BR_S(j)}+F_{FRIC_S(j)}+F_{VELO(j)}; \text{ Static Break Force}
F_BREAK_K(j)=F_{BR_K(j)}+F_{FRIC_K(j)}+F_{VELO(j)}; \text{ Kinetic Break Force}
F_SLIP_S(j)=F_{ROT(j)}+F_{FRIC_S(j)}+F_{VELO(j)}; \text{ Static Slip Force}
F_SLIP_K(j)=F_{ROT(j)}+F_{FRIC_K(j)}+F_{VELO(j)}; \text{ Kinetic Slip Force}

% Momentum break-stick
if ts>steadystatetime && BREAKSTICK==0 && (mass_v*(vd-x1s))<(F_BREAK_K(j)-F_SLIP_K(j-1))
BREAKSTICK=1;
a_momentum_sticks=a_momentum_sticks+1;
end

% Kinetic breaking mode
for i=ts:simtime
if BREAKSTICK==1
break
end
F(j)=F_BREAK_K(j);
time(j)=ts+1;
xd(i)=xds+v_d*(i-tds);

DUHAMEL INTEGRAL:
\begin{align*}
    x(i) &= (F(j)/k_m)*(1-exp(-zeta*omega_n*(i-ts)))*(cos(omega_d*(i-ts))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts)))+exp(-zeta*omega_n*(i-ts))*(xs*cos(omega_d*(i-ts))+(x1s+zeta*omega_n*xs)/omega_d*sin(omega_d*(i-ts)));

    x1(i) &= exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*(zeta*omega_n*(cos(omega_d*(i-ts))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts))))-omega_d*xs*sin(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)));

    x2(i) &= -zeta*omega_n*exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*(zeta*omega_n*(cos(omega_d*(i-ts))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts))))-(omega_d*xs*sin(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)))+exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*(zeta*omega_n*(cos(omega_d*(i-ts))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts))))-(omega_d*xs*sin(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)))+exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*(zeta*omega_n*(cos(omega_d*(i-ts))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts))))-(omega_d*xs*sin(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)))+(-omega_d^2*xs*cos(omega_d*(i-ts))-(x1s+zeta*omega_n*xs)*omega_d*sin(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)))+(omega_d^2*xs*cos(omega_d*(i-ts))-(x1s+zeta*omega_n*xs)*omega_d*sin(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)));

    P(i) &= F(j);
end

xrel(i)=xd(i)-x(i);% Kinetic breaking mode
if ts>steadystatetime && x1(i)>(vd-vd_ss)
BREAKSTICK=1;
a_break_sticks=a_break_sticks+1;
ts=i;
x=x(i);
x1=x1(i);
break
end

if i==time(j) % 1 second impulses
    ts=i;
x=x(i);
x1=x1(i);
break
end

% Break stick event
if BREAKSTICK==1
    c_breaktstart(j,sim)=ts; % recording stick event data
    c_breakxstart(j,sim)=xs; % recording stick event data
    for i=ts:simtime
        F(j)=F_BREAK_S(j);
P(i)=F(j);
x(i)=xs+vd*(i-ts);
x1(i)=vd;
x1d(i)=xds+vd*(i-tds);
        if x(i)>(F(j)/k_m)
            BREAKSTICK=0;
ts=i;
x=x(i);
x1=x1(i);
break
end
end
    c_breaktend(j,sim)=ts; % recording stick event data
    c_breakxend(j,sim)=xs; % recording stick event data
end

% Normal slip phase
for i=ts:simtime
    F(j)=F_SLIP_K(j);
x(i)=xds+vd*(i-tds);

DUHAMEL INTEGRAL:
    x(i)=(F(j)/k_m)*(1-exp(-zeta*omega_n*(i-ts)))*(cos(omega_d*(i-ts)))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts)))+exp(-zeta*omega_n*(i-ts))*(xds*cos(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)/omega_d*sin(omega_d*(i-ts));

    x1(i)=exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*(zeta*omega_n*(cos(omega_d*(i-ts)))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts)))-(-omega_d*sin(omega_d*(i-ts)))-zeta*omega_n*(xds*cos(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)/omega_d*sin(omega_d*(i-ts)))+(-omega_d*xs*sin(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)));
\( x_2(i) = -\zeta \omega_n \exp(-\zeta \omega_n \omega_d (i-ts)) \left((F(j)/k_m) (\zeta \omega_n \cos(\omega_d (i-ts)) + (x_1s + \zeta \omega_n x_0) \omega_d / \omega_d (i-ts) - \omega_d^2 x_0 \cos(\omega_d (i-ts))) + \exp(-\zeta \omega_n \omega_d (i-ts)) \right) \)

\[ \begin{align*} 
P(i) &= F(j) \\
xrel(i) &= x(i) - x(i); \\
\end{align*} \]

if \( xrel(i) > L_{\text{break}} \)
\begin{align*} 
ts &= i; \\
x_0 &= x(i); \\
x_1s &= x_1(i); \\
td &= i; \\
xds &= x(i); \\
\text{break} \\
\end{align*} 

end

if \( ts > \text{steadystatetime} && x_1(i) > (v_d - v_d_{\text{ss}}) \) \&\& \( x(i) * k_m < F_{\text{SLIP}_K}(j) \)
\begin{align*} 
ts &= i; \\
x_0 &= x(i); \\
x_0 &= x_1(i); \\
\text{SLIPSTICK} &= 1; \\
a_{\text{SLIPSTICK}} &= a_{\text{SLIPSTICK}} + 1; \\
\text{break} \\
\end{align*} 

end

\% First Slip-stick event
if SLIPSTICK == 1;
\begin{align*} 
c_{\text{SLIPSTICK}}(j, sim) &= ts; \quad \% recording stick event data \\
c_{\text{SLIPSTICK}}(j, sim) &= x_0; \quad \% recording stick event data \\
\end{align*} 

for \( i = ts : \text{simtime} \) "STICK"
\begin{align*} 
F(j) &= F_{\text{SLIP}_S}(j); \\
P(i) &= F(j); \\
x(i) &= x_0 + v_d(i-ts); \\
x_1(i) &= v_d; \\
xds &= xds + v_d(i-td); \\
\text{if } x(i) > (F(j)/k_m) \text{ break } \\
\end{align*} 

\begin{align*} 
c_{\text{SLIPSTICK}}(j, sim) &= ts; \quad \% recording stick event data \\
c_{\text{SLIPSTICK}}(j, sim) &= x_0; \quad \% recording stick event data \\
\end{align*}
for i=ts:simtime %SLIP

SLIPSTICK=0;
F(j)=F_SLIP_K(j);
xd(i)=xds+vd*(i-tds);

DUHAMEL INTEGRAL:

\[ x(i)=\left(\frac{F(j)}{k_m}\right)\left(1-\exp\left(-\zeta \omega_n (i-t_s)\right)\right)\left(\cos(\omega_d(i-t_s))+(\zeta \omega_n/\omega_d)\sin(\omega_d(i-t_s))\right)+\exp\left(-\zeta \omega_n (i-t_s)\right)\left(\frac{x_s \cos(\omega_d(i-t_s))+(x_1s+\zeta \omega_n x_s)/\omega_d \sin(\omega_d(i-t_s))}{\omega_d}\right) \]

\[ x_1(i)=\exp\left(-\zeta \omega_n (i-t_s)\right)\left(\frac{F(j)}{k_m}\right)\left(\frac{x_s \sin(\omega_d(i-t_s))+(x_1s+\zeta \omega_n x_s)\cos(\omega_d(i-t_s))}{\omega_d}\right) \]

\[ x_2(i)=-\zeta \omega_n \exp\left(-\zeta \omega_n (i-t_s)\right)\left(\frac{F(j)}{k_m}\right)\left(\frac{x_s \sin(\omega_d(i-t_s))+(x_1s+\zeta \omega_n x_s)\cos(\omega_d(i-t_s))}{\omega_d}\right)+\exp\left(-\zeta \omega_n (i-t_s)\right)\left(\frac{\omega_d^2 \sin(\omega_d(i-t_s))+(\omega_d \cos(\omega_d(i-t_s))}{\omega_d^2}\right) \]

P(i)=F(j);

xrel(i)=xd(i)-x(i);

if xrel(i)>L_break
ts=i;
x=x(i);
x1s=x1(i);
tds=i;
xd=x(i);
break
end

if x1(i)>(vd-vd_ss) && (x(i)*k_m)<F_SLIP_K(j)
ts=i;
x=x(i);
x1s=x1(i);
SLIPSTICK=1;
a_slip_sticks2=a_slip_sticks2+1;
break
end
end

% Second Slip-stick event
if SLIPSTICK==1;
c_slip2tstart(j,sim)=ts;  % recording stick event data
end
c_slip2xstart(j,sim)=xs; % recording stick event data
for i=ts:simtime %STICK
    F(j)=F_SLIP_S(j);
P(i)=F(j);
x(i)=xs+vd*(i-ts);
x1(i)=vd;
xd(i)=dxs+vd*(i-tds);
    if x(i)>(F(j)/k_m)
ts=1;
x=xs;
x1s=x1(i);
    break
end

c_slip2tend(j,sim)=ts; % recording stick event data

c_slip2xend(j,sim)=xs; % recording stick event data
for i=ts:simtime %SLIP
    SLIPSTICK=0;
    F(j)=F_SLIP_K(j);
x(i)=dxs+vd*(i-tds); 

    DUHAMEL INTEGRAL:

    x(i)=(F(j)/k_m)*(1-exp(-zeta*omega_n*(i-ts))*(cos(omega_d*(i-ts)))+exp(-zeta*omega_n*(i-ts))*(xs*cos(omega_d*(i-ts))+x1s*zeta*omega_n*xs)/omega_d*sin(omega_d*(i-ts)));
    x1(i)=exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*(zeta*omega_n*(cos(omega_d*(i-ts)))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts)))+
    (zeta*omega_n*omega_d*sin(omega_d*(i-ts)))-(zeta*omega_n*omega_d*cos(omega_d*(i-ts)))*omega_d*xs*cos(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)));
    x2(i)=-zeta*omega_n*exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*(zeta*omega_n*(cos(omega_d*(i-ts)))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts)))-(zeta*omega_n*omega_d*(i-ts))*omega_d*xs*cos(omega_d*(i-ts)))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)));
    P(i)=F(j);
xrel(i)=xd(i)-x(i);
    if xrel(i)>L_break
ts=1;
x=xs;
x1s=x1(s);
end
tds=i;
xds=x(i);
break

dds=

if x(i)>(vd-vd_ss) && (x(i)*k_m)<F_SLIP_K(j)
ts=i;
x=x(i);
x1s=x1(i);
SLIPSTICK=1;
a_slip_sticks3=a_slip_sticks3+1;
break
end

c_slip3tstart(j,sim)=ts; \text{ % recording stick event data}
c_slip3xstart(j,sim)=xs; % recording stick event data

for i=ts:simtime \%STICK
    F(j)=F_SLIP_S(j);
P(i)=F(j);
x(i)=xs+vd*(i-ts);
x1(i)=vd;
xds=xds+vd*(i-tds);
    if x(i)>(F(j)/k_m)
ts=i;
x=x(i);
x1s=x1(i);
break
end
c_slip3tend(j,sim)=ts; % recording stick event data
c_slip3xend(j,sim)=xs; % recording stick event data

for i=ts:simtime \%SLIP

SLIPSTICK=0;
F(j)=F_SLIP_K(j);
xds=xds+vd*(i-tds);

DUHAMEL INTEGRAL:

x(i)=(F(j)/k_m)*(1-exp(-zeta*omega_n*(i-ts)))*(cos(omega_d*(i-ts))+(zeta*omega_n/omega_d)*sin(omega_d*(i-ts)))+exp(-zeta*omega_n*(i-ts))*(xs*cos(omega_d*(i-ts))+(x1s+zeta*omega_n*xs)/omega_d*sin(omega_d*(i-ts)));

x1(i)=exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*zeta*omega_n*omega_d*(i-ts)+zeta*omega_n*omega_d*(i-ts)+zeta*omega_n*omega_d*(i-ts)+zeta*omega_n*omega_d*(i-ts)+zeta*omega_n*omega_d*(i-ts)+zeta*omega_n*omega_d*(i-ts)+zeta*omega_n*omega_d*(i-ts);
\begin{align*}
x_2(i) &= -\zeta \omega_n \exp(-\zeta \omega_n (i-t_s)) \left( \frac{F(j)}{k_m} (\zeta \omega_n \cos(\omega_d(i-t_s)) + (\zeta \omega_n / \omega_d) \sin(\omega_d(i-t_s))) - \zeta \omega_n (x_s \cos(\omega_d(i-t_s)) + (x_1s + \zeta \omega_n x_s) / \omega_d \sin(\omega_d(i-t_s))) + (x_1 + \zeta \omega_n x_s) \cos(\omega_d(i-t_s))) + (\omega_d^2 \cos(\omega_d(i-t_s)) - \zeta \omega_n \omega_d \sin(\omega_d(i-t_s))) - \zeta \omega_n (x_s \cos(\omega_d(i-t_s)) + (x_1s + \zeta \omega_n x_s) \cos(\omega_d(i-t_s))) + (\omega_d^2 x_s \cos(\omega_d(i-t_s)) - (x_1s + \zeta \omega_n x_s) \omega_d \sin(\omega_d(i-t_s))) \right) + \exp(-\zeta \omega_n (i-t_s)) \left( \frac{F(j)}{k_m} (\zeta \omega_n (-\omega_d \sin(\omega_d(i-t_s)) + \zeta \omega_n \cos(\omega_d(i-t_s))) - (-\omega_d^2 \cos(\omega_d(i-t_s)) - \zeta \omega_n \omega_d \sin(\omega_d(i-t_s))) + (x_1s + \zeta \omega_n x_s) \cos(\omega_d(i-t_s))) + (\omega_d^2 x_s \cos(\omega_d(i-t_s)) - (x_1s + \zeta \omega_n x_s) \omega_d \sin(\omega_d(i-t_s))) \right).
\end{align*}

\begin{align*}
P(i) &= F(j); \\
x_{rel}(i) &= x_d(i) - x(i); \\
\text{if } x_{rel}(i) > L_{\text{break}} \\
&\quad ts = i; \\
&\quad x_s = x(i); \\
&\quad x_1s = x_1(i); \\
&\quad tds = i; \\
&\quad xds = x(i); \\
&\quad \text{break}
\end{align*}

\begin{align*}
\text{if } x_1(i) > (v_d - v_d_{ss}) \ &\& (x(i) \cdot k_m) < F_{\text{SLIP}_K}(j) \\
&\quad ts = i; \\
&\quad x_s = x(i); \\
&\quad x_1s = x_1(i); \\
\text{SLIPSTICK} &= 1; \\
a_{\text{slip}_{sticks}^4} &= a_{\text{slip}_{sticks}^4} + 1; \\
\text{break}
\end{align*}

\begin{align*}
\% \text{Fourth Slip-stick event} \\
\text{if } \text{SLIPSTICK} = 1; \\
c_{\text{slip}_4t\text{start}(j,sim)} &= ts; \quad \% \text{recording stick event data} \\
c_{\text{slip}_4x\text{start}(j,sim)} &= x_s; \quad \% \text{recording stick event data}
\end{align*}

\begin{align*}
\text{for } i = ts : \text{simtime} \% \text{STICK} \\
&\quad F(j) = F_{\text{SLIP}_S}(j); \\
&\quad P(i) = F(j); \\
&\quad x(i) = x_s + v_d(i-t_s); \\
&\quad x_1(i) = v_d; \\
&\quad x_d(i) = xds + v_d(i-tds); \\
&\quad \text{if } x(i) > (F(j) / k_m) \\
&\quad ts = i; \\
&\quad x_s = x(i); \\
&\quad x_1s = x_1(i); \\
&\quad \text{break}
\end{align*}

\begin{align*}
\text{c}_{\text{slip}_4t\text{end}(j,sim)} &= ts; \quad \% \text{recording stick event data} \\
c_{\text{slip}_4x\text{end}(j,sim)} &= x_s; \quad \% \text{recording stick event data}
\end{align*}
for i=ts:simtime %SLIP
SLIPSTICK=0;
F(j)=F_SLIP_K(j);
xd(i)=xds+vd*(i-tds);

DUHAMEL INTEGRAL:

x(i)=(F(j)/k_m)*(1-exp(-zeta*omega_n*(i-ts))*(cos(omega_d*(i-ts))+zeta*omega_n/omega_d*sin(omega_d*(i-ts))))+exp(-zeta*omega_n*(i-ts))*(xs*cos(omega_d*(i-ts))+x1s+zeta*omega_n*xs)/omega_d*sin(omega_d*(i-ts)));

x1(i)=exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*(zeta*omega_n*(cos(omega_d*(i-ts))+zeta*omega_n/omega_d*sin(omega_d*(i-ts)))-zeta*omega_n*(xs*cos(omega_d*(i-ts))+x1s+zeta*omega_n*xs)/omega_d*sin(omega_d*(i-ts))))+(-omega_d*xs*sin(omega_d*(i-ts))+(x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)));

x2(i)=-zeta*omega_n*exp(-zeta*omega_n*(i-ts))*((F(j)/k_m)*(zeta*omega_n*(cos(omega_d*(i-ts))+zeta*omega_n/omega_d*sin(omega_d*(i-ts)))-zeta*omega_n*(xs*cos(omega_d*(i-ts))+x1s+zeta*omega_n*xs)/omega_d*sin(omega_d*(i-ts)))+(-omega_d^2*cos(omega_d*(i-ts))+zeta*omega_n*omega_d*sin(omega_d*(i-ts)))-zeta*omega_n*(xs*cos(omega_d*(i-ts))+x1s+zeta*omega_n*xs)*cos(omega_d*(i-ts)))+(-omega_d^2*xs*cos(omega_d*(i-ts))-(x1s+zeta*omega_n*xs)*omega_d*sin(omega_d*(i-ts)));

P(i)=F(j);
xrel(i)=xd(i)-x(i);

if xrel(i)>L_break
    ts1;
x=x(i);
x1=x1(i);
tds1;
xd=x(i);
break
end
end
end
end

F_mooring=x*k_m;
F_mooring_NZ=nonzeros(F_mooring(steadystatetime:ts));

b_simtime_end(sim)=ts;
b_break_sticks(sim)=a_break_sticks/((ts-steadystatetime)/3600);
b_momentum_sticks(sim)=a_momentum_sticks/((ts-steadystatetime)/3600);
b_slip_sticks1(sim)=a_slip_sticks/((ts-steadystatetime)/3600);
b_slip_sticks2(sim)=a_slip_sticks2/((ts-steadystatetime)/3600);
b_slip_sticks3(sim)=a_slip_sticks3/((ts-steadystatetime)/3600);
b_slip_sticks4(sim)=a_slip_sticks4/((ts-steadystatetime)/3600);
b_total_breakstick(sim) = (a_break_sticks + a_momentum_sticks) / ((ts - steadystatet ime) / 3600);

b_total_slipstick(sim) = (a_slip_sticks + a_slip_sticks2 + a_slip_sticks3 + a_slip_sticks4) / ((ts - steadystatet ime) / 3600);

b_total_TOTAL(sim) = (a_after_sticks + a_break_sticks + a_momentum_sticks + a_slip_sticks + a_slip_sticks2 + a_slip_sticks3 + a_slip_sticks4 + a_slipbreak_sticks) / ((ts - steadystatet ime) / 3600);

end

STATISTICS & OUTPUT ANALYSIS

b_total_TOTAL_mean = mean(b_total_TOTAL);
b_total_TOTAL_std = std(b_total_TOTAL);
b_total_breakstick_mean = mean(b_total_breakstick);
b_total_breakstick_std = std(b_total_breakstick);

d_breaktstart = nonzeros(c_breaktstart);
d_breaktend = nonzeros(c_breaktend);
d_breakxstart = nonzeros(c_breakxstart);
d_breakxend = nonzeros(c_breakxend);
d_slip1tstart = nonzeros(c_slip1tstart);
d_slip1tend = nonzeros(c_slip1tend);
d_slip1xstart = nonzeros(c_slip1xstart);
d_slip1xend = nonzeros(c_slip1xend);
d_slip2tstart = nonzeros(c_slip2tstart);
d_slip2tend = nonzeros(c_slip2tend);
d_slip2xstart = nonzeros(c_slip2xstart);
d_slip2xend = nonzeros(c_slip2xend);
d_slip3tstart = nonzeros(c_slip3tstart);
d_slip3tend = nonzeros(c_slip3tend);
d_slip3xstart = nonzeros(c_slip3xstart);
d_slip3xend = nonzeros(c_slip3xend);
d_slip4tstart = nonzeros(c_slip4tstart);
d_slip4tend = nonzeros(c_slip4tend);
d_slip4xstart = nonzeros(c_slip4xstart);
d_slip4xend = nonzeros(c_slip4xend);

d_slipxstart = [d_slip1xstart; d_slip2xstart; d_slip3xstart; d_slip4xstart];
d_slipxend = [d_slip1xend; d_slip2xend; d_slip3xend; d_slip4xend];

d_breaktime = d_breaktend - d_breaktstart;
d_slip1time = d_slip1tend - d_slip1tstart;
d_slip2time = d_slip2tend - d_slip2tstart;
d_slip3time = d_slip3tend - d_slip3tstart;
d_slip4time = d_slip4tend - d_slip4tstart;

PLOTTING OF FIGURES

% Mooring Load
figure; plot(F_mooring); hold all; stairs(P,'DisplayName', 'P'); hold off; xlabel('Time [s]'); ylabel('Mooring load [N]');

% Surge Velocity:
figure; plot(x1); xlabel('Time [s]'); ylabel('Surge velocity [m/s]');
% Relative displacement (ice break length implementation):
%figure;plot(x);hold all;plot(xd);hold off;


