A Suitable Transverse Shear Stiffness Definition for Buckling of Laminates and Sandwich Plates

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ABSTRACT

This report describes two methods to improve the accuracy of the transverse shear stiffness calculation. For laminates a more feasible through the thickness shear stress distribution is applied, while for sandwich plates the effect of bending of the faces is taken into account. Both methods yield stiffnesses that can be inserted into the buckling formula for plates with finite transverse shear stiffness under compressive loads.
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NOTATION

a  length of plate
A  deflection parameter of the plate
b  width of plate
C1-C6 coefficients in calculation of $N_x$,
Cxz, Cyz coefficients in calculation of $f_x$ and $f_y$, respectively
D  bending stiffness matrix of laminate, with respect to neutral axis
D* inverse of bending stiffness matrix of laminate
E  Young’s modulus in principal directions
f  through the thickness shear stress distribution for unit total shear force
G  shear modulus
m  number of half buckling waves along the plate
n  number of half buckling waves across the plate
Q(i) extensional stiffness matrix of layer i
r  radius of bending
Sxz, Syz transverse shear stiffness
t  thickness
U  strain energy in plate
w  out of plane deflection
W  work done by applied loads
z  through the thickness coordinate
γ  shear strain
vxy Poisson’s ratio (shortening in Y-direction due to tension in X-direction)
τ  shear stress

suffixes:
b  bending
cr critical (associated with buckling)
i  layer counter
l  layer counter
s  shear
ts  transverse shear
x,y,z principal directions
1,2  1st, respectively 2nd row of matrix
11,12,22,66 specific elements of stiffness matrices
1. INTRODUCTION

Many papers have appeared recently devoted to "higher order shear theory" and related subjects [1,2,3,4,5]. The new models presented are an improvement over the standard models for plates with finite transverse shear stiffness (eg. [6,7]). The essence of the improvement varies: a better shear stress distribution through the thickness [5], or implementation of a heterogeneous (layered) cross-section, such as in sandwich plates, in a new finite element [3].

The proposed model in this report has 2 distinctive parts: for laminates the improved transverse shear stiffness is obtained by a refined shear stress distribution through the thickness (from [5]); in addition, for sandwich plates the effect of bending of the faces is included.

In this report the problem is first assessed with an example that clearly shows the necessity of an improvement.

Next a simple, refined transverse shear model is used for the calculation of a more accurate transverse shear stiffness for a laminated plate. In this model neither the shear stresses through the thickness nor the shear strain through the thickness are assumed to be constant (see fig. 1). Instead the model is based upon equilibrium of the stresses under cylindrical bending.

The following section is devoted to the calculation of the corresponding stiffnesses for sandwich plates, now including the effect of the bending of the faces. These are derived from the behaviour in transverse shear buckling, i.e. without bending deformation of the plate as a whole.

Then some results are given, using this approach for two types of sandwich plates, and finally some conclusions are drawn.

The appendices contain the derivation of the buckling formula for a homogeneous plate with finite transverse shear stiffness (see fig.2a) and the derivation of a buckling formula in transverse shear for a sandwich plate (see fig.2b).

Throughout the plates are assumed to be orthotropic, rectangular and simply-supported.

The author expresses his gratitude to Dr. K. Rohwer of DFVLR for performing some analyses with the BEOS program, and to prof. A. Rothwell for his extensive support and discussions on the subject.
2. SIGNIFICANCE OF THE PROBLEM

The existence of an inaccuracy became clear in the investigation of the compression buckling behaviour of a simply-supported, orthotropic sandwich plate, using the standard formula for plates with finite transverse shear stiffness (see app. A). The transverse shear stiffnesses were calculated with the constant stress approach (shear stress constant through the thickness; see fig.1a).

The plate and material data are as follows:

symmetric plate: (length 1200. mm, width 500. mm)
faces of unidirectional CFRP (Ex=145. GPa, Ey=7. GPa, vxy=.34, Gxy=3.5 GPa, Gxz=3.5 GPa, Gyx=1.5 GPa)
core of Nomex honeycomb (Gxz=53. MPa, Gyx=29. MPa)

A contour plot of the compressive buckling load \( P_x \) for various sandwich plates (varying face and core thickness) showed an unexpected dip in the buckling load, for plates with thick faces, when introducing a few mm of core material (see fig.3).

Fig.4 shows the buckling load for sandwich plates with 2 faces each 12. mm thick and varying core thickness. It shows that the buckling load (for a core thickness of about 10. mm or more) is equal to the value for \( S_{xz} \) (also drawn in the figure). This means that buckling occurs in a transverse shear mode, i.e. without bending of the plate (following formula (A.17)).

For this plate a comparison was made with the finite element program BEOS, at DFVLR. This program gave results differing with the modeling. The results most similar to those in fig.4 are found if in BEOS the face bending stiffnesses are neglected ('BEOS1' in fig.4). The small difference that then remains is due to the fact that for this application the wavelength in BEOS is restricted to 5 waves along the plate, whereas for the transverse shear mode deformation, which yields a lower buckling load, the equivalent of an infinitely high number of waves was used.

With the BEOS program results were also obtained for a model in which the face bending stiffnesses are included. Then the buckling load soared to unreasonably high values ('BEOS2' in fig.4). It was concluded that, for sandwich plates where bending of the faces has a significant effect, the transverse shear stiffness is probably seriously overestimated. Considering only the contribution of the core to the transverse shear stiffness (with a constant stress approach), a lower buckling load was found ('BEOS3' in fig.4).

It is true that the example investigated is not a typical sandwich plate, the faces being very thick. However, the considerable differences in the results show that a suitable model for sandwich plates is essential to the validity of the results.
3. TRANSVERSE SHEAR STIFFNESS FOR LAMINATES

This section describes a transverse shear stiffness definition for laminated plates. The associated model [5] is based on equilibrium of direct and shear stresses through the thickness in cylindrical bending. The result is meant to be used in a Mindlin type plate or shell theory.

Note that for sandwich plates this assumption still does not take into account the face bending stiffnesses (see next section). The improvement here is in the use of a more accurate transverse shear stress distribution.

The assumptions made in the present model are:
- the assumed deformation pattern is cylindrical bending;
- the transverse shear stress distribution is found by equilibrium and satisfies the following boundary conditions: shear stress τxz and τyz zero at the free surface, and equal in adjacent layers at their interfaces.
- the axes of orthotropy of the laminate and of the layers coincide with the plate axes.

The last assumption allows the original formulae of [5] to be simplified. If, on the contrary, the layer orthotropic axes would not coincide with the plate axes, there would be a coupling between τxz and τyz on one hand and Rx and Ry on the other hand. Then the shear stress τxz would depend not only on the shear force in the X-direction Rx, but also on the shear force in the Y-direction Ry. By the assumption made above τxz is a function of only Rx, and τyz of only Ry.

Probably, if the laminate orthotropy axes still coincide with the plate axes while some layer orthotropy axes do not, the error resulting from the assumption above will remain small.

Simplifying the formulae of [5] yields (note that in these formulae the coordinate z is defined relative to the neutral axes of the plate):

\[
S_{xz} = \left[ \sum_{i=1}^{n} \left( \int_{z(i)}^{z(i+1)} \frac{f_{x(i)}^{(i)}}{G_{xz(i)}} \, dz \right) \right]^{-1} \tag{1a}
\]

\[
S_{yz} = \left[ \sum_{i=1}^{n} \left( \int_{z(i)}^{z(i+1)} \frac{f_{y(i)}^{(i)}}{G_{yz(i)}} \, dz \right) \right]^{-1} \tag{1b}
\]

where:

\[
f_{x(i)} = C_{xz(i)} \cdot \frac{1}{2} Q_{(i)}^{(i)} \cdot D_{(i)} \cdot z'
\]

\[
C_{xz(i)} = \sum_{j=1}^{i} \left( Q_{(i)}^{(j)} - Q_{(i-1)}^{(j-1)} \right) \cdot \frac{1}{2} D_{(i)} \cdot z'
\]

\[
Q_{(i)}^{(i)} = \begin{bmatrix} Q_{(i)}^{(i)} & Q_{(i)}^{(i-1)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(first row of Q-matrix of specific layer of laminate)}
\]

\[
D_{(i)} = \begin{bmatrix} D_{(i)} & D_{(i)}^{*} & D_{(i)}^{*} & D_{(i)}^{*} \\ D_{(i)} & D_{(i)}^{*} & D_{(i)}^{*} & D_{(i)}^{*} \end{bmatrix} \quad \text{(first row of inverse of D-matrix of laminate)}
\]
and: \[ f_y^{(i)} = Cy_z^{(i)} \cdot \frac{1}{2} \cdot Q_{\alpha}^{(i)} \cdot D_i^* \cdot x^i \] (2b)

\[ Cy_z^{(i)} = \sum_{j=1}^{i} \left( Q_{\alpha}^{(j)} \cdot Q_{\alpha}^{(j-1)} \cdot \frac{1}{2} \cdot D_i^* \cdot x^j \right) \]

\[ Q_{\alpha}^{(i)} = \begin{bmatrix} Q_{\alpha}^{(i)} & Q_{\alpha}^{(i)} & Q_{\alpha}^{(i)} \end{bmatrix} \] (second row of Q-matrix of specific layer of laminate)

\[ D_i^* = [D_i^*, D_i^*, D_i^*] \] (second row of inverse of D-matrix of laminate)

Note that the \( f_x^{(i)} \) and \( f_y^{(i)} \) represent the shear stress distribution through the thickness, equivalent to a unit total shear force in the cross-section.

As an example the Sxz for a case in fig.3, with \( nc = 10. \text{ mm} \), is calculated (see fig.5):

\[ \bar{\mathbf{S}} = \begin{bmatrix} 465.4 & 7.640 & 0 \cdot 10^6 \text{ [Nmm]} \end{bmatrix} \]

\[ \bar{S}^* = \begin{bmatrix} 2.161 & -.7346 & 0 \cdot 10^{-9} \text{ [N/mm²]} \\ -.7346 & 44.75 & 0 \end{bmatrix} \]

\[ \bar{Q}_{1j}^{(1)} = \begin{bmatrix} 145813. & 2391.3 & 0 \cdot N/mm^3 \end{bmatrix} \]

\[ \bar{Q}_{1j}^{(2)} = \begin{bmatrix} 6.67 & 3.33 & 0 \cdot N/mm^3 \end{bmatrix} \]

\[ Q_{1j}^{(1)} = Q_{1j}^{(1)} \]

\[ Q_{1j}^{(1), D_i^*} = [145813. \text{ } 2391.3 \text{ } 0.] \cdot [2.161 \text{ } -.7346 \text{ } 0.] \cdot 10^{-9} = 313.345 \cdot 10^{-6} \text{ [1/mm]}. \]

\[ Q_{1j}^{(2), D_i^*} = [6.67 \text{ } 3.33 \text{ } 0.] \cdot [2.161 \text{ } -.7346 \text{ } 0.] \cdot 10^{-9} = 11.968 \cdot 10^{-9} \text{ [1/mm]}. \]

\[ Q_{1j}^{(2), D_i^*} = QD_{1j}^{(1)} = 313.345 \cdot 10^{-6} \text{ [1/mm]}. \]

\[ Cx^{(1)} = (313.345 \cdot 10^{-6} \cdot 0.) \cdot \frac{17}{2} = .04528 \text{[1/mm]} \]

\[ Cx^{(2)} = Cx^{(1)} + (11.968 \cdot 10^{-9} \cdot 313.345 \cdot 10^{-6}) \cdot \frac{5}{2} = .04136 \text{[1/mm]} \]

\[ Cx^{(3)} = Cx^{(2)} + (313.345 \cdot 10^{-6} \cdot 11.968 \cdot 10^{-9}) \cdot \frac{5}{2} = .04528 \text{[1/mm]} \]

\[ Cx^{(4)} = Cx^{(3)} + (0. \cdot 313.345 \cdot 10^{-6}) \cdot \frac{17}{2} = 0 \text{[1/mm]} \]

The shear stress distribution in the layers (corresponding to a unit shear force) is therefore (see fig.5):

\[ \tau_x^{(1)} = .04528 \cdot \frac{17}{2} \cdot 313.345 \cdot 10^{-6} \cdot z \] \[ \text{[N/mm²]} \cdot 17 < z < -5 \]

\[ \tau_x^{(2)} = .04136 \cdot 5.984 \cdot 10^{-9} \cdot z \] \[ \text{[N/mm²]} \cdot -5 < z < 5 \]

\[ \tau_x^{(3)} = .04528 \cdot 156.67 \cdot 10^{-6} \cdot z \] \[ \text{[N/mm²]} \cdot 5 < z < 17 \]
Note that indeed at the interfaces between the layers the shear stresses in adjacent layers are equal to each other, and that the shear stress is zero at the free surface. The transverse shear stiffness $S_{xz}$ becomes:

$$S_{xz} = \left[ \sum_{i=1}^{3} \left( \frac{\int_{z(i)}^{(i+1)} \left( \frac{f_{x(i)}^{(i)}}{G_{x(i)}} \right) dz}{2} \right) \right]^{-1} \left( \frac{0.08913}{3500.} + \frac{0.01711}{53.} + \frac{0.08913}{3500.} \right)^{-1} = 3050. \text{[N/mm]}$$

Note that this value is much lower than that found by the traditional approaches:

- **constant strain:**
  $$S_{xz}' = \Sigma G_{x(i)} t(i) = 3500.*12. + 53.*10. + 3500.*12 = 84530 \text{[N/mm]}$$

- **constant stress:**
  $$S_{xz}'' = \frac{\Sigma t(i)}{\Sigma (t(i)/G_{x(i)})} = \frac{(12+10+12)^3}{12} + \frac{10}{53.} + \frac{12}{3500.} = 5912. \text{[N/mm]}$$

That the more accurate result is lower, is logical. In the constant stress approach, for instance, the shear stress $\tau$ was constant through the thickness, so the outer layers of greater stiffness (now in the region where $\tau$ is decreasing to zero) carry more load than in the more accurate calculation. The $S_{xz}$ and the associated $N_{x_{cr}}$ will both decrease (see fig. 3).

The result of formula (1) can now be used in the model of fig.2a (i.e. as used in app.A). Although the plate is assumed homogeneous, the better transverse shear stiffness (being based on a feasible shear stress distribution through the thickness) will yield a more accurate result.

If, however, a sandwich plate is considered with thick faces, their bending stiffnesses can no longer be disregarded. The following section provides a model to take this effect into account.
4. TRANSVERSE SHEAR STIFFNESSES FOR SANDWICH PLATES

The model of appendix B describes the buckling behaviour of the plate in a very restricted buckling mode (a transverse shear buckling mode); there is no bending of the plate as a whole. However, as every separate layer has both transverse shear and bending stiffnesses, this model does include the effect of the bending of the faces (actually, of every layer bending is considered, but only the faces have non-zero bending stiffnesses).

The purpose of this section is to use the results of appendix B (i.e the buckling load in the described transverse shear mode) to derive equivalent transverse shear stiffnesses for use in the general sandwich plate buckling model (see app.A - homogeneous plate with transverse shear flexibility). By this approach the general sandwich plate buckling model is improved, now including effect of the bending of the faces.

This procedure implies that bending of the faces is associated with the transverse shear behaviour of the sandwich plate.

In appendix B a formula was derived for the compressive buckling of sandwich plates, if no overall bending deformation is allowed, but taking into account the bending stiffness of the separate faces. This formula is:

\[
N_{xy} = C_1 + \frac{C_2^2 \cdot C_6 + C_3^2 \cdot C_4 - 2 \cdot C_2 \cdot C_3 \cdot C_5}{C_4 \cdot C_5^2 - C_5^2} \tag{B.17b}
\]

Note that this formula has to be used with caution: if, for example, the plate has several different face layers, these would have to be grouped into one 'superlayer' per face (see fig.6), and its associated stiffness entered in the above formula.

Note also that the transverse shear stiffnesses of a face superlayer are calculated by the procedure of the previous section.

We shall now use this result to modify the general sandwich plate buckling model.

The general sandwich plate buckling model also contains a transverse shear mode for buckling (see section 2 of app.A), but this does not include face bending effects. The resulting buckling formula in app.A is:

\[
N_{x'}^{(M_{x'y})} + N_{y'}^{(M_{x'y})} = S_{xz}^{(M_{x'y})} + S_{yz}^{(M_{x'y})} \tag{A.17b}
\]

By equating this buckling load to the one resulting from app.B, equivalent transverse shear stiffnesses \(S_{xz}'\) and \(S_{yz}'\) can be defined:

\[
N_{x'}^{(M_{x'y})} + N_{y'}^{(M_{x'y})} = S_{xz}^{(M_{x'y})} + S_{yz}^{(M_{x'y})} \tag{3}
\]

Unfortunately we only have one equation to define both \(S_{xz}'\) and \(S_{yz}'\). Therefore the following assumption is made:
\[
Sxz' = Syz' = \frac{Nx'_{ts} \left( \frac{m x^2}{a'} \right) + Ny'_{ts} \left( \frac{n y}{b'} \right)}{\left( \frac{m x^2}{a'} \right) + \left( \frac{n y}{b'} \right)^2}
\] (4)

With these equivalent stiffnesses \(Sxz'\) and \(Syz'\) the general buckling model is improved:
- the effect of face bending is included;
- the transverse shear stiffnesses of face superlayers are defined by the procedure of section 3.

To calculate the buckling load for a sandwich plate (including effects of bending of the face), select the waveform parameters \(m\) and \(n\) and perform the transverse shear buckling analysis (B.17b) to derive the equivalent stiffnesses with (4). Use these values as \(Sxz\) and \(Syz\) in the general buckling model of appendix A, for the current \(m\) and \(n\).
Repeat this procedure for a number of \(m\) and \(n\) to define the lowest critical load and its waveform. This critical load will be the buckling load of the sandwich plate.
5. RESULTS

The plate that was examined in chapter 2, demonstrating the need for the present work, is reanalysed with the method given in the previous section.

The stiffnesses needed for the transverse shear buckling analysis of section 4 are:

<table>
<thead>
<tr>
<th></th>
<th>$D_{11}$</th>
<th>$D_{13}$</th>
<th>$D_{13}^*$</th>
<th>$D_{13}^*$</th>
<th>$S_{xz}^*$</th>
<th>$S_{yz}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[kNm]</td>
<td>[kNm]</td>
<td>[kNm]</td>
<td>[kNm]</td>
<td>[N/m]</td>
<td>[N/m]</td>
</tr>
<tr>
<td>face 1</td>
<td>20.997</td>
<td>1.014</td>
<td>0.504</td>
<td>0.345</td>
<td>35.0</td>
<td>15.0</td>
</tr>
<tr>
<td>core</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>1.59</td>
<td>0.87</td>
</tr>
<tr>
<td>face 2</td>
<td>20.997</td>
<td>1.014</td>
<td>0.504</td>
<td>0.345</td>
<td>35.0</td>
<td>15.0</td>
</tr>
</tbody>
</table>

Note that $S_{xz}$ of the faces is set to:

\[ S_{xz}^{\text{face}} = \frac{5}{6} \cdot G_{xz}^{\text{face}} \cdot t_{\text{face}} \]

in agreement with section 3. For the core the following formula is used:

\[ S_{xz}^{\text{core}} = G_{xz}^{\text{core}} \cdot t_{\text{core}} \]

Both stiffnesses are as suggested in app.B.

Taking $N_y=0$, the critical load in transverse shear $N_{x_s}$ and the associated equivalent transverse shear stiffness $S_{xz}^*$ can now be found. To find the actual buckling load several half waves $m$ along the plate are examined ($n$ is always 1):

<table>
<thead>
<tr>
<th>$m$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{x_s}$ [N/mm]</td>
<td>13812.</td>
<td>6968.</td>
<td>6875.</td>
<td>8150.</td>
</tr>
<tr>
<td>$S_{xz}^*$ [N/mm]</td>
<td>2043.</td>
<td>2856.</td>
<td>4192.</td>
<td>5993.</td>
</tr>
</tbody>
</table>

Using the general plate buckling model with these stiffnesses one can get the following critical loads as a function of the waveform:
This result, together with those for other core thicknesses, is given in fig.7. The results of section 2 are also given in this figure, for comparison.

Another sandwich plate, of the same materials, is now examined. The faces are now each 2.6mm thick (this plate withstands 3000. N/mm compressive load with a compressive strain less than .4%). If the core is again 30.mm thick, the stiffnesses for the transverse shear buckling analysis are:

<table>
<thead>
<tr>
<th></th>
<th>$D_{11}$</th>
<th>$D_{22}$</th>
<th>$D_{12}$</th>
<th>$D_{16}$</th>
<th>$S_{xz}$</th>
<th>$S_{yz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[Nm]</td>
<td>[Nm]</td>
<td>[Nm]</td>
<td>[Nm]</td>
<td>[N/m]</td>
<td>[N/m]</td>
</tr>
<tr>
<td>face 1</td>
<td>213.6</td>
<td>10.31</td>
<td>5.126</td>
<td>3.505</td>
<td>7.58</td>
<td>3.25</td>
</tr>
<tr>
<td>core</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>1.59</td>
<td>0.87</td>
</tr>
<tr>
<td>face 2</td>
<td>213.6</td>
<td>10.31</td>
<td>5.126</td>
<td>3.505</td>
<td>7.58</td>
<td>3.25</td>
</tr>
</tbody>
</table>

The transverse shear buckling analysis then yields the following critical loads and equivalent stiffnesses:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Nx_{ts}$ [N/mm]</td>
<td>7797.</td>
<td>3369.</td>
<td>2562.</td>
<td>2294.</td>
</tr>
<tr>
<td>$Sxz'$   [N/mm]</td>
<td>1153.</td>
<td>1381.</td>
<td>1562.</td>
<td>1687.</td>
</tr>
</tbody>
</table>

Using the general plate buckling model with these stiffnesses the following critical loads can be obtained as a function of the waveform:
<table>
<thead>
<tr>
<th>m</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{x_{cr}}$ [N/mm]</td>
<td>2613.</td>
<td>1737.</td>
<td>1740.</td>
<td>1809.</td>
</tr>
</tbody>
</table>

Fig.8 shows the result of this analysis and those for plates with other core thicknesses, as compared to the general buckling analysis (as in appendix A) without considering the face bending stiffnesses.

In figs. 7 and 8 the results are given for both the methods of section 2 and of section 4. The results of these models differ because the transverse shear behaviour of the plates is modeled in a different way: first, the effect of bending of the faces is included in section 4. Next, the parts of the cross-section that carry normal load do not have a constant shear stress through the thickness anymore. The first difference will increase the stiffness of the plate in transverse shear deformation, the second difference will decrease it. Note that in both cases the decreasing effect is dominating.
6. CONCLUSIONS

Both for laminated plates and for sandwich plates improved transverse shear stiffnesses have been derived. The new transverse shear stiffnesses (for laminates as well as for sandwich plates) can be used in the buckling formula derived in app.A. Note, however, that for laminates the improved transverse shear stiffness is independent of the out of plane deflection pattern of the plate. This is not the case for sandwich plates; there, the defined equivalent transverse shear stiffnesses depend on the values of \( m \) and \( n \).

For laminated plates the improvement is the use of a feasible transverse shear stress distribution through the thickness. The assumption at the base of the improvement is cylindrical bending of the laminate. Note that the formulae are derived for orthotropic laminates where the orthotropic axes of the constitutive layers coincide with those of the plate.

For sandwich plates the improvement is to take into account the effect of the bending stiffness of the faces. In this way, for the examined plate, a better agreement was found with a finite element analysis that also included face bending effects.
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b. constant shear strain through thickness

c. shear stress by equilibrium in cylindrical bending

Fig. 1 Comparison of different transverse shear deformation models
a. homogeneous transverse shear deformation of the plate, without bending of the faces

b. transverse shear buckling with bending of the faces

Fig. 2 Comparison of shear deformation of homogeneous vs. heterogeneous plates
contours for buckling load \( N_{xcr} \) [N/mm]

Fig. 3 Compressive buckling load with the general sandwich plate buckling formula for different face and core thicknesses

\( nf = \text{number of face layers (in total, for both faces); } tf = 0.2*nf \text{ [mm]} \)

\( nc = \text{number of core layers; } tc = 1.0*nc \text{ [mm]} \)
Fig. 4 Compressive buckling load $N_{xcr}$ with the general sandwich plate buckling formula and transverse shear stiffness $S_{xz}$ for a face thickness of 2*12 mm.
Fig. 5 Transverse shear stress distribution in a sandwich plate under cylindrical bending
face thickness = 2*12. mm
core thickness = 10. mm

Fig. 6 Unsuitable transverse shear deformation if face layers are taken into account separately
Fig. 7 Compressive buckling load $N_{xcr}$ with the refined sandwich plate buckling formula for a face thickness of $2\times12$ mm.
Fig. 8 Compressive buckling load $N_{xcr}$ with the refined sandwich plate buckling formula for a face thickness of 2*2.6 mm
Appendix A. BUCKLING OF RECTANGULAR ORTHOTROPIC PLATES

A.1 DERIVATION OF THE FORMULA

This appendix derives the formula for the buckling load under bi-axial compression of simply-supported, rectangular orthotropic plates, with finite transverse shear stiffness.

A Mindlin type transverse shear deformation is imposed, i.e. a cross-section (in the unloaded state flat and normal to the neutral axis) remains flat but is allowed to rotate with respect to the normal to the neutral axis. A given transverse shear stiffness represents the restraint of the plate against this rotation.

These stiffnesses are associated with the plate as a whole. Although they may result from the contributions of several layers, from this point on the plate is considered to be homogeneous.

The formula is derived by the energy method. In this method the energy $U$ required to form the deflected state is balanced by the work $W$ done by the applied loads in this deflection process.

The strain energy $U$ of a part with length $dx$, width $dy$ and full height of the plate, equals the sum of two parts: bending and shear. The bending energy equals (analogous to [6,7]):

$$dU_b = \frac{1}{2} \left[ D_1 \cdot wb^2_{x,x} + D_1 \cdot wb^2_{y,y} + 2D_1 \cdot wb_{x,xx} wb_{y,yy} + D_2 \cdot ((wb_x + wb_y)^2_{xy}) \right] \cdot dx \cdot dy$$  \hspace{1cm} (A.1)

The transverse shear energy equals (analogous to [6,7]):

$$dU_s = \frac{1}{2} \left[ (S_{xz} wb_{x,x} + S_{yz} wb_{y,y}) \right] \cdot dx \cdot dy$$  \hspace{1cm} (A.2)

Note that $wb_x$, $ws_x$, etc. are the partial deflections in bending and shear, as defined lower.

Note also that $wb_{x,x} = wb_{y,y}$, $ws_{x,x} = ws_{y,y}$.

Integrated over the entire plate the strain energy becomes:

$$U = \int_a^b \int_c^d \left[ dU_b + dU_s \right] \cdot dx \cdot dy$$  \hspace{1cm} (A.3)

The work of the applied load $W$ equals (analogous to [6,7]):

$$W = \frac{1}{2} \int_a^b \int_c^d \left[ Nx w'_x + Ny w'_y \right] \cdot dx \cdot dy$$  \hspace{1cm} (A.4)

To solve the problem an assumption must be made about the deflection function. For simply-supported plates a double sine wave satisfies the boundary conditions:

$$w = A \cdot \sin \frac{m \pi x}{a} \cdot \sin \frac{n \pi y}{b}$$  \hspace{1cm} (A.5)
This deflection is composed of a bending and shear part:

\[ w = wb_x + ws_x = wb_y + ws_y \]  

(A.6)

These partial deflections are also double sine waves with the same \( m \) and \( n \), so the following holds for the deflection parameters:

\[ A = Ab_x + As_x = Ab_y + As_y \]  

(A.7)

where the coefficient \( A \) is now separated into its appropriate parts. Substitution of this deflection in \( U \) and \( W \) yields:

\[ U = \frac{ab}{8} \left( D_1 \left( \frac{m a}{a} \right) * Ab_x + D_2 \left( \frac{n b}{b} \right) * Ab_y + 2*D_3 \left( \frac{m a}{a} \right) \left( \frac{n b}{b} \right) * Ab_x * Ab_y + D_4 \left( \frac{m a}{a} \right) \left( \frac{n b}{b} \right) * (Ab_x + Ab_y)^2 \right. \]

\[ + \left. Sxz \left( \frac{m a}{a} \right) * As_x^2 + Syz \left( \frac{n b}{b} \right) * As_y^2 \right) \]  

(A.8)

\[ W = \frac{ab}{8} \left( N_x \left( \frac{m a}{a} \right) A_x^2 + N_y \left( \frac{n b}{b} \right) A_y^2 \right) \]  

(A.9)

The equations (A.8) and (A.9) contain 5 deflection parameters \( (A,Ab_x,Ab_y,As_x \) and \( As_y) \). Because these are dependent (A.7), two can be eliminated. Which ones are eliminated is free. Here \( Ab_x \) and \( Ab_y \) are eliminated. The formula for \( U \) then yields:

\[ U = \frac{ab}{8} \left( D_1 \left( \frac{m a}{a} \right) * (A-As_x)^2 + D_2 \left( \frac{n b}{b} \right) * (A-As_y)^2 + 2*D_3 \left( \frac{m a}{a} \right) \left( \frac{n b}{b} \right) * (A-As_x) * (A-As_y) \right. \]

\[ + \left. D_4 \left( \frac{m a}{a} \right) \left( \frac{n b}{b} \right) * ((A-As_x) + (A-As_y))^2 \right) + Sxz \left( \frac{m a}{a} \right) * As_x^2 + Syz \left( \frac{n b}{b} \right) * As_y^2 \right) \]  

(A.10)

A condensed notation is introduced:

\[ D16 = D_1 \left( \frac{m a}{a} \right) + D_3 \left( \frac{m a}{a} \right) \left( \frac{n b}{b} \right) \]  

(A.11)

\[ D26 = D_2 \left( \frac{n b}{b} \right) + D_4 \left( \frac{n b}{b} \right) \]  

\[ D126 = (D_1 + D_3) \left( \frac{m a}{a} \right) \left( \frac{n b}{b} \right) \]

\[ Sx = Sxz \left( \frac{m a}{a} \right) \]

\[ Sy = Syz \left( \frac{n b}{b} \right) \]

\[ Nxy = N_x \left( \frac{m a}{a} \right) + N_y \left( \frac{n b}{b} \right) \]

The solution to the problem, i.e., values for the deflection parameters \( A, As_x \) and \( As_y \), is found by the theorem of stationary total energy. This states that at the resulting deflection the total energy of the system \( (U - W) \) is stationary. In other words:
\[
\frac{\delta(U - W)}{\delta X} = 0, \quad \text{for } X = A, A_x, \text{ and } A_y \tag{A.12}
\]

This yields:
\[
\begin{bmatrix}
(D_{16} + D_{26} + 2 \cdot D_{126} - N_{xy}) & -(D_{16} + D_{126}) & -(D_{26} + D_{126}) \\
-(D_{16} + D_{126}) & D_{16} + S_x & D_{126} \\
-(D_{26} + D_{126}) & D_{126} & D_{26} + S_y
\end{bmatrix} \cdot \begin{bmatrix}
A \\
A_x \\
A_y
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \tag{A.13}
\]

This set has a non-zero solution only if the determinant is zero. This condition yields:

\[
N_{xy} = \frac{(S_x + S_y) \cdot (D_{16} - D_{126}^2) + S_x S_y \cdot (D_{16} + D_{26} + 2 \cdot D_{126})}{(D_{16} + S_x) \cdot (D_{26} + S_y) - D_{126}}
\]

Or, divided by \((S_x, S_y)\):

\[
N_{xy} = \frac{\left(\frac{1}{S_x} + \frac{1}{S_y}\right) \cdot (D_{16} - D_{126}) + (D_{16} + D_{26} + 2 \cdot D_{126})}{(1 + \frac{D_{16}}{S_x}) \cdot (1 + \frac{D_{26}}{S_y}) - D_{126}} \tag{A.14}
\]

This formula yields the critical load for a simply-supported, rectangular orthotropic plate under compressive loads. This means that at that specific load a non-zero solution for the deflection parameters will occur. It applies to a specific waveform, defined by the used \(m\) and \(n\). To find the actual buckling load of the plate, several waveforms have to be examined, to find the lowest load for any given ratio of loads \(N_y/N_x\).

This formula gives exactly the same results as the ESDU program described in [8].
A.2. SPECIAL FORMS OF THE DERIVED FORMULA

This paragraph examines how (A.14), the final formula of the previous paragraph, reduces in some special cases. These cases are an ordinary plate (very high transverse shear stiffness) and the case where the bending stiffnesses are very high compared to the Sxz and Syz (e.g. sandwich plate with a thick, weak core).

A.2.1. Orthotropic plate
The buckling formula for orthotropic plates is a special case of formula (A.14), namely for very high Sxz and Syz. Neglecting in (A.14) all terms with Sxz or Syz in the denominator yields:

\[ N_{xy} = D_{16} + D_{26} + 2D_{126} \]  \hspace{1cm} (A.15a)

Or, in the normal notation:

\[ N_x \left( \frac{M_x}{a} \right) + N_y \left( \frac{M_y}{b} \right) = D_{11} \left( \frac{M_x}{a} \right)^2 + D_{22} \left( \frac{M_y}{b} \right)^2 + 2D_{12} \left( \frac{M_x}{a} \right) \left( \frac{M_y}{b} \right) \]  \hspace{1cm} (A.15b)

This is indeed the buckling formula for simply-supported orthotropic plates under bi-axial compressive loads.

A.2.2. Transverse shear buckling
Another special case is that where the bending stiffnesses of the plate are very high compared to the transverse shear stiffnesses. This happens in a sandwich plate with a thick and relatively weak core.

For this case the original equations (A.8) and (A.9) are modified by setting:

\[ w_{b_x} = w_{b_y} = 0. \]  \hspace{1cm} (A.16)

Note that it was not necessary in the previous section to state \( w_s = 0 \), because there the numbers that tend to infinity (Sx and Sy) were all in the denominator. Trying here to neglect all terms with \( D_{ij} \) in the denominator leaves some fractions undefined. Therefore (A.16) is used.

(A.12) then yields:
\[ S_x + S_y - N_{xy} = 0. \]  \hspace{1cm} (A.17a)

or, in the normal notation:

\[ N_x\left(\frac{\partial \theta}{\partial a}\right) + N_y\left(\frac{\partial \theta}{\partial b}\right) = S_{xz}\left(\frac{\partial \xi}{\partial a}\right) + S_{yz}\left(\frac{\partial \xi}{\partial b}\right) \]  \hspace{1cm} (A.17b)

As was already pointed out in A.1, the deflection parameters \( m \) and \( n \) have to be varied to calculate the actual buckling load. For example, if we are interested in the buckling load for \( N_y = 0 \), from (A.17b) it is clear that the lowest critical load (i.e. the buckling load) will be found for:

\[ m \to \infty, \ n \text{ undefined} \]

namely:

\[ N_x = S_{xz} \]

This is in agreement with other theories.

*Formula (A.14) (and the associated formulae (A.15b) and (A.17b)) yield the compressive buckling load of a simply-supported, rectangular orthotropic plate with finite transverse shear stiffness. Sandwich plates belong to this category, but note that for these plates, if transverse shear deformation occurs, the faces tend to bend. This effect is not taken into account here, as the plate is assumed to be homogeneous through the thickness. However, it is possible to feed into the formulae equivalent stiffnesses that incorporate in some way or other these effects.*
Appendix B. TRANSVERSE SHEAR BUCKLING OF A LAYERED PLATE

This appendix is dedicated to the derivation of a formula for the compressive buckling load of a simply-supported, rectangular orthotropic plate, with finite transverse shear stiffness, if no bending occurs of the plate as a whole. This could be called transverse shear buckling because the overall deformation is 'through the thickness' shear.

The plate is considered to be a laminate (i.e. a stacking of layers with different properties). No bending of the plate as a whole signifies in this model that no layer is stretched, only bending and transverse shear deformation of the constitutive layers are taken into account, see fig.B1.

The main difference with the model of appendix A is that here every layer of the plate is considered as a separate plate, with bending and transverse shear stiffness. These plates are connected at their interfaces.

The assumed out of plane deflection of the plate as a whole is a double sine wave:

\[ w = A \cdot \sin \frac{\pi x}{a} \cdot \sin \frac{\pi y}{b} \]  

(B.1)

Because the plate does not bend as a whole, this deflection is also the deflection of the midplane of every layer \(i\):

\[ w = w_i = (A_x^i + A_y^i) \cdot \sin \frac{\pi x}{a} \cdot \sin \frac{\pi y}{b} \]  

(B.2)

So that:

\[ A = A_x^i + A_y^i \]  

(B.3)

Note that for each layer a Mindlin type deformation is imposed (fig.B1): a cross section (initially flat and normal to the neutral axis) remains flat but no longer needs to remain normal to the neutral axis. Then the compatibility of displacements at the layer interfaces can be written as:

\[ \frac{\delta w}{\delta x} \frac{(t^i + t^{i+1})}{2} = \frac{\delta w^i_s}{\delta x} \frac{t^i}{2} + \frac{\delta w^{i+1}_s}{\delta x} \frac{t^{i+1}}{2} \]

which simplifies to:

\[ A (t^i + t^{i+1}) = A_x^i \cdot t^i + A_x^{i+1} \cdot t^{i+1} \]  

(B.4a)

This is also valid in the Y-direction:

\[ A (t^i + t^{i+1}) = A_y^i \cdot t^i + A_y^{i+1} \cdot t^{i+1} \]  

(B.4b)

For each layer \(i\) the strain energy is given by (see app. A):
\[ V^t = \frac{ab}{8} (D_{11}^t (\frac{m_n}{a} \cdot Ab_{x}^{t+1} + D_{12}^t (\frac{m_m}{b} \cdot Ab_{y}^{t+1} + 2*D_{16}^t (\frac{m_n}{a} \cdot Ab_x^{t+1} \cdot Ab_y^{t+1} + D_{15}^t (\frac{m_m}{b} \cdot (Ab_x^{t+1} + Ab_y^{t+1}) + S_{xz} (\frac{m_m}{a} \cdot As_{x}^{t+1} + S_{yz} (\frac{m_m}{b} \cdot As_{y}^{t+1}) (B.5) \]

The work of the applied load (only direct loads are taken into account) integrated over the whole plate can be written as (see app. A):

\[ W = \frac{ab}{8} (N_{x} (\frac{m_m}{a} \cdot A_x^t + N_{y} (\frac{m_m}{b} \cdot A_y^t) (B.6) \]

These are the basic equations involved. Before assembling the formula for the total energy, with the compatibility equations the number of unknowns is reduced. The aim is to retain only \( A, As_{x}^t \) and \( As_{y}^t \):

1. for \( As_{x}^t \) and \( As_{y}^t \), with the interface compatibility, out of (4a):

\[ As_{x}^{t+1} = \frac{A.(t+1)^{t+1+1}}{t^{t+1}} \cdot As_{x}^{t+1} \]

\[ As_{x}^{t+2} = \frac{A.(t+1)^{t+1+2}}{t^{t+2}} \cdot As_{x}^{t+1+1} = \frac{1}{t^{t+2}} \cdot [ A.(t^{t+1+2}) - A.(t^{t+1+1}) - As_{x}^{t+1} \] \[ = \frac{1}{t^{t+2}} \cdot [ A.(t^{t+2+1}) + As_{x}^{t+1} \]

Recursively towards \( As_{x}^t \) this yields:

\[ As_{x}^t = \frac{t^t}{t^t} \cdot [ A.(t^{t} + (t^t)) - (t^t) \cdot As_{x}^t \] \[ \text{and similarly:} \]

\[ As_{y}^t = \frac{t^t}{t^t} \cdot [ A.(t^{t} + (t^t)) - (t^t) \cdot As_{y}^t \] \[ (B.9a) \]

\[ (B.9b) \]

2. for \( Ab_{x}^t \) and \( Ab_{y}^t \): out of the displacement equality of all layers (3):

\[ Ab_{x}^t = A - As_{x}^t \quad \text{and} \quad Ab_{y}^t = A - As_{y}^t \] \[ (B.10) \]

Substitution of (9) yields:

\[ Ab_{x}^t = (-1)^{t} \cdot \frac{t^t}{t^t} \cdot [ As_{x}^t - A ] \] \[ (B.11a) \]

\[ Ab_{y}^t = (-1)^{t} \cdot \frac{t^t}{t^t} \cdot [ As_{y}^t - A ] \] \[ (B.11b) \]

Substituting (9) and (11) into (5) and (6) yields:
\[ V^t = \frac{a b}{8} \gamma^t \left[ \begin{array}{c}
\frac{D_1}{1} \left( \frac{\mu a}{a} \right) \ast (a_s^t - A)^2 + \frac{D_1}{4} \left( \frac{\mu a}{a} \right) \ast (a_s^t - A)^2 + 2 \ast \frac{D_1}{4} \left( \frac{\mu a}{a} \right) \ast (a_s^t - A)^2 \ast (a_s^t - A) + \\
\overline{D_2} \left( \frac{\mu a}{a} \right) \ast (a_s^t - A)^2 \ast (a_s^t - A)^2 + \\
S_{\overline{z}} \left( \frac{\mu a}{a} \right) \ast (A \frac{t}{t} + (-1)^4) \ast (-1)^4 \ast (a_s^t - A)^2 + \\
S_{\overline{y}} \left( \frac{\mu b}{b} \right) \ast (A \frac{t}{t} + (-1)^4) \ast (-1)^4 \ast (a_s^t - A)^2 \end{array} \right] \]  

\[ W = \frac{a b}{8} \cdot A \left( \frac{\mu a}{a} \ast N_x \frac{\mu b}{b} + N_y \frac{\mu b}{b} \right) \]  

Equations (12) and (13) now have only \( A, a_s^t \) and \( a_s^t \) as variables. The condition for buckling is that the total energy is stationary with respect to the remaining variables. In a formula:

\[ \delta \left( \frac{W}{\delta X} \cdot \sum V^t \right) = 0. \quad \text{for } X = A, a_s^t \text{ and } a_s^t \]  

A condensed notation is used to clarify the equations:

\[ D_{16}^t = D_1^t \left( \frac{\mu a}{a} \right) + D_4^t \left( \frac{\mu a}{a} \right) (\overline{\mu b}) \]  

\[ D_{26}^t = D_2^t \left( \frac{\mu b}{b} \right) + D_4^t \left( \frac{\mu a}{a} \right) (\overline{\mu b}) \]  

\[ D_{126}^t = D_1^t \left( \frac{\mu a}{a} \right) (\overline{\mu b}) + D_4^t \left( \frac{\mu a}{a} \right) (\overline{\mu b}) \]  

\[ S_x^t = S_{\overline{z}} \left( \frac{\mu a}{a} \right) \]  

\[ S_y^t = S_{\overline{y}} \left( \frac{\mu b}{b} \right) \]  

\[ N_{xy} = N_x \left( \frac{\mu a}{a} \right) + N_y \left( \frac{\mu b}{b} \right) \]  

This yields for the set of equations (14):

\[ \begin{bmatrix} N_{xy} - C1 & C2 & C3 \\ C2 & C4 & C5 \\ C3 & C5 & C6 \end{bmatrix} \cdot \begin{bmatrix} A \\ a_s^t_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]  

where:

\[ C1 = \frac{a}{h} \left( \frac{t}{t} \right)^4 \cdot \left( D_{16}^t \ast D_{16}^t + 2 \ast D_{126}^t + (1 + (-1)^4) \ast (S_x^t + S_y^t) \right) \]  

\[ C2 = \frac{a}{h} \left( \frac{t}{t} \right)^4 \cdot \left( D_{16}^t + D_{126}^t + (1 + (-1)^4) \ast S_x^t \right) \]  

\[ C3 = \frac{a}{h} \left( \frac{t}{t} \right)^4 \cdot \left( D_{26}^t + D_{126}^t + (1 + (-1)^4) \ast S_y^t \right) \]  

\[ C4 = -\frac{a}{h} \left( \frac{t}{t} \right)^4 \cdot \left( D_{16}^t + S_x^t \right) \]
\[ C5 = - \frac{1}{E_1} \left[ \frac{D_{126}}{t} \right] \]
\[ C6 = - \frac{1}{E_1} \left[ \frac{D_{26}}{t} + S_y \right] \]

There exists a non-zero solution to this equation if and only if the determinant of the matrix is equal zero:

\[
\begin{vmatrix}
N_{xy} - C1 & C2 & C3 \\
C2 & C4 & C5 \\
C3 & C5 & C6 \\
\end{vmatrix} = (N_{xy} - C1) \cdot C4 \cdot C6 + 2 \cdot C2 \cdot C3 \cdot C5 - (N_{xy} - C1) \cdot C5^2 - C2 \cdot C6 - C3 \cdot C4 = 0. \tag{B.17a}
\]

\[
N_{xy} = C1 + \frac{C2^2 \cdot C6 + C3^2 \cdot C4 - 2 \cdot C2 \cdot C3 \cdot C5}{C4 \cdot C6 - C5^2}. \tag{B.17b}
\]

This formula yields for a specific waveform (defined by \( m \) and \( n \)) the equivalent critical load \( N_{xy} \). To find the actual buckling load, equation (17) has to be solved for various \( m \) and \( n \) to find the lowest critical load.

Using only one layer in (17b) yields the formula of appendix A again.

However, the application of the method described here requires some care. Note that in fig.B2 the mode of deformation implies alternate layers with severe shear deformation. This can only occur if these layers have relatively low transverse shear stiffness (as compared to the layers that do not exhibit this severe shear deformation). This situation occurs in a sandwich plate with two stiff faces and a relatively weak core (the situation drawn in fig.B1). It also occurs in a more complicated sandwich plate with alternating flexible and stiff layers. It will, on the other hand, not occur in a sandwich plate with two faces each composed of, say, three individual, relatively stiff layers (as drawn in fig.B2a). In this case it is necessary to group the three layers that constitute one face into one superlayer to produce a satisfactory model of the deformation (see fig.B2b).

The stiffnesses of the 'superlayers' are input data for the method and must be considered carefully for accurate results.

For the bending stiffnesses of each face superlayer it is best to use the 'Dij' for the superlayer (with respect to the own neutral axis) that come out of a laminate stiffness calculation. For the transverse shear stiffnesses of these 'superlayers' it is possible to use the formula of section 3 of this report.

For the core superlayer it is best to assume zero bending stiffness and to use the constant stress approach for the transverse shear stiffnesses (the core is assumed not to carry bending, so the shear stress will be constant through the thickness, hence the constant stress approach).

*Formula (B.17b) yields the compressive buckling load for sandwich plates in a transverse shear mode. As such it is an improvement over (A.17), another formula for buckling in transverse shear, because here the effect of bending of the faces is considered (assuming that appropriate attention is paid to the possible definition of superlayers).*
Fig. B1. Assumed transverse shear deformation and compatibility of layer deformations

\[ \frac{\partial \omega}{\partial x} \frac{t}{4} = \frac{\partial \omega}{\partial x} \frac{t}{6} + \frac{\partial \omega}{\partial x} \frac{t}{2} \]

Fig. B2. Concept of superlayers