Stellingen


2. Getuige de uitspraak "[…] the critical layer […] is merely an unimportant part of an equilibrium layer if turbulent stresses are included […]" (A.A. Townsend, *J. Fluid Mech.* 55, 1972) is al bijna een kwart eeuw bekend waarom de theorie van Miles uit 1957 geen juiste beschrijving geeft van de groei van golven door de wind.

3. Gezien het onvoldoende van klimaatmodellen om klimaatveranderingen uit het verleden te beschrijven zouden uitkomsten van deze modellen geen rol mogen spelen bij de discussie over de terugdringing van de uitstoot van CO₂.

4. Het is nooit aangetoond dat derde-generatie golfmodellen betere golfverwachtingen leveren dan die van de tweede generatie.

5. Het verschil in kwaliteit tussen het publiciteitsmateriaal van ruimtevaartorganisaties bestemd voor een algemeen publiek en dat bestemd voor haar afnemers kan alleen begrepen worden als men weet van wie deze organisaties voor hun financiering afhankelijk zijn.


7. Het uitvoeren van huidige bouwplannen zal leiden tot een daling van de prijs van de goedkopere eengezinswoningen en een verdere stijging van die van de duurdere.

8. Het idee dat arbeidstijdverkorting tot meer banen leidt is gebaseerd op de onjuiste veronderstelling dat de hoeveelheid te verrichten werk gegeven is.

9. De internationale uitwisseling die gestimuleerd wordt door de onderzoeksprogramma's van de Europese Unie heeft vaak slechts een culinair karakter.
WIND-WAVE INTERACTION

PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus Prof. ir. K.F. Wakker,
in het openbaar te verdedigen ten overstaan van een commissie,
door het College van Dekanen aangewezen,
op donderdag 12 december 1996 te 10.30 uur

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Cornelis MASTENBROEK

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geboren te Maasdam
Dit proefschrift is goedgekeurd door de promotor:
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Chapter 1

Introduction

Wind blowing over the ocean generates waves. The rate at which energy and momentum is transferred from the wind to the waves is characterized by the so-called growth rate of the waves. The growth rate plays an important role in the study of the interaction between the atmosphere and the ocean. First of all the growth rate determines the timescale at which wind-sea is generated. Therefore the growth rate, along with the dissipation rate of the waves, is an important parameter in the numerical models that are used to describe and forecast the sea state. Second, as the waves extract momentum from the atmospheric boundary layer, they make the surface appear considerably rougher as it would have been without waves. In this way the waves have a major impact on the dynamics of the atmospheric boundary layer over the oceans. By determining the structure of the boundary layer, the growth rate of the waves also influences the exchange of heat, moisture and gases between the water and the air. The understanding of these exchange processes is important for climate studies, as they couple the oceans to the atmosphere. Because of the importance of the growth rate, numerous studies have been dedicated to determine the dependence of this parameter on the length of the waves, the windspeed and other environmental parameters.

Attempts to describe even the main features characterizing the air-sea interface are hampered by the complexity of this system. The lengths of the wind-generated waves present on the ocean can span more than five orders of magnitude: from more than a kilometre to less than a centimetre. It is not reasonable to assume that the interaction with the atmosphere of all these waves is governed by the same physical mechanisms. The shortest waves have a length comparable to the thickness of the viscous sublayer, whereas the length of the longest waves is comparable to the thickness of the atmospheric boundary layer. Yet all these waves interact with each other, hydrodynamically as well as via the atmospheric boundary layer. Owing to the turbulent nature of, in particular, the atmospheric boundary layer, even the description of the transformation of the boundary layer as the air flows over a train of monochromatic waves is difficult.

With his sheltering hypothesis, Jeffreys (1925) proposed that separation of the air flow from the water surface played an important role in the wave growth mechanism. However, this picture failed to explain the observed growth rates. The first to give a quantitative description of wave growth by wind was Miles (1957). His main assumption is that the turbulence in the air is not affected by the presence of a wave on the surface. This leads to a singularity in the governing equations at the height where the windspeed matches the phase speed of the wave. A resonance between the air flow traveling at the same speed as the wave was believed
to be responsible for the energy flux from wind to wave. The theory of Miles has become very popular because the growth rates it predicts closely match the observations, and it is still used in numerical wave prediction models (Komen et al., 1994). When the effect of turbulence was taken into account in later studies, e.g. Townsend (1972), Gent and Taylor (1976), Chalikov (1978) and many others, no resonance mechanism was found to occur. All calculations showed that the wave induced turbulence is responsible for the phase shift in the pressure that is required for an energy flux to waves. The growth rate predicted by the different theories and numerical models therefore depend strongly on the type of turbulence closure scheme that is used.

Recently Belcher and Hunt (1993) identified two layers in the air flow over waves. The turbulence in the layer closest to the surface, in the so-called inner region, is thought to be in equilibrium with the local velocity gradients. In the inner region the level of turbulence can be expressed in the local velocity gradients, as is done in schemes based on the eddy viscosity concept. Above this layer, in the outer region, the advection of turbulence can no longer be ignored. The effects of straining on the turbulent eddies in this layer is described by rapid distortion theory (Batchelor and Proudman, 1954). The study of Belcher and Hunt is confined to waves moving slowly compared to the wind. Though these waves are responsible for the bulk of the momentum transfer between atmosphere and ocean (Donelan, 1990), the interaction of the atmosphere with faster waves is of greater practical importance. The waves of interest in numerical wave prediction models are those with phase speeds comparable with the windspeed. For these waves the analysis of Belcher and Hunt is not valid.

The aim of this study is to describe the interaction of the atmosphere and waves with phase speeds comparable to or faster than the wind.

To study the effects of waves on the turbulence in the air flow close to the water surface a numerical model is used. This model, described in chapter 2, is an extension of the model described by Makin (1979) and Burgers and Makin (1993). It is equipped with a hierarchy of turbulence parameterizations, ranging from the relatively simple mixing length scheme to the more general, and hence more complex, second-order Reynolds-stress model. Calculations show that the energy flux from the air flow to the waves depends strongly on the type of turbulence closure scheme that is used. Owing to their simplicity, the turbulence schemes based on the eddy viscosity concept, such as the mixing length model, have been widely used in the past in this problem. Results following from the different closure hypotheses are compared with detailed observations of the turbulence in chapter 3. The comparison clearly demonstrates that the local equilibrium assumption underpinning the eddy viscosity models do not hold in the air flow over waves. In the bulk of the flow the advection of turbulence disturbs the local balance of production and dissipation that is essential for the eddy viscosity models to be valid.

To apply the results from studies of the air flow over monochromatic waves to the interaction of ocean waves with the wind, several assumptions have to be made. The relative importance of some of these is discussed in chapter 4. One of the strongest assumptions is that the
aerodynamical roughness is distributed uniformly over the surface of the waves. For waves longer than several meters this assumption certainly does not hold, as these waves are known to modulate the amplitudes of shorter waves. In earlier studies it has been shown that the air flow over the longer wave, and hence the energy flux to this wave, can be significantly altered by the presence of a non-uniform surface roughness. However, the interaction between the long modulating wave and the shorter modulated waves is two-way. Not only does the variable presence of short waves alter the air flow over the long wave, the long wave also distorts the air flow felt by the shorter waves on its surface. The air flow over the long wave and the presence of short waves on the surface of the long wave are two elements of a coupled system, that have to be described simultaneously. In chapter 5 an attempt is made to formulate such a description.

An interesting by-product of the coupled model is that it shows that the gravity-capillary waves, with lengths of a few centimetres, are effectively modulated via the air flow. This has important implications for the understanding of the imaging mechanism of radars that are used to detect waves from remote platforms. The electromagnetic microwaves emitted by radars such as the SAR mounted on the ERS satellites scatter from waves in the gravity-capillary range. The reason that these radars can detect waves several orders of magnitude longer is that these longer waves affect the short gravity-capillary waves by displacing and tilting them, and by modulating their amplitude. It is generally believed that waves longer than, say, a decimetre are modulated by the gradients in the horizontal orbital velocity induced by a long wave. However, the timescales on which gravity-capillary waves are generated and dissipated are too short for this mechanism to be effective for these waves. The same short timescale makes that the gravity-capillary wave are capable of adapting to the varying conditions of the boundary layer along a long wave. When the coupling between the short waves and the air flow is taken into account some hitherto unexplained features in the observations can be understood.

In chapter 6 we turn the attention to the effects of short waves on the energy balance of the modulating long wave. The short wave modulation can affect the long wave energy in three ways: by exerting a radiation stress (Hasselmann, 1971), via a correlation between the short wave growth and the long wave phase (Garrett and Smith, 1976, Valenzuela and Wright, 1979) and by affecting the air flow over the long wave (Gent and Taylor, 1976). Using the short wave modulations calculated in chapter 5, the contributions of these three mechanisms are compared. It is found that the last mechanism, where the short waves affect the air flow by modulating the aerodynamical surface roughness, can be very effective. Calculations show that the growth rate can double compared to the case where a uniform surface roughness is assumed.
Chapter 2

The modelling of air flow over waves

To describe the interaction between the wave-induced motions in the air flow and the turbulence, models of increasing complexity have been used. In the classical theory of Miles (1957) this interaction is neglected completely, and turbulence serves only to maintain a logarithmic wind profile. Jacobs (1987) and van Duin and Janssen (1992) use a mixing length type approach to calculate the modulations of the Reynolds stress caused by the wave-induced motions. The earliest numerical simulations of the air flow over waves, e.g. Gent and Taylor (1976) and Chalikov (1978) used a one-equation $\epsilon$-model. Later Al-Zanaidi and Hui (1984) used a more general two-equation closure scheme. However, they all took the eddy viscosity concept as their starting point.

For the related problem of wind over hills, Britter et al. (1981) suggested that the eddy viscosity concept can only be valid in a thin layer adjacent to the surface, which was termed the inner region. Outside this layer, in the outer region, the turbulence is distorted too rapidly for the eddy viscosity concept to be valid, and the rapid distortion theory as described by Batchelor and Proudman (1954) should be applied. In his numerical calculations Townsend (1972) took into account the finite relaxation time of the turbulent eddies. Later this model was refined by Townsend (1980) to account for the effects of the rapid distortion mechanism. Hunt et al. (1988) developed a four-layer asymptotic model to describe the changes to the air flow passing over a hill. Recently this model was extended by Belcher and Hunt (1993) to the case of shear flow over slowly moving waves.

In this chapter a numerical model is presented that describes the air flow over a prescribed wave. In this numerical model, that is an extension of the model used by Burgers and Makin (1993), different closure schemes can be used to parameterize the turbulence. Two of the turbulence closure schemes are based on the eddy viscosity concept. In the first, the mixing length model, the eddy viscosity is calculated from the local velocity shear and the height above the surface. In the second, the $\epsilon$-$\epsilon$ model, the turbulent kinetic energy $\epsilon$ and the dissipation $\epsilon$ are used to calculate the eddy viscosity. The turbulent kinetic energy and the dissipation rate are calculated with separate balance equations. For the third closure scheme the balance equations for the six independent Reynolds stress components are solved. The higher-order correlations appearing in these equations are parameterized in the way proposed by Lauder, Reece and Rodi (1975), hence we refer to this scheme as the LRR model. The numerical model gives a detailed description of the mean flow field above the prescribed waves, as well as a description of the distribution of the turbulent moments. When the wind speed and the surface roughness are specified, the model can calculate the flux of energy and momentum to a given
wave.

Following the description of the numerical model, the model is validated against earlier analytical and numerical calculations of the dimensionless form drag induced by a periodic sinusoidal hill. Subsequently it is shown that the energy flux calculated with the numerical model is strongly dependent on the choice of closure scheme. Scaling arguments given by Belcher and Hunt (1993) offer an explanation for this dependence. Belcher and Hunt argue that it is only in a layer close to the surface that the turbulence can be calculated from the local, instantaneous velocity shear. This layer is termed the inner region. Outside the inner region the changes in the shear experienced by the turbulent eddies are too rapid for the eddies to reach equilibrium with the shear. The advection of the eddies over the wave tends to smear out the effects of the shear modulation, limiting the stress variations compared to calculations assuming local equilibrium. As it is the stress modulation that is responsible for the downwind phase shift of the undulating flow, and hence for the out-of-phase pressure component, the limitation of the stress modulation reduces the energy flux to the wave. Of the three turbulence schemes used in this study, two depend on the eddy viscosity hypothesis, which implies local equilibrium. In the third scheme, the LRR model, the advection of the second-order stresses is accounted for.

In the analysis of Belcher and Hunt (1993) the thickness of the inner region is calculated from a scaling argument. Inside this layer a simple mixing length model is used to calculate stress perturbations induced by the presence of waves, and outside this layer the stress perturbations are put to zero. In calculations with the LRR turbulence scheme the thickness of the inner region does not need to be specified a priori. This closure scheme is general enough to handle both the local equilibrium case, and the case where the history of the forcing of the eddies is important. Given a criterion that determines whether turbulence is in local equilibrium with the shear or not, calculations with the LRR can be used to determine the depth of the inner region. A comparison with the inner region depth calculated this way with the relation used by Belcher and Hunt shows excellent agreement.

### 2.1 The Reynolds equations

The velocity and pressure distribution of the air flow over waves is governed by the Reynolds-averaged Navier-Stokes equations for an incompressible fluid:

\[
\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho_a} \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \bar{u}_i \bar{u}_j'}{\partial x_j}, \tag{2.1}
\]

\[
\frac{\partial \bar{u}_i}{\partial x_i} = 0. \tag{2.2}
\]

Here the overbar denotes an ensemble average, \( \bar{u}_i, i = 1, 2, 3, \) are the Reynolds-averaged velocity components of the air flow, \( t \) is time, \( x_i, i = 1, 2, 3, \) are the spatial coordinates, \( \rho_a \) is the density
of air, $\bar{p}$ is the Reynolds-averaged deviation from hydrostatic pressure and $u_i' u_j'$, $i, j = 1, 2, 3$, are correlations that give rise to Reynolds stresses. Throughout this work repeated indices imply a summation, and two equivalent notations for both the spatial coordinates $(x, y, z) = (x_1, x_2, x_3)$ and velocities $(u, v, w) = (u_1, u_2, u_3)$ are used. The air is assumed to be flowing over a second-order Stokes wave propagating along the $x$-axis:

$$
\eta(x, y, t) = a \cos(kx - \omega t) + \frac{1}{2} ka^2 \cos(2kx - 2\omega t),
$$

(2.3)

where $\eta(x, y, t)$ is the elevation, $a$ is the amplitude of the wave, $k$ is the wavenumber and $\omega$ is the angular frequency. The orbital velocities $u_0$, of this wave are used as lower boundary conditions at $z = \eta(x, y, t)$ for the air flow. At the top of the domain $z = h$ we prescribe a constant horizontal wind speed $U_h$, which makes an angle $\theta$ with the positive $x$-axis. The vertical velocity at the top of the domain is set to zero.

Since none of the boundary conditions depends on the $y$ coordinate, neither will the solution for the averaged properties. So the numerical model needs to take into account only two spatial coordinates: $x$ and $z$. All derivatives with respect to $y$ appearing in (2.1) and (2.2) can be ignored.

### 2.2 The turbulence closure schemes

Before the four equations (2.1) and (2.2) can be solved for $\bar{u}_i$ and $\bar{p}$, expressions for the Reynolds stress terms have to be supplied. In the model described here, three different closure schemes are used. Two of these, the mixing length and the $e$-$\epsilon$ schemes, are based on the concept of eddy viscosity. In the third, dynamical equations for the second-order correlations $u_i' u_j'$ are solved, using parameterizations for third-order correlations that appear in these equations.

#### 2.2.1 The mixing length model

In analogy with the molecular viscosity, the eddy viscosity is introduced by assuming that the traceless part of the Reynolds stress tensor $-\rho u_i' u_j'$ is proportional to the rate of strain tensor. The resulting relation is known as the closure hypothesis of Boussinesq:

$$
-\bar{u}_i' \bar{u}_j' + \frac{2}{3} \epsilon \delta_{ij} = 2K S_{ij},
$$

(2.4)

where $K$ is the eddy viscosity and $\epsilon = \bar{u}_i' \bar{u}_i'/2$ is the turbulent kinetic energy. The symmetrical rate of strain tensor $S_{ij}$ is defined as

$$
S_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right).
$$

(2.5)

In the simplest of the three turbulence schemes we consider, the eddy viscosity $K$ is itself also a function of the local rate of strain, and of the height $z$ above the surface:

$$
K = (\kappa z)^2 (2S_{ij} S_{ij})^{1/2},
$$

(2.6)
where \( \kappa \approx 0.41 \) is the von Kármán constant. The closure hypothesis (2.4) in combination with (2.6) is called the mixing length scheme. Note that this turbulence parameterization does not allow the calculation of the turbulent kinetic energy \( \epsilon \). By redefining pressure as \( \bar{p} + 2\epsilon/3 \) the turbulent kinetic energy is eliminated from the equations, and the resulting set of equations is closed.

2.2.2 The \( \epsilon-\epsilon \) model

This scheme also uses the Boussinesq closure hypothesis (2.4), but differs in the parameterization of the eddy viscosity \( K \). Unlike in the mixing length model, the eddy viscosity is now a function of the turbulent kinetic energy \( \epsilon \) and the dissipation rate \( \epsilon \) (hence the name \( \epsilon-\epsilon \) model):

\[
K = c_\mu \frac{\epsilon^2}{\epsilon},
\]

(2.7)

where \( c_\mu \) is a dimensionless constant. The turbulent kinetic energy \( \epsilon \) and the dissipation rate \( \epsilon \) are calculated dynamically:

\[
\frac{\partial \epsilon}{\partial t} + \bar{u}_i \frac{\partial \epsilon}{\partial x_i} = -\frac{\partial \bar{e}u_i}{\partial x_i} - \frac{1}{\rho_a} \frac{\partial \bar{p}u_i}{\partial x_i} + P - \epsilon;
\]

(2.8)

\[
\frac{\partial \epsilon}{\partial t} + \bar{u}_i \frac{\partial \epsilon}{\partial x_i} = -\frac{\partial \bar{e}u_i}{\partial x_i} + \frac{1}{\epsilon} \left( c_{1\epsilon} P - c_{2\epsilon} \epsilon \right),
\]

(2.9)

where \( P \) is the production of turbulent kinetic energy:

\[
P = -\bar{u}_i \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j}.
\]

(2.10)

The turbulent fluxes appearing in (2.8) and (2.9) are parameterized as

\[
\bar{e}u_i' + \frac{1}{\rho_a} \bar{p}'u_i' = -\frac{K}{\sigma_x} \frac{\partial \epsilon}{\partial x_i},
\]

(2.11)

\[
\bar{e}u_i' = -\frac{K}{\sigma_x} \frac{\partial \epsilon}{\partial x_i}.
\]

(2.12)

In this study we use the standard values for the five dimensionless constants used in the \( \epsilon-\epsilon \) model (Jones and Lauder, 1972). The value of 1.92 for \( c_{2\epsilon} \) follows from observations of the decay rate of grid turbulence. The value for \( c_\mu \) follows from the observation that \( \epsilon/u_x^2 \approx 3.7 \) in boundary layers, where the friction velocity is defined as \( u_* = (\tau/\rho_a)^{1/2} \). This implies \( c_\mu = 0.073 \). For the proportionality factors of the turbulent transport terms the values \( \sigma_x = 1.0 \) and \( \sigma_x = 1.3 \) are usually taken. The requirement that the logarithmic velocity profile \( \bar{u}(z) = (u_*/\kappa) \ln(z/z_0) \) is a solution of the model in the absence of a wave leads to \( c_{1\epsilon} = 1.44 \). The four boundary conditions that are needed to solve (2.8) and (2.9) are discussed in section 2.3.2 on page 17.
2.2.3 The second-order model

In a second-order model the dynamical equations for the second-order correlations are solved. Like the Reynolds equations, the equations for the second-order correlations follow from the Navier-Stokes equations combined with a (Reynolds) averaging procedure. This set of equations will contain third-order correlations, which need to be expressed in known quantities in order to get a closed set of equations. In this work we use the parameterizations introduced by Launder, Reece and Rodi (1975), hence we refer to this scheme as the LRR second-order scheme. The balance equations for the six independent correlations $\bar{u}_i^* u_j^*$ read:

$$\frac{\partial \bar{u}_i^* u_j^*}{\partial t} + \bar{u}_k^* \frac{\partial \bar{u}_i^* u_j^*}{\partial x_k} = P_{ij} + T_{ij} + \Pi_{ij} - \epsilon_{ij},$$  \hspace{1cm} (2.13)

where the right-hand-side terms represent the production, the turbulent diffusion, the work of pressure-velocity correlations and the molecular dissipation.

The functional dependence of the production term follows from the basic equations and needs no parameterization:

$$P_{ij} = -\bar{u}_l^* u_l^* \frac{\partial \bar{u}_j^*}{\partial x_k} - \bar{u}_j^* u_l^* \frac{\partial \bar{u}_i^*}{\partial x_k},$$  \hspace{1cm} (2.14)

The turbulent transport of the Reynolds stresses is parameterized as

$$T_{ij} = c_s \frac{\partial}{\partial x_k} \left\{ \frac{\epsilon}{\epsilon} \left( \bar{u}_l^* u_m^* \frac{\partial \bar{u}_j^*}{\partial x_k} + \bar{u}_j^* u_m^* \frac{\partial \bar{u}_i^*}{\partial x_k} + \bar{u}_k^* u_m^* \frac{\partial \bar{u}_i^*}{\partial x_m} \right) \right\}.$$  \hspace{1cm} (2.15)

The pressure-velocity correlation terms are thought to relax the stresses towards isotropy, and to reduce the effectiveness of the production. For the first part of the pressure-velocity correlation term, $\Pi_{ij}^{(1)}$, the Rotta hypothesis is used:

$$\Pi_{ij}^{(1)} = -2c_1 \epsilon b_{ij},$$  \hspace{1cm} (2.16)

where $b_{ij}$ is the stress-anisotropy tensor defined as

$$b_{ij} = \frac{\bar{u}_i^* u_j^*}{2\epsilon} - \frac{1}{3} \delta_{ij}.$$  \hspace{1cm} (2.17)

The second part of $\Pi_{ij}$ is called the rapid term, and it is parameterized as

$$\Pi_{ij}^{(2)} = 4\epsilon \left\{ \frac{1}{5} S_{ij} + \frac{9c_2 + 6}{22} \left( S_{ik} b_{kj} + S_{kj} b_{ik} - \frac{2}{3} S_{kl} b_{kl} \delta_{ij} \right) + \frac{10 - 7c_2}{22} \left( R_{ik} b_{kj} + R_{kj} b_{ik} \right) \right\},$$  \hspace{1cm} (2.18)

where $R_{ij}$ is the mean vorticity tensor:

$$R_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \bar{u}_j}{\partial x_i} \right).$$  \hspace{1cm} (2.19)

The dissipation is assumed to be isotropic:

$$\epsilon_{ij} = \frac{2}{3} \delta_{ij} \epsilon.$$  \hspace{1cm} (2.20)
The dissipation rate $\epsilon$ is calculated with the same dynamical equation as in the $e-\epsilon$ scheme, i.e. equation (2.9). The only difference is the parameterization of the turbulent transport term: in order to eliminate the eddy viscosity completely from the second-order model, (2.12) is replaced by

$$\overline{e' u'} = -c_\epsilon e \overline{u'_i u'_k} \frac{\partial \epsilon}{\partial x_k}.$$  (2.21)

The LRR scheme needs six constants. Again we use the standard values for these parameters in this study. The values for $c_{1e}$ and $c_{2e}$ are the same as for the $e-\epsilon$ scheme (1.44 and 1.92, respectively). The proportionality factor for the turbulent transport term is usually taken as $c_s = 0.11$. The constants entering the parameterizations of the pressure-velocity correlation terms, $c_1$ and $c_2$, are determined from the observed anisotropy of the Reynolds stress in homogeneous shear flow. This leads to the values $c_1 = 1.5$ and $c_2 = 0.4$. For a flow in the $x$-direction over a flat plate in the $(x, y)$-plane, the LRR model now produces the following stress ratios:

$$\overline{u' u'}/u_*^2 = 2.6, \quad \overline{v' v'}/u_*^2 = 1.7, \quad \overline{w' w'}/u_*^2 = 1.3.$$  (2.22)

The constraint that the logarithmic velocity profile is a solution in the absence of a wave leads to $c_s = 0.267$.

2.3 The numerical model

2.3.1 The flow configuration

The numerical model described here will be used to assess the influence of a monochromatic wave on the air flow. The wave is propagating in the direction of the positive $x$-axis and is assumed to be invariant under translations in the $y$-direction. For this reason the air flow is independent of this coordinate. To simplify the solution of the equations, the remaining coordinates $(x, z)$ are expressed in $(\chi, \xi)$, where $\chi = (kx - \omega t)/2\pi$ and $\xi = (z - \eta)/(h - \eta)$. The domain is now $0 \leq \chi, \xi \leq 1$. The velocities and stresses are not transformed. Note that the water surface in the new coordinate system is located at one value of the vertical coordinate ($\xi = 0$). This simplifies imposing the surface boundary conditions. As the surface is assumed to be periodic in the $x$-direction, we impose periodic boundary conditions for the air flow in this direction also. The height $h$ of the domain is set equal to the length of the wave $\lambda = 2\pi/k$. At this height the wave-induced perturbations are small enough to be ignored.

The water wave is assumed to be a gravity wave in deep water, for which the dispersion relation $\omega^2 = gk$ is valid, where $g$ is the gravitational acceleration. The system is completely determined by specifying the following parameters: the wind speed $U_\lambda$ at height $\lambda$, the wavenumber $k$, the amplitude $a$, the surface roughness $z_0$ and the gravitational acceleration $g$. If we replace $g$ by the phase speed $c = (g/k)^{1/2}$, we find that the system is determined by the three dimensionless variables $U_\lambda/c$, $ak$, and $kz_0$. When we want to calculate the growth rate of the wave, the ratio of the density of air and water $\rho_a/\rho_w$ enters as a proportionality factor. We conclude
that in the numerical model presented here, the growth rate is a function of four dimensionless parameters. The dependence on the density ratio is trivial since it factorizes.

2.3.2 Boundary conditions

Lower boundary conditions. At the air-sea interface the orbital velocities \( u_0, v_0, w_0 \) and the surface roughness distribution \( z_0(x) \) have to be specified. Surface values for the \( \epsilon \) and the second-order moments are calculated from the tangential stress at the surface using their equilibrium values. For the \( \epsilon-\epsilon \) scheme \( \epsilon = 3.7u_*^2 \) is used, and for the LRR scheme the value given in the equations (2.22). The tangential stress at the surface is calculated from the velocity, assuming the tangential velocity profile is logarithmic with height between the surface and the first model layer. This assumption is also used to calculate the tangential velocity gradients, since the standard finite difference formula is inaccurate due to the large velocity gradient between the surface and the first model level.

The treatment of the dissipation rate \( \epsilon \) requires some care. In boundary layers \( \epsilon \) is inversely proportional to the distance from the surface. This makes it difficult to impose a sensible surface boundary condition. In nature the magnitude of the dissipation rate very close to the surface is limited by the presence of a viscous sublayer. However, in the model described here the viscous sublayer is not resolved. To limit the dynamical range of the variables in the model, the \( \epsilon \)-equation is rewritten in terms of the variable \( \epsilon z \). This is done by replacing all vertical derivatives in (2.9) by:

\[
\frac{\partial \epsilon}{\partial z} = \frac{1}{z} \frac{\partial (\epsilon z)}{\partial z} - \frac{\epsilon z}{z^2}.
\]

The resulting equation is then multiplied by height. The equilibrium value of this new variable \( \epsilon z = u_*^2/\kappa \) is imposed at the surface, where \( u_* \) is again calculated from the local tangential surface stress.

Upper boundary conditions. The velocities at the top of the domain are prescribed: the horizontal velocities \( u_\lambda \) and \( v_\lambda \) force the flow, and the vertical velocity \( w_\lambda \) is set to zero. The upper boundary conditions for \( \epsilon, \epsilon z \) and the turbulent moments are found by setting their vertical gradients to zero at the top.

2.3.3 Discretization

The model uses a standard finite difference scheme, with second-order accuracy in both space and time. The grid is staggered: the pressure and the stresses are calculated at half grid points, centred between four velocity points. The resolution close to the surface is increased by discretizing the vertical coordinate as

\[
\xi_j = \frac{\zeta_j - 1}{\zeta_m - 1}, \quad j = 0, 1, .., m,
\]
Fig. 2.1: The form drag of a hill $S$ vs. the surface roughness $kz_0$. The lines indicate numerical results using different closure schemes, the symbols are obtained from theories found in literature.

where $\zeta$ is the vertical stretching parameter and $m$ is the number of layers in the vertical. Starting from the surface ($j = 0$), the spacing between the vertical layers increases with a factor $\zeta$ per layer. A more detailed discussion on the discretization can be found in Makin (1979) and Burgers and Makin (1993).

When we imposed the surface boundary conditions, we assumed that between the first model layer and the surface the stress is constant with height at any given point along the wave surface. The asymptotic analyses of Jackson and Hunt (1975) and Belcher and Hunt (1993) provide an estimate for the height up to where the assumption of a constant stress layer holds for the case of flow over a hill. The scaling arguments that are the foundation of these analyses will be discussed in detail in section 2.4.4 on page 24. Here it suffices to note that the vertical resolution has to increase when the dimensionless surface roughness $kz_0$ decreases. The smaller $kz_0$, the lower the first model layer has to be located. This can be achieved by increasing the stretching parameter $\zeta$ or by increasing the number of layers in the vertical. Both options give rise to an increase in computation time. For $kz_0 = 10^{-4}$ the vertical domain of the numerical model spans roughly three orders of magnitude: from $kz \simeq 4 \times 10^{-2}$ to $kz = 2\pi$. For dimensionless roughnesses smaller than $10^{-5}$ the computation time becomes impractically long. To simulate the air flow over a wave with roughness $kz_0 = 10^{-4}$ typically 80 layers in the vertical are used, with a 6% increase in spacing per vertical layer ($\zeta = 1.06$). The lowest layer is then located at $kz = 3.7 \times 10^{-3}$. A further increase of the resolution does not change the results. A more detailed discussion of the effects of the vertical resolution can be found in section 4.2.1 on page 50.
The model uses a second-order time integration method. The difference with results from a time step with first-order accuracy is used to estimate the optimal size of the next time step. The elliptical equation for pressure is solved with the successive over-relaxation (SOR) method using Chebyshev acceleration. A good description of this method can be found in Press et al. (1992).

Earlier versions of the numerical model used in this work are described by Makin (1979) and Burgers and Makin (1993). Compared to earlier variants of the model, several changes have been made: (i) the $\epsilon-\epsilon$ and the second-order turbulence models have been implemented, where previously only a mixing length model and a one-equation $\epsilon$-model were used; (ii) the way in which the bottom boundary conditions are imposed has changed; (iii) an equation for the $y$-component of the velocity was introduced in the 2D model to enable calculations for waves propagating at an angle to the wind. Apart from these major changes several minor modifications have been made, mainly to make the code run faster on a computer. This has resulted in a code that is about two orders of magnitude faster than the one used by Burgers and Makin (1993).

### 2.4 Effects of turbulence closure on wave growth

The choice of the turbulence scheme has a large impact on the results for the energy flux from wind to waves. Not only does the growth rate change quantitatively, but also its dependence on the parameters mentioned above changes. Before discussing these changes and their cause, first the numerical model is validated by comparing calculations for air flow over hills with results from literature.

#### 2.4.1 Validation of the model

To validate the model the drag caused by a periodic sinusoidal hill is calculated, and compared to results from earlier studies. The boundary conditions for a hill can be obtained by putting $\omega = 0$ in (2.3), and by putting the orbital velocities to zero. We will compare calculations of the dimensionless form drag $S$, defined as

$$
S = \frac{2\pi \langle p\eta_x \rangle_{x=\eta}}{\rho_0 u_0^2 (ak)^2},
$$

(2.25)

where $\eta_x = \partial \eta / \partial x$. The dimensionless form drag for flow over hills is the equivalent of the growth rate parameter in the case of waves. In figure 2.1 on the preceding page the form drag $S$ is presented as a function of $kz_0$. Results from all three turbulence parameterizations are compared, both with other numerical models and with analytical calculations. The agreement between the results from the numerical model presented here and the other numerical models is excellent. The analytical results for the mixing length closure are obtained with the Van Duin and Janssen model, modified to include also the effect of the wave-induced stress in the inner
region (see discussion in Belcher et al., 1994, and in Wood and Mason, 1993). The analytical model of Belcher et al. (1993) should be comparable with results from the second-order closure schemes, since both take into account that the turbulence in the outer region is distorted rapidly. As can be seen in figure 2.1, this is indeed the case for small roughnesses. For larger roughnesses, $kz_0 > 10^{-3}$, the analytical model yields larger form drag than both numerical models. In the analytical approach of Belcher and Hunt (1993), the solution of the equations is found under the assumption that the inner region is thin. As will be discussed in section 2.4.4 on page 24 the depth of the inner region increases with the dimensionless roughness $kz_0$. For $kz_0 > 10^{-3}$ the breakdown of the assumption that the inner region is thin becomes a major source of error in the analytical theory.

From this comparison we conclude that the numerical model described here shows excellent agreement with results from earlier studies for the calculation of the form drag induced by a hill. The strong dependence of the form drag on the choice of the turbulence scheme is well reproduced, which is important for the remainder of this study.

### 2.4.2 Visualization of air flow over waves

Before we will analyse the flow over waves in detail, it is instructive to look at the main features. To this end the distributions of the streamlines, the wave-induced pressure and the turbulent kinetic energy above a wave are plotted in figure 2.2. In this example the wind speed at height $\lambda$ is five times the phase speed of the wave ($U_\lambda/c = 5$). The dimensionless roughness of the wave surface is $kz_0 = 10^{-4}$, and its steepness is $ak = 0.2$.

For a two dimensional, incompressible flow a function $\Psi$ can be found such that:

$$u = \frac{\partial \Psi}{\partial z}, \quad w = -\frac{\partial \Psi}{\partial x}$$

(2.26)

For a flow over a wave, the most relevant streamlines are those defined in a frame of reference moving with the wave. As the water surface is itself a streamline, the stream function can be found by integrating the horizontal velocity field over the vertical. Putting the value of stream function to zero at the surface $\xi = 0$, the value of the stream function at the point $(\chi, \xi)$ is found by:

$$\Psi(\chi, \xi) = (h - \eta(\xi)) \int_0^\xi (u(\chi, \xi') - c)d\xi'.$$

(2.27)

In figure 2.2 the horizontal coordinate $\chi$ is related to the phase of the long wave. On the vertical axis the height $z$ relative to a fixed frame of reference is used, scaled with the wavenumber $k$. The values of the stream function are normalized with $au_*$, where $a$ is the wave amplitude.

In the example shown here, which is typical for a wave that is growing by the wind, the flow is accelerated above the crest. This induces a pressure minimum above the crest. The pressure maximum, located close to the trough, is shifted upwind by roughly $5^\circ$. As the upwind slope of the wave is moving down, the phase shift means that the pressure is doing work on the wave. This is the main mechanism by which waves receive energy from the wind. In the lowest
Fig. 2.2: The stream function, pressure and turbulent kinetic energy distribution over a wave of $U_\lambda/c = 5$ and steepness $ak = 0.2$. The values in the plots are normalized with $au_*$, $ak\rho_u u_*^2$ and $u_*^2$ respectively.
Fig. 2.3: The growth rate parameter $\beta$ vs. the ratio $c/u_*$ calculated with the numerical model using three different closures: dash-dot, mixing length; dashed, the $c$-$\epsilon$ scheme; solid line, the LRR model. The dimensionless roughness $kz_0 = 10^{-4}$.

panel the distribution of the turbulent kinetic energy $e$ is plotted. In the LRR second-order model, which is used in this example, the equilibrium value of the turbulent kinetic energy is $e \simeq 2.8u_*^2$ (see the end of section 2.2.3). Close to the surface above the crest the presence of a wave enhances this value. At a larger distance from the surface the maximum of the enhancement shifts towards the trough. The details of distribution of the turbulence will be discussed below, but from figure 2.2 it is clear that the phase of the wave-induced turbulent kinetic energy strongly depends on height. In contrast, the phase of the wave-induced pressure seems to be constant with height.

2.4.3 The energy flux to waves

The energy flux due to surface stresses to a wave with energy $E$ per unit area is given by:

$$\dot{E}_{\text{wind}} = \langle \tau_\perp u_\perp \rangle_{z=\eta} + \langle \tau_\parallel u_\parallel \rangle_{z=\eta}, \quad (2.28)$$

where $\tau_\perp (u_\perp)$ and $\tau_\parallel (u_\parallel)$ are the normal and tangential components of the stress (velocity) close respectively. The average $\langle \cdot \rangle_{z=\eta}$ is performed along the surface of the wave $\eta$, which corresponds to $\xi = 0$. Transforming the normal and tangential stresses and velocities to the Cartesian frame of reference used in the model yields:

$$\dot{E}_{\text{wind}} = \frac{1}{\lambda} \int_0^\lambda \left\{ -p(w_0 - \eta z u_0) \dd x \right\} \left\{ \begin{array}{l} -\rho_a(u^2 w^0 w_0 - w^0 u^0 \eta_z u_0) \\ -\rho_a u^0 w^0 (u_0 - \eta_z w_0) \end{array} \right\} \dd x. \quad (2.29)$$
Following Belcher and Hunt (1993) the energy flux is separated into three parts, where for the different parts the work is done by (i) the pressure, (ii) the velocity variances or (iii) the shear stress. The relative contributions of these three terms are discussed below.

The growth rate due to the wind is defined as $\gamma = (1/E)\dot{E}_{\text{wind}}$. The energy of a gravity wave is taken to be $E = \rho_a g (\eta^2)$. Following Miles (1957) and Townsend (1972), growth rates will be presented in terms of the parameter $\beta$, defined as

$$\frac{\gamma}{\omega} = \beta \frac{\rho_a}{\rho_w} \left( \frac{u_*}{c} \right)^2 .$$

(2.30)

Based on a compilation of experimental data from various sources, Plant (1982) concluded that for waves that are slow compared to the wind $\beta = 32 \pm 16$.

The growth rate calculated with the numerical model depends strongly on the type of turbulence closure used. In figure 2.3 the growth rate parameter $\beta$ is shown as a function of the ratio $c/u_*$ for waves with a dimensionless roughness of $kz_0 = 10^{-4}$. For waves that are slow compared to the wind ($c/u_* < 13$, or equivalently $U_\lambda/c > 2$) the mixing length scheme yields much larger growth rates than the LRR model. The decay rate for waves travelling faster than the wind ($c/u_* > 22$, $U_\lambda/c < 1.2$) is also reduced in the Reynolds stress model. However, for waves just slower than the wind ($1.4 < U_\lambda/c < 2$) the growth rate is enhanced in the Reynolds stress model. In all three regimes the growth rate calculated with the $e-e$ scheme is located between results from the mixing length model and the Reynolds stress model. Note that for slow waves $\beta$ calculated with the Reynolds stress model is almost independent of the ratio $c/u_*$, consistent with the experimental data presented by Plant (1982). The same result is reported by Townsend (1972) and Belcher and Hunt (1993). However, all these models underestimate the value proposed by Plant by a factor more than 2.

In table 2.1 the relative contributions of the parts (i) to (iii) denoted in (2.29) are listed for a slow moving wave $U_\lambda/c = 5$, a fast moving wave ($U_\lambda/c = 1.5$) and a wave propagating faster than the wind ($U_\lambda/c = 0.80$). For slow waves, $U_\lambda/c > 2.7$, all contributions to the energy flux are positive, and the contribution from the pressure is dominant. When the ratio $U_\lambda/c$ drops below this value, the work of the tangential stress on the orbital velocities, contribution (iii), becomes negative. Due to the work of the pressure the total energy flux to the waves remains positive as long as $U_\lambda/c > 1$. When the ratio $U_\lambda/c$ drops below 1, the contribution due to the pressure perturbation is no longer dominant. The negative energy flux for these waves is maintained mainly by the work of the shear stress on the orbital velocities (contribution (iii)).
Fig. 2.4: Overview of the vertical structure of the flow over waves as a function of $U_\lambda/C$ for $kz_0 = 10^{-4}$. The solid line indicates the depth $kl$ of the inner region. The depth $k\delta$ of the inner surface layer (ISL) is shown as a dashed line. The remaining part of the inner region is called the shear stress layer (SSL).

2.4.4 Turbulence in the outer region

The reason for the large differences in growth rate calculated with the three turbulence closure schemes can be understood with a physical scaling argument regarding the response of turbulence to a rapidly changing forcing. This scaling argument is discussed at length in Belcher et al. (1993). An eddy advected over a wave experiences changes in velocity shear at a timescale that is proportional to the time it takes the eddy to pass over a wave:

$$T_D \sim \frac{\lambda}{|U_\lambda - c|}. \quad (2.31)$$

The timescale on which the eddy can adjust itself to changes in the shear is believed to be proportional to the eddy turnover time, which is equal to the ratio of the typical size of the eddy to the friction velocity. The typical size of eddies in a boundary layer is $\kappa z$, so it follows that the timescale of eddy adjustment to a change in the forcing is proportional to the height above the surface:

$$T_L \sim \frac{\kappa z}{u_*}. \quad (2.32)$$

For waves with a phase speed smaller than the wind speed, the vertical domain is now divided into two regions: the inner region, where $T_L < T_D$, and the outer region, where $T_L > T_D$. An estimate of the depth of the inner region $l$ can be found from (2.31) and (2.32) by putting $T_D \sim T_L$. For historical reasons (Jackson and Hunt, 1975) the $O(1)$ proportionality constant
is chosen such that

\[ kl = 2\kappa \frac{U_*}{|U_l - c|}. \]  

(2.33)

In the inner region, \( z < \ell \), the turbulence is in local equilibrium with the velocity shear. At each point the production of turbulent kinetic energy is balanced by the dissipation. It follows that an eddy viscosity closure scheme of the type (2.4) can be used to model the stress in this region. In the outer region, \( z > \ell \), the eddies are distorted more rapidly than they can react, and they will not be in local equilibrium with the shear. Turbulent kinetic energy will be advected away from regions with excess production, before a local balance with the dissipation can be reached. The eddy viscosity hypothesis, which implies local equilibrium, no longer holds. A second-order Reynolds stress model, like the LRR scheme, incorporates advection of turbulent moments. Therefore it is expected that this turbulence scheme is able to cope with this aspect of rapidly distorted turbulence. In section 2.4.6 the scaling of the inner region depth is discussed in more detail.

In figure 2.4 the depth of the inner region above a wave with a surface roughness of \( k z_0 = 10^{-4} \) is plotted as a function of \( U_\lambda / C \). Belcher and Hunt (1993) identified several other layers in the air flow over waves. These are also shown in figure 2.4. In the lowest part of the inner region, the so-called inner surface layer (ISL), the shear stress perturbation can be assumed constant with height. The depth of this layer scales as \( \delta \sim (l z_0)^{1/2} \). The remaining part of the inner region is termed the shear stress layer. It is in this layer that the velocity perturbation reaches its maximum.

Also shown in figure 2.4 is the height where the wind speed matches the phase speed of the wave, the so-called critical height. It is evident that the advection of wave induced turbulence relative to the wave is zero at the critical height. For this reason one would expect the critical height to be located inside the inner region. For waves slow compared to the wind this is indeed the case. For faster waves, i.e. those with \( 0 < U_\lambda / c < 1.2 \), the vertical structure of the flow is at first sight more complicated. For these waves two separate regions can be found where the advection of wave induced turbulence can be neglected, which is reflected by the fact that for these waves (2.33) yields two solutions for \( kl \). The lowest of the two regions is located close to the surface, the second in the vicinity of the critical height. However, for waves with a phase speed comparable to the windspeed, \( 1 < U_\lambda / c < 1.2 \), the advection of turbulence is of minor importance throughout the vertical domain. For waves faster than the wind, \( 0 < U_\lambda / c < 1 \), the critical height is located too far from the surface to be of any relevance to the problem. Therefore only the smallest of the two solutions of (2.33) is plotted in figure 2.4.

Figure 2.5 on the next page illustrates how the second-order turbulence scheme handles the transition from local equilibrium close to the surface, to rapidly distorted turbulence at some distance from the surface. In this figure the contributions to the turbulent kinetic energy of the three dominant terms of the budget equation for \( e \) are shown in the \((\chi, 2\pi \xi)\) plane for \( U_\lambda / c = 5 \). The equation for the turbulent kinetic energy can be obtained from the balance equations for the turbulent moments (2.13) by adding the equations for the velocity variances divided by two.
Fig. 2.5: Balance of the production, advection and dissipation terms in the equation for the turbulent kinetic energy over a wave with $U_{A}/c = 5$ and $k z_0 = 10^{-4}$. All quantities are normalized with the equilibrium value for the dissipation $u_2^2/\kappa z$. Solid contour lines are drawn around areas where the wave increases the contribution of the respective term.

As the contributions from the traceless pressure-velocity correlation tensors $\Pi_{ij}^{(1)}$ and $\Pi_{ij}^{(2)}$ drop out, (2.8) is obtained, obviously without the parameterization (2.11) for the diffusion term. Solid contour lines in figure 2.5 denote areas where the specific term (i.e. production, advection or dissipation) makes a larger than average contribution, the dashed lines mark the areas of a smaller than average contribution. In the plots all terms are scaled with the equilibrium value $u_2^2/\kappa z$ of the production and dissipation over a flat plate.

The first contour plot shows that the maximum of the production close to the surface is located above the crest. When the height increases, the phase of this maximum shifts to the trough. Close to the surface, for heights $k z < 0.1$, the modulation in the production is balanced by the modulation of the dissipation. In this layer the turbulence is said to be in local equilibrium. Further away from the surface, the advection becomes increasingly important in balancing the variations in the production rate. The assumption that the turbulence is in equilibrium with the local shear is no longer valid: the importance of the advection implies that the turbulence is also a function of the shear it experienced in its past. As is obvious from the Boussinesq hypothesis (2.7), eddy viscosity closures assume that the turbulence is in local equilibrium with the shear. In figure 2.5 it is shown that, above a certain height, this assumption is violated in the flow over waves. Below we will show that this breakdown of the eddy viscosity hypothesis has important consequences for the calculation of the energy flux from the air flow to the wave.

The modification of the turbulence in the outer region is described by rapid distortion theory (Batchelor and Proudman, 1954). In Britter et al. (1981), Zeman and Jensen (1987) and Belcher et al. (1993) a detailed description of the effects of rapid distortion on turbulence in the outer region can be found. Their conclusions are that due to the irrotational straining of the initially anisotropic turbulence, the horizontal velocity variance $\overline{u'v'}$ is reduced above the
crest. The effect on $\overline{w'w'}$ depends on the anisotropy $A = \overline{u'u'}/\overline{w'w'}$ of the mean flow: if $A < 3$ the vertical velocity variance $\overline{w'w'}$ is reduced due to the irrotational strain effect, for $A > 3$ the effect reverses sign. The curvature of the mean streamlines in the outer region rotates the stress tensor. Zeman and Jensen (1987) find that this effect is important at the top of the inner region. A part of the observed reduction of the shear stress above the top of the Askervein hill could be attributed to the curvature effect. The ability of second-order models like the LRR scheme to reproduce rapid distortion effects depends on the parameterization of the rapid term $\Pi^{(2)}$. Though the LRR model gives the right sign and order of magnitude of both rapid effects, alternatives to the LRR parameterization exist, and they may yield different results. A comparison of such parameterizations with direct numerical simulations can be found in Shih and Lumley (1993).

2.4.5 The relation between growth rate and turbulence model

As can be seen in figure 2.3 on page 22, the growth rate for slow waves calculated with the mixing length scheme is significantly larger than the one following from the LRR closure, with the growth rate from the $e$-$e$ scheme in between. In Belcher et al. (1993) it is shown in detail that this is a direct consequence of the difference in handling the rapidly distorted turbulence in the outer region. Since the link between turbulence models and growth rates is a central issue of this chapter, their analysis will be briefly recaptured here.

To simplify the analytical description we will assume that the wave-induced flow is potential. In Belcher et al. (1993) it is shown that this is true only in the upper part of the outer region for $z > h_m$, where $h_m$ follows from the implicit relation:

$$kh_m = \left(\frac{u_*}{\kappa U_m}\right)^{1/2}. \quad (2.34)$$

Here $U_m$ is the mean wind speed at the height $h_m$. The flow in the part of the outer region below $h_m$ is still inviscid, but rotational. In the following, a wave-induced property, or perturbation, is defined as the deviation of this property from the mean at a given height, i.e. $\bar{u} = u - \langle u \rangle$. If the flow is potential it follows that the velocity perturbations can be written as

$$\bar{u} = \text{Re}[u_m e^{ikx-kz}], \quad (2.35)$$

$$\bar{w} = \text{Re}[i u_m e^{ikx-kz}]. \quad (2.36)$$

Here $u_m$ is the complex amplitude of the wave-induced velocities at the bottom of the upper layer. For the eddy viscosity closures the Reynolds stress perturbation follows from the Boussinesq hypothesis:

$$\tilde{\tau}/\rho_a = 2\tilde{\mathcal{R}}(S_{zz}) + 2\langle K \rangle \tilde{S}_{zz}. \quad (2.37)$$

If the velocity perturbations (2.35) and (2.36) are substituted in the equations (2.5) and (2.6) that enter (2.37), an expression for the stress perturbation $\tilde{\tau}$ in terms of $u_m$ can be found
when the mixing length scheme is used. The mixing length scheme gives a stress perturbation that is 180° out-of-phase with the velocity perturbations. To calculate the impact of the stress perturbation on the out-of-phase pressure component, the linearized vertical momentum equation has to be integrated from infinity down to $h_m$:

$$\text{Im}[\hat{\rho}_m] = \rho_a \int_{h_m}^{\infty} (U_x - c) \text{Re}[\hat{u}] k \, dz - \int_{h_m}^{\infty} \text{Re}[\hat{\tau}] k \, dz. \quad (2.38)$$

Here the hat indicates an amplitude of a perturbation: $\hat{\rho} = 1/2(\rho \exp(ikx - kz) + c.c.)$. The first term on the right hand side of (2.38) represents the contribution to the growth of an undulating flow that is in phase with the wave. In terms of the potential flow approximation (2.35) and (2.36) this contribution is proportional to the imaginary part of the wave-induced velocity perturbation at the bottom of the upper layer. Due to the second term on the right-hand side of (2.38) the wave-induced Reynolds stress $\hat{\tau}$ has a direct impact on the wave growth. The Reynolds stress perturbation in the upper layer calculated with the mixing length scheme will contribute:

$$\hat{\rho}^{\text{mix}} = 4 \frac{\text{Re}[u_m] \kappa U_m}{akU_m} \frac{1}{u_*} (1 + \mathcal{O}(\varepsilon^{1/2})), \quad (2.39)$$

where $\varepsilon = u_*/\kappa U_m$ is believed to be small. The total growth rate will include contributions of the stress modulation in the inner region and the rest of the outer region, and from the term involving $\text{Re}[\hat{u}]$ in (2.38).

In the LRR model, the velocity perturbations (2.35) and (2.36) modulate the production terms. In the outer region the advection terms will balance this modulation. This results in a modulation of the shear stress that is reduced by a factor $\varepsilon$ compared to the mixing length result. The contribution to the growth rate of the stress modulation in the LRR model is:

$$\hat{\rho}^{\text{LRR}} = -4 \frac{\text{Im}[u_m]}{akU_m} \frac{1}{3u_*^2} (1 + \mathcal{O}(\varepsilon^{1/2})). \quad (2.40)$$

Even if $\text{Im}[u_m] = \text{Re}[u_m]$, meaning that the undulating flow in the outer region is 45° out-of-phase with the wave, the contribution of (2.40) is smaller than (2.39) by a factor of $\varepsilon$.

In the mixing length model, both terms in equation (2.37) contribute equally to the stress modulation. In the $e$-$\varepsilon$ scheme, where the eddy viscosity $K$ is calculated from the dynamical variables $e$ and $\varepsilon$, the modulation of the eddy viscosity will be suppressed. This leaves only the second term in (2.37) to modulate the stress. It follows that the contribution to the wave growth of stress contributions in the outer region in the $e$-$\varepsilon$ scheme will be half that of the mixing length scheme.

The above analysis explains why, for slow waves, the eddy viscosity closures give rise to higher growth rates than the LRR scheme. In figure 2.6 calculated profiles of the wave-induced horizontal velocity, stress and pressure are plotted. In this example the wind speed at height $\lambda$ is 5 times larger than the phase speed of the wave. In agreement with the analysis presented above, the mixing length model produces a stress modulation 180° out-of-phase with the wave that is much larger than the modulation following from the LRR model. This results in a
Fig. 2.6: Profiles of $\text{Re}[ar{u}]$, $\text{Re}[ar{r}]$ and $\text{Im}[ar{p}]$ above a slow wave ($U_s/c = 5$, $kz_0 = 10^{-4}$). Dash-dotted line, mixing length; dashed line, $\epsilon$-$\epsilon$ model; solid line, LRR turbulence scheme.

significantly larger out-of-phase pressure component for the mixing length model, and hence in a larger energy flux to the wave.

2.4.6 The depth of the inner region

The implicit relation (2.33) for the depth of the inner region was found by scaling arguments involving the dynamics of the turbulence in the flow over waves. In the analysis of Belcher and Hunt (1993) this height is used to determine to which height the turbulence can be considered in local equilibrium with the shear. Below this height the wave-induced Reynolds stress is parameterized with a mixing length closure, above this height the dynamics of the turbulence is assumed to be governed by rapid distortion theory. Given the fact that the calculated growth rate is sensitive to the type of closure, as we have seen above, one might expect that the results following from the analysis of Belcher and Hunt (1993) strongly depend on the height where the mixing length closure is switched off. On the basis of numerical simulations and the interpretation of observations over hills, Beljaars and Taylor (1989) suggest that the inner region depth scales different than (2.33). They find that the inner region depth collapses more rapidly with increasing $|U_i - c|/u_*$ than can be explained from the scaling arguments that lead to (2.33). However, the experimental data shown by Beljaars and Taylor show a formidable scatter, which makes it very difficult to draw firm conclusions from them. The results obtained by Beljaars and Taylor might also be influenced by their definition of the inner region, which Beljaars and Taylor define as the height where the velocity perturbation has a maximum. For the numerical simulations Beljaars and Taylor used an $\epsilon$-$\epsilon$ model, which is not valid in the outer region. Here we will use the more general LRR model, in combination with a more physical definition of the inner region depth, to check the scaling of the inner region depth as found in (2.33).

By definition, the turbulence in the inner region is in local equilibrium with the velocity
shear. The turbulence is produced and dissipated at the same point relative to the surface disturbance, with the advection or diffusion of turbulence playing only a minor role. In the outer region the advection of turbulence by the mean flow becomes important, preventing a local equilibrium between shear and turbulence. Therefore it seems reasonable to define the depth of the inner region as the height where the dissipation and the advection are equally important in balancing the perturbations in the production. Unlike the definition used by Beljaars and Taylor, this definition of the inner region depth is directly related to the physical meaning of the separation of the flow in two regions.

In figure 2.7 the amplitudes of the four terms of the turbulent kinetic energy equation are plotted as a function of the height $kz$ for the case $U_\lambda/c = 5$ and $kz_0 = 10^{-4}$. In this case, the amplitude of the dissipation (dash-dotted line) and the advection (dashed) are equal at the height $kz \approx 0.1$. In figure 2.8 the inner region depths determined in this way for different values of $U_\lambda/c$ and $kz_0$ are collected. The lines in this figure are the inner region depths calculated with (2.33), where the factor $2\kappa$ in (2.33) has been replaced by $4\kappa$. This modification is of minor importance, as the scaling argument cannot fix the $O(1)$ factor. The resemblance between the values of $kl$ determined with the numerical model and those following from the scaling argument in figure 2.8 is striking. The relation (2.33) not only holds for waves growing by the wind ($0 < c/U_\lambda < 1$), but also for waves propagating faster than the wind ($c/U_\lambda > 1$) and against the wind ($c/U_\lambda < 0$).

We conclude that the scaling of the inner region depth in the air flow over waves as proposed by Belcher and Hunt (1993) (equation 2.33) is consistent with calculations using the LRR
Fig. 2.8: The inner region depth calculated with the LRR model (symbols) and relation (2.33) (lines). The surface roughness $kz_0$ is $10^{-3}$ for the solid line and the pluses, $10^{-4}$ for the dashed line and the squares and $10^{-5}$ for the dashed-dotted line and the diamonds.

scheme. The deviations from (2.33) reported by Beljaars and Taylor (1989) are probably due to their indirect definition of the inner region, or they may be caused by the fact that they used the inadequate eddy viscosity closure to model the air flow.

2.5 Conclusions

Calculations show that the energy flux from wind to waves is sensitive to the choice of closure scheme that is used to parameterize turbulent stresses. The scaling arguments used by Belcher and Hunt (1993) offer an explanation for this dependence. The turbulence schemes that rely on the eddy viscosity closure hypothesis give rise to larger modulations of the stress outside the inner region than a model that takes into account the advection of turbulence by the mean flow. As an example of such a scheme the turbulence model of Launder, Reece and Rodi (1975) has been used. The scale height of the inner region over waves calculated with this closure scheme shows excellent agreement with the relation proposed by Belcher and Hunt.
Chapter 3

Wave tank observations

The aim of this chapter is to compare detailed observations of the air flow velocity, pressure and turbulent moments over waves with model calculations. Two complementary data sets will be used for this purpose, made in two different wind-wave tunnels. The first set of observations was made in the facility of IRPHE in Marseille (Mastenbroek et al., 1995). Owing to the long fetch (28 m) in combination with a relatively short paddle wave (0.80 m) the height to which the boundary layer is developed extends well into the outer region. This makes that the data from this experiment are particularly well suited to study the dynamics in the outer region of the flow. As discussed in the previous chapter the three turbulence schemes differ mainly in the description of the turbulence in the outer region. The closure schemes that rely on the eddy viscosity hypothesis assume that the turbulence at any given point and time is in balance with the local velocity shear. In their analysis of air flow over waves, Jackson and Hunt (1973) presented a scaling argument that divides the flow into two regions, where only in the layer closest to the surface the assumption of local balance holds. This argument was extended to slowly moving waves by Belcher and Hunt (1993). When an eddy viscosity closure is used to model the stress in the outer region, it can be expected to overestimate the amplitude of the stress modulation. It is on this phenomenon that we will concentrate in our analysis of the data.

Simultaneous with the velocity measurements the wave-induced pressure was observed. To avoid acoustical contamination of these observations by the driving mechanism of the wave paddle the wave paddle was switched off before the data acquisition started. As the pressure perturbation has only a small vertical gradient close to the surface, comparison of the pressure measurements in the outer region with model calculations gives an indication of the ability of the model to describe the energy flux to waves.

In an earlier experiment, Hsu and Hsu (1983) used a wave follower to make observations very close to the water surface. In an attempt to observe the resonance phenomena predicted by Miles’ 1957 theory, the parameters of the experiment were chosen such that observations were made both below and above the critical height. This resulted in the choice of a relatively long paddle wave (1.6 m) and low wind speeds. Though the scientific interest in the critical height has faded, the observations are still very valuable as they scan through the transition from local balance in the inner region, to the rapid distortion in the outer region. Comparing model calculations with these observations, as is done in section 3.2 below, shows that the second order model is well suited to reproduce the main features of this transition.

In a pioneering experiment, Steward (1970) was the first to make detailed air flow mea-
measurements above water waves. In this experiment paddle waves with a length of 40 cm were used, in combination with low wind speeds (1-3 m/s). In section 4.2.4 will be argued that the Reynolds number of this flow is too small for the present models to be applicable. Therefore the comparison of the numerical model with laboratory data is limited to the datasets from IRPHE and Hsu and Hsu.

3.1 Observations in the IRPHE wind-wave tunnel

3.1.1 The experimental set-up

The experiments were conducted in the large IRPHE Wind Wave Tank described in detail by Favre and Coantic (1974). This facility consists of a water tank which is 40 m long, 3 m wide and 1 m deep. Above the water surface a closed loop wind tunnel produces an air flow with wind speeds of up to 16 m/s. The height of the air column is close to 2 m. The experiments are done at a fetch of 28 m. At this fetch the wind profile is observed to be logarithmic up to 40 cm above the water surface.

The experiment consisted of three runs. During each run the mean value of the wind velocity in the potential flow above the turbulent boundary layer was kept constant at 6.3, 5.4 and 3.5 m/s, respectively. The paddle waves were generated by means of a wave maker at the entrance of the water tank. The wave maker was completely submerged, in order not to disturb the air flow.

The longitudinal and the vertical turbulent velocity fluctuations were measured by means of a X-wire, with both wires mounted at a 45° angle relative to the mean wind direction. The hot wires were calibrated before and after the experiments in a small wind tunnel for mean velocities ranging from 3 up to 18 m/s. This large range is necessary because of the large fluctuations in wind speed in the turbulent air flow, particularly close to the surface. The calibration procedure is described in detail in Giovanangeli (1980). The systematic error in the velocity observations with the X-wires is estimated to be less than 5 × 10^{-3} m/s.

The pressure fluctuations were measured using a new method (Giovanangeli, 1988). In this method the static pressure is determined from the difference of the observed total pressure and the dynamical pressure derived from velocity measurements. The velocity measurements were done with the X-wires described above. The total pressure was measured using a bleed type pressure sensor. In this sensor a flow of helium gas through a capillary tube is created by connecting one side of the tube to a reservoir of helium at constant pressure. The other side of the tube is located inside a total pressure sensing head. The flow rate in the capillary tube depends on the pressure difference between the total pressure in the sensing head and the constant pressure in the reservoir. The flow rate is measured with a hot film sensor located near the exit of the capillary tube. The transfer function (gain and phase shift) of the pressure probe have been determined in detail in previous studies (e.g. Giovanangeli and Chambaud,
1987). It was shown that the pressure probe in combination with the method used here allows for measurements of the static pressure fluctuations in air flow, particularly close to the waves, with an accuracy of 0.05 Pa.

Other investigators have shown that the driving mechanism of the paddle can induce acoustic pressure fluctuations inside the wave tank (Latif, 1974; Papadimitrakis et al., 1986; Banner, 1990). They used different methods to correct for these effects on the pressure fluctuations estimates. Preliminary experiments in the IRPHE wind tunnel confirmed that when the wave maker is running it induces a strong acoustic pressure fluctuation. Rather than trying to correct for this contamination, we chose to avoid the effect completely by turning off the wave maker before the data acquisition starts. Therefore we used the following procedure in the experiments presented here. First a continuous wave field is generated with the wave maker in the presence of wind. Then the wave paddle is suddenly stopped and the data-acquisition is started. During about 30 s, corresponding to the time required for the end of the wave field to reach the experimental section, the pressure above the paddle wave can be observed without any acoustic contamination.

### 3.1.2 Averaging and decomposition

Let $q_i, i = 1, \ldots, N$ represent one series of observations of a certain quantity at a given height. In the present experiment, this quantity can be a velocity component or the pressure. As described above, after every 30 s of observations the data acquisition is stopped and the wave maker is started to generate new paddle waves. After switching off the wave maker the data acquisition is continued. For every series this procedure was repeated ten times. With a sampling rate of 150 Hz it follows that one series consists of 45000 individual observations. Simultaneously with the velocity and pressure, the water elevation $\eta$ was measured. The upward zero crossings in $\eta_t$ are used to mark the border between two consecutive waves. Each individual wave is then divided into $N_b$ bins. After sampling all data in these bins, we get a set $q_{jk}, j = 1, \ldots, N_b, k = 1, \ldots, N_j$, where $N_j$ is the number of observations that fall into bin $j$. Following Hsu et al. (1981) we now define two averaging procedures. The phase or ensemble average of property $q$ is given by:

$$\bar{q}_j = \frac{1}{N_j} \sum_{k=1}^{N_j} q_{jk}. \quad (3.1)$$

The average over a wave period, or simply the mean, is given by:

$$\langle \bar{q} \rangle = \frac{1}{N_b} \sum_{j=1}^{N_b} \bar{q}_j. \quad (3.2)$$

The deviation of an individual observation from its phase average is called the fluctuating part: $q'_{jk} = q_{jk} - \bar{q}_j$. As these fluctuations are mainly caused by turbulence (Hsu et al., 1981), the phase average (3.1) can be regarded as an ensemble average. Therefore we will use the same
Table 3.1: Parameters for the three sets of observations. The friction velocity \( u_* \) is determined from the mean velocity profile. \( N_h \) is the number of heights at which observations were made. In the last column the dimensionless depth \( k_l \) of the inner region is given.

symbol for the phase averaging procedure defined above and the Reynolds-average. The \( \text{wave-induced} \) part of quantity \( q \) is defined as the deviation of the phase average from the mean:  
\[
\tilde{q}_j = q_j - \langle q \rangle.
\]

Three Reynolds stress components can be found from the observations by applying the phase average (3.1) to the products \((u'v')_{jk}, (u'w')_{jk}\) and \((w'v')_{jk}\). In this paper we will refer to \( u'v' \) and \( w'v' \) as the horizontal and vertical velocity variance, respectively. The cross correlation is used to define the (turbulent) stress:  
\[
\tau = -\rho u'v'.
\]

To simplify the analysis we will often look at the amplitude of the first harmonic of the wave-induced part. Therefore we define the complex amplitude \( \tilde{q} \) such that:

\[
\tilde{q}_j = \frac{1}{2} \left[ \tilde{q} e^{i(2\pi j/N_h + \phi_r)} + \text{c.c.} \right] + \text{harmonics.} \tag{3.3}
\]

The phase \( \phi_r \) is chosen in such a way that the amplitude of the elevation \( \tilde{\eta} \) is real. This greatly simplifies the interpretation of the other (complex) amplitudes: the real part \( \text{Re}[\tilde{q}] \) gives the amplitude of \( \tilde{q} \) in phase with the elevation, and the imaginary part \( \text{Im}[\tilde{q}] \) the amplitude 90° out-of-phase with the elevation. A positive value of \( \text{Im}[\tilde{q}] \) corresponds to an enhancement of \( q \) above the windward slope of the wave.

There are two possible sources of errors in the averaged observations. The first is due to random errors that have not averaged out due to the finite length of the observation series, the second is caused by systematic errors of the probe. Based on the calibration, we believe that the systematic error is less than 0.005 m/s for the hot wires, and less than 0.1 Pa for the pressure probe. The uncertainty in the Reynolds stress components due to the systematic error in the hot wire is estimated as \( 4 \Delta U u_* \), where \( \Delta U \) is the systematic error of the hot wire. The friction velocity \( u_* \) is used as a measure of the turbulent fluctuations in the velocity observations. It follows from (3.3) that the error in the components of the amplitude \( \tilde{q} \) is \( \sqrt{2} \) times the error in the mean. In the figures presented in this chapter, error bars indicate the maximum of the random error and the estimated systematic error. For the amplitudes of the horizontal and vertical velocity components the contribution of the random error dominates; for the Reynolds stresses and the pressure amplitudes the limited accuracy of the probes determines the accuracy.
3.1 Observations in the IRPHE wind-wave tunnel

Fig. 3.1: Profiles of mean turbulent stress components for run 35 ($U_\lambda/c = 5.3$). The solid squares and the error bars denote observations and their estimated uncertainties. Model results with different turbulence closures are also shown: dash-dotted line, mixing length (shear stress only); dashed line, e-e; solid line, LRR turbulence scheme.

3.1.3 Procedure

The parameters for the three sets of observations are listed in Table 3.1. To simulate these experiments with the numerical model, four quantities need to be specified: the wavelength $\lambda$, the amplitude $a$, the wind speed $U_\lambda$ and the surface roughness $z_0$. The first two are easily obtained from the observations. The surface roughness $z_0$ and the wind speed $U_\lambda$ are determined by fitting a logarithmic profile through the observed mean wind speeds using a least-squares method.

The observations are performed at a fixed height above the mean water level. Necessarily the average $\langle \rangle$ over a wavelength for the observations has to be performed for constant $z$. In the numerical model the (Cartesian) velocities, pressure, stresses and variances are calculated on the wave-following $(\chi, \xi)$-grid. To comply with the procedure followed with the observations, the results are transformed to the $(x, z)$-frame (using linear interpolation between the grid points) before they are decomposed into mean and wave-induced parts. From the wave-induced properties a complex amplitude of the first harmonic is determined according to (3.3).

The velocities in the plots are normalized with the wind speed in the middle of the wave flume $U_\infty$, velocity variances with $u_\lambda^2$ and the stress $\tau$ and the pressure $p$ with $\rho_a u_\lambda^2$. The wind speed $U_\infty$ and the friction velocity $u_*$ can be found in Table 3.1. For the density of the air $\rho_a$ the value 1.3 kg m$^{-3}$ is used. Additionally, (amplitudes of) wave-induced quantities are normalized with the steepness of the paddle wave $ak$, which is also listed in Table 3.1.

3.1.4 Comparison

Mean turbulent stress and velocity variances. The friction velocity derived from the mean velocity profile is in good agreement with the observed mean turbulent stress $\langle \tau \rangle =$
Fig. 3.2: Profiles of the wave-induced amplitudes of the horizontal velocity, vertical velocity and pressure for run 35 ($U_\lambda/c = 5.3$). The solid squares and the error bars denote observations and their estimated uncertainty. Model results with different turbulence closures are also shown: dash-dotted, mixing length; dashed, $\epsilon$-$\epsilon$; solid, LRR turbulence scheme.

$-\rho_s \langle u'w' \rangle$. This can be seen in figure 3.1, where profiles of turbulent stress and velocity variances for run 35 are shown. The stress values, normalized with the friction velocity obtained from the velocity profile, are scattered around unity. From figure (3.1) it is also clear that the turbulent stress decays with height. This is a consequence of the horizontal pressure gradient that is forcing the wind in the wind tunnel. As the flow in the numerical model is forced by a constant velocity at the top of its domain, the stress in the model is not decaying with height.

The mean variance of the observed horizontal velocity $\langle u'u' \rangle$ also decays with height. As the variance of the vertical velocity is constant with height, the anisotropy $A = \langle u'u' \rangle / \langle w'w' \rangle$ increases towards the surface. Though this trend is again not reproduced by the model, the vertically averaged anisotropy in the velocity variances is reasonably reproduced by the LRR model. The $\epsilon$-$\epsilon$ scheme, which is quasi isotropic as it gives $\langle u'u' \rangle \simeq \langle w'w' \rangle \simeq 2\epsilon/3$, overestimates the turbulent fluctuations of the vertical velocity by a factor 2. Since the mixing length scheme does not provide the turbulent kinetic energy $\epsilon$, the horizontal and vertical velocity variances cannot be calculated with this scheme.
The wave-induced velocity and pressure. The profiles of the amplitudes of the wave-induced velocities, figure 3.2, show that the numerical model reproduces well the real part of the horizontal amplitude \( \text{Re}[\bar{u}] \) and the imaginary part of the vertical velocity \( \text{Im}[\bar{w}] \). This is the part of the undulating flow in phase with the wave. Due to the work of stresses close to the surface the phase of the undulating flow is shifted downwind, giving rise to non-zero values for \( \text{Im}[\bar{u}] \) and \( \text{Re}[\bar{w}] \). The numerical simulations show a smaller downwind shift of the undulating flow than the observations. This picture is consistent with the observations of the pressure perturbations. The real part of the amplitude of the pressure variation \( \text{Re}[\bar{p}] \) is slightly overestimated in the numerical simulations. However, the imaginary part of the pressure amplitude is underestimated by a factor 2 to 4, depending on the closure scheme. So both the velocity and the pressure observations indicate that the numerical model reproduces reasonably the in-phase part of the undulating flow, but it underestimates the out-of-phase part. Possible explanations for this underestimation are discussed in the next chapter. The eddy viscosity closures yield larger out-of-phase pressures, and hence show better agreement with the observations than the LRR scheme. As we have shown in the previous chapter, the larger values for the out-of-phase pressures obtained with the eddy viscosity closures are entirely due to modulations of the turbulent stress in the outer region. By comparing these stress modulations with observations, it will be shown below that these modulations are indeed erroneous. First we will use the observed values of \( \text{Im}[\bar{p}] \) to estimate which fraction of the surface stress is sustained by the paddle waves.

The phase shifts in the pressure observed in runs 35 and BA imply that the bulk of the momentum passes through the surface via the paddle waves. If we assume that most of the work on the wave is done by the out-of-phase component of the pressure, the fraction of the momentum flux supported by the paddle wave is (Townsend, 1972):

\[
\frac{\tau_w}{\langle \tau \rangle} = \frac{1}{2} (ak)^2 \beta. \tag{3.4}
\]

Extrapolating the observed amplitudes of the out-of-phase component of the pressure to the surface we find that the paddle wave is responsible for 80% of the momentum flux in the case of run 35, and for 55% in the case of run BA. If such large fractions of the downward momentum flux are supported by organized wave-induced motions, the turbulent momentum flux close to the surface must be significantly reduced. As the growth rate of slow waves is coupled to the surface stress (Plant, 1982), this means that paddle waves are able to suppress the growth of ripples in wind-wave tanks. The fact that the presence of paddle waves seems to reduce the level of wind ripples has been observed both in the IRPHE wind tunnel, and in others (e.g. Donelan, 1987).

The modulation of the turbulence. The differences between the three closure schemes are most apparent in the amplitudes of the Reynolds stress modulation (figures 3.3 and 3.4). As expected on the basis of the physical scaling argument given above, the mixing length
Fig. 3.3: Profiles of the wave-induced amplitudes of the stress, the horizontal and the vertical velocity variance for run 35 ($U_\lambda/c = 5.3$). Line styles as in figure 3.2.

The model overestimates the real part of the stress amplitude. In the LRR closure the amplitude in this part of the vertical domain is reduced by more than an order of magnitude. For all three runs, the observations show a much smaller amplitude than the corresponding mixing length calculation. The amplitudes obtained with the $e-\epsilon$ scheme, roughly one third of the mixing length amplitudes, are still significantly larger than the observed values. This is the first observational evidence of the failure of eddy viscosity closures in modelling the turbulence in the outer region. Even the $e-\epsilon$ scheme, which takes into account advection of turbulent kinetic energy, can be clearly seen to overestimate the turbulent stress modulation. It is this excess stress modulation that gives rise to the larger values for the out-of-phase pressure that follow from the eddy viscosity closures (see figure 3.2). Hence the better agreement that the results of the eddy viscosity closures show with the observed out-of-phase pressure amplitude is caused by an erroneous effect.

In the previous chapter it was noted that according to rapid distortion theory the irrotational straining of the anisotropic turbulence should reduce the turbulent horizontal velocity fluctuations above the crest. In the figures 3.3 and 3.4 this would be reflected in negative values for $Re[w'w']$. Compared to the quasi isotropic $e-\epsilon$ scheme the LRR scheme indeed shows
a reduction of $\text{Re}[u'u']$. The estimated size of the uncertainty in the observations, indicated in the figures with error bars, is far larger than the straining effect calculated with the LRR model.

For the vertical velocity variances rapid distortion theory predicts an increase above the crest if the turbulence is sufficiently anisotropic ($A > 3$). In run BA the lowest two observations show such an increase. As we have seen above in figure 3.1 the anisotropy of the turbulence increases towards the surface. In the case of run BA the observed anisotropy reaches a value of just larger than 3. The numerical model does not reproduce the observed increase in $\text{Re}[w'w']$. This may be a consequence of the fact that the (height independent) anisotropy of the turbulence in the LRR model is smaller than 3. The observations in the present data set do not come close enough to the surface to note the possible extra reduction of the stress due to the curvature effect. This effect is known to be effective at the top of the inner region, and the data points presented here fall well outside this layer.

**The height dependence of the wave stress.** Far from the surface, at heights where the flow distortions due to the waves can be neglected, the vertical momentum flux is maintained by turbulence. Closer to the surface correlations of organized wave-induced motions also contribute to the momentum flux. In a stationary and spatially homogeneous boundary layer the vertical
Fig. 3.5: The mean wave stress as a function of height. According to the mixing length model the wave stress decays slightly faster than \( \exp(-2kz) \). When the LRR model is used the wave stress goes to zero in the outer region. The solid squares indicate observed wave stresses in run BA.

Momentum flux must be constant with height. It follows that the turbulent momentum flux must be reduced close to the waves. This fact is used in most recent theories that try to give a description of the effects of waves on the atmospheric boundary layer. One of the many assumptions that enters these models is how the wave stress decays with height. Some argue that as the amplitudes of both \( u \) and \( w \) decay as \( \exp(-kz) \), the wave stress will decay like \( \exp(-2kz) \). Others have assumed it decays with height on a scale comparable to the amplitude \( a \) of the wave. Here we will show that the height dependence of the wave stress calculated with the numerical model depends on the closure model that is used. We will also make a comparison with the wind tunnel observations presented above. First we will discuss the importance of choosing the proper averaging procedure when calculating the wave stress.

When the horizontal averaging \( \langle \cdot \rangle \) is performed at a constant height, the wave stress is equal to \( \tau_w = -\rho_a \langle \bar{u}\bar{w} \rangle \). Since this flux is not defined for heights smaller than the amplitude of the wave, it is often considered convenient to define a flux travelling through a plane that follows the undulations of the surface. One option is to transform the \( x \)-momentum balance equation to the wave-following \( (\chi, \xi) \)-coordinate system, and to perform the averaging \( \langle \cdot \rangle_\xi \) keeping \( \xi \) rather than \( z \) constant. This leads to

\[
\langle \frac{\partial(h - \eta)\bar{u}}{\partial t} \rangle_\xi = \frac{\partial}{\partial \xi} \left( \frac{-\langle \bar{u}'\bar{w}' \rangle_\xi}{\tau_w/\rho_a} - \langle \bar{u}\bar{W} \rangle_\xi + \frac{\eta_a(\bar{u}' + \bar{w}'\bar{w}')(1 - \xi)_\xi}{\tau_w/\rho_a} \right),
\]

(3.5)
where

\[ W = \bar{\omega} - (1 - \xi)(\bar{u} - c)\eta_z. \] (3.6)

With the assumption that the flow is stationary, the temporal derivative on the left hand side of (3.5) becomes zero. Consequently the sum of the terms between the brackets is independent of height, though each term individually may depend on height. The second and third term between the brackets together represent the wave stress \( \tau_w \). The first of these two is zero at the surface \( \xi = 0 \) as due to the boundary conditions \( W = 0 \) at the surface. The second contribution, which has the shape of a pressure-slope correlation, arises purely because we average over an undulating surface. At the top of the domain, \( \xi = 1 \), where the surface over which we average is flat, the pressure-slope contribution disappears. In other words: when the wave stress through an undulating surface is calculated, correlations between the pressure (and velocity variances) and the slope of the wave surface \( \eta_z \) will arise.

In figure 3.5 calculated and observed profiles of the wave stress are shown for run BA. It is clear that the different types of closure give rise to different height dependencies. The mixing length model produces a wave stress that decays exponentially with height. The decay is slightly faster than \( \exp(-2kz) \), which is shown as a dotted line. In the LRR scheme the wave stress decays from its surface value to zero at the top of the inner region \( (kz = 0.09) \). At the bottom of the outer region the wave stress shows an overshoot and reverses sign to become negative. Townsend (1972) found a similar behaviour from his analysis. The observations of Hsu, Hsu and Street (1981) also show the overshoot, though this may be coincidental because Hsu et al. calculate the wave stress, which they termed wave-associated Reynolds stress, like \( \tau_w = \rho_a(\bar{u}w)_\xi' \), where \( \xi' \) is a transformed vertical coordinate not unlike our \( \xi \). Hence they neglected the contribution from the pressure-slope correlation, which is quite substantial close to the surface in the wave-following coordinates.

To enable a comparison with the observed wave stresses, the model results in figure 3.5 have been normalized to yield the same surface value of the wave stress as the observations. The observations, also plotted in figure 3.5, are not close enough to the surface to distinguish between the turbulence parameterizations. This is of course a consequence of the fact that the observations are made at a fixed height, and that we have to clear of the crests.

### 3.2 Comparison with wave-follower observations

In the laboratory experiment of Hsu and Hsu (1983) observations have been made of the air flow close to the surface of a mechanically generated wave. The velocity of the air is observed with X-wires mounted on a vertically moving wave follower to enable observations very close to the surface. In this way the air flow could be monitored as close as 1.1 cm from the average surface of the paddle wave, where the paddle wave had an amplitude of 2.7 cm. Keeping the length of the paddle wave constant at 1.6 m, observations were made for four different wind speeds. In the first two cases the wind speeds in the middle of the tunnel are almost equal to
Table 3.2: The wind speed, friction velocity and wave steepness for the four runs of the experiment of Hsu and Hsu (1983). The inner region depth $kl$ is estimated with (2.33).

<table>
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<th>Run</th>
<th>$U_\infty$ (m/s)</th>
<th>$u_*$ (m/s)</th>
<th>$ak$</th>
<th>$U_\lambda$ (m/s)</th>
<th>$u_*$ (m/s)</th>
<th>$kl$</th>
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<td>1.1</td>
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<tr>
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<td>0.073</td>
<td>0.105</td>
<td>2.42</td>
<td>0.081</td>
<td>0.31</td>
</tr>
<tr>
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<td>0.110</td>
<td>0.115</td>
<td>3.51</td>
<td>0.126</td>
<td>0.14</td>
</tr>
</tbody>
</table>

the phase speed of the paddle wave. Both the observations and the model calculations show that the air flow in that case is only weakly affected by the presence of the paddle wave. For this reason we will concentrate on the two cases where the wind speeds are larger than the phase speed of the paddle wave.

The most interesting aspect of the Stanford experiment is that the observations cover both the inner and the outer regions. This is achieved by the combination of the use of a wave follower to get close to the surface, and by making the observations for moderate values of $U_\lambda/c$. As was shown in the previous chapter, the depth of the inner region is largest when $U_\lambda/c$ is just larger than one. In the two runs we will consider here $U_\lambda/c = 1.5$ and 2.2. The observations can therefore be used to study the performance of the model in the transition region between the inner and the outer region. In this transition region the model has to find the right balance between the local effects important in the inner region, and the rapid distortion effects dominant in the outer region. As the comparison with the IRPHE data has convincingly shown that the eddy viscosity closures fail in the outer region, we will concentrate in this section on the performance of the LRR second-order scheme.

### 3.2.1 Procedure of comparison

For each of the four wind speeds used in the Hsu and Hsu experiments, $U_\lambda$ and $u_*$ were determined by fitting a logarithmic profile to the observed mean wind speed. The resulting friction velocities differ somewhat from the values obtained by Hsu and Hsu (see table 3.2), and are similar to those reported by Harris et al. (1996) who performed a similar analysis. Hsu and Hsu obtained their (lower) estimates of the friction velocities from the cross correlation $\overline{u'w'}$. For the simulations with the numerical model we want the mean wind profile to match the observed wind. Therefore we use the friction velocity derived from the wind profile to calculate the surface roughness.

To enable observations of the air flow close to the surface, the X-wires were mounted on a wave follower. This is done by keeping the probe at a constant $z^*$, where $z^*$ is a wave-following
vertical coordinate:

\[ z = z^* + f(z^*) \eta(x, t), \quad \text{with} \quad f(z^*) = \frac{\sinh(kH - kz^*)}{\sinh kH}, \quad (3.7) \]

where \( kH = 3.9 \). Hsu and Hsu present their results as amplitudes and phases of wave-induced velocities or stresses. The horizontal averaging procedure ( ) used to distinguish between mean and wave-induced motions were performed keeping \( z^* \) constant. To match this condition, the results from the numerical model were also averaged in this way. This is done by first interpolating the velocities and stress from the \((\chi, \xi)\) model grid to the \((\chi, z^*)\) wave follower grid and then average the result over horizontal grid lines. The velocities Hsu and Hsu report are the Cartesian velocities relative to a fixed frame of reference. To obtain the vertical component of the wind velocity the motion of the wave follower is added to the apparent vertical velocity observed by the probe.

### 3.2.2 Comparison with model calculations

Hsu and Hsu performed observations for four different wind speeds. The ratio of the wind speed and the phase speed of the paddle wave for the first two runs is almost equal to 1 \((U_\lambda/c = 1.1\)
Fig. 3.7: Amplitudes of velocity and stress perturbations for run 4 \((U_{\lambda}/C = 2.2)\) of the Stanford experiment. Symbols and lines as the previous figure.

and 1.2 respectively). Both the model calculations and the observations show little impact of the wave on the air flow in that case. Therefore we concentrate on the two runs with larger wind speeds. The wind speed to phase speed ratio in run 3 and 4 is 1.5 and 2.2, respectively. The profiles of the real and imaginary parts of the wave-induced velocity components and the stress are plotted in the figures 3.6 and 3.7. The observations are shown as solid squares and the lines indicate model calculations. The inner region depths for run 3 and 4, estimated with (2.33) and the parameters listed in table 3.2, are also shown in figures 3.6 and 3.7. As for \(U_{\lambda}/c > 1\) the inner region depth reduces when \(U_{\lambda}/c\) is increased, the inner region depth for run 3 is larger than for run 4. In the latter run most observations fall in the outer region, whereas in run 3 about half the observations fall in the inner region. So the observations of run 3 are particularly interesting for studying the behaviour of the numerical model as it makes the transition from inner to outer region.

In the previous chapter it was noted that the top of the inner region is characterized by a maximum in the velocity speed-up above the crest and a stress maximum above the trough (see for instance figure 2.6). Both these features are present in the observations shown in 3.6 and 3.7. The height where the maximum velocity speed-up occurs decreases when the wind speed is increased from run 3 to 4, as does the height of the shear stress maximum above the trough. Both phenomena are related to the reduction of the inner region depth as
the windspeed is increased. The model reproduces the amplitude of both the velocity speed-up and the shear stress perturbation very well. The heights where both maxima occur is shifted downward compared to the observations. Apparently the thickness of the inner region is underestimated by the model. Despite this mismatch in the apparent thickness of the inner region, the comparison shows that the main features in the observations are reproduced by the model.

The largest deviation from the observations for the velocity components is found for the imaginary part of the vertical velocity, $\text{Im}[\tilde{w}]$. The profile resulting from the simulation does not depend strongly on the particular combination of $U_\lambda$ and $u_*$ which is chosen. Calculations show that a variation of 20% in both the wind speed $U_\lambda$ and in the friction velocity $u_*$ does not alter the results for $\text{Im}[\tilde{w}]$ significantly. Since the model agrees rather well with other three velocity components, and the model may be supposed to obey the continuity equation, the observations seem to violate this constraint. In contrast, the comparison of numerical simulations with the IRPHE experiment above showed excellent agreement between calculated and observed profile of $\text{Im}[\tilde{w}]$. These observations were made at a fixed height, where Hsu and Hsu employed a wave follower. Since the wave follower moves up and down with the elevation of the water surface, an apparent vertical velocity is introduced 90° out-of-phase with the elevation. This apparent vertical velocity was subtracted from the observations by Hsu and Hsu to obtain the Cartesian velocities. The only velocity component affected by this operation is $\text{Im}[\tilde{w}]$. We will now show that the correction is of the same order of magnitude as the discrepancy between model and observations. With the probe motion given by (3.7), it follows that the imaginary part of the vertical velocity amplitude of the probe is:

$$\text{Im}[\tilde{w}_p] = -akcf(z^*). \quad (3.8)$$

To obtain the Cartesian vertical velocity, Hsu and Hsu added the velocity of the probe to the apparent velocity observed by the moving probe. Since (3.8) yields a negative number, this reduced $\text{Im}[\tilde{w}]$. What we find is that the discrepancy between model and observations largely disappears if we subtract twice the correction (3.8) from the vertical velocity as reported by Hsu and Hsu. For the runs 3 and 4 the velocity components obtained in this way are shown as open squares in the figures 3.6 and 3.7. For the two runs with the low wind speeds, not shown here, a similar improvement is found. This shows that the difference for $\text{Im}[\tilde{w}]$ between model and observations is of the same order as the correction due to the vertical motion of the probe. Why the discrepancy seems to be exactly equal to minus 2 times the correction (3.8) remains an open question.

Harris et al. (1996) also analyse the Hsu and Hsu observations by comparing them to model calculations. To model the effects of the waves on the air flow they use a modified $e-\varepsilon$ scheme. In their approach the stress perturbations resulting from the use of an eddy viscosity closure were suppressed in the outer region using a damping function. In the more general second-order model used here, no such damping function is required. Harris et al. also took into account the
drift velocity of the water, the effect of the horizontal pressure gradient, the dynamical effects of the molecular viscosity and the geometry of the wind tunnel. These effects seem to be of minor importance judging from the overall similarity of our calculations with their results.

3.3 Conclusions

The comparison of observations in the IRPHE facility with numerical simulations shows that turbulence closure schemes based on the eddy viscosity concept overestimate the modulation of the turbulent shear stress in the outer region. A similar overestimation was found by Belcher et al. (1993) in stress profiles observed over hills. Eddy viscosity schemes imply that the turbulent eddies are in equilibrium with the instantaneous shear at each point in the flow. Physical scaling arguments explain that this assumption is violated in the outer region of the flow over waves. Numerical simulations with the LRR model, which does not rely on an eddy viscosity, show good agreement with the observations of the shear stress modulation in the outer region.

In the same experiment not only the velocity and stresses, but also the pressure was observed. The calculated amplitudes of both the pressure and the velocity perturbations show excellent agreement with the observations. Comparison of the observed phase of the pressure and velocity perturbations with numerical simulations reveals that the model underpredicts the downwind phase shift of the undulating flow by approximately a factor 4. As the down-wind shift induces the out-of-phase pressure on the surface of the wave which makes the wave grow, this result implies that the model seems to underestimate the growth rate of the paddle wave by the wind.

In another laboratory experiment, Hsu and Hsu (1983) mounted their sensors on a wave follower to enable observations close to the water surface. The main features in their observations, a velocity maximum above the crest at the top of the inner region and a maximum of the stress above the trough at the same height, are well reproduced by the numerical model. The observations also show that the heights at which these maxima occur are reduced when $U_\lambda/c$ is increased. Though the model reproduced this trend, the actual heights are underestimated by the model. The model shows good agreement with three out of four wave-induced velocity amplitudes. In all four experiments reported by Hsu and Hsu the observed and calculated value for the imaginary part of the vertical velocity amplitude differ by twice the vertical velocity of the wave-following probe. The reason for this coincidence is not known.
Chapter 4

The growth rate of waves

One of the most important aims of studies of the air flow over waves is to calculate the growth rate of waves due to the wind. Knowledge of the dependence of the growth rate of a wave on the windspeed and other environmental parameters is of great practical importance for the modelling of waves on the oceans. In the numerical models that are used to describe and predict the sea state, the energy input from wind to waves is parameterized in terms of a growth rate (e.g. Komen et al., 1994). Here a brief overview of the available measurements of this growth rate will be presented. Though these observations show a considerable scatter, they seem to indicate that the growth rates as they are calculated with the present model are too small. In the calculations presented so far, we have made some strong assumptions concerning the wave surface and the turbulence, that may have affected the results. The effect on the growth rate of some of these assumptions will be investigated here. Finally, the growth rates calculated with the present model are compared with several theories found in the literature.

4.1 Growth rate observations

Since the late 1960's, attempts have been made to measure the energy flux from wind to waves. Shemdin and Hsu (1967), Larson and Wright (1975) and Wu et al. (1977, 1979) measured the rate at which wind waves grow in a wave tank. After compensating for the wave damping due to viscosity in the water, they obtained the growth rate of waves due to the wind. Snyder et al. (1981) present observations of the growth rate made on the open ocean. They deduced the growth rate from observations of the wave-induced pressure close to the water surface. Plant (1982) collected all these observations (reproduced in figure 4.1). He concluded on the basis of these observations that the growth rate for a wide range of relative wind speeds, $1 < c/u < 20$, was $\beta = 32 \pm 16$ (shown as dotted lines in figure 4.1). Later Hasselmann and Bösenberg (1991) repeated the experiment of Snyder et al. (1981) for slightly different conditions. Their observations for the growth rate are in agreement with the parameterization of Plant (1982).

In the previous chapter we have concluded that, of the three turbulence parameterizations considered here, the LRR second-order model gives the best agreement with observations of the air flow over waves. In figure 2.3 on page 22 the growth rate calculated with this scheme is shown as a solid line. Below we will show that these growth rates are in agreement with those following from the analysis of Belcher and Hunt (1993). When we compare these theoretical growth rates with the observations in figure 4.1 we see that the model seems to underestimate the growth rates for waves that are strongly forced by the wind. For these slow waves, with
Fig. 4.1: Growth rate observations collected by Plant (1982) (after Belcher et al., 1994).

$c/u_* < 10$, the model gives $\beta \approx 13$, which is slightly lower than the lower limit allowed by the parameterization of Plant. The growth rates calculated for faster waves show better agreement with the observation. The growth rates in figure 2.3 are calculated for a dimensionless roughness of $kz_0 = 10^{-4}$. The growth rates calculated with the LRR model show only a weak dependence on this parameter: when $kz_0$ is varied between $10^{-5}$ and $10^{-3}$ the growth rates for slow waves varies only by 20%.

The calculations shown in figure 2.3 are made for a strongly idealized case. In order to simplify the model, and to minimize the number of parameters, many approximations were made. Not all of these approximations may be valid for the conditions under which the growth rate observations were made. In the remainder of this chapter we will examine some of these assumptions, and try to assess the implications these assumptions may have on the calculated growth rate.

4.2 Assumptions in growth rate calculations

4.2.1 Resolution

The computation time required by a model run is primarily determined by the vertical resolution. A single calculation with 80 vertical layers and a stretching of 6%, integrated over the time it takes the boundary layer to settle, requires several hours to complete on the hardware available for this study (a SUN SPARC-10 workstation). In practice the smallest difference between the surface and the first model layer that could be achieved is $kz_1 = 3.7 \times 10^{-3}$. As we
4.2 Assumptions in growth rate calculations

Fig. 4.2: The effect of varying the vertical resolution on the growth rate, using the mixing length (left) and the LRR second-order (right) closure schemes. Dotted lines: low resolution (lowest layer at $kz = 0.03$); solid lines: higher resolution (lowest layer at $kz = 0.0075$); dashed lines, high resolution (lowest layer at $kz = 0.0037$).

will show below, this is low enough not to influence the results.

In figure 4.2 the growth rate calculated with different vertical resolutions is plotted. For the left panel the mixing length scheme is used, for the right panel the LRR second-order scheme. Three different vertical resolutions were used. For the calculation with the lowest resolution (dotted lines with open squares) 20 layers were used, with a stretching of 19%. In that case the lowest layer is located at $kz_1 = k\eta + 0.03$. Increasing the resolution to 60 layers, with a stretching of 8%, ($kz_1 = 5 \times 10^{-3}$, solid lines in figure 4.2) does not affect the growth rates calculated with the mixing length scheme significantly. Therefore we may conclude that for this turbulence scheme 20 vertical layers is sufficient. This resolution was used for the calculations presented in Burgers and Makin (1993). However, when the second-order scheme is used (right panel of figure 4.2), the situation is different. Here the increase in resolution from 20 to 60 vertical layers had a profound impact on the calculated growth rates. The reason for this difference in behaviour between the two turbulence schemes is that the dominant growth mechanism for these two schemes is different.

In section 2.4.5 on page 27 the relation between turbulence scheme and growth mechanism was examined. There it was concluded that when the mixing length model is used, the modulation of the stress in the outer region is the dominant contribution to the growth rate. As the lowest layer in the low resolution run is located at $kz_1 = k\eta + 0.03$, the bulk of the outer region is resolved in this calculation. Increasing the resolution therefore has little or no effect on
the growth rate. For the second-order model the situation is different. The contribution of the shear stress modulation in the outer region to the growth rate is insignificant. In this model the downwind shift of the undulating flow is predominantly caused by the shear stress modulation in the inner region. The depth of the inner region for slow waves is of the order $kl = 10^{-1}$ (see figure 2.4). Therefore this region is not resolved in the low resolution run, leading to a near zero growth rate for these waves. For fast waves, which can have an inner region as deep as $kl = 1$, a part of the inner region is resolved by the low resolution run. This explains that for these waves the effect of increased resolution is smaller.

The above analysis leaves the question open how much of the inner region should be resolved. To tackle this problem we have to examine the approximations we make in applying the surface boundary conditions. As described in section 2.3.2 on page 17 we assume that the tangential stress is constant with height between the surface and the first model layer. From the analysis of Belcher and Hunt (1993) and the preceding studies we know that this assumption holds in the lowest part of the inner region. This sublayer has been termed the inner surface layer (ISL). In figure 2.4 it can be seen that the thickness $k\delta$ of this layer varies between $10^{-3}$ and $10^{-2}$. Therefore a resolution of 60 layers, with the lowest layer at $kz_1 = 5 \times 10^{-3}$, is sufficient for waves with a roughness larger than $kz_0 = 10^{-5}$. Calculations with a smaller dimensionless roughness, which may be relevant for the flow over hills, would require a finer resolution.

This discussion also illustrates the reason why attempts to model the flow over hills and waves with large eddy simulation (LES) models have failed. As in the model used here the lowest layer in the LES model has to be located at a distance $k\delta$ from the surface. The influence of a wave or hill stretches to about $kz \simeq 10$, which determines where the upper boundary of the model has to be located. In an LES model the possibilities of varying the resolution of the model in the vertical are limited. Therefore the number of grid points required to model a flow over hills or waves with a, necessarily three dimensional, LES model is:

$$ N \sim \left( \frac{10}{k\delta} \right)^3. $$

(4.1)

Even for a relatively rough wave with $kz_0 \sim 10^{-3}$, and an inner surface layer depth of $k\delta \sim 10^{-2}$, the number of grid point would be as large as $10^8$.

### 4.2.2 Angle between wind and wave

In their analysis of the growth rate measurements, Snyder et al. (1981) assumed that the growth rate depends only on the ratio of the component of the wind speed in the propagation direction of the wave to the phase speed of the wave. The numerical model used here is general enough to check this hypothesis. This is done by prescribing a non-zero velocity component in the $y$-direction at the upper boundary of the model: $v_y \neq 0$. This velocity component will influence the air flow in the along wave direction via the isotropic part of the turbulent kinetic energy production $P$, and via the $\overline{w'v'}$ contribution to the turbulent kinetic energy $\epsilon$. In figure
4.2 Assumptions in growth rate calculations

Fig. 4.3: The relative growth rate as a function of the angle between wind and wave for two cases.

4.3 the angular distribution of the growth rate is plotted for two cases. Here the absolute wind speed $U_\lambda$ is kept constant, and the direction between wind and wave is varied. The distribution for a wave with $U_\lambda/c = 10$ is typical for waves that are much slower than the wind. The growth rate for waves travelling at an angle less than 90° to the wind shows a $\cos^2 \theta$ dependence on the angle $\theta$ between wind and wave. For waves that are almost as fast as the wind the angular distribution is much narrower. In the example shown in figure 4.3 $U_\lambda/c = 1.5$ (dashed line). When in this case the wind makes an angle of 30° with the propagation direction of the wind, the along wave component has reduced enough to make the growth rate zero.

The results in figure 4.3 are consistent with the hypothesis that the ratio of the along wave wind speed component to the phase speed is of overriding importance for the growth rate. In both cases that are shown, it is when this ratio comes close to unity when the wave growth quenches. To check whether the cross wind component is indeed irrelevant for the wave growth, the growth rate $\gamma/\omega$ is plotted in figure 4.4 as a function of the ratio of the along wave wind speed to phase speed ratio. In this plot the growth rate for five sets of calculations is shown, where each set has a different value for $U_\lambda/c$. The angle between wind and wave is varied between zero and 60°. Figure 4.4 clearly shows that all the calculated growth rates collapse onto a single curve when plotted as a function of the along wave wind speed to phase speed ratio. This implies that the size of the cross wind component is of secondary importance for the growth rate, and hence that the assumption of Snyder et al. (1981) is consistent with the calculations presented here.

Related to the subject of angular dependence of the growth rate, is the issue of the damping rate of both upwind travelling waves, and of waves travelling faster than the wind. On the
The growth rate $\gamma/\omega$ as a function of the component of the wind speed in the propagation direction of the wave for four cases of $U_\lambda/c$.

basis of their pressure observations, Hasselmann and Bösenberg (1991) found that both up and downwind travelling waves with phase speeds exceeding the wind, i.e. $-1 < U_\lambda/c < 1$ are not damped by the wind. Given the finite accuracy of their observations, they concluded that the absolute value of the growth rate $(\rho_w/\rho_a)\gamma/\omega$ should be smaller than 0.03. Our model calculations show that waves travelling faster than the wind are damped, but the damping rate is within the margin given by Hasselmann and Bösenberg (see figure 4.4). A damping rate this small is of little practical importance: a value of -0.03 for $(\rho_w/\rho_a)\gamma/\omega$ implies that a wave is damped on a timescale roughly equal to 5000 times the period of the wave. For a 10 second wave this is almost half a day.

In figure 4.5 the growth rate $\beta$ for both up and downwind travelling waves is plotted as a function of $c/U_\lambda$. The calculations show that all upwind travelling waves are damped by the wind. In this respect the numerical model results differ qualitatively from the Miles (1957) theory. In the latter theory the interaction between waves and atmosphere is governed by the so-called critical layer. This is the layer where the wind speed matches the phase speed of the wave. As no critical layer exists for upwind travelling waves, these waves do not interact with the atmosphere according to the Miles theory. In the present numerical model, as well as in the analysis of Belcher and Hunt (1993) the critical layer plays no special role. Rather than in the critical layer, the interaction between wind and waves is governed by the wave-induced turbulence in the inner region. This region exists both for downwind and for upwind travelling waves, as well as for stationary waves or hills. Therefore the upwind travelling waves and hills can be treated within the same theoretical framework as the downwind travelling waves.
4.2 Assumptions in growth rate calculations

Fig. 4.5: Contributions of the tangential (dash-dotted line) and normal (dashed) stress to the growth rate, both for up and downwind travelling waves. The square corresponds to the sheltering coefficient (divided by \( \pi \)) for the hill.

The results in figure 4.5 show that for the waves shorter than, say, a few meters waves the damping can be significant. A 10 cm wave running against a 10 m/s wind is damped on a timescale of 2 s, and a 1 m wave in the same conditions will lose most of its energy to the atmosphere in about a minute. The calculations show that for upwind travelling waves, roughly 30% of the energy is transferred by the tangential stresses. In the case of slow, downwind travelling waves the contribution of these stresses was negligible.

In the laboratory experiment of Plant and Wright (1980) the phase speeds of upwind and downwind travelling waves were measured using microwave Doppler spectrometry. The upwind travelling waves were caused by the reflection of wind generated downwind travelling waves from the beach at the far end of the wave flume. No upwind travelling waves of 4 cm could be detected at a distance of several meters from the downwind beach, whereas enough waves of 10 cm and longer survived to be observed. Using the damping rate from figure 4.5, we find that for a typical wind speed of 5 m/s an upwind travelling 4 cm wave is damped on a time scale of 2 s, in which it can travel 20 cm. A 10 cm subject to the same wind is damped on a length scale of 1 m. Though this estimate is crude and ignores possible wave-wave interactions, it explains why only upwind travelling waves longer than 10 cm could be detected by Plant and Wright.

Above it was stated that upwind travelling waves and hills can be treated with the same theory as the downwind travelling waves. In that case one would expect to see a some continuity in the results going from very slow downwind travelling waves via stationary waves to very slow waves moving upwind. In all three cases the phase speed is non-existent or negligible compared
to the wind speed, so its exact size and direction should not influence the results. To show that this continuity in trends indeed exists, the growth rates for upwind travelling waves are folded around the axis \( \beta = 0 \) in figure 4.5. This operation is equivalent to plotting the out-of-phase pressure (as well as the in-phase tangential stress) as a function of \( c/U_\lambda \) keeping the phase speed \( c \) constant. Both the tangential and the normal stress contributions to the damping of upwind travelling waves can now be seen to continue trends visible in the downwind travelling branch of the plot. Using the fact that both the growth rate due to pressure, and the sheltering coefficient \( S \) for hills can be expressed in terms of the imaginary part of the pressure, we also include the results for hills in figure 4.5. For gentle waves the growth rate \( \beta \) due to the pressure is \( \text{Im}[\tilde{p}]/\rho_a u_*^2 \). With the definition of \( S \) given by (2.25), we find that \( S/\pi \) is corresponds to the growth rate \( \beta \) of a wave with zero phase speed. The value for \( \beta \) calculated from the sheltering coefficient lies exactly between the two branches of the normal stress contributions to the growth rates. As a hill has no orbital velocity, no equivalence for the tangential stress contribution to the sheltering coefficient exists.

4.2.3 Steepness

In the calculations shown in this chapter, the wave steepness was chosen so small, that its actual value, \( ak = 0.05 \), did not influence the results. Here we will consider what will happen if we increase the wave steepness. In figure 4.6, where the growth rate \( \beta(c/u_*) \) is plotted for three different steepnesses, we see that the growth rate for slow waves \( c/u_* < 10 \) increases with increasing steepness. The reverse is happening for the fast waves \( 10 < c/u_* < 20 \): here
4.2 Assumptions in growth rate calculations

Fig. 4.7: The effects of a varying steepness on the surface pressure distribution for $U_\lambda/C = 10$ (upper panel) and $U_\lambda/C = 2$ (middle panel). The wave steepness $ak$ is varied from 0.1 to 0.4. The surface elevation is plotted in the bottom panel.
Fig. 4.8: The distribution of the stream function and the pressure over a wave with a low steepness ($ak = 0.1$). The wind speed to phase speed ratio is $U_\lambda/c = 2$. 
4.2 Assumptions in growth rate calculations

Fig. 4.9: Same as previous figure, but for a steeper wave. Here \( \alpha k = 0.4 \).
the growth rate reduces when the steepness increases. The fact that the presence of these steep wave does not only affect the wave-induced properties, but also the mean wind and stress profiles, makes these results hard to interpret. To find out at what steepness the air flow starts to respond in a qualitatively different way, it is more instructive to look at the surface pressure exerted on the wave. In figure 4.7 the surface pressure, as a function of the phase of the wave, is shown for two different ratios of $U_\lambda/c$. The first case, $U_\lambda/c = 10$, is typical for a slow wave. Increasing the steepness from $ak = 0.1$ to $0.3$ has no dramatic effect on the surface pressure distribution. This is not true for waves that have a phase speed comparable to the wind speed. In the example shown in figure 4.7, $U_\lambda/c = 2$, a distinct secondary pressure maximum develops on the downwind slope of the wave when the steepness is increased from $0.1$ to $0.4$. At the same time the pressure maximum located upstream of the trough shifts towards the windward side of the crest and reduces in magnitude. To find out what the origin of these peculiar pressure distributions is we have to look at the flow pattern over this wave.

In the figures 4.8 and 4.9 the streamlines and pressure distributions are shown for the case of a gentle slope ($ak = 0.1$) and a steep slope ($ak = 0.4$), respectively. For these calculations the wind speed at height $\lambda$ was twice the phase speed of the wave. In this case increasing the steepness has two effects. By extracting more momentum from the air flow, the steep wave decelerates the mean wind speed in the layer between the surface as $z = \lambda$. This reduces the amplitude of the pressure perturbation (when scaled with $ak \rho u^T$). At the same time a large pressure gradient develops just downwind of the crest. This shock-wave like feature is located at the point where the return flow from behind the crest pushes the air flow up, thereby accelerating the oncoming flow.

### 4.2.4 Reynolds number

In the present study we assume that the Reynolds number of the flow over the wave we consider is large, as the effects of the molecular velocity on the turbulence are ignored. For short waves, or for low enough wind speeds, this assumption can no longer be valid. When the inner region and the viscous sublayer overlap too much, viscous effects will start to affect the dynamics of the flow over the wave. The interaction between the wave and the turbulence takes place in the upper part of the inner region, in the so-called shear stress layer. In the lower part, the inner surface layer, the stresses are constant with height. Hence we conclude that viscous effects can be ignored if the shear stress layer is well clear of the viscous sublayer:

$$\delta > 100 \nu / u_*.$$  \hspace{1cm} (4.2)

Here $\delta$ is the depth of the inner surface layer. For waves with $kz_0 = 10^{-3}$ we know that $k\delta = O(10^{-2})$. Taking this estimate we find that for the Reynolds numbers $R_\lambda = \lambda u_* / \nu$ smaller than $6 \times 10^4$ viscous effects can no longer be ignored. Harris et al. (1996) made calculations with a model that includes low Reynolds number effects. They found that these
4.2 Assumptions in growth rate calculations

![Graph showing varying anisotropy with different values of \( A \)](image)

**Fig. 4.10:** The effect of changing the anisotropy of the turbulence in the LRR scheme on the growth rate.

Effects become important for \( R_\lambda < 2 \times 10^4 \), which agrees reasonably with the estimate given above.

The importance of low Reynolds number effects on the dynamics in wind tunnel experiments depends on the combination of paddle wave length and wind speed that is used. The first to make detailed observations of the airflow over waves was Stewart (1970). He used a relatively short paddle wave (0.41 m) and low wind speeds (1-3 m/s), so the Reynolds number \( R_\lambda \) was of the order \( 10^3 \) in this experiment. A proper analysis of this experiment therefore has to take into account the fact that the inner region and the viscous sublayer overlap. In the two wave tank experiments analysed in the previous chapter, in the IRPHE and Stanford facilities, the wave Reynolds number is about an order of magnitude larger. Therefore we expect that for these experiments the low Reynolds number effects on the dynamics of the turbulence in the inner region is marginal.

The modelling of turbulence close to a surface is notoriously difficult. In the context of a Reynolds stress model, the parameterizations of the pressure-velocity correlation terms \( \Pi_{ij}^{(1)} \) and \( \Pi_{ij}^{(2)} \) used in the LRR model are not valid close to a solid wall. It is also believed that the dissipation is not isotropic close to a surface. Several modifications to the parameterization of the pressure-velocity correlation terms have been proposed, to enable the model to reproduce the well-known distributions of velocity and velocity variances above a flat plate (e.g. Hanjalić and Launder (1975) and Durbin (1993)). However, these methods differ widely and no consensus has been reached on what is the best approach.

It is well known that close to a surface the anisotropy of the turbulence is enhanced: near
Table 4.1: Equilibrium values of the three velocity variances for different values of $c_2$ and $c_\epsilon$ in the LRR second-order turbulence scheme. Also listed is the anisotropy $A = \overline{u'w'/u'_w}^2$.

<table>
<thead>
<tr>
<th>$c_2$</th>
<th>$c_\epsilon$</th>
<th>$\overline{u'u'/u'_w}^2$</th>
<th>$\overline{v'v'/u'_w}^2$</th>
<th>$\overline{w'w'/u'_w}^2$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.267</td>
<td>2.6</td>
<td>1.7</td>
<td>1.3</td>
<td>2.0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.225</td>
<td>3.0</td>
<td>2.0</td>
<td>1.3</td>
<td>2.4</td>
</tr>
<tr>
<td>0.6</td>
<td>0.174</td>
<td>3.5</td>
<td>2.5</td>
<td>1.2</td>
<td>2.8</td>
</tr>
<tr>
<td>0.7</td>
<td>0.113</td>
<td>4.4</td>
<td>3.2</td>
<td>1.3</td>
<td>3.5</td>
</tr>
</tbody>
</table>

By varying the constants $c_2$ and $c_\epsilon$ of the LRR model, it is possible to vary the anisotropy of the turbulence in this model. The parameter $c_2$ governs the behaviour of the rapid part $\Pi^{(2)}$ of the pressure-velocity correlation term, and thereby influences the anisotropy of the turbulence in the model. When $c_2$ is increased, the anisotropy of the turbulence also increases (see table 4.1). The parameter $c_\epsilon$ is the factor before the diffusive transport term of the balance equation for $\epsilon$. This parameter is calculated from the constraint that the logarithmic velocity profile is a solution of the model in the absence of a wave. From this figure it is clear that the growth rate of slow waves ($c/u_w < 5$) is insensitive to changes in the anisotropy. The growth rate of the faster waves increases with increasing anisotropy. This seems to indicate that the growth of the faster waves is more sensitive to changes in the parameterization of the rapid term $\Pi^{(2)}$, possibly because of a relatively large contribution of rapid distortion effects in the outer region to the growth mechanism.

### 4.2.5 Non-uniform roughness distribution

Waves of a given length are known to modulate the energy of shorter waves. The uneven distribution of these shorter waves over the long wave surface will influence the air flow over this long wave. When the short waves are considered as roughness elements, their modulation amounts to a modulation of the surface roughness. Both Gent and Taylor (1976) and Belcher and Hunt (1993) found that a variation in the surface roughness could have a significant effect on the growth rate. We can also use the present model to assess the sensitivity to changes in the roughness distribution. We will do that by modulating the drag coefficient $c_\lambda$, which is related to the surface roughness:

$$c_\lambda = \frac{\kappa^2}{\ln^2 \lambda/z_0}. \quad (4.3)$$

This drag coefficient is modulated by:

$$c_\lambda(\chi) = \tilde{c}_\lambda(1 + a k \tilde{c}_\lambda e^{i \chi}), \quad (4.4)$$
3. Effect of a non-uniform surface roughness distribution on the growth rate. As a reference the growth rate for a uniform roughness is shown ($\hat{c}_\lambda = 0$). For the dotted line a roughness modulation $\hat{c}_\lambda = 1$ was imposed, for the dashed line $\hat{c}_\lambda = -i$.

where $\hat{c}_\lambda$ is the complex amplitude of the drag coefficient modulation. The roughness distribution along the wave surface follows from the two above equations:

$$z_0(x) = 2\pi e^{-x/c_\lambda(x)^{1/2}}.$$  \hspace{1cm} (4.5)

In figure 4.11 the effect of two different surface roughness modulations is compared with a reference calculation with a uniform roughness distribution. The dotted line shows the effect of a roughness distribution with a maximum on the crest of the wave ($\hat{c}_\lambda$ real). Compared to the reference calculation the growth of the waves that are slow compared to the wind is enhanced, whereas the growth of fast waves is slightly reduced. When the maximum of the surface roughness is shifted to the downwind slope of the wave ($\hat{c}_\lambda = -i$) the growth of all waves is enhanced.

4.3 Comparison growth rate with other models

In numerical models of the waves on the oceans the energy flux from the wind to the waves is parameterized in terms of a growth rate $\gamma$. The growth rate used in the most recent version of one of these models, the WAVE Model WAM, is based on a parameterization of the theory of Miles (1957) proposed by Janssen (1991):

$$\beta = \frac{1.2}{K^2} \mu \ln^4 \mu,$$  \hspace{1cm} (4.6)
The growth rate of waves

\[ \beta \]

where \( \mu \) is the dimensionless critical height:

\[
\mu = \min \left[ 1, k z_0 \exp \left( \frac{\kappa}{u_*/c + \alpha} \right) \right]. \tag{4.7}
\]

Relative to the parameterization of the growth rate obtained with Miles' theory (\( \alpha = 0 \) in the above equation), the growth rate used in the WAM model (\( \alpha = 0.011 \)) results in a considerably larger energy input to the waves with a phase speed comparable to the wind speed. Both the growth rate for \( \alpha = 0 \) and the one used in the WAM model is plotted in figure 4.12. For slow waves, \( 2 < c/u_* < 15 \), the growth rate \( \beta \) for both of these versions has a constant value of about 30. The physical basis of the theory of Miles is not very sound. In this theory it is assumed that the effects of the turbulence modulation in the air flow over waves can be neglected. As a result the equations for the pressure perturbation become singular at the height where the windspeed matches the phase speed of the wave. A resonance mechanism at this so-called critical height causes the pressure perturbation to shift out-of-phase, and hence to generate an energy flux from wind to wave. Scaling arguments (e.g. Belcher and Hunt, 1993), model calculations with reasonably general turbulence parameterizations (e.g. this work) and wind tunnel observations (e.g. Hsu and Hsu, 1983, see also the figures 3.6 and 3.7 in section 3.2.2 on page 45) have shown that the turbulence modulation is of first order importance in the inner region of the flow. The turbulence modulation prevents the governing equations form becoming singular at the critical height, which precludes the occurrence of the Miles resonance mechanism at this height. Note that Miles' theory does not yield negative growth rates for waves faster than the wind (no critical height) and that the growth of very short waves quenches as the critical height goes to zero.
Also plotted in the figure 4.12 are the growth rates obtained with the analytical theory of Belcher and Hunt (1993), kindly provided by the first author. This theory is only valid for slow waves with \( c/u_* < 10 \) and if the inner region is thin enough, which imposes the condition \( kz_0 < 10^{-3} \). The contribution from the shear stress modulation at the surface, responsible for a significant part of the energy flux according to the article of Belcher and Hunt (1993), is discarded here. As was the case for the sheltering coefficients of hills (see figure 2.1 on page 18) the theory of Belcher and Hunt is in excellent agreement with the LRR model.

### 4.4 Conclusions

The observations of the growth rate, both at open sea and in the lab, show a large scatter. From a compilation of such measurements Plant (1982) concluded that for a wide range of windspeed to phase speed ratios \( 1 < c/u_* < 20 \) the growth rate is \( \beta = 32 \pm 16 \). The growth rates calculated with the LRR for slow waves \( c/u_* < 15 \) are approximately 12 to 15, and hence fall just outside the experimental range set by Plant.

In calculations of the growth rate strong assumptions are made concerning the shape of the wave, the roughness distribution over the wave, the Reynolds number of the air flow and the effect of a non-zero angle between the wave and the wind. Earlier studies have indicated that the growth rate is sensitive to the Reynolds number of the air flow over the wave. The model calculations presented here show that the growth rate is also sensitive to the distribution of the aerodynamical roughness over the phase of the wave. It seems reasonable to believe that short waves modulated by a longer wave induce a non-uniform roughness distribution. In the next two chapters a theory is developed that aims to describe the interaction between the short waves and the air flow over a long wave.
Chapter 5

Short wave modulation by long waves

In this chapter we will study the behaviour of short surface waves riding on longer wind waves. The longer waves typically are the energetic wave components near the peak of the spectrum, or part of a swell system. These long waves act as a large-scale, slowly varying medium in which the short waves evolve. The short waves are assumed to be at least an order of magnitude shorter than the long wave under consideration. Observations have shown that in such a system, the long waves modulate the energy of the short waves riding on them. This phenomenon is interesting for two reasons. The first is that the variable presence of short waves will affect the energy budget of the long wave. This may have implications for numerical ocean wave forecasting models, which calculate the sea state by solving the energy balance for the waves in the energetic part of the spectrum. The second reason is that the short (Bragg) waves efficiently reflect radar waves. In this way surface waves with lengths several orders of magnitude longer than the electromagnetic radar waves can be detected with radar.

Keller and Wright (1975) measured the modulation of backscattered radar waves by a mechanically generated paddle wave in a wind-wave tunnel. They showed that for the moderate incidence angle they used, the centimetre radar waves scatter primarily from wind-generated gravity-capillary waves. After removing the modulation due to the tilting of the surface, they found that the maximum of the short wave energy was located just ahead of the crest of the paddle wave. It was known that the long wave orbital velocities could compress and expand short waves riding on them (Longuet-Higgins, 1963). Assuming that the short waves would relax back to an equilibrium scale on a timescale of the order of their growth time, Keller and Wright (1975) could not explain the observed amplitude and the phase of the short wave modulation. The same conclusion was reached in field experiments when a centimetre radar was used (Wright et al., 1980). However, the theory was able to explain the observed modulation when lower frequency radars were used, with wavelengths in the order of a few decimetres.

In the theories as formulated by Keller and Wright (1975) and Alpers and Hasselmann (1978) the short wave modulation is caused by the long wave orbital velocity. Other processes, as short wave growth and dissipation and nonlinear interactions among the short waves, are assumed to relax the short wave energy to an equilibrium level. The rate at which this relaxation takes place is of the same order as the growth rate, hence it increases with both wavenumber and wind speed. When the relaxation rate becomes larger than the long wave frequency, the modulation by the orbital velocities is no longer effective. In general this is the case for waves in the gravity-capillary range (centimetre waves). These waves are not modulated by a large scale current, as they are, loosely speaking, generated locally.
Raders can also be used to detect internal waves (Hughes, 1978) and variations in water depth (de Looir, 1978). Both internal waves and topographical changes can induce current variations at the water surface. As was the case with long waves, these current variations are thought to expand and compress the water waves propagating over them. In the analysis of radar images of internal waves and topographical variations, the same discrepancy was found as in the detection of long water waves. Theory can explain the cross section modulation observed with L and P-band radar, which have wavelengths of several decimetres. But again the modulation observed with centimetre radars, like in the C and X-bands, exceed the theoretical estimates by more than an order of magnitude (Thompson and Gasparovic, 1986; Caponi et al., 1988).

Several attempts have been made to explain the mechanism by which the short gravity-capillary waves are modulated. Valenzuela and Wright (1979) checked the assumption that the modulation is linear in the long wave steepness, and found it to be justified. Smith (1990) concluded that the modulation of the wave dissipation could not account for the discrepancy between theory and observations. In several studies the possibility was mentioned that the shear stress variation might affect the gravity-capillary waves (Wright et al., 1980; Schröter et al., 1986; Smith, 1990). The growth rate of these short waves is known to be roughly proportional to the surface stress (Plant, 1982). Therefore a modulation of the shear stress will give rise to a modulation of the growth rate. Given the short timescale on which these waves react, this could be an effective modulation mechanism for these waves. Hara and Plant (1994) actually calculated the surface stress variation which they needed to explain the modulation observed with an X-band radar.

Here a theory will be presented that takes into account the two-way coupling between the modulated air flow over the long wave and the short wave modulation. This description follows Kudryavtsev et al. (1996) on the main points. The shear stress near the surface affects the short waves by modulating their growth rates. The effect of the short wave modulation on the air flow is also accounted for. A key element in this description is the fact that only a part of the flow is affected by the modulation of the short waves. As will be shown, only the flow in the inner region adjusts to the variable roughness induced by the modulated short waves. Comparison of the model with laboratory (Miller and Shemdin, 1991) and the field data of the MARSEN and SAXON experiments shows that the model is capable of reproducing both the phases and the amplitudes of the modulated centimetre waves. For moderate and high wind speeds we find that the growth rate modulation mechanism is dominant for the gravity-capillary waves.

In the theory to be presented here the modulation of the short gravity-capillary waves is linked to the modulation of longer waves via the air flow. Waves longer than a decimetre are not affected by the variations in their growth rate: their response time is too long, or, equivalently, their relaxation rate is too small. Due to this small relaxation rate, these waves are effectively modulated by the long wave orbital velocity. As these waves have their maximum energy at the
Fig. 5.1: Schematic representation of the indirect modulation of short gravity-capillary waves by a long wave. (a) This simplified system consists of a long wave (dotted), a short gravity wave (dashed) and a gravity-capillary wave (solid line). (b) The long wave orbital velocity modulates the short gravity wave adiabatically: the maximum amplitude of the gravity waves is located on the crest of the long wave. The relaxation rate of the gravity-capillary waves is too large to be modulated effectively by the orbital velocities. (c) The increase of the surface stress at the crest due to the large amplitude of the short gravity waves at the crest enhances the growth of the gravity-capillary waves.
crest, their interaction with the air flow will induce a shear stress maximum at the crest. This stimulates the growth of short gravity-capillary waves. Their large relaxation rate, which prohibited their modulation by the orbital velocities, enables them to react quasi-instantaneously to this variation in their growth rate. A graphical representation of this indirect modulation mechanism of the gravity-capillary waves is given in figure 5.1. This mechanism explains why, despite their large relaxation rates, the short gravity-capillary waves are sometimes modulated as effectively as longer waves.

In this chapter the coupled model short waves-air flow is presented. A comparison with laboratory and field observations of short wave modulations is made. In the next chapter the effect of short wave modulation on the energy budget of the long wave is examined.

5.1 The governing equations for the short waves

To describe the ensemble averaged evolution of the short waves, we use the equation for the conservation of wave action:

$$\frac{\partial N}{\partial t} + \frac{\partial \sigma}{\partial k} \frac{\partial N}{\partial x} - \frac{\partial \sigma}{\partial x} \frac{\partial N}{\partial k} = q, \quad (5.1)$$

where $N(\vec{k})$ is the wave action, $t$ is time, $\sigma = \omega + k u_o$ is the apparent frequency, where the dispersion relation $\omega^2 = gk + Tk^2$ gives the intrinsic frequency $\omega$ of the short waves. Here the gravitational acceleration $g = 9.81 \text{ m/s}^2$ and surface tension $T = 74 \times 10^{-6} \text{ m}^3/\text{s}^2$. The source function $q$ allows for changes in wave action due to wave growth, nonlinear interactions between wave components and dissipation.

We want to describe the response of the wave action $N(\vec{k})$ to the presence of a long wave with wavenumber $K \ll k$. We will assume that, in a frame moving with the phase speed $C$ of the long wave, the distribution of the wave action is stationary. Therefore we transform the balance equation to a reference frame moving with the phase speed $C$ of the long wave. We also assume that the only varying velocity is the horizontal component of the orbital velocity $u_0$ of the long wave which is propagating in the $x$-direction:

$$(u_0 - C) \frac{\partial N}{\partial x} - k_x \frac{\partial u_0}{\partial x} \frac{\partial N}{\partial k_x} = q. \quad (5.2)$$

Here we neglected the group velocity of the short waves $c_g$ compared to the long wave phase velocity $C_r$, as we assumed that the modulating wave is much longer than the short waves ($K \ll k$). Now we expand the wave action in the steepness $\varepsilon = AK$ of the long wave:

$$N = \langle N \rangle + \varepsilon \text{Re}[\tilde{N} e^{iKx}] + \mathcal{O}(\varepsilon^2). \quad (5.3)$$

The source function $q$ and the orbital velocity $u_0$ are expanded likewise. The amplitudes $\tilde{N}$, $\tilde{q}$ and $\tilde{u}_0$ are complex numbers. The origin of the spatial coordinate $x$ is chosen in such a way that the amplitude of the orbital velocity is real: $u_0 = \varepsilon \text{Re}[\tilde{u}_0 \exp(iKx)] = \varepsilon \text{Re}[C \exp(iKx)]$. 

Hence it follows that \( \tilde{u}_0 = C \). When these expansions are substituted in (5.2) the terms of order zero in \( \varepsilon \) give \( \langle q \rangle = 0 \). To get a spatially homogeneous and stationary solution for \( \langle N \rangle \) the source terms have to balance. For the terms linear in \( \varepsilon \) we get:

\[
\tilde{N} + k_x \frac{\partial \langle N \rangle}{\partial k_x} = \frac{i\tilde{q}}{\Omega},
\]

where \( \Omega = CK \) is the angular frequency of the long wave.

We will now make a distinction between contributions to the source term \( q \) that are directly affected by the wind, and the rest: \( q = q_{\text{wind}} + q_{\text{rest}} \). The rest term represents the dissipation and the interactions between wave components with different \( \tilde{k} \). Note that both \( q_{\text{wind}} \) and \( q_{\text{rest}} \) are assumed to be modulated, but only the first is affected by modulations of the air flow. The wind term is written as \( q_{\text{wind}} = \gamma N \), where \( \gamma \) is the wind dependent growth rate. Changes in the source function can be caused by variations in the action spectrum \( \tilde{N} \) and by variations in the growth rate \( \gamma \). Assuming that the response of the source function is linear to these changes in \( N \) and \( \gamma \), we can write:

\[
\tilde{q} = \frac{\partial q}{\partial N} \tilde{N} + \frac{\partial q}{\partial \gamma} \dot{\gamma}.
\]

This approximation holds as long as \( \varepsilon |\tilde{N}| \ll 1 \) and \( \varepsilon |\dot{\gamma}| \ll 1 \), where the hat indicates amplitudes normalized with the mean value, e.g. \( \tilde{N} = \tilde{N}/\langle \langle N \rangle \rangle \). The wave spectrum perturbation normalized in this way is known as the hydrodynamic part of the modulation transfer function (MTF).

The dependence of \( \dot{q} \) on the spectral level \( N \) determines the timescale on which the spectrum relaxes back to equilibrium after being disturbed. The rate at which the spectral level \( N(\tilde{k}) \) relaxes back to its equilibrium level after being disturbed is given by

\[
\Gamma(\tilde{k}) = -\frac{\partial q}{\partial N}.
\]

Here we assumed that the dependence of \( N \) on \( q \) is local in \( \tilde{k} \). The reason that \( \Gamma(\tilde{k}) \) is always positive is related to stability of the equilibrium spectrum \( \langle N \rangle \). With the help of the above definition of \( \Gamma \), and \( \partial q/\partial \gamma = N \), the equation for the modulation of the short wave spectrum by a long wave can now be written as

\[
\tilde{N} = -\left(1 - i\mu\right) \frac{\partial \ln \langle N \rangle}{\partial \ln k_x} + \left(1 - \frac{1 - i\mu}{1 + \mu^2}\right) \frac{\langle \gamma \rangle}{\Gamma} \dot{\gamma},
\]

where \( \mu = \Gamma/\Omega \) is the dimensionless relaxation rate.

The first term on the right-hand side of (5.7) gives the modulation due to the straining effect of the orbital velocities. Given the fact that the slope of the spectrum is roughly constant throughout the spectrum \((-5 < \partial \ln \langle N \rangle/\partial \ln k_x < -4)\), the amplitude and phase of this modulation are governed by the value of \( \mu \). If a short wave relaxes to equilibrium on a timescale that is long compared to the period of the modulating long wave, i.e. \( \mu \ll 1 \), it will be modulated adiabatically by the orbital velocities. In that case the maximum amplitude is given by the slope of the undisturbed spectrum and is located on the crest of the long wave. When the short
wave relaxes back to equilibrium instantaneously, $\mu \gg 1$, the amplitude of the orbital velocity modulation becomes small and the phase shifts to the convergence zone $90^\circ$ leading the crest.

The modulation due to variations in the growth rate of the short waves is given by the second term on the right-hand side of (5.7). Contrary to the modulation by the orbital velocity, this mechanism is important for waves with $\mu \gg 1$, provided that the factor $(\gamma)/\Gamma$ is not too small. In section 3.2 this factor will be related to the power of the wind dependence of the spectrum. In Smith (1990) a relation similar to (5.7) is derived (his equation 3.14). Like in the present work, the growth rate is modified by an external parameter: the surface stress. Additional to this forcing, Smith takes into account the modulation of the dissipation rate, by assuming the dissipation is affected by a modulated drift current. Here we will assume that the only modulation in the dissipation and the wave-wave interaction is caused by a modulation in the spectral level, and that these modulations can be modelled with (5.5).

### 5.2 Calculation of the relaxation rate

To calculate $\langle N \rangle$ from the zeroth order problem $\langle q \rangle = 0$, and to calculate the relaxation rate with (5.6), the functional dependence of the source terms that make up $q$ have to be known. However, very little is known about the balance between the growth, dissipation and wave-wave interaction terms for short gravity and gravity-capillary waves. Therefore we adopt a different strategy to determine $\langle N \rangle$ and $\Gamma$. For the former we take parameterizations (in terms of wavenumber, wind speed and wave age) from literature that are based on observations of the wavenumber spectrum with radar scatterometry and optical devices. The relaxation rate $\Gamma$ is derived from the wind speed dependence of these spectra in the following way (Kudryavtsev, 1994). For given wind forcing, characterized by the friction velocity $u_*$, the spectral level following from $\langle q(N(\tilde{k}), u_*) \rangle = 0$ is $N(\tilde{k})$. When the wind forcing is changed by $\delta u_*$, the spectral level will change to a new equilibrium value $N(\tilde{k}) + \delta N(\tilde{k})$ for which again $\langle q(N(\tilde{k}) + \delta N(\tilde{k}), u_* + \delta u_*) \rangle = 0$. Therefore, for small changes in the wind forcing and the spectrum we can write:

$$ q(u_*, N(\tilde{k})) + \frac{\partial q}{\partial u_*} \delta u_* + \frac{\partial q}{\partial N} \delta N = 0. \tag{5.8} $$

The only source term that depends explicitly on the wind forcing is the growth rate term $q_{wind} = \gamma N$. Combined with the definition of $\Gamma$ in (5.6), the above equation can be written as

$$ \Gamma = \frac{\langle N \rangle \partial (\gamma)/\partial u_*}{\partial \langle N \rangle/\partial u_*}. \tag{5.9} $$

With this equation we can calculate the relaxation rate from the dependence of both the spectral level and the growth rate on the wind forcing. The wind dependence of the spectral level of short gravity and gravity-capillary waves is known from observations with radar and laser-slope gauges.

Plant (1982) made a compilation of the observed growth rates in several experiments. He found that the growth rate of short gravity waves is roughly proportional to the ratio of the
friction velocity to the phase speed squared: \( \gamma \sim (u_\ast/c)^2 \). Theoretical arguments agree with this dependence (Belcher and Hunt, 1993; section 2.4.3 on page 22 of this thesis).

All parameters entering (5.7) for the modulation of the spectrum are now known, except the modulation of the growth rate \( \dot{\gamma} \). This growth rate modulation will depend on the response of the boundary layer, which in turn will depend on the presence of modulated short waves. In the remaining part of this section we will derive an expression for the growth rate modulation \( \dot{\gamma} \) in terms of the modulated spectrum \( \dot{N} \). The resulting coupled set of equations can then be solved for \( \dot{N} \) and \( \dot{\gamma} \).

5.3 The effects of short waves on the air flow

Experiments (see Plant, 1982, for a compilation) and theory (Belcher and Hunt, 1993; the present study) show that the growth rate \( \gamma \) is roughly proportional to \( (u_\ast/c)^2 \). Therefore we assume that the normalized growth rate modulation is equal to the normalized surface stress modulation:

\[
\dot{\gamma} = \dot{\tau}.
\]  

In this approximation, the growth rate modulation is determined completely by the stress modulation. To calculate the stress modulation we need a model of the air flow over the long wave. In this section such a model is presented.

To investigate the effect of the short wave modulation on the air flow over a long wave the following approach is used. The numerical model of the air flow over a wave described in chapter 2 is used to assess the influence of the short waves on the air flow qualitatively. This is done by assuming that the net effect of the presence of the modulated short waves will be that the aerodynamical roughness of the long wave surface will be modulated.

5.3.1 The effects of a varying roughness on the boundary layer

To illustrate the influence of a varying surface roughness on the air flow over waves we will use the two dimensional numerical model described in chapter 2. For this experiment we use the second-order Reynolds stress closure proposed by Launder, Reece and Rodi (1975). In chapter 3 it was shown that this model, which does not rely on the eddy viscosity hypothesis, shows good agreement with velocity and stress perturbations observed in a wave tank. Now we will use the model to see qualitatively how the air flow reacts to variations of the surface roughness. These observations will lead to a parameterization of this behaviour that will be used to calculate the effect of short wave modulations on the stress.

In Figure 5.2 the amplitudes of the wave-induced horizontal velocity and the wave-induced Reynolds stress are plotted as a function of height. The velocity amplitudes are normalized with \( \varepsilon U_\lambda \) (\( \lambda \) is the length of the wave), the stress amplitudes with \( \varepsilon \langle \tau \rangle \). The ratio of the wind speed to the phase velocity is \( U_\lambda/C = 4 \), the steepness \( \varepsilon = 0.02 \). The solid lines indicate the
Fig. 5.2: The effect of a modulated surface roughness on the velocity and stress modulation. The real parts of the horizontal velocity (left) and the stress amplitude are shown for a uniform roughness (solid lines) and a varying roughness (dashed lines). \((U_\lambda/C = 4, \hat{c}_\lambda = 1, Kz_0 = 10^{-4}, \varepsilon = 0.02)\)

perturbations calculated with a uniform roughness of \(Kz_0 = 10^{-4}\). The dashed lines in figure 5.2 indicate the velocity and stress amplitudes calculated with a varying surface roughness. The surface roughness distribution is given by the drag coefficient amplitude \(\hat{c}_\lambda = 1\). The relation between the drag coefficient variation and the surface roughness distribution was discussed in section 4.2.5. As the amplitude \(\hat{c}_\lambda\) is real, the maximum of the surface roughness is located on the crest of the wave.

In agreement with the analysis presented in Belcher and Hunt (1993), we see that the changes in the wind speed perturbations are confined to the inner region. Therefore we conclude that the velocity and stress perturbations induced by short wave modulations will be limited to the inner region of the long wave. The velocity and stress perturbations in the outer region are hardly affected by the roughness variations. The model calculations show that this important conclusion holds also for different values of the parameters \(U_\lambda/c\) and \(Kz_0\). This result will simplify the task of modelling the interaction of modulated short waves with the air flow above them, as the modulated waves will only affect the flow in the inner region. Below we will derive the governing equations for the air flow in this region.

5.3.2 Model of the air flow in the inner region

The aim of this section is to build a simple model that describes the response of the turbulent boundary layer above a long wave to the presence of modulated short waves. From the numerical simulations of the air flow over water waves we learned that the disturbances of the velocity and the turbulent stresses due to variations of the surface roughness distribution are confined
to the inner region. This suggests that we can limit our simplified model to the inner region. Therefore we will adopt the following strategy. We will use the 2D numerical model to calculate the velocity perturbation at the top of the inner region. This velocity perturbation will depend on the ratio of wind speed to phase speed, and on the average roughness of the surface. The results of the numerical model runs will be parameterized in terms of these two parameters. As this velocity perturbation will not be influenced by modulations in the surface roughness, it can be used as a constraint on the simple model of the inner region.

From the analysis of the air flow over waves presented in section 2.4 we know that the turbulence in the inner region is in local equilibrium with the velocity shear. This means that we can parameterize the turbulent stress $\tau_t$ in the inner region ($z < L$) with a mixing length model:

$$\tau_t = \rho_0(\kappa z)^2 \left( \frac{\partial u}{\partial z} \right)^2,$$  \hspace{1cm} (5.11)

The total momentum flux $\tau$ is the sum of the turbulent momentum flux $\tau_t$ and the momentum flux $\tau_w$, that is maintained by organized motions associated with the waves. In the air, the wave stress decays with height on a scale comparable to the inner region depth $l$ of the wave that extracts the momentum (see discussion of figure 3.5). For a spectrum of waves, the height dependent wave stress can be written as

$$\tau_w(z) = \int_0^\infty \int_0^\infty e^{z/l} \gamma k_z N \tilde{k}$$  \hspace{1cm} (5.12)

To simplify our model of the inner region response, we will model the height dependence of the wave stress schematically. We will assume that in the layer below the height $z_m$ the wave stress has its surface value, and that above this height the wave stress is zero. As the sum of the wave stress and the turbulent stress is constant with height, the turbulent stress profile will be as follows:

$$\tau = \begin{cases} \tau_t(z) + \tau_w & \text{if } z < z_m \\ \tau_t(z) & \text{if } z > z_m \end{cases}$$  \hspace{1cm} (5.13)

Here $\tau_w = \tau_w(z = 0)$ is the surface value of the wave stress given by (5.12). The scale $z_m$ on which the wave stress varies close to the surface is calculated from (5.12) with:

$$z_m = \frac{\tau_w(z = 0)}{|\partial \tau_w/\partial z|_{z=0}}.$$  \hspace{1cm} (5.14)

The resulting estimate of the scale height $z_m$ has the form of a weighted average of the inner region depths:

$$z_m^{-1} = \frac{1}{\tau_w} \int_0^\infty \int_0^\infty l^{-1} \gamma k_z N \tilde{k}.$$  \hspace{1cm} (5.15)

Finally, we will assume that the stress profile in the inner region is constant with height:

$$\frac{\partial \tau(x, z)}{\partial z} = 0.$$  \hspace{1cm} (5.16)
From figure 5.2 it is clear that the modulated part of the stress is constant only in the lower part of the inner region. So the velocity perturbations following from (5.16) in combination with the mixing length closure (5.11) will deviate from the velocity perturbations calculated with the full 2D model. This has some effect on the stress perturbations calculated with our simplified model of the response of the inner region. Comparison with the stress modulation calculated with the 2D model shows that the idealizations (5.13) and (5.16) introduce an error less than 20% of the total stress modulation.

The above equation form a closed set of equations that describe the response of the flow over a wave to modulations of shorter waves on its surface. When (5.13) and (5.11) are substituted in (5.16) a second-order ordinary differential equation is found for the velocity profile. The effect of the modulated short waves on the air flow enter via the wave stress term in (5.13). Note that in the absence of waves, when \( \tau_w = 0 \), the solution of this equation is the well-known logarithmic velocity profile. As the equation for the velocity profile is of second-order, two boundary conditions are required.

Before supplying the boundary conditions for the velocity, all dependent variables (the velocity \( u \) and the stresses \( \tau, \tau_w \) and \( \tau_t \)) are expanded in the long wave steepness \( \varepsilon \) like in (5.3), e.g.:
\[
    u = \langle u \rangle + \varepsilon \text{Re}[\bar{u}e^{iKz}] + \mathcal{O}(\varepsilon^2).
\] (5.17)

After substitution the terms with equal powers of \( \varepsilon \) are collected and solved separately. The modulation is described by the equations that follow from collecting the terms of order 1 in \( \varepsilon \).

The zeroth order solution. Substitution of the expansions of \( u, \tau, \tau_w \) and \( \tau_t \) in the equations (5.11) to (5.16), and collecting the terms that are independent of \( \varepsilon \) yields:
\[
    \langle \tau \rangle - \langle \tau_w \rangle = \rho_a (\kappa z)^2 \left( \frac{\partial \langle u \rangle}{\partial z} \right)^2 \text{ if } z < z_m
\] (5.18)
\[
    \langle \tau \rangle = \rho_a (\kappa z)^2 \left( \frac{\partial \langle u \rangle}{\partial z} \right)^2 \text{ if } z > z_m
\] (5.19)

For the boundary conditions of this mean flow we can take \( \langle u(z_w) \rangle = 0 \) and \( \langle u(10 \text{ m}) \rangle = U_{10} \) (equation (5.16) is valid throughout the surface layer of the atmospheric boundary layer when applied to the mean stress \( \langle \tau \rangle \)). The roughness length due to molecular viscosity \( z_v \simeq 0.1\nu/u_* \), where \( u_* \) is the scale of the turbulent velocity fluctuations near the surface. In (5.13) we assumed that the turbulence close to the surface is reduced by the organized motions induced by the presence of waves: \( \rho_a u_*^2 = \langle \tau \rangle - \langle \tau_w \rangle \). With the two boundary conditions the stress \( \langle \tau \rangle \) can be found from (5.18) by making the velocity continuous at \( z = z_m \):
\[
    \langle \tau / \rho_a \rangle^{\frac{1}{2}} = \frac{1}{c_{10m}^{-1} - c_{muv}^{-1}} \left( \frac{U_{10}}{c_{10m}^{1/2}} - \frac{\langle \tau_w / \rho_a \rangle}{c_{10m}^{-1} - c_{muv}^{-1}} \right)^{\frac{1}{2}}.
\] (5.20)

where
\[
    c_{10m} = \frac{\kappa^2}{\ln^2 z_{10} / z_m}; \quad c_{muv} = \frac{\kappa^2}{\ln^2 z_m / z_v}.
\] (5.21)
5.3 The effects of short waves on the air flow

Equation (5.20) gives the mean stress \( \langle \tau \rangle \) as a function of the wind speed \( U_{10} \), the mean wave stress \( \langle \tau_w \rangle \) and the weighted average \( z_m \) of the inner region depths.

**The first-order solution.** The terms which are first-order in \( \varepsilon \) yield:

\[
\hat{\tau} - \hat{\tau}_w = 2\rho_0 \frac{1}{2} \kappa z \left( \langle \tau \rangle - \langle \tau_w \rangle \right) \frac{\partial \hat{u}}{\partial z} \quad \text{if} \quad z < z_m \tag{5.22}
\]
\[
\hat{\tau} = 2\rho_0 \frac{1}{2} \kappa z \langle \tau \rangle \frac{\partial \hat{u}}{\partial z} \quad \text{if} \quad z > z_m \tag{5.23}
\]

The boundary conditions for the velocity perturbations are given by: \( \hat{u}(z_w) = \hat{u}_0 \) at the surface and \( \hat{u}(L) = \hat{u}_L \) at the top of the inner region of the long wave. Here we use the fact that the velocity perturbation at the top of the inner region \( z = L \) is not affected by the surface stress perturbation. Below, in (5.27), a parameterization of the velocity perturbation at \( z = L \) is given in terms of the (mean) wind speed and the phase speed. The stress perturbation \( \hat{\tau} \) is found from (5.22) and (5.23) by patching the solutions for \( \hat{u}(z) \) of both equations at \( z = z_m \):

\[
\hat{\tau} = \alpha M (\hat{\tau}_w - 2\hat{u}_L) + 2\hat{u}_L, \tag{5.24}
\]

where the fraction \( \alpha \) of the total stress that is supported by the waves follows from the zeroth order solution: \( \alpha = \langle \tau_w \rangle / \langle \tau \rangle \). The velocity perturbation \( \hat{v}_L \) in (5.24) is equal to the velocity perturbation at \( z = L \) relative to the surface velocity, normalized with the mean wind speed at \( z = L \): \( \hat{v}_L = \hat{v}_L / \langle u_L \rangle = (\hat{u}_L - \hat{u}_0) / \langle u_L \rangle \). The factor \( M \) contains the thicknesses of the relevant vertical scales \( z_m, z_v \) and \( L \):

\[
M = \left( 1 + \sqrt{\frac{c_{mu}}{c_{Lm}}} (1 - \alpha) \right)^{-1}, \tag{5.25}
\]

with

\[
c_{Lm} = \frac{\kappa^2}{\ln^2 L / z_m}. \tag{5.26}
\]

The velocity perturbation at the top of the inner region \( \hat{v}_L \) will be prescribed by a parameterization of values calculated with the 2D numerical simulation model described in chapter 2. Based on calculations with this model with parameters in the range \( 0.4 \leq U_{10}/C \leq 5 \) and \( 10^{-5} \leq K z_0 \leq 10^{-3} \), we find that:

\[
\hat{v}_L = \frac{U^2}{U_L^2} - 2.25 \frac{C}{U_L} \tag{5.27}
\]

is a good approximation of the perturbation of the wind speed at \( z = L \) relative to the surface. In figure 5.3 a comparison is made between the parameterization 5.27 and the velocity perturbation calculated with the 2D model.

When the wave stress perturbation \( \hat{\tau}_w \) is specified, the equations (5.7), (5.10), (5.24) and (5.27) form a closed set of equations for \( \hat{N}(\hat{k}), \hat{\gamma} \) and \( \hat{\tau} \). The perturbation of the wave stress follows from (5.1), by substituting the perturbation expansions of \( N \) and \( \gamma \). This is the subject of the next section.
5.3.3 The growth rate modulation

In (5.12) the wave stress $\tau_w$ is written as the growth rate times the spectral momentum density $\tilde{N}$ integrated over the spectrum. It follows that:

$$\tilde{\tau}_w = \int_{k_{\text{mod}}}^\infty \int_{k_{\text{mod}}}^\infty \left( \langle \gamma \rangle \tilde{N} + \langle N \rangle \tilde{\gamma} \right) k_x d\tilde{k} + \mathcal{O}(\varepsilon^2).$$

(5.28)

Only waves with wavenumbers $k > k_{\text{mod}}$ are allowed to contribute to the surface wave stress modulation, as our expression (5.7) for the modulation of the spectrum was derived assuming that $k \gg K$. In the calculations presented below we will take $k_{\text{mod}} = 10K$. Note that waves with wavenumbers smaller than $k_{\text{mod}}$ are allowed to exist in this description (the modulating long wave itself is such a wave), but only the waves with wavenumbers larger than $k_{\text{mod}}$ are assumed to be modulated by a wave with wavenumber $K$. The expression for $\tilde{\tau}_w$ given above can be used to eliminate $\tilde{\tau}_w$ from (5.24). Combined with $\tilde{\gamma} = \dot{\gamma}$ (equation (5.10)) this leads to:

$$\dot{\gamma} = 2(1 - \Phi(p^{-1} - 1))\dot{\nu}_L + \frac{\Phi}{P(\tau_w)} \int_{k_{\text{mod}}}^\infty \int_{k_{\text{mod}}}^\infty \langle \gamma \rangle \langle N \rangle \tilde{N} k_x d\tilde{k},$$

(5.29)

where $p$ is the fraction of the wave stress supported by waves with $k > k_{\text{mod}}$ and $\Phi$ is the so-called feedback function:

$$\Phi = \frac{M\alpha}{1 - M\alpha p}.$$ 

(5.30)

The growth rate modulation described by the first term of (5.29) is caused by two mechanisms: by the modulation of the relative wind speed, and by variations in the wave stress caused by variations of the growth rate. The second term in (5.29) gives the modulation of the growth rate due to the modulation of the short wave amplitudes. As the growth rate modulation itself
5.3 The effects of short waves on the air flow

Fig. 5.4: Diagram of the calculation of the ripple modulation. The long wave (LW) affects the short wave energy \( \dot{N} \) directly via the orbital velocities (see (5.7)), and indirectly via the modulation of the airflow \( \dot{v}_L \). The airflow modulation causes a stress modulation \( \dot{\tau} \), which in its turn modulates the growth rate \( \dot{\gamma} \). The feedback occurs because the short waves are able to modulate the surface stress via the wave stress \( \tilde{\tau}_w \).

affects the short wave amplitudes (see (5.7)), this opens the possibility of a positive feedback on the short wave modulation. The feedback function \( \Phi \) gives the sensitivity of the stress for variations of the modulated part of the spectrum. The importance of the feedback mechanism will be discussed below.

5.3.4 Discussion

The equations (5.7) and (5.29) can now be solved for \( \dot{N}(\tilde{k}) \) and \( \dot{\gamma} \). The relative variation of the wind speed at the top of the inner region, \( \dot{v}_L \), is given by (5.27). The flow diagram 5.4 illustrates that the long wave modulates the short waves both directly, via the orbital velocities, and indirectly, via the air flow. The modulation by the orbital velocities is given by the first term on the right-hand side of (5.7). The modulation of the short waves is affected by the air flow via the modulation of their growth rate \( \dot{\gamma} \), which corresponds to the second term on the right-hand side of (5.7). The growth rate is modulated because the surface stress \( \dot{\tau} \) is modulated. The modulation of the surface stress is caused by two effects: by the air flow modulation \( \dot{v}_L \), and by the variation of the stress supported by waves \( \tilde{\tau}_w \). As the wave stress perturbation depends on the modulation of the short waves, this dependence gives rise to a feedback mechanism.
Fig. 5.5: Drag coefficient (left panel) and $\alpha$ as a function of wind speed, calculated with the Donelan-Pierson spectrum (solid line) and the Apel spectrum (dashed line). The drag coefficients calculated with the DP spectrum are larger, but the relative contribution of the waves to the drag is roughly the same for both spectra.

5.4 Calculation of mean quantities

5.4.1 Calculation of the drag coefficient with two different spectra

With the zeroth order model presented in section 5.3.2 it is possible to calculate the drag coefficient as a function of the wind, provided the undisturbed spectrum $\langle N \rangle$ is known as a function of the wind speed. This approach to calculate the aerodynamic roughness of the sea was followed also by Caudal (1993) and Makin et al. (1995). In the present study two alternative parameterizations for the spectrum are used. The first one was proposed by Donelan and Pierson (1987). This spectrum, which we will denote by DP, consists of two parts: a low wavenumber part to which at some wavenumber a tail is patched. The second spectrum, proposed by Apel (1994), shows better agreement with the laser slope observations (Klinke and Jähne, 1992) for waves in the range $50 < k < 1500$ rad/m. In appendix A the parameterizations of both spectra are given, followed by a discussion of their main differences.

In figure 5.5 the drag coefficient calculated with the DP spectrum (solid line) and the Apel spectrum (dashed line) is plotted as a function of wind speed. In this calculation we assumed that the spectra are fully developed, so that we can take $U_{10}/c_p = 0.83$ in the definitions of the spectra as given in the appendix. The results in figure 5.5 show that the DP spectrum gives rise to larger drags, specially for the higher wind speeds. Also shown in this figure is the ratio $\alpha$ of the wave stress at the surface to the total stress as a function of the wind speed at 10 metres. From this figure we learn that for wind speeds larger than 10 m/s more than 90% of the stress is supported by the waves. For a given wind speed, the relative contribution of the wave stress
5.4 Calculation of mean quantities

Fig. 5.6: Spectral contribution $I(k)$ to the wave stress $\tau_w$ for two different wind speeds. Solid line, Donelan-Pierson spectrum; dashed line, Apel spectrum.

to the total stress is the same for both spectra. What is different, is which waves contribute to the wave stress. In figure 5.6 the spectral distribution $I(k)$ of the wave stress is plotted. The function $I(k)$ gives the contribution to the wave stress in the domain $\ln k$ and $\ln k + d\ln k$, or, equivalently:

$$I(k) = \frac{1}{\tau_w} \int_{-\pi/2}^{\pi/2} \gamma k^3 N(k, \theta) \cos \theta d\theta.$$  \hspace{1cm} (5.31)

Due to the higher level of the gravity-capillary waves in the Apel spectrum (see figures A.1 and A.2 in appendix A), these waves contribute more to the wave stress than these waves do when the Donelan-Pierson spectrum is used. This trend is particularly evident for the high wind speed case. When $U_{10} = 15$ m/s the peak of the distribution $I(k)$ for the Donelan-Pierson spectrum is located at $\lambda = 40$ cm, whereas in the Apel case it is located near waves with a length of 2 cm. This large difference gives an indication of the uncertainties associated with the calculation of the wave supported stress from spectra.

5.4.2 The relaxation rate

To a large extent the modulation of the spectrum, given by (5.7), is determined by the relaxation rate $\Gamma(\tilde{k})$. The ratio $\mu$ of the relaxation rate to the angular frequency of the long wave determines the amplitude and phase of the modulation by the orbital velocities. If $\mu \ll 1$, i.e. if $\Gamma \ll \Omega$, the modulation is adiabatic: the amplitude of the modulation is determined by the slope of the spectrum $\partial \ln (N)/\partial \ln k_x \approx -4.5$, with the maximum located on the crest. If the relaxation rate is much larger than the frequency of the wave, $\mu \gg 1$, the amplitude of the modulation due to the orbital velocities becomes small, and its maximum shifts to 90° leading the crest.
Fig. 5.7: The ratio of the growth rate $\langle \gamma \rangle$ to the relaxation rate $\Gamma$ for two wind speeds: left panel, 5 m/s; right panel, 15 m/s. Due to the wind independence of the short gravity waves in the Apel spectrum, the value of $\langle \gamma \rangle / \Gamma$ becomes small for $k < 50$ rad/m (dashed lines).

The growth rate modulation is important for those wave that relax on a too short timescale to be effectively modulated by the orbital velocities. For these waves, which have $\mu \gg 1$, the second term on the right-hand side of (5.7) becomes important, provided that the ratio $\langle \gamma \rangle / \Gamma$ is not too small. Here we will relate this ratio to the wind speed dependence of the equilibrium wave spectrum.

If the growth rate is assumed to be proportional to the friction velocity squared, and the wind speed dependence of the spectrum is written as $\langle N \rangle \sim u_*^n$, (5.9) yields:

$$\frac{\langle \gamma \rangle}{\Gamma} = \frac{n}{2}. \quad (5.32)$$

In figure 5.7 the ratio $\langle \gamma \rangle / \Gamma$ calculated with the DP and the Apel spectra are plotted for two different wind speeds. For both wind speeds, $U_{10} = 5$ and 15 m/s, the ratio $\langle \gamma \rangle / \Gamma \approx 0.2$ for $k < 100$ rad/m in the DP case, which corresponds to $n \approx 0.4$. The much smaller value of $\langle \gamma \rangle / \Gamma$ for the Apel spectrum is a consequence of the fact that this spectrum is virtually independent of the wind for $k < 50$ rad/m. This means that the relaxation rates for waves with $k < 50$ rad/m calculated with the Apel spectrum will be much larger than those following from the DP spectrum. As argued above, this will have a significant impact on the results for the modulation of the spectrum.

Based on stereo-photogrammetric observations of short gravity waves ($4 < k < 30$ rad/m), Banner et al. (1989) conclude that the level of these waves depends, weakly, on the wind speed ($n = 0.18 \pm 0.18$). The backscattered microwave power levels in the L-Band (which reflect from Bragg waves with a wavenumber $k \approx 40$ rad/m for moderate incidence angles) show a slightly stronger dependence on wind speed: Thompson et al. (1983) report a value for the
wind speed dependence exponent $n = 0.5$. Both these observations roughly agree with the wind speed dependence of the short gravity waves as modelled by the DP spectrum. The spectrum proposed by Apel seems to underestimate this dependence. In the work of Apel, the attention is focused on representing the data of Klinke and Jähne (1992), which cover the shorter waves: $50 < k < 1500$ rad/m. In the present work the actual level of these gravity-capillary waves is only important for details in the modulated spectrum. The fact whether or not the longer waves are modulated adiabatically or not determines the character of the solution. And this is determined by the relaxation rate.

The importance of the relaxation rate of the waves in the equilibrium range is illustrated by the following example. The equilibrium range is the part of the spectrum between 10 times the wavenumber of the peak of the spectrum and the gravity-capillary range, which starts around 50 rad/m. In figure 5.8 the amplitude and phase of the modulation of the spectrum by a long wave which is part of the spectral peak is shown. In this example the Donelan-Pierson spectrum is used to calculate the wave stress and the relaxation rate. The wind speed is taken to be 15 m/s and the spectrum is assumed to be fully developed ($U_{10}/c_{p} = 0.83$). For the calculation denoted with the dashed line only the modulating effect of the orbital velocities is taken into account, i.e. only the first term of the right-hand side of (5.7). The results in figure 5.8 show that the waves with $k < 10$ rad/m are modulated adiabatically: the amplitude of the modulation is equal to the slope of the spectrum, and the maximum of the short waves is located near the crest. As the relaxation rate increases with wavenumber, above a certain
wavenumber the forcing by the orbital velocities is balanced by the relaxation. As this happens, the amplitude of the modulation decreases, and the phase shifts to 90° leading the crest.

In the second calculation shown in figure 5.8 both modulation mechanisms are taken into account. The figure shows that the adiabatically modulated waves are not strongly affected by the growth rate modulation. However, the modulation of the gravity-capillary waves is strongly affected. Their modulation amplitude is enhanced considerably, and their phase is shifted by more than 60° towards the crest. The reason that the gravity-capillary waves are susceptible to growth rate variations is the same that prevented them from being effectively modulated by the orbital velocities: their relaxation rate is much larger than the angular frequency of the long wave ($\mu \ll 1$). The fact that the maximum of the modulation of the gravity-capillary waves shifts towards the crest is caused by the adiabatically modulated waves. These waves, which are not sensitive to the growth rate modulation themselves, do extract momentum from the air flow. This induces a surface stress modulation, and hence a growth rate modulation, in phase with the long wave. The adiabatically modulated waves are coupled to the gravity-capillary waves via the air flow.

This example shows that, even when the feedback effect is strong, the role of the adiabatically modulated waves is very important. They provide a starting point for the stress modulation, and thereby fix the phase of the modulation. When the relaxation rate is calculated with the Apel spectrum, there are no such waves available. Due to their insensitivity to the wind, the relaxation rate of the waves in the equilibrium range ($10k_p < k < 50 \text{ rad/m}$) becomes very large. This prevents them from being modulated adiabatically. Hence the feedback effect misses its nucleation point and behaves like an unguided missile. Since observations clearly show that waves in the equilibrium range depend on the wind speed, we do not use the Apel spectrum for further calculations of the modulation of short waves.

5.5 Comparison with observations

5.5.1 Laboratory measurements

In the laboratory experiment of Miller and Shemdin (1991) the modulation of waves with wavenumbers $300 < k < 700 \text{ rad/m}$ by a long wave with a period of 2 s ($K = 1.0 \text{ rad/m}$) was observed with a laser-slope gauge. Keeping the length and amplitude of the paddle wave constant, Miller and Shemdin made observations for four different wind speeds. They transformed the observations from frequency to wavenumber space taking into account the Doppler shift induced by the orbital velocities of the long wave. The observations show little dependence on the wavenumber of the short waves. For simplicity we therefore averaged the results for the different wavenumbers, and compared them with the calculated modulation for a short wave with $k = 450 \text{ rad/m}$. The background spectrum, which is used to estimate $\alpha$, $p$ and $\Gamma(\tilde{k})$, is parameterized with the DP spectrum. Since the fetch in a wave tank is limited, in this case 24
5.5 Comparison with observations

| $U_\infty$ (m/s) | $U_{10}$ (m/s) | $U_{10}/c_p$ | $|\hat{N}|$ (degrees) | phase |
|------------------|----------------|---------------|-----------------------|-------|
| 4                | 5.2            | 11            | 4.5                   | 47    |
| 6.4              | 8.3            | 13            | 3.7                   | 39    |
| 9                | 13.0           | 18            | 6.5                   | 5     |
| 10               | 14.4           | 20            | 6.6                   | 7     |

Table 5.1: Experimental results from Miller and Shemdin (1991) for the modulation of wind SW's by a mechanically generated wave. Here $U_\infty$ is the wind speed in the middle of the tank, $U_{10}$ is the equivalent 10 meter wind speed and $U_{10}/c_p$ is the observed wave age of the windsea spectrum.

The windsea spectrum is underdeveloped. In Miller and Shemdin the peak of this windsea spectrum is reported (see third column in table 5.1). It is of paramount importance that the location of the peak of the background DP spectrum used in the calculations equals the observed peak frequency, as the shape of the background spectrum determines which fraction $p$ of the waves present is subject to modulation by the paddle wave.

In figure 5.9 the observed and modeled amplitudes and phases are shown. The calculation with the orbital velocity effect only (dashed lines) shows a decrease in amplitude with increasing wind speed. For the low wind speeds ($U_{10}/C < 2.5$) the short waves with $300 < k < 700$ rad/m are modulated adiabatically, hence their maximum is located within 45° of the crest. With increasing wind speed the relaxation rate increases, and the short wave modulation goes from an adiabatic to a local balance. The effectiveness of the modulation due to the orbital velocities only reduces, and the maximum shifts to the forward face of the wave. Both these effects are absent in the observed modulation. The observations show a clear increase of the modulation with wind speed, after an initial decrease. The location of the maximum shifts to the crest rather than to the forward slope with increasing wind speed. The modulation due to the orbital motion alone clearly cannot explain both these trends.

When the modulation of the growth rates of the short waves is taken into account (solid lines in figure 5.9), both observed trends are reproduced. The effect of the growth rate modulation is most pronounced in the high wind speed cases. In these cases ($U_{10}/C > 2.5$) the relatively long short waves ($10 < k < 100$ rad/m) are still modulated adiabatically (see figure 5.10). Since their modulation is in phase with the long wave, they increase the surface stress at the crest. This favours the growth of the gravity-capillary waves ($k > 200$ rad/m) at the crest.

The reason that this mechanism of surface stress modulation is so effective is that all wind waves are subject to modulation by the paddle wave. As there is a considerable spectral gap between the paddle wave ($k = 1$ rad/m) and the peak of the windsea spectrum ($5 < k_{peak} < 55$ rad/m), the paddle wave can modulate virtually all wind waves present. As the contribution of the waves to the total stress increases with wind speed (see figure 5.5), the paddle wave modulates an increasing part of the surface stress when the wind speed increases. It follows
Fig. 5.9: Amplitudes and phases of the short wave modulation $\hat{N}$ of waves with $300 < k < 700$ rad/m by a paddle wave with a period of 2 s in a wave tank. The solid squares indicate the modulation averaged over the short wavenumber as observed by Miller and Shemdin (1991); the error bars give the spreading observed in this wavenumber interval. The calculations with the growth rate modulation (solid lines) reproduce both the observed amplitudes and phases much better than the calculations with the straining effect only (dashed lines).

that the growth rate modulation becomes increasingly effective. Combined with the increasing relaxation rate for the gravity-capillary waves this results in an effective modulation of these short waves. From the comparison with the laboratory experiment of Miller and Shemdin we conclude that the growth rate modulation mechanism explains both the amplitude and the phase of the observed short wave modulation. When this effect is not taken into account, the amplitude is significantly underestimated, and the maximum is shifted from the crest to the forward slope of the paddle wave.

### 5.5.2 Field observations

In several field experiments the radar cross section modulation was measured simultaneously with the modulating long waves and the meteorological conditions. The cross section modulation has two main contributions: the tilt modulation and the hydrodynamical modulation. By tilting the water surface the long wave modulates the incidence angle of the radar wave. For moderate incidence angles the electromagnetic radar waves are known to scatter from the water surface via the Bragg mechanism. As this modulation mechanism is well understood, it can be subtracted from the observed radar cross section modulation. The residual is equal to the hydrodynamical modulation. Hara and Plant (1994) used this procedure on the data from the MARSEN (Plant et al., 1983) and SAXON (Hara and Plant, 1994) experiments. In these
experiments the modulation of the backscatter of X-band radar waves by a 20 m long wave was measured. By subtracting the modulation effect due to the tilting of the short waves, Hara and Plant (1994) compiled estimates of the hydrodynamical modulation of the short waves from which the radar waves are scattered. The wavenumber of these Bragg waves is 280 rad/m. In figure 5.11 a comparison is made between calculations and the modulation of the Bragg waves as derived from the observation. Again two calculations are shown: one that includes the effect of the orbital velocities and the growth rate modulation (solid lines) and one that includes only the orbital velocity effect (dashed lines). As was the case in the laboratory experiment, the latter calculation does not reproduce the observed phase of the modulation for the larger wind speeds. This is caused by the increase in the relaxation rate of the Bragg wave considered. Above a certain windspeed it becomes too large for the Bragg waves to be modulated adiabatically. This results in a small modulation amplitude (|\( \tilde{N} \) | < 1) and a phase of 90° leading the crest. The observations show no such behaviour. According to both the MARSEN experiment (triangles in figure 5.11) and the SAXON experiment (squares) the phase of the modulation is almost independent of wind speed, with the location of the maximum within 45° of the crest. When the modulation of the growth rate is included in the calculations the phase of the modulation is shifted towards the crest. The longer, adiabatically modulated waves increase the effective roughness of the crest. This enhances locally the growth of the wind sensitive gravity-capillary waves.

It is interesting to note that in the laboratory experiment the amplitude of the short wave modulation increases with \( U_{10}/c \), where in the field experiments the trend is downward. In both cases the long wave phase speed is kept constant, and variations in \( U_{10}/c \) are caused purely by
Fig. 5.11: Amplitude and phase of the hydrodynamic part of the modulation versus wind speed. The symbols indicate radar observations (with the tilt modulation subtracted) in the X-band in two experiments: squares, SAXON; triangles, MARSEN. The dashed lines show the calculated modulation of X-band Bragg waves due to the orbital velocities only, for the solid lines the growth rate modulation is also taken into account.

variations in the wind speed. There is, however, an important difference between the laboratory and the field experiments that explains the different behaviour of the modulation with wind speed. Due to the spectral gap between the paddle wave and the wind generated waves in the laboratory experiment, the paddle wave is able to modulate all wind waves. As the fraction of the surface stress supported by the wave increases with wind speed, so does the effectiveness of the paddle wave to modulate the surface stress. In the field experiments the situation is different. Here the modulating long wave is part of a continuous spectrum, but it can modulate only those waves that are at least an order of magnitude shorter. However, the length of the longest wave that supports the wave stress increases with wind speed, so the fraction of the wave stress that can be modulated by a given long wave decreases. This limits the effectiveness of the growth rate modulation mechanism in the field experiments analysed here.

In a similar experiment as the ones described above, Schrütter et al. (1986) observed the modulation of backscattered C-band radar waves. For the incidence angle used (40°) the Bragg wavenumber is 120 rad/m. The modulating long waves range in length from 10 to 160 m. In figure 5.12 the observed modulation is plotted against $U_{10}/C$. The lines indicate calculations where both the orbital velocity effect and the growth rate modulation has been taken into account, with the tilt modulation added. The calculations for the two extreme wind speeds (2.5 m/s and 17.5 m/s) show reasonable agreement with the observed modulation. For the intermediate case, 10 m/s, the calculated phase is shifted forward some 45° compared to the observations, and the amplitude of the modulation by the longer waves is underestimated. This
5.5 Comparison with observations

Fig. 5.12: Amplitude and phase of the radar MTF (C-band, VV-pol, incidence angle 40°) versus $U_{10}/C$. The observations (squares) are compiled from Schröter et al. (1986) (their figure 6). Solid lines, $U_{10} = 2.5$ m/s; dashed lines, 10 m/s and dash-dotted line, 17.5 m/s.

suggests that the roughness modulation in this case is not strong enough. Hence the calculated roughness modulation can be considered as a conservative estimate. When no roughness modulation is taken into account, the calculations (not shown here) give a decrease in amplitude and a pronounced shift of phase towards 90° leading the crest. Figure 5.12 illustrates that the calculated modulation is not only a function on the wind speed relative to the long wave phase speed, but that it also depends on the absolute value of the wind speed. This strong dependence on the wind speed enters mainly because the wave stress fraction of the total surface stress is a function of wind speed. For low wind speeds the surface stress is predominantly viscous, with the waves supporting only a small fraction of the total stress. In that case the modulation of these waves does not affect the surface stress very much, which in turn means that the growth rate of the shortest waves is not effectively modulated.

Schmidt et al. (1995) measured the modulation of the radar backscatter by 70 m long wave in three different radar frequency bands: L, C and X. The found that the coherence between the backscattered radar power and the long wave height depends strongly on the radar frequency. The coherence was highest for the lowest radar frequency (L-band), and lowest in the X-band. This difference can be qualitatively explained when the modulation of the Bragg waves due to the wind is taken into account. The modulation of the waves of several decimetres that reflect the L-band radar waves is directly linked to the long wave orbital velocities, as these waves are modulated adiabatically. The shorter gravity-capillary waves that reflect the X-band radar waves have a too large relaxation rate to be modulated by the orbital velocities directly. For these waves the indirect modulation via the air flow over the wave is more important. Owing to this less direct link to the long wave, the coherence of this modulation is smaller than is the
case for the L-band Bragg waves.

5.6 Conclusions

A coupled model of the spectral evolution of short waves on the surface of a long wave and of the air flow has been developed. The interaction between the modulated waves and the air flow in this model is two-way: the waves affect the air flow by modulating the surface stress, and the air flow affects the waves by modulating their growth rate. A new element in this theory is the way in which the effect of the modulated short waves on the air flow over the long wave is modeled. Theoretical arguments and numerical simulations show that this influence is limited to the inner region of the flow over the long wave. This important result is used to construct a simple air flow model. The modulation of the short wave spectrum is calculated in the relaxation time approximation. The wavenumber dependent relaxation rate is calculated from the wind speed dependence of the undisturbed wave spectrum.

Calculations show that for wind speeds above 5 to 10 m/s the growth rate modulation can become dominant. For these wind speeds the orbital velocity mechanism puts the maximum of the gravity-capillary waves on the forward face of the long wave, whereas both laboratory and field observations show that the maximum is located closer to the crest. When the growth rate modulation is taken into account this discrepancy disappears. The longer gravity waves, which have relaxation rates small enough to be modulated effectively by the orbital velocities, are predominantly located on the crest of the long wave. Due to the interaction with the air flow they increase the surface stress on the crest, which in turn enhances the growth of short gravity and gravity-capillary waves on the crest.
Chapter 6

The effect of the short wave modulation on the long wave growth

In the previous chapter a model was developed that coupled the modulated air flow over a long wave to the short wave modulation. The key idea behind the model was that the influence of the short wave modulation on the air flow was limited to the inner region. This was illustrated by results from numerical simulations, where it was assumed that the presence of modulated short waves could be modeled by a variable surface roughness. In this chapter we will show that this is indeed the case. In fact, we will show that, with the theory presented in the previous chapter, it is possible to calculate the roughness distribution. This is an important result. Via the variation of the surface roughness, the short waves are able to influence the air flow over the long wave. In this way they can alter the energy flux from the wind to the long wave. In this chapter this effect will be studied, and it will be compared with other mechanisms by which the short waves influence the long wave energy.

In the past, short waves were thought to play an important role in the energy balance of long waves. Phillips (1963) argued that, as the short waves have their maximum amplitude at the crest, they were likely to break there. As this damped a modulation caused by the long wave, this breaking represents an energy loss to the long wave. Longuet-Higgins (1969) noted that, as the short waves carry momentum, they exert a variable surface stress on the long wave. If the short waves break preferentially near the crest, the long wave gains energy. Only a few years later Hasselmann (1971) pointed out that the mass flux associated with the short wave momentum cancels this so-called maser mechanism. Owing to the variation of short wave amplitudes with the long wave phase, the short waves exerted a radiation stress on the long wave. Substituting observed modulations, Hasselmann found that this mechanism lead to a small energy loss of the long wave.

Garrett and Smith (1976) and Valenzuela and Wright (1979) noticed that Hasselmann neglected the effect of a correlation between the surface stress induced by the short wave growth and the long wave phase. As the maximum of the short waves is observed near the crest of the long wave, the surface stress modulation induced by the short waves could sustain a positive energy flux to the long wave. At about the same time, Gent and Taylor (1976) showed that, if short waves are able to modulate the aerodynamical roughness of the long wave surface, this could have a large impact on the most efficient wave growth mechanism: the pressure-slope correlation. Since then no theory has been formulated that links the modulated short waves to a variable surface roughness. One of the aims of the present study is to provide this link, and
to calculate the effects of the presence of short waves on the air flow over the long wave.

In this chapter we will investigate the influence of the short wave modulation on the energy budget. A general formula is derived, which identifies the three interaction mechanisms mentioned above: (i) via the radiation stress; (ii) via the modulation of the surface stress and (iii) via the pressure-slope correlation. To calculate the short wave modulation by a given long wave, subject to a given wind speed, we will use the theory developed in the previous chapter. From this short wave modulation the energy flux due to the radiation stress can be calculated immediately. As the theory also provides the surface stress modulation, the calculation of the energy flux caused by this modulation is also straightforward. To estimate the effect of the short wave modulation on the surface pressure distribution, the following, two-step approach is used. From the short wave modulation the surface roughness distribution is calculated. This roughness distribution is substituted in the 2D model of the air flow over waves which was described in chapter 2. This model is used to calculate the surface pressure, and hence the energy flux due to the pressure-slope correlation. The results of this model run are compared with those from one where a uniform roughness distribution was used.

We find that short waves have a significant impact on the energy flux to long waves. The largest effect is caused via the pressure-slope correlation. The energy flux caused by the short wave radiation stresses is only a tiny fraction of the total energy flux. The calculations also show that the effect of the short waves on the long wave energy depend on the length of the long wave and on the wind speed. In general the effects increase when the length of the long wave increases. We also find that the effects are larger for higher wind speeds.

In the remaining part of this chapter we will first outline the theory of how we take into account the effects of the short wave modulation on the energy budget of the long wave. Subsequently we will examine the effect on the modulating waves for the laboratory and field cases used in the previous chapter. Finally the theory is applied to the experiment of Snyder et al. (1981). In this experiment the growth rate of waves was estimated from observations of the pressure perturbations in the air. Though the wind conditions during this experiment were light to moderate, we find that the short waves increase the calculated growth rate by 30%.

6.1 The energy transfer to the long wave

6.1.1 The energy transfer mechanisms

In section 2.4.3 on page 22, expression (2.29) was derived for the energy transfer from the air flow to a surface wave. In the derivation of this equation it was assumed that the stresses exerted by the air flow act directly on the wave we consider. For the shortest waves this may be an adequate picture, but when the wave we consider is covered by shorter waves, the situation becomes more complex. In that case the tangential stresses exerted by the air flow are partially caused by the shorter waves that extract momentum from the air flow. Before dissipating,
these waves move their energy and momentum around with their group speed. Compared to the situation where the tangential stress is thought to act directly on the orbital velocity of the long wave, this causes an imbalance which can be calculated with the radiation stress (Hasselmann, 1971). As shown in appendix B, the energy budget of the long wave reads (to lowest order in the steepness):

\[
\frac{\partial E}{\partial t} = \left( S_{xx} \frac{\partial u_0}{\partial x} \right) + \left( u_0 \tau \right) + \left( -p_0 \frac{\partial \eta}{\partial t} \right) - D, \tag{6.1}
\]

where \( E \) is the energy of the long wave, \( u_0 \) is the horizontal orbital velocity of the long wave at the surface, \( \tau \) is the surface stress, \( S_{\alpha\beta} \) is the radiation stress tensor (\( \alpha, \beta = x, y \)), \( p_0 \) is the surface pressure, \( \eta \) is the elevation of the water surface and \( D \) is the energy loss of the long wave due to dissipation. The effect of the surface tension on the pressure is ignored. The brackets \( \langle \ \rangle \) denote an average over the long wave.

As shown in the derivation in the appendix, the surface stress \( \tau \) in (6.1) is the sum of the viscous stress \( \tau_v \) and the wave stress \( \tau_w \). Both the wave stress \( \tau_w \) and the radiation stress \( S_{\alpha\beta} \) can be expressed in the wave action spectrum \( N(\vec{k}) \) of the short waves, where \( \vec{k} \) is the wavenumber vector:

\[
\tau_w = \int_{k_{\text{min}}}^{k_{\text{max}}} \int_{k_{\text{min}}}^{k_{\text{max}}} \gamma k_z N d\vec{k}; \tag{6.2}
\]

\[
S_{\alpha\beta} = \frac{1}{2} \int_{k_{\text{min}}}^{k_{\text{max}}} \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{k_\alpha k_\beta}{k^2} \omega N d\vec{k}. \tag{6.3}
\]

Here \( \omega \) is the angular frequency of the short waves, and \( \gamma(\vec{k}) = (1/N)N_{\text{wind}} \) is the growth rate due to the wind. Hence a modulation of the short wave spectrum \( N(\vec{k}) \) will affect the energy of the long wave via the mechanisms (a) and (b). It is clear that, if the short waves affect the shear stress close to the surface, they will affect the air flow over the long wave as a whole. We will show that this has a large impact on the distribution of the surface pressure, and hence on growth mechanism (c).

### 6.1.2 Distribution of the effective roughness

The three possible ways in which modulated short waves can influence the energy flux to the long wave are given in (6.1). With the theory outlined in the previous chapter the first two terms of the right-hand-side can be calculated, provided that the length of the long wave, the wind speed and the undisturbed wave spectrum are known. This could be done by solving (5.7) and (5.29) for \( N(\vec{k}) \) and \( \gamma \). Given the stress variation \( \hat{\tau} \) and the spectral variation \( \hat{N}(\vec{k}) \) the contributions (a) and (b) to the long wave energy budget can now be calculated. To calculate the effect of the short wave modulation on mechanism (c), the pressure-slope correlation term, we have to evaluate the response of the air flow over the long wave in more detail. Below we will show that without additional assumptions the surface roughness distribution \( z_0(x) \) caused
by the short waves can be calculated. When this surface roughness distribution is substituted in the 2D numerical model of the air flow over the long wave, the response of the pressure perturbation on the presence of the short waves can be calculated.

The modulation of the relative velocity \( \tilde{v}_L \) and wave stress \( \tilde{\tau}_w \) alters the velocity profile along the long wave surface. Due to the assumptions made above in the equations (5.11) to (5.16), the velocity profile in the inner region consists always of two parts that are logarithmic with height. These two chunks are patched at the height \( z = z_m \). The effective roughness length \( z_0(x) \), defined as the height where the extrapolated velocity profile for \( z > z_m \) equals the local orbital velocity, is now a function of the phase of the modulating long wave. In general the interaction of a wave with the wind depends on the vertical profile of the wind speed, up to a height comparable with the length of the wave. In particular we know from theory (Belcher and Hunt, 1993) that the distortion of the turbulence in the inner region of the flow is important for the interaction. This interaction will not be affected by the actual velocity and stress values in a layer that is very thin compared to the thickness of the inner region. The height \( z_m \), which is a weighted average of the inner region depths of the short waves, is always much smaller than the inner region depth of the long wave. It follows that we can replace the actual velocity profile by the logarithmic profile that follows from extrapolating the profile for \( z > z_m \) to the effective roughness length \( z_0(x) \).

In the 2D numerical model described in chapter 2 a surface roughness length is used to calculate the surface stress between the surface and the first model layer, assuming the velocity profile is logarithmic. The distribution of the surface roughness is usually assumed to be uniform, though the results for the growth rate are known to be sensitive for roughness variations. In the present study we will take into account the variation of the roughness length due to the presence of modulated short waves by imposing the effective roughness length distribution \( z_0(x) \) as roughness. The effective roughness is defined as the height where the extrapolated velocity profile for \( z > z_m \) equals the local orbital velocity. From the equations (5.11) to (5.16) it follows that the velocity profile for \( z_m < z < L \) can be written as

\[
    u(z) = \frac{\sqrt{\tau/\rho_a}}{\kappa} \ln \frac{z}{L} + u_L, \tag{6.4}
\]

where the boundary condition \( u(L) = u_L \) has been used. The effective roughness \( z_0 \) is defined such that \( u(z_0) = u_0 \). With the definition:

\[
    c_L = \frac{\kappa^2}{\ln^2 L/z_0} \tag{6.5}
\]

it follows that \( \tau/\rho_a = c_L v_L^2 \). Substituting expansions for \( \tau, c_L \) and \( v_L \) we find:

\[
    \tilde{\tau}_L = \tilde{\tau} - 2v_L\tilde{v}_L. \tag{6.6}
\]

The surface roughness distribution can be found from \( \tilde{\tau}_L \) by inverting (6.5):

\[
    z_0(x) = Le^{\kappa^2/\sqrt{c_L(x)}}, \tag{6.7}
\]
where \( c_L(x) = (c_L)(1 + \epsilon \text{Re} [\hat{c}_L \exp(iKx)]) \). So the effective surface roughness distribution \( z_0(x) \) follows, via \( \hat{c}_L \), from the stress modulation \( \hat{\tau} \) and the relative wind speed modulation \( \hat{\theta}_L \). To find the influence of the roughness variation on the pressure-slope correlation growth mechanism, the calculated growth rate with the variable roughness will be compared with a calculation using a uniform roughness distribution. This constant roughness is calculated with (6.7) taking \( c_L(x) = \langle c_L \rangle \).

6.1.3 Discussion of the method

In figure 6.1 a schematic overview is given of the dependencies between the different parameters. In the complementary figure 5.4 on page 79 it was shown how the effect of the long wave on the short waves was modeled. In figure 6.1 the reverse is shown: how the short waves influence the energy flux to the long wave. The calculations illustrated in the diagrams 5.4 and 6.1 are both relevant for this chapter. With the theory outlined in the previous chapter the short wave modulation \( \hat{N} \) is calculated. As a by-product, this theory yields the surface stress modulation \( \hat{\tau} \). From this surface stress modulation the distribution of the surface roughness \( z_0(x) \) is calculated. This roughness distribution is substituted in the 2D numerical model, which subsequently yields the surface pressure.

As mentioned above, there is a two-way coupling between the air flow and the short wave modulation. Nevertheless, in the scheme outlined in the figures 5.4 and 6.1 the system is solved in two subsequent steps. The reason why we can split the coupled system into two parts is the following. In the inner region of the flow the inertial terms are balanced by the stress. Hence in
Fig. 6.2: The effect of the roughness variation on the growth of the paddle wave in the case of the Miller and Shemdin experiment. Solid line, total growth; dashed line, contribution tangential stress; dashed-dotted line, contribution pressure-slope correlation; dotted line, contribution radiation stress.

In this part of the flow the pressure modulation is of secondary importance. As the perturbations of the stress and the velocities due to the short wave modulation are confined to the inner region, we can limit ourselves in the first step to the calculation of the effect of the short wave modulation on the flow in this layer and vice versa. In a later stage the implications for the pressure perturbation can be calculated, as the pressure perturbation is of minor importance in the balance in the inner region.

6.2 The effect on the growth of the modulating wave

6.2.1 The laboratory case

In section 5.5.1 on page 84 the laboratory experiment of Miller and Shemdin (1991) was discussed. It was shown that when the modulation of the growth rate of the short waves was taken into account, the observed increase of the modulation depth with wind speed could be explained. The spectrum modulation \( \hat{N}(\vec{k}) \) and the surface stress modulation \( \hat{\tau} \) can now be used to calculate the contributions of the terms (a) and (b) of (6.1) to the energy of a paddle wave. To calculate the effect of the surface roughness modulation on mechanism (c) the numerical simulation model of air flow over waves is used. As a reference the growth rate for the same wave with a uniform roughness distribution, i.e. \( c_L = \langle c_L \rangle \) in (6.7), is calculated. The result of both calculations is shown in figure 6.2 in terms of the dimensionless growth parameter \( \beta \). Figure 6.2 shows that the growth rate of paddle waves with \( U_{10}/C > 2 \) is enhanced taking the effect of the short waves into account. This enhancement is mainly due to an increase in the
6.2 The effect on the growth of the modulating wave

Fig. 6.3: Hara and Plant (1994) estimated the shear stress modulation to explain the observed modulation in the SAXON (open squares) and the MARSEN (solid triangles) experiments. The stress modulation calculated with the model described in this paper is drawn as a solid line.

pressure-slope correlation (term (c) in (6.1), shown as a dash-dotted line in figure 6.2). The extra surface stress modulation due to the short waves also contributes to an enhancement of the growth rate. The contribution of the radiation stress term (dotted line) is negligible.

The reason for the increase in growth rate for \( U_{10}/C > 2 \) can be easily understood. The main effect of the short waves in this case is to increase the surface stress at the crest through an increase in the effective surface roughness. The enhanced surface stress modulation in phase with the wave works directly on the orbital velocity (term (a) in equation 6.1) to increase the growth rate. A surface roughness modulation in phase with the wave acts to slow down the air flow over the crest by increasing the stress over the crest. This was illustrated in figure 5.2 on page 74. As discussed in Belcher and Hunt (1993) it is the stress modulation in phase with the wave that shifts the undulating flow downstream. The modulation of the pressure, which would be exactly 180° out-of-phase with the wave in the inviscid case, is also shifted downstream by the in-phase stress modulation. As the short waves increase the surface stress modulation in phase with the wave, the phase shift of the pressure increases also.

6.2.2 The effects on the growth of ocean waves

The comparison with the field data from the MARSEN and SAXON experiments (section 5.5.2 on page 86) showed that the growth rate modulation could explain the fact that the maximum of the gravity-capillary waves is observed near the crest, rather than on the forward slope of
Fig. 6.4: The effect of modulated short waves on the growth parameter of a 20 m wave, corresponding to the MARSEN and SAXON experiments. The increase of $\beta$ for the higher wind speeds is primarily caused by the effect of the roughness modulation on the out-of-phase pressure.

the wave. Before we will consider what the effect of the short waves modulation has on the energy flux to the long wave, we will compare our estimate of the stress perturbation caused by the short wave with that given by Hara and Plant (1994).

On the basis of an equation similar to (5.7), Hara and Plant estimated the surface stress variation required to explain the observed hydrodynamical modulation in the MARSEN and SAXON experiments. From the observed values of $\hat{N}$, the factor $\langle \gamma \rangle \hat{\gamma} / \Gamma$ was determined. To do so, the relaxation rate was taken equal to twice the growth rate: $\Gamma = 2\langle \gamma \rangle$. The result was termed $m_\infty$:

$$m_\infty = \frac{\langle \gamma \rangle \hat{\gamma}}{\Gamma},$$

(6.8)

as it is equal to the hydrodynamical modulation in the case $\mu \to \infty$. With (5.32) and $\hat{\gamma} = \hat{\gamma}$ it follows that $m_\infty = (n/2)\hat{\gamma}$. With a reference to experimental work of Jahne and Riemer (1990) they assumed that $n = 2.5$, so that the hydrodynamical modulation $m_\infty$ now becomes 1.25 times the stress modulation. In figure 6.3 the stress modulation Hara and Plant find in this way is compared with the result from our calculation. The different dependence of the modulation of the stress on the wind speed between both estimates is mainly due to the difference in relaxation rate. In our calculation the ratio $\langle \gamma \rangle / \Gamma$ is wind speed dependent (as illustrated in figure 5.7 on page 82), where Hara and Plant assume this ratio is always equal to 0.5. It is important to stress that Hara and Plant derived the stress perturbation from the observed hydrodynamical modulation, whereas in this work a model is presented to calculate this stress perturbation.
6.2 The effect on the growth of the modulating wave

Fig. 6.5: Effect of a varying surface roughness on the growth parameter of waves with $10 < \lambda < 160$ m for three different wind speeds: solid lines, 2.5 m/s; dashed lines, 10 m/s and dash-dotted line, 17.5 m/s.

In figure 6.4 the effect of the modulated ripples on the energy flux to the 20 m long wave is shown. In the uniform roughness case, the wave is growing when $U_{10}/C > 1$, mainly by the pressure-slope correlation (mechanism (iii)). The decay of the wave for lower wind speeds can be mainly attributed to the surface stress modulation (mechanism (i)). When the effect of the modulated short waves is taken into account, the growth of the wave for $U_{10}/C > 2$ is strongly enhanced. This is mainly caused by the indirect effect the short waves have on the pressure distribution. Due to the enhanced surface stress modulation mechanism (i) also transfers more energy to the long wave, but its contribution remains relatively small compared to the growth due to the pressure. The energy flux due to the radiation stress (mechanism (ii)) is negligible.

In figure 6.5 the effect of the short wave modulation on the modulating wave in the experiment of Schröter et al. (1986). During this experiment, where the modulating long waves varied in length from 10 to 160 m, measurements were performed for a wide range of wind speeds. As in section 5.5.2 three wind speeds are considered here: 2.5, 10 and 17.5 m/s. In figure 5.12 on page 89 it was shown that the coupling of the short waves and the air flow produces realistic short wave modulations. The effect on the energy flux (figure 6.5) is similar to the previous case. The growth of waves with phase speeds smaller than the wind speed is strongly enhanced. Again this is mainly due to an increase in the pressure-slope mechanism. It is also interesting to note that for a fixed value of $U_{10}/C \approx 2$, the effect of the short wave modulation on the growth rate is much larger for the high wind speed of 17.5 m/s than for intermediate wind of 10 m/s. This is due to the fact that for high wind speeds the fraction of
The effect of the short wave modulation on the long wave growth

Fig. 6.6: Observed and modelled growth parameter. The lines indicate calculations with uniform roughness (dashed line) and varying roughness (solid line) for a wind speed $U_{10} = 8$ m/s. The length of the long wave decreases from $\lambda = 40$ m on the left to $\lambda = 3$ m on the right. The growth parameter derived from the observations of Snyder et al. (1981) and Hasselmann and Bösenberg (1991) fall inside the shaded area.

The total stress supported by waves goes to one, and hence that their modulation in that case results in an effective modulation of the surface stress. For low wind speeds a significant part of the surface stress is supported by the viscous stress, which is not strongly dependent on the presence of short waves.

6.2.3 Relevance to growth rate observations

As a last example of the effect of short wave modulation on the growth of the modulating long wave, we will estimate the effects short waves might have had on the growth rate observations of both Snyder et al. (1981) and Hasselmann and Bösenberg (1991). The wind speed in these experiments ranged from $5 < U_5 < 10$ m/s. The grey area in figure 6.6 indicates the range in which most of the observations in both experiments fell (see also figure 4.1 on page 50). In figure 6.6 two calculations are shown: one with a uniform surface roughness (dashed line), and one with a varying surface roughness caused by the modulation of short waves (solid line). For the calculations we assumed a constant wind speed of $U_{10} = 8$ m/s, with a drag coefficient of $C_{D10} = 1.1 \times 10^{-3}$. Though the growth rate is underestimated by the model in both cases, the short wave modulation has a significant impact on the calculated growth rate. Compared to the uniform roughness case, the growth due to the pressure-slope correlation is increased by 30% by the modulated short waves.
6.3 Conclusions

The coupled model of the short wave modulation and the air flow allows for the calculation of the distribution of the effective surface roughness induced by the modulated waves. By substituting this variable surface roughness into a 2D numerical model of the air flow over the long wave, it is found that the growth rate of the long wave is significantly altered compared to the case with a uniform roughness distribution. The short waves have the largest impact on the pressure-slope energy transfer mechanism; the contribution of the surface stress modulation to the growth remains small. For constant wind speed-phase speed ratio, the effect on the growth rate increases with wind speed. In the light wind conditions ($5 < U_5 < 10$ m/s) that prevailed during the observations of Snyder et al. (1981) the modulated surface roughness increases the calculated growth rate up to 50%, compared to a calculation with a uniform surface roughness distribution. In agreement with the conclusions stated in earlier work (Hasselmann, 1971, Garrett and Smith, 1976) we find that the contribution of the short wave radiation stress on the growth of the long wave is negligible.
Appendix A

Wave spectrum parameterizations

A.1 The Donelan-Pierson Spectrum

The wave age dependent Donelan-Pierson spectrum used in this paper consists of two parts:

\[
S(\kappa) = \begin{cases} 
S_e(\kappa) & \text{if } k < k_r \\
S_r(\kappa) & \text{if } k \geq k_r 
\end{cases} \tag{A.1}
\]

Here \( S_e \) is the low-frequency energy containing part of the elevation spectrum \( S(\kappa) \), and \( S_r \) is the high frequency part. In chapter 4 and 5 the wave action spectrum \( N(\kappa) \) rather than the elevation spectrum \( S(\kappa) \) is used. They are related by:

\[
N(\kappa) = \rho c^2 \frac{\omega}{k} S(\kappa). \tag{A.2}
\]

The wavenumber \( k_r \) where the low and high frequency parts are patched is given by:

\[
k_r = \frac{10g}{(1.2U_{10})^2} \left( \frac{1.2U_{10}}{c_p} \right)^{3/2}, \tag{A.3}
\]

where the wave age \( U_{10}/c_p \) and the wind speed \( U_{10} \) at 10 meters are the two parameters on which the spectrum depends. \( c_p \) is the phase speed at the peak of the spectrum, which is located at the wavenumber \( k_p = g/c_p^2 \). The energy containing part \( S_e \) is wave age dependent:

\[
S_e(k, \theta) = \frac{1}{4} \alpha_p k^{-4} \sqrt{k/k_p} e^{-\left( k/k_p \right)^2} \gamma \Gamma h \cosh^{-2}(h\theta). \tag{A.4}
\]

where

\[
\alpha_p = 6 \times 10^{-3} \left( \frac{U_{10}}{c_p} \right)^{0.55}; \tag{A.5}
\]

\[
\gamma = \begin{cases} 
1.7 & \text{if } U_{10}/c_p < 1 \\
1.7 + 6 \log(U/c_p) & \text{if } U_{10}/c_p \geq 1
\end{cases} \tag{A.6}
\]

\[
\ln \Gamma = \frac{(\sqrt{k/k_p} - 1)^2}{2\sigma^2} \tag{A.7}
\]

\[
\sigma = 0.08 \left( 1 + 4(U_{10}/c_p)^{-3} \right)^{-1} \tag{A.8}
\]

\[
h = \begin{cases} 
1.24 & \text{if } k/k_p < 0.31 \\
2.61(k/k_p)^{0.65} & \text{if } 0.31 \leq k/k_p < 0.90 \\
2.28(k_p/k)^{0.65} & \text{if } k/k_p \geq 0.90
\end{cases} \tag{A.9}
\]
Fig. A.1: Saturation spectra of the Donelan-Pierson spectrum for three windspeeds ($U_{10} = 5$, 10 and 20 m/s). All spectra are assumed to be fully developed ($U_{10}/c_p = 0.83$).

The high frequency part of the spectrum $S_r$, which is wave age independent, is given by:

$$S_r(k, \theta) = k^{-4} \left( \frac{\beta - \beta_\nu}{a} \right)^{1/n} \cosh^{-2}(h_1 \theta) \quad (A.10)$$

where

$$\ln a = (\ln a_1 - \ln a_2) f + \ln a_2 \quad (A.11)$$

$$n = (n_1 - n_2) f + n_2 \quad (A.12)$$

$$f = \frac{|g - \gamma_w k^2|^3}{|g + \gamma_w k^2|^3} \quad (A.13)$$

$$\beta = 0.194 \frac{\rho_w}{\rho_c} \left( \frac{U_{x/k}}{c} - 1 \right)^2 \quad (A.14)$$

$$\beta_\nu = 4 \nu_w k^2 / \omega \quad (A.15)$$

$$U_{x/k} = U_{10} (1 + \frac{\sqrt{C_d}}{\kappa} \ln(\frac{\pi}{10k})) \quad (A.16)$$

$$C_d = (0.96 + 0.041 U_{10}) \times 10^{-3} \quad (A.17)$$

$$h_1(k) = \frac{0.46}{\sqrt{2(1 - 0.8^n/2)}} \quad (A.18)$$

For the constants in the definition of the high frequency part of the spectrum, Donelan and Pierson (1987) give:

$$\ln a_1 = 22$$

$$\ln a_2 = 4.6$$

$$n_1 = 5$$

$$n_2 = 1.15$$
A.2 The Apel spectrum

In 1994, Apel proposed a parameterization of the wavenumber spectrum that complied with the observations of Klinke and Jähne (1992):

\[ S(k, \theta) = A \cdot L_0(k) \cdot \gamma^\Gamma \cdot k^{-4} \cdot H_i(k) \cdot D(k, \theta). \]  
(A.19)

The proportionality constant \( A = 1.95 \times 10^{-3} \). The different functions that make up the spectrum parameterization are given by:

\[ L_0(k) = \exp[-(k_p/k)^2], \]  
(A.20)

For fully developed spectra Apel proposes \( \gamma = 1.7 \). For \( \Gamma \) equation (A.7) is used again, with \( \sigma = 0.40 \) for fully developed seas. To extend the Apel spectrum to non-equilibrium cases the wave age dependent parameterizations (A.6) and (A.8) as proposed by Donelan and Pierson can be used.

The behaviour of the Apel spectrum in the gravity-capillary range is governed by:

\[ H_i(k) = [R_{ro} + S_u \cdot R_{res}] \cdot V_{diss}, \]  
(A.21)

where:

\[ R_{ro} = \frac{1}{1 + (k/k_{ro})^2}, \]  
(A.22)

with \( k_{ro} = 100 \) rad/m;

\[ S_u = \exp\left\{[s_1 + s_2(1 - e^{-U_{10}/U_\eta})] \ln 10\right\}, \]  
(A.23)

with \( s_1 = -4.95 \), \( s_2 = 3.45 \) and \( U_\eta = 4.7 \) m/s;

\[ R_{res} = a_1 k \frac{1}{\cosh[(k - k_{res})/k_w]}, \]  
(A.24)

with \( a_1 = 0.8 \), \( k_{res} = 400 \) rad/m and \( k_w = 450 \) rad/m;

\[ V_{diss} = \exp(-k^2/k_{diss}^2), \]  
(A.25)
with \( k_{\text{dis}} = 6283 \text{ rad/m} \).

The angular dependence of the Apel spectrum is governed by the factor \( D(k, \theta) \):

\[
D(k, \theta) = \exp \left( \frac{-\theta^2}{2\theta_0^2(k)} \right),
\]

\( \text{(A.26)} \)

where:

\[
2\theta_0^2(k) = [0.14 + 5.0(k/k_p)^{-1.3}]^{-1}.
\]

\( \text{(A.27)} \)

### A.3 Discussion

In the figures A.1 and A.2 saturation spectra of the Donelan-Pierson and the Apel parameterization are plotted for three windspeeds. The saturation spectrum \( D(\tilde{k}) \) is defined as:

\[
D(\tilde{k}) = k^4 S(\tilde{k}).
\]

\( \text{(A.28)} \)

In the figures A.1 and A.2 the spectral level of the waves in the wind direction are shown, i.e. \( \theta = 0 \). From the figure two differences between the parameterizations are apparent. In the Apel spectrum the spectral level of the waves in the equilibrium range, \( 10k_p < k < 50 \text{ rad/m} \), does not depend on the windspeed. As discussed in section 5.4.2 on page 81 this is in conflict with the observations of Banner et al. (1989), who observed a weak windspeed dependence in the range \( 4 < k < 30 \text{ rad/m} \). The level of the L-band Bragg waves \( k \approx 40 \text{ rad/m} \) is also known to depend weakly on windspeed (Thompson et al., 1983). In the Donelan-Pierson parameterization the equilibrium range is windspeed dependent.

The second important difference between the two parameterizations is the level of the gravity-capillary waves (\( k > 100 \text{ rad/m} \)). Donelan and Pierson, who lacked accurate observations in this range, calculated the level of the gravity-capillary waves by assuming a balance between growth by wind and (viscous) dissipation. In contrast, Apel made a fit to the observations of Klinke and Jähne (1992), which cover the range \( 50 < k < 1500 \text{ rad/m} \). From the figures shown here it is clear that this leads to a higher cut-off frequency, and to much higher levels for high windspeeds.
Appendix B

The energy budget of a ripple covered wave

In Kudryavtsev (1994) the energy budget of a long internal wave subject to the effects of the shorter surface waves is derived. This derivation can be easily transformed to the situation of a long surface wave, which modulates the presence of much shorter waves on its surface. First the horizontal momentum budget integrated over the layer \(-h < z < \eta\) is considered (see also Phillips, 1977). Here we assume that \(h\) is much smaller than the wavelength of the long wave, so that the orbital velocities of the long wave can be considered constant with depth in the layer \(-h < z < \eta\). We also assume that \(h\) is much larger than the wavelengths of the short waves, so that the fluctuations of the velocities and pressures due to the short waves can be ignored at \(z = -h\). This of course imposes the condition that the long wave is much longer than the short waves. After averaging over many short wave scales (spatial or temporal), we find:

\[
\frac{\partial}{\partial t} \left( \int_{-h}^{\eta} u_\alpha dz + m_\alpha \right) + \frac{\partial}{\partial x_\beta} \left( \int_{-h}^{\eta} u_\alpha u_\beta dz + m_\alpha u_\beta + m_\beta u_\alpha + S_{\alpha \beta} + \frac{1}{2} \delta_{\alpha \beta} g(\eta + h)^2 - T_{\alpha \beta} \right) = \tau^w_\alpha + \tau^w_\alpha - F_\alpha, \tag{B.1}
\]

where \(m_\alpha\) is the momentum of the short waves, \(S_{\alpha \beta}\) is the radiation stress due to the short waves and \(T_{\alpha \beta}\) is the Reynolds stress tensor integrated over depth:

\[
T_{\alpha \beta} = \int_{-h}^{\eta} \tau_{\alpha \beta} dz. \tag{B.2}
\]

The three terms on the right hand side of (B.1) represent the momentum flux through the surface via the short waves (\(\tau^w_\alpha\)) or otherwise (\(\tau^w_\alpha\)) and the flux through the plane \(z = -h\), respectively. Equation (B.1) is the vertically average horizontal momentum budget in the layer \(-h < z < \eta\). The momentum in this layer can be caused by the orbital velocity of the long wave (drift currents are neglected), or by the presence of short waves.

The momentum balance of the short wave only can be written as:

\[
\frac{\partial m_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} \left( u_\beta m_\alpha + S_{\alpha \beta} \right) + m_\beta \frac{\partial u_\beta}{\partial x_\alpha} = \tau^w_\alpha - d^w_\alpha, \tag{B.3}
\]

where \(d^w_\alpha\) is the short wave momentum loss due to dissipation. The momentum budget of the short waves (B.3) can be subtracted from the total momentum budget in the layer \(-h < z < \eta\) to obtain the momentum budget of the mean flow and the long wave in this layer:

\[
\frac{\partial}{\partial t} \int_{-h}^{\eta} u_\alpha dz + \frac{\partial}{\partial x_\beta} \left( \int_{-h}^{\eta} u_\alpha u_\beta dz + \frac{1}{2} \delta_{\alpha \beta} g(\eta + h)^2 - T_{\alpha \beta} \right) +
\]

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\[ m_\beta \left( \frac{\partial u_\alpha}{\partial x_\beta} - \frac{\partial u_\beta}{\partial x_\alpha} \right) + u_\alpha \frac{\partial m_\beta}{\partial x_\beta} = \tau_\alpha^\nu + a_\alpha^w - F_\alpha, \quad (B.4) \]

By multiplying (B.4) by \( u_\alpha \), adding \( w \) times the vertically integrated vertical momentum balance and using continuity, we get the energy balance of the long wave contained in the layer \(-h < z < \eta\). We will assume that the spatial distribution of the long waves is homogeneous, so that if we average over many long waves the spatial derivatives may be dropped. The averaging procedure over an ensemble of long waves is denoted by the brackets \( \langle \rangle \). With the energy of the long wave contained in the surface layer \(-h < z < \eta\) defined as:

\[ E^{st} = \int_{-h}^\eta \left( \frac{1}{2} u_\alpha^2 + \frac{1}{2} w^2 + gz \right) dz, \quad (B.5) \]

we find:

\[ \frac{\partial E^{st}}{\partial t} = -\langle g\eta \frac{\partial m_\alpha}{\partial x_\alpha} \rangle + \langle u_\alpha^\nu (\tau_\alpha^\nu + a_\alpha^w - F_\alpha) \rangle - \langle p^\nu \frac{\partial \eta}{\partial t} \rangle - \langle p^-h \frac{\partial h}{\partial t} \rangle - D^{st}. \quad (B.6) \]

Here \( D^{st} \) is the loss of long wave energy due to friction in the surface layer:

\[ D^{st} = \int_{-h}^\eta \tau_\alpha^\rho \frac{\partial u_\alpha}{\partial x_\beta} dz. \quad (B.7) \]

The energy budget of the long wave in the layer \(-h < z < \eta\), given by equation (B.6), is subsequently added to the energy budget of the long wave integrated over the bulk layer \(-H < z < -h\), which is given by:

\[ \frac{\partial E^{bl}}{\partial t} = \langle p^-h \frac{\partial h}{\partial t} \rangle + \langle F_\alpha u_\alpha \rangle - D^{bl}, \quad (B.8) \]

to obtain the energy budget equation of the layer \(-H < z < \eta\). Here we assume that \( H \) is much larger than the wavelength of the long wave, so that all the energy of the long wave is contained in the layer \(-H < z < \eta\). Adding (B.6) and (B.8) we get the total energy budget of the long wave:

\[ \frac{\partial E}{\partial t} = -\langle g\eta \frac{\partial m_\alpha}{\partial x_\alpha} \rangle + \langle u_\alpha (\tau_\alpha^\nu + a_\alpha^w) \rangle - \langle p^\nu \frac{\partial \eta}{\partial t} \rangle - D. \quad (B.9) \]

The equation (6.1) can be found by eliminating the term that contains the short wave dissipation \( d_\alpha^w \). By multiplying the momentum equation for the short waves (B.3) by \( u_\alpha \), and by averaging over many long waves, we find:

\[ \langle u_\alpha d_\alpha^w \rangle = \langle \tau_\alpha^\nu u_\alpha \rangle + \langle g\eta \frac{\partial m_\alpha}{\partial x_\alpha} \rangle + \langle a_\alpha^w \frac{\partial u_\alpha}{\partial x_\beta} \rangle. \quad (B.10) \]

Here we used that, to first order in the steepness of the short waves, \( \partial u_\alpha / \partial t = g \eta / \partial x_\alpha \).
Bibliography


Summary

In this work a numerical model is described that calculates the effects of a water wave on the air flow close to the water. The model is equipped with a hierarchy of turbulence closure schemes, ranging from the relatively simple mixing length scheme to the more general second-order Reynolds-stress model. Calculations for a wide range of parameters show that the energy flux from wind to wave, characterized by the growth rate, depends strongly on the choice of turbulence scheme.

To determine which type of closure is appropriate for this problem results of various models are compared with detailed wind-tunnel observations of the distribution of the turbulent stress above water waves. The comparison shows that the closure schemes based on the eddy viscosity concept overestimate the amplitude of the wave-induced Reynolds stress in a part of the vertical domain. The second-order Reynolds-stress model, that takes into account advection of turbulent moments, does not suffer from this defect. These results confirm the conjecture made by Belcher and Hunt (1993) that at some distance from the surface the Reynolds stress is no longer in equilibrium with the time-varying local velocity gradients. In this part of the domain the effect of the straining on the turbulent eddies is described by the rapid distortion theory (Batchelor and Proudman, 1954).

A comparison of the growth rates calculated with the second-order Reynolds-stress model with a compilation of wave growth measurements (Plant, 1982) shows that the model tends to underestimate the growth rate. Several assumptions which are commonly made in calculations of the growth rate are examined to find the reason for this underestimation. The effects of a finite steepness of the waves and the assumptions made concerning the dependence of the growth rate on the angle between wind and wave seem to have little influence on the results. For wave longer than a few decimetres, viscosity plays no role either. When the Reynolds number, based on the wavelength and the friction velocity, becomes smaller than several times $10^4$ the dynamical effects of the molecular viscosity should be taken into account. The experiments analysed here fall just outside this region.

The model calculations show that the growth rate depends on the distribution of the aerodynamic roughness over the surface of the wave. As no data on the actual roughness distribution is available, it is usually assumed that this distribution is uniform. For waves longer than a few meters this seems a doubtful assumption, as these waves are known to modulate the amplitudes of shorter waves that act as roughness elements. A theoretical framework is proposed.
to account for this effect.

The description of the modulation of the short waves by a longer wave takes into account the fact the interaction between the air flow and the short waves is two-way. Acting as roughness elements, the short waves influence the air flow. However, the short waves themselves respond to changes in the air flow, which in turn is also affected by the long wave. This two-way interaction leads to a new modulation mechanism of the short waves, additional to the modulation by the long wave orbital velocities. The modulation via the air flow is particularly effective for the waves in the gravity-capillary range. It offers an explanation for the fact that the gravity-capillary can be effective modulated despite their high relaxation rate.

Model calculations show that the surface roughness modulation due to short waves can have a significant effect on the growth rate of a long wave. As the effectiveness of the coupling between short waves and the atmosphere increases with windspeed, the effect on the growth rate of the modulating wave also increases with windspeed. The field measurements of Snyder et al. (1981) were done under light to moderate wind conditions (5-10 m/s). Under these conditions the modeled growth rate increases by 30% by taking into account the effects of short waves.
Samenvatting

In dit proefschrift wordt een numeriek model beschreven van de verstoring van de luchtstroming vlak boven het water veroorzaakt door de aanwezigheid van een golf. Het model is uitgerust met verschillende sluitingsmodellen die de turbulentie parameteriseren. Deze sluitingsschema’s variëren van het relatief simpele mengwegmodel tot een meer algemeen tweede-orde Reynoldsspanningsmodel. Berekeningen die zijn gedaan met een grote verscheidenheid van parameters laten zien dat de energieflux van de wind naar de golven sterk afhingt van het gekozen turbulentieschema.

Om te bepalen welk turbulentie schema het meest relevant is voor dit probleem zijn gedetailleerde meetingen van de turbulentie boven golven vergeleken met modelberekeningen. Uit de vergelijking blijkt dat sluitingsschema’s die zijn gebaseerd op de Boussinesq hypothese (ook wel turbulente viscositeitstheorie genoemd) de amplitude van de Reynoldsspanning in een deel van het verticale domein overschatten. Als het tweede orde Reynoldsspanningsmodel gebruikt wordt, gebeurt dit niet. Dit wordt veroorzaakt door het feit dat dit model de effecten van de advectie van turbulentie in rekening brengt. Deze resultaten bevestigen de veronderstelling van Belcher en Hunt (1993) dat vanaf een zekere hoogte boven het wateroppervlak de turbulentie niet langer in evenwicht is met de lokale snelheidsgradiënt. Een vergelijking van de groeiselheid volgens het tweede orde Reynoldsspanningsmodel met een keur aan metingen van de groeiselheid (Plant, 1982) laat zien dat het model de groeiselheid neigt te onderschatten.

Een aantal veel gemaakte vooronderstellingen bij de berekening van de groeiselheid zijn onderzocht om de reden voor deze onderschatting te achterhalen. De effecten van een eindige steilheid van de golven en van een hoek tussen de wind en de golf blijken weinig invloed te hebben op de resultaten. Voor golven langer dan een paar decimeter speelt de moleculaire viscositeit ook geen rol. De directe effecten van de moleculaire viscositeit zijn alleen van belang als het Reynoldsetal, gebaseerd op de golfengte en de krijvingssnelheid, kleiner wordt dan enkele tienduizenden. De experimenten die in dit proefschrift worden geanalyseerd vallen buiten dit gebied.

Modelberekeningen tonen aan dat de groeiselheid afhankelijk is van de verdeling van de aërodynamische ruwheid over de oppervlakte van de golf. Aangezien er geen metingen beschikbaar zijn van de daadwerkelijke ruwheid, neemt men meestal aan dat de ruwheid uniform verdeeld is. Voor golven langer dan een paar meter lijkt dit een twijfelachtige vooronderstelling. Deze golven moduleren immers de energie van kortere golven die als ruwheid optreden. In dit
proefschrift wordt een theoretisch kader gepresenteerd om dit effect in rekening te brengen.

De beschrijving van de modulatie van korte golfjes door een langere golf houdt rekening met de interactie tussen de korte golfjes en de luchtstoming. De korte golfjes beïnvloeden de luchtstoming door als aerodynamische oppervlakte ruwheid op te treden. De korte golfjes worden echter zelf ook weer beïnvloed door de luchtstoming, die op zijn beurt weer beïnvloed wordt door de aanwezigheid van de lange golf. Deze wederzijdse beïnvloeding leidt tot een nieuw modulatie mechanisme van de korte golfjes, naast de modulatie door de orbitaalsnelheden van de lange golf. De modulatie via de luchtstoming is met name effectief voor korte golfjes in het gravitatie-capillaire gebied. Deze theorie biedt een verklaring voor het feit dat gravitatie-capillaire golfjes effectief gemoduleerd kunnen worden, ondanks hun grote relaxatie snelheid.

Modelberekeningen laten zien dat als de variatie in oppervlakteruwheid veroorzaakt door de korte golfjes in rekening wordt gebracht dit een grote invloed op de groeisnelheid van de lange golf kan hebben. Omdat de koppeling tussen de korte golfjes en de luchtstoming toeneemt met toenemende windsnelheid, neemt het effect van de gemoduleerde korte golfjes op de groeisnelheid van de lange golf ook toe met toenemende windsnelheid. De veldmetingen van de groeisnelheid van lange golven door Snyder et al. (1981) zijn gedaan voor matige windsnelheden (5 tot 10 m/s). Onder deze condities nemen de gemoduleerde groeisnelheden ongeveer 30% toe door het effect van korte golfjes in rekening te brengen.
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