ERRATA

p. 25  CC\textsubscript{1} → CC\textsubscript{14}

p. 34, line 2 from bottom  9.83 → 7.09

p. 37, Table 1.7  9.87 MPa → 10.0 MPa

p. 41, line 1  24.6 MPa → 25.0 MPa

, line 3  22.7 MPa → 23.0 MPa

, text below Table 1.8  which was measured with a rolling ball viscometer, is 0.65 cP. + was measured with a rolling ball viscometer.

p. 56, bottom  2.7 MPa → 23.0 MPa

p. 56, bottom  displaced → displacing

p. 75, middle  displacing → displaced

Σ → Σ

m=1 → m=0

p. 88, line 9  microscopic → microscopic convective

p. 92, line 2  k_{o} → \overline{k}_{o}

, Appendix 2B  implicitly → implicitly

p. 95, below expr. (2C.6c)  \omega = \frac{W}{r} + \frac{1}{2} = \frac{U}{U_{c}} \frac{(1-M)x_{1} + M}{(1-M)U_{t} + M} \overset{!}{\rightarrow} \omega = z + \frac{1}{2}

boundary → boundary and initial

p. 104, line 8  Appendix 1C → Appendix 1B

p. 115, title Fig. 3.4a  pattern → pattern at I=0.7

p. 117, title Fig. 3.4d  N_{G}=0 → N_{G}=0

, scale y-axis Fig. 3.4d  N=20

250 → 200

100 → 80

200 → 160

50 → 40

150 → 120

p. 158, bottom  Axness → Axness (without the correction factor \overline{F}(\sigma))

p. 161, legend Fig. 4.3  experiment (4.4) → equation (4.1)

p. 161, title Fig. 4.3  Heterogeneity divided → Transition (4.9)

p. 165, title Fig. 4.4  visualised as heterogeneity divided

p. 165, title Fig. 4.4  Dispersivity ratio α → Dispersivity ratio α given by random-walk model

p. 166, line 5 from bottom  Dispersivity ratio α → Dispersivity ratio α given by random-walk model

p. 167, title Fig. 4.7  aspect ratio L/W → aspect ratio λ/λ' (and L/W)

p. 168, lines 4, 5, 7  run 124 → run 124 (with S=1.05, M=5, N_{G}=0)

p. 183, legend Fig. 4.11  squares → rectangles

p. 184, legend Fig. 4.12  Transition (9) → Transition (4.9)

Transition (9) → Transition (4.9)
1. In ondergrondse processen, waarbij een vloeistof of gas een andere, mengbare vloeistof of gas als een zuiger verdringt wordt de menging tussen de twee stoffen voor een groot deel bepaald door heterogeniteiten in het poreuze medium die een afmeting hebben van 0.1 tot 10 meter in de horizontale richting en 0.01 tot 1 meter in de verticale richting. Het maakt daarbij meestal weinig uit of de verdringing stabiel of instabiel is.

Dit proefschrift.

2. Macroscopische dispersiviteit is niets anders dan een pseudo-eigenschap van een poreus medium, die nuttig is voor reservoir simulatie.

3. De benadering van het menggebied tussen zout en zoet water als een scherpe overgang, zoals toegepast door Bear & Verruijt in de behandeling van het probleem van invasie van zeewater in het grondwater, wordt ondersteund door de bevinding van dit proefschrift dat de dispersediecoëfficiënt verkleind wordt onder invloed van het dichtheidsverschil tussen zout en zoet water.


4. De betekenis van de term "Taylordispersie" zoals door Lake & Hirasaki gebruikt voor de transversale dispersie tussen geologische lagen die in permeabiliteit verschillen komt niet overeen met de oorspronkelijke betekenis.


5. De combinatie van gravitationsegregatie met het Soret effect kan in een vloeistofkolom, die bovenaan kouder is dan onderaan, een convectiestroom op gang brengen. Dit is van belang voor de berekening van de verandering van de oliecompositie met de diepte in een ondergronds olie reservoar.

6. De oplossing van de partiële differentiaalvergelijking

\[
\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} = \frac{a}{x} \frac{\partial^2 C}{\partial x^2} \quad (x > 0, \ t > 0, \ a > 0)
\]

met begin- en randvoorwaarden \( C(x,0) = 1 \) en \( C(0,t) = 0 \) en \( C(\infty, t) = 1 \), is de incomplete gamma functie \( \Gamma(1+a, ax/Ut) \) gedeeld door de complete gamma functie \( \Gamma(1+a) \):

\[
\frac{ax}{t} \int_0^\infty e^{-s} s^{a-1} ds = \frac{1}{\Gamma(1+a)} \int_0^\infty e^{-s} s^{a-1} ds.
\]

7. De "arrogantie van de fysicus", die door Lagendijk aan de kaak gesteld werd in verband met de idee van veel fysici dat met slechts enkele fysische hoofdwerken ieder fysisch, chemisch en biologisch verschijnsel voorop kan worden, kan ook in verband gebracht worden met het feit dat de uit fysici bestaande Verkenningscommissie Natuurkundig Onderzoek bij haar berekening van de aantallen gepromoveerden en promovendi onder de in Nederland opgeleide fysici de promoties van fysici aan andere faculteiten dan faculteiten Natuurkunde verontwaardigd heeft.

Lagendijk, A., Inaugurele rede, Universiteit van Amsterdam, 1989.


8. De jaarlijkse "Fortune" ranglijst van 's werelds grootste industriële ondernemingen geeft een vertekend beeld van hun onderlinge betekenis; de lijst zou realistischer en stabielere zijn indien de rangorde wordt vastgesteld op basis van netto toegevoegde waarde (rolosom + interest + pachtsom + bruto winst) in plaats van op basis van omzet.


De nederlandse jachttouw: perspectieven en gewenste structurele aanpassingen, Berenschot en Horringa & De Koning (1972).

DISPERSE MIXING
IN STABLE AND UNSTABLE
MISCIBLE DISPLACEMENTS

Proefschrift

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft
op gezag van de Rector Magnificus Prof. drs. P.A. Schenck
in het openbaar te verdedigen ten overstaan van een commissie
door het College van Dekanen daartoe aangewezen
op donderdag 17 januari 1991 te 16.00 uur

door

Leonardus Johannes Theodorus Maria Kempers

geboren te Alphen aan den Rijn

natuurkundig ingenieur
Dit proefschrift is goedgekeurd door de promotor Prof. dr. ir. J. Hagoort

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Prof. ir. C. den Ouden, destijds leraar natuurkunde
Ir. J. de Rooij, leraar natuurkunde
Dr. M. Rappaport, Universiteit van Tel Aviv
Dr. R.J.J. Jongschaap, Universiteit Twente
Prof. dr. P.P. van der Wallen Mijnlieff, Universiteit Twente
Prof. dr. ir. L. van Wijngaarden, Universiteit Twente
Cover page

The photographs, taken at equal time steps, show a stable displacement of a fluid by a more viscous, miscible fluid through a porous medium with a large number of local-scale permeability variations. It can be seen that fingerlike disturbances between the fluids successively grow, split, merge and disappear. After some evaluation of the various stages, it can be demonstrated that the mixing zone between the fluids expands with the square root of the displacement distance. This phenomenon is typical for dispersive mixing. Dispersive mixing, however, is not restricted to stable displacements: the mixing zone can also be dispersive in unstable displacements, in which viscous fingers occur.

The photographs, which were obtained from a numerical simulation, were used to investigate how much the dispersive mixing zone is affected by the viscosity contrast between the fluids and their density contrast.
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PREFACE

This thesis is of interest to those involved in underground, miscible displacement processes such as gas flooding, mixing between gases in underground gas storage and in gas recycling, ground water flow and to simulator users and developers involved in modelling sub-grid processes. In each chapter the subject of the thesis is highlighted from a different perspective. Chapter 1 treats dispersive mixing as it is seen in the effluent from the porous medium. Chapter 2 quantifies the mechanism of dispersive mixing; in particular, a random-walk model for the effect of the viscosity contrast between the displacing fluid and the displaced fluids and their density contrast on dispersive mixing is combined with an existing model for the effect of small-scale permeability variations. Chapter 3 presents a very detailed means of observation of dispersive mixing; numerical simulation using a very large number of grid blocks. Chapter 4 presents an unconventional application of dispersive mixing to unstable displacements.

The work described in this thesis has been carried out at the Koninklijke/Shell Exploratie en Productie Laboratorium (KSEPL) at Rijswijk, the Netherlands, as part of the Shell research programme. It started late in 1984 with the preparation of core-flooding experiments of stable displacements of oil by injection of miscible gas and ended late in 1989 with numerical simulations of unstable, miscible displacements. Since an investigation is never a uni-directional, steady activity in time from problem to solution, the discussions with and help from my colleagues at KSEPL and elsewhere turned my unsteady steps in the investigation process into a drift movement with less dispersion superimposed.

In particular, the following persons, in chronological order, made a contribution. In the origin of the investigation process it was dr. ir. Londe Jong, at the time research leader at KSEPL. The very accurate core-flooding equipment was designed by John Coenen and dr. Johan van Luijk and was built by the construction department of KSEPL. Preliminary experiments were performed by Jos Bogaard as trainee from the H.T.S. Dordrecht. The experimental results were gained in an indispensible cooperation with Henk Haas and Hans Groeneweg; Henk Haas contributed to the explanation of the experiments and wrote the computer program in Appendix 2D. The simulator was obtained from dr. Jim Crump of Shell Development Company in Houston; Simon Verdouw and Erik Kater made the simulator running on the CRAY-XMP at KSEPL. The post-processing software was developed by ir. Michiel van Rijen, at the time student at the Technical University of Delft. Andre van den Berg assisted me in the transformation of numbers between 0 and 32676 into the photographs shown on the cover page and in Chapters 3 and 4. The discussions with dr. ir. Dick Ligthelm and dr. John van Wunnik very much contributed to dispersion suppression during the investigation; they also critically examined parts of early versions of the manuscript. Ir. Henk van Engen and ir. Jan van den Bergh also examined parts of the manuscript and gave useful suggestions. With dr. Gokhan Coskuner of the Calgary Research Centre I had some lively discussions on various aspects of viscous fingering. The criticism and advice of my promoter Prof. Jacques Hagoort gave significant added value to the results of the research and to the manuscript. Finally, the management of Shell Internationale Research Maatschappij B.V. provided me with facilities and made it possible the results of the research to be published in this thesis.

Leiden, December 1990
SUMMARY

Mixing between miscible fluids is of interest in various underground processes, e.g. in oil recovery by injection of miscible gas; in gas storage in underground reservoirs when the injected gas differs in quality from the cushion gas, and in the disposal of polluted or salt water in an underground freshwater reservoir. In these processes one fluid displaces another, miscible fluid through a porous medium; at the interface between the fluids a mixing zone develops. We study fluid mixing in a piston-like displacement, in which the mixing is caused by an often occurring and somewhat idealised type of permeability heterogeneity within a reservoir layer: the length over which the permeability is correlated (of the order of 0.1 - 10 metres in the areal directions and 0.01 - 1 metres in the vertical direction) is small in comparison with the size of the reservoir layer, the statistical properties of the function that defines the permeability in every location do not vary with location and the distribution of the log-permeability values is normal. This type of heterogeneity causes a dispersive mixing zone between the fluids; the length of the dispersive mixing zone depends on the distance travelled by the fluids and on the 'dispersivity'. In case the displacing and displaced fluids have equal viscosity and density, standard theory shows that dispersivity is a property of the porous medium solely. In this study we investigate the dependence of dispersivity on the viscosity contrast between the displacing and displaced fluids, on their density contrast and on displacement velocity, both in the stable regime (Chapters 1-3) and the unstable regime (Chapter 4). To account for this dependence, we introduce a new quantity: 'effective dispersivity'.

In Chapter 1, measurements of effective dispersivity are discussed and analysed. These are measurements that either are reported in literature or are our own experiments conducted in a 1.8 metre long, Berea sandstone core. In this core, oil was displaced by gas, and in another series of experiments, water was displaced by brine and brine by water. From the experiments we found:
* a strong dependence of effective dispersivity on displacement velocity in the conditionally stable regime;
* a very small density difference, such as between water and brine, can suppress effective dispersivity significantly.

All the measurements on effective dispersivity reported in literature and performed by ourselves have been summarised in general observations by means of a framework of (dimensionless) similarity groups, the most important of which are mobility ratio and gravity number. A practical consequence of the framework is that a number of recommendations are given for the measurement of the dispersivity of a geological unit or a core.

In Chapter 2, a random-walk model for effective dispersivity of stable displacements is presented. The model is based on the assumption of a random-walk process of fluid parcels travelling through the porous medium, and it is an extension of the approach of Scheidegger. The model provides a multiplication factor between 0 and 1 for the effective dispersivity.

The length of a dispersive mixing zone can be predicted if the following parameters are known: permeability (effective permeability and geometric standard deviation), correlation length of permeability in the displacement direction, viscosities and densities of fluids, displacement velocity and dip.
In Chapter 3, the validation of the analytical model of Chapter 2 is discussed. First, it is shown that the qualitative observations of Chapter 1 agree with the predictions of the analytical model and that the quantitative agreement is reasonable. Second, the growth of mixing zones in the stable displacement of a fluid by another, miscible, fluid has been investigated by carrying out numerical experiments. In the presence of a viscosity contrast and a density contrast, the evolution of the mixing zone is shown to be dispersive, i.e., the length of the mixing zone increases with the square root of time. The mixing zone length agrees accurately with the analytical model of Chapter 2.

A practical consequence for the numerical simulation of a displacement in which dispersive mixing is the dominant mechanism at the displacement front is that the numerical dispersion can be employed to represent the macroscopic dispersion in a wide range of mobility ratios and gravity numbers. The number of grid blocks should then be selected such that the numerical dispersion equals the value that is predicted by the analytical model of Chapter 2.

In Chapter 4, we show that in the unstable displacement of miscible fluids two flow regimes are present: one in which the displacement is dominated by viscous fingers and one in which the displacement is dominated by dispersive mixing due to the local-scale permeability variations. There is, qualitatively, no distinct difference between stable and unstable displacement, if an unstable displacement is dispersive-mixing-dominated. In the viscous-finger-dominated regime the mixing zone expands linearly in time; in the dispersive-mixing-dominated regime the mixing zone expands as the square root of time. We have estimated the condition of transition between the two flow regimes and have validated it by monitoring the development of the mixing zone in numerical simulations of our own and by evaluating simulations reported by Araktingi & Orr, Crump, Moissis et al. and Waggoner. In addition, we found that when the unstable displacement is dominated by dispersive mixing, the length of the dispersive mixing zone can be calculated according to the random-walk model of Chapter 2. This model describes the expansion of the mixing zone between the fluids better than the conventional Koval and Todd & Longstaff models, because the random-walk model provides for an expansion of the mixing zone with the square root of time, whereas the conventional models provide for an expansion linearly in time.

The results of the work described here can be of importance to slug design in miscible flooding and in polymer flooding, to modelling mixing between gases in underground gas storage and to modelling mixing in condensate recovery by gas recycling.
SAMENVATTING IN HET NEDERLANDS

DISPERSIEVE MENGING IN STABILE EN INSTABILE MENGbare VERDRINGINGEN

Menging tussen een vloeiostof of gas en een andere vloeiostof of gas is van belang in enkele ondergrondse processen, bijvoorbeeld in de oliewinning met behulp van injectie van mengbaar gas; in de ondergrondse opslag van aardgas, dat in verbrandingswaarde verschilt van het al aanwezige buffer gas, en in de afvoer van vervuild of zout water in een ondergronds zoetwater reservoir. In deze processen verdringt een geïnjecteerde vloeiostof of geïnjecteerd gas een andere, mengbare vloeiostof of mengbaar gas door een porueus medium; tussen de vloeistoffen ontwikkelt zich een mengzone. In dit werk bestuderen we de menging veroorzaakt door een veelvoorkomende en enigszins geïdealiseerde vorm van permeabiliteits heterogeniteit binnen een reservoir laag; de lengte waarover de permeabiliteit is gecorreleerd (in de orde van 0,1 - 10 meter in de horizontale richting en 0,01 - 1 meter in de vertikale richting) is klein ten opzichte van de afmetingen van de reservoir laag, de statistische eigenschappen van de functie die op iedere plaats de permeabiliteit definiert hangen niet af van de plaats en de log-permeabiliteits waarden zijn normaal verdeeld. Dit type heterogeniteit veroorzaakt dispersieve menging tussen de vloeistoffen en/of gassen; de lengte van de dispersieve mengzone wordt bepaald door de afstand die de vloeistoffen en/of gassen hebben afgelegd en de 'dispersiviteit'. Volgens de huidige kennis, is de dispersiviteit een eigenschap van enkel het poreuze medium. In dit werk onderzoeken we in welke mate de dispersiviteit ook afhankelijk is van het viscositeits contrast tussen de twee stromende stoffen; van hun dichtheids contrast en van de snelheid van verdringing. Dit onderzoeken we zowel voor stabiele (hoofdstukken 1 - 3) als instabiele verdringingen (hoofdstuk 4). Om met deze afhankelijkheid rekening te kunnen houden, hebben we een nieuwe grootheid geïntroduceerd: "effectieve dispersiviteit".

In hoofdstuk 1 worden metingen van effectieve dispersiviteit besproken en geanalyseerd. Dit zijn zowel metingen die zijn gerapporteerd in de literatuur als eigen metingen die zijn uitgevoerd in een 1,8 meter lange zandsteen kern. In deze kern werden twee series metingen verricht: de verdringing van olie door gas; en de verdringing van zout door zout water en van zout door zout. Deze metingen laten zien dat de effectieve dispersiviteit sterk afhankelijk is van de verdringingssnelheid in het conditioneel stabiele regime en tevens dat een klein dichtheidsverschil, zoals tussen zout en zout water, de effectieve dispersiviteit aanmerkelijk kan onderdrukken. Alle metingen die gepresenteerd zijn in de literatuur en onze eigen metingen zijn samengevat in een raamwerk van dimensieloze, gelijkvormigheidselementen, waarvan de belangrijkste zijn: mobiliteitsverhouding en gravitatietegoel. Een praktisch gevolg van dit raamwerk is dat een aantal aanbevelingen wordt gegeven voor het meten van de dispersiviteit van een geologische eenheid of gesteente kern zonder last te hebben van de invloed van de vloeiostof eigenschappen.

In hoofdstuk 2 hebben we een random-stap model gepresenteerd voor de effectieve dispersiviteit van stabiele verdringingen. In het model nemen we aan dat de vloeiostofdeeltjes een random-stap proces ondergaan wanneer ze zich door het poreuze medium verplaatsen. Het model is een uitbreiding van het model van Scheidegger en levert een vermenigvuldigingsfactor tussen 0 en 1 voor de effectieve dispersiviteit.
De lengte van een dispersieve mengzone kan nu berekend worden als de volgende parameters bekend zijn: permeabiliteit (met name de effectieve permeabiliteit en de geometrische standaarddeviatie), de correlatielengte van de permeabiliteit in de verplaatsingsrichting, de viscositeiten en dichtheden van de vloeistoffen en/of gassen en de hellingshoek.

In hoofdstuk 3 hebben we de validatie van het analytische model van hoofdstuk 2 besproken. Allereerst wordt aangetoond dat de qualitatieve waarnemingen van hoofdstuk 1 overeenkomen met de voorspellingen door het analytische model en dat de quantitatieve overeenkomst bevriddigend is. In de tweede plaats wordt door middel van numerieke experimenten de groei bestudeerd van dispersieve mengzones in stabiele verdringing. We tonen aan dat, in de aanwezigheid van een viscositeitscontrast en een dichtheidscontrast, de uitbreiding van de mengzone dispersief is, dat wil zeggen dat de lengte van de dispersieve mengzone toeneemt met de wortel uit de afgelegde afstand. De lengte van de mengzone komt nauwkeurig overeen met de voorspelling door het analytische model van hoofdstuk 2.

Een praktische consequentie voor de numerieke simulatie van een verdringing waarin dispersieve menging het belangrijkste mechanisme is aan het verdringingsfront, is dat de numerieke dispersie gebruikt kan worden als macroscopische dispersie. Het aantal benodigde gridblokken moet dan zo gekozen worden dat de numerieke dispersie overeenkomt met de waarde berekend met het analytische model van hoofdstuk 2.

In hoofdstuk 4 hebben we getoond dat er twee stromings regimes te onderscheiden zijn in de instabiele verdringing van mengbare vloeistoffen of gassen: één waarin de verdringing door viskeuze vingers en één door dispersieve menging wordt gedomineerd. Er is, qualitatief gezien, geen scherp verschil tussen een stabiele en een instabiele verdringing als een verdringing dispersieve-menging-gedomineerd is. In een viskeuze-vingering-gedomineerde verdringing expandeert de mengzone lineair met de tijd; in een dispersieve-menging-gedomineerde verdringing is dit met de wortel uit de tijd. We hebben de overgang tussen de twee stromingsregimes afgeschat en tevens gevalideerd met numerieke experimenten. Deze betreffen zowel eigen numerieke experimenten als numerieke experimenten die zijn gerapporteerd door Araktingi & Orr, Crump, Moissies et al. en Waggoner. Daarbij vonden we dat de lengte van de dispersieve mengzone berekend kan worden met het random-stap model van hoofdstuk 2. Omdat een dispersieve mengzone uitbreidt met de wortel uit de tijd, beschrijft dit model de lengte beter dan de conventionele modellen van Koval en van Todd & Longstaff.

De resultaten van dit hoofdstuk kunnen van belang zijn voor het ontwerpen van slugs in oilwinning door polymeerinjectie, voor het modelleren van de menging tussen gassen in ondergrondse gasopslag, en ook voor genoemde menging in condensaat winning door gascirculatie.
INTRODUCTION: DISPERSIVE MIXING

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1. DESCRIPTION OF DISPERSIVE MIXING

In a porous medium in which one fluid (injected fluid F) displaces another, miscible fluid (produced fluid P) in a piston-like manner, the interface between the fluids, which is originally sharp and flat (Fig. 1a, top), becomes diffuse and ragged during the displacement (Fig. 1b, top). This phenomenon is the result of many rapid, irregular spatial variations in the flow velocity, both in direction and in magnitude, superimposed on the average displacement velocity $V_{av}$. These flow velocity variations are caused by the tortuosity of the flow paths in the porous medium and by rapid, irregular spatial variations in the permeability of the porous medium.

In many cases, the calculation of the location of such a diffuse and ragged interface is not possible. The reason is that either detailed knowledge about the spatial variation of the permeability is lacking or the computational effort is too large. A way to handle the diffuse and ragged interface is to consider the concentration of the injected fluid only, averaged over a cross-section perpendicular to the flow direction. The

---

**FIG. 1a:** Sharp and flat interface between fluids at time $\tau = 0$ (areal cross-section).

---

**FIG. 1b:** Different and ragged interface between fluids at time $\tau > 0$ (areal cross-section).
spatial variation of this concentration C, which would be a step if the interface were sharp and flat (Fig. 1a, bottom), is in many cases smooth and S-shaped (Fig. 1b, bottom) and is commonly described by a dispersive mixing zone. The calculation of a dispersive mixing zone can be done in one dimension and thus requires less computational effort.

Dispersive mixing zones are of importance if the displacement is stable. In an unstable displacement, perturbations of the interface develop into viscous fingers and/or a gravity tongue, and these may be dominant.

Types of dispersive mixing

Permeability variations are present in underground reservoirs on many scales: reservoirs consist of different geological units, variations are present within a geological unit and there are variations on a centimetre scale. The magnitude of the dispersive mixing zone thus depends on the length scale. A practical approach to defining reservoir heterogeneity at different length scales for reservoir engineering applications has been introduced by Allen Alpay and used by others. Three elementary length scales are introduced, each with its own type of dispersion:

1. Microscopic dispersion at laboratory scale. The areal extent of the flow domain is about $10^{-1}$ m. On this scale, microscopic heterogeneities occur owing to pore geometry (Fig. 2); frequency distributions of log-permeability are of the type depicted in Fig. 3a. This type of frequency distribution is denoted as: uniform and homogeneous. Dispersion due to heterogeneities at pore level is called microscopic, convective dispersion. Two other types of microscopic dispersion are molecular diffusion, resulting from the random thermal motion of molecules, and Taylor dispersion. Taylor dispersion is due to the non-uniform distribution of the fluid flow velocity within a pore channel. This type of dispersion is present only when the radial diffusion in the pore channel is too small to cause a uniform concentration in a cross-section of a pore channel. This is the case at high fluid velocities. The three types of dispersion are summarised as microscopic dispersion.
(2) Macroscopic dispersion at local scale. The areal extent of the flow domain is about $10^0 - 10^3$ m. On a local scale, heterogeneities occur within a reservoir layer on an areal correlation scale of the order of $10^{-1} - 10^1$ m and a vertical correlation scale of the order of $10^{-2} - 10^0$ m. For a reservoir layer, frequency distributions of log permeability are expected to be of the type depicted in Fig. 3c. This type of frequency distribution is denoted as non-uniform and homogeneous: there is a distribution of permeabilities, but the statistical properties of the distribution do not vary within the layer ('statistically homogeneous'). An example is shown in Fig. 4.

In numerical simulations of displacement processes at field scale, local-scale heterogeneity is usually not modelled because either sufficient data about the local-scale heterogeneity are not available or the modelling requires too many grid blocks. Macroscopic dispersion, in contrast, requires only a few input data and it can be modelled with less grid blocks.

A statistically homogeneous permeability distribution is of course a somewhat idealised type of local-scale heterogeneity, but it seems to be a good approximation for many depositional settings; Gelhar\textsuperscript{10} gives many examples. However, before approximating local-scale heterogeneity in a particular case as statistically homogeneous, advice of a geologist is required.

\begin{figure}[h]
\centering
\begin{subfigure}{.4\textwidth}
\centering
\includegraphics[width=0.8\textwidth]{uniform_homogeneous.png}
\caption{Uniform, homogeneous}
\end{subfigure}\hfill
\begin{subfigure}{.4\textwidth}
\centering
\includegraphics[width=0.8\textwidth]{non_uniform_homogeneous.png}
\caption{Non-uniform, homogeneous}
\end{subfigure}
\begin{subfigure}{.4\textwidth}
\centering
\includegraphics[width=0.8\textwidth]{uniform_heterogeneous.png}
\caption{Uniform, heterogeneous}
\end{subfigure}\hfill
\begin{subfigure}{.4\textwidth}
\centering
\includegraphics[width=0.8\textwidth]{non_uniform_heterogeneous.png}
\caption{Non-uniform, heterogeneous}
\end{subfigure}
\caption{Frequency distributions for log-permeabilities, illustrating definitions of uniformity and homogeneity (from Ref. 4)}
\end{figure}
FIG. 4: Permeability (millidarcy) and porosity data from laboratory analyses of cores from a borehole in the Mt. Simon aquifer in Illinois (after Ref. 9).
(3) Megascopic dispersion at field scale. The areal extent of the flow domain is about $10^3$ to $10^5$ m. On a field scale, megascopic heterogeneities occur owing to the presence of various geological units. Frequency distributions of log permeability will be of the type depicted in Fig. 3d. This type of frequency distribution is denoted as non-uniform and heterogeneous: the statistical properties of the distribution are different for the various geological units. An example is a layered reservoir with permeability variations within the layer (Fig. 5).

![Diagram showing flow of labelled water in a permeable stratified reservoir](image)

**FIG. 5:** Flow of labelled water in a permeable stratified reservoir (after Ref. 11)

**Result of different types of mixing on dispersive mixing zone**

Macroscopic dispersion and megascopic dispersion permit regions of no mixing, in contrast with microscopic dispersion that is complete to molecular level. With only microscopic dispersion present in a linear displacement through a homogeneous porous medium, the interface between the fluids which is originally flat and sharp, remains flat and becomes diffuse. With only macroscopic dispersion present, the interface becomes ragged and remains sharp. The combination of microscopic dispersion with macroscopic dispersion makes a flat and sharp interface ragged and diffuse (Fig. 1b).
2. SIGNIFICANCE OF DISPERSIVE MIXING

Dispersive mixing zones occur in many reservoir processes. Examples are:

1. Secondary oil recovery: gravity-stable displacement of oil by a miscible solvent\(^1\).

2. Underground gas storage: storage of gases of different quality, for example, the injection of natural gas in a gas reservoir with a cushion gas of inert gas, such as nitrogen or carbon dioxide\(^{13-16}\).

3. Disposal of polluted water in an aquifer: the aquifer water mixes with polluted water\(^2\).

4. Reservoir souring: hydrogen sulphide is produced upon mixing between injected water and aquifer water\(^{17,18}\) as a result of a reaction between organic compounds in aquifer water and bacteria and sulphates in injected water.

In these examples, the mixing causes a problem:

1. In secondary oil recovery, dispersive mixing causes dilution of solvent slugs and loss of miscibility.

2. In underground gas storage, the mixing between natural gas and inert cushion gas causes a loss of quality of the natural gas.

3. When polluted water is disposed of in an aquifer, the aquifer water gets polluted.

4. In reservoir souring, the extent of mixing between injected water and aquifer water determines the quantity of produced hydrogen sulphide.

In the first three examples, macroscopic and megascopic dispersion are the most important type of dispersion; in the last example, microscopic dispersion is important too. For the understanding of these underground processes and for the design of facilities at surface, it is of interest to know what the extent of the dispersive mixing zone is.

3. EXTENT OF DISPERSIVE MIXING ZONE

In the past, numerous experiments on the linear and stable displacement of miscible fluids in a porous medium have been observed by sampling the
effluent and measuring the concentration of the displacing fluid in the
samples (for a review see Perkins & Johnston\textsuperscript{19}). These experiments have
shown an S-shaped function for the concentration C that can be described
by\textsuperscript{1,2,19}

\[ C = \frac{1}{2} \text{erfc}\left(\frac{L - Ut}{2\sqrt{Kt}}\right) \]  

(1)

where C is the concentration of the injectant in the effluent
L is the length of porous medium
t is time
U is the displacement velocity (equal to Darcy velocity divided by
porosity \( \phi \))
K is the dispersion coefficient.

This is the same expression as in case of mixing due to molecular diffusion,
with K replaced by the molecular diffusion coefficient D and with the limit
U \( \to 0 \) applied:

\[ C = \frac{1}{2} \text{erfc}\left(\frac{L}{2\sqrt{Dt}}\right) \]

The diffusion-like coefficient K in (1) is called the dispersion
coefficient.

The dispersive mixing zone is commonly defined\textsuperscript{20} as the zone in which
the concentration, averaged over a cross-section perpendicular to the flow
direction, varies from 10\% to 90\%. The dispersive mixing zone length follows
from (1):

\[ X_{10} - X_{90} = 3.62 \sqrt{Kt} \]  

(2)
in which \( X_{10} \) and \( X_{90} \) are the locations at which C is 10\% and 90\%
respectively. The expression shows that the length of the dispersive mixing
zone increases with the square root of time. The expansion of the mixing
zone is further described by a factor \( \sqrt{K} \). The magnitude of the dispersion
coefficient K depends on the length scale, and is discussed below.
(1) **Microscopic dispersion coefficient.** In the early sixties, Perkins & Johnston formulated for the microscopic dispersion coefficient \( \kappa \) for fluids that have equal viscosity and equal density:

\[
\kappa = \frac{D}{F_R \phi} + 0.5 \ s \ d_g \ U
\]

on the condition of absence of Taylor dispersion: \( \frac{s \ d_g \ U}{D} < 50 \)

where \( D/F_R \phi \) = molecular diffusion coefficient \( D \), divided by the formation resistivity factor \( F_R \) and the porosity \( \phi \) to account for the tortuosity of the grain pack.

\( d_g = \) grain size

\( s = \) inhomogeneity factor of porous medium.

The empirical relation includes both mixing by diffusion and mixing by rock inhomogeneity \( (0.5 \ s \ d_g \ U) \). The diffusion term \( D/F_R \phi \) is small at field scale \(^3\) (and/or at high displacement velocity) compared with the convective term \( 0.5 \ s \ d_g \ U \), but has to be taken into account in experiments at laboratory scale (and not too high a displacement velocity). For gas injection in gas reservoirs, the diffusion term may be important at field scale as well \(^1,21\).

The second term in the dispersion coefficient \( (0.5 \ s \ d_g \ U) \) is called the convective dispersion coefficient, denoted here as \( K_c \). The inhomogeneity factor \( s \) is not precisely defined and must be measured. As shown in the above expression from the standard theory, the proportionality constant of \( U \) depends on properties of the porous medium alone. It is called the dispersivity and is denoted here as \( \beta_0 \). It has the dimension of a length.

The linear dependence of the dispersion coefficient on the displacement velocity \( U \) was confirmed by Scheidegger \(^22\), who assumed that the fluid parcels undergo a random-walk process. Scheidegger did not quantify the proportionality constant of \( U \). The random-step process in his model refers to flow variations on a pore scale, but can be applied to flow variations at other scales too.
(2) **Macroscopic dispersion coefficient.** Preliminary results on macroscopic dispersion were obtained by Warren and Skiba\(^8\) from Monte-Carlo-type calculations. In the late seventies and early eighties, hydrologists in particular have developed analytical models of macroscopic dispersion by stochastic modelling of groundwater flow, making use of analytical spectral representation methods\(^{23-36}\). Of these models, the one presented by Gelhar and Axness in Ref.\(^\text{27}\) is the most complete. Gelhar & Axness\(^\text{27}\) made a calculation of the dispersion tensor based on stochastic spectral theory, assuming a statistically homogeneous geological layer with dimensions large compared with the correlation length of the permeability and for a frontal advance of several times the correlation length. They used a log-normal distribution of the permeability and an autocorrelation function \(R(\vec{r})\), which is defined in Appendix 1B, for the log-permeability with an exponential decay with distance:

\[
R(\vec{r}) = \sigma^2 e^{-\frac{\lambda}{\lambda}} \tag{4}
\]

where \(\sigma\) is the standard deviation of the log-permeability distribution and \(\lambda\) is a characteristic correlation length. The interpretation of \(\sigma\) is as follows: the characteristic permeability ratio between adjacent areas of uniform permeability is a factor \(S = e^\sigma\) (\(S\) is the geometrical standard deviation). The expression that they derived for the longitudinal dispersion of a two-dimensional isotropic material is

\[
K_L = U \lambda \sigma^2 \exp\left(-\sigma^2 \frac{\lambda-\lambda'}{\lambda+\lambda'}\right) \tag{5a}
\]

where \(\lambda'\) is a characteristic correlation length in the transverse flow direction. Since the theory of Gelhar et al. is based on perturbation theory, a limitation to the theory is the restriction of the variance \(\sigma^2\) of the log-normal distribution to a maximum of about 1. One can also observe in expression (5a) that, beyond \(\sigma = \left(\frac{\lambda+\lambda'}{\lambda-\lambda'}\right)^{1/2}\), \(K_L\) decreases with increasing \(\sigma\), which is unrealistic.

In the three-dimensional case, they calculated for the longitudinal dispersion coefficient

\[
K_L = U \lambda \sigma^2 \left(1+\frac{1}{6}\sigma^2\right)^{-2} \tag{5b}
\]
As in (5a), the most important term in the dispersivity given by (5b) is \( \lambda \sigma^2 \). The correction term \((1 + \frac{1}{6} \sigma^2)^{-2}\) is of minor importance in the range of validity of the theory. The same applies to the correction term \(\exp(-\sigma^2 \frac{\lambda - \lambda'}{\lambda + \lambda'})\) in expression (5a) for the two-dimensional case. We shall abbreviate both correction terms with the function \( F(\sigma) \).

The theory can be applied widely, as shown by Gelhar\(^{10}\), who provided some data on variances and correlation scales for various geological settings. Gelhar\(^{34}\) has shown for a field case that the expression \( \lambda \sigma^2 \) for the dispersion coefficient satisfies beyond \( \sigma = 1 \). (Any future theoretical result that describes the relation between \( K_C \) and \( \sigma \) for large \( \sigma \) values can be implemented in the rest of this work by adaptation of the correction factor \( F(\sigma) \).)

**3 Megascopic dispersion coefficient.** Megascopic dispersion modelling is often done for heterogeneous media consisting of uniform and homogeneous layers\(^{11}\) (Fig. 3b). A possible technique to handle this type of dispersion is the Stiles method\(^{35}\). The mixing zone length is proportional to distance travelled; the megascopic dispersion coefficient is not a constant but increases with distance travelled\(^{36}\).

On the basis of the relation of Perkins & Johnston one would expect a mixing zone between two fluids of 3 cm after flooding in a 0.1 metre core (in this estimation a high displacement rate is assumed so that diffusion can be ignored). In a 1000 metres long reservoir layer the microscopic mixing zone would have a length of 3 metres. However, from data from well tracer tests, mixing zone lengths are found that are orders of magnitude larger\(^{3}\). This means that, on a field scale, microscopic dispersion is negligible compared to dispersion at local scale and field scale.

**Independence of extent of dispersive mixing zone on displacement velocity:**
When molecular diffusion is neglected and \( \beta_0 = K_C/U \) and \( t = X/U \) are substituted in (2), in which \( X \) is the distance travelled by the fluids, the mixing zone length between fluids that have equal viscosity and equal density is independent of the displacement velocity:

\[
X_{10} - X_{90} = 3.62 \sqrt{(\beta_0 X)}
\]  \( \text{(6)} \)
4. OBJECTIVE OF THIS STUDY

The theory of dispersive mixing has been developed by hydrologists mainly. In most hydrological processes, the displacing and displaced fluids are miscible and have about the same properties, for example in the displacement of water by brine. In many of the miscible displacement processes relevant to the oil industry, however, there may be a significant viscosity contrast and density contrast between the displacing and displaced fluids.

Results of some core-flooding experiments on stable displacement of miscible fluids found in literature\textsuperscript{37-41} and discussed in this study show that the length of the dispersive mixing zone is dependent not only on properties of the porous medium and the distance travelled as in eq. (6) but also on the viscosity contrast between the displacing and displaced fluids, on the density contrast and on the displacement velocity. For the understanding of the reservoir processes listed in Section 2, it is of interest to know if the expansion of the mixing zone is indeed dispersive in the presence of a viscosity contrast and a density contrast and to what extent the mixing zone length depends on properties other than reservoir properties and distance travelled, such as viscosity contrast, density contrast and displacement velocity.

The objective of this work is therefore to determine the extent and the expansion of the mixing zone in the presence of a viscosity contrast and a density contrast between the displacing and displaced fluids. This is done for piston-like, miscible displacements in which the mixing is caused by local-scale heterogeneity. To achieve this objective we have:
1. evaluated experiments in literature and performed by ourselves, which demonstrate the dependence,
2. quantified the dependence by means of a random-walk model,
3. validated the assumptions in the model by means of numerical simulations,
4. investigated dispersive mixing in unstable displacement.

Dispersive mixing at field scale (megascopic dispersion) falls thus outside the scope of this study. We mainly deal with dispersive mixing at local scale (macroscopic dispersion). In the interpretation of the core-flooding experiments (in Chapter 1) we also deal with dispersive mixing at laboratory scale (microscopic dispersion), but not with Taylor dispersion, which plays a role only at high displacement velocities near wells\textsuperscript{1}. 
5. ORGANISATION

In Chapter 1, experimental observations on the dispersive mixing zone in the presence of a viscosity contrast and a density contrast are treated. The observations are based on core-flooding experiments and one numerical experiment in the literature and on our own core-flooding experiments. The measurements are evaluated in terms of dimensionless similarity groups. The result of the chapter is a list of trends of the effective dispersivity with the similarity groups.

In Chapter 2, an analytical model is presented. The model is based on the assumption of a random walk process of the fluid parcels. With this model the length of a dispersive mixing zone can be predicted when a viscosity contrast and a density contrast are present. The input parameters of the model are the standard deviation of the permeability variation, the correlation length of the permeability, the absolute permeability, the fluid viscosities and the fluid densities, the displacement velocity and the dip angle.

In Chapter 3, detailed numerical simulations of stable, linear displacements in an reservoir with local-scale heterogeneity are used to show that, in the presence of a viscosity contrast and a density contrast, the mixing zone is dispersive and that the model of Chapter 2 is valid. The simulations are checked by comparing simulation runs of displacements without viscosity contrast and density contrast with a standard theory. The model of Chapter 2 is also checked against the experiments discussed in Chapter 1.

Finally, in Chapter 4 it is demonstrated that unstable displacements sometimes do not exhibit the viscous fingers predicted by criteria in the literature, but instead, these displacements show a dispersive mixing zone, as in stable displacements. Detailed numerical simulations show that the length of the dispersive mixing zone can be calculated with the model of Chapter 2.

In Chapters 1 - 3 only stable displacements are discussed. Unstable displacements are subject of chapter 4. After the Introduction, all chapters can be read independently. Each chapter gives a consequence of the results for practical use.
6. SYMBOLS AND UNITS

The symbols are in line with the convention of the Society of Petroleum Engineers. The unit system is SI; data from external references are listed in the units of the reference.

REFERENCES


CHAPTER 1

OBSERVATIONS FROM EXPERIMENTS

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      1.3.1.1. Brigham
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1.8. RECOMMENDATION FOR MEASUREMENT OF DISPERSIVITY

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APPENDIX 1A: INSPECTION ANALYSIS OF BASIC EQUATIONS
APPENDIX 1B: PERMEABILITY VARIATION IN BEREA CORE
APPENDIX 1C: MEASUREMENT OF DISPERSION IN DEAD VOLUME
1.1. **INTRODUCTION**

The objective of this chapter is

1. to present a coherent evaluation of experimental observations on dispersive mixing zones for a wide variety of properties of fluids and porous media
2. to provide recommendations for a reliable measurement of the dispersivity of a porous medium without effect of fluid properties.

The discussion is limited to experiments in which

1. the displacing and displaced fluids differ in density and/or viscosity
2. the porous medium is non-uniform and homogeneous (see Section 1 from Introduction)
3. the displacement is stable.

It appears that the experiments reported in literature were conducted in the unconditionally stable regime, i.e. the regime in which the displacement is stable irrespective of the displacement velocity. To have observations in the conditionally stable flow regime, i.e. the regime in which the displacement is stable on the condition that the displacement velocity is below a critical velocity, we have conducted experiments ourselves in this regime.

To obtain a coherent evaluation of the experiments, which were conducted with a wide variety of properties of fluids and porous media, we used a framework of dimensionless similarity groups.

The method of observation is sampling the effluent from the porous medium and measuring the fraction of the displacing and displaced fluids in each sample. Because the expansion of the mixing zone during its travel through the porous medium is not monitored, it is assumed that the mixing zone is dispersive. To account for the dependence of the mixing zone length on viscosity contrast between the displacing and displaced fluids, on density contrast, on gravity, on permeability and on displacement velocity, we define a new quantity, the "effective dispersivity", denoted by $\beta$. The effective dispersivity equals the dispersivity of the porous medium, $\beta_0$, in absence of a viscosity contrast and a density contrast.
1.2. SIMILARITY GROUPS

Experimental observations on effective dispersivity are discussed in terms of similarity groups, which are dimensionless, instead of density difference, displacement velocity, etc. These groups are derived from the basic equations for miscible flooding and the boundary and initial conditions. The way to derive the dimensionless similarity groups is to bring the independent variables of the equations in a dimensionless form by dividing them by a characteristic value and to substitute the dimensionless variables in the basic equations and initial and boundary conditions. The constant terms in the equations are called similarity groups. The method for deriving similarity groups is called inspection analysis. More information on inspection analysis, focussed on immiscible and miscible flooding in petroleum reservoirs, can be found in a paper of Geertsma et al.¹

We use the similarity groups to compare the experimental results of various investigations on dispersivity that were conducted with a wide variety of cores and fluids. Another reason for evaluating the experimental results in terms of similarity groups is that the number of independent parameters is smaller than without the results of inspection analysis.

The equations are restricted to linear displacements in a two-dimensional, tilted, isotropic porous medium. The reason for restricting the equations to linear displacements in an isotropic porous medium is that all laboratory experiments were conducted in that way. The restriction to a two-dimensional porous medium serves to show the essentials of the inspection analysis; no other similarity group will appear in an inspection analysis in a three-dimensional porous medium apart from an extra aspect ratio.

The main difference between our inspection analysis, which is presented in Appendix 1A, and the one by Geertsma et al.¹ is that we treat the permeability not as a constant but as a place-dependent quantity. The reason is that we are interested in dispersion that originates from irregular permeability variations. More specifically, we consider the permeability to be correlatable with itself over a certain distance: distance λ in the longitudinal direction and distance λ' in the transverse direction.

The similarity groups derived from the dimensionless equations are

\[
\frac{k}{k_0} \quad (1.1)
\]
mobility ratio = \frac{\text{viscous force in fluid } P}{\text{viscous force in fluid } I} \quad M = \frac{\mu_P}{\mu_I} \quad (1.2)

grid number = \frac{\text{gravity force}}{\text{viscous force}} \quad N_G = \frac{k_0 g (\rho_P - \rho_L) \sin \theta}{\mu_P \phi U} \quad (1.3)

Peclet numbers at laboratory scale = \frac{\text{convective transport}}{\text{dispersive transport at lab. scale}} \quad \frac{N_{pe}}{\kappa} = \frac{UL}{\kappa} \quad \frac{N_{pe}'}{\kappa'} = \frac{UL}{\kappa'} \quad (1.4)

aspect ratio \quad \frac{W}{L} \quad (1.5)

number of random permeabilities in longitudinal direction \quad \frac{L}{\lambda} \quad (1.6)

number of random permeabilities in transverse direction \quad \frac{W}{\lambda'} \quad (1.7)

where \( g = \) gravitational acceleration
\( k = \) local permeability
\( k_0 = \) effective permeability
\( L = \) length of porous medium
\( U = \) displacement velocity
\( W = \) width of porous medium
\( \theta = \) dip angle with horizontal
\( \kappa = \) longitudinal dispersion coefficient at laboratory scale
\( \kappa' = \) transversal dispersion coefficient at laboratory scale
\( \lambda = \) correlation length of permeability in main flow direction
\( \lambda' = \) correlation length of permeability in transverse flow direction
\( \mu = \) dynamic viscosity
\( \rho = \) density
\( \phi = \) porosity
subscript I refers to displacing fluid (the injected fluid)
subscript P refers to displaced fluid (the 'produced' fluid)
We consider $k/k_0$, $M$ and $N_G$ as the most important similarity groups. We remark that similarity in $k/k_0$ between two porous media means that the ratio of the permeability field and the average permeability is the same. This poses a problem: in reality, permeability is not known at every place in the porous medium; at most only statistical parameters describing the spatial variability of the permeability are known. This has a consequence for the method of comparison of displacement experiments in two systems. We cannot compare values of the concentration $c$ at a specific place and time between the systems, but only $C$ values that are the average of a certain region. In practice, this is always done: the flowing concentration of the injectant in the effluent of a displacement experiment is the average flowing concentration of a transverse cross-section.

By an appropriate correction the Peclet numbers at laboratory scale, $N_{pe}$ and $N_{pe'}$, can be left out of consideration when comparing different experiments in a porous medium. According to Perkins & Johnston (eq. (3) from the Introduction) the effects of microscopic convective dispersion and molecular diffusion on total microscopic dispersion are additive. After subtracting the diffusion term $D/F = \theta$ from the total microscopic dispersion, $N_{pe}$ and $N_{pe'}$ are constant in the entire velocity range.

The other similarity groups are less important for the following reasons. The rescaled transport equation for the longitudinal transport, (1A.3c), which is considered to be the most important transport equation, does not contain the aspect ratio $W/L$. It is therefore not unreasonable to assume that the aspect ratio affects the effective dispersivity much less than mobility ratio and gravity number.

The numbers of random permeabilities in both directions, occur in the permeability field and not in the equations. When the numbers are high, the system can be considered as infinitely large and the numbers have no effect. Therefore the numbers are unimportant.

1.3. Measurements of Effective Dispersivity from Literature

The experiments discussed in this section were conducted in the unconditionally stable flow regime, i.e. in the regime were the displacement is stable for all displacement velocities.
1.3.1. Dependence of effective dispersivity on mobility ratio

1.3.1.1. Brigham

An increase of effective dispersivity with mobility ratio was experimentally shown by Brigham. He measured, in a glass pack placed in horizontal position, that dispersion is suppressed when the displacing fluid is less mobile than the displaced fluid. At a mobility ratio of 0.175 the dispersion was suppressed by a factor of 5 compared with a dispersion at a mobility ratio of 0.998. (On basis of just these two datapoints, Stalkup presented a graph for the complete M range below 1.)

1.3.1.2. Giordano et al.

Giordano et al. found suppression of dispersion at a decrease of the mobility ratio in a few numerical simulations of miscible displacement in an non-uniform, homogeneous, square reservoir. The simulation configuration consisted of 10 by 10 squares of uniform permeability. Each square was divided in 4 by 4 grid blocks. These authors designed the permeability variation by assigning to each square a random permeability taken from a normal distribution. The standard deviation was equal to 0.5 times the mean. The fluid densities were the same. They found that the profile of the concentration of displacing fluid in the effluent was steepened when the mobility ratio was decreased from 1 in the first simulation run to 0.1 in the second run. Further decrease of the mobility ratio to 0.01 in a third run did not affect the effluent profile. Figure 1.1, which has been taken from their paper, shows the effluent profiles.

1.3.2. Dependence of effective dispersivity on mobility ratio, displacement velocity and density difference

An increase in effective dispersivity with displacement velocity has been demonstrated by Slobod & Howlett, by Newberg & Foh and by Ben Salah. Slobod & Howlett and Newberg & Foh found no change in effective dispersivity with displacement velocity at high displacement velocity. In addition, Slobod & Howlett observed a constant effective dispersivity when the fluid densities are the same.
1.3.2.1. Slobod & Howlett

Slobod & Howlett\(^6\) conducted 48 experiments in a core 1.22 m long, mounted vertically. Since Slobod & Howlett have explored the stable region and the unstable region, we have taken from their 48 experiments only those experiments that were stable. We applied the following criteria:

- Stable during displacement. In conditionally stable displacement the velocity should be below the Dietz critical velocity for gravity-stable displacement \( U_c \), which is\(^4\)

\[
U_c = \frac{k g (\rho_p - \rho_I) \sin \theta}{\phi (\mu_p - \mu_I)}
\]  

(1.8a)

- Stable at rest (some experiments had the heavier fluid on top of the lighter fluid. At rest this could have led to the intrusion of the fingers of heavier fluid into the lighter fluid).

- Locally stable. To ensure that the displacement is nowhere locally unstable, the velocity should be below the Dumoré critical velocity\(^7\), given by

\[
U_{cD} = \frac{k g}{\phi} \left( \frac{\partial \rho}{\partial \mu} \right)_{\text{min}} \sin \theta
\]

(1.8b)
The velocity $U_{CD}$ is a fraction $\frac{M-1}{M \ln M}$ of $U_C$ when a linear mixing rule is assumed for both $\rho$ and $\ln \mu$.

Applying these criteria left us with the 16 experiments listed in Table 1.1. The data of the fluids used are listed in Table 1.2. The edges of the mixing zone were defined by Slobod & Howlett as the 5% concentration and 95% concentration. From the mixing zone length $X_5 - X_{95}$ thus obtained, given in Table 1.1 in pore volumes, we calculated the inverse of the Peclet number at local scale, which is defined by

$$N_{Pe} = \frac{UL}{K_C}$$  \hspace{1cm} (1.9)

where $K_C$ is the convective dispersion coefficient, i.e. the total dispersion minus the contribution of molecular diffusion. (Note that the inverse Peclet number is equal to the effective dispersivity $\beta$ divided by the core length $L$.) It was assumed that the concentration history in the experiments of Slobod & Howlett could be described by an error function (eq. (1)). With the assumption, it follows from (1) that the inverse Peclet number is given by:

$$\frac{1}{N_{Pe}} = \left( \frac{X_5 - X_{95}}{2 \times 1.65 \times L} \right)^2$$  \hspace{1cm} (1.10)

No correction for the diffusion coefficient has been made because it was not available in the paper of Slobod & Howlett and is expected to be small compared with the lowest dispersion coefficient measured (1.3 x 10^{-8} m^2/s).

The results are listed in Table 1.1 and shown graphically in Fig. 1.2. The data points with $N_G = 0$ are represented on the logarithmic $N_G$-scale as $N_G = 0.01$; those with $M = 0.466$ and $M = 0.477$ are all represented as $M = 0.47$. 
### TABLE 1.1: SOME EXPERIMENTAL DATA OF SLOBOD & HOWLETT WITH CALCULATED VALUES OF $M$, $N_G$, $1/N_{Pe}$ ($\phi = 0.37$, $k = 18$ D, $L = 1.22$ m)

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>Displaced fluid</th>
<th>Displacing fluid</th>
<th>U (ft/day)</th>
<th>M</th>
<th>$N_G$</th>
<th>$\frac{X_5 - X_95}{L}$</th>
<th>$1/N_{Pe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>ethanol</td>
<td>Soltrol 170</td>
<td>25</td>
<td>0.477</td>
<td>0</td>
<td>0.082</td>
<td>3.11$\times10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>+ 1.96% CCl</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td>0.082</td>
<td>3.11$\times10^{-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td>0.082</td>
<td>3.11$\times10^{-4}$</td>
</tr>
<tr>
<td>5a</td>
<td>naphtha + ethanol</td>
<td>25</td>
<td>0.437</td>
<td>0</td>
<td>0.078</td>
<td>2.82$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.48% CCl</td>
<td>50</td>
<td></td>
<td></td>
<td>0.078</td>
<td>2.82$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td>0.078</td>
<td>2.82$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>2a</td>
<td>ethanol + Soltrol 170</td>
<td>25</td>
<td>0.466</td>
<td>0.30</td>
<td>0.058</td>
<td>1.56$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.11% CCl</td>
<td>50</td>
<td></td>
<td>0.15</td>
<td>0.077</td>
<td>2.74$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td></td>
<td>0.074</td>
<td>0.080</td>
<td>2.96$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>3a</td>
<td>ethanol + Soltrol 170</td>
<td>25</td>
<td>0.466</td>
<td>0.59</td>
<td>0.057</td>
<td>1.50$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>14.26% CCl</td>
<td>50</td>
<td></td>
<td>0.29</td>
<td>0.069</td>
<td>2.20$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td></td>
<td>0.15</td>
<td>0.079</td>
<td>2.89$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>4a</td>
<td>ethanol + Soltrol 170</td>
<td>25</td>
<td>0.374</td>
<td>0.76</td>
<td>0.057</td>
<td>1.50$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>22.39% CCl</td>
<td>50</td>
<td></td>
<td>0.38</td>
<td>0.069</td>
<td>2.20$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td></td>
<td>0.19</td>
<td>0.077</td>
<td>2.74$\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>12a</td>
<td>ethanol + naphtha</td>
<td>25</td>
<td>2.275</td>
<td>1.03</td>
<td>0.072</td>
<td>2.40$\times10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 1.2: FLUID DATA OF EXPERIMENTS OF SLOBOD & HOWLETT

<table>
<thead>
<tr>
<th>Fluids</th>
<th>Density at 30°C ($10^3$ kg/m$^3$)</th>
<th>Viscosity at 30°C ($10^{-3}$ mPa.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ethanol</td>
<td>0.7819</td>
<td>1.030</td>
</tr>
<tr>
<td>naphtha</td>
<td>0.7172</td>
<td>0.448</td>
</tr>
<tr>
<td>Soltrol 170</td>
<td>0.7660</td>
<td>2.220</td>
</tr>
<tr>
<td>naphtha + 7.48% CCl$_4$</td>
<td>0.7819</td>
<td>0.450</td>
</tr>
<tr>
<td>Soltrol 170 + 1.96% CCl$_4$</td>
<td>0.7819</td>
<td>2.160</td>
</tr>
<tr>
<td>ethanol + 6.11% CCl$_4$</td>
<td>0.8307</td>
<td>1.035</td>
</tr>
<tr>
<td>ethanol + 14.26% CCl$_4$</td>
<td>0.8954</td>
<td>1.033</td>
</tr>
<tr>
<td>ethanol + 16.26% CCl$_4$</td>
<td>0.9113</td>
<td>1.020</td>
</tr>
<tr>
<td>ethanol + 22.39% CCl$_4$</td>
<td>0.9601</td>
<td>0.830</td>
</tr>
</tbody>
</table>

### FIG. 1.2: Evaluation of experiments of Slobod & Howlett: reciprocal of Peclet number versus gravity number

- $M = 0.47$
- $M = 0.374$
- $M = 0.437$
- $M = 2.275$
1.3.2.2. Newberg & Foh

Newberg & Foh, who were interested in the mixing between inert gas and natural gas in underground gas storage projects, conducted nitrogen/methane displacement tests in various cores. Most of their cores were layered, either in the longitudinal flow direction or in the transverse flow direction. Since data on the heterogeneities are lacking in the paper, we evaluate only their experiments on an unlayered, short Berea sandstone core of 9.5 cm. The experiments were conducted vertically in the upward direction, with either methane displacing nitrogen or nitrogen displacing methane. Since the tests with the lighter methane injected at the bottom to displace the heavier nitrogen could have been unstable at rest, we evaluated only the nitrogen-displacing-methane tests.

Two series of nitrogen-displacing-methane tests were conducted, one series at a pressure of 3.5 MPa (500 psi) and one at 6.9 MPa (1000 psi). Fluid viscosities and densities are listed in Table 1.3. In the first series, the mobility ratio was 0.643, in the second 0.667. The exact values of the dispersion coefficients $K$ determined from the experiments are not tabulated in the paper, but instead a non-linear fitting relation between dispersion coefficient and displacement velocity is presented. The relations are:

series at 3.5 MPa: $K = 1.515 \times 10^{-7} + 0.117 \, U^{1.660} \, [m^2/s]$

(U-range: $2.5 \times 10^{-5} - 1.3 \times 10^{-4} \, m/s$)

series at 6.9 MPa: $K = 5.899 \times 10^{-8} + 0.8243 \times 10^{-3} \, U^{1.063} \, [m^2/s]$

(U-range: $4.2 \times 10^{-5} - 2.7 \times 10^{-4} \, m/s$)

The constant terms in the relations should be due to molecular diffusion in the porous medium. The second term in the relations is $U$ multiplied by the effective dispersivity. The relations are represented as inverse Peclet number, defined by $(1.9)$, versus gravity number in Fig. 1.3.
TABLE 1.3: SOME EXPERIMENTAL DATA OF NEWBERG & FOH ($\phi = 0.194$, $k = 0.102$ D, $L = 0.095$ m)

<table>
<thead>
<tr>
<th>Gas</th>
<th>Pressure (psi)</th>
<th>Density at 75°F (lbm/ft$^3$)</th>
<th>Viscosity at 75°F (cP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>methane</td>
<td>1000</td>
<td>3.144</td>
<td>0.0126</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.484</td>
<td>0.0117</td>
</tr>
<tr>
<td>nitrogen</td>
<td>1000</td>
<td>4.898</td>
<td>0.0189</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>2.453</td>
<td>0.0182</td>
</tr>
</tbody>
</table>

FIG. 1.3: Evaluation of experiments of Newberg & Foh: reciprocal of Peclet number versus gravity number
1.3.2.3. Ben Salah

Ben Salah conducted vertically upward displacements of pure water by a mixture of water and 45 %w glycerine. The mobility ratio was 0.21. Different glass bead packs were used; each pack was characterised by an average bead diameter. The data of the glass packs and the fluids are summarised in Table 1.4. In each glass pack four or five displacements were conducted at different velocities. The measured dispersion coefficients are listed in Table 1.5. From the data in the paper we calculated the effective dispersivity and the gravity number for each experiment.

In some experiments, the displacement velocity was so high that Taylor dispersion played a role. According to Perkins & Johnston, experiments with Taylor dispersion are characterised by a value greater than 1 for the ratio $\beta U/25D$. Table 1.5 shows which experiments have a value greater than or close to 1 for this ratio. For the evaluation of the trend with gravity number we have left out the experiments with a value greater than 0.9. The remaining experiments were taken up in Fig. 1.4, in which the inverse Peclet number, defined by (1.9), is plotted versus gravity number.

![Graph showing evaluation of experiments of Ben Salah: reciprocal of Peclet number versus gravity number (excluding the experiments with Taylor dispersion)]
TABLE 1.4: SOME EXPERIMENTAL DATA OF BEN SALAH

Properties of porous media

<table>
<thead>
<tr>
<th>Glass pack no.</th>
<th>Length (m)</th>
<th>Average bead size (μm)</th>
<th>Porosity</th>
<th>Permeability (μm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>0.50</td>
<td>9</td>
<td>0.428</td>
<td>3.9</td>
</tr>
<tr>
<td>22</td>
<td>0.75</td>
<td>14</td>
<td>0.371</td>
<td>11.2</td>
</tr>
<tr>
<td>23</td>
<td>1.00</td>
<td>18</td>
<td>0.377</td>
<td>15.2</td>
</tr>
<tr>
<td>24</td>
<td>1.50</td>
<td>28</td>
<td>0.368</td>
<td>39.6</td>
</tr>
<tr>
<td>25</td>
<td>2.50</td>
<td>45</td>
<td>0.374</td>
<td>92</td>
</tr>
</tbody>
</table>

Properties of fluids at 20 °C and atmospheric pressure:

<table>
<thead>
<tr>
<th>Fluids</th>
<th>Viscosity (mPa.s)</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water + 45%w glycerine</td>
<td>4.7</td>
<td>1113</td>
</tr>
<tr>
<td>distilled water</td>
<td>1.002</td>
<td>998</td>
</tr>
</tbody>
</table>

No value for the formation resistivity factor $F_R$ is given by Ben Salah. A reasonable estimate of the term $1/F_R$ is 0.7 (Perkins & Johnston). The diffusion constant $D$ is strongly dependent on the glycerine concentration (Ben Salah). In units of $10^{-9}$ m²/s it varies from 0.82 at 0 %w to 0.325 at 45 %w glycerine; Ben Salah took the value halfway the concentration interval, but we think that the harmonic average of the maximum and minimum is more realistic: the value is then $2 (1/0.82 + 1/0.325)^{-1} = 0.47$. So the term $D/F_R$ is $0.33 \times 10^{-9}$ m²/s.
<table>
<thead>
<tr>
<th>Sample No.</th>
<th>$U$ (m/s)</th>
<th>$K$ (m$^2$/s)</th>
<th>$1/N_{Pe}$</th>
<th>$\beta U$</th>
<th>$D_{25}$</th>
<th>$N_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>2.8 $\times 10^{-4}$</td>
<td>2.38 $\times 10^{-8}$</td>
<td>1.7 $\times 10^{-4}$</td>
<td>2.9</td>
<td>0.0366</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.5 $\times 10^{-5}$</td>
<td>1.839 $\times 10^{-9}$</td>
<td>8.6 $\times 10^{-5}$</td>
<td>0.18</td>
<td>0.293</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.8 $\times 10^{-5}$</td>
<td>1.512 $\times 10^{-9}$</td>
<td>8.4 $\times 10^{-5}$</td>
<td>0.14</td>
<td>0.366</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7 $\times 10^{-6}$</td>
<td>5.985 $\times 10^{-10}$</td>
<td>7.6 $\times 10^{-5}$</td>
<td>0.03</td>
<td>1.47</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2 $\times 10^{-6}$</td>
<td>4.25 $\times 10^{-10}$</td>
<td>-</td>
<td>0.00</td>
<td>9.33</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>1.8 $\times 10^{-4}$</td>
<td>1.95 $\times 10^{-8}$</td>
<td>1.43 $\times 10^{-4}$</td>
<td>2.3</td>
<td>0.189</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.47 $\times 10^{-5}$</td>
<td>9.719 $\times 10^{-9}$</td>
<td>1.48 $\times 10^{-4}$</td>
<td>1.1</td>
<td>0.401</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.8 $\times 10^{-5}$</td>
<td>1.418 $\times 10^{-9}$</td>
<td>8.05 $\times 10^{-5}$</td>
<td>0.13</td>
<td>1.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.69 $\times 10^{-5}$</td>
<td>1.333 $\times 10^{-9}$</td>
<td>7.88 $\times 10^{-5}$</td>
<td>0.12</td>
<td>2.01</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5 $\times 10^{-6}$</td>
<td>5.59 $\times 10^{-10}$</td>
<td>6.0 $\times 10^{-5}$</td>
<td>0.03</td>
<td>7.09</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>1.4 $\times 10^{-4}$</td>
<td>1.575 $\times 10^{-8}$</td>
<td>1.10 $\times 10^{-4}$</td>
<td>1.9</td>
<td>0.324</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.8 $\times 10^{-5}$</td>
<td>2.282 $\times 10^{-9}$</td>
<td>6.97 $\times 10^{-5}$</td>
<td>0.24</td>
<td>1.62</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.4 $\times 10^{-5}$</td>
<td>1.140 $\times 10^{-9}$</td>
<td>5.78 $\times 10^{-5}$</td>
<td>0.10</td>
<td>3.24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.1 $\times 10^{-6}$</td>
<td>7.62 $\times 10^{-10}$</td>
<td>5.3 $\times 10^{-5}$</td>
<td>0.05</td>
<td>5.73</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>3.39 $\times 10^{-4}$</td>
<td>7.566 $\times 10^{-8}$</td>
<td>1.48 $\times 10^{-4}$</td>
<td>9.1</td>
<td>0.358</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9 $\times 10^{-5}$</td>
<td>1.01 $\times 10^{-8}$</td>
<td>7.3 $\times 10^{-5}$</td>
<td>1.2</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.78 $\times 10^{-5}$</td>
<td>7.945 $\times 10^{-9}$</td>
<td>7.5 $\times 10^{-5}$</td>
<td>0.92</td>
<td>1.79</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 $\times 10^{-5}$</td>
<td>1.72 $\times 10^{-9}$</td>
<td>4.63 $\times 10^{-5}$</td>
<td>0.17</td>
<td>6.32</td>
<td></td>
</tr>
</tbody>
</table>
1.4. INTERPRETATION OF MEASUREMENTS FROM LITERATURE WITH SIMILARITY GROUPS

There is one drawback of the experiments reported in literature: we have to compare experiments in various cores, glass packs, etc., without detailed knowledge of the statistical parameters of the spatial variability of the permeability, because these data are not reported in the papers. Hence, we cannot evaluate the relation between dispersivity and \( k/k_0 \). There is no information on any anisotropy of the porous media, so we assume that the ratio \( \frac{\lambda'}{\lambda} \) is 1. Furthermore, correction of the dispersion for the diffusion term is not always possible because diffusion data and values for the formation resistivity factor \( F_R \) are lacking.

The remaining similarity groups, the mobility ratio \( M \) and the gravity number \( N_G \), are used for the comparison of experiments in different experimental systems.

1.4.1. Effect of mobility ratio

1.4.1.1. Brigham

The two data points of Brigham suggests that effective dispersivity increases with mobility ratio. Because of the horizontal orientation of the glass pack, the gravity number was 0 in his experiments. The trend observed is: increase in dispersivity with mobility ratio \( (M < 1 \ , \ N_G = 0) \). The explanation of this trend is probably that the larger viscosity behind the displacement front is compensated by a smaller magnitude of the fluctuations in the fluid velocity.

1.4.1.2. Giordano et al.

The finding of Brigham is confirmed by the results of Giordano et al.. Because gravity was not present in their simulation, the gravity number was zero. Further, these authors demonstrate that if the effluent profiles of Fig. 1.2 are the result of dispersion, a non-zero effective dispersivity remains at very small mobility ratio and \( N_G = 0 \).

1.4.1.3. Slobod & Howlett

The relation between effective dispersivity and mobility ratio \( M \) in the experiments of Slobod & Howlett is less clear, because the spread between some data points is significant. For example, the effective dispersivity for
data point 3b ($N_G = 0.29, M = 0.466$) is 50\% higher than that for data point 2a ($N_G = 0.30, M = 0.466$). This difference cannot be explained by the difference in gravity number of 0.01.

The conclusion is that we cannot find a firm trend in the relation between effective dispersivity and mobility ratio from their experiments.

1.4.2. Effect of gravity number

1.4.2.1. Slobod & Howlett

The first point to be noted in Fig. 1.3 is that data points with equal mobility ratio and no density difference but conducted at different rates (experiments no. la,b,c and 5a,b,c) coincide; with the help of the similarity groups we can now understand why. The $N_G$ value is 0 for these data points, hence all similarity groups remain constant, in spite of the velocity change. Or in words: the flow is dominated by the viscous force and the gravity force is negligible, so gravity has no suppressing effect on dispersion. (In the figure, data points with $N_G = 0$ are plotted at $N_G = 0.01$ because of the logarithmic scale.)

Figure 1.3 shows a decrease in effective dispersivity with increasing gravity number. This can be understood as follows: at increasing gravity force, differences in progression of different parts of the displacement front are levelled off more. In fact, the decrease in effective dispersivity takes place between $N_G = 0.1$ and $N_G = 1$.

The figure suggests that the inverse Peclet number asymptotically tends to a constant value at small gravity number (high displacement velocity and/or zero density difference): a velocity increase by a factor 2 at small gravity number (from $N_G = 0.15$ to $N_G = 0.074$) affects the Peclet number (and hence the effective dispersivity) by 2.5\% only. This explains why the displacement velocity has hardly any effect on the effective dispersivity: the flow is dominated by the viscous force and the gravity force is almost negligible.

No information about the trend of the effective dispersivity with gravity number at $M > 1$ can be derived from the data point with $M = 2.275$, so in the conditionally stable flow regime, because we have no datapoints at other $N_G$ values.

The conclusion from the above evaluation of the experiments of Slobod & Howlett is that their observations can be summarised as: effective
dispersivity decreases with increasing gravity number, somewhere between $N_G = 0.1$ and $N_G = 1$ ($M < 1$).

1.4.2.2. Newberg & Foh

The relation for the tests conducted at 3.5 MPa shows an increase in effective dispersivity with $U$: the effective dispersivity is $0.117 U^{0.660}$ [m]. The effective dispersivity in the relation for the tests conducted at 6.9 MPa is $8.243 \times 10^{-4} U^{0.063}$ and thus practically independent of $U$, which contradicts the result of the tests at 3.5 MPa. This contradiction becomes less severe when the inverse Peclet number (= effective dispersivity divided by core length) is plotted versus gravity number (see Fig. 1.3) and is compared with the data points of Slobod & Howlett with about the same mobility ratio (between 0.37 and 0.47). Both curves of Newberg & Foh fall within the gravity-number range of 0.04 to 0.27. If we compare the curve belonging to the experiments conducted at 6.9 MPa ($M = 0.667$) to the data points of Slobod & Howlett in Fig. 1.3, we see that the data points of Slobod & Howlett indicate a somewhat higher relative decrease of inverse Peclet number (and hence effective dispersivity) with gravity number in the gravity number range 0.04 - 0.27. In contrast, the relative decrease of inverse Peclet number with gravity number in the data points of Slobod & Howlett is less than the relative decrease in the curve belonging to the experiments conducted by Newberg & Foh at 3.5 MPa ($M = 0.643$). Thus, their experiments generally correspond with those of Slobod & Howlett.

An explanation for the difference between the two curves belonging to the experiments of Newberg & Foh is that the data points at 3.5 MPa can probably be better fitted by a nearly linear relation between dispersion and $U$ if Newberg & Foh had chosen a lower value for the interception of their fitting curve with the vertical axis.

The conclusion from the experiments of Newberg & Foh is that their experiments show a discrepancy. Generally, they confirm the observation derived from the experiments of Slobod & Howlett that effective dispersivity decreases with increasing gravity number.

1.4.2.3. Ben Salah

Fig. 1.4 demonstrates that the effective dispersivity decreases with gravity number. In the $N_G$ range investigated (between 0.293 and 9.83) the dependence on gravity number is however not strong. This dependence would be
much stronger if we had taken the experiments exhibiting Taylor dispersion into account: Ben Salah found a strong dependence on his gravity number but he did not exclude Taylor dispersion from his experimental data.

Since Ben Salah defined a gravity number, he was able to present his results in a more general way than Slobod & Howlett and Newberg & Foh. However, he arrived at a different expression for the gravity number \( d_p^2 \), where \( d_p \) is average bead diameter, instead of our \( k/\phi \), because he did not derive his gravity number from the basic equations of flow through porous media but went back to the Navier Stokes equations for flow between the grains. His conclusion that the mixing zone length decreases with his gravity number \( d_p^2 g (\rho_p - \rho_I)/\mu_I U \) is therefore true only if there is a monotone increasing relationship between \( d_p^2 \) and \( k/\phi \). This type of relationship does for example not exist between two media with the same average bead diameter but different bead sorting: the one with the good sorting has a much higher permeability and a little bit higher porosity than the one with the bad sorting.

The conclusion is that the experiments of Ben Salah demonstrate a small decrease in effective dispersivity with gravity number at \( M \) of 0.21 and \( N_g \) between 0.2 and 10.

1.5. EXPERIMENTAL PROGRAMME ON EFFECTIVE DISPERSIVITY IN BEREA CORE

1.5.1. Definition

The objectives of the experimental programme were to measure the dispersivity of the porous medium (in this case a Berea sandstone core) and to investigate the dependence of effective dispersivity on gravity number in the conditionally stable regime (i.e. in the regime where the displacement is stable on the condition that the displacement velocity does not exceed the critical velocity).

The length of the Berea core was 1.82 m. We selected such a long core because we could then carefully monitor the mixing zone between the displacing and displaced fluids (the length of the dispersive mixing zone between the fluids is roughly proportional to the square root of the core length). Another reason for choosing a long core is to minimise the relative contribution of the dispersion in the inlet and outlet tubes of the
equipment to the dispersion in the core. Table 1.6 lists the properties of the core.

**TABLE 1.6: DATA OF BEREA CORE**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1.829 m</td>
</tr>
<tr>
<td>W</td>
<td>0.051 m</td>
</tr>
<tr>
<td>d_p</td>
<td>28*10^{-6} m</td>
</tr>
<tr>
<td>( \phi )</td>
<td>water/brine 0.230</td>
</tr>
<tr>
<td></td>
<td>gas/oil 0.242</td>
</tr>
<tr>
<td>( F_R )</td>
<td>water/brine 13.8</td>
</tr>
<tr>
<td></td>
<td>gas/oil 12.1</td>
</tr>
<tr>
<td>( V_p )</td>
<td>0.835 dm^3</td>
</tr>
<tr>
<td>k</td>
<td>brine 0.45 ( \mu ) m^2</td>
</tr>
<tr>
<td></td>
<td>n-decane 0.62 ( \mu ) m^2</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.18</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>2.3*10^{-2} m</td>
</tr>
<tr>
<td>CEC</td>
<td>0.392 meq/100 g</td>
</tr>
</tbody>
</table>

In the core-flooding equipment the dispersive mixing zone between displacing and displaced fluids in the core is continuously monitored by measuring the density of the effluent by means of a vibrating U-tube (Anton Paar) with accuracy of 0.05%. To prevent fluid movement by thermal expansion, the core holder and fluid vessels are located in a thermostatic cabinet. In all experiments the core was set in the vertical position to guarantee a one-dimensional displacement, with the lighter fluid injected at the top or the heavier fluid injected at the bottom.

1.5.1.1. **Dispersivity of core**

The measurements were performed on one core. To determine the dispersivity of the core, we conducted a series of five water/brine
displacements in which \( M \) was close to 1 and \( N_G \) varied from almost zero (0.066) to 1.05. The \( N_G \) number was varied to make it possible to find the dispersivity from the measurements by extrapolation of the measured dispersion when \( N_G \) goes to 0. The brine (which has a high NaCl concentration of 10% w) had a viscosity and density that was a little different from the viscosity and density of the water (which has not a zero but a low NaCl concentration of 0.5 % w to prevent permeability impairment). The mobility ratio was 1.21 or 0.83 (depending on the displacement direction). In view of the observation of Brigham in an experiment with \( M = 0.175 \), we judged the values of 0.83 and 1.21 sufficiently close to 1 to expect just a small effect of viscosity contrast. Properties of the water and the brine are listed in Table 1.7.

**TABLE 1.7: PROPERTIES OF WATER AND BRINE AT EXPERIMENTAL CONDITION**

(9.87 MPa and 55°C)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Water</th>
<th>Brine</th>
<th>( % ) NaCl</th>
<th>( \text{kg/m}^3 )</th>
<th>( \text{mPa.s} )</th>
<th>( \text{m}^2/\text{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C concentration</td>
<td>0.5</td>
<td>10</td>
<td>10%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho ) density</td>
<td>993</td>
<td>1059</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu ) viscosity</td>
<td>0.48</td>
<td>0.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D diffusion coefficient</td>
<td>( 3.0 \times 10^{-9} )</td>
<td>( 2 \times 10^{-9} )</td>
<td>( 2 \times 10^{-9} )</td>
<td>( 2 \times 10^{-9} )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Correlations of the density and the viscosity of NaCl solutions, that are accurate within 0.2%, resp. 0.4%, are given by Numere et al. (Ref. 10). The water density at 55°C (131°F) and 9.87 MPa (1450 psi) is 990 kg/m³ (Fig. 3 in Ref. 10); the water viscosity, taken from Fig. 1 in the same reference, is 0.473 mPa.s.

The diffusion coefficient of NaCl in water as a function of the concentration at 25°C is taken from a figure in Ref. 11. The diffusion coefficient varies by about 2% between the low (0.5%) and the high (10%) NaCl concentration. We selected the value at 5% NaCl concentration. The diffusion coefficient at the experimental temperature can be estimated with the Stokes-Einstein equation (Ref. 12) with an input value for the viscosity of the water/brine mixture equal to the average value of the viscosity of water and viscosity of brine (at experimental condition it is 0.49 mPa.s; at 25°C it is 0.89 mPa.s).
To check the value of the dispersivity of the core as obtained from the water/brine displacements, we also measured the permeability variation. This was done with a mini-permeameter. This instrument measures the permeability locally; no plugs have to be taken from the core. Measurements were taken at 180 different locations at intervals of 1 cm. The permeability as a function of location (Fig. 1.5a) reveals a rapid permeability variation. The histogram (Fig. 1.5b) shows a log-normal permeability distribution. The correlation function of the permeability \( k \) (in fact of \( \ln(k/k_0) \)), shown in Fig. 1.5c and in Fig. 1.5d with a match line, gives a standard deviation \( \sigma \) of the log-permeability of 0.18 (corresponding to geometrical standard deviation of the permeability \( S = e^{\sigma} \) of 1.20) and a correlation length of the permeability of 2.3 cm. (More detailed information is in Appendix 1B.)

1.5.1.2. Effect of gravity number at \( M \gg 1 \)

We investigated the dependence of the effective dispersivity on displacement velocity (and hence gravity number) in the conditionally stable regime, where \( M > 1 \), for the following reason. According to our interpretation of the experiments of Slobod & Howlett and Newberg & Foh, the displacement velocity has no effect on the effective dispersivity at high displacement velocities. The experiments on which this observation is based were conducted in the unconditionally stable regime, i.e. at \( M < 1 \). However, in secondary oil recovery by miscible gas injection, or in displacement of an inert cushion gas (e.g. \( N_2 \) or \( CO_2 \)) by methane in underground gas storage projects, the mobility ratio is larger than one. The question is then whether the effective dispersivity is dependent on the displacement velocity at high displacement velocities below the critical velocity \( U_c \), or is not dependent on displacement velocity as in the case of \( M < 1 \).

We conducted three gas/oil displacements with \( M = 22 \). The displacement direction was downwards. In the series of experiments, the gravity number varied from 1.9 to a (large) value of 12, the latter for extrapolation to infinite \( N_G \) (corresponding to zero displacement velocity). (This is about the maximum \( N_G \) range that is practically possible. For the slowest experiment with \( N_G = 12 \) it takes 17 days to inject 1.2 pore volume. On the other end of the \( N_G \) range, experiments with lower \( N_G \) values would have been desirable, but close to \( N_G = 0.95 \) a downward displacement is locally unstable.) We used a mixture of methane (mole fraction 0.75) and propane as
FIG. 1.5a: Air permeability along the Berea core measured with the mini-permeameter at 1 cm interval

FIG. 1.5b: Permeability distribution of the Berea core
FIG. 1.5c: Correlation function of permeability in the Berea core

FIG. 1.5d: Determination of correlation length of permeability in the Berea core
the gas, and n-decane as the oil. The pressure was 24.6 MPa (250 bar) and the temperature was 55°C so the pressure was above the first-contact-miscibility pressure of 22.7 MPa (230 bar). The values of the density and viscosity of the gas mixture and the oil are listed in Table 1.8. The mobility ratio was 22.

**TABLE 1.8: PROPERTIES OF GAS AND OIL AT EXPERIMENTAL CONDITION**
*(24.7 MPa and 60°C)*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>x mole fraction</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>density</td>
<td>250</td>
</tr>
<tr>
<td>viscosity</td>
<td>0.030</td>
</tr>
<tr>
<td>diffusion coefficient</td>
<td>4.9*10^{-9}</td>
</tr>
<tr>
<td></td>
<td>709</td>
</tr>
<tr>
<td></td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>mPa.s</td>
</tr>
</tbody>
</table>

D is given by the empirical correlation of Renner (14) (with standard deviation of 1.0x10^{-9} m²/s). An equation-of-state program (with Redlich-Kwong-Soave equation of state (Ref. 15) with density correlations from Ref. 16) provides the following input data: molecular weight 23.07 g/gmol and specific volume of the gas phase 92.1 cm³/gmol. The viscosity of n-decane at experimental condition, which was measured with a rolling ball viscometer, is 0.65 cP. The gas/oil mixture is miscible at 2.7 MPa at every gas fraction, according to the pressure-composition diagram in Fig. 1.11 (calculated with equation-of-state program).

1.5.2. Processing of the experimental data

The effluent density, measured at regular intervals of 5 or 10 minutes, was converted to concentration using standard correlations (14,15). The resulting plots of concentration are shown in Figs. 1.6a-e (water/brine) and Figs. 1.7a–d (gas/oil). A match was made between concentration and equation.
FIG. 1.6: Concentration of displacing fluid in effluent as a function of dimensionless time (water/brine displacements).
### TABLE 1.9: MEASURED DISPERSION OF WATER/BRINE DISPLACEMENTS

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>U (m/s)</th>
<th>M</th>
<th>N\textsubscript{G}</th>
<th>(K\text{measured}) (m(^2)/s)</th>
<th>N</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aa</td>
<td>2.06(\times10^{-6})</td>
<td>1.21</td>
<td>1.05</td>
<td>1.17(\times10^{-9})</td>
<td>71</td>
<td>0.9959*</td>
</tr>
<tr>
<td>Ab</td>
<td>3.70(\times10^{-6})</td>
<td>1.21</td>
<td>0.57</td>
<td>1.22(\times10^{-9})</td>
<td>55</td>
<td>0.9954</td>
</tr>
<tr>
<td>Ac</td>
<td>8.24(\times10^{-6})</td>
<td>1.21</td>
<td>0.26</td>
<td>5.72(\times10^{-9})</td>
<td>286</td>
<td>0.9981</td>
</tr>
<tr>
<td>Ad</td>
<td>2.00(\times10^{-5})</td>
<td>0.83</td>
<td>0.13</td>
<td>1.32(\times10^{-8})</td>
<td>59</td>
<td>0.9948</td>
</tr>
<tr>
<td>Ae</td>
<td>3.94(\times10^{-5})</td>
<td>0.83</td>
<td>0.066</td>
<td>2.56(\times10^{-8})</td>
<td>54</td>
<td>0.9997</td>
</tr>
</tbody>
</table>

continued

<table>
<thead>
<tr>
<th>(K\text{tube}) (m(^2)/s)</th>
<th>(D/F\text{R})(\phi) (m(^2)/s)</th>
<th>(K\text{c}) (m(^2)/s)</th>
<th>(N_{Pe})^(-1)</th>
<th>(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0(\times10^{-11})</td>
<td>5.20(\times10^{-10})</td>
<td>2.7 (\times10^{-10})</td>
<td>0.73(\times10^{-4})</td>
<td>0.21</td>
</tr>
<tr>
<td>3.5(\times10^{-11})</td>
<td>5.20(\times10^{-10})</td>
<td>6.65(\times10^{-10})</td>
<td>0.98(\times10^{-4})</td>
<td>0.28</td>
</tr>
<tr>
<td>2.4(\times10^{-11})</td>
<td>5.20(\times10^{-10})</td>
<td>5.18(\times10^{-9})</td>
<td>3.44(\times10^{-4})</td>
<td>0.99</td>
</tr>
<tr>
<td>1.2(\times10^{-10})</td>
<td>5.20(\times10^{-10})</td>
<td>1.26(\times10^{-8})</td>
<td>3.45(\times10^{-4})</td>
<td>0.99</td>
</tr>
<tr>
<td>8.8(\times10^{-11})</td>
<td>5.20(\times10^{-10})</td>
<td>2.50(\times10^{-8})</td>
<td>3.47(\times10^{-4})</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\(N\): number of data points between \(C = 0.1\) and \(C = 0.9\)
\(R^2\): correlation coefficient

* : match between \(C = 0.1\) and \(C = 0.65\)

\[ K = K_{measured} - K_{tube} - D/F_R\phi \]
FIG. 1.7: Concentration of displacing fluid in effluent as a function of dimensionless time (gas/oil displacements).
TABLE 1.10: MEASURED DISPERSION OF GAS/OIL DISPLACEMENTS

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>U (m/s)</th>
<th>M</th>
<th>N&lt;sub&gt;G&lt;/sub&gt;</th>
<th>U&lt;sub&gt;c&lt;/sub&gt;/U</th>
<th>K&lt;sub&gt;measured&lt;/sub&gt; (m&lt;sup&gt;2&lt;/sup&gt;/s)</th>
<th>N</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ba</td>
<td>1.50*10^-6</td>
<td>22</td>
<td>12</td>
<td>12</td>
<td>2.30*10^-9</td>
<td>194</td>
<td>0.9986</td>
</tr>
<tr>
<td>Bb</td>
<td>3.00*10^-6</td>
<td>22</td>
<td>5.8</td>
<td>6.1</td>
<td>2.73*10^-9</td>
<td>52</td>
<td>0.9988</td>
</tr>
<tr>
<td>Bc</td>
<td>9.26*10^-6</td>
<td>22</td>
<td>1.9</td>
<td>2.0</td>
<td>5.98*10^-9</td>
<td>38</td>
<td>0.9999</td>
</tr>
<tr>
<td>Bd*</td>
<td>1.93*10^-5</td>
<td>22</td>
<td>0.91</td>
<td>0.96</td>
<td>4.64*10^-8</td>
<td>9</td>
<td>0.9634</td>
</tr>
</tbody>
</table>

continued

<table>
<thead>
<tr>
<th>K&lt;sub&gt;tube&lt;/sub&gt; (m&lt;sup&gt;2&lt;/sup&gt;/s)</th>
<th>D/F&lt;sub&gt;R&lt;/sub&gt; # (m&lt;sup&gt;2&lt;/sup&gt;/s)</th>
<th>K&lt;sub&gt;C&lt;/sub&gt; (m&lt;sup&gt;2&lt;/sup&gt;/s)</th>
<th>N&lt;sub&gt;Pe&lt;/sub&gt;&lt;sup&gt;-1&lt;/sup&gt;</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.7*10^-11</td>
<td>2.26*10^-9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7.2*10^-11</td>
<td>2.26*10^-9</td>
<td>3.9*10^-10</td>
<td>0.71*10^-4</td>
<td>0.20</td>
</tr>
<tr>
<td>2.2*10^-10</td>
<td>2.26*10^-7</td>
<td>3.50*10^-9</td>
<td>2.07*10^-4</td>
<td>0.60</td>
</tr>
<tr>
<td>4.5*10^-10</td>
<td>2.26*10^-9</td>
<td>4.37*10^-8</td>
<td>1.24*10^-3</td>
<td>3.6</td>
</tr>
</tbody>
</table>

*: experiment Bd was unstable

K<sub>C</sub> = K<sub>measured</sub> - K<sub>tube</sub> - D/F<sub>R</sub> #
by linear regression. In the linear regression procedure, emphasis was placed on a good match in the C interval between 0.1 and 0.9. We chose this interval because in the literature a tail effect in the concentration history has been reported, which has been attributed to diffusion from dead-end pore volumes in the sandstone. In the slowest water/brine experiment, the tail effect was apparently so large that C was matched to eq. (1) between 0.1 and 0.65. The resulting values of the total dispersion in the core and the inlet and outlet tubes of the equipment are listed in Table 1.9 (water/brine) and Table 1.10 (gas/oil), together with the value of the correlation coefficient of the linear regression. To check the value of the dispersion coefficient, C was calculated with equation (1) and was plotted in Figs. 1.6a–e (water/brine) and Figs. 1.7a–d (gas/oil) for comparison with the C values from the measurements. In these figures the front and tail effects can be observed.

Good matches between (1) and the measured concentration histories of the water/brine displacements could be obtained. Table 1.9 shows that the correlation coefficient $R^2$ is 0.9948 or more.

For the gas/oil displacements good matches between the concentration histories and the error function could be made as well. The correlation coefficient was 0.9986 or more. A point of worry is the stability of displacement $Bv$, conducted at $9.26 \times 10^{-6}$ m/s. Although the displacement velocity was below the Dietz critical displacement velocity $U_C$, the displacement could have been locally unstable because the displacement velocity was beyond the Dumoré critical velocity $U_{CD}$, given by (1.8b) $(4.22 \times 10^{-6}$ m/s). From the picture of the concentration of the displacing fluid in the effluent, we can see whether the displacement was indeed unstable. Fig. 1.7c shows that the slope of the concentration history has no discontinuities. Comparing this plot with the one (Fig. 1.7d) of an unstable displacement conducted beyond the Dietz critical rate ($U_C / U = 0.96$) shows that the displacement at $9.26 \times 10^{-6}$ m/s was stable. Another indication for stability is the correlation coefficient of 0.9999 of the match of the measured concentration history with the error function. This displacement was therefore included in our analysis of trends in effective dispersivity.

The convective dispersion $K_C$ was isolated from the total dispersion $K$ measured by subtraction of the dispersion $K_{tube}$ in the inlet and outlet tubes (dead volume) and of the molecular diffusion term. The dispersion in the inlet and outlet tubes was determined by additional displacement
experiments in which the core was bypassed (see for procedure Appendix 1C). The dispersion in the tubes was found to be small (about 1% of the total dispersion). The molecular diffusion term \( D/F_R^\phi \) was determined with measured values of \( F_R \) and \( \phi \) and some empirical correlations \(^{11,14}\) for the diffusion coefficient \( D \). For the gas/oil experiments, the value of \( D \) was not accurate enough in view of the small convective dispersion at high \( N_G \) values and therefore \( D/F_R^\phi \) was determined by equating this term to \( K_C + D/F_R^\phi \) at \( N_G = 12 \), where the convective dispersion \( K_C \) is practically zero according to Perkins & Johnston (small \( U \)). Finally, the inverse Peclet number was calculated from the dispersion coefficient \( K_C \) with equation (1.9).

Values of \( K_C \) and \( 1/N_{Pe} \) are listed in Table 1.9 (water/brine) and in Table 1.10 (gas/oil).

1.6. INTERPRETATION OF MEASUREMENT IN BEREA CORE WITH SIMILARITY GROUPS

1.6.1. Dispersivity of core

Figure 1.8 shows the measured convective dispersion \( K_C \) of the five water/brine displacements as a function of displacement velocity \( U \). The straight line in the figure represents the simple model of Perkins & Johnston, for which convective dispersion is proportional to \( U \) and intersects the vertical axis at the origin (equation (3)):

\[
K_{C,\text{Perkins}} = K_{\text{measured}} - \frac{D}{F_R^\phi} = 0.5 \text{ s d g} \cdot U.
\]

The slope 0.5 s d of the straight line, which is not predicted by Perkins & Johnston, is chosen such that the line goes through the origin and through datapoint \( A_e \). The slope of the line is then equal to the effective dispersivity at almost infinite displacement velocity and thus, since the mobility ratio is close to 1, about equal to the dispersivity of the core. We denote this dispersivity as \( \beta_{\text{ref}} \). The value of \( \beta_{\text{ref}} \), determined from the slope of the straight line, is \( 6.5 \times 10^{-4} \text{ m} \).

The dispersivity of the core can be determined also with the mini-permeameter measured values of \( \sigma \) and \( \lambda \). According to Gelhar & Axness \(^{17}\), the dispersivity of a porous medium is given by \( \lambda \sigma^2 \) multiplied by a small correction factor.
FIG. 1.8: Measured convective dispersion in core of water/brine displacements versus displacement velocity.

\[(1 + \frac{1}{6} \sigma^2)^{-2}\]. With \(\sigma=0.18\) and \(\lambda=2.3\) cm the expression of Gelhar & Axness gives 7.4*10^{-4} m, which is close to the value of 6.5*10^{-4} m determined from the core-flooding experiments.

1.6.2. **Effect of small density contrast**

The asymptotic behaviour of the relation between \(D\) and \(U\) at high \(U\) values agrees with the findings of Slobod & Bowlett that the effective dispersivity does not depend on \(U\) at high \(U\) values. The deviation of the data points from the straight line is largest at small \(U\), in accordance with the findings of Slobod & Bowlett. If we plot the relative deviation from this line defined by

\[ a = \frac{K_C}{K_{C,\text{Perkins}}} = \frac{\beta}{\beta_{\text{ref}}} \]  

(1.11)
as a function of gravity number as in Fig. 1.9, we see that between $N_G = 0.26$ and $N_G = 1.05$ a large decrease in $\alpha$ occurs from about 1.0 to 0.21. Note that although the density difference between water and brine is very small (just 6% of the average density), the effective dispersivity is much lower (in this case a factor of 5 lower) than the dispersivity of the core. In the framework of similarity groups, particularly with the use of the gravity number, this can now be understood: the effective dispersivity decreases with gravity number at $M < 1$ and the decrease is largest at $N_G$ between 0.1 and 1, in agreement with the experiments of Slobod & Howlett and Newberg & Foh.

FIG. 1.9: Dispersivity ratio $\alpha$ as a function of gravity number of water/brine and gas/oil displacements.
1.6.3. **Effect of gravity number at $M >> 1$**

The relative deviation $a$ was calculated with (1.11) using the value of $\beta_{\text{ref}}$, which was determined from the water/brine displacements. Note in Fig. 1.9 the sharp decrease of $a$ with $N_G$, in this case between $N_G = 1.9$ and $N_G = 5.8$. We have added the theoretical data point $T$: $a = 1$ at $N_G = 1 - 1/22 = 0.95$ to Figure 1.9, because we expect that the dispersivity at $N_G = 1 - 1/M$ is equal to the dispersivity at $M = 1$, $N_G = 0$. The expectation is based on the fact that the pressure gradient behind the displacement front is equal to the pressure gradient in front of it, both when $M = 1$, $N_G = 0$ and when $N_G = 1 - 1/M$ (see Appendix 1A).

We can conclude from Fig. 1.9 that there is a strong decrease of effective dispersivity with gravity number in the conditionally stable regime ($M > 1$, $N_G > 1 - \frac{1}{M}$). This decrease occurs between $N_G = 1$ and $N_G = 10$. Comparing the data points of the water/brine displacements with the data points of the gas/oil displacements shows that the dependence of effective dispersivity on gravity number at $M = 22$ is stronger than at a mobility ratio close to 1.

1.7. **AMALGAMATION OF ALL MEASUREMENTS ON EFFECTIVE DISPERSIVITY**

Although the porous media are not the same in the various investigations, the measurements performed in different porous media are consistent. As a result, we can summarise all measurements discussed in this chapter in the following list of seven observations:

1. increase in effective dispersivity with mobility ratio (at horizontal orientation and $M \leq 1$) (Brigham, Giordano et al.);
2. non-zero effective dispersivity at $M = 0$ (horizontal orientation) (Giordano et al.);
3. increase in effective dispersivity with displacement velocity ($M < 1$) (Slobod & Howlett, Newberg & Foh, Ben Salah, this work);
4. no dependence of effective dispersivity on displacement velocity at high displacement velocity ($M < 1$) (Slobod & Howlett, Newberg & Foh, this work);
5. no dependence of effective dispersivity on displacement velocity when the densities of the fluids are the same ($M < 1$) (Slobod & Howlett);
6. A small density difference between displacing and displaced fluid, such as between water and brine, suppresses the effective dispersivity significantly if the displacement velocity is sufficiently low and/or the permeability is sufficiently high (this work);

7. Strong increase in effective dispersivity with displacement velocity at high but stable displacement velocities and \( M > 1 \) (this work).

These seven observations can be translated in terms of the similarity groups, gravity number and mobility ratio as follows:

1. Increase in effective dispersivity with \( M \) (at \( N_G = 0 \) and \( M \leq 1 \));

2. Non-zero effective dispersivity at \( M = 0 \) (at \( N_G = 0 \));

3. Decrease in effective dispersivity with \( N_G \) between \( N_G = 0.1 \) and \( N_G = 1 \) (at \( M < 1 \));

4. Equal effective dispersivity at equal \( N_G \) (at \( M < 1 \));

5. Equal effective dispersivity when \( N_G \) remains 0 (at \( M < 1 \));

6. Decrease in effective dispersivity with \( N_G \) between \( N_G = 1 \) and \( N_G = 10 \) (at \( M > 1 \));

7. Strong decrease in effective dispersivity with \( N_G \) (at \( M > 1 \), \( N_G > 1 - 1/M \)).

Observations 3 - 7 can be summarised in a single observation. The result is a summary with the following three observations, which are illustrated as a function of gravity number and of mobility ratio in Fig. 1.10:

1. Increase in effective dispersivity with \( M \) (at \( N_G = 0 \) and \( M < 1 \));

2. Non-zero effective dispersivity at \( M = 0 \) (at \( N_G = 0 \));

3. Decrease in effective dispersivity with \( N_G \) in the region of \( N_G = 1 \) (independent of \( M \) and \( N_G > 1 - 1/M \) if \( M > 1 \)).

1.8. RECOMMENDATION FOR MEASUREMENT OF DISPERIVITY

The best way to measure the dispersivity is to select fluids with a mobility ratio \( M \) not too far below 1 and to exclude gravity effects by conducting the displacement such that gravity number \( N_G \) is close to 0 (say smaller than 0.1). When the displacement is conducted in a horizontal plane, the occurrence of concentration variation in the vertical direction, for example, by the formation of a gravity tongue, must be excluded.
FIG. 1.10: Trends of effective dispersivity observed in experiments of Brigham, Giordano et al., Slobod & Howlett, Newberg & Foh, Ben Salah, this work.
The decrease of effective dispersivity with gravity number, as observed in the water/brine displacements, illustrates that, when measuring the dispersivity of a porous medium, one should be careful when there is a difference in fluid properties. Although it seems that water and brine have a negligible difference in density, a measurement of the dispersivity of the core can give a wrong value for the dispersivity. Therefore: check the $N_G$ value.

A conditionally stable displacement (so $M > 1$) to measure the dispersivity should not be used, even if the value of the gravity number is such that an $a$ value of 1 is predicted. The dependence of $a$ on gravity number is so strong in the conditionally stable regime that a little uncertainty, for example, in the permeability, would affect $a$ significantly. Further, we have seen in the gas/oil displacements that a displacement conducted at a velocity close to the critical velocity for stable displacement can be locally unstable.

In the field, dispersivity is measured by adding a low-concentration tracer to the injectant. In this type of measurement, our recommendation that the mobility should be just below 1 is followed. Our other recommendation that the displacement should be conducted such that the gravity number is close to 0 is probably followed as well in most applications; by calculation of the gravity number, the designer of a tracer test can easily check this.

1.9. CONCLUSIONS

1. We have made a succesful, coherent evaluation of the measurements of effective dispersivity in stable, miscible displacements from a wide variety of sources in terms of dimensionless similarity groups.

2. The good matches between the errorfunction and the concentration profiles measured in the effluent from the Berea core indicate that dispersive mixing also occurs in the presence of a viscosity contrast and/or a density contrast between the fluids.

3. Evaluation of experiments conducted by Brigham, Giordano et al., Slobod & Howlett, Newberg & Foh, shows that the effective dispersivity is not a
function of properties of the porous medium solely. In particular, the experiments showed:
* increase in effective dispersivity with displacement velocity \((M < 1)\);
* increase in effective dispersivity with \(M\) (at horizontal orientation and \(M \leq 1\)).

In addition, their experiments showed:
* no dependence of effective dispersivity on displacement velocity at high displacement velocity \((M < 1)\);
* no dependence of effective dispersivity on displacement velocity when the densities of the fluids are the same \((M < 1)\);
* non-zero effective dispersivity at \(M = 0\) (horizontal orientation).

This last observation is based on a numerical simulation and needs further support.

4. We have found from the evaluation of our own experiments conducted at \(M = 22\) in a Berea core:
* strong decrease in effective dispersivity with displacement velocity at high (stable) displacement velocities and \(M > 1\).

Another observation was:
* a small density difference between displacing and displaced fluids, such as between water and brine, suppresses the effective dispersivity significantly if the displacement velocity is sufficiently low and/or the permeability is sufficiently high.

5. All seven of these observations can be summarised in three qualitative observations by means of gravity number \(N_G\) and mobility ratio \(M\):
* decrease in effective dispersivity with \(N_G\) in the region of \(N_G = 1\) (independent of \(M\) and \(N_G > 1 - 1/M\) if \(M > 1\));
* increase in effective dispersivity with \(M\) \((N_G = 0, M \leq 1)\);
* non-zero effective dispersivity at \(M = 0\) and \(N_G = 0\).

6. For a measurement of the dispersivity of a geological unit, a core, etc., we recommend that a fluid system with a mobility ratio just below 1 be selected and gravity effects excluded by conducting the displacement at high velocity such that the gravity number is close to 0.
REFERENCES

APPENDIX 1A

INSPECTION ANALYSIS OF BASIC EQUATIONS

Equations

The equations\(^8\) of miscible displacement at constant injection rate of incompressible fluids that exhibit no volume change upon mixing, in an isotropic, two-dimensional porous medium with a dip angle in the displacement direction are listed below. The 23 relevant physical quantities are

\[
\begin{align*}
\text{c} &= \text{local injectant concentration} \\
g &= \text{gravitational acceleration} \\
k &= \text{local permeability} \\
k_0 &= \text{effective permeability} \\
p &= \text{pressure} \\
U &= \text{average displacement velocity, measured positive in downward direction} \\
L &= \text{length of porous medium} \\
W &= \text{width of porous medium} \\
\theta &= \text{dip angle with horizontal} \\
\mu &= \text{dynamic viscosity} \\
\rho &= \text{density} \\
\sigma &= \text{standard deviation of log-normal permeability distribution} \\
\phi &= \text{porosity} \\
x, y &= \text{coordinates in respectively longitudinal and transverse directions} \\
u, v &= \text{velocity in respectively } x- \text{ and } y- \text{ directions} \\
\kappa, \kappa' &= \text{dispersion coefficient at laboratory scale in respectively } x- \text{ and } y- \text{ directions} \\
\lambda, \lambda' &= \text{correlation length of permeability in respectively } x- \text{ and } y- \text{ directions} \\
\text{subscript I refers to displaced fluid, subscript P to displacing fluid}
\end{align*}
\]

Continuity equations

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1A.1a}
\]

\[
\frac{\partial c}{\partial t} + \frac{\partial cu}{\partial x} + \frac{\partial cv}{\partial y} = \kappa \frac{\partial^2 c}{\partial x^2} + \kappa' \frac{\partial^2 c}{\partial y^2} \tag{1A.1b}
\]
Transport equations

\[ u = -k = \mu \phi \left[ \frac{\partial \rho}{\partial x} - \rho g \sin\theta \right] \quad (1A.1c) \]

\[ v = -k \mu \phi \frac{\partial \rho}{\partial y} \quad (1A.1d) \]

Mixing rule for density, e.g., \[ \rho = c \rho_I + (1-c) \rho_p \quad (1A.1e) \]

Mixing rule for viscosity, e.g., \[ \ln \mu = c \ln \mu_I + (1-c) \ln \mu_p \quad (1A.1f) \]

Initial and boundary conditions

\begin{align*}
  & t = 0 : \quad c = 0 \\
  & x = 0 : \quad c = H(t) \\
  & x = 0 : \quad u = U \quad v = 0 \\
  & x = L : \quad u = U \quad v = 0 \\
  & y = 0 : \quad v = 0 \\
  & y = W : \quad v = 0
\end{align*}

Permeability field \[ k(x,y,\lambda,\lambda',\phi) \]

Scaling

The variable \( c \) is already dimensionless and scaled between 0 and 1. \( \phi \) and \( \phi \) are dimensionless too. Scaling of the other 9 independent and dependent physical variables is as follows. The longitudinal velocity \( u \) is scaled with its average value \( U \); the transverse velocity \( v \) is also scaled with \( U \). We scale \( x \) with \( L \), \( y \) with \( W \) and time \( t \) with \( UT_L \). Dependent variables \( \rho \) and \( \ln \mu \) are scaled such that the minimum is 0 and the maximum is 1.

\begin{align*}
  x^* &= \frac{x}{L} \\
  y^* &= \frac{y}{W} \\
  t^* &= \frac{Ut}{L} \\
  u^* &= \frac{u}{U} \\
  v^* &= \frac{v}{U} \\
  \rho^* &= \frac{\rho - \rho_I}{\rho_p - \rho_I} \\
  \ln \mu^* &= \frac{\ln \mu - \ln \mu_I}{\ln \mu_p - \ln \mu_I} (1A.2a)
\end{align*}

Pressure gradient \( \frac{\partial \rho}{\partial x} \) is scaled such that at \( t = 0 \), when \( c = 0 \), no fluid properties are apparent in the dimensionless quantity:
\[
\frac{\partial p}{\partial x}^* = \frac{\frac{\partial p}{\partial x} - \rho_p g \sin \theta}{\mu_p \phi U \frac{k_0}{k}} \quad (1A.2b)
\]

At \( t = 0 \), this expression for the dimensionless pressure gradient is \(-k_0^*/k\), which is on average equal to 1. Pressure gradient \( \frac{\partial p}{\partial y} \) is scaled with the same scaling factor:

\[
\frac{\partial p}{\partial y}^* = \frac{\frac{\partial p}{\partial y}}{\mu_p \phi U \frac{k_0}{k}} \quad (1A.2c)
\]

The set of equations (1A.1) are in dimensionless form:

\[
\frac{\partial u^*}{\partial x^*} + \frac{L}{W} \frac{\partial v^*}{\partial y^*} = 0 \quad (1A.3a)
\]

\[
\frac{\partial c^*}{\partial t^*} + \frac{\partial c^*}{\partial x^*} \frac{u^*}{W} + \frac{\partial c^*}{\partial y^*} = \frac{\kappa}{UL} \frac{\partial^2 c^*}{\partial x^*^2} + \frac{\kappa'}{UL} \left( \frac{L}{W} \right)^2 \frac{\partial^2 c^*}{\partial y^*^2} \quad (1A.3b)
\]

\[
u^* = -\frac{k}{k_0} \left( \frac{\mu_p}{\mu_1} \right)^{1-\ln \mu^*} \left[ \frac{\partial p^*}{\partial x^*} + (1 - \rho^*) \frac{k_0 g (\rho_p - \rho_1) \sin \theta}{\mu_p \phi U} \right] \quad (1A.3c)
\]

\[
v^* = -\frac{k}{k_0} \left( \frac{\mu_p}{\mu_1} \right)^{1-\ln \mu^*} \left( \frac{\partial p^*}{\partial y^*} \right) \quad (1A.3d)
\]

\[
\rho^* = 1-c \quad (1A.3e)
\]

\[
\ln \mu^* = 1-c \quad (1A.3f)
\]

Initial and boundary conditions:

\begin{align*}
\text{Initial:} & \quad t^* = 0 : \quad c = 0 \\
\text{Boundary:} & \quad x^* = 0 : \quad c = H(t^*) \\
& \quad x^* = 0 : \quad u^* = 1 \quad v^* = 0 \\
& \quad x^* = 1 : \quad u^* = 1 \quad v^* = 0 \\
& \quad y^* = 0 : \quad v^* = 0 \\
& \quad y^* = 1 : \quad v^* = 0
\end{align*}
Permeability field \( k(x^*, y^*, \frac{\lambda}{L}, \frac{\lambda'}{W}, \sigma) \)

**Similarity groups**

The following 7 similarity groups are derived from the dimensionless set of equations (1A.3):

- **mobility ratio** \( M = \frac{\mu_p}{\mu_l} \)
- **gravity number** \( N_G = \frac{k_0 g (\rho_p - \rho_l) \sin \theta}{\mu_p \phi U} \)
- **Peclet numbers at laboratory scale** \( N_{pe} = \frac{UL}{\kappa} \quad N_{pe'} = \frac{UL}{\kappa} \) \( (1A.4) \)
- **permeability contrast** \( \frac{k}{k_0} \)
- **aspect ratio** \( \frac{W}{L} \)
- number of random permeabilities in the longitudinal direction \( \frac{L}{\lambda} \)
- number of random permeabilities in the transverse direction \( \frac{W}{\lambda'} \)

According to Buckingham's rule, we should have 23 - 3 = 20 dimensionless groups (23: the number of physical quantities; 3: the number of fundamental dimensions, mass, length, time). We have defined 9 independent and dependent dimensionless variables and we have found 8 similarity groups; 3 variables were already dimensionless. Hence there are no dimensionless groups lacking (20 - (9 + 8 + 3) = 0).

Substitution of (1A.3e) and (1A.3f) and of \( M, N_G \) and \( N_{pe} \) and \( N_{pe'} \) in (1A.3a) - (1A.3d) gives the following continuity and transport equations:

\[
\frac{\partial u^*}{\partial x^*} + \frac{L}{W} \frac{\partial v^*}{\partial y^*} = 0 \quad (1A.5a)
\]

\[
\frac{\partial c}{\partial t^*} + \frac{\partial c u^*}{\partial x^*} + \frac{L}{W} \frac{\partial c v^*}{\partial y^*} = N_{pe} \frac{\partial^2 c}{\partial x^*^2} + N_{pe'} \left( \frac{L}{W} \right) \frac{\partial^2 c}{\partial y^*^2} \quad (1A.5b)
\]
\[ u^* = -\frac{k}{k_0} M^c \left( \frac{\partial p}{\partial x} \right)^* + c N_G \]  
\[ v^* = -\frac{k}{k_0} M^c \left( \frac{\partial p}{\partial y} \right)^* \]  

(1A.5c)  
(1A.5d)

Initial and boundary conditions:
- \( t^* = 0 : c = 0 \)
- \( x^* = 0 : c = H(t^*) \)
- \( x^* = 0 : u^* = l \quad v^* = 0 \)
- \( x^* = 1 : u^* = l \quad v^* = 0 \)
- \( y^* = 0 : v^* = 0 \)
- \( y^* = 1 : v^* = 0 \)

Permeability field \( k(x^*, y^*, \lambda, \lambda^*, \omega) \)

Dispersivity at \( M > 1 \), \( N_G = 1 - \frac{1}{M} \)

The equations (1.5a)-(1A.5d) and the boundary conditions that depend on \( M \) and \( N_G \) are equation (1A.5c) and (1A.5d). When these equations are averaged over a transverse cross-section, then as shown below, the result of the averaging in the region where \( c = 0 \) or \( c = 1 \) is the same for case \( M > 1 \), \( N_G = 1 - \frac{1}{M} \) as for case \( M = 1 \), \( N_G = 0 \). Because, after averaging, equations (1A.5a)-(1A.5d) and the boundary conditions are the same for case \( M > 1 \), \( N_G = 1 - \frac{1}{M} \) as for case \( M = 1 \), \( N_G = 0 \), we expect that the dispersivity at \( M > 1 \), \( N_G = 1 - \frac{1}{M} \) is equal to the dispersivity at \( M = 1 \), \( N_G = 0 \).

Ahead of the mixing zone, where \( c = 0 \), (1A.5c) and (1A.5d) give:

\[ u^* = -\frac{k}{k_0} \left( \frac{\partial p}{\partial x} \right)^* \]
\[ v^* = -\frac{k}{k_0} \left( \frac{\partial p}{\partial y} \right)^* \]  

(1A.6a)

Behind the mixing zone, where \( c = 1 \), (1A.5c) and (1A.5d) give:

\[ u^* = -\frac{k}{k_0} M \left[ \left( \frac{\partial p}{\partial x} \right)^* + N_G \right] \]
\[ v^* = -\frac{k}{k_0} M \left( \frac{\partial p}{\partial y} \right)^* \]  

(1A.6b)

The injectant concentration \( C \) is defined as the transverse average of concentration \( c \). Averaging over a transverse cross-section, denoted by \(< >\),
Chapter 1

Gives thus \( <c> = C \) and further \( <u^* > = 1 \), \( <v^* > = 0 \), \( \frac{k}{k_0} = 1 \). Averaging of
(1A.6a) gives:

\[
1 = - \frac{k}{k_0} \left( \frac{\partial p}{\partial x} \right)^* \quad 0 = - \frac{k}{k_0} \left( \frac{\partial p}{\partial y} \right)^* 
\]  
\[\text{(1A.7a)}\]

Averaging of (1A.6b) gives:

\[
1 = - M \left[ \frac{k}{k_0} \left( \frac{\partial p}{\partial x} \right)^* + N_G \right] \quad 0 = - \frac{k}{k_0} \left( \frac{\partial p}{\partial y} \right)^* 
\]  
\[\text{(1A.7b)}\]

(1A.7a) and (1A.7b) are the same for two cases: case \( M = 1 \), \( N_G = 0 \) and case \( M > 1 \), when \( N_G = 1 - \frac{1}{M} \).

APPENDIX 1B

PERMEABILITY VARIATION IN BERE A CORE

Berea sandstone, which is obtained from an outcrop in Ohio, U.S.A., is known for its very homogeneous properties. Nevertheless, measurements of the permeability with a mini-permeameter show spatial permeability variations. In Fig. 1.5a the permeability measured at regular intervals of 1 cm is plotted. Note the irregular, short-distance permeability variation; there is also a long-distance permeability variation visible. The permeabilities are systematically smaller than the permeability to n-decane. This however, has, no effect on the determination of \( \sigma \) and \( \lambda \). The frequency distribution of the permeabilities is shown in Fig. 1.5b. The assumption that this distribution is log-normal is confirmed with the \( \chi^2 \) test (with level of significance of 80%; critical \( \chi^2 \) value is 6.2). The standard deviation \( \sigma \) of the distribution is 0.20. The correlation length \( \lambda \) of the log-permeability of the core can be determined from the correlation function

\[
R(x) = \frac{\int \ln \frac{k(x+\xi)}{k_{\text{average}}} \ln \frac{k(\xi)}{k_{\text{average}}} d\xi}{\int d\xi} 
\]  
\[\text{(1B.1)}\]
where \( \xi \) is the position along the core and \( x \) is the separation. Because we are interested in the permeability variations that contribute to the dispersivity, we have filtered out the slow permeability variation by taking for \( k_{\text{average}} \):

\[
k_{\text{average}}(x) = \frac{1}{20 \text{ cm}} \int_{x-10 \text{ cm}}^{x+10 \text{ cm}} k(\xi) \, d\xi
\]

(1B.2)

The correlation function in Fig. 1.5c shows a rapid decrease at small \( x \) and is practically zero at \( x \) larger than a few centimetres. On the assumption that \( R(x) \) has the form \( \sigma^2 \exp(-x/\lambda) \), \( \lambda \) can be determined exactly. From the plot of \( \log R \) versus \( x \) in Fig. 1.5d, we determined \( \lambda = 2.3 \text{ cm} \). From the same plot a new value of \( \sigma \) without effect of long distance permeability variation was determined: \( \sigma = 0.18 \).

APPENDIX 1C

MEASUREMENT OF DISPERSION IN DEAD VOLUME

The dead volume (of the tubes) was measured by saturating the core with fluid \( P \) (the displaced fluid) and then injecting fluid I (the displacing fluid) into a bypass tube over the core until 100% fluid I was measured in the density meter. The displacement experiment was then started by injecting fluid I in the top of the core to displace fluid P downwards in the core. As a result two displacement fronts were apparent in the density meter (see Fig. 1C.1). The first front is fluid P driving fluid I. The volume of fluid I produced until fluid P is detected in the density meter is then equal to the dead volume. The second front is fluid I driving fluid P. The volume of fluid P between the first and second fronts is equal to one pore volume.

This procedure also served to determine the dispersion between the fluid banks generated in the dead volume (by valves, angles in the tube, branched tubes and by diffusion in the tubes). This dispersion, \( K_{\text{tube}} \), can be determined from the first front. It was found that the concentration profile of the first front could be characterised by a diffusion-like coefficient. The value of this coefficient was subtracted from the dispersion coefficient determined from the second front, \( K_{\text{measured}} \) to
correct for the dispersion in the dead volume. The above procedure was repeated for each displacement because the dispersion caused by the tubes is dependent on displacement velocity.

In the gas/oil displacements, this procedure could not be followed because the large density difference between gas and oil (on top of the gas) could give rise to instabilities at the first front. This would then not be representative of dispersion due to the dead volume appearing in the second front. Instead, the dispersion in the tubes was taken from the measured dispersions in the water/brine displacements.

![Diagram showing injection and production flow paths, with density meter and dispersed phases](image)

**FIG. 1C.1:** Procedure to measure dead volume and dispersion in tubes
CHAPTER 2
RANDOM-WALK MODEL FOR MACROSCOPIC DISPERSION IN THE
PRESENCE OF A VISCOSITY CONTRAST AND A DENSITY CONTRAST

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2.5. DEVIATION FROM RANDOM-WALK MODEL
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REFERENCES

APPENDIX 2A: EQUATIONS FOR MISCEIBLE DISPLACEMENT
APPENDIX 2B: RELATION BETWEEN CORRELATION FUNCTIONS
APPENDIX 2C: SOLUTION TO EQUATION (2C.6)
APPENDIX 2D: LISTING OF PROGRAM FOR SOLVING IMPLICIT EQUATIONS (2C.6)
2.1. INTRODUCTION

The theory of Gelhar et al.\textsuperscript{1-7} for macroscopic dispersion is particularly useful for the study of groundwater flow in which the fluid properties of the displacing and displaced fluids are practically the same. Applications are the salting-up of potable groundwater by invading seawater and the flow of contaminants. For the displacement of oil by a gas, for instance, the difference in fluid properties has to be taken into account for a realistic prediction of the extent of mixing. In Chapter 1 we have seen that even a small density difference, such as between water and brine, can have a significant effect at low displacement velocities.

A viscosity difference between the displacing and displaced fluids, for example, has an influence on the mixing zone length in between the fluids. In an unstable displacement (in a horizontal plane the displacement is unstable when a viscous fluid is displaced by a less viscous fluid), the mixing zone caused by viscous fingers grows fastest for the case of highest viscosity difference. In a stable displacement (the displacement is stable in a horizontal plane when a fluid is displaced by a more viscous fluid), the mixing zone grows slowest for the case of highest viscosity difference.

One model has been reported that claims to quantify the dispersion in the presence of a viscosity difference. This model by Heinemann & Munka\textsuperscript{8} is a synthesis of previous models, but lacks a derivation. The model includes dispersion, fractional flow, a term for molecular diffusion, and a third, stagnant phase. Many assumptions underlie the model. In addition, the model contains four matching parameters, hence predictive power is very limited.

In this chapter a model is set up to quantify the effect of a viscosity contrast between displacing and displaced fluids and a density contrast on macroscopic dispersion. The model makes use of a random-walk concept for the movement of the fluid parcels, which enables us to calculate the dispersion coefficient as a function of the step size in the random-walk process (Section 2.3.2.1). In Section 2.3.2.2 we show a way to calculate the step size if no viscosity contrast and no density contrast are present and we demonstrate that the resulting expression for the dispersion coefficient corresponds to an expression derived by others. Finally, in Section 2.3.2.3 we extend the calculation of the step size to the presence of a viscosity contrast and a density contrast. The results of the model are discussed in Section 2.4, its shortcomings in 2.5 and a practical example in 2.6.
2.2. CONFINEMENT

The treatment here is on macroscopic dispersion. We have left microscopic dispersion out of consideration, because this part of the dispersion is not affected by the treatment presented here and is negligible on local and field scale. Hence the interface is assumed to be not diffuse but sharp. Neither do we consider megascopic dispersion, because this type of dispersion can be best modelled without a stochastic approach when the layer permeabilities are known. Within the boundaries of molecular diffusion and megascopic dispersion, the treatment is not confined to a particular length scale. We thereby assume the following conditions.

1. The permeability is randomly distributed in all directions.
2. The permeability is statistically homogeneous, i.e. the permeability frequency distribution and the permeability autocorrelation function do not depend on the position $r$ within the unit.
3. The permeability frequency distribution is log-normal.
4. The dimensions of the unit are large compared with the correlation length. This implies that, with the former condition, unique values for the dispersivity can be obtained when the distance travelled by the fluid exceeds the correlation length many times.
5. At least one of the correlation lengths perpendicular to the displacement direction is small compared with the correlation length in the displacement direction.
6. In general, the permeability and the dispersion are a tensor: in the transverse direction of the flow, permeability and dispersion are different from permeability and dispersion in the longitudinal direction. As a result, the flow is in general not parallel to the pressure gradient. Here the description is confined to displacement in one of the three principal directions of the permeability tensor.
7. The displacement velocity is held constant in time and initially the displacement front between the fluids is straight and coincides with the boundary at which the fluid is injected, so a one-dimensional description of the displacement is sufficient.

Conditions 1-6 satisfy the definition of heterogeneity at local scale (see section 1 from Introduction). Condition 3 is in many cases satisfied and this is the reason that most investigators have chosen a log-normal
distribution. Jensen et al.\textsuperscript{11} recommends a generalised distribution, which is the normal distribution of $k^P$ with $-1 \leq p \leq 1$ ($p \neq 0$) and $\ln k$ for $p = 0$. However, the question of how to calculate the effective permeability has remained for this generalised distribution. For this reason, and to adhere to the available literature, we assume a log-normal distribution.

In a great number of layered reservoirs condition 5 is satisfied because the layer thickness (which is of the same order of magnitude as the correlation length of the permeability in the across-dip direction) is much smaller than the areal correlation lengths, or in other words: the permeability changes faster in the across-dip direction than in the areal direction. This condition is discussed further in section 2.5. Condition 5 and 6 imply that the flow direction is parallel to the layering. Our treatment is applicable to both 2D and 3D configurations.

Within the confinement the relevant equations for miscible displacement are according to equations (2A.4) - (2A.6) from Appendix A:

\begin{equation}
U \phi = - \frac{k_0}{\mu(C)} \left( \frac{\partial p}{\partial x} + \rho(C) g \sin \theta \right),
\end{equation}

\begin{equation}
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = K_c \frac{\partial^2 C}{\partial x^2},
\end{equation}

where: $C$ = concentration averaged over a transverse cross-section
$g$ = gravitational acceleration
$K_c$ = convective dispersion coefficient
$k_0$ = effective permeability
$P$ = pressure
$U$ = displacement velocity
$t$ = time
$x$ = coordinate in longitudinal direction
$\theta$ = dip angle.
$\mu(C)$ = viscosity of the mixture
$\rho(C)$ = density of the mixture
$\phi$ = porosity as a function of place $\bar{r}$. 
2.3. EFFECT OF VISCOSITY CONTRAST AND DENSITY CONTRAST ON DISPERSIVITY

2.3.1. Qualitative

A simple configuration shows that dispersion decreases with increasing viscosity contrast (gravity is absent, displacement is stable). Compare for this purpose the flow in two equally long tubes in a horizontal plane. The tubes have pressure communication at both the inlet and the outlet, so that the pressure drops equally in both tubes. In the tubes one fluid displaces another, less viscous fluid. The average viscosity is higher in the tube that has the more advanced front between the fluids, and hence decelerates the flow there. As a result a difference in the advancement of the two fronts, caused by a conductivity difference between the tubes, is suppressed by the more viscous fluid. The same effect occurs in a porous medium when the displacement front has travelled further in one part of the porous medium than in another, less permeable part of the porous medium. Brigham et al.\(^{12}\), amongst a number of authors, confirmed that dispersion in a glass pack is suppressed when the displacing fluid is more viscous than the displaced fluid (see Chapter 1).

The two-tube configuration also shows in a simple way that when it is in a vertical position, a density difference between a fluid on top of a heavier fluid suppresses dispersion: a difference between the advancement of the displacement fronts in the tubes due to a conductivity difference is counteracted by the density difference. At a very low displacement velocity, any difference in advancement vanishes completely. Experimental evidence of dispersion suppression by a density difference has been reported by Slobod & Howlett\(^{13}\) and by our own experiments (see Chapter 1).

2.3.2. Quantitative

2.3.2.1. Random-walk process

A possible approach to implementing the fluid properties into the description of dispersion is to return to the level at which dispersion is generated, which is the random-walk process of the fluid parcels, caused by the spatial permeability variation. The idea of a random-walk process was introduced by Scheidegger\(^{14,15}\) and later also used by Saffman\(^{16,17}\), Nikolaevski\(^{18}\), de Josselin de Jong\(^{19}\). Scheidegger showed in a simple way the
essential features of the longitudinal dispersion coefficient. We follow a slightly adapted version of Scheidegger's approach as described by Collins.

Consider a fluid parcel that is small enough not to be separated into smaller parcels during its way through the porous medium. The time interval 0 to t is divided into N equal time steps τ. In every time step the fluid parcel travels a distance that consists of two parts: a constant part and a random part. After N time steps the average distance travelled is N λ, in which λ is the mean step size in each time step. The standard deviation of the position of the fluid parcel from the mean is N σ_λ λ. The displacement process can be envisaged as a movement with constant velocity on which is superimposed a random walk with average step size σ_λ λ. After sufficient time steps the spatial distribution of the fluid parcels around the mean location N λ can be described by a Gaussian probability function P for finding the fluid parcel between location x and location x + dx:

\[ dP = \left(2\pi N σ_λ^2 λ^2 \right)^{-1/2} \exp\left[ \frac{-(x-Nλ)^2}{2Nσ_λ^2 λ^2} \right] dx. \]

For a multitude of fluid parcels we may replace dP/dx by their concentration C. If we further substitute N = t/τ and N λ = Ut we have:

\[ C = \left(2\pi \frac{σ_λ^2 λ^2}{τ} t \right)^{-1/2} \exp\left[ \frac{-(x-Ut)^2}{2σ_λ^2 λ^2 τ} \right]. \]

Going back to the macroscopic, convection-dispersion equation (2.2a) for a tracer of concentration C, the solution to this equation when the boundary conditions are C(x,0) = 0 and C(0,t) = δ(t) is:

\[ C = \left(4\pi K_c t \right)^{-1/2} \exp\left[ \frac{-(x-Ut)^2}{4K_c t} \right]. \]

The two expressions for C are equal if

\[ K_c = \frac{1}{2} \frac{(σ_λ λ)^2}{τ}. \] (2.3)
Note that this relation corresponds to the diffusion coefficient as derived from the general theory of random-walk processes. With the displacement velocity \( U \) equal to \( \frac{N_t}{N_r} \), we have for the dispersion coefficient:

\[
K_c = \frac{1}{2} U \lambda \sigma_\lambda^2.
\]

(2.4)

This is the result Scheidegger obtained. Scheidegger did however not quantify the statistical parameters \( \sigma_\lambda \) and \( \lambda \).

2.3.2.2. Step size in random-walk process without a viscosity contrast and a density contrast

In this Section we add a new element to the Scheidegger model: we relate the step size in the random-walk process to properties of the porous medium by the simple fact that in every time step the fluid parcel travels a distance that is proportional to the local permeability. For the sake of simplicity we consider the permeability as constant over this distance; the permeability is of random value and does not correlate with adjacent permeability values. In particular, we identify \( \sigma_\lambda \) and \( \lambda \) with standard deviation \( \sigma \) and correlation length \( \lambda \) of the permeability distribution. Note that the above relation (2.4) corresponds to a more recent relation (5) of Gelhar et al. (apart from the correction factor) if we can identify \( \sigma_\lambda \) with \( \sigma \) and \( \frac{1}{2} \lambda \) with \( \lambda \) in (5).

To identify the standard deviation \( \sigma_\lambda \) in the random-walk process with the permeability standard deviation \( \sigma \), we use an approximation that we also use in Section 2.3.2.3 as the basis of our model of the effect of fluid properties on effective dispersivity.

Instead of calculating the movement of the fluid parcels in every part of the reservoir, we select one rectangular block of uniform permeability in the reservoir and consider this element as embedded in an infinite reservoir with effective properties (see Fig. 2.1). The effective permeability of the reservoir is denoted as \( k_0 \), the permeability of the selected element as \( k_\perp \). For this simple reservoir model we will calculate the distortion of a displacement front as it traverses the block of different permeability.

To calculate the distance travelled by the front in the block, \( \lambda_\perp \), we note that the pressure drop over the block is determined solely by the given displacement velocity far from the block, because a transverse flow
component is practically absent as a result of the model assumption that the block size in one of the transverse directions is small compared with its length (condition 5 of section 2.2). Intuitively, one would expect that the streamlines that circumvent a block of low permeability are not bent in a transverse direction if one transverse block size is infinitely small.
compared with the block length, regardless of the magnitude of the other transverse block sizes. Potential theory 22 confirms this expectation. For an ellipsoid embedded in an infinite medium, potential theory quantifies the velocity inside the ellipsoid in terms of the velocity at infinity, the ratio of the permeability of the surrounding medium to the permeability inside the ellipsoid, and the dimensions of the ellipsoid. In the limit of a zero transverse size, the flow velocity inside the ellipsoid is not determined by the dimension of the ellipsoid, irrespective of the magnitude of the other transverse size, but by the permeability contrast only. As a consequence, the pressure gradient is the same everywhere. In addition, the exact shape of the area of different permeability is not relevant any more. Hence, we may replace the ellipsoid by a rectangular block.

The equations describing the displacement of the front at x-coordinate \( x_0 \) in the surrounding medium with permeability \( k_0 \) and the front at \( x_1 \) in the block with permeability \( k_1 \) are thus:

\[
\Delta p = \frac{\mu}{k_0} \frac{dx_0}{dt} t = \frac{\mu}{k_1} \frac{dx_1}{dt} t,
\]

(2.5)

where \( \Delta p \) is the pressure drop over the block
\( \mu \) is the viscosity

The situation at \( t = 0 \) is depicted in Fig. 2.2a. The origin of the x-coordinate is at the upstream boundary of the block. Hence at \( t = 0 \) is \( x_0 = x_1 = 0 \). The solution to the differential equation with this boundary condition is \( x_1 = x_0 \frac{k_1}{k_0} \). When the front in the surrounding medium has reached the end of the block, as depicted in Fig. 2.2b, the front in the block has travelled a distance given by

\[
\frac{x_1}{x} = \frac{k_1}{k_0}.
\]

(2.6)

Consider an ensemble of model reservoirs, each of infinite size with effective permeability with in its centre one block of different permeability \( k_1 \), where the frequency of occurrence of the value \( k_1 \) in the ensemble is equal to the log-normal frequency distribution of the real reservoir. We find for the distance travelled in the block of permeability
FIG. 2.2: Position of displacement front between fluid I and fluid P
that (with (2.6)) \( \lambda / \lambda \) is distributed in the same way as is \( k_i/k_0 \). Hence the standard deviation of the permeability distribution, \( \sigma \), is equal to \( \sigma' \).

To identify \( 1/2 \lambda \), half the block length, with \( \lambda \), the correlation length of the permeability, we proceed as follows. The correlation function that in fact Scheidegger uses is

\[
R(x) = \sigma'^2 \left(1 - \frac{|x|}{\lambda}\right) H(\lambda - |x|),
\]

(2.7)

where \( H \) is the Heaviside function. Scheidegger does not mention this correlation function but it follows from relation (2.6), the log-normal permeability distribution with standard deviation \( \sigma' \) over blocks of length \( \lambda \) without correlation between permeabilities in adjacent blocks. With Fourier transformation (see Appendix 2B) the correlation function of Scheidegger can be expressed in a series of correlation functions of the type \( \sigma'^2 \exp(-|x|/\lambda) \) that Gelhar & Axness use:

\[
R(x) = \sigma'^2 \sum_{m=1}^{\infty} C_m \exp(-|x|/\lambda_m) \quad \text{with} \quad \sum_{m=1}^{\infty} C_m \lambda_m^{2n+1} = \frac{1}{(2n+2)!} \lambda^{2n+1},
\]

\[(n=0,1,2,\ldots).
\]

(2.8)

Because equation (2.2) is linear if \( K_C \) is not dependent on the place coordinate \( x \), the dispersivity \( K_C/U \) is now equal to \( \sigma'^2 \sum_{m=1}^{\infty} C_m \lambda_m \) according to Gelhar & Axness. This dispersivity is equal to \( \sigma'^2 \frac{1}{2} \lambda \) with (2.8). With \( \sigma' = \sigma \), this expression corresponds to expression (2.4) of Scheidegger. The conclusion is that the dispersivity of Gelhar & Axness agrees with the dispersivity of Scheidegger. A consequence is that the step size divided by two, \( \frac{1}{2} \lambda \), can be identified with correlation length \( \lambda \).

### 2.3.2.3. Step size in random-walk process in the presence of a viscosity contrast and a density contrast

To calculate the dispersivity in the presence of a viscosity difference and a density difference, we again use the random-walk model of Scheidegger. The dispersivity is given by Scheidegger as in expression (2.4). As in section 2.3.2.1 we extend Scheidegger's model to be able to calculate the standard deviation \( \sigma' \) in the random-walk process of the fluid parcels. For
this purpose, we again consider the configuration of one block of different permeability embedded in an infinite medium with effective permeability $k_0$ (Fig. 2.1). As in the previous case we calculate the movement of the displacement front between the fluids. Again we should carry out this calculation for an ensemble of reservoirs, each with one block of permeability $k_1$ taken from the permeability frequency distribution. However, it is found that, owing to gravity and viscous terms, no closed analytical expression for $\ell_1$ as a function of $k_1$, and hence between $\sigma_\ell$ and $\sigma$, can be calculated. This prohibits a direct calculation of the dispersion coefficient, $K_c$. Therefore, we allow ourselves the following extra simplification. From the manifold of permeability values we select one that is characteristic of the spatial log-permeability distribution: the log-permeability of the selected block contrasts with the log-permeability of the surrounding medium by the standard deviation $\sigma$. The permeability of the block, $k_1$, should be selected such that $\ln(k_0/k_1) = \sigma$. (This corresponds to $k_0/k_1 = e^{\sigma} = S$, where $S$ is the geometrical standard deviation of the permeability distribution.) Besides, we assume that the distribution of $\ell_1$ values is the same as that of the permeabilities. As a result, for this particular choice of $k_1$ the relation between $\ell_1$ and $\ell$ has to be determined only once. The standard deviation $\sigma_\ell$ is now equal to $\ln(\ell/\ell_1)$:

$$\sigma_\ell = \ln(\ell/\ell_1).$$  \hspace{1cm} (2.9)

This assumption has been tested extensively in a series of numerical simulations (Chapter 3). The proposed expression for the dispersivity is, with substitution of (2.9) in (2.4) and of $\lambda = \frac{1}{2} \ell$:

$$\frac{K}{U} = \lambda (\ln(\ell/\ell_1))^2.$$  \hspace{1cm} (2.10)

As shown in Section 2.3.2.2, this expression reduces to Gelhar & Axness' expression if fluid properties are not taken into account (apart from their correction factor $F(\ln S)$).

For the configuration of Fig. 2.1 and the particular choice of $k_1$, we will calculate the position of the displacement front $\ell_1$ between fluid I and fluid P with different viscosity and different density at the moment that
the displacement front in the surrounding medium has reached the end of the block (Fig. 2.2b). The situation at \( t = 0 \) is depicted in Fig. 2.2a. The positions of the fronts at an arbitrary time in between both moments is shown in Fig. 2.2c, which provides also the definition of the coordinates \( x_0 \) and \( x_1 \) of the front positions. The equations below describe the pressure drop in the displacement direction due to Darcy flow and due to hydrostatic pressure difference.

\[
\Delta p = \frac{\mu_i}{k_0} \phi \frac{dx_0}{dt} x_0 + \frac{\mu_p}{k_0} \phi \frac{dx_0}{dt} (\ell - x_0) - \rho_I g x_0 \sin \theta - \rho_p g (\ell - x_0) \sin \theta \\
= \frac{\mu_i}{k_1} \phi \frac{dx_1}{dt} x_1 + \frac{\mu_p}{k_1} \phi \frac{dx_1}{dt} (\ell - x_1) - \rho_I g x_1 \sin \theta - \rho_p g (\ell - x_1) \sin \theta,
\]

(2.11)

with initial conditions: \( x_0 = x_1 = 0 \) at \( t = 0 \), boundary condition: \( x_0 = \ell \) at \( t = \ell/U \) and definition: \( x_1 = \ell_1 \) at \( t = \ell/U \).

where \( g \) is the gravitational acceleration

\( \ell \) is the block length

\( \ell_1 \) is the value of \( x_1 \) when \( x_0 = \ell \)

\( x_0 \) is the coordinate of the displacement front in the surrounding medium

\( x_1 \) is the coordinate of the displacement front in the block

\( \Delta p \) is the pressure drop over a block

\( \theta \) is the dip angle with horizontal

\( \mu_i \) is the viscosity of the displacing fluid

\( \mu_p \) is the viscosity of the displaced fluid

\( \rho_i \) is the density of the displacing fluid

\( \rho_p \) is the density of the displaced fluid

Equation (2.11) is applicable to both downward and upward displacement if we clearly define the coordinate system. The component of the gravitational acceleration \( g \sin \theta \) is selected as the positive direction. For a displacement in the downward direction, \( U \) and the coordinates \( x_0', x_1', \ell' \) are counted as positive, while for a displacement in the upward direction, \( U, x_0', x_1', \ell' \) are counted as negative. Because the density of the fluid at the top is smallest in practical cases, the gravity number defined as
\[ N_G = \frac{k_0 g \Delta \rho \sin \theta}{\phi \mu_p U} \]  

(2.12)

where \( \Delta \rho = \rho_p - \rho_l \), is positive for displacements in either direction. We will later use the gravity number for a graphical representation of the results. For a solution of the equations (2.11) we use the definition of the Dietz critical displacement velocity for gravity-stable displacement

\[ U_c = \frac{k_0 g \Delta \rho \sin \theta}{\phi \Delta \mu} \]  

(2.13)

where \( \Delta \mu = \mu_p - \mu_l \), and we use the mobility ratio \( M \), which for miscible displacement, is

\[ M = \frac{\mu_p}{\mu_l} \]  

(2.14)

Dividing (2.11) by \( \frac{\mu_l}{k_0} \phi \), we have

\[ \frac{k_0}{\mu_l} \phi (\Delta \rho + \rho_p g l \sin \theta) \]

\[ \frac{dx_0}{dt} x_0 + M \frac{dx_0}{dt} (l - x_0) + (M - 1) U_c x_0 \]

\[ = S \frac{dx_1}{dt} x_1 + S M \frac{dx_1}{dt} (l - x_1) + (M - 1) U_c x_1. \]

(2.15)

Substituting \( \frac{dx_0}{dt} = U \) and \( x_0 = U t \) in (2.15) and dividing the equation by \( U_c \), we have from (2.15)

\[ \left( \frac{U}{U_c} - 1 \right) [(1-M) Ut + M l] = (S \frac{1}{U_c} \frac{dx_1}{dt} - 1) [(1-M) x_1 + M l]. \]

(2.16)

The solution of this differential equation is treated in Appendix 2C. The solution for \( l_1 \) is a set of implicit equations that have to be solved numerically. The Fortran program that we have written to solve these equations is listed in Appendix 2D.
2.4. **RESULTS OF RANDOM-WALK MODEL**

From (2.10) and the solution for \( t_1 \) presented in Appendix 2C (equations (2C.6)), the effective dispersivity \( \beta = K_c / U \) can be calculated. Because equations (2C.6) are implicit in \( t_1 \), we present the model results graphically. For this purpose we define a *dispersivity ratio* \( \alpha \) as the dispersivity taking fluid properties into account (effective dispersivity \( \beta \)) divided by dispersivity disregarding these (dispersivity \( \beta_0 \) of reservoir or geological unit):

\[
\alpha = \frac{\beta}{\beta_0} = \frac{(\ln t/t_1)^2}{(\ln S)^2}.
\]  

(2.17)

Substituting \( \alpha \) in (2.10) gives:

\[
K_c = U \lambda (\ln S)^2 \alpha.
\]  

(2.18)

Figure 2.3 gives a graphical representation of \( \alpha \) as a function of \( N_G \) with \( M \) and \( S \) as parameters. The figure can be used for both conditionally stable displacement (with \( M > 1 \) and \( N_G > 1 - M^{-1} \)) and unconditionally stable displacement (with \( M < 1 \)). (In Chapter 4 it is shown that that under special conditions the random-walk model is applicable to unstable, miscible displacement as well. A graphical representation of \( \alpha \) in the unstable regime is given in this chapter.)

For a conditionally stable displacement (with \( M > 1 \) and \( N_G > 1 - M^{-1} \)), \( \alpha \) lies between 0 and 1. \( \alpha \) equals 1 if the displacement velocity is equal to \( U_c \); this follows directly from equation (2.16). For an unconditionally stable displacement (\( M < 1 \)), \( \alpha \) lies between 0 and some maximum value below 1.

The figure shows that the dispersivity ratio \( \alpha \) increases with the geometrical standard deviation \( S \). What is more important is that \( \alpha \) (and hence the effective dispersivity) decreases with increasing gravity number (corresponding to decreasing displacement velocity). The magnitude of the decrease of \( \alpha \) depends on the mobility ratio: the higher the mobility ratio (hence the less viscous the displacing fluid), the greater the decrease of \( \alpha \). \( \alpha \) goes asymptotically to zero at high gravity number. The dependence of \( \alpha \)
on gravity number $\alpha$ is particularly strong at $M$ greater than, say 5, and $N_G$ between 0.8 and 4.

$\alpha$ is independent of $U$ at small $N_G$ ($N_G < 0.1$) and $M < 1$. The value of $\alpha$ then depends on $M$ and $S$ alone. The trend is: the greater $M$, the greater $\alpha$. The dependence of $\alpha$ on $M$ is weak at small $M$, hence the decrease of $M$ from 0.1 to 0.01 has a very small effect on the effective dispersivity.

**FIG. 2.3:** Dispersivity ratio $\alpha$ as a function of gravity number $N_G$ with parameters mobility ratio $M$ and geometrical standard deviation $S$
For the following special cases the explicit expressions for $a$ are:

1. No effect of gravity, for example in the case of high displacement velocity, no density difference or zero dip angle: $N_G = 0$,

\[
\lim_{N_G \to 0} a = \frac{\left(\ln \frac{M^{-1} - 1}{1 + (M^{-2} - 1) \frac{1}{S}}\right)^2}{\left(\ln S\right)^2}. \tag{2.19}
\]

This expression has been derived by solving (2.11) without the gravity terms. In Fig. 2.4 the expression for $a$ is plotted versus $M$ with $S$ as parameter. Note that $a$ is at minimum $\frac{1}{4}$, independent of the value of $S$ and $M$. We have

\[
\lim_{M \to 0} \lim_{N_G \to 0} a = \frac{1}{4}. \tag{2.20}
\]

This can also be derived by putting $\mu_p$ to zero in (2.16) and ignoring the gravity terms. Finally,

\[
\lim_{S \to 1} \lim_{N_G \to 0} a = \frac{1}{4} (M+1)^2. \tag{2.21}
\]

2. No viscosity difference: $M = 1$, for example, brine displacing water at low velocity so that gravity is important:

\[
\lim_{M \to 1} a = \frac{-N_G/S}{\left[\ln\left(1 - (1 - \frac{1}{S}) (1 - e^{-G}) S/N_G\right)\right]^2}. \tag{2.22}
\]

This expression has been derived by solving (2.11) with $\mu_I = \mu_p$. Figure 2.5 shows the course of $a$ with $N_G$ (parameter is $S$).

3. Vanishing displacement velocity: $U \to 0$. In this case gravity forces dominate over viscous forces. In equation (2.15) all terms with $\frac{dx_0}{dt}$ and $\frac{dx_1}{dt}$ vanish. The solution to (2.15) is then $x_0 = x_1$, resulting in $a = 0$. 

FIG. 2.4: Dispersivity ratio $\alpha$ as a function of mobility ratio $M$ at gravity number $N_G=0$ (parameter geometrical standard deviation $S$)

FIG. 2.5: Dispersivity ratio $\alpha$ as a function of gravity number $N_G$ at mobility ratio $M=1$ (parameter geometrical standard deviation $S$)
2.5. **DEVIATION FROM RANDOM-WALK MODEL**

Our model of section 2.3 clearly demonstrates a contrast between the longitudinal velocity inside and outside the block of different permeability. There are, however, other transverse variations in the longitudinal velocity that we have not incorporated in our one-dimensional flow model. These variations are generally small but can be important. They are present in the following cases.

A. A flux of fluid circumventing the block of lower permeability $k_1$. This occurs if both transverse correlation lengths of the permeability are finite.

B. The pressures inside and outside the block are not in equilibrium at the lateral boundaries. The fluids adapt their longitudinal velocity to restore pressure equilibrium, resulting in a velocity contrast between the velocities of the bulk fluid and the fluid close to a lateral boundary.

The random-walk model of section 2.3.2 does not deal with case A because an infinitely small correlation length in one of the transverse directions is assumed (condition 5 of section 2.2). Nevertheless, in section 2.5.1 we estimate the deviation from our model when condition 5 is not satisfied. Case B is relevant because our model assumes pressure equilibrium only at the inlet and at the outlet of the block. The deviation from our model by pressure disequilibrium is discussed in section 2.5.2.

### 2.5.1. **Transverse correlation length**

A flux circumventing the block as a result of two finite transverse correlation lengths is most apparent in case $M = 1$ and $N^X = 0$. Then the effective dispersivity is largest. In the limit of zero $a$, the flux circumventing the block becomes zero. As a result, the transverse effect decreases with decreasing effective dispersivity.

In case $M = 1$ and $N^X = 0$, potential theory\(^{22}\) shows that the flow velocity $U_1$ inside a two-dimensional ellipse of longitudinal size $\lambda$ and transverse size $\lambda'$ surrounded by an infinite medium with permeability $k_0$ is given by
where \( S \) is the ratio between the permeability inside and the permeability outside the ellipse, and \( U \) is the velocity at infinity. In the limit \( \frac{\lambda'}{\lambda} \to 0 \), equation (2.23) gives \( \frac{U}{U_1} \to S \), which is not dependent on the ellipse sizes any more. This expression equates to equation (2.6), as expected. However, for \( \lambda = \lambda' \), we have from (2.23):

\[
\frac{U}{U_1} = \frac{S + 1}{2}.
\]

(2.24)

Note that \( U \) and \( U_1 \) do not depend on time. Thus the ratio of the distances travelled by the fronts, \( \lambda/\lambda' \), is equal to the velocity ratio \( U/U_1 \). The effective dispersivity as given by (2.10) should thus be multiplied by a factor

\[
\frac{\ln(S+1)}{[\frac{\ln(2)}{\ln S}]^2}.
\]

(2.25)

For example, this factor equals 0.34 for \( S = 2 \). Hence the correction to the effective dispersivity is a factor of 0.34 at \( \alpha = 1 \). The correction changes gradually to zero correction at zero \( \alpha \).

In the three-dimensional case with an ellipsoid with \( \lambda = \lambda' = \lambda'' \) (i.e. a sphere) potential theory\textsuperscript{22} shows:

\[
\frac{U}{U_1} = \frac{2S + 1}{3}.
\]

(2.26)

The multiplication factor for the effective dispersivity is thus:

\[
\frac{\ln(2S + 1)}{[\frac{\ln(3)}{\ln S}]^2},
\]

(2.27)

which, for example, equals 0.54 for \( S = 2 \) and equals 0.44 for \( S \) close to 1. The correction changes gradually to zero correction at zero \( \alpha \).
2.5.2. **Transverse pressure disequilibrium**

In this section we demonstrate that our model predicts a jump between the pressures just inside and just outside the block of permeability $k_1$. In reality, a discontinuity in pressure cannot occur. As a result, our model is not exactly correct. It is shown below that this is particularly important at small $\alpha$.

The model prediction of the qualitative course of the pressure inside, $p_1$, and outside the block, $p_0$, is depicted in Fig. 2.6a or 6b. The figure shows that our model predicts a pressure jump between the region inside and the region outside the block. This jump occurs at the lateral boundary of the block.

Obviously, the fluids adapt their velocity in such a way that nowhere does a discontinuity in pressure exist. This is possible if there is some flow across the lateral boundaries of the block such that the pressure jump disappears. The importance of cross flow depends on the magnitude of the pressure jump, which can be quantified as follows.

The constant slope of the pressure plots in Fig. 2.6 (apart from the discontinuities in the slope at $x = x_1$ and $x = x_0$) are a result of the continuity equation for incompressible flow without a transverse component. The upward bending of the pressure plots at the fronts between fluid I and fluid P is a result of the stability of the displacement. The smaller slope for $p_1$ compared with that for $p_0$ (for $0 \leq x \leq x_1$ and $x_0 < x \leq \ell$) is a result of (2.11), from which it follows that

$$
\frac{dp_1}{dx} - \frac{dp_0}{dx} = -\mu \phi \left( \frac{U}{k_0} - \frac{1}{k_1} \frac{dx_1}{dt} \right) \quad (0 \leq x \leq x_1 \text{ or } x_0 < x \leq \ell) \quad (2.28)
$$

where $\mu$ is either $\mu_I$ or $\mu_P$. Further, if $\alpha \leq 1$, it follows from section 2.3 that

$$
\frac{1}{U} \frac{dx_1}{dt} \geq \frac{k_1}{k_0}. \quad (2.29)
$$
FIG. 2.6: Two possible courses of pressure inside and outside block of different permeability.

Hence

\[
\frac{dp}{dx}_0 - \frac{dp}{dx}_1 \geq 0. \quad (0 \leq x \leq x_1 \text{ or } x_0 < x \leq l)
\]  \hspace{1cm} (2.30)

With equal pressures at \( x = 0 \) and \( x = l \), the pressure in the block is, for example, lower than the pressure outside in the \( x \) range between 0 and \( x_1 \).

Figure 2.6 shows that a measure for the pressure jump is the pressure jump at \( x = x_0 \):

\[
\left[ \frac{dp}{dx}_0 - \frac{dp}{dx}_1 \right] x_0
\]  \hspace{1cm} (2.31)

Substitution of (2.28) and of \( S = k_0/k_1 \) gives for the pressure jump
\[- \frac{\mu \phi}{k_0} [U - S \frac{dx}{dt}] x_0 \]  

This quantity varies with time. A measure of its average value is:

\[- \frac{\mu \phi}{k_0} [U - S \frac{1}{L/U}] \frac{L}{2} \]  

which is with (2.17) equal to

\[- \frac{\mu \phi}{k_0} U [S^{1-\alpha} - 1] \frac{L}{2} \]  

If \( \alpha = 1 \) this quantity (and thus the cross flow) is 0. The cross flow increases with decreasing \( \alpha \). At \( \alpha = 0 \) the cross flow is the most important. The deviation from our model will thus be largest when \( \alpha = 0 \).

2.6. PRACTICAL EXAMPLE

As a practical example, we consider the miscible hydrocarbon gas flood in the Westpem Nisku D Reef in Canada conducted by Chevron. In a recent paper, Da Sie & Guo evaluate the performance of this vertical flood. One of the three conclusions of their paper is that the observed mixing zone between the solvent and the oil is "much smaller (< 8.5 m) than expected". Further, they mention that "dispersion of the solvent into the oil is probably not significant in a scheme where gravity segregation of gas and oil dominates flood mechanics". We show below that our model predicts a very small mixing zone length in line with their observation.

The flood started in May 1981. The displacement velocity of the flood is low: in December 1986 the solvent/oil interface was at a depth of 2129.5 m and in May 1987, it had reached a depth of 2130.6 m, so the displacement velocity was at that time 2130.6 - 2129.5 = 1.1 m per 5 months or 8.5*10^-8 m/s. With a few fluid and rock data from the paper (\( \mu_{\text{oil}} = 0.19 \) mPa.s, \( \mu_{\text{solvent}} = 0.026 \) mPa.s, \( \rho_{\text{oil}} = 540 \) kg/m³, \( \rho_{\text{solvent}} = 210 \) kg/m³, \( k_{\text{vertical}} = 0.110 \) \( \mu \)m², \( \phi = 0.12 \)) we can calculate two important parameters: mobility ratio \( M \) is 7.3 and gravity number \( N_G \) is 180. This shows that the flood is extremely gravity-dominated.
To find the dispersivity ratio $\alpha$ at these parameter values, Fig. 2.3 can be used. Interpolation between the $M = 5$ and the $M = 10$ curves in Fig. 2.3 to find $\alpha$ at $M = 7.3$, shows that $\alpha$ is practically zero at $N_G$ larger than 50. The paper does not provide information about the permeability variation but this is no problem, because $\alpha$ is practically zero for a wide range of $S$ values (between 1 and 5 at least). So at $N_G = 180$, $\alpha$ is such a low value that macroscopic dispersion is negligible in this flood.

Because of the suppression of the macroscopic dispersion, we expect that the mixing zone is dominated by molecular diffusion (microscopic dispersion is negligible because of the very low displacement velocity). For example, with a guessed value $D/F_R\phi = 10^{-9}$ m$^2$/s, the mixing zone length in May 1987 after its start in May 1981 is about $3.62\sqrt{(10^{-9} \text{ m}^2/\text{s} \times 6 \text{ years})} = 1.8$ m. This value agrees with the observation that the mixing zone length is smaller than the distance between two sampling points, which was 8.5 m.

2.7. CONCLUSIONS

1. We have developed a random-walk model to estimate dispersive mixing zones in the presence of a viscosity contrast and a density contrast.

2. The random-walk model shows that the effective dispersivity, which takes into account the effect of viscosity contrast and density contrast, is equal to the dispersivity multiplied by a second function of mobility ratio, the gravity number and the standard deviation of permeability variation. The first function is given by the theory of Gelhar et al., the second function is given by our model (the mathematical expression is (2.18)). The second function, which we call the dispersivity ratio, is a value between 0 and 1. At unit mobility ratio and zero gravity number the dispersivity ratio is 1. The dispersivity ratio is graphically presented in Fig. 2.3.

3. Some trends predicted by the random-walk model are:
   * the dispersivity ratio increases from 0.25 at zero mobility ratio to 1.0 at unit mobility ratio (at zero gravity number),
   * the dispersivity ratio tends to 0 at high values of the gravity number.
4. The model for effective dispersivity may lose its validity at small values of $a$ because of a transverse flow effect.

REFERENCES

APPENDIX 2A

EQUATIONS FOR MISCELLABLE DISPLACEMENT

If we know the value of the permeability in every part of a porous medium, the flow of two miscible, incompressible fluids is calculated from (ignoring microscopic dispersion)\(^{24}\):

\[
\vec{u} \cdot \vec{\phi}(\vec{r}) = - \frac{\vec{k}(\vec{r})}{\mu(c)} (\vec{\nabla}p + \rho(c) g \vec{v}_z), \quad (2A.1)
\]

\[
\frac{\partial c}{\partial t} + \vec{v} \cdot (c \vec{u}) = 0, \quad (2A.2)
\]

\[
\nabla \cdot \vec{u} = 0, \quad (2A.3)
\]

where:
- \( c \) = concentration of the displacing fluid in the mixture at a particular position
- \( g \) = gravitational acceleration
- \( \vec{k}(\vec{r}) \) = permeability tensor as a function of place \( \vec{r} \)
- \( p \) = pressure
- \( t \) = time
- \( \vec{u} \) = displacement velocity
- \( z \) = coordinate in vertical direction
- \( \mu(c) \) = viscosity of the mixture
- \( \rho(c) \) = density of the mixture
- \( \phi(\vec{r}) \) = porosity as a function of place \( \vec{r} \)

Because detailed knowledge of rapid, irregular spatial variations in the permeability is not normally available, average values for the permeability are used\(^{10}\). However, the permeability variation causes irregular spatial variations in the flow velocity according to (2A.1). A result is that two flowing miscible fluids develop a ragged interface. In the field or in laboratory experiments in which only the production history of the fluids is measured, individual interface distortions cannot be detected but their addition is observed as a gradual transition between the fluids in the production history of the fluids. A correct way to describe these irregular flow variations is by a random-walk process superimposed on the average displacement velocity\(^{14,15,20}\); the above equations describing the flow of
two miscible fluids are adapted by replacing the permeability by the effective permeability \( k_0 \), the porosity by the average porosity \( \phi \), and the concentration \( c \) by a volume averaged value \( C \) and by adding a diffusion-like term to equation (2A.2)\(^{25}\):

\[
\bar{\mathbf{U}} \cdot \mathbf{\phi} = - \frac{k}{\mu(c)} (\bar{\mathbf{v}}_p + \rho(c) \phi \bar{\mathbf{v}}_z),
\]

\[
\frac{\partial C}{\partial t} + \nabla (C \bar{\mathbf{U}}) = \nabla.(\bar{\mathbf{R}} \bar{\mathbf{v}}_C),
\]

\[
\nabla \cdot \bar{\mathbf{U}} = 0,
\]

where \( \bar{\mathbf{R}} \) is the dispersion tensor and \( \bar{\mathbf{U}} \) is the average displacement velocity.

**APPENDIX 2B**

**RELATION BETWEEN CORRELATION FUNCTIONS**

In this appendix a series of correlation functions of the type that is used in the theory of Gelhar et al.:

\[
\sigma^2 e^{-\frac{|x|}{\lambda}}
\]

is employed for the development of the correlation function implicitly used by Scheidegger:

\[
\sigma_{\lambda}^2 (1 - \frac{|x|}{\lambda}) H(\lambda - |x|).
\]

In section 2.3.2.1 it is shown that \( \sigma_{\lambda} \) equals \( \sigma \). The Fourier transform of \( e^{-\frac{|x|}{\lambda}} \) is equal to:

\[
F_1(\lambda, \xi) = \hat{f}(e^{-\frac{\lambda}{\lambda}}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{\lambda}{\lambda}} e^{-i\xi x} dx = \frac{\lambda}{\pi} \frac{\lambda}{1 + (\lambda \xi)^2}.
\]

The Fourier transform of \( (1 - \frac{|x|}{\lambda}) H(\lambda - |x|) \) is:
\[ F_2(\lambda, \xi) = \hat{F}(\lambda - |x|) H(\lambda - |x|) = \frac{1 - \cos \xi}{\xi^2}. \]  

Developing both Fourier transforms into Taylor series gives

\[ F_1(\lambda, \xi) = \frac{\sqrt{2}}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n \lambda^{2n+1} \xi^{2n}}{(2n+1)}, \]  

\[ F_2(\lambda, \xi) = \frac{\sqrt{2}}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n \lambda^{2n+1} \xi^{2n}}{(2n+1)!}. \]

Equating the series \( \sum_{m=0}^{\infty} C_m F_1(\lambda_m, \xi) \) to \( F_2(\lambda, \xi) \), where \( C_m \) and \( \lambda_m \) (\( m=0,1,2,\ldots \)) are unknown yet, gives

\[ \sum_{m=0}^{\infty} C_m F_1(\lambda_m, \xi) = \hat{F} \left( \sum_{m=0}^{\infty} C_m e^{-\frac{\lambda_m}{\lambda}} \right) \]

\[ = \frac{\sqrt{2}}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n (\sum_{m=0}^{\infty} C_m \lambda_m^{2n+1}) \xi^{2n}}{(2n+1)!}. \]

Hence, from (2B.5)

\[ \sum_{m=0}^{\infty} C_m \lambda_{n+1}^{2n+1} = \frac{1}{(2n+1)!} \xi^{2n+1}. \]  

\[ (n = 0,1,2,\ldots) \]  

The development of the correlation function of Scheidegger in a series of correlation functions of the type that Gelhar & Axness used is thus

\[ \sigma_{\lambda}^2 (1 - |x|) H(\lambda - |x|) = \sigma^2 \sum_{m=0}^{\infty} C_m e^{-\frac{|x|}{\lambda_m}} \]  

on condition (2B.6).
SOLUTION TO EQUATION (2.16)

The differential equation (2.16) to be solved is

\[
\frac{U}{U_C} - 1 \left[ (1-M) Ut + Mt \right] = \left( S \frac{1}{U_C} \frac{dx}{dt} - 1 \right) \left[ (1-M) x + Mt \right].
\]

Replacing \( t \) and \( x \) by new variables defined by

\[
\tau = (1-M) Ut + Mt,
\]

\[
w = S \frac{U}{U_C} \left[ (1-M) x + Mt \right] - \tau,
\]

we get

\[
- \left( 1 - \frac{U}{U_C} \right) \tau = \frac{dw}{dr} \frac{1}{S} \left( \frac{U}{U_C} \right) \left( w + \tau \right).
\]

(2C.1a)

(2C.1b)

(2C.2)

Dividing by \( \tau \), \( S \frac{U}{U_C} \) and \( w + \tau \) gives (\( \tau \) is not zero on \( \frac{Ut}{\ell} \) interval \([0,1]\) and \( w + \tau \) is not zero on \( x \ell/\ell \) interval \([0,1]\))

\[
\frac{dw}{dr} = - \left( 1 - \frac{U}{U_C} \right) \frac{S}{U_C} \frac{1}{w + \tau}.
\]

(2C.3)

Replacing \( w \) by \( z = \frac{w}{\tau} \) and substituting

\[
a = S \frac{U}{U_C} \left( 1 - \frac{U}{U_C} \right) - \frac{1}{4},
\]

(2C.4)

and separating variables, we have for the solution

\[
0 = \ln \tau + \int \frac{dz}{(a + \frac{1}{4})(z + 1)^{-1} + z} + c
\]

\[
= \ln \tau + \int \frac{(z+1) \, dz}{(z + \frac{1}{2})^2 + a} + c,
\]

(2C.5)
where \( c \) is an integration constant. In the special case that
\[
(a + \frac{1}{4})(z + 1)^{-1} + z = 0,
\]
the solution is \( z = c \). One can derive that this is the case only when \( S = 1 \), and the solution is then \( x_1 = \lambda t \).

Working out this integral and substituting \( \omega = z + \frac{1}{2} \), we find the following solution (depending on the sign of \( a \)), where \( r_o \) and \( \omega_o \) are constants.

When \( a > 0 \) then

\[
0 = \ln \frac{r}{r_o} + \frac{1}{2} \ln \left( \frac{\omega^2 + a}{\omega_o^2 + a} \right) + \frac{1}{2} \arctan \left( \frac{\sqrt{a} - \omega}{\omega_o} \right).
\]  \hspace{1cm} (2C.6a)

When \( a = 0 \) then

\[
0 = \ln \frac{r}{r_o} + \ln \left| \frac{\omega}{\omega_o} \right| - \frac{1}{2} \omega + \frac{1}{2} \omega_o.
\]  \hspace{1cm} (2C.6b)

When \( a < 0 \)

\[
0 = \ln \frac{r}{r_o} + \left( \frac{1}{2} + \frac{1}{2(\sqrt{a} - \omega)} \right) \ln \left| \frac{\omega^2 + a}{\omega_o^2 + a} \right| + \frac{1}{2(\sqrt{a} - \omega)} \ln \left( \frac{\omega_o + \sqrt{a}}{\omega + \sqrt{a}} \right).
\]  \hspace{1cm} (2C.6c)

From the definitions of \( \omega = \frac{\omega}{r} + \frac{1}{2} = \frac{U}{U_c} \frac{(1-M)x_1 + M\lambda}{(1-M)\lambda + M\lambda} \) and \( r = (1-M)\lambda t + M\lambda \),
we have by substituting the boundary conditions \( x_1 = \lambda t \) at \( t = \lambda U / U_c \) and \( x_1 = 0 \) at \( t=0 \):

\[
\omega = \frac{U}{U_c} \left[ (1-M) \frac{\lambda}{\lambda} + M \right] - \frac{1}{2},
\]  \hspace{1cm} (2C.7a)

\[
\omega_o = \frac{U}{U_c} - \frac{1}{2},
\]  \hspace{1cm} (2C.7b)

\[
\frac{r}{r_o} = \frac{1}{M}.
\]  \hspace{1cm} (2C.7c)

To calculate \( \frac{x_1}{\lambda} \) from the above equations, it is important to know how many roots can be expected when using a numerical zero-root routine. The maximum number of roots can be determined by counting the extrema and
assuming that between a maximum and the adjacent minimum a root is present.
When each of the above three equations is abbreviated as $f=0$, then from

$$0 = \frac{df(\omega)}{d\omega} \frac{d\omega}{d\epsilon_1}$$

(2C.8)

and with $\frac{df(\omega)}{d\omega} = \frac{\omega + \frac{1}{2}}{\omega^2 + a}$ for all equations and $\frac{d\omega}{d\epsilon_1} = S \frac{U}{U_c} \frac{1}{M-1}$, it follows

that there is one extremum at $\omega = -\frac{1}{2}$ corresponding to $\frac{1}{\epsilon} = \frac{M}{M-1}$, which is

negative for $M<1$ and larger than 1 for $M>1$. Hence, on the interval $0 \leq \frac{1}{\epsilon} \leq 1$ no extrema are present except for the boundaries. The number of roots is

therefore 1 at maximum.

When $a < 0$ then it is possible that $\omega^2 = -a$, corresponding to a

solution $\frac{1}{\epsilon} = 1$. In this case a boundary extremum may be found at $\omega^2 = -a$.

APPENDIX 2D

LISTING OF PROGRAM FOR SOLVING IMPLICIT EQUATIONS (2C.6)

C THIS FORTRAN 77 PROGRAM CALCULATES THE MULTIPLICATION FACTOR ALPHA FOR
C THE LONGITUDINAL DISPERSION COEFFICIENT AT A GIVEN MOBILITY RATIO,
C GRAVITY NUMBER AND GEOMETRIC STANDARD DEVIATION. THE EQUATIONS TO
C CALCULATE THE SUPPRESSION ARE GIVEN IN APPENDIX 2C.
C THE PROGRAM NEEDS AS INPUT THE MOBILITY RATIO M, THE GRAVITY NUMBER G,
C AND THE GEOMETRIC STANDARD DEVIATION S. THE VALUES SERVE AS INPUT IN
C EQUATIONS (2C.6). THESE EQUATIONS ARE SOLVED WITH A ZERO-ROOT ROUTINE
C FROM THE SCIENTIFIC LIBRARY OF IMSL (ADDRESS: NBC BUILDING
C 7500 BELLAIRE BOULEVARD, HOUSTON, TEXAS 77036-5085, USA).
C THE SOLUTION IS TRANSLATED INTO THE DISPERSIVITY RATIO ALPHA
C WITH EQUATION (2.17).
C
C DECLARATIONS
C
C REAL L,M,G,S,MMM
C CHARACTER REQ*3,OUTFILE*30,CURVENR*3
C EXTERNAL FI
C EXTERNAL FII
C EXTERNAL FIII
C COMMON M,S,G,UUC,B,OMEG0
C
C WRITE(6,'(A)')'THIS PROGRAM CALCULATES THE'
C WRITE(6,'(A)')'DISPERSIVITY RATIO ALPHA AS FUNCTION OF'
C WRITE(6,'(A)')'MOBILITY RATIO M AND GRAVITY NUMBER G,'
C WRITE(6,'(A)')'It assumes a log-normal distribution'
C WRITE(6,'(A)')'of the ratio permeability/porosity'
C WRITE(6,'(A)')'with geometric standard deviation S.')
GIVE INPUT DATA FOR THE DISPERSION PROBLEM

5 WRITE(6,'(A)')$GIVE GEOMETRIC STANDARD DEVIATION S :
READ(5,10)S
WRITE(6,'(A)')$GIVE MOBILITY RATIO M :
READ(5,10)M
WRITE(6,'(A)')$GIVE GRAVITY NUMBER G :
READ(5,10)G
WRITE(6,'(A)')$GIVE NUMBER OF BLOCKS NX :
READ(5,11)NX
WRITE(6,'(A)')$GIVE NUMBER OF UNCORRELATED BLOCKS NR :
READ(5,11)NR

10 FORMAT(G20.10)
11 FORMAT(I20)

DETERMINE SPECIAL CASES FOR WHICH ZFALSE IS NOT NEEDED

IF (ABS(M-1).LT.0.02) THEN
   L=1-(1-1/S)*(S/G)*(1-EXP(-G/S))
   UCU=G*M/(M-1)
   GOTO 111
END IF

IF (ABS(G).LT.0.02) THEN
   L=((1+(M**2-1)/S)**0.5-1)/(1/M-1)
   GOTO 111
END IF

SET VARIOUS PARAMETERS USED IN THE ZFALSE ROUTINE

EPS=1.0E-05
NSIG=5
100 ITMAX=100
XL=0.0
XR=1.0

UUC=(M-1)/(G*M)
UCU=1/UUC
OMEGA=S*UUC-0.5
B=S*UUC*(1-UUC)-0.25

IF (B.LT.0.AND.FII(XL)*FII(XR).GT.0) THEN
   XL=((0.5-(-B)**0.5)/(S*UUC) - M/(-M) + 1E-6)
END IF
IF(XL.LT.0) THEN
   XL=0.0
END IF
IF(XL.GT.1) THEN
   XL=0.0
END IF
IF (B.LT.0.AND.FII(XL)*FII(XR).GT.0) THEN
   XL=0.0
\[ \begin{align*}
XR &= (0.5 - (-B)^{0.5})/(S\times UCC) - M)/(1-M) - 1E-6 \\
\text{END IF}
\end{align*} \]

IF (XR.GT.1.0) THEN
\[ XR = 1.0 \]
END IF

IF (XR.LT.0) THEN
\[ XR = 1.0 \]
END IF

CHECK ON B

IF (B.GT.0) THEN
CALL ZFALSE (FI, EPS, NSIG, XL, XR, XAPP, ITMAX, IER)
ELSE IF (B.LT.0) THEN
CALL ZFALSE (FI1, EPS, NSIG, XL, XR, XAPP, ITMAX, IER)
ELSE
CALL ZFALSE (FI11, EPS, NSIG, XL, XR, XAPP, ITMAX, IER)
END IF

L = XAPP
IF (IER.EQ.130) THEN
ITMAX = 2 * ITMAX
GOTO 105
ELSE IF (IER.EQ.129) THEN
WRITE (6, *) 'SIGNS OF F IN LEFT AND RIGHT ROOT ARE EQUAL'
GOTO 1010
ENDIF

ENDIF

CALCULATE THE DISPERSION SUPPRESSION

SIGMA = ALOG(1/L)
SIGMA0 = ALOG(1/S)
CALL F(SIGMA0, RESULT0, NX, NR)
CALL F(SIGMA, RESULT, NX, NR)
ALPHA = RESULT/RESULT0
MMM = MIN(M, 1/M)
RATIO = (0.68 - 0.1)/(0.68 + 0.1)
PECLET = EXP(RATIO * SIGMA0**2)/RESULT

WRITE SUPPRESSION OF DISPERSION, ALPHA, AS A FUNCTION OF G

WRITE (11, 500) S, M, G, ALPHA
WRITE (6, *) 'ALPHA = ', ALPHA
WRITE (6, *) 'PECLET = ', PECLET

WRITE (6, '(A)')' DO YOU WANT TO TRY AGAIN? [Y/N]
READ (5, '(A)') REQ
IF (REQ.EQ.'Y'.OR. REQ.EQ.'y') THEN
GOTO 5
ELSE
STOP
ENDIF
C
1010 WRITE(11,'(A)')
WRITE(6,'(A)')' DO YOU WANT TO ENTER NEW PARAMETER ? [Y/N] '
READ(5,'(A)')REQ
IF(REQ.EQ.'Y'.OR.REQ.EQ.'y')THEN
  GOTO 5
ELSE
  STOP
ENDIF
END

C
SUBROUTINE F(X,Y,NUMBERX,NUMBERR)
REAL RATIO,X,Y,Z,BLOCKRATIO
INTEGER NUMBERX,NUMBERY

BLOCKRATIO=(NUMBERR+1E-6)/(NUMBERX+1E-6)
Z=0.5*(X**2)*1/(NUMBERR+1E-6)*(1-BLOCKRATIO+1)
Y=2
Y=0.5*X**2/(1+X**2)**0.5
END

C
REAL FUNCTION FI(X)
REAL M
COMMON M,S,G,UUC,B,OMEG0

OMEGA=S*UUC*((1-M)*X+M) - 0.5

C
C =-ALOG(M) - 0.5*ALOG(OMEG0**2+B)
D = -0.5*B**(-0.5)*ATAN2(B**(0.5)*(OMEG0-OMEGA),OMEG0*OMEGA+B)
FI = 0.5*ALOG((S*UUC*((1-M)*X+M)-0.5)**2+B)
1 = C + D
END

C
REAL FUNCTION FII(X)
REAL M
COMMON M,S,G,UUC,B,OMEG0

Z=(-B)**0.5

C
OMEGA=S*UUC*((1-M)*X+M) - 0.5
C = -ALOG(M) - 0.5*ALOG(ABS(OMEG0**2+B))
2 = (0.25/Z)*ALOG(ABS(OMEG0**2+B)/(OMEG0+Z)**2)
FI = 0.5*ALOG(ABS(OMEG0**2+B))
2 = (0.25/Z)*ALOG(ABS(OMEG0**2+B)/(OMEGA+Z)**2) + C
END

C
REAL FUNCTION FIII(X)
REAL M
COMMON M,S,G,UUC,B,OMEG0
OMEGA=S*UUC*((1-M)*X+M) - 0.5
C=-ALOG(M)-ALOG(ABS(OMEG0))+0.5/OMEG0
FIII = ALOG(ABS(OMEGA))- 0.5/OMEGA + C
END
CHAPTER 3
VALIDATION OF RANDOM-WALK MODEL BY PHYSICAL AND NUMERICAL EXPERIMENTS

3.1. INTRODUCTION

3.2. COMPARISON OF CORE-FLOODING EXPERIMENTS WITH RANDOM-WALK MODEL
   3.2.1. Qualitative
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3.3. SET-UP OF SIMULATIONS
   3.3.1. Simulator
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   3.3.3. Use of simulator
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   3.3.5. Example
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3.4. COMPARISON OF SIMULATIONS WITH THEORY OF GELHAR & AXNESS
   3.4.1. Simulation results
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3.5. COMPARISON OF SIMULATIONS WITH RANDOM-WALK MODEL
   3.5.1. Simulation results
   3.5.2. Discussion

3.6. PRACTICAL CONSEQUENCE FOR NUMERICAL SIMULATION

3.7. CONCLUSIONS

REFERENCES

APPENDIX 3A: DESCRIPTION OF SIMULATOR
APPENDIX 3B: AVERAGE PERMEABILITY AND CORRELATION FUNCTION OF GRID
3.1. INTRODUCTION

The experiments of stable, miscible displacements discussed in Chapter 1 show that the mixing zone between miscible fluids travelling through a porous medium is affected by a viscosity contrast and a density contrast between the fluids. In Chapter 2 this effect has been modelled for a porous medium with heterogeneity at local scale. The assumptions behind the analytical model of Chapter 2 are that:

1. the fluid parcels undergo a random walk process, as assumed by Scheidegger\(^1\) for the case of no viscosity contrast and no density contrast,

2. the step size of the random walk can be calculated from the response of the displacement front to a single block of deviating permeability that contrasts with the effective permeability of the surrounding medium by the standard deviation.

In this chapter the random-walk model is validated. This is done by testing the consequences of the assumptions, which are respectively:

1. the mixing zone between the fluids expands with the square root of time
2. the value of the dispersivity ratio \(\alpha\), which compares the dispersivity with the effect of viscosity contrast and density contrast taken into account and the dispersivity neglecting these.

For the validation we used the core-flooding experiments of Chapter 1 and detailed numerical simulations of stable displacements through non-uniform, homogeneous media. The expansion of the mixing zone with the square root of time (the dispersive behaviour of the mixing zone) cannot be tested for the core-flooding experiments, because only the concentration profile in the effluent was measured. In this respect the numerical simulation has the advantage that the mixing zone could be monitored during its travel through the porous medium.
3.2. COMPARISON OF EXPERIMENTAL OBSERVATIONS WITH RANDOM-WALK MODEL

3.2.1. Qualitative

An inspection analysis of the basic equations of miscible displacement in Chapter 1 established the most important similarity groups:

\[ M = \frac{\mu_p}{\mu_I} \]

\[ N_G = \frac{k_0 g (\rho_p - \rho_I) \sin \theta}{\mu_p U} \]

From an evaluation of the dispersion experiments reported in literature and performed by ourselves in terms of these similarity groups, we established the three following observations (section 1.6.1):

1. increase of effective dispersivity with mobility ratio \( M \) at gravity number \( N_G = 0 \) (\( M < 1 \))
2. non-zero effective dispersivity at \( M = 0 \) and \( N_G = 0 \)
3. decrease of effective dispersivity with \( N_G \) (irrespective of \( M \)).

All three observations agree with the analytical model of Chapter 2, as can be easily checked with Figures 2.3 and 2.4. Figure 2.4, in which the model results for \( N_G = 0 \) are plotted, shows an increase of dispersivity ratio \( \alpha \) (and hence effective dispersivity) with \( M \). It also shows that a non-zero dispersivity remains at \( M = 0 \) and \( N_G = 0 \). Figure 2.3 in which the model prediction of dispersivity ratio \( \alpha \) is plotted as a function of \( N_G \) and \( M \), confirms the third observation that effective dispersivity decreases with \( N_G \), irrespective of \( M \).

3.2.2. Quantitative

The measured dispersion coefficient \( K_{\text{measured}} \), which is the sum of a diffusion term and convective dispersion \( K_C \), is to be confronted with the model prediction \( K_{\text{model}} \). The random-walk model of Chapter 2 and the (small) correction factor \( (1 + \frac{1}{6} \sigma^2)^{-2} \) of Gelhar & Axness give for \( K_C \):

\[ K_{C, \text{model}} = U \lambda \frac{\sigma^2}{(1 + \frac{1}{6} \sigma^2)^2} \alpha_{\text{model}} \]
The quantitative confrontation between random-walk model and measurements of effective dispersivity is possible only for our own measurements in the Berea core, because we have measured the permeability field of the core and have determined $\lambda$ and $\sigma$.

The permeability distribution of the Berea core is log-normal; the standard deviation $\sigma$ of the log-permeability of the Berea core was determined with the mini-permeameter at 0.18, the correlation length $\lambda$ of the permeability was determined at 2.3 cm (Appendix 1C). The dispersivity ratio $\alpha$, which is a function of $\sigma$, $M$ and $N_G$, can be calculated with the FORTRAN program listed in Appendix 2D.

In Table 3.1 the measured value $K_{\text{measured}}$ and the model prediction $K_{\text{model}}$ are listed for both the water/brine and the gas/oil displacements in the Berea core. Experiment Ba, which was used to determine $D/F_R\phi$ for the gas/oil displacements is listed in the table but not used for the confrontation between experiments and random-walk model because $K_{\text{model}}$ is equal to $K_{\text{measured}}$ by definition for this experiment. In Fig. 3.1 we have plotted $K_{\text{measured}}$ against $K_{\text{model}}$ for comparison.

3.2.3. Discussion

According to Fig. 3.1, 5 of the 7 experiments agree with the model prediction. Table 3.1 shows that these 5 experiments agree within 15% with the model prediction. Of these 5 experiments, there are 2 experiments (Aa and Bb) of which the difference can be explained by the uncertainty in the diffusion term $D/F_R\phi$. The other 3 experiments (Ac, Ad, Ae) agree within 9% with the model prediction.

The other 2 of the 7 experiments (Ab and Bc) do not agree with the model prediction. The deviation of experiment Ab from the model prediction cannot be explained by an effect of the transverse correlation length or an effect of the transverse pressure disequilibrium (both treated in section 2.5). The deviation of experiment Bc from the model prediction can be explained by an effect of the transverse correlation length (treated in section 2.5.1) that is present at small $\alpha$. Furthermore, for this experiment $\alpha$ is very sensitive to $N_G$. For example, a decrease of $N_G$ from the present value 1.9 to 1.24 results in a drastic increase of $\alpha$ from the present value 0.087 to 0.51; with $\alpha = 0.51$, $K_{\text{model}}$ would be $5.98 \times 10^{-9}$ m$^2$/s which equals the measured value. Another possibility is that the experiment was locally
TABLE 3.1: COMPARISON BETWEEN DISPERSION COEFFICIENT AS MEASURED AND AS PREDICTED BY RANDOM-WALK MODEL

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>M</th>
<th>N_G</th>
<th>$K_{measured}$ (m$^2$/s)</th>
<th>$a_{model}$</th>
<th>$K_{c, model}$ (m$^2$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water/brine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aa</td>
<td>1.21</td>
<td>1.05</td>
<td>1.17*10^{-9}</td>
<td>0.44</td>
<td>6.68*10^{-10}</td>
</tr>
<tr>
<td>Ab</td>
<td>1.21</td>
<td>0.57</td>
<td>1.22*10^{-9}</td>
<td>0.68</td>
<td>1.86*10^{-9}</td>
</tr>
<tr>
<td>Ac</td>
<td>1.21</td>
<td>0.26</td>
<td>5.72*10^{-9}</td>
<td>0.92</td>
<td>5.59*10^{-9}</td>
</tr>
<tr>
<td>Ad</td>
<td>0.83</td>
<td>0.13</td>
<td>1.32*10^{-8}</td>
<td>0.77</td>
<td>1.14*10^{-8}</td>
</tr>
<tr>
<td>Ae</td>
<td>0.83</td>
<td>0.066</td>
<td>2.56*10^{-8}</td>
<td>0.80</td>
<td>2.32*10^{-8}</td>
</tr>
<tr>
<td>gas/oil</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ba</td>
<td>22</td>
<td>12</td>
<td>2.30*10^{-9}</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>Bb</td>
<td>22</td>
<td>5.8</td>
<td>2.73*10^{-9}</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>Bc</td>
<td>22</td>
<td>1.9</td>
<td>5.98*10^{-9}</td>
<td>0.087</td>
<td>5.93*10^{-10}</td>
</tr>
</tbody>
</table>

continued

<table>
<thead>
<tr>
<th>$K_{model}$ (m$^2$/s)</th>
<th>$K_{measured} - K_{model}$ (m$^2$/s)</th>
<th>$K_{measured} - K_{model}$ / $K_{measured}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.20*10^{-9}</td>
<td>0.03*10^{-9}</td>
<td>3%</td>
</tr>
<tr>
<td>2.42*10^{-9}</td>
<td>-1.20*10^{-9}</td>
<td>-98%</td>
</tr>
<tr>
<td>6.13*10^{-9}</td>
<td>-0.42*10^{-9}</td>
<td>-7%</td>
</tr>
<tr>
<td>1.20*10^{-8}</td>
<td>0.12*10^{-8}</td>
<td>9%</td>
</tr>
<tr>
<td>2.39*10^{-8}</td>
<td>0.17*10^{-8}</td>
<td>7%</td>
</tr>
<tr>
<td>2.30*10^{-9}</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.33*10^{-9}</td>
<td>0.40*10^{-9}</td>
<td>15%</td>
</tr>
<tr>
<td>3.07*10^{-9}</td>
<td>2.91*10^{-9}</td>
<td>49%</td>
</tr>
</tbody>
</table>

$K_{model} = \frac{D}{F_R \phi} + K_{tube} + K_{c, model}$
unstable because it was conducted beyond the Dumoré critical velocity, given by (1.8b) in Chapter 1, despite the very high correlation coefficient of the match between the concentration-history and convection-dispersion equation ($R^2=0.9999$).

In summary, the experiments in the Berea core all agree qualitatively with the random-walk model. The quantitative test of the random-walk model is not fully convincing, because of the 7 experiments, 2 experiments do not agree with the model prediction.

FIG. 3.1: Comparison between dispersion coefficient predicted by model and measured
3.3. SET-UP OF SIMULATIONS

To test the dispersive behaviour of the mixing zone and the model prediction, we have carried out a series of numerical simulations of stable, miscible displacements in a wide range of mobility ratios and gravity numbers. For each simulation run, we monitored the growth of the mixing zone with time and we calculated the average value of effective dispersivity in time for comparison with the random-walk model.

In this section we discuss the set-up of simulations on miscible displacements in a non-uniform, homogeneous porous medium. We also discuss the method of evaluating the simulation results.

3.3.1. Simulator

The simulator that we used is a fast miscible flood simulator for three-dimensional incompressible flow with gravity. The simulator, which has been developed by Crump\textsuperscript{2}, solves the partial differential equations that describe the miscible flood by finite differences and uses an implicit-in-pressure and explicit-in-concentration (IMPEC) numerical scheme. For the transport between adjacent blocks harmonic weighting of the mobilities is applied. By a special choice for the numerical scheme of the hyperbolic conservation-of-injectant equation, the simulator has second-order truncation error in space and suppresses oscillations. More information about the simulator can be found in Appendix 3A.

To demonstrate the ability of the simulator to simulate sharp interfaces, we have conducted a simulation run in a grid with 80 by 80 blocks without permeability variations. Figure 3.2 shows the profile of the injectant concentration over 20 grid blocks at \( t = 0.7 \) pore volumes; the other profile is the theoretical solution for the concentration of piston-like displacement. The difference between the two profiles is one and half grid block at \( C = 0.1 \) and at \( C = 0.9 \). As a result, the dimensionless numerical dispersion \( K_{\text{numerical}}/UL \) is \( 1.2 \times 10^{-4} \). In, for example, a common numerical scheme with single-point-upstream-weighting and backward-in-time\textsuperscript{3}, this value is at least \( \frac{1}{2} \frac{Ax}{L} = \frac{1}{2} \frac{1}{80} = 6 \times 10^{-3} \), so 50 times higher (\( Ax \): grid block size in longitudinal direction).
3.3.2. Grid

To study convective dispersion, we performed simulations of a two-dimensional, linear, stable displacement of two miscible fluid banks. The grid for the runs with permeability variations was designed as follows. To a grid of 20 rectangles by 20 rectangles, 400 random permeabilities from a log-normal distribution were assigned. Each rectangle was then divided into a fine mesh of 4 by 4 blocks. The result was a fine-gridded configuration containing \((20 \times 4)^2 = 6400\) blocks, with a regular pattern of 400 rectangles of uniform permeability. There is no correlation between the permeability values of adjacent rectangles. The length/width ratio of the rectangles (and also the configuration) is 6.8, so much greater than 1, because of the confinement of the model of Chapter 2 to often occurring media in which the longitudinal correlation length of the permeability is much greater than the transversal correlation length. The grid is shown in Fig. 3.3.

The geometric standard deviation \(S\) of the permeabilities is 2.60 (corresponding to a variance \(\sigma^2 = (\ln S)^2 = 0.915\)). We tested our model in a similar grid with a smaller \(S\) value \((S = 1.57)\), corresponding to \(\sigma^2 = 0.202\). The geometric average of the permeability, denoted by \(k_a\), is
FIG. 3.3: Simulation grid to scale, consisting of 20 by 20 rectangle of uniform permeability; each rectangle is divided in 4 by 4 grid blocks

respectively 10.19 $\mu m^2$ for the grid with $S=2.60$ and 10.09 $\mu m^2$ for the grid with $S=1.57$

We determined the dispersivity of the grid by expressing the spatial correlation function of the permeability of the grid in a series of correlation functions of the type that Gelhar & Axness$^4$ used. The calculation of the correlation function of the permeability of the grid (in Appendix 3B) accounts for the fact that the simulator uses harmonic weighting of mobilities in adjacent blocks. The correlation length of the grid turns out to be roughly equal to half the length of the rectangle (2 grid blocks). The number of grid blocks is high enough for the grid boundaries to have a negligible effect on the correlation function. The full expression for the dispersivity of the grid is given in (3B.12).

The gravity number $N_G$ uses the effective permeability $k_0$. For the two-dimensional grid, the relation between $k_0$ and the geometric average permeability $k_a$ (which is equal to the arithmetic average of $\log k_a$) is given by Gelhar & Axness$^4$ as

$$k_0 = k_a \exp\left(\frac{1}{2} \sigma^2 \frac{\lambda - \lambda'}{\lambda + \lambda'}\right).$$
The average permeability that the simulator uses is not exactly equal to the average permeability \( k_a \). The small difference between the value used by the simulator, \( k^\text{grid}_a \) and \( k_a \) is caused by the harmonic weighting that the simulator applies to the mobilities in adjacent blocks to calculate the transport between the blocks. The difference between \( k^\text{grid}_a \) and \( k_a \) is calculated in Appendix 3B; the result is given by equation (3B.6). In the tables that present the simulation results, \( N_{G,a} \) is the gravity number calculated with the geometric mean permeability \( k_a \), and \( N_G \) is the gravity number calculated with the effective value \( k_0 \), including the correction for the harmonic weighting. The value of \( N_G \) must be used for evaluating of the simulation results.

3.3.3. Use of simulator

The initial and boundary conditions of the rectangular grid were chosen as follows (see Fig. 3.3): at \( t = 0 \) the oil concentration is 1.0, at the injection boundary of the simulation grid the injection velocity is constant and uniform, at the production boundary (opposite the injection boundary) the pressure is constant and uniform, at the other two boundaries no flow crosses. In the simulations these boundary conditions were set by specifying all blocks at the injection boundary to be an injector with equal and constant injection rate and all blocks at the production boundary to be a producer with an equal and constant pressure.

The time step \( \Delta t \) was set by specifying a Courant Fiedrichs Levy (CFL) number of 0.4 (\( \text{CFL} = \frac{U \Delta t}{\Delta x} \)). We found that the simulation results are not sensitive to the CFL number. A decrease of the time step by a factor of 2 (CFL from 0.4 to 0.2) gives a dispersion increase of only 3%.

It has been found that in many cases the velocity field changes slowly enough compared with the pressure equation that the pressure equation need not be solved at every time step. We specified the number of time steps to skip solving the pressure equation to be 3. In tests, values of 2 to 20 have been used successfully with almost no change in the solution.

Because in our analytical model the dispersivity ratio is dependent on \( M, N_G \) and \( S \), only these parameters were varied in the numerical simulations. For each run the oil viscosity was adapted to create the desired mobility ratio. In all runs the densities were kept the same. The gravity number was set by adapting the injection velocity. In runs conducted to study the
effect of gravity, the configuration was in a vertical position (dip angle $\theta = 90^\circ$) with the gravitational acceleration parallel to the main flow direction, while in runs with $N_g = 0$ the configuration was in a horizontal position ($\theta = 0^\circ$).

3.3.4. Method of evaluation

A special computer program was written to determine the effective dispersivity from each solvent concentration pattern created by the simulator. This makes it possible to monitor the effective dispersivity in time.

The program determines the average concentration of the solvent in an array of blocks in the transverse direction:

$$C(x_i,I) = \frac{1}{N} \sum_{j=1}^{N'} c(x_i,y_j,I), \quad (i=1,...,N) \quad (3.1)$$

This average concentration is fitted by the following equation:

$$C(x_i,I) = \frac{1}{2} \text{erfc} \left( \frac{x_i - I}{\sqrt{2} \sqrt{N_{Pe}}} \right), \quad (3.2)$$

where: $L$ : the length of the total grid,  
$x$ : coordinate in longitudinal direction,  
$I$ : injected volume in pore volumes,  
$N_{Pe}$ : Peclet number,  
$U$ : displacement velocity.

Peclet is defined by

$$N_{Pe} = \frac{UL}{K_c}. \quad (3.3)$$

This is an approximated solution to the convection-diffusion equation with boundary conditions $C(0,I) = H(I)$ and $C(x,0) = 0$. The full solution is given in, for example, Bear & Verruijt.\(^5\) Equation (3.2) is a good approximation for $N_{Pe} * I >> 1$ (Ref. 5). For example, for $N_{Pe} = 30$ and $I = 0.35$, the approximation is accurate within 1%. Hence, we use the approximated
solution. For the fitting procedure the following equation, deduced from (3.2), is used:

$$\frac{x_i}{L} = \frac{2\sqrt{I}}{\sqrt{N_{Pe}}} \text{inv} \text{erfc}(2C(x_i,I)) + I, \quad (0.05 < C(x_i,I) < 0.95). \quad (3.4)$$

This is done using all transverse gridblock arrays with an average concentration between 0.05 and 0.95. The reason for selecting this interval is discussed in section 3.4.1.

In addition to $N_{Pe}$, I is also determined by fitting, although I is a given quantity. The reason for this is that the material balance is generally not exactly satisfied owing to fitting in a limited C interval. The fitted value of I never differed more than half a grid block from the given value. The small correction to I is of importance because $N_{Pe}$ is very sensitive to the value of I.

In the runs $N_{Pe}$ was determined from concentration patterns that are output every 0.01 pore volume injected. The $N_{Pe}$ values at I=0.02, I=0.03, I=0.04, etc., were plotted as a function of I to show the dispersive or non-dispersive behaviour of the displacement front. A dispersive displacement front is characterised by a constant course of $N_{Pe}$ with I. For each run an average value, $N_{Pe,sim}$ was calculated from the series of $N_{Pe}$ values and used for comparison with values in literature or our random-walk model. The initial part of $N_{Pe}$ vs. I was left out of consideration in the calculation of the average $N_{Pe}$, because it is dominated by entry effects and is not accurate.

The initial part was defined as that part of the $N_{Pe}$ vs. I curve below I = 0.35. This reflects the fact that equation (3.2) is inaccurate for small values of I and that the displacement front has travelled too few grid blocks to be dispersive. Part of the initial part is caused by the injector grid blocks: the flow to each of these blocks is not affected by the permeability of the blocks. In addition, the flow from the injector grid blocks to neighbouring grid blocks is partly affected by the forced uniform injection. The mixing zone is thus too short at small I (corresponding to too high a Peclet number).
3.3.5. Example

An example of a concentration pattern generated by the simulator appears in Fig. 3.4a, in which the concentration of the injectant is shown. The concentration averaged over the transverse cross-section is plotted in Fig. 3.4b and shows a reverse-S-shaped profile. The plot of $N_{Pe}$ determined from a series of concentration patterns of this run versus cumulative injection I is shown in Fig. 3.4c. The figure indicates that the value of $N_{Pe}$ does not fluctuate very much with I (apart from a initial part below about I=0.35). For such a curve (without the initial part) an average value of $N_{Pe}$ yielding a small standard deviation can be calculated.

The effect of the injector-grid blocks on the mixing zone length (too short a length at small I) is shown in Fig. 3.4d. This figure shows two curves for $N_{Pe}$ versus I: one is a repeat of Fig. 3.4c, the other is the result of a simulation in which the injector grid blocks were given a practically infinite permeability. A result of the infinite permeability is that the effect of the forced uniform injection is diminished. This is demonstrated by the figure: the Peclet number of the simulation with infinite permeability in the injector grid blocks is smaller in the initial part.

3.3.6. Simulation runs

To select the simulation configuration, we have conducted two series of simulation runs in which the division of a rectangle was varied. Division of a rectangle is desired to diminish the role of numerical dispersion. Furthermore, we compared the Peclet number determined from the simulations with the theory of Gelhar & Axness. We tested a configuration in which a rectangle was not divided, a configuration in which a rectangle was divided in a mesh of 2 by 2 blocks; then a division in 4 by 4 blocks and finally in 8 by 8 blocks. The spatial permeability function was the same for all these runs, the S value was 2.60. This exercise was repeated in a series of runs with another spatial permeability function and an S value of 1.57.

To check the sensitivity of the dispersivity to the ratio of the block dimensions, we carried out a few runs with a length/width two and four times that one of the reference run.
The effect of fluid properties and gravitation on the effective dispersivity were now investigated by carrying out various simulation runs in the same grid. In the first run $M$ was 1 and $N_a$ was 0; this run was taken as a reference. In the other runs in the same grid, $M$ different from 1 was selected and/or $N_g$ different from 0 (by changing the solvent viscosity and/or changing the dip angle from 0° to 90° and selecting an appropriate displacement velocity). For each run the ratio of the average effective dispersivity to the average dispersivity of the reference run was determined and compared with our model.

3.4. COMPARISON OF SIMULATIONS WITH THEORY OF GELHAR & AXNESS

3.4.1. Simulation results

Concentration patterns of the example discussed in section 3.3 are shown at every 0.05 pore volume injected in Fig. 3.5. In this run the division of a rectangle of constant permeability is 4 by 4. The series of concentration patterns displays the displacement of the mixing zone between oil and solvent from left to right and shows the development of the mixing zone into a wider zone. Besides a diffuse front in the longitudinal direction, all patterns show a ragged front in the transverse direction, which means that the front has some finger-like disturbances. For example, at time $I = 0.15$, there is a large finger-like disturbance in the middle. At $I = 0.25$ another long finger-like disturbance has developed between the middle and bottom of the figure, while the middle finger has split into two branches. The branches grow and at $I = 0.35$ they have merged into quite a thick bump. At $I = 0.60$ this bump has merged with the finger-like disturbance between the middle and bottom. One pattern earlier, at $I = 0.55$, this finger-like disturbance splits at its tip into several branches, of which two merge at $I = 0.75$. Breakthrough occurs at $I = 0.70$; at $I = 0.80$ nearly half the production boundary produces solvent; at $I = 0.90$ almost the complete production boundary produces solvent. It is remarkable that an isolated pocket of oil is left behind the mixing zone.

From these patterns one can conclude that although the front disturbances look finger-like, they do not exist a long time. Their length remains small, they split and they merge with each other. Their development
FIG. 3.4a: Example of a concentration pattern (Run 48 with $S=2.60$ $M=1$ $N_G=0$ $m=4$)
**Chapter 3**

**FIG. 3.4b:** Concentration profile at t=0.7 of run 48 with S=2.60 M=1 N_G=0 N=20 m=4

**FIG. 3.4c:** Peclet number as a function of cumulative injection of run 48 with S=2.60 M=1 N_G=0 N=20 m=4
FIG. 3.4d: Effect of injector grid blocks on transient of run 48 with $S=2.60$, $M=1$, $N_G=0$, $m=4$

... in time seems to be statistical. The result is a smooth change of the concentration $C$ in the longitudinal direction.

This can also be seen in the concentration profile at $I = 0.7$, shown in Fig. 3.4b. As a result of the finite number of random permeabilities, we see fluctuations in the profile. The figure also shows a matched concentration profile calculated with $N_{Pe} = 91.1$ and $I = 0.706$. The difference between the two profiles is at maximum three grid blocks. The correlation coefficient $R^2$ of this match is 0.993.

Although expression (3.2) (and also the full expression in Ref. 5) predicts breakthrough immediately after the start of injection, breakthrough in the simulation occurs only after about 0.7 pore volumes have been injected. We see in the simulation that an isolated pocket of oil is left behind. In Fig. 3.4b this causes a bump in the $C$ profile behind the...
FIG. 3.5a: Development of dispersive mixing zone without effect of fluid properties
(Run 48 with S=2.60  M=1  N₃ =0  m=4)
FIG. 3.5b: Development of dispersive mixing zone without effect of fluid properties
(Run 48 with S=2.60  M=1  N=0  m=4)
FIG. 3.5c: Development of dispersive mixing zone without effect of fluid properties
(Run 48 with $S=2.60$ $M=1$ $N_G=0$ $m=4$)
mixing zone. Both phenomena are caused by the limited number of random permeabilities we have used in the simulation (400), whereas expression (3.2) is based on the assumption of an infinite number of random permeabilities. However, the simulation result may be closer to reality, since the flow domain is never infinite. For this reason we have chosen to match the simulation result only in a limited C-range between $C = 0.05$ and $C = 0.95$ and not between $C = 0$ and $C = 1$.

The full results of the series with $S = 2.60$ are shown in Fig. 3.6, in which $N_{Pe}$ is plotted as a function of $I$. The figure shows four curves representing the runs with no division of a rectangle of uniform permeability, division in a 2 by 2 mesh, a 4 by 4 mesh and an 8 by 8 mesh. The figure also shows horizontal lines, each of which represents the $N_{Pe}$ value predicted by the theory of Gelhar & Axness and adapted for the correlation function of the simulation configuration (expression (3B.12)). A curve and a horizontal line of equal thickness should ideally coincide.

![Graph depicting Peclet number vs. pore volume injected](image)

**FIG. 3.6:** Comparison of Peclet number to theory of Gelhar & Axness for $S=2.60$, parameter: division of square in m°m grid blocks (horizontal lines: Gelhar & Axness; curved lines: simulation)
The figure shows that the curves do not coincide with their theoretical line in the initial part of the curve (between \( I = 0 \) and \( I = 0.35 \)). In this part of the curve we do not see a dispersive behaviour of the mixing zone as predicted by Gelhar & Axness, because the curve does not coincide with the theoretical line but has overshoot. Beyond the initial part (\( I \) larger than 0.35) we see a more or less constant course of \( N_{Pe} \) with \( I \) for all curves. Hence the mixing zone is dispersive in this part of the curve. The best relative correspondence between the curves and the theoretical lines is found for the configuration with the finest division, the worst relative correspondence is for the configuration with no division. In Table 3.2 the average \( N_{Pe} \) value, \( N_{Pe,sim} \), of each run is compared with the value predicted by Gelhar & Axness. The table shows that the finer the division, the better the comparison. The run with an 8 by 8 division of a rectangle nearly coincides with the theory of Gelhar & Axness within the error bounds of the simulation. Figure 3.7 and Table 3.3 show similar results for a series of runs with \( S = 1.57 \). For this series the 4 by 4 division and the 8 by 8 division agree with the theory of Gelhar & Axness within the error bounds of the simulation.

**Table 3.2: Comparison between average Peclet number of simulation runs with \( S = 2.60 \) and \( M = 20 \) and \( \lambda/\lambda' = 6.8 \) and theory of Gelhar & Axness (\( M = 1, N_G = 0 \)) for various divisions (Run 48 is selected as a reference run.)**

<table>
<thead>
<tr>
<th>run</th>
<th>m</th>
<th>( N_{Pe,th} )</th>
<th>( N_{Pe,sim} )</th>
<th>( \Delta N_{Pe}/N_{Pe} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>74</td>
<td>1</td>
<td>77.5</td>
<td>59.2</td>
<td>±4.7 31%</td>
</tr>
<tr>
<td>75</td>
<td>2</td>
<td>111</td>
<td>85.3</td>
<td>±8.7 30%</td>
</tr>
<tr>
<td>48</td>
<td>4</td>
<td>105</td>
<td>89.3</td>
<td>±4.8 18%</td>
</tr>
<tr>
<td>76</td>
<td>8</td>
<td>96.8</td>
<td>85.4</td>
<td>±8.9 13%</td>
</tr>
<tr>
<td>theory</td>
<td></td>
<td>86.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In all runs of this and next tables: \( W = 0.10 \) m, \( \phi = 0.35 \), \( L = \lambda/\lambda' W \), \( k = 10 \) mm², \( \rho_I = 620 \) kg/m³, \( \rho_p = 854 \) kg/m³, \( \mu_I = \mu_p/M \), \( \mu_p = 4.5 \) mPa.s, \( \theta = 0 \) in runs with \( N_G = 0 \) and \( \theta = 90^\circ \) in runs with \( N_G > 0 \).
TABLE 3.3: COMPARISON BETWEEN AVERAGE PECLET NUMBER OF SIMULATION RUNS WITH $S = 1.57$ AND $N = 20$ AND $\lambda/\lambda' = 6.8$ AND THEORY OF GELHAR & AXNESS ($M = 1$, $N_G = 0$) FOR VARIOUS DIVISIONS. (Run 32 is selected as a reference run.)

<table>
<thead>
<tr>
<th>run</th>
<th>m</th>
<th>Pe$_{\text{theory}}$</th>
<th>Pe$_{\text{sim}}$</th>
<th>$\Delta$N$<em>{\text{Pe/N}</em>{\text{Pe}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1</td>
<td>223</td>
<td>124 $\pm$ 7</td>
<td>80%</td>
</tr>
<tr>
<td>31</td>
<td>2</td>
<td>302</td>
<td>217 $\pm$ 11</td>
<td>39%</td>
</tr>
<tr>
<td>32</td>
<td>4</td>
<td>280</td>
<td>269 $\pm$ 20</td>
<td>5%</td>
</tr>
<tr>
<td>79</td>
<td>8</td>
<td>257</td>
<td>284 $\pm$ 32</td>
<td>-9%</td>
</tr>
<tr>
<td>theory</td>
<td></td>
<td>229</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIG. 3.7: Comparison of Peclet number to theory of Gelhar & Axness for $S=1.57$, parameter: division of square in m$^2$m grid blocks (horizontal lines: Gelhar & Axness; curved lines: simulation)
Table 3.4 shows the average Peclet numbers of a series of runs in which the ratio of the longitudinal correlation length to the transverse correlation length, \( \frac{\lambda}{\lambda'} \), was varied. To maintain the spatial permeability distribution, the length/width ratio of the whole configuration was in fact changed. According to the discussion of section 2.5, the Peclet number should not change if the condition \( \frac{\lambda}{\lambda'} >> 1 \) is satisfied. From Table 3.4 we conclude that this is indeed the case: all three values coincide within the error bounds of the average Peclet value.

**TABLE 3.4: EFFECT OF TRANSVERSE CORRELATION LENGTH ON AVERAGE PECLET NUMBER**

\( (S = 2.60, N = 20, m = 4, M = 1, N_G = 0) \). (Run 48 is selected as a reference run.)

<table>
<thead>
<tr>
<th>run</th>
<th>( \frac{\lambda}{\lambda'} )</th>
<th>( N_{Pe,sim} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>6.8</td>
<td>89.3 ± 4.8</td>
</tr>
<tr>
<td>92</td>
<td>13.6</td>
<td>81.5 ± 8.8</td>
</tr>
<tr>
<td>93</td>
<td>27.2</td>
<td>82.4 ±13.2</td>
</tr>
</tbody>
</table>

3.4.2. **Discussion**

From these observations one would prefer the configuration with an 8 by 8 division and consider the configuration with a 4 by 4 division as a good second candidate, while the configurations with a 2 by 2 division and the with no subdivision are not appropriate. Computer time forced us to the 4 by 4 division, which has correspondence with the theory of Gelhar & Axness within 18% for \( S = 2.60 \) and within 5% for \( S = 1.57 \). The two runs with the 4 by 4 division are thus taken as reference runs for the comparison of our model with the simulation runs with \( M \) different from 1 and/or \( N_G \) different from 0.

The length/width ratio \( \lambda/\lambda' \) was chosen at 6.8. Although we could have chosen a higher value for \( \lambda/\lambda' \), we chose this low value to test our model at the edge of the \( \lambda/\lambda' \) range in which the model is valid.
3.5. COMPARISON OF SIMULATIONS WITH RANDOM-WALK MODEL

3.5.1. Simulation results

In addition to 2 reference runs, we have conducted 14 simulation runs in a wide range of $M$ and $N_G$ values ($M$ between 0.01 and 10; $N_G$ between 0 and 10). The concentration patterns of, for example, the run with $S = 2.60$, $M = 0.2$, $N_G = 0$, are shown at every 0.05 pore volume injection in Fig. 3.8. The concentration patterns look similar to those of Fig. 3.5. The finger-like disturbances are located on the same places as in Fig. 3.5. The difference with the concentration patterns of Fig. 3.5 is that the ones of Fig. 3.8 are less ragged and that behind the mixing zone less oil is left behind (see, for example, at $I = 0.20$). The finger-like disturbances are less constricted at their base (compare Fig. 3.8 and Fig. 3.5 at, for example $I = 0.30$). Breakthrough occurs later (at $I = 0.75$) and at $I = 0.90$ a significant part of the production boundary does not produce solvent yet. As in Fig. 3.5, a little pocket of oil is left behind the mixing zone but this pocket is smaller and less oil-saturated.

The conclusion is that the mobility ratio of 0.2 has compressed the mixing zone, as seen both by a less ragged front in the transverse direction and a sharper front in the longitudinal direction. This can also be seen in the C-profile at $I = 0.7$, shown in Fig. 3.9 if we compare it with the C-profile at $M = 1$ in Fig. 3.4b.

Figures 3.10 and 3.11 show the course of the Peclet number determined from all simulations with the cumulative injection $I$. The most important feature in these figures is the constant course of $N_{Pe}$ with $I$, apart from a initial part between $I=0$ and about 0.35. The constant course of $N_{Pe}$ with $I$ shows that the mixing zone averaged over the transverse cross-section can be characterised by dispersive behaviour as we have assumed in our model. In practice this means that the mixing zone grows with the square root of time.

Although the course of $N_{Pe}$ with $I$ is constant beyond $I$ of about 0.35, we also see fluctuations. Table 3.5 (on page 131) shows that the fluctuations of the $N_{Pe}$ values after $I=0.35$ have a standard deviation of 10% or smaller, as is the case for the reference runs. Upon close examination of the curves in Figs. 3.10 and 3.11 one can see that the fluctuations are the superposition of fluctuations with two different periods: a small-amplitude
fluctuation with a period of about 1/20 pore volume and a larger-amplitude fluctuation with a much larger period. The fluctuation with a period of about 1/20 pore volume are due to an effect of the rectangles: the length of a rectangle is equal to 1/20 pore volume. The fluctuation with the much larger period is associated with the local permeability values: comparison of the curves in Fig. 3.10 shows that the characteristic behaviour of this fluctuation is evident back in almost every simulation; for example, $N_{Pe}$ has a local maximum at $I = 0.6$ in most curves of Fig. 3.10.

The dispersive behaviour of the mixing zone is further illustrated by a correlation coefficient $R^2$ of typical value 0.995 (and a minimum value 0.990) that we found between the inverse error function (3.4) and the concentration averaged over the transverse cross-section. A typical number of fitted C-values, which are the result of averaging in the transverse direction, is 20.

The dispersivity ratio $\alpha$ can be calculated from

$$\alpha_{\text{sim}} = \frac{N_{Pe,\text{ref}}}{N_{Pe,\text{sim}}},$$

where $N_{Pe,\text{ref}}$ is the average Peclet value of the reference run in the same grid with $M = 1$ and $N_G = 0$. In Table 3.5 we have listed the calculated $\alpha_{\text{sim}}$ values together with the $\alpha$ values predicted by our random-walk model ($\alpha_{\text{model}}$). In the table one can also find the relative difference $\Delta \alpha/\alpha$, calculated as $(\alpha_{\text{sim}} - \alpha_{\text{model}})/\alpha_{\text{sim}}$. The runs are listed in order of increasing value of $|\Delta \alpha/\alpha|$. The table shows that of the 14 runs, 7 agree with the model prediction within 2%. Another 3 runs agree with the model prediction within 5 to 9%. For all these 10 runs our model predicts a $\alpha$ value that agrees with $\alpha_{\text{sim}}$ within the error bounds of $\alpha_{\text{sim}}$. The remaining 4 runs differ by 24 to 88% from the model prediction.
FIG. 3.8a: Development of dispersive mixing zone with suppressing effect of fluid viscosities (Run 42 with S=2.60  M=0.2  N_G=0  m=4)
FIG. 3.8b: Development of dispersive mixing zone with suppressing effect of fluid viscosities (Run 42 with $S=2.60$ $M=0.2$ $N_G=0$ $m=4$)
FIG. 3.8c: Development of dispersive mixing zone with suppressing effect of fluid viscosities (Run 42 with $S=2.60 \ M=0.2 \ N_G=0 \ m=4$)
FIG. 3.9: Concentration profile at \( l = 0.7 \) of run 42 with \( S = 2.60 \) \( M = 0.2 \) \( N = 0 \) \( N = 20 \) \( m = 4 \).
FIG. 3.10a: Peclet number versus pore volume injected for simulation runs with $S=2.60$
FIG. 3.10b: Peclet number versus pore volume injected for simulation runs with $S=2.60$
FIG. 3.10c: Peclet number versus pore volume injected for simulation runs with S=2.60
FIG. 3.11: Peclet number versus pore volume injected for simulation runs with $S=1.57$
Table 3.5: Survey of Simulation Results with M Not Equal to 1 and/or \(N_G\) Not Equal to 0 (\(N = 20, \lambda/\lambda' = 6.8\))

<table>
<thead>
<tr>
<th>run</th>
<th>S</th>
<th>M</th>
<th>(N_{G,a})</th>
<th>(N_G)</th>
<th>(N_{Pe,sim})</th>
<th>(a_{sim})</th>
<th>(a_{model})</th>
<th>(\Delta a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>2.60</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>228 + 22</td>
<td>0.392 + 11%</td>
<td>0.390</td>
<td>0.5%</td>
</tr>
<tr>
<td>66</td>
<td>2.60</td>
<td>1</td>
<td>0.1</td>
<td>0.135</td>
<td>98.3 + 5.8</td>
<td>0.909 + 11%</td>
<td>0.917</td>
<td>-0.9%</td>
</tr>
<tr>
<td>60</td>
<td>2.60</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>285 + 26</td>
<td>0.313 + 11%</td>
<td>0.316</td>
<td>-1.0%</td>
</tr>
<tr>
<td>64</td>
<td>2.60</td>
<td>2</td>
<td>0.6</td>
<td>0.805</td>
<td>110 + 6.8</td>
<td>0.812 + 8%</td>
<td>0.801</td>
<td>1.4%</td>
</tr>
<tr>
<td>41</td>
<td>2.60</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>345 + 20</td>
<td>0.259 + 8%</td>
<td>0.255</td>
<td>1.5%</td>
</tr>
<tr>
<td>63</td>
<td>2.60</td>
<td>10</td>
<td>1</td>
<td>1.347</td>
<td>132 + 15</td>
<td>0.677 + 13%</td>
<td>0.688</td>
<td>-1.6%</td>
</tr>
<tr>
<td>58</td>
<td>1.57</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>853 + 47</td>
<td>0.315 + 9%</td>
<td>0.309</td>
<td>1.9%</td>
</tr>
<tr>
<td>81</td>
<td>1.57</td>
<td>10</td>
<td>1.3</td>
<td>1.384</td>
<td>566 + 23</td>
<td>0.475 + 8%</td>
<td>0.499</td>
<td>5%</td>
</tr>
<tr>
<td>67</td>
<td>2.60</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>152 + 12</td>
<td>0.588 + 10%</td>
<td>0.631</td>
<td>-7%</td>
</tr>
<tr>
<td>43</td>
<td>2.60</td>
<td>1</td>
<td>1.05</td>
<td>1.409</td>
<td>180 + 18</td>
<td>0.496 + 11%</td>
<td>0.453</td>
<td>9%</td>
</tr>
<tr>
<td>61</td>
<td>2.60</td>
<td>0.1</td>
<td>10</td>
<td>13.41</td>
<td>495 + 48</td>
<td>0.180 + 12%</td>
<td>0.134</td>
<td>25%</td>
</tr>
<tr>
<td>45</td>
<td>2.60</td>
<td>10</td>
<td>2</td>
<td>2.683</td>
<td>279 + 42</td>
<td>0.320 + 16%</td>
<td>0.235</td>
<td>27%</td>
</tr>
<tr>
<td>44</td>
<td>2.60</td>
<td>2</td>
<td>4.2</td>
<td>5.634</td>
<td>485 + 30</td>
<td>0.184 + 8%</td>
<td>0.060</td>
<td>67%</td>
</tr>
<tr>
<td>62</td>
<td>2.60</td>
<td>10</td>
<td>4</td>
<td>5.365</td>
<td>422 + 18</td>
<td>0.212 + 7%</td>
<td>0.026</td>
<td>88%</td>
</tr>
</tbody>
</table>

3.5.2. Discussion

Although these 4 runs do not quantitatively agree with the model prediction, they still agree with it qualitatively. For example, the \(a_{sim}\) of run 44 (\(M=2, N_G=5.634\)) is smaller than the \(a_{sim}\) of run 64 (same \(M\), smaller \(N_G\)), a trend that agrees with that predicted by the model. Another example is the \(a_{sim}\) of run 62 (\(M=10, N_G=5.365\)), which is smaller than the \(a_{sim}\) of run 63 (same \(M\), smaller \(N_G\)), and is in agreement with the trend predicted by the model.

In Fig. 3.12 the \(a\) values predicted by the model and those determined from the simulation runs are plotted. In this figure also the line \(a_{sim} = a_{model}\) is also drawn. We see in the figure that all but two data points are close to this line, illustrating the good quantitative agreement for these
data points. At small $\alpha_{\text{model}}$ (below 0.15), the data points are not close to the ideal line. We see that the $\alpha_{\text{sim}}$ value stabilises at a more or less constant value of 0.20, while the $\alpha_{\text{model}}$ value goes to zero. A possible explanation for the deviation between the two $\alpha$'s at small $\alpha_{\text{model}}$ is the transverse flow effect that is discussed in section 2.5.2. In this section it is shown that the disequilibrium in pressure in the transverse direction as predicted by the model causes the displacement front to bend at the lateral boundaries of the heterogeneity. This phenomenon occurs particularly at small $\alpha_{\text{model}}$ but is not included in the model. Part of the explanation for the deviation between the two $\alpha$'s at small $\alpha_{\text{model}}$ is a contribution of the numerical dispersion. The numerical dispersion found in the test run treated in section 3.2.1 corresponds to an $\alpha$ value of 0.07, so this is the minimum $\alpha$ value in the simulations, irrespective of $\alpha_{\text{model}}$. 

FIG. 3.12: Comparison between dispersivity ratio predicted by model and dispersivity ratio determined from 14 simulation runs
3.6. PRACTICAL CONSEQUENCE FOR NUMERICAL SIMULATION

The detailed simulations of miscible displacements, treated in the previous sections, have shown a ragged displacement front. The computational effort required for this detailed representation of the fluid distribution is considerable. In the examples of the previous sections, 6400 grid blocks were needed to calculate the effect of a statistically homogeneous porous medium with a log-normal permeability distribution. From the simulations we conclude that this number of 6400 grid blocks can be reduced, because the mixing zone between the fluid banks appeared to be dispersive in a wide range of mobility ratios and gravity numbers. As a consequence, the same production performance can be obtained in this example by replacing the two-dimensional, non-uniform, homogeneous simulation model that has 80 by 80 grid blocks by a one-dimensional, uniform simulation model that consists of only 1 by say 80 grid blocks and by adding a dispersion term to the conservation-of-injectant equation (3A.4). This can be done if the numerical dispersion is much smaller than the added dispersion term. The dispersion coefficient $K$ of the dispersion term $K \frac{\delta^2 C}{\delta x^2}$ should be calculated with the random-walk model of Chapter 2.

Alternatively, one can use a one-dimensional, uniform simulation model with coarse grid blocks and employ the numerical dispersion to represent the dispersion term. The number of (coarse) grid blocks in the uniform model should then be selected such that the numerical dispersion equals the dispersion calculated with the model of Chapter 2. For example, in a numerical scheme with single-point-upstream-weighting and forward-in-time, the numerical dispersion at $M = 1$ and $N_G = 0$ is given by $\frac{1}{2} U (\Delta x - U \Delta t)$. A first estimate of the ideal grid block size in the uniform model with coarse grid blocks can be obtained by equating this expression to the expression for the macroscopic dispersion at $M = 1$ and $N_G = 0$, which is $U \lambda \sigma^2$:

$$\Delta x = 2 \lambda \sigma^2 + U \Delta t$$  \hspace{1cm} (3.6)

A better estimate of the grid block size $\Delta x$ can then be obtained by determining the numerical dispersion in a simulation run with the right values of $M$ and $N_G$ and with the above estimate of $\Delta x$ and by comparing the value of the numerical dispersion to the macroscopic dispersion, which is $U \lambda \sigma^2 a(M, N_G, \sigma)$ after which $\Delta x$ can be adjusted.
3.7. CONCLUSIONS

1. The three qualitative observations, which summarise all measurements of effective dispersivity treated in Chapter 1, agree with the random-walk model. Of the 7 experiments in the Berea core, 5 agree quantitatively with the model prediction.

2. All simulation runs confirm the model assumption that the mixing zone, averaged over the transverse cross-section, is dispersive. This means that the mixing zone grows proportionally to the square root of time.

3. The model prediction of the dispersivity agrees with the simulation result within the error bounds of the simulations for 10 of the 14 runs.

4. The difference found between the simulation result and the model for the other 4 runs occurs at small \( \alpha \). The difference is attributed to a transverse flow effect, which is not incorporated in the random-walk model, and to the numerical dispersion of the simulations.

REFERENCES

APPENDIX 3A

DESCRIPTION OF SIMULATOR (Ref. 2)

We will first describe the implementation of physical properties and equations in the simulator and then the numerical scheme. Simulations described in this work were performed using one fluid phase (oleic) and two components (oil and solvent soluble in oil). The density and the viscosity of the oil/solvent mixture are computed with a linear mixing rule for the density and the quarter-power blending rule for the viscosity:

\[ \rho(c) = \rho_I c + \rho_P (1-c), \]

\[ \frac{1}{\mu(c)} = \left( \frac{c}{0.25} + \frac{1-c}{0.25} \right)^4. \]

The mobility \( k(r)/\mu(c)\phi \) is abbreviated as \( \lambda \). The transport between adjacent blocks is computed using harmonic weighted mobilities \( \lambda_j \) and \( \lambda_{j+1} \):

\[ \lambda_{j+\frac{1}{2}} = \frac{2}{\frac{1}{\lambda_j} + \frac{1}{\lambda_{j+1}}} . \]

Gravity dip is implemented, as is permeability heterogeneity. The latter is used both to represent the dispersion observed in the production profile of stable displacements and to generate viscous fingers in the unstable displacement of oil by solvent. The permeability variation is introduced in the grid by means of a random generator. The user has to specify a variance and a seed value.

The physical equations for miscible flow solved by the simulator are

\[ \frac{\partial c}{\partial t} + \nabla \cdot (c \vec{U}) = \kappa \frac{\partial^2 c}{\partial x^2} + \kappa' \left( \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \right) + q(r,t), \]

\[ \vec{U} = - \frac{k(r)}{\mu(c)\phi} [\vec{v}_p + \rho(c)g \vec{v}_z], \]

\[ \nabla \cdot \left( \frac{k(r)}{\mu(c)\phi} [\vec{v}_p + \rho(c)g \vec{v}_z] \right) = - q(r,t), \]
where \( z \) is the place coordinate in the vertical direction and \( q \) is the influx or efflux due to a well. The microscopic dispersion coefficients \( \kappa \) and \( \kappa' \) were set to zero.

In the numerical procedure the implicit-in-pressure and explicit-in-concentration (IMPEC) scheme is used. The pressure equation (3A.6) is solved first. The term \( \frac{\partial}{\partial x}(\lambda \frac{\partial p}{\partial x}) \), for example, is discretised at \( \bar{r} = (x_i, y_j, z_k) \) and \( t = t^n \) as

\[
\frac{1}{(\Delta x)^2} \left[ \lambda_{i+\frac{1}{2},jk} \left( p_{i+1,jk}^{n+1} - p_{ijk}^{n+1} \right) - \lambda_{i-\frac{1}{2},jk} \left( p_{ijk}^{n+1} - p_{i-1,jk}^{n+1} \right) \right].
\] (3A.7)

The resulting set of equations in \( p_{ijk}^{n+1} \) (for each grid block, one equation) is solved using a method treated in Ref. 7.

The velocity is then calculated from equation (3A.5), in the \( x \)-direction, for example, discretised as

\[
U_{x,i+\frac{1}{2},jk}^{n+1} = \lambda_{i+\frac{1}{2},jk} \frac{p_{i+1,jk}^{n+1} - p_{ijk}^{n+1}}{\Delta x}.
\] (3A.8)

The truncation error is of order \((\Delta x)^2\).

The last step is the calculation of the concentration from equation (3A.4). For this purpose a variant of the total-variation-diminishing (TVD) scheme \(^{8,9,12}\) is used for the discretisation of the convection term. This scheme is known to suppress oscillations and numerical dispersion. It is recommended for the discretisation of a hyperbolic conservation equation with a shock. The scheme uses either a two-point upstream difference or a central difference, depending on the relative magnitude of the upstream difference and the downstream difference. It has a truncation error of order \((\Delta x)^2\).
APPENDIX 3B

AVERAGE PERMEABILITY AND CORRELATION FUNCTION OF GRID

The simulator uses harmonic weighting for the mobility. If the fluid properties are equal, this means that the transport between block \( i \) and adjacent block \( i+1 \), with respectively permeability \( k_i \) and \( k_{i+1} \), is determined by a permeability (from (3A.3))

\[
K_{i+1} = \frac{2}{k_i + k_{i+1}}.
\]  
(3B.1)

If the fluid properties are not equal, this equation is a good approximation because the variation in the viscosity is on average much smaller than the variation in the permeability. For the calculation of the correlation function we therefore take (3B.1).

A consequence of this harmonic weighting is that the average value of the permeability is different from the geometric average chosen \( k_a \). This can be shown as follows.

The average value of \( \ln k \) in the main flow direction is defined as

\[
\ln k_{\text{grid}} = \frac{\int \ln k(\xi) \, d\xi}{\int d\xi},
\]  
(3B.2)

where \( \xi \) is the space coordinate in the longitudinal direction. This integral can be split into parts over which \( \ln k(\xi) \) is constant:

\[
E(\ln k_i) \frac{\Delta x}{\xi} + E(\ln K_{i+1}^\dagger) \left|_{k_i \neq k_{i+1}} \frac{\Delta x}{\xi} \right.,
\]  
(3B.3)

where \( E(x_i) \) is defined as \( \frac{1}{N} \sum_{i=1}^{N} x_i \), \( \Delta x \) is the block size and \( \xi \) is the rectangle size (both in the longitudinal direction). In the following we use the abbreviation \( m = \frac{\xi}{\Delta x} \), which is 4 in most of our simulations. \( E(\ln k_i) \) is just \( \ln k_a \). The expectation of \( \ln K_{i+1}^\dagger \left|_{k_i \neq k_{i+1}} \right. \) can be calculated as follows:
\[ E(\ln \frac{1}{k_{i+1}}) = -E(\ln \frac{1}{2} (k_{i+1} \div k_i) + \frac{k}{k_i} \div k_{i+1}), \]

\[ = -E(\frac{1}{2} (\ln \frac{k_i}{k_{i+1}} + \ln \frac{k_{i+1}}{k_i}) + \frac{1}{8} (\ln \frac{k_i}{k_{i+1}} - \ln \frac{k_{i+1}}{k_i})^2), \]

\[ = \frac{1}{2} E(\ln \frac{k_i}{k_{i+1}}) + \frac{1}{2} E(\ln \frac{k_i}{k_{i+1}}) \]

\[- \frac{1}{8} E((\ln \frac{k_i}{k_{i+1}})^2) + \frac{1}{4} E(\ln \frac{k_i}{k_{i+1}}) E(\ln \frac{k_i}{k_{i+1}}) - \frac{1}{8} E((\ln \frac{k_i}{k_{i+1}})^2), \]

\[ = \text{if } k_i \neq k_{i+1}: \quad 0 + 0 - \frac{1}{8} \sigma^2 + 0 - \frac{1}{8} \sigma^2 = -\frac{1}{4} \sigma^2, \]

\[ \text{if } k_i = k_{i+1}: \quad 0 + 0 - \frac{1}{8} \sigma^2 + \frac{1}{4} \sigma^2 - \frac{1}{8} \sigma^2 = 0. \] (3B.5)

Use has been made in the above calculation of the following mathematical rules, approximation and definitions.

* \[ E(A+B) = E(A) + E(B), \]

* \[ E(A \cdot B) = E(A) \cdot E(B) \text{ if } A \text{ and } B \text{ are uncorrelated,} \]

* \[ \ln \frac{A+B}{2} = \frac{\ln A + \ln B}{2} + \frac{(\ln A - \ln B)^2}{8} \]

* + \[ \frac{1}{16} 0((A-1)^p(B-1)^q) \text{ with } p+q = 4. \]

* Term \[ \frac{1}{16} 0((A-1)^p(B-1)^q) \text{ is ignored, which is at maximum 2% of } \ln \frac{A+B}{2} \text{ for } A, B \leq 5 \text{ and 6% for } A, B \leq 10. \]

* \[ E(\ln \frac{k_i}{k_{i+1}}) = 0, \]

* \[ E(\ln \frac{k_i}{k_{i+1}})^2 = \sigma^2. \]

The result of the above exercise is
\[ \ln \frac{k_{\text{grid}}}{k_a} = -\frac{1}{4} \frac{\sigma^2}{m}. \] (3B.6)

Hence by the procedure of harmonic weighting the average value, \( \ln k_{\text{grid}} \)' is an amount \( \frac{1}{4} \frac{\sigma^2}{m} \) smaller than the value \( \ln k_a \). This amount tends to zero when the number of grid blocks per rectangle, \( m \), becomes large. The inaccuracy in expression (3B.6) is an amount \( \frac{1}{16} \frac{\sigma^4}{m} \), which is only 0.016 for the \( \sigma \)-range \([0,1]\) and \( m=4 \).

We will now carry out a similar calculation for the correlation function defined by

\[ R(x) = \frac{\int \ln \frac{k(\xi+x)}{k_a} \ln \frac{k(\xi)}{k_a} d\xi}{\int d\xi}. \] (3B.7)

It can be calculated by integration over \( \xi \)-ranges of constant permeability \( k \). The correlation function is the sum of the following terms

\[ B_n = E\{\ln \frac{k_{i+n}}{k_a} \ln \frac{k_{i}}{k_a} \} \quad (n = 0,1,2,\ldots), \]

\[ C_n = E\{\ln \frac{k_{i+n}}{k_a} \ln \frac{k_{i+n}}{k_a} \} \quad (n = 0,1,2,\ldots), \] (3B.8)

\[ D_n = E\{\ln \frac{k_{i+n}}{k_a} \ln \frac{k_{i}}{k_a} \} \quad (n = 0,1,2,\ldots). \]

One can calculate that

\[ B_0 = \sigma^2, \]

\[ B_n = 0 \text{ for } n \geq 1, \]

\[ C_0 \approx \frac{1}{2} \sigma^2 \left(1 + \frac{5}{16} \sigma^2 \right), \]
\[ C_1 = \frac{1}{4} \sigma^2 \left( 1 + \frac{5}{16} \sigma^2 \right), \]
\[ C_n = \frac{1}{16} \sigma^4 \text{ for } n \geq 2, \quad (3B.9) \]
\[ D_0 = \frac{1}{2} \sigma^2, \]
\[ D_n = 0 \text{ for } n \geq 1. \]

The correlation function can be best represented in a graphical way. Figure 3B.1 shows the different parts of the correlation function for \( x \geq 0 \). Since \( C_n \neq 0 \) (\( n \geq 2 \)) there is correlation between \( k(\xi) \) and \( k(\xi+x) \) for large distances \( x \). This correlation is due to the difference between the geometric mean permeability \( k_a \) and the average permeability of the simulations \( k_{\text{grid}} \) given by expression (3B.6). Since dispersion is related to permeability variation and not to an average permeability, we redefine the correlation function. We define a new correlation function \( R(x)_{\text{new}} \):

\[ R(x)_{\text{new}} = \frac{\int \ln \frac{k(x+\xi)}{k_{\text{grid}}} \ln \frac{k(\xi)}{k_{\text{grid}}} \, d\xi}{\int d\xi}. \quad (3B.10) \]

From the definitions it follows that \( R(x)_{\text{new}} \) is related to \( R(x) \) as

\[ R(x) = \frac{\int (\ln \frac{k(x+\xi)}{k_{\text{grid}}} + \ln \frac{k_{\text{grid}}}{k_a})(\ln \frac{k(\xi)}{k_{\text{grid}}} + \ln \frac{k_{\text{grid}}}{k_a}) \, d\xi}{\int d\xi} = R(x)_{\text{new}} + \frac{1}{16} \frac{\sigma^4}{m^2}. \quad (3B.11) \]

The result of this rescaling is that from the old correlation function \( R(x) \) an amount \( \frac{1}{16} \frac{\sigma^4}{m^2} \) must be subtracted to obtain \( R(x)_{\text{new}} \). The rescaled correlation function is shown in Fig. 3B.2. It turns out that the dispersivity due to the terms with \( C_n \) (\( n \geq 2 \)) and the dispersivity due to the term \( - \frac{1}{16} \frac{\sigma^4}{m^2} \) cancel. Note in Fig. 3B.2 that we have decomposed the part of the correlation function with \( D_0 \) into two parts, each of the form of a triangle.
FIG. 3B.1: Graphical representation of correlation function $R(x)$ of grid

FIG. 3B.2: Graphical representation of correlation function $R_{new}(x)$ of grid
Figure 3B.2 shows that the correlation function is a sum of a function of the type of one triangle centred at \( x=0 \):

\[ A \left[ 1 - \frac{|x|}{b} \right] H(1 - \frac{|x|}{b}) \quad (b > 0) \]

of the type of two triangles centred at \( x=a \) and \( x=-a \):

\[ A \left[ \left( 1 - \frac{|x-a|}{b} \right) H(1 - \frac{|x-a|}{b}) + \left( 1 - \frac{|x+a|}{b} \right) H(1 - \frac{|x+a|}{b}) \right] \quad (b > a > 0) \]

and of a constant term \( A \).

The dispersivity due to a correlation function of the first type has been calculated in Appendix 2B, and is equal to \( A \frac{D}{2} \). The dispersivity due to a correlation function of the second type can be calculated as follows. The Fourier transform of a function of the second type is:

\[ A \frac{v^2}{\pi} 2 \cos \frac{\alpha}{\kappa} \frac{1 - \cos \frac{\kappa b}{\kappa} }{\kappa^2 b} \]

The Taylor expansion of this expression is:

\[ A \frac{v^2}{\pi} 2 \sum_{m=0}^{\infty} \frac{(-1)^m a}{(2m)!} \frac{2m}{\kappa} 2^m \frac{(-1)^n b}{(2n+1)!} \frac{2n+1}{\kappa} \]

\[ = A \frac{v^2}{\pi} 2 \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{(-1)^n a}{(2m)!} \frac{2m}{b} \frac{2(n-m)+1}{(2n+2)!} \frac{2n}{\kappa} \]

With the notation of Appendix 2B, it follows from the above that

\[ A \frac{2^n}{\pi} \sum_{m=0}^{\infty} \frac{2m}{(2m)!} \frac{2(n-m)+1}{(2(n-m)+2)!} = \sum_{m=1}^{\infty} C_m R_m \]

\[ (n=0,1,2..) \]

Hence the dispersivity is equal to

\[ \sum_{m=1}^{\infty} C_m R_m = A b \]

Finally, the dispersivity due to a constant term \( A \) can be determined by decomposing the constant \( A \) into two series of adjacent and equal triangles of amplitude \( A \); the period of the two series should be the same, the phase
difference between the series should be half a period. The dispersions can then be calculated using the dispersions due to a correlation function of the second type. The result is a dispersions of \( \frac{1}{2} A L \).

The dispersions due to correlation function \( R(x)_\text{new} \) is, with the above:

\[
\beta = \frac{1}{2} A \sigma^2 \left[ 1 - \frac{1}{m} + \frac{1}{m^2} \left( 1 + \frac{1}{8} \sigma^2 \right) \right]. \tag{3B.12}
\]

This expression is, for \( m=4 \):

\[
\beta = \frac{1}{2} A \sigma^2 \left( \frac{13}{16} + \frac{1}{128} \sigma^2 \right), \tag{3B.13}
\]

and for infinite \( m \) is equal to:

\[
\beta = \frac{1}{2} A \sigma^2. \tag{3B.14}
\]

Hence, for \( m=4 \) the ratio of two dispersions (such as \( \alpha \)) is practically equal to the ratio of their \( \sigma^2 \) values.

**Boundaries**

The presence of the boundaries is ignored in the above calculation. To calculate the effect of the boundaries on the correlation function, we should have introduced a factor \( \left[ 1 - \frac{|x|}{L} \right] H(1 - \frac{|x|}{L}) \) in the above calculation. Here we calculate the magnitude of this slovenliness.

If we ignore the variation of the factor \( 1 - \frac{|x|}{L} \) over small distances (\( x < \ell \)), we can correct \( B_1 \) and \( C_1 \) with a factor \( 1 - \frac{\ell}{L} \), correct \( C_n \) (\( n \geq 2 \)) with a factor \( 1 - \frac{n\ell}{L} \) and the constant term \( -A \) with a factor \( \frac{1}{2} \). Hence in the calculation of expression (3B.12) we have ignored terms of magnitude \( \frac{\ell}{L} B_1 \) and \( \frac{\ell}{L} C_1 \), and \( \Sigma \frac{n\ell}{L} C_n \) and \( -\frac{1}{2} A \). One can calculate that the last two terms cancel. The final result is that we have ignored a term

\[
\frac{1}{4} \frac{\ell}{L} \sigma^2 \Delta z \left( 1 + \frac{5}{16} \sigma^2 \right). \]

For our configuration of a 4 by 4 division this is equal to a very small term of \( \frac{1}{2} \sigma^2 \frac{1}{160} \left( 1 + \frac{5}{16} \sigma^2 \right) \), which is for \( \sigma = 1 \) 2.6 only \( 1 \% \) of the total dispersions.
CHAPTER 4

DISPERSE MIXING IN UNSTABLE DISPLACEMENTS

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REFERENCES
4.1. INTRODUCTION

When a fluid in a porous medium is displaced by a less viscous fluid, the displacement front may be unstable: a small perturbation of the displacement front grows. The resulting large perturbations are called viscous fingers.

Viscous fingering has been studied now over more than 30 years. A lot of insight and some practical prediction methods have been developed\(^1,2\), both for miscible and immiscible displacements. Due to its complicated character viscous fingering is still the subject of active research, see e.g. Ref. 3. For a review, see Ref. 4.

Most of the research has been devoted to viscous fingering in uniform porous media, in which the permeability of the porous medium does not vary with location. However, underground reservoirs are seldom uniform and exhibit large and numerous permeability variations on many length scales; only in the laboratory can porous media be created that are uniform. Just recently, viscous fingering in miscible displacements has been studied in combination with (large) permeability heterogeneity (Araktingi & Orr\(^5\), Moissis, Miller & Wheeler\(^6\), Crump\(^7\), Waggoner\(^8\)). The numerical simulations they present show that at large permeability heterogeneity viscous fingering is absent\(^5,7,8\) and that at small permeability heterogeneity, viscous fingering is absent during the early part of the displacement.\(^7\) One paper\(^5\) quantifies empirically the level of heterogeneity at which viscous fingering is negligible, but for only one value of the ratio between the viscosity of the displaced fluid and the viscosity of the displacing fluid. The main objective of this chapter is to quantify the set of conditions at which viscous fingering in miscible displacements is dominated by heterogeneity for a wide range of viscosity ratios.

Heterogeneity is present in many different shapes. As in Refs. 5–8 and in the rest of this work, miscible, unstable displacement is studied for an often occurring and somewhat idealised type of heterogeneity, called local-scale heterogeneity: the length over which the permeability is correlated is small in comparison with the size of the porous medium, the statistical properties of the function that defines the permeability in every location do not vary with location ("statistically homogeneous") and the distribution
of the log-permeability values is normal. To characterise the heterogeneity of the medium, we use a heterogeneity index $HI$:

$$HI = \frac{\lambda}{L} \sigma^2, \quad (4.1)$$

where $\sigma$ is the standard deviation of the log-normal permeability distribution, $\lambda$ is the correlation length of the permeability and $L$ is the length of the medium. (Note that $HI$ equals $\beta_o/L$ by approximation, with $\beta_o$ the dispersivity of the porous medium (see Chapter 2)). As shown in chapter 3 for a stable displacement, this type of heterogeneity causes a dispersive mixing zone between the fluids, which expands as the square root of time. A second objective of this chapter is to determine the mixing zone length if viscous fingering is dominated by this heterogeneity.

As outlined in the Introduction (pages 3 and 4), we discern two types of dispersive mixing: microscopic and macroscopic dispersive mixing. Macroscopic mixing is the result of the type of heterogeneity considered in this study and permits regions of no mixing. Microscopic dispersive mixing, which is present in uniform porous media or porous media with other types of heterogeneity too, is complete to molecular level and may completely suppress viscous fingers. We therefore define the unstable regime as the set of conditions at which viscous fingers are generated and are not suppressed by microscopic dispersive mixing.

In this chapter we provide a simple estimation of the condition within the unstable regime at which viscous fingering is dominated by macroscopic dispersive mixing. Gravity effects are not considered here. We thereby make use of the theory of Gelhar & Axness$^9$ for macroscopic dispersive mixing and the model of Koval$^1$ for viscous fingering. The estimated condition is then tested by conducting numerical simulations in the unstable regime and by use of the simulations of Refs. 5-8. Finally, our simulations are used to validate the application of the model of Chapter 2 to the unstable regime.

4.2. CONDITION FOR VISCOUS FINGERING

The most recent and complete treatment of unstable displacements that takes the stabilising effect of microscopic dispersive mixing into account
is the one of Coskuner & Bentsen. Their instability condition for a system with length L and transverse dimensions W and H is:

\[ \frac{U \phi \frac{d\mu}{dC} - k g \frac{d\rho}{dC} \sin\theta}{\bar{\mu} \kappa} \frac{\partial C}{\partial x} \frac{L^2}{\Omega} \left[ \frac{1}{\Omega} + \frac{\kappa'}{\kappa} \left( \frac{1}{\Omega} + 1 \right) \right]^{-1} > \pi^2 \],

(4.2)

with \( \Omega = \frac{L^2(W^2 + H^2)}{W^2 H^2} \) for a three-dimensional system;

for a two-dimensional system, \( \Omega = \frac{L^2}{W^2} \).

where \( C \) = injectant concentration
\( g \) = gravitational acceleration
\( k \) = permeability
\( U \) = displacement velocity
\( \theta \) = dip angle
\( \mu \) = viscosity of mixture
\( \rho \) = density of mixture
\( \kappa \) = microscopic dispersion coefficient in longitudinal direction
\( \kappa' \) = microscopic dispersion coefficient in transverse direction
\( \phi \) = porosity

The term \( \partial C/\partial x \) is accordingly the average concentration gradient and \( \bar{\mu} \) is the average viscosity. For \( \partial C/\partial x \) Coskuner & Bentsen take a representative value, which is the value at \( x = L/2 \) after the injection of half a pore volume:

\[ \frac{\partial C}{\partial x} = -\sqrt{\frac{U}{\pi \kappa L}} \].

(4.3)

At reservoir scale however, at which microscopic dispersive mixing is negligible, the front between the displacing fluid and the displaced fluid is relatively sharp. In the limit of vanishing microscopic dispersion, \( d\mu/dC = \mu_I - \mu_p \). Then condition (4.2) can be replaced by the well-known instability conditions that the displacing fluid (with viscosity \( \mu_I \) and density \( \rho_I \)) is less viscous than the displaced fluid (with viscosity \( \mu_p \) and density \( \rho_p \)) and that the displacement velocity exceeds a 'critical' velocity. Expressed with mobility ratio M and gravity number \( N_G \):
This aspect of the work of Coskuner & Bentsen is thus not relevant to field scale. It is useful for numerical simulation of viscous fingers because numerical dispersion may suppress fingers.

4.3. VISCOUS FINGERING IN HETEROGENEOUS POROUS MEDIA

To our knowledge, there are only a few references in which the coupling between viscous fingers in miscible displacement and permeability heterogeneity is studied. In this section these references are discussed.

To model the effect of viscous fingering, Koval used the fractional flow theory and replaced the mobility ratio \( M \) in this theory by an "effective mobility ratio" \( K \). The \( K \)-factor was empirically quantified by Koval as

\[
K = E H
\]  

with

\[
E = (0.22 M^{0.25} + 0.78)^4
\]

The \( H \)-factor, which is larger than 1, accounts for the heterogeneity. Koval introduced this factor as a small correction to his model to match his model with the measured production of displaced fluid in a displacement with \( M = 1 \) after injection of one pore volume of displacing fluid. The model further predicts that the production of displaced fluid is, after injection of one pore volume, equal to \( 2/(\sqrt{K} + 1) \) pore volume. Another result is that the front of the most advanced injectant travels with a constant velocity \( U \) \( K \) and the least advanced injectant travels with a constant velocity \( U/K \). For the type of heterogeneity that we use, Koval's model cannot be valid for all \( M \) values greater than 1, because the model predicts that, in the limit \( M = 1 \), the mixing zone between the most advanced and least advanced gas expands linearly in time, while it should be with the square root of time.
FIG. 4.1: Comparison of displacements run in same permeability field for $M = 1$ (dashed line) and $M = 20$ (solid line) at 0.2 pore volumes injected. Contour lines represent concentration of 0.2 (from Ref.5).
Araktingi & Orr simulated unstable displacements in practically uniform and in statistically homogeneous media. In the series of simulations they varied the standard deviation $\sigma$ between small (0.1) and large (2.3) and the correlation between adjacent permeability values from slightly correlated ($\lambda/L = 0.1$) to highly correlated ($\lambda/L = 0.9$). In all their simulations in the unstable regime, the mobility ratio $M$ was 20. For comparison they also conducted simulations in the stable regime at $M = 1$. They found two types of unstable displacements, depending on the heterogeneity of the medium. For small heterogeneity index ($HI < 0.3$), they found a fingering behaviour similar to the behaviour observed in homogeneous media: spreading of fingers, tip-splitting, shielding and merging of fingers. For large heterogeneity ($HI > 0.5$) they observed in their concentration patterns no clear difference between a stable displacement (with $M = 1$) and an unstable displacement (with $M = 20$) in the same porous medium; they concluded that the permeability field dominated finger formation. Figure 4.1, taken from their paper, illustrates this.

Moissis, Miller & Wheeler studied the effects of local-scale permeability variations on viscous fingering by numerical simulation too. They simulated unstable, miscible displacements at various mobility ratios (mostly 75) in a porous medium with small permeability variations. The permeability distribution was normal with a standard deviation equal to 0.063, 0.133 or 0.201 times the average permeability. They employed 80 by 80 grid blocks; squares of 2 by 2 grid blocks had a uniform permeability. Permeability values of adjacent squares were uncorrelated. From their simulations, they found that the permeability near the entrance of the medium determines the finger pattern at early times. At later times, they found that the modest heterogeneity has negligible effect on the number of active fingers, their locations and their lengths. Apparently, local-scale permeability variations dominate the displacement front only initially.

Crump tried to match numerical simulations of unstable, miscible displacements in a medium with non-uniform permeability with the mixing parameter model of Todd & Longstaff, which accounts for viscous fingers and is used to reduce the number of grid blocks in the simulation of an unstable, miscible displacement. The model of Todd & Longstaff provides an effective mobility ratio like the model of Koval, which is $M^{1-\omega}$. The mixing
parameter $\omega$ is an empirical parameter for which Crump tried to find a unique value. The porous medium he used had a log-normal permeability distribution with Dykstra-Parsons variation $V_{DP}$ of 0.7844 (corresponding to a geometrical standard deviation $S = 1/(1-V_{DP})$ of 4.64) and had a number of 100 random permeability values randomly distributed over 400 grid blocks with interpolated permeability values in between. For mobility ratios of 10 and higher he found a more or less constant value for the mixing parameter but for mobility ratios of 5 and smaller he found that the mixing parameter was strongly dependent on the mobility ratio. Crump ascribed this dependency to the relatively important contribution of heterogeneity to the mixing zone at small mobility ratio.

Perkins, Johnston & Hoffman\textsuperscript{11} present the observation, after evaluating their experiments in a transparent model, that when the linear growth of viscous fingers is extrapolated to zero, the viscous fingers had a length of zero not at the inlet to the transparent model but a small distance (a few centimetres) from the inlet. They postulated that although viscous fingers might be initiated at the inlet by permeability variations, the rate of growth in length of the viscous fingers would have to exceed the initial rate of growth of the dispersive mixing zone before fingers could be propagated. Perkins et al. developed an expression for the distance from the inlet at which viscous fingers first would be observed. Their calculations show that the initial region devoid of viscous fingers is entirely negligible on a reservoir scale and in most laboratory systems (Stalkup\textsuperscript{12}).

Recently, Waggoner\textsuperscript{8} studied, by numerical simulation, the interaction between viscous fingering and random heterogeneity. He used random permeability distributions that were extremely uniform ($V_{DP} = 0.01$) or were quite non-uniform ($V_{DP} = 0.54$ or 0.86). Adjacent permeability values were not correlated (expressed by Waggoner as $\lambda/L = 0$), slightly correlated ($\lambda/L = 0.17$) or highly correlated ($\lambda/L = 0.67$ and 1.5). Waggoner found that at a small standard deviation of the permeability variation, viscous fingering dominates the displacement front and that at large standard deviation heterogeneity dominates, but he did not give a quantitative estimation of the transition.
4.4. FLOW REGIMES IN THE UNSTABLE REGIME

4.4.1. Illustration

To illustrate the behaviour of viscous fingers in non-uniform porous media, we discuss here some finger patterns that were obtained by numerical simulation.

**Viscous fingers** are illustrated in Fig. 4.2a. The finger pattern was generated by small spatial variations in the permeability. In this case the spatial permeability variations were very small: the standard deviation $\sigma$ was only 5% of the average permeability (corresponding to a geometrical standard deviation $S = e^\sigma$ of 1.05). In fact, the porous medium was practically uniform. A uniform porous medium is a medium without permeability variation. The practically uniform permeability is illustrated by the stable displacement seen in the same grid when the displacing and displaced fluids are given the same viscosity (Fig. 4.2b).

An unstable displacement in a non-uniform and statistically homogeneous porous medium is shown in Fig. 4.2c. In this case the geometrical standard deviation $S$ is 10. The pattern of Fig. 4.2c clearly shows large finger-like disturbances of the displacement front. The fingers of this pattern are severely affected by the large permeability contrasts. This is shown by comparing Fig. 4.2c with a simulation of a stable displacement in the same grid. Unlike the stable displacement in the uniform medium (Fig. 4.2b), the stable displacement in the non-uniform medium still has large finger-like disturbances (see Fig. 4.2d), although they are somewhat shorter than the fingers in the unstable displacement. Furthermore, the fingers in the stable displacement follow the same path as in the unstable displacement. We call these fingers **heterogeneity fingers**.

In summary, in the very non-uniform medium there is, qualitatively, no distinct difference between a stable displacement and an unstable one, unlike a displacement in a uniform medium.

4.4.2. Estimation of transition

If a displacement is affected more by local-scale permeability variations than by viscous fingers, some viscous fingers may be present during the displacement but they are so small that they cannot be
discriminated from distortions of the displacement front caused by the permeability variations (heterogeneity fingers). This is not a new idea. Perkins et al., for instance, observed a small region devoid of viscous fingers at the inlet of their transparent model, which they attributed to the domination by longitudinal dispersion (read: irregular, permeability variations). Araktingi & Orr also used it to explain their simulations of unstable, miscible displacement in heterogeneous media. They discovered that if the heterogeneity index exceeds a certain value, flow is dominated by the permeability field and viscous fingers are not present. Because their simulations were conducted at one value for the mobility ratio, their results cannot be generalised.

Below we present an estimation of the condition at which a displacement front in the unstable regime is affected more by local-scale permeability variations than by viscous fingers. (The estimation is restricted to cases for which gravity can be ignored.) When the condition applies, the displacement front exhibits a dispersive mixing zone, as in the stable regime. For this reason, we refer to such a displacement as being dispersive-mixing-dominated. When the condition does not apply, that is, when the displacement front is affected more by viscous fingers than by local-scale permeability variations, we call the displacement viscous-finger-dominated.

The expression we use for the length of the mixing zone due to viscous fingers in an homogeneous, uniform medium is given by Koval as

$$X_0 - X_{100} = \left( E - \frac{1}{E} \right) X ,$$

(4.6)

where $X_C$ is the distance travelled by a concentration C averaged over a transverse cross-section (C denoted by percent), $X$ is the average distance travelled and the Koval E-factor is defined by (4.5).

The length of the mixing zone due to local-scale permeability variations in a stable displacement without viscous effects (hence $M = 1$) can be estimated with expression (6) from the Introduction with the dispersivity $\beta_0$ given by the theory of Gelhar & Axness:

$$X_{10} - X_{90} = 3.62 \sqrt{\lambda \sigma^2 X} .$$

(4.7)
FIG. 4.2: Effect of small-scale heterogeneity on unstable and stable displacement (X/L=0.5)
A comparison of equations (4.6) and (4.7) suggests an estimate of the condition at which the displacement is dispersive-mixing-dominated, namely

$$3.62 \sqrt{(\lambda \sigma^2 X)} > (E - \frac{1}{E}) X.$$  \hspace{1cm} (4.8)

The point at which the transition from dispersive-mixing-dominated to viscous-finger-dominated displacement occurs is given by, after substitution of (4.1) in (4.8):

$$\frac{X_{tr}}{L} = HI \left( \frac{3.62}{E - \frac{1}{E}} \right)^2.$$  \hspace{1cm} (4.9)

At \(X\) smaller than \(X_{tr}\), the displacement is dispersive-mixing-dominated; at larger \(X\) it is viscous-finger-dominated. The transition between dispersive-mixing-dominated and viscous-finger-dominated is gradual. If \(X_{tr} > L\), then the displacement is dispersive-mixing-dominated for all possible \(X\).

4.4.3. Discussion

As an illustration, consider a linear drive of oil by a miscible gas with a mobility ratio of 10. The reservoir has a small dip angle, so we assume that stabilisation by gravity is negligible. The main target of the gas injection is a non-layered reservoir-unit in which gravity tonguing does not occur because of a combination of a limited vertical permeability and a high displacement velocity. For the heterogeneity index of the reservoir unit we take an average value, based on the empirical relation of Arya et al.\textsuperscript{13}: this relation gives \(\rho_0 = 0.044 \text{L}^{1.13}\), so \(\text{HI} = \rho_0/L = 0.044 \text{L}^{0.13}\) which we approximate by 0.044. With \(M = 10\) and \(\text{HI} = 0.044\), our condition (4.9) predicts that there is a transition between dispersive-mixing-dominated and viscous-finger-dominated displacement at \(X_{tr} = 0.32\text{ L}\). This means that viscous fingers dominate the displacement only after \(X = 0.32\text{ L}\).

For the general case, Fig. 4.3 presents the transition between dispersive-mixing-dominated and viscous-finger-dominated displacements graphically. The figure demonstrates that a displacement tends to be dispersive-mixing-dominated instead of viscous-finger-dominated:
FIG. 4.3: Heterogeneity divided by distance travelled vs mobility ratio. The results of this work show that dispersive mixing dominates also in a part of the unstable regime. In this part viscous fingers can be neglected.

* at a moderate mobility ratio $M$
* at a large correlation length $\lambda$ (provided $\lambda \ll L$, to satisfy the condition of dispersive behaviour),
* at a small length $L$ of the medium (provided $L \gg \lambda$),
* at a large standard deviation $\sigma$ of the permeability variation,
* at a small distance travelled $X$.

Expression (4.9) clearly shows that when the heterogeneity index exceeds a certain value, the displacement is dominated by local-scale permeability variations. The expression thus agrees with the observation of Araktingi & Orr (at mobility ratio of 20) that, when the heterogeneity index $HI$ is smaller than 0.3, the displacement is dominated by viscous fingers: (4.9) predicts that, at $M = 20$, the displacement is viscous-finger-dominated during the whole displacement if $HI \leq 0.3$. 
Transition (4.9) also implies that a displacement is dispersive-mixing-dominated in the beginning (at small $X$) and becomes viscous-finger-dominated later (at large $X$), as was observed by Moissis et al. Waggoner's observation that viscous fingers dominate only at small standard deviation $\sigma$ is explained by (4.9) as well.

Crump demonstrated that the mixing parameter model of Todd & Longstaff fails at low mobility ratios if a constant value for the mobility ratio is required. This can be understood better in the light of the results presented here: if the effects of unfavourable mobility ratio on the mixing-zone length are smaller than the effects of non-uniform permeability, the displacement can be considered as dispersive-mixing-dominated.

While the treatment of Perkins et al. about suppression of viscous fingers by longitudinal dispersion is not important at reservoir scale, our treatment has implications for reservoir scale. The differences between our approach and theirs are the following:

1. Perkins et al. took microscopic dispersion into consideration. This type of dispersion is negligible at reservoir scale. We consider dispersion at local scale. The value for $X_{tr}$ is then orders of magnitude larger.

2. Perkins et al. demand that viscous fingers be totally absent and translate this demand in a comparison between the rate of growth of the fingered zone and the rate of growth of the dispersive mixing zone. We demand that viscous fingers do not dominate the displacement and translate this in a comparison between the length of the fingered zone and the length of the dispersive mixing zone. The result is that the value of $X_{tr}$ with our approach is four times theirs.

3. Perkins et al. use a conservative estimate of the length of the fingered zone that is based on a low estimate of the effective mobility ratio. We use Koval's model, which gives a higher effective mobility ratio. The result is that the value for $X_{tr}$ with our approach is about one-ninth of theirs.
4.5. **LENGTH OF MIXING ZONE IN DISPERSIVE-MIXING-DOMINATED REGIME**

4.5.1. **Application of random-walk model of Chapter 2 to unstable regime**

If a displacement is dispersive-mixing-dominated instead of viscous-finger-dominated, then the increase in the mixing-zone length with time can be better described with a model that provides for an increase of the mixing-zone length with the square root of time than one that provides for an increase that is proportional to time, such as the ones of Koval and of Todd & Longstaff. We show in this section a way of calculating the length of the mixing zone so that it is consistent with the dependence on the square root of time.

In Chapter 2, we quantified the length of a dispersive mixing zone that expands with the square root of time. In that chapter no restriction to the stable regime was made in the assumptions or in the derivation of the random-walk model, except for the assumption of the presence of a dispersive mixing zone. In Chapter 3, this assumption, though, was successfully tested for the stable regime. In section 4.4, the presence of a dispersive mixing zone in the unstable regime has been discussed. If there is a dispersive mixing zone present in the unstable regime instead of viscous fingers, it should thus be possible to calculate the length of the mixing zone with the analytical model of Chapter 2.

4.5.2. **Random-walk model in brief**

The analytical model of Chapter 2 has been set up with the assumption of a random walk for the fluid parcels. The step size of the random walk has been calculated by considering the flow inside and outside a block of lower permeability that is embedded in an infinite medium with effective permeability. The permeability of the block contrast with the effective permeability by the geometrical standard deviation $S = e^\sigma$. The block has a length of $2\lambda$ and a width of $2\lambda'$. This configuration replaces the porous medium with irregular spatial permeability variations that obey a log-normal distribution with geometrical standard deviation $S$ and a correlation length $\lambda$ in the longitudinal direction and a correlation length $\lambda'$ in the transverse direction.
The model of Chapter 2 predicts that in a linear displacement the injectant concentration, averaged over a transverse cross-section, is given by

$$C = \frac{1}{2} \text{erfc} \left( \frac{X - I}{2 \sqrt{I}} \sqrt{N_{Pe}} \right),$$

(4.10)

where $x$ = coordinate in longitudinal direction

$L$ = length of porous medium

$I$ = dimensionless time, defined by $Ut/L$ (in pore volumes), where $t$ is time

$N_{Pe}$ = Peclet number, defined by $N_{Pe} = L/\beta$

$\beta$ = effective dispersivity.

The effective dispersivity is defined by

$$\beta = \lambda \sigma^2 a$$

(4.11)

The dispersivity ratio $a$ is quantified by the random-walk model of Chapter 2. It is 1 by definition when $M = 1$ and $N_G = 0$. In Fig. 4.4 we have plotted dispersivity ratio $\alpha$ as a function of mobility ratio $M$ for $N_G = 0$ for both the stable regime and the unstable regime. Figure 4.5 presents the dispersivity ratio $\alpha$ as a function of gravity number $N_G$ with mobility ratio $M$ as parameter for both the stable and the unstable regimes.

4.5.3. Deviation from random-walk model in unstable regime

4.5.3.1. Transverse correlation length

The random-walk model of Chapter 2 is confined to the condition $\lambda' << \lambda$. Section 2.5.1 gives a calculation for the deviation from the random-walk model when this condition is not satisfied and $M = 1$ and $N_G = 0$; for other $N_G$ and $M$ values in the stable regime, the deviation is less. As a result, the random-walk model predicts a too high dispersivity ratio $\alpha$ in the stable regime. It can be shown that in the unstable regime the deviation is higher than at $M = 1$ and $N_G = 0$. This means that in the unstable regime, the minimum deviation from the random-walk model is given by expression (2.25)
FIG. 4.4: Dispersivity ratio $\alpha$ as a function of mobility ratio $M$
(Gravity number $N_G = 0$, parameter geometric standard deviation $S$)

FIG. 4.5: Dispersivity ratio $\alpha$ as a function of gravity number $N_G$
(parameter: geometric standard deviation $S$ and mobility ratio $M$)
and that a maximum deviation cannot be given. As a result, the random-walk model predicts a too low dispersivity ratio $\alpha$ in the unstable regime.

4.5.3.2. Transverse pressure disequilibrium

Another deviation is caused by the unrealistic consequence of the model of Chapter 2 of a pressure disequilibrium at the lateral boundaries of the block of lower permeability. In reality, the fluid flow is such that pressures on the two sides of the lateral boundaries are in equilibrium. In section 2.5.2 it is shown that this deviation is not important at dispersivity ratio $\alpha$ close to 1. In the unstable regime, $\alpha$ is greater than 1, so this deviation may play a role at large $\alpha$ but not as much as in the stable regime.

4.6. TEST OF TRANSITION BETWEEN FLOW REGIMES AND OF RANDOM-WALK MODEL

4.6.1. Set-up of simulations

4.6.1.1. Purpose

We carried out a series of 16 simulation runs of unstable, miscible displacement in a medium with local-scale permeability variations. The purposes of the simulations were

1. to test condition (4.9) for the transition between dispersive-mixing-dominated and viscous-finger-dominated displacement,

2. to show that the transition correctly predicts the domination of dispersive mixing over viscous fingers, and that the length of the mixing zone can be calculated with the random-walk model of Chapter 2.

The simulation runs were conducted in the unstable regime, hence the mobility ratio was larger than 1, the displacement velocity exceeded the critical velocity and the numerical dispersion did not suppress the fingers. In the simulation series we varied a number of parameters, including $S$ and $M$ as we did in Chapter 3. The lowest $S$ value was 1.05, the highest 10. $M$ varied between 1.5 and 100. The aspect ratio $L/W$ varied from 1 to 27.2.

To evaluate the effect of the contrast in fluid properties on the mixing-zone length, we conducted reference runs without a contrast in fluid properties, so $M = 1$ and $N_G = 0$. The dispersivity ratio $\alpha$ (see expression (4.11)) is 1 for the reference runs.
4.6.1.2. Simulator

The simulator (Ref. 7) that we used and today's computer capacity enabled us to simulate dispersion and to generate viscous fingers without artificial measures. Such measures were taken by, for example, Gardner & Ypma\textsuperscript{14} and Giordano & Salter\textsuperscript{15}. They incorporated a dispersion term in the conservation-of-injectant equation to model dispersion and gave one or more grid blocks a very high permeability to generate one or more viscous fingers. In our simulations, both the level of dispersion that is predicted by the theory of Gelhar & Axness and generation of viscous fingers was attained by one measure: implementing a random permeability distribution in the simulation grid.

The simulator has been validated for stable displacement in a non-uniform medium in Chapter 3 and for unstable displacement in a uniform medium by Crump. In addition, we compared our simulation of an unstable displacement at $M = 5$ in the practically uniform medium (with $S = 1.05$) discussed in section 4.3.1. with the Koval model and derived a Koval $K$-factor of 1.57 from the recovery profile. The Koval model predicts $E = 1.51$ and $H = 1$, so $K = 1.51$. Figure 4.7 shows that the difference in recovery between the simulation and the Koval model is indeed small.

![Simulation](Simulation.png)

**FIG. 4.7:** Comparison of run 124 with Koval model
4.6.1.3. Grid

The simulation runs were carried out in the same way for stable displacements in Chapter 3. Thus the grid consisted of 80 by 80 blocks, squares of 4 by 4 blocks having a uniform permeability. The permeability values were assigned randomly to the squares and taken from a log-normal permeability distribution with a specified geometrical standard deviation $S$; permeability values of squares were uncorrelated.

The solvent was injected over a complete boundary, production was on the opposite boundary; the other two boundaries were closed. (For more details see Chapter 3.) In each grid, a reference run with $M = 1$ and $N_G = 0$ was conducted.

4.6.1.4. Evaluation of a run

In each simulation run, a concentration pattern was generated every 0.01 pore volume injected. For each concentration pattern, the concentration $C$, which is the concentration of the injectant averaged over a transverse cross-section, was matched with equation (4.10), yielding a value for $N_{Pe}$. The $N_{Pe}$ values at $I=0.02$, $I=0.03$, $I=0.04$, etc. are averaged to a value, $N_{Pe, sim}$ which is used for comparison with the random-walk model. The initial part of $N_{Pe}$ vs $I$ (i.e. at $I$ below 0.35) was ignored in the calculation of the average $N_{Pe}$.

4.6.1.5. Criteria for dispersive mixing

To establish the dispersive character of the mixing zone (and the negligible presence of viscous fingers), we required the same behaviour from the mixing zone expansion as we have observed in the stable regime (in Chapter 3). This implies that we required a constant course of the Peclet number with time (within 10%). We also required a sufficiently high correlation coefficient $R^2$ (above 0.99) between expression (4.10) and the mixing zone.

Some runs with $M = 1$ and $N_G = 0$ that we used as a reference for runs in the unstable regime did not obey these criteria. Therefore, if the reference run had a Peclet variation of more than 10% (but always below 15%), we demanded for dispersive mixing that the Peclet variation not be more than 15%. If the reference run had a correlation coefficient with expression (4.10) below 0.99, we demanded for dispersive mixing that the correlation coefficient not be more than 0.01 less than the one of the reference run.
4.6.1.6. Comparison with random-walk model

For comparison of a run with the random-walk model for the mixing-zone length, the dispersivity ratio $\alpha$ was calculated from the average values of the Peclet number in time, respectively $N_{Pe, ref}$ of the reference run and $N_{Pe, sim}$ of the simulation run in the unstable regime, according to

$$\alpha_{sim} = \frac{N_{Pe, ref}}{N_{Pe, sim}} \quad (4.12)$$

This equation follows from equation (4.11). The subscript sim indicates that this $\alpha$ value has been obtained from the simulation. This $\alpha$ value is compared with the one predicted by the random-walk model.

Table 4.1 lists the runs that exhibited viscous fingers because they had a too large dependency of Peclet on $I$ or had too low a correlation coefficient with (4.10). Table 4.2 lists the other runs, which exhibited a dispersive mixing zone. Table 4.3 lists the reference runs with $M = 1$ and $N_G = 0$. The reference run with $S = 1.052$ in this table did not show a constant course of $N_{Pe}$ with time owing to the practically uniform permeability and the contribution of numerical dispersion. The value of $N_{Pe, sim}$ of this run was therefore taken from the theory of Gelhar & Axness.

4.6.2. Results

4.6.2.1. Transition between flow regimes

The simulation results sustain expression (4.9) for the transition between dispersive-mixing-dominated and viscous-finger-dominated displacement. This is shown in Fig. 4.8, in which the character of the mixing zone in the simulation runs is compared with expression (4.9). The curve in this figure, which represents (4.9), separates the region in the quadrant where, according to (4.9), a displacement exhibits a dispersive mixing zone from the region where a displacement exhibits viscous fingers. The shaded area around the curve represents the gradual transition between the flow regimes. The figure shows that nearly all points, which represent a run, are in the region where they should be according to (4.9); the single point that is in the wrong region is in the shaded area.
TABLE 4.1: RUNS THAT EXHIBIT VISCOUS FINGERING ACCORDING TO CRITERIA OF
SECTION 4.6.1.5 ($N_G = 0$).

<table>
<thead>
<tr>
<th>run</th>
<th>S</th>
<th>M</th>
<th>$\frac{L}{W}$</th>
<th>ref. $\text{N}_{Pe,\text{sim}}$</th>
<th>$R^2$</th>
<th>$\Delta I$</th>
<th>$\alpha_{\text{sim}}$</th>
<th>$\alpha_{\text{model}}$</th>
<th>$\frac{\Delta \alpha}{\alpha}$</th>
<th>$\chi^2_{tr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>94</td>
<td>2.60</td>
<td>10</td>
<td>27.2</td>
<td>93</td>
<td>24.5</td>
<td>$\pm 3.0$</td>
<td>0.95*</td>
<td>0.012</td>
<td>3.14</td>
<td>2.30</td>
</tr>
<tr>
<td>100</td>
<td>2.60</td>
<td>100</td>
<td>27.2</td>
<td>93</td>
<td>9.4</td>
<td>$\pm 1.4$</td>
<td>0.93*</td>
<td>-0.009</td>
<td>8.18</td>
<td>2.58</td>
</tr>
<tr>
<td>102</td>
<td>1.57</td>
<td>10</td>
<td>6.8</td>
<td>32</td>
<td>24.1</td>
<td>$\pm 1.4$</td>
<td>0.97*</td>
<td>-0.004</td>
<td>11.2</td>
<td>3.49</td>
</tr>
<tr>
<td>104</td>
<td>1.052</td>
<td>10</td>
<td>6.8</td>
<td>125</td>
<td>18.8</td>
<td>$\pm 3.7$</td>
<td>0.8*</td>
<td>-0.002</td>
<td>830</td>
<td>11.83</td>
</tr>
<tr>
<td>106</td>
<td>2.60</td>
<td>10</td>
<td>1.0</td>
<td>113</td>
<td>13.7</td>
<td>$\pm 1.0$</td>
<td>0.975*</td>
<td>0.004</td>
<td>4.91</td>
<td>2.30</td>
</tr>
<tr>
<td>107</td>
<td>5</td>
<td>10</td>
<td>6.8</td>
<td>108</td>
<td>10.5</td>
<td>$\pm 1.6$</td>
<td>0.93*</td>
<td>-0.023</td>
<td>3.68</td>
<td>1.82</td>
</tr>
<tr>
<td>114</td>
<td>10</td>
<td>10</td>
<td>1.0</td>
<td>118</td>
<td>5.7</td>
<td>$\pm 1.1$</td>
<td>0.93*</td>
<td>-0.055</td>
<td>3.14</td>
<td>1.60</td>
</tr>
<tr>
<td>124</td>
<td>1.052</td>
<td>5</td>
<td>1.0</td>
<td>125</td>
<td>52.9</td>
<td>$\pm 19.0$</td>
<td>0.985*</td>
<td>0.005</td>
<td>294</td>
<td>6.38</td>
</tr>
<tr>
<td>126</td>
<td>1.052</td>
<td>2</td>
<td>6.8</td>
<td>125</td>
<td>226</td>
<td>$\pm 78.7$</td>
<td>0.992</td>
<td>0.006</td>
<td>69</td>
<td>2.15</td>
</tr>
<tr>
<td>129</td>
<td>10</td>
<td>20</td>
<td>1.0</td>
<td>118</td>
<td>4.0</td>
<td>$\pm 0.8$</td>
<td>0.91*</td>
<td>-0.10</td>
<td>4.5</td>
<td>1.63</td>
</tr>
<tr>
<td>130</td>
<td>10</td>
<td>100</td>
<td>1.0</td>
<td>118</td>
<td>2.5</td>
<td>$\pm 0.4$</td>
<td>0.8*</td>
<td>-0.2</td>
<td>7.16</td>
<td>1.70</td>
</tr>
<tr>
<td>131</td>
<td>2.60</td>
<td>5</td>
<td>6.8</td>
<td>48</td>
<td>24.8</td>
<td>$\pm 3.4$</td>
<td>0.975*</td>
<td>0.005</td>
<td>3.60</td>
<td>2.02</td>
</tr>
</tbody>
</table>

* too large standard deviation of $\text{N}_{Pe,\text{sim}}$ or too low $R^2$

$\Delta I$ = deviation of $I$, as determined in match between simulation and (4.10),
from the given value of $I$ ($\Delta I = 0.0125$ corresponds to one grid block)

$$\frac{\Delta \alpha}{\alpha} = \frac{\alpha_{\text{sim}} - \alpha_{\text{model}}}{\alpha_{\text{model}}}$$
### TABLE 4.2: RUNS THAT EXHIBIT A DISPERSIVE MIXING ZONE ACCORDING TO CRITERIA OF SECTION 4.6.1.5 ($N_G = 0$).

<table>
<thead>
<tr>
<th>run</th>
<th>S</th>
<th>M</th>
<th>$L/W$</th>
<th>ref.</th>
<th>$N_{Pe,\text{sim}}$</th>
<th>$R^2$</th>
<th>$\Delta I$</th>
<th>$a_{\text{sim}}$</th>
<th>$a_{\text{model}}$</th>
<th>$\Delta a/a$</th>
<th>$X_{tr}/L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>2.60</td>
<td>2</td>
<td>27.2</td>
<td>93</td>
<td>48.7 ± 8.3</td>
<td>0.99</td>
<td>0.009</td>
<td>1.58</td>
<td>1.48</td>
<td>6%</td>
<td>2.69</td>
</tr>
<tr>
<td>139</td>
<td>1.57</td>
<td>1.5</td>
<td>6.8</td>
<td>32</td>
<td>165.0 ±13.8</td>
<td>0.996</td>
<td>0.006</td>
<td>1.63</td>
<td>1.38</td>
<td>15%</td>
<td>1.93</td>
</tr>
<tr>
<td>112</td>
<td>2.60</td>
<td>2</td>
<td>1.0</td>
<td>113</td>
<td>37.4 ± 5.5</td>
<td>0.992</td>
<td>0.006</td>
<td>1.80</td>
<td>1.48</td>
<td>18%</td>
<td>2.69</td>
</tr>
<tr>
<td>138</td>
<td>5</td>
<td>4</td>
<td>6.8</td>
<td>108</td>
<td>18.4 ± 1.7</td>
<td>0.98</td>
<td>0.006</td>
<td>2.10</td>
<td>1.61</td>
<td>23%</td>
<td>1.67</td>
</tr>
<tr>
<td>98</td>
<td>2.60</td>
<td>2</td>
<td>6.8</td>
<td>48</td>
<td>44.0 ± 2.0</td>
<td>0.99</td>
<td>0.007</td>
<td>2.03</td>
<td>1.48</td>
<td>27%</td>
<td>2.69</td>
</tr>
<tr>
<td>101</td>
<td>1.57</td>
<td>2</td>
<td>6.8</td>
<td>32</td>
<td>105.4 ±11.4</td>
<td>0.992</td>
<td>0.005</td>
<td>2.55</td>
<td>1.71</td>
<td>33%</td>
<td>0.61</td>
</tr>
<tr>
<td>117</td>
<td>10</td>
<td>5</td>
<td>1.0</td>
<td>118</td>
<td>8.0 ± 1.0</td>
<td>0.96</td>
<td>-0.017</td>
<td>2.24</td>
<td>1.50</td>
<td>33%</td>
<td>2.40</td>
</tr>
<tr>
<td>127</td>
<td>1.57</td>
<td>2</td>
<td>1.0</td>
<td>128</td>
<td>67.0 ±10.0</td>
<td>0.994</td>
<td>0.008</td>
<td>3.40</td>
<td>1.71</td>
<td>50%</td>
<td>0.61</td>
</tr>
</tbody>
</table>

### TABLE 4.3: REFERENCE RUNS ($M = 1$, $N_G = 0$)

<table>
<thead>
<tr>
<th>run</th>
<th>S</th>
<th>$L/W$</th>
<th>$N_{Pe,\text{ref}}$</th>
<th>$R^2$</th>
<th>$\Delta I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>1.57</td>
<td>6.8</td>
<td>269 ±20</td>
<td>0.998</td>
<td>0.007</td>
</tr>
<tr>
<td>48</td>
<td>2.60</td>
<td>6.8</td>
<td>89.3 ± 4.8</td>
<td>0.993</td>
<td>0.008</td>
</tr>
<tr>
<td>93</td>
<td>2.60</td>
<td>27.2</td>
<td>76.9 ± 0.4</td>
<td>0.998</td>
<td>0.007</td>
</tr>
<tr>
<td>108</td>
<td>5</td>
<td>6.8</td>
<td>38.6 ± 5.5</td>
<td>0.985</td>
<td>0.008</td>
</tr>
<tr>
<td>113</td>
<td>2.60</td>
<td>1.0</td>
<td>67.2 ±10.1</td>
<td>0.993</td>
<td>0.007</td>
</tr>
<tr>
<td>118</td>
<td>10</td>
<td>1.0</td>
<td>17.9 ± 2.6</td>
<td>0.97</td>
<td>0.007</td>
</tr>
<tr>
<td>125</td>
<td>1.052</td>
<td>1.0</td>
<td>$1.6 \times 10^4$</td>
<td>0.998</td>
<td>0.0086</td>
</tr>
<tr>
<td>128</td>
<td>1.57</td>
<td>1.0</td>
<td>228 ±24.6</td>
<td>0.996</td>
<td>0.008</td>
</tr>
</tbody>
</table>
The dispersive or non-dispersive character of the mixing zone in the simulations is illustrated in more detail in Fig. 4.9, which shows the course of the Peclet number with dimensionless time for all simulation runs. In each picture of this figure the curve representing $N_{Pe}$ vs. $I$ of an unstable run is plotted together with the curve representing $N_{Pe}$ vs. $I$ of the reference run to compare the two curves. (The plots of the reference runs belonging to the runs 104, 124 and 126 are not shown because the $N_{Pe,ref}$ value is much too large.) If the curves in one plot have more or less the same shape, then, apparently, the same mechanism (permeability contrasts) dominates the mixing zone in the unstable run as in the reference run. If the two curves are not similar, the mixing zone is clearly dominated by another mechanism (viscous fingers). The figure shows that for most curves that are dispersive-mixing-dominated according to the criteria of section 4.6.1.5 there is indeed a similarity between the curve of the unstable run and the reference run. The same figure shows that this similarity does not exist for the runs that are viscous-finger-dominated according to the criteria of 4.6.1.5.
FIG. 4.9a: Variation of Peclet number with time for simulation runs 94, 100, 102
FIG. 4.9b: Variation of Peclet number with time for simulation runs 104, 106, 107
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Run with M > 1  Reference run M = 1

Run 114: S = 10, M = 10, N_G = 0, L/W = 1

![Graph showing Peclet number variation with time for Run 114.]

Run 124: S = 1.052, M = 5, N_G = 0, L/W = 1 (P_{ref} = 1.6 \times 10^4)

![Graph showing Peclet number variation with time for Run 124.]

Run 126: S = 1.052, M = 2, N_G = 0, L/W = 6.8 (P_{ref} = 1.6 \times 10^4)

![Graph showing Peclet number variation with time for Run 126.]

FIG. 4.9c: Variation of Peclet number with time for simulation runs 114, 124, 126
Run 129: $S = 10, M = 20, N_3 = 0, L/W = 1$

Run 130: $S = 10, M = 100, N_3 = 0, L/W = 1$

Run 131: $S = 2.60, M = 5, N_3 = 0, L/W = 6.8$

FIG. 4.9d: Variation of Peclet number with time for simulation runs 129, 130, 131
FIG. 4.9e: Variation of Peclet number with time for simulation runs 99, 139, 112
FIG. 4.9f: Variation of Peclet number with time for simulation runs 138, 98, 101
Run 117: $S = 10, M = 5, N_D = 0, L/W = 1$

Run 127: $S = 1.57, M = 2, N_D = 0, L/W = 1$

**FIG. 4.9g:** Variation of Peclet number with time for simulation runs 117, 127
FIG. 4.10: Transition from dispersive-mixing dominated to viscous-finger-dominated displacement as mobility ratio $M$ increases ($S = 10$, $X/L = 0.15$).
There are a few exceptions to this general observation. For example, run 114, which is evaluated to be viscous-finger-dominated according to the criteria of 4.6.1.5, does show a similarity in the course of \( N_p \) vs. \( I \) with the reference run, which suggests dispersive-mixing domination. A dispersive-mixing domination would better agree with transition (4.9) because this condition predicts a transition to viscous-finger-dominated displacement at large \( X (X/L = 0.96) \).

For the runs that are evaluated to be dispersive-mixing-dominated, Fig. 4.9 shows that there is a transient between \( I = 0 \) and \( I \) is about 0.35, as was also observed in the simulation runs of displacements in the stable regime (see Chapter 3). The Peclet number is roughly constant at \( I \) larger than about 0.35, which means that the mixing zone expands as the square root of time, in correspondence with the simulation runs conducted in the stable regime. The fluctuations seen in this part of the curve are of the same magnitude as in the reference runs. Some simulation runs exhibit a slight increase in Peclet at \( I \) is about 0.9, which means that the mixing zone expands there somewhat less than as the square root of time.

The concentration patterns with, for example \( S = 10 \), give additional insight. They are shown in Fig. 4.10 for various mobility ratios. The concentration patterns of the simulations with \( M = 1 \) (Fig. 4.10a) and with \( M = 5 \) (Fig. 4.10b) resemble each other. In contrast, the concentration pattern of the run with \( M = 100 \) (Fig. 4.9d) contains long fingers that are almost straight. From this we conclude that the front in the displacement with a moderate \( M \) value of 5 is dominated by the same factor as is the front in the displacement with \( M = 1 \), which is the permeability field, but that this is not the case for the front with \( M \) equal to 100. This observation agrees with transition (4.9): the figure shows concentration patterns at \( I = X/L = 0.15 \) and at \( X/L = 0.15 \) and \( M = 5 \), the displacement is dispersive-mixing-dominated (\( X_{tr}/L \) is equal to 2.40 for \( M = 5 \)), but at \( X/L = 0.15 \) and \( M = 100 \) the displacement is viscous-finger-dominated (\( X_{tr}/L \) is 0.084 for \( M = 100 \)).

4.6.2.2. Random-walk model

All runs listed in Table 4.2, which exhibit a dispersive mixing zone, have a systematically higher \( a \) value (\( a_{sim} \)) than predicted by the random-walk model of Chapter 2 (\( a_{model} \)). This systematic underestimation is attributed to the violation of the condition \( \lambda' \ll \lambda \) (see section 4.5.3.1).
For one run (the one with \( L/W = \lambda'/\lambda = 27.2 \)), this condition is satisfied; this run has an \( a \) value that is just 6% higher than the model prediction. The three runs with \( \lambda/\lambda' = 1 \) have an \( a \) value that is much higher, 18 - 50%, than the model prediction. The four runs with an intermediate value \( \lambda/\lambda' = 6.8 \), have an \( a \) value that is 15 - 33% higher than the model prediction. Hence, the greater \( \lambda/\lambda' \), the more accurate the model prediction, in correspondence with section 4.5.3.1. The highest deviation in Table 4.2, 50%, is for a run for which not only does \( \lambda/\lambda' = 1 \) but also for which (4.9) predicts viscous fingers after \( X/L > 0.61 \), while a dispersive mixing zone is nonetheless observed.

### 4.6.3. Evaluation of simulation runs conducted by others

For the test of the transition between the flow regimes, (4.9), we also considered the simulation runs of Araktingi & Orr, of Crump of Moissis et al. and of Waggoner. Because neither Araktingi & Orr nor Crump conducted their runs with the aim of investigating dispersive mixing of a displacement in the unstable regime, we had to discriminate ourselves the runs exhibiting a dispersive mixing zone from the runs exhibiting viscous fingers. We assumed that a simulation run of Araktingi & Orr had a dispersive mixing zone if they evaluated the run as being dominated by the permeability field; we took only those runs into consideration that did not have an extremely large correlation length of the permeability \((\lambda/L \leq 0.6)\). A simulation run of Crump was judged to have a dispersive mixing zone if Crump found a mixing parameter that was dependent on the mobility ratio. The runs that Waggoner called 'spreading' are dispersive-mixing-dominated. We only took those runs into consideration in which numerical dispersion could not suppress viscous fingers (this is possible in his runs with \( \lambda = 0 \)) and those runs that did not have an extremely large correlation length \((\lambda/L = 0.17)\). Finally, we accounted for the fact that Waggoner evaluated his runs at \( X/L = 0.5 \).

All our runs, together with those of Araktingi & Orr, Crump, Moissis et al., Waggoner and this work, have been summarised in Fig. 4.11, in which heterogeneity index \( HI \) is plotted against \( M \). The figure shows a very good agreement between the simulation results and expression (4.9) for the transition between the flow regimes.
4.7. PRACTICAL CONSEQUENCES

4.7.1. Design of unstable displacements

The findings of this chapter provide a new perspective to the evaluation of an unstable displacement: if the mobility ratio is moderate and the reservoir (unit) has sufficiently large local-scale permeability variations such that (4.9) predicts dispersive mixing, viscous fingers can be disregarded when the displacement velocity is increased beyond $U_c$.

For the reservoir engineer who is interested in such an evaluation, transition (4.9) is an easy and time-saving criterion: detailed numerical simulations of the type shown in section 4.6 are not needed; instead, he can use transition (4.9). Furthermore, he can model the expansion of the mixing zone between the fluids more accurately by using expressions (4.10) and (4.11) instead of a viscous fingering model such as Koval's. To judge the applicability of this work to his problem, he can use Fig. 4.12, which
provides a graph for various reservoir processes as a function of mobility ratio and heterogeneity index.

4.7.2. Mixing between gases in Underground Gas Storage

As an example of an unstable displacement at moderate mobility ratio, we consider the displacement of inert cushion gas by hydrocarbon gas in underground gas storage. To save hydrocarbon gas, inert gas such as carbon dioxide is sometimes taken as cushion gas; in that case, during injection in the summer period into the storage reservoir, hydrocarbon gas displaces the cushion gas. A practical problem with different gases is that injected hydrocarbon gas becomes contaminated with inert gas, and so special measures have to be taken at surface to meet the contract specification. The designer of the process is therefore interested in the level of contamination of the hydrocarbon gas when it is backproduced (see for example Ref. 16).

Because the cushion gas is more viscous than the injected hydrocarbon gas (for example, carbon dioxide is about two times more viscous than

![Diagram showing the transition between different processes](image)

FIG. 4.12: Nomogram for application of this work to various reservoir processes
hydrocarbon gas at reservoir pressure), the displacement is conducted in the unstable regime if the stabilising effect of the density difference between the gases is negligible. On the basis of an unstable displacement, the designer would expect a high level of contamination. This may, however, not be the case if the displacement is dispersive-mixing-dominated.

For a calculation of the mixing between the gases in an homogeneous reservoir layer with local-scale permeability variations, let us assume that the reservoir has a heterogeneity index of 0.023 (σ = ln 2.6 and λ = L/40). We first calculate the mixing between the gases with the Koval model. According to the theory of Gelhar & Axness, which provides a relation between heterogeneity index and dispersivity, and to Gardner & Ypma, which provides a relation between dispersivity and Koval H-factor, HI = 0.023 corresponds to a Koval H-factor of 1.66. The Koval E-factor for M = 2 is 1.18 (expression (4.5)). The Koval K-factor is 1.66*1.18 = 1.95. The amount of inert gas that lags behind the average displacement front between the gases is a potential source of contamination of the hydrocarbon gas when the same amount of gas is backproduced as was injected (see Fig. 4.13). This amount is a fraction 1 - 2/(√K + 1) of the injected gas (this expression corresponds to the oil left behind in a reservoir in which one pore volume of solvent is injected). With K = 1.95, the fraction is 0.165.

FIG. 4.13: Unstable displacement of inert cushion gas by injected natural gas in underground gas storage.
However, (4.9) predicts dispersive mixing and therefore the contamination should be calculated with equations (4.10) and (4.11). With $HI = 0.023$ and $a = 1.48$ for $a = ln 2.6$ and $M = 2$, and with the relation in Gardner & Ypma, we calculate this fraction to be 0.102, hence a considerably lower level of contamination is indicated than is predicted with the Koval model.

In fact, we can compare these contamination values to our simulation run with $S = 2.60$ and $M = 2$ in Table 4.2. This simulation gives an amount of 0.091 pore volumes of displaced fluid that is left behind at the injection of one pore volume of solvent. This value corresponds to the fraction of inert gas that could be present in the backproduced gas. The value is 12% lower than predicted with our model but 81% lower than predicted with the Koval model.

4.7.3. Design of slugs in polymer flooding

A method for reducing the costs of polymer flooding is the stepwise reduction (grading) of the polymer concentration in successive slugs (see for example Ligthelm\(^{17}\)). Because of the unfavourable viscosity contrast between the successive slugs, special grading schemes have been proposed to guarantee that the polymer bank retains its integrity during its passage through the reservoir (such as the one of Claridge\(^{18}\)). These grading schemes are always based on the assumption that, owing to the unfavourable viscosity contrast, viscous fingers develop between the slugs.

This work shows that although the 'classical' condition (4.2) predicts viscous fingers between the slugs, the heterogeneity may contribute more to the distortion of the displacement front than viscous fingers, particularly because the viscosity contrast between the polymer slugs is moderate (say, a factor of 2). Therefore transition (4.9) may be of importance for grading in polymer flooding. This may be generally put as follows: grading schemes may be based more on the size of the heterogeneities in the field (the correlation length $\lambda$) than on a fingering model such as Koval's.

Consider the following simple example of one slug with mobility ratio 1 at the front and mobility ratio 2 at the back travelling through a reservoir unit with $HI = 0.01$ (calculated with $\lambda/L = 0.01$ and $\sigma = 1.0$). Mobility ratio 2 corresponds to a Koval E-factor of 1.18 and heterogeneity index 0.01
corresponds to a Koval H-factor of 1.5, calculated with a relationship in Gardner & Ypma. The Koval K-factor is $1.5 \times 1.18 = 1.65$. The integrity of the slug is just preserved if fingers are allowed to travel from the back of the slug to the front, but not further. This occurs, according to Koval's rule, if the slug size is 0.44 pore volumes. For the calculation, see Fig. 4.14.

However, expression (4.9) predicts dispersive mixing instead of viscous fingers for this example. According to our model for dispersive mixing, the length of the mixing zone is $3.62 \sqrt{(\text{HI} \cdot X/L)}$ pore volumes. The dispersivity ratio $\alpha$ is 1.47 for $\sigma = 1.0$ and $M = 2$. Now, the integrity is preserved if the slug size is 0.22 pore volumes. For the calculation, see Fig. 4.14.

![Diagram](image)

**FIG. 4.14:** Calculation of slug size for preserving slug integrity with Koval model and with our model.
4.8. CONCLUSIONS

1. There are two flow regimes in unstable, miscible displacement: a viscous-finger-dominated regime, and a dispersive-mixing-dominated regime due to local-scale permeability variations. In the first regime the mixing zone expands linearly in time; in the second regime the mixing zone expands as the square root of time. An expression has been determined that describes the transition between the flow regimes ((4.9)).

2. Dispersive-mixing-dominated displacement instead of viscous-finger-dominated displacement occurs particularly at a moderate mobility ratio (say, below 5) and/or large heterogeneity index (say, beyond 0.04). The relevance of this work for a particular displacement process can be established from a nomogram we have constructed from our results (Fig. 4.12).

3. An unstable displacement is dispersive-mixing-dominated in the beginning and may become viscous-finger-dominated later.

4. The transition between the flow regimes has been successfully tested in a series of numerical simulations of displacement in the unstable regime and by comparison with simulation runs of Araktingi & Orr, Crump, Moissis et al. and of Waggoner. This is shown in Fig. 4.11.

5. If (4.9) predicts dispersive mixing, the length and expansion of the dispersive mixing zone in the unstable regime can be calculated with the random-walk model of Chapter 2 (as expressed here in equations (4.10) and (4.11) and Figs. 4.4 and 4.5).

REFERENCES

LIST OF SYMBOLS

Symbols used throughout this work are:

Latin

\( C \) \hspace{1cm} \text{concentration c, averaged over a transverse cross-section}
\( c \) \hspace{1cm} \text{concentration of injectant (fluid I) at a particular position}
\( D \) \hspace{1cm} \text{molecular diffusion coefficient}
\( d_P \) \hspace{1cm} \text{grain diameter}
\( E \) \hspace{1cm} \text{Koval E factor, given by (4.5)}
\( F_R \) \hspace{1cm} \text{formation resistivity factor}
\( g \) \hspace{1cm} \text{gravitational acceleration}
\( H \) \hspace{1cm} \text{Koval H factor, given by (4.4)}
\( H(x) \) \hspace{1cm} \text{height of porous medium}
\( H(x) \) \hspace{1cm} \text{Heaviside function of x}
\( HI \) \hspace{1cm} \text{heterogeneity index, defined by (4.1)}
\( I \) \hspace{1cm} \text{cumulative injection of injectant (in pore volumes) (I is also equal to dimensionless time Ut/L)}
\( K \) \hspace{1cm} \text{total dispersion coefficient (in Introduction, Chapters 1 and 3); harmonic average of two permeabilities (in Chapter 2); Koval K factor (in Chapter 4), given by (4.4)}
\( K_c \) \hspace{1cm} \text{convective dispersion coefficient at macroscopic scale in longitudinal direction}
\( K_{\text{measured}} \) \hspace{1cm} \text{measured total dispersion coefficient}
\( K_{\text{model}} \) \hspace{1cm} \text{total dispersion coefficient predicted by random-walk model}
\( K_{\text{tube}} \) \hspace{1cm} \text{dispersion coefficient of inlet and outlet tubes}
\( k \) \hspace{1cm} \text{permeability}
\( k_a \) \hspace{1cm} \text{geometric mean permeability}
\( k_{\text{grid}} \) \hspace{1cm} \text{average permeability of grid as used by simulator, given by (38.6)}
\( k_0 \) \hspace{1cm} \text{effective permeability}
\( k_1 \) \hspace{1cm} \text{permeability of block in random-walk model (k_1 = k_0/S)}
\( L \) \hspace{1cm} \text{distance between injector and producer}
\( \text{length of porous medium} \)
characteristic length in Scheidegger's dispersion model (in Chapter 2);
rectangle size in longitudinal direction (in Chapter 3)

rectangle size in transverse direction

$x_l$ value when $x_0 = li$

mobility ratio, defined by (1.2) or (2.14)

number of grid blocks per rectangle in longitudinal direction ($m = li/\Delta x$)

number of grid blocks in longitudinal direction

number of grid blocks in transverse direction

gravity number, defined by (1.3) or (2.12)

$N_G$ with effective permeability $k_0$ replaced by geometric mean permeability $k_a$

Peclet number in longitudinal direction, given by (1.9)

Peclet number from reference simulation run with $M = 1$ and/or $N_G = 0$

Peclet number from run with $M < 1$ or $M > 1$ and $N_G > 0$

Peclet number predicted by theory of Gelhar & Axness

Peclet number at laboratory scale in longitudinal direction, given by (1.4)

Peclet number at laboratory scale in transverse direction, given by (1.4)

pressure

correlation function of permeability

correlation coefficient

place vector

geometric standard deviation of permeability distribution ($S = e^\sigma$)

inhomogeneity factor

time

displacement velocity, averaged over a transverse cross-section

displacement critical displacement velocity for gravity-stable displacement, defined by (1.8a)

displacement critical displacement velocity for gravity-stable displacement, defined by (1.8b)

displacement velocity in longitudinal direction at a particular position

displacement velocity in transverse direction at a particular position
W \quad \text{width of porous medium}
X \quad \text{average distance travelled by injectant}
X_C \quad \text{distance travelled by a concentration C (C is denoted by percent)}
X_{tr} \quad \text{X value at which transition between dispersive-mixing-dominated and viscous-finger-dominated displacement occurs}
x \quad \text{coordinate in longitudinal direction}
x_0 \quad \text{coordinate of displacement front in surrounding medium}
x_1 \quad \text{coordinate of displacement front in block with permeability } k_1
y \quad \text{coordinate in transverse direction}

\textbf{Greek}

\alpha \quad \text{dispersivity ratio, defined by (2.17)}
\alpha_{model} \quad \text{a predicted by random-walk model}
\alpha_{sim} \quad \text{a determined from simulation}
\beta \quad \text{effective dispersivity: the dispersivity of a porous medium in the presence of a viscosity contrast and/or density contrast between the displacing and displaced fluids } (\beta = \alpha \beta_0)
\beta_0 \quad \text{the dispersivity of porous medium}
\Delta t \quad \text{time step in numerical simulation}
\Delta x \quad \text{gridblock size in longitudinal direction}
\theta \quad \text{dip angle}
\kappa \quad \text{microscopic dispersion coefficient in longitudinal direction}
\kappa' \quad \text{microscopic dispersion coefficient in transverse direction}
\lambda \quad \text{mobility } (\lambda = k/\mu \phi) \text{ (in Appendix 3A)}
\lambda' \quad \text{correlation length of permeability in longitudinal direction}
\mu_{I} \quad \text{dynamic viscosity of fluid I}
\mu_{p} \quad \text{dynamic viscosity of fluid P}
\mu(c) \quad \text{dynamic viscosity as a function of } c \quad (\mu(1) = \mu_I, \mu(0) = \mu_P)
\rho_{I} \quad \text{density of fluid I}
\rho_{p} \quad \text{density of fluid P}
\rho(c) \quad \text{density as a function of } c \quad (\rho(1) = \rho_I, \rho(0) = \rho_P)
\sigma \quad \text{standard deviation of log-normal permeability distribution } (\sigma = \ln S)
\sigma_f \quad \text{standard deviation of random-walk process}
\phi \quad \text{porosity}
CURRICULUM VITAE


6. dit proefschrift