Complex Impedance and Equivalent Bolometer Analysis of a Low Noise Bolometer for SAFARI

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Abstract Transition-edge-sensor (TES) bolometers are the chosen detector technology for the SAFARI Imaging Spectrometer on the SPICA telescope. For this mission, SRON is developing bolometers, each consisting of a TiAu TES that is weakly coupled to the thermal bath through thin legs of silicon nitride. In order to understand and optimize the bolometer and to verify our detector models, we characterize the devices using a series of complex impedance measurements. We apply equivalent bolometer analysis (EBA) in combination with model fitting to these data. From this analysis, we obtain important parameters that give us confidence in our understating of the response and noise of these sensitive detectors.

Keywords Complex impedance · Equivalent bolometer analysis · Bolometer theory · Noise · Transition edge sensor · SPICA · TES · Calorimeter · NEP · Alpha · Beta

1 Introduction

As part of the Japanese-led SPICA mission, the SAFARI spectrometer images the sky using cryogenically cooled optics and a Fourier transform infrared spectrometer (FTIR). The SPICA mission has selected transition edge sensor (TES) bolometers as the technology for this application. For this mission, SRON is developing TES bolometers which consist of superconducting proximity effect Ti/Au bilayers, each

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suspended on a thin silicon nitride membrane island. The island is thermally decoupled from the bath temperature by long, thin SiN legs. The detectors are operated in the superconducting phase transition at 100 mK. Phonon noise, which is associated with heat diffusing through thermal links such as the nitride legs, is expected to be the principal thermal noise source in the detectors. A detailed description of the example bolometer, which we analyze here, is given by R. Hijmering in these proceedings [1].

The design of our detectors has been guided in part by application of a distributed model described by P. Khosropanah in these proceedings [2]. Coefficients of the linearized differential equations of the model are represented using the usual matrix formulation [3, 4].

$$M_n = \begin{bmatrix} i\omega L + R_L + R_0(1+\beta) & R_0 I_0 \alpha / T & \overbrace{\begin{matrix} 0 & 0 & \cdots & 0 \end{matrix}}^n \\ -R_0 I_0 (2+\beta) & i\omega C_{TES} + G_{TES} - P_0 \alpha / T & -G_{TES} & 0 & \cdots & 0 \\ \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} & \begin{matrix} -G_{TES} \\ 0 \\ \vdots \\ 0 \end{matrix} & \begin{matrix} i\omega C + G_{TES} + G \\ -G \\ \cdots \\ 0 \end{matrix} & \begin{matrix} -G \\ i\omega C + 2G \\ -G \\ \cdots \\ -G \end{matrix} & \begin{matrix} \vdots \\ -G \\ \ddots \\ -G \end{matrix} & \begin{matrix} 0 \\ \vdots \\ -G \\ i\omega C + 2G \end{matrix} \end{bmatrix} \quad (1)$$

The heat capacity of the electrons in the TES is represented by C_{TES} . The legs are represented by a large number of bodies with equal heat capacity C . In the simplest version, the thermal couplings, G and G_{TES} , were assumed to be equal. R_L is the shunt resistance and L is the inductance in the bias circuit. The D.C. current, temperature, resistance, and power of the TES are I_0 , T_0 , R_0 , and P_0 . The parameter $\alpha_I = (T/R)\partial R/\partial T$ is the temperature sensitivity of the TES. The parameter $\beta_I = (I/R)\partial R/\partial I$ describes the sensitivity to bias current. (In the matrix, we omit the subscript.) The temperatures of bodies are evenly distributed between the bath, typically at 30 mK, and the temperature of the TES, typically near 100 mK. Noise due to the thermal couplings, resistors, and other sources can be easily added to the model, and the matrix can be inverted to find the response of the bolometer to these [3, 4].

2 Impedance Measurements

The current, resistance and the total thermal conductance to the bath, called G_0 , were obtained from I – V curves of the bolometer. The thermal conductances of the links G and G_{TES} in the model combine to give G_0 . The remaining parameters, including, and heat capacities can be found by measuring the complex impedance of the TES bolometer [3]. In impedance measurements, a small ac stimulus is added to the bias of the bolometer and the resulting current through the TES is measured as a function of frequency. The ratio of voltage and current, called a transfer function, contains information about the impedance of TES in combination other impedances in the electronic bias circuit. The Thevenin voltage and equivalent impedance of the circuit can be obtained from measurements of transfer functions when the unbiased TES is in the superconducting and normal state. The Thevenin equivalents are then used to extract the impedance of the TES from the transfer functions [5].

Fig. 1 (Color online) Complex impedance data of TES bolometer for SAFARI with fitted model. Each curve corresponds to a different bias of the TES, with resistance ranging from 17 mΩ to 85 mΩ (as plotted in Fig. 3). Normal resistance is 103 mΩ

The impedance of the model can be computed from the upper left component of the inverse of the matrix.

$$Z(w) = M_{0,0}^{-1} - (R_L + i\omega L) \quad (2)$$

In standard practice, the model's impedance is fit to the data to extract the remaining parameters including α_I , β_I , and heat capacities [5–9]. The measured impedance of the bolometer and model fit is shown in Fig. 1.

3 Equivalent Bolometer Analysis

Model fitting can be tricky because models are approximate and the impedance data are imperfect. To mitigate these problems, we apply model fitting in conjunction with *equivalent bolometer analysis* [10] (EBA): A complex bolometer can be represented by an equivalent simple bolometer with a frequency dependent heat capacity $C_{eq}(\omega)$ and thermal conductance $G_{eq}(\omega)$. It has the same α_I , β_I , resistance, etc. as the complex bolometer. All the parameters of the equivalent bolometer can be obtained directly from the impedance data, without modeling or fitting. Because these parameters are model independent, they can be used to fix some of the model parameters and to check the validity of the impedance data.

Based on EBA, alpha and beta of a bolometer are

$$\alpha_I = \frac{G_0 T_0}{P_0} \frac{Z_0 - Z_\infty}{Z_0 + R_0}, \quad \beta_I = \frac{Z_\infty}{R_0} - 1 \quad (3)$$

And $G_{eq}(\omega) = \text{Re}[G_T]$ and $C_{eq}(\omega) = \text{Im}[G_T]/\omega$, where

$$G_T(\omega) = G_0 \frac{R_0 + Z(\omega)}{R_0 + Z_0} \frac{Z_\infty - Z_0}{Z_\infty - Z(\omega)} \quad (4)$$

$C_{eq}(\omega)$ represents the amount of heat capacity that is coupled to the TES electrons. At very low frequency, it equals the heat capacity of the whole bolometer. At very high frequency it equals the heat capacity of the electrons in the TES. If the legs have significant heat capacity or if there are other hanging heat capacities in the real bolometer then this term should decrease with frequency as the bodies become decoupled from the electron system. The heat capacity of the TES is expected to change through the phase transition, so different impedance measurements may yield different values of $C_{eq}(\omega)$.

$G_{\text{eq}}(\omega)$ represents the conductance from the electrons in the TES to other bodies in the bolometer. At high frequencies, heat capacities hanging off the TES act like thermal shorts. Such shorts reduce $G_{\text{eq}}(\omega)$. Therefore, we expect the equivalent conductance to increase with increasing frequency if there is significant heat capacity in the legs. We expect the conductance of the legs to not change significantly over the narrow range in temperature of the phase transition. Therefore, the $G_{\text{eq}}(\omega)$ should be the same for all valid impedance measurements of this bolometer. (Note that this assumption might not be valid for a bolometer in which the electron-phonon (e-p) coupling [11] in the TES is dominant.)

The equivalent bolometer parameters depend on Z_0 and Z_∞ , which are low frequency and high frequency limits of the bolometer impedance. Therefore, experiments should be designed to determine these as accurately as possible. Crucially, R_0 and Z_0 almost cancel in TESs with high loop gain, so these must be measured very precisely. In addition, we note that models that do not accurately match the true impedance at very low frequency may yield incorrect values for α_I and heat capacities. We use the measured impedance at 10 Hz to 30 Hz to set Z_0 . We fit the model to the data to find the value of Z_∞ .

In Fig. 2, we plot measurements of G_{eq} and C_{eq} . Most of the G_{eq} data lays on the bottom (low TES resistance) curves. The outlying data is from higher resistance impedance data. It is suspect because it does not produce the same thermal conductance. We attribute this to an inaccurate measurement of Z_0 in that data. (We need Z_0 with an accuracy of roughly 1 m Ω , which was near the level of our experimental errors.) The thermal conductance increases with frequency as would be suspected for a distributed thermal conductance in the legs. Except in the suspect data, the heat capacity is constant across the bandwidth of the TES at 1.2 fJ/Hz. (The heat capacity data below 100 Hz was rejected because it is susceptible to small errors in Z_0 .)

We plot noise predictions from EBA in Fig. 2. They are calculated directly from the impedance data using EBA without modeling. The total measured noise (black curve) is just above the EBA upper limit (red), which is the first order nonequilibrium Johnson noise [12] (green) added to the upper limit of the phonon noise [10] (purple curve) in the TES:

$$P_{PN} < \sqrt{4k_B T_0^2 G_{\text{eq}}} \quad (5)$$

As shown in Fig. 2, the EBA predictions almost match the measured Johnson and phonon noise. The small difference may be due to calibration errors.

4 Model Fitting

We fitted the model to data of Fig. 1 with the total heat capacity of the legs, alpha and beta as free parameters. We fixed the TES heat capacity of the TES to 1.2 fJ/K, which was obtained from EBA. The resulting alpha and leg heat capacity are plotted in Fig. 3. The resulting beta values are shown in Fig. 4. The total heat capacity of the legs in the model was found to be ~ 0.6 fJ/K (except in the suspect data at high resistance). (Note that the leg heat capacity measurement is may in part correspond heat capacity of the Si island on which the TES resides.)

Fig. 2 (Color online) Equivalent bolometer measurements: (a) thermal conductance, (b) heat capacity, and (c) noise. These are derived from data of Fig. 1. In the noise plot, the *purple curve* represents the upper limit on phonon noise from EBA. The *green curve* represents the non-equilibrium Johnson noise from EBA. The *red curve* is the sum of the two noise sources. The noise predictions have scatter due to noise in the impedance data

Fig. 3 (Color online) Results from fit for α_I and the total heat capacity of the legs

We nulled the stray magnetic field [1] at the TES and repeated the impedance measurements. The resulting data and model fits are shown in Fig. 4. The curves are shifted to the right compared to Fig. 1, because nulling the field increases α_I and the speed of the bolometer. The high frequency impedance Z_∞ is also increased

Fig. 4 (Color online) Impedance after nulling the magnetic field and the effect on β_I . The β_I parameter with the nulled magnetic field (*diamonds*) is compared to measurements with the field un-nulled (*triangles*)

because β_I is affected by magnetic field. This occurred because the un-nulled field had broadened the phase transition.

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