The Effect of Non-linear Soil Behavior under Vibrating Loads

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Abstract

Seismic vibrators are used to investigate the structure of the subsurface in the framework of soil and gas exploration, also to locate faults and rupture zones in earthquake investigations. Vibrators introduce seismic waves that propagate through the ground and are received by seismic sensors at some distance from the source. This enables interpretation of the sub-soil structure by inverse analysis. To analyze the behavior of the soil below a seismic vibrator, analytical models were developed in the past in which the behavior of the soil is simplified by linear elasticity. This assumption of linear elastic soil model does not describe real soil behaviors.

In this research, a seismic vibrator standing on a homogeneous soil is simulated by means of an axi-symmetric finite element model with an advanced soil model, i.e. Hardening Soil model with small-strain stiffness, called HS-small model in which the non-linear, irreversible soil behaviors as well as small-strain stiffness effects below a seismic vibrator are taken into account.

The sign and amplitude of displacements of the ground surface in the near field and the far field are observed for a homogeneous sand and homogeneous clay. A comparison between the linear elastic model and the HS-small model in simulation the soil response is made. The response of the sand and clay under delta-pulse load, minimum-phase-wavelet load, chirp load, and harmonic load with different frequencies (5Hz and 50Hz) and amplitudes (10kN/m² and 20kN/m²) is simulated and evaluated. The influence of stress-strain dependent soil stiffness and porewater on seismic wave velocities is evaluated by means of seismograms.
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CHAPTER 1: INTRODUCTION

1.1. Problem definition

Seismic Exploration

The seismic method is a very successful technique that has been used in many fields. Its main field of application lies in the exploration, and recently also in the exploitation, of gas- and oil-reservoirs. The seismic method is dominated by reflection seismology, i.e., where focus lies in obtaining reflections from the subsurface. The seismic reflection method consists of many steps which can roughly be divided into:

- Seismic data acquisition, the stage in which seismic data is acquired. For these data, a seismic source is used to generate the seismic waves; the seismic waves then propagate through the subsurface and arrive back at the surface where they are sensed by seismic geophones. The signals from these sensors are then digitized and stored on a device like a computer;

- Seismic data processing, the stage where the raw seismic data is converted into an image of reflectors so, say, a photograph of the subsurface. An example is given in Figure 1.1;

- Seismic interpretation, the stage where the image is interpreted in terms of rock and fluid properties, such as oil and gas. It is in this stage that it is determined whether to drill for oil or gas, or not.

Figure 1.1. An example of a seismic image of the subsurface (source; DINO data base, TNO)
The work in this thesis is focused on a seismic source that is typically used on land, called Vibroseis™ (1961). A schematic picture is given in Figure 1.2. The idea behind it is that a controlled signal is put into the ground and we can control both the amplitude and the frequency content; therefore it is better controlled than a source like dynamite. However, the Vibroseis method is not a perfect method since the signal put into the ground depends on the interaction of the vibrator with the soil beneath it. Currently, there are some ways to take it into account but also this method makes some assumptions of the ground underneath the Vibroseis truck.

![Figure 1.2: An example of Vibrator truck (Vibroseis)](image)

**Geotechnical Soil modeling**

In Geotechnical Engineering, specifically in The Netherlands, the main focus of attention is on the (dynamic) behavior of soils. Soil responses and soil–structure interaction under dynamic loads have received considerable attention in, e.g., foundation engineering. However, the focus is mostly on damage control rather than on imaging reflectors, like in seismic exploration. A dynamic-load problem is typically tackled by geotechnical methods, like the settlement of the ground, ground surface motions, and susceptibility of soil to liquefaction under dynamic loads can be evaluated.

Soil is sometimes assumed to be a purely elastic material. This assumption does not represent real soil behaviors because soil is a complex material that consists of soil particles, gas and fluid in pores between grains. The interaction between those phases under loads makes the behavior of soil to be different from other materials. Soil responds differently under variation of dynamic loads. The stress-strain relationship of soil is non-linear and hysteretic, especially at shear strains larger than $10^{-5}$ to $10^{-4}$ and below dynamic loads as indicated in the literature.
Currently, various idealized soil models and analytical methods have been developed in geotechnical engineering to simulate soil behaviors and its responses under loadings. The dynamic soil response with advanced soil models can now be obtained, taking into account the nonlinear and hysteretic behaviors of soil, i.e. the Hardening Soil model (Schanz, 1998) and the Hardening Soil model with small-strain stiffness (Benz, 2006).

Advanced soil models to investigate loads as common in seismic exploration

This study is carried out to investigate the nonlinear and hysteretic soil behaviors under dynamic loads for better understanding of loads typically encountered in seismic exploration. Soil responses under dynamic loads are simulated with an advanced geotechnical-engineering soil model rather than the simple linear-elastic soil model as commonly employed in seismic exploration.

This simulation can be done by numerical methods. A well-known numerical method is finite element method (FEM) which is often applied due to its advantages and contributions to many engineering fields such as mechanical engineering and geotechnical engineering. In this thesis we will use the Plaxis software which has the advanced soil models included in their package.

Based on the above considerations, we define the objectives of this thesis in the following section.

1.2. Main objectives

The main objectives in this study can be stated as follows:

1. Investigate and evaluate the nonlinear, hysteretic soil behavior as given by the so-called Hardening-Soil model with small-strain stiffness, under dynamic loads as typically encountered in seismic exploration with the Vibroseis method. The investigation will be purely done from a modeling point of view, using the finite-element code Plaxis.

2. Compare and evaluate the differences between the responses of the Hardening-Soil model with small-strain stiffness (as commonly used in geotechnical engineering) and the linear-elastic soil model (as commonly used in seismic exploration).

The evaluation of the responses is done via the signals obtained from the dynamic modeling, such as amplitude, frequencies and velocities of separate wave arrivals (P-waves, S-waves, Rayleigh waves)
1.3. Thesis outline

This thesis is divided into five chapters and four appendices

Chapter 1 gives an introduction about the seismic method and the definition of Vibroseis using in seismic exploration. A brief description about the linear elastic soil model using in seismic exploration and an introduction about the HS-small model are given. This chapter also describes the objectives of the research.

Chapter 2 provides the general background about Vibroseis mechanical model and the elastic wave equations. Descriptions about soil behaviors that related to the concern of the research and explanations about constitutive soil models, from the simple to the advanced, are reviewed. The reasons for using HS-small model to investigate the soil response under vibrating loads in this thesis are given.

Chapter 3 deals with modelling features, i.e. model geometry, finite element mesh, input data sets, setup of calculation and simulation cases. The chosen size of model geometry and the density of finite element mesh are explained. The selection of soil types and their properties for modelling are described. A comparison between 6-node element mesh and 15-node element mesh is made for making decision of selected finite element mesh. Four types of vibration using in the simulation are introduced, i.e. harmonic load, delta-pulse load, min-phase-wavelet load, and chirp load.

In chapter 4, a comparison between the HS-small model and the linear elastic model in simulation the soil response is made. The response of different soils under different vibrating loads is investigated. The propagation of seismic waves in the soil is shown together with their velocities. The influences of soil type and porewater in seismic wave velocities are investigated.

The final chapter, chapter 5, contains the general conclusions learned from this study, and some recommendations for future research are provided.
CHAPTER 2: BACKGROUND

2.1. Introduction

In this chapter, the mechanical model of the vibrator used in the simulation is first reviewed. Secondly, a background on elastic wave equations and wave propagation is given. Then, the principle soil behaviors that are related to this study are described. Finally, the principle of constitutive soil models is explained and discussed.

2.2. Mechanical model of the Vibroseis truck

A Vibroseis truck, as shown in figure 1.1, which has a servo-hydraulic vibrator mounted on it, normally consists of 3 components, i.e. holddown mass, reaction mass, and baseplate. Holddown mass and reaction mass represent the weight of the truck. Holddown mass, about 100kN, induces uniform static load to keep the baseplate in contact with the ground. The reaction mass allows the vibrator to exert dynamic load on the baseplate by hydraulic system. Hydraulic vibrator can generate a force of 100 kN with a frequency range of 5 to 100 Hz.

These components are related to a model containing springs, a dashpot and masses. A simplified mechanical model was introduced by Lerwill (1981) which describes all components of the vibrator. The model is shown in figure 2.1.

In the model:
- $k_1$, $k_2$ are the spring constant of suspensions in reaction mass and holddown mass, respectively.
- $c$ is the viscosity of the damper.
- $f$ is the force generated by a hydraulic ram (i) and exerted on the baseplate.
- $f_1, f_2, f_3$ are the forces due to the suspensions and dashpot in the model.
The equations of motion of the holddown mass and the reaction mass are:

\[ f_2 + f_3 = -m_h \frac{d^2u_h}{dt^2} \tag{2.1} \]

\[ f_1 + f = -m_r \frac{d^2u_r}{dt^2} \tag{2.2} \]

The total force applied on the top of the baseplate, \( F_{\text{applied}} \), is given by:

\[ F_{\text{applied}} = f + f_1 + f_2 + f_3 = -m_h \frac{d^2u_h}{dt^2} - m_r \frac{d^2u_r}{dt^2} \tag{2.3} \]

where:

- \( u_h, u_r \) are the vertical displacements (positive downwards) of holddown mass and reaction mass, respectively.
- \( m_h, m_r \) are the masses of holddown mass and reaction mass, respectively.

Since it is convenient to work in the frequency domain, a Fourier transformation is performed on the equation 2.3. The force applied on the baseplate is then obtained in frequency domain as follows.

\[ F_{\text{applied}} = \omega^2 m_h u_h + \omega^2 m_r u_r, \tag{2.4} \]

in which \( \omega \) is the frequency of motion.

2.3. Purely linear elastic soil and wave propagation

In this section, a general introduction of linear elastic soil behavior is given. Linear elastic soil behavior is a common assumption in seismic exploration. Principles of compression waves, shear waves, Rayleigh waves and general aspects of seismic wave propagation in homogeneous media are presented in this section.

2.3.1. Basic equation of elasticity

A stress vector \( \tau \) acting on each plane of a cubic body as Figure 2.2 can be decomposed in two components that are normal stress and shearing stresses. For example, the stress vector acting on the plane normal to x-axis, \( \tau_x \), can be expressed as follows:

\[ \tau = \begin{bmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{xz} \end{bmatrix}, \tag{2.5} \]
in which $\sigma_{xx}$ is the normal stress, and $\sigma_{xy}$ and $\sigma_{xz}$ are shearing stresses. The subscripts $xx$, $xy$, and $xz$ indicate the directions of the stress components. The first letter indicates the plane that the stress vector component acts on and the second letter indicates the direction of the stress vector component. These three stress components fully determine the state of stress acting on the plane normal to x-axis. They are components of the first-order stress tensor, usually called stress vector.

![Figure 2.2 Stress components in a three dimensional continuum](image)

Similarly, on the planes normal to y-axis and z-axis, the components of stress tensors are:

$$\tau_y = \begin{bmatrix}
\sigma_{yx} \\
\sigma_{yy} \\
\sigma_{yz}
\end{bmatrix}, \quad \text{(2.6)}$$

$$\tau_z = \begin{bmatrix}
\sigma_{zx} \\
\sigma_{zy} \\
\sigma_{zz}
\end{bmatrix}. \quad \text{(2.7)}$$

The total stress state acting on the body can be given as follows:

$$\sigma = \begin{bmatrix}
\sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\
\sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\
\sigma_{xz} & \sigma_{yz} & \sigma_{zz}
\end{bmatrix}, \quad \text{(2.8)}$$

where $\sigma$ is called the second-order stress tensor.

Because of the symmetry in elastic soils, $\sigma_{xy} = \sigma_{yx}$, $\sigma_{yz} = \sigma_{zy}$, and $\sigma_{xz} = \sigma_{zx}$. Thus, the second-order stress tensor is written as:

$$\sigma = \begin{bmatrix}
\sigma_{xx} & \sigma_{yy} & \sigma_{zz} \\
\sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\
\sigma_{xz} & \sigma_{yz} & \sigma_{zz}
\end{bmatrix}^\prime \quad \text{(2.9)}$$
Let us denote the sides of the cube in Figure 2.2 by $\Delta x$, $\Delta y$ and $\Delta z$ ($\Delta x = \Delta y = \Delta z$). The body is now deformed only in the $(x, y)$ plane as in Figure 2.3. Suppose that the displacements at point A are $u_x$ and $u_y$. Point B and C displace as $u_x + \Delta u_x^B$, $u_y + \Delta u_y^B$ and $u_x + \Delta u_x^C$, $u_y + \Delta u_y^C$, respectively.

The normal strains and shear strains denoted by $\varepsilon$ and $\gamma$, respectively, in $x$ and $y$ directions become

$$\varepsilon_{xx} = \lim_{\Delta x \to 0} \frac{\Delta u_x^B}{\Delta x} = \frac{\partial u_x}{\partial x},$$

$$\varepsilon_{yy} = \lim_{\Delta y \to 0} \frac{\Delta u_y^C}{\Delta y} = \frac{\partial u_y}{\partial y},$$

$$\gamma_{xy} = \gamma_{yx} = \frac{1}{2} \lim_{\Delta x \to 0, \Delta y \to 0} \left( \frac{\Delta u_x^C + \Delta u_y^B}{\Delta x} \right) = \frac{1}{2} \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right).$$

Similarly, the strains in $(x, z)$ plane and $(z, y)$ plane can be derived as follows

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z},$$

$$\gamma_{xz} = \gamma_{zx} = \frac{1}{2} \left( \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right),$$

$$\gamma_{yz} = \gamma_{zy} = \frac{1}{2} \left( \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right).$$

The volumetric strain $\varepsilon_v$ of the body is:
\( \varepsilon_v = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}. \quad (2.16) \)

For an isotropic elastic material, the stresses can be expressed in terms of the strains by the generalized form of Hooke’s law:

\[ \sigma_{xx} = \lambda \varepsilon_v + 2\mu \varepsilon_{xx} \quad \text{or} \quad \sigma_{xx} = \lambda \varepsilon_v + 2\mu \frac{\partial u_x}{\partial x}, \quad (2.17) \]

\[ \sigma_{yy} = \lambda \varepsilon_v + 2\mu \varepsilon_{yy} \quad \text{or} \quad \sigma_{yy} = \lambda \varepsilon_v + 2\mu \frac{\partial u_y}{\partial y}, \quad (2.18) \]

\[ \sigma_{zz} = \lambda \varepsilon_v + 2\mu \varepsilon_{zz} \quad \text{or} \quad \sigma_{zz} = \lambda \varepsilon_v + 2\mu \frac{\partial u_z}{\partial z}, \quad (2.19) \]

\[ \sigma_{xy} = 2\mu \gamma_{xy} \quad \text{or} \quad \sigma_{xy} = \mu \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right), \quad (2.20) \]

\[ \sigma_{yz} = 2\mu \gamma_{yz} \quad \text{or} \quad \sigma_{yz} = \mu \left( \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right), \quad (2.21) \]

\[ \sigma_{zx} = 2\mu \gamma_{zx} \quad \text{or} \quad \sigma_{zx} = \mu \left( \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right), \quad (2.22) \]

Here \( \lambda \) and \( \mu \) are the Lamé constants. They are related to the Young’s modulus \( E \) and Poisson’s ratio \( \nu \)

\[ \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad (2.23) \]

\[ \mu = \frac{E}{2(1+\nu)}. \quad (2.24) \]

The stresses in equations (2.17)-(2.22) on the body must satisfy the equations of equilibrium:

\[ \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} = 0 \quad (2.25) \]

\[ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} = 0 \quad (2.26) \]

\[ \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} + \frac{\partial \sigma_{zz}}{\partial z} = 0 \quad (2.27) \]
The equations of equilibrium in term of displacements and only $\varepsilon_v$ can be obtained by substituting Equations (2.10) to (2.22) into the equations of equilibrium (2.25), (2.26), and (2.27):

$$\left(\lambda + \mu \right) \frac{\partial \varepsilon_v}{\partial x} + \mu \nabla^2 u_x = 0 \quad (2.28)$$

$$\left(\lambda + \mu \right) \frac{\partial \varepsilon_v}{\partial y} + \mu \nabla^2 u_y = 0 \quad (2.29)$$

$$\left(\lambda + \mu \right) \frac{\partial \varepsilon_v}{\partial z} + \mu \nabla^2 u_z = 0 \quad (2.30)$$

These equations are usually called the Navier equations.

### 2.3.2. Elastic equations of motion

In isotropic elastic media, the basis equations of motion of an element can be derived from Navier equations as follows:

$$\left(\lambda + \mu \right) \frac{\partial \varepsilon_v}{\partial x} + \mu \nabla^2 u_x = \frac{\partial^2 u_x}{\partial t^2} \quad (2.31)$$

$$\left(\lambda + \mu \right) \frac{\partial \varepsilon_v}{\partial y} + \mu \nabla^2 u_y = \frac{\partial^2 u_y}{\partial t^2} \quad (2.32)$$

$$\left(\lambda + \mu \right) \frac{\partial \varepsilon_v}{\partial z} + \mu \nabla^2 u_z = \frac{\partial^2 u_z}{\partial t^2} \quad (2.33)$$

where $\rho$ is the density of material and $t$ is the time.

### 2.3.3. Elastic wave equations

In soil dynamics the propagation of waves is usefully analyzed in an elastic half space (or half plane) loaded on its surface by a time-dependent load as Figure 2.4. The load may be fluctuated with time or may be applied in a very short time. For the case of a pulse load the solution has first been given by Lamb (1904), and later by others such as Pekeris (1955) and De Hoop (1960). Their solutions are simplified by only considering vertical displacements and disregarding horizontal displacements. Barends (1980) suggested an extension to problems in which the most important aspects of the solutions, such as the magnitude of vertical displacements and the effect of damping are well approximated. There are 2 categories of seismic waves, comprising body waves, i.e. compression (P) and shear (S)
waves, and surface waves, i.e. Rayleigh (R) waves. The equations of these waves are obtained as follows:

Another form of the equations of motion can be derived by differentiating the first, the second, and the third equation in the set of equations of motion (2.31), (2.32) and (2.33) with respect to \( x, y \) and \( z \), respectively then adding the result. This gives:

\[
(\lambda + 2\mu)\nabla^2 \varepsilon_v = \rho \frac{\partial^2 \varepsilon_v}{\partial t^2} \quad (2.34)
\]

This equation has a solution of the form:

\[
\varepsilon_v = f_1(t - r / c_p) + f_2(t + r / c_p) \quad (2.35)
\]

where \( r \) is the direction of the wave and \( c_p \) is the velocity of the wave, given by:

\[
c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad (2.36)
\]

This is the velocity of a wave called a compression wave or P-wave.

Another solution of the basic equations of motion (from eq. (2.31) to (2.33)) can be obtained by differentiating the first equation with respect to \( y \), the second one with respect to \( x \), and then subtracting the result, giving:

\[
\mu\nabla^2 \sigma_{xy} = \rho \frac{\partial^2 \sigma_{xy}}{\partial t^2} \quad (2.37)
\]

where \( \sigma_{xy} \) is the rotation about the z-axis

\[
\sigma_{xy} = \left( \frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x} \right) \quad (2.38)
\]

The equation (2.35) above has again the form of a wave equation, which has a solution:

\[
\sigma_{xy} = g_1(t - r / c_s) + g_2(t + r / c_s), \quad (2.39)
\]

where \( c_s \) is the velocity of the wave, given by:

\[
c_s = \sqrt{\frac{\mu}{\rho}}. \quad (2.40)
\]

This is the velocity of a wave called a shear wave, rotational wave, or simply S-wave.

Another solution of the basic equations of motion was found by Lord Rayleigh (1885). This is a wave that propagates along the free surface of an elastic half space and decays
exponentially with depth. It is called a Rayleigh wave, surface wave, or simply R-wave. The relationship between the velocity of a Rayleigh wave, \( c_r \), and the velocity of compression and shear waves can be expressed as follows (A. Verruijt, 2006):

\[
(2 - \frac{c_r^2}{c_s^2})^2 - 4 \sqrt{1 - \eta^2} \frac{c_r^2}{c_s^2} \sqrt{1 - \frac{c_r^2}{c_s^2}} = 0 ,
\]

where

\[
\eta^2 = \frac{c_r^2}{c_s^2} = \frac{(1 - 2\nu)}{2(1 - \nu)} .
\]

P-waves, S-waves, and R-waves play an important role in earthquake and seismic exploration. These waves propagate at velocities which are a function of the density and elastic properties of the ground. In soils, the compression waves (P-waves) propagate at much higher velocities than S-waves. Rayleigh waves propagate at a velocity roughly 90% of the velocity of S waves (Telford et al, 1990). Analytical solutions for different loading conditions applied to the surface of an isotropic, homogeneous, and elastic half-space have been developed by various researchers. However, the propagation of waves is very difficult to determine theoretically, especially for large vibrations and very soft soils due to the damping which may occur in the soil because of irreversible deformations. A solution that takes into account the effect of material damping has suggested by Bornitz (Bornitz, 1931) and Barkan (Barkan, 1962). The vibrations also generate soil displacements in horizontal and vertical directions. The Figure 2.4 shows a measurement of ground wave propagation.

![Figure 2.4: The measurement of ground wave propagation](image)

The table below shows a referential range of wave velocities propagation in different soils with different stiffness (Ortigao, 2007).
<table>
<thead>
<tr>
<th>Material</th>
<th>(c_p) (m/s)</th>
<th>(c_s) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay (undrained)</td>
<td>1500</td>
<td>150</td>
</tr>
<tr>
<td>Sand (dry)</td>
<td>480</td>
<td>250</td>
</tr>
<tr>
<td>Gravel (dry)</td>
<td>750</td>
<td>180</td>
</tr>
</tbody>
</table>

Table 2.1: Range of velocities of S wave and P wave

2.4. Soil behaviors

Soil generally consists of 3 phases: grains, liquids, and gases. The microscopic interactions and reactions of these phases in a soil are complex which makes it difficult to predict the responses of soil precisely. Therefore, a description and explanation of soil behavior is needed before determination seismic soil responses.

2.4.1. Stress dependency of stiffness

There is a change in the shape and/or volume of soil under loading. The parameter that relates the stress, i.e. the intensity of loading, and the strain is termed the stiffness of the soil (Whitlow, 2001). Below a certain stress level, the relationship between stress and strain of soils may be linear. This is described by Hooke’s law and soils are then called linear elastic. However, in reality most soils do not satisfy this law, for instance soils become gradually stiffer and stronger under compression. This is mainly caused by the increase of the forces between the individual particles, which give the structure of particles an increasing strength. For example, the stresses of sand layer under a thick clay layer are high due to the weight of the clay layer. This makes the sand layer stiff and strong. Hence, deep soil layers tend to have greater stiffness than shallow layers.

The stiffness of soils is nonlinear with strain and stress level, different directions of loading and drainage conditions. Stiffness of soils at small to moderate strains is one of the most important parameters in the area of soil dynamics.

The stiffness of soil can be expressed by interrelated elastic moduli, i.e., Young’s modulus \(E\), bulk modulus \(K\), or constrained soil modulus \(D\) which can be quantified from laboratory tests, i.e. the oedometer test (Janbu, 1969), triaxial test (Lambe, 1968) and in-situ tests, i.e. pressuremeter test (Baguelin et al., 1978) or full-scale loading test (Schnaid, 1993). Young’s modulus \(E\) is commonly used in soil mechanics. This modulus can be expressed either in terms of a secant modulus \(E_{sec}\) or tangent modulus \(E_{tan}\).

The determination of soil stiffness from laboratory triaxial test is shown in Figure 2.5. In soil mechanics the tangent modulus, \(E_{tan}\), is usually indicated as \(E_0\) and the secant modulus at 50% strength, \(E_{sec}\), is denoted as \(E_{50}\).
2.4.2. Strain dependency of stiffness

Soils exhibit a very high stiffness at small strains (less than $10^{-5}$). This stiffness is considered as a maximum stiffness of soil which is represented by small-strain shear modulus $G_{\text{max}}$ or $G_0$ (Burland, 1989). The small-strain shear moduli can be obtained from seismic measurements, such as $G_0$ from shear waves (Tatsuoka et al., 1997). Small-strain stiffness is a fundamental stiffness applicable to all types of soils including clays, silts, sands, gravels, and rocks (Tatsuoka et al., 2001). This stiffness is also applicable for static and dynamic loadings (Burland, 1989) and under both drained and undrained loading conditions (Lo Presti, et al., 1996) because excess pore pressures do not develop at such small strains. Small-strain stiffness of soil is important in most soil dynamic problems because shear-stress levels are relatively low and the stresses are repeated in which the resulting strains are mostly elastic or recoverable. Elastic deformation characteristics represent one of the basic parameters required to properly describe small strain deformation characteristics of soils (Tatsuoka et al., 1997). The value of small-strain shear modulus can be measured from both laboratory and in-situ tests, e.g. resonant column test (Hardin and Drnevich, 1972) and seismic cone test (Robertson et al., 1986).

Figure 2.6 shows the definitions of the small-strain shear modulus $G_0$, secant shear modulus $G_{\text{sec}}$, tangent shear modulus $G_{\text{tan}}$ and unload-reload shear modulus $G_{\text{ur}}$ in a triaxial test. At very small strains, both secant and tangent modulus converge to the small-strain shear modulus $G_0$. 

![Figure 2.5: Definition of stiffness moduli $E_0$ and $E_{50}$ from triaxial test (Plaxis manual, 2007)](image)
Laboratory reports also indicated that stress-strain relationships for the strain levels produced by seismic loadings are nonlinear and hysteretic. That proposal has been confirmed by numerous results of cyclic triaxial test on soil samples (Hardin and Drnevich, 1972a, 1972b). The relation between stress and strain of soil under cyclic loading is given by the following equation

\[ \tau = f(\gamma) \] (2.43)

where \( \tau \) and \( \gamma \) are the shear stress and shear strain respectively. This is given in Figure 2.6. In Figure 2.7, \( G_0 \) and \( G_{\text{sec}} \) are maximum and secant shear modulus corresponding to each cyclic loop.

It can be seen in Figure 2.7 that in the \((\tau, \gamma)\) plane the behavior of soil is characterized by a hysteresis loop, the surface and inclination of which depend on the strain amplitude. The
larger shear strain the wider the hysteresis loop and the flatter it is on the horizontal axis. It means that the larger the maximum strain amplitude, the lower the secant-shear modulus $G_{sec}$ as shown in Figure 2.8.

2.4.3. Material damping

Damping is the second important parameter to characterize the soil dynamic responses together with small-strain stiffness ($G_0$). Material (or hysteretic) damping is associated with the energy that is dissipated in one cycle of deformation. The damping factor is proportional to the area enclosed by the hysteresis loop and corresponds to the energy dissipated in one cycle of motion. It depends on the magnitude of the strain for which the hysteresis loop is determined as shown in Figure 2.9.

This damping ratio ($\xi$) is defined as:

$$\xi = \frac{1}{4\pi} \frac{\Delta E}{E_{max}}$$

(2.44)

where $\Delta E$ is the energy dissipated during one cycle and $E_{max}$ is the maximum strain energy stored during that cycle.
The relationship between shear modulus, damping ratio and strain is expressed in figure 2.10

![Graph showing secant shear modulus (G/G_max) and hysteresis damping ratio (ζ) as function of maximum strain](image)

Figure 2.10: Secant shear modulus (\(G/G_{\text{max}}\)) and hysteresis damping ratio (\(\zeta\)) as function of maximum strain (Plaxis manual, 2007)

### 2.4.4. Memory of pre-consolidation stress

The behavior of soil, especially clay, is significantly affected by its stress history. At a given depth an element of soil is subjected to vertical and horizontal stresses due to the weight of overburden and any superimposed loading on the surface. Under the influence of these stresses the soil has been consolidated since its deposition. This stress state of soil is called pre-consolidated stress. A soil will be called normally consolidated if it has never been subjected to stresses greater than those presently existing. If a soil was subjected to consolidating stress greater than that presently existing, it will be called as over-consolidated soil (Whitlow, 2001). This behavior can be seen in the result from laboratory tests. When the soil specimen is loaded, it will show relatively small decrease of void ratio with load up to a maximum effective stress, \(\sigma'\), to which the soil was subjected in the past. The effective stress is the stress that causes the displacements, \(\sigma' = \sigma - u_w\), in which \(u_w\) is the porewater pressure. If the effective stress on the soil specimen is increased further, the decrease of void ratio will be larger. The effect can be demonstrated by unloading and reloading a soil specimen. Observed stiffness for unloading and reloading is much higher than that in primary loading. Figure 2.11 shows the determination of pre-consolidation stress from oedometer test (Whitlow, 2001).
The ratio of the pre-consolidation stress ($\sigma_c$) to the existing vertical effective stress ($\sigma_0'$) defines the over-consolidation ratio, OCR = $\sigma_c / \sigma_0'$. Soils that are normally consolidated have an OCR = 1 and soils that are over-consolidated have an OCR > 1. Measured magnitudes of OCR for a variety of natural soft clays are in the range if 1.2 to 3 except within the desiccated crust, where they may be much higher (Terzaghi et al., 2006). There are many reasons why a soil may be over-consolidated. It could be due to either a change in the total stress ($\sigma$) or a change in pore water pressure ($u_w$); both changes would alter the effective stress ($\sigma'$).

2.4.5. Irreversible volumetric strain due to primary loading

Figure 2.11: Unloading and reloading as a function of void ratio ($e$) and effective stress ($\sigma'$) showing pre-consolidated stress

Figure 2.12: Relation between effective isotropic stress and volume change
If a body of soil with initial volume $v_0$ at the initial isotropic stress $p_0'$ is subjected to an effective isotropic stress $p_y'$, it is expected that the particles will come closer together, increasing the number of contacts and enlarging the areas of contact. The soil will become gradually stiffer, the porosity will decrease, and the deformation in all directions will become equal. The volume of the soil will be $v_y$. The soil is then unloaded to the initial stress $p_0$. The volume of soil swells back at $v_k$ which is much smaller than $v_0$ as shown in Figure 2.12. The relative volume change ($\Delta v = v_0 - v_k$) is called irreversible volumetric deformation. This volume change divided by the initial volume is called irreversible volumetric strain, which is denoted by Equation 2.45 (Verruijt, 2006).

\[
\varepsilon_{vol}^{irr} = \frac{\Delta v}{v_0}.
\] (2.45)

Changes in volume are often accompanied by changes in density, stiffness and strength. Such changes depend on the intensity and rate of loading and on the loading history of the soil. In undrained conditions, the immediate response of the soil is an increase in pore pressure. Therefore, the change in volume of the soil is related to the stiffness of both the porewater and the soil solids.

### 2.4.6. Irreversible shear strain due to mobilizing shear strength.

When a soil is loaded by increasing shear stresses, it can be expected that in the contact points between the particles the shear forces will increase. This leads to a tendency for sliding in the contact points, and therefore there will be considerable deformations. This process is called shearing. During shearing it is expected that the soil structure is not reinstated when the load is moved. Shearing therefore is an irreversible process leading to permanent deformation. As the shear-stress levels increase, the shear strain becomes progressively larger. The deformations under shearing are often larger than those with compression and the soil steadily loses rigidity.

Numerous field and laboratory observations have been made to clarify the soil behavior under cyclic loadings. The responses of soils not only depend on the frequency content of the incident ground motion but also the amplitude of loading imposed. The higher the loading level, the larger the strain induced. Large induced strains lead to highly nonlinear soil responses (Gazetas and Stokie, 1991; Mooney et al., 2005).

At moderate and large strain levels (larger than $10^{-5}$) the response of soil is highly nonlinear and inelastic because in those strain levels the soil particles rearrange during cyclic loading (Dobry et al., 1982, Ng and Dobry, 1994) thus inducing irrecoverable shear and volumetric
strains and leading to settlements in dry soil and pore pressure buildup in saturated soils. Once the pore pressure increases, the effective stresses and shear resistance decrease. Degradation of shear stiffness and loss of shear strength drive the saturated soils to increasingly nonlinear behavior. Moreover, once the amplitude of the load is no longer constant, the soil behavior becomes more complex.

### 2.4.7. Drained and undrained soil behavior

When an external stress is applied to a soil mass that saturated with pore water, the total applied stress ($\sigma$) will be balanced by two internal stress components: pore pressure ($u_w$) and effective stress ($\sigma'$).

If the loading is applied very slowly, water will be able to seep from the soil while total stress increases. There will be no change of initial pore pressure and the volume change will follow the change of loading. The initial pore water pressure ($u_w^0$) remains constant, and the change of effective stress follows the change of total stress. When the effective stress remains constant at $\sigma'_0 + \Delta\sigma'$, the volume will remain constant at $v_0 - \Delta v$. This kind of soil behavior is called drained loading because all of the drainage of pore water takes place during the loading (Atkinson, 1993).

On the contrary, if the loading is applied so quickly there will be no time for any drainage at all, then the volume remains constant. If the loading is isotropic with no shear distortion and undrained with no volume changes then nothing happens in the soil. Therefore, the effective stress remains constant: $\Delta\sigma' = \Delta\sigma - \Delta u_w = 0$ or $\Delta\sigma = \Delta u_w$. This means that the increase of pore pressure due to the increase of stress gives rise to excess pore pressure $\Delta u_w$. The pore pressure at the end of loading is $u_w = u_w^0 + \Delta u_w$. This kind of soil behavior is called undrained loading because there is no drainage of water during the loading. The most important feature of undrained behavior is no change in volume (Atkinson, 1993).

The excess pore pressure will lead to seepage and, as time passes, there will be volume changes. The rate of volume change decreases as the excess pore pressure reduces. This leads to a change of the effective stress in the soil.

### 2.4.8. Pore water generation due to seismic loadings

The generation of pore pressure within a saturated soil during seismic loadings under undrained condition is generally assumed to be due to the shear stresses and shear strains generated by propagation of shear waves, although other forms of waves also exist in the soil (Seed, 1979). In undrained condition, the load is transferred from the soil skeleton to the
incompressible pore fluid, generating excess porewater pressure. This pore water pressure accumulates and leads to a reduction of the effective stresses in the soil. Hence, a decrease in the effective stress due to the generation of excess pore water pressure results in decreased shear strength.

Dobry (1985) investigated porewater pressure generation as a function of shear strain and number of loading cycles for sands representing a wide range of relative densities. The results indicate that there is a threshold shear strain (~0.01%) below which excess porewater pressure does not develop. At larger strains, significant porewater pressure develops at shear strains greater than 0.3% and 10 cycles of loading.

2.4.9. Dilatancy

Another feature to be considered in soil behavior is the dilatancy, first reported by Reynolds (1885). Dilatancy is described as the change in volume of soil that is associated with shearing or pore pressure change in the soil (Vermeer, 1970). Naturally, when the arrangement of the soil particles is disturbed by distortion, it is to be expected that rearrangement will be accompanied by some change in the volume of soil (Wood, 2004). A suitable parameter for characterizing dilatancy is the dilatancy angle $\psi$. This parameter was introduced by Bent Hansen (1958) and represents the ratio of plastic volume change over plastic shear strain.

Dilatancy is observed in all granular soils. In dense soils, if shear stress is applied, the relative positions of particles will change and the total volume of the soil will increase. In loose soils, the shear stress will cause a reduction of the soil volume. Dilatancy may have some unexpected results, especially when soil is saturated with water. The tendency for volume decrease in a short time may lead to a large increase in pore-water pressures so that the sand particles may start to float in the water. This phenomenon is called liquefication. For soils, the dilatancy angle $\psi$ is known to be significantly smaller than the friction angle $\phi$. The dilatancy in the repetition of load does not occur in the infinitesimal and intermediate strain ranges. Its effect begins to appear when the magnitude of shear strain increases above the level of $10^{-7}$ to $10^{-3}$ (Ishihara, 1996). The illustration for dilatancy of loose sand and dense sand by shearing is provided by Figure 2.13.
2.4.10. Time and loading speed dependent behavior

Important features of soil behavior are loading speed and time dependency of stress and strain. Laboratory experiments indicated that deformations of soil are considerably influenced by the speed of the load to which the soil is subjected. The resistance to deformations and the strength of soil increase as the speed of loading and time to failure are increased.

Straining due to drainage of water is about the dissipation of excess pore water pressure that often takes long time to dissipate. Therefore, the soil strains are not only dependent on stress state, drainage of water but also on the time.

2.5. Soil models

In this section a review of the constitutive soil models based on different fundamental theories is presented. This will show the reader how we arrive at the non-linear soil model we will finally use in our seismic modeling, namely the Hardening Soil model with small-strain stiffness.

2.5.1. Isotropic linear elastic model

This is the simplest soil model still used for geotechnical engineering applications based on the Hooke’s law in which the soil is assumed to be linear elastic. This was also discussed in
section 2.3. The stress-strain relationship of the soil can be expressed by a compliance relationship indicating a dependency of stress and strain as shown in Equation 2.46.

\[ \sigma = D \varepsilon, \quad (2.46) \]

where \( D \) is the stiffness of the soil. The components in the Equation 2.46 are second-order tensors. They are often expressed as follows:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{zx}
\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix}
1-\nu & \nu & 0 & 0 & 0 \\
\nu & 1-\nu & \nu & 0 & 0 \\
\nu & \nu & 1-\nu & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}-\nu & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}-\nu \\
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\gamma_{xy} \\
\gamma_{xz}
\end{bmatrix}, \quad (2.47)
\]

in which \( E \) is Young’s modulus and \( \nu \) is Poisson’s ratio. These results can also be obtained from eq. (2.16) to (2.24). This pair of constants is sufficient to describe the linear elastic response of isotropic soils. The elastic constants which describe the behavior of soil under special conditions can be deduced from the more fundamental effective stress constants and cannot be chosen independently.

In soil mechanics the shear and bulk moduli, \( G \) and \( K \) respectively, are preferable to Young’s modulus \( E \) and Poisson’s ratio \( \nu \) because it is important to consider separately shearing by compression and swelling. They obey:

\[ G = \frac{E}{2(1+\nu)}, \quad (2.48) \]

\[ K = \frac{E}{3(1-2\nu)}. \quad (2.49) \]

Figure 2.14 shows the definitions of elastic moduli, in which \( q \) is deviator stress, \( p \) is isotropic stress, \( \varepsilon \) is linear strain, \( \gamma \) is shear strain, and \( \varepsilon_v \) is volumetric strain.
This model can be developed for the case of anisotropic and nonlinear elastic soil in which the constitutive Equation 2.46 is defined as the relationship of stress increments ($\Delta \sigma$) and strain increments ($\Delta \varepsilon$). The stiffness of soil is depended upon the direction and level of stress and strain.

### 2.5.2. Linear-elastic perfectly-plastic model

The linear-elastic perfectly-plastic model is based on the principle of Hooke’s law combined with the Mohr-Coulomb failure criterion. This model is sometimes called the Mohr-Coulomb model. In linear-elastic perfectly-plastic model the strains ($\varepsilon$) and strain rates ($\dot{\varepsilon}$) are decomposed into a linear elastic part and a perfectly plastic part as the Equation 2.50.

$$
\varepsilon = \varepsilon^e + \varepsilon^p \quad \text{and} \quad \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p
$$  

(Hill, 1950), plastic strain rates are proportional to the derivative of the yield function with respect to the stress in plastic potential function, $g$, as follows.

$$
\dot{\varepsilon}^p = \lambda \frac{\partial g}{\partial \sigma},
$$  

in which $\lambda$ is the rate form of the so-called plastic multiplier. In the case of purely elastic behavior $\lambda$ is zero whereas in the case of plastic behavior $\lambda$ is positive (Brinkgreve, 1994). Figure 2.15 shows the stress-strain relationship of a linear-elastic perfectly-plastic model.

![Figure 2.14: Definition of elastic moduli $E$, $G$, and $K$](image_url)
Figure 2.15: Stress-strain relationship of a linear-elastic perfectly-plastic model (Potts and Zdravkovic, 1999)

The stress-strain response travels along the line ABCDCF. On the first part, AB, straining the soil behaves elastically according to Hooke’s law before the stress reaches point B. There are no permanent strains. The gradient of the line AB is given by Young’s modulus, $E$. At B the yield stress, $\sigma_y$, is reached and soil becomes plastic. If the soil is strained beyond $\varepsilon_B$, to C, there is no longer linear relationship between stress and strain and the stress remains constant and equal to $\sigma_y$. If the soil is now unloaded it becomes elastic and the stress-strain curve follows the path CD, which is parallel to the path BA. There is still a strain in the soil given by $\varepsilon_C^p = \varepsilon_C - \varepsilon_B$. The stress-strain curve re-traces the path DC until point C is reached if the soil is now reloaded. At that point the stress equals to yield stress and the soil becomes plastic again along the path CF.

### 2.5.3. Hyperbolic Duncan-Chang model

This model is a type of nonlinear elastic model which represents a hyperbolic relationship between stress and strain. The original model is attributed to Kondner (1963). He proposed a hyperbolic equation based on results from undrained triaxial tests, as follows:

$$ |\sigma_i - \sigma_3| = \frac{\varepsilon_i}{E_0} - \frac{1}{q_{ult}} $$

in which $|\sigma_i - \sigma_3|$ is deviatoric stress and $\varepsilon_i$ is axial strain, $E_0$ is the initial stiffness of the primary loading curve, corresponding to the initial Young’s modulus in Figure 2.16, $q_{ult}$ is the asymptotic value of the principle stress difference that can be expressed in terms of a friction angle, $\phi$, and cohesion, $c$, according to Coulomb’s friction law, given by:
\[
q_{ult} = \frac{1}{R_f} \frac{2c \cos \varphi - 3\sigma_3 \sin \varphi}{1 - \sin \varphi},
\]

(2.54)

where \(R_f\) is the failure ratio.

![Hyperbolic stress-strain relation in primary loading for a standard drained triaxial test](image)

The original model has been extensively developed by Duncan and Chang (1970). They reported that for a number of different soils, values of \(R_f\) were found in a range from 0.75 to 1.00 (Brinkgreve, 1994). With use, further refinements were added and the model was applied to both drained and undrained problems. The number of parameter needed to define the model also increases to nine (Seed et al. 1975).

For finite-element use, it is common to reformulate and elaborate upon Equation 2.53 to obtain the tangent stiffness modulus, \(E_{tan}\), as follows:

\[
E_{tan} = E_{tan}^r \left[ \frac{\sigma_3}{p_a} \right]^m \left[ \frac{\sigma_1 - \sigma_3}{q_{ult}} \right]^2
\]

(2.55)

where \(E_{tan}^r\) is reference Young’s modulus corresponding to the atmospheric pressure \(p_a\), \(m\) is the power in stress-dependent stiffness relation, \(\sigma_1\) and \(\sigma_3\) are the maximum and minimum principal stress, respectively.

For unloading and reloading, the stiffness is supposed to depend only on the minimum principle stress and not on the principle stress difference. Therefore, the unloading-reloading stiffness is defined as follows:
\[ E_{\text{ur}} = E_{\text{ur}}^{\text{ref}} \left[ \frac{\sigma_1}{P_a} \right]^m \] \quad (2.56)

The reference stiffness for unloading-reloading \( E_{\text{ur}}^{\text{ref}} \) is substantially higher than that for primary loading.

The hyperbolic Duncan-Chang model is capable of describing specific soil characteristics such as the effect of stress-dependent stiffness, typical unloading-reloading behavior and failure. This model is an improvement over linear elasticity but still it is incapable of representing many of the important facets of soil behavior. For example, the hyperbolic Duncan-Chang model is not capable of describing the effect of dilatancy due to the assumption of a constant Poisson’s ratio. The other limitation of this model is the inconsistency with respect to neutral loading which is defined as a constant stress ratio path without change of \( \sigma_1/\sigma_3 \).

2.5.4. Hardening Soil model

The Hardening Soil model is a further development of the hyperbolic model by using the theory of plasticity rather than elasticity theory, including soil dilatancy and introducing a yield cap. The model can simulate both shearing hardening due to primary deviatoric loading and compression hardening due to oedometer loading and isotropic loading. When subjected to primary deviatoric loading, soil shows a decreasing stiffness and simultaneously developing irreversible plastic strains. The observed relationship between the axial strain and the deviatoric stress can be well approximated by a hyperbola that can be described by:

\[ \varepsilon_1 = \frac{q_a}{2E_{50}} \frac{(\sigma_1 - \sigma_3)}{q_a - (\sigma_1 - \sigma_3)} \quad \text{for } q < q_f, \quad (2.57) \]

in which, the ultimate deviatoric stress, \( q_f \), and the asymptotic value of the shear strength, \( q_a \), are defined as:

\[ q_f = \frac{6 \sin \varphi (p + c \cdot \cot \varphi)}{3 - \sin \varphi}, \quad (2.58) \]

\[ q_a = \frac{q_f}{R_f}, \quad (2.59) \]

where \( R_f \) is the failure ratio. The above expression for \( q_f \) is derived from the Mohr-Coulomb failure criterion, which involves the strength parameters \( c \) and \( \varphi \). The friction angle, \( \varphi \), ranges from 0 to 90 degree. As soon as \( q = q_f \), the failure criterion is satisfied and perfectly
plastic yielding occurs. The failure ratio $R_f$ should be smaller than 1. This hyperbolic relationship is plotted in Figure 2.17.

![Figure 2.17: Hyperbolic stress-strain relation in primary loading for a standard drained triaxial test](Plaxis manual, 2007)

In Figure 2.17, $E_{50}$ is the confining stress dependent stiffness modulus for primary loading given by the following equation:

$$E_{50} = E_{50}^{ref} \left( \frac{\sigma_3 + c \cdot \cot \phi}{\sigma_{50}^{ref} + c \cdot \cot \phi} \right)^m,$$  \hspace{1cm} (2.60)

where $E_{50}^{ref}$ is the reference secant modulus corresponding to the reference stress, $\sigma_{50}^{ref}$, and is determined from a triaxial stress-strain curve for a mobilization of 50% of the maximum shear strength $q_f$. The actual stiffness depends on the minor principal stress, $\sigma_3$, which is the effective confining pressure in a triaxial test. The amount of stress dependency of stiffness is given by the power $m$. As observed, $m$ should be taken equal to 1 for soft clays and in the range $0.5 < m < 1$ for silts and sands (Schanz et al., 1999)

For unloading and reloading stress paths, as also indicated in Figure 2.17, another reference stiffness modulus is used:

$$E_{ur} = E_{ur}^{ref} \left( \frac{\sigma_3 + c \cdot \cot \phi}{\sigma_{ur}^{ref} + c \cdot \cot \phi} \right)^m,$$  \hspace{1cm} (2.61)

where $E_{ur}^{ref}$ is the reference Young’s modulus for unloading and reloading, corresponding to the reference pressure $\sigma_{ur}^{ref}$. Doing so the unloading and reloading path is modeled as purely elastic. The elastic components of strain $\varepsilon$ are calculated according to a Hookean type of elastic relation using following equation with a constant value for unloading and reloading Poisson’s ratio $\nu_{ur}$.
\[ G_{ur} = \frac{1}{2(1+\nu_{ur})} E_{ur}. \]  

(2.62)

For drained triaxial test stress paths with \( \sigma_2 = \sigma_3 = \) constant, the unloading-reloading elastic Young’s modulus, \( E_{ur} \), remains constant and the elastic strains are given by the equations:

\[
\varepsilon_1^e = \frac{q}{E_{ur}}; \quad \varepsilon_2^e = \varepsilon_3^e = \nu_{ur} \frac{q}{E_{ur}}. \]

(2.63)

In the Hardening Soil model, the associated flow rule is defined as a relationship between rate of plastic shear strain (\( \dot{\gamma}^p \)) and plastic volumetric strain (\( \dot{\varepsilon}_v^p \)). It has the linear form:

\[ \dot{\varepsilon}_v^p = \sin \Psi_m \dot{\gamma}^p, \]

(2.64)

where \( \Psi_m \) is the mobilized dilatancy angle, defined as:

\[ \sin \Psi_m = \frac{\sin \phi_m - \sin \phi_{cv}}{1 - \sin \phi_m \sin \phi_{cv}}, \]

(2.65)

with \( \phi_{cv} \) being the critical state friction angle, constant for a certain material, independent of the density, and \( \phi_m \) being the mobilized friction angle that can be calculated:

\[ \sin \phi_m = \frac{\sigma_1^* - \sigma_3^*}{\sigma_1^* + \sigma_3^* - 2c\cos \phi} \]

(2.66)

The Hardening Soil model accounts for stress-dependency of stiffness moduli. The model also takes into account soil dilatancy and the yield surface can expand due to plastic straining.

This soil model does not account for softening due to soil dilatancy and de-bonding effects. It is an isotropic hardening model so that it models neither hysteretic and cyclic loading nor cyclic mobility of anisotropic behavior (Brinkgreve, 2002).

### 2.5.5. Hardening Soil model with small-strain stiffness (HS-small model)

The Hardening Soil model assumes elastic material behavior during unloading and reloading. However, the strain range in which soils can be considered truly elastic is very small and soil stiffness varies nonlinearly with increasing strain amplitude. A model is therefore developed to account for very small-strain stiffness and nonlinear dependency on strain of stiffness. This is called the HS-small model. The HS-small model is based on all features of the Hardening Soil model as discussed in the previous section. In addition, there are two parameters included to describe the stiffness behavior at small strains, i.e. the initial or very
small-strain shear modulus $G_0$ and the shear strain level $\gamma_{0.7}$ at which the secant shear modulus $G$ is reduced to 70% of $G_0$.

Hardin and Drnevich (1972b) proposed the relationship between stress and strain at small strains as follows:

$$\frac{G}{G_0} = \frac{1}{1 + \frac{\gamma}{\gamma_r}}, \quad (2.67)$$

where the threshold shear strain $\gamma_r$ is quantified as:

$$\gamma_r = \frac{\tau_{\text{max}}}{G_0}, \quad (2.68)$$

with $\tau_{\text{max}}$ being the shear stress at failure.

Santos and Correia (2001) suggested using of a shear strain $\gamma_r = \gamma_{0.7}$ at which the shear modulus $G_0$ is reduced to 70% of its initial value. Then the Equation 2.67 can be rewritten as:

$$\frac{G}{G_0} = \frac{1}{1 + a \frac{\gamma}{\gamma_{0.7}}}, \quad (2.69)$$

where $a = 3/7$.

The relation between stress and strain at small strains is shown in Figure 2.18.

![Figure 2.18: The modified Hardin-Drnevich relationship (Plaxis manual, 2007)](image)

The hysteretic behavior of soil in unloading-reloading cycle is described in the HS-small model by follow a rule suggested by Masing (Masing G, 1926):
- The shear modulus in unloading is equal to the initial tangent modulus for the initial loading curve.

- Doubling the primary-loading threshold shear strain in unloading-reloading curve gives \( \gamma_{0.7 \text{ un/re-loading}} = 2 \gamma_{0.7 \text{ pre-loading}} \)

The HS-small model still has limitations as it does not incorporate a gradual softening during cyclic loading in which softening plays a role and does not take into account the softening due to soil dilatancy and debonding effects. The model also does not incorporate the accumulation of irreversible volumetric straining and liquefaction behavior with cyclic loading.

2.6. Conclusion

Soil is a complex material and its behavior is significantly influenced under changes of boundary conditions including loads, moisture content, and groundwater table. A sufficient understanding about soil behaviors is needed in designs of geotechnical application.

To model soil responses under loadings, the constitutive soil models have been developed from simple to advanced ones. In the beginning, it was assumed that soil is purely elastic, then plasticity theory was used to model soil strength at failure. Further, nonlinear stress-strain relationship, stress and strain dependency of stiffness and hysteresis damping are taken into account in advanced models.

The isotropic linear elastic model is the simplest one that is based on Hooke’s law and considers soil as a linear elastic material with constant soil stiffness. The Mohr-Coulomb model and hyperbolic model are developed by using the theory of plasticity and a nonlinear relation between stress and strain. They however are not good to simulate the soil response under dynamic loadings. The HS-small model is currently the most advanced soil model even though it still has some limitations. The HS-small model takes into account the soil behavior at small strains. The features of the HS-small model are most apparent in working load conditions and give more reliable displacements than the Hardening Soil model. They also introduce hysteretic damping when used in dynamic applications. Thus, HS-small model is used to simulate soil responses in this study.
CHAPTER 3 FINITE ELEMENT MODEL FOR VIBROSEIS TEST

3.1. Introduction

The Finite Element Method (FEM) is a numerical method for solving partial differential equations. The basic of FEM is based on the discretisation of the governing equations using basis functions, well known as the finite element shape functions. The unknown fields of the problem will be described by using the shape functions and discrete nodal values which represent the amplitude of the shape functions. The principle of finite element procedures can be found throughout the literature (G.N.Wells, 2006).

Finite element programs have been widely developed and applied in engineering problems. A commercial FEM program, Plaxis, is a two-dimensional finite element computer program that performs deformation and stability analyses for static and dynamic geotechnical applications. This program can simulate real situations either by a plane strain or an axisymmetric model. The framework for this program can be found in the Plaxis manual (Plaxis version 8, 2007).

Previous numerical studies of dynamic soil responses and wave propagation used to model soil and baseplate as linear elastic materials to simplify the analysis. The Green’s functions (elastic wave equations) are used as a tool for elastic wave propagation. The elasticity assumption has limitations such as it does not take into account the stress and strain dependency of soil stiffness. This therefore does not present the real soil behavior. In this study, wave propagation and soil responses under vibrations will be simulated by an advanced soil model, i.e. the HS-small model. A comparison is made between the results obtained from the linear elastic model and from the HS-small model.

3.2. Model

The Vibroseis truck is assumed to have a circular steel baseplate located on the ground surface. A hydraulic system induces vertical vibrations on the baseplate generating seismic waves in the soil. The baseplate has a diameter of 1.0 (m) and it is assumed to be a relatively rigid material. Its thickness is calculated from the stiffness of steel. The baseplate is axially symmetric, thus stress state and deformation are assumed to be identical in any radial direction. An axi-symmetric model is therefore used to model the problem as shown in Figure 3.1 with cylindrical coordinate $x$ (radial direction), $y$ (vertical direction) and $\theta$ (circumferential direction). A beam with length of 0.5 meter is used to model to baseplate. Receivers are placed on the ground surface along the radial direction and vertical direction.
with regular distance of 2.0 meters to measure the response of the soil. In the simulations, receivers are at selected points on the finite element mesh. These points are selected on the upper boundary of the model geometry and downward to the lower boundary at the distance of 50m from the left boundary.

Figure 3.1: Circular baseplate and modelling of the problem.

3.3. Material data sets

3.3.1 Baseplate parameters

The steel baseplate with diameter of 1.0 meter is modelled by an elastic plate of 0.5m length with parameters as shown in table 3.1 below.

<table>
<thead>
<tr>
<th>Elastic material</th>
<th>$E_A$ (kN/m)</th>
<th>$EI$ (kNm²/m)</th>
<th>$d$ (m)</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>4.20E+07</td>
<td>1.40E+05</td>
<td>0.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.1: Properties of the baseplate

In this table:

- $E_A$ is the axial stiffness and $EI$ is the flexural rigidity of steel.

- $d$ is the equivalent thickness of the plate and it is calculated from the equation:

$$
\frac{E_A}{12EI} = \frac{1}{d^2}
$$

(3.1)

- $\nu$ is the Poisson’s ratio of steel; for thin structure it is advised to set it to zero in the FEM.
3.3.2. Linear elastic soil model parameters

Simulations are conducted for very dense sand and normally consolidated clay. The selected relative density of dense sand and plasticity index of normally consolidated clay are 95 percent and 50 percent, respectively.

The linear elastic soil model parameters include unit weight, Young’s modulus, Poisson’s ratio, and permeability. They are all interpreted from the relative density \((D_r)\) of sand and the plasticity index \((PI)\) of clay.

The unit weight of dry sand, \(\gamma_{\text{dry}}\), depends upon the relative density of sand and it varies in range of 13.0 to 19.0 \((kN/m^3)\). A higher relative density of sand \((D_r)\) leads to a higher unit weight and stiffness, and a lower permeability. For very dense sand \((D_r = 95\%)\), the dry unit weight is chosen as 19.0 \((kN/m^3)\).

The plasticity index \((PI)\) determines the properties of clay soil. The lower the plasticity index, the higher the density and the stiffness, and the lower the permeability. The \(PI\) is assumed 50\%. According to literatures, the unit weight of normally consolidated clay with \(PI = 50\%\) is around 16.5 to 18.0 \((kN/m^3)\). Here, the unit weight of clay for the simulation is chosen as 17.5 \((kN/m^3)\).

A linear elastic soil model usually employs secant stiffness modulus \(E_{50}\) which is the average stiffness of the soil. In geotechnical modelling, soil stiffness is usually referred to stiffness modulus at a reference confining pressure \(p^{\text{ref}}\), called reference secant stiffness modulus \(E_{50}^{\text{ref}}\).

In Plaxis, a default setting of \(p^{\text{ref}} = 100\ kN/m^2\) is used. The stiffness of sand and clay is also dependent on the relative density and plasticity index, respectively. The higher the relative density is, the higher the stiffness and the lower the permeability are. The higher the plasticity index is, the lower the stiffness and the lower the permeability are.

According to the literature, reference secant stiffness modulus of sand ranges from 20.000 to 60.000 \(kN/m^2\) for very loose sand and very dense sand, respectively. The reference secant stiffness of clay ranges from 5.000 to 20.000 \(kN/m^2\) for low plasticity index and high plasticity index, respectively. In this study, the soil is subjected to small vibrations therefore the stiffness is rather based on unloading-reloading stiffness than secant stiffness. The unloading-reloading stiffness is approximately three times larger than secant stiffness. Hence, the reference stiffness, \(E_{\text{ref}}\), of very dense sand and normally consolidated clay is chosen as 200.000 \(kN/m^2\) and 15.000 \(kN/m^2\), respectively.
Other indirect parameters such as reference shear modulus \( G^{\text{ref}} \) and the reference oedometer modulus \( E^{\text{ref}}_{\text{oed}} \) can be derived according to Hooke’s law. The compression-wave velocity and shear-wave velocity are also obtained as listed in Table 3.2.

The reference shear modulus and oedometer modulus are determined as follows:

\[
G^{\text{ref}} = \frac{E^{\text{ref}}}{2(1+\nu)} \tag{3.2}
\]

\[
E^{\text{ref}}_{\text{oed}} = \frac{(1-\nu)}{(1+\nu)(1-2\nu)} E^{\text{ref}} \tag{3.3}
\]

in which, \( \nu \) is the Poisson’s ratio

\[
\nu = \frac{K_0}{1+K_0} \tag{3.4}
\]

\( K_0 \) is the stress ratio in primary compression, \( K_0 = 1 - \sin(\phi) \), in which \( \phi \) being the internal friction angle of soil.

The wave velocities are derived by substituting equations (2.23) and (2.24) into equations (2.36) and (2.40). The wave velocities are then derived from Young’s modulus and Poisson’s ratio by the following equations:

\[
c_p = \sqrt{\frac{E^{\text{ref}}_{\text{oed}}}{\rho}} \tag{3.5}
\]

\[
c_s = \sqrt{\frac{G^{\text{ref}}}{\rho}} \tag{3.6}
\]

where

\[
E^{\text{ref}}_{\text{oed}} = \frac{(1-\nu)E^{\text{ref}}}{(1+\nu)(1-2\nu)} , \tag{3.7}
\]

\[
G^{\text{ref}} = \frac{E^{\text{ref}}}{2(1+\nu)} , \tag{3.8}
\]

\[
\rho = \frac{\gamma}{g} , \tag{3.9}
\]

in which \( \gamma \) is the total unit weight of soil and \( g \) is the gravity acceleration (9.8 m/s\(^2\)).
<table>
<thead>
<tr>
<th>Type of soil</th>
<th>$E^{\text{ref}}$ (kN/m$^2$)</th>
<th>$\nu$</th>
<th>$G^{\text{ref}}$ (kN/m$^2$)</th>
<th>$E^{\text{ref}_\text{ocd}}$ (kN/m$^2$)</th>
<th>$c_s$ (m/s)</th>
<th>$c_p$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense sand</td>
<td>200000</td>
<td>0.26</td>
<td>79370</td>
<td>244700</td>
<td>202.3</td>
<td>355.3</td>
</tr>
<tr>
<td>N.C clay</td>
<td>15000</td>
<td>0.4</td>
<td>5500</td>
<td>24070</td>
<td>55.78</td>
<td>116.1</td>
</tr>
</tbody>
</table>

Table 3.2: Set of indirect soil parameters for very dense sand and normally consolidated clay

### 3.3.3. HS-small soil model parameters

The HS-small model requires many more input parameters in comparison to the linear elastic model, including:

- **Failure parameters:**
  - Soil cohesion ($c$)
  - Angle of internal friction ($\varphi$)
  - Angle of dilatancy ($\psi$)

- **Basic parameters for soil stiffness:**
  - Power for stress-level dependency of stiffness ($m$)
  - Reference stress for stiffness ($p^{\text{ref}} = 100$ kPa)
  - Reference secant stiffness for triaxial test ($E_{50}^{\text{ref}}$)
  - Reference tangent stiffness for primary oedometer test ($E_{\text{ocd}}^{\text{ref}}$)
  - Reference unloading-reloading stiffness ($E_{\text{ur}}^{\text{ref}}$)

- **Advanced parameters**
  - Poisson’s ratio for unloading-reloading ($\nu_{\text{ur}}$)
  - $K_0$ – value for normal consolidation ($K_0^{\text{nc}} = 1 - \sin \varphi$)
  - Reference stiffness at very small strains ($G_0^{\text{ref}}$)
  - Shear strain at $G_{\text{secant}} = 0.7G_0(\gamma_{0.7})$
  - Failure ratio ($R_f$), default $R_f = 0.9$

These parameters are approximated based on the relative density of sand and plasticity index of clay as follows.
According to many references in the literature, sand is non-cohesive soil and its internal friction angle depends on the relative density, the higher the density, the higher the friction angle and dilatancy angle. The internal friction angle ($\varphi$) of sand has a range of 30 to 40 degrees, the dilatancy angle ($\psi$) a range of 0 to 10 degrees, often smaller than the friction angle. The sand for which a relative density ($D_r$) of 95 percent is chosen, is a clean sand and non-cohesive ($c = 0$). This is very dense sand and its internal friction angle and dilatancy angle are selected at 40 degrees and 10 degrees, respectively.

Clay is a cohesive soil. The cohesion and internal-friction angle are dependent upon plasticity index: the higher the plasticity index, the lower the cohesion and friction angle. According to Brinkgreve (Brinkgreve, 2004), the friction angle of clay is in the range of 15 to 30 degrees and cohesion is in the range of 2 to 20 kPa. The dilatancy angle of clay is zero. The clay is normally consolidated and its plasticity index is 50 percent. The magnitudes of friction angle and cohesion are therefore selected at 20 degrees and 16 kPa, correspondingly.

The magnitudes of reference secant stiffness and oedometer stiffness for sands are suggested by Schanz (Schanz, 1998) as follows:

$$E_{oed}^{ref} \approx 60D_r (MPa),$$

$$E_{50}^{ref} \approx E_{oed}^{ref},$$

in which $D_r$ is the relative density of sand chosen as 0.95 and the reference stress $p^{ref}$ is chosen as 100 (kN/m$^2$).

The reference secant stiffness and oedometer stiffness for clay are suggested by Brinkgreve (Brinkgreve, 2004) based on plasticity index as follows:

$$E_{oed}^{ref} = \frac{50000}{PI} (kPa),$$

$$E_{50}^{ref} \approx 2E_{oed}^{ref},$$

in which $PI$ is the plasticity index of clay, here taken as 0.5 and the reference stress $p^{ref}$ is taken as 100 (kN/m$^2$).

Under unloading-reloading, the stiffness of soil always shows a higher value than that under primary loading. The relation between reference secant stiffness and unloading-reloading stiffness is approximated by Brinkgreve (Brinkgreve, 2004) as follows:

$$E_{ur}^{ref} = 3E_{50}^{ref}.$$
In the HS-small model, the Poisson’s ratio under unloading-reloading is selected at default value $\nu_{ur} = 0.2$ for all types of soil.

The reference shear stiffness at very small strains and shear strain at secant shear stiffness reduced to 70 percent of the maximum shear stiffness are determined from the suggestion of Plaxis (Plaxis manual, 2007) by the following equations:

$$ G_{0}^{ref} = (2.5 \text{ to } 10)G_{ur}^{ref} $$  \hspace{1cm} (3.15)

where

$$ G_{ur}^{ref} = \frac{E_{ur}^{ref}}{2(1 + \nu_{ur})} $$  \hspace{1cm} (3.16)

$$ \gamma_{0.7} = (1 \text{ to } 2) \cdot 10^{-4} $$  \hspace{1cm} (3.17)

The shear strain at which secant shear stiffness reduced to 70 percent of maximum shear stiffness is chosen as $1 \cdot 10^{-4}$.

All indirect parameters of linear elastic and HS-small model are calculated by above equations based on the direct parameters. They are all listed in the Table 3.3.
<table>
<thead>
<tr>
<th>Soil model</th>
<th>Model parameters</th>
<th>Symbol</th>
<th>Unit</th>
<th>Dense sand</th>
<th>N.C clay</th>
<th>Dense sand</th>
<th>N.C clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative density/ Plasticity index</td>
<td>Dr / PI</td>
<td>%</td>
<td></td>
<td>95</td>
<td>50</td>
<td>95</td>
<td>50</td>
</tr>
<tr>
<td>Unsaturated unit weight</td>
<td>$\gamma_{\text{unsat}}$</td>
<td>kN/m$^3$</td>
<td>19</td>
<td>17.5</td>
<td>19</td>
<td>17.5</td>
<td></td>
</tr>
<tr>
<td>Saturated unit weight</td>
<td>$\gamma_{\text{sat}}$</td>
<td>kN/m$^3$</td>
<td>21</td>
<td>20.5</td>
<td>21</td>
<td>20.5</td>
<td></td>
</tr>
<tr>
<td>Shear strain at 0.7Go</td>
<td>$\gamma_{0.7}$</td>
<td>-</td>
<td>1.0E-04</td>
<td>1.0E-04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>$\nu_{\text{ur}}$</td>
<td>-</td>
<td>0.20</td>
<td>0.20</td>
<td>0.26</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>Stiffness</td>
<td>$E_{\text{ref}}$</td>
<td>kN/m$^2$</td>
<td></td>
<td>20000</td>
<td>15000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triaxial compression stiffness</td>
<td>$E_{\text{50,ref}}$</td>
<td>kN/m$^2$</td>
<td>57000</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary oedometer stiffness</td>
<td>$E_{\text{oed,ref}}$</td>
<td>kN/m$^2$</td>
<td>57000</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unloading/reloading stiffness</td>
<td>$E_{\text{ur,ref}}$</td>
<td>kN/m$^2$</td>
<td>171000</td>
<td>6000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reference pressure</td>
<td>$p_{\text{ref}}$</td>
<td>kN/m$^2$</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Un/reloading shear stiffness</td>
<td>$G_{\text{ur,ref}}$</td>
<td>kN/m$^2$</td>
<td>71250</td>
<td>2500</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Small strain stiffness</td>
<td>$G_{\text{oed,ref}}$</td>
<td>kN/m$^2$</td>
<td>214000</td>
<td>17500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate of stress-dependency</td>
<td>$m$</td>
<td>-</td>
<td>0.4</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohesion</td>
<td>$c$</td>
<td>kN/m$^2$</td>
<td>0</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Friction angle</td>
<td>$\phi$</td>
<td>degree</td>
<td>40</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dilatancy angle</td>
<td>$\psi$</td>
<td>degree</td>
<td>10</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Failure ratio</td>
<td>$R_{\text{f}}$</td>
<td>-</td>
<td>0.9</td>
<td>0.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stress ratio in primary compression</td>
<td>$K_{\text{o, inf}}$</td>
<td>-</td>
<td>0.36</td>
<td>0.66</td>
<td>0.36</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>Initial conditions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permeability</td>
<td>$k$</td>
<td>m/s</td>
<td>1.00E-03</td>
<td>2.0E-07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Over-consolidated ratio</td>
<td>OCR</td>
<td>-</td>
<td>1</td>
<td>0.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-overburden pressure</td>
<td>POP</td>
<td>kN/m$^2$</td>
<td>20</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: Input data set of soil parameters for Linear Elastic and HS-small model
3.4. Model dimensions and boundaries

The dimensions of the model are 100 m length and 40 m depth. This size is chosen since it is a setting which can typically be used for shallow seismic exploration and it is still a size that is manageable with the finite-element calculations. Also we want to minimize the influence of reflected stress waves from the boundaries leading to distortions in the results. Absorbing boundaries or viscous boundaries at the bottom and right-hand side are aimed to absorb the increments of stresses on the boundaries caused by dynamic loading. A damper that is used in a certain direction on the boundary ensures that an increase of stress is absorbed without reflection. Otherwise, generated waves due to the increase of stress on the boundaries would be reflected to the receivers.

Figure 3.2: Geometry of the model with receiver points.

The fixities are used on the boundaries to fix the horizontal displacement at the left-hand and right-hand side boundaries and vertical displacement at the bottom of the model. Rotation
fixity is applied on the baseplate at the boundary point to fix the rotational degree of freedom of the baseplate around y-axis due to the vibration of dynamic loads.

50 receiver points are located on the ground surface along the horizontal axis and 20 vertically directional receivers at the distance of 50m from the vertical axis with regular distance of 2.0m as shown in Figure 3.2. They are set up to monitor soil responses and wave propagation in horizontal and vertical directions. Uniform static and dynamic loads are applied on the baseplate with corresponding designed values.

3.5. Finite-element meshing and element size

For simulation with small-strain analysis, the soil is modelled by 15-node triangular elements in an axisymmetric mesh. In the finite element method discretisation, the maximum element size of the mesh should be smaller than half the wavelength. In order to enhance the accuracy of calculation, the mesh is generated in fine mode with a total of 610 elements. The average element size is 2 meters, providing over 3 to 30 elements per wavelength (wavelength = \(2\pi c\_s/\text{frequency}\)) and allowing the displacement field to be well characterized. At the contact area between the baseplate and the soil, finite elements are refined with smaller size of 0.5 meter and 1 meter. This performance will increase the accuracy of the numerical integration scheme. The finite element mesh is kept the same for all computation in this study. The finite element mesh is shown in Figure 3.3.

In the testing phase, significant differences in accuracy were observed in the results from 6-node element meshes and 15-node element meshes. Simulations were conducted on both meshes with the linear elastic soil model. The results show that the relation between load and
displacement is not consistently linear with the 6-node element mesh. In contrast, the load-displacement relationship is perfectly linear in the results from the 15-node element mesh. Table 3.3 shows the vertical displacement at a point in time that is simulated with the 6-node element and 15-node element mesh. It can be seen that in the 15-node element mesh, the displacement under 10kPa and 20kPa scales with factor 2 whereas this relation is not consistently a factor 2 with the 6-node element mesh. Therefore, the 6-node element mesh does not show linear behavior for the purely linear elastic soil model, whereas it should. Hence, the 15-node triangular element axisymmetric mesh is chosen to model the problem in this study.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>6-node element mesh</th>
<th>15-node element mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic time [s]</td>
<td>Uy due to</td>
<td>Uy due to</td>
</tr>
<tr>
<td></td>
<td>dynamic load</td>
<td>dynamic load</td>
</tr>
<tr>
<td>8.00E-03</td>
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<td>1.31E-08</td>
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<tr>
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<td>1.12E-08</td>
</tr>
<tr>
<td>6.22E-01</td>
<td>5.65E-09</td>
<td>1.21E-08</td>
</tr>
<tr>
<td>7.16E-01</td>
<td>6.86E-10</td>
<td>6.10E-10</td>
</tr>
<tr>
<td>8.10E-01</td>
<td>2.06E-09</td>
<td>3.35E-09</td>
</tr>
<tr>
<td>9.02E-01</td>
<td>1.88E-09</td>
<td>2.99E-09</td>
</tr>
<tr>
<td>9.94E-01</td>
<td>2.11E-10</td>
<td>1.18E-09</td>
</tr>
<tr>
<td>9.96E-01</td>
<td>1.41E-09</td>
<td>2.06E-09</td>
</tr>
</tbody>
</table>

Table 3.3: Factor of displacement at a receiver point in time under 10kPa and 20kPa obtained from 6-node element mesh and 15-node element mesh.

A comparison between a fine mode and very fine mode meshing is made to look at the accuracy and stability of the finite element code. A very fine mesh in which the elements have half the size of the fine mesh is generated for comparison. As mention above, the fine mesh is an option that provides a required number of elements to obtain satisfactory accuracy. However, simulations show a “numerical influence” with the finite element code. The results from the very fine mesh are quite different from those of the fine mesh in their details, except for the main events, while being simulated with the same conditions as shown
in Figures 3.4 and 3.5. The test is performed with both the linear elastic model and the HS-small model for sand under a 10 kN/m² delta-pulse load and a 20 kN/m² min-phase-wavelet load. Both the two soil models show the “numerical influence”. The difference in the results could be from the integration scheme applied by the finite element code or from numerical noise. This problem needs further study.

Figure 3.4: Consistency check of the finite element code with the linear elastic simulation for sand under a 10kN/m² delta-pulse load
Although there is a difference in the results between the fine mesh and very fine mesh, the fine mesh is still chosen for simulation in this thesis because of its advantages in calculation time and computer memory storage.

### 3.6. Calculation phases and time steps

First, we establish the initial stress state of the soil, generated with the unit weight of the soil itself. The initial stresses are increased with depth. The initial porewater pressure is also generated based on the groundwater table.

Then, the calculation phases are established corresponding to the installation phases of Vibroseis test. For the numerical calculations with the Vibroseis, three phases are needed:

- **In the first phase**, static load is applied on the baseplate. Drained loading soil behavior and plastic analysis are set up for this calculation. Therefore, there is no excess porewater pressure generated in the soil. The displacements of baseplate and ground, stress-strain relationship in the soil are immediately calculated.

- **In the second phase**, the dynamic load is inserted on the baseplate together with the static load in the first phase acting on plate. The displacements from the first phase will be set to zero to avoid their influence on the results (see in chapter 4). Drained loading behavior is still kept in this phase to ignore the influences of excess porewater pressure. Calculation time is selected depending on loading conditions that are listed in the next section.

- **In the third phase**, all loads are removed from the baseplate. However, the results from this phase are not used because attention is only paid to the soil response under dynamic calculation phase.
The calculation time step will affect to the solution because the accuracy of the implicit integration scheme depends on the increment of the time step. The smaller the time step is, the higher the accuracy, but also the longer the calculation time and the larger the use of computer memory. If the time step is too large, the solution will display substantial deviations and the calculated response will be unreliable. A reasonable time, step $\Delta t = 0.002$ seconds, is used to achieve a accurate solution as suggested by the following equation (Plaxis manual, 2007):

$$
\Delta t_{\text{critical}} = \frac{B}{\alpha \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)} \left[ 1 + \frac{B^4}{4S^2} - \frac{B^2}{2S} \left[ 1 + \frac{(1-2\nu)(2S)}{4} \right] \right]}}
$$

(3.18)

where $\alpha$ is the factor that depends on element type, (for the 15-node element $\alpha = 1/(19 \sqrt{c_{15}}$ with $c_{15} = 4.479$), $\nu$ is the Poisson’s ratio, $B$ is the average length of an element, and $S$ is the surface of the element. This time step $\Delta t$ is chosen to ensure that a wave during a single time step does not move a distance larger than the minimum dimension of an element.

### 3.7. Loads and boundary conditions

If the settlement due to the static load is large, it will make the soil under the baseplate inhomogeneous. The static load therefore has to be carefully selected to avoid the induction of heterogeneity in soil. The problem will be modelled under a consistently static load of 40 kN/m² acting on the baseplate for all cases. This static load keeps the baseplate in contact with the ground. The area of baseplate is 0.785 m² and the force applied on the baseplate is approximately 30 kN, smaller than the weight of holddown mass.

The amplitude of the dynamic loads should be lower than that of the static load to avoid the soil-pulling effect under the baseplate during vibration. Therefore, the amplitudes of dynamic loads exerted on the baseplate are chosen as 10 kN/m² and 20 kN/m². The frequencies of the harmonic load to be modelled are chosen as 5 Hz and 50 Hz. The large difference in the applied frequencies is selected in order to investigate the influences of separate frequencies (low and high for seismic-exploration applications) to the soil response and wave propagation. The simulation time depends upon the type of signal generated on the baseplate, i.e. one second for impact load, harmonic load, and minimum-phase wavelet and a chirp (or sweep) signal.
In this study, the baseplate is considered as a relatively rigid steel plate and its weight is neglected. The soil response and wave propagation under dynamic loads are investigated with four types of signal, i.e. harmonic, delta pulse, min-phase wavelet, and chirp (or sweep).

- Harmonic load is applied to see what happens with the response over different cycles of the load.
- Impact load creates a delta pulse so would give the Green’s function of the earth.
- Minimum-phase wavelet is a band-limited delta pulse, more in line with reality.
- Chirp signal: signal as commonly used in seismic exploration with Vibroseis (to be compared with minimum-phase wavelet after correlation with the input signal).

In the finite-element program, the amplitude of the dynamic load is given in time and multiplier domain as shown in Figure 3.6. The actual amplitude of dynamic load acting on the baseplate is a multiplier of the input value.
The spectrum of delta-pulse load, min-phase-wavelet load, and chirp load is shown in Figure 3.7.

3.8. Cases of simulation

Two cases of earth models (half spaces) will be investigated, namely, a homogeneous very dense sand (case I), an homogeneous normally consolidated clay (case II). The aim of the
modelling with the two soil types is to investigate the transmission of seismic waves in different soils. The geometry of those cases is shown in Figures 3.8 and 3.9.

Figure 3.8: Earth-model geometry of homogeneous very dense sand

Figure 3.9: Earth-model geometry of homogeneous normally consolidated clay
CHAPTER 4: RESULTS AND DISCUSSIONS

4.1. Introduction

Numerical simulations were performed to investigate the soil response under dynamic loads as encountered in seismic exploration. Both of the two soil models, i.e. the linear elastic soil model and the HS-small soil model, were used to simulate for very dense homogeneous sand and normally consolidated homogeneous clay under four types of dynamic signal: delta pulse, minimum phase wavelet, harmonic, and chirp (or sweep). Each type of those dynamic signals was simulated with two load amplitudes, i.e. 10 kN/m² and 20 kN/m². The frequency of the harmonic signals was chosen as 5 Hz and 50 Hz for both load amplitudes.

In this chapter, a comparison between the linear elastic and HS-small soil model is made. Observations and evaluations on soil response under static load and dynamic loads are conducted by means of processing result data in tables and plotting graphs. Nonlinear soil response under dynamic loads is discussed and summarized.

4.2. Settlement of soil due to static load

In reality, soils are going to settle under static loading. The amplitude of settlement depends on the type of soil and the loading amplitude, the stiffer the soil and the lower the loading amplitude, the lower the settlement. In numerical simulations, the settlement is not only dependent on the soil stiffness but also on the assumptions of the soil model that are applied in the calculation.

For that reason, a comparison between linear-elastic-model settlement and HS-small-model settlement is made. The static-load amplitude for the simulations is chosen as 40 kN/m². Under the same amplitude of static load, the extreme settlement of the homogenous sand and the homogeneous clay simulated by HS-small model is 3 times larger than that simulated by the linear elastic model. The linear elastic model gives a very small value of the extreme settlement under static load, i.e. around $1.10^{-3}$m for both the two soils. This is due to the constant soil stiffness used in the linear elastic soil model whereas it is not a constant in reality. Nonlinear and stress-strain dependency of stiffness is applied in the HS-small model therefore the settlement under static load simulated by the HS-small model could be more realistic prediction.

A comparison on settlement between the homogeneous sand and the homogeneous clay predicted by the HS-small model is also made. The settlement of the clay is ten times larger
than that of the sand. This seems to be reasonable because the stiffness of the clay is 28 times lower than stiffness of the sand.

The extreme settlement of the soils under static load is very small, therefore it can be assumed that settlement after static load does not contribute to the heterogeneity of the soil and it can be neglected. Hence, the displacement is reset to zero in the following dynamic calculation.

The extreme settlements of the homogeneous sand and the homogeneous clay under a static load of 40 kN/m² are summarized in the Table 4.1 below.

<table>
<thead>
<tr>
<th>Simulation case</th>
<th>Extreme settlement under static load (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elastic model</td>
</tr>
<tr>
<td>Very dense sand (case I)</td>
<td>0.13*10⁻³</td>
</tr>
<tr>
<td>Normally consolidated clay (case II)</td>
<td>1.61*10⁻³</td>
</tr>
</tbody>
</table>

Table 4.1: Extreme settlement of the soils under a static load of 40 kN/m².

### 4.3. Soil responses under dynamic loads

The soil response under dynamic loads can be observed via the displacement-time history of the ground surface during loading. The observations of the soil response from dynamic loads in seismic exploration are often conducted both in the near field, i.e. close to the source (within 4 wavelengths from the source), and in the far field, i.e. greater than 4 wavelengths away from the source. Displacements in the near field are due to a mixture of P-waves, S-waves, and R-waves. In the far field, displacements are dominated by R-waves.

#### 4.3.1. Linear elastic model versus HS-small model

In numerical simulation of seismic problems, the linear elastic soil model is often employed to simplify the calculation. Many assumptions are applied in the modelling. The soil is considered as purely elastic material that is the first assumption. The second assumption is no porewater in the soil or excess-porewater influence is neglected in the simulation. It therefore has limitations in the results that do not present real soil behaviors. A comparison between the linear elastic model and the HS-small model is made in order to prove the non-linear behaviors of soil under dynamic load.

Figure 4.1 shows the displacement-time history of a point on the ground surface of the homogeneous sand at a distance of 10.0 m from the centre of baseplate under harmonic load amplitude 10 kN/m² and load frequency 5 Hz simulated with both the linear elastic model and the HS-small model. The result simulated with the HS-small model clearly shows the irreversible settlement of the soil under primary loading whereas this settlement is not seen
in the result simulated with the linear elastic modeling because of the assumption of linear elasticity. This irreversible deformation under primary loading simulated with the HS-small soil model shows real soil behavior under loading.

In the result of the HS-small model simulation, after reaching the critical settlement under the primary loading the ground surface then vibrates around that level of settlement. The ground motion at the near field is consistent with the signal of vibration generated on the baseplate in the both soil models.

![Graph](image)

Figure 4.1: Near-field displacement – time history for the sand under a harmonic load 10kN/m² at 5 Hz simulated with the linear elastic model and HS-small model

The difference between the linear elastic model and the HS-small model in simulating the irreversible deformation of a soil under dynamic loads is much more pronounced in the soil response under high loading frequency. Figure 4.2 shows the soil response under a high harmonic frequency (50 Hz) simulated with the linear elastic model and the HS-small model. The HS-small model clearly shows that the irreversible deformation still develops under loading cycles after primary loading whereas there is no settlement occurring in the soil that is modelled with linear elasticity theory. That means under each loading cycle, a plastic deformation occurs in the soil and “new” stiffness at a point is established or in other words, the stiffness of soil is stress and strain dependent.
We now investigate the soil response under dynamic loads in the far field, many wavelengths away from the source. The far-field observation is conducted at a point 80.0 m from the centre of baseplate on the ground surface. Figure 4.3 below shows the far-field displacement-time history on the surface of the homogeneous sand under a 10kN/m$^2$ harmonic load at 5Hz simulated with the linear elastic and the HS-small soil model.

The far-field signal predicted by the linear elastic model is still consistent with the signal generated from the source whereas the signal predicted from the HS-small model is no longer consistent with the signal in the near field. The signal predicted by HS-small model arrives earlier in comparison with the signal predicted by the linear elastic model. This is due
to the increase of wave velocities in depth governed by HS-small model. Moreover, the amplitude of displacement in the far field predicted by HS-small model is much smaller than that predicted by linear elastic model. This difference could result from the hysteretic damping behavior of soil and the development of plastic zone in the soil described by HS-small model.

The soil response under high loading frequency in the far field is also investigated. The result from the linear elastic model shows that the signal in the far field is consistent with the signal in the near field and there is no settlement at the far field predicted by this soil model. The HS-small soil model indicates that after some first cycles of ground motion, the signal is then consistent with the signal generated from the source and there is settlement of the ground in the far field but the amplitude is lower than that in the near field. This is due to the dissipation of energy while going through the soil.

The difference between the linear elastic model and the HS-small model in dynamic soil simulation is precisely demonstrated by plotting the development and distribution of plastic points in the soil during loading. The plastic points are the stress points in a plastic state and are indicated by small symbols with different shapes and colors, depending on the type of plasticity that has occurred, in this case, friction hardening and compaction hardening.

Figure 4.4: Far-field displacement-time history for the sand under a 20kN/m² harmonic load at 50Hz predicted by the linear elastic model and the HS-small model
Figure 4.5: Plastic points in homogeneous sand under a harmonic load of 20kN/m$^2$ at 5 Hz simulated with the HS-small soil model at time $t = 0.01s$

The linear elastic model has, by definition, no plastic point occurs during loading therefore it is not necessary to put the figure here. On the contrary, there are plastic points developing in the soil when it is simulated with the HS-small model. Figure 4.5 shows the distribution of plastic points developing in the homogenous sand under a harmonic load of 5 Hz at time $t = 0.01s$. The loading cycle is 0.2s. Thus, the soil is under primary loading at time $t = 0.01s$. As can be seen, the plastic points develop beneath the baseplate, and an area of plastic points spreads out around the baseplate. That means there is an irreversible deformation under the baseplate and a propagation of a seismic wave through the soil generating plastic deformation in the soil.

The development of irreversible deformation of the soil under high loading frequency can be seen via the distribution of the plastic points from a loading cycle to other ones in the soil as shown in Figure 4.6.
The distribution of plastic points increases from the second loading cycle (Figure 4.6a) to the third loading cycle (Figure 4.6b) within the harmonic loading frequency of 50 Hz acting on the sand. That means the amplitude of irreversible deformation increases under each loading cycle. The propagation of seismic waves induces strains in the soil therefore we can see the movement and expansion of plastic points from the second loading cycle (t = 0.03s) to the third loading cycle (t = 0.05s).

The hysteretic damping behavior of the soil can be seen once we look at the time-displacement history of ground surface at different distances from the baseplate. The response of the soil under a dynamic load simulated with the HS-small soil model is highly damped out far from the source. The hysteretic damping behavior of the clay is much higher than that of the sand. The response signal of the clay is no longer consistent with the input signal at distance of 18 meters from the source under a 20 kN/m² harmonic load at 50 Hz as shown in Figure 4.7. Due to the damping behavior, the ground motion in the far field consists with eigenfrequency of the soil.
Figure 4.7: The time-displacement history at different points on the ground of the homogeneous clay under a harmonic load of 20kN/m² at 50Hz simulated with the HS-small model.

4.3.2. Response of soil types under different loading amplitudes

A comparison between the response of the sand and clay is made. Results from the HS-small simulations indicate that the clay settles much larger than the sand under the same harmonic load as shown in Figure 4.8. This is reasonable if we look at the soil stiffness of the sand and the clay. The stiffness of the sand is much larger than that of the clay. The amplitude of vertical displacement in the clay is also higher than that in the sand.
A comparison of soil response for the sand and the clay in the far field is also made. Under the same amplitude and frequency of dynamic load, the vertical displacement of the clay is much higher than that of the sand.

The Figure 4.9 shows the displacement-time history in the far field for the sand and the clay under a 20 kN/m² harmonic load at 50 Hz. There are many ground-motion cycles on the surface of the sand than on the surface of the clay. However, the magnitude of the ground motion in the sand is lower than in the clay. This is because the stiffness of the clay is much smaller than that of the sand. Therefore, the wave velocity of the clay is smaller and the travel time longer. We can see that the amplitude of the vibration in the clay is increasing over the cycles. That means there is “overlap of waves” in the clay under this high loading frequency. The signal in the far field of the clay is not consistent with the input signal due to the hysteretic damping behavior of the clay.
The response of a soil under different loading amplitudes is also investigated. The result pronounces that the higher the loading amplitude, the larger the displacement as shown in Figure 4.10. There is a difference in the amplitude of the settlement but the signals of vibration are in phase with each other.

Figure 4.10: Displacement-time history in the near field (a) and far field (b) for the sand under 20 kN/m² and 10 kN/m² harmonic loads at 5 Hz simulated with the HS-small model.

Simulations with the HS-small model also indicate that a soil under a higher frequency of the dynamic load has a higher settlement. Figure 4.11 shows a comparison of the displacement-time history for the sand under 50Hz and 5Hz at the same loading amplitude of 10kN/m². In the near field, the increase of settlement due to high frequency loading is clearly shown.
In conclusion, the HS-small soil model simulations give reasonable and feasible settlement. The extreme settlement increases with the increase of the loading frequency, the higher the frequency and loading amplitude, the larger the settlement. The clay shows higher damping behavior than the sand. Under the same dynamic loading, the stiffer the soil, the higher the settlement is.

4.3.3. Soil response under different dynamic loads

Section 4.2.2 above shows the soil response under a standard signal, i.e. harmonic load. In order to have a better view of dynamic soil response, three normally dynamic loads, i.e.
delta-pulse load, min-phase wavelet load, and chirp load, with different amplitudes are simulated. Comparisons of soil responses under those dynamic loads are made.

1. **Delta-pulse versus min-phase wavelet:** The delta pulse is a standard-dynamic load. However, in seismic exploration band-limited pulses are used. A min-phase wavelet is a type of band-limited pulses which is compared with the delta-pulse load. Figure 4.12 shows the displacement-time history in the near field and far field for the sand under a delta pulse and a min-phase wavelet with the same 10 kN/m² loading amplitude. It can be seen that the displacement of ground surface due to the min-phase-wavelet load is in phase with the ground motion generated by the delta-pulse load. However, the settlement induced by the min-phase wavelet is larger than that caused by the load delta-pulse load.

![Figure 4.12: Displacement-time history for the sand in the near field (a) and far field (b) under a min-phase-wavelet load and a delta-pulse load with the same amplitude 10kN/m² as predicted by HS-small soil model.](image)
In homogeneous clay, the signal induced from the min-phase-wavelet load is also in phase with that induced by the delta-pulse load. However, the difference in settlement due to the two loads is not as large as for the sand. Figure 4.13 shows the near-field and far-field time-displacement of the clay under a 20kN/m² delta-pulse load and min-phase-wavelet load.

![Figure 4.13: Displacement-time history for the clay in the near field (a) and far field (b) under a delta-pulse load and a min-phase-wavelet load with the same 20kN/m² amplitude as predicted by HS-small soil model.](image)

2. Min-phase-wavelet load versus chirp load

A sweep or chirp is a signal in which the frequency increases with time. This can be seen as an “extended” min-phase-wavelet signal. What in special about the sweep or chirp is that cyclic loading on the soil takes place and therefore hysteretic damping should occur. For the min-phase-wavelet, this damping would not occur. Therefore, after correlation of the sweep or chirp signal with the received response, a different result than with the min-phase-wavelet load should be obtained. The algorithm for calculation the correlated chirp signal is based on
the usual Fourier transform approach. This is not the main purposes of the study so it will not be presented here, details can be found in the literature. In this section, we investigate these differences between soil response under the min-phase-wavelet load and the correlated-chirp load.

The resulting data for the soil response under the chirp load is correlated or deconvoluted to investigate the correlation between chirp load and the min-phase-wavelet load. As can be seen in Figure 4.14, the displacement-time history of the ground surface under a chirp load after correlation is totally different from the displacement-time history of ground surface under the min-phase-wavelet load in both the near field and the far field with the same loading amplitude predicted by the HS-small model.

![Displacement-time history for the sand in the near-field (a) and far field (b) under a correlated chirp load and a min-phase-wavelet-load with the same amplitude 10kN/m² as predicted by HS-small model](image)

Figure 4.14: Displacement-time history for the sand in the near-field (a) and far field (b) under a correlated chirp load and a min-phase-wavelet-load with the same amplitude 10kN/m² as predicted by HS-small model

The amplitude of displacement of the ground surface under the min-phase-wavelet load is larger than that under the correlated chirp load. Wave components
can be clearly distinguished in the displacement-time history of soil under correlated chirp load. In the far field, simulations indicate that the signal from the correlated chirp load is damped in comparison with the signal from the min-phase-wavelet load. The amplitude of displacement of the ground under the correlated chirp signal is lower than that under min-phase-wavelet load even those loads have the same amplitude.

Under the same loading amplitude, the min-phase-wavelet load induces much settlement on the clay whereas there is no settlement induced by the correlated chirp load. The magnitude of displacement on ground surface generated from the correlated chirp load is also much lower than that generated from the min-phase-wavelet load and the signal from the correlated chirp load is also more strongly damped in the far field compared to the signal from the min-phase-wavelet load as shown in Figure 4.15.

![Displacement-time history](image-url)

Figure 4.15: Far-field displacement-time history of the clay under delta-pulse load (a) and correlated chirp load (b) with the same amplitude 10kN/m² predicted by HS-small soil model
4.4. Seismograms

The seismogram that shows the overall soil response and wave propagation through the soil with separated wave arrivals, such as P-waves, S-waves, and R-waves can be obtained by extracting the displacement-time history for all selected points on the ground surface and in the borehole.

A testing phase is performed with the finite difference method (FDM) to compare to the finite element method (FEM). Simulations were performed with the two methods for the same soil and loading conditions. The linear soil model and delta-pulse load are applied in the simulations. The seismogram simulated with the FDM (Figure 4.16a) shows smooth results. However, the wave arrival is a curvature in the near field. This could be the influences due to the mixture of different ground motions, whereas it should be a straight line like the result from the FEM as shown in Figure 4.16b. This is because the applied soil stiffness in both two methods is constant therefore the wave velocity should also be constant. The noise in FEM seismogram could result from the “numerical influence” of the finite element code as explained in section 3.5.

Figure 4.16: Adapt test FDM versus FEM both on the linear elastic soil model
In spite of the advantages of the FDM, the available FDM codes only use to the linear elastic soil model. Therefore, it can not take into account the nonlinear and hysteretic behavior of real soils. Hence, the FEM code is employed for simulation in this thesis.

**4.4.1. Linear elastic model versus HS-small model.**

In order to show the difference between the linear elastic model and the HS-small model in the dependence of seismic wave velocity with depth, a diagram about the increment of compression wave velocity with depth for the clay was made. Figure 4.17 indicates that the compression wave velocity in the elastic model is a constant, independence with depth, due to the assumption of constant soil stiffness in the linear elastic model, whereas it nonlinearly increases with depth in the HS-small model. This is because the HS-small model simulates that the soil stiffness is stress-strain dependent.

![Figure 4.17: Increment of compression wave velocity with depth in the clay simulated with the linear elastic model and the HS-small model.](image)

A comparison between the linear elastic model and the HS-small model is once again made by looking at the seismograms of the soils.

As can be seen in the seismograms of the soil simulated with the linear elastic model and the HS-small model, wave components propagating in the soil, i.e. the P-wave, S-wave, and R-wave, can be well discerned as shown in Figure 4.18. The direction wave arrivals of these waves are denoted by color lines along the “peaks” of displacement amplitudes in the seismograms. One can distinguish between P-wave, S-wave, and R-wave by determining the velocity of these waves.
Moreover, the linear elastic model predicts that the velocity of these waves are constant in time and space, as seen in the seismogram, the train of “peak” displacement amplitudes is a straight line. This is due to the assumption of constant soil stiffness in the linear elastic model.

On the contrary, the seismogram from the HS-small model shows that the trains of “peak” displacement amplitudes are curved. This means that the velocities of these waves are nonlinear in time and space. This is because the wave velocity is dependent on soil stiffness, which is stress-strain dependent, in other words, at depth the HS-small model predicts a
higher stiffness. The velocities of P-wave and S-wave predicted by the HS-small soil model are higher than those given from the linear elastic soil model. The velocity of R-wave is the same in both the two soil models. Last but not least, one can see that there is a dispersion of the S-waves and R-waves in the seismogram simulated with the HS-small soil model.

In both the two seismograms, there are still wave reflections occurring at the boundary of the model even there is a viscous boundary is applied. This shows that the absorbing boundary condition in Plaxis finite element code is not performing perfectly.

Figure 4.18b above shows the seismogram on ground surface of the sand under a 10 kN/m\(^2\) delta-pulse simulated with the HS-small model. Three wave velocities are determined, based on the slopes of colored lines indicated in Figure 4.19b, at about 500 m/s, 250m/s, and 190 m/s corresponding to P-wave, S-wave, and R-wave, respectively. Whereas, the predicted values are 355 m/s and 202 m/s for P-wave and S-wave, respectively. That means the HS-small model gives higher wave velocities than predicted by the linear elastic model.

Another comparison between the linear-elastic and HS-small simulations is made by plotting seismic simulations at different depths, simulating a borehole. A series of points in the vertical direction is selected at a horizontal distance of 50 m from the centre of the baseplate. Seismograms show that the wave velocities are assumed to be constant in the linear elastic soil model; therefore, the curvature of “peaks” downwards with depth as shown in Figure 4.19a, whereas the HS-small simulations produce seismograms in which the wave velocity increases with depth, as shown by the upward trend of the curvature (Figure 4.19b). This is due to the stress-dependence of soil stiffness, i.e. the deeper, the higher the stresses and higher the soil stiffness and the higher the wave velocity.

![Figure 4.19: Seismograms in a borehole for the sand under a 10 kN/m\(^2\) delta-pulse load simulated with the linear elastic soil model (a) and the HS-small soil model (b).](image)
4.4.2. Different soils

In order to investigate the effects of nonlinear and hysteretic soil behaviors, seismograms of different soil types are made. The first thing that can be seen from Figure 4.20 is the difference in wave velocities between the sand and the clay. Seismic waves propagate at higher velocities in sand and at lower velocities in clay under the same amplitude and frequency of the dynamic load. The seismograms clearly show that wave velocities are nonlinear in the clay. The second thing that can be seen from the seismograms is the dependence of wave velocities on the soil stiffness. Figure 4.20 shows the surface seismograms of the sand and the clay under a 10 kN/m² harmonic load at 5 Hz simulated with the HS-small soil model.

Figure 4.20: Seismograms on surface for the sand (a) and the clay (b) under a 10 kN/m² harmonic load at 5 Hz simulated with the HS-small soil model.
Seismograms for both the sand and the clay at different depths are also simulated with the HS-small model. They also indicate that wave velocities increase with depth and wave velocities in the sand is higher than those in the clay as shown in Figure 4.21 below.

Figure 4.21: Seismograms in borehole for the sand (a) and the clay (b) under a 10 kN/m² harmonic load at 5 Hz simulated with the HS-small soil model.

Instead of using a constant of soil stiffness in the linear elastic model, an increment of soil stiffness with depth was applied in the linear elastic model to investigate the changing of wave velocities. The incremental input values were chosen to get the same elastic stiffness values predicted by the HS-small model at the depth of 40 meters. Figures 4.22, 4.23 and 4.24 show the increment of elastic stiffness, compression and shear wave velocities with depth, respectively, in the sand and clay simulated with the two soil models.

Figure 4.22: Increment of elastic stiffness with depth for the linear elastic model with the input value of $E_{\text{increment}} = 16500$ kN/m²/m and 35000kN/m²/m for sand (a) and clay (b), respectively, in comparison with the HS-small model.
It can be seen that the stiffness of clay is highly dependent on the depth. With the same elastic stiffness, the compression wave velocity predicted by the linear elastic model is higher than predicted by the HS-small model. The shear wave velocity is predicted the same in both the two models.

A simulation was performed for the linear elastic model in case the soil stiffness increasing with depth. An incremental input value of elastic stiffness was used for clay in the linear elastic model, i.e. $E_{\text{increment}} = 3500\,\text{kN/m}^2/\text{m}$. At the depth of 40 meters, the stiffness of clay predicted by the linear elastic model is the same as predicted by the HS-small model, showing in Figure 4.22b. The corresponding wave velocities are shown in Figures 4.23b and 4.24b. The seismogram of this simulation is shown in Figure 4.25 below. From the seismogram, it can be see that the compression wave velocity predicted by the linear elastic model with an increment of soil stiffness with depth is also nonlinearly increasing as indicated by curvatures of wave arrivals.
Another simulation was conducted with the clay by applying an increment of elastic stiffness in the linear elastic model to get the same compression wave velocity as predicted by the HS-small model. In that case, the elastic stiffness at the depth of 40 meters predicted by the linear elastic model is lower than predicted by the HS-small model. The shear wave velocity in linear elastic model is also lower than that in the HS-small model as shown in Figure 4.26, 4.27 and 4.28.

Figure 4.25: Seismogram of the clay under a harmonic load simulated with the linear elastic model with $E_{\text{increment}} = 3500 \text{ kN/m}^2/\text{m}$

Figure 4.26: Increment of elastic stiffness with depth for the input value of $E_{\text{increment}} = 1740 \text{ kN/m}^2/\text{m}$ in the linear elastic model in comparison with the HS-small model for clay.
Figure 4.27: Increment of $C_p$ with depth for the input value of $E_{\text{increment}} = 1740 \text{kN/m}^2/\text{m}$ in the linear elastic model in comparison with the HS-small model for clay.

Figure 4.28: Increment of $C_s$ with depth for the input value of $E_{\text{increment}} = 1740 \text{kN/m}^2/\text{m}$ in the linear elastic model in comparison with HS-small model for clay.

The seismogram of the clay also indicates that the compression wave velocity predicted by the linear elastic model is the same as predicted by the HS-small model (Figure 4.29).

Figure 4.29: Seismogram of the clay simulated with the linear elastic model with $E_{\text{increment}} = 1740 \text{kN/m}^2/\text{m}$.
4.4.3. Different loading amplitudes and frequencies

The difference in wave velocity between low amplitude and high amplitude loads can be hardly seen and therefore not shown here. However, the difference in wave velocity between low frequency and high frequency is shown in the velocity of the R-wave. Under high loading frequency, the velocity of R-wave is higher than that under low loading frequency, while the P-wave and S-wave velocities appear the same. The seismogram shows some noises in the result of soil response under high loading frequency. This could be due to the numerical noise. Figure 4.30 shows the seismograms on the surface of the sand under a 20 kN/m² harmonic load at frequencies of 5 Hz and 50 Hz.

Figure 4.30: Seismograms on surface for the sand under 5 Hz (a) and 50 Hz (b) harmonic loading frequencies with the same 20 kN/m² amplitude simulated with the HS-small soil model.
4.4.4. Min-phase-wavelet load and correlated chirp load

There is a large difference that can be seen in the seismograms of a soil under a min-phase-wavelet load and correlated chirp load, not only in the amplitude of wave velocities but also in the shapes of arrivals as shown in Figure 4.31. Under the same amplitude of dynamic load, the wave velocities under correlated chirp load are lower than that under min-phase-wavelet load as indicated by the angle of the colored lines. This is due to the damping of the signal while traveling in the soil.

Figure 4.31: Seismograms for the sand under min-phase-wavelet load (a) and correlated-chirp load (b) with the same amplitude 20 kN/m² simulated with the HS-small soil model.
In order to see the development and propagation of seismic waves through the soil, one can look at the development and movement of plastic points in the soil in different time steps.

Figure 4.32 shows the propagation of seismic waves via development and movement of plastic points in the sand under a 50 Hz harmonic-load frequency in two loading cycles. As can be seen in the figure, the area of plastic points spreads in the soil due to the movement of seismic waves which induce shear strain in the soil.

![Figure 4.32: Development and movement of plastic points in two loading cycles in the sand under a 50 Hz harmonic load at 20 kN/m² simulated with the HS-small soil model.](image)

(a) $t = 0.20s$

(b) $t = 0.22s$

4.5. The role of porewater

As mentioned in chapters 1 and 2, porewater plays an important role in the soil especially in dynamic soil responses. The influence of porewater on the soil response and wave
propagation will be investigated by simulating the sand saturated with groundwater at 2 meters below the ground surface and fully saturated sand in undrained loading condition. The permeability of the sand is indicated in Table 3.3.

Simulations with HS-small model show that the amplitude of displacement in case of soil saturated with porewater is lower than that in case of dry soil in both the near field and the far field as shown in Figure 4.33. The higher the groundwater table is, the higher the amplitude of ground motion. The figure also shows that the signals are out of phase in the far field whereas they are in phase with each other in the near field. That means material damping occurs in the soil saturated with water.

Figure 4.33: Displacement-time history in the near field (a) and far field (b) of the sand under 20 kN/m² harmonic load at 5 Hz simulated with the HS-small soil model in cases with and without porewater.

The seismograms of the two cases also indicate the considerable influences of porewater in the transmission of seismic waves. The first thing that can be seen is the difference in wave velocities. The seismic waves transmit at much higher velocities when the soil saturated with
water in compared with the case of no water in the soil. The higher the ground water table is, the higher the wave velocities. The second thing is that the amplitude of vibration close to the source is much lower than that below the groundwater table as seen in Figure 4.34b. That is reasonable because water is non-compressive material therefore it well transmits seismic waves.

Figure 4.34: Seismogram of the dry sand (a) and saturated sand (b) under a 20 kN/m² harmonic load at 5 Hz simulated with the HS-small soil model.
CHAPTER 5: CONCLUSION AND RECOMMENDATION

5.1. Conclusions

In this thesis we investigated the effect of non-linear soil behavior on responses commonly obtained in seismic exploration. For the non-linear soil model we chose the HS-small model. The HS-small soil model shows the nonlinear relationship between stresses and strains whereas this relationship is linear in the elastic soil model. The HS-small soil model describes real soil behaviors much better, for example the stress-strain dependent soil stiffness, hysteretic damping behavior, soil dilatancy, that are not included in the linear elastic soil model commonly used in seismic exploration.

For the simulations, we used the FEM as implemented by Plaxis. We found that the accuracy of dynamic soil responses with the FEM is highly dependent on the type of finite element mesh which is chosen. The 15-node finite element mesh gives better results in comparison to the 6-node element mesh. The accuracy of calculation is also dependent on the size of the element. The smaller the element size is, the higher the accuracy but the longer the calculation times and higher the memory storage.

We simulated the “seismic” responses under different circumstances, namely:

- Different soil types: very dense sand and normally consolidated clay;
- Different types of loads: impact loads, min-phase-wavelet loads, harmonic loads and chirp/sweep loads;
- Different amplitudes and frequencies of loads;
- Different regions of observation: near field and far field;

The results of these simulations were compared with the responses from the linear elastic soil model. First of all, it needs to be said that the differences between the responses of the two soil models are generally different, and therefore the nonlinear soil behavior has a significant effect. Simulation of the soil response under dynamic loads with the HS-small soil model shows irreversible soil deformation under primary loading whereas the linear elastic soil model cannot show this soil behavior. The responses generated by dynamic loading predicted by the linear elastic and the HS-small model are in phase in the near field but dephased in the far field due to the hysteretic damping soil behavior simulated with the HS-small model.
Based on the responses at different distance from the load, apparent velocities can be determined. In general, the seismic wave velocities predicted by the HS-small soil model are higher than that predicted by the linear elastic soil model, particularly in the homogeneous clay because the stiffness of clay is higher dependent on the stress than the homogeneous sand. Also the loading frequency has a positive influence on the R-wave, the higher the frequency, the higher the R-wave velocity. Loading frequency does not show its influence on the P-wave and S-wave velocities. Loading amplitude has a positive effect on the seismic wave velocities however under a small range of loading amplitude the difference in wave velocity is not much shown. Due to the damping behavior, the velocities of seismic waves generated by chirp load are lower than that generated by min-phase-wavelet load.

5.2. Recommendations

This study considers only the response of the homogeneous soil. Therefore, the transmission and reflection of seismic waves at the interface between soil layers would be the topic for further study.

It is recommended to investigate the effects of loading amplitude on wave velocities by applying a large range of loading amplitudes.

The reflections of seismic waves on the boundary of the model still occur even though the viscous-boundary conditions are applied. This is due to the numerical method used in the finite element code.

The noise and “numerical influence” of the finite element code should be improved to achieve smooth calculation results.
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APPENDIX

A. Finite element method for geotechnical applications

A.1. Introduction

The Finite Element Method (FEM) is a numerical method for solving partial differential equations. It is particularly suited for solving partial differential equations on complex geometry, as are commonly encountered in geotechnical engineering. The procedure can be largely automatised, making it well suited to efficient computer implementation. The application of FEM often leads to very large systems of linear equations which can be solved using a computer.

A.2. Basic concepts of FEM

The basic of FEM is based on the discretisation of the governing equations using basis functions, well known as the finite element shape functions. The unknown fields of the problem will be described by using the shape functions and discrete nodal values which represent the amplitude of the shape functions.

The simplest shape functions are continuous, linear functions. There are more complex shape functions which are encountered in many problems. Therefore, it is useful to define shape functions on finite elements. A mesh divides the geometry of the problem into a number of elements (Figure A.1). The mesh consists of nodal points and elements. Nodes are points in space, while elements are lines (in 1D), surfaces (in 2D) or volumes (in 3D) which are constructed by joining nodal points. The finite elements in 2D problems are usually triangular or quadrilateral in shape.

Figure A.1: A typical two dimensional finite element mesh and 2D finite elements
In Plaxis finite element code, the 15-node and 6-node triangular finite elements are used. A 15-node element consists of 15-nodes and a 6-node triangle is defined by 6 nodes. The distribution of nodes over the elements is shown in Figure A.2. Adjacent elements are connected through their common nodes. During a finite element calculation, displacements \((u_x, u_y)\) are calculated at the nodes. In contrast to displacements, stresses and strains are calculated at individual Gaussian integration points (or stress points) rather than at the nodes. A 15-node triangular element contains 12 stress points as indicated in Figure A.2a and a 6-node triangular element contains 3 stress points as indicated in Figure A.2b.

The 15-node element provides an accurate calculation of stresses and failure loads. The 6-node triangles are available for a quick calculation.

Each node has a number of degrees of freedom that correspond to discrete values of the unknowns in the boundary value problem to be solved. Associated with each node is a hat-like basis function, called shape function, which has a value of one at the node and zero at all other nodes as shown in Figure A.3.

In axisymmetric finite element analysis, it is usual to adopt cylindrical coordinate \(x\) (radial direction), \(y\) (vertical direction) and \(\theta\) (circumferential direction). Cartesian coordinates used for plane strain analysis. There are four non-zero stresses \((\sigma_x, \sigma_y, \tau_{xy} \text{ and } \sigma_\theta)\), and two
displacements \((u_x \text{ and } u_y)\) in the \(x\) and \(y\) direction, respectively. Therefore, the strains reduce to:

\[
\varepsilon_x = -\frac{\partial u_x}{\partial x}; \quad \varepsilon_y = -\frac{\partial u_y}{\partial y}; \quad \gamma_{xy} = -\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}; \quad \varepsilon_\theta = -\frac{u_x}{x}; \quad \gamma_{x\theta} = \gamma_{y\theta} = 0
\]

(A.1)

Figure A.4: Examples of axisymmetric problem

Each unknown field is interpolated using the polynomial shape functions.

The displacement field for an element is given by:

\[
\begin{bmatrix}
u_x \\
u_y \\
u_x^2 \\
u_y^2 \\
\vdots \\
u_x^n \\
u_y^n
\end{bmatrix} = [N]
\begin{bmatrix}
u_{x1} \\
u_{y1} \\
u_{x2} \\
u_{y2} \\
\vdots \\
u_{xn} \\
u_{yn}
\end{bmatrix},
\]

(A.2)

in which, \([N]\) is the matrix contained the element shape functions. It has the form:

\[
N = \begin{bmatrix}
N_1 & 0 & N_2 & 0 & \ldots & N_n & 0 \\
0 & N_1 & 0 & N_2 & \ldots & 0 & N_n
\end{bmatrix}
\]

(A.3)

\(n\) is the numbers of nodes of the element,
\[ \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ \vdots \\ u_{x_n} \\ u_{y_n} \end{bmatrix} \] is the vector storing the nodal degrees of freedom of \( n \) nodes.

The strain field is given by:

\[ \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varepsilon_{\theta} \end{bmatrix} = [B] \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ \vdots \\ u_{x_n} \\ u_{y_n} \end{bmatrix}. \tag{A.4} \]

The matrix \([B]\) contains derivatives of the shape functions \( N_i \). It has the form:

\[ [B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \cdots & \frac{\partial N_n}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & \cdots & 0 & \frac{\partial N_n}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_n}{\partial y} & \frac{\partial N_n}{\partial x} \\ \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_n}{\partial x} & \frac{\partial N_n}{\partial y} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{A.5} \]

The stress field is given by:

\[ \{\sigma\} = [D] \{\varepsilon\} \tag{A.6} \]

where

\[ \{\sigma\}^T = \begin{bmatrix} \sigma_x & \sigma_y & \tau_{xy} & \sigma_{\theta} \end{bmatrix}. \tag{A.7} \]

\([D]\) is the stiffness matrix.

**A.3. Analytical steps of FEM**

The FEM analysis involves the following steps:
Element discretisation: This is the process of modelling the geometry of the problem under investigation by an assemblage of finite elements. These elements have nodes defined on the element boundaries, or within the element.

Primary variable approximation: A primary variable is selected (e.g. displacements, stresses, etc.) and rules as to how it should vary over a finite element established. This variation is expressed in terms of nodal values.

Element equations: Use of an appropriate variational principle (e.g. minimum potential energy) to derive element equations: \([K_E]\Delta d_E = \{\Delta R_E\}\). Where, \([K_E]\) is the element stiffness matrix, \(\{\Delta d_E\}\) is the vector of the incremental element nodal displacements and \(\{\Delta R_E\}\) is the vector of incremental element nodal forces.

Global equations: Combine element equations to form global equations: \([K_G]\Delta d_G = \{\Delta R_G\}\). Where, \([K_G]\) is the global stiffness matrix, \(\{\Delta d_G\}\) is the vector of all incremental nodal displacements and \(\{\Delta R_G\}\) is the vector of all incremental nodal forces.

Boundary conditions: Formulate boundary conditions and modify global equations. Loadings \(\{\Delta R_G\}\) (e.g. line and point loads, pressures and body forces) affect, while the displacements \(\{\Delta d_G\}\) affect.

Solve the global equations: The global equations are in the form of a large number of simultaneous equations. These are solved to obtain the displacements \(\{\Delta d_G\}\) at all the nodes. From these nodal displacements, stresses and strains are evaluated (Potts and Zdravkovic, 1999).

B. Series of simulations

In order to sort out the work, a matrix of simulations is made as indicated in the following table.
### B.1. Number of simulations for the linear elastic soil model

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Table B.1: Series of simulations for the linear elastic soil model

### B.2. Number of simulation for HS-small soil model

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Table B.2: Series of simulations for the HS-small soil model
C. Seismograms

C.1. Linear elastic simulations

1. Delta-pulse loads

Figure C.1: Seismograms for the sand under a 10kN/m² delta-pulse load

Figure C.2: Seismograms for the sand under a 20kN/m² delta-pulse load

Figure C.3: Seismograms for the clay under a 10kN/m² delta-pulse load
2. Min-phase-wavelet loads

Figure C.4: Seismograms for the clay under a 20kN/m² delta-pulse load

Figure C.5: Seismograms for the sand under a 10kN/m² min-phase-wavelet load

Figure C.6: Seismograms for the sand under a 20kN/m² min-phase-wavelet load

Figure C.7: Seismograms for the clay under a 10kN/m² min-phase-wavelet load
3. Harmonic loads

Figure C.8: Seismograms for the clay under a 20kN/m^2 min-phase-wavelet load.

Figure C.9: Seismograms for the sand under a 10kN/m^2 harmonic load at 5 Hz.

Figure C.10: Seismograms for the sand under a 10kN/m^2 harmonic load at 50 Hz.

Figure C.11: Seismograms for the sand under a 20kN/m^2 harmonic load at 5 Hz.
Figure C.12: Seismograms for the sand under a 20kN/m$^2$ harmonic load at 50 Hz.

Figure C.13: Seismograms for the clay under a 10kN/m$^2$ harmonic load at 5 Hz.

Figure C.14: Seismograms for the clay under a 10kN/m$^2$ harmonic load at 50 Hz.

Figure C.15: Seismograms for the clay under a 20kN/m$^2$ harmonic load at 5 Hz.
4. Correlated chirp loads

Figure C.16: Seismograms for the clay under a 20kN/m² harmonic load at 50 Hz.

Figure C.17: Seismograms for the sand under a 10kN/m² correlated chirp load

Figure C.18: Seismograms for the sand under a 20kN/m² correlated chirp load.

Figure C.19: Seismograms for the clay under a 10kN/m² correlated chirp load
Figure C.20: Seismograms for the clay under a 20kN/m² correlated chirp load

C.2. HS-small simulations

1. Delta-pulse loads

Figure C.21: Seismograms for the sand under a 10kN/m² delta-pulse load

Figure C.22: Seismograms for the sand under a 20kN/m² delta-pulse load
2. Min-phase-wavelet loads
3. Harmonic loads

Figure C.27: Seismograms for the clay under a 10kN/m² min-phase-wavelet load

Figure C.28: Seismograms for the clay under a 20kN/m² min-phase-wavelet load

Figure C.29: Seismograms for the sand under a 10kN/m² harmonic load at 5Hz

Figure C.30: Seismograms for the sand under a 10kN/m² harmonic load at 50Hz
Figure C.31: Seismograms for the sand under a 20kN/m² harmonic load at 5Hz

Figure C.32: Seismograms for the sand under a 20kN/m² harmonic load at 50Hz

Figure C.33: Seismograms for the clay under a 10kN/m² harmonic load at 5Hz

Figure C.34: Seismograms for the clay under a 10kN/m² harmonic load at 50Hz
4. Correlated chirp loads

Figure C.35: Seismograms for the clay under a 20kN/m² harmonic load at 5Hz

Figure C.36: Seismograms for the clay under a 20kN/m² harmonic load at 50Hz

Figure C.37: Seismograms for the sand under a 10kN/m² correlated chirp load.

Figure C.38: Seismograms for the sand under a 20kN/m² correlated chirp load.
D. Displacement-time history

Figure D.1: Near-field displacement-time history for the sand under a 10kN/m² harmonic load at 5Hz and 50Hz simulated with the HS-small model.
Figure D.2: Far-field displacement-time history for the sand under a 10kN/m² harmonic load at 5Hz and 50Hz simulated with the HS-small model.

![Graph showing displacement-time history for sand under 10kN/m² load at 5Hz and 50Hz](image)

Figure D.3: Near-field displacement-time history for the sand under 10kN/m² and 20kN/m² harmonic loads at 5Hz simulated with the HS-small model.

![Graph showing displacement-time history for sand under 10kN/m² and 20kN/m² loads at 5Hz](image)

Figure D.4: Far-field displacement-time history for the sand under 10kN/m² and 20kN/m² harmonic loads at 5Hz simulated with the HS-small model.

![Graph showing displacement-time history for sand under 10kN/m² and 20kN/m² loads at 5Hz](image)

Figure D.5: Near-field displacement-time history for the sand and the clay under 10kN/m² harmonic loads at 5Hz simulated with the HS-small model.

![Graph showing displacement-time history for sand and clay under 10kN/m² load at 5Hz](image)
Figure D.6: Far-field displacement-time history for the sand and the clay under 10kN/m² harmonic loads at 5Hz simulated with the HS-small model.

Figure D.7: Near-field displacement-time history for the clay under 10kN/m² harmonic loads at 5Hz and 50Hz simulated with the HS-small model.

Figure D.8: Far-field displacement-time history for the clay under 10kN/m² harmonic loads at 5Hz and 50Hz simulated with the HS-small model.
Figure D.9: Near-field displacement-time history for the clay under 10kN/m$^2$ and 20kN/m$^2$ harmonic loads at 50Hz simulated with the HS-small model.

Figure D.10: Far-field displacement-time history for the clay under 10kN/m$^2$ and 20kN/m$^2$ harmonic loads at 50Hz simulated with the HS-small model.

Figure D.11: Near-field displacement-time history for the clay under a 20kN/m$^2$ delta-pulse load and a 20kN/m$^2$ harmonic load at 50Hz simulated with the HS-small model.
Figure D.12: Far-field displacement-time history for the clay under a 20kN/m$^2$ delta-pulse load and a 20kN/m$^2$ harmonic load at 50Hz simulated with the HS-small model.

Figure D.13: Near-field displacement-time history for the clay under a 20kN/m$^2$ delta-pulse load and a 10kN/m$^2$ harmonic load at 50Hz simulated with the HS-small model.

Figure D.14: Far-field displacement-time history for the clay under a 20kN/m$^2$ delta-pulse load and a 10kN/m$^2$ harmonic load at 50Hz simulated with the HS-small model.