Mechanisms of shear transfer 
in cracks in concrete

A survey of literature

by

Ir. J. C. Walraven
MECHANISMS OF SHEAR TRANSFER IN CRACKS IN CONCRETE

A survey of literature

Ir. J.C. Walraven
Stevin Laboratory
University of Technology
Delft - Holland

Report 5-78-12
Foreword

This study on "Mechanisms of shear transfer in cracks in concrete" has been performed within the research project "Betonmechanica" (means concrete mechanics). The aim of the project is to develop computer programs based on the finite element method which are able to analyse arbitrary plane structures of reinforced or prestressed concrete with any type of loading until failure. Besides all, the shear behaviour of structures should receive the main attention.

For this purpose, several research institutions in the Netherlands are working closely together, each of them dealing with a special part of the whole project. One of the special tasks of the working group at the Delft University of Technology is the experimental investigation of the concrete behaviour in the vicinity of a crack. The result of these efforts should lead to a material description which is suitable and sufficient for the insertion into the computer programs.

During the preparation of the experimental work, an extensive literature survey has been carried out in which all available experimental results and the respective theories are critically studied and compared. Because we thought that this study would be a useful contribution to the present discussion of this topic we decided to publish it.

The project "Betonmechanica" is financially supported and organized by the CUR, the research committee in the Dutch Concrete Association. We are greatly indebted to the CUR and her members who have contributed to this issue.

Delft, December 1978

Dr.-Ing. H.W. Reinhardt
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. NOTATIONS</td>
<td>1</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>3</td>
</tr>
<tr>
<td>2. SHEAR TRANSFER BY AGGREGATE INTERLOCK</td>
<td>9</td>
</tr>
<tr>
<td>2.1. Introduction</td>
<td>9</td>
</tr>
<tr>
<td>2.2. Shear transfer in cracks under monotonic loading</td>
<td>10</td>
</tr>
<tr>
<td>2.3. Shear transfer in cracks under cyclic loading</td>
<td>18</td>
</tr>
<tr>
<td>2.4. Shear transfer in cracks under impact loading</td>
<td>27</td>
</tr>
<tr>
<td>3. SHEAR TRANSFER IN CRACKS AND JOINTS BY DOWEL ACTION</td>
<td>30</td>
</tr>
<tr>
<td>3.1. Introduction</td>
<td>30</td>
</tr>
<tr>
<td>3.2. Dowel action in planar elements under monotonic loading</td>
<td>31</td>
</tr>
<tr>
<td>3.2.1. Theoretical approximation of the load-displacement relation, based on the model of a beam on an elastic foundation</td>
<td>34</td>
</tr>
<tr>
<td>3.2.2. Mechanisms developed to predict the ultimate bearing capacity of dowels</td>
<td>47</td>
</tr>
<tr>
<td>3.3. Dowel action in planar elements under cyclic loading</td>
<td>53</td>
</tr>
<tr>
<td>3.4. Comparison between the contribution of aggregate interlock and dowel action to the total shear transfer in cracks</td>
<td>55</td>
</tr>
<tr>
<td>3.5. Dowel action in beams</td>
<td>58</td>
</tr>
<tr>
<td>3.5.1. Investigations in order to assess the dowel cracking load in beams</td>
<td>59</td>
</tr>
<tr>
<td>3.5.2. Investigations to study the behaviour of the dowel mechanism after cracking, when stirrups supply an additional support</td>
<td>65</td>
</tr>
<tr>
<td>4. SHEAR TRANSFER IN CRACKS CROSSED BY REINFORCEMENT</td>
<td>73</td>
</tr>
<tr>
<td>4.1. Introduction</td>
<td>73</td>
</tr>
<tr>
<td>4.2. Reinforced cracked concrete under monotonic loading</td>
<td>73</td>
</tr>
</tbody>
</table>
4.2.1. Prediction of the ultimate shear bearing capacity of reinforced cracks

4.2.1.1. Reinforcement normal to the crack plane

4.2.1.2. Reinforcement inclined to the crack plane

4.2.2. Fundamental behaviour of reinforced cracked concrete under a monotonic increasing load

4.3. Reinforced cracked concrete under cyclic loading

4.4. Reinforced cracked concrete subjected to impact loading

5. CONCLUSIONS

6. REFERENCES
0. **NOTATIONS**

All values used in this report are expressed in N, mm, N/mm², unless indicated otherwise.

- \( b \) width of a section
- \( b_n \) width of a section minus the sum of the bar diameters
- \( d \) depth of a section
- \( f \) "free-length" of a bar (not supported by bearing stresses)
- \( f'_{cc} \) cube compression strength
- \( f'_{cyl} \) cylinder compression strength
- \( f_{ctu} \) concrete splitting strength
- \( f_{sy} \) yielding strength of steel
- \( v \) shear stress in a section
- \( v_u \) ultimate shear stress in a section
- \( s \) distance of steel bars
- \( w \) crack width
- \( y_o \) sum of dowel displacements at one side of a crack
- \( y_{oc} \) value of \( y_o \) for which a dowel crack occurs
- \( A \) area of a section
- \( A_c \) concrete area
- \( A_s \) steel area
- \( E \) modulus of elasticity
- \( F_d \) dowel force
- \( F_{dcr} \) dowel cracking force
- \( F_{du} \) ultimate dowel resistance
- \( G \) shear stiffness modulus of uncracked concrete
- \( G^* \) shear stiffness modulus of cracked concrete
- \( G_f \) foundation modulus of concrete
\( \alpha \)  
factor reducing the shear stiffness modulus after cracking \((\alpha = G^*/G)\)

\( \Delta \)  
shear displacement

\( \gamma \)  
shear deformation

\( \varepsilon \)  
strain

\( \varepsilon_{cr} \)  
strain at which cracking occurs

\( \theta \)  
inclination of bars against plane of cracking

\( \phi \)  
opening direction of a crack

\( d \)  
bar diameter

\( \rho \)  
reinforcement ratio

\( \rho_0 \)  
percentage of reinforcement

\( \sigma_r \)  
radial stress

\( \sigma_n \)  
normal stress

\( \sigma_\psi \)  
tangential stress

\( \mu \)  
coefficient of friction

\( \tau \)  
bond stress
INTRODUCTION

By virtue of the development of new numerical techniques in general, also a strong impulse has been given to the development of particular calculation methods, describing the behaviour of structures. The most important step extending the reach of the calculations was the introduction of the finite element method, by means of which - initially only on basis of assumed linear elastic behaviour - much more complicated structures could be analyzed than before. The actual development of non-linear calculation procedures is of special importance for concrete structures, which exhibit usually a strong non-linear character over a considerable range of loading. However, it must be realized, that the accuracy of any calculation method depends greatly on the reliability of the material properties inserted. This report focuses on one of these properties, about which a lot of confusion is met, namely the shear stress - shear displacement relation of cracked reinforced concrete.

Shear displacements in cracks can occur for several reasons:

- due to anisotropical properties of the crack itself, if the direction of the reinforcement is not normal to the crack plane,

- due to redistribution of forces in adjacent parts of the structure as a result of cracking,

- due to modifications in the external loading acting on the structure.

Furtheron, interface shear transfer in cracks is not only important since it has to be involved in the formulation of the stress-strain behaviour of cracked concrete, but also because it may contribute significantly to the bearing capacity of structures. As an example, the contribution of the shear transfer in the cracks in a beam without stirrups, subjected to shear, may amount to 50 - 90% of the total resistance, according to experiments of FENWICK [20], TAYLOR [78] and SWAMY [74] (Fig. 1.1). Also CEDOLIN [10] showed in a numerical analysis of a similar structure, that slight modifications in the formulation of the shear stress-displacement relation in the cracks resulted in significant differences in the total shear resistance.
The opinions of several authors, about the way in which the mechanism of interface shear transfer in cracks has to be introduced into calculation, vary widely. NGO/SCORDELIS [60], NILSON [62] and STAUDER [72] introduced cracks in their non-linear FEM-programs by disconnecting the nodes between elements. In this way discrete cracks are obtained, each side of which is considered to be an independent external boundary, unable to transfer shear stresses (Fig. 1.2). The disadvantage of this method is that the crack directions are confined to the direction of the element edges. Another, more generally applied method, which is not submitted to this restriction, is the system of cracks, smeared out over the elements. It is assumed that, if the tensile strength of the concrete in an element is reached, an infinite number of identical cracks with the same direction occurs (Fig. 1.3).

The elements are then considered as continua with anisotropical behaviour. The assumption that these cracks behave as smooth surfaces, unable to transmit shear stresses, was done by CERVENKA [11] and LOOV [47]. Inherent to this assumption is that after cracking, the main stress directions are fixed in the direction of the cracks; hence a redistribution of forces after cracking cannot be expected.
The shear stiffness modulus, which was equal to $G$ before cracking, is after cracking immediately reduced to zero (Fig. 1.4).

Fig. 1.4 Crack characteristics according to [11, 47].

Maintainence of the full shear resistance after cracking was proposed in the non-linear finite element programs developed by FRANKLIN [24], ISENBERG/ADHAM [35], ZIENKIEWICZ/PHILLIPS/OWEN [91], SWOBODA [75], and MÜLLER [58, 59] (Fig. 1.5). The same assumption is also used in some analytical methods, predicting the shear strength of structural members, on basis of a plasticity model (BRAESTRUP/NIELSEN [6, 61], THUERLIMANN [82], CEE-Model Code [8]). Also EBBINGHAUS [16] takes a fully maintained $G$-value after cracking into account, but adds the condition that this value is reduced to zero, if the shear stress in the crack exceeds a certain limit. This limit decreases for increasing total crack width (Fig. 1.6).

Fig. 1.5 Crack characteristics according to [6, 8, 24, 35, 58, 59, 61, 75, 82, 91].

Fig. 1.6 Shear modulus of cracked concrete as a function of total crack width and shear stress according to [16].
A formulation which is often used implies, that the shear stiffness of an element is reduced after cracking to a lower value, greater than zero, according to the equation:

\[ v = a \cdot G \cdot \gamma \]

in which \( a \) is a predetermined constant with \( 0 < a < 1 \) (Fig. 1.7).

This type of formulation was used by HAND/PECKNOLD/SCHNOBRICH [31] \((a = 0.4)\), YUZUGULLU/SCHNOBRICH [89] \((a = 0.2)\), SUIDAN/SCHNOBRICH [73] \((a = 0.5)\), LIN/SCORDELIS [45], CEDOLIN/DEI POLI [9], and KRISHNAMURTHI/PANEERSELVAM [42]. In a later publication CEDOLIN and DEI POLI improved this formulation relating \( a \) to the strain normal to the direction of the cracks (Fig. 1.8) [10]. The suggested expression for \( G \) is:

\[ G = C \left( 1 - \frac{\varepsilon_\perp}{\varepsilon_p} \right) \quad \text{if} \quad \varepsilon_\text{cr} < \varepsilon_\perp < \varepsilon_p \]

\[ G = 0 \quad \text{if} \quad \varepsilon_\perp > \varepsilon_p \]
in which C is a numerical constant (suggested value 0.1 E), $\varepsilon_1$ is the fictitious strain (the contribution of the concrete to the strain is put equal to zero) in the direction normal to the crack and $\varepsilon_\text{p}$ is a limit value, after which interface shear transfer becomes 0 ($0.0035 < \varepsilon_\text{p} < 0.0045$). Also SCHIMMELPFENNIG [70] proposed a decreasing value of $\alpha$, in combination with a maximum shear stress, depending on the total crack width of the element (Fig. 1.9).

![Fig. 1.9 Shear modulus of cracked concrete as a function of crack width and ultimate stress values according to [70].](image)

However, although relating the value $\alpha$ to the development of the crack widths results in an improved expression for $\alpha$, also this formulation is rather provisional, since it seems to be obvious that other factors may influence the shear stiffness as well, as the distribution of crack widths and distances, the concrete quality, the value of the shear displacement and the load history. These aspects are taken into account by SCHÄFER [69], who stated that the initial resistance of two crack interfaces against shear displacement can be neglected over a certain length (Fig. 1.10), while after this "free sliding" a linear relation between shear stress and shear displacement was assumed.

![Fig. 1.10 Free sliding according to SCHÄFER [69].](image)
For the free sliding theoretical assumptions were made, the linear relation afterwards was based on test results [34]. In combination with formulas for the average crack width and distance, a relation was derived which relates \( \alpha \) both to the strain normal to the crack direction and to the shear displacement (Fig. 1.11).

![Fig. 1.11 Reduction factor \( \alpha \) as a function of the strain distribution \[69\].](image)

GEISTEFELD [27] deals, in his two-dimensional tension stiffening model, with the shear stiffness in a similar way, but does not take into account the free sliding.

None of the authors takes the effect of wedging action in the cracks into account, which may develop when a shear displacement of the crack interfaces relative to each other occurs, and which results in compressive stresses normal to the crack plane. It is obvious, considering the great scatter in opinions on this subject, that the basic properties of the mechanism are generally not very well known, and that a study on this field is worthwhile. In the next chapters of this report a survey of literature is presented, focusing on the shear transfer in cracks.

Chapter 2 deals with experiments on pure "aggregate interlock", an expression introduced by FENWICK [20] for the phenomenon of shear transfer from concrete to concrete. In chapter 3 the mechanism of dowel action, transfer of forces due to the stiffness of reinforcing bars normal to their longitudinal axis, is treated and in chapter 4 the combined action of both mechanisms is discussed. In all chapters different types of loading are considered: monotonic increased loading, cyclic loading and impact loading.
2. SHEAR TRANSFER IN CRACKS BY AGGREGATE INTERLOCK

2.1. Introduction.

In order to avoid confusion when talking about crack width, it is emphasized, prior to dealing with the phenomenon of aggregate interlock, that the crack width, as defined in this report, is just the displacement, measured normal to the crack plane, that the parts of the construction, separated by the crack, have undergone in relation to each other. This definition is necessary, since on micro-scale the local crack widths vary considerably when a shear displacement has occurred (Fig. 2.1). As defined here, the crack width is independent on the shear displacement.

![Fig. 2.1 Local variations of crack width after a shear displacement.](image)

Because the strength of the matrix-material in normal concrete is generally lower than the strength of the aggregate particles, a crack develops, intersecting the matrix but avoiding the aggregate, which results in a crack plane that is not smooth, but exhibits a certain degree of roughness (Fig. 2.2). The roughness may be influenced by the gradation of the aggregate and by the ratio of strength between matrix and aggregate: a lightweight concrete, in which the aggregate particles are relatively weak, and a high strength concrete, in which the matrix is relatively strong exhibit a smoother crack plane than concretes with average strengths.

![Fig. 2.2 Roughness of crack plane.](image)
It is obvious that the shear stress-strain relation must depend on the total contact area between the crack interfaces. Therefore the shear resistance of the crack may be expected to be enhanced if the crack width is decreased and the shear displacement is increased. Furtheron it must be realized that a shear displacement, if the crack width is kept constant, does not result only in shear stresses, but also, due to the wedging action between the particles, in a reaction normal to the crack plane. For the description of the behaviour of cracked reinforced concrete this action is of importance, most of all because it enables a link between the action of the reinforcement, which can be expressed as a normal force on the crack plane, and the stresses in the crack itself. Next, an important feature is that due to high stress concentrations in the cracks a deterioration of the material may occur, which makes it necessary to take also the aspect of load history into account. Since considerable differences may be expected between the behaviour under monotonic increased loading and repeated loading those subjects are dealt with separately.

2.2. Shear transfer in cracks under monotonic loading.

FENWICK [20] conducted tests on specimens as represented in Fig. 2.3. The specimen was cracked prior to testing by an external tensile force. The crack plane was predestinated by a groove along the outline of the specimen. After cracking the crack width was kept constant on values ranging from 0.06 mm to 0.38 mm.

![Shear transfer in cracks under monotonic loading](image-url)

Fig. 2.3 Test equipment used by FENWICK and PAULAY [20].
To obtain a constant crack width during a test, after every load increment the crack width was adjusted by means of an external force normal to the crack plane. Values for this force are not given. The shear displacements as well as the width of the crack were measured by mechanical strain gages on the top and bottom surfaces of the block. In the first series of tests the concrete strength was kept constant on $f'_{ccyl} = 33$ N/mm$^2$, and the width of the preformed crack was varied. The average curves obtained are represented in Fig. 2.4. In the second series of tests the influence of the concrete strength was investigated for a constant crack width of $w = 0.19$ mm. The results of this series are shown in Fig. 2.5. Due to the small dimensions of the test specimens a considerable scatter in readings was obtained, for which reason the tests were repeated five to six times for each set of variables. The specimens failed all as a result of flexural tension cracking of the concrete. In no case was the aggregate interlock action observed to break down, so that the relations are only valid for a restricted area (dotted line in Fig. 2.4), in which the block did not exhibit flexural cracks. From a regression analysis of the test results as a relation between shear stress and shear displacement was derived:

$$
\tau = \frac{0.928}{w} - 0.6584 \left( \sqrt{f'_{ccyl}} - 1.4465 \right) \left( \Delta - 0.0436 \, w \right).
$$

**Fig. 2.4** Shear stress - displacement relations for constant crack widths according to [20].

**Fig. 2.5** Shear stress - displacement relations for different concrete qualities, according to [20].
HOUDE and MIRZA \cite{34} carried out an investigation on specimens which were to a certain extent comparable with those of FENWICK (Fig. 2.6).

![Test equipment as used by HOUDE/MIRZA \cite{34}](image)

Fig. 2.6 Test equipment as used by HOUDE/MIRZA \cite{34}.

In the test series the influence of the crack width, the concrete strength and the maximum aggregate particle size were the object of investigation. Varying the concrete compression strength $f'_{ccyl}$ between 16.5 and 51 N/mm$^2$, the influence of this strength on the shear stress-displacement relation was found to be proportional with $\sqrt{f'_{ccyl}}$. A set of experimentally obtained shear stress-displacement curves is represented in Fig. 2.7.

![Shear stress-displacement relations for constant crack widths according to HOUDE/MIRZA \cite{34}](image)

Fig. 2.7 Shear stress-displacement relations for constant crack widths according to HOUDE/MIRZA \cite{34}.
The influence of the maximum aggregate particle size appeared to be negligible when compared with the influence of the crack width. For a concrete strength $f'_{ccyl} = 31 \text{ N/mm}^2$ the shear stress–displacement relation was expressed as:

$$\tau = 2.06 \left(\frac{1}{W}\right)^{3/2} \Delta$$

The disadvantages concerned with the test arrangements used by FENWICK/PAULAY and HOUDE/MIRZA were avoided in experiments carried out by LOEBER/PAULAY [65]. The type of specimen used in their tests is represented in Fig. 2.8.

![Fig. 2.8 Test arrangement according to LOEBER/PAULAY [65].](image)

The lower part of the specimen was completely fixed, while the upper part could move freely. The crack width could be adjusted with an accuracy of 2%. Crack width and shear displacement were measured at both sides of the specimen. The test results were not influenced by the development of secondary cracks in the specimens. The tests were carried out with a constant concrete strength equal to $f'_{ccyl} = 37 \text{ N/mm}^2$, since the influence of the concrete strength was established in an earlier program [20]. Object of investigation were the influence of the crack width, the effect of the aggregate shape and size and the effect of load history – on the one side increase of load for a constant crack width and on the other side increase of load for a constant ratio.
between crack width and shear stress. The maximum values of the shear stresses were much higher than obtained in the tests of \([20, 34]\).

The upper limit of shear transfer at \(v = 7 \text{ N/mm}^2\) was not reached due to failure of the aggregate interlock in the crack itself. Either failure occurred due to local crushing of the concrete in the top or bottom section of the specimens, or further increase of load was not possible with the loading arrangement used. Also LOEBER and PAULAY observed that the shear stress-displacement relation is essentially depending on the crack width. Shape and size of the aggregate particles had for the range tested (\(D_{\text{max}} = 9.5 - 19 \text{ mm}\), round and crushed) no noticeable influence. The relation between shear stress and displacement is represented in Fig. 2.9, and the stresses normal to the crack plane, necessary to keep the crack width constant, are represented as a function of the shear stress in Fig. 2.10.

Fig. 2.9 Shear stress-displacement ratio for constant crack width, acc. to \([65]\). Fig. 2.10 Mean shear stress - restraining normal stress relations for crack widths 0.25 - 0.51.

For the last curves no significant influence of aggregate type or crack width was observed. It is seen in Fig. 2.9 that the shear stress-displacement relations are composed of two straight lines. This is in agreement with the physical behaviour, as it can be expected that due to an increase of shear displacement the total contact area between the crack interfaces
increases too (crushing of matrix material), which results in a greater resistance against shear displacement. The influence of load history is shown in Fig. 2.11. In this figure the mean experimental curve obtained in separate tests, carried out with a constant ratio between shear stress and crack width, is given, with at both sides the interval of scatter of ±13% (shaded area). Upon this curve the results from the previous "constant crack width tests" (Fig. 2.9) have been transposed to enable a comparison to be made. The dotted line connects the appropriate stress values for the three distinct crack widths used in the tests. It reveals the same form as the relationship obtained from the "variable crack width test".

![Fig. 2.11 Mean experimental curve for shear stress-displacement relationship with constant shear stress to crack width ratio, according to 65.](image)

TAYLOR [77, 78] pointed out that cracks do not open to their final width and shear then, but open and shear simultaneously. Therefore it was doubted whether FENWICKS and LOEBERS results are immediately applicable to analyse real beam behaviour. Observations on beams without shear reinforcement seemed to show a constant ratio of crack width / shear displacement during crack opening. Therefore aggregate interlock tests were carried out, not with constant crack widths as in [20, 34, 65], but with constant crack width / shear displacement ratios (Fig. 2.12). The ratio of normal to shear displacement could be changed between the tests but was constant.
during a test, simulating the real beam-crack behaviour. The test specimen, an unreinforced concrete block with a pre-cracked section 127 mm long and 140 wide, was bolted to a pair of linked crossheads. The lower crosshead was bolted to the floor and the upper crosshead was pulled horizontally. The linkages were made in the form of a parallel ruler system so that vertical and horizontal displacements were induced simultaneously. By casting test blocks with crosshead fixing grooves at various spacing, it was possible to set up the specimen with varying values of the angle $\phi_1$; each value, for the small displacements used in the test, defined a different ratio of normal to shear displacement (Fig. 2.12).

![Fig. 2.12 Schematic illustration of block test, according to TAYLOR (77, 78).](image)

The forces in the system were measured by a load cell in the pulling system and by strain gauges fixed to the freely pivoting ties between the crossheads. The major parameters that were considered to affect the problem were the displacement ratio $\frac{dw}{ds}$, the concrete strength, the aggregate size and the aggregate type. However, although a good approximation of real crack behaviour in constructional situations seemed to be obtained, it is felt that also objections might be raised, due to which no general validity may be contributed to the results. At first only the normal displacements at the crack were measured: therefore it is not sure that the ratio of normal to shear displacement, which was supposed to be introduced by the test arrangement, is also obtained at the level of the crack. This may only be considered as true if it were sure that the crack interfaces have no internal resistance
against displacements, so that no preference for any direction of crack opening would exist. Secondly, the observations that there is a linear proportionality between normal and shear displacements in cracks in beams without shear reinforcement were based on measurements by means of strain gauges, which were stuck into the beam after the cracks had formed, so that a certain interval of displacement was not measured. Experiments, in which the measurements on similar type of beams were carried out from the beginning of loading, so that a complete picture of the behaviour after cracking was obtained, revealed an increasing ratio between shear and normal displacements of the cracked interfaces \[85\]. As could be expected the ultimate shear stress in the tests decreased as a function of \(\frac{dw}{d\Delta}\) (Fig. 2.13), while the same was true for the crack width concerned with it (Fig. 2.14).

Unfortunately the influence of the maximum aggregate size could not be assessed, as the values obtained by the tests meant to study this effect were disturbed by local gripping of the blocks. The test results showed that the type of aggregate is important and the parameter is probably the relative strength of aggregate and matrix within the concrete. If the aggregate is relatively weak compared with the matrix, the aggregate fails when the crack is formed, which results in a smoother crack surface than would be obtained if the aggregate were stronger than the matrix, which is generally the case; then the bond between the aggregate

---

**Fig. 2.13** Ultimate shear stresses as a function of the direction of crack opening, according to TAYLOR \[77, 78\].

**Fig. 2.14** Crack width at ultimate shear stress as a function of the direction of crack opening, according to TAYLOR \[77, 78\].

---

Bibliotheek
gld. Civiele Techniek T.H.
Stevinweg 1 - Delft
and the matrix fails when the crack is formed, which results in the
maximum roughness that is possible. This was demonstrated by means of
a concrete with a relatively weak aggregate (limestone). Fig. 2.15
shows that for approximately equal concrete strengths (49-53 N/mm²)
lower ultimate shear values were observed for limestone (●) than for
a concrete with a stronger aggregate (○). It could also be observed
that a reduction of the strength of the limestone concrete
to 30 N/mm² resulted in a higher shear resistance (compare the ●-values),
since in this case the aggregate-matrix bond failed before the aggregate
itself was splitted and a rougher crack surface was obtained.

Fig. 2.15 Relation between ultimate shear stress and concrete strength [78].

2.3. Shear transfer in cracks under cyclic loading.

COLLEY and HUMPHREY [13] carried out tests on aggregate interlock
joints in concrete pavements, subjected to cyclic loading, in order
to study the influence of the joint width, the level of loading and
the aggregate type on the shear transfer capacity of the joints. The
specimens tested consisted of two slabs with a joint in between, the
width of which could be governed by axial forces at the end of the
slabs. A transverse contraction joint, as created in the specimen, is
normally constructed in concrete pavements to release tensile stresses,
and when properly designed they control the location of transverse
cracks. They are most frequently constructed by sawing or forming a
narrow groove in the pavement to the depth required to produce a plane of weakness. At this plane, restrained contraction forces produce a crack below the groove. Also in the specimens the joint was formed in this way, so that the results of the tests are also directly applicable to describe the behaviour of cracks under cyclic loading in general. The slabs were further based on a subsoil, the quality of which was also a variable in the program. The dimensions of the specimens are represented in Fig. 2.16. The slabs were loaded in a way, comparable with the loading of a highway joint by heavy traffic. To simulate a wheel approaching a joint, the load applied to the approach slab was increased from zero to full load in 1/4 sec. (Fig. 2.17).

![Diagram of test slab and loading cycle](image)

Fig. 2.16 Plan of test slab and instrumentation according to [13].

Fig. 2.17 Loading cycle according to tests of [13].

The load on the approach slab was then released to zero and simultaneously the load on the departure slab increased from zero to full load in 0.02 sec. Then the load on the departure slab was reduced to zero in 1/4 sec. This action was followed by an interval of approximately 1 sec., in which the slab returned to a no load position. A joint under test received about 50000 of these 1.5 sec.-loading cycles a day. At the end of a loading day, static load test data were obtained. The load transfer effectiveness was rated using a method, devised by TELLER and SUTHERLAND [81]. In this method, joint effectiveness, $E_j$, is computed using the formula:

$$E_j(\%) = \frac{25!}{\delta_j + \delta_j} (100)$$
where $\delta_j^u$ is the deflection of the unloaded slab and $\delta_j$ is the deflection of the loaded slab. If load transfer at the joint were perfect, the deflections of the loaded and unloaded slabs would be equal and the effectiveness would be 100%. If, however, there were no load transfer at the joint, only the loaded slab would deflect and the effectiveness would be zero. All effectiveness values were computed from measured deflections obtained with a 40 kN static load, applied to the approach slab. Fig. 2.18 shows the influence of joint opening on effectiveness in a 225 mm thick concrete slab on a gravel subbase. It is seen that effectiveness decreased as the joint opening became wider. Effectiveness also decreased with additional load applications, although more than 90 percent of the decrease occurred during the first 500000 repetitions.

![Fig. 2.18 Influence of joint opening on effectiveness, for a 225 mm concrete slab, gravel subbase, $v_{max} = 0.19$ N/mm$^2$](image1)

![Fig. 2.19 Influence of load on effectiveness, for a 175 mm slab, 0.9 mm joint, gravel subbase](image2)

The influence of load level on the joint effectiveness is shown in Fig. 2.19. The data are for 175 mm thick slabs on gravel subbases with joint openings of 0.9 mm. Prior to repetitive loading the effectiveness of the three joints was about the same, ranging from 87% to 95%. Significant differences in effectiveness developed under the action of repetitive loading and became more pronounced as the tests continued. However, light loads cause little or no wear. It must be emphasized, when interpreting these values, that the load intensity in these tests is comparably low (ranging from 0.1 - 0.2 N/mm$^2$). In Fig. 2.20 the influence of the aggregate shape on effectiveness is shown. Crushed gravel and crushed stone had the same gradation, so that a fair comparison could be made.
It is seen that joint effectiveness values for the crushed stone were larger than values for the natural gravel. The crushed gravel, obtained by crushing the natural gravel and rearranging it to its original gradation, showed an increase in effectiveness from 50 to 84 percent.

![Diagram showing joint effectiveness values for crushed gravel, crushed stone, and natural gravel.]

Fig. 2.20 Influence of aggregate shape on joint effectiveness, for a 225 mm thick slab, 0.9 mm joint width, gravel subbase [13].

Thus, for the same hardness, the effectiveness increased with increasing angularity of the coarse aggregate.

Tests with higher stress intensities were carried out by WHITE and HOLLEY [87], who developed a program to study the effect of seismic loading on the behaviour of cracks in reinforced concrete. These tests produced valuable preliminary assessment of the behaviour of the models and established a viable testing scheme, on which an extension of the program was based, carried out by LAIBLE, WHITE and GERGELY [43]. The experimental results obtained in these programs had to be directly applicable to the design and calculation of concrete nuclear containment vessels, so that the loading conditions were adapted to this requirement. The tests were carried out on specimens as represented in Fig. 2.21.

![Diagram showing test specimen used in nuclear containment vessels.]

Fig. 2.21 Test specimen used in [43, 87].
Dowel action was excluded by using external restraint bars. These bars have a negligible shear stiffness but act to hold the specimen halves together when shearing and overriding occurs. The specimens were cracked at mid-depth prior to testing by forcing cracking angles into the sides of the specimen. The desired initial crack width was then set by positioning the upper half of the specimen with respect to the lower half by adjustment of the nuts on the restraint rods that passed through the upper restraint beams. The horizontal shearing surface had a net cross-sectional area of 194000 mm². Fully reserving cyclic shear stresses of about 1.24 N/mm² were applied across initial crack widths of 0.25, 0.51 and 0.76 mm by hydraulic rams. Most of the specimens were subjected to 25 cycles of shear. Two types of loading were performed: (1) incremental, where the load was increased in increments of about 15% of the peak load, and (2) direct loading and unloading to and from the peak stress level. The incremental type of loading was done for the first and fifteenth cycles on nearly all tests while the direct loading was used for all other cycles. As variables were studied a.o.: the width of the preset crack, the concrete strength, the aggregate size and quality and the degree of transverse reinforcement restraint. A result of a specimen with an initial crack width of 0.76 mm and a restraint stiffness of 600 kN/mm, is represented in Fig. 2.22. The result may be considered to be representative for the generally observed behaviour, although there is a marked difference in behaviour between the specimens with 0.25 mm initial crack widths and the other groups, as will be discussed later. The most significant aspect of the behaviour is that during any of the load cycles the total load-slip relationship was highly non-linear. A considerable amount of hysteresis existed when the initial crack width was 0.51 or 0.76 mm. Although the loading portion of the load-slip curve during the first cycles is nearly linear, the very next cycle of all specimens demonstrated a marked degree of non-linearity similar to the results for cycle 15 in Fig. 2.22 a. In only a few cases did the slip freely return to as little as 50% of the maximum value. Usually the return slip was in the range of 0-20% of the maximum slip, which was believed to be caused by the locking effect of contact areas, creating a high degree of resistance. However, since hardly any
shear stress was necessary to bring the specimen back to the neutral position, it seems to be more obvious that this irreversibility is due to local crushing of the concrete. Fig. 2.22 d illustrates that the maximum shear displacement value for each cycle increased most rapidly during the initial cycles but the increase tended to stabilize as cycling proceeded.

![Graphs illustrating shear stress vs. displacement](image)

Fig. 2.22 Typical tests results: specimen with initial crack width 0.76 mm and restraint stiffness = 600 kN/mm, according to [43].

Fig. 2.23 shows the influence of the initial crack width on the specimen behaviour. It appears that specimens with initial crack widths of 0.25 mm behave much differently from the other specimens, in that both the initial ratio of shear displacement to crack width and the rate of increase of this value with cycling are much less severe than for the specimens with initial crack widths of 0.51 mm.
or higher (also for lower shear stresses in the tests of COLLEY and HUMPHREY [13] a marked difference as a function of crack width was observed). The effect of concrete strength was particularly evident in the first cycle when the lower strength cement paste leads to easier crushing.

Fig. 2.23 Shear displacement as a function of crack width and cycling, restraint 600 kN/mm, according to [43].

A specimen which only developed 75% of its design strength, exhibited slip values which were 40 and 10% greater than normally for the 1st and 15th cycles respectively. It was also observed that the shear capacity is enhanced substantially by the use of larger aggregate size. Specimens with 13 mm maximum aggregate diameter showed an increase in horizontal slip of 50 and 30% over specimens with 38 mm maximum diameter, during the 1st and 15th cycles respectively. The effect of the restraint stiffness normal to the crack plane for the cycles 1 and 15 is given in Fig. 2.24 for specimens with initial crack widths of 0.76 mm. In this figure also some results of LOEBER [65] are projected for tests with a constant crack width of 0.76 mm and a shear stress of 1.17 N/mm². Constant crack width is equivalent to infinite restraining stiffness.
Fig. 2.24 Shear displacement versus restraint stiffness; initial crack width 0.76 mm, shear stress = ± 1.26 N/mm², according to [43].

This line, plotted in Fig. 2.24, appears to be a good lower bound for the experimental data. Also tests on small scale specimens with very high restraints (reported in [88]) fit the overall trend of the data. It is seen in Fig. 2.24 that for each value of crack width, there will be a certain amount of shear displacement that occurs with infinite stiffness, because the two sides of the open crack must come into bearing to transfer shear force. The maximum stiffness used in this study (1340 kN/mm), which is equivalent to 1% steel with about 150 mm unbonded length on each side of the crack appears for this crack width to be nearly as effective as infinite restraint, at least for the first cycle of loading. To explain the behaviour of the specimens the roughness of the crack plane was subdivided into a local roughness and a general roughness, as is illustrated in Fig. 2.25.

Fig. 2.25 Subdivision into roughness levels according to [43].

The initial loading causes deterioration of the local roughness which results in an increased gap between opposite areas of contact. In
subsequent loadings the two surfaces must travel this distance before contact is made. This movement is called free slip. It was stated that there are two modes of behaviour which determine the resistance of a surface to slip: (1) bearing and crushing of adjacent pieces of aggregate and mortar, and (2) frictional resistance due to the normal force provided by the reinforcement bars across the crack plane. The degree to which each of these modes of action occurs will determine the stiffness characteristics of the surface. The bearing mode of behaviour predominates in the early cycle loadings, particularly in the first cycle. It is the primary mode of shear transfer in all cycles when the initial crack width is low (0.25 mm). Local concrete crushing occurs and there is relatively little overriding. Consequently the increase in crack width and bar forces are not as large as in the friction mode of behaviour. The resistance in this mode is highly dependent on the local roughness of the surface and the amount of initial crack width and less dependent on the amount of normal restraint. The friction mode of behaviour occurs primarily in the later cycle loadings when the initial crack width is 0.51 mm or greater. There is a little crushing of concrete and appreciable overriding. Consequently the increase in crack width and bar forces is larger than in the bearing mode of behaviour. Once this mode of shear transfer is developed, the resistance is more dependent on the general roughness of the surface and is less dependent on the crack width. The dominant feature of the continued cycling is however that the initial stiffness for each cycle is reduced because of the crushing and compaction of the opposing interfaces.

LOEBER and PAULAY subjected specimens as represented in Fig. 2.8, in which the crack widths were kept constant during loading, to repetitive loading of high intensity ($v_{max} = 6 \text{ N/mm}^2$).

Fig. 2.26 Response to cyclic loading for crack widths 0.13 mm, 0.25 mm and 0.51 mm, according to LOEBER/PAULAY.
Also here both an increase in tangential stiffness of the mechanism was observed as cyclic loading progressed and a decreasing stiffness for greater crack widths (Fig. 2.26). The crack surfaces after testing were, as expected, heavily striated. A large quantity of crushed matrix and smaller aggregate particles could be seen. The surface irregularities were worn down and edges of aggregate indentations were rounded off. As in the static tests no difference in behaviour of the specimens with different coarse aggregate size was observed.

The most characteristic feature concerned with response to cyclic loading is that the system is hardening, that is, the stiffness increases with deformation. Most other material properties are softening, because of the material stiffness decreases with deflections, as a result of accumulation of damage. In a hardening system the forces may be very sensitive to the displacements.

2.4. Shear transfer in cracks under impact loading.

CHUNG [12] carried out an investigation to establish the shear resistance of concrete joints subjected to impact loading. The effects of repeated load on the dynamic strength of the joints formed also a part of the study. The test specimens were push-off specimens (Fig. 2.27). Each specimen measured 125 x 200 mm in cross section and 660 mm in length.

![Fig. 2.27 Test specimen subjected to impact loading according to CHUNG [12].](image)

It was composed of two parts - a "precast part" of 45 N/mm² and a "cast in situ" part of 38 N/mm² concrete, which was subsequently added. The interface between the two parts, 150 mm long and 125 mm wide, was a
rough surface produced by exposing the coarse aggregate on the precast part by water hosing the top surface before it set. Hence, although there is no crack, a plane of potential cracking is procured: all specimens sheared off along the full length of the joint. The specimens were divided in groups. The first group was used to assess the ultimate static load. The specimens in the second group were subjected to a dynamic load, the intensity of which was so chosen that failure occurred in one blow. The specimens in the third and the fourth group were, prior to impact loading, subjected to two million cycles of repeated loading at 400 cycles/min. The minimum load was 10% of the static ultimate load of the control specimens. The maximum load was 55% of the static ultimate load for the specimens of the third group and 66% for the specimens of the fourth group. The maximum load in the second group was reached in 0.8 millisecond. In term of shear stress, the rate of stressing was about 1200 N/mm²/sec. Typical force-time curves for the second group are presented in Fig. 2.28.

![Force-time curves for specimens under impact loading](image)

Fig. 2.28 Force-time curves for specimens under impact loading [14].

The dynamic increase factor, defined as the ratio between the dynamic shear strength and the static shear strength, had for the second group of specimens an average value of 1.80, for the third group 1.84 and for the fourth group 1.75. The impulse capacity for the second group was 9.36.10⁻³, for the third group 9.86.10⁻³ and for the fourth group 8.09.10⁻³ N·sec/mm². Hence it can be concluded that repeated loading of 55% intensity did not have significant detrimental effect, while repeated loading of 66% intensity reduced the impulse capacity by 14%.
It is fortunate that repeated loading exceeding half the static strength is seldom encountered in the normal service of a structure. It is only in abnormal cases that the effect of repeated loading would cause concern.
3. SHEAR TRANSFER IN CRACKS AND JOINTS BY DOWEL ACTION

3.1. Introduction.

Dowel action can be defined as the capacity of reinforcing bars to transfer forces perpendicular on their axis (Fig. 3.1).

Fig. 3.1 Example of pure dowel action.

For a thin slice of an elastic material, loaded in a way as indicated in Fig. 3.2 the stresses in the material can be determined using plane stress elasticity methods [71, 88].

Fig. 3.2 Thin slice of an elastic material, loaded by a dowel force.

In this way it was found that the radial stresses are equal to:

\[ \sigma_r = \frac{F}{\pi R} \cos \psi \]  \hspace{1cm} (3.1)

Values of the circumferential tensile stresses in the concrete are:

\[ \sigma_\psi = 0.34 \frac{2F}{\pi R} \quad \psi = 0 \]  \hspace{1cm} (3.2)

\[ \sigma_\psi = 0.637 \frac{2F}{\pi R} \quad \psi = \frac{\pi}{2} \]  \hspace{1cm} (3.3)
Thus the tensile stress in the direction of the dowel force is highest and the tensile stress normal to the dowel force, which tends to produce a wedging splitting action is only 54% of the maximum. This trend was experimentally confirmed by WEAVER and CLARK [86]. When the tensile strength of the concrete is reached and a crack is formed, an adjustment in the load carrying system may be expected.

In an in-plane loaded planar element a redistribution of stresses occurs, resulting in higher stresses under the bar (Fig. 3.3a). When the dowel load is increased an on-going deterioration of the concrete under the bar occurs, resulting in a gradually decreasing stiffness till the ultimate load is reached (curve a in Fig. 3.3c).

\[ \text{Fig. 3.3 a.} \quad \text{Fig. 3.3 b.} \quad \text{Fig. 3.3 c.} \]

\( a.b: \) Dowel cracking in a planar structure and in a beam.
\( c. : \) Load-deflection curves for both cases.

In a beam however, after the formation of a crack generally no redistribution of stresses is possible, and a rigorous extension of the crack along the bar axis, resulting in failure must be expected; only when the beam is reinforced with stirrups, the dowel crack may be stopped and a completely different mechanism is activated to transfer dowel forces. Because the mechanisms, occurring in planar structures and beams are basically different, they will be treated separately.

3.2. **Dowel action in planar elements under monotonic loading.**

The deflection of the dowel will be defined as the total distance between the axes of the undeformed parts of the bars at both sides of the crack or joint (Fig. 3.4).
As is indicated the total deflection is both a result of the deformation of the part of the bar embedded in the concrete and the part which is free over a certain length.

For the deformation over the free length, PAULAY [6] distinguished three mechanisms (Fig. 3.5):

- **Load transfer by bending**: the capacity of this mechanism is limited by the formation of plastic hinges in the bar.

- **Load transfer by pure shear**.

- **Load transfer by kinking**: if there is a considerable shift between the two main bar axes, for instance as a result of plastic deformations, the axial force in the local deviation results in a component perpendicular to the main bar axis. On this subject several discussions have been held. Often it was stated that this contribution could not be great, since the bar diameter is normally very great with regard to the crack width (Fig. 3.6).
However, it has to be realized that due to the crushing of the concrete, great deformations can occur, resulting in a considerably increased "free length". Most of all for thin bars, due to kinking, an important increase in load can be reached after the formation of plastic hinges. The deformations necessary to develop this force are relatively big, so that for the transfer of forces in cracks in planar structures, kinking doesn't result in a significant dowel contribution. The phenomenon is predominantly interesting for the failure load of joints, in which great deformations can take place. PAULAY, PARK and PHILLIPS \(^\text{[64]}\) demonstrated this by tests, conducted on a corbel which was connected to the rest of the specimen by reinforcing bars, crossing a smooth contact area, to exclude all possible load bearing components but dowel action (Fig. 3.7).

![Smooth contact area diagram](image)

Fig. 3.7 Dowel test, conducted by PAULAY, PARK, PHILLIPS \(^\text{[64]}\).

After loading up to yielding in the bars, by virtue of the kinking effect an additional shear force could be resisted, which was for the bars \(\phi 6.4\) mm, 9.5 mm and 12.7 mm respectively equal to 88%, 43% and 9%. The \(\tau-\Delta\) relations are represented in Fig. 3.8.

![\(\tau-\Delta\) relation diagram](image)

Fig. 3.8 \(\tau-\Delta\) relation for the tests represented in Fig. 3.7.
For the derivation of the load-displacement relation of bars, subjected to dowel action, both the deformations of the bar over the free length and over the embedded length have to be taken into account. With regard to the description of the behaviour the assumed mechanisms can be subdivided into two groups:

- Mechanisms to predict the load-displacement relation for the dowel in constructional situations, where only relatively small displacements have to be expected, as for instance in in-plane loaded planar elements. In these cases the materials behave approximately linearly elastic, which makes it possible to use the model of a beam on elastic foundation. For low stresses this system gives good results, but for higher stresses soon a redistribution of forces occurs, so that additional considerations have to be taken into account.

- Mechanisms to predict the ultimate bearing capacity of the dowel. This ultimate capacity, however, can only be reached when high shear stresses are active and great deformations can occur, for instance in joints. Before the ultimate loading stage is reached, the stresses have been redistributed to such an extent, that the original model of a beam on elastic foundation is too far from reality to give reliable results. Therefore other mechanisms are adopted, taking into account plastifications of the materials. Those models are generally not fit to predict the great displacements, attended with this degree of loading.

3.2.1. Theoretical approximation of the load-displacement relation, based on the model of a beam on an elastic foundation.

The description of the dowel load-displacement relation can be based on the theory of beams on an elastic foundation, as published by TIMOSHENKO and LESSELS [83] (Fig. 3.9).

Fig. 3.9 Dowel considered as a beam on an elastic foundation.
The first known application of this principle on the mechanism of dowel action is a publication of FRIBERG [25], who tried to calculate the load bearing capacity of steel dowels in joints in concrete pavements. The derivation of the load-displacement relation, both taking into account the deformations in the concrete and in the steel, is given, based on the publications mentioned before. The model and the different partial displacements attended with it, are represented in Fig. 3.10.

![Fig. 3.10 Calculation model and partial displacements.](image)

The total deflection in the centre is the sum of three components:

\[ y_{\text{tot}} = y_o + y'(-f) + y_F \]  

(3.4)

According to FINNEY [21] \( y_o \) can be expressed as:

\[ y_o = F_d \frac{1}{283EI} (1 + \beta f) \]  

(3.5)

in which

\[ \beta = \sqrt[4]{\frac{G_f \phi}{4 EI}} \]

\[ EI = E_s \frac{P^4}{64} \]  

(flexural stiffness of the bar)

\[ G_f = \text{Foundation modulus of concrete} \]

\[ y' = - F_d \frac{1}{283EI} \beta (1 + 2\beta f) \]  

(3.6)

\( y_F \) is the deflection of a bar, which is fixed-ended at \( x = 0 \) and subjected to a load \( F_d \) at \( x = -f \):

\[ y_F = \frac{F_d f^3}{3 EI} \]  

(3.7)
The relation between the dowel force $F_d$ and the total deflection $y_o$ is:

$$y_o = \frac{F_d}{6G^3EI} \left( 3 + 68f + 68^2f^2 + 28^3f^3 \right)$$  \hspace{1cm} (3.8)$$

The shear deformation of the steel over the free length is neglected in this formulation. STANTON \cite{71} demonstrated that this contribution to the total displacement is always $< 4\%$, independent on the bar diameter. The relation between the dowel force and the total dowel deflection, which is equal to the total shift between the two parallel bar axes ($\Delta = 2y_o$) is then:

$$F_d = \frac{38^3EIa}{3 + 68f + 6(8f)^2 + 2(8f)^3}$$  \hspace{1cm} (3.9)$$

As has been stated before, the validity of this model is restricted to the elastic stage. A prediction of the ultimate bearing capacity on basis of constant elastic material properties is doomed to fail. MARCUS \cite{49} demonstrated with the results of his experiments that, if the elastic model would be valid, the concrete stress under the bars in the ultimate loading stage would reach values up to 2.6 times the concrete compression strength. Between the initial (elastic) loading stage and the ultimate (plastic) loading stage a transition range with changing material properties exists. Attention must be paid to two important variables, used in formulation (3.9): the value $G_f$, involved in the foundation modulus of the concrete, and the value of the free length $f$.

For the foundation modulus of the concrete $G_f$, a lot of different values are encountered in literature. A survey of values, given by FINNEY \cite{21} is represented in Table 3.1.
Table 3.1. Values for $G_f$ according to a survey of FINNEY [21].

<table>
<thead>
<tr>
<th>$G_f$ Range, N/mm</th>
<th>Average $G_f$, N/mm</th>
<th>Source</th>
<th>Remarks</th>
<th>Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>834 - 417</td>
<td>-</td>
<td>Grinner</td>
<td>Estimation</td>
<td>[30]</td>
</tr>
<tr>
<td>max. 695</td>
<td>19.7 $f_c'$</td>
<td>Friberg</td>
<td>Tests on embedded dowels - 1938</td>
<td>[25]</td>
</tr>
<tr>
<td>198 - 325</td>
<td>256</td>
<td>MSHD *</td>
<td>Load-deflection test 1947</td>
<td>[22]</td>
</tr>
<tr>
<td>217 - 1637</td>
<td>639</td>
<td>MSHD *</td>
<td>Tests on embedded dowels</td>
<td>Not published</td>
</tr>
<tr>
<td>247 - 2307</td>
<td>712</td>
<td>Marcus</td>
<td>Dowels with uniform bearing pressure</td>
<td>[49]</td>
</tr>
<tr>
<td>Not known</td>
<td>681</td>
<td>Loe</td>
<td>Load-deflection tests 1952</td>
<td>[46]</td>
</tr>
<tr>
<td>250 - 2391</td>
<td>695</td>
<td>MSHD *</td>
<td>Tests on embedded dowels</td>
<td>Not published</td>
</tr>
</tbody>
</table>

* Michigan State Highway Department

There are several reasons which can be advanced in order to explain the scatter in the values.

- At first the value $G_f$ is strongly related to the quality of the concrete immediately under the bar. So, even when the same concrete composition is used, a scatter is obtained, depending on the position of the bar during casting. When the direction of the bar is parallel to the direction of casting (Fig. 3.11 a) a greater value of $G_f$ can be expected than in the case of a bar perpendicular to this direction (Fig. 3.11 b), since during the vibration a local segregation of water under the bar can be expected, resulting in a lower concrete quality. Also for bars, situated nearer to the bottom of the construction (Fig. 3.11 c) a greater value for $G_f$ may be expected, since from the top to the bottom an increasing density of the concrete is obtained.
Furtheron it is possible that, when dowel action is combined with an axial tensile force in the bar, due to bond action between concrete and steel, in several directions tensile stresses in the concrete around the bar are activated (Fig. 3.12), resulting in microcracks and a lower value of the foundation modulus $G_f$.

Next it is obvious, that the value of $G_f$ must decrease with increasing dowel force. As was stated in 3.1., at first small cracks will occur, parallel to the bar axis (Fig. 3.3 a). As a result the concrete compressive stresses under the bars are enhanced. For a higher degree of loading also crushing of the concrete under the bars occurs. As a result a gradually decreasing value of $G_f$ for increasing dowel action may be expected. This is confirmed by a comparison between the theoretical values, obtained by eq. (3.9) and the experimental values, obtained in the tests of PAULAY, PARK and PHILLIPS [64] (Fig. 3.7 and 3.8). To get an agreement between (3.9) and the experiments, $G_f$ must decrease as a function
of the increasing dowel displacement: the results of this calculation are represented in Fig. 3.13. Apparently the bar diameter is not a significant parameter in this respect. This agrees with observations done by MARCUS [49] and ELEIOTT [18].

![Fig. 3.13](image)

Fig. 3.13 $G_f$ as a function of the dowel displacement, deduced from the experiments described in [64].

The value of $G_f$ obtained in this way is in fact a function of both changes in material properties and redistribution of stresses. $G_f$ may only be considered as the value, which has to be inserted in (3.9) to get the same load-deflection curve as is experimentally obtained due to a complex physical mechanism (Fig. 3.14).

![Fig. 3.14](image)

Fig. 3.14 Stress distribution, actual and modelled.

Summarizing, it is obvious that $G_f$ may not be considered as a uniform value, only depending on the concrete quality, but that further considerations have to be taken into account. Therefore, it is not surprising that the values for $G_f$ represented in Table 3.I scatter over a wide range.

- The second variable in the formulation (3.9) which has to be treated with caution is the free length $f$. At first sight it seems to be logical
to insert for this variable the crack- or joint width. However, there are circumstances, that may lead to free lengths which are considerably greater.

- When bars cross a crack not perpendicularly, the concrete adjacent to the bars may locally crack off (Fig. 3.15).

![Diagram of crack and inclination angle](image)

Fig. 3.15 Increase in free length for bars inclined to a crack.

The free length, which is caused in this way, must depend on the angle $\theta$ and the bar diameter $d$. SCHÄFER [69] suggested the relation:

$$f = C_r \cdot d \cdot \tan \theta$$  \hspace{2cm} (3.10)

in which $C_r$ is a constant. This relation gives $f = 0$ for $\theta = 0^\circ$ and $f = \infty$ for $\theta = 90^\circ$.

- As has been mentioned before, due to an axial tensile force in a bar great bond stresses occur, resulting in microcracks as represented in Fig. 3.16.

![Diagram of microcracks](image)

Fig. 3.16 Microcracks according to [29].

The existence of these cracks was at first experimentally proved by GOTO [29]. Due to this crack development cone shaped concrete elements are extracted, resulting in an increase of the free length. This increase ($= 2 l_o$) depends on the value $\Delta \sigma_s$, which is the difference between the steel stress in the crack and in the undisturbed area, the bond properties
and the diameter of the bar. LEONHARDT \cite{44} gave as an approximation for deformed bars (estimation on basis of centrical tensile tests):

\[
\ell_0 = \frac{\Delta \sigma}{\frac{3}{4}\varepsilon} \phi \quad (N/mm^2 \text{ and mm}) \quad (3.11)
\]

See also Fig. 3.17.

![Diagram](image)

Fig. 3.17 Distribution of steel stresses and bond stresses over and besides the free lengths.

The influence of the axial tensile force was experimentally confirmed by ELEIOTT \cite{18, 88}. He carried out tests on specimens as represented in Fig. 3.18. The specimen was so designed, that it enabled both a test on pure aggregate interlock and a test on dowel action alone.

![Diagram](image)

Fig. 3.18 Test specimen used by ELEIOTT \cite{18}.

The tests on pure dowel action were carried out with embedded bars of different sizes, stressed to different levels of axial stress. The tests were principally bound to study the behaviour under cyclic loading.
However, already during the first cycles a pronounced influence of the axial stress level was observed. Some of the results are represented in the figures 3.19-21.

Fig. 3.19 shows the load-deflection curves for the first load cycle for two tests on bars $\phi$ 12.8 mm; one test was carried out without an axial tensile stress (a), the other with an axial stress of 175 N/mm² (b).

![Fig. 3.19 Test results of (18) for a stressed and an unstressed dowel.](image)

![Fig. 3.20 Test results of (18) for a dowel subjected to an increased axial stress during cycling.](image)

The severe loss of stiffness due to an increased axial stress is obvious. Fig. 3.20 gives the load-deflection curves for three cycles of a test on a bar $\phi$ 12.8 mm; during the first cycle the axial tensile stress was $\sigma_a = 175$ N/mm², which was increased to 350 N/mm² for the second and the third cycle. Also here a severe loss of stiffness was observed. Other tests in which the axial stress was kept constant on the lower level showed a much smaller deflection after the same number of cycles.

![Fig. 3.21 a, b. The effect of an increased axial stress during cycling on the dowel stiffness according to (18, 88).](image)
The same phenomenon was observed comparing the two tests represented in Fig. 3.21 a and b. In the test without axial stress (a) the deflection stabilized after five cycles (the deflection after 10 cycles was approximately the same as after 5 cycles); however, an increase of the axial stress after the fifth cycle leaded to an increasing deflection and a reduced stiffness.

STANTON calculated the effect of the value of the free length on the dowel stiffness on basis of the formula (3.9) for bar diameters $\phi$ 57.4 mm and $\phi$ 35.1 mm, for several values of $\beta$. The results of this calculation are represented in Fig. 3.22.

Recapitulating it may be stated that the deflection of a bar, subjected to a dowel force, is partially a result of the deformation of the concrete around the bar and partially of the deformation of the steel over a free length. When the theory of a beam on an elastic foundation is used to calculate the load-deflection relation, some parameters have to be handled with care. The value $G_f$, necessary to calculate the deformations in the concrete is not a constant, but decreases with increasing deflection. The position of the bar in the construction and the eventual bond stresses may have an influence on $G_f$, the bar diameter has apparently no significant influence on this value. The free length of the steel, and as a result the contribution of this part to the total dowel deflection, increases if the axial tensile stress increases; also for bars inclined to the crack- (joint) plane an enhancement of the free length is possible, this being due to the local cracking off of the concrete. For relatively small deflections
the deformation of the steel over the free length has mainly to be attributed to bending action; shear deformations may be neglected. Kinking of the bars can only occur for relatively great deflections. A prediction of the load-deflection relation only as a function of the deformations in the steel is an unrealistic approximation, despite of arguments, sometimes encountered in literature, which seem to confirm the opposite. This will be shown, investigating a statement of PAULAY [65], who concluded on basis of his tests (Fig. 3.7) that the dowel action was approximately proportional to the reinforcement ratio, independent on the size of the bars used (Fig. 3.23). Therefore, taking into account the three mechanisms for the shear transfer, being possible for the steel over the free length (Fig. 3.5), it was concluded that shear and kinking would predominantly be responsible for the behaviour. This would be contrary to the tendencies, emerging from other tests and theories treated in this chapter. However, it can be demonstrated that these results do not violate the assumption of the behaviour according to the model of a beam on an elastic foundation. According to formula (3.9) for a free length $f = 0$ the dowel action of one bar can be written as:

$$F_d = \beta^3 EI Z \gamma \phi = \beta^3 EI A$$

with

$$\beta = \sqrt{\frac{6 G_f}{EI}}$$

(3.12)

Substitution of $\frac{EI}{64}$ for $I$ results in the relation:

$$F_d = 3.56 \phi^{1.75} G_f^{0.75} \delta = 0.01 \phi^{1.75} A$$

(3.13)

so that a proportionality with $\phi^{1.75}$ is obtained.

If a number of $n_1$ bars with a diameter $\phi$ in a joint results in a reinforcement ratio $\rho_o$, this can be written as:

$$n_1 \pi \phi^2 = 0.01 \rho_o bd$$

(3.14)

If a number of $n_2 (< n_1)$ bars with a greater diameter $a\phi_1(a > 1)$ results in the same reinforcement ratio, this can be written as:

$$n_2 \pi (a\phi_1)^2 = 0.01 \rho_o bd$$

(3.15)

From (3.14) and (3.15) it can be derived that:
\[ n_2 = \frac{n_1}{\alpha^2} \quad (3.16) \]

So the total dowel force for the bars with the smaller diameter is:
\[ \Sigma F_d = n_1 \cdot \phi_1^{1.75} \cdot c \cdot \Delta \quad (3.17) \]

and for the bars with the greater diameter:
\[ \Sigma F_d = \frac{n_1}{\alpha^2} \cdot \alpha^{1.75} \cdot \phi_1^{1.75} \cdot c \cdot \Delta = \frac{1}{\alpha^{0.25}} \cdot (n_1 \cdot \phi_1^{1.75} \cdot c \cdot \Delta) \quad (3.18) \]

This implies that, if the reinforcement ratio in a joint is the same, on basis of the model of a beam on an elastic foundation, greater bars would give a slightly lower total dowel force than smaller bars (since \( \alpha > 1 \)). This is confirmed by Fig. 3.23. If these curves (from [64]) are corrected by means of a reduction factor \( \alpha^{0.25} = (\frac{25}{20})^{0.25} \), it appears that a comparison on basis of the model of a beam on an elastic foundation gives a surprisingly good result.

Fig. 3.23 Total dowel force for a joint with bars of several diameters, but the same reinforcement ratio, according to tests of [64].

Fig. 3.24 The same lines after a correction on basis of the model of a beam on an elastic foundation. (According to this theory the curves must cover each other).

MEHLHORN [40] carried out tests on specimens with inclined bars with special attention to another phenomenon concerned with dowel action,
namely the influence of the concrete stress concentrations, adjacent to the crack plane on the bond slip relation of the embedded bars. This aspect of the behaviour is the more important, since the crack width, which is one of the most important variables of the whole mechanism of shear transfer, is directly related to this bond slip relation. The type of specimen used in the tests is represented in Fig. 3.25.

Fig. 3.25 Test specimen as used by MEHLHORN [40].

Tests were carried out on specimens with reinforcing bars \( \phi 10 \) and \( \phi 16 \text{ mm} \), for angles \( \theta = 45^\circ, 60^\circ \) and \( 90^\circ \). Each test was repeated several times in order to obtain representative results. Since the results of the investigation were principally bound to be applied to reinforced slabs, in which the cracked parts are connected over the uncracked compression area, only displacements normal to the crack plane were imposed. On the bars strain gauges were stuck over a length of 360 mm. For the \( \phi 16 \text{ mm} \) bars this length appeared to be insufficient, so that some extrapolation of the measurements was necessary. The results of the measurements were used to reconstruct the bond stress distribution over the length of the bars. It was also possible to deduce the basic bond slip relation for each reinforcement geometry. These relations are represented in Fig. 3.26 for all angles and for both diameters. It is seen that in the case of \( \phi 10 \text{ mm} \) bars no systematic variation with the angle of inclination
could be observed. An upper limit to the validity of the measurements was obtained due to yielding at one side of the bar. In the case of \( \phi \) 16 mm bars a deterioration of bond quality with decreasing angle of inclination (greater stress concentrations) was observed, which has probably to be attributed to the formation of longitudinal and transverse cracks, which were not observed in the specimens with the \( \phi \) 10 mm bars.

Fig. 3.26 Basic bond-slip curves for several angles of inclination for bars \( \phi \) 10 mm (a) and \( \phi \) 16 mm (b), deduced from tests conducted by MEHLHORN.

3.2.2. Mechanisms developed to predict the ultimate bearing capacity of dowels.

After a great deflection has occurred, the redistribution of stresses has been developed to such an extent, that the model of a beam on an elastic foundation is not fit any more to describe the behaviour. After a considerable crushing of the concrete under the bar, the stresses in the steel reach the yielding limit, so that plastic hinges in the bar are formed. Based on this mechanism, represented in Fig. 3.27, RASMUSSEN \([67]\) proposed the formula:

\[
F_{du} = C \cdot \phi^2 \cdot \sqrt{f_{sy} \cdot f'_{ccyl}} \tag{3.19}
\]

With the value \( C \) as an experimental constant, with proposed value 1.3.
Tests of BENNETT and BANERJEE \[1\], on specimens as represented in Fig. 3.28, seem to confirm the validity of formula (3.19).

It has to be noted that the formula (3.19) could only be applied to the bars at the bottom of the connection, since the resistance of the top bars was limited by the tensile stresses at the sides rather than by the compression bearing stresses beneath the bars. DULACSKA \[15\] developed a theory, based on comparable considerations as on which Rasmusens theory was founded. It was supposed that, due
to crushing of the concrete under the bar, the support is removed over a certain length $c_o$ (Fig. 3.30).

Fig. 3.30 Assumed mechanism according to DULACSKA [15].

A plastic hinge is formed at the place, where the shear force $D$ is equal to 0. As a result:

$$M_u = (t + \frac{1}{2}c_o) F_{du}$$

in which for $F_{du}$ is substituted:

$$F_{du} = t.\delta \left(n.f'_{cc}\right)$$

The value of the plastic moment in the bar can be calculated on basis of the stresses acting on the bar. These considerations result in an ultimate dowel resistance equal to:

$$F_{du} = \rho.\phi^2.c_o.f_{sy}.n\sin\theta \left[\sqrt{1 + \frac{f'_{cc}}{3\rho.c_o^2.f_{sy}.n\sin^2\theta}} - 1\right]$$ (3.20)

in which

$$\rho = 1 - \frac{N^2}{N_y}$$

$N = Axial force in the bar$

$N_y = Axial yielding force in the bar$
The unknown values of $c_0$ and $n$ were experimentally determined from tests on specimens as represented in Fig. 3.31.

It was assumed that $n = 4$, while the test results indicated that $c_0 = 0.05$. The value of $\rho$ in the tests was equal to $\rho = 1$. Substitution of these values in (3.20) results in the relation:

$$F_{du} = 0.2 \rho^2 f_{sy} \sin \theta \sqrt{(1 + \frac{f_{cc}}{0.03 f_{sy} \sin ^2 \theta}) - 1} \quad (3.21)$$

From the load-deflection measurements emerged an almost rigid plastic behaviour, which seems to confirm the presence of plastic hinges (Fig. 3.32). However, no simple basic relationship similar in form to the static equilibrium equations for the load capacity was found for the deformations. This was attributed to the fact that slip takes place mostly as a result of plastic deformations, and the shear deformations of the reinforcement are not negligible. As a relation fitted to the test data was given:

$$\Delta = \frac{11.35 F_d}{\phi 1.04} \sqrt{\frac{1}{f_{cc}}} \tan (\frac{F_d}{F_{du}} \cdot \frac{\pi}{2}) \quad (N, \text{mm}) \quad (3.22)$$
Furtheron it was observed that the ultimate resistance and the corresponding deflection increased with increasing value of the angle $\theta$ between the reinforcing bars and the joint plane, for the range of values investigated. Some results are represented in Fig. 3.33.

Also MILLS \[57\] developed a calculation model to predict the ultimate bearing capacity taking plastifications of the concrete into account. For the elastic loading stage it was shown that the support length $x_e$ is approximately equal to (Fig. 3.34):

$$x_e = 1.6 \cdot f'_{ccyl} \cdot \theta^{-1/8} \quad (3.23)$$
To calculate the stress distribution for a higher load the stress-strain diagram represented in Fig. 3.35 was taken into account. When the ultimate strain $\varepsilon_u$ is reached, the support length is equal to $(x_e + L)$ (Fig. 3.36).

![Fig. 3.35 Assumed stress-strain diagram acc. to [57].](image)

![Fig. 3.36 Support length in the plastic loading stage.](image)

Fig. 3.35 Assumed stress-strain diagram acc. to [57].

Fig. 3.36 Support length in the plastic loading stage.

$L$ can be calculated from:

$$\frac{L + x_e}{x_e} = \frac{\varepsilon_u E_c}{\alpha f'_{ccyl}}$$

(3.24)

or:

$$L = x_e \left( \frac{\varepsilon_u E_c}{\alpha f'_{ccyl}} - 1 \right)$$

Substitution of $x_e$ from (3.23) gives:

$$L = 1.6 \phi f^{-1/8} \left( \frac{\varepsilon_u E_c}{\alpha f'_{ccyl}} - 1 \right)$$

(3.25)

For the concrete compression stresses a diagram as represented in Fig. 3.37 is assumed.

![Fig. 3.37 Stress distribution at ultimate loading according to [57].](image)

Fig. 3.37 Stress distribution at ultimate loading according to [57].
The total ultimate dowel resistance is then equal to:

\[ F = 1.6 \alpha \varepsilon_{u} f_{ccyl}^{7/8} (\frac{\varepsilon_{u} E_{c}}{\alpha f'_{ccyl}} - 0.5) - 0.5 \alpha \varepsilon_{u} f'_{ccyl} \]  

(3.26)

In this expression a lot of parameters are, or may be, dependent on \( f'_{ccyl} \). It can be stated that: \( E_{c} \sim f'_{ccyl} \), \( \varepsilon_{u} \), and \( \alpha \sim f'_{ccyl} \). Therefore a simplified general formula was preferred:

\[ F = \phi^{2} \left[ k_{1} f'_{ccyl} \frac{1}{\varepsilon_{u}} - k_{2} f'_{ccyl} \tan \theta \right] \]  

(3.27)

For \( k_{1} = 20 \) and \( k_{2} = 1.5 \) good approximations were found for the tests of DULACSKA and three own tests, according to the principle represented in Fig. 3.38.

Fig. 3.38 Experimental set-up according to MILLS.

3.3. **Dowel action in planar elements under cyclic loading.**

In order to find characteristics to simulate earthquake conditions, tests with cyclic loading have been carried out at Cornell University, Ithaca, USA. Also cyclic loading tests on pure dowel action formed a part of the program. In the Fig. 3.40 - 3.43 load-slip curves are represented for tests on specimens as in Fig. 3.18. The same tendencies emerged from tests on other types of specimens, also on larger scale (Fig. 3.39).

Fig. 3.39 Tests specimens of tests by STANTON (a), and JIMENEZ (b).
Fig. 3.40 Dowel action under cyclic loading; ø 12.8 mm, axial stress $\sigma_s = 0$, §87.

Fig. 3.41 Dowel action under cyclic loading; ø 19.2 mm, axial stress $\sigma_s = 0$, §87.

Fig. 3.42 Effect of axial stress on dowel action under cyclic loading; bar ø 12.8 mm §87.

Fig. 3.43 Effect of axial stress on dowel action under cyclic loading; bar ø 19.2 mm §87.
It can be observed that the load-slip curves for dowel action alone are qualitatively similar to those for aggregate interlock, except the return to the neutral slip position after unloading is more complete for dowel action. The second and subsequent cycle behaviour differs considerably from that of the first cycle. A large increase of free slip occurs because of the concrete crushing action of the first cycle. This concrete crushing is mentioned by STANTON, who reports: "The complete splitting failure of specimen 3 (Fig. 3.39 a) permitted good inspection of damage to the concrete around the bar at the shear plane. A funnel-shaped volume of crushed concrete was observed, with visible damage extending about 25 mm in each direction from the slip plane". The initial stress concentrations in the concrete are as a result reduced by localized failures, thus, after free slip occurs, the elastic curve of the bars has a better contact with the compacted concrete, and the shear stiffness increases. After a certain number of cycles there is no crushing action any more: the bar can firmly bed itself in the concrete without producing any significant concrete crushing and a stabilization of the shear displacement is obtained (Fig. 3.40-3.41). If axial stresses in the bar are active, the number of cycles after which stabilization is reached is increased (Fig. 3.42-3.43). A survey of the work carried out in this field at Cornell University, Ithaca, is presented in [87].

3.4. Comparison between the contribution of aggregate interlock and dowel action to the total shear transfer in cracks and joints.

From a comparison between the experimental values obtained in tests on pure aggregate interlock and on pure dowel action separately, it appears that the contribution of aggregate interlock dominates over that of dowel action if the crack widths are not too large. In Fig. 3.44 a comparison is represented between the load-deflection relations for aggregate interlock, found experimentally by PAULAY and LOEBER [65] and those for dowel action from the tests of PAULAY, PARK and PHILLIPS [64].
Fig. 3.44 Comparison between aggregate interlock and dowel action on basis of \[ 64, 65 \], for crack widths \( w = 0.125, 0.25 \) and 0.51 mm, and reinforcement percentages of 0.65, 1.31 and 1.96%; \( f'_{cc} = 30 \) N/mm\(^2\).

It must be realized that the contribution of dowel action may be overestimated, since no axial tensile force was present in the tests of PAULAY. The ratio between the contributions of aggregate interlock and dowel action has also been directly assessed in other tests. In the experiments of \[ 64 \] also joints with various surface preparation have been investigated. The pure aggregate interlock contribution, obtained by substracting the dowel component from the total resistance, is represented for every type of joint in Fig. 3.45.

Fig. 3.45 Load-displacement curves for shear transfer in a joint for various surface preparations according to PAULAY, PARK, PHILLIPS \[ 64 \].

From all curves, except of course the lower one, the contribution of dowel action has been substracted.
Also in the tests of ELEIOTT [18] a direct comparison was possible between two specimens, according to the principle represented in Fig. 3.18; in one of the specimens the aggregate interlock was eliminated by means of greased plates, in the other both aggregate interlock and dowel action were active. The reinforcement in both cases was the same (1 bar $\phi$ 12.8 mm) just like the axial tensile stress $\sigma_s = 175$ N/mm$^2$ for the first loading cycles. The results of the first test were represented already in Fig. 3.42; the results of the last test are shown in Fig. 3.46 (Note the different scale of the horizontal axes).

Fig. 3.46 Combined aggregate interlock and dowel action under cyclic loading, acc. to [18, 88]; $f_{ccy} = 21$ N/mm$^2$, $\phi$ 12.8 mm, $\sigma_s = 175$ N/mm$^2$.

A comparison of the stiffnesses for the case under consideration ($\phi$ 12.8 mm, $\rho_o = 1.33\%$, $\tau_{max} = 1.05$ N/mm$^2$, concrete area = 9525 mm$^2$)
learns that about 12% of the shear stiffness was provided by dowel action and about 88% by aggregate interlock of the concrete surfaces.

3.5. **Dowel action in beams.**

The mechanism of dowel action in beams can be subdivided into three phases (Fig. 3.47). The load deflection relation in phase A differs not very much from that observed for dowels in planar elements in the uncracked phase. When, however, in the direction parallel to the bar a tensile crack is formed, a redistribution of stresses as in planar elements is not possible, since the concrete cover tends to split off (Fig. 3.48).

![Diagram](image)

**Phase A:** before dowel crack along axis

**Phase B:** after dowel crack

**Phase C:** after dowel crack, if a stirrup is near to the vertical crack.

**Fig. 3.47** Different phases in the behaviour of dowel action in beams.

![Diagram](image)

**Fig. 3.48** Distribution of (tensile-)stresses in a beam just before a dowel crack is formed.
After the formation of a dowel crack there are two possibilities; either there are no stirrups in the direct vicinity of the flexural (vertical) crack, so that no potential support is available, which results in a definite disappearance of the dowel force (Phase 3, extended to the lower axis), or there are stirrups in the neighbourhood which act, after being crossed by the dowel crack, as a support, by virtue of which the dowel force, after a falling branch, is developed again. Most investigations in this field are carried out on specimens without stirrups, with the aim to assess the dowel cracking load. TAYLOR [79] proved with beam tests that the dowel force-component in the shear capacity of beams without stirrups is considerable. The formation of dowel cracks results in this type of constructions generally immediately in total failure.

3.5.1. Investigations in order to assess the dowel cracking load in beams.

JONES [39] was the first who tried to assess the dowel cracking force by means of experiments. The tests were carried out on a beam-model as represented in Fig. 3.49.

![Fig. 3.49 Experiment carried out by JONES [39].](image)

As an empirical formula for the dowel cracking load was given:

\[ F_{d,cr} = 0.7 f_{ctu} \sqrt{\frac{z}{I_c b_n}} \]  

(3.28)

in which

- \( F_{d,cr} \) = dowel cracking load
- \( z \) = distance between compressed and tensioned bars
- \( I_c \) = moment of inertia of concrete cover
- \( b_n \) = beam width minus the sum of the bar diameters.
FORSELL [23], LORENTSEN [48] and ARROYO [2] carried out tests on specimens as represented in Fig. 3.50.

![Test specimen diagram](image)

**Fig. 3.50 Test specimen as used by FORSELL [23], LORENTSEN [48], ARROYO [2].**

The compression area was over a certain length replaced by a not supported reinforcement bar, which can, due to its small flexural stiffness, only resist a slight shear force. The test results are difficult to interpret; the displacements do not agree very well with those to be expected in a beam and the shear component of the compression reinforcement had to be estimated. The results may therefore not be considered as very reliable.

FENWICK [20] carried out tests on both short and long dowels (Fig. 3.51).

![Specimens and load-displacement curves](image)

**Fig. 3.51 Type of specimens and load-displacement curves for tests carried out by FENWICK [20].**
The short dowels simulated the situation in a beam between two flexural cracks and the long dowels the conditions in a beam-end between the support and the last crack. A disadvantage of this experimental set-up is that there is no tensile stress in the steel; a direct comparison with other tests is therefore hard to give. It is striking that there is a great difference in bearing capacity between the upper- and the lower bars. Due to sedimentation under the upper bars the dowel strength was only 60% of that of the lower bars. The empirical expression for the dowel cracking load derived from the experiments is for the upper bars:

\[ F_{d,cr} = \frac{1}{3} \cdot b_n \cdot l \cdot f_{ctu} \left( \frac{1}{1 + R} \right) \]  

with \( l = \) embedment length of the bar \( R = \) ratio between the displacements at the end of the dowel \( \approx 1.42 \)

and for the lower bars:

\[ F_{d,cr} = \frac{1}{3} \cdot b_n \cdot l \cdot f_{ctu} \left( \frac{R}{1 + R} \right) \]

with \( R \approx 1.75. \)

KREFELD and THURSTON carried out 9 tests on specimens as represented in Fig. 3.52. The dowel action was tested by pressing the central part of the beam, which was separated from the rest by a pre-formed crack, in downward direction until a dowel crack developed.

Fig. 3.52 Test specimen used by KREFELD and THURSTON.
After cracking the test was continued in a deflection controlled way, so that also the behaviour after dowel cracking (phase B, Fig. 3.47) could be studied. The conditions in the test specimen are a good approximation of the reality, since dowel action is combined with an axial tensile force in the longitudinal steel. For the calculation of the dowel force after dowel cracking (phase B) the next formula was given:

\[ F_d = \frac{c \sqrt{f'_{ccyl}} b^3}{2 + a \sqrt{d \cdot I_c \cdot b_n}} \]  

(3.30)

with \( b_n = \) beam width minus the sum of the bar diameters

\( a = \) length of dowel crack

\( c = \) empirical constant.

The other symbols are equal to those, given in the investigation of JONES [39]. For \( a = 0 \) the expression becomes equal to that given by JONES:

\[ F_d = c \sqrt{f'_{ccyl}} \sqrt{d \cdot I_c \cdot b_n} \]  

(3.31)

BAUMANN and RÜSCH [3] carried out tests on the same type of specimens as used by KREFELD and THURSTON. However, also the behaviour after dowel cracking in the case that stirrups are present was investigated (this will be treated in 3.5.2.). 26 Tests were carried out, in which 4 without stirrup reinforcement. One of the most important variables was the diameter of the longitudinal steel; 17 specimens contained \( \phi 20 \text{ mm} \) bars, 3 had \( \phi 16 \text{ mm} \) bars and 6 had \( \phi 26 \text{ mm} \) bars. The evaluation of the results was conducted on basis of the model of a beam on an elastic foundation. For the calculation the use of an effective length was proposed, which is defined such, that the rectangular stress block, see Fig. 3.53, would give the same vertical reaction as would be obtained by integration of stresses over the length of the bar.
According to this definition the dowel cracking load can be expressed as:

\[ F_{d,cr} = f_{ctu} \cdot b \cdot \ell_{e} \]  

(3.32)

Empirically it was assessed that:

\[ l_{e} = \frac{66 \, \phi}{\sqrt{\tau'_{cc}}} \]  

(3.33)

A combination of those formulas results in:

\[ F_{d,cr} = 1.64 \, b_{n} \, \phi \, \sqrt{\tau'_{cc}} \, (N) \]  

(3.34)

When the first crack developed, the displacement \( \Delta \) was between 0.06 and 0.16 mm. The relation between \( F_{d} \) and \( \Delta \) could, for this series of tests, be approximated with (Fig. 3.54):

\[ \Delta = 0.08 \, \frac{F_{d}}{F_{d,cr}} \, (\text{mm}) \]  

(3.35)

Fig. 3.54 Load-deflection relation for dowel action in the uncracked phase (A), according to BAUMANN and RÜSCH [3].
This relation agrees reasonably well with expressions given by ELLIOTT [18] and TELLER and CASHEL [80].

TAYLOR [78] carried out tests also on the same type of specimens as KREFELD/THURSTON and BAUMANN/RÜSCH [3], some on the same scale, other scaled to 2/7 of the original size. The total number of tests was 46, from which 34 were on model scale. As variables were introduced the concrete strength, the number and the position of the bars, the span, the cover and the width of the preformed crack. The tests were conducted in a deformation-controlled way. An example of a load-deflection curve obtained in this investigation is represented in Fig. 3.55.

\[ F_{d,cr} = 11.95 + 0.001b_n f_{ctu} \Delta (kN) \] (3.36)

TAYLOR derived with the aid of a regression analysis of test results of his own tests and those of [3, 41] an expression for the dowel cracking load \( F_{d,cr} \). This relation is:

\[ F_{d,cr} = 4.95 + 0.001b_n f_{ctu} \phi \] (kN) \] (3.36)

The shape of the curve is defined by:

\[ F_d = 1.55 F_{d,cr} \Delta^{0.25} \] (3.37)

The predicted value of \( F_{d,cr} \) on basis of the first formula deviates at most 35% from the measured values.

HOUDE and MIRZA [34] carried out 32 tests on specimens as represented in Fig. 3.56.
The tests were carried out with and without axial tensile force. The embedment length varied between 380 and 790 mm and the specimen width between 100 and 150 mm. Two different bar diameters were used. The dowel cracking load appeared to be directly dependent on the width minus the sum of the bar diameters and on the concrete tensile strength. The bar diameter and the embedment length seemed not to have any influence. Also longitudinal stresses under the yielding limit did not result in other behaviour than was observed for bars without stress. On basis of the test results the dowel cracking load was expressed as:

$$F_{d,cr} = 37 b_n f'_{ccyl}^{1/3}$$

(3.38)

In fact this is the same formula as given by BAUMANN/RÜSCH (3.34), but for the influence of the bar diameter, which is to the opinion of the investigators neglectable. For the load-deflection relation is given:

$$\frac{F_d}{F_{d,cr}} = 78 \Delta \quad (\Delta \text{ in mm})$$

(3.39)

The dowel stiffness agrees well with that derived by TAYLOR [78], which gives 10% higher values. However, the dowel deflection at failure is according to TAYLOR about 10 times as great.

3.5.2. Investigations to study the behaviour of the dowel mechanism after cracking, when stirrups supply an additional support.

BAUMANN and RÜSCH [3] studied not only the dowel behaviour before cracking, but also after the development of a dowel crack, if the bar
is supported by a neighbouring stirrup. Therefore stirrups were provided in the specimens, which were activated after a certain extension of the crack (Fig. 3.57).

![Diagram](image)

Fig. 3.57 Test specimen used by BAUMANN/RÜSCH [3].

On basis of their tests the investigators derived that the load-deflection relation in phase C (Fig. 3.47) can be described by the relation:

\[ \Delta (\text{mm}) = \gamma \cdot F_d^2 \]  \hspace{1cm} (3.40)

in which

- \( F_d \) = dowel force in tf
- \( \gamma \) (mm/tf²) = 0.45 f²/I_v
- \( f \) (cm) = distance from stirrup to vertical crack
- \( I_v \) (cm⁴) = moment of inertia of longitudinal bars + cover
  (see Table 3.II).
\[ I_v = \text{Moment of inertia of} \]
\[ \text{the total cross-section of} \]
\[ \text{longitudinal reinforcement} \]
\[ + \text{shaded concrete area.} \]

<table>
<thead>
<tr>
<th>Longitudinal reinforcement</th>
<th>( c_1/c_2 )</th>
<th>( I_v ) (cm(^4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 ( \varnothing ) 16</td>
<td>3.8</td>
<td>56.2</td>
</tr>
<tr>
<td>2 ( \varnothing ) 20</td>
<td>4.0</td>
<td>82.8</td>
</tr>
<tr>
<td>4 ( \varnothing ) 20</td>
<td>4.0</td>
<td>165.6</td>
</tr>
<tr>
<td>1 ( \varnothing ) 26</td>
<td>4.3</td>
<td>76.4</td>
</tr>
<tr>
<td>2 ( \varnothing ) 26</td>
<td>4.3</td>
<td>152.8</td>
</tr>
<tr>
<td>2 ( \varnothing ) 20</td>
<td>8/4</td>
<td>690.0</td>
</tr>
<tr>
<td>2 ( \varnothing ) 20</td>
<td>8/4</td>
<td>1380.0</td>
</tr>
</tbody>
</table>

Table 3.11. Auxiliary values for the load-deflection relation according to BAUMANN/RÜSCH (3.40).

TAYLOR [78] derived an expression for the ultimate dowel force in phase C by considering the bar length between the vertical crack and the nearest stirrup as a fixed-ended beam, which gives a maximum reaction if at both ends plastic hinges are formed (Fig. 3.58).
The value of the yielding moment is to a certain amount reduced by the tensile force in the steel. On basis of plastic analysis (Fig. 3.59) TAYLOR derived the interaction diagram presented in Fig. 3.60.

Fig. 3.59 Plastic analysis of stresses in a reinforcing bar subjected to an axial tensile force and a moment.

Fig. 3.60 Interaction between moment and tensile force in a reinforcing bar in the plastic loading state according to TAYLOR.
For the effect of a stirrup near to the vertical crack on the dowel action after cracking three possibilities can be distinguished:

- If the stirrup is very near to the vertical crack, the dowel strength after the development of a dowel crack is equal to the strength of the stirrup.

- If the stirrup is too far away from the vertical crack, the reaction delivered by the plastic hinges ($= M_p/l$) is smaller than the load at which the dowel crack was formed, so that the presence of the stirrup does not influence the dowel behaviour.

- If the stirrup is in between, the dowel strength after cracking is enhanced to the plastic beam strength of the reinforcing steel.

The transition value between the possibility that the stirrup strength is decisive and the possibility that the plastic beam strength of the longitudinal steel is decisive can be assessed by assuming that the stirrup strength is equal to $M_p/l$. In Fig. 3.61 the result of this calculation is given. The distance $l$, expressed in a number of bar diameters, is calculated for the most practical values of $d_1/d_s$, the ratio between the diameters of longitudinal- and stirrup bars, and $f_{ly}/f_{sy}$, the ratio between the yielding stresses of both reinforcement types.

![Fig. 3.61 Determination of the distance l between stirrup and vertical crack, below which the stirrup strength and above which the "plastic beam strength" is decisive for the maximum dowel force, according to [78].](image)
It is seen in Fig. 3.61 that only in the case of small stirrup distance the ultimate dowel capacity can become equal to the stirrup strength. JOHNSTON and ZIA [38] developed a theory according to which the behaviour after dowel cracking is based on the model of a beam on an elastic foundation, with also the stirrups forming part of the support mechanism. They distinguished several possible failure mechanisms (Fig. 3.62).

<table>
<thead>
<tr>
<th>Failure modes</th>
<th>Failure criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dowel support failures</td>
</tr>
<tr>
<td>A</td>
<td>Horizontal cracking of cover</td>
</tr>
<tr>
<td>B</td>
<td>Horizontal cracking of cover at first stirrup</td>
</tr>
<tr>
<td>C</td>
<td>Yielding of first stirrup</td>
</tr>
<tr>
<td>D</td>
<td>Horizontal cracking of cover</td>
</tr>
<tr>
<td>E</td>
<td>Yielding of first stirrup</td>
</tr>
</tbody>
</table>

Fig. 3.62 Summary of failure modes and failure criteria according to JOHNSTON and ZIA [38].

In this figure the hierarchy of failure modes is A-B-C or D-E; D and E are special cases of A and C, respectively, when r = 0. Furthermore dowel material failures can only occur when encountered prior to dowel support.
failure. As pointed out in Fig. 3.62 there are four failure criteria to be considered in the analysis. The first failure criterion, horizontal cracking of the cover, is primarily a function of the tensile strength of the concrete and the state of combined stresses at the level of the tensile reinforcement. This criterion is related to the critical deflection of the composite bar, $y_{oc}$, which will cause horizontal cracking of the concrete cover. The value $y_{oc}$ has to be determined from tests. The second failure criterion, yielding of the dowel under combined shear and tension, can be determined from properties of the reinforcement. The same is true for the third failure criterion, yielding of the dowel under combined bending and tension. The final failure criterion, yielding of the stirrup, is a function of the concrete properties as well as the stirrup properties. Calculations indicate that the deflections of the dowel are damped very rapidly by the concrete. Therefore, with realistic stirrup spacings, only the first stirrup is generally considered to be active in providing support to the dowel. The results of the calculations are presented as a series of interaction curves, as given in Fig. 3.63.

In Fig. 3.63a the typical interaction curve (abcd) is composed of several segments, one for each type of failure. If the example of Fig. 3.63b is considered it is seen that if the critical deflection $y_{oc}$ of the dowel that would cause cracking of the concrete cover is 0.1 mm, the interaction curve is represented by aef. It can be seen that for $T/T_y (= \text{ratio tensile force in the steel/yielding force}) < 0.8$ approximately, there is
no reduction of dowel capacity and the capacity is controlled by cover cracking. On the other hand, for $T/T_y > 0.8$, there is a sharp reduction of dowel capacity controlled by yielding of the dowel under combined shear and tension, prior to cover cracking. If $y_{oc} = 0.025$ mm, the interaction curve is represented by abcd. For $T/T_y < 0.65$, there is a steady decrease in dowel capacity due to yielding of the dowel under combined bending and tension. For $0.65 < T/T_y < 0.95$, the dowel capacity is governed by cover cracking and is virtually constant. For $T/T_y$ approaching 1, the dowel capacity reduces very rapidly as the result of yielding due to combined shear and tension. However, the calculation method can only be used with good result, if a reliable value of $y_{oc}$ can be inserted, which is difficult, since no adequate data can be found in literature.
4. SHEAR TRANSFER IN CRACKS CROSSED BY REINFORCEMENT

4.1. Introduction.

An important mechanism of shear transfer in concrete planar constructions is shear transfer in cracks crossed by reinforcement. The shear resistance in a "reinforced crack" is the sum of the contributions of a number of individual components: aggregate interlock, dowel action and eventually axial forces in bars, inclined to the plane of cracking. As has been demonstrated before the contribution of aggregate interlock is related to a number of factors, among which the crack width is one of the most important. When cracked reinforced concrete is subjected to a given combination of stresses, the crack width depends both on geometrical factors (reinforcement ratio) and on physical factors (bond properties); furthermore, the dowel contribution is influenced by the level of axial stress in the reinforcing bars and as a result by the reinforcement ratio. So the reinforcement crossing a crack, which can be considered as a non-linear spring, has an important influence on the shear resistance in a crack; as a result the shear transfer in a crack has to be considered as a three-dimensional mechanism. Schematically this mechanism can be represented as in Fig. 4.1.

![Fig. 4.1 Mechanism of shear transfer in cracked concrete (schematically),](image)

4.2. Reinforced cracked concrete under monotonic loading.

Since firstly in recent years an increased interest developed into the shear stress-displacement behaviour of cracked concrete, in the utmost part of the experiments, carried out in order to establish the ultimate bearing capacity as a function of several parameters, not much attention has been paid to this aspect. However, the mechanical models that were
developed to predict the shear strength of "reinforced cracks", contain several features which are characteristic for the overall behaviour. Because of their illustrating character these models are dealt with prior to discussing the fundamental aspects.

4.2.1. Prediction of the ultimate shear bearing capacity of "reinforced cracks".

4.2.1.1. Reinforcement normal to the crack plane.

To predict the ultimate shear bearing capacity of reinforced cracked concrete with reinforcement normal to the crack plane often use is made of the "shear-friction" analogy. Since its introduction about ten years ago (MAST [50], BIRKELAND [5]), the shear friction concept has been widely used as a relatively simple analytical tool for the design of various types of connections and real and potential crack planes in corbels, ledger beam bearings, composite beams and the like. The method takes into account that the shear resistance is developed by virtue of the restraining action of the reinforcement against displacements normal to the crack plane, due to wedging action in the crack (Fig. 4.1). Under ultimate conditions the tensile force in the reinforcement is equal to the yielding force \( A_s f_{sy} \). Hence, in the original concept, the ultimate shear resistance was formulated as:

\[
V_u = \mu A_s f_{sy}
\]  

(4.1)

in which \( \mu \) is a coefficient of friction, taken 1.4 in design codes [1, 66]. This value for \( \mu \) corresponds with a mechanism as represented in Fig. 4.2.

![Shear friction analogy for \( \mu = 1.4 \)](image)

Fig. 4.2 Shear friction analogy for \( \mu = 1.4 \).

The applicability of the model, its restrictions and its physical background were the object of an extensive experimental study of MATTOCK et al. [51 - 56, 84] on test specimens as represented in Fig. 4.3.
Fig. 4.3 Test specimens as used by MATTOCK in several experiments.

A part of the specimens was cracked prior to loading by line loads, applied to opposite faces at the location of the shear plane, but also specimens without a preformed crack were tested. Some typical features of the test series are seen in Fig. 4.4a and b, in which a lot of ultimate shear values both for cracked and uncracked specimens are projected as a function of the total maximum force normal to the crack plane (a). The behaviour of specimens without a preformed crack is principally different from that exhibited by the specimens which were cracked prior to testing.

Fig. 4.4 Shear resistance as a function of the total normal stress perpendicular to the shear plane for cracked and uncracked specimens; a. Experimental results. b. Interfering mechanisms.
The difference in mechanisms is explicated by means of the schematical representation in Fig. 4.4b. If there is no preformed crack, the shear is initially transmitted across the uncracked plane. At higher loads (point a) short diagonal cracks form across the shear plane (Fig. 4.5).

![Fig. 4.5 Diagonal cracks in an initially uncracked shear plane.](image)

Steel bars and compression struts between the diagonal cracks form a truss, which is able to resist further loading. Failure can occur due to yielding of the reinforcing bars (traject a-b in Fig. 4.4b) or due to crushing of the compression diagonals under the combined action of axial and shear forces (traject b-c). If, however, an initially cracked specimen is tested the irregularities of the two sides of the crack ride up on each other and this tends to open the crack and create forces in the transverse steel. If the reinforcement ratio is not too high, the ultimate resistance is reached when the steel is at the onset of yielding (traject d-e). If the shear plane is heavily reinforced or subjected to a high normal compressive stress, the shear resistance of the specimen may reach the shear corresponding to failure of an initially uncracked specimen having the same characteristics. In such a case the crack locks and the behaviour and strength are similar to those for an initially uncracked section (traject e-c). In the experiments of MATTOCK however, more important features of the behaviour were established. At first it was shown that it is appropriate to add the external normal stress on the crack plane, $\sigma_N$, to the reinforcement parameter $\rho_{sy}$. Both components appeared to be equivalent and exchangable. According to this conclusion, the shear friction equation could be generalized to:

$$
\nu_u = \mu (\rho_{sy} + \sigma_N)
$$

(4.2)
However, it emerged also from the tests, that the shear friction equation may be a safe lower limit design equation, but does not reflect completely the mechanisms of shear transfer in a cracked section; for low values of \((\rho_{f_{sy}} + \sigma_N)\) the equation is too conservative and for relatively high values the shear resistance is overestimated (Fig. 4.4b). TASSIOS \([76]\) stated, that for low values of the normal force an additional friction has to be taken into account, due to which a certain resistance against sliding exists even in absence of any external normal force (Fig. 4.6).

![Graph showing shear force and displacement](image)

**Fig. 4.6 Additional fraction in shear transfer according to TASSIOS \([76]\).**

MATTOCK \([51]\) proposed a changed inclination of the teeth, from \(54^\circ\) (= arctg. 1.4) to \(38.7^\circ\) (= arctg. 0.8), which is more in agreement with experimental observations (section 4.2.2.), combined with a constant cohesion:

\[
V_u = \frac{V_u}{bd} = 2.8 + 0.8 (\rho_{f_{sy}} + \sigma_N)
\]

However, the variables in the equation, the cohesion and the tooth inclination, are still principally chosen so, that the best agreement with experimental results is obtained. FATTAH-SHAIK \([19]\) proposed, as long as the physical backgrounds of shear transfer are not sufficiently clarified, to maintain the original shear friction equation, but to insert all frictional mechanisms in one value \(u\). It was proposed to use the relation, originally suggested by RATHS \([68]\):

\[
V_u = c_R u (\rho_{f_{sy}})
\]
with:  

\[ c_r = \text{capacity reduction factor (equal to 0.85 for shear)} \]

\[ \mu_e = \text{effective coefficient of friction, calculated from:} \]

\[ \mu_e = 7 \left( \frac{C_s^2 \mu}{\nu} \right) \]

, where:

\[ C_s = \text{constant used for effect of concrete density (based on MATTOCK's tests), with:} \]

- \( C_s = 1.0 \) for normal weight concrete
- \( C_s = 0.85 \) for sand-lightweight concrete
- \( C_s = 0.75 \) for all-lightweight concrete

\[ \mu = \text{coefficient for static friction (}= 1.4 \text{ for concrete to concrete).} \]

According to several authors different expressions for \( \mu_e \) are obtained:

**MATTOCK**

\[ \mu_e = \frac{2.8}{\rho f_{sy} + \sigma N} + 0.8 \]  \[ (51) \]  \[ (4.5) \]

**BIRKELAND**

\[ \mu_e = \frac{2.8}{\rho f_{sy} + \sigma N} \]  \[ (u = 1.4) \]  \[ (5) \]  \[ (4.6) \]

**RATHS**

\[ \mu_e = \frac{3.13}{\sqrt{\rho f_{sy} + \sigma N}} \]  \[ (u = 1.4) \]  \[ (68) \]  \[ (4.7) \]

Comparison of these values with test results are given in Fig. 4.7 and 4.8.

![Fig. 4.7 and 4.8 Comparison of values for the effective coefficient of friction, given by several authors, with experimental results](image-url)
4.2.1.2. Reinforcement inclined to the crack plane.

MATTOCK [52] carried out further tests in order to be able to predict also the shear resistance of cracked planes reinforced with inclined parallel reinforcement or orthogonal arrays of reinforcing bars. In this approximation the improved shear friction relation (4.3) was used, with a tooth inclination of $38.7^\circ$.

Fig. 4.9 Shear transfer in initially cracked concrete with orthogonal reinforcement, according to MATTOCK [52].

When it is supposed that the shear displacement is really caused by sliding of the (smooth) tooth-surfaces (Fig. 4.9), the strain in the steel can be expressed as:

$$\varepsilon_s = \delta_u \cos (\theta + \phi)$$  \hspace{1cm} (4.3)

$\delta_u$ is defined as the relative displacement which would lead to yielding of the steel if this were directed normal to the crack plane. So, if $\theta = 90^\circ$, then $\varepsilon_s = \varepsilon_{sy}$. So for an arbitrary inclination of the bars (4.8) can be written as:

$$\varepsilon_s = -\varepsilon_{sy} \csc \phi \cos (\theta + \phi)$$  \hspace{1cm} (4.9)

Substitution of $\phi = 38.7^\circ$ in this equation results in:

for $0 < \theta < 12.7^\circ$ \hspace{1cm} $\sigma_s = -f_{sy}$ \hspace{1cm} (yielding under compression)

for $12.7^\circ < \theta < 90^\circ$ \hspace{1cm} $\sigma_s = -1.6 f_{sy} \cos(\theta + 38.7^\circ)$  \hspace{1cm} (4.9b)

for $90^\circ < \theta < 180^\circ$ \hspace{1cm} $\sigma_s = f_{sy}$ \hspace{1cm} (yielding under tension)
For reinforcement normal to the crack plane MATTOCK gave, as a prediction of the average values, the relation, repeated here:

\[ V_u = 2.8 \, b \, d + 0.8 \, A_s \, f_{sy} \]  

(4.10)

in which the first term was considered as the contribution of dowel action. As this value is a constant (not influenced by the number of bars, the diameter and the concrete quality) it can at most be considered as a global average value over many tests. In the case of orthogonal arrays of reinforcement, the value \( V_u \) is increased because the axial forces in the bars contribute to the shear resistance, so:

\[ V_u = 2.8 \, b \, d + 0.8 \, F + F_v \]  

(4.11)

in which (Fig. 4.10):

\[ F = (\frac{d}{s} \sin \theta) \, A_s \, \sigma_{s_1} \sin \theta + (\frac{d}{s} \cos \theta) \, A_s \, \sigma_{s_2} \cos \theta \]  

(4.12)

\[ F = \frac{A_s \, d}{s} \left( \sigma_{s_1} \sin^2 \theta + \sigma_{s_2} \cos^2 \theta \right) \]

and

\[ F_v = -\left( \frac{d}{s} \sin \theta \right) A_s \sigma_{s_1} \cos \theta + \left( \frac{d}{s} \cos \theta \right) A_s \sigma_{s_2} \sin \theta \]

\[ = \frac{A_s \, d}{s} \sin^2 \theta \left( -\sigma_{s_1} + \sigma_{s_2} \right) \]  

(4.13)

Fig. 4.10 Shear transfer in initially cracked concrete with orthogonal reinforcement [52].
Combination of (4.11), (4.12) and (4.13) leads to:

\[
\tau_u = 2.8 + 0.8 \frac{A}{bs} (\sigma_{s1} \sin^2 \theta + \sigma_{s2} \cos^2 \theta) + \frac{A}{2sb} \sin^2 \theta \left( -\sigma_{s1} + \sigma_{s2} \right) \tag{4.14}
\]

in which \(\sigma_{s1}\) and \(\sigma_{s2}\) can be calculated from (4.9b).

A comparison of theoretical and experimental values is given in Fig. 4.11.

![Comparison between theoretical and experimental values](image)

Fig. 4.11 Comparison between the theoretical failure loads according to (4.14) and experimental values \([52]\).

The small contribution of the shear components of the axial forces in the inclined bars is striking. Equation (4.14) was derived from equation (4.10) (which was valid for bars perpendicular to the crack plane). In eq. (4.10) the dowel contribution was represented by the first term. This term is also considered to be valid for an orthogonal net, according to the next argument (Fig. 4.12). The dowel force (perpendicular to the bar) is small if \(\theta\) is small and maximum if \(\theta = 90^\circ\), in which a proportionality with \(\sin \theta\) is assumed. The component parallel is then equal to \(F_D \sin^2 \theta\).

![Dowel action in bars inclined and normal to the crack plane](image)

Fig. 4.12 Dowel action in bars inclined and normal to the crack plane.
Hence, for an orthogonal reinforcement, the dowel contribution can be described as:

\[ F_D (\sin^2 \theta + \sin^2 (90^\circ + \theta)) = F_D \]  \hspace{1cm} (4.15)

This is in reasonable agreement with the test results of DULACKSKA [15].

In the case of parallel arrays of reinforcement it was assumed that:

1. The crack opens in a direction \(90^\circ - \phi\) if \(\theta < 90^\circ - \phi\) and in a direction \(\phi\) if \(\theta > 90^\circ - \phi\) (Fig. 4.13).

2. Dowel action varies with \(\sin^2 \theta\).

![Fig. 4.13 Mechanism of crack opening for parallel arrays of reinforcement.](image)

This leads to stresses in the reinforcement, equal to:

\[
\begin{align*}
\text{for } 0 < \theta < \phi & \quad \sigma_s = 0 \\
\text{for } \phi < \theta < 90^\circ & \quad \sigma_s = -1.60 f_{sy} \cos (\theta + 38.7^\circ) \\
\text{for } 90^\circ < \theta < 180^\circ & \quad \sigma_s = f_{sy}
\end{align*}
\]  \hspace{1cm} (4.16)

The total shear resistance is then:

\[ V_u = 2.8 \, bd \sin^2 \theta + 0.8 \, F + F_v \]  \hspace{1cm} (4.17)

which can be written as:

\[ V_u = 2.8 \, bd \sin^2 \theta + \frac{A_{b,s}}{sb} (0.8 \, \sin^2 \theta - 0.5 \, \sin^2 \theta)bd \]  \hspace{1cm} (4.18)

A comparison of this theoretical approximation with experimental results is represented in Fig. 4.14.
Fig. 4.14 Comparison between the theoretical failure loads according to (4.18) and experimental values \[\text{[52]}\].

The formulas are based on tests without stresses normal to the crack plane and may therefore not anyhow be considered to be valid also for this case. However, in the case of reinforcement normal to the crack plane it was demonstrated that an external stress, normal to this plane, may be added to \(\sigma_{f_{sy}}\) in the calculation of the ultimate resistance. The same is probably true for inclined parallel reinforcement.

4.2.2. Fundamental behaviour of reinforced cracked concrete under a monotonic increasing shear load.

To obtain basic information on shear transfer in normal weight and lightweight reinforced concrete MATTOCK \[\text{[53]}\] carried out an experimental investigation on specimens of the type as represented in Fig. 4.3a with special attention to the relation between the load and the displacements of the crack interfaces. The total test program consisted of a number of specimens with several concrete qualities and strengths. Within every series the specimens were reinforced with a different number of closed stirrups. All specimens, which are considered here were cracked prior to testing, which resulted in an average initial crack width of 0.25 mm (so that it can be concluded that the yielding limit of the steel must have been exceeded). The specimens were subjected to a monotonic increasing load until an average shear displacement of 0.27 mm was obtained. The load required to produce this shear displacement was approximately
proportional to the ratio of reinforcement crossing the shear plane. The separation normal to the crack plane was measured by a gage located at the middle of the length of the shear plane and the shear displacement by a gage 50 mm below it (only at one side of the specimen). Fig. 4.15 gives an example of a set of shear stress-displacement curves for one of the series.

Fig. 4.15 Shear stress-displacement curves for precracked specimens with different reinforcement ratios, according to MATTOCK [53].

Subsequently Fig. 4.16 gives a survey of the displacement paths for the different series. Every diagram contains a number of curves for different reinforcement ratios, which are shifted from each other to enable a better comparison (all initial crack widths were among 0.25 mm). It is evident that the reinforcement ratio does not influence the direction of opening significantly. Furtheron it was observed, that the direction of opening was not changed after reaching the maximum shear value. However, the most interesting conclusion from the investigation was concerned with the influence of the aggregate composition. It was concluded that the only factor affecting the opening direction was the fraction of sand and fine aggregate. Low and medium strength concretes with sand exhibited an average opening direction making an angle of 30 degrees with the crack plane, independent on the quality of the greater aggregate particles: it was indeed observed, that sand-gravel concrete specimens, in which the cracks follow the interface of the aggregate, did not behave differently from the sand-lightweight concrete specimens, in which the crack does intersect the greater aggregate particles.
However, specimens made of all-lightweight concrete and others, made of high strength sanded lightweight concrete, in which it may be expected that the crack intersects the aggregate particles of all sizes, exhibited an opening direction making an average angle of only 24 degrees. Hence, it was concluded that the shear transfer behaviour is a function more of the minor roughness of the crack faces than of their major roughness or unevenness (Fig. 4.17).
This is explained by idealizing the sand particles as spheres: the large radius of curvature of the crack surface due to major roughness can be neglected relative to the small radius of curvature of the surface of the sand particles (Fig. 4.18); the direction of motion due to overriding of the minor roughness would be 30 degrees to the line of the crack, which corresponds closely to the average measured values for the sand and gravel concrete specimens with $f'_{ccyl} = 28 \text{ N/mm}^2$ and also to the sanded-lightweight concrete specimens with $f'_{ccyl} = 17.5 \text{ N/mm}^2$.

In a later investigation [54] MATTOCK carried out tests on specimens of Fig. 4.3a with a tensile force normal to the crack plane, which was kept constant during loading, but was different for each specimen. Also here the average crack width was 0.25 mm, increased by a value of up to 0.03 mm by the external tensile force. Two series were tested with different constant reinforcement ratio ($\rho_o = 1.05\%$ and 1.57\%). No systematic variation in the angle of inclination of the crack opening curves was observed (Fig. 4.19). In this case the average angle was 40 degrees, so greater than in the fore-going series.
Fig. 4.19 Crack opening directions for precracked specimens with the same reinforcement ratios but different tensile forces normal to the crack plane according to MATTOCK [54].

4.3. Reinforced cracked concrete under cyclic loading.

Tests on "reinforced cracks" subjected to shear and normal (tensile) forces were carried out on small scale specimens, as represented in Fig. 3.18 by ELEIOTT [18] and on large scale specimens, as represented in Fig. 4.20 by JIMENEZ [37, 88].

Fig. 4.20 Test specimen, subjected to cyclic loading, according to [88].

The effect of cyclic loading depends most of all on two parameters, the crack width (narrowly related to the restraint stiffness normal to the
crack plane) and the level of shear stressing. Some results of the tests on the large scale specimens, in which different bar sizes and levels of cyclic shear were considered, are represented in Fig. 4.21.

![Diagram of shear displacement and crack width](image)

Fig. 4.21 Specimens subjected to the same level of shear stress (cyclic loading). Specimen a: 4 φ 22, \( w_0 = 0.50 \) mm; Specimen b: 4 φ 19, \( w_0 = 0.50 \) mm; Specimen c: 4 φ 29, \( w_0 = 25 \) mm [36].

The first and the second specimen contained 4 φ 22 and 4 φ 19 mm reinforcing bars respectively, which were stressed to such a level that an initial crack width of 0.50 mm was obtained; the reinforcement in the third specimen was equal to that of the second, but stressed to a lower level, so that the initial crack width was only 0.25 mm. The peak shear displacement decreased and the initial stiffness increased with increasing bar sizes. (Compare the specimens a and b in Fig. 4.21). Specimen a required a shear displacement about twice that of specimen b to mobilize the shear transfer mechanism. While specimen b maintained a constant crack width during the first 15 cycles, specimen a exhibited a decreasing rate of increase in both displacements. Next the favourable influence of the
lower axial stress level is seen comparing the results of the specimens b and c. In Fig. 4.22 the results are shown for a specimen with two bars $\phi$ 45 mm with an initial crack width of 0.5 mm, subjected to increasing shear stress levels.

![Graph showing shear stress levels and crack faces](image)

Fig. 4.22 Results for a specimen, reinforced with two bars $\phi$ 45 mm, subjected to increasing shear stress levels [88].

It is seen that an increase in shear level results in an enhanced overriding of the crack faces, which is visible in the increasing values for the shear displacement and the crack width. Because a lot of combinations of parameters are possible (axial restraint stiffness, axial tensile force, concrete strength) and the number of test results is small, only global tendencies can be given.

4.4. Reinforced cracked concrete subjected to impact loading.

CHUNG [12], who carried out impact loading tests on unreinforced joints, (section 2.4), conducted also tests on reinforced specimens. In this case the specimens were reinforced with two 5 mm mild steel stirrups, perpendicular to the joint (Fig. 4.23).

![Diagram of specimen](image)

Fig. 4.23 Specimen subjected to impact loading according to CHUNG [12].
Again the specimens were subdivided in four groups. The first group was used to establish the static ultimate load; the other three were only impact-loaded. The specimens of group three and four were, prior to loading, subjected to two millions cycles of repeated loading at 400 cycles/min, the peak load of which was 55% and 66% of the static resistance. Without repeated loading the dynamic increase factor was equal to 2.02 (group two), while repeated loading to 55% and 66% before testing resulted in dynamic increase factors of 1.98 and 1.80. Also here it was seen that repeated loading of 55% intensity did not have a significant detrimental effect, while repeated loading of 66% intensity reduced the impulse capacity (20%). A difference in behaviour between reinforced and unreinforced specimens was the ductility of the joint and its impulse capacity, which was appreciably greater for the reinforced joints. A survey of the total series is represented in Table 4.I and II.

<table>
<thead>
<tr>
<th>Specimen group</th>
<th>Number of specimens</th>
<th>Shear reinforcement</th>
<th>Loading history</th>
<th>Applied load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rise time (10^(-3) sec)</td>
<td>Duration (10^(-3) sec)</td>
</tr>
<tr>
<td>A1</td>
<td>4</td>
<td>0</td>
<td>S</td>
<td>-</td>
</tr>
<tr>
<td>A2</td>
<td>8</td>
<td>0</td>
<td>D</td>
<td>0.80</td>
</tr>
<tr>
<td>A3</td>
<td>8</td>
<td>0</td>
<td>R_1 + D</td>
<td>0.91</td>
</tr>
<tr>
<td>A4</td>
<td>4</td>
<td>0</td>
<td>R_2 + D</td>
<td>0.68</td>
</tr>
<tr>
<td>B1</td>
<td>4</td>
<td>0.43%</td>
<td>S</td>
<td>-</td>
</tr>
<tr>
<td>B2</td>
<td>8</td>
<td>0.43%</td>
<td>D</td>
<td>1.01</td>
</tr>
<tr>
<td>B3</td>
<td>8</td>
<td>0.43%</td>
<td>R_1 + D</td>
<td>0.98</td>
</tr>
<tr>
<td>B4</td>
<td>4</td>
<td>0.43%</td>
<td>R_2 + D</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Table 4.I. Summary of test results of CHUNG [12].
<table>
<thead>
<tr>
<th>Specimen group</th>
<th>Rate of stressing (N/mm²/sec)</th>
<th>Shear strength (N/mm²)</th>
<th>Dynamic increase factor *</th>
<th>Impulse capacity (10⁻³ N·sec/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>-</td>
<td>4.96</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A2</td>
<td>11 100</td>
<td>8.91</td>
<td>1.80</td>
<td>9.36</td>
</tr>
<tr>
<td>A3</td>
<td>10 000</td>
<td>9.12</td>
<td>1.84</td>
<td>9.86</td>
</tr>
<tr>
<td>A4</td>
<td>12 800</td>
<td>8.69</td>
<td>1.75</td>
<td>8.09</td>
</tr>
<tr>
<td>B1</td>
<td>-</td>
<td>5.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B2</td>
<td>10 300</td>
<td>10.45</td>
<td>2.02</td>
<td>12.90</td>
</tr>
<tr>
<td>B3</td>
<td>11 500</td>
<td>10.24</td>
<td>1.98</td>
<td>12.90</td>
</tr>
<tr>
<td>B4</td>
<td>12 800</td>
<td>9.33</td>
<td>1.80</td>
<td>10.32</td>
</tr>
</tbody>
</table>

* Dynamic increase factor = Dynamic strength / Static strength

Table 4.11. Shear strength and impulse capacity according to the test results of CHUNG [12].
5. CONCLUSIONS

1. To describe the shear transfer in cracks a lot of formulations of basically different type are encountered, several of which would result in erratical results, if applied in calculations concerning shear critical constructions.

2. When the interfaces of a crack are submitted to a shear displacement, not only a shear stress is developed, but also wedging action, resulting in compressive stresses normal to the crack plane. Due to this wedging action, reinforcement perpendicular to the crack plane or other restraint forces are activated, which are directly related to the shear displacement, necessary to provide equilibrium. Although the direct resistance against shear displacement of concrete interfaces, generally indicated with the term 'aggregate interlock', has been investigated by a number of authors in several ways, hardly any attention has been given to the phenomenon of wedging action, which provides the link between the normal and the shear stresses and deformations in a cracked structure.

3. Upon the fundamental mechanism of shear transfer in cracks several opinions, based on different test results exist. In [43] two levels of crack roughness are distinguished governing the behaviour of crack interfaces subjected to a shear displacement: a global roughness, leading to overriding and a local roughness, producing an initially great resistance against shear displacement, which disappears due to crushing under increasing stresses. In [53] it is stated that the level of the sand particles is essential for the transfer of stresses in cracks, and only overriding is considered. In [78] the ratio between aggregate strength and matrix strength is considered to be the most important variable influencing the degree of roughness and as such the behaviour.

4. The crack width is generally considered as the most important variable influencing the shear stress-displacement ratio. In general also the concrete strength is believed to be an important factor. About the influence of maximum particle size and type (natural, crushed) several opinions are met.
5. Although experiments, carried out by several authors to study the transfer of stresses in cracks, reveal a lot of tendencies of the behaviour, the data available are or incomplete or not accurate enough to be used for deriving basic characteristics.

6. For cyclic loading a pronounced difference was observed between the first and the subsequent cycles. In general in the first cycle an approximately linear relation between shear stress and shear displacement was observed, for the subsequent cycles a severe non-linear, hardening type, relation was measured. However, for small crack widths (< 0.25 mm) a different behaviour was observed. The most important parameters concerned with cyclic loading are: the concrete quality, the crack width, the number of cycles and the maximum shear level. Although a lot of important work has been done on this field at Cornell University, Ithaca, a systematic investigation of these parameters, especially for lower crack widths, still stands out.

7. Repeated loading under service conditions did not influence the impact-shear resistance of closed cracks in concrete specimens with a strength of $f'_{cyl} \geq 40 \text{ N/mm}^2$.

8. If the relation between dowel stress and displacement is described by a model of a beam on an elastic foundation, the critical variables are the free length of the dowel - depending on the bond properties of the steel, the concrete properties and the inclination of the reinforcing bars to the crack plane - and the foundation modulus of the concrete, which decreases as a function of shear and axial deformations. It was shown that an axial stress in the dowel bar reduces the resistance against shear displacement.

9. The shear stress-displacement relations for aggregate interlock and dowel action have a similar form. In general it was observed that, for small crack widths aggregate interlock dominates over dowel action.

10. More experimental results are necessary to be able to insert appropriate basic material properties concerning the transfer of stresses in cracks in modern non-linear numerical calculation procedures.
6. REFERENCES


7. Building Code Requirements for Reinforced Concrete (ACI 318-71), American Concrete Institute, Michigan, 1971.


91. ZIENKIEWICS, O.C., PHILLIPS, D.V. and OWEN, P.R.J., "Finite element analysis of some concrete non-linearities - Theory and examples", Proceedings of the Seminar 'Concrete Structures subjected to triaxial stresses', May 1974, Bergamo, Italy.