Modelling, simulation and identification of a steering system actuator

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Modelling, simulation and identification of a steering system actuator

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by

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in partial fulfillment of the requirements for the degree of

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Abstract

The Automotive Development Center of SKF has been developing models for sensor bearings targeting brushless electric motor control. Since brushless electric motors need a rotor position measurement in order to be able to determine the stator currents, SKF comes up with a solution that integrates this angular position sensor in the motor bearings. Specifically, SKF is interested in developing sensor bearings using magnetic sensing technology used in actuators for steering systems such as Active Front Steering and Electric Power Steering, which nowadays for economic and environmental reasons use electric power. These sensors have to be very accurate, since the error in angle measurement causes the torque ripple (the variation in torque, typically high-frequent) to increase which is undesired since this causes audible noise and vibrations which can be felt in the steering wheel of the car. Nowadays there is no validated model available of such an analogue sensor, and there is a lack of knowledge with respect to the interaction of the sensor with the motor and the entire Electric Power Steering system.

Therefore a master thesis subject was defined on this matter, with as objective to deliver a validated model of a bipolar and multipolar sensor. The sensor works as follows: a number of Hall-effect cells capture the magnetic field intensity of a magnetic ring fixed to the inner ring of the bearing, which is thus rotating with the motor shaft. These cell signals are used to construct a sine and cosine signal, which deliver the angle using the arctangent function. The analogue sine and cosine signals are the output of the sensor. If a bipolar magnet is used, the output signals are absolute which signifies that they denote a mechanical revolution. However, for high performance applications a multipolar magnetic ring is used, which delivers directly the electrical angle if the number of pole pairs of the magnetic ring is equal to the number of permanent magnets in the rotor of the brushless motor. Since this electrical angle is used for determination of the stator currents, in the case a bipolar magnet is used to control the same motor, the signal has to be multiplied in order to obtain the electrical angles. Therefore the hypothesis is that for sensors having comparable performances (having the same or similar specifications) the multipolar sensor will deliver a torque ripple $N$ times smaller...
than the bipolar sensor, when used for control of a motor with $N$ permanent magnets. The torque ripple is the variation in torque which needs to be minimized.

Simulations have been done with a model of a bipolar and multipolar sensor, which has been validated using measurements done on prototypes having both a bipolar and multipolar ring. The model can be used to investigate the performance of the sensors which is measured by the amplitude variation of the sine and cosine, the offset (difference with zero) of the sine and cosine and the nonlinearity, which is the error in angle measurement. Using this sensor model, two important basic parameters have been identified which determine the nonlinearity of the sensor: the mechanical design (notably the number of Hall-effect cells that is used) and the quality of the magnetic ring. Subsequently the sensor model has been integrated in a brushless electric motor model to investigate the torque ripple in the system present for the same motor equipped with a bipolar and multipolar sensor. It is found that the theory is confirmed, but that the torque ripple is not $N$ times as small for multipolar sensors but only twice, and that the variation of the torque ripple among the collection of sensors that is used for simulation is much smaller for multipolar sensors than for bipolar sensors. This signifies that multipolar sensors are more robust with respect to nonlinearity differences among the sensor collection, since the torque ripple only slightly increases when the results of the simulation done with the worst sensor are compared with the results of the simulation done with the best sensor from the collection.

Lastly, to be able to investigate further the influence of the sensor on the system and to give more insight in the way the mechanical system is built up, a modular model is developed of a rack type belt-driven Electric Power Steering (EPS) system. An important feature of this model is the fact that it is flexible: because of the use of modular components other architectures of EPS systems can easily be built as well. The model has been partly validated using reference values found in literature. It would be interesting to continue improving this model by performing validation with the use of a physical test set-up, in order to investigate the accuracy of the model. Also some suggestions are done to improve the sensor performance and the sensor modelling.
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Delft, University of Technology

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Susanne Blokland
The time when a car was steered using only human power at the steering wheel moving the wheels through mechanical transmissions is long gone. Most cars are nowadays equipped with power steering. This permits the driver to obtain the same wheel steer angle with less effort which significantly enhances the comfort of the driver. Not only comfort is important though; also safety plays a big role in the implementation of assisted steering. Active front steering is introduced to be able to change the front wheel steer angle of the car independently from the driver. On the road nowadays are only cars with a mechanical link between the steering wheel and the wheels since Steer-by-Wire (where the link is removed) is too expensive due to the need for redundancy related to safety. The first power steering systems were hydraulic. Mainly because of legislation with respect to the emission of CO$_2$ and because of the simplification of the implementation of the assisted steering for car manufacturers, the trend is to have brushless electric motors delivering the extra power needed. For choosing the motor for these steering applications the torque ripple is an important criterion since it denotes the quality of the output torque.
In fact, torque ripple is the variation in torque delivered by the motor. It causes vibrations in the system which can be felt by the driver in the steering wheel, and it creates audible noise. This torque ripple and its analysis is the subject of this master thesis. An introduction to the subject is given in section 1-1. The detailed problem definition is given in section 1-2 and subsequently, in section 1-3 the objectives of this master thesis are defined. The chapter concludes with the contents of the master thesis in section 1-4 and some details on the company where the master thesis is performed in section 1-5.

1-1 Introduction to the subject

A schematic view of a brushless motor is seen in figure 1-2.

![Figure 1-1: Definition torque ripple](image1)

![Figure 1-2: Schematic overview of a brushless electric motor](image2)

In this figure the motor, sensor and control are included. A brushless motor con-
1-2 Problem definition

To obtain a smooth torque without a lot of torque ripple, an accurate rotor position is needed with a maximum error of approximately $\pm 1^\circ$. The current sensing technology using a bipolar magnet does not deliver a sufficient accuracy for brushless motors having multiple permanent magnets for performance reasons. It speaks for itself that if for a brushless motor consisting of a single permanent magnet on the rotor a bipolar sensor is used, for multiple permanent magnets a multipolar sensor can be used to improve the accuracy of the sensed rotor position.

Nowadays within ADC-SI at SKF there is a need for a simulation tool predicting the possible performance and accuracy of these multipolar sensors. In general, since the focus within ADC-SI is on the sensors itself, there is a lack of prior knowledge as well from the system point of view. This includes the interaction of the sensor with the motor and with the entire electric power steering system.

1-3 Objectives

An overview of the objectives of this master thesis is given in figure 1-3. The overview also includes the objectives of the preceding literature thesis. These objectives are based on the problem definition given in section 1-2. In this overview, the path that is
followed during the master thesis is denoted. The bipolar sensor is the currently used analogue magnetic sensor. The model of the bipolar sensor that already partly exists is named SOFIAs. For validation of this model the bipolar and multipolar prototype TSxC can be used. The sensor model is coupled with an high-resolution converter model called SAHRAs. To obtain an actuator model, the complete model is integrated in an existing and validated brushless electric motor model named SiMBBaS. In order to be able to get an overview of the impact of the sensor on the steering system, a basic model of an EPS is developed in which the electric motor model is contained. From this approach clearly the modular structure of the models is seen, for the models will all be integrated in each other. For convenience, all existing models and those being developed within ADC-SI are denoted in figure 1-4 as well as their relations and connections with each other.

Figure 1-3: Overview of objectives and stages of literature study and master thesis
The end goals of the master thesis are:

- Provide a validated bipolar and multipolar sensor model
- Develop an EPS model that:
  - Includes the electric motor model including different sensor models
  - Can be used with any model of an electric motor, sensor, etc.
- Investigate the performance of the multipolar sensor and prove the improvement of the motor control when using the multipolar sensor
- Provide an EPS model that can be used to investigate the impact of different sensors and electric motor specifications on the entire EPS system

In figure 1-4 the different software models are seen that are mentioned and their relation towards one another is shown. The focus is on delivering models that are able to predict sensor and system behavior within a certain confidence level. These models can be used to provide specification guidelines for customers and to demonstrate the potential of SKF products.

1-4 Contents

The thesis is build up as follows. First, the component that represents the SKF product is modelled: the sensor. From that point of view, the system is step by step extended...
by plugging this model into other modular models. The same approach is taken to structure the thesis. To begin with, the sensor modelling is described and from there on, each chapter denotes a step higher in the integration of this model:

Chapter 2: Modelling of the sensor
Chapter 3: Sensor performance
Chapter 4: Integration sensor and motor
Chapter 5: Modelling of an Electric Power Steering system

These constructive chapters are followed by the conclusions and recommendations:

Chapter 6: Conclusions
Chapter 7: Recommendations

Furthermore, appendices are added with measurement results and other extra information as well as a bibliography and glossary.

1-5 The company

SKF Group is the leading global supplier of products, solutions and services within rolling bearings, seals, mechatronics, services and lubrication systems. Not only provides SKF their customers with high quality products, there are also Service units that provide technical support, maintenance services, condition monitoring and training. The company was founded in 1907 in Sweden and quickly became a worldwide company with establishments in Europe, America, Asia, Australia and Africa by 1920. Today SKF is represented in more than 130 countries including 110 manufacturing sites. In total SKF Group employs almost 45,000 people. The abbreviation of SKF stands for Svenska KullagerFabriken in Swedish, which means Swedish roller bearing factory. The SKF business is organized into three divisions: Industrial, Automotive and Service. This thesis is performed at the Automotive Development Center which has locations in the Netherlands (Nieuwegein) and in France (Saint-Cyr sur Loire). The department where the thesis is performed is Automotive Development Center - Sensor Integration (ADC-SI) in France, which is occupied with the development of mechatronic solutions for sensor bearings. At the site in Saint-Cyr sur Loire also production lines are present of the Automotive and Industrial Division. Integrated wheel bearings for the automotive industry and deep groove ball bearings and sensor bearings for the industry are manufactured in Saint-Cyr sur Loire among other products. SKF is as first bearing production company ISO 14001 certified since 1998, which is an environmental certification. Sustainability with respect to the environment is an important topic for SKF, which is also found back in the Vision, Mission, Drivers and Values of the company and in specific commitments such as "BeyondZero" that states that SKF will realize
business objectives in such a way that negative environmental impact is minimized.

SKF is working with Six Sigma which is a systematic and disciplined approach to achieve excellence in all existing and new processes within the company. In using the Six Sigma tool SKF is able to provide solutions for reoccurring problems without a known solution. In the research and development area, Design for Six Sigma is used to ensure a robust design of new products. For the development of simulation tools, the Plan-Do-Check-Act (PDCA)-method is used at ADC-SI. This means that an entire project is divided into four phases: the Plan phase where the necessary information is gathered and where a planning is made. In general, in this phase the progress of the rest of the project is defined. In the second phase, the Do phase, the modelling of the tool is actually carried out and in the third phase, the Check phase, the work is examined by means of a validation procedure. This is done to obtain confidence in the simulation tool. Finally, the project enters a phase that is repeated: the Act phase. Here ameliorations are defined and a review is done on the project and the project phases. This phase enables the amelioration of the tool by partly following the same method again to improve the simulation tool.
Chapter 2

Modelling of the sensor

In this chapter the basic analytic model of the rotor position sensor will be described. This model will be used as basis for the modelling in MATLAB. The model includes both the use of bipolar and multipolar rings. Since for the development of the model the PDCA-method was used (Plan-Do-Check-Act), this approach is also used as structure for this chapter. In section 2-1 a start will be made with a general description of the working principle of the sensor and the design. The Plan phase of the modelling will be the subject of section 2-2. Here the objectives, name and logo of the simulation tool are presented as well as the model architecture. In the Do phase, the modelling will be described extensively (section 2-3). Subsequently, the validation of the simulation tool is the subject of section 2-4 (Check phase) and in the Act phase, possible ameliorations of the simulation tool are presented (section 2-5).

2-1 General description sensor

The basic working principle of the SxC is as follows: a magnetic ring consisting of at least one north pole and one south pole is mounted on the inner ring of a motor bearing. This bearing is mounted on a motor shaft so when this shaft is rotating, a sinusoidal magnetic field is generated with respect to the outer ring of the bearing. This rotating magnetic field is denoted by $B(\omega t)$ with $\omega$ the rotational velocity of the shaft. This is visualized in figure 2-1. If a bipolar ring is used, one mechanical revolution yields one

![Figure 2-1: Rotating magnetic field visual representation](image-url)
period of the sinusoidal magnetic field. The south pole of the magnet generates the positive magnetic field, and the north pole the negative magnetic field intensity. This rotating magnetic field is subsequently captured by a certain number of Hall-effect cells which measure the magnetic field intensity. Hall-effect cells are ratiometric; i.e. if the magnetic field intensity becomes twice as large, the output of the Hall-effect cells also becomes twice as large (keeping in mind the limits). The working principle of the cells is based on the potential difference that is produced when a magnetic field is applied perpendicular to the current flow in the Hall-effect cell. The current flow will bear off as a result of the Lorentz force and there will be a potential difference over the conductor. This is illustrated in figure 2-2. The cells all capture the same magnetic field but at a different position and at a slightly different distance from the ring (the airgap). The mathematical expression for one cell signal is:

\[ U_i = o_i + g_i B (\omega t - \varphi_i) \cdot e^{-k \cdot \text{airgap}} \]

with \( o_i \) the offset of the Hall-effect cell, \( g_i \) the sensitivity of the Hall-effect cell, \( \varphi_i \) the placement angle of the cell and \( k \) the convexity constant which defines the exponential decrease of the magnetic field with the radial distance (airgap). This can be visualized as in figure 2-3. The output voltage signals of the Hall-effect cells are added to each

Figure 2-2: Hall-effect cell: with (right) and without magnetic field applied

Figure 2-3: Visualization of the cell signals and their relative phase shifts
other taking into account the placement angle of the corresponding cell. The sensor output signals that are obtained are a sine and a cosine, which form the analogue output of the sensor. The analytical expressions for these operations are:

\[
\begin{align*}
sine (t) &= \sum_{i=1}^{N} U_i (t) \cos \varphi_i \\
\cosine (t) &= \sum_{i=1}^{N} U_i (t) \sin \varphi_i
\end{align*}
\]

These two signals together form the analogue output of the sensor. To obtain the angle from these signals, the arctangent formula can be used as can be seen in figure 2-1. So the expression for the angle is:

\[
\theta (t) = \text{atan2}(\text{sine} (t), \text{cosine} (t))
\]

where the \text{atan2} function is used since this function is defined on the domain \(-\pi, +\pi\).

The physical design of the sensor can be seen in the exploded view in figure 2-4. In this figure, (1) denotes the bearing, (2) is the outside cover, (3) represents the magnetic ring attached to the inner ring of the bearing, (4) is the element at which the Hall-effect cells are mounted as well as the remaining electronics, and (5) is the cover for the electronics.

Summarizing, to obtain a model of the sensor the following consecutive steps have to be taken into account:

- Generating the rotating magnetic field (output: \( B (t) \));
- Capturing the magnetic field by Hall-effect cells (output: \( U_i (t) \));
• Reconstruction of the sine and cosine using the cell signals (output: \( \text{sine}(t) \) and \( \text{cosine}(t) \));

• Post-conditioning of the sine and cosine (output: \( \text{sine}(t) \) and \( \text{cosine}(t) \)).

In the following sections, the modelling of the sensor will be described. This will be done with the help of the approach used at SKF, the PDCA-method as mentioned before. This method used four project phases which are explained in the following sections.

### 2-2 Plan phase

In the Plan phase of the PDCA-method the preparation is done for the modelling of the sensor. This includes the definition of the objectives of the model, the establishment of a time planning for the project and the gathering of all the information needed for the second project phase where the simulation tool is actually built.

**Objectives** The objectives of the model are to obtain an accurate description of a bipolar and multipolar SxC, signifying that the important outputs of the system are well estimated. The model should also permit the simulation of 'collections' of sensors, which means that the simulation tool takes into account the variation of all parameters and is capable to estimate the minimum, mean, maximum and standard deviation of the important outputs. These important outputs will be regarded later in this section. The model will have to be validated in a certain parameter range, which will permit a prediction of sensor performance within a certain confidence level for a larger parameter range since the nature of a simulation tool is to predict the performance of real parts.

**Name and logo** To give the simulation tool a recognizable identity, a name and logo are chosen in the Plan phase. Since there was already an existing non-validated model of a bipolar sensor partly available, this was already done. Therefore the new logo of the multipolar sensor is an extension to this logo. The name of the simulation tool is SOfware For Integrated Angle Sensor-bearing (SOFIAs). The logo for the multipolar model is shown in figure 2-5 and consists of an "m" in superscript signifying that this model works with multipolar rings as well as bipolar rings.

![SOFIAs](image)

**Figure 2-5:** Logo of SOFIAs, multipolar version
Model architecture  To obtain a model of the sensor, first the important inputs and outputs of the simulation tool have to be defined. The sensor performance is defined using the nonlinearity. The nonlinearity is basically the ‘error’ of the measurement of the rotor position: it is the difference between the calculated angle using the sine and cosine output of the sensor and the ‘true’ angle (see figure 2-8). The need for this error measurement makes that not only the sensor output has to be calculated but also (part of) the ECU has to be simulated to obtain the nonlinearity. This is denoted in figure 2-6. Both modules denoted in figure 2-6 are needed for the control of

the brushless electric motor; the difference is that the first part denotes the complete product of SKF whereas the second part (the ECU) is done by the customer. In the ECU the outputs of the sensor are used to obtain the required currents. This means that certain knowledge has to be available on how the analogue signals are used for motor control. Other important outputs of the sensor that have to be predicted with a certain maximum error are the amplitude and the offset of the sine and cosine, the phase shift between the signals, the offset of the nonlinearity (defines as the mean of the nonlinearity over a mechanical turn) and the delay. These parameters are all characteristics of the two output signals and of the nonlinearity as is shown in figure 2-7 and figure 2-9. The nonlinearity is more complex to determine, especially regarding bipolar

and multipolar rings. In this simulation tool, the nonlinearity is calculated using the
difference between the calculated angle with the sine and cosine and an ideal reference (see figure 2-8). But how to interpret this nonlinearity? Therefore the application is important, though it is not desirable to include the application in the simulation tool. Therefore it is chosen to use a mechanical and electrical nonlinearity, signifying that the nonlinearity is not only calculated over one mechanical turn of the motor shaft but also per period of the output signals. Logically, this yields the same result for a bipolar sensor, since one mechanical turn is denoted by one period of a sine and cosine. But for multipolar rings, there is a difference between these nonlinearities since the multipolar ring has multiple periods (depending on the number of pole pairs) covering one mechanical turn. The difference between the mechanical and electrical nonlinearity is visualized in figure 2-9. Per sensor, a nonlinearity is thus obtained which evaluates over one revolution of the shaft of the motor as seen in figure 2-9. To give a certain performance level to the sensor, the peak-to-peak nonlinearity is introduced. Basically, per sensor there exists one mechanical peak-to-peak nonlinearity. In addition to that, for the multipolar sensor a number of electrical peak-to-peak nonlinearities equal to the number of pole pairs of the magnetic ring of the sensor exist. These are calculated as the difference between the maximum and minimum value of the nonlinearity.

Now that the inputs and outputs of the model are known the model architecture can
be defined which is logically based on the fact that the model consists of the two parts defined before: the sensor part and the ECU part. These parts, or blocks contain the analytical models for the various submodels of the sensor. Apart from this, a user interface has to be defined to obtain a structured way of defining the variables. Also the simulation results have to be processed in order to be able to draw conclusions on the sensor performance. The model structure for the proposed MATLAB model is shown in figure 2-10. In figure 2-10, the main blocks are shown as well as the functions combining them. For instance, a function is the Input definition, which consists of the blocks Interface, Init and partly, StatParameters. In the block Interface all the sub-blocks defining the interface are contained. In Init (short for initialization), the user inputs are collected from the interface and saved in structures. These structures correspond to the sub-blocks where they are mainly used. For instance, in the block where the cell output signals are defined (Block_cells, which is a sub-block of the Sensor block in figure 2-10) the parameters that are necessary to define these Hall-effect cell output signals (such as the cell offset and sensitivity) are taken from the structure cells. In StatParameters, the statistical parameters are defined that are different for each simulation in a cycle (unless the variation is set to zero, of course). These statistical parameters are defined through a normal distribution. In figure 2-10, the distinction between the sensor block and the ECU block is clearly visible. The contents of the blocks shown will be further explained in section 2-3, where the next phase is described. This phase is the 'Do' phase, where the actual model is realized. In this phase also the limits of the simulation tool are explicitly defined.

2-3 Do phase

The analytic model of the actual sensor and ECU will now be described as well as certain limits and assumptions that are made regarding the model. The sensor model will be described in subsection 2-3-1 and subsequently the ECU model will be regarded in subsection 2-3-2. The interface and results blocks that form the bridge between the model and the user are described in subsection 2-3-3.
2-3-1 Sensor model

The sensor model exists of three parts. Schematically, the block looks as denoted in figure 2-11.

*Figure 2-11: Contents block sensor*

**Magnetic field description** The limits on this part of the simulation are first of all that there is a limited number of ways to describe the magnetic field. These are different for bipolar and multipolar rings and derived from a number of measurements of magnetic rings. Two examples of such measurements are shown in figure 2-12.

*Figure 2-12: Two examples of ring measurements - bipolar and multipolar ring*

To characterize the magnetic field, approximations based on different approaches are used. These are:
• Collections of rings
• Single rings

For collections of rings, the Six Sigma approach is used. This method is based on defining a mean value for the parameter and a variation. Obviously, when only a single ring is used no variation is taken into account and exactly the same ring characterization is used for all simulations.

The following magnetic ring models are included in the SxC simulation tool SOFIAs:

• Perfect sine model;
• Pole length model;
• Pole pair length model (only for multipolar rings);
• Combination of harmonics model;
• Material model (only for bipolar rings);
• Real measurement model.

The most obvious characterization of the magnetic field which can also serve as a reference is to use a perfect sine with as period the number of pole pairs. This model is visualized in figure 2-13. Secondly, the amplitudes of the different poles or pole pairs (in the case of a multipolar ring) can differ as well as the length of one pole (pair). The definitions of these lengths are given in figure 2-12. This leads to another description of the magnetic field: the mean amplitudes of each south and north pole can be defined as well as the pole (pair) lengths of each pole (pair) independently. With this information, two models can be defined. The first model varies the mean amplitude per pole pair (same mean amplitude for the south and north pole) and the length per
pole pair. The lengths of the pole pairs are chosen through a normal distribution from the mean value. Of course these lengths are chosen in such a way that the total mechanical length of 360° is guaranteed, and it is noted that this model is only applicable to multipolar rings. The second model varies the mean amplitude and length per pole and keeps them constant over the pole pairs. This means that all pole pairs have the same length which is equal to 360° divided by the number of pole pairs. The models are visualized in figure 2-13 and figure 2-14. Another description is given by the harmonic decomposition of the magnetic field. Every signal can be approximated by a sum of sines with certain amplitude and phase. Regarding figure 2-12, the bipolar ring’s magnetic field intensity is not a proper sine as can be especially well viewed from the north pole. This form of the magnetic field intensity can be described by adding a certain harmonic. The model is visualized in figure 2-13. A collection of harmonic models is obtained by adding a standard deviation to each harmonic amplitude. Because of the manufacturing process of multipolar rings, their shape is sometimes topped off at the peaks. This is clearly seen in figure 2-12 and it should be noted that it is definitely not a cause of saturation of the cells (used by the measurement), which gives the same shape but at far larger magnetic field intensities. This phenomenon is implemented in the multipolar pole pair model.

Since there is an interest in using a fixed magnetic ring profile to for instance see the influence of certain ring parameters on the variation with the design, or to reproduce with the simulation tool existing sensors there is a magnetic model added to the simulation tool that uses a measurement as input magnetic field. Since this measurement is done at a certain airgap and the magnetic material has a certain exponential decrease of the intensity, these two parameters are also an input to this model to be able to simulate at a different and varying airgap. It is noted though that every harmonic of the magnetic field decreases exponentially with a different factor. So to obtain reliable results, a measurement should be used which is done at an airgap close to the desired nominal airgap value of the design.

Finally there is one other bipolar model that already existed implemented in the simulation tool. This material model determines the magnetic field properties $k$ and $B_0$ as

$$[B_0, k] = f(\text{thickness}, \text{airgap})$$

Figure 2-14: Magnetic ring models - Part 2
function of the thickness of the ring and the airgap (the distance between the ring and the cells).

**Cells** The modelling of the cell signals is done as follows. Through the selection of the cell type, the nominal value and the standard deviation of the cell offset and sensitivity are known which are different per cell. Per simulation for each cell a sensitivity and offset value is taken from the collection described by the nominal value and standard deviation. Regarding the cell placement, two default placements are possible for the multipolar rings, which are denoted in figure 2-15. The main difference is that for Design 1 (at the left) the cells take their signals from different pole pairs, while Design 2 takes the signals from the same pole pair. For a perfect sinusoidal-shaped magnetic field, both methods will yield the same results if the cells are perfectly placed. But depending on various parameters such as the deviation in pole (pair) length, it is possible that obtaining the cell signals from one pole pair can be more convenient than obtaining them from different pole pairs.

For each cell, the mean placement angle and the standard deviation are independent inputs. The magnetic field intensity that is seen by the cells is dependent on the airgap, which is equal to the distance between the active element of the cell and the magnetic ring surface. There are in fact two deviations that determine this distance eventually. The package airgap, i.e., the distance from the ring to the Hall-effect cell package is defined using a mean value and standard deviation depending on mechanical tolerances. This yields one package airgap per sensor. The airgap varies among the number of cells, since the active element within the package is not always placed exactly in the same way. This tolerances are taken into account when calculating the airgap per cell.

Furthermore, noise can be added to each of the signals with a certain amplitude given in [mV/V] of the cell signal. Filtering can be applied to filter out high-frequency noise. The filters implemented in the model are a first-order RC-filter and a Butterworth filter. In practice, the mean value and standard deviation of the cell offset and amplitude are dependent on temperature and supply voltage. In the model, the values are taken for the reference temperature and supply voltage. It is assumed that the amplitude of the magnetic field varies linearly with temperature, and that the cell offset and amplitude vary linearly with supply voltage. In figure 2-16 this is denoted for the offset. In
Modelling of the sensor

Figure 2-16: Extract from cells datasheet: offset varies with temperature and supply voltage

Figure 2-16, it is clearly seen that the offset is dependant of temperature. For instance, when the temperature is 85°, the offset of the regarded cell type (see red dot) is maximal 3% higher or lower than the offset at 25°. With supply voltage, the difference between the minimum and maximum offset becomes larger (and so the standard deviation) and the mean shifts up. In green the assumptions of SOFIAs are denoted. The same is done for the sensitivity in figure 2-17. Because the variation of the cell sensitivity and offset is neglected in the model, the results of the simulations will only be reliable at reference temperature of 25° and for a supply voltage of 5 Volt.

Reconstruction sine and cosine  The reconstruction of the sine and cosine signals, the actual output of the sensor denoting the angle, is modeled using the analytical relation described in section 2-1. This method yields as resulting output signals with an amplitude twice multiplied by the number of cells (due to the use of multiple cells and the placement angle) and an offset of zero. The output amplitude is in fact desired to be equal to the input amplitude of each of the cell signals, thus for the analytical method, the output amplitude is multiplied by $\frac{2}{N}$ where $N$ is the number of cells. The equiva-
lent of this method that is used in reality is an electric scheme consisting of resistances and op-amps. The output voltage would drop to zero if all input signals approach zero, since using the ideal, analytical description mentioned above yields an offset of zero. To avoid this, a signal is added which adds as offset half the supply voltage of the electric scheme (op-amps). This is denoted in figure 2-18. The branches on the bottom con-

![Figure 2-18: Example of the electric scheme of a sine signal of an S5C](image)

nected to the supply voltage and the ground are the branches that are added to create the offset. The signals denoted with “U” are the cell signals. The resistances are chosen in such a way that the signals are added according to their placement angles around the magnetic ring. Of course, these resistance values have been standardized. That is the main reason why this electric scheme can never result in a perfect sine or cosine output signal. The other reason is the fact that per component, the resistance values differ slightly from the value indicated. These values are all lumped in the model, as a sigma for all resistance values. The resistance values are also a function of temperature but since the temperature influence on the cells is not taken into account as well, there is no dedicated definition of the temperature influence on the resistance values admitted to the simulation tool. The electric scheme that is used to reconstruct the sine and cosine is added to the simulation tool as a second manner of simulating the reconstruction. The resistance values can be chosen manually. The analytical method is named “ideal” in the simulation tool because it does not deal with the resistance values which are rounded to the nearest possible resistance value. The variation of the resistances is also not taken into account. When the electric scheme is used for reconstruction, these two things are being taken into account at the expense of the simulation time. Since for the electric scheme the Simscape Toolbox of MATLAB is used, and the program thus switches to the Simulink environment to perform the simulation the simulation time increases significantly when this more realistic method is used.

Now that all blocks of the sensor are known, the ECU modelling is next in section 2-3-2.
2-3-2 ECU model

First the contents of the ECU block are shown in figure 2-19. The post conditioning consists of the rescaling of the sine and cosine signal. In the ECU this is done by dividing the signal by a constant value, which is determined from the specifications of the sensor. It is thus very important that the amplitude of the sine and cosine do not have a large variation.

In simulation, for every simulation within a cycle the factor is derived once per mechanical revolution. For bipolar sensors this means that the amplitude is determined of the sine and cosine and the signals are scaled with this amplitude. For multipolar sensors, it means that the maximum amplitude of the sine and cosine are used to scale the signal. The differences in amplitudes between the pole pairs will thus stay present in the result. The offset is removed by means of analytically subtracting the minimum value plus the amplitude of the signal.

The angle calculation can be done by two analytical methods: an algorithm using the arctangent delivering the mechanical angle on a domain \([0, 2\pi]\) (without the algorithm the function would only be defined on the domain \([-\pi/2, +\pi/2]\)) and the \(\text{atan2}\) function which is defined on the domain \([-\pi, +\pi]\). The latter yields the fastest results and both methods are ideal.

In addition to this method of simulating the ECU and thus calculating the angle, another method can be used which uses an interpolator. This interpolator model is developed within SKF and it is called SAHRAs (see figure 1-4). An interpolator uses the analogue output signals of the sensor to transform them into a certain number of pulses per revolution. The higher this number of pulses, the higher also the accuracy of the output. The interpolator integrated in SOFIAs targets only bipolar rings. The two models are coupled in the sense that SOFIAs is the active model which calls the model of SAHRAs from within the simulation loop. The outputs of SAHRAs are subsequently transported to SOFIAs. As already said, instead of having analogue signals as output, SAHRAs generates 4096 pulses per mechanical revolution. These pulses are used to drive the motor as well.
2-3-3 User interface and results

To be able to use the model of the sensor, a user interface is created which looks as in figure 2-20. The different stages of defining the sensor are denoted in the figure. If all parameters have been defined, pushing the Simulation button will start the simulation. An example of a simulation cycle and the results is given in figure 2-21. For a cycle of 10 simulations, in the yellow field various plots of the outputs can be shown. Also the statistic results are plotted for certain outputs. In the red field, the results for the cycle are shown. The minimum, mean, maximum value and standard deviation are displayed and also the simulation in which they occurred. In the green field finally, the results for one simulation in the cycle are shown. If the intermediate results are saved, the results of all simulations in the cycle can be viewed here. Also the harmonics of the nonlinearities can be saved by checking the corresponding checkbox.

![User interface](image-url)
In the Check phase of the project, the objective is to validate the model with as goal to obtain a certain confidence level. This can be done using measurements of prototypes. The prototype testing method that is proposed is described here and the measurement results will be presented. Subsequently, conclusions will be drawn on the modelling of the sensor.

2-4 Measurements

The prototype that is used for the measurements is shown in figure 2-22. In figure 2-22, (A) denotes one of the Hall cells on the PCB, (B) is the outside cover for the electronics and (C) is the magnetic ring. Four different placements are possible with the TSxGe (as the prototype is named) and it can be used with both bipolar and multipolar (six pole-pair) rings. The possible configurations for testing with the TSxGe are denoted in figure 2-23. Measurements are done with these possible configurations and different magnetic rings. To validate the simulation tool, the same configurations and magnetic
Figure 2-22: Prototype TSxC

Figure 2-23: Possible configurations of the TSxC

ring profiles will be simulated and the results will be compared. The outputs that will be compared are: the amplitude and offset of the sine and cosine, the phase shift between the sine and cosine and the nonlinearity. This nonlinearity is defined as:

- For both bipolar and multipolar sensors: difference between the minimum and maximum nonlinearity measured over one revolution, referenced as mechanical nonlinearity
- For multipolar sensors only: difference between the minimum and maximum nonlinearity per period of the sine and cosine sensor output signals, referenced as electrical nonlinearity

Since for the multipolar sensor six electrical nonlinearities will be obtained per revolution of the shaft (one for each period), the minimum, mean and maximum nonlinearity will be regarded here.

The result analysis exists of the following steps:
Modelling of the sensor

- Normalizing sine and cosine (remove offset, and make amplitude equal to one);
- Calculate angle with the \texttt{atan2} function;
- Calculate the nonlinearity using the reference angle, and removing the peaks;
- Determining the phase shift and peak-to-peak nonlinearities.

The results of the measurements can be found in appendix A: in table A-1 and A-2 for bipolar rings and in table A-3, A-4 and A-5 for multipolar rings. Note that the mechanical nonlinearities (mechanical NL) in table A-3, A-4 and A-5 are the nonlinearities measured over one mechanical period of $360^\circ$. The electrical nonlinearities are the nonlinearities per period of the sine and cosine. Obviously, the higher the nonlinearity, the larger the error in the rotor position is.

\section*{2-4-2 Simulations}

To validate the simulation tool using the TSxC results, the measurement results presented in section 2-4-1 have to be compared to simulation outcomes. Therefore the simulation parameters have to be determined which represent the measurements as well as possible.

In general, the two main variables in SOFIAs are the magnetic ring and the mechanical design. For the measurements with the TSxC, two rings for both magnetic ring types are used. This means that these parameters are known quite accurately (there are measurements available of the rings that were used). Of the mechanical design only the mean values and standard deviations are known. This means that the simulation outcomes will cover a sample of sensors with the two rings, and all possible placements of the cells.

To determine the design parameters (airgap, placement of the cells and cell sensitivity), the design of the TSxC is regarded as well as the chosen cell type. The nominal value and variation of the airgap can be deduced from the tolerances indicated on the mechanical drawing of the PCB and the tolerances in placement of the active element within the Hall-effect cell. The first value of the airgap signifies the difference between the ring radius and the bearing outer ring radius (simply said). It comprises all the mechanical variances that exist per sensor. The second value of the airgap signifies the differences per cell: in one sensor, more than one cell is placed and their mutual placements differ. This deviation is though much smaller than the deviation per sensor. The mean value for the airgap is now defined by the so-called package airgap with its variation together with the crystal airgap (per cell) and its variation. Per simulation in one cycle, there will be chosen a package airgap taking its variation, and added to that package airgap there will be added the crystal airgap using its variation to generate an different crystal airgap for each cell. Furthermore, for the radial placement of the cells a tolerance is given of 0.167. The cells that are used are the Honeywell SS496A1 cells [1] with variation in sensitivity and offset.
For the simulation are used: a speed of 10 [rpm] (same speed at which the measurements were done) and a calculation step size of 0.01°. Since for several parameters their variations are taken into account in the simulations such that a collection of sensors is simulated, several simulations have to be done in order to get a collection of possible sensors at the output. In one simulation cycle (with for each simulation the same variables, but different statistical parameters) 1000 simulations are done and thus 1000 sensors are simulated. The temperature and supply voltage are kept constant at their reference values which are equal to 25° and 5 Volt. To determine the nonlinearity, the ideal angle calculation is used (using the \( \text{atan2} \)-function). In table A-6, A-7, A-8, A-9 and A-10 in appendix A, all simulation results can be found.

### 2-4-3 Validation

Now that measurements and simulations are available concerning the same configuration, the validation of a part of the simulation tool can be done. Because of the predicting nature of the simulation tool, the goal of validation is not to test every possible configuration in hardware, but to obtain a sufficient number of measurements that correspond to simulation outcomes so that the confidence in the model increases.

Validation of the model can be extended at every moment by comparing measurements of future prototypes to simulations comparing the amplitude and offset of the sine and cosine, the phase shift between them, and the error in angle measurement.

To obtain a sufficiently validated model, outcomes of SOFIAs need to be compared to a number of measurements corresponding to a large batch of prototypes. Nowadays, only a few prototypes are available for testing and their number is not sufficient to predict the correct minimum and maximum values of output variables, since the variation is large to such a degree that a larger sample size is needed.

Nevertheless, for now the configurations that are tested with the TSxC can be regarded and some conclusions on the validation of the model can be drawn.

Regarding tables A-6, A-7, A-8, A-9 and A-10, the objective should be that the measured values fall within the interval posed by the simulations. This is the case since the same rings are used, but a collection of placements since the exact placement of the cells within each configuration is not known.

The simulation outcomes signify in fact the nonlinearities that can occur when the same ring is used with different placements. The fact that for each configuration two rings are used gives us an impression of the possible differences among the entire collection (that is, all possible magnetic rings in a batch and all possible placements).

*In fact, the measured values all fall within the limits posed by the simulations.*
Furthermore, to increase the confidence in the model, different ring models can be used that represent an entire collection of rings and they can be compared to the measurements to have an idea whether the specific model can be a good description of the magnetic ring. Also the form of the nonlinearity will be regarded.

**Simulations with different ring models**  For bipolar S5C sensors, several simulation cycles have been carried out with different magnetic ring models. The design parameters are the same for each simulation, such that the differences per simulation cycle can be put down to the magnetic ring models. The models that have been tested are:

- **Perfect sine model**: $B_0$ and $\sigma$ are chosen according to measurements done on 10 rings.
- **Combination of harmonics model**: $B_0$ and $\sigma$ of both the south- and the north-pole are chosen according to measurements done on 10 rings. The relative amplitudes of the harmonics are taken as the average amplitude of all 10 measurements. The sigma’s have been chosen according to the measurements.
- **Material model**: the mean thickness is taken from the mechanical drawing and the material is known.
- **Pole length model**: $B_0$ and $\sigma$ of both the south- and the north-pole and the pole length and $\sigma$ are chosen according to measurements done on 10 rings.

![Distribution nonlinearities for different magnetic ring models - S5C bipolar rings](image)

**Figure 2-24: Distribution nonlinearities for different magnetic ring models - S5C bipolar rings**

The results of the simulations can be found in appendix B, table B-1. It is seen that all measurements fall within the limits posed by the simulations. In fact, all models predict a similar amplitude of the sine and cosine. This holds also for the offsets of the sine and cosine except for the pole length model. Apparently the mean value of the offset diminishes when the south pole is smaller than 180°. Regarding the nonlinearity and the phase shift, the standard deviations are
comparable among the magnetic ring models and for the nonlinearity, the most important differences are seen. The perfect sine and material model’s minimum nonlinearity is zero and the pole length model has a very small minimum nonlinearity, whereas the combination of harmonics model yields a minimum nonlinearity of 0.5°. Therefore the mean values differ quite a lot, but the maximum nonlinearities are similar again. These differences are visualized in figure 2-24, where of all models the distribution of the nonlinearity is plotted. Apparently, the characterization of the magnetic model has quite a large influence on the shape of the distribution function of the nonlinearities.

**Nonlinearity shape** Also visually, the level of confidence of the simulation tool outcomes can be increased. This is done by comparing the shapes of the nonlinearity of the measurement with the simulated nonlinearity. The simulation outcome that is used for this comparison is taken from the results presented in section 2-4-2. In figure 2-25 the simulation and measurement results are shown for the S3C bipolar sensor (ring 1). In figure 2-25 can be seen that the nonlinearity is well-predicted although the simulation is taken from a cycle of simulations and the placement of the cells and the offset and sensitivity of the cells are not exactly known for the configuration. A similar result is obtained for the measurements and simulations with five cells and a bipolar ring. This result can be seen in figure 2-26. In this figure, the measurement shows more high-frequency fluctuation (for example at t=2.2) but the main characteristics are well-estimated. Again it is noted that the design corresponding to this simulation outcome does not necessarily coincide with the
design used for the measurement, which explains the difference between the two.

For the multipolar sensors, one example is shown here. This again concerns a measurement of a fixed ring and design, while the simulation cycle yields all results with this fixed ring and all possible designs within the specification. A resembling output is searched for and plotted in figure 2-27 alongside the measurement output. It is very clear when regarding figure 2-27 that the simulation result and measurement result have the same shape. The amplitudes of the dif-

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**Figure 2-26:** Simulation and measurement of one particular sensor (S5C) with the same bipolar magnetic ring

**Figure 2-27:** Simulation and measurement of one particular sensor (S5C) with the same multipolar magnetic ring
ferent harmonics differ since the exact correct design used in the measurement is not known.

**Conclusion of validation** Since not many prototypes were available for validation of the model, it is hard to say something about the fact if the limits posed by the simulation tool (the minimum and maximum values of the important parameters) are correct. Therefore there is a need for a range of prototype measurements with different designs (not such as the TSxCS, where the rings can be replaced but the placement of the cells for each design is logically fixed) and different rings. But what is important, is that the measurements done now are all contained in the ranges of output provided by the simulations. For now the measurement results do not nonvalidate the model, in other words, there are no measurement results up until now that decrease the confidence in the simulation tool.

### 2-5 Act phase

In the Act phase of the project, the simulation tool itself (the use) and the model used in the simulation tool is examined thoroughly to find possible ameliorations. One of them includes the addition of temperature and voltage dependency to the cell parameters, as described in 2-3. Other possible improvements are:

- Adding the influence of an axial misalignment, as indicated in figure 2-28;
- Implement the possibility of negative speeds (investigate non-symmetrical behavior);
- Implement a Sensitivity Analysis tool with as reference collections of sensors (see below).

The Sensitivity Analysis implemented nowadays in the tool uses the ideal sensor as a reference. In fact, what is more useful to use as reference to investigate the

![Figure 2-28: Magnetic field intensity axial variation](image)
influence of certain parameters on the outputs (such as the nonlinearity, but also the phase shift between the sine and cosine) is the distribution of that output taking into account all variations. For example, to investigate the influence of temperature on the nonlinearity a number of simulation cycles will be performed with the temperature evolving. All other parameters will vary as normal. In this way, the obtained result will exist of an evaluation of the distribution of the nonlinearities caused by the temperature change. In figure 2-29 this approach is visualized. It is noted that to obtain simulation results with the simulation tool, this approach is followed. To be more efficient, this method can be implemented as Sensitivity Analysis since in this way, the parameters will be varied automatically and the impact of the varying parameter on the output can be directly observed in the simulation tool itself.
Chapter 3

Sensor performance

After the modelling the sensor is done and the model has been validated in the process of the simulation tool development, the simulation tool can be used to investigate the possible achievements of the sensor and the sensibility of the sensor to the variation of certain input parameters.

In this chapter, only the sensor performance itself is regarded. The combination with the application (brushless motor control) is the topic of chapter 4. First, the simulations in general are described in section 3-1. Subsequently, the simulation results and analysis are presented in section 3-2. Conclusions on the performance of the sensors are drawn in section 3-3. Finally, in section 3-4 some recommendations are given on how to improve further the performance of the sensors.

3-1 Simulation methodology

The simulations are done for both bipolar and multipolar sensors. To be able to simulate entire collections of sensors, a collection of rings and a collection of mechanical designs have to be chosen. Subsequently, one parameter is varied to see the influence on the nonlinearity. This can be done in two ways: regarding the arithmetic results (the minimum, mean and maximum values) and the distribution of the nonlinearities. Of course the other output parameters such as the
sine and cosine amplitude and offset can be regarded as well.

The general methodology that is followed to carry out simulations and interpret the results is depicted in figure 3-1.

![Figure 3-1: Simulation methodology](image)

3-2 Simulation results

To investigate the performance of the sensor, several parameters can be varied to see the influence on the outputs of the sensor and the nonlinearity. In the following subsections, simulations have been carried out following the methodology depicted in figure 3-1. First of all, a harmonic analysis is done to investigate how the nonlinearity is build up. Subsequently, simulations have been done with bipolar and multipolar sensors to be able to investigate the performance of these sensors. Because of the large number of inputs that are possible, the results presented in these subsections should be regarded with care; they do not apply to the general case but only for the specified magnetic rings and designs.

3-2-1 Harmonics analysis

To obtain more insight in how the nonlinearity is build up, 100 simulations have been done with a collection of bipolar and multipolar sensors. The resulting nonlinearities are regarded, but in a special way. They are analysed using the Fast Fourier Transform (FFT), which yields a harmonic decomposition of the signal. In fact, this is the same principle as described in section 2-3 for the Combination of harmonics magnetic ring model. This model also uses an FFT, but then obviously the harmonic decomposition of the ring.

The harmonics of the minimum and maximum nonlinearity resulting from the
Simulations done for a bipolar sensor are displayed in figure 3-2. The simulations have been done with three and five cells, and for the rest all input parameters are held constant. The first thing that is very clear is that three important harmonics exist in the nonlinearity: the first, the second and the Xth and its multiples, where X is the number of cells. This is most clearly seen for the simulation results of the S5C. From previous studies (see [2]) it is known that the Xth harmonic in the nonlinearity is caused by the harmonic X-1 and X+1 of the ring. Harmonic two was discovered to be related to the design, that is the airgap, cell sensitivity and radial cell placement. This was shown by simulations with a perfect ring and a varying design. In the simulation results, the nonlinearity only existed of the harmonic two. On the other hand, when the magnetic field is varied but the design is ideal, it is seen that the standard deviation of the nonlinearity is smaller. Thus the design is increasing the deviation of the nonlinearity. The nonlinearities are thus mainly determined by the harmonic one, two and three in case of a bipolar S3C. The higher the number of cells, the more influence the design can have since then the ring’s harmonics and thus the Xth harmonic are lower.

For multipolar rings, the harmonics of the minimum and maximum nonlinearity are displayed in figure 3-3. Something similar to the results seen for bipolar sensors is seen for multipolar sensors. The important harmonics are all multiples of the number of pole pairs and the number of cells. For both three and five cells, the harmonic 15 and 16 are the largest. It seems that the difference between the minimum and maximum nonlinearity comes from different harmonics.

The harmonic analysis will be used as a tool to better interpret the results of the simulations. This will be especially useful in the analysis of the torque ripple, the variation in torque when the sensor is regarded as part of the application, brushless electric motor control in chapter 4.
3-2-2 Bipolar simulations

Influence number of cells on nonlinearity  To investigate the relation between the number of cells and the resulting nonlinearity, simulations are done with the same collection of rings and mechanical designs with their respective variations and for each simulation, the number of cells is varied in a range from three to nine cells. These cells, as for all bipolar designs, are placed uniformly around the magnetic ring. The arithmetic results of the simulations can be found in table 3-1. In

<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>1.61</td>
<td>2.79</td>
<td>0.27</td>
<td>0.35</td>
<td>0.14</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>mean</td>
<td>7.40</td>
<td>4.86</td>
<td>1.69</td>
<td>1.44</td>
<td>0.69</td>
<td>0.47</td>
<td>0.44</td>
</tr>
<tr>
<td>maximum</td>
<td>17.59</td>
<td>7.05</td>
<td>4.20</td>
<td>2.62</td>
<td>1.95</td>
<td>1.40</td>
<td>1.41</td>
</tr>
<tr>
<td>standard deviation</td>
<td>2.44</td>
<td>0.67</td>
<td>0.67</td>
<td>0.37</td>
<td>0.29</td>
<td>0.21</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 3-1: Results of simulations with different numbers of cells

Figure 3-3: Harmonics of minimum and maximum nonlinearity for a multipolar S3C and S5C
nonlinearity and the mean is approximately equal to the difference between the maximum and the mean, the distribution is normal. If this is not the case - such as for sensors using seven until nine cells in figure 3-4 - the distribution resembles more a normal distribution with at the positive side the outliers. These outliers represent combinations of designs and ring characteristics that accumulate to enlarge the nonlinearity.

It is found that for the highest four nonlinearities of the S5C, the phase shift between the sine and cosine output signals is in two cases significantly higher or lower than the mean of 90°. The phase shift deviation from its mean value is typically related to the design. It is found that for phase shifts which differ significantly from the mean phase shift, the second harmonic is large in the harmonic decomposition of the nonlinearity. This second harmonic is in fact introduced by all design parameters: sensitivity of the cells, airgap variation and variation in cell placement. The airgap variation becomes more and more significant when the number of cells increases and thus the nonlinearity variation caused by the cell parameters decreases.

**Influence temperature on nonlinearity** Even though in SOFIA's the effect of temperature variation on the standard deviation of the cell sensitivity and offset is not taken into account, still simulations can be done to investigate the influence of this parameter. To do this, the standard deviation of the sensitivity and of the offset of the cells is altered manually in the program, such that the deviation at reference temperature is increased to match the deviation at the minimum and
maximum temperature (-40 and 125°C respectively). This is done for one cell: the Honeywell SS496A1 which is the most used cell nowadays. The minimum and maximum deviation are denoted in figure C-1 in appendix C. Simulations have been done using the Combination of harmonics model using the mean harmonic decomposition of a collection of bipolar rings. The magnetic ring denoted by this harmonic decomposition is varied throughout the simulation by defining a standard deviation for each harmonic, in correspondence to the measurements done on the magnetic rings. The resulting nonlinearity distribution is seen in figure 3-5. The distributions differ from each other by means of their shape, mean value and variance. In fact, the distributions at a temperature of -40 and 25°C (reference temperature) are quite similar. For an ambient temperature of 125°C the standard deviation has become quite large. In the table with the nonlinearities’ minimum, mean and maximum values (table 3-2) the arithmetic results for the different ambient temperatures are displayed. For the rings and the cells used in this simulation, the maximum temperature yields a larger mean value for the nonlinearity. The minimum temperature of -40°C yields actually a smaller mean value which in fact is logical, since the variation of the cell sensitivity and offset

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>25</th>
<th>125</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>0.30</td>
<td>0.35</td>
<td>0.59</td>
</tr>
<tr>
<td>mean</td>
<td>1.55</td>
<td>1.63</td>
<td>2.53</td>
</tr>
<tr>
<td>maximum</td>
<td>3.91</td>
<td>4.05</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Table 3-2: Minimum, mean and maximum nonlinearities for different temperatures

Figure 3-5: Nonlinearity distributions for different temperatures for S5C sensors
might be larger than at the reference temperature but the magnetic field intensity is lower and thus the resulting cell amplitudes and offsets yield a smaller variation.

**Influence supply voltage cells on nonlinearity** In the same manner as the temperature influence is investigated, the supply voltage of the cells can be regarded to see how the sensor performs when the supply voltage drops above and below the nominal value of 5V. The minimum and maximum supply voltage that are regarded are small but common variations around the nominal value of 5.0 Volt: minimum 4.9 Volt and maximum 5.1 Volt. In figure C-2 in appendix C, the information concerning the influence of the supply voltage on the cell sensitivity and offset can be found which are taken from the datasheet of the cells (Honeywell SS496A1[1]). What is interesting to see is that in figure C-2 on the left, the nominal sensitivity of the cells decreases with decreasing supply voltage but the maximum sensitivity increases. For supply voltages under the recommended supply voltage of 5.0 Volt apparently the Hall-effect sensor reacts very differently than for supply voltages above the recommended voltage. The resulting nonlinearity distributions of the simulations done with three different supply voltages can be found in figure 3-6. The distribution denoted in figure 3-6 is a measure for the sensitivity of the entire sensor collection to small variations of the supply voltage. It is clearly seen in this figure that the nonlinearity variation as a result of the supply voltage variation is negligeable for this collection of bipolar S5Cs, especially when regarding the influence of the temperature. The nonlinearity changes only when there is a variation in amplitude difference of the sine and cosine, and this variation is very small. This does not mean that the variation of the amplitude of the sine and cosine itself does not vary much. The distribution of the offsets of the sine and cosine actually changes quite a lot with the supply voltage as can be observed from figure 3-7. This change is attributed to the mean value of both the amplitudes and offsets. This mean value drops significantly with supply voltage drop and increases with
an increase in the supply voltage. For the offset, it is easily seen that this is related to the supply voltage since the mean offset is always approximately half the supply voltage value. The standard deviation of both the amplitudes and offsets does not change a lot.

Influence accuracy cell placement on nonlinearity Simulations have also been done to investigate the influence of the cell placement on the S5C sensor. The cells are placed around the ring on the PCB with a certain radial tolerance. This tolerance is given in terms of a standard deviation on the placement angle of the cell. The value of this standard deviation is varied in five simulation cycles from zero to 0.3. The arithmetic results of these simulation cycles are given in table 3-3. As can be seen in table 3-3, the cell placement variation does not have a significant influence on the nonlinearity of this collection of cells.

3-2-3 Multipolar simulations

Unless otherwise noted, the multipolar simulations are based on measurements on eight pole pair magnetic rings having specifications as displayed in table C-1 in appendix C.
Influence number of cells on nonlinearity  For a project within SKF, there is an interest to simulate with magnetic rings having seven pole pairs targeting brushless electric motor control for a motor with seven permanent magnets in the rotor. There are no measurements of such rings available and thus the results of the measurements on rings with eight pole pairs described in table C-1 have to be transformed to be used with seven pole pairs.

The mean pole pair length will be just the mechanical angle divided by the number of pole pairs. This yields a mean pole pair length of 25.714°. Unfortunately, this is not a round number such as 22.5° for eight pole pairs and thus in the simulation, errors will be introduced due to the round-off of the pole pair length. These errors are visible when the ring in combination with a 'perfect' sensor design is simulated. The amplitude of the north and south pole are then equal and constant, and also the pole length is constant over the simulations and equal to the rounded value above. In table 3-4, the results are shown. Obviously, the nonlinearity displayed in table 3-4 should all be equal to zero. This means that when in the simulation cycle with the entire collection the perfect sensor would be simulated, the nonlinearity is not zero but 0.30° in the case of an S3C. Since the mean value only decreases due to this rounding error, it is not taken into account in the rest of the simulations below.

The difference between the minimum pole length and the mean pole length is 2.22% of the mean value. For seven pole pairs, this would yield a minimum of 25.14°. In the same manner the maximum is derived: 26.17°. The standard devi-
Another point taken into consideration when formulating a description for the ring with seven pole pairs is the fact that the ring will be used in a sensor with a very small diameter. This means that the poles, since there are fourteen, will be close to each other. The perturbing magnetic field of the other poles will thus have more influence which will result in the fact that the magnetic field intensity will resemble more a sine than when the diameter is large and the pole number small. This is visualized in figure 3-8. For this reason, the choice has been made to not include the effect of capping in the simulations for this ring.

The magnetic ring collection obtained is simulated for different numbers of cells. The results can be found in table 3-5 and 3-6.

<table>
<thead>
<tr>
<th>Cell design 1</th>
<th>Number of cells</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>minimum</td>
<td>1.864</td>
</tr>
<tr>
<td>mean</td>
<td>4.407</td>
</tr>
<tr>
<td>maximum</td>
<td>8.155</td>
</tr>
</tbody>
</table>

Table 3-5: Simulation results (mechanical nonlinearity)

<table>
<thead>
<tr>
<th>Cell design 1</th>
<th>Number of cells</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>minimum</td>
<td>0.638</td>
</tr>
<tr>
<td>mean</td>
<td>3.067</td>
</tr>
<tr>
<td>maximum</td>
<td>8.155</td>
</tr>
</tbody>
</table>

Table 3-6: Simulation results (electrical nonlinearity)
In figure 3-9 the distributions of the mechanical nonlinearities are plotted. The electrical nonlinearity distributions can be found in figure 3-10. Comparing figure 3-9 with 3-10 and table 3-5 and 3-6, it is clear that the electrical nonlinearity...
has a smaller standard deviation and mean. If a specification is given on the maximum peak-to-peak nonlinearity that is allowed, the empirical cumulative distribution function of the simulations in one cycle can be estimated. This cumulative distribution, the word already says it, displays the proportion of data less than each value of the nonlinearity with a confidence interval of 95%. In figure 3-11, such a plot is displayed. In this figure it can be seen that for an S3C only very little sensors will have a nonlinearity equal or lower than $2^\circ$ peak-to-peak. But for an S8C for example approximately 70% of the sensors are predicted to have a nonlinearity smaller or equal to $2^\circ$ peak-to-peak.

**Figure 3-11: Estimated cumulative distribution function of multipolar sensors**

Influence cell sensitivity on nonlinearity To investigate the influence of the cell sensitivity on the nonlinearity, S5C sensors are simulated with a multipolar ring. This multipolar ring has eight pole pairs and is modeled using the harmonic decomposition measured over 550 rings out of one fabrication series and its characteristics have been described in table C-1 in appendix C. Simulations are done with different cells which are presented in table 3-7 (Remark: all values are at the reference supply voltage of 5V and the reference temperature of 25°C). For accuracy reasons, nowadays the preferred cell is the Honeywell SS496A1 because of its low sensitivity variation. But it is interesting to investigate whether a low-cost Hall-effect sensor such as the Micronas HAL1823 can compete with this sensor in terms of performance. The cells in table 3-7 having a high mean value for the sensitivity are particularly useful if the magnetic field amplitude is low. The use of such cells than implies a higher amplitude of the sine and cosine at the output such that noise has less influence on the signals. For the results, the similar cells
are grouped. That is, four different 'sets' of cells are defined: the first contains the currently used cell, the Honeywell SS496A1 with its cheaper alternatives from Allegro and Micronas (the A1223 and HAL1823), the second contains the two Honeywell cells (the high accuracy A1 and the standard A cell), the third contains the cells with a sensitivity of 3.125 [mV/G] and the fourth the cells with a sensitivity of 5 [mV/G]. The results are printed in table 3-8. Only the electrical nonlinearities are printed since they are the main interest. The distributions of the nonlinearities can be found in figure 3-12. It is clear from table 3-8 and figure 3-12 that the change of the cell does not change a lot in the nonlinearities. The mean values of the electrical nonlinearity increase at the most approximately 0.1°. The maximum values are defined by the outliers, and thus these values can be lower than the reference cell (Honeywell SS496A1) since for each simulation cycle 1000 sensors are simulated. It appears that the maximum values are defined by combinations that do not appear often and thus, in one simulation cycle or the other, some 'worst case' scenarios might not occur. A way to investigate this...
would be to use more simulations in one cycle but this is not carried out here, since the difference is already proven to be very small.

**Influence north and south pole amplitudes on nonlinearity**  In contrast with bipolar rings, for multipolar rings there are several ways the amplitude of the magnetic field can be of influence on the nonlinearity of the sensor. In general, multiplying the magnetic field intensity by a factor does not increase or decrease the nonlinearity, only the amplitude of the sine and cosine are affected. But in case the amplitudes of the north- and south poles change independently from each other, the nonlinearity is affected. But the ratio between the north- and south pole does not only vary per pole, but also per pole pair. To investigate the differences, various simulations have been done. The results are presented below. In figure 3-13, the distribution of the nonlinearities is shown for a nominal simulation (with the absolute north and south amplitudes of all poles equal) and two simulations where respectively the south pole amplitudes and the north pole amplitudes are all 20 [G] higher (keeping the other amplitudes constant, see table C-2 in appendix C). In figure 3-13 it is seen that when the two distributions with respect to the nominal value are regarded, that both amplitude variations affect the mean value negatively but the south pole amplitude variation has more influence on the nonlinearity. It is also clearly seen that the minimum nonlinearity in both cases increases. For the same increase in amplitude, the south pole increase yields an
increase of the mean nonlinearity of 7.4% and the north pole variation increases the mean nonlinearity with 3.3%.

In figure 3-14, the nonlinearity distributions are given for simulations done with

Figure 3-13: Simulation results for larger south and north poles

Figure 3-14: Simulation results for one pole with equally larger and smaller south and north pole
one pole pair having a smaller or larger amplitude compared to the other pole pairs. This is denoted in table C-2 in appendix C in which a star denotes the nominal value of 850 [G]. When comparing figure 3-13 and 3-14 it is seen that the two last simulations yield a larger mean nonlinearity. The impact of a difference between one pole pair and the other pole pairs in amplitude is thus higher than when all south poles or all north poles are varied.

3-3 Conclusions on sensor performance

The sensor performance is measured by means of the amplitude and offset of the sine and cosine and the nonlinearity. The performance is highly dependant on the application and the desired accuracy and there is an infinite number of bipolar and multipolar sensors possible in practice. In this section, some conclusions are thus given with respect to the general findings regarding the sensor.

The sensor performance is determined by two main factors: the mechanical design (placement cells, etc.) and the quality of the magnetic ring. Regarding the nonlinearity:

- The higher the number of cells, the more the nonlinearity is determined by the quality of the magnetic ring
- Positive temperature changes increase and negative temperature changes decrease the nonlinearity’s mean value and standard deviation
- Small supply voltage changes do not influence the nonlinearity significantly
- For multipolar sensors: the electrical nonlinearity has a smaller mean value than the mechanical nonlinearity
- Using cells with different sensitivity variation does not increase the nonlinearity significantly
- The amplitude difference between the pole pairs has more influence on the nonlinearity than the difference per pole

3-4 Recommendations on sensor performance

To increase the sensor performance, there are several possibilities to obtain a better accuracy. Eliminating the second harmonic in the nonlinearity would yield a better result since the variation of the nonlinearity will decrease. Within SKF a method is developed to accomplish the elimination of this harmonic by means of a phase shift compensation method. This method is not regarded here due to confidentiality. Furthermore, the nonlinearity can obviously be decreased by choosing a suitable magnetic ring with respect to the number of cells since then the harmonics related (the number of cells and its multiples) will decrease. Also the process of manufacturing can be regarded to be able to work with lower mechanical tolerances which will decrease the variation of the nonlinearity. Furthermore,
harmonic filtering can probably be used to filter out a specific harmonic, but cost calculations will have to be performed to investigate the feasibility.

In the simulations in this chapter only the nonlinearity caused by the ring and design are regarded. Simulations have not been done taking into account noise on the signals. When the measurement results are regarded, this noise is always present and makes that even in the case of a perfect ring and perfect design, there is a certain nonlinearity in the angle calculation due to this noise. Simple first-order filtering can be applied to diminish this noise, but this will introduce phase shift of the output as well as a delay. It is recommended to also perform simulations taking these effects into account.
Chapter 4

Integration sensor and motor

The sensor that is the subject of chapter 2 and 3, the SxC, is developed targeting brushless electric motor control. The sensor in combination with this application is the subject of this chapter. Taking into account the application, theoretically it is favored to use multipolar rings instead of bipolar rings in certain cases. Why this is the case is the topic of section 4-1. The brushless electric motor model as well as the integration of SOFIAs within this model is described in section 4-2. Simulations are done to test the hypothesis in section 4-3, and in section 4-4 conclusions will be presented on the performance of multipolar sensing with respect to bipolar sensing taking into account the application.
4-1 Hypothesis

The hypothesis is:

Given two sensors for a brushless electric motor with \( N \) permanent magnets in the rotor. To control this motor, the sensed mechanical angle of the bipolar sensor is multiplied by \( N \) whereas the sensed electrical angle of a multipolar sensor corresponds directly to the electrical period of the motor. Therefore, using the multipolar sensor the nonlinearity will not be multiplied whereas for the bipolar case, multiplication is needed. This means that when using a multipolar sensor, provided that the performance of the sensors is comparable, the torque ripple is expected to be \( N \) times lower than when a bipolar sensor is used.

This originates from the difference between the electrical and mechanical angle. The electrical angle is defined from the point of view of the motor. Each permanent magnet of the rotor adds 360° of electrical angle. Since a bipolar sensor only provides one mechanical angle, for the electrical angle it is needed to multiply by the number of permanent magnets. For a multipolar sensor with \( N \) electrical periods in one mechanical angle the electrical angle is already provided and thus multiplication is not needed in the ECU. This is visually explained in figure 4-2.

In order to be able to compare the two sensors, the same reference has to be taken. This reference is the electrical angle, and thus for honest comparison the bipolar errors have to be multiplied by \( N \) and compared with the multipolar electrical errors. This is done in the modelling of the brushless electric motor, which will be the subject of the next section.

The performance of the sensors has to be comparable. Therefore, existing sensor designs have been selected with similar mechanical designs. The rings are of course different for multipolar and bipolar sensors. It has to be noted that for
every application, the motor parameters are in fact chosen to obtain the desired performance. Therefore the absolute performance of the motor is not regarded in this chapter, but only the relative performance using different sensors in the same motor.

4-2 Description of the model

To simulate the performance of the bipolar and the multipolar sensor integrated in the motor, the SxC model developed in chapter 2 has to be integrated in the brushless electric motor. First in subsection 4-2-1, a brief description of the within ADC-SI already existing brushless motor model is given. Then in section 4-2-2, the Real-Time model of SOFIAs is presented which was used for the simulations of the motor and the sensor together.

4-2-1 Brushless motor model

A schematic view of a brushless motor is seen in figure 1-2 in section 1-1. In this figure the motor, sensor and control are included. A brushless motor exists of a stator and a rotor. The stator consists basically of electrical windings that can be fed with current by the controller and the rotor consists basically of permanent magnets. When the motor is turned on, the stator currents generate a magnetic field, which is controllable by varying the stator currents. In this way, at every rotor position, a magnetic field exactly tuned to the magnetic field generated by the permanent magnets of the rotor can be created in the sense that where the rotor has a north pole, the stator currents must generate a north pole as well. This will cause the rotor to turn since two equal magnetic poles repel. Logically, to determine the stator currents the rotor position is needed, in order to create the magnetic field that will repel the magnetic field of the rotor. This rotor position is provided by the SxC of which the modelling is described in chapter 2 and the performance in chapter 3.
The brushless motor model developed for the purpose of performance testing on rotor position sensors is named Simulation for Motor Brushless Bench and Sensor-bearing (SiMBBaS). It is a model of a Permanent Magnet Synchronous Motor (PMSM) with a star coupling (wye-configuration) of the three stator windings as visualized in figure 4-4 on the right side. This star coupling is normally more efficient than the delta configuration shown on the left in figure 4-4. The delta configuration is used for high torque applications, whereas the star coupling is used in motors designed for high speed applications. A schematic view of the contents of the electric motor model is seen in figure 4-3. It consists of an electrical model which includes the equations of the stator windings which use the voltages across the phases to determine the electromagnetic torque. This torque is the input of the mechanical model where the resistive torque is added and the motor states are determined (angular position, speed and acceleration) as well as the resulting output torque. The rotor position is the input of the magnetic model which determines the magnetic flux as function of the rotor position generated by the permanent magnets of the rotor. Different parameters can be chosen to optimize the performance of the motor including the number of permanent magnets of the rotor. In fact, the motor model can work with two current control methods:

- Six-step control;
- Sinusoidal control.

The first method is the least accurate method but also the cheapest and simplest method since it only uses a sector where the rotor is in. Each mechanical revolution is divided into six sectors when a bipolar rotor is used. The second method uses sinusoidal control. The difference between the two methods can be seen in figure 4-5, where the functional description is given for each control method. The difference depicted in figure 4-5 consists of the current control: for the six-step method, there is only one active current, whereas for the sinusoidal control method all three phase currents evolve at the same time. For a bipolar rotor, the rotor position is the mechanical angular position, for multipolar rotors the rotor position is the electrical equivalent of the rotor position as is explained in section 4-1.

**Figure 4-4:** Schematic view for delta and wye stator winding configurations
The model that is used for the simulations is the model using the sinusoidal control method. This model is chosen since it is typically used for high-performance applications such as the steering systems regarded in this thesis. The realization of the motor model in Simulink is depicted in figure 4-6. In this figure, the sensor
4-2-2 Real-Time sensor model

To implement the model of the SxC in the motor model, a real-time determination of the rotor position is needed within the simulation loop of the electric motor model. There are two possible ways to implement the SxC model in the motor model:

- Predetermine the measured rotor position over one mechanical revolution of the shaft before motor simulation and use a look-up table in the simulation loop;
- Determine the measured rotor position in the simulation loop using the previous measured angle and the simulation time.

The SxC model SOFIAs itself works with predetermined vectors which are processed through the simulation loop. This signifies that for the entire time span selected in SOFIAs first the magnetic model is generated. Subsequently, all cell signals are created over one (or more) mechanical turns, etc. If this has to be transformed to a real Real-Time model which does not run through the simulation loop once per simulation, but once at every time step, the model needs to be significantly changed in the sense that once the input is defined, all parameters are known. The simulation is then runned for each time step with scalars instead
Both methods will have the same output. The advantage of the second modelling method is that there is no need to store the look-up tables before starting simulation. If simulations will be done to investigate the performance of the motor (in terms of torque quality or the amount of torque ripple on the output) there is no need to store the output of for example 10,000 sensors before simulation.

A visual representation of the simulation loop of the motor including the SxC sensor is shown in figure 4-7. Depending on which SOFIAs Real-Time model is implemented in the motor model, the angle output which is fed back to the motor is either taken from the look-up table or determined through a simulation in real-time. For the first method the two blocks 'Sensor' and 'ECU' are placed in the 'Parameter' block which changes the look-up table after a certain number of revolutions. The SOFIAs block then exists of only a look-up table block.

In the interface of the motor simulation the choice for a single-sensor simulation or a collection-of-sensors simulation has to be implemented. The number of revolutions of the motor after which the sensor is changed has to be defined too.

The outputs of the combined model are just the outputs of the normal motor model (that is, torque and the speed) and other desired outputs, for each sensor. For instance, the maximum torque ripple can be regarded: a measure of the worst performance that this particular motor can achieve when it is equipped with sensors from the specified collection.

It is noted here that the model seen in figure 4-7 is just a proposition and the interface and simulation loops are not yet implemented in the motor model. For now, simulations are done using a look-up table for a single sensor.

4-3 Simulations

Regarding the hypothesis set in section 4-1, it will have to be investigated whether the performance of a motor equipped with a bipolar sensor compared to the same motor equipped with a multipolar sensor is in fact at least \( N \) times worse in terms of torque ripple, the variation in output torque amplitude (see also chapter 1). Since the torque ripple signifies unwanted vibrations in the mechanical system which the driver can feel in the steering wheel as well as audible noise which the driver and his or her passengers can possibly hear in the passenger compartment, it can be taken as reference to compare the sensors. Since it is favourable to reduce the torque ripple as much as possible, the peak-to-peak torque ripple will be regarded, which is defined as the difference between the maximum torque attained and the minimum. In order to investigate the torque ripple better, the harmonics of the torque output of the brushless electric motor will be regarded as well. In any case, the lower the torque ripple on the torque output for a certain sensor,
the better is the performance of the motor.

In order to have an equal comparison between the bipolar and multipolar sensors, a motor configuration is chosen having eight pole pairs. The bipolar sensors used in the simulation will be based on measurements of bipolar rings, and the multipolar sensors will be based on measurements on rings with eight pole pairs, corresponding to the number of permanent magnets of the motor. The sensor output will be transformed in the ECU to an angle range which is directly used for current determination. There is thus a need for multiplication in the motor’s ECU.

Before the simulations can be done with the motor model to investigate the performance of the motor using the different sensors, a load on the motor has to be defined as well. The load model that is used for the simulations is in fact the model of the load of the physical test bench that is used for validation of the brushless electric motor model. A short component description and illustration of this physical test bench can be found in appendix D. The schematic view of the bench is depicted in figure 4-8. The model of this physical bench is thus used for simulation. The three important inputs are noted at the bottom: current request which is a measure for the output torque and speed, the sensor, and load selection. The simulations have been done simulating a medium constant load, a current request of 10 [A] and as already said, sinusoidal control. The motor characteristics used in the simulation can be found in table D-1 in appendix D. The most important characteristics of this set-up with respect to the simulations done here are the stiffness and damping of the coupling between the EPS motor and the motor used to simulate the load. It is important to have a high resonance frequency to avoid that the lower harmonics present in the sensor nonlinearities cause resonance in the motor torque output. Therefore the coupling has to be chosen in a certain way. Simulations will be done with two stiffness values: one
of 70 [Nm/rad] and a stiffness coefficient of 200 [Nm/rad]. The latter will enlarge the resonance frequency with respect to the lower value but dependant on the harmonics present in the nonlinearity, it will yield better or worse results. Simulations have been done using these two stiffness values for bipolar as well as multipolar sensors. These simulations are the subject of the next paragraph.

**Simulations for motor having eight permanent magnets** To investigate the performance of both sensors implemented in the same motor, simulations have been done using the simulation outcomes of SOFIAs. Simulation cycles are conducted for bipolar sensors and for sensors using a magnetic ring with eight pole pairs, corresponding to the number of permanent magnets of the targeted motor application.

<table>
<thead>
<tr>
<th>Test</th>
<th>70 [Nm/rad]</th>
<th>200 [Nm/rad]</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Torque [Nm]</td>
<td>Torque [Nm]</td>
</tr>
<tr>
<td></td>
<td>Speed [rad/s]</td>
<td>Speed [rad/s]</td>
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<tr>
<td></td>
<td>Power [W]</td>
<td>Power [W]</td>
</tr>
<tr>
<td>ideal angle</td>
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<td>0.0676</td>
</tr>
<tr>
<td>min NL</td>
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<td>0.0588</td>
</tr>
<tr>
<td>max NL</td>
<td>0.2920</td>
<td>0.1019</td>
</tr>
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</table>

**Multipolar sensors - Design 1**

<table>
<thead>
<tr>
<th>Test</th>
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<th>200 [Nm/rad]</th>
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</thead>
<tbody>
<tr>
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<td>Torque [Nm]</td>
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<td>Speed [rad/s]</td>
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<tr>
<td></td>
<td>Power [W]</td>
<td>Power [W]</td>
</tr>
<tr>
<td>ideal angle</td>
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<td>0.0571</td>
</tr>
<tr>
<td>min NL</td>
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<td>0.0493</td>
</tr>
<tr>
<td>max NL</td>
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<td>0.0711</td>
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</table>

**Multipolar sensors - Design 2**

<table>
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<th>200 [Nm/rad]</th>
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<tbody>
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<td>Torque [Nm]</td>
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<td></td>
<td>Power [W]</td>
<td>Power [W]</td>
</tr>
<tr>
<td>ideal angle</td>
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</tr>
<tr>
<td>max NL</td>
<td>0.2924</td>
<td>0.0750</td>
</tr>
</tbody>
</table>

**Table 4-1: Results of simulations S5C**

The parameters of the simulation cycles are chosen such that the results yield representative collections of sensors. Subsequently, of these collections, the sensors yielding the minimum and maximum nonlinearity are stored and subsequently used in simulations with the brushless electric motor model set-up. This yields an image of the torque (ripple) obtained for the best and the worst sensor. The motor simulation parameters are denoted in table D-1. The graphical results of the simulations done with a stiffness value for the coupling of 70 [Nm/rad] can be
found in figure 4-9 and 4-10. For the stiffness value of 200 [Nm/rad] the results can be found in appendix D, in figure D-4 and D-5. The torque ripple peak-to-peak amplitudes can be found in table 4-1 for the simulations done with sensors having five cells, as well as the average torque, the speed and the power. To investigate the results first of all the torque ripple in table 4-1 is regarded. It is seen that the torque ripple amplitude for the ideal angle and the sensors with the minimum nonlinearity are similar; this is logical since the sensor with the smallest nonlinearity approaches the ideal sensor. For bipolar sensors, the torque ripple is twice as large for the maximum nonlinearity if compared with the minimum nonlinearity outcomes: this is not the case for the multipolar sensors. In fact, for multipolar sensors with Design 2, the torque ripple peak-to-peak amplitude is smaller for the maximum nonlinearity than for the minimum nonlinearity sensor. This signifies that for multipolar sensors, the torque ripple is similar even for sensors with higher nonlinearities, while for bipolar sensors, the torque ripple increases significantly. To observe the possible causes of this difference, the harmonic decompositions of the torque outputs in figure 4-9, D-4, 4-10 and D-5 are regarded. The torque harmonics for an ideal sensor are displayed in blue. It is seen that for both stiff-
ness values the preponderant harmonics in the output are the harmonics 6, 48 and 18 (in that order). For multipolar ideal sensors, the preponderant harmonics are 6, 18, 24/25 and 48. The main difference is the harmonic 48 which is much smaller for multipolar rings, and the absence of harmonic 42 but introduction of the harmonics 24/25. In short: the multiplication of bipolar angles seems to have as result that the preponderant harmonics are pushed towards the lower region.

Furthermore, the differences between the minimum and maximum nonlinearity sensors are regarded. For bipolar sensors the difference is determined by the amplitude of the harmonic corresponding to the number of cells (the harmonic 5) and to a lesser degree by its multiples such as the harmonic 25. When the nonlinearity harmonics of these bipolar sensors are regarded (see figure D-2) it is clear that the same differences between the minimum and maximum nonlinearity sensors can be seen here. The nonlinearity difference between the minimum and maximum is mainly due to the harmonic five. For multipolar sensors, the nonlinearity harmonics of the sensors used in the simulation are shown in figure D-3. Here, is it seen that the main difference between the minimum and maximum nonlinearity
is mainly due to the harmonic 16 which corresponds to the second harmonic since the sensors have eight pole pairs. The harmonic two was found to be related to the design. So when now the torque harmonics are regarded again (see figure 4-10 and D-5) it can be seen that also here, the differences between the minimum nonlinearity and maximum nonlinearity sensors are directly transformed to the torque harmonics, in other words the harmonic 16 yields the largest difference in torque harmonic amplitude. Thus for both bipolar and multipolar sensors, the torque harmonics for the minimum and maximum nonlinearity sensors differ in the same way as the nonlinearity harmonics themselves.

The influence of the changing harmonics though is not the same and this is why the torque ripple is considered almost constant for multipolar rings even though the nonlinearity increases, while it increases when the nonlinearity of the bipolar sensor increases. The influence of the harmonic 16 is very small as can be seen in figure 4-10. The harmonics 6 and 18 have still much larger amplitudes. For bipolar sensors on the other hand, the harmonic five has become the largest harmonic when the differences between minimum and maximum nonlinearity are regarded (see figure 4-9). Also harmonic 25, a multiple of the harmonic five, is larger than the harmonic 18 and a lot of other harmonics are introduced when the nonlinearity of the bipolar sensor increases. Therefore we can say that the nonlinearity harmonics of the bipolar sensor have much more influence on the torque harmonics and thus the torque ripple than the multipolar sensor nonlinearity harmonics. The effect is so strong, since the preponderant harmonic five is even multiplied by $N$, the number of permanent magnets in order to obtain electrical angles which are needed to control the motor currents.

Now the influence of the resonance is regarded. When comparing both figure 4-9 to 4-10 and figure D-4 to D-5 (where thus the stiffnesses are the same, but bipolar and multipolar sensors are used), the influence of the resonance can be seen. The resonance peaks are visible around harmonic 24 and 42 respectively. It is seen that for both bipolar and multipolar sensors, the resonance seems worse for the higher stiffness value. The amplitude of the harmonics of the resonance frequency are higher for bipolar sensors which is logical, since the amplitudes present in the nonlinearity harmonics are increased with a factor eight (the multiplication). To illustrate again the difference between the use of a bipolar sensor versus a multipolar sensor, figure 4-11 is introduced. In this figure, two examples of the nonlinearities in the angles provided by the sensors (through their sine and cosine outputs) are shown. One contains the angle of a bipolar sensor, which is multiplied to obtain in fact eight electrical angles. The multipolar sensor already has these electrical angles which are denoted in the figure by the horizontal lines. What is clearly visible and already stated, is that the nonlinearity is indeed higher for the bipolar sensor than for the multipolar sensor in every sector of 360° electrical, but that the frequency is much lower.

The torque ripple is in fact found not $N$ times as low (with $N$ the number of
permanent magnets in the rotor) using multipolar sensors but a lower value. This is due to the presence of the ‘motor’ harmonics: these harmonics related to the system are preponderant and thus, the harmonics of the sensor nonlinearities do have an influence on the torque harmonics but they are not significantly larger than the motor harmonics. In fact, the bipolar sensors with maximum nonlinearity have a torque ripple twice as large as comparable multipolar sensors with maximum nonlinearity. For minimum nonlinearities, the torque ripple is comparable. Furthermore, higher harmonics which are present in the nonlinearity play an important role in the torque harmonics. The torque ripple is significantly increased when harmonics around the resonance harmonic are present; this is illustrated for a multipolar sensor in figure 4-12. To obtain the results displayed in figure 4-12, simulations are done using an ideal multipolar angle with a certain harmonic with an amplitude of one degree added. In this way, the influence of a single harmonic on the torque output can be regarded. The results are obvious: for the lower and higher harmonics the torque harmonics are not particularly affected, but for harmonic 20 which is close to the resonance harmonic 24 the influence is significant. These harmonics cause large peaks in the torque output which are undesirable. In fact, it seems that these effects of the mechanical system are significant for both bipolar and multipolar sensors, since for bipolar sensors the higher harmonics are usually low compared to multipolar sensors but they are multiplied by the number of permanent magnets to obtain the electrical angle from the mechanical angle. From certain motor manufacturers the objective is known to prefer sensors with a low derivative of the nonlinearity over one turn which signifies that high-frequency harmonics in the nonlinearity of the sensor are indeed undesirable, as also found.

**Figure 4-11:** Example of bipolar and multipolar nonlinearity shapes
4-4 Conclusions

Regarding the simulations that have been done, in general it can be concluded that the use of multipolar sensors instead of bipolar sensors for brushless electric motors having multiple permanent magnets is favourable in terms of torque ripple. Since the maximum electric nonlinearity of a multipolar sensor compared to a comparable bipolar sensor are in general lower than the mechanical nonlinearity of the bipolar sensor, and since the use of a bipolar sensor signifies the need for multiplication of the angle in order to obtain the electrical angle the multipolar sensor will yield a better accuracy on the electrical angle. This result is confirmed with respect to the torque ripple measured as a result of the use of different sensors in the brushless electric motor. The hypothesis though in section 4-1 stated that the torque ripple would be at least $N$ times lower when using multipolar sensors, since the multiplication by the number of permanent magnets, $N$, is not needed. But this turns out to be not true: the torque ripple peak-to-peak amplitude is only twice smaller for the simulated multipolar sensors. This is due to the presence of significant harmonics in the torque output of the motor related to the mechanical system (load, coupling).

For bipolar sensors, the peak-to-peak amplitude of the torque ripple is mainly determined either by:
- The motor characteristics when almost ideal sensors are used
- The harmonic corresponding to the number of cells for sensors with lower performance levels
- Resonance

For multipolar sensors, the peak-to-peak amplitude of the torque ripple is mainly determined by:

- The motor characteristics
- Resonance

This signifies that for bipolar sensors, there will be a more significant difference in the torque ripple among the sensors taken from a collection than when multipolar sensors are used instead.
Modelling of the EPS system

After the modelling of the sensor targeting brushless electric motor control (see chapter 2) and integration of this sensor model into a model of a brushless electric motor (see chapter 4), in this chapter a model is proposed that uses this complete brushless electric motor model in an Electric Power Steering model. This chapter is build up in the same way as chapter 2 since for creating the model the same method is used: the PDCA-method. First a general description will be given of the chosen EPS type that will be modelled and this choice will be argumented. Subsequently, the four phases of the PDCA-method are regarded in section 5-2, 5-3, 5-4 and 5-5.
5-1 General description EPS

Several EPS systems with different characteristics are implemented in cars. The used EPS type depends mostly on the targeted car type and the rack load. The rack load is the force exerted on the rack, which is used to turn the wheels. Typically, the larger and heavier the vehicle, the higher the rack load. There exist several types of EPS which are briefly described here.

- Column driven
- Rack driven
- Pinion driven

For the column driven EPS, the electric motor is coupled to a reduction gear which transfers the output torque of the motor to the steering column. The rack driven EPS distinguishes itself from the column type EPS by the fact that here the electric motor is not coupled to the steering column, but the rack. This yields a higher possible rack load but the column type EPS is easier to install and because of that, among other reasons, cheaper. Several reduction gears can be used in combination with the rack type EPS such as belt-driven EPS where a belt-and-pulley system is used, or a harmonic rack in case a concentric motor is used. The third type, the pinion driven EPS, uses either the pinion of the traditional pinion-and-rack transmission between the steering column and the rack or an additional pinion at the rack to transmit the assisted torque to the rack.

The EPS type chosen for modelling is the belt-driven rack type EPS of which an example is shown in figure 5-1. This type of EPS is chosen since it is most chosen by manufacturers that search for high performance. The rack types of EPS permit the highest rack loads, and thus can achieve the needed assistance even for larger vehicles. In figure 5-2 a detailed view is seen of the system. In figure 5-2 the

Figure 5-2: Detailed view of a rack type belt-driven Electric Power Steering
steering column is attached at the pinion-and-rack transmission at (6). The rack-
and-pinion system transforms the rotational displacement of the steering wheel to
the horizontal displacement of the rack. If the steering wheel is turned, the tooth
at the pinion push the tooth at the rack away and thus converts rotational motion
into translational motion. The rack is attached to the tie rods and the movement
of the rack provokes there a moment around the wheel center, through which the
wheels start turning. This is all part of the conventional steering system, with as
only difference that more torque has to be exerted on the steering wheel to obtain
the desired motion of the front wheels.

The assisted power is delivered through a brushless electric motor seen at (7) and
in more detail with the transmission to the rack at (1). This motor is connected
to the rack through a reduction gear (pulley and belt system, see (2), (3) and
(4)) and a ballscrew transmission at (5). This ballscrew transmission converts
rotational motion of the lowest pulley into translational motion of the rack with
a ballnut thread. The needed motor power is determined in an ECU which uses as
inputs the measured torque at the pinion (the difference between the input angle
and realized angle) and the velocity of the vehicle. In this way, the assistance
can be varied with car velocity to make steering much easier at low speeds (for
parking) and harder at high speeds (to prevent instability at lane driving).

Typically, the brushless electric motor used for this type of EPS works at high
speeds, therefore the reduction gear is needed. The gear ratio of this belt-and-
pulley system is approximately 3:1.

5-2 Plan phase

Objectives  A mechanical model of an Electric Power Steering system is developed
to be able to investigate the influence of the SKF product, the sensor bearing, in
the electric motor powering the EPS system on the total steering system as well
as to get insight into how the system is build up. As already seen in chapter 2
and 3, the sensor delivers a nonperfect angle and the various possible shapes and
amplitudes of this nonlinearity are expected to have different effects on the be-
havior of the whole EPS system.

It is known that brushless electric motor manufacturers have an interest to use
sensors in their motors which are characterized using the derivative of the non-
linearity, thus, the slope of the nonlinearity of the sensor. What the differences
are for different $\frac{dN\text{NL}_a}{dt}$ of the sensors in the entire system can be investigated with
this model. Also, the basic understanding of the mechanical system for which the
sensor is basically designed is an objective for developing this model.

An important criterium of the model is that it is able to be used in cooperation
with the brushless electric motor model which is already developed. This model
is made in Simulink, so for the mechanical model MATLAB and/or Simulink can
best be used. To obtain a flexible, easily changable model (for instance, that can
be changed to a different EPS architecture), modularity of the subcomponents is very important. The following practical objectives of the model will have to be taken into account:

- To obtain a simplified but robust static and dynamic model of a rack EPS from input at pinion to tires
- To use as much as possible modules that have been developed within SKF already: focus on block integration

For now, the model will be developed and validated in order to be able for SKF to do simulations with it. Therefore the entire model can be used, but also subcomponents of it in combination with a Real-Time software program and hardware to couple it to existing parts of an EPS steering system. It is noted that this is not the objective here; the goal is to deliver a validated model.

**Name and logo**  To define a identity of the simulation tool which makes it more recognizable, a name and logo are chosen. The name is: SIMMoE which stands for “Sensor Impact Modular Model of an EPS” and the logo is depicted in figure 5-3.

![SIMMoE Logo](image)

**Figure 5-3:** Logo of the EPS model simulation tool

**Model architecture**  The model architecture is based on the transmission of movements of the mechanical system. The movement of this system is initiated by an input. The input here is the torque measured by the torque sensor at the pinion,
which is basically a measurement for the difference between the desired angle (thus, the steering wheel angle) and the realized angle (the angle at the pinion). One part of the movement is realized by the mechanical link between the steering wheel and the other part is realized by the motor via the reduction gear and ballscrew. The model structure is shown in figure 5-4. In fact, the architecture depicted in figure 5-4 is based on the general description of a mechanical system consisting of a motor. The system is viewed as a motor having a certain input, a transmission and a load. The load is at one hand the mechanical steering system and at the other hand, the steering linkage and the wheels. The impact of this steering system is fed back to the motor as being the load, taking into account the load inertia as well. So in fact, the entire system is being looked at from the point of view of the motor. The mechanical steering system block in figure 5-4 consists of the subblocks denoted in figure 5-5. The motor and sensor block are directly derived from the models that already exist and described in chapter 2 and 4. In the motor model, also other sensors can be included which have been modeled within ADC-SI. These can be retraced in figure 1-4 in section 1-3. For this project, the focus is on using the model of the SxC that is developed. The ECU block is described later. The transmission block is visually described in figure 5-6.

5-3 Do phase

In this section, the model of the belt-driven rack EPS is described. The modelling is done combining several existing blocks from the SKF Mechatronics Toolbox [3, 4, 5] in accordance with the objectives. Some blocks had to be created such as the Wheels block and the ECU block [6, 7, 8]. In general, the following structure for the components is used: a block consisting of an input torque or force from the block successing the block, denoting the torque or force fed back, an input state vector, an output state vector and an output torque or force to the predecessor of the block. This is denoted in figure 5-7. In this way, the load inertia is taken into account when the motor torque output is determined by means of the input...
Modelling of the EPS system

torque measured at the pinion since for each block, the resulting force or torque is fed back to the previous block.

In figure E-1 in appendix E the entire Simulink model is depicted. The sub-blocks denoted in the figure are explained below.

**Mechanical steering system** The intended movement of the driver of the car is determined by the torque sensor. If there is a discrepancy between the steering wheel angle and the pinion angle (where the sensor is located), there is a torque exerted on the system and thus the assisted power is needed to help get the steering wheels in the desired position with reduced effort for the driver. The measured steering torque by the torque sensor is equal to:

\[ \tau_s = K (\theta_{sw} - \theta_p) \]

where \( \theta_{sw} \) is the steering wheel angle determined by the driver and \( K \) the stiffness of the shaft. The resulting pinion angle \( \theta_p \) is defined in terms of the displacement of the rack and the radius of the pinion:

\[ \theta_p = \frac{y}{R} \]

where \( y \) is the rack displacement and \( R \) the pinion radius. If the measured steering torque is nonzero the electric motor needs to deliver extra power to reach the desired wheel steering angle.

**Motor and sensor** The motor and sensor sub-block denoted in figure 5-4 are described in chapter 2 and 4. The inputs of the block are:

- Load torque in [Nm]
- Current request in [A]

The outputs of the combined motor/sensor block are:

- Output rotation vector in [rad], [rad/s] and [rad/s²]
Motor torque in [Nm]  
Actual current in [A]

These in- and outputs are equal for every motor and sensor combination used. The sensor is completely embedded in the motor model. There are in fact two motor models implemented in the simulation tool: the SiMBBaS model developed within ADC-SI and a more simple motor model, included in the SKF Mechatronics Toolbox. The advantage of the latter is the reduced simulation time.

**ECU** In the ECU the current request of the motor is calculated. This determines the output speed of the motor and thus the assisted power fed to the rack to reach the desired front wheel steer angle. The calculation of the current request is done using two inputs. These are:

- Speed of the car
- Torque measured by torque sensor

When the torque measured is nonzero, the motor needs to deliver extra power. When the torque is high with respect to the maximum torque, the assisted torque must be higher. The closer the torque is to zero, the slower the engine will react in order to reduce the overshoot. The output current request is also dependant on the speed of the car. This is visualized in figure 5-8. The car should react faster and more direct when the speeds are low, to enable easy parking. For high speeds, the torque exerted on the steering wheel should have less effect in order to improve safety.

**Transmission** The rack displacement which denotes in fact the displacement of the load, is determined by the following equation of motion:

\[ \sum F = m \ddot{y} \iff m \ddot{y} = -2F_{sl} + F_m - F_p \]
where $F_{sl}$ the resulting steering linkage force, $F_{m}$ the force delivered by the motor and $F_{p}$ the resulting force on the rack by the pinion. The motor force in this equation comes from the combined system of the brushless motor and the transmission. The transmission consists of a belt-and-pulley system which is modeled as follows:

$$\left( I_{\text{out}} + \frac{R_{\text{in}}}{R_{\text{out}}} I_{\text{in}} \right) \ddot{\theta} + \left( \frac{R_{\text{in}}}{R_{\text{out}}} F_{m,\text{in}} + F_{m,\text{out}} \right) \dot{\theta} = \frac{R_{\text{in}}}{R_{\text{out}}} \tau_{m} - \tau_{bs}$$

where $I_{\text{in}}$ and $I_{\text{out}}$ denote the input and output pulley inertia respectively, $R_{\text{in}}$ and $R_{\text{out}}$ the input and output pulley radii and $F_{m,\text{in}}$ and $F_{m,\text{out}}$ the friction coefficients of the input and output pulley. The quantities $\tau_{m}$ and $\tau_{bs}$ stand for the torque delivered by the motor and the torque delivered to the ballscrew, which is the component that transforms the rotational motion of the belt-and-pulley system into translational rack motion. A more detailed view of a ballscrew can be found in figure 5-9. As said, the system transforms rotational motion into translational motion by means of the recirculating ball principle. When the nut turns as a result of the motor movement, the rack equipped with ballnut thread is moved with minimal friction thanks to the balls which form the contact between the nut and the ballnut thread on the rack. The equation of motion of this ballscrew is:

$$I_{bs} \dddot{\theta} + B_{bs} \ddot{\theta} = \tau_{bp} - F_r \frac{p}{2\pi}$$

where $I_{bs}$ is the inertia of the ballscrew, $B_{bs}$ its friction coefficient, $p$ the ballscrew pitch (lead per turn) and $\tau_{bp}$ and $F_r$ the torque from the belt-and-puleys system and the force from the rack, respectively.

**Wheels** The steering linkage force is approximated by an ideal transmission:

$$F_{sl} = K_{sl} \left( y - L\theta_w \right)$$

where $L$ is the steering linkage rate and $\theta_w$ the front wheel steer angle. The equation of motion of one front wheel is:

$$\sum M = I \ddot{\theta} \Leftrightarrow I_{w} \dddot{\theta}_w + b_{w} \ddot{\theta}_w = LF_{sl} - \tau_w$$
where $\tau_w$ is the self-aligning torque of the wheel as a result of slip, $I_w$ the inertia of the front wheel and $b_w$ the damping coefficient. The tire characteristics are neglected, that is, the self-aligning torque working against the turning of the wheels is not taken into account. This choice is made since it involves the modelling of elements that are not part of the steering system at all: the tires. Tire characteristics are in general nonlinear and very complex to determine. Also, when the turning of the car is done smoothly, the self-aligning moment stays small and therefore it is assumed zero.

**Interface and results** Now that all parts of the model itself are known, the interface is described. In order to be able to work easily with the EPS model proposed, an interface is developed which permits to select certain components of the model and the characteristics of these components. For instance, the motor can be chosen and the corresponding parameters. An example of the interface is shown in figure 5-10. The results for a simulation are shown in the top right panel of the interface. Some examples of results obtained with the model can be found in the next section, where the subject is the validation of the model.

**5-4 Check phase**

The model of the EPS steering system has been validated in the following manner. First, it is checked whether all the links with the interface are correct, in other words if the selected parameters are actually loaded in the model. This is the case,
so that the second phase can be started: the validation of the modelling. This cannot be done completely for this system since there is no corresponding physical EPS system present of which the measurement results can be compared with the simulation results. Therefore it is chosen to perform the validation partially as follows. Two different values for the input are set on the system and two outputs are regarded. The input is the steering wheel angle, defined in the real system by the driver. The outputs regarded are:

- Displacement of the rack
- Front wheel steer angle

These two outputs are chosen, since in literature values [9] have been found which can be used as a reference. The information used as reference is:

- Maximal displacement of the rack: 140 - 200 [mm]
- Lock-to-lock rotation steering wheel: 4 turns
- Maximal rotation of front wheel: 37 - 45° in both directions

The mechanical parameters are chosen according to the models used in [7, 8]. The motor parameters and sensor parameters are chosen as follows: the motor parameters are selected for a typical EPS motor and correspond to the configuration used for simulation in chapter 4. These characteristics can be found in table D-1 in appendix D. As a sensor, a perfect sensor is taken. This means that the nonlinearity of this sensor is zero; the angle used for determining the phase currents of the stator is perfect. For the first input a constant angle of 90° is put on the system. The results of the simulation with this input are seen in figure 5-11 and 5-12. In these figures, the reference values for this input are also plotted. They are determined by assuming that four turns of the steering wheel yields the maximal displacement of the rack and the maximal rotation of the front

![Figure 5-11: Displacement of the rack for constant input of 90°](image.png)
wheel. Since for both two values are given, the result should be between the two limits. In figure 5-11 and 5-12 it is clearly seen that the system reaches the desired value very fast for the speed of the car being zero, but slower for 100 [km/h] as desired. For both the displacement of the rack and the rotation of the front wheel, the response is within the limits of the expected result. Now the second input is set on the system. This input is a sinusoidal input which signifies that the driver steers fully to the right, back to the center, fully to the left, and back to the center again. This is defined by one period of a sine, and the maneuver is performed in four seconds and the results can be found in figure 5-13. Here the influence of the friction can be seen: the return position is not exactly zero since energy is lost. But again the maximum values are within the limits and both the front wheel angle and rack displacement variables yield the expected form of output. Therefore it is concluded that for the regarded inputs and parameters, the model yields the expected results but it is highly dependant on the choice of

![Figure 5-12: Rotation of the wheel for constant input of 90°](image1)

![Figure 5-13: Displacement of the rack and front wheel angle for full lock right and left](image2)
parameters such as the motor characteristics and the friction values if the output will be useful.

5-5 Act phase

Some recommendations can be made regarding the proposed modelling of the EPS system. At first, to include the tire-road interaction, a model of a tire can be added. This could be done as follows. Since the displacement of the rack in combination with the steering linkage has as output the rotation of the wheel around its vertical axis, a lateral force and self-aligning moment are generated since slip is introduced as a result of the car making a turn. The self-aligning moment is the quantity of interest here. The relationship between slip and this self-aligning moment is approximated by:

\[ M_y = C\alpha t \]

where \( C \) is the cornering stiffness in \([N/\text{rad}]\) and \( t \) the trail which signifies that the lateral force does not work in the center of the wheel but at an offset. The assumption holds for small slip angles (up to \( 10^\circ \)). The slip angle is caused by the rotation of the wheel and can be written as follows:

\[ \alpha = \theta_w + \tan^{-1} \left( \frac{-v - xr}{V} \right) \]

for a front wheel. In this equation \( v \) stands for the lateral velocity of the car, \( x \) for the distance from the center of gravity of the car to the front axle, \( r \) for the rotational velocity of the center of gravity of the car and \( V \) for the forward velocity (a constant value). Since in this formula car characteristics are involved, a way to obtain the variables is to couple a simplified car model to the steering system model.

What would also be possible is to build in the same way, using the same modular components EPS models with different architectures as described in section 5-1. The differences in vibrations in the system can be evaluated to see for which system, which parameters are of importance.

Another interesting addition would be to investigate different ECUs of different EPS systems to see how the motor output is controlled by the torque that is measured, since this is of great influence on the end result. The current request is determined by the vehicle speed and the torque measured by the torque sensor, but each manufacturer tunes the output in a certain way to obtain sporty, or for instance more comfortable responses. It means that different control systems that determine the current request can be evaluated.
Furthermore it would be interesting and recommended to perform a better validation with the help of a physical model of an EPS. In this way, experimental results can be linked to the simulation results to be able to improve the modelling and to really define the parameters of interest and importance. This can also be done with the use of parts of a steering system coupled with a Real-Time software program simulating the load or other parameters, as desired.
Conclusions will be drawn on two subjects: the modelling techniques that were used to obtain the sensor model and the Electric Power Steering model and the performance of the sensor itself and with the application taken into account.

Modelling of the sensor First of all, a validated bipolar and multipolar sensor model is obtained as was stated in the objectives of this master thesis. Also a (partly) validated model of an EPS system is obtained. The sensor model provides a way of predicting the performance of a collection of sensors, taking into account the variations of all parameters involved. The validation was done using several measurements on prototypes that were built of the sensor, with both bipolar and multipolar sensors.

Modelling of the EPS system For the modelling of the EPS system a physical test system was not available so that reference values found in literature are used to obtain an expected behavior of the system. These expectations are matched with the outcomes of the simulation tool for two different inputs, so that there is a level of confidence in the model.

Overall, the two models give SKF the opportunity to better predict the SxC sensor collection performances. The EPS model can be used to do simulations to investigate the impact of the sensor on the entire system.

Performance of the sensor Furthermore, conclusions are drawn regarding the performance of the sensor itself and the sensor integrated in its application, which is brushless electric motor control. Concluding from simulations with both bipolar and multipolar sensors done with the developed model, the performance level
of the sensor is dependant on several parameters of which the most important
are the number of Hall-effect cells used and the quality of the magnetic ring.
The more Hall-effect cells are used, the more the nonlinearity of the sensor (that
is, the maximum error in angle measurement over one mechanical revolution) is
determined by the magnetic ring. In general, the mean nonlinearity of a multipolar
sensor is smaller than the mean nonlinearity of the bipolar sensor, due to the
harmonic content of the two sensors. This is explained in section 3-2.

**Performance of the sensor in combination with the motor** When the sensor is in-
tegrated in the brushless electric motor model, it is proven that the multipolar
sensor performs better than the bipolar sensor when it is integrated in the same
motor having a number of permanent magnets equal to the number of pole pairs
of the magnet used in the multipolar magnet. This is due to the direct electrical
angles that the multipolar sensor provides, which eliminates the necessity of
multiplying in the ECU (see section 4-4) which is necessary for bipolar sensors
where the electrical period is equal to a part of the mechanical angle. Further-
more, the simulations with the sensor integrated in the motor model show that
the harmonic decomposition of the sensors nonlinearity is very important for the
end result, since resonance in the torque output can be caused by certain harmon-
ics present in the sensor nonlinearity. Lastly the difference between the best and
worse sensor in a collection yields in the case of a bipolar sensor a larger increase
in torque ripple (variation in torque) than in the case of a multipolar sensor. Mul-
tipolar sensors thus prove to be more robust when the entire collection is regarded
than bipolar sensors for the same motor.

In general, for motors having more than one permanent magnet, the use of mul-
tipolar sensors is preferred over the use of bipolar sensors due to the smaller
variation in torque ripple among the sensor collection. The worst multipolar sen-

susanne blockland
Chapter 7

Recommendations

As with the conclusions, there are two types of recommendations that can be done: with respect to the modelling of both the sensor and the Electric Power Steering system and with respect to the performance of the sensor and the sensor and motor combined. At first, the recommendations with respect to the modelling are given.

Modelling of the sensor

In order to improve the sensor model that has been developed, the improvements described in section 2-5 can be implemented. These include the addition of the axial alignment errors that are present in the mechanical design of the sensor and the extension of the model with a sensitivity analysis tool. This will permit the user to investigate the sensitivity of a sensor collection to a certain parameter change, without having to process all simulation data by hand as is the case for the current model.

Modelling of the EPS system

For the EPS model the recommendations with respect to the modelling are described in section 5-5. They include the addition of a tire model using for instance a module which contains a simple car model. These additions can be made in order to include the tire-road interaction which gives a better view of the real front wheel steer angle when actually driving the car at a certain speed. Other suggestions are to include other architectures of EPS systems, such as the column drive EPS and to obtain more information about the motor control in terms of the determination of the current request. This can be implemented in the current model of the ECU to obtain different possible controls. Lastly, the validation of the motor needs to be extended with measurements done on a physical EPS system in order to compare the model outcomes with the real system. This will greatly increase the confidence in the model.
In general, two models are obtained with a limited confidence level, since both models cannot be validated completely. The large number of variables for the sensor model plays a role in this, while for the EPS system the lack of a physical set-up limits the validation.

**Performance of the sensor**  The sensor performance can be divided into two parts: the improvement of the design and the improvement of the magnetic ring. In order to eliminate the second harmonic which mainly causes the variation in the nonlinearities (see chapter 3) a phase shift compensation method is developed. The nonlinearity can further be decreased by carefully selecting a harmonic profile of the ring that will be used in a particular sensor design, such that the harmonics in the nonlinearity will be minimal. Other solutions to improve the performance of the sensor include the use of a look-up table, filtering of harmonics and noise filtering.

**Performance of the sensor in combination with the motor**  Regarding the performance of the sensor in combination with the motor, it can be stated that since the performance is highly dependant on the motor and sensor characteristics it is hard to say something about the absolute performance of the motor. Here is room for improvement; if there is more data available of EPS motors, this data can be implemented in the system and for these motors, the theory can be tested as well. The goal is to be able to quickly integrate the SKF sensing solution in a motor of the client, and get an idea of the performance level that can be obtained.

In general, the performance of the sensor itself and the sensor in combination with the application can be improved by carefully looking at the possibilities. For instance, if the choice is made (for accuracy reasons) to use five cells, the magnetic ring profile has to be chosen such that it is in accordance with the harmonic five that will be large in the nonlinearity. In this way, sensors are obtained that are designed using the information that is already available to improve the design step by step.
### A-1 Bipolar S3C measurements

<table>
<thead>
<tr>
<th></th>
<th>Sine</th>
<th>Cosine</th>
<th>Phase shift [°]</th>
<th>Mechanical NL [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude</td>
<td>Offset</td>
<td>Amplitude</td>
<td>Offset</td>
</tr>
<tr>
<td>ring 1</td>
<td>0.865</td>
<td>2.517</td>
<td>0.841</td>
<td>2.520</td>
</tr>
<tr>
<td>ring 2</td>
<td>0.880</td>
<td>2.533</td>
<td>0.859</td>
<td>2.517</td>
</tr>
</tbody>
</table>

**Table A-1:** Results of the measurements on the bipolar S3C
A-2 Bipolar S5C measurements

<table>
<thead>
<tr>
<th></th>
<th>Output signals</th>
<th></th>
<th>Nonlinearity</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sine</td>
<td>Cosine</td>
<td>Phase shift [°]</td>
<td>Mechanical NL [°]</td>
</tr>
<tr>
<td></td>
<td>Amplitude</td>
<td>Offset</td>
<td>Amplitude</td>
<td>Offset</td>
</tr>
<tr>
<td>ring 1</td>
<td>0.857</td>
<td>2.492</td>
<td>0.863</td>
<td>2.552</td>
</tr>
<tr>
<td>ring 2</td>
<td>0.868</td>
<td>2.491</td>
<td>0.879</td>
<td>2.549</td>
</tr>
</tbody>
</table>

Table A-2: Results of the measurements on the bipolar S5C

A-3 Multipolar S3C Design 2 measurements

<table>
<thead>
<tr>
<th></th>
<th>Output signals</th>
<th></th>
<th>Nonlinearity</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sine</td>
<td>Cosine</td>
<td>Phase</td>
<td>Mech.</td>
</tr>
<tr>
<td></td>
<td>Amplitude</td>
<td>Offset</td>
<td>Amplitude</td>
<td>Offset</td>
</tr>
<tr>
<td>ring 1</td>
<td>1.459</td>
<td>2.542</td>
<td>1.442</td>
<td>2.475</td>
</tr>
<tr>
<td>ring 2</td>
<td>1.441</td>
<td>2.529</td>
<td>1.420</td>
<td>2.473</td>
</tr>
</tbody>
</table>

Table A-3: Results of the measurements on the multipolar S3C Design 2

A-4 Multipolar S5C Design 1 measurements

<table>
<thead>
<tr>
<th></th>
<th>Output signals</th>
<th></th>
<th>Nonlinearity</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sine</td>
<td>Cosine</td>
<td>Phase</td>
<td>Mech.</td>
</tr>
<tr>
<td></td>
<td>Amplitude</td>
<td>Offset</td>
<td>Amplitude</td>
<td>Offset</td>
</tr>
<tr>
<td>ring 1</td>
<td>1.373</td>
<td>2.528</td>
<td>1.371</td>
<td>2.489</td>
</tr>
<tr>
<td>ring 2</td>
<td>1.359</td>
<td>2.531</td>
<td>1.351</td>
<td>2.496</td>
</tr>
</tbody>
</table>

Table A-4: Results of the measurements on the multipolar S5C Design 1
A-5 Multipolar S5C Design 2 measurements

S5C Design 2

<table>
<thead>
<tr>
<th></th>
<th>Sine</th>
<th>Cosine</th>
<th>Phase shift [°]</th>
<th>Mech. NL [°]</th>
<th>Electrical NL [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude</td>
<td>Offset</td>
<td>Amplitude</td>
<td>Offset</td>
<td>nonlinearity</td>
</tr>
<tr>
<td>ring 1</td>
<td>1.440</td>
<td>2.498</td>
<td>1.448</td>
<td>2.532</td>
<td>91.303</td>
</tr>
<tr>
<td>ring 2</td>
<td>1.421</td>
<td>2.501</td>
<td>1.427</td>
<td>2.528</td>
<td>91.293</td>
</tr>
</tbody>
</table>

Table A-5: Results of the measurements on the multipolar S5C Design 2

A-6 Bipolar S3C simulations

S3C Simulations

<table>
<thead>
<tr>
<th></th>
<th>Sine</th>
<th>Cosine</th>
<th>Phase shift [°]</th>
<th>Mechanical NL [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude</td>
<td>Offset</td>
<td>Amplitude</td>
<td>Offset</td>
</tr>
<tr>
<td>min</td>
<td>0.833</td>
<td>2.407</td>
<td>0.835</td>
<td>2.427</td>
</tr>
<tr>
<td>mean</td>
<td>0.876</td>
<td>2.491</td>
<td>0.874</td>
<td>2.512</td>
</tr>
<tr>
<td>max</td>
<td>0.921</td>
<td>2.575</td>
<td>0.921</td>
<td>2.598</td>
</tr>
</tbody>
</table>

Table A-6: Simulation results of the bipolar S3C
A-7 Bipolar S5C simulations

<table>
<thead>
<tr>
<th>S5C</th>
<th>Simulations</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output signals</td>
<td>Nonlinearity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sine</td>
<td>Cosine</td>
</tr>
<tr>
<td></td>
<td>Amplitude</td>
<td>Offset</td>
</tr>
<tr>
<td>min</td>
<td>0.843</td>
<td>2.428</td>
</tr>
<tr>
<td>mean</td>
<td>0.876</td>
<td>2.488</td>
</tr>
<tr>
<td>max</td>
<td>0.907</td>
<td>2.555</td>
</tr>
</tbody>
</table>

*Table A-7: Simulation results of the bipolar S5C*

A-8 Multipolar S3C Design 2 simulations

<table>
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<tr>
<th>S3C Design 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output signals</td>
<td>Nonlinearity</td>
</tr>
<tr>
<td></td>
<td>Sine</td>
</tr>
<tr>
<td></td>
<td>Amplitude</td>
</tr>
<tr>
<td>min</td>
<td>1.394</td>
</tr>
<tr>
<td>mean</td>
<td>1.444</td>
</tr>
<tr>
<td>max</td>
<td>1.498</td>
</tr>
</tbody>
</table>

*Table A-8: Simulation results of the multipolar S3C Design 2*
## A-9 Multipolar S5C Design 1 simulations

### S5C Design 1

<table>
<thead>
<tr>
<th>Output signals</th>
<th>Nonlinearity</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td>Cosine</td>
<td>Phase shift [°]</td>
</tr>
<tr>
<td>Amplitude</td>
<td>Offset</td>
<td>Amplitude</td>
</tr>
<tr>
<td>min</td>
<td>1.313</td>
<td>2.438</td>
</tr>
<tr>
<td>mean</td>
<td>1.349</td>
<td>2.497</td>
</tr>
<tr>
<td>max</td>
<td>1.388</td>
<td>2.558</td>
</tr>
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</table>

*Table A-9: Simulation results of the multipolar S5C Design 1*

## A-10 Multipolar S5C Design 2 simulations

### S5C Design 2

<table>
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<tr>
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<th>Design</th>
</tr>
</thead>
<tbody>
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<td>Sine</td>
<td>Cosine</td>
<td>Phase shift [°]</td>
</tr>
<tr>
<td>Amplitude</td>
<td>Offset</td>
<td>Amplitude</td>
</tr>
<tr>
<td>min</td>
<td>1.350</td>
<td>2.441</td>
</tr>
<tr>
<td>mean</td>
<td>1.387</td>
<td>2.499</td>
</tr>
<tr>
<td>max</td>
<td>1.438</td>
<td>2.559</td>
</tr>
</tbody>
</table>

*Table A-10: Simulation results of the multipolar S5C Design 2*
Appendix B

Sensor simulations using other models

Figure B-1: Simulation with model families - S5C bipolar rings
Appendix C

Information for sensor performance simulations

C-1 Hall-effect cell Honeywell SS496A1: temperature influence

Figure C-1: Sensitivity and offset vs. temperature [1]
C-2  Hall-effect cell Honeywell SS496A1: supply voltage influence

![Graphs showing sensitivity and offset vs. supply voltage](image)

**Figure C-2:** Sensitivity and offset vs. supply voltage [1]

C-3  Specifications rings with eight pole pairs

<table>
<thead>
<tr>
<th></th>
<th>South pole amplitude</th>
<th>North pole amplitude</th>
<th>Pole length</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>799.9</td>
<td>-900.3</td>
<td>22.0</td>
</tr>
<tr>
<td>mean</td>
<td>862.1</td>
<td>-847.1</td>
<td>22.5</td>
</tr>
<tr>
<td>maximum</td>
<td>918.3</td>
<td>-788.9</td>
<td>22.9</td>
</tr>
<tr>
<td>standard deviation</td>
<td>19.7</td>
<td>18.6</td>
<td>0.11</td>
</tr>
</tbody>
</table>

**Table C-1:** Magnetic ring measurement results - eight pole pairs
## C-4 Amplitudes per simulation cycle

<table>
<thead>
<tr>
<th>Simulation cycle</th>
<th>Pole pair 1</th>
<th>Pole pair 2</th>
<th>Pole pair 3</th>
<th>Pole pair 4</th>
<th>Pole pair 5</th>
<th>Pole pair 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S</td>
<td>N</td>
<td>S</td>
<td>N</td>
<td>S</td>
<td>N</td>
</tr>
<tr>
<td>nominal</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>1</td>
<td>870</td>
<td>*</td>
<td>870</td>
<td>*</td>
<td>870</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
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<td>*</td>
<td>*</td>
<td>830</td>
<td>830</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

Table C-2: Amplitudes used for simulations

Note: a star in the table denotes the nominal amplitude of 850 [G].
Appendix D

Motor simulations

D-1 Motor bench

The motor bench used to validate is depicted in figure D-1. The motor model has been made using the motor characteristics of the motor in this experimental set-up. The setup consists of a typical EPS brushless electric motor, two different ways of motor control (selectable): six-step control and sinusoidal control, another brushless motor serving as load, a torque transducer, two SKF sensor bearings and an optical encoder for the rotor position angle.

Figure D-1: Test-bench for validating motor model
**D-2 Motor simulation tool configuration**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Motor</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque constant</td>
<td>[Nm/A]</td>
<td>0.0073</td>
<td>0.00564</td>
</tr>
<tr>
<td>Friction</td>
<td>[Nm/rad/s]</td>
<td>0.00025</td>
<td>0.0001</td>
</tr>
<tr>
<td>Inertia</td>
<td>[kgm(^2)]</td>
<td>7.19 (\cdot) 10(^{-5})</td>
<td>2.49 (\cdot) 10(^{-5})</td>
</tr>
<tr>
<td>Pole pairs</td>
<td>[-]</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>Load resistance</td>
<td>[]</td>
<td>-</td>
<td>0.115</td>
</tr>
</tbody>
</table>

**Table D-1: Motor configuration parameters**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Simulation 1</th>
<th>Simulation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness</td>
<td>[Nm/rad]</td>
<td>70</td>
<td>200</td>
</tr>
<tr>
<td>Damping</td>
<td>[Nm/rad/s]</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>
D-3 Nonlinearity harmonics of sensors used for simulation

Figure D-2: Harmonics of the nonlinearity of bipolar S5C sensors

Figure D-3: Harmonics of the nonlinearity of multipolar S5C sensors
D-4 Torque harmonics of motor with coupling stiffness 200 [Nm/rad]

Figure D-4: Harmonics of the output motor torque for bipolar S5C sensors - stiffness 200 [Nm/rad]
Figure D-5: Harmonics of the output motor torque for multipolar S5C sensors - stiffness 200 [Nm/rad]
At the next page the Simulink model of the EPS, SIMMoE, is found.
Figure E-1: Simulink model of SIMMeE


[6] L. Gasc, “Conception d’un actionneur à aimants permanents à faibles ondu-


Glossary

List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC-SI</td>
<td>Automotive Development Center - Sensor Integration</td>
</tr>
<tr>
<td>PDCA</td>
<td>Plan-Do-Check-Act</td>
</tr>
<tr>
<td>SxC</td>
<td>Sensor x Cells</td>
</tr>
<tr>
<td>TSxC</td>
<td>Test SxC</td>
</tr>
<tr>
<td>SOFIAs</td>
<td>SOftware For Integrated Angle Sensor-bearing</td>
</tr>
<tr>
<td>SiMBBaS</td>
<td>Simulation for Motor Brushless Bench and Sensor-bearing</td>
</tr>
<tr>
<td>SAHRAs</td>
<td>SoftwAre of High Resolution for Angle Sensor-bearing</td>
</tr>
<tr>
<td>SIMMoE</td>
<td>Sensor Integration Modular Model of an EPS</td>
</tr>
<tr>
<td>ECU</td>
<td>Electronic Control Unit</td>
</tr>
<tr>
<td>PCB</td>
<td>Printed Circuit Board</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>EPS</td>
<td>Electric Power Steering</td>
</tr>
<tr>
<td>AFS</td>
<td>Active Front Steering</td>
</tr>
<tr>
<td>PMSM</td>
<td>Permanent Magnet Synchronous Motor</td>
</tr>
</tbody>
</table>

List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>Rotational velocity of the shaft in [rad/s]</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>Self-aligning torque of wheel in [Nm]</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Front wheel steer angle in [rad]</td>
</tr>
<tr>
<td>$\theta_{sw}$</td>
<td>Steering wheel angle in [rad]</td>
</tr>
<tr>
<td>$\varphi_i$</td>
<td>Placement angle of a Hall-effect cell in [$^\circ$]</td>
</tr>
<tr>
<td>airgap</td>
<td>Distance from center Hall-effect cell to surface ring in [mm]</td>
</tr>
<tr>
<td>$B(\omega t)$</td>
<td>Rotating magnetic field in [G]</td>
</tr>
<tr>
<td>$B_0$</td>
<td>Magnetic field intensity at ring surface in [G]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Damping coefficient of front wheel in [Nm/rad]</td>
</tr>
<tr>
<td>$B_{bs}$</td>
<td>Friction coefficient ballscrew in [Nm/rad/s]</td>
</tr>
<tr>
<td>$F_m$</td>
<td>Force on rack delivered by motor in [N]</td>
</tr>
<tr>
<td>$F_p$</td>
<td>Force on rack delivered by pinion in [N]</td>
</tr>
<tr>
<td>$F_{m,in}$</td>
<td>Input pulley friction coefficient in [Nm/rad/s]</td>
</tr>
<tr>
<td>$F_{m,out}$</td>
<td>Output pulley friction coefficient in [Nm/rad/s]</td>
</tr>
<tr>
<td>$F_{sl}$</td>
<td>Steering linkage force in [N]</td>
</tr>
<tr>
<td>$g_i$</td>
<td>Gain or sensitivity of a Hall-effect cell in [V/G]</td>
</tr>
<tr>
<td>$I_w$</td>
<td>Inertia of the front wheel in [kgm$^2$]</td>
</tr>
<tr>
<td>$I_{bs}$</td>
<td>Inertia of ballscrew in [kgm$^2$]</td>
</tr>
<tr>
<td>$I_{in}$</td>
<td>Input pulley inertia in [kgm$^2$]</td>
</tr>
<tr>
<td>$I_{out}$</td>
<td>Output pulley inertia in [kgm$^2$]</td>
</tr>
<tr>
<td>$K$</td>
<td>Stiffness of the shaft in [Nm/rad]</td>
</tr>
<tr>
<td>$k$</td>
<td>Exponential coefficient for degradation of the magnetic field intensity</td>
</tr>
<tr>
<td>$L$</td>
<td>Steering linkage rate in [m]</td>
</tr>
<tr>
<td>$o_i$</td>
<td>Offset of a Hall-effect cell in [V]</td>
</tr>
<tr>
<td>$p$</td>
<td>Ballscrew pitch (lead per turn) in [m]</td>
</tr>
<tr>
<td>$R$</td>
<td>Pinion radius in [m]</td>
</tr>
<tr>
<td>$R_{in}$</td>
<td>Input pulley radius in [m]</td>
</tr>
<tr>
<td>$R_{out}$</td>
<td>Output pulley radius in [m]</td>
</tr>
<tr>
<td>$y$</td>
<td>Rack displacement in [m]</td>
</tr>
</tbody>
</table>