Project: Reduced models in DIANA

A fast method for preliminary assessment of concrete structures with nonlinear finite element analysis

Report #2

Shear beams in finite element modelling: software implementation and validation

Faculty of Civil Engineering and Geosciences
Structural Mechanics Section
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1 INTRODUCTION

Models for beam and shell elements allow for relatively rapid finite element analysis of concrete structures. This project involves the partnership of TNO DIANA BV, TU Delft and Rijkswaterstaat and aims at the development of such elements with shear capabilities and implementation in DIANA software.

1.1 Background and overview

The reduction of calculation time and degrees of freedom and the few required input variables are advantages of the reduced models, such as beam formulations, making them especially popular FEM options in the engineering practice. Standard nonlinear fiber beam formulations do not account for shear effects and cannot capture all failure modes (like shear failure) and hence cannot be used in the assessment of structures with shear problems, as it may result in large overestimations of ultimate carrying capacities. Solving this handicap would provide a tool for faster and simpler nonlinear analysis that is advantageous for pre- and quick scan assessment stages. The shear-sensitive beam model to be implemented in DIANA is based on the PhD work of D. Ferreira (2013) developed at Universitat Politècnica de Catalunya, BarcelonaTECH (UPC).

In 2012, the document RTD 1016: 2012 “Guidelines for Nonlinear Finite Element Analysis of Concrete Structures. Scope: Girder Members” (RTD1016 2012) was completed. With these guidelines it is possible to perform sophisticated nonlinear finite element analyses (NLFEA) and determine, as far as possible, existing residual capacities in structures. In 2015, the document ‘Validating the Guidelines for Nonlinear Finite Element Analysis of Concrete Structures’ was completed. It includes the validation of the guidelines by means of various benchmarks of RC beams, prestressed beams and slabs. Numerical simulations were performed with 2D (plane stress) and 3D (solids) continuum nonlinear finite element models.

The first part of this project (from September 2014 to June 2015) was mainly focused on the following tasks:

- Calculation of existing and new benchmarks with the fiber beam model (with own code from UPC - CONSHEAR) and comparison with plane stress models in DIANA performed by (RTD1016b, Hendriks et al. 2015) and with standard beam elements in DIANA: Report #1 ‘Calculation of benchmarks with a shear beam model’. Report number: TUD/CITG/B&I/CM-2015-13.
- Preliminary implementation of the fiber beam model in DIANA software.
- Expansion of the original 2D beam formulation to the 3D case to be implemented in DIANA.
- Study the possibility of extending the shear beam formulation shells.

The second part of the project (from July 2015 to February 2016) is focused on the following tasks:

1. Implementation and validation of 2D and 3D beam elements including shear effects in DIANA software;
2. Application of the beam models to the structural analysis of bridges;
3. Feasibility study of the extension of the formulation to shell elements.
The ultimate goals of this project are to make these models (beams and shells) available for the DIANA users and to use it for quick scan analysis in assessment of existing infrastructure and other suitable applications.

This report relates to the implementation and validation of 2D and 3D beam elements including shear effects in DIANA software.

1.2 Scope and objectives

The present report relates to the implementation of 2D and 3D shear-sensitive beam elements in DIANA source code and its validation with various benchmarks of RC and PC beams. The case-studies include the ones previously analysed in the ambit of the validation of the Guidelines for Nonlinear Finite Element Analysis of Concrete Structures. Scope: Girder Members" (RTD1016 2012) and also presented in Report #1 ‘Calculation of benchmarks with a shear beam model’ (TUD/CITG/B&I/CM-2015-13).

The goal of this work is to develop a new tool in DIANA software related with capturing shear effects with beam elements (2D and 3D). In this manner, transversal reinforcement is considered and the beam model is able to compute multiaxial strains and stresses in concrete, as well, as inclined cracking. Shear failures can be captured by failure of transversal reinforcement or crushing of the diagonal concrete strut. These capabilities enlarge the range of application of the standard beam elements, which were limited to bending problems.

This report is also a basis for the DIANA manual and examples to be included in relation to this model.

1.3 Outline

The present document is divided into 6 chapters.

Chapter 1 is the present introduction.

Chapter 2 offers a brief description of the theoretical background of the shear fiber beam elements for the 2D and 3D cases. This chapter refers only to the fundamentals of the model as the detailed formulations are available in published works, included as references along the text.

Chapter 3 explains the implementation of the model into DIANA source code. The 2D and 3D beam models are included in DIANA 10 development version.

Chapter 4 discusses the pre- and post-processing requirements for this model in DIANA Interactive Environment (DianaIE).

Chapter 5 relates to the recalculation of benchmarks of RC and PC beams. Here the results of the new shear fiber beam model in DIANA are compared with the results of simulations made with the original code (CONSheAR) and with 2D plane stress analyses with DIANA performed by Hendriks and Belletti et al. (2014) and with experimental data.

The case studies considered in the validation are listed in the following.

RWS Benchmarks: Reinforced concrete beams:
- Case RB1: Vecchio & Shim 2004 – Beam C3 (bending)
- Case RB2: Collins & Kuchma 1999 – beam SE-50A-45 (shear)
- Case RB3: Grace 2001 – Control beam category II (bending)
- Case RB3A: Grace 2001 – Control beam category I (shear)

RWS Benchmarks: Prestressed concrete beams
- Case PB1: Leonhardt, Koch et al. 1975 – Beam IP1 (bending)
- Case PB2: Sun & Kuchma 2007 – Girder 3 (shear)
- Case PB3: Runzell et al. 2007 – Specimen I (shear)
- Case of the DIANA Users Contest / TU Delft 2014 – Mid beam, Code101 (shear – bending)
  and Edge beam
- Case of the DIANA Users Contest / TU Delft 2014 – Edge beam, Code201 (no symmetrical, shear – bending)

The following cases were included in the DIANA test library: RB1, RB2, PB1 for 2D beam and PB2, PB3 for 3D beam.

/Test/nl/fiber/beam2d/RB1  Fiber beam RB1  Vecchio and Shim 2004 C3
/Test/nl/fiber/beam2d/RB2  Fiber beam RB2  Collins and Kuchma 1999 SE-50A-45
/Test/nl/fiber/beam2d/PB1  Fiber beam PB1  Leonhardt, Koch et al 1975 IP1
/Test/nl/fiber/beam3d/PB2  Fiber beam PB2  Sun and Kuchma 2007 Girder3
/Test/nl/fiber/beam3d/PB3  Fiber beam PB3  Runzell et al 2007 Specimen I

Chapter 6 resumes the results and the main conclusions of the work.
2 THEORETICAL BACKGROUND

This section presents the key aspects of the theoretical background of the model developed by Ferreira, Bairán & Marí (UPC 2013). Details on the formulation, validation and application of the model can be found in the following references:


- PhD thesis of D. Ferreira, UPC (2013) ‘A model for the nonlinear, time-dependent and strengthening analysis of shear critical frame concrete structures’, presents the development and implementation of the beam model, as well as, its validation with classical shear beam tests and some applications;


- The model was applied to several different studies of the concrete structural behaviour in which shear effects are relevant:
  - Assessment of existing structures:
    o Ferreira D., Bairán J., Marí A., Efficient 1D model for blind assessment of existing bridges: simulation of a full scale loading test and comparison with higher order continuum models, Structure and Infrastructure Engineering, 2014, published online.
  - Strengthening of structures:
    o Ferreira D., Bairán J., Marí A., Shear strengthening of RC beams by means of vertical prestressed reinforcement: numerical studies, Structure and Infrastructure Engineering, 2015, published online.
  - Service state analysis:
  - Structural response since early ages:
2.1 Fundamentals of the model

The model is a displacement-based fiber beam FE formulation for the nonlinear, time-dependent and phased analysis of reinforced and prestressed concrete frame structures. The model idealizes three dimensional RC frames with 1D beam elements of arbitrary cross section interconnected by nodes. The cross section is discretized into fibers or layers of concrete and filaments longitudinal steel. Transversal reinforcement is accounted as smeared in the concrete fibers.

Axial force-bending-shear force interaction is accounted allowing for the nonlinear analysis of shear critical concrete frame structures. The fundamentals of the model are represented in Figure 2.1 concerning different levels of analysis: structure, element, section and fiber. The Timoshenko beam theory is linked with a shear-sensitive sectional model (element integration point level) that associates the Bernoulli-Navier plane section theory with an assumption of fixed shear stress pattern (constant in the integration points along the height of the cross section). Cracking is simulated through the smeared and rotating crack approach. The effects of shear and its interaction with normal forces are accounted from SLS and ULS. This allows including the effects of shear in deflections, strains in concrete and reinforcement and cracking behaviour in addition to capture shear failure mechanisms. The time step-by-step analysis allows the simulation of segmental construction procedures and subsequent later changes, in which repair and strengthening interventions are included. The nonlinear analysis is performed within a Newton-Raphson framework.

The fixed stress approach used in the model, although not guaranteeing compatibility between the fibers, gives satisfactory results in the simulation of the shear-resistant mechanism of reinforced cracked concrete cross-sections at a low computational and modelling cost, as concluded in (Bairán and Marí 2007).

Cracking is simulated through the smeared and rotating crack approach. The Hognestad parabola is assumed for concrete in compression. Softening (Vecchio and Collins 1986) and strength enhancement (Kupfer, Kilsdorf et al 1969) factors are included for the respective states of compression-tension and biaxial compression. For concrete in tension a linear response is assumed before cracking and a tension stiffening curve (Cervenka 1985) is considered in the cracked stage. Figure 2.1-2 represents the constitutive models originally used, however any other constitutive law based on total rotating crack approach can be used. Steel and FRP are only submitted to axial strains and stresses by means of uniaxial constitutive laws.
The inclusion of shear in beams by means of the assumption of shear stress pattern in a cross section is resumed in Table 2.1-1. Comparison is made between the standard beam elements and the new formulation with consideration of shear effects.

The parameters compared are:
- degrees of freedom in the finite element,
- generalized strains in the integration point,
- conditions assumed in the layer (kinematic and force based),
- strains and stresses in a layer,
- imposed conditions in the state determination of the layer,
- strains corrected in the iterative procedure at the layer level required to fulfil the imposed conditions,
- order of the material model for concrete needed in the model.
<table>
<thead>
<tr>
<th>MODEL</th>
<th>DOF/node</th>
<th>Generalized strains in Gauss Point, $e_0$</th>
<th>Kinematic/force conditions in layer</th>
<th>Layer strains $\varepsilon$ / stresses $\sigma$</th>
<th>Imposed conditions in the layer</th>
<th>Iterative strains</th>
<th>Concrete material model</th>
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<tr>
<td>Standard flexural 3D beam</td>
<td>${u, v, w, \theta_x, \theta_y, \theta_z}$</td>
<td>${e_{0x}, \gamma_{0xy}, \gamma_{0xz}, \phi_x, \phi_y, \phi_z}$</td>
<td>$e_s = e_{0x} + z\phi_y - y\phi_z$</td>
<td>${e_s}$</td>
<td>-</td>
<td>-</td>
<td>[D] (1x1) 1D model</td>
</tr>
<tr>
<td>3D Beam with shear in 2D M-N-V</td>
<td>${u, v, w, \theta_x, \theta_y, \theta_z}$</td>
<td>${e_{0x}, \gamma_{0xy}, \gamma_{0xz}, \phi_x, \phi_y, \phi_z}$</td>
<td>$e_s = e_{0x} + z\phi_y - y\phi_z$</td>
<td>${e_s, \varepsilon, \gamma_{xz}}$</td>
<td>$\sum \sigma_z = 0$</td>
<td>$\tau_{xz} = \tau_{xz}^*$</td>
<td>[D] (3x3) 2D plane strain model</td>
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<tr>
<td>3D Beam with shear in 3D M-N-V</td>
<td>${u, v, w, \theta_x, \theta_y, \theta_z}$</td>
<td>${e_{0x}, \gamma_{0xy}, \gamma_{0xz}, \phi_x, \phi_y, \phi_z}$</td>
<td>$e_s = e_{0x} + z\phi_y - y\phi_z$</td>
<td>${e_s, \varepsilon, \gamma, \varepsilon_y, \gamma_{xz}}$</td>
<td>$\sum \sigma_y = 0$</td>
<td>$\tau_{xy} = \tau_{xy}^*$</td>
<td>[D] (5x5) 3D model</td>
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2.2 Original version: 3D beam element with 2D shear-bending interaction (My-N-Vz)

2.2.1 Element level

A 2-noded Timoshenko 3D FE with linear shape functions is implemented (Figure 2.2-1). Nonlinear interaction of axial force and bending moment is accounted at the full 3D level (N-My-Mz). Nonlinear interaction of normal and tangential forces is accounted only at 2D level (N-My-Vz). Timoshenko beam theory states that non-deformed plane sections perpendicular to the beam axis remain plane but not necessarily normal to the longitudinal axis after deformation. An average rotation of the section due to distortion $\gamma_0$ is considered in order to maintain valid the plane-section assumption.

The generalized strains are determined in a Gauss point in the middle of the FE through the displacement fields adopting the Timoshenko beam element formulation. The generalized strains in Gauss point of a 3D beam are:

\[
e_s = \begin{cases} 
\varepsilon_0 &= \frac{\partial u}{\partial x} \\
\gamma_\gamma &= \frac{\partial v}{\partial x} - \theta_z \\
\gamma_\gamma &= \frac{\partial w}{\partial x} - \theta_y \\
\phi_\gamma &= \frac{\partial \theta_y}{\partial x} \\
\phi_\gamma &= \frac{\partial \theta_z}{\partial x} 
\end{cases}
\]

one axial strain ($\varepsilon_0$), two shear strains ($\gamma_\gamma$ and $\gamma_\gamma$), two bending curvatures ($\phi_\gamma$, $\phi_\gamma$) and one torsion curvature.
Theoretical background

The 3D bending analysis (N-My-Mz) implements the standard formulation and, for that reason, will not be described here. Considering the 2D case for interaction of bending and shear forces (N-My-Vz) the kinematic equations that relate the nodal displacements \( u_j \) to the generalized strains in the Gauss points \( \varepsilon_0 = [\varepsilon_0 \gamma_0 \phi_y] \) are given by the transformation matrix B:

\[
B = \begin{bmatrix}
\frac{\partial N_i}{\partial x} & 0 & 0 \\
0 & \frac{\partial N_i}{\partial y} & N_i \\
0 & 0 & \frac{\partial N_i}{\partial z}
\end{bmatrix}
\]

where \( \varepsilon_0 \) is the axial strain, \( \gamma_0 \) is the shear rotation, \( \phi_y \) is the curvature of the cross-section and \( N_i \) are the shape functions.

At the sectional level, the relationship between the generalized strains \( \varepsilon_0 \) determined in the axis of the element and the strains in each fiber \( \varepsilon_{\text{fiber}} = [\varepsilon_x \gamma_z] \) is given by the transformation matrix \( T \), accordingly to Eq. (31). Filaments of longitudinal steel are assumed to be submitted only to axial strains as set by the transformation matrix \( T_{sl} \) given by Eq. (32).

After the state determination of the fibers (concrete and transversal steel - denoted as c+st) and the filaments (longitudinal steel – denoted as sl), the section stiffness matrix \( K_{sec} \) and the internal load vector \( \mathbf{S}_{sec} = [M V_z M_y] \) are given by the summation of both contributions:

\[
K_{sec} = K^{c+st} + K^{sl} : \quad K^{c+st}_{sec} = \int T^T K_{\text{fiber}, c} T \, dA, \quad K^{sl}_{sec} = \int T^T E^{sl} T_{sl} \, dA_{sl}
\]

\[
\mathbf{S}_{sec} = \mathbf{S}_{c}^{c+st} + \mathbf{S}_{sl}^{sl} : \quad \mathbf{S}_{c}^{c+st} = \int T^T \mathbf{S}_{\text{fiber}, c} T \, dA, \quad \mathbf{S}_{sl}^{sl} = \int T^T \sigma_{sl} \, dA_{sl}
\]

where \( A \) represents the area of each fiber and \( A_{sl} \) the area of each filament.

Making use of the presented sectional concept, the classical FEM equations (Zienkiewicz and Taylor 2004) of the element stiffness matrix \( K_{elem} \) and the internal resistant load vector \( \mathbf{F}_{elem} \) can be written as:

\[
K_{elem} = \int B^T K_{sec} B \, dx
\]

\[
\mathbf{F}_{elem} = \int B^T \mathbf{S}_{sec} dx
\]

The former integrals are solved through the Gaussian Quadrature Method using a reduced integration rule in order to avoid shear locking; in this case one integration point is considered.
2.2.2 **Integration point level (section and fiber levels)**

The model uses a simplified sectional formulation that links the plane section theory with the assumption of fixed shear stress along the cross section. This assumption results into a hybrid approach as the input variables comprises both kinematical quantities, in terms of curvature and axial beam’s strain, and the applied shear force. The output of the sectional model is the axial force, bending moment and shear deformation. Figure 2.2-2 presents a scheme of the inputs and outputs of the sectional model.

![Sectional model diagram](image)

**Figure 2.2-2: Sectional model**

Axial strain orthogonal to the cross-section ($\varepsilon_x$) is computed, in all fibers, by means of the Navier-Bernoulli plane section assumption:

$$\varepsilon_x(z) = \varepsilon_0 + \phi_y z$$

where $\varepsilon_0$ is the axial strain of the reference axis of the section and $\phi_y$ is the curvature of the cross-section with respect to the $y$-axis. In the shear resistant fibers a constant shear stress $\tau^*$ flow along the section is assumed as:

$$\tau^* = G^* A^* \gamma_0$$

where $G^*$ is the transversal modulus, $\gamma_0$ is the distortion at the neutral axis and the effective shear area $A^*$ is given by the summation of the areas of the shear resistant fibers. By these means and using the equilibrium, compatibility and constitutive equations, the complete 2D stress-strain state and the stiffness matrix of the fiber are determined.

The concrete part of the shear resistant fiber is submitted to a 2D stress-strain state (Eq. 15); after rotating the principal stiffness to the local referential, a $3 \times 3$ stiffness matrix $D_c$ is obtained. A concrete fiber can have different $n_k$ configurations of transversal steel (and different material properties) that are accounted for in the model through its volumetric ratio $\rho_{st,k}$ and are submitted to axial stresses $\sigma_{z,k}$ (along direction $z$). According to Eq. (16) the total transversal steel is taken into account by the summation of the contributions of the different stirrups configurations ($A_{st,k}$ is the area of transversal steel, $b_k$ is the width of the cross-section and $s_k$ is the longitudinal spacing of each configuration of stirrups $k$). Compatibility requirements impose that the vertical strain $\varepsilon_z$ in concrete is equal to the strain in the transversal reinforcement.

$$
\begin{pmatrix}
\Delta \sigma_x \\
\Delta \sigma_z \\
\Delta \tau_{xz}
\end{pmatrix}^c = D_c
\begin{pmatrix}
\Delta \varepsilon_x \\
\Delta \varepsilon_z \\
\Delta \gamma_{xz}
\end{pmatrix}
$$

$$
D_c =
\begin{pmatrix}
D_{11} & D_{12} & D_{13} \\
D_{21} & D_{22} & D_{23} \\
D_{31} & D_{32} & D_{33}
\end{pmatrix}
$$

(11)
Along the z direction the incremental tensile stresses in the transversal steel $\Delta \sigma_z^{st}$ must equilibrate the incremental compression stresses in concrete $\Delta \sigma_z^{c}$:

$$\Delta \sigma_z^{c} + \rho_c \Delta \sigma_z^{\sigma} = 0, \quad \rho_c \Delta \sigma_z^{\sigma} = \sum_{k=1}^{n_c} \left( \frac{A_{st,k}}{s_k b_k} \Delta \sigma_{z,k}^{\sigma} \right)$$

In addition to this equilibrium requirement, the other condition to be fulfilled in order to determine the fiber state is: the computed increment of shear stress $\Delta \tau_{xz}$ must equate the impose shear stress given by the fixed stress constraint $\Delta \tau_{xz}^*$ as

$$\Delta \tau_{xz}^* - \Delta \tau_{xz} = 0$$

By solving the system composed by Eqs. (13) and (14) – and making use of Eqs. (15) and (16) - the unknown increments of vertical axial strain $\Delta \varepsilon_z$, and shear strain $\Delta \gamma_{xz}$ are determined as functions of the increments of the longitudinal axial strain $\Delta \varepsilon_x$, the shear stress $\Delta \tau^*$ and the material stiffness matrix $D_{fiber}$.

$$\Delta \varepsilon_z = f \left( \Delta \varepsilon_x, \Delta \tau^*, D_{fiber} \right) = \frac{\left( D_{22} D_{33} - D_{23} D_{32} \right) \Delta \varepsilon_x - D_{23} \Delta \tau^*}{D_{22} D_{33} - D_{23} D_{32}}$$

$$\Delta \gamma_{xz} = f \left( \Delta \varepsilon_x, \Delta \tau^*, D_{fiber} \right) = \frac{\left( D_{23} D_{31} - D_{21} D_{33} \right) \Delta \varepsilon_x + \bar{D}_{22} \Delta \tau^*}{D_{22} D_{33} - D_{23} D_{32}}$$

To achieve both requirements along the vertical and transversal directions (Eqs. (11) and (12), respectively), an innermost iterative procedure within the fiber level is needed. After computation of the 2D fiber strain and stress states, Eqs. (13) and (14) are checked and the unbalanced vertical $\delta \varepsilon_z$ and tangential $\delta \tau_{xz}$ stresses are respectively computed as

$$\delta \varepsilon_z = -\rho_c \Delta \sigma_z^{\sigma} + \Delta \varepsilon_z$$

$$\delta \tau_{xz} = \Delta \tau^* - \Delta \tau_{xz}$$

The increment of longitudinal axial strain $\Delta \varepsilon_x$ is kept fixed and the iteration goes through the correction of the vertical $\delta \varepsilon_z$ and transversal $\delta \gamma_{xz}$ strains, which are computed through the following expressions as functions of the unbalanced stresses $\delta \sigma_z$ and $\delta \tau_{xz}$:

$$\delta \varepsilon_z = f \left( \delta \tau_{xz}, \delta \sigma_z, D_{fiber} \right) = \frac{\delta \sigma_z D_{33} - D_{23} \delta \tau_{xz}}{D_{22} D_{33} - D_{23} D_{32}}$$

$$\delta \gamma_{xz} = f \left( \delta \tau_{xz}, \delta \sigma_z, D_{fiber} \right) = \frac{\delta \tau_{xz} D_{22} - D_{23} \delta \sigma_z}{D_{22} D_{33} - D_{23} D_{32}}$$

The strain corrections ($\Delta \varepsilon_z^{\sigma}$, $\Delta \gamma_{xz}^{\sigma}$) are introduced in the next iteration until both the unbalanced vertical $\delta \varepsilon_z$ and tangential stresses $\delta \tau_{xz}$ vanish:

$$\Delta \varepsilon_z^{\sigma} = \Delta \varepsilon_z + \delta \varepsilon_z$$

$$\Delta \gamma_{xz}^{\sigma} = \Delta \gamma_{xz} + \delta \gamma_{xz}$$
Convergence is checked by comparing the norm of residual forces with the norm of the total applied forces. A tolerance rate of 1% is considered. This is checked for both requirements:

i. Equilibrium in the vertical direction: convergence rate \( \lambda_z^u \) is the ratio between the increment of stresses in relation to the past iteration \( \Delta \sigma_z^u \) and the total stress \( \sigma_z^u \)

\[
\lambda_z^u = \frac{\Delta \sigma_z^u}{\sigma_z^u} < 0.01
\]

ii. Equality between shear stresses: convergence rate \( \lambda_{xz}^u \) is the ratio between the shear stress outputted in the fiber \( \tau_{xz} \) and the imposed shear stress \( \tau^* \)

\[
\lambda_{xz}^u = \frac{\tau_{xz}}{\tau^*} < 0.01
\]

Once this iteration procedure is finished and convergence is achieved (i.e. \( \delta \varepsilon_z \approx 0 \) and \( \delta \tau_{xz} \approx 0 \)) the state determination of the fiber is accomplished. As stress \( \sigma_z \) is null and the section model does not include \( \varepsilon_z \), a static condensation may be applied:

\[
\left( \begin{array}{c}
\sigma_x \\
0 \\
\tau_{xz}
\end{array} \right) =
\left( \begin{array}{ccc}
D_{11} & D_{12} & D_{13} \\
D_{21} & D_{22} & D_{23} \\
D_{31} & D_{32} & D_{33}
\end{array} \right)
\left( \begin{array}{c}
\varepsilon_x \\
\varepsilon_z \\
\gamma_{xz}
\end{array} \right)
\]

and thus

\[
\left( \begin{array}{c}
\sigma_x \\
\tau_{xz}
\end{array} \right) = K_{ fibre} \left( \begin{array}{c}
\varepsilon_x \\
\gamma_{xz}
\end{array} \right)
\]

\[
K_{ fibre} = \left( \begin{array}{cc}
D_{11} - \frac{D_{12} D_{21}}{D_{22}} & D_{13} - \frac{D_{12} D_{23}}{D_{22}} \\
D_{31} - \frac{D_{32} D_{21}}{D_{22}} & D_{33} - \frac{D_{32} D_{23}}{D_{22}}
\end{array} \right)
\]

where \( K_{ fibre} \) is the condensed stiffness matrix of the fiber. The shear modulus in Eq. (10) is given by

\[
G^* = D_{33} - \frac{D_{12} D_{23}}{D_{22}}
\]

The complete mathematic derivations of these equations can be found in the previously listed references.

2.2.3 Definition of shear resistant areas of the cross section

The cross-section is discretized into two types of fibers as presented in Figure 2.2-3: a) non shear resistant ones, submitted only to 1D axial stresses, and b) shear resistant fibers submitted to a multiaxial stress-strain state (see Figure 3.2-3). This division is performed considering the following criteria:

i) for traditional cross section geometries, such as, rectangular, T-shape and I-shape, it is considered that the fibers that pertain to the web (disregarding the cover area) are 2D fibers;

ii) particularly for the T-shape and I-shapes cross sections, an effective area of the compressive flanges can be considered to contribute to the shear-resistance mechanism and assigned as 2D fibers - the effective width of the flange \( b_{ef} \) can be determined accordingly to (Zararis, Karaveziroglou et al. 2006);

iii) for complex geometries a more sophisticated analysis with the model TINSA (Bairán and Marí 2006, Bairán and Marí 2006) is required in order to determine the portion of section that is preponderant for resisting shear forces; however in these cases, existent
recommendations in some design codes (for example, the parameter $b_w$ in EC2) can be used as a simplified criterion.

Option i) is the default in the model.

**Figure 2.2-3**: Types of fibers in the sectional level

Figure 2.2-4: Stress state in the fibers: a) shear resistant and b) non shear resistant

### 2.2.4 Uncoupled torsion

Torsion is assumed uncoupled through a nonlinear torque-strain curve. A trilinear model proposed by Chan (1982) is used to simulate the sectional response in terms of torsion-twist relationship. This torsion-twist model is presented in Figure 2.2-5: Model adopted for torsion: torque-twist relationship (Mari 2000) and is defined through the following parameters: $T_{cr}$ and $\alpha_{cr}$ are, respectively, the torque and the twist at first cracking; $T_{yp}$ and $\alpha_{yp}$ are the torque and correspondent twist at first full yielding of all the reinforcement; $\alpha_u$ is the twist at ultimate failure. The unloading in each phase is assumed to be elastic considering the initial stiffness.
2.3 3D beam element extended to 3D shear-bending interaction (My-Mz-N-Vy-Vz)

2.3.1 Element level

The 2-noded Timoshenko 3D FE with linear shape functions is also considered. Nonlinear interaction of normal and tangential forces is accounted at the full 3D level (N-My-Mz-Vy-Vz). The degrees of freedom in a 3D beam element are represented in Figure 2.3-1, including the location of one integration point.

![Figure 2.3-1: Degrees of freedom of a 3D beam element](image)

There are 6 DOF per node, three displacements and three rotations:

\[
\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}, \quad \mathbf{d}_1 = \begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix}, \quad \mathbf{d}_2 = \begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix}, \quad \mathbf{d}_3 = \begin{bmatrix} \theta \end{bmatrix}
\]

The shape functions are linear; the equations of the shape functions and its derivatives are given by:
Theoretical background

\[ N_i(x) = 1 - \frac{x}{L} \quad N'_i(x) = -\frac{1}{L} \]
\[ N'_i(x) = \frac{1}{L} \quad N'_i(x) = 1 \]

The discretization of the displacement field can be written as:

\[ u(x) = N_i(x)u_1 + N_2(x)u_2 \]
\[ v(x) = N_i(x)v_1 + N_2(x)v_2 \]
\[ w(x) = N_i(x)w_1 + N_2(x)w_2 \]
\[ \theta_x(x) = N_i(x)\theta_x + N_2(x)\theta_{x2} \]
\[ \theta_y(x) = N_i(x)\theta_y + N_2(x)\theta_{y2} \]
\[ \theta_z(x) = N_i(x)\theta_z + N_2(x)\theta_{z2} \]

or in the matrix form,

\[ u = \begin{bmatrix} N_i & N_2 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \]

\[ N_i = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \]

The discretization of the strain field is given by the cinematic equations of the Timoshenko beam theory as:

\[ \varepsilon_x = \begin{bmatrix} \varepsilon_0 \\ \gamma_x \\ \gamma_y \\ \gamma_z \\ \phi_x \\ \phi_y \\ \phi_z \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} - \theta_x \\ \frac{\partial w}{\partial x} - \theta_y \\ \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial x} \\ \frac{\partial \theta_z}{\partial x} \end{bmatrix} \]

Or in a matrix form:

\[ \varepsilon_x = Lu \]
\[ u = Nd \]
\[ \varepsilon_x = LN\overrightarrow{d} = B\overrightarrow{d} \]
\[ B = LN \]

In which the intervenient matrices and vectors are:
Chapter 2

The discretization of the generalized strains is given by:

\[
L = \begin{pmatrix}
  \frac{\partial}{\partial x} & 0 & 0 & 0 & 0 \\
  0 & \frac{\partial}{\partial x} & 0 & 0 & -1 \\
  0 & 0 & \frac{\partial}{\partial x} & 0 & 1 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & \frac{\partial}{\partial x}
\end{pmatrix}
\]

(32)

\[
\mathbf{u} = \begin{bmatrix}
  u(x) \\
  v(x) \\
  w(x) \\
  \theta_x(x) \\
  \theta_y(x) \\
  \theta_z(x)
\end{bmatrix}
\]

\[
B_i = \begin{pmatrix}
  N'_i & 0 & 0 & 0 & 0 & 0 \\
  0 & N'_i & 0 & 0 & 0 & -N_i \\
  0 & 0 & N'_i & 0 & N_i & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & N'_i & 0 \\
  0 & 0 & 0 & 0 & N_i
\end{pmatrix}
\]

(33)

The discretization of the generalized strains is given by:

\[
\varepsilon_i = [B_i B_2] \begin{bmatrix}
  \frac{d_1}{d} \\
  \frac{d_2}{d}
\end{bmatrix}
\]

(34)

With the generalized strains in the Gauss point the analysis passes to the fiber level. Each fiber (integration point) is determined according to the formulation presented in Section 2.3.2. After this determination, the stiffness matrix and internal load vector of the element are computed by the common FEM equations and assembled into the structural system.

2.3.2 Integration point level

The cross section (or Gauss point in the length of the element) is divided into fibers (or integration points along the height of the element) with correspondent coordinates \((y_i, z_i)\) located in the centre of each fiber as represented in Figure 2.3-2.
Theoretical background

Figure 2.3-2: Division of the cross section into fibers under two shear forces $V_y$ and $V_z$.

The kinematic / force-based assumptions in the fiber combine the plane section theory with the assumption of the constant shear stress along the cross section, as in the same manner as presented previously for the 2D case, but now considering the coupling of the two shear forces $V_y$ and $V_z$:

$$\varepsilon_x = \varepsilon_0 + z\phi_y - y\phi_z \quad (35)$$

$$V_y^* = G_y A_y^* \Rightarrow \tau_y^* = \frac{V_y^*}{A_y^*}, \quad A_y^* = \sum A_y^{\text{shear}} \quad (36)$$

$$V_z^* = G_z A_z^* \Rightarrow \tau_z^* = \frac{V_z^*}{A_z^*}, \quad A_z^* = \sum A_z^{\text{shear}} \quad (37)$$

The shear areas $A_y^*$ and $A_z^*$ correspond to the summation of all the fibers that are shear resistant in $y$ and $z$ directions, respectively, as schematically represented in Figure 2.3-3. Transversal reinforcement is considered smeared in two directions also, $\rho_{yw}$ and $\rho_{zw}$, as represented in Figure 2.3-4.

Figure 2.3-3: Shear resistant areas in the two directions $A_y^*$ and $A_z^*$
Figure 2.3-4: Smeared transversal reinforcement in the two directions $\rho_{y}^{sw}$ and $\rho_{z}^{sw}$

The state determination of the fiber has the following inputs and outputs, in terms of strains, stresses and stiffness:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_y \\
\tau_z \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_y \\
\gamma_z \\
\end{bmatrix}
\]

and the strain-stress state in the fiber (concrete and smeared stirrups) (Figure 2.3-5) can be described as (all the formulation is incremental, but the sign $\Delta$ is not used here for the sake of simplicity):

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_y \\
\tau_z \\
\end{bmatrix} =
\begin{bmatrix}
D_{11} & D_{12} & D_{13} & D_{14} & D_{15} \\
D_{21} & D_{22} & D_{23} & D_{24} & D_{25} \\
D_{31} & D_{32} & D_{33} & D_{34} & D_{35} \\
D_{41} & D_{42} & D_{43} & D_{44} & D_{45} \\
D_{51} & D_{52} & D_{53} & D_{54} & D_{55} \\
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_y \\
\gamma_z \\
\end{bmatrix}
\]

In which the contribution of the stirrups is given in the $y$ and $z$ direction as (considered orthogonal in this initial formulation, which can be extrapolated to generally inclined reinforcement):

\[
\bar{D}_{22} = D_{22} + \rho_{y}^{sw}E_{y}^{sw}
\]
\[
\bar{D}_{33} = D_{33} + \rho_{z}^{sw}E_{z}^{sw}
\]
The concrete state (stresses and stiffness) are determined in principal directions and a 3D constitutive law is required. In the principal directions, the 3D strain-stress state is written as \((\gamma_{12}=0)\) (assuming Poisson null):

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{13} \\
\tau_{23}
\end{bmatrix} = D_{12}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{13} \\
\gamma_{23}
\end{bmatrix}
\]

where \(E_1, E_2\) and \(E_3\) are the stiffness modulus in each principal direction and \(G_{13}\) and \(G_{23}\) are the transversal modulus. The referential transformations are performed through the rotation matrices \(T_\varepsilon\) and \(T_\sigma\) by means of the directional cosines:

\[
\varepsilon_{\nu} = T_\nu^T \varepsilon_{12} \quad ; \quad \varepsilon_{12} = T_\varepsilon \varepsilon_{\nu}
\]

\[
\sigma_{\nu} = T_\nu^T \sigma_{12} \quad ; \quad \sigma_{12} = T_\sigma \sigma_{\nu}
\]

\[
D_v = T_\varepsilon^T D_{12} T_\sigma
\]

The shear modulus is determined accordingly to the requirement that the angles of the principal directions of the stresses and strains are the same (Bazant 1983):

\[
G_{13} = \frac{\sigma_1 - \sigma_3}{2(\varepsilon_1 - \varepsilon_3)} \quad (45)
\]

\[
G_{23} = \frac{\sigma_2 - \sigma_3}{2(\varepsilon_2 - \varepsilon_3)} \quad (46)
\]

The concrete material stiffness matrix \(D_v\) in the orthogonal directions is given by:

\[
D_v = \begin{bmatrix}
D_{11} & D_{12} & D_{13} & D_{14} & D_{15} \\
D_{21} & D_{22} & D_{23} & D_{24} & D_{25} \\
D_{31} & D_{32} & D_{33} & D_{34} & D_{35} \\
D_{41} & D_{42} & D_{43} & D_{44} & D_{45} \\
D_{51} & D_{52} & D_{53} & D_{54} & D_{55}
\end{bmatrix}
\]

\[
(47)
\]
Four conditions are imposed in the state determination of the fiber:

1) Equilibrium in y-direction:

\[ \sum \sigma_y = 0 \quad ; \quad \sigma_y^e + \rho_{yw} \sigma_y^{sw} = 0 \]

\[ \sigma_y^e = D_{24} \varepsilon_y + D_{25} \varepsilon_z + D_{24} \gamma_y + D_{25} \gamma_z \]

\[ \sigma_y^{sw} = E_y \varepsilon_y \]

(48)

2) Equilibrium in z-direction:

\[ \sum \sigma_z = 0 \quad ; \quad \sigma_z^e + \rho_{zw} \sigma_z^{sw} = 0 \]

\[ \sigma_z^e = D_{34} \varepsilon_y + D_{35} \varepsilon_z + D_{34} \gamma_y + D_{35} \gamma_z \]

\[ \sigma_z^{sw} = E_z \varepsilon_z \]

(49)

3) Imposed condition for shear stress \( \tau_{yz} \):

\[ \tau_{yz}^* = \tau_{yz}^e \]

(50)

\[ \tau_{yz} = D_{41} \varepsilon_x + D_{42} \varepsilon_y + D_{43} \varepsilon_z + D_{44} \gamma_y + D_{45} \gamma_z \]

4) Imposed condition for shear stress \( \tau_{xz} \):

\[ \tau_{xz}^* = \tau_{xz}^e \]

(51)

\[ \tau_{xz} = D_{51} \varepsilon_x + D_{52} \varepsilon_y + D_{53} \varepsilon_z + D_{54} \gamma_y + D_{55} \gamma_z \]

These conditions give a system of linear equations with 4 equations and 4 unknowns:

- Unknowns to determine:

\[ (\varepsilon_y, \varepsilon_z, \gamma_y, \gamma_z) \]

(52)

- Parameters known:

\[ (\varepsilon_x, \tau_{xy}^*, \tau_{xz}^*, [D]) \]

(53)

So, the unknowns are determined as:

\[
\begin{bmatrix}
\varepsilon_y \\
\varepsilon_z \\
\gamma_y \\
\gamma_z
\end{bmatrix} = f \left( \varepsilon_x, \tau_{xy}^*, \tau_{xz}^*, [D] \right)
\]

(54)
The systems of equations can be written as:

\begin{align}
a) \quad & D_{21} \varepsilon_x + D_{22} \varepsilon_y + D_{23} \varepsilon_z + D_{24} \gamma_y + D_{25} \gamma_z = 0 \\
b) \quad & D_{31} \varepsilon_x + D_{32} \varepsilon_y + D_{33} \varepsilon_z + D_{34} \gamma_y + D_{35} \gamma_z = 0 \\
c) \quad & D_{41} \varepsilon_x + D_{42} \varepsilon_y + D_{43} \varepsilon_z + D_{44} \gamma_y + D_{45} \gamma_z = \tau_{xy} \\
d) \quad & D_{51} \varepsilon_x + D_{52} \varepsilon_y + D_{53} \varepsilon_z + D_{54} \gamma_y + D_{55} \gamma_z = \tau_{xz}
\end{align}

and organized in an AX=B system (where X are the unknowns) as:

\[
\begin{bmatrix}
D_{22} & D_{23} & D_{24} & D_{25} \\
D_{32} & D_{33} & D_{34} & D_{35} \\
D_{42} & D_{43} & D_{44} & D_{45} \\
D_{52} & D_{53} & D_{54} & D_{55}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_y \\
\varepsilon_z \\
\gamma_y \\
\gamma_z
\end{bmatrix}
= 
\begin{bmatrix}
-D_{21} \varepsilon_x \\
-D_{31} \varepsilon_x \\
\tau_{xy} - D_{41} \varepsilon_x \\
\tau_{xz} - D_{51} \varepsilon_x
\end{bmatrix}
\] (56)

and solved numerically by Gauss elimination method and back substitution.

To comply with all the four conditions, an iterative procedure is needed. The unbalanced stresses, in each iteration, are given as:

1) \( \delta \sigma_y = \sigma_y - \rho_{yw} \sigma_{yw} \)
2) \( \delta \sigma_z = \sigma_z - \rho_{zw} \sigma_{zw} \)
3) \( \delta \tau_{xy} = \tau_{xy} - \tau_{xy}^* \)
4) \( \delta \tau_{xz} = \tau_{xz}^* - \tau_{xz} \)

The axial strain \( \varepsilon_x \) remains constant during this iteration procedure; it is not corrected. The correction of the strains \( (\varepsilon_y, \varepsilon_z, \gamma_y, \gamma_z) \) is performed until the four unbalanced stresses vanish \( (\delta \sigma_y, \delta \sigma_z, \delta \tau_{xy}, \delta \tau_{xz}) \) as:

\[
\begin{aligned}
\varepsilon_y &= \varepsilon_y + \delta \varepsilon_y \\
\varepsilon_z &= \varepsilon_z + \delta \varepsilon_z \\
\gamma_y &= \gamma_y + \delta \gamma_y \\
\gamma_z &= \gamma_z + \delta \gamma_z
\end{aligned}
\] (58)

The strain corrections are found through the strain-stress relationships of the fiber, in which the components correspondent to the axial strain and stress are not included as they are maintained constant:

\[
\begin{bmatrix}
\delta \sigma_y \\
\delta \sigma_z \\
\delta \tau_{xy} \\
\delta \tau_{xz}
\end{bmatrix}
= 
\begin{bmatrix}
D_{22} & D_{23} & D_{24} & D_{25} \\
D_{32} & D_{33} & D_{34} & D_{35} \\
D_{42} & D_{43} & D_{44} & D_{45} \\
D_{52} & D_{53} & D_{54} & D_{55}
\end{bmatrix}
\begin{bmatrix}
\delta \varepsilon_y \\
\delta \varepsilon_z \\
\delta \gamma_y \\
\delta \gamma_z
\end{bmatrix}
\] (59)

There is again a system of four equation and four knowns that can be written in a system as AX=B:
and solved numerically through Gauss elimination method also. Convergence is checked by comparing the norm of residual forces with the norm of the total applied forces. A tolerance rate of 1% is considered. This is checked for the four requirements:

i. Equilibrium in the y-direction: convergence rate \( \lambda_{y'} \) is the ratio between the increment of stresses in relation to the past iteration \( \Delta \sigma_y \) and the total stress \( \sigma_y' \)

\[
\lambda_{y'} = \frac{\Delta \sigma_y}{\sigma_y'} < 0.01
\]

ii. Equilibrium in the z-direction: convergence rate \( \lambda_{z'} \) is the ratio between the increment of stresses in relation to the past iteration \( \Delta \sigma_z \) and the total stress \( \sigma_z' \)

\[
\lambda_{z'} = \frac{\Delta \sigma_z}{\sigma_z'} < 0.01
\]

iii. Equality between shear stresses \( V_y \): convergence rate \( \lambda_{xy} \) is the ratio between the shear stress outputted in the fiber \( \tau_{xy} \) and the imposed shear stress \( \tau_{xy'} \)

\[
\lambda_{xy} = \frac{\tau_{xy}}{\tau_{xy'}} < 0.01
\]

iv. Equality between shear stresses \( V_z \): convergence rate \( \lambda_{xz} \) is the ratio between the shear stress outputted in the fiber \( \tau_{xz} \) and the imposed shear stress \( \tau^* \)

\[
\lambda_{xz} = \frac{\tau_{xz}}{\tau^*} < 0.01
\]

After completion of the fiber determination a static condensation is performed in order to eliminate \( \epsilon_y \) and \( \epsilon_z \) that are not included in the element formulation.
3 IMPLEMENTATION IN DIANA SOURCE CODE

3.1 Overview

The model described previously is implemented in the development version of DIANA 10.

Key general notes about this implementation are listed in the following:

- 2-noded Timoshenko beam elements 2D (L6BEA) and 3D (L12BEA) with one integration point (Gauss point) along the beam axis were implemented in DIANA.

- The fiber integration format was implemented in a general manner; not necessarily linked to the L6BEA and L12BEA elements. The shear-bending state determination in each integration point (fiber), which leads to the multiaxial determination of strains and stresses in concrete including the effects of transversal reinforcement, is available only in the elements L6BEA and L12BEA.

- At the moment, it is not possible to consider the shear distribution in a phased analysis of the cross section. This needs to be solved in the future in order to allow for simulation of strengthening measures. Also, phased analysis for the stirrups is not yet available, which will require the consideration of different types of stirrups in a cross section (in terms of material properties and quantities).

- The activation of the shear formulation is given by a material type in 'MATERI' definition in which the following parameters are given by the user. The meaning and definition of the input parameters are explained in detail in the next chapter.

  STIRRU.d(13) = "Stirrup data", (vertical area, horizontal area, horizontal transversal spacing, vertical transversal spacing, longitudinal spacing, Young’s modulus, yielding stress, ultimate stress, ultimate strain, lower vertical limit, upper vertical limit, lower horizontal limit, upper horizontal limit)

- For beams without shear reinforcement, a STIRRU definition is still needed in order to activate the shear effects; in this case, the same iterative formulation is applied in the fiber considering null quantity of transversal reinforcement.

- The choice for the fiber integration is made in 'DATA', for example:
  THINTE 11
  NUMINT GAUSS FIBER

- Stirrups may only be vertical and have one configuration in the cross-section; but this can be extended in the future.

- This model is associated with Total Strain Rotating Crack Models for the concrete. Specific material functions used in the original version of the model were implemented for direct verification (Hognestad parabola and Cervenka tension stiffening curve); however, its use is not limited to these functions, any other predefined curves available in DIANA can be used.

- The set of shear- and non-shear resistant fibers is made through the vertical and horizontal limits defined in the properties of the stirrups. Zones can also be used to define shear resistant & non shear resistant areas by setting different materials to each zone; DIANA does not allow for different integrations points in the different zones. These definitions are explained in detail in the next chapter.

- No bond slip model is associated with the transversal reinforcement.
- Plasticity of transversal reinforcement is considered locally in the code. Yielding and failure of stirrups considered by means of an elastic-perfect plastic function. Extending the plasticity of the stirrups to a more generic feature is a consideration for the future.

- The fiber state determination is implemented in the loop of each integration point along the height of the cross section correspondent to the Gauss point in the middle to the axis of the element in DIANA, adapting the existing subroutine.

- The new implementation is within the general flow of DIANA, so all types of analysis procedures available can be used with this model.

- One of the key aspects of the new implementation is the fact that DIANA beam elements only allow for vertical cracks, which had to be changed to accommodate the present model. The 2D fiber beam element can be compared to a membrane-like behaviour and the 3D fiber beam element to a near solid-like behaviour without the interaction of the torsion rotation (this last one needs to be solved in the code as it is implemented in a very provisional way).

- DIANA does not compute plastic strains in compression, as CONSHEAR does, and this might be the reason for the slight differences observed in the preliminary tests.

- Longitudinal reinforcement bars in DIANA are compatible with the new formulation.

- By default DIANA assumes a shear stress correction factor of 1.2; this parameter was here set to 1.

- The shear modulus G is computed in the principal directions assuming equal angles for strains and stresses, in accordance to the proposal of (Bazant 1983). The shear retention factor is not used.

- The constant shear stress \( V_{\text{fixed}} = G_{\text{eq}} \gamma_{\text{elem}} \) is assumed in each fiber without the use of the sectional concept.

The changes in DIANA in order to implement the new model are represented in a global view in the flowchart of Figure 3.1-1 and within the incremental-iterative solution procedure in Figure 3.1-2.
Figure 3.1-1: DIANA model. Global flowchart for incremental – iterative solution procedure and changes (in red)

Figure 3.1-2: DIANA model. Flowchart for iterative solution procedure and changes (in red)
3.2 2D beam element

3.2.1 Implementation in the 2-noded Timoshenko 2D FE (L6BEA)

The fiber state determination is implemented in L6BEA element: straight 2-node, 2-dimensional class-III beam element, with linear interpolation functions and 1-point Gauss integration scheme along the bar axis.

![Figure 3.2-1: DIANA manual. L6BEA](image)

The displacement field is a function of two displacements, axial $u$ and vertical $w$, and a rotation $\theta$. The generalized strains in the cross section are the axial strain $\varepsilon_0$, the shear rotation $\gamma_0$ and the curvature $\phi$: 

$$
\varepsilon_0 = \frac{\partial u}{\partial x} \\
\gamma_0 = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} + \theta \\
\phi = \frac{\partial \theta}{\partial x}
$$

The kinematic equations that relate the nodal displacements $a_j=[u_j, w_j, \theta_j]^T$ on each node $j$ to the generalized sectional strains in the Gauss points $\varepsilon_0 = [\varepsilon_0, \gamma_0, \phi]^T$ are given by the transformation matrix $B$ as:

$$
\begin{pmatrix}
\varepsilon_0 \\
\gamma_0 \\
\phi
\end{pmatrix} =
\begin{bmatrix}
B_1 & B_2
\end{bmatrix}
\begin{pmatrix}
a_1 \\
a_2
\end{pmatrix} ;
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} =
\begin{pmatrix}
\frac{\partial N_i}{\partial x} & 0 & 0 \\
0 & \frac{\partial N_i}{\partial x} & N_i \\
0 & 0 & \frac{\partial N_i}{\partial x}
\end{pmatrix}
$$

The relationship between the generalized strains $\varepsilon_0$ determined on the axis of the FE and the strains on each fiber $\varepsilon_{fiber}= [\varepsilon_s, \gamma_{sc}]^T$ is given by the transformation matrix $T$ as:

$$
\begin{pmatrix}
\varepsilon_s \\
\gamma_{sc}
\end{pmatrix}_{fiber} = T
\begin{pmatrix}
\varepsilon_0 \\
\gamma_0 \\
\phi
\end{pmatrix} ;
T =
\begin{bmatrix}
1 & 0 & z \\
0 & 1 & 0
\end{bmatrix}
$$

3.2.2 Algorithm for the fiber state determination

The algorithm for the fiber state determination, described in the following, is implemented in subroutine `ynclbe.f`; each fiber composed by concrete and smeared stirrups is determined in the general loop through the integration points in DIANA.

Changes to allow for rotating cracks in the beams were made in the subroutines of the directory `totstr`. 
The key aspects of the implementation are:
- The strain-stress treatment of the integration points in beam elements, that was only in the longitudinal direction ($\varepsilon_x, \sigma_x$) (Figure 3.2-2) was changed to accommodate a 2D strain-stress state like a membrane ($\varepsilon_x, \varepsilon_z, \gamma_{xz}, \sigma_x, \sigma_z, \tau_{xz}$) (Figure 3.2-3) and rotating cracks.
- Each fiber has a full stiffness matrix (3x3), instead of only the standard $E$ modulus.
- Condensation of the remaining degree of freedom ($\varepsilon_z$) is a standard procedure in DIANA, so no changes were required here.
- The stiffness of the stirrups was added to the concrete stiffness in the correspondent direction.
- The assumed constant shear stress along the height of the cross section (Figure 3.2-4) is considered by setting a constant $\tau^*_{xz}$ for all the fibers in the mid-length Gauss point of each element, determined as $V^* = G_{eq, \gamma_{xz,ele}}$. The equivalent transversal modulus, $G_{eq}$, is the average shear modulus in a cross section.

![Figure 3.2-2: Stress state in the fiber in a flexural beam model](image1)

![Figure 3.2-3: Stress state in the fiber in the shear beam model](image2)

![Figure 3.2-4: Shear fiber beam model assumptions](image3)
The flow of the algorithm for fiber state determination involves the following steps:

1. Determine axial strain and imposed shear stresses through the generalized strains and shear forces $(\varepsilon_x, \tau_{xz})$

2. Determine first trial of the strain tensor of the fiber: $\varepsilon = (\varepsilon_x, \varepsilon_z, \gamma_{xz})^T$

3. Compute updated stress tensor and stiffness matrix (to store): $\sigma = (\sigma_x, \sigma_z, \tau_{xz})^T$ & $[D]$ 
   Intermediate steps:
   - Transform strains to principal directions;
   - Determine stresses and stiffnesses in principal directions;
   - Transform back stresses and stiffness matrices to orthogonal directions.

4. Check imposed equations:
   i. $\sum \sigma_z = 0$
   ii. $\tau_{xz} = \tau_{xz}^*$

5. Compute unbalanced stresses: $\delta \sigma = (\delta \sigma_x, \delta \sigma_z, \delta \tau_{xz})^T$

6. Check if unbalanced stresses are small enough (comparing norms as NR):
   i. $\delta \sigma_x \approx 0$ (?)
   ii. $\delta \tau_{xz} \approx 0$ (?)
   - If convergence is achieved (all the four imposed equations are fulfilled) go to 9
   - If convergence is not achieved (continue)

7. Compute unbalanced strains: $\delta \varepsilon = (\delta \varepsilon_x, \delta \gamma_{xz})^T$

8. Update strain tensor in the fiber: $\varepsilon = \varepsilon + \delta \varepsilon$ and go to step 3

9. Perform static condensation of the fiber and storage.

### 3.3 3D beam element

#### 3.3.1 Implementation in the 2-noded Timoshenko 3D FE (L12BEA)

The fiber state determination is implemented in L12BEA element: straight 2-node, 3-dimensional class-III beam element, with linear interpolation functions and 1-point Gauss integration scheme along the bar axis.

![Diagram](Figure 3.3-1: DIANA manual. L12BEA)

The generalized strains are determined in a Gauss point through the displacement fields adopting the Timoshenko beam element formulation, as presented in Section 2.3. The generalized strains in Gauss point of a 3D beam are:
Implementation in DIANA source code

one axial strain \((\epsilon_0)\), two shear strains \((\gamma_y, \gamma_z)\), two bending curvatures \((\phi_y, \phi_z)\) and one torsion curvature.

The relationship between the generalized strains \(\mathbf{\epsilon}_0\) determined on the axis of the FE and the strains on each fiber \(\mathbf{\epsilon}_{\text{fiber}}=[\epsilon_x, \gamma_{xy}, \gamma_{xz}]^T\) is given by the transformation matrix \(T\) as:

\[
\begin{pmatrix}
\epsilon_x \\
\gamma_{xy} \\
\gamma_{xz}/_{\text{fiber}}
\end{pmatrix}
= T
\begin{pmatrix}
\epsilon_0 \\
\gamma_y \\
\gamma_z
\end{pmatrix}; \quad T = \begin{bmatrix}
1 & 0 & 0 & z & y \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

3.3.2 Algorithm for the fiber state determination

The algorithm for the fiber state determination, described in the following, is implemented in subroutine \textit{yncl3d.f}. Each fiber composed by concrete and smeared stirrups is determined in the general loop through the integration points in DIANA.

Changes to allow for rotating cracks in the beams were made in the subroutines of the directory \textit{totstr}.

The key aspects of the implementation are:
- The shear resistant fibers are under a multiaxial strain-stress state \((\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{xz}, \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz})\).
- Each fiber has a full stiffness matrix (5x5), instead of only the standard \(E\) modulus.
- Condensation of the remaining degrees of freedom \((\epsilon_y, \epsilon_z)\) is a standard procedure in DIANA, so no changes were required here.
- The stiffness of the stirrups is added to the concrete stiffness in the correspondent directions \((y, z)\).
- The assumed constant shear stresses are considered by setting constant \(\tau^*_{xy}\) and \(\tau^*_{xz}\) for the correspondent shear resistant fibers in the mid-length Gauss point of each element, determined respectively as \(V_{y}^* = G_{\text{eq}}\gamma_{xy,\text{ele}}\) and \(V_{z}^* = G_{\text{eq}}\gamma_{xz,\text{ele}}\).
- The equivalent transversal modulus, \(G_{\text{eq}}\), are the average shear modulus in a cross section.

The flow of the algorithm for fiber state determination involves the following steps:
1. Determine axial strain and imposed shear stresses through the generalized strains and shear forces \(\left(\epsilon_x, \tau_{xy}, \tau_{xz}\right)\).
2. Determine first trial of the strain tensor of the fiber: \(\mathbf{\epsilon} = \left(\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{xz}\right)^T\).
3. Compute updated stress tensor and stiffness matrix (to store): \( \sigma = (\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz})^T \) & \( D \)

Intermediate steps:
- Transform strains to principal directions;
- Determine stresses and stiffnesses in principal directions;
- Transform back stresses and stiffness matrices to orthogonal directions;

4. Check imposed equations:
   i. \( \sum \sigma_y = 0 \)
   ii. \( \sum \sigma_z = 0 \)
   iii. \( \tau_{xy} = \tau_{xy}^* \)
   iv. \( \tau_{xz} = \tau_{xz}^* \)

5. Compute unbalanced stresses: \( \delta \sigma = (\delta \sigma_y, \delta \sigma_z, \delta \tau_{xy}, \delta \tau_{xz})^T \)

6. Check if unbalanced stresses are small enough (comparing norms as NR):
   i. \( \delta \sigma_y \approx 0 \) (?)
   ii. \( \delta \sigma_z \approx 0 \) (?)
   iii. \( \delta \tau_{xy} \approx 0 \) (?)
   iv. \( \delta \tau_{xz} \approx 0 \) (?)
   - If convergence is achieved (all the four imposed equations are fulfilled) go to 9
   - If convergence is not achieved (continue)

7. Compute unbalanced strains: \( \delta \epsilon = (\delta \epsilon_x, \delta \epsilon_y, \delta \gamma_{xy}, \delta \gamma_{xz})^T \)

8. Update strain tensor in the fiber: \( \epsilon = \epsilon + \delta \epsilon \) and go to step 3

9. Perform static condensation of the fiber and storage.

### 3.4 Material model

A Total Strain Rotating crack model is used. The constitutive equations used in the PhD work were included in DIANA, for direct comparison while testing the implementation. The shear implementation can be associated to any other constitutive function for tension and compression behaviour of concrete predefined in DIANA.

The constitutive curves used in the implementation are:
- The Hognestad parabola (Hognestad, Hanson and McHenry 1955) for the concrete in compression:
  \[
  \sigma_z = f_p \left( 2 \left( \frac{\epsilon_z}{\epsilon_p} \right) - \left( \frac{\epsilon_z}{\epsilon_p} \right)^2 \right)
  \]
  where \( \epsilon_p \) is the strain at the peak stress \( f_p \).
For concrete in tension, before onset of cracking concrete performs with a linear-elastic behaviour.

After cracking tensile softening is represented by the Cervenka 1985 curve (slightly changed to avoid the abrupt jump down):

\[
\sigma_1 = f_t \left[ 1 - \left( \frac{\varepsilon_0 - \varepsilon_{cr}}{c - \varepsilon_{cr}} \right) \right] \quad \varepsilon_0 \geq \varepsilon_{cr}
\]

where \(c\) is the strain at which principal stress goes to zero (taken as the yielding strain of steel, 0.002 and \(k_2\) is a parameter that defines the shape of the softening curve considered as 0.5 by default); \(\varepsilon_{cr}\) is the strain that corresponds to the peak tensile stress \(f_t\).

- Longitudinal steel is modelled with Von Mises plasticity and hardening models for embedded reinforcements.
4 IDEAS FOR PRE- AND POST-PROCESSING

4.1 Overview

The fiber beam element requires the definition of new input parameters related with:
- Geometry properties of the transversal reinforcement;
- Material properties of the transversal reinforcement;
- Division of the cross section into fibers;
- Definition of the shear resistant and non-shear resistant areas in the cross section.

These elements generate an output that is not the typically produced by beam elements:
- 2D beam elements generate a membrane-like output ($\varepsilon_x, \varepsilon_y, \gamma_{xz}, \sigma_x, \sigma_y, \tau_{xz}$);
- 3D shear beam element generates a solid-like output ($\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}, \sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}$);
- Cracks are able to rotate and principal strains and stresses present different angles with damage;
- Strains and stresses in the transversal reinforcement.
- Critical fibers may no longer be located in the extremities (as in the case of bending problems), it is important to know the strain and stress state in all the fibers along the height of the cross section for the 2D beams, and also along the width for the case of the 3D beams.

These aspects ask for new developments on the pre and post-processing features in DianaIE in order to accommodate all the potentialities of the new model.

4.2 Input

Activation of the shear formulation is related to the material properties (STIRRU); by assigning a concrete material with STIRRU definition to an element (only possible with L12BEA and L6BEA)

4.2.1 Geometric properties of the transversal reinforcement

Information for stirrups geometry definition (STIRRU) is (Figure 4.2-1):

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTELV</td>
<td>Area of the vertical transversal reinforcement</td>
</tr>
<tr>
<td>ASTELH</td>
<td>Area of the horizontal transversal reinforcement</td>
</tr>
<tr>
<td>HOSPA</td>
<td>Spacing in the horizontal direction</td>
</tr>
<tr>
<td>VESPA</td>
<td>Spacing in the vertical direction</td>
</tr>
<tr>
<td>LOSPA</td>
<td>Longitudinal spacing</td>
</tr>
</tbody>
</table>

This information allows computing the ratio of transversal reinforcement smeared in the concrete fibers.

For the 2D beam the required information can be limited to:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTELV</td>
<td>Area of the vertical transversal reinforcement</td>
</tr>
<tr>
<td>HOSPA</td>
<td>Spacing in the horizontal direction</td>
</tr>
<tr>
<td>LOSPA</td>
<td>Longitudinal spacing</td>
</tr>
</tbody>
</table>

The number of input parameters asked to the user can be adapted to the type beam element (2D or 3D beam).
4.2.2 Material properties of the transversal reinforcement

Information for stirrups material definition (STIRRU) is (Figure 4.2-2):

- **YOUNGS**  Young’s modulus, $E_s$
- **YIELDS**  Yielding stress, $f_{sy}$
- **ULTIMS**  Ultimate stress, $f_{su}$
- **ULTIME**  Ultimate strain, $\varepsilon_{su}$

![Figure 4.2-2: Material definition of the stirrups](image)

For beams without shear reinforcement, a STIRRU definition is still needed in order to activate the shear effects; in this case, null quantity of transversal reinforcement is considered in the fiber determination.

Different quantities or configurations of transversal reinforcement in a beam can be reproduced by assigning different material properties with respective STIRRU definitions to elements.
4.2.3 Definition of shear resistant areas of the cross section

Remarks about the division of the cross section into shear and non-shear resistant areas are listed in the following:

- Considering only the web of the cross section is a conservative and easier procedure that can be used for the majority of cases;
- If more realistic computations are aimed, for T- and I-shaped cross sections, parts of the flanges contribute to shear resistance (so called, effective width of the flange for shear), accounted for through an equivalent area as represented in Figure 4.2-3;
- The effective flange width $b_{ef}$ can be calculated through empirical expressions present in literature (e.g. Zararis 2006);
- The definition of these parts of the flange as shear and non-shear resistant is made by limitations given by the user;
- Zones definition can be used. In this case, the user can also set a material without stirrups to the zone that is non shear resistant, instead of using these limitations.

Limits for setting shear resistant & non-shear resistant areas (STIRRU):

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>VLLIM</td>
<td>Lower vertical limit</td>
</tr>
<tr>
<td>VULIM</td>
<td>Upper vertical limit</td>
</tr>
<tr>
<td>HLLIM</td>
<td>Lower horizontal limit</td>
</tr>
<tr>
<td>HULIM</td>
<td>Upper horizontal limit</td>
</tr>
</tbody>
</table>

*Figure 4.2-3: Definition of shear resistant areas of the cross section*
Example of material definition with STIRRU:

`MATERI`
1 NAME "Concrete"
MCNAME CONCR
MATMDL TSCR
YOUNG 2.570E+10
POISON 0.00
DENSIT 2.50E+03
TOTCRK ROTATE
TENCVR CERVEN
KPOWER 5.00E-01
TENSTR 2.00E+06
EPSULT 2.00E-03
COMCRV HOGNES
COMSTR 2.770E+07
STIRRU 0.000 0.0 0.3 0.5 0.2 2.E+11 5.E+8 5.E+8 0.05 -.25 +.250 -1.5 +1.5
SHEAR 1.0

4.2.4 Division of the cross section into fibers

The following points must be considered:
- The cross section is divided into a number of fibers defined in THINTE.
- For the 2D beams THINTE represent the number of divisions along the height.
- For the 3D beams THINTE represent the number of divisions along the height and the width.

The choice for the fiber integration is made in 'DATA':

`DATA`
1 NAME "BEAM"
THINTE 50
NUMINT GAUSS FIBER

4.2.5 D-regions

In D-regions of frame elements, the shear effects should not be activated. The D-region (i.e., the locations of the model were shear effects are not activated and the elements act as standard beams) are considered at a distance ranging from 1d - 2d of localized disturbed points, such as load points and supports. This represents the areas where beam formulations (suitable for B-regions) are not applied. The user can do this, by not activating the shear effects in the elements in these locations. This must be explained in the manual.
4.3 Ideas for pre-processing in DianaIE

Some ideas for pre-processing treatment of this model in Diana Interactive Environment are listed in the following:

- In the manual, state that this model is only available for L12BEA and L6BEA and make the following options only available for these elements.

- Set an option to activate the shear effects in the definition of the geometry of the cross section. If this option is activated, the geometric properties of the stirrups are asked, as well as, the limitations for the locations of the shear resistant fibers.

- In case of no transversal reinforcement, this option still needs to be activated, but with zero area of transversal reinforcement (it can have an extra option for no transversal reinforcement).

- Different quantities or configurations of transversal reinforcement in a beam can be reproduced by assigning different material properties with respective STIRRU to elements.

- The transversal reinforcement (that is treated as smeared in the computations) could be represented as individual rebars along the beam in the pre-processing interface.

- Explain in the manual that in D-regions of frame elements, this option should not be activated. (at a distance of aprox. 1d - 2d of localized disturbed points, such as load points and supports). The user can do this, by not activating the shear effects in the elements in these locations. I think this will be difficult to put in an automatic manner.

- About the limitations of shear resistant and non shear resistant fibers, we can say in the manual that considering only the web of the cross section is conservative; if more realistic computations are aimed, we can propose some formulas to calculate the web widths that collaborate to shear resistance. I think is very difficult to put this in an automatic way.

- For analysis of strengthened structures, need to define n STIRRU configurations in a concrete material; code needs to be adapted to accommodate this.

The following pictures represent ideas for pre-processing commands in DianaIE for inclusion of this model.
Shear effects / smeared transversal reinforcement (option only available for L6BEA & L12BEA)

Add one more option: If user chose Shear effects has to fill information about smeared transversal reinforcement.

Ask for:
Geometry: ASTELV, ASTELH, HOSPA, VESPA, LOSPA
Material: YOUNGS, YIELDS, ULTIMS, ULTIME

Add one more option: If user chose Shear effects / smeared transversal reinforcement has to fill information about shear resistant areas of the cross section.

Ask for:
VLLIM, VULIM, HLLIM, HULIM
4.4 Output

The results given by the shear fiber beam elements are listed in the following.

- Strains and stresses in concrete for all the fibers:
  - 2D beams - $\varepsilon_x$, $\varepsilon_y$, $\gamma_{xz}$, $\sigma_x$, $\sigma_y$, $\tau_{xz}$
  - 3D beams - $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$, $\gamma_{xy}$, $\gamma_{xz}$, $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{xz}$
- Principal strains and stresses in concrete for all the fibers:
  - 2D beams - $\varepsilon_1$, $\varepsilon_2$, $\gamma_{12}$, $\sigma_1$, $\sigma_2$, $\tau_{12}$
  - 3D beams - $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, $\gamma_{12}$, $\gamma_{13}$, $\sigma_1$, $\sigma_2$, $\sigma_3$, $\tau_{12}$, $\tau_{13}$
- Cracking with the correspondent principal directions
- Strains and stresses in the transversal reinforcement:
  - 2D beams - $\varepsilon_z$, $\sigma_z$
  - 3D beams - $\varepsilon_y$, $\sigma_y$, $\sigma_z$
The output for these elements in DianaIE should include them; some suggestions for accomplish this are listed in the following:

- For the 2D beams a colour map representation like a membrane could be done, showing the strains and stresses in all fibers along the height of the cross section and along the longitude of the beams.

- For the 3D beam the colour map could like a solid, showing the strains and stresses in all fibers along the height and width of the cross section and along the longitude of the beams.

- The transversal reinforcement (that is treated as smeared in the computations) could be represented as individual rebars along the beam, as well as, in the cross-section. Strains and stresses in the stirrups could be visualized as in the same manner as embedded reinforcements.

- Clipping planes to see results inside: Cross section results of strains and stresses could be given by cuts in the beam in the positions chosen by the user. Here the strains and stresses in the stirrups (in the cross section) could also be represented in individual rebars in the positions defined by the user.

- For tabular results, the user must be able to select a particular fiber and element to ask results related with concrete (strains, stresses, cracking, etc.) and with transversal reinforcement (strains, stresses, plasticity).

![Figure 4.4-1: Suggestion for output visualization of the model in DianaIE](image)
5 RECALCULATION OF BENCHMARKS WITH DIANA

The benchmarks were previously analysed with the original code CONSHEAR and reported in Report #1 of this project.

The benchmarks were analysed with the model implemented in Diana10 and compared with experimental data and other computation results:

- 2D plane stress from (RTD1016b, Hendriks et al. 2015)
- CONSHEAR (Report #1, Ferreira 2015).
- Fiber beam elements in DIANA10 with bending behaviour only (without STIRRU definition).

The meshes and characteristics of the model were maintained as similar as possible to the models made with CONSHEAR, for direct comparison.

The material models were the same in the analysis according to the characteristics presented in the previous chapters:

Concrete:
- Total strain crack model
- Rotating crack orientation
- Tension: Cervenka curve
- Compression: Hognestad parabola

Steel:
- Class: reinforcement and pile foundations
- Material model: Von Mises plasticity
- Plastic hardening: yield stress-plastic strain

5.1 Case RB1: Vecchio & Shim (2004)

The series of beams tested by (Vecchio and Shim 2004) in Toronto were a reproduction of the experiments by (Bresler and Scordelis 1963). In total, four series of three beams with rectangular cross sections subjected to point loads were tested. They differed from each other in the amount of shear reinforcement, span length, cross-section dimensions and concrete compressive strength. The measured experimental data were the applied load and the displacement at mid-span.

From this set of experiments, specimen C3 is selected as this beam has the longest span (6400 mm) and a flexure-compressive failure mechanism.

5.1.1 Experimental setup and results

The beam has a total length of 6.840 m, a depth of 0.552 m, and a width of 0.152 m. The characteristics of the beam in terms of geometry, reinforcement, loading, boundary conditions and experimental setup are presented in Figure 5.1-1. The bottom longitudinal reinforcement is extended outside the beam and welded to thick plates.

The beam exhibited a flexural-compressive failure mode (Figure 5.1-2) with a clear maximum in the load-deflection response. The experimental ultimate value of applied load was equal to $P_{\text{EXP}} = 265$ kN at a deflection of 44.3 mm.
Recalculations of benchmarks with DIANA

5.1.2 Beam finite element model in DIANA 10

The characteristics of the model are presented in Figure 5.1-3; the beam was discretized into 32 3D beam FEs (L6BEA). The cross section was divided into 41 fibers; steel filaments were simulated according to their positions in the beam (3xM10, 2xM25 and 2xM30) and shear reinforcement was considered smeared in the shear resistant fibers ($\rho_{sw}=0.2\%$). Apart from the concrete cover, all the fibers were considered shear resistant.

Regarding the material properties, the values given in the paper for the concrete and steel mechanical properties were used in the model as resumed in Table 5.1-1 for concrete and in Table 5.1-2 for steel.

Load (P) was applied as a nodal force, with automatic increments controlled by an arc-length procedure. An energy tolerance criterion was considered with a tolerance of $1\times10^{-3}$.

Computation time takes around 1-2 minutes.

<table>
<thead>
<tr>
<th>Table 5.1-1: Case RB1. Constitutive properties for concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{cm}$ (N/mm$^2$)</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Mean measured values</td>
</tr>
</tbody>
</table>

Figure 5.1-1: Case RB1. Dimensions (in m), reinforcements and experimental setup (Vecchio and Shim 2004)

Figure 5.1-2: Case RB1. Failure mechanism observed at ultimate load (Vecchio and Shim 2004)

Figure 5.1-3: Case RB1. Mesh of the model
Table 5.1-2: Case RB1. Reinforcement properties

<table>
<thead>
<tr>
<th>Bar</th>
<th>$\Phi$ (mm)</th>
<th>$A_s$ (mm$^2$)</th>
<th>$E_s$ (N/mm$^2$)</th>
<th>$f_{ym}$ (N/mm$^2$)</th>
<th>$f_{um}$ (N/mm$^2$)</th>
<th>$\varepsilon_{su}^*$</th>
<th>$E_{sy}^*$ (N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M10</td>
<td>11.3</td>
<td>100</td>
<td>200000</td>
<td>315</td>
<td>460</td>
<td>0.025</td>
<td>6000</td>
</tr>
<tr>
<td>M25</td>
<td>25.2</td>
<td>500</td>
<td>220000</td>
<td>445</td>
<td>680</td>
<td>0.05</td>
<td>6000</td>
</tr>
<tr>
<td>M30</td>
<td>29.9</td>
<td>700</td>
<td>200000</td>
<td>436</td>
<td>700</td>
<td>0.05</td>
<td>6000</td>
</tr>
<tr>
<td>D4</td>
<td>3.7</td>
<td>25.7</td>
<td>200000</td>
<td>600</td>
<td>651</td>
<td>0.0112</td>
<td>6000</td>
</tr>
</tbody>
</table>

assumed values

5.1.3 Nonlinear finite element analyses

Load – deflection response

The load – deflection curves are presented in Figure 5.1-4:
- DIANA L6BEA is the fiber beam model with bending behaviour;
- DIANA L6BEA STIRRU is the fiber beam model with shear capabilities;
- DIANA plane stress results from Hendriks et al 2014;
- Basic model CONSHEAR performed by Ferreira 2015, Report #1;
- Experimental results.

The results of the nonlinear analysis determined by the different models are resumed in Table 5.1-3. The deflection of the beam for peak load in DIANA L6BEA STIRRU is presented in Figure 5.1-5

![Figure 5.1-4: Case RB1. Load-deflection curves](image-url)
Recalculations of benchmarks with DIANA

Figure 5.1-5: Case RB1. Deflection for peak load (DIANA10, L6BEA STIRRU)

Table 5.1-3: Case RB1. Results of the NLFEAs (kN)

<table>
<thead>
<tr>
<th>Level of damage</th>
<th>DIANA Plane stress</th>
<th>CONSHEAR</th>
<th>DIANA10 L6BEA STIRRU</th>
<th>DIANA10 L6BEA</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Load</td>
<td>277</td>
<td>270</td>
<td>269</td>
<td>270</td>
<td>265</td>
</tr>
<tr>
<td>Start yielding long. reinforcement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M30 (bottom 2nd layer reinf.)</td>
<td>260</td>
<td>242</td>
<td>248</td>
<td>247</td>
<td>No data</td>
</tr>
<tr>
<td>M10 (top reinf.)</td>
<td>227</td>
<td>244</td>
<td>244</td>
<td>243</td>
<td>No data</td>
</tr>
<tr>
<td>M25 (bottom 1st layer reinf.)</td>
<td>268</td>
<td>262</td>
<td>263</td>
<td>263</td>
<td>No data</td>
</tr>
<tr>
<td>Start yielding stirrups D4</td>
<td>231</td>
<td>270</td>
<td>225</td>
<td>-</td>
<td>No data</td>
</tr>
<tr>
<td>Start crushing of concrete</td>
<td>272</td>
<td>270</td>
<td>270</td>
<td>270</td>
<td>No data</td>
</tr>
<tr>
<td>Computation time</td>
<td>1h</td>
<td>2 minutes</td>
<td>8 minutes</td>
<td>1-2 minutes</td>
<td></td>
</tr>
</tbody>
</table>

Consistently with the plane stress model, the fiber beam model in DIANA (L6BEA STIRRU) determined bending failure with yielding of the longitudinal and transversal reinforcement and crushing of concrete for a similar peak load level. As this is a bending problem, both beam models (with and without shear) present similar results. Still, capturing the yielding of stirrups is only possible with the shear beam model. The arc-length procedure allowed for post peak computations.

Convergence behaviour
The energy criterion is used in the global NR iteration procedure with a tolerance of $1 \times 10^{-3}$. Whenever the energy norm reaches this value (represented by a red line in the graph) the analysis proceeds to the next load step. The number of iterations vs. $\log(\text{energy norm})$ is presented in Figure 5.1-6 for the fiber beam models in DIANA.
Figure 5.1-6: Case RB1. Energy norm vs. global iterations at the NR level, DIANA L6BEA and DIANA L6BEA STIRRU

Strains and stresses

For the case of the shear fiber beam model (L6BEA, STIRRU), the stresses in longitudinal reinforcement are presented in Figure 5.1-7 for peak load; where yielding of top and bottom reinforcement is perceived.

The longitudinal stresses in concrete, in the bottom layer (cracked) and top layer (compression) are presented in Figure 5.1-8 and Figure 5.1-9, respectively, for peak load. Stress levels in the top concrete layer are near the compressive strength for which crushing occurs.

Figure 5.1-7: Case RB1. Stresses in longitudinal reinforcement for peak load (DIANA10, L6BEA STIRRU)
These results of crushing of concrete and yielding of longitudinal reinforcement demonstrate the bending-related failure mechanism of this case.

Strains and stresses in transversal reinforcement are not yet possible to be visualized in DianaIE.

5.1.4 Concluding remarks

From the analysis of the RB1 test (benchmark failing in bending) with fiber beam elements in DIANA and comparison with DIANA plane stress (Hendrik, Belletti et al. 2014) and experimental results (Vecchio and Shim 2004) the following conclusions are pointed out:

- Fiber beam elements in DIANA present similar results to DIANA plane stress model until peak load in terms displacements, failure mechanism and propagation of damage throughout the nonlinear analysis;
- Shear effects are not very relevant in this case, so there is no significant difference in the results of the fiber beams with and without shear effects; with exception that strains and stresses in the transversal reinforcement are only computed by the shear fiber beam model.
- Post-peak response of the fiber beam models differs from the plane stress model and from the experimental data.
- Computation time in the shear fiber beam elements is approximately 10% of the plane stress elements.
5.2 Case RB2: Collins & Kuchma (1999)

Case RB2 considers beam SE-50A-45 of the experimental program of (Collins and Kuchma 1999) and was reported in (CEB Bulletin N0 237 1997). The beam was labelled beam 8 in an international workshop on shear force held in Rotterdam, the Netherlands in 2007. Beam RB2 has been selected because it is characterized by a diagonal-tension failure mechanism.

5.2.1 Experimental setup and results

The beam has a total length of 5.0 m, a depth of 0.5 m, and a width of 0.169 m. The geometry and reinforcement are presented in Figure 5.2-1 and Figure 5.2-2.

![Figure 5.2-1: Case RB2. Dimensions (in mm), reinforcements layout and loading. (Hendriks, Belletti et al. 2015)](image)

![Figure 5.2-2: Case RB2. Cross section details (dimensions in mm), (Hendriks, Belletti et al. 2015)](image)

Four #15 bars are placed as tensile and compressive reinforcement along the entire length of the beam. Additional four #15 bars are placed as tensile reinforcement over a length of 1 m in correspondence of Section B-B and Section C-C characterized by the maximum value of applied moment. The beam has no transversal reinforcement.

The experimental set up is shown in Figure 5.2-3. The beam is loaded by two point loads - P at the top left and 2P at the middle right loading point - as represented in Figure 5.2-1. The beam exhibited a typical brittle diagonal-tension failure mode, shown in Figure 5.2-4. The specimen was tested twice: after the first test, the beam was strengthened and tested with failure in the opposite side. The load-deflection curve is not available the references; the experimental ultimate load values are $P_{\text{EXP}} = 69 \text{ kN}$ for the first test and $P_{\text{EXP}} = 81 \text{ kN}$ for the second test.
5.2.2 Beam finite element model in DIANA 10

The characteristics of the model are presented in Figure 5.2-5; the beam was discretized into 49 Fes; the cross section was divided into 47 fibers; steel longitudinal filaments were simulated according to their positions in the beam. There is no shear reinforcement ($\rho_{sw} = 0$). Apart from the concrete cover, all the fibers were considered shear resistant.

Regarding the material properties, the values given in the paper for the concrete and steel mechanical properties were used in the model; the others were considered the same as in DIANA plane stress.
model (Hendriks, Belletti et al. 2015). The material properties used in the model are listed in Table 5.2-1 for concrete and in Table 5.2-2 for steel.

Loads (P and 2P) were applied as nodal forces, with automatic increments controlled by an arc-length procedure. An energy tolerance criterion was considered with a tolerance of $1 \times 10^{-3}$. Computation time takes around 10 minutes.

**Figure 5.2-5:** Case RB2. Mesh of the model

<table>
<thead>
<tr>
<th>Table 5.2-1: Case RB2. Constitutive model parameters for concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{cm}$ (N/mm$^2$)</td>
</tr>
<tr>
<td>Mean measured values</td>
</tr>
</tbody>
</table>

* assumed / determined values (same as DIANA plane stress, Hendriks et al. 2014)

<table>
<thead>
<tr>
<th>Table 5.2-2: Case RB2. Reinforcement properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar</td>
</tr>
<tr>
<td>#15</td>
</tr>
</tbody>
</table>

* assumed / determined values (same as DIANA plane stress in Hendriks, Belletti et al. 2015)

5.2.3 Nonlinear finite element analyses

**Load-deflection response**

The load – deflection curves are presented in Figure 5.2-6:

- DIANA L6BEA is the fiber beam model with bending behaviour;
- DIANA L6BEA STIRRU is the fiber beam model with shear capabilities;
- DIANA plane stress results from Hendriks et al 2014;
- Basic model CONSHEAR performed by Ferreira 2015, Report #1;

The experimental values of ultimate load are included in the graph.

In Figure 5.2-7 the same results are presented excluding the DIANA fiber beam model without shear effects, in order to zoom in into the graph.

The deflection of the beam for peak load in DIANA L6BEA STIRRU is presented in Figure 5.2-8.

In the case of the beam models, default parameters for concrete in tension were used in the computations (Cervenka equation $k_2=0.002$ and $c=0.5$).

The results of the nonlinear analysis determined by the different models are resumed in Table 5.2-3.
Figure 5.2-6: Case RB2. Load-deflection curves

Figure 5.2-7: Case RB2. Load-deflection curves from the shear fiber beam models
Figure 5.2-8: Case RB2. Deflection for peak load (DIANA10, L6BEA STIRRU)

Table 5.2-3: Case RB2. Results of the nonlinear finite element analysis (kN)

<table>
<thead>
<tr>
<th>Level of damage</th>
<th>DIANA Plane stress</th>
<th>CONSHEAR Beam element (default TS)</th>
<th>CONSHEAR Beam element (reduced TS)</th>
<th>DIANA10 L6BEA STIRRU</th>
<th>DIANA10 L6BEA</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak load (kN)</td>
<td>74.9</td>
<td>94.0</td>
<td>74.0</td>
<td>95.2</td>
<td>323.4</td>
<td>75 (average)</td>
</tr>
<tr>
<td>Yielding long. reinf.</td>
<td>No yielding</td>
<td>No yielding</td>
<td>No yielding</td>
<td>No yielding</td>
<td>241.2</td>
<td>No data</td>
</tr>
<tr>
<td>Yielding stirrups</td>
<td>No stirrups</td>
<td>No stirrups</td>
<td>No stirrups</td>
<td>No stirrups</td>
<td>-</td>
<td>No data</td>
</tr>
<tr>
<td>Crushing concrete</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>323.43</td>
<td>No data</td>
</tr>
<tr>
<td>Computation time</td>
<td>2h</td>
<td>3 minutes</td>
<td>3 minutes</td>
<td>10 minutes</td>
<td>5 minutes</td>
<td></td>
</tr>
</tbody>
</table>

Shear failure mechanism, without yielding of longitudinal reinforcement, is given by the shear fiber beam model and the plane stress model, for load levels similar to the experimentally observed.

The use of arc-length procedure in the shear fiber beam model in DIANA is not enough to capture the post peak response. Divergence occurred (iteration method failed) in the last load step.

The fiber beam model in DIANA without shear effects (L6BEA), acts as the standard are pure flexural beam elements that are not able to capture shear failure. This model predicted ductile bending failure, with yielding of longitudinal reinforcement, at nearly three times the ultimate load measured in the experimental test. This value is consistent with the analytical calculations for the bending capacity of the beam, presented in Hendriks et al 2014. The flexural beam model is detached from the experimental response.

The beam is critical to shear and has no shear reinforcement presenting a brittle shear failure. It demonstrates how standard flexural beam elements overcome shear failures, giving unsafe predictions of loading capacity, by continuing the analysis until maximum bending capacity. The fiber beam model with shear captures shear failure.

**Convergence behaviour**

Figure 5.2-9 represents the energy norm versus the number of iterations throughout the nonlinear analysis until failure; the red line sets the norm for which convergence is achieved and the analysis continues with the next load step.
Recalculations of benchmarks with DIANA

Figure 5.2-9: Case RB2. Energy norm vs. global iterations at the NR level

Figure 5.2-10: Case RB2. Energy norm vs. global iterations at the NR level

Strains and stresses
For the case of the shear fiber beam model (L6BEA, STIRRU), the strains and stresses in longitudinal reinforcement are presented in Figure 5.2-11 and Figure 5.2-12 for peak load; where it can be observed that it remains elastic until ultimate load.

The longitudinal strains and stresses in concrete for peak load level, in the bottom and top fibers, are presented in Figure 5.2-13 and Figure 5.2-14, where can be observed that compression stress levels are not high, being far from ultimate strength of concrete and crushing.
The failure mechanism is shear driven by diagonal tension. The principal strains and stresses in concrete and the strains and stresses in the vertical direction (EZZ, SZZ) in concrete are not yet possible to be visualized in DianaIE.

In contrast to the shear fiber beam model, the flexural fiber beam model shows yielding of longitudinal reinforcement at peak load level (very high ultimate load, approximately 3 times the experimentally observed) demonstrated in Figure 5.2.15.

This difference of results at failure between the flexural and shear fiber beam models demonstrates the relevance of the new implementation by capturing shear-related failure mechanism with the beam elements.

![Figure 5.2-11: Case RB2. Strains in longitudinal reinforcement for peak load P=95kN (DIANA10, L6BEA STIRRU)](image1)

![Figure 5.2-12: Case RB2. Stresses in longitudinal reinforcement for peak load P=95kN (DIANA10, L6BEA STIRRU)](image2)

![Figure 5.2-13: Case RB2. Longitudinal strains in concrete bottom fiber for peak load P=95kN (DIANA10, L6BEA STIRRU)](image3)
5.2.4 Concluding remarks

From the analysis of the RB2 test (benchmark failing in shear diagonal-tension) with fiber beam elements in DIANA and comparison with DIANA plane stress (Hendriks, Belletti et al. 2015) and experimental observations (Collins & Kuchma 1999) the following conclusions are pointed out:

- Shear fiber beam elements in DIANA (L6BEA STIRRU) give similar predictions of the ultimate load, load-deflection curve and failure mechanism in comparison with the DIANA plane stress model;
- Post-peak response is not reproduced with the shear fiber beam model and divergence occurred in the last load step;
- The results of ultimate load and shear failure mechanism are consistent with the experimental observations;
- Flexural fiber beam model is not capable of capturing shear failure; the analyses overpass shear failure and continue until ultimate bending capacity is reached resulting in unsafe estimations of the capacity of beams critical in shear.
- Computation time taken by shear fiber beam elements is approximately 10% of the plane stress elements.
5.3 Case RB3: Grace (2001)

The experimental program of (Grace 2001) studied the effect of strengthening using fiber-reinforced polymer (FRP) strips. The control beam of the category II (group of beams that fail in bending) from this program is used as a case study. The control beam is not strengthened with FRP.

5.3.1 Experimental setup and results

The beam has a total length of 8.230 m, depth of 0.457 m and width of 0.250 m. The geometry, boundary, loading conditions and test setup are represented in Figure 5.3-1. The sectional geometry and reinforcement is represented in Figure 5.3-2. The beam is reinforced with three #8 bars (Φ=25.4 mm) at the top and bottom. Transversal reinforcement consists in #3 (Φ=9.525 mm) stirrups spaced of 152 mm. The concrete cover is 51 mm. Both longitudinal and transversal reinforcement are constant thought the longitude of the beam. There are some doubts about the construction of the supports, as there is no description in the reference.

The beam exhibited a flexural failure mode, as can be observed in Figure 5.3-3, for an ultimate load of $P_{EXP} = 141.9$ kN.

![Figure 5.3-1: Case RB3. Loading and boundary conditions (dimension in m), (Hendriks et al. 2014)](image1)

![Figure 5.3-2: Case RB3. Cross section details (dimensions in mm), (Hendriks et al. 2014)](image2)
5.3.2 Beam finite element model in DIANA 10

The characteristics of the model are presented in Figure 5.3-4; the beam was discretized into 43 FEs; the cross section was divided into 45 fibers; steel longitudinal filaments were simulated according to their positions in the beam, constant along the beam (3+3 Ø25.4mm); transversal reinforcement (Ø9.2//152mm) is simulated as smeared in concrete (ρsw = 0.37%); apart from the concrete cover, all the fibers were considered shear resistant.

Regarding the material properties, the values given in the paper for the concrete and steel mechanical properties were used in the model; the others were considered the same as in DIANA plane stress model (Hendriks, Belletti et al. 2015). The material properties considered are listed in Table 5.3-1 for concrete and in Table 5.3-2 for steel (longitudinal and transversal steel present equal values).

Load P was applied as nodal force in node 6, with automatic increments controlled by an arc-length procedure. An energy tolerance criterion was considered with a tolerance of 1×10^-3. Computation time takes around 10 minutes.

![Figure 5.3-4: Case RB3. Mesh of the model](image)

**Table 5.3-1**: Case RB3. Constitutive properties for concrete

<table>
<thead>
<tr>
<th></th>
<th>$f_{cm}$ (N/mm²)</th>
<th>$f_{ctm}$ (N/mm²)</th>
<th>$E_c^*$ (N/mm²)</th>
<th>$\varepsilon_{cu}$ * (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean measured values</td>
<td>31.2</td>
<td>2.44</td>
<td>31297</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

* assumed / determined values (same as DIANA plane stress, Hendriks, Belletti et al. 2014)

**Table 5.3-2**: Case RB3. Reinforcement properties.

<table>
<thead>
<tr>
<th>Bar</th>
<th>$\Phi$ (mm)</th>
<th>$A_s$ (mm²)</th>
<th>$E_s$ (N/mm²)</th>
<th>$f_{ym}$ (N/mm²)</th>
<th>$f_{um}$ * (N/mm²)</th>
<th>$\varepsilon_{su}$ *</th>
<th>$E_{sy}$ * (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#3 (stirrups)</td>
<td>9.5</td>
<td>71</td>
<td>200000</td>
<td>414</td>
<td>500</td>
<td>0.05</td>
<td>1794</td>
</tr>
<tr>
<td>#15 (long. reinf.)</td>
<td>25.4</td>
<td>509</td>
<td>200000</td>
<td>414</td>
<td>500</td>
<td>0.05</td>
<td>1794</td>
</tr>
</tbody>
</table>

* assumed / determined values (same as DIANA plane stress, Hendriks, Belletti et al. 2014)
5.3.3 Nonlinear finite element analyses

Load-deflection response
The load – deflection curves are presented in Figure 5.3-5:
- DIANA L6BEA is the fiber beam model with bending behaviour;
- DIANA L6BEA STIRRU is the fiber beam model with shear capabilities;
- DIANA plane stress results from Hendriks et al 2014;
- Basic model CONSHEAR performed by Ferreira 2015, Report #1.
The experimental values of ultimate load are included in the graph.

The results of the nonlinear analysis determined by the different models resumed in Table 5.3-3.

![Figure 5.3-5: Case RB3. Load-deflection curves](image)

<table>
<thead>
<tr>
<th>Level of damage</th>
<th>DIANA Plane stress</th>
<th>CONSHEAR Beam element</th>
<th>DIANA10 L6BEA STIRRU</th>
<th>DIANA10 L6BEA</th>
<th>Exp.</th>
</tr>
</thead>
</table>
| Peak load                        | 149                | 148                   | 149                   | 149           | 141.9| 519
| Start yielding long. reinf.      | 127                | 128                   | 138                   | 141           | No data |
| Start yielding stirrups          | 135                | 132                   | 120                   | -             | No data |
| Start crushing of concrete       | 104                | 137                   | 144                   | 145           | No data |
| Computation time                 | 1h                 | 2 minutes             | 10 minutes            | 3 minutes     |      |

Consistently with the plane stress model, the shear fiber beam model in DIANA (L6BEA STIRRU) determined bending failure with yielding of the longitudinal and transversal reinforcement and crushing of concrete for equal peak load level. This is a bending problem so both fiber beam models in DIANA (with and without shear) present similar results. Capturing the yielding of stirrups is only possible with the shear beam model, predicting its start for a slight lower load level than the plane stress Fe model. The basic fiber beam model and the implemented in DIANA give very similar results. Computation time decreases significantly with the fiber beams when compared with the plane stress computation in DIANA.
Convergence behaviour
The energy criterion, with a tolerance of $1 \times 10^{-3}$, controls the global NR iteration procedure. Figure 5.3-6 represents the energy norm versus the number of iterations throughout the nonlinear analysis until failure; the red line sets the norm for which convergence is achieved.

![Figure 5.3-6: Case RB3. Energy norm vs. global iterations at the NR level](image)

Strains
For the case of the shear fiber beam model (L6BEA, STIRRU), the strains and stresses in longitudinal reinforcement are presented in Figure 5.3-8 and Figure 5.3-7 for peak load; where it can be observed that the top reinforcement is yielded.

The longitudinal strains and stresses in concrete for peak load level are presented in Figure 5.3-9 and Figure 5.3-10 for the bottom and Figure 5.3-11 and Figure 5.3-12 for the top fibers, where can be observed that compression stress levels are high; in the bottom concrete layer near the left support are near the compressive strength for which crushing occurs.

These results represent the bending failure mechanism predicted by the model.

![Figure 5.3-7: Case RB3. Strains in longitudinal reinforcement for peak load (DIANA10, L6BEA STIRRU)](image)
Figure 5.3-8: Case RB3. Stresses in longitudinal reinforcement for peak load (DIANA10, L6BEA STIRRU)

Figure 5.3-9: Case RB3. Longitudinal strains in concrete bottom fiber for peak load (DIANA10, L6BEA STIRRU)

Figure 5.3-10: Case RB3. Longitudinal stresses in concrete bottom fiber for peak load (DIANA10, L6BEA STIRRU)
5.3.4 Concluding remarks

From the analysis of the RB3 test (benchmark failing in bending) with fiber beam elements in DIANA and comparison with DIANA plane stress (Hendriks, Belletti et al. 2015) and experimental observations (Grace 2001) the following conclusions are pointed out:

- Shear fiber beam elements in DIANA (L6BEA STIRRU) gave very similar predictions of the ultimate load, load-deflection curve and failure mechanism in comparison with the DIANA plane stress model;
- The results of ultimate load and failure mechanism are consistent with the experimental observations;
- The start of yielding of reinforcement, both longitudinal and transversal, is similarly predicted by both models;
- Flexural fiber beam element in DIANA (L6BEA), presented similar response to the other computations because this is a bending problem; however the model does not give information about the state of damage of the transversal reinforcement;
- Computation time is decreased in approximately 80% with the shear fiber beam elements in comparison with plane stress elements.
5.4 Case RB3A: Grace (2001)

The experimental program of (Grace 2001) studied the effect of strengthening using fiber-reinforced polymer (FRP) strips. The control beam of the category I (group of beams that fail in shear) from this program is used as a case study. The control beam is not strengthened with FRP.

5.4.1 Experimental setup and results

The only difference between RB3A and RB3 (described in point 5.5) is that the stirrups have a spacing of 0.457 m instead of 0.152 m. All the other dimensions and parameters remain the same.

The beam exhibited a shear failure mode as represented in Figure 5.4-1 for an ultimate load of $P_{\text{EXP}} = 155.7 \text{ kN}$.

![Figure 5.4-1: Case RB3A. Failure mechanisms observed at ultimate applied load, (Grace 2001)](image)

5.4.2 Beam finite element model in DIANA 10

The model is exactly the same as in case RB3 (presented in Section 5.3, see Figure 5.3-1 and Figure 5.3-2); the only difference is the quantity of stirrups that instead of $\rho_{\text{sw}} = 0.37\%$ (Ø9.2//152mm) in case RB3 is now $\rho_{\text{sw}} = 0.12\%$ (Ø9.2//457mm) for case RB3A. Load is applied in 75 load steps. All the other mesh and material properties are the same.

As transversal reinforcement is not included in the flexural fiber beam model (L6BEA) the computation is the same for the cases RB3A and RB3. The shear fiber beam model (L6BEA STIRRU) includes the difference in the quantity of transversal reinforcement of both cases.

5.4.3 Nonlinear finite element analyses

Load – deflection response

The load – deflection curves are presented in Figure 5.3-5:
- DIANA L6BEA is the fiber beam model with bending behaviour;
- DIANA L6BEA STIRRU is the fiber beam model with shear capabilities;
- DIANA plane stress results from Hendriks et al 2014;
- Basic model CONSHEAR performed by Ferreira 2015, Report #1.

The experimental values of ultimate load are included in the graph.

The load – deflection curve is presented in Figure 5.4-2 and compared with the DIANA plane stress FE model performed by Hendriks, Belletti et al. (2015). The ultimate load observed experimentally is indicated in the graphic. The results of the nonlinear analyses for peak load, failure mechanism and the start of different levels of damage resumed in Table 5.4-1.

The deflection of the beam for peak load in DIANA L6BEA STIRRU is presented in Figure 5.4-3.
Figure 5.4-2: Case RB3A. Load-deflection curves

Table 5.4-1: Case RB3A. Results of the NLFEAs (kN)

<table>
<thead>
<tr>
<th>Level of damage</th>
<th>DIANA Plane stress</th>
<th>CONSHEAR</th>
<th>DIANA10 L6BEA STIRRU</th>
<th>DIANA10 L6BEA</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak load</td>
<td>146</td>
<td>134</td>
<td>141</td>
<td>149</td>
<td>155.7</td>
</tr>
<tr>
<td>Start yielding long. reinf.</td>
<td>126</td>
<td>127</td>
<td>132</td>
<td>141</td>
<td>No data</td>
</tr>
<tr>
<td>Start yielding stirrups</td>
<td>96.2</td>
<td>71.1</td>
<td>72.5</td>
<td>-</td>
<td>No data</td>
</tr>
<tr>
<td>Start crushing of concrete</td>
<td>96.2</td>
<td>130</td>
<td>131</td>
<td>145</td>
<td>No data</td>
</tr>
<tr>
<td>Computation time</td>
<td>1h</td>
<td>2 minutes</td>
<td>12 minutes</td>
<td>2 minutes</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.4-3: Case RB3A. Deflection for peak load (DIANA10, L6BEA STIRRU)

Failure mechanism is predicted to be shear-bending related by the shear fiber beam model, with yielding of stirrups, yielding of longitudinal reinforcement and crushing of concrete. This damage is consistent with the results of DIANA plane stress model (Hendriks, Belletti et al. 2015).

With the decrease of transversal reinforcement in beam RB3A in comparison with the case RB3, it presents a lower ultimate load capacity and failure mechanism changed from bending to shear-bending related. This difference in response between cases RB3 and RB3A is captured by the shear
fiber beam model in DIANA (L6BEA STIRRU). In contrast, the bending fiber beam model in DIANA (L6BEA) is impermeable to this difference, as the computation is exactly the same.

Yielding of transversal reinforcement is predicted by the shear fiber beam model in DIANA for lower load level than DIANA plane stress and crushing of concrete for lower load level. Start of yielding of longitudinal reinforcement is similarly predicted by both models. Pertaining to the ultimate load, the shear fiber beam model predicts a slightly lower value than the plane stress model.

The experimental value for ultimate load of case RB3A (P=155.7kN) is higher than for case RB3 (P=141.9kN), which is unexpected has the only difference between the two is that beam RB3A has more stirrups than RB3; this difference must be related with experimental randomness. The numerical models that account for shear effects predicted lower ultimate loads for RB3A in comparison with case RB3.

**Strains**

For the case of the shear fiber beam model (L6BEA, STIRRU), the strains and stresses in longitudinal reinforcement are presented in Figure 5.4-4 and Figure 5.4-5 for peak load; where it can be observed that the top reinforcement is yielded.

The longitudinal strains and stresses in concrete for peak load level are presented in Figure 5.4-6 and Figure 5.4-7 for the bottom and in Figure 5.4-8 and Figure 5.4-9 for the top fibers, presenting similar results as the RB3 case.

These results represent the shear-bending failure mechanism, with extensive yielding of transversal reinforcement and yielding of longitudinal reinforcement.

---

**Figure 5.4-4:** Case RB3. Longitudinal strains in concrete bottom fiber for peak load (DIANA10, L6BEA STIRRU)

**Figure 5.4-5:** Case RB3. Longitudinal stresses in concrete top fiber for peak load (DIANA10, L6BEA STIRRU)
Figure 5.4-6: Case RB3. Longitudinal strains in concrete top fiber for peak load (DIANA10, L6BEA STIRRU)

Figure 5.4-7: Case RB3. Longitudinal stresses in concrete top fiber for peak load (DIANA10, L6BEA STIRRU)

Figure 5.4-8: Case RB3. Strains in longitudinal reinforcement for peak load (DIANA10, L6BEA STIRRU)
5.4.4 Concluding remarks

From the analysis of the RB3A test (benchmark failing in shear) with fiber beam elements in DIANA and comparison with DIANA plane stress (Hendriks, Belletti et al. 2015) and with experimental data (Grace 2001) the following conclusions are pointed out:

- Shear fiber beam elements in DIANA (L6BEA STIRRU) model gave very similar predictions of the ultimate load, load-deflection curve and failure mechanism in comparison with the DIANA plane stress model;
- Both models slightly underestimated the experimental failure load;
- The start of yielding of longitudinal reinforcement is very similarly predicted by both models;
- Shear fiber beam elements in DIANA (L6BEA STIRRU) predicts start of yielding of stirrups sooner than DIANA plane stress model;
- As transversal reinforcement is not included in the flexural fiber beam model (L6BEA) the computation is the same for the cases RB3A and RB3. The shear fiber beam model (L6BEA STIRRU) includes the difference in the quantity of transversal reinforcement of both cases.
- Computation time is decreased in approximately 80% with the shear fiber beam elements in comparison with plane stress elements.
5.5 Case PB1: Leonhardt, Koch et al. (1975)

The case PB1 is a prestressed beam tested by (Leonhardt, Kock et al. 1973), in an experimental campaign involving the testing of ten beams. Beam IP1 (original name in the experiments) is selected as case study presenting flexural-compressive failure mechanism.

5.5.1 Experimental setup and results

The beam has a total length of 7.0 m, span of 6.5 m and depth of 0.9 m with variable thickness of the web. The geometry, cross-section and reinforcement details and characteristics of the experimental setup are shown in the original draws presented in Figure 5.5-1, Figure 5.5-2 and Figure 5.5-3; and resumed in the scheme of Figure 5.5-4. The beam has longitudinal reinforcement (bars of diameters Φ8 and Φ14) and non-symmetric stirrups of Φ16 on the left hand-side and Φ12 on the right hand-side distanced of 140mm. The prestressing reinforcement consists of 2 post-tensioned tendons prestressed at both sides and made of 12Φ12.2 strands each; the initial stress in each tendon is equal to 635 MPa. The applied prestressing force at each cable corresponded to 995 kN and after the losses the prestressing force measured was of 891 kN in each cable. An assumed cover of 20 mm is considered as in Hendriks, Belletti et al. (2015).

A point load equal of 196 kN was applied at the same time of the prestressing of the tendons. After that the beam was loaded in a 3-point loading scheme until failure. Loading and boundary conditions are represented in Figure 5.5-5 and the global experimental set up in Figure 5.5-6.

The beam exhibited a flexural-compressive failure mode for the maximum load of 1897.5 kN, when the concrete failed. The observed crack pattern near failure (P=1765 kN) is presented in Figure 5.5-7. Crack patterns and deflection with increasing load are the experimental data used for comparison with the numerical results.

![Figure 5.5-1: Case PB1. Elevation and cross-sectional details (in cm) (Leonhardt et al. 1973)](image1)

![Figure 5.5-2: Case PB1. Reinforcements (in cm) (Leonhardt et al. 1973)](image2)

![Figure 5.5-3: Case PB1. Reinforcement cage and prestressing cables (Leonhardt et al. 1973)](image3)
Figure 5.5-4: Case PB1. (a) Variation of the web thickness along the length of the beam (b) transversal cross section (dimensions in mm) (Hendriks & Belletti et al. 2014)

Figure 5.5-5: Case PB1. Loading and boundary conditions (Leonhardt et al. 1973)
5.5.2 Beam finite element model in DIANA 10

The characteristics of the model are presented in Figure 5.5-8; the beam was discretized into 38 FEs and 39 nodes. The cross section was divided into 77 fibers. The thickened web zone at the end of the beam was simulated by varying the web width of the various elements in that area; for this 5 different cross sections were considered in the beam. Passive steel longitudinal filaments were simulated according to their positions, using one constant configuration along the beam: 12xØ14 top bars, 4xØ8 web bars and 12xØ8 bottom bars. Transversal reinforcement is considered smeared in the cross-sections, with different quantities in the left and right half parts of the beam: Ø16//=140mm in the left half and Ø12//=140mm in the right half. The web of the cross section (discounting the concrete cover) and the top flanges were considered shear resistant.

The 2 prestressing cables were simulated by means of prestressed reinforcement elements in DIANA with their respective eccentricity, taking into account the slight curvature in both ends.

Regarding the material properties, the values given in the original report for the concrete and steel mechanical properties were used in the model; the others were considered the same as in DIANA plane stress model (Hendriks, Belletti et al. 2015). The material properties used in the model are listed in Table 5.5-1 for concrete and in Table 5.5-2 for steel.

The beam is simple supported, with constraint nodes 3 and 37. The analysis was performed in two phases:

1) Dead weight, prestressing force of P=891 kN in each cable and application of point load of 196 kN in the mid-span (node 20);
2) Application of incremental load with automatic increments controlled by an arc-length procedure.
Energy tolerance considered was $1 \times 10^{-3}$. Computation time takes around 9 minutes.

Figure 5.5-8: Case PB1. Mesh of the model

Table 5.5-1: Case PB1. Constitutive properties for concrete

<table>
<thead>
<tr>
<th></th>
<th>$f_{cm}$ (N/mm$^2$)</th>
<th>$f_{ctm}^*$ (N/mm$^2$)</th>
<th>$E_c^*$ (N/mm$^2$)</th>
<th>$f_{cu}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean measured values</td>
<td>25.3</td>
<td>2.16</td>
<td>26675</td>
<td>0.0035</td>
</tr>
<tr>
<td>* assumed / determined values (when possible, same as DIANA plane stress, Hendriks, Belletti et al 2014)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5-2: Case PB1. Reinforcement properties

<table>
<thead>
<tr>
<th>Bar</th>
<th>$\Phi$ (mm)</th>
<th>$A_s$ (mm$^2$)</th>
<th>$E_s$ (N/mm$^2$)</th>
<th>$f_{ym}$ (N/mm$^2$)</th>
<th>$f_{um}$ (N/mm$^2$)</th>
<th>$\varepsilon_{su}^*$</th>
<th>$E_{sy}^*$ (N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_{12}$</td>
<td>12.0</td>
<td>113</td>
<td>203000</td>
<td>500</td>
<td>611</td>
<td>0.05</td>
<td>2335</td>
</tr>
<tr>
<td>$\Phi_{16}$</td>
<td>16.0</td>
<td>201</td>
<td>195000</td>
<td>400</td>
<td>512</td>
<td>0.05</td>
<td>2336</td>
</tr>
<tr>
<td>$\Phi_{8}$</td>
<td>8.0</td>
<td>50</td>
<td>197000</td>
<td>460</td>
<td>567</td>
<td>0.05</td>
<td>2245</td>
</tr>
<tr>
<td>$\Phi_{14}$</td>
<td>14.0</td>
<td>154</td>
<td>207000</td>
<td>397</td>
<td>517</td>
<td>0.05</td>
<td>2496</td>
</tr>
<tr>
<td>$\Phi_{12.2}$</td>
<td>12×12.2</td>
<td>12×117</td>
<td>207000</td>
<td>1225</td>
<td>1363</td>
<td>0.02</td>
<td>9799</td>
</tr>
</tbody>
</table>

* assumed / determined values

5.5.3 Nonlinear finite element analyses

Nonlinear-deflection response

The load – deflection curves are presented in Figure 5.5-9:

- DIANA L6BEA is the fiber beam model with bending behaviour;
- DIANA L6BEA STIRRU is the fiber beam model with shear capabilities;
- DIANA plane stress results from Hendriks et al 2014;
- Basic model CONSHEAR performed by Ferreira 2015, Report #1;
- Experimental curve (Leonhardt, Kock et al. 1973).

The results of both nonlinear analyses for the start of different levels of damage are marked along the load-deflection curve and also resumed and compared in Table 5.5-3.

The deflection of the beam for peak load in DIANA L6BEA STIRRU is presented in Figure 5.5-10.
Recalculations of benchmarks with DIANA

Figure 5.5-9: Case PB1. Load-deflection curves

<table>
<thead>
<tr>
<th>Level of damage</th>
<th>DIANA Plane stress</th>
<th>CONSHEAR</th>
<th>DIANA10 L6BEA STIRRU</th>
<th>DIANA10 L6BEA</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start yielding long. reinf. (bottom)</td>
<td>1320</td>
<td>1656</td>
<td>1528</td>
<td>1540</td>
<td>No data</td>
</tr>
<tr>
<td>Start yielding long. reinf. (web)</td>
<td>1340</td>
<td>1836</td>
<td>1902</td>
<td>1912</td>
<td>No data</td>
</tr>
<tr>
<td>Start yielding stirrups</td>
<td>1728</td>
<td>1856</td>
<td>1956</td>
<td>-</td>
<td>No data</td>
</tr>
<tr>
<td>Start yielding long. reinf. (top)</td>
<td>1756</td>
<td>1956</td>
<td>1948</td>
<td>1953</td>
<td>No data</td>
</tr>
<tr>
<td>Crushing of concrete</td>
<td>1872</td>
<td>1896</td>
<td>1997</td>
<td>1998</td>
<td>No data</td>
</tr>
<tr>
<td>Peak load</td>
<td>1948</td>
<td>1961</td>
<td>1997</td>
<td>1999</td>
<td>1897.5</td>
</tr>
</tbody>
</table>

Computation time 1h30m 2 minutes 9 minutes 3 minutes

The fiber beam model in DIANA (L6BEA STIRRU) presented a similar response to DIANA plane stress model, which also agrees with the experimental results. Similar values for ultimate load were predicted and for the same failure mechanism related to bending, with yielding of reinforcement (transversal and longitudinal) and crushing of concrete. The sequence of the development of damage is similarly predicted by the fiber beam and the plane stress model; however the fiber beam model presents later yielding of longitudinal and transversal reinforcement. Both models predicted well the failure mechanism and peak load.
Convergence behaviour

The energy criterion, with a tolerance of $1 \times 10^{-3}$, controls the global NR iteration procedure. Figure 5.5-11 represents the energy norm versus the number of iterations throughout the nonlinear analysis until failure; the red line sets the norm for which convergence is achieved.

Strains

For the case of the shear fiber beam model (L6BEA, STIRRU), the strains and stresses in longitudinal reinforcement (active and passive) for peak load level are presented in Figure 5.5-12 and Figure 5.5-13; where yielding is observed.

The longitudinal strains in concrete, in the top and bottom layers are presented in Figure 5.5-14 and Figure 5.5-15, respectively, for peak load. Stress levels in the top concrete layer are near the compressive strength for which crushing occurs.

These results of crushing of concrete and yielding of longitudinal reinforcement demonstrate the bending-related failure mechanism of this case.
Figure 5.5-12: Case PB1. Strains in longitudinal reinforcement for peak load (DIANA10, L6BEA STIRRU)

Figure 5.5-13: Case PB1. Strains in longitudinal reinforcement

Figure 5.5-14: Case PB1. Longitudinal strains in concrete top fiber for peak load (DIANA10, L6BEA STIRRU)
5.5.4 Concluding remarks

From the analysis of the PB1 test (benchmark failing in bending) with fiber beam elements in DIANA and comparison with DIANA plane stress (Hendriks, Belletti et al. 2015) and with experimental data (Leonhardt et al. 1973) the following conclusions are pointed out:

- Fiber beam and plane stress models in DIANA presented quite similar results in terms of load-deflection curve, maximum load and bending failure mechanism.
- The results were consistent with the experimental measurements.
- Computation time is decreased in approximately 90% with the shear fiber beam elements in comparison with plane stress elements.
5.6 Case PB2/NSEL: Sun and Kuchma (2007)

The work of (Sun and Kuchma 2007) consisted in a large experimental campaign to study the shear behaviour and resistant mechanism of high strength concrete prestressed girders of large dimensions. The high strength concrete prestressed girders were designed according to AASHTO LRFD Bridge Design Specifications and tested at the Newmark Structural Laboratory. From this experimental campaign, Girder 3 is selected as case study presenting shear - compressive failure mechanism.

5.6.1 Experimental setup and results

The beam has a T-shape cross section, total length of 15.85 m and depth of 1.6 m; a concrete deck is casted on top of the girder. The cross-section, geometry and reinforcement details are shown in the original draws of Figure 5.6-1; and also resumed in Figure 5.6-2.

Figure 5.6-1: Case PB2. Cross section details (dimensions in inch) (Sun and Kuchma 2007)

Figure 5.6-2: Case PB2. (a) dimensions of the beam and loading scheme, (b) reinforcement and cross section (dimensions in mm) (Hendriks, Belletti et al 2014)

Main transversal reinforcement consists of #4 double legged deformed bars with different spacings - 203 / 305 / 610 mm. In both ends this shear reinforcement is increased in #5 double legged bars at 50
mm spacing in a length of 500 mm from each ends. The passive longitudinal reinforcement consisted in two pairs of #3 longitudinal bars in the web (spaced at 458 mm) and one layer of 6x#8 bars in the top flange. The East end was designed to satisfy LRFD requirements while the West end region contained addition reinforcement: distributed horizontal, vertical and confinement reinforcement. In this region, passive longitudinal reinforcement in the web was increased to 6 pairs of #3 bars (spaced at 152.4mm), in a length of 3.408m. The concrete deck has two layers of 5#6 bars.

Pertaining to prestressing steel, 44 seven-wire low-relaxation straight strands (with diameter of Φ15.2 mm each) were used: 42 strands in the bottom bulb and 2 strands in the top flange. The effective initial stress in the prestressing steel, measured before testing and after immediate losses, was 1068 N/mm².

The test consists in a simple supported configuration with distributed load in a length of 13.41 m as represented in Figure 5.6-3. The beam exhibited a shear-compression failure mechanism with crushing of concrete at web-bulb interface, as represented in Figure 5.6-4. The beam was tested twice: after the East end failed, this side was repaired by adding reinforcement and concrete and vertical post-tensioning; the beam was then reloaded in the same loading and support conditions until failure of the West end. This case study relates to the first test only: failure was achieved for a load level of 6984 kN at a deflection of 76.2mm.

Experimental data available in the reference consists on deflections, crack patterns, strains in concrete and reinforcement.

![Figure 5.6-3: Case PB2. Loading, boundary conditions and dimension (in ft) (Sun and Kuchma 2007)](image)

![Figure 5.6-4: Case PB2. Failure mechanisms at ultimate load of East end (Sun and Kuchma 2007)](image)

### 5.6.2 Beam finite element model in DIANA 10

The characteristics of the model are presented in Figure 5.6-5; the beam was discretized into 52 FEs. The cross section was divided into 50 fibers and defined with zones. Passive steel longitudinal filaments were simulated according to their positions in the beam using 2 different configurations in the girder: \( A_{s1} = 6\#8 \) top bars, 6x2#3 web bars; \( A_{s2} = 6\#8 \) top bars, 2x2#3 web bars. The deck part has two layers of 5#6 bars.

Transversal reinforcement is considered smeared in the cross-sections, with different quantities along the beam: \( A_{s1} = 2\#4/203\text{mm}, A_{s2} = 2\#4/305\text{mm}, A_{s3} = 2\#4/610\text{mm} \). The web of the girder and the top flanges were considered shear resistant; the fibers of the deck and the fibers of the bottom flange are non-shear resistant.
The 44 prestressing cables were assembled into 8 groups of bars with the same eccentricity and modelled with prestressed reinforcement elements in DIANA.

Regarding the material properties, the values given in the report for the concrete and steel mechanical properties were used in the model; the others were considered the same as in DIANA plane stress model (Hendriks, Belletti et al. 2015). The material properties used in the model are listed in Table 5.6-1 for concrete and in Table 5.6-2 for steel. Lateral reduction through the equation of Vecchio and Collins (1986) was considered.

The beam is simple supported, with constraint nodes 2 and 52. The analysis was performed in two phases:
1) Dead weight and prestressing force of $P=149.52$ kN per cable;
2) Application of incremental distributed load (between nodes 5 to 49) with automatic increments controlled by an arc-length procedure.

Energy tolerance considered was $1 \times 10^{-3}$. Computation time takes around 30 minutes (very difficult and slow to converge in the last load steps, near failure).

Figure 5.6-5: Case PB2. Mesh of the model

Table 5.6-1: Case PB2. Constitutive properties for concrete

<table>
<thead>
<tr>
<th></th>
<th>$f_{cm}$ (N/mm$^2$)</th>
<th>$f_{ctm}^*$ (N/mm$^2$)</th>
<th>$E_c^*$ (N/mm$^2$)</th>
<th>$\varepsilon_{cu}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder: Mean measured values</td>
<td>109.63</td>
<td>5.28</td>
<td>52710</td>
<td>0.0026</td>
</tr>
<tr>
<td>Deck: Mean measured values</td>
<td>24.82</td>
<td>2.65</td>
<td>25076</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

* assumed / determined values (when possible, same as DIANA plane stress, Hendriks, Belletti et al. 2014)

Table 5.6-2: Case PB2. Reinforcement properties.

<table>
<thead>
<tr>
<th>Bar</th>
<th>$\Phi$ (mm)</th>
<th>$A_s$ (mm$^2$)</th>
<th>$E_s$ (N/mm$^2$)</th>
<th>$f_{ym}$ (N/mm$^2$)</th>
<th>$f_{um}$ (N/mm$^2$)</th>
<th>$\varepsilon_{su}^*$</th>
<th>$E_{sy}^*$ (N/mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#3</td>
<td>9.5</td>
<td>71</td>
<td>200000</td>
<td>413.7</td>
<td>620.5</td>
<td>0.05</td>
<td>4315</td>
</tr>
<tr>
<td>#4</td>
<td>12.7</td>
<td>129</td>
<td>200000</td>
<td>467.5</td>
<td>731.5</td>
<td>0.05</td>
<td>5539</td>
</tr>
<tr>
<td>#6</td>
<td>19.1</td>
<td>284</td>
<td>200000</td>
<td>413.7</td>
<td>620.5</td>
<td>0.05</td>
<td>4315</td>
</tr>
<tr>
<td>#8</td>
<td>25.4</td>
<td>510</td>
<td>200000</td>
<td>413.7</td>
<td>620.5</td>
<td>0.05</td>
<td>4315</td>
</tr>
<tr>
<td>strands,7wire</td>
<td>15.24</td>
<td>140</td>
<td>196500</td>
<td>1675</td>
<td>1862</td>
<td>0.02</td>
<td>16295</td>
</tr>
</tbody>
</table>

* assumed / determined values
5.6.3 Nonlinear finite element analyses

Load – deflection response
The load – deflection curves are presented in Figure 5.6-6:
- DIANA L12BEA is the fiber beam model with bending behaviour;
- DIANA L12BEA STIRRU is the fiber beam model with shear capabilities;
- DIANA plane stress results from Hendriks et al 2014;
- Basic model CONSHEAR performed by Ferreira 2015, Report #1;
- The experimental curve from (Sun and Kuchma 2007).

The numerical deflections were corrected from the initial results related to the dead weight and application of prestressing forces.

The results of the nonlinear analysis determined by the different models are resumed in Table 5.6-3.

The deflection of the beam for peak load in DIANA L6BEA STIRRU is presented in Figure 5.6-7.

![Figure 5.6-6: Case PB2. Load-deflection curves](image-url)

### Table 5.6-3: Case PB2. Results of the NLFEAs and experimental data (kN)

<table>
<thead>
<tr>
<th>Level of damage</th>
<th>DIANA Plane stress</th>
<th>DIANA L6BEA STIRRU</th>
<th>DIANA L6BEA</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start yielding stirrups #4</td>
<td>4678.3</td>
<td>-</td>
<td>-</td>
<td>No data</td>
</tr>
<tr>
<td>Start yielding long. reinf. #3 (web)</td>
<td>5402.4</td>
<td>7374</td>
<td>7562</td>
<td>No data</td>
</tr>
<tr>
<td>Crushing of concrete</td>
<td>6797</td>
<td>7502</td>
<td>7569</td>
<td>No data</td>
</tr>
<tr>
<td>Peak load</td>
<td>7279.8</td>
<td>7502</td>
<td>7569</td>
<td>6984</td>
</tr>
<tr>
<td>Failure mechanism</td>
<td>Shear-compression</td>
<td>Shear-Bending</td>
<td>Bending</td>
<td>Shear-compression</td>
</tr>
<tr>
<td>Computation time</td>
<td>5h</td>
<td>20 minutes</td>
<td>2 minutes</td>
<td></td>
</tr>
</tbody>
</table>
The initial elastic response predicted by the fiber beam models fits the results of the plane stress model; which are stiffer than the experimental behaviour. The fiber beam models present a ductile response with yielding of longitudinal reinforcement; the inclusion of shear effects diminishes the ultimate load. Transversal reinforcement remains elastic. Computation time diminishes significantly in comparison with the plane stress model.

**Convergence behaviour**
The energy criterion, with a tolerance of $1 \times 10^{-3}$, controls the global NR iteration procedure. It represents the energy norm versus the number of iterations throughout the nonlinear analysis.

**Strains and stresses**
For the case of the shear fiber beam model (L6BEA, STIRRU), the stresses in longitudinal reinforcement (active and passive) for peak load level are presented in Figure 5.6-9; where yielding is observed.

The longitudinal strains in concrete, in the top and bottom layers are presented in Figure 5.6-10 and Figure 5.6-11, respectively, for peak load. Stress levels in the top concrete layer in Figure 5.6-12 are near the compressive strength for which crushing occurs.
Figure 5.6-9: Case PB1. Stresses in longitudinal reinforcement for peak load (DIANA10, L6BEA STIRRU)

Figure 5.6-10: Case PB2. Longitudinal strains in concrete bottom fiber for peak load (DIANA10, L6BEA STIRRU)

Figure 5.6-11: Case PB2. Longitudinal strains in concrete top fiber for peak load (DIANA10, L6BEA STIRRU)
Figure 5.6-12: Case PB2. Longitudinal stresses in concrete bottom fiber for peak load (DIANA10, L6BEA STIRRU)

5.6.4 Concluding remarks

From the analysis of the PB2 test (benchmark failing in shear-compression) with fiber beam elements in DIANA and comparison with DIANA plane stress model (Hendriks, Belletti et al. 2015) and experimental data (Sun and Kuchma 2007) the following conclusions are pointed out:

- The shear fiber beam model is consistent with the plane stress model and with experimental observations in terms overall response;
- No yielding of transversal reinforcement was observed in this case, in contrast with plane stress model;
- Computation time is decreased to approximately 10% with the shear fiber beam elements in comparison with plane stress elements.
5.7 Case PB3/MnDOT: Runzell et al. (2007)

Case PB3 is a prestressed beam from the experiments of (Runzell, Shield et al. 2007), in which two ends of a 26.822 m girder removed from a bridge (Mn/DOT Bridge No. 73023) were tested. The dismantled bridge was around 20 years old; meaning that the girder was designed according to either the 1983 Standard or 1979 Interim specifications. The experimental tests were carried out in the University of Minnesota Structures Laboratory. The specimens were tested without and with the deck, referred in the report as Specimens I and II, respectively. PB3 relates to Specimen I (without the deck).

5.7.1 Experimental setup and results

The original length of the girder was 26.82 m. The specimens cut from the girder were 9.29 m as represented in Figure 5.7-1. The experimental test was designed in order to fulfill the laboratory characteristics and capacities in terms of maximum length and maximum load possible to be applied and also to meet the objective of studying the shear behavior and resistance of the girders. The experiment was designed in order to present a shear span-to-depth ratio of 2.7. The specimen was modified by increasing its length from 9.29 m to approximately 12.116 m by adding a cast-in-place beam extension onto the original prestressed beam as presented in Figure 5.7-2 and Figure 5.7-3. This cast-in-place beam extension was designed to resist the maximum bending moment and shear force possible to be applied in the lab.

Details on the decisions of the design of the experiments can be found in Runzell et al. (2007) and in Hendriks, Belletti et al (2014).

Figure 5.7-1: Case PB3. Girder of Mn/DOT Bridge No. 73023 (dimensions in m) (Hendriks, Belletti et al 2014)

Figure 5.7-2: Case PB3. Modified specimen (Runzell et al. 2007)
The geometry of the I-shape cross section is presented in Figure 5.7-4. The girder has 10 inclined and 33 straight strands of 13 mm diameter each. Prestressing stress was measured in laboratory, presenting a value after all losses of 864 N/mm². Details on the prestressing geometry, in terms of longitudinal and sectional configuration are presented in Figure 5.7-5 and Figure 5.7-6, respectively. The girder has no longitudinal passive reinforcement. In terms of transversal reinforcement there are double led stirrups of 13mm of diameter spaced of 531mm throughout the beam, with closer spacing at the left end of the girder, corresponding to the anchorage zone. The bridge deck presents two layers of 13mm longitudinal bars, spaced of 457mm (at the top) and 229mm (at the bottom), as presented in Figure 5.7-4b. The cross section geometry and reinforcement of the extension beam is presented in Figure 5.7-7.
Figure 5.7-6: Case PB3. (a) Prestressing Strand Pattern at Girder End, (b) Prestressing Strand Pattern at harp point (Runzell et al. 2007)

Figure 5.7-7: Case PB3. Geometry and reinforcement of the extension beam (dimensions in inches) (Runzell et al. 2007)

The loading and boundary conditions of the experimental setup are shown in Figure 5.7-8. The beam was submitted to a three-point loading scheme, with increasing load until failure. A diagonal tension failure mechanism was observed in the experimental test, involving crushing of concrete at web-bulb interface with spalling of concrete in the web, as can be seen in Figure 5.7-9. The first cracks were due to bending in bottom flange below the point of load application. Several large web cracks were formed for higher load levels.

Figure 5.7-8: Case PB3. Loading, boundary conditions and dimension (dimension in ft) (Runzell et al. 2007)
Figure 5.7-9: Case PB3. Failure mechanisms at peak load of Specimen I, (a) cracking, (b) web crushing (Runzell et al. 2007)

5.7.2 Beam finite element model in DIANA 10

The characteristics of the model are presented in Figure 5.7-10; the beam was discretized into 48 FE. The cross section was defined with zones divided in 7 fibers each. Two types of concrete cross section were considered; one for the I-shaped girder and other for the rectangular extension beam. No passive steel reinforcement was considered. Specimen I, without the top deck, was simulated. Transversal reinforcement is considered smeared in the cross-sections, with a constant quantity along the beam of 2Φ13//531mm. The web of the girder and the top flanges were considered shear resistant; the fibers of the bottom flange are non-shear resistant.

The 43 prestressing cables were assembled into 2 groups of bars with the same eccentricity, one linear and one with inclined configuration, modelled as prestressed reinforcement elements in DIANA. A prestressing force of P=85.337kN / cable was considered, referring to the reported stress in the cable at the time of testing (864MPa).

Regarding the material properties, the values given in the report for the concrete and steel mechanical properties were used in the model; the others were considered the same as in DIANA plane stress model (Hendriks, Belletti et al. 2015). The material properties used in the model are listed in Table 5.7-1 for concrete and in Table 5.7-2 for steel. Regarding the tension stiffening curve, the default parameters were used; k=0.5 and c=0.002. At lack of more information, the same concrete material properties were used for the girder and for the extension beam (DIANA 2D simulations considered elastic concrete material in the extension part of the beam). Lateral reduction through the equation of Vecchio and Collins (1986) was considered.

The beam is simple supported, with constraint nodes 2 and 48. The analysis was performed in two phases:

1) Dead weight and prestressing force of P=85.337x10=853.37kN for the group 1 with 10 cables and P=85.337x33=2816.13kN for group 2 with 33 cables;
2) Application of incremental point load in node 20 with automatic increments controlled by an arc-length procedure.

Energy tolerance considered was 1×10⁻³. Computation time is around 15 minutes.
Table 5.7-1: Case PB3. Constitutive properties for concrete

<table>
<thead>
<tr>
<th></th>
<th>( f_{cm} ) (N/mm(^2))</th>
<th>( f_{ctm}^* ) (N/mm(^2))</th>
<th>( E_c^* ) (N/mm(^2))</th>
<th>( \varepsilon_{cu}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean measured values</td>
<td>69.84</td>
<td>4.54</td>
<td>34819</td>
<td>0.0031</td>
</tr>
</tbody>
</table>

* assumed / determined values (when possible, same as DIANA plane stress, Hendriks, Belletti et al. 2014)

Table 5.7-2: Case PB3. Reinforcement properties.

<table>
<thead>
<tr>
<th>Bar</th>
<th>( \Phi ) (mm)</th>
<th>( A_e ) (mm(^2))</th>
<th>( E_e ) (N/mm(^2))</th>
<th>( f_{rm} ) (N/mm(^2))</th>
<th>( f_{um} ) (N/mm(^2))</th>
<th>( \varepsilon_{su}^* )</th>
<th>( E_{sy}^* ) (N/mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>strands ( \Phi 11.2 )</td>
<td>43x11.2</td>
<td>43x98.77</td>
<td>196500</td>
<td>1675</td>
<td>1862</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

assumed / determined values

5.7.3 Nonlinear finite element analyses

Load – deflection curve

The load – deflection curves are presented in Figure 5.7-11:
- DIANA L12BEA is the fiber beam model with bending behaviour;
- DIANA L12BEA STIRRU is the fiber beam model with shear capabilities;
- DIANA plane stress results from Hendriks et al 2014;
- The experimental curve from (Runzell et al. 2007).

Basic model CONSHEAR performed by Ferreira 2015, Report #1 is not included because it had an error.

The numerical deflections were corrected from the initial results related to the dead weight and application of prestressing forces.

The results of the different nonlinear models are resumed in Table 5.7-3.

The deflection of the beam for peak load in DIANA L6BEA STIRRU is presented in Figure 5.7-12.
Shear fiber beam models and plane stress models present very similar response until peak load. The shear fiber beam model is not able to continue to post-peak response, as divergence occurs in the last
load step. The ultimate load and failure mechanism is consistent with the experimental values. The shear fiber beam model reached failure by crushing of concrete and yielding of stirrups, while longitudinal prestressing remained elastic, representing a shear mechanism. The flexural fiber beam model predicted bending ductile failure for a higher load level, demonstrating the relevance of accounting for shear effects in this case.

**Convergence behaviour**
The energy criterion, with a tolerance of $1 \times 10^{-3}$, controls the global NR iteration procedure. Figure 5.7-13 represents the energy norm versus the number of iterations throughout the nonlinear analysis.

![Figure 5.7-13: Case PB3. Energy norm vs. global iterations at the NR level](image)

**Strains**
For the case of the shear fiber beam model (L6BEA, STIRRU), the stresses in longitudinal reinforcement (active) for peak load level are presented in Figure 5.7-14; where it can be observed that remains in elastic range.

The longitudinal strains in concrete, in the top and bottom layers are presented in Figure 5.7-15 and Figure 5.7-16, respectively, for peak load. Stress levels in the top concrete layer in are far from compressive ultimate strength.

These results demonstrate that the failure mechanism is not related to bending and shear effects are important.
Recalculations of benchmarks with DIANA

Figure 5.7-14: Case PB1. Stresses in longitudinal reinforcement for peak load (DIANA10, L6BEA STIRRU)

Figure 5.7-15: Case PB2. Longitudinal strains in concrete bottom fiber for peak load (DIANA10, L6BEA STIRRU)

Figure 5.7-16: Case PB2. Longitudinal strains in concrete top fiber for peak load (DIANA10, L6BEA STIRRU)
5.7.4 Concluding remarks

From the analysis of the PB3 test (benchmark failing in shear) with fiber beam elements in DIANA and comparison with DIANA plane stress model (Hendriks, Belletti et al. 2015) and experimental data (Runzell et al. 2007) the following conclusions are pointed out:

- Failure mechanism and peak load predicted by the shear fiber beam model is consistent with experimental observations and computations performed by the plane stress model;
- Computation time taken by shear fiber beam elements is approximately 25% of the plane stress elements.
- Shear effects are relevant in this case; disregarding it in the analysis results in important overestimation of peak load.
5.8 Cases of the DIANA User’s Contest 2014: large T-shaped prestressed concrete girders tested at TU Delft

A prediction contest for the results of T-shaped prestressed girders tested in the Stevin Laboratory of TU Delft was organized by TU Delft, Rijkswaterstaat and University of Parma. The blind predictions were submitted by the participants until August 2014, the experimental tests were carried out during autumn 2014 and the Workshop was held in Parma in November 5th 2014.

A blind prediction was carried out with the model CONSHAR for participating in the contest (work included in Report #1 of this project). This case was reproduced with the fiber beam models in DIANA.

5.8.1 Experimental setup and results

The experimental campaign consisted on 4 T-shaped girders, 2 mid beams (b_f=750mm) and 2 edge beams (b_f=875mm), casted in May 2012 (Figure 5.8-1). The width of the top flange is the only difference between the two types of girders. The girders have a total length of 12.0 m and a depth of 1.3 m. The geometry and general reinforcement of the girders are represented in Figure 5.8-2 and Figure 5.8-3.

Each girder is pre-tensioned with 24 strands of Φ15.7mm (150 mm² per strand) of steel type FeP1860. Concrete is of type C53/65. The measured prestressing force per strand before casting was of 214kN; and the mean cubic concrete compressive strength at time of prestressing was 54 MPa.

Passive reinforcement consisted of stirrups Φ10 mm distanced of 120 and 80 mm and 10xΦ8 mm longitudinal bars. The type of steel is B500A for the longitudinal passive reinforcement and B500B for the stirrups.

The test setup is represented in Figure 5.8-4: the loading jack is located at 2.950 m from the centre of the support. Data on the geometry, reinforcement, materials, support and loading systems were provided to the participants and available in www.dianausers.nl.

![Figure 5.8-1](Image)

**Figure 5.8-1**: Case of DIANA User’s Contest. T-shaped prestressed girders tested in the Stevin Laboratory at TU Delft (picture given in the contest, www.dianausers.nl)
Figure 5.8-2: Case of DIANA User’s Contest. Dimensions of the girders (draws given in the contest, www.dianausers.nl)

(a) Mid-beam
(b) Edge-beam
Edge cross section
Edge cross section
Mid cross section
Mid cross section
Recalculations of benchmarks with DIANA

**Figure 5.8-3**: Case of DIANA User’s Contest. Dimensions and reinforcement of the girders (draws given in the contest, www.dianausers.nl)

**Figure 5.8-4**: Case of DIANA User’s Contest. Experimental test setup

The questions of the contest were:
- Maximum (and) minimum load at failure;
- Failure mechanism;
- Cracking pattern at 75% and 100% of the failure load;
- Crack width at 75% of failure load;
- Load – displacement diagram at position of the load.

The failure mechanism was localized around the point of load application by crushing in compression of the empty ducts localized in the top flanges of the girders (see in Figure 5.8-1). These empty ducts acted as a weak spot that triggered the failure mechanism. The experimental results in terms of ultimate load, ultimate displacement and crack width are resumed in Table 5.8-1 (according to what was divulged in the Workshop in Parma, www.dianausers.nl). In the edge-beams it was observed a twist of the cross section that resulted in lower ultimate loads in comparison with the mid-beams.

**Table 5.8-1**: Case of DIANA User’s Contest. Experimental results divulged in Parma, November 5

<table>
<thead>
<tr>
<th></th>
<th>Mid-beam 1 (bf=750mm)</th>
<th>Mid-beam 2 (bf=750mm)</th>
<th>Edge-beam 1 (bf=875)</th>
<th>Edge-beam 2 (bf=875)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate load (kN)</td>
<td>2696</td>
<td>2540</td>
<td>2378</td>
<td>2575</td>
</tr>
<tr>
<td>Ultimate displacement (mm)</td>
<td>≈ 42</td>
<td>≈ 60</td>
<td>≈ 37</td>
<td>≈ 43</td>
</tr>
<tr>
<td>Failure mechanism</td>
<td>Bending local*</td>
<td>Bending local*</td>
<td>Bending local*</td>
<td>Bending local*</td>
</tr>
<tr>
<td>$w_{cr}$ (mm) (at 75% Pu)</td>
<td>0.10</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*Crushing compression flange, localized zone in the empty ducts (weak spot)*

5.8.2 **Beam finite element model in DIANA 10**

The characteristics of the model are presented in Figure 5.8-5: the beam was discretized into 54 FEs and the cross section was defined with zones with 40 fibers. The thickened web zone at the end of the beam was simulated by varying the web width of the cross section of the elements in that area. The beam is simple supported.

All the fibers in the flange where considered shear resistant. The transversal empty ducts in the flanges of the beams where not included in the numerical model. The passive reinforcement was accounted as constant along the entire beam: longitudinal reinforcement as 10xØ8 filaments and transversal reinforcement as Ø10/114.3mm (average of sequence 6Ø10/120 + 2Ø10/80, corresponding to a quantity of $\rho_{sw} = 0.92\%$). A prestressing force of $P=214$ kN was applied in the model.
Regarding the concrete properties, the average experimental properties provided for the contest are listed in Table 5.8-2. The inputs of the model refer to the values at 273 days of age.

For the steel properties the only information given was the class of the materials (B500B for passive reinforcement and FeP1860 for active reinforcement); there was no experimental data available. The mechanical properties considered in the model are listed in Table 5.8-3 for passive reinforcement and in Table 5.8-4 for active reinforcement. The average values were considered as 1.1 x characteristic values.

Girder I (mid beam) is symmetric and was simulated with 2D fiber beam elements in DIANA. Girder II (edge beam) is not symmetric and was simulated with 3D fiber beam models in DIANA.

The computation time is approximately 9 minutes.

Table 5.8-2: Case of DIANA User’s Contest 2014. Experimental mechanical properties of concrete provided in the contest and considered in the model

<table>
<thead>
<tr>
<th>Experimental properties</th>
<th>t = 28 days</th>
<th>t = 273 days (9 months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{cm,cube}$ (MPa)</td>
<td>83</td>
<td>89.83</td>
</tr>
<tr>
<td>$f_{cm,cylinder}$ (MPa)</td>
<td>68</td>
<td>74.83</td>
</tr>
<tr>
<td>$f_{ck,cube}$ (MPa)</td>
<td>77</td>
<td>-</td>
</tr>
<tr>
<td>$f_{ck,cylinder}$ (MPa)</td>
<td>62</td>
<td>-</td>
</tr>
<tr>
<td>$f_{ctm,sp}$ (MPa)</td>
<td>-</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Table 5.8-3: Case of DIANA User’s Contest 2014. Mechanical properties of steel considered in the model (passive reinforcement (Ø8, Ø10) - B500B)

<table>
<thead>
<tr>
<th>Properties in the model</th>
<th>Characteristic values</th>
<th>Average values (1.1fs,m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{sy}$ (MPa)</td>
<td>500</td>
<td>550</td>
</tr>
<tr>
<td>$f_{su}$ (MPa)</td>
<td>540</td>
<td>594</td>
</tr>
<tr>
<td>$E_s$ (GPa)</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>$\varepsilon_{su}$</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Table 5.8-4: Case of DIANA User’s Contest 2014. Mechanical properties of steel considered in the model (active reinforcement (Ø15.7, 150mm2) – FeP1860)

<table>
<thead>
<tr>
<th>Properties in the model</th>
<th>Characteristic values</th>
<th>Average values (1.1fp,m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_y (MPa)</td>
<td>1640</td>
<td>1804</td>
</tr>
<tr>
<td>f_u (MPa)</td>
<td>1860</td>
<td>2046</td>
</tr>
<tr>
<td>E_s (GPa)</td>
<td>195</td>
<td>195</td>
</tr>
<tr>
<td>ε_u</td>
<td>0.025</td>
<td>0.025</td>
</tr>
</tbody>
</table>

5.8.3 Nonlinear finite element analyses

The load – deflection curves for the Girder 1 (mid-beam, code 101) for original load position are presented in Figure 5.8-6:
- DIANA L6BEA is the fiber beam model with bending behaviour;
- DIANA L6BEA STIRRU is the fiber beam model with shear capabilities;
- Basic model CONSHEAR performed by Ferreira 2015, Report #1;
- The experimental results at failure.

The load – deflection curves for the Girder 1 (mid-beam, code 101) for second load position are presented in Figure 5.8-7:
- DIANA L6BEA is the fiber beam model with bending behaviour;
- DIANA L6BEA STIRRU is the fiber beam model with shear capabilities;
- Basic model CONSHEAR performed by Ferreira 2015, Report #1;
- The experimental results at failure.

The load – deflection curves for the Girder 2 (edge-beam, code 201) are presented in Figure 5.8-8:
- DIANA L12BEA is the fiber beam model with bending behaviour;
- Basic model CONSHEAR performed by Ferreira 2015, Report #1.

The initial displacements due to prestressing and dead weight were discounted.
Figure 5.8-6: Case of DIANA User’s Contest 2014. Load-deflection curves for Girder I (mid-beam, code 101, bf=750mm) (original load position)

Figure 5.8-7: Case of DIANA User’s Contest 2014. Load-deflection curves for Girder I (mid-beam, code 101, bf=750mm) (new load position)
The numerical predictions are consistent with the available experimental data on the ultimate failure stage. The beam models predicted a flexural-shear failure mechanism including yielding of bottom longitudinal prestressed reinforcement (strands) near the area of load application, concrete crush in the web at shear span, yielding of stirrups in the web at shear span, severe diagonal cracking, and failure of the stirrups.

Information about peak loads, failure mechanisms and load levels for the start of yielding of transversal reinforcement are listed in Table 5.8.5. The transversal reinforcements yields for lower load levels in the case of the new position of the applied load, as shear forces increase due to the decrease of the shear span. This is captured by the shear fiber beam model. The flexural fiber beam model predicts pure bending failure mechanism.

**Table 5.8-5**: Case of DIANA User’s Contest. Results of the NLFEAs and experimental data (kN)

<table>
<thead>
<tr>
<th>Girder I, mid-beam (original position load)</th>
<th>DIANA10 L6BEA STIRRU</th>
<th>L6BEA</th>
<th>$P_{u,exp}$ (kN) (average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yielding stirrups</td>
<td>2840</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Peak load</td>
<td>2880</td>
<td>2880</td>
<td>2617</td>
</tr>
<tr>
<td>Failure mechanism</td>
<td>Flexural-shear</td>
<td>Flexural</td>
<td>Local</td>
</tr>
<tr>
<td>Computation time</td>
<td>9 minutes</td>
<td>3 minutes</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Girder I, mid-beam (new position load)</th>
<th>DIANA10 L6BEA STIRRU</th>
<th>L6BEA</th>
<th>$P_{u,exp}$ (kN) (average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yielding stirrups</td>
<td>2600</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Peak load</td>
<td>3320</td>
<td>3360</td>
<td>-</td>
</tr>
<tr>
<td>Failure mechanism</td>
<td>Flexural-shear</td>
<td>Flexural</td>
<td>Local</td>
</tr>
<tr>
<td>Computation time</td>
<td>9 minutes</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
5.8.4 Concluding remarks

From the analyses of the T-shaped prestressed girders and comparison with the experimental data provided in the Workshop, the following conclusions are drawn:

- These cases present a bending-related failure mechanism; hence, there is little difference in consider or not shear effects in the failure loads and displacements computed by the fiber beam models.
- Numerical ultimate load and displacement levels are coherent with the experimental observations.
- Shear fiber beam models predict yielding of transversal reinforcement.
- Experimental failure mechanism was localized around the point of load application by crushing of the empty ducts.
- The empty ducts were not accounted in the model; the fiber beam model is not able to capture localized failure modes; for that it is needed to go to 2D and 3D analyses;
6 CONCLUSIONS

This report resumes the theoretical formulation of the shear-sensitive fiber beam model, explains its implementation in the DIANA 10 and outlines the requirements for pre-and post-processing in Diana Interactive Environmental.

The verification of the new implementation is performed by a systematic comparison between computations of the new beam model (with and without consideration of shear effects) and experimental data for various cases of beam tests. The comparison was made in terms of load-displacement curves until failure, failure mechanisms and development of damage.

The analysed benchmarks include the cases already analysed with DIANA plane stress models by Hendriks, Belletti et al. 2015 and with the basis fiber beam model in Report #1 of this project. Results of the computations made with DIANA plane stress models were included in this report for comparison purposes.

Table 6-1 presents a resume of results at failure, comparing numerical (from DIANA fiber beam and plane stress models) and experimental data. The failure mechanism is presented with a symbol representing its fitting with the experimental observations: ✓ in case of correct fitting, ✗ in case of incorrect fitting. Shear DC stands for shear diagonal compression, and Shear DT means shear diagonal tension.

<table>
<thead>
<tr>
<th>Case study</th>
<th>Results at failure</th>
<th>DIANA10 Fiber beam STIRRU</th>
<th>DIANA Plane stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>RB1: Vecchio &amp; Shim 2004</td>
<td>Load ($P_{u,num}/P_{u,exp}$)</td>
<td>1.01</td>
<td>1.05</td>
</tr>
<tr>
<td>Mechanism</td>
<td>Bending ✓</td>
<td>Bending ✓</td>
<td></td>
</tr>
<tr>
<td>Computation time</td>
<td>8 minutes</td>
<td>1h</td>
<td></td>
</tr>
<tr>
<td>RB2: Collins &amp; Kuchma 1999</td>
<td>Load ($P_{u,num}/P_{u,exp}$)</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>Mechanism</td>
<td>Shear DC ✓</td>
<td>Shear DC ✓</td>
<td></td>
</tr>
<tr>
<td>Computation time</td>
<td>10 minutes</td>
<td>2h</td>
<td></td>
</tr>
<tr>
<td>RB3: Grace 2001</td>
<td>Load ($P_{u,num}/P_{u,exp}$)</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>Mechanism</td>
<td>Bending ✓</td>
<td>Bending ✓</td>
<td></td>
</tr>
<tr>
<td>Computation time</td>
<td>10 minutes</td>
<td>1h</td>
<td></td>
</tr>
<tr>
<td>RB3A: Grace 2001</td>
<td>Load ($P_{u,num}/P_{u,exp}$)</td>
<td>0.91</td>
<td>0.94</td>
</tr>
<tr>
<td>Mechanism</td>
<td>Shear-bending ✓</td>
<td>Shear-bending ✓</td>
<td></td>
</tr>
<tr>
<td>Computation time</td>
<td>12 minutes</td>
<td>1h</td>
<td></td>
</tr>
<tr>
<td>PB1: Leonhardt, Koch et al. 2007</td>
<td>Load ($P_{u,num}/P_{u,exp}$)</td>
<td>1.05</td>
<td>1.03</td>
</tr>
<tr>
<td>Mechanism</td>
<td>Bending ✓</td>
<td>Bending ✓</td>
<td></td>
</tr>
<tr>
<td>Computation time</td>
<td>9 minutes</td>
<td>1h30min</td>
<td></td>
</tr>
<tr>
<td>PB2/NSEL: Sun and Kuchma 2007</td>
<td>Load ($P_{u,num}/P_{u,exp}$)</td>
<td>1.07</td>
<td>1.04</td>
</tr>
<tr>
<td>Mechanism</td>
<td>Shear ✓</td>
<td>Shear DC ✓</td>
<td></td>
</tr>
<tr>
<td>Computation time</td>
<td>20 minutes</td>
<td>5h</td>
<td></td>
</tr>
<tr>
<td>PB3/MnDOT: Runzell et al. 2007</td>
<td>Load ($P_{u,num}/P_{u,exp}$)</td>
<td>0.83</td>
<td>0.91</td>
</tr>
<tr>
<td>Mechanism</td>
<td>Shear ✓</td>
<td>Shear DT ✓</td>
<td></td>
</tr>
<tr>
<td>Computation time</td>
<td>15 minutes</td>
<td>1h30min</td>
<td></td>
</tr>
<tr>
<td>DIANA User’s Contest (1) Mid-beam</td>
<td>Load ($P_{u,num}/P_{u,exp}$)</td>
<td>1.1</td>
<td>-</td>
</tr>
<tr>
<td>Mechanism (local)</td>
<td>Shear-bending ✗</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
From the several analyses performed, the following general conclusions are drawn:

- Computation time decreases significantly with the fiber beam models, in average 85% in comparison with the correspondent plane stress models in DIANA.
- Even for the bending cases, the use of the shear fiber beam models in DIANA allows to compute the state of the transversal reinforcement and to include the bending-shear interaction in the response, with the consequent increment of strains in the longitudinal reinforcement due to shear force.
- For the shear cases, the use of standard beam models results into computations detached from the structural behaviour, as shear failure mechanism is not represented. Hence, ultimate load capacities are overestimated.
- Shear fiber beam models are able to reproduce with an acceptable accuracy the response of reinforced and prestressed beams with bending and shear failure mechanisms.

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REFERENCES


