Salt intrusion in the river Maputo; field survey and one-dimensional modelling

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ABSTRACT

The aim of the study was to give an indication of the accuracy and applicability of one-dimensional salt intrusion models in estuaries in Mozambique. Attention is mainly centered on tidally averaged models, although some preliminary testing of a real-time model is included as well. Testing of the models is done with data, obtained from a field study in the river Maputo in Mozambique.

The results show that, notwithstanding the many limitations, tidally averaged, one-dimensional models can be an indicatory tool for description of salt intrusion in the estuary considered. Furthermore the study reveals that description of dispersion mechanisms, solely on the basis of theoretical considerations, is still not feasible but always needs some sort of empirical relationship. It was found that during preliminary investigation of estuaries more emphasis has to be laid on measuring salinity distributions under varying circumstances in the estuary than on detailed measuring of the geometry.
CONTENTS

I. INTRODUCTION

II. THEORETICAL BACKGROUNDS OF SALT-INTRUSION
   II.1 Introduction
   II.2 Salt balance equations
   II.3 Causes of dispersion
   II.4 Dispersion in estuaries; dispersion coefficient values

III. ONE-DIMENSIONAL SALT INTRUSION MODELS
   III.1 Introduction
   III.2 Tidally averaged models
   III.3 High water slack/Low water slack models
   III.4 Real-time models
   III.5 Selection of models for further testing

IV. SURVEY RESULTS OF THE RIVER MAPUTO
   IV.0 General description of the river
   IV.1 River geometry
   IV.2 Tidal Phenomena
   IV.3 Fresh water discharges
   IV.4 Salinity measurements

V. CALIBRATION AND TESTING OF THE MODELS
   V.1 Introduction
   V.2 Van der Burgh model
   V.3 Combined model
   V.4 Penpas model

VI. COMPARISON OF THE MODELS

VII. CONCLUSIONS AND RECOMMENDATIONS

NOTATION

REFERENCES
APPENDICES

I. Classification of estuaries; estuary parameters
II. Tidal analysis in Maputo Bay
III. Tidal prism computation
IV. Geometry input Penpas model
V. Measuring instruments used
VI. Method of Heun
VII. Numerical methods Penpas model

FIGURES

READERS' GUIDE

All symbols used in the report can be found after Chapter VI. Two types of figures are used in the report:

- figures within the text parts. Numbers of these figures always start with the chapter number in Roman numerals (I, II, VI and so on).

- figures left out of the text parts (mainly presentations of measuring results, maps and so on). This type has numbers, not preceded by a Roman numeral. These figures can be found at the end of the report.
I. INTRODUCTION

All over the world rivers flow through countries and end in seas and oceans. The water in seas and oceans is always salt, the salinity ranging from 25 - 35 % whereas the discharge coming down the river is fresh. This implies there is a mixing or transition zone at the downstream end of a river where salt and fresh water come together. Part of the fresh water discharge is quite often used by the population for various purposes. The purposes we are interested in, in this report, are those posing limits to the salt content, i.e., irrigation and drinking water supply. For this use of the river water it is important to be able to predict the length of the fresh-salt-transition zone and the salinity distribution within this zone. Investing in expensive irrigation schemes or pumping stations is never wise unless some reasonably accurate predictions can be provided about

   a. the point the salt will reach under specific extreme natural circumstances (e.g., a dry period of a number of months). Usually these circumstances have never been adequately investigated.

   b. the changes in the salt-intrusion which will take place due to the lowering of the discharge by extractions for irrigation or drinking water supply (minus the increased drainage caused by the irrigation).

In order to obtain predictions of salinity intrusion under circumstances that have not yet been observed, we normally turn to models in order to simulate reality. One class of models uses a, - to a certain extent simplified-, on-scale reproduction of the river at hand, in order to simulate future circumstances. These models are named scale-models. Another class limits itself to a description of the stream area by means of mathematical equations. This class is referred to as mathematical models. In this report we will limit ourselves to one group of models only, the one-dimensional (cross-sectionally averaged) mathematical type. One of such one-dimensional models was tested provisionally by the Mozambiquean Water Service. Results were reasonable in a few estuaries but questions were raised at its accuracy and applicability in other rivers. The present study is intended to investigate the subjects mentioned above for one-dimensional models and to hand further recommendations concerning such models to the Water Service in Maputo, Mozambique.
A field study was set up in one of the Mozambiquean estuaries to provide data for the testing of a few one-dimensional models. The river selected for this purpose was the river Maputo, the most southern river in the country. The reasons for this were mainly

- some measurements were already performed in this estuary
- the estuary is quite easily attainable for cars and (small) measuring boats point
- the river has a quite simple geometry, without large side-bays or tributaries for the first 90 km

The Maputo river is a not too large river with fresh water discharges ranging from zero to approximately 300 m³/s. In the wet season the river is often stratified or partially mixed, in the dry season it usually changes into a well-mixed state.

The field-survey took place from March up to June 1984, including a transition period from wet to dry season. During the measuring campaign attention was paid to the various aspects of the river, like the geometry, the tidal regime in the whole estuary, the fresh water discharges and the salt distributions along the river axis, and within cross-sections.

The data obtained was used as input and testing data for a few one-dimensional models. In this study the emphasis is laid on testing of two tidally averaged models, one with a semi-empirical relationship for the salt dispersion mechanisms (Van der Burgh model), the other with a little more theoretical basis (designated here as the Combined model). The tidally averaged models run with tidal mean variables, leaving out the motions and salinity variations within a tidal cycle.

Besides the two tidally averaged models, some preliminary tests were done, using a real-time salt intrusion model developed by the Delft Hydraulics' Laboratory, the Netherlands, with the same data (Penpas model). Real-time models do not average the variables over a tidal cycle but are able to compute variations during a tidal cycle.

The first part of the report contains a description of the various mechanisms of salt intrusion, the mathematical equations used to describe them and the resulting salt-intrusion model, based on these equations (chapters II and III).

After that, the field study results and the calibration and testing of three models are presented (chapter IV and V). Chapter VI deals with the comparison of the various models and the possibilities to apply them in other estuaries. Conclusions are given in chapter VII.
CHAPTER II  THEORETICAL BACKGROUND OF SALT INTRUSION

II.1. SALT INTRUSION MECHANISMS IN ESTUARIES

Basically movement of salt in water is caused by two mechanisms:

- molecular diffusion
- transport by local, instanteneous velocities

Real estuary-flows are always turbulent because of the very large Reynolds numbers involved. If mixable fluids are present, like fresh and salt water, molecular diffusion is always negligible compared to the velocity transport, the advection. From now on it is therefore assumed that any movement of salt results from the watermotion. This implies that if we talk about the causes of mixing or dispersion of salt we basically talk about the causes of watermotion. Later on we will see that a specific distribution of the salt is by itself one of the causes of watermotion. Thus the interactions between salt distribution and watermotion have to be studied.

Unfortunately it is not yet possible to describe any turbulent water motion in full detail. Even in steady uniform flow in a channel with smooth, rectangular cross-section, turbulence causes an unpredictable component of the local, instantaneous velocity profile, let alone if we turn to natural streams with irregular shaped cross-sections, varying roughness parameters, tidal flows, wind influences and so on.

We therefore have to take refuge in describing mean flows and everything that is left out is lumped together in parameterized transport terms using coefficients. Such coefficients can never be predicted in a 100% accurate way, but if we analyze what part of the actual motion we exclude by averaging over certain timespans or length-scales, it might be possible to arrive at some indicatory values. Nevertheless, measurements are needed to calibrate every transport model.

In this chapter the first thing we will do is to have a closer look at the salt balance in an estuary. With this salt balance we will be able to identify the various
mechanisms of salt transport (section II.2). In section II.3 the various mechanisms of salt transport will be covered and theoretical and empirical formulae will be provided to give an indication of the magnitude of the various transports. In section II.4 we will turn to the contribution of the various mechanisms to the dispersion coefficients in real estuaries, as obtained from various research studies.
II.2. SALT BALANCE EQUATIONS

Intrusion of salt into estuaries can be described by salt balance equations that cover all ingoing and outgoing movement of salt at a specific point. With these equations we can also identify and compare mechanisms of salt mixing and intrusion. Therefore we will have a closer look at salt balances in this section and derive simplified equations in case certain factors can be simplified or averaged.

The general salt balance equation of an elementary volume element is:

$$\frac{dc}{dt} + \frac{dc}{dx} + \frac{dc}{dy} + \frac{dc}{dz} = 0 \quad \text{II.1}$$

whereby $u, v, w$ are the $x, y, z$ components respectively of the local, instantaneous velocity and $c$ is the salinity. As these instantaneous velocities are very difficult to work with, because of the large and rapid, turbulent fluctuations the first step is usually to divide the velocity in a turbulent mean and a fluctuating part

$$u = \bar{u} + u'$$
$$v = \bar{v} + v'$$
$$w = \bar{w} + w'$$
$$c = \bar{c} + c'$$

= averaged over a timespan $\Delta t$, whereby $\Delta t$ is the time needed to filter out turbulent fluctuations

Eq. II.1 becomes

$$\frac{dc}{dt} + \frac{d\bar{u}c}{dx} + \frac{d\bar{v}c}{dy} + \frac{d\bar{w}c}{dz} = - \frac{d\bar{u}'c'}{dx} - \frac{d\bar{v}'c'}{dy} - \frac{d\bar{w}'c'}{dz} \quad \text{II.2}$$

For the turbulent fluctuating expressions can be written, in
analogy with Fick's law for molecular diffusion (Fick, 1882)

\[
-u'c' = \xi_{xx} \frac{\partial c}{\partial x} + \xi_{xy} \frac{\partial c}{\partial y} + \xi_{xz} \frac{\partial c}{\partial z}
\]

\[
-v'c' = \xi_{yx} \frac{\partial c}{\partial x} + \xi_{yy} \frac{\partial c}{\partial y} + \xi_{yz} \frac{\partial c}{\partial z}
\]

\[
-w'c' = \xi_{zx} \frac{\partial c}{\partial x} + \xi_{zy} \frac{\partial c}{\partial y} + \xi_{zz} \frac{\partial c}{\partial z}
\]

where \( \xi_{ij} \) is a turbulent diffusion tensor

If the main flow directions are parallel to the \( x, y, z \)-axes this can be reduced to

\[
-u'c' = \xi_{xx} \frac{\partial c}{\partial x}
\]

\[
-v'c' = \xi_{yy} \frac{\partial c}{\partial y}
\]

\[
-w'c' = \xi_{zz} \frac{\partial c}{\partial z}
\]

If the river axis is taken as the \( x \)-axis the coefficients can be written as

\[ \xi_{xx} = \xi_{\text{longitudinal}} = \xi_l \]

\[ \xi_{yy} = \xi_{\text{transverse}} = \xi_t \]

\[ \xi_{zz} = \xi_{\text{vertical}} = \xi_v \]

These coefficients are referred to as turbulent diffusivities, as they describe the small scale mixing due to turbulent eddies.

The equation for the salt balance is now

\[
\frac{\partial c}{\partial t} + \frac{\partial (u \cdot c)}{\partial x} + \frac{\partial (v \cdot c)}{\partial y} + \frac{\partial (w \cdot c)}{\partial z} = \frac{\partial}{\partial x} \left( \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial c}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial c}{\partial z} \right) \]

The time derivative now refers to low frequent, non-turbulent variations in time.

In this study we limit ourselves to one-dimensional representations of the river-flow. This implies that we integrate the three dimensional equation over the depth and width in order to arrive at an equation with only \( x \) and \( t \) as independent variables

\[
\frac{d}{dt} \left( \int_A c \cdot dA \right) + \frac{d}{dx} \left( \int_B u \cdot c \cdot dA \right) = \frac{d}{dx} \left( \int_A \xi_l \cdot \frac{\partial c}{\partial x} \cdot dA \right) \]
we put \( \int \tilde{z} \, dA = A \tilde{z} \) \\
\( \sim = \) average over cross-section \\
and \( \int \tilde{u} \tilde{z} \, dA = A \tilde{u} \tilde{z} + \int (\tilde{u} - \bar{u}) (\tilde{z} - \bar{z}) \, dA \)

The equation is then rewritten as

\[
\frac{d}{dt} (A \tilde{z}) + \frac{d}{dx} (A \tilde{u} \tilde{z}) = - \frac{d}{dx} \left( \tilde{u} \tilde{z} \right) dA + \frac{d}{dx} \left( A \tilde{u} \frac{d\tilde{z}}{dx} \right).
\]

The terms on the right hand side of this equation are usually difficult to determine and are therefore usually lumped together to give

\[
\frac{d}{dt} (A \tilde{z}) + \frac{d}{dx} (A \tilde{u} \tilde{z}) = \frac{d}{dx} \left( A D_x \frac{d\tilde{z}}{dx} \right).
\]

Where \( D_x \) is a longitudinal dispersion coefficient

This simplification implies that the longitudinal dispersion term on the right hand side of II.8 represents the phenomena

- turbulent transports
- transports originating from the averaging over the cross-section

In the next section the separate mechanisms of salt intrusion and mixing will be covered and an indication will be given of the relative magnitude of the various contributions.

As we are quite often not interested in the to and fro movement of the tide, but want to know what the maximum intrusion will be or the tidal average, we can modify II.8 to our needs. In case of the tidal average interest for instance we can average II.8 over a tidal cycle period, \( T \), resulting in (in case of steady state, so no changes from one tidal cycle to another)

\[
\frac{d}{dx} < A \tilde{u} \tilde{z} > = \frac{d}{dx} < A D_x \frac{d\tilde{z}}{dx} > \quad < > = \text{tidal average}
\]

or

\[
< \tilde{u} > \cdot < \tilde{z} > = D_{xt} \frac{d}{dx} < \tilde{z} >
\]

In II.9 the influence of the tidal averaging is again lumped in a dispersion coefficient, \( D_{xt} \). This \( D_{xt} \) is therefore not equal to \( D_x \) in formula II.8.
Not all estuaries can be represented by one-dimensional models. One dimensionality mainly demands

1. no large variations over the cross-section of the salinity

2. the $v$ and the $w$ must be close to zero, meaning that the river indeed flows in the $x$-axis direction

3. no large variations over the cross-section of the velocity component in $x$-direction, $u$.

Regarding the demands for one-dimensional flow we can state some of the prerequisites to be able to represent an estuary as a one-dimensional flow system

- salt must be well-mixed over the cross-section. Estuaries with salt wedges, which quite often appear, are not suited for a one-dimensional approach. In this case it is better to use a two-dimensional model with a vertical and a longitudinal axis ($x$ and $z$-axis). In such cases with large depth variations over the width even a two-dimensional model will not suffice any more.

- estuaries must not be too wide. Too wide estuaries usually do not have a one-dimensional flow pattern any more owing to depth differences (gulleys, shoals), windforces and density differences. Sometimes it is still possible to represent a very wide estuary in two or more gulleys with an additional storage area. This means we divide the river into more than one, separate one-dimensional flows.

It is difficult to say what is "too" wide for a single channel representation since this depends on many factors like type and number of gulleys, ebband flood-flow streamlines, magnitude of discharges, tidal amplitudes and velocities, upstream storage areas and so on. Every (part of an) estuary has to be considered separately, and one has to resort to measurements in the estuary under consideration. In case of too wide estuaries two-dimensional models with longitudinal and lateral axes are most suitable in well mixed estuaries, but again lose most of their value when two or more layers of distinct salinities develop.
II.3. CAUSES OF DISPERSION

The causes of dispersion of salt in estuaries will be discussed for vertical, lateral and longitudinal directions respectively. Although most models of (not too wide and reasonably well mixed) estuaries are based on one-dimensional analysis of salt intrusion it is enlightening to see what happens within a cross-section. With this knowledge in mind we will be able to determine whether such a reduction towards one dimension is allowed and what is the influence of the transverse and vertical mixing on the one dimensional dispersion coefficients, $D_x$ and $D_{xy}$.

Besides, we will indicate how the main mechanisms of salt intrusion and mixing are caused and in what types of estuaries they will be important.

II.3.1. cross-sectional mixing

In straight rivers of constant depth with a steady, uniform flow and without any tidal or densimetric influences, mixing is caused by the turbulent motions, generated by bottom and bank friction. To be able to quantify some factors we assume that the mixing can be described by the turbulent diffusivities of section II.2, $\varepsilon_v$, $\varepsilon_f$, and $\varepsilon_t$.

From experimental studies with flumes [12] it is found that in the above mentioned case with a logarithmic velocity profile over the depth the $\varepsilon_v$ amounts to

$$\varepsilon_v = 0.067 \cdot a \cdot u^*$$

$$u^* = \sqrt{\frac{\tau_b}{\rho}}$$

II.10.

$\tau_b$: bottom shear stress, $\rho$: density

$u^*$: shear velocity, $a$: depth

A similar expression was found in rectangular, straight laboratory channels for the transverse mixing coefficient

$$\varepsilon_t \approx 0.15 \cdot a \cdot u^*$$

II.11.

although the coefficient varies over a range of approximately 50%. The $\varepsilon_v$ and $\varepsilon_t$ do not have the same value as the turbulence is non-isotropic.

Natural streams, still without any tidal or densimetric
influences -, are different because of

- irregularly varying cross-section (bank irregularities, changes in depth)

- channel curves in longitudinal direction and so a constantly changing velocity profile over the cross-section

Measurements prove that these factors do not have much influence on the value of $\varepsilon_v$, but they certainly have an impact on $\varepsilon_t$, as they cause a variety of transverse motions. The range of $\varepsilon_t$ in natural streams is more like [2]

$$\varepsilon_t \approx 0.6 \cdot a \cdot u^* \pm 50\%$$

II.12.

Everything turns out to be even more complicated if the river has tidal flows, with water movement alternatingly in upstream and downstream direction.

In case only constant density flows are regarded, the vertical mixing is again, predominantly caused by turbulence, generated by bottom shear stresses and we would expect equation II.10. to be valid. In this case however the $u^*$ varies from nearly zero at slack tide to a maximum at the time of the highest tidal velocity. Since a shear flow velocity is difficult to measure in an unsteady flow some research-workers relate the $\varepsilon_v$ to the depth mean velocity amplitude, $u_3$, in this case [13]

$$\varepsilon_v \approx 0.0025 \cdot a \cdot u_3$$

II.13.

In transverse direction also other effects play a role, when tidal movement is introduced. Those effects, like pumping, trapping (see definitions in section II.3.2.b) and wind influences all cause extra transverse mixing of salt. Values taken from on-site measurements in constant density reaches of estuaries show

$$\varepsilon_t \approx (0.45 \text{ to } 1.60) \cdot a \cdot u^*$$

II.14.

The last factor on cross-sectional mixing that will be discussed briefly is the stratification of the depth into layers of different densities (i.e. salinities). In case an estuary is completely at rest the natural situation would be that the heavier salt water of sea or ocean lays on the bottom and the less dense, fresh water from tributaries for instance is situated on top of that. It can be understood that whenever one or both layers start to move the interface between them will be disturbed and because of the shear generated, mixing between the layers occurs. The increase in potential energy of the system suppresses the turbulence and therefore the motions of the
water particles will be reduced in a vertical way. This implies reduction of the salt mixing over the depth. Indeed from on-site measurements in salt-wedged estuaries (e.g. [14]) can be seen that \( f_v \) is reduced to about 0.1 to 0.01 of its value in non-stratified flows.

The transverse mixing, however, is usually enhanced by stratification [15]. Especially in case of rough side walls or of variations in roughness within one cross-section transverse circulations are driven by stratification. This is caused by the so called transverse gravitational circulation, discussed in section II.3.2.c. So far, however, only experiments in test flumes have been carried out; experimental verification in real estuaries is not yet available.

One-dimensional models all assume well-mixed circumstances over the cross-section. High vertical and transversal mixing coefficients shape good conditions for this of course. They provide an indication how fast the salinity can be smoothed out over the cross-section in case of changes (tide moving in, river discharge coming down). Another way to reach a well mixed situation is the increase of the timespan in which the flow is approximately steady. Even with small mixing coefficients a substance can be mixed quite well over the cross-section if the flow remains steady for a long enough period. In case of tidal estuaries, this steady mixing period is naturally imposed by the tidal cycle period.

Some researchers have tried to find a relationship for the time that is required for a concentration profile to adopt itself to a new velocity profile. In [2] the following formula is mentioned:

\[
T_c = \frac{X^2}{D} \quad \text{(II.15)}
\]

\( D \) is the overall dispersion coefficient in the direction of mixing.
\( X \) is a characteristic length over which mixing is viewed.

For the Maputo estuary (under well mixed circumstances)

mixing over depth:
\[
X \approx a \approx 2.5 - 4.0 \text{ m} \\
D_v \approx f_v \approx 0.005 - 0.02 \text{ m}^2/\text{s}
\]

\( T_c \approx 300 - 3000 \text{ s} \)

mixing over width:
\[
X \approx B \approx 100 - 500 \text{ m} \\
D_t \approx \ell_t \approx 0.01 - 0.05 \text{ m}^3/\text{s}
\]

\( T_c \approx 3000 - 10000 \text{ s} \)

\( \ell_t \) is the characteristic length over which mixing is viewed.
II.3.2. longitudinal dispersion

In this section several causes of salt mixing in longitudinal direction will be discussed, being

a. shear flow effects
b. tidal movement
c. density differences
d. wind

II.3.2.a shear flow effects

Like in the discussion of the cross-sectional mixing we first consider the mixing in case of a steady, uniform flow in a straight, smooth channel. In this case the turbulent mixing coefficient $\varepsilon_L$ will, most probably be in the same order of magnitude as the $\varepsilon_T$ for transverse mixing, although some actual measurements tend to produce higher values [2]. In any case mixing in longitudinal direction by turbulent eddies is generally unimportant because of the dominancy of other factors ($\varepsilon_L \ll \varepsilon_T$).

One of those important factors is the dispersion caused by the shear flow (shear flow being defined as flow with velocity gradients), causing particles at different positions in the cross-section to move at different speeds and so spreading out in a longitudinal direction (fig. II.2).

![Diagram](image)

fig.II.2.

A well-known, theoretically derived formula, describing the
dispersion, assuming a logarithmic velocity profile is (Elder, [16])

\[ D_x = 5.93 \cdot a.u^x \]

Further studies proved, however, that this derivation is not correct, not even as a first estimate in real rivers. In natural streams values have been found [2] in the range of

\[ \frac{D_x}{a.u^x} = 20 - 7500 \]

The reason for this appears to be the variation of velocity in transverse direction in the cross-section. Besides the variation of the \( u \)-component of the velocity in the vertical direction, as was covered by Elder [16], there is often a more pronounced variation in transverse direction in natural streams. This is caused by irregularities, bends, man-made structures etc. From the theoretical derivations of Elder we can see that the longitudinal dispersion coefficient is proportional to the square of the distance over which the shear profile extends, so for the transverse and vertical directions the width and depth respectively. This implies that the contribution of the transverse velocity profile is often much more important than the vertical one since width to depth ratios in rivers usually range from 10 to over 100.

In real streams there are of course also more irregular riverbanks and bottoms due to bends, dead zones, sandbars or -dunes, bridgepiers and so on. Every irregularity, introducing velocity differences, contributes to the dispersion, the spreading of salt in the longitudinal direction. Therefore estimates in real streams are difficult to make.

As the value of this longitudinal dispersion coefficient in non-tidal reaches of rivers can be quite important (eg. in studies to spreading of pollutants) quite some efforts have been undertaken to find a formula, predicting the \( D \) on the basis of simple cross-sectional mean parameters. Two of those attempts will be mentioned here.

The first [24] is only valid in case the tidal flows are quite small (<1.0 m/s) and the estuary has a simple cross-sectional shape along its length. Harleman then gives an estimate of \( D_x \) in homogeneous tidal zones of rivers and estuaries [24]

\[ D_x = 6.93 \cdot n \cdot \bar{u} \cdot R^{5/6} \]

\( n \) = Manning's coefficient of roughness
R = hydraulic flow radius
\( \bar{U} \) = tidal mean flow

The formula is based on a logarithmic distribution of flow velocities.

Fischer [17] arrived at

\[ D_x = 0.001 (\bar{u})^2 \frac{B^2}{(a \cdot \bar{u})} \quad [m^3/s] \quad \text{II.16.d} \]

B = width  \( a \) = depth
\( \bar{u} \) = mean cross-sectional value

This formula provides a rough estimate only. It depends for instance on formula II.12 for an estimate of \( E_t \), it assumes a certain velocity profile and it does not explicitly reflect the presence of flow irregularities such as dead zones.

Observations in real rivers show a maximum deviation of both formulas of a factor four.

II.3.2.b. Tidal movements

A second factor influencing the longitudinal mixing of salt is the tidal movement (for the moment still in constant density situations). At first sight we can regard the tidal movement as a constantly reversing uniform flow, with velocities in the form of

\[ u = \hat{u} \sin \left( \frac{t \cdot 2\pi}{T} \right) \]

\( \hat{u} = \hat{u}(x,t) \) = velocity amplitude

This would imply that the resulting dispersion coefficients would be of the kind described above for steady, one directional river flow, at least if the tidal period is long enough so that the velocity profile can adjust itself easily to the slowly varying flow situation.

This means that the dispersion coefficient in such a tidal flow can never be any larger than that of a stream with a constant flow of \( u = \hat{u} \).

Indeed evidence exists that, in constant density reaches of relatively smooth rivers, values of \( D \) occur, clearly below that of the uniform shear flow [18]. One prediction of \( D \) under these circumstances is [2]

\[ D = 0.1 \cdot \frac{u^2}{T} \cdot T \cdot \left[ \frac{1}{T} \cdot \tilde{f}(t) \right] \quad \text{AND} \quad \frac{T'}{T} = \frac{T}{T_c} \]

\[ T_c = \frac{x^2}{D} \quad \text{II.17.} \]

for values of \( \left[ \frac{1}{T} \cdot \tilde{f}(t) \right] \) see fig.II.3

\[ T_c^{\text{transv.}} = \frac{v}{E_t} \quad T_c^{\text{vert.}} = \frac{a}{E_v} \]
This formula is only valid for channels that
- are uniform over a reach much longer than the channel width
- are more wide than deep
- have a constant density
- have no phase shift along the channel in the velocity profile

The maximum $D_{xt}$ that can be found from the above mentioned equation is, appr. 60 - 100 m²/s, under normal conditions. The fact that in a wide variety of estuaries dispersion coefficients of much higher values have been found is caused by other effects of the tidal motion, that are not so obvious at first sight and of course by density differences, to be discussed in section II.3.2.c.

These, dispersion enhancing effects of tidal motion are mainly

1. tidal pumping
2. trapping

ad 1. tidal pumping

The effect that the local tidally and depth-averaged velocities in the estuary are not zero, even if the fresh water riverdischarge would be zero and the successive tides are exactly the same is called tidal pumping. This effect will contribute to $D_{xt}$ (eq II.9).

Quite often there are clear paths and parts of the cross-section of predominant ebb-flow (seaward) and flood flow (landward). Clear examples are the distinct flood and ebb gulleys in some estuaries. This is mainly induced by the shape of the estuary (see example in fig II.4.). This phenomenon takes care that salt can intrude further in the estuary than could be expected on the basis of tidal averaged flows and velocities. The effect of the phenomenon is further increased by
- the earth's rotation deflecting the ebb and flood flow to different directions, in wide estuaries
- the fact that friction retards the ebb flow (relatively low tidal elevations) more than the flood flow (relatively high water elevations)
- irregular bottom topography

ad 2. Trapping

This effect results from the phase difference of tidal water-elevations and flows. These phase differences are caused by special features of the geometry of the river, like sidebays, small tributaries, bends and bank-irregularities combined with the inertia of flow (bigger masses of flow possess a bigger inertia).

The moment the floodtide starts the whole estuary is filled with the incoming water including side irregularities like bays and sidechannels. At a certain moment around high water slack the pressure gradient, causing the water to flow, reverses, but owing to the momentum of the watermass, the water still flows upstream for a while against the opposing pressure gradient. However the momentum of the main channel flow is generally much larger than that of the short sidechannels and bays. Therefore the flow in these side channels reverses before the main flow does.

The sidechannel water, coming into the main channel is caught in this main flow and mixes with the water coming in from the sea in the main channel at that time. As can be seen from the figure this has a distinct dispersive effect. This effect can be observed more or less in every estuary. One noticable effect is the fact that the highest salinities do never occur exactly at high water slack but a little while before (5-20 minutes).

An attempt to quantify the dispersive effect was undertaken by Okubo [25]. The effective longitudinal dispersion coefficient for a uniform velocity of flow in the main channel \( u = \tilde{Q} \cos (\pi x / L) \) and a ratio of "trap" volume (sidechannel & bay volume) to main channel volume of \( r \) is
\[
\begin{align*}
\mu & = \text{WATER OF EQUAL SALINITY} \\
\rightarrow & = \text{FLOW DIRECTION} \\
\rightarrow \rightarrow & = \text{PRESSURE GRADIENT} \\
\text{UPSTREAM SIDE} \\
\text{OCEAN SIDE}
\end{align*}
\]

\[D = \frac{D'}{1+k} + r. \frac{\hat{u}^2}{2k(1+r)^2(1+r+\sigma/k)}\]

\[\frac{1}{k} = \text{characteristic exchange time between traps and main flow (5-20 minutes)} \]
\[D' = \text{longitudinal dispersion coefficient in main channel}\]
\[\sigma = \frac{2\pi}{T}\]

Extensive experimental verification has not yet been undertaken, but in some estuaries II.18 proved to hand a useful indication [25].

II.3.2.c. Density differences

A last causal factor of mixing and transport of salt in the longitudinal direction is the difference in density of the salt sea water and the fresh river water flowing into the estuary. At first a short description has to be given of the behaviour of the two types of water, when brought together. When the flow would be zero, so no shear stresses are generated, a natural equilibrium would be reached if the isohalines (lines of constant salinity, so density in this case) in a salt/fresh water basin would be horizontal. This means two distinctive layers on top of each other, with a sharp interface (complete stratification) (see fig II.5.b). However if, by whatever means, eg. shear at the interface, the isohalines are not exactly horizontal but curved (partially mixed) to almost vertical (well mixed) there will be longitudinal pressure gradients that cause circulational flows, usually referred to as gravitational circulation.
In general
\[
\frac{dP}{dz} = -\rho(x,z,t)\cdot g
\]
\[\text{or}\quad P = \frac{h}{g} \int_{z}^{h} \rho(x,z,t) \cdot dz\]

\[
\frac{dP}{dx} = g \left[ \rho(x,h,t) \cdot \frac{dh}{dx} + h \frac{d}{dx} \left( \frac{\partial \rho(x,z,t)}{\partial x} \cdot dz \right) \right]
\]

In case of a well-mixed situation (then \(\rho(x,h,t) = \rho(x,t)\)) eq. II.19.a is

\[
\frac{dP}{dx} = \rho(x,t) \frac{dh}{dx} + \frac{\partial \rho(x,z,t)}{\partial x} \cdot g \cdot (h-z)
\]

The density gradient term is zero at \(z=h\) (surface) and has a maximum at \(z=h_b\) (bottom). In other words, the longitudinal pressure gradient becomes \(z\)-dependent. Resulting from this difference in pressure-gradient there will be a net flow in the direction of decreasing density (landwards) over the bottom and a net flow seawards at the surface (fig.II.6.b).

Hansen and Rattray [19],[20] analysed the gravitational
circulation in a vertical two dimensional plane, \((x,z)-axes\) assuming no variation across the channel. They produced graphs from which can be seen what parts of the salt-transports in an estuary are maintained by the density driven circulation mechanism (see fig II.7.). A severe limitation of the graphs and the analysis, however, is the two dimensionality.

\[ F_m = \text{densimetric Froude number} \]
\[ R = \text{Richardson number} \] (see appendix I)
\[ \bar{c} = \text{average salinity} \]
\[ \Delta c = \text{salinity difference bottom-free surface} \]
\[ U_s = \text{residual velocity at surface} \]
\[ \nu = \text{fraction landward salt-transport caused by all dispersion mechanisms other than the density driven circulation} \]

fig.II.7

The same mechanism of longitudinal density gradients also induces net transverse flows in non-rectangular cross-sections, like shown in figure II.8. This results in transverse as well as longitudinal mixing. The net flow upstream is usually in the deeper parts of the crosssection where the pressure gradient is biggest; the net flow downstream occurs in the more shallow parts.

fig.II.8.

In section II.1.3.1. we showed that the transverse mixing is often much more effective in dispersing than the vertical one, owing to the difference in "length scale". Therefore it
seems logical that something is missing in the two dimensional analysis of Hansen and Rattray. However in case the rivers at hand have a constant-depth cross-section, Hansen and Rattray might provide reasonable estimates.

Fischer [21] analyzed the transverse gravitational circulation in channels with non-rectangular cross-sections (so with varying depth over the cross-sec.). From tests in a triangular cross-section he tried to quantify the effect on the longitudinal dispersion coefficient caused by the transverse gravitational circulation. He arrived at the following formula (based only on the test results in his laboratory flume)

\[ D = 1.9 \times 10^5 \left( \frac{g}{\rho} \frac{dp}{dx} \right)^2 \frac{a^6 B^2}{E_0^6 \kappa t} \]

\( E_0 = \) vertical mixing coefficient for momentum

Up to now there is no single formula found yet that describes the effect of all density differences on the dispersion coefficient. From various test flumes and some real estuaries however data on density driven circulations have been found and graphed (fig.II.9., taken from[2]). The graph will give an insight in the order of magnitude of the gravitational mixing.

In order to present all different measurements in one graph the ordinate was chosen as \( L_i \cdot u/y (a.u^*) \). This is proportional to \( D/au^* \), which provides an estimate for the \( D_0 \) of density driven currents.

The value of \( L_i \) found from such figures should not be taken as a maximum intrusion-length value, as some mechanisms are not represented by measurements in laboratory flumes. The flumes are always relatively narrow, have rectangular cross-sections and certainly cannot represent all aspects of tidal and wind driven-mixing. The graph therefore only gives an indication about the intrusion length, to be expected at least.

II.3.2.d. Windeffects

As a last cause of mixing, windshear-forces must be mentioned. In case of large waterbasins with little watermotion (artificial lakes behind dams for instance) this might well be the most important mixing mechanism available. Also in case large dead zones exist in an estuary, wind can play an important role in dispersion.
At first the wind drag causes turbulence in the upper layers. These layers in turn will affect the section immediately below and if the wind remains constant for some time, the depth affected by turbulency will become quite large. Of course this turbulence is a source of mixing. Secondly the wind is able to produce large-scale rotational currents, so called gyres, especially in case the water depth over the basin is not constant (the same drag force has more effect on a shallow layer of water with a small mass than on a very deep layer).

Wind effects are difficult to present in a dispersion coefficient form, owing to the grossly varying strength and direction of the wind in reality. Besides it is very difficult to measure wind effects in real estuaries as the other factors cannot be eliminated from the measurements in a satisfactory way.

\[ \frac{L_i}{a.u^*} \]

\[ R_{ie} = \frac{\Delta p \cdot g \cdot \xi_f}{B \cdot u^3} \]

fig. II.9.

\[ Li = \text{minimum intrusion length} \]

\[ R_{ie} = \text{estuarine Richardson number, see appendix I} \]
In this section we will try to estimate the (relative) magnitudes of the various mechanisms of salt mixing and intrusion. We therefore return to our basic salt balance, integrated over the cross-section (I.8). For the sake of simplification we will only consider steady situations in this section. For the tidal averaged models this means no changes from one tide to another as far as tidal amplitude and discharges are concerned. Due to the spring-tide/neap-tide cycle this will never be exactly correct (see for instance fig.5.C, where the tidal amplitude in the Maputo river is graphed for some neap-tide/ spring-tide cycles).

We will discuss the case of tidally averaged flow and indicate what sort of terms are lumped together in the dispersion coefficient, \( D_{XT} \). If other cases are considered, the real time approach for instance, the same discussion will be possible, leaving out the terms with the extra average deviations. For the real time models we can leave out the part that defines deviations from the the tidal average values, for instance.

If a steady state is reached and the streamlines of the mean flow are more or less parallel to the x-axis, eq. II.9 is valid
\[
\langle \tilde{u} \rangle \langle \tilde{c} \rangle = D_{XT} \frac{d \langle \tilde{c} \rangle}{d X} \tag{II.21}
\]
\( \langle \rangle = \text{tidal average} \)

Or in words:
the downstream advection caused by the fresh water discharge equals the transport caused by all other mechanisms, represented by the dispersion coefficient, \( D_{XT} \).

In order to identify the mechanisms we divide the instantaneous velocity and salinity in components. The components describe the different averaged values
\[
\begin{align*}
\tilde{u}(x,y,z,t) &= \langle \tilde{u} \rangle + \tilde{u}_c(x,t) + \tilde{u}_s(x,y,z) + \tilde{u}'(x,y,z,t) \tag{II.22} \\
\tilde{c}(x,y,z,t) &= \langle \tilde{c} \rangle + \tilde{c}_c(x,t) + \tilde{c}_s(x,y,z) + \tilde{c}'(x,y,z,t) \tag{II.23}
\end{align*}
\]
\( \sim = \text{cross-sectional averaged value} \)
\( \langle \rangle = \text{tidal cycle averaged value} \)
in this way for example is
\[ <\tilde{u}> = \frac{1}{T} \int_{0}^{T} (\frac{1}{h} \int u \, dy \, dz) \, dt \]

\( u_{c}(x,t) \) is cross-sectional average at any time during the tidal cycle minus the tidal cycle average value, so
\[ u_{c}(x,t) = \tilde{u} - <\tilde{u}> \]

\( u_{s}(x,y,z) \) is tidal cycle averages at any point minus the cross-sectionally averaged tidal cycle average, so
\[ u_{s}(x,y,z) = <u> - <\tilde{u}> \]

\( u'(x,y,z,t) \) is the remainder, so
\[ u'(x,y,z,t) = u(x,y,z,t) + <\tilde{u}> - <u> - \tilde{u} \]

This division is illustrated in fig.II.11.a+b. for a u,z-plane, but is also valid in transverse direction.

\[ \begin{align*}
\text{fig.II.11.a} & \quad \text{fig.II.11.b}
\end{align*} \]

The total salt transport through a cross-section over a whole tidal cycle is
\[ T_{s} = A. <\tilde{u} \cdot c> = Q_{1} <\tilde{c}> + A. [<u_{c}c_{c}> + u_{c}c_{s} + <u'c_{c}>] \]

Note that all cross-products of terms, for instance, equal zero.

The first term on the right hand side of eq II.24 is the mean advection, the other three terms are caused by various mechanisms of dispersion.

The total salt transport in a cross-section that has reached a steady state is zero, meaning that the salinity distribution at that point, at the beginning and end of the observation period is the same. This implies, using eq. II.22. to substitute the mean salt advection
\[ -D \frac{d <\tilde{c}>}{dx} = <u_{c}c_{c}> + u_{c}c_{s} + <u'c_{c}> \]

Analyzing the three dispersion terms on the right hand side of this equation we arrive at the following conclusions.
1. $<\mathbf{u}_c, \mathbf{c}_e>$

If we consider the tidal flow as a pure sinusoid flow, transporting a certain salinity distribution back and forth and we assume a cosine for the salinity at a point

$$u_c = \hat{u}_c \cdot \sin \left( \frac{2\pi t}{T} \right)$$

$$c_e = \hat{c}_e \cdot \cos \left( \frac{2\pi t}{T} \right)$$

the term $<\mathbf{u}_c, \mathbf{c}_e>$ would equal

$$\hat{u}_c \cdot \hat{c}_e \int \left[ \sin \left( \frac{2\pi t}{T} \right) \cos \left( \frac{2\pi t}{T} \right) \right] dt = 0$$

Indeed, in some studies this term is considered negligibly small. However, deviations from this cos/sin assumptions occur. An example is the fact that the maximum salinity is often reached before high water slack and the minimum before low water slack (see fig.II.12).

![fig.II.12](image)

The explanation for this is usually sought in the trapping mechanism. Water in small, side tributaries will contain water of a somewhat lower salinity as the main channel, during and shortly after the flood tide. The flow reverses a bit earlier in these "traps" and the low salinity water is taken upstream in the main channel, lowering the salinity there.

2. $\mathbf{u}_s, \mathbf{c}_s$

This term represents the residual circulation. It is customary to divide this term in a lateral and a vertical direction to discern the transverse and vertical circulations.

$$\mathbf{u}_s \cdot \mathbf{c}_s = \mathbf{u}_{sT} \cdot \mathbf{c}_{sT} + \mathbf{u}_{sv} \cdot \mathbf{c}_{sv}$$

The transverse residual circulation term is caused by several mechanisms,

- transversal gravitational circulation (fig.II.8)
- tidal "pumping" (fig II.4)
- wind driven gyres in $x,y$-plane (horizontal)

In every estuary these three mechanisms will contribute in a
different way. Unfortunately it is very difficult to separate the various causes in a numerical way and therefore to relate the observed transverse currents to their cause. This is also valid, though in a less severe way, for the vertical residual circulation. Mechanisms causing this term 2.b are

- vertical gravitational circulation (density differences)(fig.II.6)
- wind driven turbulence in x,z plane (small scale rotations)

3. $<u'.c'>$ 

This term expresses all sorts of phenomena with timescales much less than the period cycle,

- windvariations
- turbulent motions due to the oscillatory shear flow in the estuary
- human activity, ships, etc.

Table II.1 presents some actual estimates of the various terms in real estuaries all over the world [23],[24]. From this table we can see that

- in strongly stratified estuaries (eg Vellar) the relative importance of the vertical residual (and shear flow) components becomes bigger
- in less stratified estuaries transverse variations become progressively more important
- trapping is a term that is certainly not always to be neglected and is non-negligible in stratified as well as moderately mixed estuaries
- residual circulations are dominant over the shear flow terms, in general
- large variations occur from estuary to estuary, but even within one estuary in different seasons of the year (Columbia: high and low discharge).

Note: remember that all these values are given under the assumption of steady state situations, so no changes from one tidal cycle to another. In reality this causes deviations, the magnitude depending on the magnitude of the actual change from tide to tide. Those deviations are in case of the steady state approach also lumped in the dispersion coefficient.

Now that we had a look at the order of magnitude of the various mechanisms of salt dispersion in an estuary, we
<table>
<thead>
<tr>
<th>River</th>
<th>STATE</th>
<th>(-A_o )</th>
<th>(\langle u_c, c \rangle)</th>
<th>(\langle \dot{u}_s, C_s \rangle)</th>
<th>(\langle u_r, C_v \rangle)</th>
<th>(\langle \dot{u}_r, C_v \rangle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vellar estuary</td>
<td>strongly stratified</td>
<td>-75</td>
<td>-14</td>
<td>-105</td>
<td>-4</td>
<td>-42</td>
</tr>
<tr>
<td>(9/2/1977)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Southampton water</td>
<td>partially stratified</td>
<td>-25</td>
<td>250</td>
<td>102</td>
<td>-20</td>
<td>-50</td>
</tr>
<tr>
<td>Transect A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transect B</td>
<td></td>
<td>0</td>
<td>-220</td>
<td>-200</td>
<td>-20</td>
<td>-12</td>
</tr>
<tr>
<td>Mersey estuary</td>
<td>partially stratified</td>
<td>2200</td>
<td>-300</td>
<td>-350</td>
<td>-220</td>
<td>-280</td>
</tr>
<tr>
<td>Columbia</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>high discharge</td>
<td>stratified</td>
<td>-1.23</td>
<td>10</td>
<td>-0.37</td>
<td>10</td>
<td>-0.01</td>
</tr>
<tr>
<td>low discharge</td>
<td>partially stratified</td>
<td>-0.44</td>
<td>10</td>
<td>-0.39</td>
<td>10</td>
<td>-0.07</td>
</tr>
<tr>
<td>Gironde</td>
<td>stratified</td>
<td>-5850</td>
<td>-550</td>
<td>-250</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

salt fluxes in kg/s; positive values indicate downstream salt flux. Data from [22] and [23].

table II.1

return to our overall salt-balance equations to have a look at the practical use of the equations (II.8 and II.9). A large field survey would be required to obtain estimates of the magnitude of the various separate mechanisms; an estimate of the overall dispersion coefficient, \(D_{xt}\) is usually much easier to obtain from measurements. Starting from eq. II.9

\[
D_{xt} = \frac{\langle \ddot{u} \rangle \langle \tau \rangle}{\frac{d}{a \times}}
\]

The \(D_{n}\) can be obtained by observing a longitudinal salinity gradient and simultaneously measuring the fresh water discharge, \(Q_f\). The result is still influenced by the fact that this equation is based on a steady state of the estuary. If this is not completely true the time derivative in eq II.8 has to be included as well, which makes the estimate of \(D_{xt}\) a lot more difficult.

In several studies dispersion coefficients have been calculated in this way. Care has to be taken though that in some literature dispersion coefficients are not measured as tidal average values but as high slack or low slack values.
In that case the salinity is not equal to $c$, but the value at the time of the slack-tides. Naturally we cannot compare these values directly with the tidally averaged values.

To give an idea of the order of magnitude of dispersion coefficients in real estuaries we included table II.2, taken from [2] with some extensions from other publications.

From table II.2 can be seen that most dispersion coefficients are in the range of 100 - 300 m$^2$/s with lower values in the constant density reaches of the rivers. Data found in the Maputo survey fit well inside this range. Naturally this table does not provide a basis for predictions in other estuaries.
<table>
<thead>
<tr>
<th>Estuary</th>
<th>charact. value, or range of observed values of disp. coeff. ($m^3/s$)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hudson</td>
<td>160</td>
<td>Thatcher &amp; Harleman 1)</td>
</tr>
<tr>
<td>Rotterdam waterway</td>
<td>280</td>
<td></td>
</tr>
<tr>
<td>Potomac</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>Delaware</td>
<td>500-1500</td>
<td></td>
</tr>
<tr>
<td>San Francisco bay</td>
<td>200</td>
<td>Glenne &amp; Selleck (1969)</td>
</tr>
<tr>
<td>Severn</td>
<td>10-100</td>
<td>Stommel (1953)</td>
</tr>
<tr>
<td>Potomac</td>
<td>20-100</td>
<td>Hetling &amp; O'Connell 2) (1966)</td>
</tr>
<tr>
<td>Delaware</td>
<td>100</td>
<td>Paulson (1969)</td>
</tr>
<tr>
<td>Mersey</td>
<td>160-360</td>
<td>Bowden (1963)</td>
</tr>
<tr>
<td>Rio Quayas</td>
<td>760</td>
<td></td>
</tr>
<tr>
<td>(Equador)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Severn (winter)</td>
<td>54-122</td>
<td>Bowden (1963)</td>
</tr>
<tr>
<td>Severn (summer)</td>
<td>124-535</td>
<td></td>
</tr>
<tr>
<td>Thames (low Q)</td>
<td>53-84</td>
<td>Bowden (1963)</td>
</tr>
<tr>
<td>Thames (high Q)</td>
<td>338</td>
<td>Bowden (1963)</td>
</tr>
<tr>
<td>Tees</td>
<td>86</td>
<td>Farraday (1973)</td>
</tr>
<tr>
<td></td>
<td>108</td>
<td></td>
</tr>
</tbody>
</table>

notes
1) Their D is defined in a different way. The difference is usually no more than a factor three [26]
2) from dye experiment
3) computed from data of Murray et al
4) mid flood tide
5) mid ebb tide

table II.2.
CHAPTER III. ONE-DIMENSIONAL SALT INTRUSION MODELS

In chapter II we have tried to explain the mechanisms of longitudinal salt transportation and salt-mixing in general and attempted to give estimates for the magnitude of the effect of the various mechanisms. This served mainly as an illustration of the variety of mechanisms of salt dispersion and of the danger of lumping all various types of transportation together in one single coefficient, a dispersion coefficient, $D_x$.

However, as previously mentioned in trying to predict salt-intrusion it is impossible to separate the various causes or at least it would require too elaborate a field survey, which is usually neither feasible nor economical. Therefore in predicting salt intrusion with numerical models we always start from the equations II.8 or II.9, with one single dispersion coefficient. This does not yet mean that the dispersion coefficient is taken as a constant in time and space. The difference between the various models is mostly dependent on the difference in the factors determining the value of $D_x$ in the estuary and time.

In this report only one-dimensional models are reviewed. This poses a severe limitation on the applicability and accuracy of all models tested. In section II.2 we already discussed what type of estuaries might be acceptable to be represented as a one-dimensional flow system. Summarized we can say:

- salinity in cross-sections of the estuary must be reasonably mixed, meaning no real stratified flow must be present
- the river must have constant, well defined stream areas, all through the tidal cycle and yearly seasons (dry and wet seasons)

Within the class of one-dimensional models we can still make a further subdivision into

1. tidal average models
   a. steady state
   b. non steady
2. high water slack/low water slack models
3. real-time models

In the following sections each class of models mentioned above will be discussed briefly and specific examples will be mentioned. In the final section (III.5) we will select models for further testing and comparison using the data of the Maputo river field study.
The class of tidally averaged models can further be subdivided into steady and non-steady models. In working with the first type we are only interested in the steady state solutions, for instance the stationary situation under the lowest fresh water discharge to be expected. These steady state models are especially useful in estuaries that adjust themselves quite rapidly to changing conditions, so that equilibria can be reached within a single dry season for instance.

The steady state models can also be used in case there is uncertainty whether a steady state will actually be reached but the consequences are so big it is better to be prepared for the worst, so in our case for the furthest salt-intrusion possible.

The non-steady class is useful in case the river does not adjust itself very quickly to varying circumstances. Some rivers never reach equilibria within one dry season, for instance, and a steady state approach could be misleading or give very conservative results (e.g., by having to install irrigation pumps further upstream than really necessary). An example of such a river is the Gambia estuary (see study Sanmuganathan [27]). In fig. III.1. we can observe the ever increasing intrusion of a 1.5% salt front into the estuary, even at the end of the dry season.

As is common in all estuaries, the moment the fresh water discharge increases (transition between dry season and wet season), the salt is pushed back instantaneously.
III.2.1. Steady state, tidally averaged models

The backbone for this class of models is formed by eq II.9

\[ \langle \vec{u} \rangle \cdot \langle \vec{c} \rangle = D_{xT} \frac{d}{dx} \langle \vec{c} \rangle \]

or with all average signs left out

\[ \frac{Q_f}{A} \cdot \frac{dc}{dx} = D_{xT} \frac{dc}{dx} \]

all variables cross-sectionally averaged and averaged over the periodic, tidal cycle, therefore \( \langle \vec{u} \rangle = \) fresh water velocity = \( \frac{Q_f}{A} \)

In this way the complex system of salt transports and mixing is reduced to a simple equation whereby all effects of averaging and measurement errors are lumped in the dispersion coefficient, \( D_{xT} \). This coefficient is not necessarily constant in the estuary.

To work with the models, the only input needed is

- a. fresh water discharge, \( Q_f \)
- b. salinity in the rivermouth (tidally averaged value), \( C_0 \)
- c. input for the geometry of the estuary
- d. dispersion coefficient, \( D_{xT}(x) \)

The fresh water discharge is usually measured or given in case of predictions. The value of the downstream salt...
boundary, the salinity in the rivermouth, $c_o$ is not so
difficult to measure or can be estimated, for extreme
situations from relationships that will be derived in
section IV.4 ($c_o - Qf$ relationship).

Thus there remains only the geometry input and the $D_{xT}(x)$. As
far as the geometry is concerned there are two
possibilities. Either the cross-sectional area ($A$) can be
measured in a lot of places in the estuary and given as
input (eq III.1.a), or the surface width ($B$) and average
depth ($H_o$) can serve this purpose (III.1.b). At first sight
we may say that the first possibility, using the $A$, will be
the most accurate in describing the geometry of the river.
However measuring $A$ over a whole estuary requires an
extensive, expensive field survey. Besides, the
simplification of the description of the salt-transport is
already so great that a bit smaller accuracy may not even be
noticed in running the model.
The width of the estuary however can easily be measured.
Even a large scale map of the area under consideration, if
available, may serve this purpose. The average depth ($H_o$)
can be estimated from a few cross-sectional measurements
only.

This report will pay more attention to the selection of
geometry input in section V.2 at the calibration and testing
of the first tidally averaged model. The accuracy of the two
possibilities will then be compared.

Whatever option is chosen, it is always useful to reduce
all geometry input measured to simple input formulæ in
order to avoid large input data sets. The way geometry is
usually represented is by means of mathematical (quite often
exponential) functions $A(x)$ and $B(x)$, found with a
regression analysis. The river Maputo input is given in the
form

$$A(x) = A_o \cdot \exp(A_r x) \quad \text{III.2.a}$$
$$B(x) = B_o \cdot \exp(B_r x) \quad \text{III.2.b}$$

For the values of the constants, we refer to section IV.1
and figure 4.A and B.

The input factor remaining is the $D_{xT}(x)$, the dispersion
coefficient along the $x$-axis, the main streamline of the
estuary.

The elements of tidally averaged models discussed so far are
valid for all models. The only difference is usually found
in the assumption for the $D_{xT}(x)$. Up to now no relationship
has been found we can apply in all estuaries over the world.
Most models are tested and "made" applicable for one or at
the most a relatively small number of estuaries only, by
adjusting $D_{xT}(x)$ functions.

We will, in this section, discuss a few different $D_{xT}(x)$
assumptions, which result in different models.

1. Van der Burgh

Van der Burgh [6] assumed a semi-empirical relationship for the $D_{xt}(x)$

$$\frac{dD_{xt}}{dx} = -K \cdot U_f, \quad D_{xt}(0) = D_0 \quad (3.3)$$

The coefficient $K$ is assumed to be a constant in one specific estuary but can vary from one estuary to another. To run the model this $K$ is needed together with a value for $D_0$, the dispersion coefficient in the river mouth. This $D_0$ is not constant under varying circumstances. Therefore the only way to be able to make predictions with the model based on this form of $D_0$-relationship is to find a link between the value of the $D_0$ and one of the overall parameters that express the specific conditions of the estuary (estuary parameters see appendix I).

This implies that the model has to be calibrated with data taken from quite a few observations made under varying circumstances in order to establish a relationship $D_0$ versus estuary parameter.

Then again, even if such a relationship can be found (or assumed) extrapolation outside the range of actual observations is still dangerous as the $D_{xt}(x)$ function has no theoretical basis and therefore will not react on a shift in salt-transport mechanism that might occur when the conditions change substantially (eg. very low fresh water discharge).

Despite that Van der Burgh formulated his D-assumption as being purely empirical, nowadays some theoretical studies show that the relationship is not altogether wrong. On basis of the analysis of Hansen and Rattray [19], for instance, we can derive that

$$\frac{dD_{xt}}{dx} = -U_f$$

However, Van der Burgh tried to establish a relationship between the value of the dispersion coefficient in the mouth, $D_0$, and an overall value of one of the estuary parameters, the flood number. In this study the flood number is replaced by more sophisticated estuary parameters, available nowadays. Moreover, in a later model (Combined model) we notice that the overall estuary parameter is replaced by a local one, whereby values are used of the local variables instead of the ones in the rivermouth only.

If the two alterations stated here would be made in the Van der Burgh model, we would see that it quite resembles a model, discussed in chapter V, a modified Combined model.
In order to find a $D_{X\Gamma}(X)$ relationship that has a bit more theoretical basis and that therefore might be able to make more accurate predictions, also in case extrapolation is needed, the following discussion will be held, combining the ideas and formulations of various more or less theoretical studies of salt intrusion.

For this we start with the analysis of the gravitational circulation of Chatwin [9], who derived an equation for the ratio $c/c_0$.

Presented in its simplest form

$$u_f \cdot \frac{c}{c_0} = K_3 \left( \frac{\rho_0 \cdot g \cdot H}{\rho c^2 \cdot u_t} \right)^3 \left[ - \frac{d}{dx} \left( \frac{c}{c_0} \right) \right]^{3/2}$$

III.4.

(symbols see end of report)

where all variables are averaged over cross-section and tidal cycle.

The equation is valid under the following conditions

- wide and prismatic estuary
- turbulent viscosity and coefficient of diffusision proportional to $(V \cdot Ut)h$

According to Chatwin, the coefficient $K_3$ has a value around 3.7. The equation can be solved analytically, resulting in

$$\frac{c}{c_0} = \left[ 1 - \frac{3}{K_3} \cdot \frac{u_f}{(\rho_0 \cdot g \cdot H)^{3/2}} \cdot \frac{x^{3/2}}{H} \right]^{3/2}$$

III.5.

Other studies, i.e. Smith [10] cover a somewhat broader and more theoretical field, however eq III.5. seems to provide fair approximations, at least under the condition that the depth in a cross-section does not vary very much. In case of varying depths also the transverse gravitational circulation gets involved; especially the value of the coefficient $K_3$ will be dependent on the ratio $h/B$, see Fischer [2].

As rivers are never completely prismatic, eq III.5. can be transformed for the non-prismatic case to

$$\frac{c}{c_0} = \left[ 1 - \frac{3}{K_3} \cdot \frac{x^{3/2}}{(\rho_0 \cdot g \cdot H)^{3/2}} \cdot \int dx \right]^{3/2}$$

III.6

A generalisation of this equation, involving part of the theory of Smith [10] is
\( \frac{c}{co} = \left[ 1 - \frac{\nu_i}{K_3} \int u_i^{k-1} \left( \frac{\alpha_p \rho \cdot g \cdot H}{f_i^{\nu_i \cdot u_i}} \right)^{3k-1} \cdot dx \right]^{1/k_i} \) III.7

(eq. III.6 is a special case with \( K_3 = 2/3 \) and \( k = 2/3 \))

The last step is to rewrite this equation in a form that resembles the salt balance equations of tidally averaged steady models (II.9)

\[ u_i \cdot c = \left[ -k_3 \cdot u_i \cdot \left( \frac{\alpha_p \rho \cdot g \cdot H}{f_i^{\nu_i \cdot u_i}} \right)^{k-1} \cdot \frac{d\left( \frac{c}{co} \right)}{dx} \right] \cdot co \] III.8.a.

The dispersion coefficient is a bit difficult to spot but can be written as

\[ D_x = \left[ k_3 \cdot u_i \cdot \left( \frac{\alpha_p \rho \cdot g \cdot H}{f_i^{\nu_i \cdot u_i}} \right)^{k-1} \cdot \frac{d\left( c/co \right)}{dx} \right] \cdot \left( \frac{k_3}{co} \right) \] III.8.b.

Or, eliminating \( d(\frac{c}{co})/dx \) between III.8.a. and III.8.b

\[ D_x = k_3 \cdot u_i \cdot \left( \frac{\alpha_p \rho \cdot g \cdot H}{f_i^{\nu_i \cdot u_i}} \right)^{k-1} \cdot \left( \frac{d\left( c/co \right)}{dx} \right) \] III.8.c

Equations III.8 are taken as the backbones of the second tidal averaged model. It still has three empirical coefficients, \( k, K_1 \) and \( K_3 \). Contrary to the Van der Burgh model the value of these factors ought to be constant under varying circumstances, like discharge and tidal amplitude changes, according to the theory.

The relationship coefficient - estuary parameter is already included in the main equation itself. The estuarine Richardson number (\( R_{ie} \), see Appendix I) is clearly visible in the "dispersion coefficient" part!

This implies that calibration of the model consists of finding the optimal \( k, K_1 \) and \( K_3 \) from the available salt measurements. Checking of the model can be done by comparing these values for the various measurements and to express their variation in specific statistical parameters and/or reliability fields.

3. River Kuantan Model

Thompson, Neville-Jones and Shahabudin [28] use the following \( D_x (x) \) relationship

\[ D_x = D_1 \cdot T_r + D_2 \cdot (E_{no})^n \] III.9
\[ n = -2.5 \]
\[ T_r = \text{tidal discharge ratio} = \frac{A_u A_{10}}{A_{u10}} \]
\[ E_{no} = \text{local estuary number} = \frac{\rho \mu c}{Q_1 T g H (1 - \%)} \]

Again two constants, \( D_1 \) and \( D_2 \) have to be found by calibration with actual measurements.

If all input data, so geometry, \( Q_f, c_o \) and \( D_{xf}(x) \) are known it will be possible to write equation III.1.a or b in the form of a finite difference equation. Then, starting from the river mouth we can compute the salinity in the next point upstream over a certain distance, \( \Delta x \), with the help of this difference equation. This is repeated for the following upstream points up to the point where the salinity equals the natural fresh water salinity.

For the numerical method of this process see appendix VI.

The longitudinal salinity profile computed in this way is a tidally averaged one. If we are interested in the profiles at high water slack or low water slack we have to shift the tidally averaged profile over a certain distance. Therefore it is assumed that

- the tidally averaged profile is exactly halfway between the profiles of high water slack or low water slack. This will, in most cases be an approximation.

- the shape of the profile during the periodic tidal cycle does not change noticeably.

The measurements in the river Maputo (see section IV.4) show that this is a fair assumption as long as the estuary is well mixed. In case of (partially) stratified flow this is definitely not true, but under those circumstances it is not advisable to use the one dimensional model for other reasons as well.

The distance from high water slack profile to low water slack profile is referred to as tidal excursion length (\( \text{Lt} \)). With the aforementioned assumptions this means that the distance over which the tidally averaged profile has to be shifted to obtain the high water slack or low water slack profile is \( \frac{1}{2}.\text{Lt} \), upstream or downstream respectively. Values for the tidal excursion length are given in section IV.4.
III.2.2 tidally averaged, non-steady models

Not many examples of this class of tidally averaged, non-steady models are known. The class is also referred to as quasi non-stationary models, as they disregard the short term non steadiness during the tidal period.

The models use eq. II.8 as their starting point, however, integrated over the periodic tidal cycle.

\[
A \frac{dc}{dt} + \frac{d}{dx}(\partial_t c) = \frac{d}{dx} \left( A \partial_t d\frac{dc}{dx} \right)
\]

The numerical solution of the equation is possible, although a bit more difficult, requiring the same input data as mentioned for steady state tidally averaged models, together with

- initial conditions of salt distribution
- \(D_{xt}(x,t)\) so not only a function of place but also time dependent

An example of this type of models is given by Schönfeld and van Dam [29]. They found the dispersion coefficient from actual on-site measurements along the estuary and did not try to find a mathematical relationship as such. Furthermore they assume the dispersion coefficients to be independent of time, although they use different values for wet seasons and dry seasons in the estuaries they look at.

III.3. High water slack/ low water slack models

The next class of models can be described as instantaneous steady state models. The models use eq. II.8, so an equation with variables averaged over a small timespan \(\Delta t\) of a few minutes in order to leave out the fluctuating part of the turbulent motions.

This is the "real-time" approach. However, the models reviewed here, only regard the situation exactly at high water slack or low water slack, which implies that the time derivative \(\frac{d}{dt}\) is approximately zero, due to the absence of water motion.
\[ \psi, c = D_{x,ST} \frac{\partial c}{\partial x} \]  

\( D_{x,ST} \) = slacktime dispersion coefficient

where all variables being averaged over cross-section and timespan \( \Delta t \)

The equation is tackled exactly in the same way as discussed at the tidally averaged steady state models with the following differences

- geometry input should now be given either as the values of \( A \) or \( B \) and \( H_0 \) at high slack or low slack tide

- salinity in the mouth should be given at high water or low water. This is definitely one of the advantages of, especially the high water slack models as the value around slacktide is not as difficult to determine as the tidally average one. Around high water slack can be assumed that the salinity is equal to the salinity in the sea or ocean outside the river, which in general is a fairly constant value.

- \( D_{x,ST} \) is, seen from its definitions clearly different from the \( D_{x,T} \).

The different models using the slack-approach again differ only in their approximations of the dispersion parameters. Although several slack-tide models have been developed, only two will be mentioned; one low water slack and one high water slack model.

Low water slack model - Ippen and Harleman[30]/ Harleman and Abraham [31]

--------------------------

Ippen and Harleman assumed the dispersion coefficient \( D_{x,LWS} \) to be an inverse function of \( x \), the distance to the river mouth

\[ D_{x,LWS} = \frac{D_{0,LWS} \cdot L_B}{x + L_B} \]  

\( L_B \) is the distance from the mouth to the point where the salt content deviates from the sea-salt concentration.

By assuming this relationship it is possible to solve eq. III.11 analytically resulting in

\[ \frac{c}{c_0} = \exp \left[ -\frac{\psi}{2 D_{0,LWS} \cdot (x + L_B)^2} \right] \]  

III.13.
(only valid for a constant cross-sectional area $A$)

Harleman and Abraham found, from an analysis of the model-tests by Ippen and Harleman and measurements in the Rotterdam Waterway that the $D_{O,\text{LWS}}$ and the $D_{O}$ are correlated to the estuary number, $E$, \( \frac{D_{O,\text{LWS}}}{u_{L, T}} = 0.055 \left( \frac{H_0}{\Delta H} \right)^{2.7} E^{0.2} \) \[III.14.a\]

\[\frac{2\pi \cdot L_{a}}{u_{o} \cdot T} = 0.70 \cdot E^{-0.2} \] \[III.14.b\]

Besides the constraints of steady models this model has the severe limitation that it is valid for constant cross-sectional area rivers only, although it can most probably be generalized if needed.

This model is not one of much practical use, but introduced the estuary parameter $E$, a number that is still used (in a slightly modified form) in many studies.

**High water slack model - Gambia estuary; Sanmuganathan [27]**

Sanmuganathan works with eq. III.11 at the moment of high water slack. In his analysis he tries to fit his measured longitudinal salinity curves with the help of a Fourier analysis.

The dispersion coefficient $D_{X,HWS}$ is assumed to have a similar relationship as was assumed by Thompson/Neville/Shahabudin (eq III.9)

\[D_{X,HWS} = D_{1} \cdot T + D_{2} \cdot (E_{no})^{n} \frac{c}{c_{o}} \cdot L \] \[III.15\]

$L$ = tidal intrusion length  
$E_{no}$ = local estuary number

**III.4. Real-time models**

The most detailed models covered in this review are the real-time models. This class is always non-steady in estuaries as it describes the complete course of water velocities, water-levels, salt-transports and salt-concentrations during
the tidal cycle. The group of real-time models has the advantage that they are able (if well calibrated and tested)

- to provide longitudinal salt profiles at any time during the tidal cycle. This means we do not have to work with assumptions of profile shifts or tidal excursion lengths as in all models discussed previously.

- to handle non-stationary phenomena, like changes in tide, due to the neap-tide/spring-tide cycle and changes in fresh water discharge.

- to give more accurate results, as they leave out the averaging over the tide, a term that introduces extra errors in the dispersion coefficient.

A real-time model is especially useful if you know that the points of the river, you are interested in, might be salt for part of the tidal cycle only, so that extractions of water for drinking water supplies or irrigation can take place for part of the day. This situation requires extensive knowledge about the to and fro movement of the salt which can be provided by a real time model.

The real-time models usually operate in two parts. One calculates the realtime water-motion and -levels, the second part covers the superimposed salt-transports and -concentrations. Both parts have interactions of course which will be made clear during the discussion of the basic equations of the models.

watermotion; basic equations

The first section of the model starts from the two basic water motion equations

continuity equation \[ \frac{\partial a}{\partial t} + \frac{\partial Q}{\partial x} = 0 \] III.16

equation of motion \[ \frac{du}{dt} + u \frac{\partial u}{\partial x} + g \frac{\partial a}{\partial x} + g i + g \frac{u vl}{C^2 R} + \frac{\partial a}{\partial x} \frac{\partial P}{\partial x} = 0 \] III.17

(Symbol: see end of report)

Note the last term in the equation of motion, that represents the influence of the density differences (so salinity differences) on the watermotion in non-constant density reaches of the estuary.

The second term in III.17, the convective term \( u \frac{\partial u}{\partial x} \) is usually left out during calculations. In tidal flows this results in small deviations only (\( \frac{\partial u}{\partial t} \) is dominant in tidal flows) and it has a positive effect on the stability of the
numerical process.

salt-transport; basic equations

This section has a set of equations similar to that of the water motion section

salt balance equation $\frac{dc}{dt} + u \frac{dc}{dx} = \frac{1}{A} \frac{d}{dx} (A \cdot dx \cdot \frac{dc}{dx})$ III.19

Combining III.19 and III.16 we arrive at an equation that is quite often used to compute the salt transport, $T_s$, in a direct way

salt transport equation $\frac{dT_s}{dx} + B \cdot \frac{d(u \cdot c)}{dt} = 0$ III.18

Equation III.19 is based on the salt-balances we have seen so far (compare eq II.8). In order to be able to compute salt-movement with III.18 and III.19 (approximations of) the results of the water motion section must be known. With eq III.16 to 19 we can compute all variables mentioned, i.e. water-levels, velocities (discharges), salt-transportation and concentrations, provided the necessary input is given.

Input consists of

a. river geometry
b. friction coefficients
c. boundary conditions
d. initial conditions
e. dispersion coefficients

a. River geometry

Real time models have a much greater need for data on river geometry than do all other models covered. In tidally averaged and slack tide models it was sufficient to know the
cross-sectional area (or surface width and mean depth) at one time during the tide (at an average level or at high/low water slack respectively). Real-time models must know how cross-sectional profiles change with the variation in water level.

b. Roughness coefficients

An essential part of the water motion equation consists of the friction term \( \frac{u''}{c} \). The factor C is the so-called Chézy friction coefficient. This factor has to be measured directly, which is quite difficult and inaccurate or can be found from calibrations of the watermotion. The factor changes along the estuary, usually starting at very low values in the upstream parts (=high friction: 25-35 m²/s) and ending at high values (=low friction: 70-85 m²/s) near the ocean/sea end of the rivers.

c. Boundary conditions

Boundary conditions can be separated as well in a part needed for the watermotion and the ones needed for the salt computations. Four boundary conditions are needed. The standard boundary conditions in most real time salt-intrusion models are

watermotion

- known discharge at the upstream end of the estuary; if possible outside the tidal range in order to have a fairly constant discharge over the day

- known waterlevel at downstream end, so in the river mouth. This waterlevel must be given all through the periodic cycle. It can be obtained by direct recording, Fourier analysis of a few measurements, using the tidal components given in tide tables or from nearby waterlevel gauges.

saltmotion

- salttransport at the upstream end; if possible in a constant density reach of the river where the salinity is close to zero throughout the tidal cycle

- salinity in the river mouth. This usually is one of the sources of uncertainty. At first maximum and minimum salinities during the tidal period are influenced by the fresh water discharge from the river (contrary to the watermotion boundary, where the tidal waterlevels are hardly influenced by changes in fresh water river discharges).
Therefore measurements of maximum and minimum salinities may lose their validity under different circumstances. Furthermore the course of the salt concentrations in the mouth is often not exactly known, in between high and low water slack. Different models present this downstream salt boundary in a different form. At the discussions of the various existing realtime models this part will be covered more extensively.

d. initial conditions

To start a real-time model initial conditions must be known. This is necessary to avoid instabilities and to reach a higher level of accuracy in further results. For this purpose starting values have to be provided for all water-levels, discharges and salt concentrations in the whole estuary. These initial values may have been measured directly or just estimated. Wrong or inaccurate estimates are not so disastrous however since, during the running of the model their influence is slowly lost due to the friction and dispersion terms. The more inaccurate the initial conditions the longer the model has to run before results can be interpreted as a good simulation of reality. The initial values of salinities however will take a longer period to loose their influence on present situations than do the waterlevels and discharges. We have to pay attention to this factor during calibration of the model. The best thing to do is therefore to start at least from a measured longitudinal salt profile, while the watermotion initializations can be estimated more roughly.

e. dispersion coefficients

Dispersion coefficients $D_x$ have to be known at all times during the periodic cycle and at all places in the estuary. This term is, again, usually one of the factors that distinguishes one model from another. Other factors are concerned with different assumptions in downstream saltboundary and tackling of the differential equations, whereby various numerical schemes can be used for the solution. We will briefly discuss these distinguishing factors for three existing real time models.

Thatcher and Harleman [26]

Thatcher and Harleman apply a finite difference scheme to solve the differential equations.
They assume a $D_x$ relationship of the form

$$D_x = k \left[ \frac{dC}{dx} \right] + D_G$$

where $\dot{C} = \frac{C}{C_0}$ and $\frac{x}{L}$

The $D_G$ is the term that represents dispersion in constant density reaches. Harleman [24] gives an expression for this coefficient mentioned in section II.3.2.a (eq.II.16.d)

$$D_G = 6.93 \cdot n \cdot \bar{u} \cdot k^{0.5} \left[ m^3/s \right]$$

The term $k \left[ \frac{dC}{dx} \right]$ provides for additional dispersion due to density differences. The parameter $K$ is said to depend on the degree of stratification in an estuary, expressed by the internal estuary number, $E_i$ (see appendix I). Thatcher and Harleman deduced a relationship between the dimensionless parameter $k/u_0 L$ and $E_i$, from numerous model tests and actual measurements [26]

$$\frac{k}{u_0 L} = 0.002 \cdot E_i^{-0.25}$$

The formulae derived by Thatcher and Harleman are averages of a lot of measurement data. However, using the model in a specific estuary means calibrating the constants in eq. II.16.d and III.17 in such a way that the computation results match the actual measurements.

Thatcher and Harleman tackled the downstream salt boundary in an original way. They divided the tidal cycle into two parts, a flood part and an ebb part. During the flood part the value of the salinity is assumed to be equal to that of the seawater, during the ebb period, the salinity is computed from a transport/mass balance in the most downstream part of the estuary, based on salt concentrations, one time step before.

At the end of the ebb period the salinity has usually dropped clearly below the ocean/sea salinity. Therefore at the start of the flood tide there is a need of a transition period for the salinity to rise from this low "end ebb value" up to the ocean/sea salinity.

Thatcher and Harleman assume a linear interpolation between the "end-ebb-value" and the ocean salinity over a certain timespan. They normally work with a timespan of $\frac{1}{2} T$ ($T =$ tidal period), but in some applications of the model (eg. Hudson estuary) this timespan had to be adjusted up to $\frac{1}{4} T$ (see fig.III.3).

In applying this method the only input data required by the model are the maximum salinity (sea/ocean salinity) and the transition period length, $T_t$. The first can be measured or estimated from relationships between $C_{\text{max}}$ and $Q_f$, the fresh water discharge. Such a relationship will be derived in
chapter IV.4 for the Maputo estuary.

The $\tau_\gamma$ can be estimated from values mentioned in literature, while a sensitivity analysis will show the correct value.

In general it can be said that rivers, ending in bays, like the Maputo for instance, have a transition period ranging from $\%T$ to $\%T$. Of course, exceptions are always present (Hudson estuary) in case the bay and/or river has a geometry very different from the "normal rivers".

**Penpas model [33]**

Penpas is a model developed by the Delft Hydraulics Laboratory. The model is based on the model of Thatcher and Harleman with some small modifications in salt boundary and dispersion coefficients.

The dispersion coefficient is given by a formula, based on eq.III.16

$$D_x = D_1 \cdot a \cdot u + D_2 \cdot \frac{dc}{dx}$$

Also in this case estimates are given, based on the work of Thatcher and Harleman, but the values of the constants $D_1$ and $D_2$ have to be found from calibration with actual measurements in the estuary at hand.

The salt boundary at the river mouth is tackled in the same way as in Thatcher and Harleman model. Only the shape of the transition curve is different. Instead of a linear
interpolation from "end-ebb-salinity" to $c_{\text{MAX}}$. Penpas uses a cosine curve to connect the two levels (fig III.4)

![Diagram](https://via.placeholder.com/150)

fig. III.4

Also Penpas only asks for two input values, being the maximum salinity, $c_{\text{MAX}}$ and the length of the transition period, $T_f$.

III.5. Selection of models for further testing and comparison

We have already mentioned, in the introduction to this report that this study of one dimensional salt-intrusion models was started as a result of a discussion with and within the National Waterservice of Mozambique. The Waterservice had made an attempt to apply a tidally averaged model, based on Van der Burgh in several estuaries [1], but some questions arose about

- the applicability of the model and possibilities for use in other rivers

Since the dispersion coefficient - distance to mouth relationship of Van der Burgh is purely empirical, changes in salt transport mechanisms will not be followed by the coefficient. In estuaries where other salt intrusion mechanisms prevail, this will not be noted by the Van der Burgh model.

- the accuracy of predictions

Most of all whether this accuracy could be improved by using other one-dimensional models,
either with a better theoretical basis or a more detailed description of the processes

- suitability of surface-width input, with a constant depth, for the geometry

the question was whether the prediction accuracy would be hampered by the use of the surface width and constant mean depth as input in comparison to the cross-sectional area input. If no or relatively small differences would exist, the second way of providing geometry input would be much more cost-effective, since a much smaller number of field measurements are needed.

For these three main reasons the project was started consisting of

* an extensive field study in one of the rivers in Mozambique, the river Maputo (data presented in chapter IV)

* a study testing and comparing a several classes of models with the measurements obtained in the Maputo river, in order to arrive at conclusions about applicability and accuracy of various model-types

As a result, three models were chosen for this test-study:

1. tidally averaged, steady model, based on the Van der Burgh dispersion assumption. From now on to be called the Van der Burgh model. This model will be tested with different inputs for the geometry as well.

   The model is chosen because the Waterservice of Mozambique already had some experience with it and because of its relative simplicity.

2. a tidally averaged, steady model with a more theoretical basis of the dispersion coefficient (see section III.2.1; combined model). This model is referred to as the Combined model and is chosen to see the effects of a more theoretical dispersion coefficient approach on the accuracy of the resulting prediction.

   A slightly modified version of this model will also be considered.

3. real-time model PENPAS

   The Delft Hydraulics Laboratory kindly assisted the study by granting the use of the real-time model PENPAS, developed at the DHL. A real-time
model was chosen in order to notice the effects on accuracy of the predicted results if a more detailed description of the processes was made (including a more detailed input of course).

The calibration and testing of these three models will be presented in chapter V. The actual comparison between them can be found in chapter VI.
CHAPTER IV. SURVEY RESULTS OF THE RIVER MAPUTO

IV.0 General description of the river Maputo

The river Maputo is the most southern river in Mozambique, running, roughly in an west-easterly direction. The total section-length within Mozambique is approximately 150 km. The last part (upstream) serves as a natural border with its neighbouring countries, the republic of South Africa and Swaziland. The springs of the river are situated in two mountain ridges in the neighbouring countries. The river has a few glacial sources but is mainly fed by rainfall and landdrainage.

This implies that the highest discharges arrive in and shortly after the rainy seasons of the regions it flows through. The main rain-season in the area under consideration is between January and April. For discharge reactions on rainy periods and forming of discharge equilibrium refer to part IV.3.

The river ends in the Maputo bay, the same bay where the capital of the country is situated. The bay has many shoals and gulleys which make it difficult to define clear riverboundary-conditions concerning geometry, tidal waterlevels and salinities.

Close to the bay the river broadens in a marked way, its width increasing from 2 up to 8 km at the mouth. A clear distinction between river and bay is therefore difficult, if not impossible to make (see fig 2.a).

The river does not have any major side branches up to some 140 km, so can be considered as a single channel. At a few points, though, islands are situated in the river.

In the whole study only the first 75 km were considered. From historical observations it is known that salt never intruded any further than 70 km upstream [30]. This situation occurred after a two year drought period, a very rare case. It is therefore justified to limit ourselves to this 75 km unless the general circumstances would change very much (e.g., the minimum discharge or the geometry of the river).

For most of its length the river runs in a valley with a width of about 2 - 5 km and hills on both sides. Close to the bay (first 10 km) the landscape flattens out. For the first part (12 km) the river is quite straight with mildly
curving bends only and widths in the order of 500-1200 m (apart from the wider reach near the bay). After a transition part of about 30 km the river becomes quite meandering with a steep bottom slope \((i \approx 5 \times 10^{-4})\), having lots of bends and steep banks and a very well defined stream profile, widths being 50-100 m. The depth of the river ranges locally from zero up to eleven metres. Cross-sectional average values range from approximately 2.0 m in the upstream part up to 4.5 m, at mid tide near Maputo bay.

The geology of the valley can be seen from figure IV.1., whereby a geological profile is shown at km 38, that can be considered quite characteristic for the region. The profile was made for a drinkwater study. The underlying chalk-formation shows up at the surface in some places for instance at km 8.23 and 37 (chalk quarry). These clear rock outcrops usually show at one of the banks only, in general the northern one.

![Geological profile](image)

Fresh water discharges of the river usually vary between 10 and 300 m³/s. In the years of 1982 and 83 the discharge came down to almost zero, after a period of severe drought in the area. The maximum discharge ever measured (estimated with the "slope-area"-method) is 16,000 m³/s. This discharge occurred in February 1984, after a heavy cyclone, Domoine, had travelled exactly parallel to the axis of the river. This can be considered as a very rare case, however (once in 40-50 years).

The tidal waterlevel amplitude in the bay of Maputo varies between almost zero (neap tide) and 1.5 metres (spring tide). In minimum discharge periods the tidal movement causes the waterlevel to vary during the day up to km 65-70 (Santaca). Up to Salamanga (km 41) the periodic waterlevel amplitude is still in the same order of magnitude as in the bay of Maputo. Between Salamanga and Santaca the amplitude is damped very quickly. Explanations for this can be found in the steep bottom slope, the quite narrow and constant cross-sectional river profile and possibly the large number of bends.

As far as salinity is concerned, there was some data.
available, even before the measurements had commenced. There had been some three measurements of salinity intrusion during the period of '82-'83, all showing reasonably well mixed situations at fresh water discharges ranging from zero to twenty m3/s.

Besides this it was noticed that, starting from Bela Vista (km 23) onwards the whole Maputo valley had once been irrigated with the river water, as could be seen from the old irrigation houses and pumps along the river. However, since most pumps did not work anymore for more than 8-10 years, no major conclusions can be drawn about salt intrusion lengths as the average discharges have been lowered drastically in the last 10 years due to irrigation water abstractions in Swaziland and South-Africa. Besides that, it is not exactly known whether the pumps were needed during the whole dry season or only at the beginning (one year or half year crop production), when there is still a high enough fresh water discharge to push the salt back.

During the survey it was observed that the river was quite stratified during periods of relatively small waterlevel amplitudes (neap tide periods) for up to two or three days at a time. The rest of the 14 day tidal cycle showed well mixed situations, due to the then greater mixing capacity of the tide.

At neap tide periods, the fresh water that turned salt, disposed of its suspended sediment load and became very clear. A very distinct transition line between clear and turbid water could be seen moving up- and downstream, lagging a little (300-600m) behind the actual salt-fresh water interface, most probably because of the time needed for the particles to settle (fig IV.2). In well-mixed situations this phenomenon did not take place, probably due to the high turbulence level in the water, caused by the higher tidal velocities.
IV.1 RIVER GEOMETRY

For all types of salinity intrusion models, the geometry of the river is one of the main input factors. River geometry has a direct influence on fresh water velocities, upstream storage, tidal velocities, cross-sectional mixing of salt, etc. It is known that wrong, incomplete or inaccurate river geometry input will result in false values of salinity intrusion lengths and/or salinity profiles.

The question arises, however, how accurate the description of the river geometry should be. A too rough measuring grid might result in inaccurate results, a too detailed grid in too long field measuring and computing time and work.

It was decided to measure river cross-sections approximately every 3 kilometers. The general opinion was that this grid was a little too detailed for the river at hand, but in this way it would be possible to observe the inaccuracy introduced by leaving out some of the results. Besides this, it was considered wise to have the salt-profile measurements in places where the bottom profile of the river was known and a spacing of 3 km for these measurements seemed quite feasible.

After a general survey the position of the cross-section measurement stations was determined. In doing this care was taken that:

- the position was quite random, so not all stations would be in the middle of river curves or in perfectly straight reaches

- it would not be too difficult to find the exact position of the stations again (± 50 m) during the salt measurements, relying on natural identification marks, like rock outcrops, prominent trees or shrubs, etc.

The positioning of the stations is shown in figure 2.a.

The distance to the river mouth, along the main streamline of the river was measured by:

- a rough grid of fixed points, their position determined by means of available maps or surveying marks. Four of these fixed points were chosen at the spots of the available or newly erected waterlevel gauges.

- the detailed station grid by measuring the time laps between stations, when navigating with one of the measuring boats at constant speed along the river. This was repeated a couple of times and
averages were taken.

It is expected that the distance between two rough-grid points, obtained from map measurements might have a maximum absolute error of appr. 400 m, considering the change in river course, since the drawing of the map (1956) and the uncertainty about the "centre" of river streamlines along which was measured.

About four to six stations were situated in between two rough-grid points that implies that the error in distance between two stations has a maximum of 100 m. The time lapse measurements had an higher accuracy and the error in distance as a result of these can be considered as being 20-30 m. All in all the distance between two stations has an absolute maximum error of \( \pm 120 \) m, which results in relative errors of appr. 4%.

cross-sectional profiles

The actual cross-sectional profiles were taken by

1. navigating between the two banks at a constant speed, with a running echo sounder (type see, appendix V).
2. measuring of the width at that point and with the obtained value determining the length scale of the echo sounder print out. The measuring of the width was done in two ways:
   a. angle determination using a levelling instrument and measuring the angle between two erect poles a known distance apart on the opposite bank (fig IV.3.a)
   b. measuring the vertical angle between two readings of a measuring scale erected at the other bank (fig IV.3.b)
3. Determining the value of the absolute waterlevel at the time of the echo sounding, by relating it to periodic waterlevels at one of the installed gauges along the river (fig. IV.4.b)

maximum errors in:
- depth reading by echo sounder: \( 0.20 \) m
- determining absolute waterlevel at time of sounding \( 0.10 \) m
- width determination:
  with method a (\( B>300 \) m.) \( \pm 3\% \)
  with method b (\( B<300 \) m.) \( \pm 2\% \)
  error in width because of uncertainty in exact average waterlevel \( 10 \) m
- total width error: \( B<300 \) m. \( 15\% \) or \( 5\% \)
  \( B>300 \) m. \( 40\% \) or \( 6\% \)
- tidal average area \( = (B+dB)*(Q+Q_B) \)
  \%dA = %dB + %dQ = 6\% + 9\% = 15\%
The profiles measured are represented in fig.4.d to i. These figures served as the basis for the input of the geometry in all models tested.

From the figures we can read:

- the cross-sectional area at tidal averaged waterlevel
- the width at tidal averaged waterlevel
- the mean depth at tidal averaged waterlevel
- the absolute level of the tidal averaged waterlevel.

Now that we have obtained the values for some cross-sectional parameters, we have to find a way to convert them to suitable model input data. It is not very convenient to use the measured values of the parameters as direct input data as:

1. it would call for long data input lists because of the large number of stations

2. the parameters are all measured in the stations. In case you want to use the station parameters in a direct way you can only use grids with stations on every node. For most numerical models this is a severe disadvantage because it would mean that the branch-lengths would vary between 1.8 and 3.4 km, like the distance between two measuring stations. However in most models we want a smaller and fairly constant branch length, which improves efficiency and accuracy.

For these reasons a different approach was sought. For the two tidal averaged models used (see chapter III) the
simplest way in which to provide the geometry input is to establish a mathematical relationship between the geometry parameter at hand (tidal averaged area $A$, width $B$ or depth $H$) and the distance to the river mouth (along the streamline axis of the river, $x$). For this purpose a regression analysis was used to establish such a relationship. It turned out that for both the $A-x$ and $B-x$ relationships an exponential function could be taken, except for the first two kilometers, due to the large deviations in the river mouth. The best-fit functions are presented in fig. 4. $A+B$, together with the actual measurements. The calculated functions, derived here are subsequently used as geometry input in the two stationary models.

It proved to be a little more difficult for the depth-distance relationship ($H-x$). The regression analysis showed only a very weak relation and therefore a direct function $H=f(x)$ could not be established. To represent the depth at the various places in the models we are left with two possibilities. One is to calculate the depth at every place from $H = A/B$, the most logical approach. The second one is to assume the depth to be (fairly) constant all over the estuary, at a value of $H_0$, in the Maputo case 3.6 m. This last method seems unrealistic at first sight, but is used by some researchers.

In the Van der Burgh stationary model both options were tested to get an idea of the validity of the last assumption in the Maputo case.

In working with the second tidally averaged model only the first $H$-representation was used.

In using the aforementioned relationships the nodes in numerical models can be chosen in an optimal way, on the basis of numerical criteria only.

A non-steady, real time model, like the Penpas model also used, cannot use the simple functions, derived above as this model does not work with tidal averaged values. For this model a different approach was chosen. At first the length of the branches and the positioning of the nodes were chosen on the basis of numerical stability and accuracy, taking care that at least the fixed, rough grid points would coincide with a model node.

Within a model-branch there were usually more than one measuring station with its measured river profile (fig IV.5). Subsequently all known river profiles in or near the model-branch were given weighting factors on the basis of their "influence-length" within the branch. In the case of the branch drawn in fig IV.5 the present profiles obtain the weighting factors noted there.

The profiles are combined to one characteristic branch profile all contributing according to their weighting.
factors. This process of combining is illustrated with an example in appendix IV.

Bed slope

The river bottom slope was determined using the four available waterlevel gauges, installed in Antunes (km 8.0), Bela Vista (km 23.0), Salamanga (km 41.0) and Santaca campomento (km 70.0) (fig. 2.a).

The levels of three of these gauges (all except Antunes) were related to the so-called MGM, the standard level in Mocambique for landsurveying. This level is roughly equal to the average sea level around Maputo. The level of the forth, Antunes had to be estimated, because of some difficulty in surveying its level.

Besides this, at all four points the cross-section of the river was measured at a certain waterlevel, Ha, indicated by the gauge.

From this echo sounding profile an average depth was determined, say Hd. As it was known at what level the zero of the gauge was situated, say Hp, the average bottom level could be determined by

$$Z_b = H_p + H_a - H_d \quad \text{(in meters + MGM)}$$

The bottom slope was determined by means of the formula:
\[ i_4 = \frac{(Z_b)_{i+} - (Z_b)_i}{(X)_{i+} - (X)_i} \]

\( i_b = \text{BOTTOM SLOPE} \)

\[ A = \text{cross-sectional area}, \quad B = \text{surface width}, \quad Z_b = \text{average bottom level} \]

\( (\ldots)_i = \text{VALUE AT GAUGE} \ i \)

**FIG. IV.6**

Of course this method only provides rough estimates as at the spot of the level gauges dips and dunes may be present in the riverbottom.
IV.2 TIDAL PHENOMENA

periodic waterlevels

The river Maputo ends in the bay of Maputo (see fig.1), which is a tidal area directly connected to the Indian ocean. This implies that all tidal waves coming in from the ocean first pass over, and are deformed by, all shoals and banks in the bay before reaching the river mouth. There was no waterlevel-gauge installed exactly at the river mouth because of

- difficulties in defining the exact position of the river mouth
- difficulties in reaching the river mouth over land
- no absolute necessity for one being there as it turned out that the salt boundary condition was not really fixed at the mouth either (see section IV.4).

Reasonable estimates can be made for the amplitude at the mouth though from the data of the Antunes gauge and the tide table of the port of Maputo (calculated estimates).

As there were permanently manned camps in Bela Vista and Santaca, these gauges were the ones that were read continuously during the whole field study. The other two were only occasionally read. The results of the gauge readings in Bela Vista are presented in the graphs 5.a to 5.d. It can be seen that around neap tide the curves are almost perfect sine-functions. However with bigger amplitudes the curves are deformed because of the different relative values of friction at high and low tide. Especially around low tide the depth of the river becomes so small that friction becomes relatively more important compared to the inertia forces. This can be seen from the deformation of the waterlevel curves towards a more flattened shape around low tide (fig IV.7).

The same phenomenon is present when the intrusion of a single tidal wave is represented for three places along the river (fig 7). In the upstream places the friction deformation plays a more important role due to the larger length of travel of the incoming tidal wave.
The obtained data from the gauge in Bela Vista were analyzed in a more detailed way as well. In fig 5.e the maximum, minimum and the tidal averaged levels of the gauge as well as the amplitude are graphed for two months, April and May 1984. The periodicity, of approximately 14 days, due to the moon-cycle is clearly visible. Since the gauge of Bela Vista was the only tidal one continuously read we will base a lot of calculations on these measurements. For instance in all estuary parameters, discussed in appendix I, values of the tidal amplitude at the river mouth are required. As those were not measured we will try to relate the amplitudes in the river mouth to those found in Bela Vista. From this relationship we will be able to predict the values in the river mouth on the basis of the Bela Vista values.

Furthermore we tried to establish a relationship between the tidal amplitude in Bela Vista and the estimated one of the port of Maputo (appendix VI)(fig,6). This was done in order to be able to estimate missing or future values at the Bela Vista gauge.

**Tidal prism**

A parameter that is frequently used in mathematical salinity intrusion models is the tidal prism, the total amount of water that enters the estuary between low water and high water slack. If there was no phase difference in tidal waterlevels all over the estuary, this prism would be equal to

\[ Pt = 2 \int_a^b h(x) \cdot B(x) \cdot dx \]  

\(\Delta h(x)=\)amplitude of waterlevel as a function of \(x\)
B(x) = width of the river as a function of x
L = length of tidal influence

However, with the shifting of phase, the total tidal prism will be lower than this value. In appendix III a calculation is set up to determine the difference of the theoretical maximum and the real value, using data obtained from an actual measurement at 2/5/1984. It is seen that the maximum error in using the theoretical maximum is about 14%, justifying the use of this maximum value in models.

The tidal prism is expected to have a more or less linear relationship with the tidal amplitude at, for instance, the river mouth or the better known values in Bela Vista. Roughly speaking this relationship has the form

\[ P_t = N_2 (\Delta H) \text{bela vista} \]  \hspace{1cm} (IV.2.2) \hspace{1cm} (see APP. I)

where \( N_2 \) is in the order of \( 44 \times 10^6 \) - \( 49 \times 10^6 \) m\(^3\). For the model calculations a \( N_2 \) value of \( 46 \times 10^6 \) m\(^3\) will be used.

One day in the field study, an attempt was made to measure the actual inflow in the estuary by means of discharge measurements every half hour. This was done by the "moving boat" method at Antunes, the place with the first gauge. The results are shown in fig. 8. The results provide very rough estimates only due to certain circumstances like the large width (appr. 1 km) and the inexperience of the measuring crew. These factors caused large errors in the discharge measurements, that could only be corrected to some extent in the analysis of the data. The calculated tidal prism on that day turns out to be much smaller than expected by the formula derived from the waterlevel amplitudes along the estuary.

Tidal velocities

Maximum tidal velocities are also frequently used in salinity intrusion models. They indicate the magnitude of the shear velocities \( (u^* : = \sqrt{\tau \cdot u}) \) and so the relative importance of the mixing in an estuary (see chapter II). High tidal velocities indicate better vertical mixing than lower ones, due to the bottom (and banks) shear stresses.

A method to estimate the maximum tidal velocities
is based on
- the calculated tidal prism
- the assumption of more or less periodic discharges
in the form of sine functions, say

\[ Q = \hat{Q} \cdot \sin\left(\frac{2\pi t}{T}\right) \]

\[ T = \text{tidal period} \]

In approximately half a tidal cycle the total volume of Pt enters so

\[ \int_0^{\frac{T}{2}} Q \, dt = P_t \quad \Rightarrow \quad \left[-\hat{Q} \cdot \frac{T}{2\pi} \cdot \cos\left(\frac{2\pi t}{T}\right)\right]_0^{\frac{T}{2}} = P_t \]

This ends in the following relationship:

\[ \hat{u}(x) = \hat{\delta}(x) \cdot \frac{P_t(x)}{A(x)} = \frac{P_t(x)}{T \cdot A(x)} \quad \text{IV.2.3} \]

If functions are available for \( P_t(x) \) and \( A(x) \), the maximum tidal velocities at every spot can be estimated from IV.2.3.

The root mean square values can then be found from

\[ u_{rms} = \sqrt{\frac{T}{\pi} \hat{u}^2} \approx 0.886 \cdot \hat{u} \]

This method is of course as accurate as the assumptions it is based on. It will be reasonably accurate in cases, where the tidal amplitudes are quite small. The assumption that discharges and waterlevels can be represented by sinusoides is then fair. If the situation at or around spring tide is considered however we notice that deviations from these assumed functions occur that can be substantial.

Furthermore it is based on the assumption that the fresh water river discharge is negligible compared to the tidal discharges.
The river Maputo is one of the rivers encountered in the hydrological network of the Direccao Nacional de Aguas (national water service) and therefore sufficient data is available, of recent years, about its discharges and levels at the fixed gauges in Bela Vista, Salamanga and especially the non-tidal gauges in Santaca (km 70) and Madebula (km 120).

During the field survey of March - June 1984 a permanently manned camp was set up in Santaca. The purpose of this camp was

1. to measure the discharge in the river on a daily basis, in order to use this data in salt studies

2. to determine a waterlevel/discharge relationship in this place in order to determine the river discharge if only waterlevel readings are available

The discharges measured are shown in fig 10. The method of measuring is shown in fig IV.8.

Fig. IV.8

At each point, distance xi from the bank, the depth di was measured.

An impeller (app.V) was lowered to 0.6*ai and the rotations counted per minute. From the manufacturers table the average velocity can be determined. The velocity thus measured, at 0.6*ai is considered to be the average for the whole area Ai (=Bi * ai).

The total discharge was calculated by means of the formula

\[ Q = \sum_{i=1}^{n} \left( \sum_{j=1}^{m} A_i \right) \]

As far as the level/discharge relationship is concerned; the
data, obtained during the three-month field study forms a graph like fig.11. From this graph a level-discharge formula can be determined, by means of a regression analysis. In the Santaca case this formula is

$$\log(H+0.6) = -0.197 + 0.381 \log(Q)$$

correlation coefficient $r^2 = 0.973$ with 47 data pairs

$H$ in m, $Q$ in m$^3$/s

After a few heavy showers at the end of March and beginning of April, the rainfall stopped around April 11th. From fig. 10 it can be seen that the discharge adjusts itself within two to three weeks to this new situation, a time lapse that might be important when we discuss salt-distribution equilibria, later on in section IV.4.

One of the questions still remaining is how fast a change in the discharge measured in the non-tidal area at Santaca will affect the downstream parts. Especially in periods of rapidly changing discharges it is uncertain which discharge-value needs to be used in calculations about salt-intrusion in the part say downstream of Bela Vista. Waves (changes in discharge), can react in different ways, depending on the importance of friction, their amplitude and period etc.

In case friction could be neglected and the amplitude is not too large, in comparison to the waterdepth the velocity of a wave would be close to:

$$c = U_f \pm \sqrt{g.a} \approx \sqrt{g.a}$$

for instance with: $Q_f = 100$ m$^3$/s  

<table>
<thead>
<tr>
<th>Santaca</th>
<th>Bela Vista</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_f = 1$ m/s</td>
<td>$U_f = 0.065$ m/s</td>
</tr>
<tr>
<td>$a = 2.5$ m</td>
<td>$a = 3.5$ m</td>
</tr>
</tbody>
</table>

$$\frac{1+\sqrt{0.025+0.065+0.025}}{2} = 6 \text{ m/s}$$

average travelling time between Santaca and Bela Vista is then approximately

$$\frac{70,000-23,000}{6} = 7830 \text{ s} = 2.18 \text{ hr.}$$

On the other hand if the change in discharge simply reacts as a floodwave, its travelling speed would equal

$$c = \frac{3/2 \times 1 + 3/2 \times 0.065}{2} = 0.80 \text{ m/s}$$

with a travelling time of $$\frac{47,000}{0.8} = 6.3 \text{ hour.}$$

In reality the situation is somewhere in between these two
limits.
In both cases however the change in discharge will be felt in Bela Vista within one day.

For a discharge of 50 m³/s the respective travelling times Santaca - Bela Vista become
floodwave dynamic wave
c = 0.4 m/s c = 5.75 m/s
average average
T = 32.6 hrs T = 2.3 hrs
IV.4 SALINITY MEASUREMENTS

Within the field survey, several types of salt measurements were undertaken.

a. Longitudinal salinity profile measurements at either:
   - high slack
   - low slack
   This is the main set of measurements.

Starting from the river-mouth, the measuring boat tried to keep up with the slack tide and stopped at every measuring station on the way to take a vertical salt profile at the deepest point of the cross-section. From these verticals we can see whether the river was well mixed at that point and find the average salinity value.

Due to the phase shift in tide along the river it was possible to travel with the slack tide upstream. At low-water slack this was fairly easy, due to the low tidal wave velocity (very low depths at that time so velocities greatly reduced by friction).

At high tide slack it proved to be a little more difficult, especially when the salt intruded over a long distance, so that measurements had to be taken over a long reach.

At such slack measurements the salinity was measured in the vertical in the deepest point of the cross-section. This vertical profile was averaged and the value derived from this was assumed to be the cross-sectional average.

These average salt contents are plotted against the distance to the mouth. Results are shown in figures 12.a to 12.m. Each measurement done in this way has a code name (F3 - F15).

In Table IV.1 details of each measurement are brought together including dates of measurement, river discharge at that time, tidal amplitude in Bela Vista and estuary parameters derived from these data. When the profiles were measured at high and low slack tide, it was possible to estimate a tidally average longitudinal profile by taking the averages of the two forementioned curves.

b. Hourly measurements of the salinity during the day at one fixed point

The usual procedure was to take sample bottles from...
<table>
<thead>
<tr>
<th>measurement</th>
<th>date</th>
<th>$Q_f$ (m$^3$/s)</th>
<th>$\Delta H_{BV}$</th>
<th>$e/C_1$</th>
<th>$R_{ie}/C_2$</th>
<th>$E_{i}/C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F3</td>
<td>3-4-1984</td>
<td>190</td>
<td>1.71</td>
<td>111</td>
<td>0.20</td>
<td>5.0</td>
</tr>
<tr>
<td>F4</td>
<td>6-4-1984</td>
<td>160</td>
<td>1.48</td>
<td>108</td>
<td>0.26</td>
<td>3.8</td>
</tr>
<tr>
<td>F5</td>
<td>11-4-1984</td>
<td>175</td>
<td>1.45</td>
<td>121</td>
<td>0.30</td>
<td>3.3</td>
</tr>
<tr>
<td>F6</td>
<td>19-4-1984</td>
<td>115</td>
<td>1.65</td>
<td>69.7</td>
<td>0.14</td>
<td>7.1</td>
</tr>
<tr>
<td>F7</td>
<td>25-4-1984</td>
<td>75</td>
<td>0.65</td>
<td>115.4</td>
<td>0.088</td>
<td>11.4</td>
</tr>
<tr>
<td>F8</td>
<td>2-5-1984</td>
<td>60</td>
<td>1.67</td>
<td>22.5</td>
<td>0.054</td>
<td>18.5</td>
</tr>
<tr>
<td>F9</td>
<td>10-5-1984</td>
<td>55</td>
<td>0.84</td>
<td>62.8</td>
<td>0.49</td>
<td>2.0</td>
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<tr>
<td>F10</td>
<td>17-5-1984</td>
<td>50</td>
<td>1.70</td>
<td>16.2</td>
<td>0.04</td>
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<td>F11</td>
<td>23-5-1984</td>
<td>50</td>
<td>0.60</td>
<td>83</td>
<td>1.23</td>
<td>0.8</td>
</tr>
<tr>
<td>F12</td>
<td>29-5-1984</td>
<td>50</td>
<td>1.48</td>
<td>16.9</td>
<td>0.056</td>
<td>17.9</td>
</tr>
<tr>
<td>F13</td>
<td>20-10-1983</td>
<td>0.05</td>
<td>1.56</td>
<td>0.03</td>
<td>0.0001</td>
<td>1000.0</td>
</tr>
<tr>
<td>F14</td>
<td>28-4-1982</td>
<td>21</td>
<td>1.73</td>
<td>6.9</td>
<td>0.025</td>
<td>40.0</td>
</tr>
<tr>
<td>F15</td>
<td>15-7-1982</td>
<td>4.8</td>
<td>1.04</td>
<td>2.2</td>
<td>0.015</td>
<td>67.0</td>
</tr>
</tbody>
</table>

$C_1 = 4.0 \times 10^{-3}$ s/m$^3$  $C_2 \approx 1925$ s  $C_3 \approx 1.9$ 1/s

table IV.1
the surface in Bela Vista every hour from an old pier, extending about 15 meters into the river (fig 13).
On some occasions a whole vertical profile was taken every hour at the deepest point of the cross-section, in Bela Vista and Antunes to be able to see the way the salt ingressed (see fig. 14 + 15).

c. measurements of a few verticals within one cross-section at the same time

This was done to find out if it could be justified to represent the river in a two or even one dimensional form.
When judged on the basis of figure 16 we can say that lateral salt gradients are generally small and can be neglected, especially in comparison to the longitudinal ones.

Looking at the salt measurements, some conclusions can be drawn about the salt distribution in the Maputo estuary

stratification and vertical mixing

The Maputo estuary is definitely not always of well mixed type. Particularly in the beginning of the field study (high discharges >100 m3/sec), and during all neap tide periods (even at the lower fresh water discharges) the estuary was vertically stratified and sometimes a very clear interface could be identified. Besides this it was noticed that in case of such stratified situations the salt often intruded further into the estuary than under well-mixed circumstances.
The older measurements however, performed in 1982 and '83, with discharges below 20 m3/sec prove that the river becomes well mixed throughout the cycle of neap and spring tides, if the discharge is low enough.
This implies it is not realistic to use all available data on salt distributions in one-dimensional models that all assume a well-mixed situation. It might be better to use the well-mixed estuary data only as input for such models. Of course this will limit the prediction capability of the models to those situations where well-mixed profiles can be expected. On the other hand, the cases we are most interested in, the ones with the very low discharges during dry periods, seem to be of well-mixed type.
If we want to predict salt-intrusion under stratified circumstances it is advisable to turn to "two-layer models".

69.
These models, however, will not be discussed in this report.

---

movement during tidal cycle

Only at the time of a well-mixed estuary (eg. dd. 17/5/1984, 29/5/1984) the shape of the longitudinal salt distribution curve is almost the same during the whole tidal cycle. The only difference between high and low tide curve is a shift over a certain distance, $L_{HN}$.

The shift distance is plotted in fig IV.10 against the amplitude of the waterlevel in Bela Vista. Too little data is available to determine a formula, valid for all situations, but a reasonable indication about the travelling distance of salt particles during the tidal period can be obtained from the graph.

All tidal-averaged models calculate average salinity distributions between low and high tide. By using fig. IV.10 and this average profile, maximum and minimum salt intrusions can be determined.

During really stratified periods, - extending from approximately one day before until two days after neap tide-, the water at different levels in the cross-section does not move as one at the same speed. Especially in the bottom layer outward velocities are hindered, or almost zero, as seen from the fairly constant concentrations of salt at the bottom during a tidal cycle.

This is probably caused by an almost stationary salt wedge. Because of the density difference the salt wedge would like to intrude further, but it is stopped by the relatively small shear force produced at the interface of the two layers, by the top layer movement.
This shear force also provides the energy for the mixing of salt and fresh water-layers. Since the mixing rate is not so great at neap tide, with lower top velocities, the salt wedge is able to intrude and remain there.

Salt content of river water

The natural salt load of the river is always below 0.3 % (150 mg Cl/l). This indicates that the fresh river water is suitable for drinking water supply as well as irrigation, as far as salinity is concerned.

Salt boundary at river mouth

The boundary condition for the salt intrusion at the river mouth is quite difficult to determine. At high slack periods the salinity at the river mouth cannot be considered constant but varied within the range of 28-32 %, for the field study period (high tide measurements, averaged over the cross-section). From the measurements in 1982 - '83 it can be seen that this value was much higher (up to 35 %) in those, very dry periods.

The most probable reason for this is that the river is not directly connected to the indian ocean, but ends in the bay of Maputo. This bay acts as a continuation of the river and thus changes the salinity.

The change most probably depends on

- tidal amplitude in the ocean
- salinity in the ocean (fairly constant over the year)
- discharge of all rivers ending in the bay, being the Maputo, the Incomati and the Umbeluzi.

In the very dry seasons of '81/'82 and '82/'83 the discharge of these rivers was so low that the water entering the bay from the ocean, mixed with only a negligible amount of fresh water.
Figure IV.12 provides an indication of the relationship between salinity in the mouth and the fresh water discharge respectively. Of course this figure provides no more than a rough indication and we have to be very cautious in applying it to future situations. The fact that the salinity in the bay does not depend on the discharge of one river only, but on two others as well (Incomati, Umbeluzi), with different catchment area characteristics makes predictions on the basis of the discharge of the river Maputo only haphazard.

Input data for salt intrusion models

This report deals with one-dimensional models only. This makes sense if the river under consideration can be represented as a one-dimensional flow system. This means no large variations in velocity and salinity over the cross-section may occur. As can be seen from the salt measurements, some days this condition is certainly fulfilled. At other times, a stratified situation develops with two (or three) layers, reacting in a different way. In such cases one-dimensional models cannot be used.

From the measurements shown we can see that there is no fixed-shape salt profile during the day in these cases (e.g. Fl1, dd. 23/5/1984) and discussions about averages over the day and tidal excursion
lengths are completely useless. As we can assume that such models are mainly used for low(er) discharge circumstances and we know from earlier studies that at those times the estuary is well-mixed all through the neap-tide/spring-tide cycle, it seems logical to include the available well-mixed measurements only in the calibration and running of the models. This implies we will limit ourselves to working with the measurements F8, F10, F12, F13, F14 and F15 in all tidal averaged models. In the non-stationary model an attempt will be made to run the model over a certain timespan including one neap-tide time period in order to discover how much the results will deviate, because of the change in stratification.

Besides these six longitudinal salinity profiles used for calibration and checking of the models, the relationship of fig.IV.12 will be used to predict the maximum and average salinity in the Maputo bay at a certain discharge of the river. The maximum is needed as input for the nonsteady, the tidal average for the two steady models.
CHAPTER V  CALIBRATION AND TESTING OF THE MODELS

V.1 Introduction

The three models selected for further testing have been described in chapter III. In this chapter we will concentrate ourselves on calibration of the various models using observations of the Maputo fieldsurvey. After calibration of each model, they will be tested on their prediction possibilities, within the measurements range (interpolation) and outside it (extrapolation). The general strategy is to try to predict the longitudinal salinity profile under the conditions of measurement F15. In this way it is possible to compare the predictions of the various models.

In comparing the models we will mainly look at

- the maximum deviation in intrusion distance of a specific salinity front
- the deviation in intrusion distance of the 2% front. Salinities of 2% can be used neither in irrigation nor in watersupply any more

V.2 Van der Burgh model

Calibration of the Van der Burgh, tidally averaged, steady model (see III.2.1) means finding

- a constant \( K \)-value, the factor that decreases the dispersion coefficient while running upstream (eq. III.3)

- a linear relationship between the dispersion coefficient in the rivermouth, \( D_0 \), and one of the estuary parameters, the floodnumber, \( e \), the estuarine Richardson number, \( R_{e} \), or the densimetric estuary number, \( E_e \).

Measurements used for the calibration were the longitudinal salinity profiles F8, F10, F12, F13, F14 and F15 (see fig 12.a to .m).
In order to perform the calibration, a small computer programme was used to scan what value of $K$ and $D_o$ would result in the smallest differences between measured and computed salinities. The curve with the smallest differences was named the "best-fit curve".

From the first round of rough optimization attempts it became clear that all measurements could be fitted reasonably with a $K$-value of around 0.3, except F13. This measurement taken under very exceptional circumstances (at the end of a two year drought period) could only be fitted with a $K$ of 0.1 or less. It was decided to leave out the F13 measurement for the time being and to run the model from now on with $K=0.3$.

This $K$-value fixed, optimal values of $D_o$ were found for each of the five remaining measurements. This was done using both ways of geometry input being:

- cross-sectional area $A$ input
- surface width $B$ input combined with a constant depth $H_o$, assumed to be 3.6 m; the mean depth in the estuary

All results are shown in table V.1. Before we discuss the results we first have to mention two factors having an influence on the determination of the optimal $D_o$ values of a measurement.

<table>
<thead>
<tr>
<th>measurement</th>
<th>Optimal $D_o$ ($m^2/s$)</th>
<th>$A$-input</th>
<th>$B$-input</th>
<th>$e/C1$</th>
<th>$R_e/C2$</th>
<th>$E_i/C3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F8</td>
<td>145.2</td>
<td>162.4</td>
<td>22.5</td>
<td>0.054</td>
<td>185</td>
<td></td>
</tr>
<tr>
<td>F10</td>
<td>149.6</td>
<td>165.6</td>
<td>16.2</td>
<td>0.040</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>F12</td>
<td>183.4</td>
<td>203.2</td>
<td>16.9</td>
<td>0.056</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>F14</td>
<td>72.3</td>
<td>79.1</td>
<td>6.9</td>
<td>0.025</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>F15</td>
<td>35.4</td>
<td>36.9</td>
<td>2.2</td>
<td>0.015</td>
<td>67</td>
<td></td>
</tr>
</tbody>
</table>

$C1=4.0 \times 10^{-3}$ s/m$^2$ $C2=1925$ s $C3\approx 1.9$ s$'$

Table V.1
In order to find the best-fit curves it was first decided upon what was meant with "best-fit". The simplest way would be to have a parameter indicating the degree of difference between the two curves, the observed and the computed. Two useful parameters used in other studies are

1. accumulated squares of the absolute difference between $c_m$ (measured salinity) and $c_c$ (computed salinity) (see fig V.1)
   \[ AS = \sum_{i=1}^{n} (c_c - c_m)^2 \]
   to obtain the best-fit curve AS must be minimized

2. accumulated squares of the relative differences between $c_c$ and $c_m$
   \[ ARS = \sum_{i=1}^{n} \left( \frac{c_c - c_m}{c_m} \right)^2 \]
   to obtain the best-fit curve ARS must be minimized

The use of $AS$ provides reasonable fits, especially for the high and medium salinities. At the foot of the curve however, salinity differences do not contribute so much to the total sum but can have high relative difference values (100-200% deviation or more still results in small absolute deviations). On the other hand, using $ARS$ in fitting of the curves lays undue emphasis on the fitting of especially the low salinities (an equal absolute deviation gives a much higher relative difference in case $c=2.0\%$ than in case $c=30\%$).

The ideal weighting parameter will be something in between

---

fig. V.1

The use of $AS$ provides reasonable fits, especially for the high and medium high salinities. At the foot of the curve, however, salinity differences do not contribute so much to the total sum but can have high relative difference values (100-200% deviation or more still results in small absolute deviations). On the other hand, using $ARS$ in fitting of the curves lays undue emphasis on the fitting of especially the low salinities (an equal absolute deviation gives a much higher relative difference in case $c=2.0\%$ than in case $c=30\%$). The ideal weighting parameter will be something in between.
AS and ARS, so no emphasis on either foot nor top-part of the curves. In this study was chosen for a weighting factor AMS

\[ AMS = \sum_{i=1}^{n} \left( \frac{C_{ci} - C_{mi}}{\sqrt{C_{mi}}} \right)^2 \]

This factor usually provided curves that looked well fitted.

Fig V.2 shows the best-fit curves under the conditions of measurement F15, with weighting parameters AM, ARS and AMS respectively, together with the actual observation curve.

minimum dispersion coefficient, \( D_{nim} \)

Owing to the factor \( K \), the dispersion coefficient will decrease, running upstream. This is not too bad an assumption as the dispersion, due to the salinity differences decreases landwards (chapter II). In running the model however it quite often happened that the \( D_{nT} \) approached zero or even tended to become negative, owing to the constant diminishing by eq III.3. In reality this is sheer impossible. A more realistic scheme would be that the \( D \) would decrease to a certain value, \( D_{nim} \) already present without the presence of density differences, so caused by shear flow dispersion.

For this reason an additional, conditional equation was introduced in the model, fixing the \( D \) at a certain value, \( D_{nim} \), when the \( D \) tended to become less than \( D_{nim} \). This parameter was estimated from eq. II.17, giving values for dispersion in tidal, constant density flows.

\[ D_{nim} = 0.1 \cdot \bar{u}^2 \cdot T \cdot \left[ \frac{1}{T} \int f(T') \right] \]

(for values of \( \frac{1}{T} \int f(T') \) see fig II.3)

Figure V.3 shows the estimated values of the \( D_{nim} \) along the estuary. A cause of errors in this estimate is the value of \( \bar{u}^2 \). Usually the tidally averaged value of the velocity is known (fresh water velocity) but the flow during the tidal cycle has to be measured in situ for all points, information that was not present in the Maputo study.

During calibration of the Van der Burgh model with the six measurements, it was seen that quite often a \( D_{nim} \) of 10 m²/s gave good results. This value differs a factor 4 - 6 from the estimated values of fig V.3. As the calculation of \( D_{nim} \) with eq II.17 needs extra data (flow in the river all through the tidal cycle) and the deviation is still quite large it is recommended to find \( D_{nim} \) by means of a sensitivity analysis and to calculate the
value given by II.17 at some places in the estuary only, to
give an indication of its magnitude.

In running of the model a value of 10 m²/s was used as the
fixed value for the $D_{\min}$ all over the estuary.

As mentioned in the beginning of this section the purpose of
the calibration is to establish a linear relationship
between $D_0$ and one of the estuary parameters. Therefore the
optimal $D_0$ -values found, are graphed, against $e$, $R_{ie}$ and $E_i$
in fig V.4.a,b and c respectively.

A statistical elaboration shows the correlation of the
various realtionships, expressed in the correlation
coefficient $r (-1 < r < +1)$. The closer the absolute value
of $r$ approaches 1, the better is the correlation between the
two parameters (in case of equal numbers of observations).
With this $r$-value and the number of observations included we
can say something about the certainty that the correlation
is real ($\alpha\%$), from tables in statistical handbooks (for
instance [8]).
The relationships, the correlation coefficients and the
 corresponding confidence percentages are given in table V.2.
The relationships themselves are also graphed in figures
V.4).
\[ \varepsilon = \text{flood number} \quad \frac{Q \cdot T}{P} \approx C_1 \cdot \frac{Q}{\Delta H_{\text{eq}}} \quad C_1 \approx 4 \times 10^{-3} \text{ (s/m^3)} \]

**Fig. V. 4.a.**

\[ R_{i_{\text{e}}} = \frac{\Delta \rho \cdot g \cdot h \cdot u^4}{(c_{1 \cdot u_c})^3} \approx C_1 \cdot \frac{\Delta \rho \cdot Q^4}{(\Delta H_{\text{eq}})^2} \]

\[ \varepsilon = \text{A-input} \quad x = \text{B - Ho = 3.6 m - input} \]

**Fig. V. 4.b.**

\[ R_{i_{\text{e}}} / C_2 \quad (\frac{1}{s}) \quad C_2 \approx 1925 \text{ (s)} \]
$$E_l = \text{dendimetric estuary number} = \frac{Pr \cdot F_i^2}{Q_i \cdot T} = C_3 \cdot \left(\frac{\Delta H_{0w}}{Q_{f, \text{app}}}\right)^3 (s)$$

$$C_3 \approx 1.9 \left(\frac{L}{S}\right)$$

Fig E.4c
The various results lead to some conclusions already

- the relatively simple estuary parameter e, the flood number of Canter Cremers does not provide as good a correlation as the other two parameters do (correlation coefficient deviates more from 1 and is smaller)

- Richardson number and densimetric Estuary number give equal correlative results

- The correlation between $D_0$ and estuary parameters is not influenced by the kind of geometry input used. This is definitely a positive point for the B-H$_o$-input as this kind of input is much easier to obtain. Most probably, the quite rough model smooths out all inaccuracies that arise because of the constant depth assumption.

Interpolation

The first test of the Van der Burgh model is to find out how it predicts the tidally averaged longitudinal salinity profiles under circumstances that still are within the range of observations made. Therefore we use one of the relationships, given in table V.2, and try to reproduce the salinity profile under the F15 circumstances.
The relationship used is the $D_0 - R_{ie}$ formula with A-input. This is one of the equations that might offer the most accurate prediction, seen from the correlation coefficient. Other relationships with equal r's will result in similar results.

$$D_0 = 3258 \frac{R_{ie}}{C_2} - 6.60 \quad C_2 \approx 1925 \text{ s}$$

Measurement F15 has an $R_{ie}/C_2$ of 0.0149 (see table IV.1), so $D_0$ is 41.9 m$^2$/s. The best fit curve gave a $D_0$-value of 35.4 m$^2$/s, a relative deviation of 16 %. What this means for the longitudinal salinity profile can be seen in figure V.5, where the curves, calculated with both $D_0$'s are graphed. Furthermore a statistical approach of the same $D_0 - R_{ie}$ relationship shows us that we cannot talk about a sharp straight prediction line, but only about confidence intervals, areas where the prediction will be in for a specific percentage of certainty. Returning to our F15 measurement conditions we find from such a statistical elaboration that the predicted $D_0$ has 90% certainty of being in the range of

$$31.4 \text{ m}^2/\text{s} < D_0, \text{F15} < 52.4 \text{ m}^2/\text{s}$$

The longitudinal salt profiles with these values are graphed as well in fig V.5. From these graphs we can now find the maximum deviations between predictions and actual observations. Maximum deviation in intrusion distance of a specific salt front is for instance 7.4 km (9% -front), while the 2% -front has a 7 km deviation. The low accuracies are definitely not only caused by the non-existence of a linear relationship between $D_0$ and $R_{ie}$ but also by the small number of measurements used to establish this relationship. All equations were based on five measurements only!

extrapolation

Keeping all the observed uncertainties in mind we will now attempt to say something about the extrapolating capabilities of the Van der Burgh model. We already noticed that under extreme circumstances, like those of measurement F13, the model lacks the ability to describe the situation in the same way as was done for the other measurements. This implies there is a lower limit to the conditions that can be described, a severe limitation for models that are usually needed to describe extreme, low discharge situations. Later on we will see however that not only the Van der Burgh
\( x = \text{observations (high + low water slack)} \)

○ \text{Observation F15 (tidally averaged long sal. profile)}

1 \quad \text{Prediction Do - Re - relationship}

2+3 \quad \text{Lower + upper boundaries 90\% confidence interval}

\text{Van der Burgh model interpolation results}

\text{Fig. IV.5}
model has difficulties in describing the F13 circumstances.

Another measurement under fairly extreme circumstances is F15 \((Q_f=5\ \text{m}^3/\text{s})\). This measurement is also well out of range of all other measurements \((Q_f > 21\ \text{m}^3/\text{s})\) and therefore F15 was chosen as test case.

We will leave out all data of F15 and construct the Do-estuary parameter relationships again, with the data of the remaining observations only. The relationships, slightly different from the ones with the F15 data included are given together with their Do prediction under F15 conditions in table V.3.

<table>
<thead>
<tr>
<th>equations</th>
<th>r</th>
<th>(D_o) predicted ((\text{m}^3/\text{s}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A geometry input</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D_o = 5.50\ e/C1 + 51.86)</td>
<td>0.76</td>
<td>63.9</td>
</tr>
<tr>
<td>(D_o = 2939\ R_{i\infty}/C2 + 8.30)</td>
<td>0.90</td>
<td>52.1</td>
</tr>
<tr>
<td>(D_o = -4.27\ E_i/C3 + 246)</td>
<td>-0.94</td>
<td>-40.3</td>
</tr>
<tr>
<td>(D_o) of best fit curve</td>
<td></td>
<td>35.4</td>
</tr>
<tr>
<td>B-Ho geometry input</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D_o = 6.24\ e/C1 + 55.2)</td>
<td>0.77</td>
<td>68.8</td>
</tr>
<tr>
<td>(D_o = 3311\ R_{i\infty}/C2 + 6.88)</td>
<td>0.91</td>
<td>56.2</td>
</tr>
<tr>
<td>(D_o = -4.81\ E_i/C3 + 275)</td>
<td>-0.94</td>
<td>-47.8</td>
</tr>
<tr>
<td>(D_o) of best fit curve</td>
<td></td>
<td>36.9</td>
</tr>
</tbody>
</table>

\[C_1 \approx 4 \times 10^{-3}\ \text{s/m}^2\quad C_2 \approx 1925\ \text{s}\quad C_3 \approx 1.9\quad \text{s}^{-1}\]

| table V.3. |

Deviations between \(D_o\) prediction and \(D_o\) best fit is quite large with a minimum of 50% in case of the \(D_o-R_{i\infty}\) relationships.

The \(D_o-E_i\) relationship even provides negative \(D_o\)-values, something that is physically impossible.

The prediction results for \(D_o-R_{i\infty}\) and \(D_o-e\) relationships are graphed in fig. V.6.a, while for the \(D_o-R_{i\infty}\) predictions a reliability interval is constructed in fig V.6.b in a similar way as was done with the interpolation testing.

From fig V.6.a and V.6.b we can conclude that the predictions are definitely not accurate. In case of the best predictive relationship, the \(D_o-R_{i\infty}\) function, specific predicted salinity fronts may be shifted from the measured ones as much as 10 km (within 90% reliability of the prediction - function). The 2% salinity front deviates up to 7 km.

The other relationships, \(D_o-e\) and \(D_o-E_i\), are even more inaccurate to completely useless. Besides this we also have to remember that these predictions do not yet cover extreme
○ measurement F15
prediction with A-input:
1 $D_o - R_{15}$
2 $D_o - \varepsilon$

prediction with B-H$_2$-input:
3 $D_o - R_{15}$
4 $D_o - \varepsilon$

5 Best Fit (A-input)

VAN DER BURGH MODEL
EXTRAPOLATION RESULTS

Fig. V. 6.a.
Observation F15
(Tidally averaged long sal. profile)

Prediction $D_0 - V_{Re}$ relationship

Lower + Upper boundary
90% Reliability interval

Van der Burgh model
Extrapolation results

Fig. V.6.b.
situations in the estuary but only go down as far as fresh water discharges of 5 m³/s.

V.3 Combined model

Calibration of the Combined, tidally averaged, steady model asks for the determination of the three constants, Kₖ, K₁ and K₃. The same approach was used as with the Van der Burgh model.

Best-fit curves were found for the longitudinal salinity profiles F₈, F₁₀, F₁₂, F₁₃, F₁₄ and F₁₅. Also in this model, the F₁₃ measurement posed the model for problems, only providing very inaccurate best-fit curves. It was clear that the model could not really handle this type of estuary conditions.

The same weighting parameter was used to determine best fit computations. For the minimum dispersion coefficient a value of 10 m²/s was assumed, however this value was never reached during operation of the model. This already indicates a better simulation of natural circumstances, as the dispersion never tends to become lower than possible in practical circumstances. This is one of the factors we have to keep in mind when comparing the Combined and Van der Burgh models.

The values for the constants in the best-fit curves are gathered in table V.4

Some conclusions drawn from table V.4 are

- the k-value always tends to become 1.0 in the best-fitting process. Seen from eq.III.7, this would mean that the influence of the fresh water velocity, uₖ, would disappear altogether from the computations. This is very unlikely, seen from actual observations, whereby the variation in fresh water discharge is a main factor in the salt intrusion distance.

- Although theoretically said to be fairly constant, especially the K₃ has quite some deviation. Moreover we notice a clear correlation between the K₃ value and for instance the fresh water discharge (Qₖ) or an estuary parameter.
### Table V.4

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Optimal Value of k</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kl</td>
</tr>
<tr>
<td>F8</td>
<td>1.0</td>
</tr>
<tr>
<td>F10</td>
<td>1.0</td>
</tr>
<tr>
<td>F12</td>
<td>1.0</td>
</tr>
<tr>
<td>F14</td>
<td>1.0</td>
</tr>
<tr>
<td>F15</td>
<td>1.0</td>
</tr>
<tr>
<td>Data F15 included average standard dev.</td>
<td>1.0</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Data F15 excluded average standard dev.</td>
<td>1.0</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

This, and the first mentioned conclusion indicate that the model has to be modified, with an adjusted set of constants, in order to make it represent reality in a better way (see immediately after the testing of the non-modified model).

Kl sometimes has a negative value, what means it passed zero. Seen from eq III.7 the salinity distribution exactly at Kl=0 does not exist. However, rewriting eq III.7 to eq III.8 takes away the problem, while it can be proved that even in eq III.7 the distribution of salt is determined if one looks at the right or the left limit of Kl nearing to zero.

Physically, the only restriction on Kl is that it has to be smaller than 1.0, seen from eq III.8. If not, the physical situation will not be correctly represented.

First of all we will now continue testing the Combined model, described in section III.2.1. Afterwards we will also try to modify the model in such a way that the deficiencies, mentioned in the first two conclusions will disappear.

**Interpolation predictions**

Again interpolation capabilities will be tested by including all measurements in the calibration and after calibration.
take a look at what results for the prediction of the F15 conditions of the estuary, being just inside the measuring range.

If we set off from the original set up of the model, the various K-values must be constant. This implies that we arrive at the following averaged values for $k$, $K_l$ and $K_3$

<table>
<thead>
<tr>
<th>variable</th>
<th>value</th>
<th>90% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>$K_l$</td>
<td>0.137</td>
<td>0.064 - 0.200</td>
</tr>
<tr>
<td>$K_3$</td>
<td>4.070</td>
<td>3.316 - 4.823</td>
</tr>
</tbody>
</table>

In figure V.7 we have graphed the curves of the F15 measurement conditions, resulting from computations with

- all mean values of $k$, $K_l$ and $K_3$
- all lower, 90% conf. interval limits
- all upper, 90% conf. interval limits

We notice that the interpolated mean prediction of the F15 conditions deviate up to 7 km at especially the lower salinity-fronts, with the actual observed intrusions. If we consider the outer limits of the 90% confidence interval this maximum deviation is even increased up to 9 km at the 2% front. The complete observed longitudinal, tidally averaged salinity profile does not fit within the 90% confidence area what means that the prediction is very inaccurate.

extrapolation
---------

In case the F15 measurement is left out in compiling the average values of the various constants the K-values become

<table>
<thead>
<tr>
<th>variable</th>
<th>value</th>
<th>90% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>$K_l$</td>
<td>0.151</td>
<td>-0.047 - 0.286</td>
</tr>
<tr>
<td>$K_3$</td>
<td>4.427</td>
<td>2.882 - 5.972</td>
</tr>
</tbody>
</table>

The graphs, computed with these K-values, under F15 conditions are shown in fig V.8. It is clear that the results are even more inaccurate than the interpolation predictions, as could be foreseen of course. Maximum deviations at the 2% front are

- 7.5 km with the mean-prediction curve
- 11.0 km with the upper limit curve of the 90% confidence area

90.
OBSERVATION F15
(TIDALLY AVERAGED LONG. SAL. PROF)

PREDICTION \( D_o - R_{ib} \) - RELATIONSHIP

90\% CONFIDENCE INTERVAL
LOWER + UPPER BOUNDARY

NON-MODIFIED COMBINED MODEL
INTERPOLATION RESULTS

Fig. V. 7.
OBSERVATION F5
(TIDALLY AVERAGED LONG. SAL. PROFILE)

PREDICTION
①②③ LOWER + UPPER BOUNDARIES
90% CONFIDENCE INTERVAL

NON-MODIFIED COMBINED MODEL
EXTRACTION RESULTS

Fig. V.8.
It is quite logical that these predictions outside the observation range are quite useless, with this non-modified Combined model. Besides this we also have to remember that we did not even include the very low discharges, say lower than 2 - 5 m3/s.

Modified Combined model

From the best-fit curve values of the three constants, \( k \), \( K_1 \) and \( K_3 \), we noticed that

- the influence of the fresh water velocity (discharge) would officially disappear

- the \( K_3 \) value is not really constant but, at first sight has a correlation with the fresh water discharge or one of the estuary parameters

For these reasons we slightly adjusted the Combined model to improve its prediction capabilities. These adjustments are

- fixing of the \( k \) value at 1.0, something that clearly emerged from the non-modified model

- assuming the \( K_3 \), in eq.III.8, to have a linear relationship with one of the estuary parameters, \( R_{e} \). This parameter was selected as it gave the best results in the Van der Burgh model as well. Besides this we can already state beforehand that it will represent physical situations in a better way.

Predictions will now be made by taking the average value of \( K_1 \), found from the non-modified results and estimating \( K_3 \) from the \( K_3 - R_{e} \) relationship to be established. Again we will test inter- and extrapolation capabilities in the same way as was done in the non-modified model case.

Fig V.9 shows the various values of \( K_3 \) and \( R_{e} / c^2 \). We also included the correlation lines, calculated with a statistical analysis, including and excluding the Fl5 data. The equations of the relationships are given by

data Fl5 included

\[
K_3 = 66.25 \frac{R_{e}}{c^2} + 1.552 \quad \text{V.1}
\]

\[ c^2 = 1925 \quad s \]

\[ r = 0.89 \]
Figure 5.9

\[ \frac{R_{ie}}{C_2} \]  

\( C_2 = 192.5 \) (s)
data F15 excluded
\[ K3 = 70.30 \frac{R_{ie}}{c^2} + 1.333 \]  
\[ \text{correlation coef. } r = 0.82 \]  

\[ V.2 \]

interpolation

\[ K3_{FIS} = 2.539 \]  
\[ (90\% \text{ confidence interval } 2.170 < K3 < 2.908) \]

The values for the Kl are taken from the non-modified case, so
\[ K1_{FIS} = 0.137 \]  
\[ (90\% \text{ confidence interval } 0.064 < K1 < 0.200) \]

The longitudinal salinity profiles computed with this coefficient input are graphed in fig V.10, again together with the actual observations of F15. It will be clear that, thanks to the modification of the model, the results have improved substantially, with maximum deviations in intrusion of specific salt fronts
1.6 km for 3\%_f-front  
3.0 km for 2\%_f-front.

extrapolation

\[ K3_{FIS} = 2.381 \]  
\[ (90\% \text{ confidence interval } 1.536 < K3 < 3.226) \]

Again the corresponding Kl values are taken from the non-modified model
\[ K1_{FIS} = 0.151 \]  
\[ (90\% \text{ confidence interval } -0.047 < K1 < 0.286) \]

The longitudinal salinity profiles computed with these coefficients are graphed in fig V.11, together with the actual F15 measurement. Results are less accurate than in the interpolation case, as could be expected with a smaller number of measurements included, but still offers a substantial improvement compared to the non-modified model case. Maximum deviations intrusion distances
4.8 km at 3\%_f front

\[ 95. \]
- Observation F15
- Prediction with K3-Re-relationship
- Lower and upper boundaries
- 90% confidence interval

Modified combined model interpolation results

Fig. V. 10
MODIFIED COMBINED MODEL
EXTRAPOLATION RESULTS

Fig. V. 11
7.0 km at 2‰ front

The modified model can definitely make estimates of the intrusion for values of the fresh water discharge down to 2-5 m³/s. Below this it suffers from the same hindrance as all other (versions of) models, or put in a better way the estuary does not react in the same way anymore as it does under higher fresh water discharge cases.
The last model to be calibrated was the real-time model Penpas (for description see section III.4).
The approach in testing was a bit different from that of the tidally averaged models in this respect that Penpas has two parts to be calibrated, the watermotion and the salt-transport.
Calibration of the two parts will be discussed in separate sections, although they are naturally intertwined.
In the end some problems arose during testing of the salt part, which resulted in incomplete calibration. Reasons for this and further test possibilities are given in the section at hand.

Calibration of watermotion part

In order to simulate the watermotion, this part had to be calibrated with some actual observations. The unknown variables which have to get a value in this calibration process are, in our case

- friction coefficients (k-values) for all branches
- amplitude and course of the waterlevel variation at the downstream boundary, the rivermouth.
The waterlevel course during a tidal cycle was never directly measured in the rivermouth; the first point where waterlevels were measured was at the gauge in Antunes (km 8.0).

To tackle the calibration in a systematical way the river was divided in four sections, with a waterlevel gauge or one of the riverboundaries at either end (see fig V.12).

The main calibration was done with data from 29/5/1984, the day when all waterlevel gauges had been recorded continuously on one and the same day. Due to the uncertainty in the downstream section-boundary (1), we started calibration at the other end of the estuary, in section (4).
The section was considered as a separate river system with a known waterlevel boundary in Salamanga (km 41.7) and a known discharge boundary in Santaca (km 70). The k-values of the three branches within this section were adjusted in such a way that the waterlevel variation in Santaca, calculated by the model coincided with the actual observations.
After this, the following sections downstream were added to the flow system, one at the time and the roughness coefficients in the added parts were adjusted in the same way.

The waterlevel boundary in the Maputo bay was found by assuming a roughness coefficient for the first 8 kilometres and correcting the waterlevel variations in the bay, so that the waterlevels at the various gauges corresponded again. The assumption of the friction coefficient in that part does not introduce large errors as

- the roughness in the part upstream of Antunes (km 8.0 - km 15.0) was already fairly constant and very low, owing to the smooth muddy bottom and banks (sediment deposits)

- in this section the friction is already so low that the influence on the watermotion is not dominant any more

The shape of the boundary condition is shown in fig V.13, for the observation at 29/5/1984. In fig. V.14 the difference is shown between measured waterlevels at that date and the ones calculated with the model and the final friction coefficients.

The results of the watermotion calibration are shown in table V.5, where all k-values are listed together with the corresponding (tidally averaged) values of the Chezy coefficient.

The influence of the saltdistribution on the watermotion was found by including an estimated longitudinal salt profile in the calculations. It was found that waterlevels varied up to 5 cm (in Salamanga, km 41.7) when the density term in eq III.17 was included.
WATERLEVEL (m) MGM.

CORRECTED WATERLEVEL BOUNDARY

COSINE - CURVE

M.G.M. = AVERAGE MOZAMBIQUEAN SEA-LEVEL

Fig. V.13

29/5/1984
WATER LEVEL
in m + MGM

WATER MOTION CALIBRATION

M_A MEASUREMENT AT 29/5/1984 IN ANTUNES
M_B " " " " " BELA VISTA
M_S " " " " " SALAMANGA
P_A,B,S COMPUTED CURVES WITH PenPAAS PROGRAMME

Fig. V. 14
<table>
<thead>
<tr>
<th>branch (km)</th>
<th>k-value (m)</th>
<th>Chezy coef. (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 8.0</td>
<td>0.001</td>
<td>83.8</td>
</tr>
<tr>
<td>8.0 - 15.5</td>
<td>0.001</td>
<td>83.2</td>
</tr>
<tr>
<td>15.5 - 23.0</td>
<td>0.001</td>
<td>80.3</td>
</tr>
<tr>
<td>23.0 - 32.4</td>
<td>0.04</td>
<td>54.4</td>
</tr>
<tr>
<td>32.4 - 41.7</td>
<td>0.06</td>
<td>50.6</td>
</tr>
<tr>
<td>41.7 - 49.7</td>
<td>0.25</td>
<td>39.5</td>
</tr>
<tr>
<td>49.7 - 57.7</td>
<td>0.60</td>
<td>32.0</td>
</tr>
<tr>
<td>57.7 - 65.7</td>
<td>0.60</td>
<td>30.1</td>
</tr>
<tr>
<td>65.7 - 70.0</td>
<td>0.60</td>
<td>30.0</td>
</tr>
</tbody>
</table>

table V.5

Salt-transport calibrations

Once the watermotion was calibrated we turned to the salttransports. The adjustable parameters in this part were

- a. dispersion coefficient
- b. waterlevel amplitude in river mouth

a. dispersion coefficient
The dispersion coefficient in Penpas is split up in two parts, like described in equation III.23. Officially the two coefficients $D_1$ and $D_2$ must be fairly constant over the estuary and for all circumstances. Only in that case it is possible to make good predictions with the model, for conditions outside the measuring range.

b. waterlevel amplitude in river mouth, $\Delta H_0$
The shape of the waterlevel variation in the river mouth was assumed to be constant (see fig V.13). The amplitude could only be estimated from

- waterlevel amplitude in Bela Vista and Antunes
- tide-table of Maputo harbour

The calibration was done for one day (tidal cycle) at the time. This implies we wanted the model to simulate the salinity variations during a tidal cycle between the measured high and low water slack-profiles. The coefficients
are well calibrated when the curves of minimum and maximum salinity during a tidal cycle coincide with the measured profiles at the slack-tides.

In order to lose influences of wrong estimates of the initial conditions the tidal cycle of the specific day was repeated a few times to let the model adapt itself to the conditions. Usually about three to four cycles were necessary as adaption phase.

As input was given

- waterlevel variation in the rivermouth at the particular day, starting at high water slack
- upstream fresh water discharge, $Q_f$
- measured longitudinal salinity profile at high water slack as initial salt conditions
- estimated dispersion coefficient

During the calibration some questions arose about the accuracy of the numerical methods of Penpas (for numerical schemes see appendix VII). The accuracy is, of course, directly related to the branch length, $\Delta x$ and the time step $\Delta t$, used in the programme execution. In the end it was seen that values of $\Delta x$ of 2 - 2.5 km and of $\Delta t$ of 250 sec were sufficiently small to limit inaccuracies, although these values do not exactly meet the conditions posed for the accuracy (see appendix VII). To prove this, figure V.15.a presents the low and high water slack salinity profiles, computed by the programme under similar physical circumstances but with different branch lengths and time steps. It can be seen that decreasing of the values of $\Delta x$ and $\Delta t$ will not result in a much different curve, indicating a reasonable, numerical accuracy (note: nothing is yet said about the ability to describe the real physical circumstances).

Rough calibration was done for a few measuring days. An example, of measurement F14 is shown in figure V.15.b. The optimal coefficients, found so far are listed in table V.6.

Detailed calibration and the consequent testing turned out to be a laborious job, something that would fall outside the scope of this particular study. Therefore it was decided to stop after the rough calibration had been done and leave the remaining parts for further study. This implies we will not discuss the results obtained so far in detail, as they are only indicatory values, but will
INPUT DATA

F12

D1 = 2.5  D2 = 70.0

Penpas Model

Fig. V.15.a.
Input Data F14 (28-4-1982)

\[\begin{array}{ccc}
\theta & D_1 & D_2 & \Delta H_{\text{MOUTH}} \\
\hline
1 & 1.0 & 25.0 & 1.31 \\
2 & 2.0 & 30.0 & 1.31 \\
3 & 1.0 & 25.0 & 1.55 \\
\end{array}\]

Fig. VI 15 b.
limit ourselves to some main conclusions.

1. The longitudinal salt intrusion profiles are more sensitive to a small change in the waterlevel amplitude in the river mouth than a change in dispersion coefficients (see fig V.15.b). This is quite unfortunate as especially the waterlevel in the river mouth was never recorded and had to be estimated. This introduces a fairly large inaccuracy in the computations.

2. Although the condition, to avoid unstable oscillations is

$$\Delta x \leq \frac{2D_x}{u}$$

the $\Delta x$ in the river Maputo was taken around 2 - 2.5 km, about ten times the permissible value. This seemed to give reasonable results, but further investigation to those oscillations in the parts where the salt gradients are small (low Dvalue) is definitely needed.

3. The programme is not really designed for the combination of

- requirement of small branch lengths, $\Delta x$
- a quite rough grid of measured river profiles

Each and every geometry input profile for the programme (see appendix III) has to be calculated by hand. The programme has no facility to interpolate between two measured profiles, so that large sets of input data can not be avoided.

4. In rivers with small depths the dispersion coefficient will be quite small (river Maputo order of magnitude 50 -200 m²/s). This means, according to the accuracy conditions that small branch lengths and time steps are needed, which results in long computing times.
Conclusion number 2 and the detailed calibration definitely need some further study. This might be worthwhile to do, in order to be able to compare the prediction results of real-time models with those of the tidally averaged models. Mainly due to time-constraints, these elements could not be presented in this study any more.
Before we go into the actual comparison of the results of the various models, we again want to make some important, general remarks about the class of models at hand, some of which were already brought forward in an earlier stage.

**one-dimensional models**

First of all we still have to remember the one-dimensionality of the models. In section II.2 it has been discussed already what limitations this creates and under what circumstances this type of models can successfully be applied. One of the key factors is formed by the cross-sectional salinity variations. A wide variation in salinity over the depth or width induces low accuracy in all calculations and predictions.

**accuracy field survey measurements**

It must be realized that the accuracy of the models can never be any higher than the accuracy of the input data found from the field-survey in the river Maputo. Some remarks can be made about this:

1. accuracy of the field survey measurements is not very high, partly due to the poor quality of the topography of the waterlevel gauges, partly due to the rough measuring methods and the scarcity of discharge and velocity measurements in the tidal part of the river. Maximum errors in the geometry, for instance, might end up as high as 15%.

2. Variations in salinity over the cross-section are not always small (see fig. 13 to 16 and vertical profiles 12.a to .m). Especially during neap tides, the estuary was (partially) stratified. During the study cross-sectional average values were computed by assuming them equal to the average salinity of the vertical profile, measured in the deepest point of the cross-section. Fig. 16 shows that this is not always
correct, although deviations in salinity around slack tides were, in general, less.

tidally averaged versus real-time

It must be clear that tidally averaged models are not selected for their high accuracy in describing natural processes. By averaging over the tidal period we lump all deviations from tidal averaged flow into one term, the dispersion term. As all estuaries have different characteristics, that determine these deviations, but are not exactly known or taken into account (trapping volume, dead zones, bank irregularities, human activity), this is definitely a term that causes an lower accuracy in the model results. An advantage of the real time-models is that they do not average over the tidal period and therefore leave out one inaccurate factor at least.

A second advantage of the real-time models is the fact that the situation is computed not only as a tidal average but at any time during the tidal cycle. That might be important if for instance water of low salinity reaches water inlets only part of the day. This advantage should not be exaggerated though. This and some other studies give good hope to believe that, in well mixed estuaries the longitudinal salinity profile is shifted along the estuary at the speed of the cross-sectional averaged tidal velocity. This means that a watermotion model would be sufficient to determine the shifting or, if only the maximum intrusion is wanted, an empirical relationship between tide characteristics and tidal excursion length (see section IV.2).

Furthermore, the accuracy of the real-time models depends heavily on the input available. This implies, if we want to talk about good representation of natural situations

- a more detailed river geometry has to be known, with measuring profile distances much less than needed for tidally averaged models (note: we are not talking about numerical inaccuracies yet)

- more knowledge of the tidal regime along the estuary is required.
   This means
   - more measurements of tidal discharges to check the watermotion part of the programme with
   - measurement of waterlevel amplitude and variation during the tidal cycle in the river mouth

These factors make a full-scale field survey a necessity.
An other limiting factor, that, perhaps in future will gradually disappear is the numerical operation of real-time models. The models have more severe demands like

- more knowledge about complex numerical schemes (or more expensive standard computer programme packages)

- a more powerful computer (higher running cost).

The Penpas model for instance needed a computer which had at least a 200 Kbyte capacity, the Van der Burgh and the Combined model can be run on any 16 Kbyte micro-computer, if necessary.

Unfortunately it was not possible to compare the results of the Penpas real-time model with measurements and tidally averaged models. Generally speaking they need to be a great deal more accurate to be competitive to the simple tidally averaged models, under the circumstances in the Maputo river. Besides, nothing is yet known about its ability to describe the estuary conditions in more extreme situations, for instance with discharges less than 5 m3/s. Theoretically, if one model can describe all situations it will be a real-time model. In practice, however, it might be that even such a model cannot describe such exceptional circumstances, as other mechanisms might prevail at that time or, in this case it might be that such extreme observations are the result of different degrees of state of equilibrium (FL3 measurement was taken after a very long period of drought, so perhaps under real equilibrium circumstances).

It could be useful to finish the calibration and testing of the Penpas model in a correct way and draw some further conclusions about the accuracy and ability to cope with wide-ranged measurements.

Various tidally averaged models

Comparing the results of the various tidally averaged models, we arrive at the following remarks

- all models have severe problems in describing extreme circumstances, like the very low discharges in the Maputo river

- seen from the interpolation results only the modified combined model is able to give a reasonably accurate representation of salt intrusion within the range of measuring conditions

- although the Combined model has a somewhat better
theoretical basis than Van der Burgh, as far as the formulation of the dispersion coefficient, D is concerned, this does not show in the accuracy of the predictions.

In case of the extrapolation, both models have salt intrusion distance deviations of 7 - 11 km. The Combined model needs a purely empirical relationship as well (K3-R2 relationship) to improve the predictions a little. The modified Combined model gives maximum deviations from intrusion distances around 5 - 7 km.

Even then it is very remarkable that the deviations of the intrusion of low-salinity fronts (0.5-2.0%) of the van der Burgh and the modified Combined model are almost equal or even slightly less in the Van der Burgh model (both appr. 7 km). This is especially important as they are usually the salinities we are interested in as they form the limits for human consumption and irrigation.

- The purely empirical relationships in both the Van der Burgh model (eq III.3) and the modified Combined model (eq.V.1 and V.2) already warn us against direct application of the models in other estuaries. No real theoretical basis exists for the assumption that these relationships are of the same form in other estuaries.

- In the application of tidally averaged models in the river Maputo there is almost no loss of accuracy in case a constant, mean depth is assumed all over the estuary. As mentioned before this means a substantial saving on field surveys. Of course it can not be assumed to be valid in all sorts of estuaries. Seen from this study and the results of earlier studies in Mozambique [1], we can state some conditions under which it seems reasonable to assume a constant depth over the estuary

  - tidally and cross-sectional averaged depth variation over the estuary less than a factor three to four

  - a river geometry whereby both cross-sectional area and width can be presented in a more or less exponential function over the tidal intrusion length

This part leads to the conclusion that a detailed knowledge of the geometry is not needed. The results did not notably improve in case of a more accurate geometry input.
data requirements for Van der Burgh and Combined models are almost equal, at least if the $R_{he}$ is used as estuary parameter to establish the $D_o$-estuary parameter relationship. The Combined model, however, requires values for rms. velocities and friction coefficients all along the estuary, while Van der Burgh only needs them in one location, for the determination of the estuary parameter. In other words, the Combined model works with something like a local estuary parameter.
CHAPTER VII  CONCLUSIONS AND RECOMMENDATIONS

VII.1  Conclusions

This report mainly deals with the testing of three tidally averaged, one-dimensional salt intrusion models. The models are distinguished by their formulation of the dispersion coefficient, a factor that plays a crucial role in the representation of salt intrusion. The three models are

- the Van der Burgh model
  A model based on a semi-empirical formulation of the dispersion coefficient (section III.2.1).

- the Combined model
  A model whereby the dispersion coefficient is obtained from a somewhat generalized combination of theories (section III.2.1).

- the Modified Combined model
  A model based on the Combined model, but with an empirical formula to predict one of the factors that contribute to the dispersion coefficient (section V.3).

For the testing of the models a field survey was set up in the river Maputo (Chapter IV). Main conclusions of the field survey were

1. accuracy of most measurements was not very high, partly due to difficulties in surveying the topography of the river, partly due to the methods of measuring.

2. Considering the limitations of the tidally averaged models in other fields, the geometry measuring grid, with measured cross-section distances of 2 - 3 km, was detailed enough in the case of the rather simple geometry of the river Maputo.

The testing of the three models results in the following conclusions.
3. one-dimensional salt intrusion models suffer from inaccuracies and can be used in reasonably well mixed estuaries only;

4. none of the models tested could cope with extreme situations, like very low fresh-water river discharges;

5. prediction of salinity intrusion in estuaries is quite inaccurate in both the Van der Burgh model and the theoretically based Combined model. Deviations in salt intrusion-length are of the order of 7 - 11 km in the Maputo river (sections V.2 and V.3) in both models;

6. The Modified Combined model is a little more accurate for the higher salinities (c > 3 %); at low salinities, however, the deviations became just as large as with the Van der Burgh model (about 7 km);

7. the models considered are quite insensitive to changes in the accuracy of the river geometry input;

8. the empirical character of the models used implies that applicability to other estuaries is questionable;

Besides the testing of the tidally averaged models some preliminary calibrations and tests were made with a real-time, one-dimensional model, developed by the Delft Hydraulics Laboratory (section III.4). Due to time constraints the tests were not completed. Despite this some conclusions can be drawn already

9. real-time models need much more input data than tidally averaged models and therefore require extensive field surveying;

10. low-depth rivers need more computing time in the Penpas model than rivers with large depths;

11. the Penpas model, like all real-time models was very sensitive to errors in the water motion part. Thus, if we want to work with real-time models more information is needed about the tidal discharges and waterlevels, all through the estuary.
VII.2 Recommendations

Starting from the conclusions, the following recommendations can be given:

1. use one-dimensional, tidally averaged models for indicatory, qualitative studies, but not for accurate salt intrusion predictions;

2. put more emphasis on salinity measurements, such as vertical and longitudinal profiles under various conditions than on detailed description of the geometry in the river to be studied;

3. be cautious when applying the relationships found for the dispersion coefficients in the river Maputo. These relationships are semi-empirical and therefore may be different for other estuaries.

Furthermore a recommendation for further study of one-dimensional salt intrusion models is given:

4. it would be very useful to test a real-time model, eg. the Penpas' model, in more detail than done in this particular study using the Maputo data, in order to obtain an indication of the accuracy of the predictions of real-time models (especially in comparison with tidally averaged models) (see section V.4).
NOTATIONS

A  cross-sectional area
a  waterdepth
B  cross-sectional width
C  Chézy-coefficient
Cl,C2,C3 constants (appendix I)
c  salinity = mass of salt in kg per 1000 kg of water
D  dispersion coefficient
E  estuary number
Ei  internal estuary number
Eo  external estuary number
e  flood number of Canter-Cremers
F  Froude number
Fr  internal Froude number = densimetric Froude number
fₑ  roughness coefficient = g/C²
fₑ  roughness coefficient
ɡ  gravitational constant
H  cross-sectional mean depth
Ho  mean depth over estuary
AH  tidal waterlevel amplitude
i  bed slope of river
K,K1,K3 constants
k  characteristic exchange time between traps and main channel; roughness coefficient in Penman's Combined model
L  length of tidal influence in estuary
Lₑ  distance from river-mouth to point where salt deviates from seawater salinity
l  length of section
N1,N2 constants (appendix I)
n  Mannings coefficient of roughness
Pₑ  tidal prism
Qₑ  fresh water river discharge
q  discharge
R  hydraulic flow radius
Rₑ  Richardson number
r  ratio trap volume to main channel volume
T  tidal period
Tₑ  adoption period concentration profile
Tₛ  salt transport over total width
Tₜ  transition period
T'/Tₑ  time
V₁,V₂,V₃ constants (appendix I)
u,v,w  velocity components
 uf  fresh water velocity = Qₑ/A
 uᵣ  residual velocity at surface
 uᵣ  rms tidal velocity (rms. = root mean square)
x, y, z spatial axes

\( \alpha \) angle
\( \epsilon \) turbulent diffusivity
\( v \) fraction of landward transport not caused by density differences
\( \rho \) density
\( \sigma \) \( 2\pi/T \)
\( \tau \) salt transport per unit width

SUBSCRIPTS etc.

\( \bar{\cdot} \) averaged over a timespan of a few minutes
\( \hat{\cdot} \) averaged over cross-section
\( \langle \cdot \rangle \) averaged over tidal cycle period
\( \prime \) fluctuating part
\( l \) longitudinal
\( v \) vertical
\( t \) transverse (lateral)
\( ^{\wedge} \) or \( \Delta \) amplitude
\( ^{o} \) values in the river-mouth
\( x \) longitudinal
\( x_{T} \) longitudinal and tidally averaged
\( x_{t} \) longitudinal, real time
\( x, LWS \) longitudinal at low water slack
\( x, HWS \) longitudinal at high water slack
\( x, ST \) longitudinal at slack tide
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In an estuary, relatively low-density riverwater flows into higher-density seawater. When no flow is present, a natural equilibrium would be that the river water would be stationed on top of the salt seawater and the interface would be sharp and horizontal. In order to reach that equilibrium, the salt water has the tendency to intrude into the estuary, in the form of a salt-wedge over the bottom (fig.app.I.1). The fresh water has a tendency to float on top of the salt waterbody of the sea. The above described situation sometimes occurs, especially in estuaries that end in a sea or bay with little or no tide (e.g., part of the Mediterranean sea). High fresh water velocities also stabilize salt wedge situations.

In case large tidal motions exist, however, the kinetic energy of the to and fro movement is sufficient to break up the salt wedge bit by bit and mix the salt and freshwater over the cross-section. If the mixing is strong enough, it might result in an almost completely mixed situation, whereby there is hardly any variation of salinity over the depth (fig. II.5.b). If not, than an equilibrium will be established somewhere in between the stratified and well-mixed situations (fig II.5.b).

The mixing-state of the cross-section has a great influence on processes like dispersion and salt-intrusion. Therefore attempts have been made to classify the conditions of an estuary (like fresh water discharge, tidal amplitude and other parameters influencing the mixing state) with one single parameter, the estuary parameter. The goal of all parameters is to represent the state of an estuary with over-all parameters only, like average depth, velocities, densities and so on.

Many parameters have been developed, already an indication that up to now there is no single estuary parameter yet that can be used for the description of all classes of estuaries.

In this report we will concentrate on three of such parameters, which are frequently used in one form or another, the flood number, \( e \), the estuarine Richardson number, \( R_{ie} \) and the densimetric Estuary number, \( E_l \).
Flood number

Introduced by Canter-Cremers, definition

\[ e = \frac{Q_f \times T}{P_t} \]

whereby \( P_t \) = tidal prism, the volume of water (in m³) entering the estuary during flood tide

\( Q_f \) = fresh water river discharge

\( T \) = tidal period.

High \( e \)-values represent strongly stratified estuaries
Low \( e \)-values represent well mixed estuaries

Examples

Rotterdam waterway \( 0.5 < e < 5 \); stratified
Maputo river \( 0.01 < e < 0.04 \); well mixed – partially mixed.

Estuarine Richardson number

First introduced in this form by Fischer [21], definition

\[ R_i = \frac{\Delta \rho \cdot Q \cdot a \cdot u_f}{\left( \frac{u_f}{u_i} \cdot a \right)^3} \]

\( u_f \) = rms. tidal velocity
\( B \) = channel width
\( a \) = water depth
\( u_f \) = fresh water velocity
\( f_i \) = friction coefficient = \( g/C \)

The numerator stands for an input of buoyancy per unit width of channel, originating from the fresh water river discharge, the denominator expresses the mixing power of the tide;

low values (\(<0.08\)) represent well mixed situations,
high values (\(>0.8\)) represent strongly stratified situations,
range observed in river Maputo \( 0.03 < R_i < 0.3 \).

densimetric Estuary number

Introduced by Thatcher and Harleman [26], definition

\[ E_i = \frac{P_t \cdot F_i}{Q_f \cdot T} \]

\( P_t \) = tidal prism
\( F_i \) = densimetric Froude number \( = \frac{u_f}{\sqrt{\frac{\Delta \rho}{\rho} \cdot g \cdot a}} \)
\( u_t \) = rms. tidal velocity

Low \( E_t \) values indicate strongly stratified estuaries (<0.03)
high values indicate well mixed situations (>2).
In between those two limits is a transition range.
Example: Maputo river \( 1.7 < E_t < 6.8 \).

In the Maputo study we will use the parameters in a slightly different way. To characterize an estuary usually the values in the rivermouth are substituted in the above mentioned formulae, like \( P_t \) , \( B \) , \( u_t \) and \( F_t \). However, in the Maputo river it was difficult:

a. to identify the exact position of the river mouth,
b. measure in that area because of the large width and the uncertainty in differentiating between stream areas and storage areas of the cross-sections.

There was one station in the river where all the variables used in the estuary parameters were measured continuously, Bela Vista (23.0 km). It was decided to compute all estuary parameters in this point and to assume the following relationships

\[
\begin{align*}
E_{MOUTH} &= V1 \cdot E_{BELA VISTA} \\
F_{MOUTH} &= V2 \cdot R_{BELA VISTA} \\
N_{MOUTH} &= V3 \cdot E_{BELA VISTA}
\end{align*}
\]

\( V1, V2 \) and \( V3 \) are constants.

If the linear assumptions are correct, it will be allowed to use the Bela Vista estuary parameters in relationships with other parameters (see Van der Burgh for instance), without disturbing correlations.
The assumptions were checked later on with the watermotion part of the Penpas model and turned out to be quite reasonable, with the following \( V \)-values

\[
\begin{align*}
V1 &= 0.22 - 0.26 \quad \text{average} \ 0.24 \\
V2 &= 0.012 - 0.016 \quad \text{average} \ 0.015 \\
V3 &= 0.55 - 0.67 \quad \text{average} \ 0.6
\end{align*}
\]

As it is difficult to measure parameters like the tidal prism and the rms tidal velocity, also these two variables were related to another parameter which was continuously measured in Bela Vista, the waterlevel variation

\[
P_t = N2 \cdot (\Delta H_{BV})
\]

124.
\[ u_t = N3^*(\Delta H_{BV}) \]

\((\Delta H_{BV}) = \text{tidal waterlevel-amplitude in Bela Vista}\)

The range of \(N2\) and \(N3\) were found to be

\[ N2 = 10.8 - 11.9 \times 10^6 \text{ (m2)} \ (\text{average } 11.2 \times 10^6 \text{ m2}) \]
\[ N3 = 0.45 - 0.50 \ (1/\text{s}) \ (\text{average } 0.48 \ 1/\text{s}) \]

In using the estuary parameters other simplifications were introduced as well, splitting them up in a part, independent of the estuary conditions and a variable part.

\[ e_{\text{BV}} = C_1 \times \frac{Q_f}{\Delta H_{BV}} \quad C_1 = \frac{T}{N_2} \approx 4.0 \times 10^{-3} \ \text{s/m}^2 \]
\[ R_{i,E_{\text{BV}}} = C_2 \times \frac{\Delta \rho \cdot Q_f}{(\Delta H_{BV})^3} \quad C_2 = \frac{g}{\beta_{\text{BV}} \cdot \sigma_{\text{BV}} \cdot (N_2)^2} \approx 1925 \ \text{s} \]
\[ E_i_{,\text{BV}} = C_3 \times \frac{(\Delta H_{BV})^3}{\rho \cdot Q_f} \quad C_3 = \frac{N_2 (N_3)^2}{g \cdot a \cdot T} \approx 1.9 \ \frac{1}{\text{s}} \]

In finding correlations with for instance dispersion coefficients in the rivermouth (Van der Burgh) or \(K_3\) values (modified combined models) we normally will use the "stripped" estuary parameters, so \(e_{\text{BV}}/C1, R_{i,E_{\text{BV}}}/C2\) and \(E_i_{,\text{BV}}/C3\).

Summarizing, if we want to transform the stripped estuary parameters into (estimates of) the original defined estuary parameters we can use the following relationships.

\[ E = e_{\text{BV}} \times V_i = \frac{Q_f}{\Delta H_{BV}} \times V_i \times C_1 \approx g \times 6 \times 10^{-4} \times \frac{Q_f}{\Delta H_{BV}} \]
\[ R_{i,E} = R_{i,E_{\text{BV}}} \times V_2 = \frac{\Delta \rho \cdot Q_f}{(\Delta H_{BV})^3} \times V_2 \times C_2 \approx 2.8 \cdot g \times \frac{\Delta \rho \cdot Q_f}{(\Delta H_{BV})^3} \]
\[ E_i = E_i_{,\text{BV}} \times V_3 = \frac{(\Delta H_{BV})^3}{\rho \cdot Q_f} \times V_3 \times C_3 \approx 1.14 \times \frac{(\Delta H_{BV})^3}{\rho \cdot Q_f} \]

**********************************************************************************

125.
APPENDIX II

TIDAL ANALYSIS IN THE BAY OF MAPUTO

This appendix presents an analysis of tidal wave components, calculated with observations of the year 1974. Only the most important components are included. Analysis method was that of Doodson. For the location of the stations, see fig 2.b.

STATION 1 (ferryboat to Catembe)

<table>
<thead>
<tr>
<th>Wavetype</th>
<th>M2</th>
<th>S2</th>
<th>N2</th>
<th>K2</th>
<th>K1</th>
<th>O1</th>
<th>P1</th>
<th>M4</th>
<th>MS4</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₀</td>
<td>0.93</td>
<td>0.54</td>
<td>0.16</td>
<td>0.15</td>
<td>0.04</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>g</td>
<td>123</td>
<td>164</td>
<td>110</td>
<td>160</td>
<td>201</td>
<td>004</td>
<td>228</td>
<td>164</td>
<td>266</td>
</tr>
</tbody>
</table>

STATION 2 (Shallows of Inhaca)

<table>
<thead>
<tr>
<th>Wavetype</th>
<th>M2</th>
<th>S2</th>
<th>N2</th>
<th>K2</th>
<th>K1</th>
<th>O1</th>
<th>P1</th>
<th>M4</th>
<th>MS4</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₀</td>
<td>0.72</td>
<td>0.46</td>
<td>0.12</td>
<td>0.12</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>g</td>
<td>109</td>
<td>149</td>
<td>102</td>
<td>149</td>
<td>148</td>
<td>350</td>
<td>149</td>
<td>211</td>
<td>258</td>
</tr>
</tbody>
</table>

The following page offers an abstract from the tide tables of station 1 for the year 1984. The zero waterlevel of the tables is 2 metres below the average annual sea level (MGM-level).

Dependent on wind, airpressure and other meteorological factors the predicted values in the tide tables can vary to a maximum of approximately 0.3 - 0.4 m plus or minus from the real values, encountered in the Maputo bay.

************************************************

126.
<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Level</th>
<th>Date</th>
<th>Time</th>
<th>Level</th>
<th>Date</th>
<th>Time</th>
<th>Level</th>
<th>Date</th>
<th>Time</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984-03-01</td>
<td>14:16</td>
<td>16</td>
<td>1984-03-02</td>
<td>14:16</td>
<td>16</td>
<td>1984-03-03</td>
<td>14:16</td>
<td>16</td>
<td>1984-03-04</td>
<td>14:16</td>
<td>16</td>
</tr>
<tr>
<td>1984-03-05</td>
<td>14:16</td>
<td>16</td>
<td>1984-03-06</td>
<td>14:16</td>
<td>16</td>
<td>1984-03-07</td>
<td>14:16</td>
<td>16</td>
<td>1984-03-08</td>
<td>14:16</td>
<td>16</td>
</tr>
<tr>
<td>1984-03-09</td>
<td>14:16</td>
<td>16</td>
<td>1984-03-10</td>
<td>14:16</td>
<td>16</td>
<td>1984-03-11</td>
<td>14:16</td>
<td>16</td>
<td>1984-03-12</td>
<td>14:16</td>
<td>16</td>
</tr>
</tbody>
</table>

**Note:** The table above contains data for various dates and times, likely related to water levels or similar measurements. The specific context is not clear from the image.
APPENDIX III

TIDAL PRISM

The steady state, tidally averaged models sometimes use the tidal prism in their calculations. In chapter IV it was shown that we can compute this parameter by applying

\[ P_r = 2 \int \Delta H(x) \cdot B(x) \cdot dx \]

with \( L \) = distance to mouth where vertical waterlevel variations during the tidal cycle are reduced to a few percent of the waterlevel amplitude in the rivermouth.

\( \Delta H(x) \) = waterlevel amplitude along the river, \( B(x) \) = width along the estuary

In fact, this is not exactly correct, due to the phase shift in waterlevels and discharges. The maximum waterlevel will not be reached for all points along the estuary at the same time, meaning that the total volume of water entering between low slack and high slack (at the rivermouth) will be less than the value, given by equation IV.2.1. The deviation from the formula (assuming the waterlevel amplitude presentation to be correct) can be estimated. The phaseshift along the river is determined by knowing the tidal wave period \( T \) and wave velocity \( c \), also referred to as celerity. Tidal waves can be assumed to intrude with a velocity close to

\[ c = \sqrt{g \cdot a} \]

high water slack: \( c = 6.6 - 7.0 \text{ m/s} \)
low water slack: \( c = 4.4 - 4.9 \text{ m/s} \)

The tidal wave length amounts to

\[ \lambda = c \cdot T \]

high water slack: \( \lambda = 295 - 313 \text{ km} \)
low water slack: \( \lambda = 197 - 219 \text{ km} \)

The phase shift from mouth to upstream end in the estuary will be bigger if the wave length is smaller. If we talk about maximum deviations we therefore have to consider the
low water case. We will do this for the F8-measurement.

If we assume the water level in the river mouth to be at its minimum, we can compute the water level which occurs upstream with

\[ H(x) = H_a(x) + \Delta H(x) \cdot \cos(2\pi \frac{t - t_o}{T}) \]

\( H_a(x) \) = tidally averaged water level
\( \Delta H(x) \) = water level amplitude

\( t_o \) can be computed by \( t_o = \frac{x}{c} \).

Table app.III.1 shows the deviations from tidally averaged values, fig. app.III.1 does the same together with the water level amplitudes.

<table>
<thead>
<tr>
<th>point (km)</th>
<th>( t_o ) (s)</th>
<th>( \cos(\frac{2\pi (-t_o)}{T}) )</th>
<th>( \Delta H ) (F8 meas.) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0</td>
<td>1.0</td>
<td>1.79</td>
</tr>
<tr>
<td>8.0</td>
<td>1800</td>
<td>0.968</td>
<td>1.84</td>
</tr>
<tr>
<td>23.0</td>
<td>5200</td>
<td>0.744</td>
<td>1.83</td>
</tr>
<tr>
<td>41.7</td>
<td>9300</td>
<td>0.261</td>
<td>1.28</td>
</tr>
<tr>
<td>60.0</td>
<td>13600</td>
<td>-0.334</td>
<td>0.30</td>
</tr>
</tbody>
</table>

If we use linear interpolation between the points included in table app.III.1 we can calculate the deviation in the situation with and without phaseshifts, equalling the volume at average levels in all places minus the volume at the time of low water slack in the river mouth.
V average \_V
between \( m^3 \)

<table>
<thead>
<tr>
<th>Range</th>
<th>No Phase Shift</th>
<th>With Phase Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 8.0 km</td>
<td>17.6 * 10^6</td>
<td>17.1 * 10^6</td>
</tr>
<tr>
<td>8.0 - 23.0 km</td>
<td>19.1 * 10^6</td>
<td>17.1 * 10^6</td>
</tr>
<tr>
<td>23.0 - 41.7 km</td>
<td>8.9 * 10^6</td>
<td>5.9 * 10^6</td>
</tr>
<tr>
<td>41.7 - 70.0 km</td>
<td>1.7 * 10^6</td>
<td>0.4 * 10^6</td>
</tr>
<tr>
<td>Total</td>
<td>47.3 * 10^6</td>
<td>40.5 * 10^6</td>
</tr>
</tbody>
</table>

Deviation = \[ \frac{47.3 - 40.5}{47.3} \] = 14%

Of course this figure provides an indication only. At first, the conditions at high water slack cause less deviations (longer tidal wave-length). Furthermore, measurement F8 had a waterlevel amplitude at Bela Vista of 1.83 m, a spring tide measurement (see fig.5.c). At other times during the spring tide/neap tide cycle the deviation will be less, due to the higher low water levels and therefore higher celerities of the tidal waves. The figure of 14% can therefore be considered as a maximum.
In real-time models geometry input is needed in every schematized branch and at every timestep. The length of the branches is usually not equal to the distance between the profiles measured in situ, and therefore we have to provide geometry input at places where no geometry was measured. The following describes how we arrive at our geometry input in every branch, starting from the measured cross-sections.

The geometry in the Penpas model is considered to be constant in one branch, which means we have to combine the various measured cross-sections to one characteristic branch cross-section. This is done in the following way. We start from the measured profiles of the various cross-sections in and close by the branch under consideration (i) (see fig app.IV.2).

We have to take care that we know the absolute level of each point in the cross-section, related to the Mozambiquean average sea level (MGM) for instance. Furthermore we have to estimate the part that was not directly measured with the echo-sounder, but that might become flooded during flood- or spring-tide.

The next step is tabulating the width of the cross-sections at metre or half metre depth intervals, dependent on the (ir-)regularity of the profiles. An example of this is given in table app.IV.1.
In chapter IV.1 and fig IV.5 we arrived at some weighting coefficients for each cross-sectional measurement being the relative weight that a specific cross-sectional measurement contributes to a branch. These weighting coefficients are used to multiply the widths at all levels resulting in table app.IV.2.

<table>
<thead>
<tr>
<th>level (m)</th>
<th>cross-sectional width (m) of cross-section no.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>j</td>
</tr>
<tr>
<td>8.0</td>
<td>100</td>
</tr>
<tr>
<td>7.0</td>
<td>80</td>
</tr>
<tr>
<td>6.0</td>
<td>20</td>
</tr>
<tr>
<td>5.0</td>
<td>10</td>
</tr>
<tr>
<td>4.0</td>
<td>0</td>
</tr>
<tr>
<td>3.0</td>
<td>0</td>
</tr>
<tr>
<td>2.0</td>
<td>0</td>
</tr>
</tbody>
</table>

In cross-sections j j+1 j+2 j+3 j+4, the weighted widths (m) in cross-sections at different levels are given in the following table:

<table>
<thead>
<tr>
<th>levels (m)</th>
<th>weighted widths (m) in cross-sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>10 25 70 65 70 231</td>
</tr>
<tr>
<td>7.0</td>
<td>8   21 50 45 51 175</td>
</tr>
<tr>
<td>6.0</td>
<td>2   21 37.5 50 40 150.5</td>
</tr>
<tr>
<td>5.0</td>
<td>1   10.5 25 25 32.5 94</td>
</tr>
<tr>
<td>4.0</td>
<td>0   4.5 12.5 12.5 12.5 42</td>
</tr>
<tr>
<td>3.0</td>
<td>0   0   0   5   7.5 12.5</td>
</tr>
<tr>
<td>2.0</td>
<td>0   0   0   0   0   0</td>
</tr>
</tbody>
</table>

Table app.IV.2
The weighted widths are added up to form the total width at one level. Consequently we draw the widths at the various levels in one graph (fig.app.IV.3).

The last step is to smooth the obtained graph a little in order to be able to represent it by at the most, three straight lines.

The model asks for the nodes at the beginning and end of each straight part. Every level above the highest given to the model is assumed to have the same width as that highest given level (node (3) in fig app.IV.3).
APPENDIX V

MEASURING INSTRUMENTS USED IN FIELD SURVEY

Levelling instrument

---
type: Carl Zeiss Jena NI 030 10-Gl04e tilting level
mean square error per 1 km double levelling 2 - 3 mm

Echosounders

---
for profile measurements
type: Raytheon survey fathometer model DE - 719c (self registering)
for small use only
type: Euromarine digital echo sounder

Sextant

---
type: Master deluxe mark 25

Salinity meters

---
for measuring of sample bottles

type: Yellow Spring Instruments model 33 S-C-T meter
(Salinity - Conductivity - Temperature)
for profile measuring
type: Beckman RS5-3 S-C-T meter electrodeless induction salinometer

Impeller

---
type: Gurley no.622 current meter, suspended by cable

******************************************************************************************************
APPENDIX VI

METHOD OF HEUN

In both steady state, tidally averaged models we arrive at one-dimensional, differential equations (eq. III.1 and III.8 respectively). Both equations can be written as

\[ \frac{dc}{dx} = f(c) \]

This type of differential equations can be solved numerically with the method of Heun. Starting from a point \( x_1 \), with a known salinity \( c_1 \) we compute the salinity in a point \( x_2 \), with a distance \( \Delta x \) from \( x_1 \) in two steps

**step 1** \[ c_2^* = c_1 + f(c_1) \Delta x \]

**step 2** \[ c_2 = c_1 + \frac{f(c_1) + f(c_2^*)}{2} \Delta x \]

The first step provides a first estimate based on linear extrapolation, the so-called predictor formula. The second corrects the first estimate, the so-called corrector formula. This relatively simple scheme has a numerical error, per step \( \Delta x \), of the order of magnitude \( (\Delta x)^2 \).
The Penpas model has two separate calculation schemes, one for water motion, the other for the salt transport calculations.

**Watermotion**

Differential equations III.16 and III.17 are solved by means of a finite difference scheme. The method used is a Leap Frog scheme on a staggered grid. This implies that discharges and water levels are computed at different places and at different times.

In fig. app.VII.1 is shown from what places in the grid information is obtained to compute a discharge in a point \( t=(n+1)\frac{t}{2} \) and \( x=x_{m+n} \) and a water level in a point \( t=(n+2)\frac{t}{2} \) and \( x=x_m \) (\( t \) is time, \( x \) is distance to river mouth).

\[
\text{TIME} \quad \frac{\Delta t}{k} \quad n+1 \\
\text{nodes} \\
n+2 \\
n+1 \\
n \\
n-1 \\
n-2 \\
\text{NUMER OF NODE} \\
\text{TIME} \quad \frac{\Delta t}{k} \\
\text{nodes} \\
\text{X = points in space and time where H is calculated} \\
\text{O = points in space and time where Q is calculated}
\]

For stability and accuracy some conditions have to be fulfilled:

- **Stability**: \( \Delta t < \Delta x/c \)  
  \( c = \text{celerity } (\approx \sqrt{g \alpha}) \)

- **Accuracy**: \( \Delta t \approx \Delta x/c \)
Salt transports

Differential equation III.18 which describes salt transport in the estuary is solved with an implicit finite difference scheme. As the results of the watermotion part are needed as input, the points where and when the salt transports and concentrations are to be calculated depend on the grid already used in the watermotion calculation. It is possible however to use a different timestep. As far as accuracy and stability is concerned a larger timestep is usually permitted in comparison to the watermotion.

The salt transports are always calculated in the points where the discharges are known. The salinities can only be computed in the waterlevel points.

In fig app.VII.2 we have indicated where the calculation obtains its input from

for a case of \( \Delta t = 3 \times \Delta t \).

![Diagram](image)

\( \times \) = points where \( H \) and \( c \) are calculated
\( \circ \) = points where \( Q \) and \( T \) are calculated

fig. app.VII.2

Salt calculations are made with the so-called Crank-Nicholson implicit scheme. The implicit equations are solved by means of direct iterations.

Conditions that have to be fulfilled for the permissible timestep (\( \Delta t \)) and branchlength (\( \Delta x \))

- the one used in the watermotion calculation, so \( \Delta t_{\text{waterm.}} < \frac{\Delta x}{C} \) cond.2
- for convergence of the direct iterations \( \Delta t < \frac{(\Delta x)^2}{D} \) cond.3
- for accuracy

137.
\[ \Delta x < \frac{2D}{u} \quad \text{cond 4} \]

\[ u = \text{real time velocity} \]

Although there are many problems related to the accuracy of such schemes, this condition is usually sufficient.
Figures

1. Map of Southern Mozambique
2.a Map of river Maputo
2.b Map of Maputo Bay
3. Average waterlevel in river Maputo
4.a A - x - figure
4.b B - x - figure
4.c H - x - figure
4.d to 4.i measured cross-sectional profiles
5.a to 5.d Tidal waterlevels Bela Vista
5.e analysis tidal waterlevel Bela Vista
6. relationship tidal waterlevel amplitude Bela Vista and Maputo harbour
7. tidal wave intrusion in river Maputo at 29/5/1984
8. moving boat method results
9. bathymetry of river mouth of Maputo
10. Fresh water discharge
11. Discharge - waterlevel relationship Santaca
12.a to 12.m longitudinal and vertical salinity profiles
13. salinity during four weeks at Bela Vista
14. salinity during a day at Antunes
15. salinity during a day at Bela Vista
16. vertical salinity profiles within one cross-section at one time (Antunes)
Fig. 1.
MAPUTO BAY

SCALE 1: 250,000

Fig. 2.b.
Average Water- Level

$X = \text{Measurements}$

$--- = \text{Interpolation}$

$i = \text{Slope Water Level (Tidally Averaged)}$

$i = 0.8 \times 10^{-5}$

$i = 2 \times 10^{-5}$

$i = 8 \times 10^{-5}$

Santraca- Level at $Q_f \approx 50 \text{ m}^3/\text{s}$

Graph showing distance to river mouth (km) vs. water level.

Fig. 3.
Average Cross Sectional Area (AAv)

Equations used in models:

\( x < 2 \text{ km.} \)  \( AAv = 16,000 - 5111 \times x \) \( [m^2] \)

\( x > 2 \text{ km.} \)  \( AAv = 6549 \times \exp(-0.0626 \times x) \) \( [m^2] \)

\( x = \text{measured point} \)

Fig. 4.2.

\( x, \text{distance to river mouth} \) \( (km) \)
AVERAGE WIDTH - EQUATIONS (BAV) 
USED IN MODELS:

\[ x < 2 \text{ km} : \quad B_{AV} = 8000 - 3340 \times x \quad [\text{m}] \]
\[ x > 2 \text{ km} : \quad B_{AV} = 1470 \times \exp(-0.0538 \times x) \quad [\text{m}] \]

\[ x = \text{MEASURED POINT} \]

Fig. 4.6
**Fig. 4. d.**
Estação I (8.0 km)

Estação II (9.0 km)

Estação III (13.2 km)

Fig. 4 e.
Estação IV (17.1 km)

\[ A\text{av} = 3215 \text{ m}^3 \]
\[ B\text{av} = 545 \text{ m} \]

Estação V (19.7 km)

\[ A_{\text{av, tot}} = 1804 \text{ m}^3 \]
\[ B_{\text{av, tot}} = 625 \text{ m} \]

Estação VI (25.0 km)

\[ A\text{av} = 1654 \text{ m}^3 \]
\[ B\text{av} = 313 \text{ m} \]

Fig. 4.5
Estação VII (26.5 km)

Estação VIII (30.1 km)

Estação IX (32.9 km)

Fig. 4, p.
Estação X (36.0 km)

\[ A_{av} = 743 \text{ m}^3 \]
\[ B_{av} = 180 \text{ m} \]

HHW = EXTREME HIGH WATER
LLW = EXTREME LOW WATER
AV = AVERAGE LEVEL

Estação XI (39.0 km)

\[ A_{av} = 513 \text{ m}^3 \]
\[ B_{av} = 200 \text{ m} \]

Estação XII (41.7 km)

\[ A_{av} = 506 \text{ m}^3 \]
\[ B_{av} = 150 \text{ m} \]

Estação XIII (44.5 km)

\[ A_{av} = 314 \text{ m}^3 \]
\[ B_{av} = 114 \text{ m} \]

Fig. 4. h.
**Estação XIV** (47.1 km)

HHW = Extreme High Water  
LLW = Extreme Low Water  
AV = Average Level

$A_{HH} = 222 \, m^3$  
$B_{HH} = 96 \, m$

**Estação XV** (48.3 km)

$A_{AV} = 634 \, m^3$  
$B_{AV} = 164 \, m$

**Estação XVI** (51.0 km)

$A_{LV} = 208 \, m^3$  
$B_{LV} = 77 \, m$

*Fig. 4.i*
Tidal Analysis
Bela Vista
April + May 1984
Fig. 5.6.
FIG. 6

TIDAL AMPLITUDE
BELA VISTA
IN M

TIDAL AMPLITUDE
(PREDICTED)
PORT OF MAPUTO
IN M

= ASSUMED RELATIONSHIP
IN MODELS
WATERLEVEL
in m + MGM

TIDAL WAVE PROPAGATION ON 29/5/1984

SANTACA (70.0 km)

MGM-LEVEL

SALAMANGA (41.7 km)

ANTUNES (40 km)

BELA VISTA (23.0 km)

TIME (in hours of 29/5/1984)

FIG. 7.
Discharge measurements with "moving-boat" method

Antunes (8.0 km to rivermouth)

Date 15/5/1984

$\times$ = Discharge measurement (Positive disch. = upstream)

$\rightarrow$ = Waterlevel in Antunes

Discharge

\( (m^3/s) \)

Waterlevel

\( (m) \)

MGM

0

1000

2000

3000

4000

-1.0

-2000

-1000

0.0

1.0

2.0

3.0

10.00

11.00

12.00

13.00

14.00

15.00

16.00

17.00

18.00

Time (hrs)

Fig. 8.
BATHYMETRY

MAPUTO RIVERMOUTH

LINES OF EQUAL DEPTH (m)

DEPTHs TAKEN AT WATER
LEVEL OF -0.90 m MGM.

X = SALT MEASURING STATION

— — = CROSS-SECTIONS WHERE COMPLETE PROFILES WERE MEASURED
Fig 10

Discharge (m³/s)

- Measurements in Madebula (km 121)
- Measurements in Santaca (km 70)
zero gauge Santaca = +0.60 m MGM

\[ \log (H+0.6) = -0.197 + 0.381 \log (Q) \]

Fig. 11.
Fig. 12.a.
Fig. 12.b.
Figure 12.d: Vertical Profiles (High Water Slack)

- High Water Slack
- Low Water Slack
- Tidal Average

Salinity (%)

Distance to River Mouth (km)

F6
19-4-1984
High + Low Water Slack
Figure 12.e.

**Vertical Profiles (H.W. Slack)**

**F 7**

25-4-1984

High Water Slack

**Salinity at Bottom**

**Depth Averaged Salinity**

**Salinity at Surface**

**Distance to Mouth (km)**
Fig. 12.9. Distance to Rivermouth (km)

Salinity (‰)

CROSS SECTORIAL AVERAGE AT HIGH WATER SLACK

CROSS SECTORIAL AVERAGE AT LOW WATER SLACK

Fig. 9
10-5-1984
HIGH & LOW WATER SLACK

VERTICAL PROFILES
A: HIGH WATER & LOW WATER
Fig. 12.1

Salinity at different depths and distances to mouth on 23-5-1984.
**Fig. 14**

**Vertical Salinity Profile at Deepest Point in C.S. during the Day**

**Antunes (km 8.0), 2/5/1984**

**Fig. 15**

**Vertical Salinity Profile at Deepest Point in C.S. during the Day**

**Bela Vista (km 23.0), 1/5/1984**

**NOTE:** C.S. = Cross-Section
SALINITY (%)

VERTICAL SALINITY PROFILES
DATE: 12/4/1984
TIME: 14.30 - 14.55 hr

ISOHALINES (%)
DATE: 12/4/1984
TIME: 14.30 - 14.55 hr

ISOHALINES (%)
DATE: 2/5/1984
TIME: 11.48 - 12.00 hr

ALL MEASUREMENTS TAKEN IN ANTUNES (km 8.0)

Fig. 16