ABSTRACT

Underwater noise from offshore pile driving gained considerable interest in the recent years mainly due to the large scale construction of offshore wind farms. The most common foundation type of a wind turbine is a monopile, upon which the wind tower rests. The pile is driven into place with the help of hydraulic hammers. During the hammering of the pile, high levels of noise are generated. In this work, a linear semi-analytical model is presented for predicting the levels of underwater noise for a wide range of system parameters. The total system under consideration consists of the following sub-systems: a hydraulic hammer, a circular cylindrical shell, a compressible fluid and a water-saturated seabed. Strong and weak points of the present model are discussed and the influence of a number of parameters on the levels of noise generated is examined.

INTRODUCTION

Vibrations and acoustic radiation of submerged thin elastic shells has been the subject of study of numerous researchers in the past few decades. The methods which are used for solving the coupled vibro-acoustic problem can be divided into three main groups: Finite Element methods, Boundary Element methods and Semi-analytical methods. In cases where the geometry of the system is relatively simple, i.e. shells of revolution, the analytical or semi-analytical formulations are preferred due to reduced computational time.

Concerning the dynamic behaviour of thin elastic structures, most of the publications deal with the analytical approach to the vibro-acoustics of shells submerged in unbounded fluid domains. Junger and Feit (Miguel C. Junger and David Feit, 1986) have proposed a solution for the case of an infinite cylinder submerged in an infinite fluid domain. Stephanisen (P.R. Stephanisen, 1981) extended the method to the case of a finite length cylinder with a number of different edge boundary conditions. Laugnanet and Guyader (B. Laulagnet and J. Guyader, 1989) examined the problem for the case of a shell submerged in light and heavy fluids and pointed out the differences between the two cases. All the above studies were limited to the case of shells immersed in infinite fluid domains.

In this study, we present a method to examine the vibrations and acoustic radiation of a thin circular cylindrical shell partially submerged in a fluid and partially embedded into the soil. The fluid is bounded in the direction along the length of the shell whereas it extends to infinity in the radial direction. The shell is also filled with fluid, allowing thus one to examine the influence of the inner fluid on the sound radiation at the exterior fluid domain.

The pile is described with an appropriate shell theory including the effects of both shear deformation and rotation inertia (J. Kaplunov et al, 1993 and Belov et al, 1999). The fluid is
treated as a three-dimensional compressible medium, with a pressure release boundary describing the sea surface and a rigid boundary describing the seabed. The model allows also the substitution of the rigid boundary condition with the more general case of an impedance boundary. The soil is modelled with distributed uncoupled massless springs and dashpots along the length of the shell which is subjected to the soil reaction in all directions.

The solution of the coupled problem is based on the expansion of the response of the immersed system on the modal basis of the in vacuo shell structure. Unlike several other works (J. Guyader and B. Laulagnet, 1994) in which the fluid problem is treated by boundary integral equations, the pressure of the inner and outer fluid domains is expressed here in terms of the governing equations alone. The coupling of the shell vibrations with the surrounding fluid is accomplished in the modal domain, i.e. the fluid pressure acting on the surface of the shell is expressed in terms of the in vacuo shell modes. Since the modal basis is that of the in vacuo system and not the one of the actual immersed structure, the fluid pressure couples all shell modes of the same circumferential configuration, whatever their axial wavenumber.

The influence of the intermodal coupling on the generated pressure levels is discussed and the difference between a heavy and a light fluid case is highlighted. Emphasis is placed on the examination of the influence of the spring and dashpot coefficients used for the soil description on the generated pressure levels at the exterior fluid domain. The present model is developed in such a way that it can be readily extended in order to account for more advanced elements, i.e. a 3D description of the soil.

MODEL AND GOVERNING EQUATIONS

Geometry of the model

The geometry of the model is shown in Fig. 1.
The pile is modelled as a thin elastic circular cylindrical shell of constant thickness and finite length. The shell description includes the effects of both shear deformation and rotation inertia. The hammer is modelled as a force applied at the top of the shell with a small inclination. The inclination of the force allows the activation of axially asymmetric vibration modes. The fluid is modelled as a three-dimensional inviscid compressible medium. The soil is substituted by distributed springs and dashpots along the height of the shell ranging from \( z \leq z < L \) in all directions. The sea surface is modelled as a pressure release boundary. The sea bottom is modelled either as a rigid boundary or as an impedance boundary. The constants \( E, \nu, \rho \) and \( h \) correspond to the complex modulus of elasticity, the Poisson’s ratio, the density and the thickness of the shell respectively. The shell is filled with fluid at \( z \leq z < L \) and surrounded by fluid at \( z_i \leq z < z_2 \). For the case where the shell is filled with fluid, the boundary condition for the inner fluid in the bottom of the pile always represents a rigid boundary.

**Vibrations of the shell structure**

The following set of coupled equations describes the vibro-acoustics of the system:

\[
\begin{align*}
\frac{\partial T_i}{\partial z} + \frac{1}{R} \frac{\partial S_{ij}}{\partial \theta} - I_{tg} \mathbf{u}_{\Phi} &= -\hat{k}_{z,\text{soil}} u_z(z,\theta,t) \cdot H(z-z_2) + F_z(z,\theta,t) \\
\frac{1}{R} \frac{\partial T_i}{\partial \theta} + \frac{1}{R} N_{\Phi} - I_{tg} \mathbf{u}_{\Phi} &= -\hat{k}_{\theta,\text{soil}} u_{\theta}(z,\theta,t) \cdot H(z-z_2) + F_{\theta}(z,\theta,t) \\
\frac{T_{\Phi}}{R} \frac{\partial N_{\Phi}}{\partial z} - \frac{1}{R} \frac{\partial N_{\Phi}}{\partial \theta} + I_{n} u_{n} &= -\hat{k}_{r,\text{soil}} u_r(z,\theta,t) \cdot H(z-z_2) + \\
+ P_{1}(R,\theta,z,t) \cdot H(z-z_0) - P_{2}(R,\theta,z,t) \cdot (H(z-z_1) - H(z-z_2)) + F_r(z,\theta,t) \\
N_{\xi} &= \frac{1}{R} \frac{\partial G_{\xi}}{\partial \theta} + \frac{H_{\xi}}{\partial z} \\
N_{\theta} &= \frac{1}{R} \frac{\partial G_{\theta}}{\partial \theta} + \frac{H_{\theta}}{\partial z} \\
N_{r} &= \frac{1}{R} \frac{\partial G_{r}}{\partial \theta} + \frac{H_{r}}{\partial z}
\end{align*}
\]

In the above system of equations \( T_i \) are the longitudinal stress resultants, \( S_{ij} \) are the membrane shear forces, \( H_{ij} \) are the twisting stress couples, \( N_i \) are the transverse shear forces and \( G_i \) are the bending stress couples, with \( i \neq j = z, \theta \). The constitutive and geometric relations, as well as the modified inertial terms \( I_{tg} \mathbf{u}_{\Phi} \), \( I_{tg} \mathbf{u}_{\Phi} \) and \( I_{n} u_{n} \) are given in Appendix A.

Soil reaction is characterised by the following operators

\[
\hat{k}_{z,\text{soil}} = k_z + c_z \frac{\partial}{\partial t} \quad \hat{k}_{\theta,\text{soil}} = k_{\theta} + c_{\theta} \frac{\partial}{\partial t} \quad \text{and} \quad \hat{k}_{r,\text{soil}} = k_r + c_r \frac{\partial}{\partial t},
\]

in which the terms \( k_z, k_{\theta} \) and \( k_r \) correspond to the soil stiffness and the terms \( c_z, c_{\theta} \) and \( c_r \) correspond to the equivalent viscous soil damping coefficients along \( z, \theta \) and \( r \) coordinates respectively. The terms \( P_{1}(R,\theta,z,t) \) and \( P_{2}(R,\theta,z,t) \) in (3) correspond to the inner and outer fluid pressure respectively. For the high order theories, a small correction factor is included into the pressure components, to account for the fact that the fluid pressure is not applied at the mid-surface of the shell but at the outer and inner surfaces. This corresponds to the terms proportional to \( h^2 \) in the following expressions:
\[ P_1(R, \theta, z, t) = \left( 1 - \frac{8 - 3v}{10(1-v)} \cdot h^2 \left( \frac{\partial^2}{\partial z^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} \right) \right) p_1(r, \theta, z, t) \]  
\[ P_2(R, \theta, z, t) = \left( 1 - \frac{8 - 3v}{10(1-v)} \cdot h^2 \left( \frac{\partial^2}{\partial z^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} \right) \right) p_2(r, \theta, z, t) \]  

By substituting the geometrical and constitutive relations, the modified inertia terms and the fluid pressure components into the equations of motion, the following coupled system of partial differential equations can be obtained:

\[ \{L\} u(z, \theta, t) + \{I_{\text{mod}}\} u(z, \theta, t) = -\{K_{\text{soil}}\} u(z, \theta, t) \cdot H(z-z_c) + p_1(R, \theta, z, t) \cdot H(z-z_0) - p_2(R, \theta, z, t) \cdot \left[ H(z-z_c) - H(z-z_0) \right] + F_{\text{ext}}(z, \theta, t) \]  

The operators \( \{L\} \) and \( \{I_{\text{mod}}\} \) correspond to the stiffness and modified inertia shell matrices based on the chosen shell theory. The operator \( \{K_{\text{soil}}\} \) accounts for the soil dynamic stiffness. The springs and dashpots are assumed uncoupled in the various directions, i.e. the operator for the dynamic soil stiffness matrix is diagonal and given as:

\[ \{K_{\text{soil}}\} = \begin{bmatrix} k_z + c_z \frac{\partial}{\partial t} & 0 & 0 \\ 0 & k_\theta + c_\theta \frac{\partial}{\partial t} & 0 \\ 0 & 0 & k_r + c_r \frac{\partial}{\partial t} \end{bmatrix} \]

The vectors \( p_1(R, \theta, z, t) \), \( p_2(R, \theta, z, t) \) and \( F_{\text{ext}}(z, \theta, t) \) correspond to the inner fluid pressure, the outer fluid pressure and the external force respectively. The fluid pressure is coupled only to the radial motion of the shell, i.e. vectors \( p_1(R, \theta, z, t) \) and \( p_2(R, \theta, z, t) \) contain only the radial component. The soil and the fluid pressures are active only within a certain part of the shell. This is taken into account with the use of the Heaviside step functions \( H(z-z_c) \).

**Description of the outer fluid domain**

We consider a pressure variation of the fluid around its equilibrium position. The motion of the outer fluid is described by the velocity potential \( \phi_2(r, \theta, z, t) \). The equation of motion for the outer fluid domain reads:

\[ \nabla^2 \phi_2(r, \theta, z, t) - \frac{1}{c_{f,2}^2} \cdot \phi_2(r, \theta, z, t) = 0 \]

where \( c_{f,2} \) is the sound speed in the exterior fluid domain and the Laplacian operator is defined in the cylindrical coordinate system as:

\[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \]

We search for a solution in the frequency domain in the form of:

\[ \phi_2(r, \theta, z, t) = \tilde{\phi}_2(r, \theta, z, \omega) \cdot \exp(\omega t) \]

Our goal is to derive the complex amplitude \( \tilde{\phi}_2(r, \theta, z, \omega) \). The tilde over the potential function denotes the complex amplitude in the frequency domain. In the bounded fluid domain considered here, the fluid motion must satisfy the homogeneous Helmholtz equation, the
radiation condition at $r \to \infty$, the boundary conditions at $z = z_1$, $z = z_2$ and the interface condition at the fluid-shell contact surface. The sea surface is always defined as a pressure release surface, whereas for the seabed two cases are considered: a rigid boundary case and an impedance boundary case (Fig. 2). Note that the first is actually a special case of the latter for very high values of the impedance.

This implies the following mathematical formulation for the outer fluid domain:

**Helmholtz equation:**

$$\nabla^2 \tilde{\phi}_2 + k_{f,2}^2 \cdot \tilde{\phi}_2 = 0,$$

with:

$$k_{f,2}^2 = \frac{\omega^2}{c_{f,2}^2}$$  \hspace{1cm} (11)

**Radiation condition at infinity:**

$$\lim_{r \to \infty} r \left( \frac{\partial \tilde{\phi}_2}{\partial r} - k_{f,2} \cdot \tilde{\phi}_2 \right) = 0$$  \hspace{1cm} (12)

**Boundary condition at the fluid surface:**

$$z = z_1: \quad \tilde{p}_2(r, \theta, z, \omega)_{z=z_1} = 0$$  \hspace{1cm} (13)

**Impedance boundary condition at the fluid bottom (case 1):**

$$z = z_2: \quad \tilde{v}_{r,2}(r, \theta, z, \omega)_{z=z_2} = \frac{\tilde{p}_2(r, \theta, z, \omega)_{z=z_2}}{Z_{\text{soil}}(\omega)}$$  \hspace{1cm} (14-1)

**Rigid boundary condition at the fluid bottom (case 2):**

$$z = z_2: \quad \tilde{v}_{r,2}(r, \theta, z, \omega)_{z=z_2} = 0$$  \hspace{1cm} (14-2)

**Interface condition with the shell, $r = R$:**

$$\tilde{v}_{r,2}(r, \theta, z, \omega)_{r=R} = i \cdot \omega \left( 1 + \frac{v \cdot h^2}{2 \cdot (1-v)} \left( \frac{\partial^2}{\partial z^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} \right) \right) \tilde{u}_r(z, \theta, \omega)$$  \hspace{1cm} (15)

The term $Z_{\text{soil}}(\omega)$ corresponds to the equivalent point impedance of the soil surface. The **acoustic fluid velocity vector** is related to the scalar velocity potential with the following expression:

$$\mathbf{v}_f(r, \theta, z, t) = \nabla \phi(r, \theta, z, t),$$

where:

$$\nabla = \frac{\partial}{\partial r} + \frac{1}{r} \frac{\partial}{\partial \theta} + \frac{\partial}{\partial z}.$$

The **acoustic pressure** is related to the scalar velocity potential as:

$$p_f(r, \theta, z, t) = -\rho_{f,2} \cdot \frac{\partial \phi(r, \theta, z, t)}{\partial t}$$
Description of the inner fluid domain

The motion of the inner fluid is described by the velocity potential \( \varphi_i(r, \theta, z, t) \). The inner fluid must satisfy the homogeneous Helmholtz equation, the condition for limited values of the pressure amplitude at \( r = 0 \), the boundary conditions at \( z = z_0, z = L \) and the interface condition at the fluid-shell surface. This implies the following mathematical formulation for the inner fluid domain:

\[
\text{Helmholtz equation: } \nabla^2 \tilde{\varphi}_i + k_i^2 \cdot \tilde{\varphi}_i = 0, \quad \text{with: } k_i^2 = \frac{\omega^2}{c_i^2},
\]

\[
\text{Bounded values of the potential at } r = 0
\]

\[
\text{Boundary condition at the fluid surface } z = z_0: \quad \tilde{p}_i(r, \theta, z, \omega)_{|z=z_0} = 0
\]

\[
\text{Boundary condition at the fluid bottom } z = L: \quad \tilde{v}_i(r, \theta, z, \omega)_{|z=L} = 0
\]

\[
\text{Interface condition with the shell at } r = R:
\]

\[
\tilde{v}_{i,l}(r, \theta, z, \omega)_{|r=R} = -i \cdot \omega \left( 1 + \frac{\nu \cdot h^2}{2 \cdot (1-\nu)} \left( \frac{\partial^2}{\partial z^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} \right) \right) \tilde{u}_r(z, \theta, \omega)
\]

SOLUTION OF THE SYSTEM OF COUPLED EQUATIONS

The response of the immersed system is based on the modal expansion over the in vacuo modes of the shell. The semi-analytical approach proposed here is based on the following steps:

1) The evaluation of the in vacuo response of the shell without the presence of the fluids and the soil;
2) The solution of the governing equations for the two fluid domains by expressing the interface conditions at the shell-fluid contact surface in the modal domain;
3) The solution of the coupled system of equations resulting from the substitution of the obtained solutions for the shell and the fluid domains into the interface conditions.

In vacuo response of the system

The procedure for calculating the eigenfrequencies and eigenmodes of a circular cylindrical shell with arbitrary edge constraints will not be described in detail here since it is a routine process in structural dynamics. The analytical solution is based on a coupled system of partial differential equations, describing the free vibrations of the shell, which includes the effects of both shear deformation and rotation inertia. The partial differential equations are transformed into a system of algebraic equations which can be solved with high accuracy (A. Ludwig and R. Krieg, 1981). The final solution per vibration mode can be expressed in the following manner:

\[
u_{nm}(z, \theta, t) = A_{mn} \cdot U_{nm}(z) \cdot \cos(n\theta) \cdot \exp[i \cdot \Omega_{mn} \cdot t]
\]

\[
u_{nm}(z, \theta, t) = A_{nn} \cdot U_{nn}(z) \cdot \sin(n\theta) \cdot \exp[i \cdot \Omega_{nn} \cdot t]
\]

\[
u_{nm}(z, \theta, t) = A_{mn} \cdot U_{mn}(z) \cdot \cos(n\theta) \cdot \exp[i \cdot \Omega_{mn} \cdot t]
\]
where \( n = 0,1,2,\ldots,\infty \) is the circumferential order and \( m = 1,2,\ldots,\infty \) is the axial order.

The functions \( U_{znm}(z) \), \( U_{\theta nm}(z) \) and \( U_{rnm}(z) \) describe the axial distribution for the axial, circumferential and radial displacement fields respectively; \( \Omega_{nm} \) is the eigenfrequency. The trigonometric functions describe the circumferential distributions of the axial, circumferential and radial displacement fields in a similar manner. The integers \( n \) and \( m \) indicate that an infinite set of eigensolutions may exist. The deformation shapes together with the eigenfrequencies are the required eigensolutions of the problem. The unknown modal factors \( A_{nm} \) are determined by satisfying the boundary condition at the source of the external load.

**Solution of the governing equations for the two fluid domains**

**Outer fluid domain**

A solution to the set of equations (11)-(13) and (14-2) can be expressed in the following form:

\[ \tilde{\phi}_n(r,\theta,z,\omega) = -2 \cdot j \cdot \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} C_{np} \cdot H_n^{(2)}(k_{n,\theta,p} \cdot r) \cdot \sin(k_{n,z,p} \cdot (z - z_1)) \cdot \cos(n\theta) \]

\[ k_{n,z,p} = \left( \frac{2p + 1}{2} \right) \cdot \left( z_2 - z_1 \right) \quad \text{and} \quad k_{n,\theta,p} = \left( \sqrt{k_{f,2}^2 - k_{n,z,p}^2} \right)^{1/2} \cdot z_1 \leq z \leq z_2, \quad p = 0,1,2,\ldots,\infty \]  \hfill (24)

The summation index \( p \) refers to the fluid modes for the outer fluid domain. The pressure of the fluid and the velocity component normal to the surface of the shell, based on (24) become:

\[ \tilde{p}_n(r,\theta,z,\omega) = -2 \cdot \omega \cdot \rho_{f,2} \cdot \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} C_{np} \cdot H_n^{(2)}(k_{n,\theta,p} \cdot r) \cdot \sin(k_{n,z,p} \cdot (z - z_1)) \cdot \cos(n\theta) \]  \hfill (25)

\[ \tilde{v}_{n,2}(r,\theta,z,\omega) = -2 \cdot j \cdot \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} C_{np} \cdot H_n^{(2)}(k_{n,\theta,p} \cdot r) \cdot \sin(k_{n,z,p} \cdot (z - z_1)) \cdot \cos(n\theta) \]  \hfill (26)

The function \( H_n^{(2)}(k_{n,\theta,p} \cdot r) \) denotes the derivative of the Hankel function with respect to radial coordinate. The undetermined modal coefficients \( C_{np} \) will be expressed in terms of the modal coefficients of the vibrating shell from the equilibrium of the velocities at the shell-fluid interface. By expressing equation (15) in the modal domain i.e. in terms of the (21)-(23) and (26), and by making use of the orthogonality property of the fluid modes, the unknown complex modal coefficients \( C_{np} \) can be expressed in the following manner:

\[ C_{np} = \frac{\omega}{(z_2 - z_1) \cdot H_n^{(2)}(k_{n,\theta,p} \cdot R)} \cdot \sum_{m=1}^{\infty} A_{nm} \cdot \left[ \frac{U_{rnm}(z)}{z_1} \right] \cdot \left( \frac{v}{2 \cdot (1 - v)} \cdot \left( \frac{d^2 U_{rnm}(z)}{dz^2} + \frac{n^2 \cdot U_{rnm}(z)}{R^2} \right) \right) \cdot \sin(k_{n,z,p} \cdot (z - z_1)) \, dz \]

The pressure at the outer shell-fluid interface can now be expressed in terms of the modal amplitudes of the in vacuo shell structure:

\[ \tilde{p}_n(R,\theta,z,\omega) = \frac{2 \cdot \alpha^2 \cdot \rho_{f,2}}{(z_2 - z_1)} \cdot \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} A_{nm} \cdot \tilde{p}_{2,mp}, \quad \text{with:} \]  \hfill (27)
The solution to the complete system of equations

Following a procedure similar to that described for the outer fluid domain, the following set of equations can be derived for the case of the inner fluid:

\[
\tilde{p}_i(R, \theta, z, \omega) = -\frac{2 \cdot \omega^2 \cdot \rho_i}{(L - z_0)} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_{nm} \cdot \tilde{p}_{j,nm}, \quad \text{with:}
\]

\[
\tilde{p}_{j,nm} = \frac{J_1(k_{n,z,j} R)}{J_0^2(k_{n,z,j} R)} \sin(k_{n,z,j} (z - z_0)) \cdot \cos(n \theta) \cdot \int_{z_0}^{z} F_{nm}(z) \cdot \sin(k_{n,z,j} (z - z_0)) dz
\]

\[
F_{nm}(z) = U_{nm}(z) + \left( \frac{v \cdot R^2}{2(1-v)} \left( \frac{d^2 U_{nm}(z)}{dz^2} \right) - \frac{n^2}{R^2} \cdot U_{nm}(z) \right)
\]

\[
k_{n,z,j} = \left( \frac{2(j+1) \pi}{2 \cdot (L - z_0)} \right) \text{ and } k_{n,z,j} = \left( k_{j,z}^2 - k_{n,z,j}^2 \right)^{\frac{1}{2}}, \quad z_0 \leq z \leq L, \quad j = 0, 1, 2, ..., \infty
\]

\[8\]

**Solution to the complete system of equations**

Since the in vacuo normal modes and the natural frequencies can be obtained via the procedure described previously and the fluid pressure on the surface of the shell can be expressed in the modal domain, the solution of the inhomogeneous problem, equation (9), can now be addressed. We expand equation (9) in the modal domain by substituting equations (21) - (23) and (27) - (28). The resulting equation in the frequency domain reads:

\[
- \left( \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_{nm} \cdot \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} U_{nm}(z) \cdot \cos(n \theta) \\ U_{nm}(z) \cdot \sin(n \theta) \\ U_{nm}(z) \cdot \cos(n \theta) \end{bmatrix} + \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_{nm} \cdot \begin{bmatrix} \tilde{T}_{11} & \tilde{T}_{12} & 0 \\ \tilde{T}_{21} & \tilde{T}_{22} & 0 \\ 0 & 0 & \tilde{T}_{33} \end{bmatrix} \begin{bmatrix} U_{nm}(z) \cdot \cos(n \theta) \\ U_{nm}(z) \cdot \sin(n \theta) \\ U_{nm}(z) \cdot \cos(n \theta) \end{bmatrix} = -H(z - z_2) \cdot \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_{nm} \cdot \begin{bmatrix} k_z + i \cdot \omega \cdot c_z & 0 & 0 \\ 0 & k_\theta + i \cdot \omega \cdot c_\theta & 0 \\ 0 & 0 & k_r + i \cdot \omega \cdot c_r \end{bmatrix} \begin{bmatrix} U_{nm}(z) \cdot \cos(n \theta) \\ U_{nm}(z) \cdot \sin(n \theta) \\ U_{nm}(z) \cdot \cos(n \theta) \end{bmatrix} - \left( \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \sum_{m=1}^{\infty} A_{nm} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \tilde{F}_{nm}(\omega) \cdot \delta(z - z_0) \end{bmatrix} \right)
\]

\[8\]
In (29) the external force is decomposed into its Fourier coefficients per circumferential mode. The tilde over the modified inertia operators is included to illustrate that the time dependence is dropped and the frequency of the excitation enters the operators. The above system of equations is reduced to a system of algebraic equations by using the orthogonality property of the in vacuo shell modes. The resulting system of coupled equations reads:

\[
A_{nm} - \sum_{k=1}^{3} M_{nm,k} \cdot (\omega^{2k} - \omega^{2k}_{nm}) = A_{nm} \cdot K_{nm,vol} + R_{nm} - i \cdot \omega \cdot (A_{nm} \cdot Z_{1,nm} + A_{nm} \cdot Z_{2,nm}) \]

(30)

In expression (30) the following notations are introduced:

- The terms \(M_{nm,k}\) with \(k = 1, 2, 3\) correspond to the generalized modal masses. The expressions are quite lengthy to be shown explicitly here. If we define the generalized modal inertia as \(I_{nm} = \sum_{k=1}^{3} M_{nm,k} \cdot \omega^{2k}_{nm}\), terms proportional to \(\omega^{2k}_{nm}\) refer to the low order theory whereas terms proportional to \(\omega^{4k}_{nm}\) and \(\omega^{6k}_{nm}\) reflect the contribution of the higher order theory adopted here.

- Modal soil reaction:

\[
K_{nm,vol} = -\int_{Z} \left( \left( k_{m} + i \cdot \omega \cdot c_{m} \right) \cdot U^{2}_{nm}(z) + \left( k_{p} + i \cdot \omega \cdot c_{p} \right) \cdot U^{2}_{nm}(z) + \left( k_{g} + i \cdot \omega \cdot c_{g} \right) \cdot U^{2}_{nm}(z) \right) \delta(z) \, dz
\]

- Modal force:

\[
R_{nm} = \int_{0}^{L} \left[ \left( F_{n}^{(n)}(z) \cdot U_{nm}(z) + F_{dm}(z) \cdot U_{dm}(z) + F_{rn}(z) \cdot U_{rn}(z) \right) \right] \, dz
\]

- Inner fluid modal cross-impedance:

\[
Z_{1,nmq} = \frac{2 \cdot \omega \cdot \rho_{f,1}}{i \cdot (L - z_{0})} \int_{z_{0}}^{z_{1}} U_{nm}(z) \cdot \left[ \sum_{j=0}^{3} \left( \frac{8 - 3v}{10 \cdot (1 - v)} \cdot h^{2} \cdot \left( k_{n,c,j}^{2} + \frac{n^{2}}{R^{2}} \right) \right) \right] \, dz
\]

- Inner fluid self-impedance:

\[
Z_{1,nmm} = \frac{2 \cdot \omega \cdot \rho_{f,1}}{i \cdot (L - z_{0})} \int_{z_{0}}^{z_{1}} U_{nm}(z) \cdot \left[ \sum_{j=0}^{3} \left( \frac{8 - 3v}{10 \cdot (1 - v)} \cdot h^{2} \cdot \left( k_{n,c,j}^{2} + \frac{n^{2}}{R^{2}} \right) \right) \right] \, dz
\]

- Outer fluid modal cross-impedance:

\[
Z_{2,nmq} = \frac{2 \cdot \omega \cdot \rho_{f,2}}{i \cdot (z_{2} - z_{1})} \int_{z_{1}}^{z_{2}} U_{nm}(z) \cdot \left[ \sum_{p=0}^{3} \left( \frac{8 - 3v}{10 \cdot (1 - v)} \cdot h^{2} \cdot \left( k_{n,c,p}^{2} + \frac{n^{2}}{R^{2}} \right) \right) \right] \, dz
\]
Outer fluid self-impedance:

\[
Z_{2,\text{num}} = \frac{2 \cdot \omega \cdot \rho_{f,2}}{i \cdot (z_2 - z_1)} \int_{z_1}^{z_2} \frac{1}{U_{\text{num}}(z)} \left\{ \sum_{p=0}^{\infty} \frac{H_n^{(2)}(k_{n,r}, p \cdot R)}{H^{(2)}_{n, r}(k_{n,r}, p \cdot R)} \cdot \sin(k_{n,c}, p \cdot (z - z_1)) \cdot \left[ F_{\text{num}}(z) \cdot \sin(k_{n,c}, p \cdot (z - z_1)) \right] \right\} dz
\]

It is worth mentioning a number of points with respect to equation (30). At first, we note that the integral over the circumference drops simultaneously from all terms because of the axial symmetry of the system. This reflects the fact that modes of different circumferential configuration are completely decoupled.

Secondly, terms proportional to \( h^2 \) which enter the impedance coefficients \( Z_{i,\text{num}} \) \((i=1,2)\), correspond to the small correction, which we have already discussed for the position of the application of the inner and outer fluid pressures. It is clear that those terms are of a magnitude smaller than unity for the range of acoustic frequencies examined here.

Thirdly, the intermodal coupling is clearly shown in (30). We denote the modes entering the integral representation of the impedance with the index \( q \) in order to distinguish them from the shell modes given by the index \( m \). Terms \( Z_{i,\text{num}} \) correspond to the mutual impedances or modal cross-impedances, whereas terms \( Z_{i,\text{num}} \) correspond to the self-impedances. We also modify the modal amplitudes \( A_{\text{num}} \) accordingly, to distinguish the two separate cases:

- \( q = m \): \( A_{nq} = A_{nm} \), which represents the combination of the loading on mode \((m,n)\) due to the pressure generated by the mode \((m,n)\).
- \( q \neq m \): \( A_{nq} \neq A_{nm} \), which represents the combination of the loading on mode \((m,n)\) due to the pressure generated by another mode \((q,n)\).

Thus, even though modal coefficients of different circumferential wavenumbers are completely decoupled, all modes of a specified circumferential configuration are coupled via the mutual impedances.

Finally, material damping can be incorporated in the form of a complex modulus of elasticity for the shell structure. In this case equation (30) can be modified accordingly to give:

\[
A_{\text{num}} = \sum_{k=1}^{3} M_{\text{num},n,k} \cdot \left( \omega^{2k} - \omega^{2k}_{\text{num}} \cdot (1 + i \cdot \eta_{\text{num}}) \right) = A_{\text{num}} \cdot K_{\text{num, soil}} + R_{\text{num}} +
\]

\[
- i \cdot \omega \cdot \left( A_{\text{num}} \cdot Z_{1,\text{num}} + A_{\text{num}} \cdot Z_{2,\text{num}} \right) - i \cdot \omega \cdot \sum_{q=m}^{\infty} A_{nq} \cdot Z_{1,\text{num}} + \sum_{q=m}^{\infty} A_{nq} \cdot Z_{2,\text{num}}
\]

\[
\eta_{\text{num}} = \frac{C_{\text{num}}}{K_{\text{num}}} \quad \text{corresponds to the structural loss factor of the chosen material. By neglecting the mutual impedances, an approximation which is often used in the case of light fluids, equation (31) can be solved directly to give the unknown modal amplitudes:}
\]

\[
A_{\text{num}} = \frac{R_{\text{num}}}{i \cdot \omega \cdot \left( Z_{1,\text{num}} + Z_{2,\text{num}} \right) - K_{\text{num, soil}} + \sum_{k=1}^{3} M_{\text{num},n,k} \cdot \left( \omega^{2k} - \omega^{2k}_{\text{num}} \cdot (1 + i \cdot \eta_{\text{num}}) \right)}
\]
Modelling of the hammer-force at the top of the shell

The hammer is modelled as an impact force exerted at the pile head. In contrast to other studies (G. Reinhall and H. Dahl, 2011), the force here is allowed to take a small inclination in order to excite axially asymmetric modes. The force can take the form of an exponential pulse, a triangular pulse or any other form depending on the hammer characteristics. In a first approximation, the distribution of the force along the circumference of the shell is assumed constant. For such a uniform distribution it can easily be shown that only the two lowest circumferential modes of the shell are excited. The total force is further decomposed into two components (see Fig. 3):

\( F_z(t) \): Total vertical load pointing towards the positive \( z \)-direction at the pile head

\( F_x(t) \): Total horizontal load pointing towards the positive \( x \)-direction at the pile head (taken as a percentage of the vertical load, e.g. 2%).

For the purposes of the present model, a transformation of the force in the frequency domain is needed:

\[
\bar{F}_z(\omega) = \int_{-\infty}^{\infty} F_z(t) \exp(-i \omega t) \, dt \quad \text{and} \quad \bar{F}_x(\omega) = \int_{-\infty}^{\infty} F_x(t) \exp(-i \omega t) \, dt
\]

Note that the force in the frequency domain is defined as a complex function, carrying both amplitude and phase information. This is necessary when one would like to calculate the response of the system back to the real time domain with the use of the Inverse Fourier Transform. The horizontal force is further decomposed into a radial and a tangential component. Based on the above considerations the modal force to be inserted into (31) can be decomposed into Fourier series per \( n \)-mode to give:

\[
R_{2n} = \frac{\bar{F}_z(\omega)}{2 \cdot \pi \cdot R} : U_{2n}(0) \quad \text{(for } n = 0) \quad \text{and} \quad R_{n+1} = \frac{\bar{F}_x(\omega)}{2 \cdot \pi \cdot R} : [U_{2n+1}(0)-U_{2n+1}(0)] \quad \text{(for } n = 1)
\]
NUMERICAL AND OTHER ISSUES

Matrix formulation of the problem

For a given force amplitude all quantities entering equation (31) are known except from the modal coefficients $A_{nm}$ which remain undetermined. After rearranging some terms in (31), the following matrix equation needs to be solved with respect to the unknown vector $A$:

$$ C \cdot A = R , \quad \text{where:}$$

- $C$ is the given coefficient matrix of order $m \times m$, 
- $A$ is the unknown vector of order $m \times 1$, 
- $R$ is the given external force vector of order $m \times 1$.

In the case where the mutual impedances are neglected, all the non-diagonal terms of the coefficient matrix are set equal to zero and the problem is reduced to a set of uncoupled algebraic equations given as:

$$ C_d \cdot A = R , \quad \text{where:}$$

- $C_d$ is taken from $C$ after setting all the non-diagonal terms equal to zero.

Equations (35) and its simpler form (36) allow us to make some important conclusions. The physical significance of neglecting the cross-coupling impedance terms, i.e. use of equation (36), has to do with the fact that the modes of the immersed system are expected to be only slightly different from the in-vacuo modes when the fluid is light. Therefore we can assume that the immersed structure will keep its in-vacuo mode shapes when vibrating in a light fluid medium (i.e. a steel circular cylindrical shell vibrating in air). On the contrary, equation (35) shows that a structure immersed in a heavy fluid medium (i.e. a steel cylindrical shell in water) will not be able to retain its in-vacuo mode shapes. Obviously, the resulting system of linear equations will not be diagonal in the case of a heavy fluid.

Spring and dashpot coefficients

A correct description of the soil should include springs and dashpots which depend both on the frequency of the excitation and the wavelength (modal shape) of the structure. The magnitude of the total damping usually depends on a number of parameters such as the excitation frequency, the geometry of the soil-pile interface, the geometrical boundaries of the examined system, the mode of oscillation and the stress-strain characteristics of the soil.

In this work, the concept of distributed springs and dashpots is introduced for the description of the soil surrounding the pile. Note that the same concept has been introduced for the description of the interface at the seabed level, with the introduction of the complex seabed soil impedance $Z_{soil}(\omega)$ in (14-1). In cases where the soil impedance is very large, the boundary condition tends asymptotically to the rigid boundary case. In this paper, we will limit ourselves only to the discussion of the rigid boundary case.

The presence of distributed springs and dashpots, although very practical from an engineering point of view, introduces large uncertainty into the calculations. A realistic estimation of the spring and dashpot constants can be achieved within certain confidence limits. For this reason, we decide here to examine the response of the total system from a different point of
view. Instead of trying to get an accurate estimation of parameters of the springs and dashpots, we will examine the influence of realistic but somewhat arbitrarily chosen coefficients on the pressure levels. In this way we aim to show whether the contribution of the soil is of substantial importance for the calculation of the underwater noise levels or not.

As a starting point, the soil material damping is expressed in terms of an equivalent damping ratio throughout the examined frequency range for a certain strain rate (George Gazetas and Richardo Dobry, 1984). The soil radiation damping is incorporated into the model in a simplified manner as described in a previous work by Gazetas et al. (George Gazetas and Richardo Dobry, 1983). The spring coefficients are assumed constant throughout the frequency range with average values retrieved from the correspondent static stiffness.

INFLUENCE OF VARIOUS PARAMETERS ON THE PRESSURE LEVELS

Shell geometry and material constants
A shell with a certain geometry and material constants is chosen for numerical evaluation. The parameters used hereafter correspond to a real case where measurements on the levels of underwater noise were also conducted. The material properties and the shell geometry are summarized in table 1. The reference spring and dashpot coefficients are calculated with the method described previously. As we have already mentioned, for the purposes of the present work, the spring and dashpot coefficients do not necessarily need to correspond to the actual soil parameters. We will refer to these values hereafter as the ‘set of reference values’ for the soil material.

<table>
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<th>Parameter</th>
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<th>Units</th>
<th>Parameter</th>
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<th>Units</th>
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<td>Cex</td>
<td>3,00E+05</td>
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</tr>
<tr>
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<td>m</td>
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<td>L</td>
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</tbody>
</table>

Table 1. Material constants, geometrical parameters and soil properties (reference values) for the examined case

Influence of the inner fluid on the radiation at the exterior fluid domain
It would be interesting to examine the influence of the inner fluid on the pressure levels at the exterior fluid domain. We limit ourselves to the examination of the maximum pressure amplitudes. Since the model is linear and the seabed is defined as a rigid boundary, it is reasonable to assume that the maximum pressure level will be a good indicator for the pressure levels elsewhere in the fluid domain. The applied force is chosen as: 
\[ \vec{F}(\omega) = A \text{ and } \vec{F}(\omega) = 0,01 \cdot A , \text{ with } A = 1 \text{ MN} \]
The set of reference values for the soil material is applied. The results of the analysis are shown in Fig. 4. The vertical axis shows the maximum pressure level in $dB$ re $1\mu Pa$ plotted versus the excitation frequency. Frequencies between 28 Hz and 4000 Hz with a frequency step of 28 Hz are chosen.

![Fig. 4 Comparison of the maximum pressure levels with and without the presence of the inner fluid](image)

The cut-off frequency is clearly seen at 130 Hz. For frequencies lower than 1.9 kHz, the filled shell gives in general higher pressure levels with a maximum difference of 20 $dB$ re $1\mu Pa$. For frequencies above the 1.9 kHz the empty shell gives in general higher pressures with a maximum difference of about 15 $dB$ re $1\mu Pa$. By this comparison we aim only at showing that the inner fluid can influence the pressure levels at the exterior fluid domain considerably, since differences of 15-20 $dB$ re $1\mu Pa$ are not small.

**Influence of the intermodal coupling**

Extensive studies have been carried out in the past for the determination of the influence of mutual impedances on the radiated power levels in the exterior fluid domain. The majority of those studies though was limited to the case of infinite fluid domains. In our case, the fluid domain is bounded and therefore it would be interesting to check to what extent the intermodal coupling influences the radiation at the exterior fluid. We will examine the influence of intermodal coupling for two separate cases: a shell vibrating in air (*light fluid*) and a shell vibrating in water (*heavy fluid*). The applied force and soil properties are the same as in the previous case. We limit ourselves to the case of the empty shell.

The results of the analysis for the shell vibrating in water (see Fig.5) indicate that the maximum pressure levels calculated with the two methods can differ significantly. For the lower frequency range and up to 1.6 kHz, the intermodal coupling does not influence the pressure levels considerably. For frequencies between 1.6-2.0 kHz the modal cross-coupling tends to lower the maximum pressure levels considerably. For higher frequencies the difference between the two curves is about 2-7 $dB$ re $1\mu Pa$. When modal cross-coupling is excluded the input modal energy is dissipated and radiated by the same shell mode. On the
contrary, when modal cross-coupling is included the input modal energy is dissipated, radiated but also transmitted to other shell modes via the mutual impedances. This energy redistribution among different shell modes tends to weaken the pressure levels. To conclude with, we can say that the intermodal coupling should be considered when a shell vibrates in a heavy fluid if meaningful results are to be obtained.

The results of the analysis for the shell vibrating in air (see Fig.6) indicate that the maximum pressure levels are identical for both analyses. This confirms our physical expectation, i.e. when a shell vibrates in a light fluid the intermodal coupling can be completely neglected.
In this section the influence of the chosen soil parameters is discussed. We start with the hypothetical case where the dashpot coefficients of the soil can be estimated within certain confidence limits, i.e. \((10^{-2} \times \text{reference value}) \div (10^{2} \times \text{reference value})\), whereas the spring coefficients can be determined with high accuracy. In other words, we start with the examination of the influence of the dashpot coefficients alone.

In Fig. 7 the maximum pressure amplitude is shown for five different cases of varying dashpot coefficients. The influence of the dashpot coefficients is shown to be frequency dependent, i.e. the increase in the pressure amplitude is neither uniform nor proportional throughout the frequency range to the decrease of the magnitude of the dashpot coefficients. In general it can be seen that an increase in the value of the dashpot coefficients results in a decrease of the maximum pressure amplitudes. An increase of the dashpot coefficients by a factor of 100 reduces the maximum pressure levels by as much as 50 dB re 1\(\mu\)Pa for certain excitation frequencies. This general trend can be explained from the fact that a larger portion of the input energy is absorbed by the soil and consequently a smaller portion irradiates in the form of pressure waves at the exterior fluid domain.

![Fig. 7 Maximum pressure levels for constant spring coefficients and varying dashpot coefficients](image)

We proceed now with the case where there is a large uncertainty in the estimation of the spring coefficients. In Fig. 8, the maximum pressure amplitude is shown for four different cases with varying values of the spring coefficients. One can notice once more that the variation of the spring coefficients influences the maximum pressure levels. Characteristic is the case where the spring stiffness increases by a factor of \(10^{3}\). In this particular case, the pressure levels vary up to 50 dB re 1\(\mu\)Pa when compared to the reference case.

In contrast to our approach, where the soil is substituted by uncoupled elastic springs and viscous dampers in all directions, a common modelling approach widely adopted in practice consists of the substitution of the soil by a fluid medium. This offers considerable advantages for Finite Element simulations since non-reflecting boundaries can be constructed relatively easy to account for the radiation condition at infinity. On the contrary, such a description
neglects very basic characteristics of the soil medium, as for example its capacity to provide shear resistance and therefore to support shear waves too. With the present work we tried to show that the soil parameters and, consequently the soil description itself, can greatly influence the generated pressure levels in the exterior fluid domain. Therefore careful choice is needed with regard to the modelling technique used for the soil simulation.

Fig. 8 Maximum pressure levels for varying spring coefficients and constant dashpot coefficients

**PRESSURE PLOTS FROM A REAL CASE**

**Pressure levels in the frequency domain**

Until now we have examined the influence of a number of parameters on the underwater pressure levels. In all the previous cases, the applied force was a fictitious one. In this chapter we will examine the response of the system to a force taken from a real case. For such a force, the Fourier transformation will be complex-valued carrying both amplitude and phase information. Consequently, we will be able to trace back the response in the real time domain by superimposing the various harmonics with their correspondent phase delay.

In Fig. 9, the maximum pressure levels as well as the ones calculated at two different distances from the shell surface are shown. As we can see, the assumption we made previously that the maximum pressure levels can be representative of the pressure levels elsewhere in the fluid domain was indeed correct. The maximum pressure levels decrease rapidly for the first 5 meters from the surface of the shell, whereas for larger distances the decrease is smaller. Note also that the decrease in amplitude is frequency dependent.

In Fig. 10 to 12, the pressure levels are shown for three different excitation frequencies. The vertical axis of each graph shows the depth of the fluid starting from the sea surface and the horizontal axis shows the radial distance from the surface of the pile. The different colours correspond to different pressure levels in dB re 1µPa. The pressure levels are calculated in the
three-dimensional space. Here the results for the plane at a circumferential angle $\theta = 0^\circ$ are shown.

![Graph showing pressure levels at different distances from the shell surface for a depth of 2 meters below the sea surface](image)

**Fig. 9** Pressure levels at different distances from the shell surface for a depth of 2 meters below the sea surface

The results of the pressure plots are fully intuitive. The pressure field for very large wavelengths of the vibrating structure consists of almost radially spreading cylindrical fronts of varying amplitude with depth as shown in Fig. 10. As we move towards higher frequencies, the pressure waves propagate in the form of cones which after continuous reflections at the top and bottom surfaces result at a pressure field as shown in Fig. 11. Once the wavelengths of the vibrating structure become very small, the pressure field tends towards a standing wave.

![Graph showing pressure levels plot for $f = 267$ Hz](image)

**Fig. 10** Pressure levels plot for $f = 267$ Hz
at the proximity of the shell surface and the pressure levels reduce rapidly as we move away from the source.

**Evolution of the pressure field with time**

The actual pressure field in the time domain is evaluated with the use of an Inverse Fourier transformation of the complex amplitudes obtained in the frequency domain. For example, once the complex amplitude \( \bar{p}_2(r, \theta, z, \omega) \) is evaluated from (27), after substitution of \( R = r \) into the Hankel function at the numerator, the inverse transformation yields the required pressure field in the time domain:
\[ p_2(r, \theta, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{p}_2(r, \theta, z, \omega) \cdot \exp(i \omega \cdot t) \, d\omega \] (37)

The evolution of the generated pressure field with time for a radial distance up to 10 m is shown in Fig. 13. The time step between the different pictures is taken equal to 0.4 msec. During this time interval the waves propagating downwards at the shell can cover approximately a distance equal to 2 m. The regions with red colour are regions of compression in the fluid medium whereas the regions with blue colour are regions of rarefaction. White colour corresponds to very high levels of compression.

Fig. 13 Pressure field for different time steps starting from 2 msec (upper-left) until 4.4 msec (lower-right)
When the surface of the shell is displaced towards the fluid, the fluid is compressed. This region of compression is followed by a region of rarefaction. The development of this phenomenon can be clearly seen in the shown pictures.

Note that pressure levels which are higher than $2 \cdot 10^5$ Pa were calculated close to the pile surface. It is worth mentioning that the very high values of pressure decay rapidly as one moves away from the pile. A decrease of the pressure amplitude by a factor of 10 is not uncommon for distances larger than 10 meters from the pile surface. As we have already discussed, the field generated by the higher-frequency components decays rapidly with distance from the pile surface, see Fig. 11, and therefore only the low-medium frequency components carry energy for larger distances away from the pile. Similar pressure levels ($4 \cdot 10^4$ Pa) where recorded at a distance equal to 5 m from hydrophones positioned in the water at two different depths. This is in total agreement with the results of the present analysis.

Although the near-pressure field is quite complex, the Mach-cones developed from the compressional waves travelling with supersonic speed downwards at the shell’s surface can easily be distinguished. Mach-cones are formed with an angle of $16.8^0$ with respect to the pile. This phenomenon is also observed in other studies (G. Reinhall and H. Dahl, 2011) and is well known in the field of wave dynamics.

CONCLUSIONS

In this paper an attempt is made to develop a computationally inexpensive, yet realistic model for estimating the levels of underwater noise from offshore pile driving. The model incorporates all major parts of the system: the hydraulic hammer is substituted by a force, the pile is described as a thin circular cylindrical shell, the water is described as a compressible fluid medium and the water-saturated seabed is substituted by uncoupled springs and dashpots in all directions. The solution of the coupled vibro-acoustic problem is based on the expansion of the response of the complete system on the modal basis of the in vacuo shell structure. Modes of different circumferential order remain uncoupled due to the axial symmetry of the system. On the contrary, all modes of a specified circumferential configuration are coupled via the mutual impedances of the fluid.

With a simple example, it has been shown that the intermodal coupling should be considered in cases where the shell vibrates in a heavy fluid. For the case of a light fluid the intermodal coupling can be neglected. It is also shown that the presence of the inner fluid affects the generated pressure field in the exterior fluid. In contrast to other works where the soil is modelled as a fluid, we introduced here the concept of elastic springs and viscous dampers for the soil description. Although very advantageous from a practical point of view, such a description introduces a serious challenge of a realistic estimation of the equivalent spring and dashpot coefficients. These coefficients will generally depend on the excitation frequency, the geometry of the soil-pile interface, the geometrical boundaries of the examined system, the mode of oscillation and the stress-strain characteristics of the soil. It is shown that the pressure levels can be greatly affected by the choice of these coefficients. The influence is shown to be frequency dependent, i.e. the increase/decrease in the pressure amplitude is not proportional to the decrease/increase of the magnitude of the dashpot coefficients. The results indicate that more investigation is needed with respect to the contribution of the soil in the overall response of the immersed system, a contribution which is very often overlooked in practice.
The present model allows the substitution of the rigid boundary condition at the seabed level with a generalized impedance boundary characterized by a complex soil impedance \( Z_{\text{soil}}(\omega) \). In cases where the ratio of the soil to the fluid impedance is much larger than unity, i.e. \( Z_{\text{soil}}(\omega)/Z_{\text{fluid}}(\omega) >> 1 \), the boundary condition will tend asymptotically to the rigid boundary case. The results presented here are thus valid only in the case of relatively stiff soil configurations.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the contribution of IHC Merwede which provided valuable sets of data during this process.

REFERENCES


APPENDIX A

Constitutive relations

\[ T_z = \frac{2 \cdot E \cdot h}{1 - v^2} (\varepsilon_z + v \cdot \varepsilon_\theta), \quad T_\theta = \frac{2 \cdot E \cdot h}{1 - v^2} (\varepsilon_\theta + v \cdot \varepsilon_z), \quad S_{z\theta} = \frac{E \cdot h}{1 + v} \cdot \varepsilon_{z\theta}. \]

\[ G_z = \frac{2 \cdot E \cdot h^3}{3 \cdot (1 - v^2)} (\kappa_z + v \cdot \kappa_\theta), \quad G_\theta = \frac{2 \cdot E \cdot h^3}{3 \cdot (1 - v^2)} (\kappa_\theta + v \cdot \kappa_z), \quad H_{z\theta} = \frac{2 \cdot E \cdot h^3}{3 \cdot (1 + v)} \cdot \kappa_{z\theta}. \]

Geometrical relations

\[ \varepsilon_z = \frac{\partial u_z}{\partial z}, \quad \varepsilon_\theta = \frac{1}{R} \frac{\partial u_z}{\partial \theta} + \frac{u_z}{R}, \quad \varepsilon_{z\theta} = \frac{1}{R} \frac{\partial u_z}{\partial \theta} + \frac{\partial u_z}{\partial z}, \quad \gamma_z = \frac{\partial u_z}{\partial z}, \]

\[ \gamma_\theta = \frac{1}{R} \left( \frac{\partial u_z}{\partial \theta} - u_\theta \right), \quad \kappa_z = - \frac{\partial^2 u_z}{\partial z^2}, \quad \kappa_\theta = - \frac{1}{R} \left( \frac{\partial^2 u_z}{\partial \theta^2} - \frac{\partial u_z}{\partial z} \right), \quad \kappa_{z\theta} = \frac{1}{R} \left( - \frac{\partial^2 u_z}{\partial \theta \partial z} + \frac{\partial u_z}{\partial z} \right). \]

Modified inertia terms

\[ I_{u \sigma u\sigma} = 2 \cdot \rho \cdot h \left[ \frac{\partial^2 u_\sigma}{\partial t^2} + \frac{1}{3} \frac{h^2 \cdot v^2 \cdot \left( R \cdot \frac{\partial^6 u_\sigma}{\partial z^2 \partial t^4} + R \cdot \frac{\partial^4 u_\sigma}{\partial z \partial \theta \partial t^3} \right)}{(1 - v^2) \cdot R} \right] + \frac{2}{45 \cdot (1 - v)^3 \cdot R \cdot E}. \]

\[ = \left( h^4 \cdot v^2 \cdot (3 - 5 \cdot v - v^2) \right) \left( R \cdot \frac{\partial^6 u_\sigma}{\partial z^2 \partial t^4} + R \cdot \frac{\partial^4 u_\sigma}{\partial z \partial \theta \partial t^3} \right) \cdot (1 + v) \cdot \rho + \frac{1}{315 \cdot (1 - v)^3 \cdot R \cdot E^2}. \]

\[ I_{u \kappa u\kappa} = 2 \cdot \rho \cdot h \left[ \frac{\partial^2 u_\kappa}{\partial t^2} + \frac{1}{3} \frac{h^2 \cdot v^2 \cdot \left( R \cdot \frac{\partial^6 u_\kappa}{\partial \theta^2 \partial t^4} + R \cdot \frac{\partial^4 u_\kappa}{\partial z \partial \theta \partial t^3} \right)}{(1 - v^2) \cdot R} \right] + \frac{2}{45 \cdot (1 - v)^3 \cdot R \cdot E}. \]

\[ = \left( h^4 \cdot v^2 \cdot (3 - 5 \cdot v - v^2) \right) \left( \frac{\partial^6 u_\kappa}{\partial \theta^2 \partial t^4} + \frac{\partial^4 u_\kappa}{\partial z \partial \theta \partial t^3} \right) \cdot (1 + v) \cdot \rho + \frac{1}{315 \cdot (1 - v)^3 \cdot R \cdot E^2}. \]

\[ I_{u v u v} = 2 \cdot \rho \cdot h \left[ \frac{\partial^2 u v}{\partial t^2} + \frac{1}{3} \frac{h^2 \cdot (7v - 17) \cdot \left( R \cdot \frac{\partial^6 u v}{\partial z^2 \partial t^4} + R \cdot \frac{\partial^4 u v}{\partial z \partial \theta \partial t^3} \right)}{(1 - v)^2 \cdot R} \right] + \frac{2}{1050 - 1050v} \cdot \frac{h^2 \cdot (1 + v) \cdot \rho}{E}. \]

\[ \frac{\partial^2 u v}{\partial t^2} + \frac{1}{3} \frac{h^2 \cdot (1 + v) \cdot \rho \cdot (32 - 96 \cdot v + 261 \cdot v^2 - 197 \cdot v^3)}{7875 \cdot (1 - v)^2 \cdot R \cdot E} \cdot \left( \frac{R \cdot \frac{\partial^6 u v}{\partial z^2 \partial t^4} + R \cdot \frac{\partial^4 u v}{\partial z \partial \theta \partial t^3}}{E} \right) \]