MOUND BREAKWATERS
UNDER WAVE ATTACK

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INTRODUCTION

Until well into the 60's the only possible option to calculate a rubble mound breakwater was to use the deterministic classical formulae and to carry out experiments with regular waves. At that time laboratories began to incorporate techniques for the generation of irregular waves in their experiments regarding breakwater stability. This new technique was not accompanied by a critical analysis of results; specially in the wide variation which must be expected as a consequence of the randomness of the structural response and wave attack.

It is common practise in laboratory reports to say that a specific section of the breakwater is stable under the action of a sea state defined by certain determined parameters. This conclusion has normally been obtained after the carrying out, in the best cases, of three experiments keeping the same sea state parameters.

It is clear that these results, obtained from a very small number of tests on an infinite sample space, does not provide guarantees of stability of the construction under the action of this same sea state and even less generalisation of conclusions. This state of the art will continue to exist, on the other hand, for as long as causes of the phenomenon are unknown.

The attitude adopted by the designer of rubblemound breakwaters when choosing among existing calculus methods, should be compatible with the actual phenomenon, bearing in mind the errors and imperfections of the method that he uses and attempting to evaluate the degree of uncertainty in his results.

We do not intend here to recommend the use of any one specific calculus method. This decision is the designer's priority. However,
in view of the most recent research and results of this study, it appears reasonable to emphasise the high failure risk which exists calculating rubblemound breakwaters with the deterministic classical formulae.

Bruun (1979) has synthesised possible causes of failure or breaking in rubblemound breakwaters. Localization in the construction of the eleven possible causes of failure can be seen in figure 1.1. Then eleven possible resulting damage types are the following:

1) Knock-outs of single main layer units due to wave action. These knock-outs can cause breaking in the armour units with a consequent weight loss and, therefore, jeopardise stability.

2) Lift-outs of main layer armour units by wave action, this being the commonly considered damage.

3) Slides of the entire main layer as a whole owing to lack of friction with underlying layers. This damage will occur, in general, on very steep slopes.

4) Gradual breakdown or failures due to fatigue because of continuous movements of the main layer armour units without great displacements, this means that small initial damages can be important for the definitive stability of the structure.

5) Scouring of the crown base or wave screen.

6) Damages in the inner layer (protected zone) due to overtop of the breakwater and overwash onto it.

7) Damage due to a lack of compactness in underlying layers which allows water to flow inside, causing great lift-out forces on the crown and inner layers.

8) Toe erosion scouring of the lower part of the main layer.

9) Soil failures due to the low bearing capacity area.
Fig. I.1. - Location of failures in a rubble mound breakwater. Bruun (1979)
10) Discrepancies in the characteristics of materials used in construction. When quarry stones are used fundamentally, what can occur is that the armour units are of different sizes, specific weights, etc.

11) Failure due to poor construction, which can leave weak points in the structure.

It is obvious that there is no reason why these types of damage should occur in isolation, but that several of them could arise at the same time.

The subject to be discussed below, is flow characteristics on the slope and its structural stability from the point of view of damage 2, although damage types 1 and 4 will also be considered.

The analysis method to be followed consists of the following steps:

1) Analysis of the idealised model of the rubble mound breakwater (undefined slope) under regular wave action. This analysis will be basically on an empirical nature, being based on experimental laboratory results.

2) Application of regression techniques in order to obtain some behaviour models.

3) Determination of interaction curves, which quantify the phenomenon in a diagramme (H,T).

4) Application of the hypothesis of equivalence between regular waves and irregular waves.

5) Probabilistic analysis of the phenomena when the wave attack is a sea state.
INTRODUCTION

Use of Iribarren's number, Iribarren and Nogales (1949), in the analysis of flow characteristics on sloping structures, was introduced by Battjes (1974a and b), who applied it to the case of flat slopes.

Ahrens and McCartney (1975) verify, by means of experimental results, the usefulness of Iribarren's number to describe run-up and stability on rough, permeable slopes, and furthermore propose a fit model.

This line of work was later followed by Bruun and Günbak (1976, 1977, and 1978) and Bruun and Johannesson (1977). Losada and Giménez-Curto, (1979a), propose an exponential-type model, as a function of Iribarren's number, for stability of rubble mound breakwaters, introducing, on the other hand, interaction curves for direct analysis of stability as a function of wave height and period.

In this chapter an exponential model which depends upon Iribarren's number is presented, for analysis of flow characteristics -run-up, run-down, reflexion and transmission- on rough, permeable slopes under regular wave action. Also interaction curves are defined, which quantify the phenomenon as a function of wave height and wave period.

Analysis of flow characteristics on sloping structures under irregular wave action has been carried out following a semi-empirical or an empirical line by Saville (1962), Van Oorschot and d'Angremond (1968), Battjes (1974a), Bruun and Johannesson (1977), Bruun and Günbak (1978), Kamphuis and Mohamed (1978).

By assuming the hypothesis of equivalence, introduced by Saville (1962), and using the interaction curves, a new probabilistic approach to flow characteristics on sloping structures under a sea state is obtained in this work.
FLOW CHARACTERISTICS UNDER REGULAR WAVES

When a regular wave train reaches an undefined slope the following physical phenomena are produced: breaking, run-up and run-down, reflection and transmission. Taking $Z$ as a variable to measure the phenomenon, it would be a function of the following parameters:

1. Parameters of the medium
   - Depth at the toe of the slope, $d$
   - Bottom slope, $\theta$
   - Specific weight of the water, $\gamma_w$
   - Acceleration due to gravity, $g$
   - Kinematic viscosity, $\mu$

2. Parameters of incident waves
   Given that we are dealing with regular waves, these will be:
   - Wave height, $H$
   - Wave period, $T$
   - Wave approach angle, $\theta$

3. Parameters of the structure
   a) Geometry. It is defined by two parameters:
      - Slope angle, $\alpha$
      - A characteristic width, $\lambda$
   b) Characteristics of roughness permeability, which will depend on the following parameters:
      - Type of armour units of the main layer
      - The way in which they have been placed on the slope
Size of the armour units, which may be characterised by the length of the side of the equivalent cube, \( l = \frac{3W}{\gamma_r} \), \( W \) being the weight of the armour units and \( \gamma_r \) their specific weight.

- Characteristics of the underlying layers.

Regarding rubblemound breakwaters designed according to traditional criteria, it is acceptable to say that the mean characteristics of roughness and permeability depend only on the type of the armour units and their size.

From experiments described by the Technical Advisory Committee on Protection Against Inundation (1974), we may conclude that when incident wave height is considerably greater than the side of the equivalent cube the magnitude of the physical phenomena associated with the breaking of the wave on the slope are independent of the height of roughness and therefore of the size of the blocks.

On the other hand, when the wave breaks on smooth, impermeable slopes, experiments confirm that depth does not influence run-up, Saville (1956), Battjes and Roos (1975) and Hunt (1959), nor the reflection coefficient, Moraes, (1970).

The aforementioned considerations suggest that flow characteristics on a rough, undefined slope under the action of a regular wave train can be approximately governed by an expression of the type:

\[
f(z, \text{type of armour unit}; \alpha, \lambda, \gamma_w, \mu, g, H, T, \theta) = 0 \quad (1)
\]

Taking the following dimensionless monomials:

- \( z \): generic dimensionless variable expressing flow characteristics
- \( \alpha \)
- \( \theta \)
- \( H/L_0 = 2\pi H/gT^2 \)
- \( H^2/\mu T \)
- \( \lambda/H \)
expression (1) remains, for each type of armour unit:

\[ z = f(\alpha, \Theta, \frac{H}{L_0}, \frac{H^2}{\mu T}, \frac{\lambda}{H}) \] (2)

According to Battjes (1974b), Reynolds' number \( \frac{H^2}{\mu T} \) holds, for smooth slopes, normally over a minimum threshold above which it has no influence, and assuming perpendicular incidence (\( \Theta = 0 \)) and neglecting the variable \( \frac{\lambda}{H} \) (relative width of the breakwater) which only has appreciable influence on the phenomenon of transmission, expression (2) can be further reduced to:

\[ z = f(\alpha, \frac{H}{L_0}) \] (3)

As is shown below, this function can be reduced to a simpler one of a single variable:

\[ z = f(Ir) \] (4)

\( Ir = \tan \alpha / \sqrt{\frac{H}{L_0}} \) being Iribarren's number. This function holds for each type of armour unit and perpendicular incidence.

**Breaking**

In classical literature four types of breaking are defined, as points of reference, Iversen (1952), Patrick and Wiegel (1954), Galvin (1968): surging, collapsing, plunging and spilling.

Iribarren's number, which originated as an indication of whether or not wave breaking occurred on a flat slope, Iribarren and Nogales (1949), produces not only this information but also how the wave breaks, Battjes (1974a, 1974b).

Table 1.1 shows the results of Günbak (1976) on a rough slope, with \( \cot \alpha = 2.5 \).
TABLE 1.1

Breaking on rip-rap slope (Günbak 1976).

<table>
<thead>
<tr>
<th>TYPE OF BREAKING</th>
<th>IRIBARREN NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spilling</td>
<td>—</td>
</tr>
<tr>
<td>Plunging</td>
<td>$Ir &lt; 2.0$</td>
</tr>
<tr>
<td>Plunging or collapsing</td>
<td>$2.0 &lt; Ir &lt; 2.6$</td>
</tr>
<tr>
<td>Collapsing or surging</td>
<td>$2.6 &lt; Ir &lt; 3.1$</td>
</tr>
<tr>
<td>Surging</td>
<td>$3.1 &lt; Ir$</td>
</tr>
</tbody>
</table>
In figure 1.1 the variables $Z = Ru$ and $Z = Rd$ are defined, these being the maximum and minimum levels referring to the S.W.L. reached by the water on the slope.

With reference to equation (3) and taking $Ru/H$ and $Rd/H$ as the dimensionless variable, we can, as a result, express:

$$Ru/H = f(\alpha, H/L_0^0)$$  \hspace{1cm}  (5a)  
$$Rd/H = f(\alpha, H/L_0^0)$$  \hspace{1cm}  (5b)

Figures 1.2 to 1.7 show, against Iribarren's number, data corresponding to rough, permeable slopes taken from Ahrens and McCartney (1975) and Günbak (1979). All data correspond to perpendicular wave incidence.

These figures also show the exponential model, Giménez-Curto (1979):

$$Ru/H = A \{1 - \exp(B.Ir)\}$$  \hspace{1cm}  (6a)  
$$Rd/H = A \{1 - \exp(B.Ir)\}$$  \hspace{1cm}  (6b)

$A$ and $B$ being fit coefficients calculated by the least squared method.

A generalised correlation coefficient has also been calculated, defined by:

$$\rho^2 = 1 - \sigma_C^2/\sigma_y^2$$  \hspace{1cm}  (7)

where $\sigma_C^2$ and $\sigma_y^2$ are variance with respect to the fitted curve, $y = f(x)$, and with respect to the mean value $\bar{y}$. That
Fig. 1.1.- Definition sketch of wave run-up and run down
Fig. 1.2: Relative run-up versus Iribarren's Number on rip-rap slope. (Experimental data of Ahrens, 1975)
RUN-UP AND RUN-DOWN ON RIP-RAP SLOPE
GÜNBAK'S DATA (1976)
cot \( \alpha = 2.50 \)

Fig. 1.3. - Relative run-up and run-down versus Iribarren's Number on rip-rap slope
(Experimental data of Günbak, 1976)
Fig. 1.4 - Relative run-up and run-down versus Iribarren's Number on quarrystone slope.
(Experimental data of Dai and Kamel, taken from Günbak, 1979)
RUN-UP ON TETRAPODS SLOPE
JACKSON'S DATA (1958)
cot α = 1.25, 1.50, 2.00, 3.00
d/H > 3.00

Fig. 1.5.- Relative run-up versus Iribarren's Number on tetrapods slope.
(Experimental data of Jackson, taken from Günbak, 1979)
RUN-UP AND RUN-DOWN ON DOLOS SLOPE
WALLINGFORD'S DATA (1970)

Δ cot α = 1.50
⊕ cot α = 2.00
* cot α = 3.00
d/H > 1.60

Fig. 1.6 - Relative run-up and run-down versus Iribarren's Number on dolos slope.
(Experimental data of Hydraulics Research Station, Wallingford, taken from Günbak, 1979)
Fig. 1.7.- Relative run-up and run-down versus Iribarren's Number on quadripods slope.
(Experimental data of Dai and Kamel, taken from Günbak, 1979)
Fig. 1.8 - Comparison of relative run-up and run-down for several types of slope
\[ \sigma_c^2 = \frac{1}{N} \sum_{i=1}^{N} (y_i - f(x_i))^2 \] (8)

\[ \sigma_y^2 = \frac{1}{N} \sum_{i=1}^{N} (y_i - \bar{y})^2 \] (9)

and where \((x_i, y_i)\) and \(N\) are the points to be fitted and the number of them, and \(f(x)\) the curve being fitted.

Tables L2 and L3 give the results obtained in the fit.

Figure 1.8 shows the best fit curves together with a curve which represents the trend of these data for smooth, impermeable slopes, based on data of Technical Advisory Committee on Protection against Inundation (1974), Battjes and Roos (1975) and Günbak (1976). It is concluded from this figure that it is not correct to apply a reduction factor depending only on the type of armour unit to the run-up on smooth, impermeable slopes in order to obtain the run-up on rough slopes, as is recommended by the Technical Advisory Committee on Protection against Inundation (1974), PIANC (1976), Stoa (1979).

**Reflection**

Defining the reflection coefficient \(c_r\) as the reflected wave height \(H_r\) and the incident wave height \(H\) ratio, equation (3) is applied for reflection as:

\[ c_r = f(\alpha, H/L_0) \] (10)

Figures 1.9, 1.10 and 1.11 represent reflection data on rough, permeable slopes taken from Günbak (1979), Hydraulics Research Station (1970) and Sollitt and Cross (1972).
REFLEXION ON RIP-RAP SLOPE
GÜNBAK'S DATA (1976)

\[
\triangle \cot \alpha = 1.50 \\
\star \cot \alpha = 2.50
\]

Fig. 1.9.- Reflexion coefficient versus Iribarren's Number on rip-rap.
(Experimental data of Günbak, 1976)
REFLEXION ON QUARRYSTONE SLOPE
SOLLITT & CROSS' DATA (1972)

\[ \cot \alpha = 1.50 \]
\[ d/H > 3.95 \]

**Fig. 1.10.** - Reflexion coefficient versus Iribarren's Number on quarrystone.
(Experimental data of Sollitt and Cross, 1972)
REFLEXION ON DOLOS SLOPE
WALLINGFORD'S DATA (1970)

Iribarren's Number, Ir

Fig. 1.11.- Reflexion coefficient versus Iribarren's Number on dolos.
(Experimental data of Hydraulics Research Station, 1970)
TABLE 1.2

Fit and generalised correlation coefficients of the model defined in equation (6a) for run-up of water on rough, permeable slopes.

<table>
<thead>
<tr>
<th>TYPE OF ARMOUR UNIT</th>
<th>A</th>
<th>B</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rip-rap (Ahrens &amp; McCartney)</td>
<td>1.7887</td>
<td>-0.4552</td>
<td>0.96</td>
</tr>
<tr>
<td>Rip-rap (Günbak)</td>
<td>1.4510</td>
<td>-0.5230</td>
<td>0.81</td>
</tr>
<tr>
<td>Rubble (Dai &amp; Kamel)</td>
<td>1.3698</td>
<td>-0.5964</td>
<td>0.61</td>
</tr>
<tr>
<td>Tetrapods (Jakson)</td>
<td>0.9341</td>
<td>-0.7502</td>
<td>0.74</td>
</tr>
<tr>
<td>Dolos (Wallingford)</td>
<td>1.2158</td>
<td>-0.5675</td>
<td>0.74</td>
</tr>
<tr>
<td>Quadripods (Dai &amp; Kamel)</td>
<td>1.5382</td>
<td>-0.2483</td>
<td>0.86</td>
</tr>
</tbody>
</table>
TABLE 1.3

Fit and generalised correlation coefficients of the model defined in equation (6b) for run-down of water on rough, permeable slopes.

<table>
<thead>
<tr>
<th>TYPE OF ARMOUR UNIT</th>
<th>A</th>
<th>B</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubble (Dai &amp; Kamel)</td>
<td>-0.8523</td>
<td>-0.4256</td>
<td>0.60</td>
</tr>
<tr>
<td>Rip-rap (Günbak)</td>
<td>-6.2204</td>
<td>-0.0398</td>
<td>0.93</td>
</tr>
<tr>
<td>Dolos (Wallingford)</td>
<td>-1.0607</td>
<td>-0.2659</td>
<td>0.83</td>
</tr>
<tr>
<td>Quadripods (Dai &amp; Kamel)</td>
<td>-0.7952</td>
<td>-0.4481</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Fig. 1.12.- Comparison of reflection coefficient for several types of slope.
These figures also show the exponential model, Giménez-Curto (1979):

\[ c_t = A \left\{ 1 - \exp(B \cdot r) \right\} \quad (11) \]

Table 1.4 gives the values of coefficients obtained in the same way as for run-up and run-down. The resulting curves are represented in Figure 1.12 together with a curve which shows the trend of data for smooth, impermeable slopes, Moraes (1970) and Battjes (1974a, 1974b).

**Transmission**

Having defined the transmission coefficient \( c_t \) as the transmitted wave height \( H_t \) and the incident wave height \( H \) ratio, equation (3) is applied for transmission, for a specific breakwater width, as:

\[ c_t = f (c, H/L_o) \quad (12) \]

Figure 1.13 represents the results of transmission experiments carried out by Sollitt and Cross (1972) as well as the exponential model, Giménez-Curto (1979):

\[ c_t = A \left\{ 1 - \exp(B \cdot r) \right\} \quad (13) \]

with the resulting fit coefficients: \( A = 1.64 \), \( B = -0.01278 \) and \( \rho = 0.87 \).

**INTERACTION CURVES**

Interaction curves of each phenomenon analysed are defined as the sets of points on the plane \( (H, T) \) which produce the same quantitative manifestation of the phenomenon, Giménez-Curto (1979).
TRANSMISSION THROUGH
A QUARRYSTONE BREAKWATER
SOLLITT & CROSS' DATA (1972)
cot \alpha = 1.50
d/H = 3.95

Fig. 1.13 - Transmission coefficient for a quarrystone breakwater
(Sollitt and Cross, 1972)
TABLE 1.4

Fit and generalised correlation coefficients of the model defined in equation (11) for the reflection coefficient on rough, permeable slopes.

<table>
<thead>
<tr>
<th>TYPE OF ARMOUR UNIT</th>
<th>A</th>
<th>B</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dolos (Wallingford)</td>
<td>3.9990</td>
<td>-0.0197</td>
<td>0.83</td>
</tr>
<tr>
<td>Rubble (Sollit &amp; Cross)</td>
<td>0.5030</td>
<td>-0.1248</td>
<td>0.70</td>
</tr>
<tr>
<td>Rip-rap (Günbak)</td>
<td>1.3508</td>
<td>-0.0710</td>
<td>0.97</td>
</tr>
</tbody>
</table>
For perpendicular incidence and a specific type of armour unit the exponential model can be used:

\[ Z = AH \left\{ 1 - \exp(B \cdot Ir) \right\} \quad (14) \]

and expressing Iribarren's number as:

\[ Ir = \sqrt{g/2\pi} \tan \alpha T/\sqrt{H} \quad (15) \]

a function of type:

\[ f(Z, H, T, \alpha) = 0 \quad (16) \]

can be obtained, on elimination of \( Ir \). Thus allowing representation of the curves \( Z = \text{cte} \) on the plane \((H, T)\) for each slope.

Figures 1.14 and 1.15 show some examples of interaction - curves of breaking and run-down phenomena. These curves are limited by the curve which defines the wave stability limit. Maximum value of the wave slope has been taken as \((H/L)_{\text{max}} = 0.142 \) which corresponds to \( Ir^0 = 2.654 \tan \alpha \).

FLOW CHARACTERISTICS UNDER IRREGULAR WAVES

In the case of a sea state the variables type of breaking, \( Ru, Rd, H_f, H_t \) may be considered random variables which acquire a value for every wave of the sea state.

The distribution function of these random variables may be obtained by assigning to each individual, irregular wave the same phenomenon value which would be produced by a periodic - wave train of the same height and the same period.
Fig. 1.14: An example of interaction diagram of breaking
Fig. 1.15 - An example of interaction curves of wave run-down
It is important to note the statistical nature of this hypothesis, which does not necessarily imply that each individual wave produces the same phenomenon manifestation as the equivalent regular wave train, but is less restrictive; it refers to averages of many values rather than to individual values.

This hypothesis, known as the hypothesis of equivalence, was introduced by Saville (1962) and was empirically proven by Van Oorschot and d'Angremond (1968) and Battjes (1974a) for run-up on smooth slopes, and by Bruun and Johannesson (1977) and Bruun and Günbak (1978) for run-up on rough, permeable slopes.

Taking into account that the distribution function of \( Z \) is defined by:

\[
F_Z(x) = \text{Prob}[Z \leq x]
\]  

and that \( Z = x \) is an interaction curve, the result is:

\[
F_Z(x) = \int_D p^*(H,T) \, dH \, dT
\]  

where the integration domain, \( D \), is the shaded area in Figure 1.16 and \( p^*(H,T) \) is the joint probability density function of wave heights and periods in the place occupied by the slope.

Thus, knowing the joint probability density function of wave heights and periods and interaction curves, by means of numeric integration it is possible to obtain the distribution of type of breaking, \( Ru, Rd, H_t \) of \( H_t \) in a sea state.

Figures 1.17 and 1.18 show examples of distribution functions of type of breaking and \( H_t \).

It has been assumed that:

\[
p^*(H,T) = \frac{p(H,T)}{\int_0^\infty \int_0^{H_p} p(H,T) \, dH} \]  

\[
\int_0^\infty \int_0^{H_p} p(H,T) \, dH
\]
Fig. 1.16. – Definition sketch of the integration domain in eq. (18)
Fig. 1.17. - Example of distribution of type breaking. Rip-rap slope, \( \cot \alpha = 3.00 \), horizontal bed with \( d = 10.0 \text{ m} \).
Fig. 1.18.- Example of distribution of reflected wave height.

Dolos slope, cota = 2.00, horizontal bed with d=10.0 m.
where \( p(H,T) \) is a theoretical distribution, Longuet-Higgins (1975), (L.H.), Cavanié, Arhan and Ezraty (1976), (C.A.E.), and \( H_b \) is breaking-wave height defined by the breaking limit curve. This curve has been defined in the examples in accordance with Mitchell (1893), Miche (1944), Goda (1970) and Reid and Bretschneider (1953) as follows:

\[
H_b = 0.142 \frac{gT^2}{2\pi} \quad \text{Deep water} \quad \frac{d}{L} > 0.5 \quad (20)
\]

\[
H_b = 0.142 L \tanh \left( \frac{2\pi d}{L} \right) \quad \text{Transitional water} \quad 0.1 \leq \frac{d}{L} < 0.5 \quad (21)
\]

\[
H_b = \frac{bd}{1+ad/gT^2} \quad \text{Shallow water} \quad \frac{d}{L} < 0.1 \quad (22)
\]

where \( L \) is the wavelength and \( a \) and \( b \) are coefficients defined by (Goda, 1970)

\[
a = 43.75 \{ 1 - \exp(-19m) \} \quad (23)
\]

\[
b = \frac{1.56}{1+\exp(-19.5m)} \quad (24)
\]

\( m = \tan \beta \) being bottom slope.

Distribution of the maximum value of variables \( R_u, R_d, H_r, H_t \) in a sea state may be obtained by accepting statistical independence among the successive values acquired by each of these variables within the sea state. Note that this hypothesis does not necessarily imply statistical independence among successive wave heights and periods.

Taking \( F_M(z) \) as the distribution function of maximum value for \( Z \) in the sea state, it follows that:

\[
F_M(z) = \left( F_z(x) \right)^N \quad (25)
\]

\( N \) being the number of waves in the sea state.
**TABLE 1.5**

Data of experiments from Kamphuis and Mohamed, 1978, on smooth, impermeable slopes and $d = 0.90$ m.

<table>
<thead>
<tr>
<th>CASE</th>
<th>SPECTRUM</th>
<th>$H_s$ (m)</th>
<th>$T_p$ (sg)</th>
<th>$T_z$ (sg)</th>
<th>$\text{ctg} \alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Bretschneider</td>
<td>0.0528</td>
<td>1.59</td>
<td>1.30</td>
<td>1.50</td>
</tr>
<tr>
<td>B</td>
<td>Bretschneider</td>
<td>0.0755</td>
<td>1.43</td>
<td>1.20</td>
<td>2.00</td>
</tr>
</tbody>
</table>
Equation (25) shows the influence of duration on maximum values, Losada and Giménez-Curto (1979b).

An experimental comparison

The probabilistic method of analysis described here has been compared with experiments carried out by Kamphuis and Mohamed (1978) regarding run-up on smooth, impermeable slopes. Two cases have been chosen, details of which are given in table 1.5, in which \( H_s \) is the significant wave height, \( T_z \) the mean zero-upcrossing period and \( T_p \) the spectral peak period.

The theoretical method has been applied with the following criteria:

a) run-up on smooth, impermeable slopes is represented by the following model:

\[
\begin{align*}
    \frac{R_u}{H} &= I_r & \text{for } I_r &< 2.5 \\
    \frac{R_u}{H} &= 2.5 - \frac{(I_r-2.5)}{3} & \text{for } 2.5 &< I_r &< 4.0 \\
    \frac{R_u}{H} &= 2.0 & \text{for } 4.0 &\leq I_r
\end{align*}
\]  

which is based on data from the Technical Advisory Committee of Protection Against Inundation (1974), Battjes and Roos (1975) and Günbak (1976).

b) On the basis of this model, interaction curves have been obtained which, together with the breaking limit, are shown in figures 1.19 and 1.20.

c) Using equations (18) and (19) with joint distributions of wave heights and periods of Bretschneider (1959), Longuet-Higgins (1975) and Cavanié, Arhan and Ezraty (1976), run-up distribution functions have been obtained.

Parameters of joint distributions of wave heights and period obtained from the spectral moments, are given in Table 1.6.

Figures 1.21 and 1.22 represent the values obtained in the experiments and the results obtained by the method described above, together with Rayleigh distribution, for cases A and B respectively.
Fig. 1.19 - Interaction curves of run-up on smooth, impermeable slope based on eqs. (26). \( \cot \alpha = 1.50 \)
Fig. 1.20 - Interaction curves of run-up on smooth, impermeable slope based on eqs. (26). \( \cot \alpha = 2.00 \)
TABLE 1.6

Values of parameters used in distributions for experimental comparison.

<table>
<thead>
<tr>
<th>CASE</th>
<th>DISTRIBUTION</th>
<th>PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>m₀</td>
</tr>
<tr>
<td>A</td>
<td>Bretschneider</td>
<td>1.74·10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>Longuet-Higgins</td>
<td>1.74·10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>Cavanié, Arhan and Ezraty</td>
<td>1.74·10⁻⁴</td>
</tr>
<tr>
<td>B</td>
<td>Bretschneider</td>
<td>3.55·10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>Longuet-Higgins</td>
<td>3.55·10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>Cavanié, Arhan and Ezraty</td>
<td>3.55·10⁻⁴</td>
</tr>
</tbody>
</table>
Joint distribution \( (H, T) \)

1. Bretschneider
2. Longuet - Higgins
3. Cavanié, Arhan and Ezraty

Fig. 1.21.- Experimental comparison of the probabilistic method described herein (Caso A)
Joint distribution, \((H, T)\)

1. Bretschneider
2. Longuet-Higgins
3. Cavanié, Arhan and Ezraty

Fig. 1.22.- Experimental comparison of the probabilistic method described herein (Case B)
CONCLUSIONS

1) For rough, permeable slopes, flow characteristics under the action of a regular wave train may be represented by a function of the type:

\[ z = A \{1 - \exp(B.Ir)\} \] (27)

2) Run-up on rough, permeable slopes cannot be calculated on the basis of run-up on smooth, impermeable slopes by application of a reduction coefficient that only depends on the type of armour unit.

3) Accepting the hypothesis of equivalence, distribution of flow characteristics in sea state can be obtained on the basis of interaction curves and joint probability density function of wave heights and periods.
CHAPTER 2 MOUND STABILITY

INTRODUCTION

Ahrens and McCartney (1975) introduced the use of Iribarren's number in the analysis of stability of rip-rap slopes.

Bruun and Johannesson (1976) emphasise the influence of the wave period and conclude that Iribarren's number is a determining factor for stability of the main layer of the mound and for flow characteristics.

Bruun and Johannesson (1977) and Bruun and Günbak (1977, 1978) insist on using Iribarren's number in the study of stability and propose using the hypothesis of equivalence for analysis under irregular waves.

Losada and Giménez-Curto (1979a) present an exponential model as a function of Iribarren's number to represent rubblemound breakwater stability under regular waves, introducing, on the other hand, interaction curves for analysis of stability as a function of wave height and period.

Finally, Losada and Giménez-Curto (1979b) propose a method for evaluating failure probability in marine constructions under a sea state, this being based on the interaction curve and on the hypothesis of equivalence, and also analyse the influence of sea state duration.

This chapter presents all of these ideas, indicating in the first place a general statement of the problem under regular waves, which leads to the concept of stability function. Subsequently -
this function is calculated by means of the model introduced by Losada and Giménez-Curto (1979a) and finally the problem of stability under irregular waves is dealt with by way of a probabilistic approximation based on interaction curves and the hypothesis of equivalence.

STABILITY UNDER REGULAR WAVES

General statement

This considers the problem of stability of main layer armour units of an undefined mound under regular wave action.

This chapter deals almost exclusively with the type of damage (2) of those considered by Bruun (1979), that is, lift-outs of armour units caused by wave action. The following variables play a role, initially, in this phenomenon:

1. Parameters of the Medium: \( d, \beta, \gamma, \rho \) (see chapter 1)
2. Parameters of incident waves: \( H, T, \theta \) (see chapter 1)
3. Parameters of the Structure:
   - slope angle \( \alpha \)
   - weight of the armour units \( W \)
   - specific weight of the armour units \( \gamma_f \)
   - thickness of the armour layer \( e \)
   - interaction forces among armour units (friction and interlocking)
   - characteristics of roughness and permeability of the armour layer
   - characteristics of the underlying layers

Regarding rubble mound breakwaters designed in accordance with traditional criteria, it can be accepted that characteristics of underlying layers depend on those of the main layer (type of armour units, \( W, e \)). On the other hand, one may accept that average
characteristics of friction and interlocking, as well as roughness and permeability, of the armour layer depend on:

- type of armour units
- size
- way of placing them onto the slope

The size of armour units may be characterised by means of the length of the side of the equivalent cube:

\[ L = (W/\gamma_w)^{1/3} \]  

(1)

The thickness of the armour layer tends to be a specific number of times the side of the equivalent cube:

\[ e = nL \]  

(2)

Assuming that depth is moderately great, such that the influence of \( d \) and \( \beta \) can be ignored, and that the way of placing the armour units onto the slope is at random, it turns out that for each type of armour unit stability conditions are governed by a function of the type:

\[ f(\alpha, W, \gamma_f, H, T, \theta, \gamma_w, \nu, g) = 0 \]  

(3)

The equality expressed by this equation means the stability limit, that is, that for the wave conditions defined by \( H \), \( T \) and \( \theta \), and given \( \alpha, \gamma_f, \gamma_w, \nu \) and \( g \), weight \( W \), which satisfies equation (3) is the minimum that the armour units must possess in order to be stable. Therefore, function "f", carries with it, tacitly, a stability criterion (damage or failure criterion).
Taking the following six dimensionless monomials:

\[ \begin{align*}
\alpha &= \frac{W}{\gamma_w H^3} \\
S_T &= \frac{\gamma_T}{\gamma_w} \\
\theta &= \frac{H^2}{\mu T} \\
H/L_0 &= 2\pi \frac{H}{gT^2}
\end{align*} \]

equations (3) remains:

\[ f(\alpha, W/\gamma_w H^3, S_T, \theta, H^2/\mu T, H/L_0) = 0 \quad (4) \]

Accepting, as for flow characteristics, that Reynolds' number, \(H^2/\mu T\), is kept above a minimum threshold so that variations in its value do not significantly affect the resulting phenomenon, and assuming normal incidence of waves (\(\theta = 0\)), equation (4) becomes:

\[ f(\alpha, W/\gamma_w H^3, S_T, H/L_0) = 0 \quad (5) \]

which can also be written as:

\[ W = \gamma_w H^3 f(\alpha, S_T, H/L_0) \quad (6) \]

On the other hand, all theoretical formulae existing at present to calculate the weight of main layer armour units of a rubblemound breakwater, can be written in the following manner (PIANC (1976)):

\[ \frac{W(S - 1)^3}{H^3 \gamma_x} = \psi \quad (7) \]
or else as:

$$W = \gamma W H^3 \frac{S_r}{(S_r - 1)^3} \psi$$

(8)

where $\psi$ is a dimensionless function depending on $\alpha$ in all cases, and on other parameter such as type of armour unit, $H$, $T$, $d$, $L$, friction coefficients and other empirical coefficients, according to the different formulations.

It can be seen that equation (6) has the same formal structure as equation (8). Comparing both expressions we can write:

$$f(\alpha, S_r, H/L_0) = R(S_r) \psi (\alpha, H/L_0)$$

(9)

where function $R$:

$$R = S_r/(S_r - 1)^3$$

(10)

encompasses the effect of $S_r$ (specific relative weight of armour units) in the function $f$, in accordance with the generality of existing theories, PIANC (1976).

Lastly, it is concluded that for each type of armour unit, having established a stability criterion, and in the case of normal incidence of regular waves, function $\psi$ depends solely on $\alpha$ and $H/L_0$ (accepting the aforementioned hypotheses). This function will be known here as the stability function.

The weight that the armour units must possess, $W$, in order to strictly fulfil the criterion which the stability function implicitly carries with it, is expressed, bearing in mind equations (6), (9) and (10), by

$$W = \gamma W H^3 R \psi$$

(11)

In the following paragraphs the stability function is studied, using experimental results of Iribarren, Ahrens and McCartney and Hudson.
Stability criterion

The first problem faced in analysis of stability conditions is the definition of a stability of damage criterion.

A breakwater under specific wave conditions, can be stable or unstable. It is said to be unstable when incident waves produce a loss of armour units on its main layer (damage). The breakwater is stable if the waves are incapable of extracting any armour unit from its main layer.

Traditionally damage has been defined as the percentage of armour units displaced with respect to the total number of armour units used in the construction of the main layer (classical definition).

This definition is inconsistent, given that damage depends therefore on the size of the main layer. If the dimensions of the latter were to be standardised in relation to the size of armour unit the classical definition of damage would be consistent.

One way to avoid the existing inconsistency in the previous criterion would be to define damage as the percentage of displaced armour units with respect to the number of them contained initially in a band of specific width around the S.W.L. (Van de Kreeke (1969), Ouellet (1972), Günbak (1978)).

The main disadvantage of these definitions is that they do not provide clear information about the situation of the breakwater with respect to its total destruction.

Iribarren (1965) proposes a definition of breaking which gives a clear picture of the breakwater's future. We can express his criterion in the following way:

A rubble mound breakwater has reached its breaking level when the depth of damage on its main layer is equal to the length of the side of the equivalent cube L.
This is to say that the first layer of armour units of the main layer has been lifted (and displaced) in an area sufficiently so as to expose at least one armour unit of the second layer of the main layer to direct wave action. Once this situation has been reached the breakwater would be severely damaged and it can be said that its total destruction is merely a question of time. (Note that interlocking of second layer armour units is much lower than that of outer armour units).

In figure 2.1 two examples are shown, from experiments carried out by Iribarren, which correspond to this breaking situation (unpublished data).

To reach this state the waves must exceed a certain threshold. In order to displace an armour unit integrated in the main layer the wave must overcome friction and interlocking existing among armour units.

Generally speaking friction refers to resistance to extraction of a microscopic type, due to roughness of armour units. Interlocking refers to resistance of a macroscopic type, depending upon the shape of the armour units.

Once this threshold has been exceeded, the only resistance which the armour unit can offer against its extraction by waves is its own weight.

Taking this into account it is understood that a main layer designed with a type of armour units which develop a great deal of interlocking among each other, will possess a much higher threshold of the aforementioned nature, and as a consequence will require less weight in order to resist wave action. However if this level is exceeded the armour unit is deprived of the contribution of adjacent armour units and as its weight is low it is easily displaced by wave action. Also if the armour unit breaks into various pieces, each one will weigh even less and they will, as a consequence, be more vulnerable, Magoon and Baird (1977).
Fig. 2.1.- Slope profiles in breakage point. (Iribarren's criterion)
When the waves overcome the friction and interlocking that exists among the armour units of the main layer, the only resistance they can offer is their own weight. Under such conditions they are easily displaced, even by lower waves, so that damage of type 4 can occur, caused by small displacements, Bruun (1979). In these movements some armour units will hit against others and this can give rise to damages of type 1, Bruun (1979). Consequently it seems reasonable to use this threshold in the design, as a level of the damage starting point.

Therefore initiation of damage is defined as the minimum wave height, for constant period, which is capable of overcoming friction and interlocking among the armour units.

When interpreting experimental results the problem arises in distinguishing, once the first armour units fall, which of them were part of the main layer, and therefore have been extracted, and which were not, with a view to finding out whether friction and interlocking have been overcome. For this reason it is necessary to establish a clear criterion.

Bearing in mind these aforementioned points, the initiation of damage situation is defined, in this study, as minimum wave height, for constant period, capable of producing extraction of at least ten per cent of the total number of armour units which are displaced until breaking of the breakwater by Iribarren's criterion is reached.

Regarding experiments by Ahrens and McCartney and Hudson, results of which are also used in this study, there has not been sufficient information available in order to apply the aforementioned criterion, and therefore criteria used by the authors
themselves will be employed. In the case of Ahrens and McCartney that of zero damage, which corresponds to a situation where there has been hardly any stabilisation of armour units that were in very unstable positions in construction, without appreciable armour unit displacement. In the case of Hudson, that of no damage, which corresponds to a maximum displacement of one per cent of the armour units used in construction of the main layer (classical definition).

THE STABILITY FUNCTION

Experimental data

The experimental results used in this study are those obtained by Iribarren (1965), Ahrens and McCartney (1975) and Hudson (1958), taken from Bruun and Johannesson (1977).

All of these tests are with regular waves and perpendicular incidence.

Regarding Iribarren's results, 93 experiments have been selected on the basis of a homogenization, (31 with quarry stones, 40 with parallelopipedic blocks and 22 with tetrapods). These tests were carried out with a wave flume of 31.5 m. long, 1.0 m wide and 1.5 m deep. Characteristics of the model, of waves and experimental technique are given in tables 2.1, 2.2 and 2.3.

Ahrens and McCartney (1975) conducted experiments on stability of rip-rap slopes. These tests were done with a wave flume of 193.55 m. long, 4.57 m wide and 6.10 m deep. The average weight, \( W_{50} \), varies from 12 to 55 Kgs. Armour units used were of diorite with specific weight of 2.71 t/m\(^3\). 48 experiments have been used (16 with \( \cot\alpha = 2.50 \), 19 with \( \cot\alpha = 3.50 \) and 13 with \( \cot\alpha = 5.00 \)). Wave heights vary from 0.55 to 1.83 m. and periods from 2.8 to 11.3 sg. Water depth is 4.57 m.
### TABLE 2.1

**Characteristics of the model in Iribarren's tests**

<table>
<thead>
<tr>
<th>Armour units</th>
<th>Slopes</th>
<th>( P (g) )</th>
<th>( \gamma_r (g/cm^3) )</th>
<th>Thickness of armour layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarry stones</td>
<td>1.50</td>
<td>12.5</td>
<td>2.53</td>
<td>&gt;3( \ell )</td>
</tr>
<tr>
<td>Parallelopiped blocks</td>
<td>1.50</td>
<td>12.5</td>
<td>15.7</td>
<td>2.10</td>
</tr>
<tr>
<td>Tetrapods</td>
<td>1.33</td>
<td>12.5</td>
<td>2.10</td>
<td>2( \ell )</td>
</tr>
</tbody>
</table>

Undefined slope; length of the equivalent cube: \( l = (P/\gamma_r)^{1/3} \); weight of the units of secondary layer \( \sim P/20 \); variable core. \( P \) is the weight of armour units, \( \gamma_r \) is the specific weight of armour units. The weight of the armour units is uniform; it is included in the interval \( P \pm 0.1P \) at a probability level of 95%. The numbers in brackets refer to the number of experiments considered in this work.

### TABLE 2.2

**Characteristics of the waves in Iribarren's tests**

<table>
<thead>
<tr>
<th>Regular waves</th>
<th>Normal incidence</th>
<th>Relative depth (( d/L ))</th>
<th>Depth (( d ))</th>
<th>Wave heights (( H ))</th>
<th>Wave periods (( T ))</th>
<th>Wave steepness (( H/L_\alpha ))</th>
<th>Ratio (( d/H ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.05</td>
<td>0.21</td>
<td>0.03</td>
<td>0.91</td>
<td>0.0015-0.1027</td>
<td>&gt;2.3</td>
</tr>
</tbody>
</table>

\( H, T, L, d \) are the wave height, wave period, wave length and depth at toe of the breakwater. \( L_\alpha \) is the deep-water wave length.

### TABLE 2.3

**Experimental techniques in Iribarren's tests**

<table>
<thead>
<tr>
<th>Objective</th>
<th>Method adopted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration of wave heights</td>
<td>Before fitting the model in the flume a wave absorbing ramp was placed at the end; wave heights were then measured at the model site.</td>
</tr>
<tr>
<td>Control of reflected waves</td>
<td>Tests were conducted with sets of waves, stopping the wave generator before the doubly reflected wave returned to the breakwater. A guillotine was dropped near the model breakwater, so that the higher waves caused by stopping the generator did not affect the model.</td>
</tr>
<tr>
<td>Breakage criterion</td>
<td>The breakwater is considered to be critically damaged when the depth of the damage in the cover layer reaches the length of the equivalent cube ( (P/\gamma_r)^{1/3} ). At this stage the first layer of the cover layer is completely broken and the stability of the breakwater is endangered.</td>
</tr>
<tr>
<td>Standardisation of all tests</td>
<td>Each experiment commences with a wave height sensibly lower than that of the initiation of the fall of armour units. The wave height is increased by small steps until it reaches the breakage point. The model is not rebuilt for each new wave height. For each wave height, ten sets of waves are applied to the model.</td>
</tr>
</tbody>
</table>

If a stable profile has not been established, the series of ten sets is repeated. When close to breakage, 50 sets of waves are applied and, with profiles taken directly, tests are made to see if breakage point has been reached.
Hudson's results have been taken from Bruun and Johanesson (1977), considering 21 experiments all with quarry stones (9 with \( \cot \alpha = 1.50 \), 5 with \( \cot \alpha = 2.0 \) and 7 with \( \cot \alpha = 4.0 \)). Equipment used, characteristics of the model and the waves are very similar to those of Iribarren's experiments except that the latter experimented with a wider range of wave characteristics than Hudson. The greatest difference lie in experimental technique. In Hudson's experiments waves occurring due to stoppage of the generator, which are higher, affected the breakwater, while in Iribarren's, thank to the guillotine, this did not happen. Furthermore the model was reconstructed after the attack of every size of wave. These two differences in experimental technique make Hudson's experiments very much on the safe side in comparison to Iribarren's, as on the one hand the breakwater is struck by some waves which are higher than those considered in the analysis and on the other the stabilising effect that low waves have on the mound is not taken into account.

The fit model

It has already been seen that once a stability criterion has been established, for each type of armour unit and in the case of normal incidence of regular waves, the stability function depends solely on the slope angle \( \alpha \) and on the wave steepness \( \frac{H}{L} \).

\[
\psi = \psi(\alpha, \frac{H}{L}) \quad (12)
\]

The previous chapter shows that flow characteristics on rough, permeable slopes are well represented by Iribarren's number as the
sole parameter:

\[ \text{Ir} = \tan \alpha / \sqrt{H / L_0} \]  \hspace{1cm} (13)

consequently it is reasonable to believe that this parameter must play an important role in rubblemound breakwater stability.

Losada and Giménez-Curto (1979a) propose an exponential model as a function of Iribarren's number for analysis of stability of a rubblemound breakwater under regular waves.

Using this fit model we can write:

\[ \psi = A(\text{Ir} - \text{Ir}_0) \exp\{ B(\text{Ir} - \text{Ir}_0) \} \quad \text{ Ir} \geq \text{Ir}_0 \]  \hspace{1cm} (14)

where

\[ \text{Ir} = \tan \alpha / \sqrt{H / L_0} \]  \hspace{1cm} (13)

\[ \text{Ir}_0 = 2.654 \tan \alpha \]  \hspace{1cm} (15)

A and B are fit coefficients which depend on the type of armour unit and slope angle.

Figures 2.2, 2.3 and 2.4 show the stability function drawn against Iribarren's number for various slope angles and types of armour unit according to Iribarren's experiments. The corresponding values of fit coefficients A and B and of \text{Ir}_0 are given in table 2.4 (Losada and Giménez-Curto (1979a)).

On the other hand figures 2.5 and 2.6 represent the stability function against Iribarren's number for experiments by Ahrens and McCartney and Hudson.

It is interesting to point out the great quantitative differences existing between Hudson's results and the equivalent ones of
Fig. 2.2 - The stability function of quarry stones. Initiation of damage. After Iribarren's data, 1955.

\[ \psi \]

Stability function, \( \psi \)

- \( \cot \theta = 1.50 \)
- \( \cot \theta = 2.00 \)
- \( \cot \theta = 3.00 \)
- \( \cot \theta = 4.00 \)
Fig 2.3 - The stability function of parallelepiped blocks. Initiation of damage.

(After Irribarrin’s data, 1965)
Fig. 2.4.- The stability function of tetrapods. Initiation of damage. (After Iribarren's data, 1965)
Fig. 2.5 - The stability function of rip-rap. Zero damage. (After Ahrens and McCartney's data, 1975)
Fig 2.6 - The stability function of quarry stones. No damage. (After Hudson's data, 1958)

- cota = 4.00
- cota = 3.00
- cota = 1.50
TABLE 2.4

Values of $A$, $B$ and $I_{r0}$. Initiation of damage.
(After Iribarren's data).

<table>
<thead>
<tr>
<th>Armour unit</th>
<th>Cota</th>
<th>$A$</th>
<th>$B$</th>
<th>$I_{r0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarry stones</td>
<td>1.50</td>
<td>0.09035</td>
<td>-0.5879</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>2.00</td>
<td>0.05698</td>
<td>-0.6627</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>3.00</td>
<td>0.04697</td>
<td>-0.8084</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>4.00</td>
<td>0.04412</td>
<td>-0.9339</td>
<td>0.66</td>
</tr>
<tr>
<td>Paralleloipedic</td>
<td>1.50</td>
<td>0.06819</td>
<td>-0.5148</td>
<td>1.77</td>
</tr>
<tr>
<td>blocks</td>
<td>2.00</td>
<td>0.03968</td>
<td>-0.6247</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>3.00</td>
<td>0.03410</td>
<td>-0.7620</td>
<td>0.88</td>
</tr>
<tr>
<td>Tetrapods</td>
<td>1.33</td>
<td>0.03380</td>
<td>-0.3141</td>
<td>1.99</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>0.02788</td>
<td>-0.3993</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>2.00</td>
<td>0.02058</td>
<td>-0.5078</td>
<td>1.33</td>
</tr>
</tbody>
</table>

TABLE 2.5

Values of $A$, $B$ and $I_{r0}$ for rip-rap. Zero damage.
(After Ahrens and McCartney's data).

<table>
<thead>
<tr>
<th>Cota</th>
<th>$A$</th>
<th>$B$</th>
<th>$I_{r0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td>0.1834</td>
<td>-0.5764</td>
<td>1.06</td>
</tr>
<tr>
<td>3.50</td>
<td>0.1819</td>
<td>-0.6592</td>
<td>0.76</td>
</tr>
<tr>
<td>5.00</td>
<td>0.1468</td>
<td>-0.6443</td>
<td>0.53</td>
</tr>
</tbody>
</table>
Iribarren. Hudson's results are very much on the safe side compared with Iribarren's, and even on the safe side regarding those of Ahrens and McCartney, which is very difficult to explain if one takes into account the fact that rip-rap should be less stable than uniform quarry stones, Shore Protection Manual (1977). These differences can be explained, in part at least, by the different experimental criteria used and by the use of different stability criteria.

Tables 2.5 gives the values of A and B and \( I_r \) that are the result of the fit of data from Ahrens and McCartney (Losada and Giménez-Curto, 1979a).

It should be pointed out that the stability function presents a maximum, which corresponds to a minimum in breakwater stability. The values of Iribarren's number produced by the most unfavourable situation is given by:

\[
I_{r\text{crit}} = I_r^{-1/B} \tag{16}
\]

and the maximum value is:

\[
\psi_{\text{max}} = -A/Be \tag{17}
\]

The existence of a stability minimum for specific wave steepness or combinations of wave heights and periods has already been pointed out by Ahrens and McCartney (1975) and shown by Bruun and Günbak (1977, 1978).

Randomness in the structural response. Confidence bands.

In the previous paragraph it was seen that in experimental data corresponding to the stability function variable scatter exists, which in some cases is relatively important. How can this scatter be explained when according to the general statement already esta-
lished, the stability function should depend solely on \( a \) and \( H/L_0 \).

When in the general statement we spoke of characteristics of friction, interlocking, roughness and permeability, considering that they depend solely on the type of armour unit, we referred to average characteristics, given that the situation is variable from one armour unit to another, and every armour unit is found to be in different stability conditions.

What happens is that the behaviour of the main layer, as a discontinuous granular system, introduces a random component into the system (see Price (1979)). Owing to the fact that in the structural response itself there exists an important random component it turns out that stability conditions of main layer armour units are variable from one armour unit to another and from every one after a length of time.

Fit coefficients \( A \) and \( B \) defined in tables 2.4 and 2.5 have been obtained by means of linear regression using the change of variable:

\[
X = I_r - I_{r0} \\
\xi = \ln \left\{ \frac{\psi}{(I_r - I_{r0})} \right\}
\]

through which the fit model defined in equations (13) (14) and (15) is transformed into the straight line:

\[
\xi = mX + n
\]

where

\[
m = B \\
n = \ln A
\]
Scatter existing in experimental results can be accounted for by modifying expression (20) in the following way:

\[ \xi = m \chi + n + S \]  

(23)

where \( S \) is a random variable of zero average.

It seems reasonable to admit that distribution of \( S \) would not depend on wave characteristics but solely on structure, and accordingly its distribution would be the same for any value of \( \chi \).

Assuming on the other hand that \( S \) is gaussian, and estimating its variance \( (S^2) \) by means of experimental results \( (\chi_i, \xi_i) \) according to

\[ S^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\xi_i - m \chi_i - n)^2 \]  

(24)

where \( N \) is the number of data, confidence bands may be defined that measure scatter of the data.

Figure 2.7 shows resulting control curves for the case of parallelepipedic blocks and \( \cot \alpha = 2.0 \). These control curves are obtained, in \((Ir, \psi)\) axes by multiplying the best fit curves; equation (14) by the factors given in tables 2.6 for Iribarren's experiments and 2.7 for those of Ahrens and McCartney.

From this study it is deduced that the highest control curve for 95% confidence level, which could be taken as a stability function of design, \( \psi_D = f (Ir) \), can be obtained approximately by multiplying coefficient \( A \), given in tables 2.4 and 2.5, by the following factors:
Fig. 2.7 - Control curves for confidence levels of 90%, 95% and 99%
### TABLE 2.6

Confidence band factors for initiation of damage (after Iribarren's data)

<table>
<thead>
<tr>
<th>Armour units</th>
<th>Cota</th>
<th>Confidence level</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarry stones</td>
<td>1.50</td>
<td>1.34</td>
<td>1.14</td>
<td>1.58</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.75</td>
<td>0.71</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.00</td>
<td>1.37</td>
<td>1.46</td>
<td>1.64</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.73</td>
<td>0.69</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.00</td>
<td>1.29</td>
<td>1.35</td>
<td>1.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.77</td>
<td>0.74</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.00</td>
<td>1.51</td>
<td>1.64</td>
<td>1.91</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.66</td>
<td>0.61</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>Parallelopipedic blocks</td>
<td>1.50</td>
<td>2.71</td>
<td>3.28</td>
<td>4.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.37</td>
<td>0.30</td>
<td>0.21</td>
<td></td>
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<tr>
<td></td>
<td>2.00</td>
<td>2.06</td>
<td>2.37</td>
<td>3.11</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.48</td>
<td>0.42</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.00</td>
<td>1.62</td>
<td>1.77</td>
<td>2.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.62</td>
<td>0.56</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>Tetrapods</td>
<td>1.33</td>
<td>1.51</td>
<td>1.64</td>
<td>1.91</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.56</td>
<td>0.61</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>1.99</td>
<td>2.27</td>
<td>2.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.50</td>
<td>0.44</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.00</td>
<td>1.73</td>
<td>1.93</td>
<td>2.37</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.58</td>
<td>0.52</td>
<td>0.42</td>
<td></td>
</tr>
</tbody>
</table>

Upper factor is for upper control curve and lower factor for lower control curve.

### TABLE 2.7

Confidence band factors for rip-rap. Zero damage criterion (after Ahrens and McCartney's data)

<table>
<thead>
<tr>
<th>Cota</th>
<th>Confidence level</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td></td>
<td>1.46</td>
<td>1.57</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.69</td>
<td>0.64</td>
<td>0.55</td>
</tr>
<tr>
<td>3.50</td>
<td></td>
<td>1.40</td>
<td>1.50</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.71</td>
<td>0.67</td>
<td>0.59</td>
</tr>
<tr>
<td>5.00</td>
<td></td>
<td>1.42</td>
<td>1.52</td>
<td>1.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.70</td>
<td>0.66</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Upper factor is for upper control curve and lower factor for lower control curve.
Rip-rap 1.5  
Quarry stones 1.5  
Parallelopipedic blocks 2.5  
Tetrapods 2.0

This factor can be interpreted as a safety coefficient with regards the structural response of the mound. It is seen that the random component is much more important in parallelopipedic blocks.

INTERACTION CURVES

The variable which definitely governs stability of main layer armour units of a rubblemound breakwater is weight $W$. Given the type of armour unit, slope ($\alpha$) and specific weights ($\gamma_r$ and $\gamma_w$), stability conditions produced by the preestablished criterion strictly hold for:

$$W = \gamma_w H^3 R \psi$$  \hspace{1cm} (11)

although, for each combination of wave height and period the value of $W$ defined by equation (11) is different. If this value is less than the actual weight of the armour units, $W_0$, the breakwater will be stable (according to the criterion already established). Consequently it is useful to know the curves $W = \text{cte}$ on the plane $(H,T)$, that is, the set of points on the plane $(H,T)$ which produce the same value of $W$ (interaction curve).

Given the type of armour unit, $W$, $\alpha$, $\gamma_r$ and $\gamma_w$ of the expressions

$$W = \gamma_w H^3 R \psi$$  \hspace{1cm} (11)
\[ \psi = A(Ir - Ir_0) \exp \{ B(Ir - Ir_0) \} \quad Ir > Ir_0 \quad (14) \]

\[ Ir = \sqrt{g/2\pi \tan \alpha} T/\sqrt{H} \quad (25) \]

\[ Ir_0 = 2.654 \tan \alpha \quad (15) \]

\[ R = S_r/(S_r-1)^3 \quad (10) \]

\[ S_r = \gamma_r/\gamma_w \quad (26) \]

a function:

\[ f_0(H,T) = 0 \quad (27) \]

can be obtained which defines the corresponding interaction curve.

Figure 2.8 shows an example of a stability interaction curve - for the case of rip-rap, \( W = 3.0 \text{ ton} \), \( \cot \alpha = 3.50 \), \( \gamma_r = 2.6 \text{ t/m}^3 \) and \( \gamma_w = 1.0 \text{ t/m}^3 \).

The concept of stability interaction curves, introduced by Losada and Giménez-Curto (1979a), clearly reflects the joint influence of period and wave height on stability of the breakwater, separating on plane \((H,T)\) stability and instability zones.

The minimum wave height which can produce damage in the breakwater corresponds to Iribarren's number defined by

\[ Ir_{\text{crit}} = Ir_0 - 1/B \quad (16) \]

and the minimum period is defined by Iribarren's number:

\[ Ir = \frac{1}{2B} \left\{ 5 + B \left[ Ir_0 - \sqrt{B^2 - 14B Ir_0 + 25} \right] \right\} \quad (20) \]
Fig. 2.8—An example of interaction curve defining the stability area.
Fig. 2.9.- Variation of interaction curves with weight of armour units.
Fig. 2.10. - Variation of interaction curves with specific weight of armour units.
Fig. 2.11.- Interaction curves for different types of armour units

- $W = 10$ ton
- $\gamma_r = 2.4$ t/m$^3$
- $\gamma_w = 1.0$ t/m$^3$
It is seen that the three parabolas which define Iribarren's numbers $I_{r0}$, $I_{r1}$, $I_{rcrit}$ depend solely of $I_{r0}$ and $B$, that is, the type of armour units and slope angle. Therefore all resulting interaction curves for the different values of $W$ (or $W_{50}$), $\gamma_r$ and $\gamma_w$ have points corresponding to minimum period on the parabola $I_r = I_{r1}$ and corresponding to minimum wave height on the parabola $I_r = I_{rcrit}$.

Figures 2.9, 2.10 and 2.11 represent some examples of how interaction curves vary with the weight of armour units, their specific weight and the type of armour unit.

VARIATION OF STABILITY WITH THE SLOPE ANGLE

According to experimental results considered until now, it seems that stability of main layer armour units of a rubblemound breakwater improves as the slope becomes milder.

Starting with a very steep slope and progressively making it milder, stability conditions improve and in fact the slope becomes more stable. But this happens up to a specific slope angle limit and once this is exceeded stability conditions worsen. Iribarren (1965) discovered through experiments the reality of this phenomenon.

Figure 2.12 represents the variation of stability function with the slope angle for the case of tetrapods (Fig. 2.4 to which have been added results of 7 experiments carried out by Iribarren with $\cot \alpha = 2.5$). The great loss of stability on passing from $\cot \alpha = 2.0$ to $\cot \alpha = 2.5$ can be observed.

Two factors enter into extraction of armour units from the main layer - wave attack, forces which the waves exercise against the armour units, and the structural response of the system itself to these forces, a balance existing, of necessity, between them.
Fig. 2.12. - Stability function for tetrapods. Initiation of damage. (After Iribarren's data, 1965)
Forces exercised by the waves against armour units depend on flow characteristics and, more specifically, on the field of velocities and accelerations which is established on the slope.

Battjes and Roos (1975) conducted experiments on smooth, impermeable slopes from which it was concluded that both maximum and minimum velocity in run-up are proportional to \(\tan\alpha\). On the other hand the time during which the water runs up the slope turns out to be inversely proportional to the square root of \(\tan\alpha\). As a result, on making the slope milder velocities decrease and times increase, in other words velocities and accelerations in run-up decrease.

It appears reasonable to assume that this phenomenon is produced in qualitatively the same way in the case of rough, permeable slopes.

On the other hand, if breaking of the wave has occurred run-down will be free, higher velocity and acceleration being attained as down height increases \((R_u+R_d)\). Given that this height decreases on making the slope milder, it turns out that the milder the slope the lesser the down velocities and accelerations also.

It can be concluded that the same wave produces forces against the armour units which become less as the slope becomes milder.

Therefore the change of stability which is produced from a specific angle of the slope, Iribarren (1965), does not appear to have its explanation in a change of flow characteristics which would give rise to a variation in the forces that waves exercise against armour units, but that the explanation of this phenomenon should lie in the structural behaviour itself of the granular system, that is, in the variation of the armour unit stability level with the slope angle.

The idea seems to be confirmed, as will be shown later, by experimental results of Brebner (1978) and Price (1979).

The question, then, is: How does the stability level of armour units change when the slope angle is varied?

Considering an isolate armour unit placed onto a slope, the only force on which it can count to oppose and action (wave attack)
is its own weight. When this armour unit forms part of the main layer it is clear that it is capable of opposing a far greater force owing to collaboration that it receives from adjacent armour units, that is, to interlocking of the system.

Evidently an armour unit that is isolated on a smooth slope is more or less stable depending on the mildness of the slope, its stability being maximum when the slope is horizontal.

On the other hand, in order for there to exist collaboration among armour units a certain compactness of the main layer is necessary, and contact among armour unit to facilitate transmission of forces from one to another is also required.

If armour units are placed on a horizontal plane and little by little they are tilted (a becomes greater) what is attained is an increase in the number of contacts and in the stresses in these contacts, which brings about an increase in collaboration among the armour units.

It seems that collaboration among armour units increases as the slope angle increases. Brebner (1978) has shown, for the case of dolos, that a certain slope angle is required in order to attain the entire interlocking level which these armour units are capable of developing.

Bearing in mind the abovementioned reasoning, it is possible to explain the presence of the maximum in stability for a specific slope by assuming that stability conditions for each armour unit are determined by two factors, the armour units own weight and collaboration on adjacent armour units.

Naturally this collaboration will depend largely on the type of armour unit.

Figure 2.13 shows schematically how stability varies with the slope angle when the type of armour unit is such that collaboration among armour units has very little importance.
Fig. 2.13.- Representation sketch of stability versus slope for armour units of very little interlocking.

Fig. 2.14.- Representation sketch of stability versus slope angle for armour units of high interlocking.
TABLE 2.8

Optimum stability slope

<table>
<thead>
<tr>
<th>TYPE OF ARMOUR UNIT</th>
<th>$\cot \alpha_{opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>rip-rap</td>
<td>&gt; 5.00</td>
</tr>
<tr>
<td>quarry stones</td>
<td>4.00 to 6.00</td>
</tr>
<tr>
<td>parallelopipedic blocks</td>
<td>3.00 to 4.00</td>
</tr>
<tr>
<td>stabits</td>
<td>2.00 to 2.50</td>
</tr>
<tr>
<td>tetrapods</td>
<td>2.00 to 2.50</td>
</tr>
<tr>
<td>dolos</td>
<td>1.75 to 2.00</td>
</tr>
</tbody>
</table>
Figure 2.14 represents stability variation with the slope angle for armour units with great interlock, that is, capable of developing very important collaboration among themselves.

Finally it can be concluded that all those types of armour unit capable of developing a minimum amount of collaboration among themselves will present a maximum in stability for a specific slope.

Price (1979) detected this maximum for dolos and stabits with a simple static extraction experiment.

This optimum slope will be steeper and the maximum of stability sharper as the level of collaboration that can be developed among armour units becomes more important, that is, depending on the amount of interlock.

Taking into account the experimental results of Iribarren (1965), Ahrens and McCartney (1975) and Price (1979) table 2.8 has been drawn up which fixes the maximum stability slope angle for various types of armour unit.

STABILITY UNDER A SEA STATE. THE FAILURE PROBABILITY

In the case of regular waves the weight of armour units required for the breakwater to be stable is given by

\[ W = y_w H^3 R \psi \]  \hspace{1cm} (11)

It is evident that once slope characteristics (type of armour unit, \( \alpha \), \( y_L \), \( y_w \)) are fixed this weight depends exclusively on \( H \) and \( T \)

\[ W = f_0(H,T) \]  \hspace{1cm} (29)

In the case of irregular waves the variable \( W \) may be considered a random variable which takes on a value for each wave.
Assuming the hypothesis of equivalence (see chapter 1), it can be accepted that the distribution function of $W$ within a sea state is obtained by assigning to each irregular wave of the sea state the same value of $W$ which is produced by a periodic wave train of the same height and the same period.

Applying this hypothesis we can write:

$$ F_W(x) = \text{Prob} \{ W \leq x \} = \int_D p^*(H,T) \, dH \, dT \quad (30) $$

where the integration domain, $D$, is the shaded area in figure 2.1, defined by the breaking limit and interaction curve $W = x$.

Defining that a rubblemound breakwater under the action of a sea state fails if at least one wave exists within the sea state which produces a value of $W$ greater than the actual weight of the armour units, $W_0$, and admitting furthermore the statistical independence among successive values of the variable $W$ within a sea state, failure probability can be calculated as follows:

$$ f = 1 - \left( F_W(W_0) \right)^{3600t/T_z} \quad (31) $$

where $t$ is the duration of the sea state in hours, $T_z$ is mean zero-upcrossing wave period in seconds and $f$ failure probability.

By way of an example, failure probability of a rubblemound breakwater of the following characteristics has been analysed:

Type of armour unit: parallelopipedic blocks
$W_0 = 50.0 \text{ ton}$
$cota = 2.0$
$\gamma_{z} = 2.3 \text{ t/m}^3$
$\gamma_{w} = 1.0 \text{ t/m}^3$
Depth at the toe of breakwater : $d = 20.0 \text{ m}$
Horizontal bottom
Fig. 2.15.- Definition sketch of integration domain, D
Fig. 2.16. - Breaking limit and interaction stability curve for the proposed example.
Fig. 2.17.- Failure probability for the proposed example.
(Sea state duration 1 h.)
Fig. 2.18. - Failure probability for the proposed example. (sea state duration 6 h.)
Fig. 2.19. - Failure probability versus duration for several distributions of wave heights and periods.

1. Bretschneider
2. Longuet-Higging, \( v = 0.2 \)
3. \( v = 0.3 \)
4. \( v = 0.4 \)
5. Cavanié, Arhan y Ezraty, \( \varepsilon = 0.4 \)
6. \( \varepsilon = 0.6 \)
7. \( \varepsilon = 0.8 \)

Sea state duration \( t \) (hours)

Failure probability

\( H_s = 7.0 \) m.

\( T_z = 12.0 \) sc.
Figure 2.16 represents the corresponding breaking limit and interaction curve.

Figure 2.17 and 2.18 show results obtained for variable sea states and durations of 1 and 6 hours respectively.

Distribution of wave heights and periods used in this examples is that of Longuet-Higgins (with \( \gamma = 0.25 \)) and applying expression (19) from chapter 1.

Figure 2.19 shows, on the other hand, failure probability against duration for a sea state defined by \( H_s = 7.0 \), \( T_z = 12.0 \) seconds and for various distributions of wave heights and periods.

CONCLUSIONS

1) Stability conditions of an undefined, rough, permeable slope under regular wave action are governed by the stability function which depends on:

- Type of armour unit
- Slope angle
- Wave steepness
- Incidence angle

for a preestablished stability criterion.

2) In the case of normal wave incidence and given the type of armour units and slope, the stability function depends only on Iribarren's number.

3) Randomness in the structural behaviour can be accounted for by using confidence bands for the stability function.

4) Given the type of armour units, their weight, slope angle and specific weights of water and armour units, a curve exists as an interaction curve - which determines stability conditions of the slope as a function of incident wave height and period.
5) For each type of armour unit, an optimum slope of maximum stability exists. The greater the interlocking among armour units the steeper the optimum slope and the more peaked the stability maximum, therefore stability losses for small variations in the slope angle with regard to the optimum, will be greater.

6) By means of the hypothesis of equivalence and interaction curves failure probability of a slope under action of a sea state can be obtained.

7) Given a rubble mound breakwater a minimum sea state exists which produces a significant failure probability. If a sea state is presented which is the same or higher than this minimum, failure of the structure is only a question of the duration of the sea state.
CHAPTER 3 OBLIQUE WAVE ATTACK

INTRODUCTION

At the International Navigation Congress in Rome, Abecasis (1953) pointed out the no concordance of calculus formulae with the experience for oblique wave attack. Since the few contributions in this field have been made, Iribarren (1965), Van de Kreeke (1969), Whillock (1977). However, it is generally accepted among engineers and researchers that the conventional calculus formulae do not adequately quantify oblique wave attack.

This chapter presents a first approach to the analysis of the behaviour of rubble mound breakwaters under oblique wave attack. In the same way as in previous chapters, firstly flow characteristics on the slope with regular wave attacking obliquely are analysed. Afterwards slope stability under regular waves is studied, and finally a probabilistic approximation is given for the case of irregular waves.

FLOW CHARACTERISTICS ON A SLOPE UNDER OBLIQUE REGULAR WAVE ATTACK

The Technical Advisory Committee (1974), suggests that the run-up on a impermeable smooth slope under oblique wave attack is \( \cos \theta \) times the run-up under perpendicular wave attack, where \( \theta \) is the angle of incidence. One interpretation of this criterion can be given by observing that the slope of the breakwater in the direction of propagation is \( \tan \theta \cos \theta \) and for smooth impermeable slopes and \( Ir < 2.3 \), Hunt (1959), it can be written:

\[
\frac{R_u}{H} = Ir \quad (1)
\]

For \( \theta < 50^\circ \) a comparison of this result with experimental data of Hosoi and Shuto, indicates a reasonable degree of agreement, Battjes (1974), at least for breaking waves.
Regarding equation (27) of chapter 1, flow characteristics on a rough permeable slope under periodic waves with oblique incidence can be estimated, taking Iribarren's number in the direction of propagation as follows:

\[ Z = A \left(1 - \exp(B \cdot \text{Ir \cos} \theta)\right) \]  

(2)

\( A \) and \( B \) being the values for perpendicular incidence.

Figures 3.1, 3.2 and 3.3 show Hudson's data (1962), taken from Günbak (1979), for run-up and run-down on a rip-rap slope with incidence angles of \( \theta = 0^\circ \), \( 30^\circ \) and \( 60^\circ \). The best fit curves are drawn according to equation (27) of chapter 1. Also the curves corresponding to equation (2) are shown.

The coefficients \( A \), \( B \) and the correlation coefficient, \( \rho \), are given in table 3.1.

From these figures it can be concluded that for periodic waves with small incidence angle (\( \theta < 45^\circ \)), equation (2) gives a reasonable degree of agreement with the data. For higher incidence angles equation (2) does not give great enough values of run-up and run-down, denoting a significant effect of the wave refraction on the slope and other unknown effects.

**STABILITY OF RUBBLE MOUND BREAKWATERS UNDER OBLIQUE REGULAR WAVE ATTACK**

The weight of the armour unit, \( W \), necessary for it to be stable on the slope under the attack of regular waves with perpendicular incidence is defined by:

\[ W = \gamma_w H^3 \rho \]  

(3)
RUN-UP AND RUN-DOWN ON RIP-RAP SLOPES.
HUDSON'S DATA (1962)
PERPENDICULAR INCIDENCE

Δ COT α = 2.00
+ COT α = 3.00

Fig. 3.1—Run-up and run-down on rip-rap slope. Perpendicular incidence
Fig. 3.2. - Run-up and run-down on rip-rap slope. $\theta = 30^\circ$
RUN-UP AND RUN-DOWN ON RIP-RAP SLOPES.
HUDSON'S DATA (1962)
OBLIQUE INCIDENCE $\theta = 60^\circ$
$\triangle \cot \alpha = 2.00$
$\triangle \cot \alpha = 3.00$

Fig. 33.- Run-up and run-down on rip-rap slope. $\theta = 60^\circ$
TABLE 3.1

Coefficients of the best fit curves for Hudson's data.

<table>
<thead>
<tr>
<th>θ</th>
<th>A</th>
<th>B</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Ru</td>
<td>1.3219</td>
<td>-0.9654</td>
</tr>
<tr>
<td></td>
<td>Rd</td>
<td>1.7404</td>
<td>-0.1395</td>
</tr>
<tr>
<td>30</td>
<td>Ru</td>
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<tr>
<td></td>
<td>Rd</td>
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</tr>
<tr>
<td>60</td>
<td>Ru</td>
<td>0.9084</td>
<td>-0.8824</td>
</tr>
<tr>
<td></td>
<td>Rd</td>
<td>0.7738</td>
<td>-0.3245</td>
</tr>
</tbody>
</table>
The stability of a rubble mound breakwater under oblique wave attack can also be represented by equation (3), taking into account that \( \psi \) also depends on the wave incidence angle, \( \theta \) (see chapter 2).

In the following study it will be assumed that the incidence angle only affects the slope angle. Secondary phenomena, refraction, longitudinal currents and others, are not considered; therefore the conclusions herein derived are valid for small incidence angles (\( \theta < 45^\circ \)).

A wave with oblique incidence breaking on a slope reduces its Iribarren's number compared to the same wave with perpendicular incidence; moving as a consequence the breaking type in the direction surging-collapsing-plunging, and decreasing run-up and run-down.

The component of the slope in the direction of propagation is \( \tan \theta \cos \theta \). If the design slope is near the optimum stability slope, and if the breakwater is structurally isotropic, the stability of the breakwater decreases as the incidence angle increases. If the design slope is much steeper than the optimum stability slope, the stability of the breakwater under oblique wave attack increases as the incidence angle increases.

Expressing the stability of the breakwater under oblique and perpendicular wave attack through their respective maxima of the stability function, \( \psi_{\text{max}}(\theta) \) and \( \psi_{\text{max}}(0) \), in figures 3.4 to 3.8 the variation of the ratio \( \psi_{\text{max}}(\theta)/\psi_{\text{max}}(0) \) with the incidence angle, \( \theta \), for different kinds of armour units and design slopes are shown. The stability functions have been taken from Losada and Giménez-Curto (1979a). In some cases the curves are schematic (dashed curves) due to the lack of experimental data.

The small amount of experimental data available on oblique wave attack,
Fig. 3.4.- Effect of oblique wave attack on the stability of rip-rap
Fig. 3.5. – Effect of oblique wave attack on the stability of uniform quarry stones.
Fig. 3.6. Effect of oblique wave attack on the stability of parallelopipedic blocks.
Fig. 3.7: Effect of oblique wave attack on the stability of tetrapods
Fig. 3.8. - Effect of oblique wave attack on the stability of dolos blocks
roughly agrees with the results herein obtained. Van de Kreeke (1969) concluded, after experiments on rip-rap on a small scale, that for angles of incidence less than 45° the stability of the breakwater is similar to that of perpendicular incidence. Whillock (1977) presents results on stability of dolos blocks under oblique wave attack. Despite the fact that the amount of data is small, it can be appreciated that stability decreases as the angle of incidence increases, at least up to 60°.

THE STABILITY OF RUBBLE MOUND BREAKWATERS UNDER A SEA STATE WITH OBLIQUE INCIDENCE

On the basis of stability functions for oblique wave incidence, stability interaction curves can be obtained. The area between the interaction curve and breaking limit is the failure area.

Under general conditions the failure probability of the rubble mound breakwater under a sea state with oblique incidence can be calculated by considering the failure area and joint probability distribution of wave heights and periods.

Figure 3.9 shows the failure area for tetrapods and several angles of incidence.

CONCLUSIONS

1) There is a dangerous lack of experimental data on flow characteristics and stability of rubble mound breakwaters under oblique wave attack.
Fig. 3.9. - Effect of the wave incidence angle on the failure area.
2) Run-up and run-down on rip-rap under small oblique incidence of waves ( $\theta < 45^\circ$) are functions of $I_r \cos \theta$. For higher incidence angles the hypothesis is unreliable.

3) The stability of steep slopes of rip-rap, quarry stones, and parallelopipedic blocks under oblique wave attack is not worse than under perpendicular wave incidence. For milder slopes the opposite may be true.

4) The stability of steep slopes with armour units of high interlocking under oblique wave attack is hazardously worse than under perpendicular incidence.

5) The failure probability of a rubble mound breakwater under a sea state with oblique incidence, can be calculated by taking into account the breaking limit, the interaction curve and a joint distribution of wave heights and periods.

6) It seems to be imprudent, until reliable experimental information is obtained, to use units of high interlocking when oblique wave incidence with high energy is a possibility.
REFERENCES


