Michelson wide-field stellar interferometry
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Summary

The main goal of this thesis is to develop a system to permit wide field operation of Michelson Interferometers. A wide field of view is very important in applications such as the observation of extended or multiple objects, the fringe acquisition and/or tracking on a nearby unresolved object, and also to reduce the observation time. For ground-based arrays, the field of view should be at least equal to the isoplanatic patch.

Optical Stellar Interferometry consists of using two or more telescopes to collect the light from a distant object in order to obtain information with very high angular resolution. When two apertures are separated by a distance B, called baseline, the flat wavefront that comes from a distant source at a certain angle $\theta$ from the baseline does not reach the apertures at the same time. This delay will introduce an Optical Pathlength Difference (OPD) at both arms of the interferometer; if this path length difference is larger than the coherence length, then the light from both apertures will not interfere. In order to detect fringes over an extended field of view the OPD needs to be compensated before the beam combination takes place. In most interferometers nowadays this is done by means of delay lines. The light coming from an off-axis direction has a different delay than the light coming on-axis, referred to as differential delay. In so-called Fizeau interferometers, the synthetic aperture of the telescope array is exactly reproduced in a down-scaled version at the recombination optics; this recombination scheme has intrinsic path length compensation and a correspondingly wide field. This technique is very promising, but it is not useful if the baselines are very large compared to the single collector's aperture. In this case, the central peak is narrow and the energy is spread over the sidelobes of the interference pattern, limiting the sensitivity of the instrument. For non-Fizeau interferometers, the beams from the telescope array are simply overlapped (pupil-plane recombination) or combined in the image plane without maintaining the input pupil configuration. At angles where the differential delay becomes higher than the coherence length, the fringes disappear and the high-resolution information on the objects that are off-axis is lost. One way to solve this problem and acquire a wide field of view is to introduce a correction to the OPD for every angle in the telescope's field.

In order to avoid the serious drawback of Fizeau interferometry at a large ratio of baseline over aperture size, we thought of a new approach to the problem, i.e., a system that could use a Michelson pupil-plane combination scheme but acquiring a
wide field of view in one shot, saving also observation time. The functional principle of our approach is the introduction of an equalised OPD. This extra OPD can be translated in first-order approximation in steps of constant width and variable height which can be achieved by setting a stair-shaped mirror in an intermediate image plane of the interferometer. The focal plane has the characteristic that the light from different parts of the sky is focused separately, and for this reason we use it to introduce the equalization of the OPD. An extra OPD is introduced as a function of the field angle, so that coherent interference over a wide field of view can be obtained. The dimensions of the steps and the orientation of the mirror depends on the baseline and the pointing direction. Because the projection of the baseline vector on the entrance pupil changes with the hour angle during an astronomical observation, the mirror has to be actuated to follow these changes: it has to be rotated to follow the rotation of the projected baseline in order to maintain the steps perpendicular to it, and the depth must vary as it has to be adapted to the modulus of the projected baseline. In a system formed by more than two telescopes, it is necessary to have a step mirror at the focal plane of each telescope and a common reference point for the different baselines. Each mirror will be perpendicular to the projection of its baseline on the entrance pupil of its telescope. In this thesis we have studied the problem of the field of view for non-Fizeau arrays analytically and experimentally. The complete analytical description of a pupil-plane interferometer with a staircase mirror in the focal plane of one of its arms is developed, and the results are compared with the experiments. We have designed a stair-shaped mirror that was placed in the focal plane of one of the arms of a two-arm Michelson-type interferometer. With a Xenon arc-lamp and a starmask we simulate different configurations of objects in the sky, with several off-axis objects with a differential delay that should be compensated by the staircase mirror. For all configurations, the experimental results followed the analytical predictions, and the visibility of the on-axis and off-axis objects was retrieved simultaneously. Special attention was given to the case of a star focused on the edge of a step. Analytical calculations and experiments show that in that case two sets of fringes, each corresponding to one step, are detected. By adding the information contained in this two fringes it is possible to retrieve the visibility of the source, meaning that no information is lost due to the discontinuous nature of the mirror, and a continuous wide field can be reconstructed.
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"Once in Hawaii I was taken to see a Buddhist temple. In the temple a man said, "I am going to tell you something that you will never forget." And then he said "To every man is given the key to the gates of heaven. The same key opens the gates of hell."

Richard P. Feynman, "The Meaning of it All"

CHAPTER 1

Introduction

Optical aperture synthesis has become one of the most important techniques for astronomy in the last decades. It has overcome the problem of resolution related to single telescope diameter, opening new possibilities for high resolution imaging, microarcsecond astrometry and planet detection to a level that is difficult to achieve in classical astronomy. In Section 1.1 of this introduction we briefly overview the history of optical aperture synthesis from Fizeau [Fizeau1867] and the first measurements of star radii performed by Michelson [Michelson1920, MichelsonPease1921] to the last instruments like the Very Large Telescope Interferometer (VLTI), the Large Binocular Telescope (LBT) and the Keck Interferometer. In Section 1.2, the main principles of interferometry, from the perspective of the theory of partial coherence, are explained and related to the usual notations and definitions used in astronomy. The general description of an optical interferometer is addressed in Section 1.3, and in Section 1.4 the reasons for applying interferometry to astronomy are described. The main topic of this thesis, the problem of the field of view in a non-homothetic interferometer, is introduced in Section 1.5. Finally, the outline and goal of the thesis are described in Section 1.6.

1.1 Brief historical overview

This historical overview is largely based on [Lawson1997], where the interested reader can find most of the references. The application of optical interferometry to astronomy was independently developed by Fizeau, in France, and Michelson, in the United States, during the late 19th century. First suggestion of this application was done by Fizeau in 1867 [Fizeau1867] when he announced the results of the Prix Bordin competition, and was followed in 1874 by the measurements of
Stephan ([Stephan1873] and [Stephan1874]), who, masking the 80 cm Foucault telescope in the Observatoire de Marseille, concluded that the stars had diameters smaller than 0.158 arcsec. In the United States, and supposedly not having knowledge of the French efforts in the field, Michelson set in 1890 the mathematical principles of stellar interferometry ([Michelson1890] and [Michelson1891a]) and in 1891 he measured the angular diameters of the satellites of Jupiter placing two slits separated 4 inches in the 12-inch equatorial telescope at Mount Wilson [Michelson1891b]. A few years later, Schwarzschild used his method to study binary stars [Schwarzschild1896], studies that were continued by Anderson [Anderson1920], who measured the orbit of Capella. The latter results encouraged Michelson to speed up the building of the 20-feet interferometer (see Fig. 1.1) [Michelson1920] that he and Pease used to measure the diameter of Betelgeuse ($\alpha$-Orionis) [MichelsonPease1921]; the first measurement of a stellar diameter performed with an interferometer. The technological limit was reached in 1931, when Pease commissioned his 50-feet interferometer; this instrument was very difficult to build and to operate given the available technology at that time [Pease1931].

The success of radio interferometry in the 1930’s brought further development to the technique, giving inspiration to scientists to develop intensity interferometry and infrared heterodyne interferometry. Hanbury Brown and Twiss developed their intensity interferometer in 1956 [BrownTwiss1956a] and measured the diameter of Sirius at Jodrell Bank [BrownTwiss1956b]. The Intensity Interferometer at the Narrabri Observatory was used to measure the angular diameters of 32 stars [Brown1974]. Again two groups, working independently, developed infrared

![Fig. 1.1](image-url)
1.1 Brief historical overview

heterodyne interferometry: the French one in 1979 at the C.E.R.G.A (Centre d’Etudes et de Recherches Géodinamiques et Astronomiques) in the Observatoire du Plateau de Calern [Assus1979], and C. H. Townes in the United States. The latter was the first to detect fringes from separated telescopes at infrared wavelengths using the ISI (Infrared Spatial Interferometer) at Mount Wilson [Bester1990].

By the 1970’s, the technology had advanced enough to permit a continuation of the early work of Pease and Michelson. The first fringes with two separate telescopes at visible wavelengths were obtained by Labeyrie in 1974 at the Observatoire de Nice with a prototype of the I2T (Interféromètre à 2 Télescopes) [Labeyrie1975] with a 12 meter baseline. The interferometer was later moved to C.E.R.G.A where the baseline was increased to 137 meters [Koechlin1988]. This was the only modern stellar interferometer in function for many years, until the commission of the prototype Stellar Interferometer of the Sydney University ([DavisTango1985a] and [DavisTango1986]) and the SUSI (Sidney University Stellar Interferometer) in 1985 [DavisTango1985b]. In the meantime another interferometer with smaller apertures was built for astrometry purposes, the Mark I, that was able to measure and track atmospheric turbulence in real time ([ShaoStaelin1977] and [ShaoStaelin1980]). Labeyrie was the pioneer in the use of large aperture telescopes with the 1.5 m diameter telescopes of the GI2T (Grand Interféromètre à 2 Télescopes) that was equipped with a grating spectrometer capable of measuring fringe visibility with a very high spectral resolution ([Labeyrie1986] and [Mourard1994]). Shao and Staelin continued with their programme and developed the Mark III, which was the first fully automated interferometer and was intended as an astrometry instrument but was also capable of measuring star’s diameters and binary orbits [Shao1988]. Some of this instruments are still in operation. A list with the main interferometers that are actually in operation or design can be found in Table 1.1.

The possible science to be performed with a stellar interferometer became more ambitious when in 1985 Baldwin and his co-workers were able to measure the closure phase at optical wavelengths [Baldwin1986], opening the possibilities for optical synthesis imaging [Haniff1987]. The initial observations were made using aperture masks, but later arrays with 4 or 6 beams were designed and the first optical synthesis image [Baldwin1996] was obtained in 1995 with COAST (Cambridge Optical Aperture Synthesis Telescope) and one year later with the NPOI (Naval Prototype Optical Interferometer) [Benson1997]. The NPOI was able to perform imaging co-phasing its 6 stations in 2002 [Benson2003].

A very interesting innovation that was introduced to stellar interferometry in the late 1980’s was the use of optical fibres. Shaklan and Roddier demonstrated in 1987
the applicability of single-mode fibres to interferometry ([ShaklanRoddyer1987] and [ShaklanRoddyer1988]). V. Coude du Foresto with S. Ridgway demonstrated for the first time separated-telescope interferometer operation with single-mode fibres combination, with the FLUOR (Fibre Linked Unit for Optical Recombination) recombination unit at the McMath Solar Observatory in 1991 [Du Foresto1992]. The FLUOR unit is currently in use at the IOTA (Infrared-Optical Telescope Array) ([Du Foresto1998] and [Du Foresto2001]). An even more ambitious project is the OHANA (Optical Hawaiian Array for Nanoradian Astronomy) that intends to link all the telescopes existing on the top of Mauna Kea by means of optical fibres, to create an interferometer of unprecedented sensitivity an angular resolution [Mariotti1998]. Most of the research performed in recent years for this purpose is devoted to the development of efficient fibres and waveguides for the infrared. Among the telescopes on the Mauna Kea Observatory there are the two 10 meter diameter Keck Telescopes. Their construction was finished in 1993 and they were not specifically built for interferometric operation but were linked to perform direct detection of “hot Jupiter” planets, and astrometric detection of smaller ones [ColavitaWizinowich2003]. The first fringes of the Keck Interferometer were obtained in 2001, and in 2003 the observation of the galaxy NGC 4151, 12.26 Mparsec far from earth [Swain2003], was achieved. An array of large aperture telescopes that was indeed built to perform interferometric measurements is the VLTI (Very Large Telescope Interferometer) [Glindemmann2001]. It has been built by ESO (European Southern Observatory) and its first fringes, combining the light from two telescopes, were obtained in 2001 [Glindemmann2003]. The array is composed of four Unit Telescopes of 8 meters diameter and four Auxiliary Telescopes of 1.8 meters diameter. Currently, the four Unit Telescopes are in operation, and the light from the four of them was first combined, two by two, in 2003. With baseline lengths up to 200 meters, the VLTI combines the high resolution of a large baseline interferometer and a good u-v coverage with the power of its large collecting area, being able to detect fringes of weak stars. The VLTI is designed to perform high angular resolution measurements and high accuracy astrometry. The present instruments are MIDI (MIDinfrared Instrument), AMBER (Astronomical Multi BEam combineR) and PRIMA (the Phase Referenced Imaging and Micro-arsec Astrometry instrument). In 2003 MIDI obtained interferometric fringes of NGC 1068, performing the first detection by infrared interferometry of an extragalactic object. The imaging instrument [Lardiere2003] is currently under study and the planet detection module [Gondoin2003] is currently being tested in order to enlarge the scientific possibilities of the instrument.
## 1.1 Brief historical overview

### Table 1.1 Main optical interferometers in operation or design.

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<thead>
<tr>
<th>Name</th>
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<th>Diameter (m)</th>
<th>Maximum baseline (m)</th>
<th>Date of operation</th>
<th>$\lambda$</th>
<th>Location</th>
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<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>CHARA</td>
<td>6</td>
<td>1</td>
<td>350</td>
<td>2000</td>
<td>vis–K</td>
<td>Mt. Wilson, California</td>
</tr>
<tr>
<td>COAST</td>
<td>5</td>
<td>0.4</td>
<td>22</td>
<td>1992</td>
<td>J-K</td>
<td>Cambridge, UK</td>
</tr>
<tr>
<td>GI2T</td>
<td>2</td>
<td>1.5</td>
<td>65</td>
<td>1990</td>
<td>vis</td>
<td>Calern, France</td>
</tr>
<tr>
<td>IOTA</td>
<td>3</td>
<td>0.4</td>
<td>38</td>
<td>1995</td>
<td>H-K</td>
<td>Mt. Hopkins, Arizona</td>
</tr>
<tr>
<td>NPOI</td>
<td>6</td>
<td>0.5</td>
<td>435</td>
<td>1998</td>
<td>vis</td>
<td>Flagstaff, Arizona</td>
</tr>
<tr>
<td>PTI</td>
<td>3</td>
<td>0.4</td>
<td>110</td>
<td>1995</td>
<td>H-K</td>
<td>Mt. Palomar, California</td>
</tr>
<tr>
<td>SUSI</td>
<td>2</td>
<td>0.14</td>
<td>640</td>
<td>1990</td>
<td>vis</td>
<td>Narrabri, Australia</td>
</tr>
<tr>
<td><strong>Large ground-based interferometers</strong></td>
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<td></td>
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<td></td>
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<td></td>
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<tr>
<td>VLTI</td>
<td>4+4</td>
<td>8.2+1.8</td>
<td>130 to 202</td>
<td>2003</td>
<td>J-N</td>
<td>Cerro Paranal, Chile</td>
</tr>
<tr>
<td>Keck-I</td>
<td>2+4</td>
<td>10+1.8</td>
<td>85 to 135</td>
<td>2001</td>
<td>J-N</td>
<td>Mauna Kea, Hawaii</td>
</tr>
<tr>
<td>LBTI</td>
<td>2</td>
<td>8.4</td>
<td>22.8</td>
<td>2002</td>
<td>vis-N</td>
<td>Mt. Graham, Arizona</td>
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<td></td>
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<td>OHANA</td>
<td>&lt;7</td>
<td>3.6 to 10</td>
<td>800</td>
<td>?</td>
<td>I–K</td>
<td>Mauna Kea, Hawaii</td>
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<tr>
<td><strong>Space interferometers</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIM</td>
<td>2</td>
<td>0.3</td>
<td>10</td>
<td>2009</td>
<td>0.4–0.9 $\mu$m</td>
<td>Earth drift-away</td>
</tr>
<tr>
<td>IRSI (Darwin)</td>
<td>4 to 6</td>
<td>1.5</td>
<td>25 to 1000</td>
<td>2014</td>
<td>10 $\mu$m</td>
<td>L2</td>
</tr>
<tr>
<td>TPF</td>
<td>4</td>
<td>3.5</td>
<td>75 to 1000</td>
<td>2020</td>
<td>3–30 $\mu$m</td>
<td>L2 or Earth drift-away</td>
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Other ground-based interferometers currently in operation are the CHARA (Center for High Angular Resolution Astronomy) array, the PTI (Palomar Testbed Interferometer) and the LBT (Large Binocular Telescope). This last one is unique in the sense that it is composed of two telescopes that are mounted together, and it will be the only interferometer performing direct imaging with a large field of view (approx. 1 arcmin). All this arrays, specially the ones with large apertures, have to fight the effect of the atmosphere, which introduces phase fluctuations, in order to perform interferometric measurements. One solution thought to avoid this problem was to send the interferometers into space. Several missions are currently in design to build interferometers in space. The first one to be launched will be the Space Interferometry Mission (SIM). It is designed as a 10-m baseline optical Michelson interferometer operating at visible wavelengths. Its aim is to achieve an accuracy of 1 μas over a field of view of 1°, searching for planetary companions to nearby stars by detecting the astrometric “wobble” relative to reference stars. The other two main missions, one european and another one from the United States, are the Infrared Space Interferometer (IRSI) DARWIN mission, by the European Space Agency (ESA), and the Terrestrial Planet Finder (TPF), by the NASA. The Darwin mission [LundBonnet2001] consists of 4 to 6 free-flying telescopes and a central hub for beam combination and its primary science goal is the direct detection and characterization of extra-solar planets orbiting nearby stars by nulling the light of the star with a rejection ratio of at least $10^5$ [Ollivier2001]. A secondary goal is the direct interferometric imaging combining the light of the telescopes in Fizeau mode [D'Arcio2003]. It is planned to be launched in 2014. The TPF mission is in its study phase and it has not yet been decided whether it will be an infrared interferometer or a visible coronagraph; the decision will be taken in 2005. The interferometer candidate consists of 3 or 4 telescopes that will be on a fixed structure or free-flying. Like the DARWIN mission, its main scientific goal is the detection and study of exoplanets by means of nulling interferometry with, in the case of the interferometer concept, a rejection ratio of at least $10^6$.

### 1.2 Principles of interferometry

In this section we describe the basic principles of interferometry using the quasi-monochromatic theory of partial coherence. We describe what is visibility, why it is the primary observable in stellar interferometry and how it is related to the observed source. This section is largely based on [BornWolf1999] and [CalvoPadilla2002].
1.2 Principles of interferometry

1.2.1 Interference of two partially coherent beams

We want to study the correlation that may exist between the vibrations at two arbitrary points in the wave field produced by a finite source with a finite spectral range. A suitable measure for this correlation is suggested by a two-beam interference experiment, analogue to the Young interferometer (Fig. 1.2). An extended polychromatic source $\sigma$, produces a wave field represented by the analytic signal $\Psi(P, t)$ which is a function of position, $P$, and time, $t$, where polarization effects have been neglected. The observable power $I(P)$ is proportional to the mean value of $\Psi(P, t)\Psi^*(P, t)$. Now consider two points in the wave field, $P_1$ and $P_2$. In order to study the interference effects arising from the superposition of the vibrations from this points, an opaque screen is placed across the field with pinholes at positions $P_1$ and $P_2$. The complex disturbance produced at a point $Q$, separated by a certain axial distance $z$ from the screen, is

$$\Psi(Q, t) = K_1\Psi(P_1, t - t_1) + K_2\Psi(P_2, t - t_2),$$

(1.1)

where $t_1$ and $t_2$ are the times needed for the light to travel from $P_1$ and $P_2$ to $Q$, respectively, and $K_1$ and $K_2$ are complex constants inversely proportional to $z$.

The power at the screen is defined by

$$I(Q, t) = \langle \Psi(Q, t)\Psi^*(Q, t) \rangle.$$  

(1.2)

---

![Fig. 1.2 Scheme of the Young interferometer: the interferometer plane is an opaque screen with two small pinholes, $P_1$ and $P_2$, separated a distance $S$. The interference produced by the superposition of the two secondary wavefronts generated at $P_1$ and $P_2$ when they are illuminated by the radiation emitted by the extended source is observed at a plane situated at an axial distance $z$ from the pinholes.](image-url)
Operating and assuming stationarity† conditions in the process of superposition of the two wavefronts, we have that

\[
l(Q) = |K_1|^2 l_1 + |K_2|^2 l_2 + 2 \text{Re} \{ K_1^* K_2 \langle \psi^*(P_1, t-t_1) \psi(P_2, t-t_2) \rangle \},
\]

where \( l_1 \) and \( l_2 \) are the powers associated to the radiation generated at the secondary sources \( P_1 \) and \( P_2 \), respectively. The third term on the right in Eq. (1.3) is

\[
\langle \psi^*(P_1, t-t_1) \psi(P_2, t-t_2) \rangle = \lim_{T \to \infty} \frac{1}{T/2 - T/2} \int_{T/2}^{T/2} \psi^*(P_1, t-t_1) \psi(P_2, t-t_2) dt.
\]

By changing variables according to \( t - t_2 = t' \) in Eq. (1.4) we obtain

\[
\langle \psi^*(P_1, t-t_1) \psi(P_2, t-t_2) \rangle = \lim_{T \to \infty} \frac{1}{T/2 - T/2} \int_{T/2}^{T/2} \psi^*(P_1, t+t') \psi(P_2, t') dt'.
\]

Eq. (1.5) is called *mutual coherence function* and represents a temporal complex cross-correlation between the functions \( \psi(P_1, t-t_1) \) and \( \psi(P_2, t-t_2) \) during the time interval \( T \). This function depends on the time \( \tau = t_2 - t_1 \) and on the separation between \( P_1 \) and \( P_2 \), \( S \), as a result of the stationarity and ergodicity‡ conditions imposed to the process of superposition of both signals. It is a fundamental function in the theory of optical coherence and is defined as

\[
\langle \psi^*(P_1, t-t_1) \psi(P_2, t-t_2) \rangle = \Gamma(S, \tau) = \Gamma_{12}(\tau).
\]

Rewriting Eq. (1.3) in terms of \( \Gamma_{12}(\tau) \) we obtain

\[
l(Q) = |K_1|^2 l_1 + |K_2|^2 l_2 + 2 \text{Re} \{ K_1^* K_2 \Gamma_{12}(\tau) \}.
\]

This equation gives the resulting power when two stationary optical fields interfere. When \( t_1 = t_2 \) and \( P_1 = P_2 \), we obtain

\[
\langle \psi^*(P_1, t-t_1) \psi(P_1, t-t_1) \rangle = \Gamma(P_1, P_1, 0) = \Gamma_{11}(0),
\]

which is denominated the optical power associated to source \( P_1 \) and analogously for \( \Gamma_{22}(0) \) for the source \( P_2 \). Using these definitions we have that

\[
\hat{I}^{(1)}(Q) = |K_1|^2 l_1 = |K_1|^2 \Gamma_{11}(0),
\]

\[
\hat{I}^{(2)}(Q) = |K_2|^2 l_2 = |K_2|^2 \Gamma_{22}(0).
\]

† In an ensemble of functions, stationarity implies that all the ensemble averages are independent of the origin of time.

‡ In an ensemble of functions, ergodicity implies that each ensemble average is equal to the corresponding time average involving a typical member of the ensemble.
1.2 Principles of interferometry

We then normalize Eq. (1.7) obtaining

\[ I(Q) = i^{(1)}(Q) + i^{(2)}(Q) + 2 \sqrt{i^{(1)}(Q)I^{(2)}(Q)} \text{Re}[\gamma_{12}(\tau)], \tag{1.10} \]

where:

\[ \gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}}, \tag{1.11} \]

is denominated complex degree of coherence, and is expressed as

\[ \gamma_{12}(\tau) = |\gamma_{12}(\tau)| \exp\{j\text{arg}[\gamma_{12}(\tau)]\}. \tag{1.12} \]

The complex degree of coherence is characterized by the following properties:

- It is a function with a maximum value at the origin for \( \tau = 0 \).
- As a consequence of the Cauchy-Schwartz inequality its value is \( 0 \leq |\gamma_{12}(\tau)| \leq 1 \).
- It is a complex analytic signal.
- As we will explain in the next section, the modulus of the complex degree of coherence is proportional to the contrast or visibility of the interference fringes, therefore by measuring it one can obtain information about the “quality” of the illuminating source of the interference system.

We define three operating regimes for the interference system as a function of the value of \( |\gamma_{12}(\tau)| \): if \( |\gamma_{12}(\tau)| = 1 \), the system is operating in the coherent limit, and the vibrations at \( P_1 \) and \( P_2 \) may be said to be coherent; if \( |\gamma_{12}(\tau)| = 0 \), the system operates in the incoherent limit and the superposition of the two beams do not give rise to any interference effect; if \( 0 < |\gamma_{12}(\tau)| < 1 \) the vibrations are said to be partially coherent and the source operates with partial degree of coherence. In general, this last condition applies to a natural source, and the value \( |\gamma_{12}(\tau)| \) is associated to the degree of coherence.

1.2.2 Interference with quasi-monochromatic light

Suppose that the light emitted by \( \sigma \) is quasi-monochromatic, with a mean frequency \( \nu_0 \), and that the spectral range fulfils \( \Delta \nu \ll \nu_0 \). Then Eq. (1.12) can be expressed in a different way:

\[ \gamma_{12}(\tau) = |\gamma_{12}(\tau)| \exp\{j(\alpha_{12}(\tau) - 2\pi \nu_0 \tau)\}, \tag{1.13} \]

where

\[ \alpha_{12}(\tau) = 2\pi \nu_0 \tau + \text{arg}[\gamma_{12}(\tau)], \tag{1.14} \]
and Eq. (1.10) becomes

\[ I(Q) = I^{(1)}(Q) + I^{(2)}(Q) + 2\sqrt{I^{(1)}(Q)I^{(2)}(Q)}|y_{12}(\tau)|\cos(\alpha_{12}(\tau) - 2\pi \nu_0 \tau). \] (1.15)

Because of the quasi-monochromatic condition, \([y_{12}(\tau)]^2\) and \(\alpha_{12}(\tau)\) will vary slowly with \(\tau\) in comparison to \(\cos 2\pi \nu_0 \tau\) and \(\sin 2\pi \nu_0 \tau\), and if the openings at \(P_1\) and \(P_2\) are sufficiently small the power distribution at the vicinity of \(Q\) will consist of an almost uniform background \(I^{(1)}(Q) + I^{(2)}(Q)\) with a superimposed sinusoidal power distribution, as shown in Fig. 1.3. Defining the contrast of the fringes as

\[ V(Q) = \frac{l_{\text{max}} - l_{\text{min}}}{l_{\text{max}} + l_{\text{min}}}, \] (1.16)

where \(l_{\text{max}}\) and \(l_{\text{min}}\) are the power maxima and minima near \(Q\), respectively

\[ l_{\text{max}} = I^{(1)}(Q) + I^{(2)}(Q) + 2\sqrt{I^{(1)}(Q)I^{(2)}(Q)}|y_{12}(\tau)|, \] (1.17)

\[ l_{\text{min}} = I^{(1)}(Q) + I^{(2)}(Q) - 2\sqrt{I^{(1)}(Q)I^{(2)}(Q)}|y_{12}(\tau)|, \]

we have that the contrast of the fringes can be easily related to the degree of coherence of the interfering light beams by

\[ V(Q) = \frac{2\sqrt{I^{(1)}(Q)I^{(2)}(Q)}}{I^{(1)}(Q) + I^{(2)}(Q)}|y_{12}(\tau)|. \] (1.18)
1.2 Principles of interferometry

In the special case when \( f^{(1)}(Q) = f^{(2)}(Q) \), the contrast

\[
V(Q) = |\gamma_{12}(\tau)|,
\]

is equal to the degree of coherence of the source. Moreover, when the path difference between the light paths through both pinholes \( |s_2 - s_1| = c|\tau| \) is small compared to the coherence length of the light, defined by \( L_c = c/\Delta \nu \), the function \( \gamma_{12}(\tau) \) is hardly different from \( \gamma_{12}(0) \), and, defining the complex coherence factor, \( \mu_{12} \), as \( \gamma_{12}(0) = \mu_{12} \), Eq. (1.15) can be written as

\[
I(Q) = f^{(1)}(Q) + f^{(2)}(Q) + 2\sqrt{f^{(1)}(Q)f^{(2)}(Q)}|\mu_{12}|\cos[\text{arg}(\mu_{12})].
\]

Eq. (1.20) represents the basic formula of the quasi-monochromatic theory of partial coherence. The complex coherence factor is usually called visibility by the astronomers.

1.2.3 Relation with the source: the van Cittert-Zernike theorem

We want to determine the complex coherence factor for two points \( P_1 \) and \( P_2 \) on a screen illuminated by an extended quasi-monochromatic source \( \sigma \), as represented in Fig. 1.4, whose dimensions are small compared to the distance to the screen. If the source is divided into small elements \( d\sigma_j \), centred at \( S_j \), which are mutually incoherent, and of linear dimensions small compared to the mean
wavelength $\lambda_0$, the complex disturbance due to element $d\sigma_m$ at a point $P_i$ in the screen is

$$\Psi_{mi}(t) = A_m(t - \frac{r_{mi}}{c}) \frac{\exp(-2\pi j \nu_0(t - r_{mi}/c))}{r_{mi}},$$  

(1.21)

where the strength and phase of the radiation coming from element $d\sigma_m$ are characterized by the modulus of $A_m$ and its argument, respectively, and $r_{mi}$ is the distance from the element $d\sigma_m$ to the point $P_i$.

As, to date, no extended astronomical source is known that is spatially coherent because of an internal physical process [Anantharamaiah1994], and not considering the case of scintillation, we assume that the elements are mutually incoherent and that the distance $r_{m2} - r_{m1}$ is small compared to the coherence length of the light. The mutual coherence function of $P_1$ and $P_2$ is then:

$$\Gamma_{12}(0) = \sum_m \langle A_m(t) A_m^*(t) \frac{\exp(2\pi j \nu_0(r_{m1} - r_{m2})/c)}{r_{m1}r_{m2}} \rangle.$$  

(1.22)

Considering a source with a total number of elements so large that it can be regarded as continuous, the sum in Eq. (1.22) can be replaced by the integral

$$\Gamma_{12}(0) = \frac{1}{\mu_{12}} \int l(S) dS,$$  

(1.23)

where $r_1$ and $r_2$ denotes the distances from a typical source element at $S$ to the points $P_1$ and $P_2$, respectively, and $\kappa_0 = 2\pi \nu_0/c$.

The complex coherence factor is therefore

$$\mu_{12} = \frac{1}{\mu_{12}} \int l(S) \frac{\exp(j \kappa_0(r_1 - r_2))}{r_1r_2} dS,$$  

(1.24)

where:

$$l(P_j) = \Gamma_{ij}(0) = \frac{1}{\mu_{12}} \int \frac{l(S)}{r_j} dS.$$  

(1.25)

This result is known as the van Cittert-Zernike theorem. Observing Fig. 1.4, and assuming that the linear dimensions of the source and the distance between $P_1$ and $P_2$ is small compared to the distance of these points from the source, we can approximate

$$r_1 - r_2 \sim \frac{(X_1^2 + Y_1^2) - (X_2^2 + Y_2^2)}{2r} = \frac{(X_1 - X_2)\xi + (Y_1 - Y_2)\eta}{r}.$$  

(1.26)
1.3 General description of an optical stellar interferometer

Defining

$$\frac{X_1 - X_2}{r} = p \quad \frac{Y_1 - Y_2}{r} = q$$

we rewrite Eq. (1.25) as

$$\varphi = \frac{k_0 \left( (X_1^2 + Y_1^2) - (X_2^2 + Y_2^2) \right)}{2r}.$$  \hspace{1cm} (1.27)

Eq. (1.28) shows that the degree of coherence is equal to the absolute value of the normalized Fourier transform of the intensity function of the source. This form of the van Cittert-Zernike theorem is widely used in stellar interferometry, since the stellar sources are supposed to be at a distance very large compared to the separation of the telescopes and the size of the source itself, and are also supposed to be two-dimensional objects. The limit of applicability of the two-dimensional Fourier transform is studied in [Perley1994], where is given an estimate of the maximum undistorted field of view in a two-dimensional image,

$$\theta_{max} = \sqrt{\frac{\lambda_0 B}{P_1 P_2}}.$$  \hspace{1cm} (1.29)

where $B = [(X_1 - X_2)^2 + (Y_1 - Y_2)^2]^{1/2}$ is the distance between $P_1$ and $P_2$, and $\lambda_0$ is the mean wavelength. For example, observing an object in the sky at 2.2 μm for B=200 meters, its image can be considered two-dimensional on a field of view of approximately 1 arcmin diameter.

1.3 General description of an optical stellar interferometer

Before we apply the van Cittert-Zernike theorem to astronomy, it is illustrative to first shortly describe a general stellar interferometer and some common terms that are widely used in aperture synthesis. Fig. 1.5 is a schematic drawing with two telescopes observing a distant stellar source situated in a direction given by the pointing vector $\mathbf{s}$. The plane waves from the source are collected by the
telescopes that are separated by a distance $B$, with $B$ being the modulus of the baseline vector $B$ with coordinates given by

$$B_{ij} = (B_{x_i}, B_{y_i}) = (X_i - X_j, Y_i - Y_j).$$

(1.30)

As can be seen in the figure, the light from the source is not collected at the same time by both telescopes. There is a time difference called delay or, equivalently, an Optical Pathlength Difference (OPD) that is simply given by the scalar product of the baseline and the pointing vector:

$$OPD = B \cdot s.$$  

(1.31)

The collected light is collimated and directed towards a beam combiner. There are two ways to combine the beams: re-focus the beams in a common focal plane (image plane or multiaxial beam combination, see Fig. 1.5a), or overlap the beams by means of a beam splitter (pupil-plane or coaxial beam combination, Fig. 1.5b). Depending on the method used to interfere the light, spatial or temporal fringes will be observed as result of the interference of the beams. Of course, if we want to observe temporal fringes, the OPD has to be corrected before combination takes place to assure coherent interference of the light and the optical path has to be modulated. This is done using a delay line, that basically consists of a cat’s-eye retroreflector or a rooftop mirror on a rail. There are two levels of correction: coherencing, when the error in the correction of the OPD is smaller than the coherence length but larger than the central wavelength of the detected light, and cophasing, when the error in the correction of the OPD is smaller than the central wavelength. In the first case, reliable measurements of the fringe contrast can be done, but to measure the fringe phase it is necessary to cophase the beams.

1.4 Application to astronomy: why use interferometers?

As explained before, in astronomical observations performed with single telescopes, the angular resolution of the measurements is limited by the diameter of the aperture. Moreover, increasing the size of the single aperture is not useful unless this is accompanied by a powerful adaptive optics system, as the perturbation produced by the atmosphere limits the coherence cross-section of the optical beam to a patch which diameter is given by the Fried parameter, $r_0$, that is proportional to $\lambda^{6/5}$. The Fried parameter is typically 10 cm at visible wavelengths. But in an interferometer, the angular resolution depends on the separation of the collectors, as can be seen in Eq. (1.28). The primary observable in a stellar
1.4 Application to astronomy: why use interferometers?

Fig. 1.5 Illustration of a general interferometer: two or more telescopes separated a distance $B$ are used to collect the light from a distant stellar source. (a) The light is combined by refocusing the beams in a common focal plane giving as result spatial fringes. (b) The beams are combined with a beam splitter resulting in temporal fringes. The contrast and phase of these fringes is measured and the result is the complex visibility for baseline $B$, one particular component of the spatial Fourier transform of the source’s intensity distribution.
interferometer is $\mu_{ij}$, called the visibility of the fringe pattern for the two telescopes situated in points $P_i$ and $P_j$. Using the definition of baseline, we can rewrite Eq. (1.27) as

$$u_{ij} = \frac{B_{\xi}}{\lambda}, \quad v_{ij} = \frac{B_{\eta}}{\lambda},$$

(1.32)

where $u$ and $v$ are the coordinates in the Fourier plane that depend on the separation of the telescopes projected in the direction of observation. The van Cittert-Zernike theorem is now written as

$$\mu_{ij} = \frac{\iint l(\xi, \eta) \exp(-2\pi j(u_{ij}\xi + v_{ij}\eta)) \, d\xi \, d\eta}{\iint l(\xi, \eta) \, d\xi \, d\eta} = \frac{\iint l(s) \exp(-j\kappa B_{ij} \cdot s) \, ds}{\iint l(s) \, ds},$$

(1.33)

where $\mu_{ij}$ is the visibility of the fringes for telescopes $i$ and $j$, and $l(\xi, \eta)$ is the intensity distribution of the observed source in angular coordinates. We see that for every baseline, the contrast of the fringe pattern is giving the modulus of one component of the Fourier transformation. In stellar interferometry, the Fourier plane is called the $uv$-plane. Ideally, when the modulus and phase of a sufficient number of components in the $uv$-plane are retrieved, the intensity distribution of the source can be derived by means of a deconvolution, obtaining interferometric images with high angular resolution. But, in reality, the atmosphere introduces optical path delays and tilts in the wavefront even before the light arrives at the telescope, and imaging is a feasible but not easy task, requiring other procedures, like closure phase, to recover the phase of the fringe in order to obtain images [Monnier1999]. Besides, to obtain an image of decent quality and reliability, the coverage of the $uv$-plane has to fulfil the Nyquist sampling theorem, requiring a high number of baselines. Still, high angular information is scientifically very useful for other fields of astronomy, like stellar astrophysics, to measure stellar diameters, study star formation, resolve binaries, and of course, for the detection and characterization of planets outside our solar system. And the cost of acquiring that high angular information with an interferometer is still cheaper than the single aperture telescope that would be needed to have the equivalent angular resolution.

### 1.5 The field of view problem

In the field of optical interferometry several aspects need to be improved or newly implemented. One of these aspects is the extended field of view: what can be done
in order to get a wide interferometric field of view and what are the possible applications of such a goal.

A wide field of view is very important in applications such as the observation of extended or multiple objects or the fringe acquisition and/or tracking on a nearby unresolved object [Beckers1990], and also for the not-trivial task of reducing the observation time. For ground-based arrays the field of view should at least be equal to the isoplanatic patch [Beckers1986].

Many observational studies in astronomy such as studies on galaxy formation and kinematics, star formation, stellar evolution and circumstellar physics as well as astrometric detection of extra-solar planets and binaries require a field of view larger than the point spread function of a single telescope which is of the order of 0.06 arcsec for an 8 meter telescope working at 2 μm.

As explained in Section 1.3, to measure at least the contrast of the fringes, the \( \text{OPD} \) error has to be smaller than the coherence length. In an interferometer, this correction is done for the pointing direction of the telescope, but for any other angle the \( \text{OPD} \) will be different. If this difference is larger than the coherence length, coherent interference of the beams will not take place. Therefore, the interferometric field of view is limited by the spectral resolution, \( \lambda_0/\Delta \lambda \). In fact, it is the product of the spectral by the spatial resolution, \( \lambda_0/B \) [Perrin2001]. For example, the field of view of an interferometer with a baseline of 200 meters observing at 2.2 μm with a spectral resolution of 10 is of approximately 23 mas. A low spectral resolution also produces an effect called “bandwidth smearing” [Thompson1994]. The interferometer observes a finite bandwidth \( \Delta \lambda \), but the external geometrical \( \text{OPD} \) is compensated for the central wavelength \( \lambda_0 \). The averaging of the visibility over the bandwidth produces a radial blurring of the image, the so-called “bandwidth smearing”, a sort of chromatic aberration. One way to increase the field of view is to increase the spectral resolution, but a high spectral resolution may be incompatible with the observation of faint objects. It is thus desirable to keep a moderate spectral resolution, and to find alternative ways to increase the field of view.

In this section we describe how the two different types of interferometers deal with the field-of-view problem depending on the way in which the light beams are combined. We also explain the differences between both approaches and give the reason why we decided to choose the field extension of non-homothetic interferometers as the main topic of this thesis.

### 1.5.1 Existing techniques

There are two main technology concepts to observe a large field of view, depending on the type of beam combination selected: for multiaxial beam
combiners, homothetic mapping, and for coaxial beam combiners, wide field mosaic imaging. They are both shortly described in this section where we also show the drawbacks and advantages of each of them. Finally, in the last part of this section, our new approach is assessed.

**Homothetic mapping**

In homothetic mapping, the configuration of the telescopes as seen from the science source is re-imaged to a smaller scale, but maintaining orientation and relative separations, before the beams interfere. A Fizeau-type instrument is intrinsically a homothetic mapper, where the beam combination scheme has a natural wide field of view only limited by atmospheric anisoplanatism and the correcting adaptive optics system. A Michelson-type telescope array can also be used as a homothetic mapper when the images are recorded in the focal plane and if the exit pupil after the telescopes is an exact demagnified replica of the input pupil as seen by the incoming wavefront. To express it in a simple way, if $M$ is the angular magnification of the telescope, then the effective baseline at the exit pupil has to be $B_0 = B/M$, as illustrated in Fig. 1.6. Pupil rotation has to be accurately controlled to maintain the orientation of the exit pupils. To cophase the longest VLTI baselines to better than 300 nm over a continuous field of view of 4 arcsec the baseline has to be known with an accuracy of a few tens of microns and pupil

![Fig. 1.6](image.png)

Simplified illustration of the homothetic principle: (a) homothetic and (b) non-homothetic output pupil configuration. In the first case, the fringe pattern of an off-axis star is centered in its envelope, while in the second one, the fringe pattern of the off-axis star is shifted from its envelope, limiting the field of view of the non-homothetic interferometer.
1.5 The field of view problem

rotation to within 8 arcsec [D'Arcio1999]. The interferometer then behaves like one huge telescope of which only the fraction of its surface that contains the telescopes is being used. This is the case of the Large Binocular Telescope Interferometer, which actually has a continuous field of view of 1 arcmin operating at 2.2 μm. Homothetic mapping has been anticipated by the designers of the VLT Interferometry Laboratory by providing a pit with a diameter of 2 meters in the interferometric laboratory. This technique is also being developed in the Delft Testbed Interferometer (DTI) [van Brug2003] at the Knowledge Centre of Aperture Synthesis, a collaboration between the Institute for Applied Physics (TNO) and the Delft University of Technology (TUDelft). The DTI is a Fizeau-type interferometer designed to acquire an imaging angle proportional to 2 arcsec within the VLTI setup.

Wide field mosaic imaging

Wide field mosaic imaging is being developed at the Wide-field Imaging Interferometry Testbed [Leisawitz2003]. The technique is analogous to the mosaicing method employed in radio astronomy [Cornwell1994], adapted to a Michelson pupil-plane beam combiner with detection in the image plane. A delay line is used to scan the optical path length through the sky and an N×N pixels array detector records simultaneously the temporal fringe patterns from many adjacent telescope fields. The recorded data needs to be jointly deconvolved to reconstruct the image. If a detector with a large number of pixels is used, and the image plane is sampled at high enough spatial frequency, then this technique could in principle be used to multiply the field size by a factor of N/2, reaching a field of several arcmin. This technique is under development at NASA for the Space Infrared Interferometric Telescope (SPIRIT) and the Submillimeter Probe of the Evolution of Cosmic Structures (SPECS).

1.5.2 Equalised wide-field for non-homothetic arrays

The two methods presented above have some restrictions. Homothetic mapping is related to Fizeau or Michelson-imaging interferometry, and both techniques present the problem that the complexity of the system increase with the field-to-resolution ratio (FRR), which is the number of resolved elements in the desired field [Rousset2001]. For values of the FRR of the order of 100 the requirement of the lateral homothetic pupil mapping is sufficient but for larger fields the longitudinal homothecy must also be considered. Furthermore, Fizeau interferometry has the problem that the signal-to-noise ratio decreases with the
number of telescopes, limiting the number of baselines that can be used in a system. Besides, when the size of the baseline is large compared to the single telescope diameter, the energy is spread over the sidelobes of the diffraction pattern limiting the sensitivity of the instrument [Labeyrie1996]. This makes the solution a good one for instruments with many telescopes and short baselines, but it is not the optimal solution for an interferometer with only three or four telescopes and baselines of hundreds of meters. The mosaicing method can acquire a total field of view of several arcmin, but not in one shot. Actually the observing time is also multiplied by the N/2 factor. The latter is not a technical but a practical problem, because every observation requires long exposures.

In order to avoid these drawbacks we developed a new approach to the problem, i.e., a system that could use a Michelson pupil-plane combination scheme but acquires a wide field of view in one shot, saving also observation time. We introduce this new technique in this thesis, called equalised wide-field approach because it consists of positioning a stair-shaped mirror into an intermediate image plane for each telescope in the array. This allows to correct for the differential delay for off-axis positions, as is illustrated in Fig. 1.7. The shape of the mirror depends on the baseline and the pointing direction, and, as the entrance pupil varies with hour angle [Schoeller2000], it must be actuated during the observation.
1.6 Outline of this thesis

This thesis is focused on the study of a novel method to permit wide-field imaging using a Michelson stellar interferometer. The thesis is organized as follows:

In Chapter 2 the differential Optical Pathlength Difference ($\Delta OPD$) is defined and its dependence on the field angle is determined. Calculations based on a real instrument, the Very Large Telescope Interferometer (VLTI), show that we do not need second order corrections to the $\Delta OPD$ to reach a cophased field of view of approximately 1 arcmin on the sky.

In Chapter 3, the shape of the mirror required to correct the differential OPD in the focal plane of an optical system is calculated.

In Chapter 4, the performance of the staircase mirror is described analytically. The complete process, from an object in the sky to the visibility retrieval, is studied taking into account the effects of placing the mirror in the focal plane. The analytical results of some simple cases, i.e., an off-axis binary star and a disk, are compared to the theoretical visibility curves provided by the van Cittert-Zernike theorem. The effect of the mirror is that not only the modulus of the visibility of the different sub-fields is retrieved, but also the phase relative to the centre of the field. The similarity with the case of radio-interferometric mosaic imaging suggests the use of the same algorithms to obtain images of large fields with this technique. This subject is briefly introduced and its application to optical interferometry is explained. A detailed description of the MATLAB program used to obtain the analytical results is given in Appendix A.

In Chapter 5 the design and performance of the Wide Field Interferometry breadboard is described. Some of the most important factors that can affect the measurement of the visibility are described, and the experimental results proving the feasibility of the concept are presented.

In Chapter 6 we investigate how to retrieve the visibility when a star is focused on the edge of a step. Even though the Optical Pathlength Difference correction is discontinuous, the analytical description of the problem shows that the visibility can be completely recovered, so that no information is lost. Experimental results, demonstrating that the visibility is recovered within a 1% error, are presented. In this chapter we only pay attention to the effect on the visibility because of the edge. A more detailed study on the general effect of the edge on the interferometer can be found in Appendix B, where the diffraction effects due to the edge are studied using Fourier analysis to determine the limits of the approximation presented in Chapter 6.
1.7 References

1.7 References


Introduction


"Most people die of a sort of creeping common sense, and discover when it is too late that the only things one never regrets are one's mistakes.”

Oscar Wilde

CHAPTER 2

Differential Optical Path Difference

In this Chapter, the differential Optical Path Difference ($\Delta OPD$) is defined and its dependence on the field angle is determined. In Section 2.4 some calculations are performed based on a real instrument, the Very Large Telescope Interferometer (VLTI), showing that second order corrections to $\Delta OPD$ are not needed to reach a cophased field of view of approximately 1 arcmin on the sky.

2.1 Introduction

Optical Stellar Interferometry [Labeyrie1978] consists of using two or more telescopes that collect light from a distant object in order to obtain information with very high angular resolution. When two apertures are separated by a distance $B$, called baseline, the flat wavefront that originates from the distant source at a certain angle $\theta$ does not reach the apertures at the same time (see Fig. 2.1). This delay will introduce an Optical Path length Difference ($OPD$) between both arms of the interferometer. If the $OPD$ is larger than the coherence length, the light from both apertures will not interfere [TallonTallon-Bosc1994]. In order to ensure that interference will occur, an extra path in one of the arms of the interferometer should be introduced. When the interferometer is not a Fizeau-type [Beckers1986], an increase in resolution implies a decrease in the field of view. When detecting light coming from a distant stellar source using two telescopes separated a distance $B$, the light arrives first at one of the telescopes. In order to have coherent interference between the beams from each telescope, we need to introduce in one of the rays an extra optical path which will correct the Optical Path
Difference (\(OPD\)), shown in Fig. 2.1. This \(OPD\) is the projection of the baseline \(B\) in the pointing direction \(s\) of the incoming light beam:
\[
OPD = B \cdot s.
\] (2.1)

For the light coming from a different direction \(s'\) the \(OPD'\) is:
\[
OPD' = B \cdot s'.
\] (2.2)

and the differential Optical Path Difference (\(\Delta OPD\)) is the difference between both optical path length differences, i.e,
\[
\Delta OPD = B \cdot (s' - s).
\] (2.3)

The useful interferometric field of view corresponds to the case when the \(\Delta OPD\) is smaller than the coherence length. It is the area of the sky that contains objects whose rays are combined coherently in the combined focus [Beckers1990]. For example, for the Very Large Telescope Interferometer (VLTI) the unvignetted field of view is 2 arcsec [Glindemann2000] and the interferometric field of view is typically smaller than 50 mas. Using the dual-feed facility PRIMA (Phase Referenced Imaging and Micro-arcsec Astrometry) [Glindemann2001a] it will be possible to pick two stars each in a 2 arcsec field of view and separated up to 1 arcmin.
In order to proceed with the calculation of the $\Delta OPD$ as a function of both the pointing direction of the telescopes and the field angle, we need to define next a common reference coordinate system.

### 2.2 Reference coordinate system

As we can see in Fig. 2.2 [Thompson1991], in the equatorial system every object in the sky has a position given by two celestial coordinates that are declination ($\delta$) and right ascension ($RA$). Declination corresponds to latitude projected on the sky. It is measured from $-90^\circ$ (projected south pole) to $+90^\circ$ (projected north pole). Right ascension is the azimuthal angle at which the hour circle of a celestial object is located, the rotation axis taken as the direction of the celestial pole (P). It is measured in units of time. The zero point of $RA$ is the point where the sun crosses the celestial equator in the day of the vernal equinox. The difference between the Local Sidereal Time ($LST$) and the right ascension is called the Hour Angle ($H$). It tells us how much time has to pass or has passed since the source crossed the meridian.

The extra external $OPD$ for any off-axis direction has been calculated as a function of the pointing direction and the baseline vector as shown in Fig. 2.2. We suppose an object S, in a position given by $s$ with declination $\delta$ and hour angle $H$. Two telescopes are considered, T1 and T2, separated by a distance $B$, and positioned at the origin and at the end of baseline vector $B$, respectively. The declination of a vector $s'$, slightly different from the pointing direction, can be represented as $\delta + \Delta \delta$ with hour angle $H + \Delta H$, where the difference can be both positive or negative. The coordinates of the baseline vector $B$ are defined in the local horizon Cartesian system: the XY plane is the observer’s horizon and the North is the X-direction. The Z-direction points to the zenith. The scalar product of the baseline and the pointing vectors is the $OPD$. To calculate this scalar product, we express $B$ and $s$ in the same coordinate system. We define a new Cartesian system $uvw$, where $w$ is parallel to $s$, $u$ is perpendicular to the plane defined by $w$ and the pole (P) and points towards the positive hour angle direction, and $v$ is perpendicular to the plane defined by $u$ and $w$. The following rotation matrix gives the change from $XYZ$ to $uvw$ coordinates:

$$
R = \begin{bmatrix}
\cos H & \sin H \sin L & -\sin H \cos L \\
-sin \delta \sin H & \sin \delta \cos H \sin L + \cos \delta \cos L & -\sin \delta \cos H \cos L + \cos \delta \sin L \\
\cos \delta \sin H & -\cos \delta \cos H \sin L + \sin \delta \cos L & \cos \delta \sin H \cos L + \sin \delta \sin L
\end{bmatrix}
$$

(2.4)
where \( L \) is the latitude of the site of observation. The coordinates of \( B \) in the \( uvw \) coordinate system are:

\[
\begin{bmatrix}
    B_u \\
    B_v \\
    B_w
\end{bmatrix} = R
\begin{bmatrix}
    B_X \\
    B_Y \\
    B_Z
\end{bmatrix},
\]

(2.5)

and the coordinates of \( s \):

\[
\begin{bmatrix}
    s_u \\
    s_v \\
    s_w
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0 \\
    1
\end{bmatrix}.
\]

(2.6)
2.3 Calculation of the differential OPD

As can be seen in Fig. 2.1, the OPD and OPD' at T1 with respect to T2 for the direction given by \( \mathbf{s} \) and \( \mathbf{s}' \) are, respectively, given by:

\[
\text{OPD} = B_w = B_X \cos \delta \sin H + B_Y [\cos L \sin \delta - \sin L \cos \delta \cos H] + B_Z [\cos \delta \cos H \cos L + \sin \delta \sin L] \tag{2.7}
\]

and:

\[
\text{OPD}' = B'_w = B_X \cos(\delta + \Delta \delta) \sin(H + \Delta H) + B_Y [\cos L \sin(\delta + \Delta \delta) - \sin L \cos(\delta + \Delta \delta) \cos(H + \Delta H)] + B_Z [\cos(\delta + \Delta \delta) \cos(H + \Delta H) \cos L + \sin(\delta + \Delta \delta) \sin L] \tag{2.8}
\]

The projection of \( \mathbf{B} \) on the entrance pupil plane of T1 is

\[
\mathbf{B}_p = \begin{bmatrix} B_u \\ B_v \\ 0 \end{bmatrix}. \tag{2.9}
\]

The difference between OPD' and OPD is the differential optical path difference, or differential delay, \( \Delta \text{OPD}_{\text{ext}} = B'_w - B_w \). Considering only the second-order terms of the Taylor’s expansion of the cosine and the sine in Eq. (2.8), we obtain the following expression for the first-order terms of \( \Delta \text{OPD}_{\text{ext}} \):

\[
\Delta \text{OPD}_{\text{ext}} = \Delta \delta (\cos \delta \sin H + B_Y (\cos L \cos \delta \sin L \sin \delta \cos H) - B_Z (\cos L \sin L \cos \delta - \sin L \sin \delta \cos H)) \tag{2.10}
\]

The second-order terms are given by

\[
\Delta \text{OPD}''_{\text{ext}} = \frac{\Delta \delta^2}{2} [-B_X \sin H \cos \delta + B_Y (\sin L \cos \delta \cos H - \cos L \sin \delta)] \tag{2.11}
\]

In practice, a linear approximation of \( \Delta \text{OPD}_{\text{ext}} \) is valid because, in general, for a field of view of several arcmin the second-order terms of the expression are several orders of magnitude smaller than the first-order terms, as will be shown in next section. For example, for the typical parameters used by the VLTI instruments (baseline of 200 meters, central wavelength of 2.2 \( \mu \)m, bandwidth of 0.22 \( \mu \)m) correcting only the first-order terms, the cophased field of view (where the OPD
errors are smaller than the wavelength) is 1 arcmin and the coherenced field of view (where the \( \text{OPD} \) errors are smaller than the coherence length) is 2 arcmin. We have to decide if the \( Z \)-component to \( \Delta \text{OPD}_{\text{ext}} \) has to be taken on account or can be dismissed. Or, equivalently, whether we can consider both telescopes to be coplanar or not. The case of non-coplanar arrays has been extensively studied in radio interferometry [Perley1994]. Two conditions have to be met to consider the array coplanar: the field of view has to be limited to a small enough angular region (at 2.2 \( \mu \text{m} \) for a baseline of 200 meters to approximately 1 arcmin diameter) and the telescopes have to lie on a common plane. In general, when talking of wide field in optical interferometry we refer to something smaller than the arcmin, and usually the telescopes are built on a flat surface so, theoretically (if the engineers do not fail in calculating the position of the telescopes), an optical interferometer can be considered coplanar.

Dismissing the \( Z \)-component in Eq. (2.10), we have plotted the resultant \( \Delta \text{OPD}_{\text{ext}} \) as a function of the declination and hour angle. We can see that the function depends not only on the baseline vector but also on the declination and the hour angle of the pointing vector. The shape of the linearized \( \Delta \text{OPD}_{\text{ext}} \) function is a tilted plane where the constant \( \text{OPD} \) zones, as shown in Fig. 2.3, are perpendicular to \( B_p \). If \( B \) points from T1 to T2, the projection of the baseline in the entrance pupil is parallel to the gradient of \( \Delta \text{OPD}_{\text{ext}} \) at T1.

![Fig. 2.3](image)

**Fig. 2.3** Differential external \( \text{OPD} \) shape; the contour lines are lines of constant \( \text{OPD} \).
2.4 Application to the VLTI

In this section we extrapolate the previous results to the case of the VLTI. First we shortly introduce the instrument, and in the last part of the section we calculate the first and second-order terms of $\Delta OPD_{\text{ext}}$ for two of the telescopes that are composing the interferometer, in order to study the contribution of the last ones.

2.4.1 The VLTI and its instruments

The ESO Very Large Telescope Interferometer (VLTI) is located on Cerro Paranal, in northern Chile. As described in [Richichi2002], this facility consists of four fixed 8.2 m Unit Telescopes (UT), and four 1.8 m Auxiliary Telescopes (AT) which can be moved over an array of 30 stations (see schematic representation in Fig. 2.4). All the AT stations, as well as the UTs, are connected by a network of underground light ducts. Central to the facility is the delay line tunnel, where optical path differences are continually adjusted to correct for both long-range effects (such as those due to sidereal motion) and fast, short-range variations such as those due to differential atmospheric piston. A number of relay mirrors feed the light from the telescopes into the tunnel, and from there to a central underground laboratory, where the beams from two or more chosen telescopes are brought together and coherently combined. Each identical UT is a Ritchey-Chretien type and can operate in cassegrain, nasmyth and coudé focus. The optical parameters of the

Fig. 2.4 VLTI system, from [Glindemann2000].
2 Differential Optical Path Difference

We have calculated the first and second order terms of $\Delta OPD_{\text{ext}}$ for two of the UT telescopes of the VLTI, UT2 and UT3, with respect to the origin of coordinates of VLTI’s reference system, observing an object situated at declination $\delta=-50^\circ47'1''$ and right ascension $RA=18h36m56s$, for two different moments of the Local Sidereal Time.
2.4 Application to the VLTI

Case 1: UT2

The results for UT2 are shown in Fig. 2.5. For a field of view of 1 arcmin angular diameter, one can see that the maximum first order correction is approximately 4.5 mm, and the maximum second order correction is 600 nm, with the latter being of the order of magnitude of the wavelength in the visible spectrum, and at least an order of magnitude smaller than the wavelength in the near-infrared spectrum. Translating the contribution of the second order term to the angle in the sky, for the reference baseline of UT2 (the baseline with respect to the origin of coordinates), it corresponds to 5 milliarcsec.

Fig. 2.5 First and second order corrections of the OPD for UT2 for a field of view of 1 arcmin diameter observing a star at two different moments, separated by 6 hours.
Case 2: UT3

The results for UT3 are shown in Fig. 2.6., also for a field of view of 1 arcmin angular diameter. In this case, the maximum first order correction required is 10 mm. The maximum second order correction is 1400 nm, that, for the reference baseline of UT3, corresponds to 3.5 milliarcsec in the sky.

Fig. 2.6  First and second order corrections of the OPD for UT3 for a field of view of 1 arcmin diameter observing a star at to different moments, separated 6 hours.
2.5 Conclusions

In this chapter, we have defined the differential Optical Path Difference ($\Delta OPD$), and the equation to calculate it as a function of the field angle has been presented. We have shown that we only need to correct the first order terms of the differential $OPD$, because the second order terms of the $OPD$, even for large baselines, corresponds to milliarcsec in the sky within a field of view of 1 arcmin diameter. The shape of the linearized $\Delta OPD_{ext}$ function is a tilted plane where the constant $OPD$ zones are perpendicular to the projection of the baseline on the entrance pupil of the telescope.

2.6 References


 Equalization of the differential Optical Path Difference

In this chapter, the shape of the mirror required to correct the differential OPD in the focal plane of an optical system is calculated. The general calculation of the shape of the mirror is addressed in Section 3.1. A more detailed study, for the particular case of the VLTI is presented in Section 3.2.

3.1 The staircase mirror

As described in Chapter 2 (Eq. (2.10)), a linear approximation of the geometrical external OPD was given by:

\[
\Delta OPD_{ext} = \Delta \delta - B_x \sin \delta \sin H + B_y (\cos L \cos \delta + \sin L \sin \delta \cos H) \\
- B_z (\sin \delta \cos H \cos L - \cos \delta \sin L) + \Delta H (B_x \cos \delta \cos H \\
+ B_y \sin L \cos \delta \sin H - B_z \cos \delta \sin H \cos L)
\]  

The first-order corrections for the OPD can be introduced by means of a staircase mirror situated in the focal plane of the optical subsystems before the beam combination. The focal plane has the characteristic that the light from different parts of the sky is focused separately (Fig. 3.1); that is why we intend to use it to introduce the equalization of the OPD. The correction introduced by one step is:

\[
\Delta OPD = 2d \cos \alpha,
\]

where \(\alpha\) is the angle between the normal to the mirror surface and the optical axis of the system, and \(d\) is the depth of the step. In an intermediate focal plane, with focal length \(F\), the angular coordinates, \(\Delta H\) and \(\Delta \delta\), expressed in radians, correspond to focal plane coordinates \(u = t\Delta H\) and \(v = t\Delta \delta\), where \(t\) is the scale
in the focal plane, in meters per radian. Using this conversion and Eq. (3.1) and Eq. (3.2), we can derive the equation of the planar surface $OPD_{\text{int}}$, to which the shape of the mirror has to be adjusted, in order to correct $\Delta OPD_{\text{ext}}$ in the focal plane:

$$OPD_{\text{int}} = \frac{1}{2\cos \alpha} \left[ u\left[ -B_X \sin \delta \cos H + B_Y \left( \cos L \cos \delta + \sin L \sin \delta \cos H \right) \right] - B_Z \left( \sin \delta \cos H \cos L - \cos \delta \sin L \right) + v \left( B_X \cos \delta \cos H \right. \right.$$  

$$\left. + B_Y \sin L \cos \delta \sin H - B_Z \cos \delta \sin H \cos L \right) \right]$$

where $B_X$ and $B_Y$ are the coordinates of the telescope in the local horizon system, in meters, from an external reference point.

The mirror consists of steps of fixed width and adaptable depth. The depth of the steps must vary, as it has to be adapted to the changes of $B_w$. The mirror has to be rotated to follow the rotation of $B_p$ so that the steps are always perpendicular to the projected baseline. As we explained in Section 2.3, $B_p$ is parallel to the gradient of

Fig. 3.1  OPD equalization; the light from different directions in the field is focused on different steps and this effect introduces an extra OPD that depends on the field angle.
3.1 The staircase mirror

the $OPD_{int}$ function. This property will determine the relation between the width of the steps, $w$, and the depth, $d$. The gradient of the $OPD_{int}$ function is given by the vector $\nabla OPD_{int} = \partial OPD_{int}/\partial u, \partial OPD_{int}/\partial v$. The width of the steps, $w$, is set to be equal to the modulus of a vector $p = a\nabla OPD_{int}$, where $a$ is a distance in meters, and is given by:

$$w = a\left[\left(\frac{\partial OPD_{int}}{\partial u}\right)^2 + \left(\frac{\partial OPD_{int}}{\partial v}\right)^2\right]^{1/2}.$$  \hspace{1cm} (3.4)

The depth of the steps, $d$, is equal to the value of the function $OPD_{int}$ at the point determined by the coordinates of $p$,

$$d = \nabla OPD_{int} \cdot p = a\left[\left(\frac{\partial OPD_{int}}{\partial u}\right)^2 + \left(\frac{\partial OPD_{int}}{\partial v}\right)^2\right].$$  \hspace{1cm} (3.5)

Therefore, $w$ and $d$ are related by the following equation:

$$\frac{w}{d} = \left[\left(\frac{\partial OPD_{int}}{\partial u}\right)^2 + \left(\frac{\partial OPD_{int}}{\partial v}\right)^2\right]^{-1/2}.$$  \hspace{1cm} (3.6)

As $w$ is fixed, it has to fulfil two conditions: it has to be bigger than one Airy disk so that the standard interferometric field of view is not cut, and it has to be valid for correcting the $OPD$ at any pointing direction. We calculate $w$ as follows. The maximum depth is a function of the coherence length $L_c$, the angle $\alpha$ and depends on the maximum value of the optical path for a given scan distance. If the maximum optical path is, for example, $3/2$ of the coherence length, then the maximum depth is $d_{max} = 3L_c/(2\cos\alpha)$. Secondly, given the values of the maximum depth and the baseline, we calculate the minimum width for any pointing direction, $w_{min}$, and check that it is effectively bigger than one Airy disk. If it is not, we choose a larger $d_{max}$. The position of the minimum is given by $\delta_{min}$ and $H_{min}$, solutions of the following system:

$$\frac{\partial w}{\partial \delta_{min}}(\delta_{min}, H_{min}) = 0$$

$$\frac{\partial w}{\partial H}(\delta_{min}, H_{min}) = 0$$

$$\frac{\partial^2 w}{\partial \delta_{min} \partial H}(\delta_{min}, H_{min}) > 0.$$  \hspace{1cm} (3.7)
With the width $w_{\text{min}}=w(\delta_{\text{min}}, H_{\text{min}})$, the expression for $B_p$, and the equation for $\text{OPD}_{\text{int}}$, we can write the function $M(u,v)$ that determines the shape of a mirror, with $(2N+1)$ steps, as follows

$$M(u,v) = \sum_{n_s=1}^{N_s} n_s \cdot \text{rect}\left(\frac{B_u u + B_v v}{w_{\text{min}} B_p} - n_s\right),$$  \hspace{1cm} \text{(3.8)}$$

where $\text{rect}(x) = 1$ when $|x| \leq 1/2$ and zero elsewhere, and

$$d = w_{\text{min}} \left[ \left(\frac{\partial}{\partial u} \text{OPD}_{\text{int}}\right)^2 + \left(\frac{\partial}{\partial v} \text{OPD}_{\text{int}}\right)^2 \right]^{1/2}. \hspace{1cm} \text{(3.9)}$$

In a system consisting of more than two telescopes, as plotted in Fig. 3.2, it is necessary to have a step mirror at the focal plane of each telescope. We need a common reference point for the different baselines and the line parallel to the edge of the steps of each mirror should be perpendicular to the projection of the baseline on the entrance pupil of the particular telescope.

![Fig. 3.2 Mirror position for a telescope system. Telescopes $T_1$, $T_2$ and $T_3$ have different mirrors $m_1$, $m_2$ and $m_3$ with different widths $w_1$, $w_2$, $w_3$ and depths $d_1$, $d_2$, $d_3$. The dividing lines of each staircase mirror should be perpendicular to the projection of each baselines on the entrance pupil of each telescope, $B_{p1}$, $B_{p2}$, $B_{p3}$, defined by the pointing vector $s$.](image-url)
3.2 Application to the VLTI

One interferometer consisting of more than two telescopes is the VLTI, described in the previous chapter. We suppose here the combination of the beams coming from UT2 and UT1 and calculate the shape of the mirror that would be required to correct the $\Delta OPD_{ext}$ in the coudé focus of each Unit Telescope. In the VLTI system, the coordinates of the telescopes are given with respect to a common reference point. The distance from this reference point, what we have called the reference point.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.3}
\caption{Mirror width as a function of declination and hour angle for a given baseline and a maximum depth. The areas with low $w$ values correspond to areas of low gradient of the $OPD$ function.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.4}
\caption{Mirror depth as a function of declination and hour angle, for a given baseline and a minimum width.}
\end{figure}
baseline, for UT2 is 24x21/2 meters. Using a wavelength of 2.2 µm and a bandwidth of 0.22 µm, the coherence length of the light is 22 µm. In this case, we consider the telescopes to be coplanar and we therefore dismiss the term given by $B_z$. If the normal of the mirror is at an angle of 60° with the optical axis the maximum depth of the steps is 66 µm. With this depth, we plot the width depending on declination and hour angle as shown in Fig. 3.3. We take the minimum of this function as width of the steps. In the case of UT2, the minimum width is 802 µm. The size of the PSF of a single UT at a wavelength of 2.2 µm in the coudé focus is 136 µm, therefore, the minimum width we have calculated is valid. With this width,

![Graph](image)

**Fig. 3.5** (a) Mirror depth plotted as a function of the Local Sidereal Time for a given width, declination and baseline, for UT2. (b) Projection of the baseline vector on the entrance pupil. Due to the rotation of the earth, the telescope has to change the pointing direction to follow an object in the sky, resulting in the rotation of $B_p$. 
we obtain the depth of the steps as a function of declination and hour angle, as shown in Fig. 3.4. Note that this function never takes on a value larger than 66 μm for any direction in the sky. We now point UT2 to an object situated at declination \( \delta = -50^\circ 47' 1'' \) and right ascension \( RA = 18h36m56s \). Celestial objects are at constant \( RA \), but their hour angle \( H \) changes as time proceeds, observing the relation \( H = LST - RA \), where \( LST \) is the Local Sidereal Time [Thompson1991]. Using a mirror with \( w = 802 \) μm, the variation of the depth as a function of the Local Sidereal Time and the rotation of \( B_p \) are shown in Fig. 3.5a and Fig. 3.5b, respectively. The shape of the mirror is shown in Fig. 3.6 for three different moments during the observation, with a difference of 3 hours between them.
Next, we also show the calculations for the UT1. Its coordinates in the VLTI system, from the common reference point, are (-16,-16), and its reference baseline is $16 \times 2^{1/2}$ meters. If the normal of the mirror is at an angle of 60° with the optical axis the maximum width of the steps is 1203 µm, again smaller than the PSF of a single UT. Following the same calculations as for the UT2, we show in Fig. 3.7 and Fig. 3.8 conditions for step depth and mirror rotation, respectively, for the UT1.

![Graph](image)

**Fig. 3.7** (a) Mirror depth plotted as a function of the Local Sidereal Time for a given width, declination and baseline, for UT1. (b) Projection of the baseline vector on the entrance pupil.
3.3 Conclusions

In this chapter the general procedure has been given to calculate the shape of the mirror required to correct the differential OPD in the focal plane of a telescope in an array. The mirror consists of steps of fixed width and adaptable depth. The depth of the steps must vary, as it has to be adapted to the changes of the pointing vector, and the mirror has to be rotated in order to follow the rotation of the baseline projected in the entrance pupil, so that the steps remain perpendicular to

Fig. 3.8 Simulation of a staircase mirror placed in the coudé focus of UT1. The shape of the mirror is shown for three different values of the Local Sidereal Time, (a), (b) and (c), at intervals of 3 hours. The uv plane is the focal plane. The black lines are the projections of the mirror steps on the uv plane. The arrows are the gradient of the OPD, and are perpendicular to the steps. The baseline projected at the entrance pupil is parallel to the gradient of the OPD. During observation, as the projection of the baseline on the entrance pupil rotates, the mirror follows this movement.
Equalization of the differential Optical Path Difference

The width of the steps is fixed, and the procedure to calculate its value has been shown in this chapter. One way to build this variable mirror would be to have several reflecting strips placed on top of, e.g., piezo actuators. Two sets of actuators would be needed: a set to provide long strokes, and a set to provided fine positioning. The required accuracy in the fine positioning of the steps should be in the order of tens of nanometers, which, for a baseline of 100 meters, translates to 100 \( \mu \)arcsec on the sky. This requirement is met by commercial piezo actuators. The steps must be maintained parallel to each other. This can be done by placing at least two actuators in each mirror strip. If the actuators are 10 millimeters apart, an accuracy of 10 nm in the positioning of each actuator brings a parallelism of 1 \( \mu \)rad.

3.4 References

Michelson Wide-Field Interferometry

In this chapter the performance of the staircase mirror in a wide-field imaging interferometer is described analytically. The complete process, from an image in the sky to the visibility retrieval, is studied taking into account the effects of placing the mirror in the focal plane. The analytical description of the interferometer is assessed in Section 4.1. The most important effect, the equalisation of the optical path and, through this equalisation, the simultaneous correction of the path for the off-axis and the on-axis light, is described in Section 4.2. The retrieval of the visibilities for these off-axis sources is explained in Section 4.3, first for simple cases, an off-axis binary star and a disk, and secondly for an image composed of two disks in different sub-fields, and the analytical results are compared to the theoretical visibility curves provided by the van Cittert-Zernike theorem. Finally radio-interferometric mosaic deconvolution is briefly introduced and its application to optical interferometry is explained.

4.1 Analytical description of the interferometer

We first describe the case of a system consisting of a two-arm interferometer, like the one shown in Fig. 4.1. The apertures $A$ and $A'$ are in the plane $\Sigma(\xi, \eta)$ at positions $(\xi_A, \eta_A)$ and $(\xi_A', \eta_A')$, and their radii are $\rho_A$ and $\rho_{A'}$, respectively. The apertures of both arms are separated by a distance $B$, called baseline, given by a vector $B(\xi_B, \eta_B, 0)$, with $\xi_B = \xi_{A'} - \xi_A$ and $\eta_B = \eta_{A'} - \eta_A$. This interferometer is used to observe a monochromatic object in the sky represented by its specific power, or power at a frequency $\nu$, $L_\nu(\text{WHz}^{-1})$ [Karttunen1994]. We also define the flux density, or flux at frequency $\nu$, $F_\nu(\text{Wm}^{-2}\text{Hz}^{-1})$. In astronomy, the observed flux densities are generally small, so that they are often expressed in
Janskys (Jy), with 1 Jy corresponding to $10^{-26}$Wm$^{-2}$Hz$^{-1}$. The relation between the flux density and the specific power is given by:

$$L_{\nu} = \iint_{A} F_{\nu} d\xi d\eta = \pi \rho_{A}^{2} F_{\nu}. \quad (4.1)$$

The object subtends a solid angle $S$ and its flux density can be described as the integration of the contributions from the solid angle element $d\omega$ centred at $(\theta, \varphi)$ (see Fig. 4.1), in spherical coordinates as

$$F_{\nu} = \frac{1}{S} \left[ I_{\nu}(\theta, \varphi) \cos(\theta) d\omega \right] \frac{1}{S} \left[ I_{\nu}(\theta, \varphi) \cos(\theta) \sin(\theta) d\theta d\varphi \right], \quad (4.2)$$

where $I_{\nu}(\theta, \varphi)$, expressed in units of Wm$^{-2}$sterad$^{-1}$Hz$^{-1}$, is the specific intensity of the element $d\omega$.

The geometrical external Optical Path Difference for arm $A'$ with respect to arm $A$ is given by the scalar product of the baseline vector $B = (\xi_{B}, \eta_{B}, 0)$ and the unit vector pointing to the element $d\omega$ in cartesian coordinates $s = (\xi_{s}, \eta_{s}, \zeta_{s})$, with $\xi_{s} = \sin \theta \cos \varphi$, $\eta_{s} = \sin \theta \sin \varphi$ and $\zeta_{s} = \cos \theta$. The light from each arm is focused and then recollimated to reduce the size of the pupil. In the intermediate focal plane of the arm from aperture $A'$ we place the staircase which introduces a field position dependent extra path length corresponding to a phase shift. After recollimating the beams, these are combined in the pupil plane by means of a beam splitter, and the combined beam is focused again on a CCD camera or detector so that the effect of phase shifting can be studied.

According to the Huygens-Fresnel principle [Goodman1996], the diffracted amplitude $U(x, y)$ created by an aperture $\Sigma(\xi, \eta)$, illuminated by monochromatic light of frequency $\nu$, at a plane $P(x,y)$ situated at a distance $z$ is

$$U_{\nu}(x, y) = \int_{-\infty}^{\infty} h(x, y; \xi, \eta) u_{\nu}(\xi, \eta) d\xi d\eta, \quad (4.3)$$

where, in accordance to Kirchhoff boundary conditions, $u(\xi, \eta)$ is identically equal to zero outside the aperture $\Sigma$. Assuming that the distance between aperture and observation planes is much greater than the maximum linear dimension of the aperture and that, in the plane of observation, only a small region around the $z$ axis is of interest and, finally, that the distance $z$ is much greater than the maximum linear dimension of this region, the weighting function $h$ is approximated by

$$h(x, y; \xi, \eta) \approx \frac{1}{j\lambda z} \exp(j\kappa r_{01}), \quad (4.4)$$

with

$$r_{01} = \sqrt{Z^2 + (x^2 - \xi^2) + (y^2 - \eta^2)} \quad (4.5)$$
Fig. 4.1 Light coming from an object situated at a large distance from the apertures $A$ and $A'$ is collected by them and focused by an optical system. In the focal plane of $A'$ a mirror is equalising the optical path. Light is then collimated again, combined by overlapping the beams, shown schematically in the figure, and focused at plane $(x,y)$. In the simulations, all the lenses have the same focal length $f$ and the system can be simplified to the one described in the picture.
If we assume that the distance \( z \) is large enough to use the Fraunhofer approximation, the diffraction pattern produced by each individual telescope for a wavelength \( \lambda \) at a distance \( z \) will be given by

\[
U_\nu(x, y) = \frac{\exp(jkz)\exp\left[\frac{jk}{2z}(x^2 + y^2)\right]}{j\lambda z} \int_0^\infty \int_{-\infty}^{\infty} u_\nu(\xi, \eta)\exp\left(-\frac{jk}{2z}(\xi x + \eta y)\right)d\xi d\eta. \tag{4.6}
\]

That is just the Fourier transform of the aperture distribution with respect to the angular frequency coordinates \( (x/\lambda z, y/\lambda z) \). For a circular aperture of radius \( \rho_A \), using radial coordinates \( \rho = \sqrt{\xi^2 + \eta^2} \) and \( r = \sqrt{x^2 + y^2} \), we define the aperture transmission function

\[
t_t(\rho) = T_A c \text{irc} \left( \frac{\rho}{\rho_A} \right), \tag{4.7}
\]

where \( T_A c \) is a frequency-dependent amplitude transmission factor. Because of the circular symmetry, the Fourier transform can be rewritten as a Fourier-Bessel transform and, in radial coordinates:

\[
U(r) = \frac{\exp(jkz)\exp \left( \frac{k r^2}{2z} \right)}{j\lambda z} \Im \left\{ u_\nu(\rho) \right\} |_{r = r/(\lambda z)}, \tag{4.8}
\]

where \( \Im \) denotes the Fourier transform and \( f_r \) the radial angular frequency with respect to which the transformation is carried out. The function \( u_\nu(\rho) \) is the transmitted flux density, i.e., the transmission function of the telescope multiplied by the square root of the flux density of the observed source. The contribution to the flux density of the solid angle element situated in \((\theta, \varphi)\) is

\[
d^2 F_\nu = I_\nu(\theta, \varphi) \cos(\theta) \sin(\theta) d\theta d\varphi, \tag{4.9}
\]

and the corresponding expression for the complex amplitude due to the infinitely small source element is

\[
du_\nu(\rho) = \sqrt{d^2 F_\nu} \cdot t(\rho). \tag{4.10}
\]

The contribution of the solid angle element situated in \((\theta, \varphi)\) to the complex amplitude in the image plane \([W^{1/2}\text{m}^{-1}\text{sterad}^{-1}]\) of the telescope is given by

\[
dU_\nu(x, y; \theta, \varphi) = \frac{CT_A c}{j\lambda f} \sqrt{d^2 F_\nu} \pi \rho_A \begin{bmatrix} J_1 \left( \kappa \sqrt{((x-x_f)^2 + (y-y_f)^2) \frac{\rho_A}{f}} \right) \\ \frac{\kappa \sqrt{((x-x_f)^2 + (y-y_f)^2) \frac{\rho_A}{f}}} \end{bmatrix} \sqrt{d\theta d\varphi}. \tag{4.11}
\]
4.1 Analytical description of the interferometer

Where $f$ is the focal length of the telescope, $C = \exp(j\kappa f)\exp\left(\frac{kr^2}{2f}\right)$, and $(x_f, y_f)$ is the position in the focal plane of telescope A of the element at $(\theta, \varphi)$, and is given by $x_f = -\xi_f = -f\theta\cos\varphi$; $y_f = -\eta_f = -f\theta\sin\varphi$, where, for small angles $\theta$, we have considered $\tan\theta \approx \theta$.

The infinitesimal contribution to the modulus squared of the complex amplitude in the image plane of the telescope is given by

$$dD_{\nu}(x, y; \theta, \varphi) = \frac{T_A^2}{(\lambda f)^2} \pi \rho_A^2 \left[ \frac{J_1\left(\kappa_{\nu A}\left((x-x_f)^2 + (y-y_f)^2\right)\rho_A\right)}{\kappa_{\nu A}\left((x-x_f)^2 + (y-y_f)^2\right)\rho_A f} \right]^2 dL_{\nu}, \quad (4.12)$$

with

$$dL_{\nu} = \pi \rho_A^2 I_{\nu}(\theta, \varphi) \cos(\theta) \sin(\theta) d\theta d\varphi. \quad (4.13)$$

To obtain the total flux density for frequency $\nu$ in the image plane of the telescope we have to integrate Eq. (4.12) with respect to the solid angle subtended by the source:

$$D_{\nu}(x, y) = \frac{T_A^2}{S} \pi \rho_A^2 \int \sum_{\nu} I_{\nu}(\theta, \varphi) \cos(\theta) \sin(\theta) d\theta d\varphi \quad (4.14)$$

with $Q_A = \pi N_A^2 \rho_A^2 / \lambda^2$ and $N_A = \rho_A / f$. If the source is not monochromatic but emitting in a certain bandwidth $\Delta \nu$, the total flux is

$$D_A(x, y) = \int_{\Delta \nu} D_{\nu}(x, y) d\nu. \quad (4.15)$$

In the special case when the observed source can be described as a set of Dirac’s delta functions positioned at $(\theta_i, \varphi_i)$, with the property $\int \delta(\theta, \varphi) d\theta d\varphi = 1$, we obtain the specific intensity

$$I_{\nu}(\theta, \varphi) = \sum_{i=1}^n F_{\nu i} \delta(\theta - \theta_i, \varphi - \varphi_i). \quad (4.16)$$
The flux density in the image plane of the telescope is

\[ D_{A'}(x, y) = T_{A'}^2 Q_{A'} \sum_{i=1}^{n} F_{vi} \left[ 2 \frac{J_1(\kappa N_{A'}((x-x_i)^2 + (y-y_i)^2))}{\kappa N_{A'}} \right]^2, \]

(4.17)

with \( x_i = -f_i \cos \phi_j \) and \( y_i = -f_i \sin \phi_j \).

We are interested in studying the interference of the light transmitted through the two telescopes \( A \) and \( A' \). The interference pattern is the coherent superposition of the complex amplitudes of the two beams that are combined at the beam splitter. The flux density resulting from this coherent superposition is

\[ D_v(x, y) = \]

(4.18)

\[ \sum_{i=1}^{n} \left[ T_{A'}^2 Q_{A'} E_{A'i} + T_{A'}^2 Q_{A'} E_{A'i}^2 + 2 T_{A'} Q_{A'} E_{A'i} T_{A'} Q_{A'} E_{A'i} \cos(\psi_{A'A}(x_i, y_i) + \phi_{A'A}) \right] \]

where \( \psi_{A'A}(x_i, y_i) \) is the difference in phase for element \( i \) for both telescopes with respect to a common reference point, \( \phi_{A'A} \) is a phase term that contains any possible phase shift due to transmission or reflection effects, and, assuming that both telescopes have the same focal length and that the optics used to focus the combined light has the same focal length as the telescopes, \( E_{A'i} \) is

\[ E_{A'i} = F_{vi} \left[ 2 \frac{J_1(\kappa N_{A'}((x-x_i)^2 + (y-y_i)^2))}{\kappa N_{A'}} \right]^2, \]

(4.19)

and a similar expression can be written for \( E_{A'i} \). Assuming also that both telescope apertures have the same radius, so \( N_{A'} = N_{A} = N \), we have that \( E_{A'i} = E_{A'i}' = E_i \), and we can rewrite Eq. (4.18) as

\[ D_v(x, y) = Q_v \sum_{i=1}^{n} E_i \left[ T_{A'}^2 + T_{A'}^2 Q_{A'} E_{A'i} + 2 T_{A'} Q_{A'} E_{A'i} \cos(\psi_{A'A}(x_i, y_i) + \phi_{A'A}) \right]. \]

(4.20)

As explained before, the external phase difference \( \psi_{A'A}(x_i, y_i) \) is given by the scalar product of the baseline vector \( B = (\xi_B, \eta_B, 0) \) and the unit vector pointing to the element \( i \), \( s_i = (\xi_{si}, \eta_{si}, \zeta_{si}) \):

\[ \psi_{A'A}(x_i, y_i) = \kappa B \cdot s_i = \kappa(\xi_B \sin \theta_i \cos \phi_j + \eta_B \sin \theta_i \sin \phi_j). \]

(4.21)
4.2 Differential OPD correction

In general, \( \theta_i \) will be smaller than one arcmin, and in that case, \( \sin \theta_i \approx \theta_i \). Changing to the coordinates in the image plane, the phase difference in the image plane will be

\[
\psi_{A'A}(x_fi, y_fi) = \kappa(\xi_B x_{fi} + \eta_B y_{fi})^\frac{1}{f}.
\]  

(4.22)

Because we combine both amplitudes in the pupil plane along a common axis, the combined interference pattern will not be spatially modulated and in order to observe fringes and calculate the visibilities we need to modulate it temporally. We will introduce in arm \( A' \) a phase difference \( \psi(t) = \kappa \Delta(t) \).

If we want to see constructive interference at the centre of the field, which coordinates are \( (x_{fn/2}, y_{fn/2}) \), we need to make \( \psi(t) = -\psi_{A'A}(x_{fn/2}, y_{fn/2}) - \psi_{A'A} \).

In our case \( (x_{fn/2}, y_{fn/2}) = (0,0) \), so \( \psi_{A'A}(x_{fn/2}, y_{fn/2}) = 0 \). When observing with non-monochromatic light, with coherence length \( L_c = \lambda^2/\Delta \lambda \), if the maximum path scanned to observe temporal fringes is a factor \( a \) of the coherence length \( \Delta(t)_{\text{max}} = a L_c \), coherent interference do not occur for the points with \( k\psi_{A'A}(x_{fi}, y_{fi}) > a L_c \) and their visibilities can not be retrieved.

4.2 Differential OPD correction

Our aim is to correct this differential OPD that depends on the field position. This is done by introducing an extra internal Optical Path Difference that depends on position in the focal plane. As explained in Chapter 3, a staircase-shaped function is chosen. The internal phase function introduced by a stair-shaped mirror with \((2N+1)\) steps placed in the focal plane is given by

\[
\psi_{\text{int}}(x, y) = \sum_{n_s=-N}^{N} \kappa n_s \text{rect}\left(\frac{\xi_B x + \eta_B y}{w_B} - n_s\right),
\]  

(4.23)

with the function \( \text{rect}(x) \) given by \( \text{rect}(x) = 1 \) when \( |x| \leq 1/2 \) and zero elsewhere. The width of the steps of the mirror, \( w \), is calculated as explained in Section 3.1, and the depth \( d \) is given by

\[
d = \frac{w}{2 \kappa \cos \alpha} \left[ \left( \frac{\partial}{\partial x} \psi_{A'A}(x, y) \right)^2 + \left( \frac{\partial}{\partial y} \psi_{A'A}(x, y) \right)^2 \right]^{1/2},
\]  

(4.24)

\[
d = \frac{w}{2 f \cos \alpha} \left( \xi_B^2 + \eta_B^2 \right) = \frac{w}{2 f \cos \alpha} \left( \xi_B^2 + \eta_B^2 \right).
\]
where $\alpha$ is the angle that the normal to the surface of the mirror forms with the 
optical axis of the system. With the temporal modulation and the staircase mirror in 
the focal plane of arm $A'$, the flux density of the combined light is 

$$D_v(x, y, t) = Q_v \sum_{i=1}^{\infty} E_i [T_{A^v}^2 + T_{A'^v}^2]$$  

(4.25) 

$$2T_{A^v}T_{A'^v} \cos[\psi_{A^v}(x_{fB}, y_{fB}) + \phi_{A^v} + \psi_{int}(x_{fB}, y_{fB}) + \psi(t)]$$ 

with the conditions given by Eq. (4.23). The external OPD is completely corrected 
for the source points focused in the middle of the steps, where 

$$\xi_B x + \eta_B y = n_s w \quad n_s = 0, \pm 1, \pm 2 \ldots \pm N.$$  

(4.26) 

For the points not located on the middle of the steps, the external OPD is not 
completely corrected, but the difference is always smaller than $\Delta(t)_{\text{max}}$, assuring 
partially coherent interference for the entire field determined by the field of view 
and the number of steps of the staircase mirror.

The integration of the flux density over the spectrum of the observed source gives 
the flux 

$$D(x, y, t) = \int D_v(x, y, t)dv.$$  

(4.27) 

Integrating the flux in a region $R$, the power, $L(t)$, of the object that is in that region 
is obtained, where the time dependence is due to the temporal modulation of the 
optical path 

$$L(t) = \int \int D(x, y, t)dxdy.$$  

(4.28) 

The power is a temporal fringe pattern, and is used to calculate the visibility. 
The modulus of the visibility $\mu(B)$ of the object situated in region $R$ is calculated using 
the relation 

$$\mu(B) = \frac{L_{\text{max}} - L_{\text{min}}}{L_{\text{max}} + L_{\text{min}}},$$  

(4.29) 

where $L_{\text{max}}$ is the maximum power, obtained when the temporal modulation of the 
internal optical path has a value such that the phase difference between the arms 
of the interferometer is 0 and $L_{\text{min}}$ is the minimum power, obtained when the phase 
difference is $\pi$.

The position of an object relative to a reference source, equivalent to the relative 
phase of the visibility, $\beta$, can be calculated by measuring the distance from the 
peak of the power fringe pattern of the object to the peak of the power fringe
4.3 Wide-field imaging

4.3.1 Introduction

The main goal of a wide-field interferometer is to recover the visibility curves of extended objects, or for objects that are out of the standard interferometric field of view. In our approach to wide-field interferometric imaging, we aim at a “one-shot” full-field coverage, using all the available photons. For the reasons mentioned in [Glindemann2001], we prefer not to use homothetic mapping and we avoid the sequential “mosaicing” technique [Zhang2001] because of speed and efficiency reasons. Referring to radio interferometry techniques, we choose for the optical equivalent to the solution suggested in [Cornwell1994] where an array of detectors is placed in the focal plane of the interferometer antenna array. Simulations have been carried out to study the performance of our wide field interferometer to retrieve the sampled visibility function of objects out of the narrow interferometric field and of large objects. Using a MATLAB program, visibilities have been calculated for single objects located at different steps of our staircase correction element. A description of the program, as well as the results from the simulations, are provided in Appendix A. In Section 4.3.2, the theoretical visibility, calculated with the van Cittert-Zernike theorem [BornWolf1999], is compared with the analytically calculated one. The last one corresponds to the Michelson contrast, defined with the maximum and minimum calculated by integration of the flux when interference is totally constructive or destructive, respectively, as shown in Eq. (4.29). All the objects are non-resolved by the single aperture. The width of the steps is chosen based on the maximum scanned path and the maximum baseline, and is fixed for all the baselines; only the depth of the steps is adapted. As expected, for very extended objects, the results are not good when the size of the object convolved with the PSF is comparable to the width of the step, but anyway these extended structures can be resolved with a single telescope and have no special interest for an interferometer, because the information relative to large
structures is mostly present in the short spatial frequencies. That is why only small structures are studied (Section 4.3.2), or extended objects consisting of many small structures (Section 4.3.3). In the last section the mosaicing procedure for the reconstruction of these large images built up from small structures is explained, along with the possibility of combining the short spacing information [EkersRots1979].

4.3.2 Off-axis small structures

Off-axis binary star

The first object studied is a binary star situated at “one field” distance from the phase reference object (this means, if the reference object is on-axis, the binary is focused at the centre of the first step, where \( n_s = 1 \), according to Eq. (4.23)). It is composed of two stars separated by an angular distance smaller than the diffraction limit, thus the binary is not resolved by the single aperture, and the single stars are point sources for the interferometer

\[
I_\nu(\theta, \varphi) = \sum_{i=1}^{2} F_{\nu i} \delta(\theta - \theta_i, \varphi - \varphi_i),
\]

where \((\theta_1, \varphi_1)\) and \((\theta_2, \varphi_2)\) are the angular positions of the stars forming the binary with respect to the phase reference object, and \(F_{\nu i}\) is the specific flux for each star at frequency \(\nu\). The complex degree of coherence of an object, calculated from the van Cittert-Zernike theorem [BornWolf1999], as explained in Chapter 1 (Section 1.4), is:

\[
\mu_\nu(B, \nu) = \frac{\int I(\theta, \varphi) \exp(-2\pi i (\nu \gamma_0 \sin \theta \cos \varphi + \nu \gamma_1 \sin \theta \sin \varphi)) d\theta d\varphi}{\int I(\theta, \varphi) d\theta d\varphi}. \tag{4.32}
\]

If \(\varphi_1 = \varphi_2 = 0\), the phase centre of the binary is \(\bar{\theta} = (\theta_1 + \theta_2)/2\). The coordinates of the projection of the baseline vector in the plane perpendicular to the direction given by \(\bar{\theta}\) are used to calculate \(u_{ij}\) and \(v_{ij}\), as shown in Eq. (1.31). In Fig. 4.2 we have plotted the case when \(\xi_B = B; \eta_B = 0\). Then we have that \(u_{ij} = \nu \xi_B \cos \bar{\theta} / c\) and \(v_{ij} = 0\). Using Eq. (4.31) we obtain

\[
\mu_\nu(B, \bar{\theta}, \nu) = \frac{F_{\nu 1} \exp(-j\kappa \theta_1 B \cos \bar{\theta}) + F_{\nu 2} \exp(-j\kappa \theta_2 B \cos \bar{\theta})}{F_{\nu 1} + F_{\nu 2}}, \tag{4.33}
\]
where $vB \cos \tilde{\theta}/c$ is the angular spatial frequency observed for an effective baseline $B \cos \tilde{\theta}$ at frequency $v$. We want to calculate the visibility integrated over a bandwidth. For simplicity we will consider the bandwidth pattern as a “top hat” function centered on $v_0$, and the spectral power to be constant over the bandwidth. The integration respect to frequency yields:

$$
\mu(B, \tilde{\theta}) = \int_{\Delta \nu} \mu_v(B, \nu) d\nu = \int_{v_0 - \Delta \nu/2}^{v_0 + \Delta \nu/2} \sum_{i=1,2} F_{\nu_i} \exp(-j \kappa_0 B \cos \tilde{\theta}) \sum_{i=1,2} F_{\nu_i}.
$$

(4.34)

And changing to wavelength notation the theoretical visibility of the binary is

$$
\mu(B, \tilde{\theta}) = \frac{\sum_{i=1,2} F_i \exp(-j \kappa_0 B \cos \tilde{\theta}) \sin c(\pi \kappa_0 B \cos \tilde{\theta}/L_c)}{\sum_{i=1,2} F_i}.
$$

(4.35)

where $\kappa_0 = 2\pi/\lambda_0$ is the wave number for the central wavelength $\lambda_0$, $F_i = \Delta \lambda F_{\lambda_i} = \Delta \nu F_{\nu_i}$, and $L_c = \lambda_0^2/\Delta \lambda$ is the coherence length.

We want to compare the visibility obtained in Eq. (4.35) with the one obtained with Eq. (4.29). We consider $F_{\lambda_i} = F_{\nu_i}$ for $i = 1, 2$. As described before (see Fig. 4.2 Illustration of the calculation of the projected baseline.)
Eq. (4.25)), considering two telescopes with same collecting area, the flux density at the focal plane for a frequency $\nu$ is

$$D_\nu(x, y, t) = 2 T^2_{A'} Q^2_{\nu} \sum_{i=1}^{2} E_i \left[ 1 + \cos \psi_i \right],$$

(4.36)

with $\psi_i = \psi_{A'A}(x_{fi}, y_{fi}) + \phi_{A'A} - \psi_{int}(x_{fi}, y_{fi}) + \psi(t)$ and, for the particular case of the binary star we are studying

$$E_i = F \nu \left[ \frac{2 J_1 \left( \kappa N_{\nu} \left( \frac{(x-x_{fi})^2 + y^2}{\kappa N_{\nu}} \right) \right)^2}{\kappa N_{\nu} \left( \frac{(x-x_{fi})^2 + y^2}{\kappa N_{\nu}} \right)} \right],$$

(4.37)

with $x_{fi} = -f_{0j}$. Considering again a "top hat"-shaped bandwidth and a constant spectral power, changing to wavelength notation and integrating with respect to wavelength like previously, we obtain

$$D(x, y, t) = 2^{2} \sum_{i=1}^{2} E_i \left[ 1 + \cos \psi_i \sin c(\pi \psi_i / \lambda_c) \right],$$

(4.38)

where $P = \Delta \lambda T^2_{A \lambda} Q_{\lambda} = \Delta \nu T^2_{A \nu} Q_{\nu}$. As explained in the previous section, integrating the flux in the region of the focal plane where the binary is focused, the power, $L(t)$, of the binary is obtained, where the time dependence is due to the temporal modulation of the optical path introduced in order to observe fringes. With the maximum of this temporal fringe pattern, $L_{\text{max}}$, and the minimum, $L_{\text{min}}$, the Michelson contrast is obtained, equivalent in this case to one component of the visibility curve of the binary:

$$\mu(B, \bar{\nu}) = \frac{L_{\text{max}} - L_{\text{min}}}{L_{\text{max}} + L_{\text{min}}}. \quad (4.39)$$

In Fig. 4.3, and Fig. 4.4, the visibility calculated according to van Cittert-Zernike and the one obtained from the contrast of the fringe pattern are compared for different binary stars and different bandwidths. We use the parameters of the experiment described in the next section: focal length $f=6563$ mm, apertures with radius $r_A=10$ mm, baseline varying from 20 to 60 mm and central wavelength $\lambda_0 = 575$ nm. The size of the PSF of the system is approximately 300 $\mu$m. The first binary system has an angular separation of 2.20x10^{-5} rad, and a bandwidth of 10 nm has been used for the calculation. For this bandwidth, if the maximum optical path that we want to scan is $2L_c$, the width of the steps of the mirror, calculated using the maximum baseline, is 3.6 mm. The second system has a larger angular separation, 2.54x10^{-5} rad and a bandwidth of 130 nm. For this bandwidth, the width of the steps of the mirror is 278 $\mu$m. As can be seen in Fig. 4.3, and 4.4, the
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Fig. 4.3 Visibility calculated according to van Cittert-Zernike (solid black line) and the one obtained from the contrast of the fringe pattern (black dots) of a binary star with an angular separation of $2.20 \times 10^{-5}$ rad, focused at a distance of one field size from the phase reference. The spectral bandwidth is 10 nm.

Fig. 4.4 Same as Fig. 4.3, but now with an angular separation of $2.54 \times 10^{-5}$ rad. The spectral bandwidth is 130 nm.
two results are in good agreement, not only in the ideal case of quasi-
monochromatic light, but also in the case of a broad bandwidth, when the width of
the step is of the order of the PSF. This shows that it is possible, using the
staircase mirror, to extend the interferometric field of view and recover the
information from objects that are not at the pointing centre.

Off-axis uniform disk

Considering that the width of the step in the staircase mirror is calculated using the
maximum baseline, this width is only 1.5 times the width of the PSF for a maximum
baseline of 60 cm. A uniform disk that is partially resolved by the single aperture is
chosen. A disk of angular radius $\rho$, situated at an angular position $(\theta = \theta_0, \phi = 0)$
from the phase reference, is described by

$$I_{\nu}(\theta, \phi) = I_{\nu}^{\text{circ}} \left( \frac{\sqrt{(0 - \theta_0)^2 + \phi^2}}{\rho} \right). \quad (4.40)$$

The visibility calculated with the van Cittert-Zernike theorem, is

$$\mu_{\nu}(B, \theta_0, \nu) = 2 \frac{J_1(\pi \rho \nu B \cos \theta_0 / c)}{\pi \rho \nu B \cos \theta_0 / c} \exp(-2\pi\nu \theta_0 B \cos \theta_0 / c). \quad (4.41)$$

The spectral response is considered constant over the bandwidth, again a “top hat”
function centered at $\nu_0$. Integrating for the bandwidth we obtain

$$\mu(B, \theta_0) = \int_{\nu_0 - \Delta\nu / 2}^{\nu_0 + \Delta\nu / 2} 2 \frac{J_1(\pi \rho \nu B \cos \theta_0 / c)}{\pi \rho \nu B \cos \theta_0 / c} \exp(-2\pi\nu \theta_0 B \cos \theta_0 / c) d\nu. \quad (4.42)$$

Assuming $F_\nu$ constant over the spectral range and evaluated at a wave number
$\kappa_0$ which is the average wave number of the source, we obtain

$$\mu(B, \theta_0) = 2 \frac{J_1(\pi \rho \nu_0 B \cos \theta_0 / c)}{\pi \rho \nu_0 B \cos \theta_0 / c} \int_{\nu_0 - \Delta\nu / 2}^{\nu_0 + \Delta\nu / 2} \exp(-2\pi\nu \theta_0 B \cos \theta_0 / c) d\nu. \quad (4.43)$$

The final result, expressed again as a function of the coherence length reads

$$\mu(B, \theta_0) = 2 \frac{J_1(\pi \rho B \cos \theta_0 / \lambda_0)}{\pi \rho B \cos \theta_0 / \lambda_0} \sin\left(\frac{\pi \theta_0 B \cos \theta_0}{L_c}\right) \exp\left(-\frac{2\pi \theta_0 B \cos \theta_0}{\lambda_0}\right). \quad (4.44)$$
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Fig. 4.5 Visibility calculated according to van Cittert-Zernike (solid black line) and the one obtained from the contrast of the fringe pattern (black dots) of a disk with an angular radius of $1.98 \times 10^{-6}$ rad, focused at a distance of one field size from the phase reference. The spectral bandwidth is 10 nm.

Fig. 4.6 Same as Fig. 4.5, but now with an angular radius of $1.53 \times 10^{-6}$ rad. The spectral bandwidth is 130 nm.
The visibility calculated as previously using Eq. (4.38) and Eq. (4.39), results that
the flux in the combined focal plane is
\[ D(x, y, t) = 2P \sum_{i=1}^{n_p} E_i \left[ 1 + \cos \psi_i \sin c \left( \pi \psi_i L_c / L \right) \right], \] (4.45)
where \( n_p \) is the number of points that form the disk. As in the previous case, the
flux in the region of the focal plane where the disk is focused is integrated and the
power, \( L(t) \), of the disk is obtained. The maximum of the temporal fringe pattern,
\( L_{\text{max}} \), and the minimum, \( L_{\text{min}} \), give the visibility
\[ \mu(B, \theta_0) = \frac{L_{\text{max}} - L_{\text{min}}}{L_{\text{max}} + L_{\text{min}}}. \] (4.46)
In Fig. 4.5, and Fig. 4.6 the results of the different calculations are plotted. In the
first one, a bandwidth of 10 nm was used, and the disk had an angular diameter of
1.98x10^-5 rad. The width of the steps of the mirror for this bandwidth, again
calculated for a maximum scanned path of 2\( L_c \) and the maximum baseline, is 3.6
mm. The results are in good agreement with the theoretical prediction. Finally we
use a wider bandwidth, 130 nm. For this bandwidth the width of the step is 278 \( \mu \)m.
The results observed are good for a small disk of 1.53x10^-6 rad, almost unresolved
by the single aperture, which is quite positive considering the width of the step.

### 4.3.3 Large objects consisting of small structures

As we explained before in this thesis, the main reason to apply interferometry to
astronomy was to obtain images of unprecedented high angular resolution. Large
baselines provide sampling of the high spatial frequencies in the Fourier plane.
Information about extended structures is mostly contained in the low frequencies.
The high frequency components of an object resolved by a single aperture are
typically so small that they are filtered out by the interferometer. The interesting
information is the one contained in the detail, related to the small structures. What
happens then with a large object that is composed of many small structures? The
ultimate goal of a stellar interferometer is to obtain wide-field images with high
angular resolution. But the only way to do it with an interferometer is by obtaining
the images of all the small structures that compose the large object and combine
them to reconstruct the complete object. This has been called “mosaicing” in radio
interferometry. Conventional interferometers have a narrow field of view and, to
obtain a wide-field image, it would be necessary to point the array in many different
directions and combine the images. But in this thesis we have developed a
technique to obtain all the information related to different directions in the sky.
simultaneously. Once all the data are available, one has to decide about the best way to combine all the information to create the wide-field image of the large object. As F. Gueth explains in [Gueth1998], there are basically two ways to “mosaic” a large image: linear and non-linear mosaicing, which are schematically described in Fig. 4.7. The latter method is described in more detail in this section.

In the linear mosaicing approach each field is deconvolved using classical methods, and a mosaic is reconstructed afterwards joining the adjacent images. Non-linear mosaicing consists in performing a joint deconvolution of all the fields so that the reconstruction and deconvolution of the mosaic is done simultaneously. The first authors that explained the principles of mosaicing where Ekers and Rots [EkersRots1979]. In their paper they refer to the problem of the missing low spatial frequencies that make that large structures cannot be mapped when reconstructing interferometric images. The solution they suggested is to combine the observations made with the interferometer with observations of the same object made with a single aperture to retrieve the spatial frequencies from \(-D/\lambda\) to \(D/\lambda\), \(D\) being the diameter of the dish. The same procedure could be applied to an optical interferometer.

A proper and detailed study and further comparison of both mosaicing algorithms have to be implemented before choosing any of them. But that is out of the scope of this thesis. As we want to provide the reader with an outline of the composition of wide-field images, we have chosen one of the two methods to reconstruct the wide-field visibility function in order to compare it with the one provided by the van Cittert-Zernike theorem. We have chosen the non-linear mosaicing method because, as Cornwell writes in [Cornwell1988], since the two deconvolution algorithms that dominate in radio interferometry, CLEAN and MEM (Maximum Entropy Method), are quadratic, therefore non-linear, we might expect to obtain better results combining all the data before the deconvolution phase.

Next, we briefly describe the non-linear mosaicing technique introduced by Cornwell in radio interferometry and explain how to apply it to our wide-field interferometer to calculate the visibility function of an object composed of several unresolved structures.

**Introduction to radio-interferometric non-linear mosaicing**

The mosaicing technique was developed in radio interferometry to solve the problem of imaging extended sources. In order to understand the application of mosaicing to optical interferometry, it is necessary to explain first how the principle was developed for radio interferometers. Cornwell, in both [Cornwell1994] and [Cornwell1988], presents a very good introduction to the problem of imaging
Fig. 4.7  Schematic illustration of linear vs non-linear mosaicing. (a) Each independent field (1) and (2) gives a visibility $\mu_1$ and $\mu_2$. (b) Linear mosaicing consists in deconvolving each field separately, and reconstruct a mosaic afterwards joining the adjacent images. (c) Non-linear mosaicing consists in performing a joint deconvolution of all fields. The main advantage for the wide-field interferometer is that for objects inside the isoplanatic patch only the phase of the reference object has to be calculated, and the relative phases can be used to reconstruct the field.
extended sources in radio interferometry. He suggested to do a joint deconvolution of all data instead of a linear combination. By looking at his equations it is straightforward to develop an extension to the mosaicing combination that we want to perform with our interferometer.

As Cornwell explains in his paper, the information collected by a single aperture can be understood by splitting the aperture into sub-apertures, each of which forms an interferometer with every other sub-aperture. The output power of a single aperture is a linear combination of all the sub-interferometers. With \( s_p \) being the average pointing vector of the elements in the sky, and \( u \) the vector separation of the telescopes (or sub-apertures) measured in wavelengths, \( u = B(s)/\lambda \), where \( B(s) \) is the effective baseline for the pointing position \( s \), we have that the visibility function sampled at pointing position \( s_p \) is basically the Fourier transform of the sky brightness \( I(s) \) weighted by a taper function, \( A(s - s_p) \):

\[
\int \int A(s - s_p)I(s)\exp(2\pi iu s)\,ds,
\]

where

\[
I_0 = \int \int A(s - s_p)I(s)\,ds,
\]

and \( A(s - s_p) \) is determined by the field of view of the interferometer, which tends to be zero for large offsets from the pointing centre \( s_p \).

If the coverage of the uv-plane is complete, Eq. (4.47) can be inverted to obtain the sky brightness distribution around pointing \( s_p \):

\[
\int \int \mu(u,s_p)\exp(-2\pi iu s)\,du = \frac{A(s - s_p)I(s)}{I_0}.
\]

In practice there is a limited number of baselines and only limited sampling of the uv-plane is possible. The function that samples the visibility is \( g(u,s_p) \). The Fourier inversion of the sampled visibility is:

\[
\int \int g(u,s_p)\mu(u,s_p)\exp(-2\pi iu s)\,du = \frac{G(s,s_p) \otimes [A(s - s_p)I(s)]}{I_0},
\]

with \( G(s,s_p) \), the Fourier transform of \( g(u,s_p) \) with respect to \( u \). The result is that we have the sky brightness distribution but convolved with a point spread function \( G(s,s_p) \). Linear methods cannot recover the non-sampled spacings and one must use a non-linear deconvolution algorithm, like CLEAN or the Maximum Entropy Method (MEM) [CornwellBraun1994].
The procedure to image a large field necessarily requires the sampling of many pointing centres and further deconvolution to remove the effects of the point spread function. This can be done linearly for each pointing centre and later a full image is formed taking the sum of the images for the individual pointing centres. But it could be improved by taking into account the deconvolution of the neighboring images (Fig. 4.7). The deconvolution of all the pointing centres should be performed jointly rather than separately, with a non-linear method. The idea is to find an estimate of the sky brightness consistent with the visibilities from many pointings. Therefore, by inverse Fourier transforming the observed visibility as a function of the offset, the visibilities of the sub-interferometers are obtained. A way to obtain an estimate of the complete visibility function of a selected sky brightness area is to scan over the required region, and then apply a Fourier inversion with respect to the scan position:

\[
\mu(u, u_p) = \frac{\int \int \mu(s_p) \exp(-2\pi j u_p s_p) ds_p}{\int \int A(s_p) ds_p},
\]

(4.51)

where \(u_p\) is the Fourier pair of \(s_p\), \(u_p = B(s_p)/\lambda\). According to the Whittaker-Shannon theorem [Calvo Padilla 2002], there is no need to scan for complete sampling of all spatial frequencies. To recover the visibility over the wide field we need samples spaced in \(s_p\) by a minimum increment \(\Delta s_p\) given by the Nyquist frequency, that is equal to half the bandwidth of the function to be recovered. A two-telescope interferometer, in one dimension, can in principle be sensitive to frequencies in the range \(((B-D)/\lambda, (B+D)/\lambda)\), so to avoid aliasing, the sampling limit is \(\Delta s_p \leq \lambda/D\). If \(O(s_p)\) is the sampling function in the \(s_p\)-space, typically a collection of Dirac delta functions, the normalized Fourier transform of the sampled visibility function is

\[
\mu_s(u, u_p) = \frac{\int \int O(s_p) \mu(s_p) \exp(-2\pi j u_p s_p) ds_p}{\int \int A(s_p) ds_p}.
\]

(4.52)

By inverse Fourier transforming this expression we would be able to obtain the complete sky brightness distribution but since the sampling of the uv-plane is not complete, there will be many images consistent with the observed visibility data. As
explained before, linear methods cannot recover the non-sampled spacings and one must use a non-linear deconvolution algorithm.

In this thesis, we have focused on the technique to recover the visibilities \( \mu(\mathbf{u,s_p}) \) for many pointing positions simultaneously, equalizing the differential optical path with the staircase mirror. If the pointing centre is \( s \) and if we express the pointing direction \( \mathbf{s_p} \) as a function of \( s \) \( \mathbf{s_p}(s) = s + \Delta \mathbf{s_p} \), we can retrieve the relative angular position with respect to the pointing centre \( s \), \( \mathbf{u_p} \Delta \mathbf{s_p} = (n_p d_{eff} + n_{p_p - p_p})/B \), where \( d_{eff} = 2d \cos \alpha \). This quantity is obtained from Eq. (4.30) with the distance \( n_p - p_p \) equal to the distance from the peak of the fringe pattern corresponding to pointing direction \( \mathbf{s_p} \) to the peak of the reference fringe pattern corresponding to \( s \).

The joint visibility function obtained from the data of the wide-field interferometer now yields

\[
\mu_p(\mathbf{u,u_p}) = \frac{1}{\int \int A(\mathbf{s_p})d\mathbf{s_p}} \int \int O(\mathbf{s_p})\mu(\mathbf{u,s_p}) \exp(-2\pi j(n_p d_{eff} + n_{p_p - p_p})/B) \exp(-2\pi j\mathbf{u_p s_p})d\mathbf{s_p} \tag{4.53}
\]

If the different pointings \( \mathbf{s_p} \) forming the wide field of view are into the isoplanatic patch, the use of the mosaic visibility \( \mu(\mathbf{u,u_p}) \) has a fundamental advantage because it implies that only the phase of the reference object has to be calculated to obtain the complete image of the extended field, instead of the closure phase of every single pointing position, as it would be the case in the linear reconstruction case. A study of the subsequent deconvolution of the wide-field visibility to obtain the image of an extended object is an important subject but it is beyond the scope of this thesis. For the interested reader, the adaptation of the Maximum Entropy Method that Cornwell proposes for the application of non-linear mosaicing can be found in [CornwellEvans1985] and [Cornwell1994].

A nice property of this algorithm is that the final result is a complete image where the sidelobes out of the primary beam are suppressed (very important for the overlapping of the fields) and spacings shorter than those directly measured are recovered. The difference in computation time compared to separate deconvolution of independent fields is small.

**Application of non-linear mosaicing to optical interferometry: two disks**

The goal of our approach to wide field interferometry is to obtain information from many adjacent fields at the same time, and process all the information together to obtain a complete image. The information is obtained at the same time because of
the presence of the steps that spatially modulate the Optical Path Difference in the focal plane. The increase in the image acquisition speed is equal to the number of steps, each step corresponding to one field. We are not going to actually deconvolve the wide-field visibility, but we want to compare the result of the visibility obtained for the whole field with the van Cittert-Zernike theorem, and the one obtained by “mosaicing” the visibilities $\mu(u, s_p)$ resulting from the simulations of the wide-field interferometer (see equation Eq. (4.52)). The input image we use is composed of two disks of angular radius $\rho_p$ equal to $1.653\times 10^{-5}$ rad for $p=1, 2$, positioned at $s_p = (\theta_p, \varphi_p)$ with $\theta_1 = 0; \varphi_1 = 0$ and $\theta_2 = 3.3 \times 10^{-3}$ rad; $\varphi_2 = 0$, and the baseline is $B = (B, 0)$. One disk is focused on the central step, the other one is focused two steps further. The visibility according to the van Cittert-Zernike theorem is

$$\mu_v (B, \theta, \nu) = \frac{1}{2} \sum_{p=1, 2} J_1(\pi \rho_p \nu B/c) \exp(-2\pi \nu \theta_p B \cos \theta/c), \quad (4.54)$$

where $\rho_p$ is the angular radius and $\theta_p$ the angular position of disk $p$. Integrating with respect to frequency, and changing notation to $u = (\nu B/c, 0)$ and $u_p = (\nu B \cos \theta_p/c, 0)$, the visibility is:

$$\mu(u, u_p) = \frac{1}{2} \sum_{p=1, 2} 2J_1(\pi \rho_p u) \pi \rho_p u \exp\left(\frac{\pi \lambda u}{L_c}\right) \exp(-2\pi j \theta_p u). \quad (4.55)$$

Next, we calculate the visibility as in the previous case, by integrating the flux at the detector to obtain the power while modulating temporally the optical path. Because of the steps, we observe fringes for the on- and off-axis disks simultaneously. The maximum and minimum of each fringe will give us the visibility for each of the two pointings, $\mu(u, s_p)$, and the peak-to-peak distance $n_{p_x - p_y}$ will give us the phase relative to the reference source, $u_p \Delta s_p = (n_s d_{eff} + n_{p_x - p_y})/B$ (see Eq. (4.30)). According to Eq. (4.53), we can easily calculate the joint visibility because we only have two pointings different from zero and the integrals are replaced by sums so that

$$\mu_s (u, u_p) = \sum_{p=1, 2} \mu(u, s_p) \exp(-2\pi j (n_s d_{eff} + n_{p_x - p_y})/B) \exp(-2\pi j u_p s)$$

$$\sum_{p=1, 2} \frac{1}{2}, \quad (4.56)$$

and, because in our case $s = (0, 0)$, the joint visibility is

$$\mu_s (u, u_p) = \sum_{p=1, 2} \mu(u, s_p) \exp(-2\pi j (n_s d_{eff} + n_{p_x - p_y})/B) \quad (4.57)$$
4.3 Wide-field imaging

The modulus of the visibility given by Eq. (4.55) for the two disks is the solid line in Fig. 4.8, the one given by Eq. (4.57) is shown by black dots. For the simulations, we chose a bandwidth of 10 nm, and we integrated over this bandwidth considering a constant spectral response. The results for the analytical joint visibility are in good agreement with the theoretical results, the deviation being smaller than 2%. This shows that it is possible to apply the mosaicing theory to more complex images, with, of course, the necessity of extending the number of pointings to the Nyquist frequency in the case of an unknown image.

Fig. 4.8 Visibility calculated according to van Cittert-Zernike (solid black line) and the one obtained “mosaicing” the visibilities resulting from the contrast of the fringe patterns (black dots) for two disks with an angular radius of $1.653 \times 10^{-5}$ rad, separated by 3.3 millirad, for a bandwidth of 10 nm.
4.4 Conclusions

In this chapter we have described analytically the complete process, from an image in the sky to the visibility retrieval, that takes place in a two-arm Michelson interferometer with a staircase mirror in the focal plane of one of its arms. The most important effect is the equalisation of the optical path and through this equalisation the simultaneous correction of the path difference for the off-axis and the on-axis light, so that it becomes possible to retrieve the visibility and relative phase corresponding to off-axis objects. The visibility obtained from the contrast of the fringe pattern agree with the theoretical visibility calculated according to the van Cittert-Zernike theorem. Therefore, wide-field imaging, within the isoplanatic patch, is possible with a Michelson pupil-plane interferometer, and we suggest to do this using the algorithms provided by the radio-interferometric non-linear “mosaicing” technique, that consists in performing a joint deconvolution of all the fields so that the reconstruction and deconvolution of the mosaic is done simultaneously. Since the deconvolution procedures are non-linear we might expect to obtain better results combining all the data before the deconvolution phase. Simulations show that the visibility of a large structure formed by two disks, calculated with the van Cittert-Zernike theorem, agrees very well with the “mosaiced” one composed from the visibilities obtained from the fringe pattern corresponding to each disk.

4.5 References

4.5 References


5.1 Goal of the Wide Field setup

The Wide Field Interferometer breadboard is designed and implemented in order to demonstrate the feasibility of our wide-field solution, i.e., the use of the staircase mirror. The main goal of the setup is to observe fringes from separated stars (on- and off-axis), in a field of view of approximately 1 arcmin angular radius, simultaneously and with comparable contrast, so that the modulus of the visibility and the relative phase of the on- and off-axis stars can be obtained.
5.2 General description

We have designed and implemented a tabletop setup consisting of a two-telescope Michelson-type interferometer. In the focal plane of one of the arms of the interferometer a staircase mirror was placed. A scheme of the setup has been plotted in Fig. 5.1. It consists of three main subsystems: the star simulator, the interferometer and the beam combiner.

The star simulator basically consists of a Xenon lamp of which the light is filtered and focused on a mask with several pinholes on it, simulating the stars. The light from the mask is collimated in order to simulate several incoherent stars at an infinite distance. Two sets of lenses, \( L_2 \) and \( L'_2 \), are the two telescopes that collect the light from the stars, each of them forming an arm of the interferometer. In the focal plane of one of the interferometer arms we place a staircase mirror. The light is collimated again with lens \( L_3 \) before entering the delay line and beam combiner block. Two mirrors on top of a piezo stage form the delay line used to modulate the path. The beams are overlapped with a beam splitter/combiner cube. The two outputs of the beam splitter are used to detect the fringes and to control the pupil position, respectively.

Fig. 5.1 Description of the setup consisting of three main subsystems: the star simulator with the white light source and the star mask, the interferometer, and the beam combiner with the delay line. The filters at the star simulator are used to narrow the bandwidth by the Xe lamp to 150 nm. A detector (PD) was placed in one of the outputs of the beam splitter and a CCD camera at the other output to control pupil position. The mirrors of the delay line were connected to a piezo driver and to a signal generator to modulate the path and observe temporal fringes. \( L_i \) are lenses.
5.3 The star simulator

The star simulator consists of a Xenon arc lamp with a set of filters that reduces its spectrum to a selected bandwidth, as can be observed in Fig. 5.2. For most of the experiments, a bandwidth of 150 nm (500-650 nm) is selected, resulting in a coherence length of 2.2 µm. The light is imaged onto the star mask, which consists of a set of pinholes of 5 µm diameter, with each pinhole defining one star. The star mask was manufactured at the Delft Institute of Microelectronics and Submicrontechnology (DIMES). It is a glass plate with a 200nm Chromium layer, where a pattern of pinholes of 5 µm has been written using the Electron Beam Pattern Generator (EBPG) technique. The background light that is transmitted through the mask is given by the absorption law

\[ I(z) = I(0) \exp[-a(\omega)z], \quad (5.1) \]

where \( I(0) \) is the power that illuminates the mask, \( I(z) \) is the transmitted power and \( a(\omega) \) is the absorption coefficient for a frequency \( \omega \), related to the extinction coefficient \( k(\omega) \) by \( a(\omega) = 2k(\omega)\omega/c \), where \( c \) is the speed of light in vacuum. For a wavelength of 575 nm, the extinction coefficient of Chromium is approximately 3.32 units, and for \( z=200 \) nm only a 5x10^{-7} percent of the power will be transmitted through the mask and contribute to background noise.

In the experiment described here the illuminated mask has four pinholes, distributed in a square with separation of 150 µm. The size of the pinholes has been chosen smaller than the airy disk of the collimating aperture, ensuring full spatial coherence for the beams over the entire bandwidth. The light from two different pinholes is not coherent, ensuring two independent sources as is the case of real stars. The light from the star mask is collimated in such a way that at the entrance pupil of the telescopes we have four beams, simulating four distant objects in the sky. Two of these objects have an external \( \text{OPD} \) equal to zero, therefore they are considered on-axis objects. The other two have an external \( \text{OPD} \) different than zero, hence for the interferometer they are off-axis objects, with the off-axis angle being approximately 1 arcmin.

5.4 The interferometer and the staircase mirror

The most important parameters of the interferometer are shown in Table 5.1. As we can see in Fig. 5.3, the two interferometer beams are obtained by wavefront division by means of two 20 mm diameter apertures. The separation between the
### Table 5.1  System parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Θ (rad) off-axis beam angle</td>
<td>0.0003</td>
</tr>
<tr>
<td>$F_1$ (mm) focal length of collimating lens $L_1$</td>
<td>500</td>
</tr>
<tr>
<td>$D_2$ (mm) diameter of telescope entrance pupil</td>
<td>20</td>
</tr>
<tr>
<td>B (mm) baseline vector</td>
<td>30</td>
</tr>
<tr>
<td>$F_2$ (mm) focal length of first lens $L_2$ in telescope objective</td>
<td>+1330</td>
</tr>
<tr>
<td>$F'_2$ (mm) focal length of second lens $L'_2$ in telescope objective</td>
<td>-100</td>
</tr>
<tr>
<td>$F_2-F'_2$ (mm) distance between $L_2-L'_2$</td>
<td>1258.67</td>
</tr>
<tr>
<td>$F$ (mm) effective focal length of telescope objective</td>
<td>6563.567</td>
</tr>
<tr>
<td>BFL (mm) back focal length of telescope objective</td>
<td>394.22</td>
</tr>
<tr>
<td>NA numerical aperture of telescope objective</td>
<td>1.25e-3</td>
</tr>
<tr>
<td>$Z_f$ (mm) focal depth of telescope objective</td>
<td>367.214</td>
</tr>
<tr>
<td>$L'_2$-image surface (mm)</td>
<td>394.218</td>
</tr>
<tr>
<td>$L_2$-image surface (mm)</td>
<td>1652.888</td>
</tr>
<tr>
<td>$F_3$ (mm) focal length of lens $L_3$</td>
<td>+1000</td>
</tr>
<tr>
<td>M magnification of the telescope</td>
<td>6.563</td>
</tr>
<tr>
<td>s (μm/arcsec) scale in the focal plane of the telescope's objective</td>
<td>32.46</td>
</tr>
<tr>
<td>δ (μm) spot size in focal plane</td>
<td>328</td>
</tr>
<tr>
<td>d (μm) separation between on and off axis beam spots</td>
<td>1948</td>
</tr>
<tr>
<td>$D_3$ (mm) diameter of telescopes exit pupil</td>
<td>3.047</td>
</tr>
</tbody>
</table>
5.4 The interferometer and the staircase mirror

Fig. 5.2  Star simulator: white light is emitted by the Xe lamp, collimated, filtered to a limited bandwidth and focused onto the star mask. A He-Ne laser is used for alignment, which can be selected to illuminate the setup by a flip mirror.

Fig. 5.3  Lens $L_1$ collimates the light from the star simulator. The two input beams to the interferometer are obtained by wavefront division. In the focal plane of the system formed by $L_2$ and $L_2'$ is set the staircase mirror.
apertures ("baseline") is 30 mm. After each aperture a telescope objective is placed, and at the focal plane of one of the interferometer arm a staircase mirror is set. A detailed draw of the staircase mirror showing its dimensions is in Fig. 5.4. The staircase mirror introduces a temporal delay to the imaged spots that depends on the incidence angle of the incoming beam in the entrance pupil of the telescope. The introduced delay depends on the tilt angle of the mirror, \( \alpha \). We have chosen a tilt of 45° so that the symmetry between both arms is easily maintained, and the lateral displacement that is introduced to the beams because of the steps is small enough as compared with the spot size. The width of the step depends on the effective focal length, \( F \), of the telescope objective, which was chosen so that the point spread function of the focused spot is several times smaller than the width of the step. The actual configuration uses a positive and a negative achromat, and the effective focal length is 6.563 m. The depth of the step, \( d \), is related to the width, \( w \), and the tilt angle, \( \alpha \), by the following equation

\[
\frac{d}{w} = \frac{B}{2F \tan \alpha},
\]  

(5.2)
where $\alpha$ is the angle that the normal to the surface of the mirror is forming with the optical axis of the system, $F$ is the effective focal length of the telescope objective and $B$ is the baseline or separation between the telescopes. The staircase mirror was manufactured at Philips Research Laboratories (Eindhoven, The Netherlands) with a diamond turning machine (DTM) using a chisel of 3 $\mu$m. The step width, chosen in order to scan a maximum path equal to $2L_c$, is 1339±6 $\mu$m and corresponds to approximately four airy disks. For this step width, both the on and the off-axis beams are focused close to the centre of their respective steps. The depth of the steps is 3±0.2 $\mu$m.

The staircase mirror holder can be rotated, so that the steps can be set either perpendicular to the baseline, in order to correct the external optical path, or parallel to it, which corresponds to an uncorrected situation (i.e., no staircase mirror).

Among several possible achromat configurations, we choose the one that presents the best chromatic behaviour. The position of the focus and the size of the spot for the whole bandwidth have been studied. The results for the actual configuration show that the system is free of lateral and axial chromatic aberration. All the wavelengths are focused in a region inside the focal depth of the central wavelength, as is illustrated in Fig. 5.5. In Fig. 5.6 we can see that the diffraction spots for the two beams do not overlap neither.

![Figure 5.5](image.png)

**Fig. 5.5** Back Focal Length vs. wavelength. The squares represent the position of the back focal plane and the bars the focal depth as a function of the wavelength. As can be seen in the figure, the position of the focus depending on the wavelength differs very little.
5.5 Beam combination, path modulation and detection

In Fig. 5.7 there is a picture of the beam combiner subsystem. The light is collimated again before entering the delay line and beam combiner block. Two mirrors on top of a piezo stage, set in one of the interferometer arms before the beam splitter, provide a delay line. The piezo is connected to the computer and a ramp voltage signal is used to modulate the optical path. The beams from the two interferometer arms overlap at a beam splitter/combiner cube. The outputs at each side of the beam splitter are detected by a low noise photo detector and a CCD camera that measures the modulated fringes and controls the pupil position, respectively. The photo detector and the camera are also connected to the computer, and the data acquisition and the optical path modulation are performed synchronously.
5.6 Factors affecting the visibility

In this section we study some of the most important factors that can affect the visibility retrieved by the wide-field interferometer: first, the background noise and power mismatch, a factor that will determine the accuracy of the measurements. Next, the wavefront error and, finally, we analyse how the lateral displacement introduced by the steps affects the visibility.

5.6.1 Background noise and power mismatch

As we saw in Chapter 1, Section 1.2.1, the power produced by the interference of two partially coherent beams is [BornWolf1999]

\[ I(t) = \langle E(t)E^*(t) \rangle, \]

(5.3)

with

\[ E(t) = E_1(t) + E_2(t), \]

(5.4)

Fig. 5.7 View of the delay line, the beam combiner and the detector.
where \( E_1(t), E_2(t) \) is the signal detected from each arm. In the analysis in Chapter 1 we did not consider the presence of noise in the detected signal. In this section we consider a signal \( E_i(t) \) represented by \( E_i(t) = E_i^s(t) + N_i(t) \), where \( E_i^s(t) \) is the signal coming from the source and \( N_i(t) \) is the background noise present in the detected signal. The power of the interference signal is

\[
l(t) = |E_1(t)|^2 + |E_2(t)|^2 + 2|E_1^s(t)||E_2^s(t)|\mu_{12}\cos\psi(t),
\]

(5.5)

considering each signal uncorrelated to the noise of the other one and the noise uncorrelated to each other, \( \langle E_1^s(t)N_2^*(t)\rangle = \langle E_2^s(t)N_1^*(t)\rangle = \langle N_1(t)N_2^*(t)\rangle = 0 \).

From the detected fringe pattern we calculate the contrast, \( V_{12} \), that is related to the visibility by

\[
|\mu_{12}| = V_{12} \frac{|E_1(t)|^2 + |E_2(t)|^2}{2|E_1^s(t)||E_2^s(t)|}.
\]

(5.6)

Because of the presence of noise and the power mismatch, the contrast \( V_{12} \) is smaller than the visibility, and we have to calibrate the fringe pattern in order to calculate the visibility \( |\mu_{12}| \). We have to measure the contrast of the interference signal with noise, and the signal from each arm with and without noise. The latter is done placing a pinhole in front of the detector that filters all the background noise and only permits the detection of the selected signal. The order of magnitude of the detected signals is about 1 Volt, and the detector noise is 0.01 Volt. Introducing these numbers in Eq. (5.6) we find that the accuracy in the calculation of the visibility is limited to 2%.

### 5.6.2 Wavefront errors

According to [Leveque1997], the loss in the visibility due to wavefront errors can be calculated by:

\[
1 - \Delta\mu / \mu = \exp\left(-\kappa^2 \sigma_w^2\right),
\]

(5.7)

where \( \Delta\mu / \mu \) is the relative visibility loss, \( \kappa \) the wave number for wavelength \( \lambda \), and \( \sigma_w \) is the rms wavefront error.

For example, using this equation we see that an rms wavefront error of \( \lambda / 30 \) at a wavelength of 575 nm can cause a visibility loss of 4%.

### 5.6.3 Chromatic dispersion

When the optical path through the various glass elements (lenses, beam splitter) is larger in one arm than in the other, the zero path position for every wavelength is
5.6 Factors affecting the visibility

different, and the result is a broader set of fringes with lower contrast. The refractive index of glasses can in general be described by the Sellmeier equation:

\[ n^2(\lambda) = 1 + \frac{C_1\lambda^2}{\lambda^2 - C_4} + \frac{C_2\lambda^2}{\lambda^2 - C_5} + \frac{C_3\lambda^2}{\lambda^2 - C_6}, \tag{5.8} \]

where \( C_i \) are the dispersion coefficients of the glass, and the wavelength \( \lambda \) is given in \( \mu \text{m} \). When the difference in optical path is \( \Delta d \), the phase difference between the beams is equal to

\[ \Delta \varphi = \kappa_0 n(\lambda_0) - 1)\Delta d. \tag{5.9} \]

The first-order terms in Eq. (5.9) produce a shift of the whole fringe pattern, and the second and higher order terms make the fringes to move relative to the envelope, degrading the contrast and producing an asymmetric fringe pattern.

5.6.4 Lateral displacement introduced by the steps

As can be seen in Fig. 5.4, the steps in the mirror introduce a lateral shift \( \Delta r \) to the beam equal to

\[ \Delta r = 2n_s d \sin \alpha, \tag{5.10} \]

where \( \alpha \) is the angle that the normal to the surface of the mirror is forming with the optical axis of the system, and \( d \) is the depth of the step. We have studied the effect in the visibility of the lateral displacement introduced by the steps in the staircase mirror by combining two interference patterns, one of them displaced by a distance \( \Delta r \). The relative visibility loss is given by:

\[ \frac{\Delta \mu}{\mu} = 1 - \frac{\mu'}{\mu}, \tag{5.11} \]

where \( \mu \) and \( \mu' \) are the modulus of the visibility calculated without and with displacement, respectively

\[ \mu = \frac{L_{\text{max}} - L_{\text{min}}}{L_{\text{max}} + L_{\text{min}}}, \quad \mu' = \frac{L'_{\text{max}} - L'_{\text{min}}}{L'_{\text{max}} + L'_{\text{min}}}. \tag{5.12} \]

The maximum and minimum power are defined by

\[ L_{\text{max}} = 2\pi \int_0^R r^2 \left| U_1(r) \right|^2 dr, \tag{5.13} \]

\[ L_{\text{min}} = 2\pi \int_0^R r \left| U_1(r)(1 + \exp(j\pi)) \right|^2 dr, \tag{5.14} \]
The Wide Field Interferometry breadboard

\[ L'_{\text{max}} = 2\pi \int_0^R |U_1(r) + U_2(r)|^2 dr, \quad (5.15) \]
\[ L'_{\text{min}} = 2\pi \int_0^R r|U_1(r) + U_2(r) \exp(j\pi)|^2 dr, \quad (5.16) \]

with \( R = 1.22\lambda/\text{NA} \), where \( U_1(r) \) and \( U_2(r) \) are the impulse responses without and with displacement, respectively:

\[ U_1(r) = \rho \frac{\exp(j\kappa F)}{j\lambda F} \exp \left( j\frac{kr}{2F} \right) \frac{2J_1(\kappa r \rho / F)}{\kappa r F}, \quad (5.17) \]
\[ U_2(r) = \rho \frac{\exp(j\kappa F)}{j\lambda F} \exp \left( j\frac{kr}{2F} \right) \frac{2J_1(\kappa(r+\Delta r) \rho / F)}{\kappa (r+\Delta r) F}, \quad (5.18) \]

where \( \rho \) is the output pupil radius, \( F \) the focal length, \( \kappa \) the wave number for wavelength \( \lambda \), and \( \rho \) a proportionality constant.

In Fig. 5.8 the relative loss in the visibility is shown as a function of the lateral displacement for the parameters of the setup. In the setup, \( \alpha \) is 45 degrees and the depth of the steps is 3 \( \mu \text{m} \). The total number of steps is 5, therefore the maximum displacement is only 0.55% of the output pupil radius. The relative loss introduced to the visibility is smaller than 0.2%.

![Graph](image_url)

**Fig. 5.8** Relative visibility loss as a function of the lateral displacement for the parameters of the setup shown in Table 5.1.
5.6 Factors affecting the visibility

Extrapolation to the VLTI

In Fig. 5.9 the relative loss in the visibility has been plotted as a function of the displacement of the output pupil, extrapolated to the case of the VLTI. The output pupil radius is 4 cm, the central wavelength is 2.2 μm, and the focal length is 408 meters. A lateral displacement of 1.5% of the radius brings a relative loss in the visibility of 1%. Translating this into field of view, the useful field of view becomes

\[
fov = \frac{(\Delta r / d \sin \alpha + 1)w}{t},
\]

(5.19)

where \( t \) is the scale in the focal plane in meters per arcsec, and \( w \) is the width of the steps.

For the parameters of UT2 from Chapter 3, we find \( w = 802 \) μm, the focal plane scale equals 2 mm/arcsec. If the normal to the surface of the mirror is forming 45 degrees with the optical axis, the total field of view covered with relative losses smaller than 1% is of 5-25 arcsec; depending on the depth of the steps. It is an important reduction of the field of view. Therefore, it is important to avoid visibility losses due to non-perfect overlapping of the beams. This can be done by cutting away the edge of the recombined beam with a diaphragm in the pupil image, with a small reduction of the power.

![Fig. 5.9](image) Relative visibility loss as a function of the lateral displacement for the parameters of the VLTI.
5.7 Experimental results

5.7.1 Assembly, alignment and calibration

The setup was assembled starting from the star simulator, then both interferometer arms, the delay line mirrors and the beam combiner. A He-Ne laser set before the star mask was used for the alignment. The piezo stage of the delay line was actuated with a triangle function of 4 Volts amplitude, equivalent to a modulation of the path of approximately $9 \mu m$. The fringes produced by the coherent interference of the beams were detected with the photo detector, as well as the power of each beam for calibration of the visibility. The maximum visibility obtained with the laser, calculated averaging the maxima and minima, was $0.92\pm0.02$, as seen in the calibrated fringe pattern shown in Fig. 5.10. Higher contrast fringes should be expected from a monochromatic point source. The limited observed contrast could be due to three reasons: wavefront errors, misalignment or the fact that the observed source is partially resolved by the baseline. In order to calibrate the instrument for the white light measurements, it is necessary to know if the object observed is not a point source. If the reason for the low contrast is misalignment, the contrast of the on-axis and off-axis objects will be different because the misalignment will affect more strongly the off-axis objects. As changing the

![Fig. 5.10](image-url)
5.7 Experimental results

Alignment did not improve the contrast, we illuminated the star mask with monochromatic light at 501 nm and 535 nm. The visibility depends on the wavelength, and if the source is partially resolved it should increase at longer wavelengths. Different values of the visibility were obtained for the three wavelengths, and from these measurements a diameter of $1.0 \times 10^{-5}$ was calculated for the source. Direct measurements of the distance from the star mask to the collimating lens, resulted in a distance of 480 mm, i.e., 20 mm shorter than the focal length of the lens. Consequently, the collimating lens is not relaying the star mask at infinity, but producing a virtual image of a star with $0.9 \times 10^{-5}$ rad angular diameter. The visibility at 632 nm, observing with a baseline of 30 mm, is expected to be 0.95. Given this number, the residual reduction can be attributed to residual misalignment or surface errors. The total reduction due to these factors adds up to about 3%. For a partially resolved source, the visibility depends on the wavelength; given the measured values of the visibility with the laser radiation, the expected visibility for the white light source, with a central wavelength of 575 nm, is about 0.90.

5.7.2 Dispersion correction

When measuring the white light fringes, first results showed that fringes for the on-axis and off-axis stars were observed simultaneously when the dividing lines of the steps of the staircase mirror are perpendicular to the baseline, showing that the staircase mirror indeed corrects the differential $\text{OPD}$. However, as can be seen in the raw data shown in Fig. 5.11, the modulus of the visibility was only 26%, a quite lower value than the one obtained with the laser. The reason for such a low contrast as compared with the contrast measured with the laser is attributed to dispersion produced by the various optical elements. There are three achromats in each arm of the interferometer, and every achromat is composed of two different types of glass, with a tolerance error of about 100 $\mu$m each one. The beam splitter is a cube made of BK7 glass, with a tolerance error, according to the manufacturer, of about 200 $\mu$m. As explained in Section 5.6.3, when the optical path through the various glasses is larger in one arm than in the other, the zero path position for every wavelength is different, and the result is a broader set of fringes with lower contrast. The different types of glass present in the setup are BK7, F2, SF2 and SF5. The refractive index of each type of glass is given by the Sellmeier equation, written in Eq. (5.8).
Table 5.2  Sellmeier dispersion coefficients from [glassbank].

<table>
<thead>
<tr>
<th></th>
<th>BK7</th>
<th>F2</th>
<th>SF2</th>
<th>SF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1.03961212</td>
<td>1.39757037</td>
<td>1.40301821</td>
<td>1.52481889</td>
</tr>
<tr>
<td>C2</td>
<td>0.231792344</td>
<td>0.159201403</td>
<td>0.231767504</td>
<td>0.187085527</td>
</tr>
<tr>
<td>C3</td>
<td>1.01046945</td>
<td>1.2686543</td>
<td>0.939056586</td>
<td>1.42729015</td>
</tr>
<tr>
<td>C4</td>
<td>0.00600069867</td>
<td>0.00995906143</td>
<td>0.0105795486</td>
<td>0.011254756</td>
</tr>
<tr>
<td>C5</td>
<td>0.0200179144</td>
<td>0.0546931752</td>
<td>0.0493226978</td>
<td>0.0588995392</td>
</tr>
<tr>
<td>C6</td>
<td>103.560653</td>
<td>119.248346</td>
<td>112.405955</td>
<td>129.141675</td>
</tr>
</tbody>
</table>

Fig. 5.11  Raw data of the white light fringes in the presence of dispersion. The black solid line is the fringe of an on-axis star and the dotted line is the fringe of an off-axis star.
The dispersion coefficients $C_{ij}$ of the various types of glasses are shown in Table 5.2. When the difference in optical path for material $i$ is $\Delta d_i$, the phase difference between the beams, $\Delta \varphi_i$, is

$$\Delta \varphi_i = \kappa_0 \sum_{j=1}^{n} (n_j(\lambda) - 1) \Delta d_i.$$  \hspace{1cm} (5.20)

As an example, in Fig. 5.12 the simulated interference pattern is shown when one of the beams experiences an additional path of 300 $\mu$m of BK7, 100 $\mu$m of F2, 100 $\mu$m of SF2 and 100 $\mu$m of SF5. As has been already explained, the second and higher order terms in the $n(\lambda)$-function make the fringes to move relative to the envelope, degrading the contrast and producing an asymmetric fringe pattern. For the given additional paths, the contrast in the simulated fringe pattern is degraded to 40%.

In order to correct the dispersion, two types of glasses, expected to give the larger contributions to the dispersion (BK7 and F2), were inserted to the interferometer’s arms. By tilting the glass plates and observing the fringes, we finally obtained an increase in the visibility to 0.82$\pm$0.02.
5.7.3 Measurements with the 30 mm baseline

After correction for the dispersion, we measured the fringes corresponding to the four stars in the star mask. In the first set of data the staircase mirror was placed with the steps parallel to the baseline, so that no correction of the external OPD is introduced. The calibrated fringes are shown in Fig. 5.13, obtained with a total scanning distance of 9 μm, approximately four times the coherence length. The fringe from the on-axis star (Fig. 5.13a) has a visibility of 0.82±0.02. As expected, the peak of the fringe from the off-axis star is not observed, since it is not in the field of view of the interferometer, and the visibility was 0.14±0.02 (Fig. 5.13b). Similar results are found with the other two on- and off-axis stars.

In the second set, we place the mirror with the steps perpendicular to the baseline, so that correction of the external OPD does take place. The results are shown in Fig. 5.14. The visibility calculated from the calibrated fringe of the two on-axis stars is again 0.82±0.02 (Fig. 5.14a and Fig. 5.14c), and the visibility of the two off-axis stars is 0.74±0.02 (Fig. 5.14b and Fig. 5.14d). This value, as compared with the visibility of the off-axis star without correction (Fig. 5.13b), clearly demonstrates the effect of the OPD correction due to the steps.

The relative angle $\varphi$ that gives the position of the off-axis star with respect to the on-axis one can be calculated by measuring the distance from the peak of the off-
Fig. 5.14  White light fringes when the steps of the mirror are perpendicular to the baseline: (a) and (c) calibrated fringes from the on-axis stars, (b) and (d) calibrated fringes from the off-axis stars. The interferometric field of view has been extended and, as compared to the results of Fig. 5.13, the peaks of the fringes of the off-axis stars are in this case observed.
axis fringe pattern to the peak of the on-axis pattern, \( n_{p-p} \), and knowing in which step the off-axis star is focused:

\[
\varphi = \frac{1}{B}(2n_s d \cos \alpha + n_{p-p}),
\]

where \( B \) is the baseline, \( d \) the depth of the steps and \( \alpha \) the angle that the mirror is forming with the optical axis; \( n_{p-p} \) can be both positive or negative. In our case, the relative angular separation between the stars measured from the fringe pattern is \((3.1 \pm 0.1) \times 10^{-4} \) rad, which closely agrees with the calculation of the angular position by direct measurement of the experimental parameters.

The visibility of the off-axis fringes is not equal to the visibility of the on-axis fringe pattern again because of dispersion effects. For the data shown in Fig. 5.14, we set the on-axis star at the centre of the achromats and the dispersion correction was optimized for this direction. Therefore, the off-axis stars hit the achromats off-axis and the dispersion correction is not optimum in this case. By moving the stars in such a way that they hit the achromats symmetrically, we observe the same visibility, 0.80\( \pm \)0.02 in both cases. This is shown in Fig. 5.15, where both stars' calibrated fringes are plotted in Fig. 5.15a and Fig. 5.15b, respectively.

![Fig. 5.15](image_url)  
Fig. 5.15 (a) and (b) White light fringes for two stars, symmetric with respect to the optical axis, hence the same dispersion correction is applied and they show similar contrast.
5.8 Conclusions

In this chapter we have described the tabletop setup designed to demonstrate the feasibility of our wide field solution. It consists of a two-telescope Michelson-type interferometer, with a staircase mirror placed in the focal plane of one of its arms. The main goal of the setup was to observe simultaneously fringes with comparable contrast from separated stars, in a field of view of approximately 1 arcmin angular radius, and to retrieve the visibility and relative phase of the off-axis stars. The experimental results using white light (with a bandwidth from 500 nm to 650 nm), and a 30 mm baseline, shown that fringes from an off-axis star separated 1.07 arcmin from the on-axis reference star were obtained simultaneously and with a contrast of 0.80±0.02, similar to the contrast of the on-axis one. The relative angular separation between the stars has been retrieved and corresponds to the expected value, 3x10^{-4} rad, to within 10^{-5} rad. The different factors that can affect the retrieval of the visibility, such as dispersion, background noise, or the effect of the lateral displacement of the pupil introduced by the steps, were also studied. The lateral displacement, a problem that is intrinsic in the staircase mirror approach, extrapolated to the VLTI parameters, introduces an error of 1% for a field of view of 5-25 arcsec, but the visibility losses due to non-perfect overlapping of the beams can be avoided by cutting away the edge of the recombined beam with a diaphragm in the pupil image. The experimental results clearly show that the equalisation of the optical path is a useful method to perform wide-field observations.

5.9 References

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“Truth indeed rather alleviates than hurts, and will always bear up against falsehood, as oil does above water”
Miguel de Cervantes

CHAPTER 6
Visibilty retrieval in the presence of discontinuous path length compensation

In this chapter we present a further analysis of the new path length compensation method for Michelson wide-field interferometry described in this thesis. We investigate how to retrieve the visibility when a star is focused on the edge of a step. Even though the Optical Pathlength Difference (OPD) correction is discontinuous, we show both numerically (Section 6.3) and analytically (Section 6.2) that the visibility can be completely recovered, so that no information is lost. Experimental results (Section 6.4) demonstrate that the visibility is recovered to within a 1% error. In this chapter we only explain the effect that the presence of an edge has in the main observable of a stellar interferometer, the visibility. A study in detail on the general effect that the presence of a step’s edge has in the interferometer can be found in Appendix B, where the diffraction effects due to the edge are studied using Fourier analysis to determine the limits of the approximation presented in this chapter.

6.1 Introduction

Until recently it has been thought that Fizeau-type interferometers were the only way to perform wide-field imaging, since in pupil-plane interferometers the field of view consists only of a few resolution elements at low spectral resolutions [Schoeller2000a]. But when the aperture is highly diluted the applicability of a Fizeau interferometer is limited by the signal-to-noise ratio, that decreases with the number of telescopes. This problem does not occur in Michelson interferometers [Labeyrie1996]. In order to use this favourable property of Michelson interferometers we have developed in this thesis a new approach to the pupil
Visibility retrieval and discontinuous OPD compensation problem, i.e., a system that uses a Michelson pupil-plane scheme with a wide field of view to be acquired in one shot [Montilla2003,Montilla2004a] using a staircase mirror to introduce an extra path length providing simultaneous correction of the Optical Path Difference (OPD). Given the discontinuous nature of the staircase mirror, it may appear that the correction of the OPD in such a way would yield a discontinuous field of view. When an object is focused on the edge of a step, the light is reflected from two steps simultaneously, implying a different phase shift for each part. The effect on the detected light distribution after combining the beams is that instead of having one fringe set, with one maximum at the zero path position, there are two fringe sets, each with a relative maximum. Each set has a relative contrast, and in this paper we study the relation of the relative contrast with the visibility of the source, and how the latter can be recovered. The addition of the contrast of the two components of the fringe results in the expected contrast of the source multiplied by a factor that depends on the bandwidth and the baseline; with the aid of this factor the visibility can be retrieved. This subject is addressed in Section 6.2, where the effect of the differential phase shift between two steps on the detected fringe pattern is described analytically.

6.2 Analytical description

The schematic representation of a two-arm Michelson-type interferometer, with a staircase mirror placed at an intermediate focal plane of one of its arms, is depicted in Fig. 6.1a. Suppose a source observed with one of the arms of the interferometer, with an aperture of radius $\rho_a$, numerical aperture $N_a$ and transmission function $T$ constant over the spectral bandwidth, i.e., a "top-hat" bandwidth of $\Delta \lambda$ centered on $\lambda_0$. The flux [Karttunen1994], in Wm$^{-2}$, in the focal plane of the objective is:

$$D_a(x,y) = \Delta \lambda \left( \frac{\pi N_a \rho_a}{\lambda_0} \right)^2 T P(x,y),$$

(6.1)

where $P(x,y)$ depends on the source geometry (a point source, a binary, a disk) and is equal to the convolution of the flux of the object with the Point Spread Function ($PSF$) of the system. A similar function is obtained for the flux at the focal plane of the other arm. In order to make the analysis simpler, we consider the same aperture radius and numerical aperture for the combined focus. If the entrance apertures of the arms of the interferometer are separated by a distance $B$, assuming their aperture radii, numerical apertures and transmission functions to...
Fig. 6.1  (a) Scheme of the experimental setup consisting of three blocks: the star simulator with the white light source and the star mask, the interferometer, with the staircase mirror in one of the arms, and the beam combiner with the delay line. At the two outputs of the interferometer a detector and a CCD camera are placed. The delay line is scanned by a piezo element. (b) Schematic representation of a source imaged on the edge between two steps in the staircase mirror.
be equal, the flux \( D(x, y, t) \) detected in the combined focal plane is given by [BornWolf1999]

\[
D(x, y, t) = 2D_d(x,y) \left[ 1 + \mu(B) \cos(\varphi_t) \sin(c\left(\frac{\pi \varphi_t}{\kappa_0 L_c}\right)) \right],
\]

(6.2)

where \( \mu(B) \) is the modulus of the complex coherence factor (visibility) of the source observed with baseline \( B \), \( L_c = \frac{\lambda_0^2}{2\Delta\lambda} \) is the coherence length and \( \kappa_0 = \frac{2\pi}{\lambda_0} \) is the wave number at the central wavelength \( \lambda_0 \). The phase difference \( \varphi_t \) between both arms has three contributions: the external geometrical optical path difference, the internal phase introduced by the steps, and the temporal phase introduced by the temporal modulation of the internal optical path. The power \( L(t) \), in units of Watts, is obtained by simply integrating the flux. The time dependence in \( L(t) \) is due to the temporal modulation of the optical path. The visibility \( \mu(B) \) is calculated using the relation

\[
\mu(B) = \frac{L_{\text{max}} - L_{\text{min}}}{L_{\text{max}} + L_{\text{min}}},
\]

(6.3)

where \( L_{\text{max}} \) and \( L_{\text{min}} \) are the maximum and minimum power, obtained when the temporal modulation of the internal optical path has a value such that the phase, \( \varphi_t = 0 \) or \( \pm \pi \), respectively.

We now describe the fringe power corresponding to a source focused on the edge of a step as shown in (Fig. 6.1b). In this case, the interference of the beams results in two sets of fringes, both with a contrast smaller than the one corresponding to the star focused in the middle of the step. In this situation, the internal phase introduced by the steps is not the same across the entire PSF. Given a step depth of \( d \) and considering that the normal to the surface of the mirror forms and angle \( \alpha \) with the optical axis, we have that the phase difference introduced by the step is given by \( \kappa_0 d_{\text{eff}} \), where \( d_{\text{eff}} = 2d \cos \alpha \). The light reflected from each step will result in two relative maxima of the power: the first one occurs when the phase is 0 and \( -\kappa_0 d_{\text{eff}} \) for the light focused on the first and second step, respectively, and the second one when the phase is \( \kappa_0 d_{\text{eff}} \) and 0 for the light focused on the first and second step, respectively. Correspondingly, there will also be two relative minima.

The resulting fringe pattern comprises two sets of fringes, each of them with visibilities \( \mu_1(B) \) and \( \mu_2(B) \), respectively. We will first calculate \( \mu_1(B) \); the maximum of this fringe set is reached when \( \varphi_t = 0 \) in the first step and \( \varphi_t = -\kappa_0 d_{\text{eff}} \) in the second one. Considering that the steps are orthogonal to the
6.2 Analytical description

x-axis and that the edge of the step is located at $x_2$, the integration of the flux gives a maximum power equal to:

$$ L_{\text{max}} = G \left\{ \left[ 1 + \mu(B) \right] \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy + \right. $$

$$ \left. \left[ 1 + \mu(B) \cos(\kappa_0 d_{\text{eff}}) \right] \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy \right\} $$

where:

$$ G = 2 \Delta \lambda \left( \frac{\pi N_{\text{eff}}}{\lambda_0} \right)^2 T^2 $$

$$ E = \sin \left( \frac{\pi d_{\text{eff}}}{L_c} \right) $$

For practical reasons, the integration domain $(x_1, x_3)$ and $(y_1, y_2)$ should be limited to the size of the diffraction image of the single aperture. In reality, the effective integration area is determined by the detector, so a sufficiently small photosensitive area has to be chosen depending on the size of the PSF of the system. The minimum of the fringe set occurs when $\varphi_t = \pm \pi$ in the first step and $\varphi_t = \pm \pi - \kappa_0 d_{\text{eff}}$ in the second one. The minimum power is:

$$ L_{\text{min}} = G \left\{ \left[ 1 - \mu(B) A \right] \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy + \right. $$

$$ \left. \left[ 1 - \mu(B) \cos(\kappa_0 d_{\text{eff}}) C_{\pm c} \right] \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy \right\} $$

with:

$$ A = \sin \left( \frac{\pi \lambda_0}{L_c} \right) $$

$$ C_{\pm c} = \sin \left( \frac{\pi d_{\text{eff}}}{L_c} \pm \frac{\pi \lambda_0}{2L_c} \right) $$
The relative visibility $\mu_1(B)$ calculated from Eq. (6.3) equals

$$\mu_1(B) = \mu(B) \left\{ \frac{\int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy + \cos(\kappa_0 d_{eff}) (E + C_{\pm c}) \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy}{2 \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy + \mu(B) R_1} \right\}$$

(6.8)

with

$$R_1 = (1 - A) \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy + \cos(\kappa_0 d_{eff}) (E - C_{\pm c}) \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy.$$  

(6.9)

Next we redo the above calculation in order to find an expression for the visibility of the second fringe set $\mu_2(B)$. The maximum of the second fringe set is reached when $\varphi_t = \kappa_0 d_{eff}$ in the first step and $\varphi_t = 0$ in the second one. The integration of the flux is:

$$L_{max} = G \left\{ \left[ 1 + \mu(B) \cos(\kappa_0 d_{eff}) E \right] \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy + \right\}$$

(6.10)

And the minimum of the second fringe set results:

$$L_{min} = G \left\{ \left[ 1 - \mu(B) \cos(\kappa_0 d_{eff}) C_{\pm c} \right] \int_{x_1}^{x_2} \int_{y_1}^{y_2} P(x, y) \, dx \, dy + \right\}$$

(6.11)
The relative visibility of the second fringe set is:

\[
\mu_2(B) = \frac{\cos(\kappa_0 d_{eff})(E + C_{+C}) \int_{x_1 y_1}^{x_2 y_2} P(x, y) dx dy + \int_{x_1 y_1}^{x_2 y_2} P(x, y) dx dy}{2 \int_{x_1 y_1}^{x_2 y_2} P(x, y) dx dy + \mu(B) R_2}
\]

(6.12)

with

\[
R_2 = \cos(\kappa_0 d_{eff})(E - C_{+C}) \int_{x_1 y_1}^{x_2 y_2} P(x, y) dx dy + (1 - A) \int_{x_1 y_1}^{x_2 y_2} P(x, y) dx dy.
\]

(6.13)

For typical values used in aperture synthesis, we have, in general, \((1 - A) \ll 1\) and \(\cos(\kappa_0 d_{eff})(E - C_{+C}) \ll 1\). These approximations are validated, for example, with the parameters of the tabletop setup described in Section 6.4; for a bandwidth \(\Delta\lambda = 150\) nm and a central wavelength \(\lambda_0 = 575\) nm, we have that \((1 - A) \approx 0.02\). The value of \(\cos(\kappa_0 d_{eff})(E - C_{+C})\) for the same spectral range, for a baseline of 30 mm is approximately 0.0025. In this way, we can consider that

\[
\mu(B) R_1, \mu(B) R_2 \approx 2 \int_{x_1 y_1}^{x_2 y_2} P(x, y) dx dy.
\]

(6.14)

Adding \(\mu_1(B)\) and \(\mu_2(B)\), we obtain:

\[
\mu_1(B) + \mu_2(B) \equiv f_\mu(B),
\]

(6.15)

![Fig. 6.2 Variation of \(\cos(\kappa_0 d_{eff})(E - C_{+C})\) with the baseline, for a bandwidth of 150 nm and a central wavelength of 575 nm.](image)
where \( f \), the so-called edge factor, is given by

\[
f = \frac{f_t + f_{-t}}{2}, \tag{6.16}
\]

and

\[
f_{\pm t} = \frac{1 + A + \cos(\kappa_0 d_{\text{eff}})(E + C_{\pm c})}{2}. \tag{6.17}
\]

Combining the measured fringe visibilities \( \mu_1(B) \) and \( \mu_3(B) \), and calculating the edge factor from Eq. (6.15) using the experimental parameters, the expected visibility can be easily recovered. As can be seen from Eq. (6.7) and Eq. (6.15), the edge factor depends on the central wavelength, the coherence length and the depth of the steps. As the depth of the steps depends on the projected baseline, for a given central wavelength and a given coherence length, the edge factor will fluctuate during observation as the baseline projection changes. In Fig. 6.3 the edge factor is plotted as a function of the baseline, for a maximum baseline of 35 mm. Note that the maximum change in the visibility is of approximately 15% of the expected value.

![Fig. 6.3 Analytical edge factor as a function of the baseline, for a spectral range of 500-650 nm.](image-url)
The relative angle, \( \varphi \), that gives the position of the star focused in the edge with respect to a reference one, can be calculated as explained in Chapter 4, by measuring the distance from one of the peaks of the fringe sets to the peak of the reference fringe pattern. If \( n_{p_1-p_n} \) is the distance from the peak of the first fringe set to the reference fringe pattern, and \( n_{p_2-p_n} \) the distance from the peak of the second fringe set to the reference fringe pattern, knowing between which steps, \( n_s \) and \( n_s+1 \), the star is focused, the relative phase is

\[
\varphi = \frac{1}{B} (n_s d_{\text{eff}} + n_{p_1-p_n}) = \frac{1}{B} ((n_s + 1)d_{\text{eff}} - n_{p_2-p_n}),
\]

(6.18)

where \( n_{p_1-p_n} \) and \( n_{p_2-p_n} \) can be either positive or negative but have always opposite sign.

### 6.3 Numerical simulations

Numerical simulations were implemented in order to study the effect of the edge of the staircase mirror in a pupil plane interferometer when several baselines are used, and to make a comparison with the analytical calculations. Given a specific power, position and bandwidth of the source, position, aperture diameter and focal length of the telescopes, and maximum and minimum baselines, we developed a program that calculates the width of the step for the maximum baseline. This width corresponds to a field of view for which the differential OPD is equal to 2 times the coherence length, and it also depends on the angle between the normal to the surface of the mirror and the optical axis. For the other baselines, the depth of the step is re-evaluated assuring the right correction for every baseline. The PSF of the single telescope is calculated and the differential optical path is corrected in the focal plane. Then the beams coming from the two telescopes are combined and focused at the detector plane. The program calculates the integration of the flux at the detector as a function of the temporal modulation of the internal optical path, and from the resulting fringe modulation it calculates the contrast which gives the visibility component corresponding to each baseline.

In the simulations, the emission bandwidth of the source is discretized, and the flux is calculated by adding the contribution of the different wavelengths:

\[
D(x, y) = \sum_{\lambda = \lambda_{\text{min}}}^{\lambda_{\text{max}}} D_{\lambda}(x, y).
\]

(6.19)

The parameters used for the simulations are shown in Table 6.1.
The temporal modulation of the path is a ramp varying from $-2L_c$ to $2L_c$, and the normal to the surface of the mirror is at an angle of $\pi/4$ with the optical axis. All the simulations were done for five different wavelengths and five baselines, with a mirror consisting of three steps in a grid of 63x63 points.

The input for the simulation is a star that is focused on the edge between two steps as illustrated in Fig. 6.1b. The results are shown in Fig. 6.4 and Fig. 6.5. In Fig. 6.4a the position of the star in the focal plane of the telescope is plotted. The total dimension of the extended field is approximately 2.4 mm, corresponding to a field of view of 1.26 arcmin. The correction of such a field of view is achieved with a mirror containing three steps of width 800 $\mu$m for a maximum baseline of 35 mm. In Fig. 6.4b, the section of the mirror parallel to the baseline is depicted, together with the profile of the PSF. The beams from the two arms of the interferometer are combined and the result from the integration of the flux is plotted in Fig. 6.5a as a function of the temporal modulation and the baseline. One can observe that the maximum of the temporal fringe pattern is translated from the zero path position and that the distance over which the maximum is translated is directly proportional to the baseline. Two local maxima are present in the fringe, and the separation of the maxima from the zero path position gives the relative position of the source. In this case, for the maximum baseline, the local maxima of the fringe pattern are reached when the temporal excursion is equal to $-L_c$ and $L_c$, respectively, and for the other baselines, the maximum is reached when the modulation is $-d_{\text{eff}}/2$ and $d_{\text{eff}}/2$, where $d_{\text{eff}}/2 = L_cB/B_{\text{max}}$. In Fig. 6.5b the fringe pattern obtained for the maximum baseline is shown. The two visibility components, calculated for the five baselines, are shown in Fig. 6.5c and are represented by black triangles and squares, respectively. Because the object is not resolved by the interferometer even with its largest baseline, the expected value of the visibility for every baseline is approximately 1. The black dots in Fig. 6.5c are the result of the addition of the two visibility components for every baseline divided by the edge factor. As can be seen, the fluctuation with respect to the expected value of the visibility is around 1%.

### Table 6.1 Simulation parameters

<table>
<thead>
<tr>
<th>$B_{\text{min}}$</th>
<th>$B_{\text{max}}$</th>
<th>$\rho_a$</th>
<th>$F$</th>
<th>$\Delta\lambda$</th>
<th>$\lambda_0$</th>
<th>$L_c$</th>
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<tr>
<td>30 mm</td>
<td>35 mm</td>
<td>20 mm</td>
<td>6563 mm</td>
<td>150 nm</td>
<td>575 nm</td>
<td>2.2 $\mu$m</td>
</tr>
</tbody>
</table>

The temporal modulation of the path is a ramp varying from $-2L_c$ to $2L_c$, and the normal to the surface of the mirror is at an angle of $\pi/4$ with the optical axis. All the simulations were done for five different wavelengths and five baselines, with a mirror consisting of three steps in a grid of 63x63 points.

The input for the simulation is a star that is focused on the edge between two steps as illustrated in Fig. 6.1b. The results are shown in Fig. 6.4 and Fig. 6.5. In Fig. 6.4a the position of the star in the focal plane of the telescope is plotted. The total dimension of the extended field is approximately 2.4 mm, corresponding to a field of view of 1.26 arcmin. The correction of such a field of view is achieved with a mirror containing three steps of width 800 $\mu$m for a maximum baseline of 35 mm. In Fig. 6.4b, the section of the mirror parallel to the baseline is depicted, together with the profile of the PSF. The beams from the two arms of the interferometer are combined and the result from the integration of the flux is plotted in Fig. 6.5a as a function of the temporal modulation and the baseline. One can observe that the maximum of the temporal fringe pattern is translated from the zero path position and that the distance over which the maximum is translated is directly proportional to the baseline. Two local maxima are present in the fringe, and the separation of the maxima from the zero path position gives the relative position of the source. In this case, for the maximum baseline, the local maxima of the fringe pattern are reached when the temporal excursion is equal to $-L_c$ and $L_c$, respectively, and for the other baselines, the maximum is reached when the modulation is $-d_{\text{eff}}/2$ and $d_{\text{eff}}/2$, where $d_{\text{eff}}/2 = L_cB/B_{\text{max}}$. In Fig. 6.5b the fringe pattern obtained for the maximum baseline is shown. The two visibility components, calculated for the five baselines, are shown in Fig. 6.5c and are represented by black triangles and squares, respectively. Because the object is not resolved by the interferometer even with its largest baseline, the expected value of the visibility for every baseline is approximately 1. The black dots in Fig. 6.5c are the result of the addition of the two visibility components for every baseline divided by the edge factor. As can be seen, the fluctuation with respect to the expected value of the visibility is around 1%.
Fig. 6.4 Effect of the staircase mirror in an off-axis star that is imaged on the edge of a step: (a) position of the off-axis star. (b) Section of the staircase mirror parallel to the baseline, and the point spread function resulting from the combination of the beams from the two arms of the interferometer.
Fig. 6.5  (a) The power, plotted as a function of time and baseline. The relative maxima of the fringe, reached when the total OPD is zero, correspond to an internal path of $d/2$ and $-d/2$, respectively. (b) The fringe corresponding to the maximum baseline ($d=2L_c$), and the relative maxima of the fringe take place when the internal OPD is $L_c$ and $-L_c$, respectively. (c) The contrast from each set of fringes plotted in black squares and triangles, respectively. The black dots represent the addition of the contrast of the two sets of fringes corresponding to every baseline divided by the edge factor.
6.4 Experimental results

The experimental results presented here have been obtained with the setup shown in Fig. 6.1a. It consists of a two-telescope Michelson-type interferometer, where in the focal plane of one of the arms a staircase mirror is placed. The setup consists of three main blocks: the star simulator, the interferometer and the beam combiner. The star simulator consists of a Xenon arc lamp, with a set of filters that reduces its spectrum to a selected bandwidth and whose light is imaged onto a star mask. The star mask is mounted in a translation stage, allowing to move the stars over the field. The light from the star mask is collimated and re-imaged in the focal plane of each of the telescopes in the interferometer block. In the beam combiner block, the light is collimated again and two mirrors on top of a piezo stage, set in one of the interferometer's arms, provide a delay line to modulate the optical path. The beams overlap at a beam splitter/combiner cube and a low noise photo detector measures the modulated fringes.

6.4.1 Experimental results with the 30 mm baseline

The fringes were measured for five different positions of a star with respect to the edge of the step. First, the star was situated on-axis and at the centre of one step, obtaining one set of reference fringes. The visibility of this set of fringes was used as reference. Then, the star mask was translated in such a way that the star was placed close enough to the edge so that a second fringe set started to appear. Subsequently, the star was moved to the edge between the steps, resulting in two sets of fringes with similar contrast. As the star is moved further towards the centre of the next step, the contrast of one of the sets increases while the other one decreases in a way that the total contrast stays constant. When the star is focused close to the centre of the next step, the interference fringes form one set again.

The measured calibrated fringes are shown in Fig. 6.6 and Fig. 6.7 for two different central wavelengths and bandwidths. Because the edge factor depends on the bandwidth and central wavelength of the detected light, measurements were taken with two different sets of filters in the star simulator. In Fig. 6.6, the results using a bandwidth from 500 nm to 650 nm are plotted. When the star is focused close to the centre of the step, the visibility is 0.82±0.02, as shown in Fig. 6.6a. In Fig. 6.6b, the star is placed close to the edge and a second set of fringes begins to appear; the addition of the two visibility components is equal to 0.92. Fig. 6.6c shows the calibrated fringe measured when the star is focused approximately at the centre of the edge, so that the two components show similar contrast, and their addition is 0.91. From Fig. 6.6d, the addition of the two components is 0.90, and finally in
Fig. 6.6 Experimental results of the effect of the edge on the visibility, with a bandwidth of 150 nm and a central wavelength of 575 nm; going from (a) to (e), the star is moved from the centre of the central step to the centre of the next step. In (a) and (e), because the star is focused in the centre of the step, one fringe set is observed. In (b), as the star is moved close to the edge, a second fringe set appears, with contrast smaller than the other component. In (c), the star is focused approximately in the middle of the edge, both fringe sets present similar contrast. In (d), as the star is moved towards the centre of the next step, the contrast of the first fringe set decreases and the contrast of the second one increases.
Fig. 6.7  Same as in Fig. 6.6, but for a bandwidth of 215 nm centered in 542 nm
Fig. 6.6 e the fringe when the star is focused in the next step is plotted, and the visibility is 0.78.

The difference in contrast from 0.82 to 0.78 between the case when the star is on-axis with respect to the case when the star is off-axis is attributed to dispersion effects. To compare the experimental results with the analytical predictions, we have to calculate the edge factor with Eq. (6.16) and Eq. (6.17). The average from the three measurements taken when the star is focused on the edge is multiplied by the edge factor to obtain the visibility. This resulting visibility is compared with the reference visibility obtained by averaging the two visibilities taken when the stars are focused on the steps. To calculate the edge factor, it is necessary to measure the depth of the steps. From a direct measurement, the depth of the steps of the staircase mirror used in the setup is 2.65 $\mu$m, which agrees quite well with the 2.64 $\mu$m calculated from the measurement of the peak-to-peak distance of the two fringe sets. The edge factor calculated with the parameters of the experiment is then 1.13±0.01. The average of the addition of the two contributions to the visibility is 0.91±0.02. From Eq. (6.15) a visibility of 0.80±0.02 is recovered, where the error is calculated by error propagation. Given that the average of the visibility measured when the star is focused on the steps is 0.80±0.02, the result is that the visibility when the star is focused on the edge is completely recovered.

The results obtained using a bandwidth from 435 nm to 650 nm are plotted in Fig. 6.7. The average of the three measurements taken when the star is focused on the edge (Fig. 6.7b, Fig. 6.7c, and Fig. 6.7d) gives an addition of the two contributions equal to 0.81±0.02. From the used parameters, the analytical edge factor is 1.03±0.01 and, from Eq. (6.15), a visibility of 0.79±0.03 is calculated. From the average of the two measurements of the fringe pattern when the star is focused on the steps (Fig. 6.7a and Fig. 6.7d) a visibility of 0.78±0.02 is obtained. The visibility is recovered with a 1% error.

### 6.4.2 Experimental results with the 34 mm baseline

For the experiment described in this section we changed the baseline of the interferometer to 34 mm. In order to use the same mirror, we also have to change the effective focal length of the setup to 7.438 m to keep the relation between the depth and the width of the steps, as shown in Section 5.4. When changing the focal length, the positions of the stars in the focal plane change, and this allows to have a reference star in a step and another star in the edge between two steps. The results are depicted in Fig. 6.8. The visibility of the reference star, plotted in Fig. 6.8a, is 0.70±0.02. It is lower than expected due to dispersion effects. The calibrated fringe from the star focused on the edge between two steps is shown in Fig. 6.8b. The addition of the two visibility components is equal to 0.80±0.02.
6.5 Conclusions

Because the edge factor depends on the depth of the steps and the bandwidth and we have not changed these parameters, it is 1.13±0.01 as in the previous case. With this factor, a visibility of 0.71±0.02 is recovered, and the relative angle is (3.0±0.1)x10^-4 rad.

Fig. 6.8  Experimental results of the effect of the edge on the visibility, with a bandwidth of 150 nm, a central wavelength of 575 nm, and a baseline of 34 mm: (a) the calibrated fringe from the reference star, (b) the calibrated fringe from the star focused on the edge between two steps.

6.5 Conclusions

We have analytically described the effect on the detected visibility due to the discontinuous correction of the OPD in a stellar interferometer containing a staircase mirror. When the star is focused on the edge between two steps in the staircase mirror, the modulated fringe pattern is split into two fringe sets. The addition of the individual contrasts of the two sets, apart from a proportionality constant, yields the expected visibility as if the star was focused only on one step. This constant is the so-called edge factor and can be obtained from the spectrum of the detected light and the mirror configuration. Theoretical analysis and numerical simulations are in good agreement with the experimental results.
Therefore, the discontinuous nature of our wide-field solution does not imply that the acquired field of view is discontinuous. Using the simple algorithm described in this chapter, we show that a continuous wide field of view can be acquired in one shot with a Michelson pupil-plane interferometer.

6.6 References


Conclusions

Optical Stellar Interferometry is a technique that makes use of the van Cittert-Zernike theorem to obtain high resolution information about stellar sources. The light beams from two or more telescopes are combined coherently and from the resulting fringe pattern information about the visibility function of the source is obtained. As explained in Chapter 1, to measure at least the contrast of the fringes, the Optical Pathlength Difference (OPD) error has to be smaller than the coherence length. In an interferometer, the correction of the OPD is done for the pointing direction of the telescope, but for any other angle the OPD will be different. If this difference is larger than the coherence length, coherent interference of the beams will not take place, and, as result, the field of view is limited. Depending on the type of interferometer, there are specific solutions to deal with the field-of-view problem. Fizeau interferometry, that basically consists on reproducing a down-scaled version of the synthetic aperture of the telescope array at the recombination optics, has the problem that the signal-to-noise ratio decreases with the number of telescopes, limiting the number of baselines that can be used in a system. Besides, when the size of the baseline is large compared to the single telescope diameter, the energy is spread over the sidelobes of the diffraction pattern limiting the sensitivity of the instrument. This principle can be a good solution for instruments with many telescopes and short baselines, but it is not the optimal solution for an interferometer with only three or four telescopes and baselines of hundreds of meters. For the latter a Michelson-type interferometer is a better option since, when working in pupil-plane combination, this type of interferometer has the advantage that all the photons are concentrated in one central peak in the fringe pattern. The mosaicing method developed by NASA to perform wide-field observations with Michelson-type interferometers can acquire a total field of view of several arcmin, but not in one shot. Actually the observing time is also multiplied by the N/2 factor.

Our goal in this thesis was to use the advantages of Michelson pupil-plane interferometers and develop a new technique to acquire a wide field of view in one shot.
In order to do that, we looked at the problem of defining and calculating the differential Optical Path Difference ($\Delta OPD_{\text{ext}}$) as a function of the baseline and pointing vectors. According to our simulations we concluded that the second order terms of the $\Delta OPD_{\text{ext}}$ function, even for large baselines, are quite small, being of the order of the milliarcsec, translated to angle in the sky, within a field of view of 1 arcmin angular diameter. Therefore, to observe a wide field of view, only correction of the first-order terms of the function is necessary. The shape of the linearized $\Delta OPD_{\text{ext}}$ function is a tilted plane where the constant OPD zones are perpendicular to the projection of the baseline on the entrance pupil of the telescope. By correcting the $\Delta OPD_{\text{ext}}$ as a function of the angle in the sky, one can obtain high resolution information from a wide field of view.

The system we have chosen consists in correcting the first-order terms of $\Delta OPD_{\text{ext}}$ in an intermediate focal plane of the optical subsystems before the beam combination takes place. The focal plane has the characteristic that the light from different parts of the sky is focused at different places in the plane. A stair-shaped mirror placed in the focal plane introduces an additional term to the optical path, $\Delta OPD_{\text{int}}$, that corrects $\Delta OPD_{\text{ext}}$ for every angle. The depth and width of the steps of the mirror depends basically on the projected baseline, the angle that the normal to the surface of the mirror forms with the optical axis of the system, the focal length and the path length scanned by the delay line. The effect of the mirror is to shift the off-axis fringes into the region scanned by the delay line, allowing their detection. From the contrast of the fringe, the modulus of the visibility of the off-axis source can be calculated. From the distance from the peak of the off-axis fringe to the peak of the on-axis reference fringe, and knowing in which step the off-axis source was focused, the relative angular separation of the off-axis star with respect to the reference source can be obtained. In Chapter 3 of this thesis, the general procedure to calculate the shape of the mirror required to correct the differential OPD in the focal plane of a telescope in an array is developed. The mirror consists of steps of fixed width and adaptable depth. The depth of the steps must vary, as it has to be adapted to the changes of the pointing vector. The mirror has to be rotated to follow the rotation of the baseline projected in the entrance pupil, in order to have the steps perpendicular to it. The width of the steps is a fixed parameter given a certain telescope array configuration.

The complete process starting from an object in the sky to the visibility retrieval, considering the case of a Michelson interferometer with a staircase mirror in the focal plane of one of its arms, has been described analytically in Chapter 4. The most important effect of the staircase mirror is the simultaneous correction of the optical path for the on- and off-axis light, so that the retrieval of the visibility and relative angular position for the on- and off-axis objects can be achieved. The
analytical results agree with the theoretical visibility curves provided by the van Cittert-Zernike theorem. Therefore, wide-field imaging, within the isoplanatic patch, with a Michelson pupil-plane interferometer is possible, and we suggest to do it using the algorithms provided for radio-interferometric mosaicing. We designed and implemented a tabletop setup consisting of a two-telescope Michelson-type interferometer to demonstrate the feasibility of our wide field solution. In the focal plane of one of the arms of the interferometer a staircase mirror was placed. The main goal of the setup was to simultaneously observe fringes with comparable contrast from separated stars, in a field of view of approximately 1 arcmin angular radius, and to retrieve the visibility and relative phase of the off-axis stars. The experimental results using white light (with a bandwidth from 500 nm to 650 nm) and a 30 mm baseline, show that fringes from an off-axis star separated 1.07 arcmin from the on-axis reference star were obtained simultaneously and with a contrast similar to the on-axis one, this contrast being 0.80±0.02. The relative angle of the off-axis star has been retrieved and corresponds to the expected value, (3.1±0.1)x10^-4 rad. The different factors that can affect the retrieval of the visibility, such as dispersion, background noise, or the effect of the lateral displacement of the pupil introduced by the steps were also studied. The lateral displacement, a problem that is intrinsic in the staircase mirror approach, extrapolated to the VLTI parameters, introduces an error of 1% for a field of view of 5-25 arsec, depending on the depth of the steps, when the normal to the surface of the mirror forms an angle of 45° with the optical axis. One way to avoid the visibility losses due to non-perfect overlapping of the beams is to cut away the edge of the recombined beam with a diaphragm in the pupil image, which would produce a small reduction of the power.

Finally, the effect on the detected visibility due to the discontinuous correction of the OPD in a stellar interferometer containing a staircase mirror has been analytically described. When the star is focused on the edge between two steps in the staircase mirror, the modulated fringe pattern is split into two fringe sets. The addition of the individual contrasts of the two sets, apart from a proportionality constant, yields the expected visibility as if the star was focused only on one step. This constant is the so-called edge factor and can be obtained from the spectrum of the detected light and the mirror configuration. Theoretical analysis and numerical simulations are in good agreement with the experimental results. It has been proved experimentally that the visibility can be recovered within 1% error. The diffraction effects due to the edge have been studied using Fourier analysis to determine the limits of this approach showing that it is valid when the diameter of the re-collimating lens is bigger than 1.5-1.7 times the output pupil. Therefore, the discontinuous nature of our wide-field solution does not imply that the acquired
field of view is discontinuous. Using a simple algorithm, we show that a \textit{continuous} wide field of view can be acquired in one shot with a Michelson pupil-plane interferometer.

The experimental results presented in this thesis have been obtained with a mirror of fixed shape, but, as has been explained, an actuated mirror is required to use the system for astronomical observations. The mirror could consist of an array of parallel strips, each of them mounted on a piezo stage, independently controlled, to actively correct the \textit{OPD}. The required accuracy in the fine positioning of the steps should be in the order of tens of nanometers. This requirement is met by commercial piezo actuators. The challenge is to find a system that combines this accuracy with a large stroke, of the order of several millimeters. One option is to use two sets of actuators: one to provide long strokes, and another one to provided fine positioning. Future work should focus on the design and test of such an actuated staircase mirror, but with the advance of this type of technology provided by, for example, adaptive optics, this task should be challenging although not impossible to achieve.

The study of the possible algorithms to deconvolve the wide-field data in order to produce images of extended objects was out of the scope of this thesis, but future research on wide-field interferometry, based in our method or in the homothetic one, should also be focused in this subject. Optical interferometry can profit from the work already developed in the field of radio interferometric imaging, but the algorithms have to be adapted and studied in order to choose which type of reconstruction applies better to the optical case.
Appendix A
Numerical Simulations

A.1 2-Dimension numerical simulation

A MATLAB program has been written to simulate the effect of the staircase mirror in a pupil plane interferometer. Given a specific luminosity, position and bandwidth of a source, position, aperture diameter and focal length of the telescopes, and maximum and minimum baselines, we developed a program that calculates the width of the step for the maximum baseline. This width corresponds to a field of view for which the differential OPD is equal to 2 times the coherence length, and it also depends on the angle between the normal to the surface of the mirror and the optical axis. For the other baselines, the depth of the step is re-evaluated assuring the right correction for every baseline. The PSF of the single telescope is calculated and the differential optical path is corrected in the focal plane. Then the beams coming from the two telescopes are combined and focused at the detector plane. The program calculates the integration of the flux at the detector as a function of the temporal modulation of the internal optical path, and from the resulting fringe modulation it calculates the contrast which gives the visibility component corresponding to each baseline. An scheme of the structure of the program can be found in Fig. A.1.

In the simulations, the emission bandwidth of the source is discretized, and the flux is calculated by adding the contribution of the different wavelengths:

$$D(x, y) = \sum_{\lambda = \lambda_{\text{min}}}^{\lambda_{\text{max}}} D_\lambda(x, y).$$  \hspace{1cm} (A.1)

The parameters used for the simulations are shown in Table A.1.

Table A.1 Simulation parameters.

<table>
<thead>
<tr>
<th>B_{\text{min}}</th>
<th>B_{\text{max}}</th>
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<td>6563 mm</td>
<td>150 nm</td>
<td>575 nm</td>
<td>2.2 \mu m</td>
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Defining setup characteristic values
Defining coordinates of the focal plane
Initialising variables
Calculating the external OPD function of the coordinate of the star
Calculating the internal OPD added by the staircase mirror
Defining the sampled object (a few points)
Defining the temporal OPD used for this source
Define the emission spectrum
For each wavelength
For each point of the source
Calculating the element of field due the current wavelength and point of the source, on each point of the focal plane for arm A
Calculating the element of field due the current wavelength and point of the source, on each point of the focal plane for arm B with temporal OPD
Calculating corresponding maximum and minimum intensities with correction and with ideal correction
Adding it to the integral
Saving Data
Saving Data
Saving Data
Displaying the results

Fig. A.1  Structure of the program used for the numerical simulations.
The temporal modulation of the path is a ramp varying from $-2L_c$ to $2L_c$, and the normal to the surface of the mirror is at an angle of $45^\circ$ with the optical axis. All the simulations were done for five different wavelengths and five baselines, with a mirror consisting of three steps in a grid of 63x63 points. For reasons of computation speed, it was chosen to set the variables in the computation as dimensions of a 4-dimensional hyper-matrix instead of using loops. The first and the second dimensions were $x$ and $y$, the coordinates in the focal plane of the telescope, the third one was the wavelength and the fourth one the temporal modulation of the optical path. As there is a limitation of four dimensions when using hyper-matrices in MATLAB, the calculation for different baselines was done in a loop. Therefore, the size of the grid, the number of wavelengths and the points calculated for the temporal modulation were limited by the virtual memory of the computer, and the number of baselines by the computing time. The results are described in the following subsections.

### A.1.1 On-axis star

We use as input a star that is exactly in the centre of the field of view, as shown in Fig. A.2. The position of the star in the focal plane of the telescope is plotted in Fig. A.2a. The total dimension of the extended field are approximately 2.4 mm, which for the focal length of 6563 mm corresponds to a field of view of 1.26 arcmin. A mirror with three steps of a width of 800 $\mu$m, as is seen in Fig. A.2b, was considered. This width was calculated for the maximum baseline of 35 mm. The light is focused on the step and the differential OPD is introduced. The light from the two arms of the interferometer is combined and the result from the integration of the flux is plotted in Fig. A.3a in function of the temporal modulation and the baseline. Because for an on-axis star the differential OPD is equal to zero, the maximum of the fringe is obtained when the temporal OPD is zero for all the baselines. In Fig. A.3b the fringe obtained for the maximum baseline is plotted. Because the star is a point source, not resolved by the interferometer even with its largest baseline, the value of the visibility for every baseline is approximately 1, as plotted in Fig. A.3c, where the solid line is the expected visibility in function of the baseline, and the black dots are the results for the five different baselines calculated.
Fig. A.2  (a) Position of the on-axis star, in the centre of the field of view. (b) Section of the staircase mirror parallel to the baseline, and the point spread function resulting from the combination of the beams from the two arms of the interferometer when the temporal modulation makes the combination constructive.
Fig. A.3  (a) The power, calculated by integrating the flux of the spot shown in Fig. A.2b, is plotted in function of time and baseline. (b) The fringe corresponding to the maximum baseline, showing that the maximum of the fringe is at zero modulation. (c) Expected visibility (solid line) compared to calculated visibility (dots) for various baselines.
A.1.2 Off-axis star not centred in the step
In Fig. A.4 the configuration when the input is an off-axis star which is not centred in a step are shown. The differential OPD for this star is $L_c/2$. In Fig. A.4a the position of the star in the focal plane is plotted, and in Fig. A.4b the shape of the staircase mirror and the Point Spread Function resulting from the combination of the beams from the two arms of the interferometer. The difference with the case of an on-axis star is clear observing Fig. A.5a. We see that the maximum of the fringe is translated from the zero position, and this translation is directly proportional to the baseline. The reason is that for an off-axis star which is not in the centre of the step the differential OPD is not completely corrected by the staircase mirror. As explained in Chapter 3, the interference of the beams over the extended field is coherenced, that means, it is corrected enough to assure coherent interference, but it is cophased only for the sources that are focused in the centre of the steps. For the other sources, the separation of the maximum of the fringe depends on the position of the source and the baseline, and it is equal to the differential OPD that remains after correction by the staircase mirror, always smaller than $L_c$. In this case, for the maximum baseline the maximum of the fringe is reached when the temporal modulation is equal to $L_c/2$, as seen in Fig. A.5b, and for the rest of the baselines is reached when it is $d_{off}/4$. As is plotted in Fig. A.5c, the visibility calculated for the different baselines is again 1.

A.1.3 Off-axis star centred in the step
The configuration of an off-axis source that is focused exactly in the middle of a step is represented in Fig. A.6. The position of the star in the focal plane is plotted in Fig. A.6a, and the combined Point Spread Function with the staircase mirror in Fig. A.6b. The results shown in Fig. A.7 coincides with the results for an on-axis star. The reason is that the differential OPD for the objects focused in the centre of the steps is completely corrected for any baseline. Because of that, the maximum of all the fringes, as seen in Fig. A.7a, is always reached for a temporal modulation equal to zero, like in the case of an on-axis star. The visibility components calculated for each baseline are plotted in Fig. A.7c.
Fig. A.4 (a) Position of the off-axis star. (b) The section of the staircase mirror parallel to the baseline, and the point spread function resulting from the combination of the beams from the two arms of the interferometer.
Fig. A.5  (a) The power plotted as a function of time and baseline. The maximum of the fringe, reached when the total OPD is zero, corresponds to an internal path of \( d_{\text{eff}}/4 \). (b) The fringe corresponding to the maximum baseline, when \( d=2L_c \), and the maximum of the fringe takes place when the internal OPD is \( L_c/2 \). (c) Same legend as in Fig. A.3c.
Fig. A.6  (a) Position of the off-axis star. (b) The section of the staircase mirror parallel to the baseline and the $PSF$. 
Fig. A.7  (a) The power plotted as a function of time and baseline. Because the external OPD is completely corrected by the mirror, the maximum of the fringe is reached when the modulated temporal OPD is zero. (b) The fringe corresponding to the maximum baseline. (c) Same as in Fig. A.3c
Fourier analysis of the effect of a staircase step on the visibility

The system we use for our theoretical analysis is represented in Fig. B.1 and consists of a two-arm ($A$ and $A'$) Michelson interferometer. At the entrance of each arm there is an aperture with a lens. In the focal plane of the lens in arm $A'$ we place the staircase device formed by steps of depth $d$. When the focal image is exactly centred on the edge of the step a wavelength-dependent phaseshift of $\delta(\lambda)$ is imparted to half the Fourier spectrum of the corresponding Point Spread Function (PSF). We collimate the beam from each arm again and after this we combine both beams. In order to study the effects that the phaseshift $\delta(\lambda)$ has on the final detected power, we perform a 1-D simulation of the system.

Fig. B.1 Schematic representation of the interferometer. $A_1$ and $A'_1$ are the diffracting apertures with lenses that produce images in the focal plane ($A_2$ and $A'_2$). A phase shift is introduced at $A'_2$ to half the area of the focused beam. The two beams are then recombined using apertures $A_3$ and $A'_3$ and the power modulation is measured at the location of $D$. 
Fourier analysis of the effect of a staircase step on the visibility

We are simulating a Michelson interferometer therefore, after recombination, the apertures are both effectively centred at the origin, and their diameters are equal and represented by $D$. The one-dimensional apertures are represented by the functions $A_1(x)$ and $A'_1(x)$,

$$A_1(x) = A'_1(x) = \text{rect}[x/D],$$  \hspace{1cm} (B.1)

with:

$$\text{rect}((x-x_0)/d) = \begin{cases} 1 & |(x-x_0)/d| \leq 1/2 \\ 0 & |(x-x_0)/d| > 1/2 \end{cases}.$$  \hspace{1cm} (B.2)

The PSF of the system is

$$A_2(\lambda, f_x) = A'_2(\lambda, f_x) = \frac{D}{\lambda F} \text{sinc} \left( \frac{D f_x}{2} \right),$$  \hspace{1cm} (B.3)

with

$$f_x = \frac{2\pi x}{\lambda F},$$  \hspace{1cm} (B.4)

and, as usually,

$$\text{sinc} (x) = \frac{\sin(x)}{x}.$$  \hspace{1cm} (B.5)

In the focal plane of arm $A'$ there is a staircase mirror that introduces a partial phase shift to half the spectrum of the PSF. It's represented by the function $S(f_x)$

$$S(f_x) = \begin{cases} 1 & f_x \leq 0 \\ \exp(-j\delta(\lambda)) & f_x > 0 \end{cases}.$$  \hspace{1cm} (B.6)

The complex amplitude $A_3(x)$ in the far-field of arm $A$ is simply given by

$$A_3(x) = A_1(x);$$  \hspace{1cm} (B.7)

in arm $A'$ we obtain

$$A'_3(\lambda, x) = \frac{1}{2\pi} \int_{-\infty}^{0} A'_2(\lambda, f_x) \exp(jfx) df_x + \frac{1}{2\pi} \int_{0}^{\infty} A'_2(\lambda, f_x) \exp(-j\delta(\lambda)) \exp(jfx) df_x.$$  \hspace{1cm} (B.8)

Using Eq. (B.3) we get

$$A'_3(\lambda, x) = A'_1(x) + \frac{1}{2\pi\lambda F} \int_{0}^{\infty} \frac{D}{2} \text{sinc} \left( \frac{D f_x}{2} \right) \left( \exp(-j\delta(\lambda)) - 1 \right) \exp(jfx) df_x.$$  \hspace{1cm} (B.9)
Introducing Eq. (B.9) in the computing code Maple we obtain the following expression:

\[ A'_3(\lambda, x) = A'_1(x) + \frac{\exp(-j\delta(\lambda)) - 1}{2\pi} \left[ \pi + 2j\arctanh\left(\frac{2x}{D}\right) \right], \quad (B.10) \]

and for the wavelength-integrated flux we obtain

\[ I_3(x) = \int_{\lambda_0 - \Delta\lambda}^{\lambda_0 + \Delta\lambda} |A'_3(x)|^2 d\lambda, \quad (B.11) \]

\[ I'_3(x) = \int_{\lambda_0 - \Delta\lambda}^{\lambda_0 + \Delta\lambda} |A'_3(\lambda, x)|^2 d\lambda, \quad (B.12) \]

where \( \lambda_0 \) is the mean wavelength and \( \Delta\lambda \) is the bandwidth. We can see the graphical representation of function \( I'_3(x) \) in Fig. B.2, when \( \delta = \kappa_0 d_{\text{eff}} \) with \( d_{\text{eff}} = 2d\cos\alpha \), where \( \alpha \) is the angle that the normal to the surface of the mirror forms with the optical axis and \( d=2.64 \mu\text{m} \), and \( \kappa_0 \) is the wave number for wavelength \( \lambda_0 \).

Now we combine the beams from arm \( A \) and \( A' \). The diameter of the collimating lenses and the diameter of the beam combination lens are both equal and

![Graphical representation](image)

**Fig. B.2** The computed flux \( I'_3(x) \). The apparent asymmetry is due to the choice of sampling points.
represented by $D_c$. In the combined focal plane the contribution of arm $A$, when $D_c>D$, is given by

$$A_d(\lambda, f_x) = A_2(\lambda, f_x), \quad (B.13)$$

and when $D_c<D$ the result is

$$A_d(\lambda, f_x) = \frac{D_c}{\lambda F} \text{sinc} \left( \frac{D_c}{2} f_x \right), \quad (B.14)$$

The contribution of arm $A'$ is given by:

$$A'_d(\lambda, f_x) = A_d(\lambda, f_x) + \{ \exp[-j\delta(\lambda)] - 1 \} G(\lambda, f_x), \quad (B.15)$$

where:

$$G(\lambda, f_x) = \int_{-D/2}^{D/2} \frac{\pi + 2j \arctanh \left( \frac{2x}{D} \right)}{2\pi} \exp(-jfx)dx. \quad (B.16)$$

Finally the combined PSF is:

$$D(\lambda, f_x) = A_d(\lambda, f_x) + A'_d(\lambda, f_x). \quad (B.17)$$

In Fig. B.3 and Fig. B.4 we have plotted the combined PSF when the OPD is zero and $\pi$, respectively, for one of the steps, and $D_c$ is infinitely large. The parameters used are the actual dimensions of the experiment, $D=2$ mm. The depth of the steps, $d$, is 2.64 $\mu$m. The result for the other step is similar. The bandwidth is 500-650 nm and the baseline 30 mm; for these values the edge factor calculated with Eq. (6.16) is 1.13. The visibility calculated with Eq. (6.15) is then 1.00, the expected value for a point source.

In the previous simulation we did not consider the effect of the finite apertures of the collimating lenses, but if we consider a finite diameter $D_c$, the high frequencies on function $A'_d(x)$ will be cut as an effect of the convolution of the Fourier transforms of the function itself and the function that defines the beam combination lens. As we are not integrating to infinity any more, the high frequencies are cut, and this will affect the final measured visibility.

In Fig. B.5 and Fig. B.6 we have plotted the combined PSF when the OPD is zero and $\pi$, respectively, for one of the steps, for a collimating lens of 30 mm aperture, the same as in the experiment. For this value, the diffraction is not affecting much the measurements and we get back a visibility equal to 0.99. The results when the edge is not in the centre are similar.
Fig. B.3 Flux profile of the combined beams when the interference is constructive for one of the steps and in the limiting case that the diameter of the collimating lens is infinitely large.

Fig. B.4 Same as in Fig. B.3, but now in the case of destructive interference.
Fig. B.5 Flux profile of the combined beams when the interference is constructive for one of the steps and the diameter of the collimating lens is 30 mm (as compared to 2 mm for the initial apertures $A_1$ and $A_1'$).

Fig. B.6 Same legend as in Fig. B.5, but now in the case of destructive interference.
Of course, in a real system the collimating lens or mirror will have a size very similar to that of the output pupil of the telescopes. We have performed simulations for different ratios $D_c/D$. We overlapped the functions in the pupil plane, and modulating the optical path we observe two sets of fringes, as explained in Chapter 6. We calculate the contrast from each fringe set, $V_1$ and $V_2$, and the power of each beam, $I_1$ and $I_2$, to calculate the visibility of each fringe set, $\mu_1$ and $\mu_2$. Because the edge is in the middle of the PSF, in this case $V_1=V_2$ and $\mu_1=\mu_2$. Finally, we add both contributions and divide over the edge factor $f$, that in this case was 1.13. As shown in Eq. (6.15), doing this we expect to recover a value approximately equal to one, or with an error of maximum 2%. The maximum power of the combined light when there is no edge has been normalized to unity. The results are presented in Table B.1.

### Table B.1

<table>
<thead>
<tr>
<th>$D_c/D$</th>
<th>0.3</th>
<th>0.5</th>
<th>0.8</th>
<th>1</th>
<th>1.3</th>
<th>1.5</th>
<th>1.7</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{\text{max}}$</td>
<td>0.20</td>
<td>0.34</td>
<td>0.55</td>
<td>0.71</td>
<td>0.74</td>
<td>0.75</td>
<td>0.755</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>$I_{\text{min}}$</td>
<td>0.04</td>
<td>0.07</td>
<td>0.11</td>
<td>0.16</td>
<td>0.20</td>
<td>0.20</td>
<td>0.21</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td>$I_1$</td>
<td>0.07</td>
<td>0.12</td>
<td>0.20</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$I_2$</td>
<td>0.04</td>
<td>0.07</td>
<td>0.12</td>
<td>0.18</td>
<td>0.21</td>
<td>0.22</td>
<td>0.22</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>$V_1$</td>
<td>0.68</td>
<td>0.67</td>
<td>0.66</td>
<td>0.62</td>
<td>0.58</td>
<td>0.57</td>
<td>0.57</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>0.70</td>
<td>0.70</td>
<td>0.68</td>
<td>0.63</td>
<td>0.585</td>
<td>0.58</td>
<td>0.57</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>$2\mu_1/f$</td>
<td>1.25</td>
<td>1.24</td>
<td>1.20</td>
<td>1.12</td>
<td>1.04</td>
<td>1.02</td>
<td>1.01</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

The results show that when cutting the tails of the function, we are actually removing the coherent background that was producing the fluctuations calculated in the edge factor. Therefore, when $D_c/D$ is smaller than approximately 1.5-1.7, according to our calculations one can not apply the procedure to calculate the expected visibility explained in Chapter 6, because the error in the obtained result is larger than 2%. On the other hand, reducing the coherent background produces an increment in the visibility of each fringe set. This is due to the effect that the minimum power decreases slower than the maximum power, as plotted in Fig. B.7. The position of the peaks of each fringe set relative to the reference remains unchanged.
Fig. B.7 Maximum power (black dots) and minimum power (black triangles) of the combined light, plotted as a function of $D_c/D$. When cutting the pupil, both the maximum and the minimum tend to zero, but the minimum decreases slower than the maximum, producing an increment of the contrast.
Samenvatting

Het belangrijkste doel van het in dit proefschrift beschreven onderzoek was de ontwikkeling van een systeem dat een brede veldhoek mogelijk maakt bij het gebruik van Michelson interferometers. Een breed gezichtsveld is belangrijk voor toepassingen zoals het waarnemen van grote of meervoudige objecten, acquisitie van interferentielijnen en/of het volgen van een nabijgelegen niet-opgelost object, en ook voor reductie van de tijd benodigd voor de waarneming. Voor op aarde geplaatste telescoopreeksen zou het gezichtsveld tenminste even groot moeten zijn als de Fried parameter.

Observatie van sterren met optische interferometrie vereist tenminste twee telescopen om het licht van een ver verwijderd object te ontvangen en zo informatie te verkrijgen met een zeer hoog hoekoplossend vermogen. Als twee aperturen zijn gescheiden door een afstand B, de zogeheten basislijn, bereikt het vlakke golfront dat afkomstig is van een ver verwijderde bron met een zekere hoek ten opzichte van de basislijn de beide aperturen op verschillende tijdstippen. Het tijdsverschil veroorzaakt een optische-weglengteverschil voor beide armen van de interferometer; als dit weglengteverschil groter is dan de coherentielengte zal het licht vanuit beide aperturen niet interfereren. Om interferentielijnen te kunnen waarnemen over een breed gezichtsveld moet het optische-weglengteverschil worden gecompenseerd voordat combinatie van de bundels plaats vindt. In de meeste tegenwoordige interferometers wordt dit gedaan door middel van vertragingslijnen. Licht dat uit een andere richting dan de optische as komt heeft een andere vertraging dan licht vanuit de richting van de optische as, hetgeen wordt aangeduid met differentiële vertraging. In zogenoemde Fizeau interferometers wordt de synthetische apertuur van de telescoop exact gereproduceerd in een teruggeschaalde versie ter plaatse van de optiek voor recombinaatie van de bundels; dit recombinaatieschema heeft intrinsieke padlengtecompensatie en een overeenkomstig groot gezichtsveld. Deze techniek is veelbelovend maar niet bruikbaar als de basislijnen zeer groot zijn in vergelijking tot de apertuur bij een enkele collector. In dat geval is de centrale piek smal en de energie gespreid over de zijlobben van het interferentiepatroon, waardoor de gevoeligheid van het instrument wordt beperkt. Voor interferometers die niet van het Fizeau-type zijn overlappen de bundels van reeks telescopen elkaar simpelweg (recombinaat in het vlak van de pupil), of worden de bundels samengevoegd in het beeldvlak zonder dat de configuratie van de intensiepupil
wordt gehandhaafd. Voor hoeken waarvoor de differentiële vertraging groter wordt dan de coherentielengte verdwijnen de interferentielijnen en gaat de hoge-resolutie informatie van niet op de optische as liggende objecten verloren. Een manier om dit probleem op te lossen en een breed gezichtsveld te verkrijgen is de introductie van een correctie voor het optische-weglengteverschil voor iedere hoek in het beeldveld van de telescoop.

Teneinde de belangrijke nadelen van Fizeau interferometrie bij een grote verhouding tussen basislijn en apertuurgrootte te vermijden hebben wij een nieuwe aanpak van het probleem onderzocht, te weten een systeem dat gebruik zou kunnen maken van een Michelson pupil-vlak combinatieschema, maar met een groot beeldveld in één enkele opname, waarbij ook op observatietijd zou kunnen worden bespaard. Het functionele principe van onze aanpak is de introductie van een gelijk gemaakt optisch weglengteverschil door gebruik van van een getrapte spiegel die in het vlak van een tussenbeeld is geplaatst. Het brandvlak heeft de eigenschap dat het licht vanuit verschillende delen van de hemel op verschillende plaatsen wordt gefocuseerd; deze eigenschap gebruiken wij voor het gelijkmaken van het optische weglengteverschil. Er wordt een extra weglengteverschil geïntroduceerd als functie van de kijkrichting, zodat coherente interferentie over een grote veldhoek kan worden verkregen. De vorm van de spiegel hangt af van de basislijn en de kijkrichting. Omdat de projectie van de basislijn vector op de intreepupil verandert met de uurhoek gedurende een astronomische waarneming moet de spiegel worden bestuurd om deze veranderingen te volgen: hij moet worden geroteerd om de draaiing van de geprojecteerde basislijn te volgen teneinde de oriëntatie van de stappen loodrecht daarop te handhaven, terwijl de diepte moet variëren aangezien deze moet worden aangepast op de modulus van de geprojecteerde basislijn. In een systeem dat wordt gevormd door meer dan twee telescoopen, dient een gestapte spiegel aanwezig te zijn in het brandvlak van elke telescoop, en moet er een gemeenschappelijk referentiepunt zijn voor de verschillende basislijnen. Iedere spiegel staat loodrecht op de projectie van zijn basislijn op de intreepupil van zijn telescoop. In dit proefschrift is het probleem van het gezichtsveld voor reeksen interferometers die niet van het Fizeau-type zijn analytisch en experimenteel bestudeerd. De volledige analytische beschrijving van een pupilvlak interferometer met een getrapte spiegel in het brandvlak van een van zijn armen is ontwikkeld, en de resultaten zijn vergeleken met de experimenten. Wij hebben een getrapte spiegel ontworpen die in het brandvlak behorende bij een van de armen van een Michelson interferometer is geplaatst. Met behulp van een Xenon lamp en een sterrenmasker zijn verschillende objectconfiguraties aan de hemel gesimuleerd, met diverse niet op de optische as gelegen objecten, met een verschillende vertraging, die moesten worden gecompenseerd door de getrapte
De experimentele resultaten waren voor alle configuraties in overeenstemming met de analytische voorspellingen, en zowel op de optische as gelegen objecten als ernaast gelegen objecten waren tegelijkertijd zichtbaar. Speciale aandacht is gegeven aan het geval waarbij een ster werd gefocussereerd op de rand van een stap. Analytische berekeningen en experimenten laten zien dat in dat geval twee stelsels interferentielinien worden waargenomen; de samenvoeging van het contrast van beide betekent dat de zichtbaarheid van de bron met een zekere factor moet worden vermenigvuldigd. Deze factor ligt dicht bij een, en hangt af van de basislijn en de coherentielengte en kan eenvoudig worden bepaald, waardoor het mogelijk wordt de zichtbaarheid van de bron uit de gegevens af te leiden. Dit betekent dat geen informatie verloren gaat als gevolg van de discontinue aard van de spiegel, en dat een continue breed beeldveld kan worden gereconstrueerd.

Icíar Montilla
“Si yo hablase lenguas humanas y angélicas, y no tengo amor, vengo a ser como metal que resuena, o címbalo que retiñe. Y si tuviese pro-fecía, y entendiese todos los misterios y toda ciencia, y si tuviese toda la fe, de tal manera que trasladase los montes, y no tengo amor, nada soy”.
1 Corintios 13:1-2

Acknowledgements

The process of writing a thesis is sometimes great and sometimes hard, but through all these years there have been many persons that shared with me the happy moments as well as helped and supported me through the difficult ones. To all these persons I dedicate my thesis, but I would like to specially thank many of them.

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About the author

The author was born in Madrid, Spain, in 1974. Since she remembers she wanted to study Physics, what she did at the Universidad Complutense de Madrid. After spending one year studying Fundamental Physics, finally she graduated with the specialization of Materials Science. In 1999 she obtained a Erasmus studentship to join the Defects of Materials group of Tom van Veen in the Interfaculty Reactor Institute at the Delft University of Technology, to do a project to study the behaviour of helium and deuterium in silicon dioxide. After her graduation she decided to stay in Delft, a quite contradictory decision considering that hiking is one of her passions and The Netherlands the flattest country in the world, and in April 2000 she joined the Optics Group at the Physics Faculty to start a PhD on Stellar Interferometry. The work carried out during the last years forms the basis of this thesis.

Publications list

Refereed publications


I. Montilla, S. F. Pereira, J. J. M. Braat, "Michelson Wide Field Stellar Interferometry: principles and experimental verification" (accepted for publication in Applied Optics).

Published conference proceedings


Other publications