Complex pupil function reconstruction at high numerical aperture using the extended Nijboer-Zernike diffraction theory

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ABSTRACT
We have derived an analytical expression for the field components in the focal region of a high-numerical-aperture imaging system using the so-called Extended Nijboer-Zernike diffraction theory. It is shown that the transmission function, aberrations and polarization properties of an imaging system with high numerical aperture can be derived from the through-focus intensity map via an inversion process based on this analysis.

Problem definition:
How to retrieve optical system properties (amplitude, phase and polarization in the exit pupil) from intensity measurements through the focal volume?

1) Intuitive picture
(based on ray optics)

2) Scalar imaging
Analytic ‘tool’ derived from the Extended Nijboer-Zernike theory, first developed for the scalar imaging case (point source)

\[ U(r, \varphi, f) = \frac{1}{\pi} \int_0^\infty d\rho \exp(i\rho f^2) \frac{1}{\sqrt{2\pi}} \sum_{l=0}^{\infty} \beta^\beta_l R^\beta_l \exp(2i\rho \beta l \cos(\varphi - \varphi')) d\rho \]

<table>
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<th>Wavefront (exit pupil)</th>
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<td>Focal volume</td>
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<td>Ray density variation in the focal volume due to wavefront aberration and transmission variation</td>
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\[ U(r, \varphi, f) = \frac{1}{\pi} \int_0^\infty d\rho \exp(i\rho f) \sum_{l=0}^{\infty} \beta^\beta_l R^\beta_l \exp(2i\rho \beta l \cos(\varphi - \varphi')) d\rho \]

3) Vectorial imaging
Complication in a high NA imaging system: vectorial calculation of the aberrated intensity distribution is needed in the focal volume!

The electric field vector produced by a point source illuminating an aberrated optical system (complex pupil function given by two sets of Zernike coefficients \( \beta^\alpha_l \), \( \beta^\beta_l \)) is given by

\[ \tilde{E}^\alpha (r, \varphi, f) = -i\gamma J_\alpha(r) \exp\left(-\frac{if}{1+1/4}\sum_{l=0}^\infty \beta^\alpha_l R^\alpha_l \exp(i\beta l)\right) \]

\[ \tilde{E}^\beta (r, \varphi, f) = -i\gamma J_\beta(r) \exp\left(-\frac{if}{1+1/4}\sum_{l=0}^\infty \beta^\beta_l R^\beta_l \exp(i\beta l)\right) \]

Energy density in the focal region
\[ \beta^\alpha_m = a\beta^\alpha_m; \quad \beta^\beta_m = b\beta^\beta_m; \quad (a^2 + b^2) = 1 \text{ for normalisation purposes} \]

Conclusions

1) A parametric analytic expression has been found for the electric field and the energy density in the focal volume of an optical system including pupil aberrations, pupil transmission variations and spatially varying birefringence of the imaging system (small source point, high NA)

2) Complex Zernike coefficients (two sets) are suitable to describe the generalized state of polarization in the exit pupil of the optical system

3) The inverse problem can be solved using a through-focus intensity map (energy density function) to solve a system of linearized equations in the two unknown sets of Zernike coefficients

4) The extended Nijboer-Zernike analysis offers an eigenfunction expansion for the vectorial propagation operator from exit pupil to image volume. This property can be exploited for the fast forward calculation of the image intensity in the focal volume.