Stellingen

behorende bij het proefschrift

Wind-induced Dynamic Behaviour of Tall Buildings

G.P.C. van Oosterhout
1. Als de huidige trend van sterkere en lichtere bouwmaterialen doorzet, zal in de komende eeuw het dynamisch gedrag het ontwerp van hoge gebouwen gaan bepalen.

2. De komst van eindige elementen methoden voor de bestudering van het dynamisch gedrag van hoogbouw heeft het verschil tussen berekend en werkelijk gedrag niet significant verkleind.

3. De meeste testresultaten ter bepaling van het dynamisch gedrag van gebouwen die in het verleden zijn gepubliceerd in de literatuur zijn onbruikbaar voor vergelijkend onderzoek. Dientengevolge dienen schatters voor dynamische eigenschappen die uit zulke onderzoek zijn voortgekomen met de nodige argwaan worden gebruikt.

4. Hoge gebouwen kunnen veel economischer worden gebouwd door de gevel en andere niet-constructieve elementen expliciet in te zetten in het beheersen van de trillingen als gevolg van wind.

5. Met de huidige stand van de technologie is het mogelijk om een hoog gebouw te ontwerpen naar analogie van het menselijk lichaam: balken en kolommen vormen het geraamte en actief aangestuurde kabels zijn de spieren van het gebouw.

6. Slankheid is een bron van trillingen.

7. De aanwezigheid van waterbedden in hoge woongebouwen kan het comfort van de bewoners op twee manieren bevorderen.

8. Een universiteit die er naar streeft om tot de top van Europa te horen, moet het ontwikkelen en beheren van elektronische middelen ter verspreiding van kennis door haar werknemers serieus nemen, bijvoorbeeld door deze activiteiten in de output-metingen mee te nemen.

9. Voor de ontwikkeling van de klimsport in Nederland is het wenselijk dat gevels met meer reliëf worden uitgevoerd dan momenteel gebruikelijk is.

10. Wielrenners en gebouwontwerpers moeten de juiste versnelling kiezen om succesvol te zijn.

11. Een verdieping is een verhoging.
Wind-induced Dynamic Behaviour
of
Tall Buildings
Wind-induced Dynamic Behaviour of Tall Buildings

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Delft, op gezag van de Rector Magnificus Prof. ir. K.F. Wakker, in het openbaar te verdedigen ten overstaan van een commissie, door het College van Dekanen aangewezen, op maandag 23 september 1996 te 16.00 uur

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Geert-Jan van Oosterhout
Delft, July 1996
# TABLE OF CONTENTS

1 Introduction .................................................................................................................. 1
   1.1 Background .............................................................................................................. 1
   1.2 Purpose and related work ....................................................................................... 1
   1.3 Scope ...................................................................................................................... 3

2 Assessment of wind-induced dynamic behaviour ...................................................... 5
   2.1 Introduction ............................................................................................................ 5
   2.2 Dynamic wind loads ............................................................................................. 5
   2.3 Response assessment ........................................................................................... 10
   2.4 Estimates of the modal properties ....................................................................... 15
   2.5 SkyDyFe .............................................................................................................. 22
   2.6 Summary .............................................................................................................. 23

3 Serviceability ........................................................................................................... 25
   3.1 Wind-induced dynamic behaviour and serviceability of tall buildings ............. 25
   3.2 Occupant comfort ............................................................................................... 26
   3.3 Serviceability and tall building design guidelines .............................................. 28
   3.4 Summary .............................................................................................................. 34

4 Sensitivity analyses ................................................................................................... 35
   4.1 Introduction ............................................................................................................ 35
   4.2 Theoretical dependencies .................................................................................... 35
   4.3 Wind parameters .................................................................................................. 37
   4.4 Structural parameters .......................................................................................... 41
   4.5 Foundation .......................................................................................................... 52
   4.6 Sources of uncertainty ....................................................................................... 53
   4.7 Summary .............................................................................................................. 55

5 Case-studies ............................................................................................................... 57
   5.1 The Cardington building ..................................................................................... 57
   5.2 The Voorhof II building ...................................................................................... 75
   5.3 Summary .............................................................................................................. 85

6 Design solutions ....................................................................................................... 85
   6.1 Introduction ............................................................................................................ 85
   6.2 Outriggers ............................................................................................................. 87
   6.3 Advanced non-structural elements ..................................................................... 103
   6.4 Summary .............................................................................................................. 121

7 Conclusions ............................................................................................................... 121
Appendix A Dutch wind climate data
Appendix B Modal analysis
Appendix C Skydyfe input
Appendix D Derivation of response equations
Appendix E Modal properties of a bending-shear parallel system
Appendix F Estimate of the lowest natural frequency

References
Summary
Samenvatting
Curriculum vitae
NOTATION

\( \bar{x} \) = mean of quantity \( x \)
\( \sigma_x \) = root mean square (rms) value of quantity \( x \)
\( \dot{x} \) = peak value/amplitude of quantity \( x \)
\( H_{xy} \) = transfer function from \( x \) to \( y \)
\( P \) = probability
\( S_{xx} \) = spectrum of quantity \( x \)
\( \dot{x} \) = the first derivative of \( x \) with respect to time
\( \ddot{x} \) = the second derivative of \( x \) with respect to time
\( \Delta x \) = variation in \( x \)

Indices:

\( b \) = building
\( equi \) = equivalent
\( h \) = at \( z=h \), at the top of the building
\( min \) = minimum value
\( max \) = maximum value
\( p \) = characteristic value
\( r \) = reduced, dimensionless
\( ref \) = reference
\( s \) = structural
\( w \) = walls, non-structural elements
\( x \) = in the direction of the (horizontal) \( x \)-axis
\( y \) = in the direction of the (horizontal) \( y \)-axis
\( z \) = in the direction of the (vertical) \( z \)-axis
\( \varepsilon \) = proportionality factor

\( A \) = cross-section, area \( \text{m}^2 \)
\( a \) = acceleration \( \text{m/s}^2 \)
\( \alpha \) = Gumbel constant \( \text{s/m} \)
\( b \) = width perpendicular to the wind \( \text{m} \)
\( C \) = costs \( \text{f.\$} \)
\( C_D \) = drag coefficient \( - \)
\( C_e \) = energy dissipation factor \( - \)
\( C_F \) = effective force coefficient \( - \)
\( C_m \) = effective mass coefficient \( \text{Nm} \)
\( c \) = rotation spring stiffness \( \text{kg/s} \)
\( c \) = viscous damping \( \text{m} \)
\( d \) = smallest width in plan \( \text{m} \)
\( d_0 \) = average height of the surrounding buildings \( \text{m} \)
\( E \) = energy \( \text{J} \)
\( E \) = Young's modulus \( \text{N/m}^2 \)
\( EA \) = axial stiffness \( \text{N} \)
\( EI \) = bending stiffness \( \text{Nm}^2 \)
\( F \) = force \( \text{N} \)
\( f \) = frequency \( \text{Hz} \)
\( f_c \) = natural frequency 
\( G \) = shear modulus 
\( GA \) = shear stiffness 
\( h \) = total height 
\( h_t \) = floor-to-floor height 
\( h_r \) = aspect ratio h/d 
\( I \) = turbulence intensity 
\( k_e \) = equivalent spring stiffness 
\( k_r \) = factor according to Eurocode 1 
\( L \) = gust length 
\( m \) = mass 
\( m \) = number of 
\( m_e \) = effective mass 
\( n \) = number of floors, number of .. 
\( P_w \) = wind pressure 
\( q \) = distributed load 
\( R \) = number of years, reference period 
\( T \) = period 
\( t \) = thickness 
\( t \) = time 
\( u \) = displacement 
\( u_{\text{ms}} \) = drift index 
\( v_R \) = Gumbel constant 
\( V \) = volume 
\( v \) = wind velocity 
\( v_* \) = shear velocity 
\( w \) = shear wave velocity 
\( z_0 \) = aerodynamic roughness 

\( \alpha \) = ratio shear/ bending stiffness 
\( \beta \) = ratio of outrigger stiffness to core stiffness 
\( \chi \) = aerodynamic admittance 
\( \phi \) = torque angle 
\( \gamma \) = shear strain 
\( \eta \) = factor according to Eurocode 1 
\( \phi \) = mode shape function 
\( \kappa \) = von Karman constant 
\( \mu \) = mass density per unit length 
\( \nu \) = Poisson’s ratio 
\( \theta \) = phase angle 
\( \rho \) = density 
\( \omega \) = angular frequency 
\( \omega_n \) = natural angular frequency 
\( \psi \) = rotation in x-z or y-z plane 
\( \zeta \) = damping ratio 

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1 INTRODUCTION

1.1 Background

For more than a century, tall buildings have been designed and erected. A tall building is not defined by its height or number of stories. The important criterion is whether or not the design is influenced by some aspects of 'tallness'. It is a building in which tallness strongly influences planning, design, construction and use and whose height creates conditions different from those that exist in 'common' buildings of a certain region or period [COU81]. The knowledge about the behaviour of tall structures has grown enormously. Most of the research has been focused on structural behaviour in ultimate limit states, as the potential hazard for occupants is evident. Nowadays, ultimate limit state design is common in the design of tall buildings.

In research and design relatively little attention has been paid to serviceability conditions, although these conditions will apply during 99% of the life-span of a building. As there are no lives at stake, the need to research and codify the behaviour of a building in everyday situations is not felt to be a pressing issue. Nevertheless, serviceability requirements have become important design issues for tall buildings. In this respect some developments in the building industry are relevant:

- Application of high-strength materials, like high-strength concrete.
- A decrease in partial safety factors for self-weight in Building Codes and guidelines.
- A decrease in the number of non-structural elements, like partition-walls.

Due to these developments, modern tall buildings are relatively light and flexible, which makes them sensitive to dynamic loads. A major concern is that wind-induced vibrations might cause discomfort for the occupants of a tall building. Therefore, attention should be devoted to the control of vibrations in the design process.

Evaluation of the dynamic behaviour is often limited to a small proportion of the total design effort, generally, because of a lack of awareness. Moreover, the dynamic performance usually is not checked until the end of the design process, when it is difficult and expensive to change the design. The importance of serviceability as a limit state can be illustrated by the costs of retrofitting. For example, Pols reported on a 17 storey residential building in Delft that had to be refurbished at a cost of 15 million Dutch guilders [POL95].

Vibrations that cause discomfort to occupants should be limited in modern tall buildings. Therefore, a reliable assessment of the wind-induced dynamic behaviour is required.

1.2 Purpose and related work

There is a need for tools that reliably assess the wind-induced dynamic behaviour of structures. These tools should support the design process of tall buildings, starting from the preliminary design stage. The development of one such tool has been the aim of this Ph.D. project.

The design tool should use a simple model for wind-induced dynamic behaviour, since it must be functional at preliminary design stages. At that stage in the design process, there is little detail in the design and modelling should be simple. However, the tool should allow for the substitution of results from more detailed calculations, so that it can improve the accuracy of the results as the building design develops. Nevertheless, the analysis of the wind-induced dynamic behaviour should be reliable at preliminary design stages within the accuracy that can
be obtained. Decisions made at these stages have a large impact on the final design of the building.

Thus, requirements for the design tool are: reliability, simplicity and flexibility. Moreover, it is desirable that the tool will be user-friendly and interactive.

The study of wind-induced dynamic behaviour has generally been divided into three fields:

- Wind engineering; the determination of wind speeds, assessment of the spatial correlation between pressures on a structure, the along- and cross-wind dynamic loads.
- Structural dynamics; the assessment of the displacement and acceleration, estimates of dynamic properties.
- Serviceability limit states; human perception of motion and the acceleration as a measure of it, (to a lesser extent) deformation due to dynamic loads.

The response to wind-induced loads on buildings can be split in three main components: along-wind, cross-wind and torsional response. The significance of each component depends on the type of building. This thesis concentrates on the along-wind response due to the turbulent fluctuations of the wind. Typically, European tall buildings have elongated plans and are slab-like structures rather than the cylinder-like structures that are common in the USA and Japan. Figure 1.1 shows an elongated plan, i.e., the width $b$ is much larger than the depth $d$. For such structures the along-wind response dominates.

For cylinder-like structures, like tall buildings with circular or square plan, cross-wind response will be the main concern of the designer. Measures to reduce the cross-wind response are most effective when the vortex-shedding process is disrupted, for example, by modification of the façade. Torsion is a particularly important loading component for buildings with asymmetry in mass or stiffness distribution. The best way to avoid significant torsional response is to design symmetrical tall buildings.

The study of the response to along-wind loads starts with the determination of the along-wind loads. In 1948, von Karman presented a spectrum of the turbulent along-wind speed fluctuations that was derived from a theoretical model that assumed that the turbulence perpendicular to the wind direction equals the turbulence in the wind direction [KAR48]. All subsequent spectra that were proposed are semi-empirical. Davenport’s spectrum is probably the most well known among these proposals [DAV67]. Davenport also developed a procedure to determine the response to along-wind forces that was based on a stochastic approach. He introduced the gust factor, which is now a widely accepted concept to handle the random character of the wind-induced response of tall buildings. Owing to work of Van Koten [BOS74], the wind loading part of the Dutch Building Code that was published in 1972 (NEN 3850) [NEN72] was already based on the principles of Davenport. A comprehensive overview of the development of the wind loading theory is given in [GEU92].

The estimation of the main dynamic parameters has been the key issue in structural dynamics. Ellis [ELL80] proposed estimates for the three fundamental natural frequencies, which were based on a large data-base of full-scale tests on tall buildings. Many others proposed rules of thumb for the fundamental natural frequency. An extensive set of estimates is collected in [OOS93].

Jeary’s research has helped to quantify the energy dissipation in tall buildings [JEA81]. He suggested an estimate for the damping ratio that is a function of the natural frequency and the peak dynamic displacement [JEA83, JEA86]. Davenport and Hill-Bradly proposed a similar expression in [DAV86].

There are four ways to control dynamic response: the shape, the mass, the stiffness and the damping. In most cases the latter three parameters are used. The shape of a building is often dictated by architectural requirements and accordingly there is little freedom for the designer to
modify the shape of a tall building. Vickery et al. studied the effectiveness of stiffness, mass and damping in reducing wind-induced dynamic response [VIC83]. Their main conclusion was that damping is the most effective of the three parameters considered in the study.

Finally, research on the human perception of low-frequency horizontal vibration started in the USA in the seventies. Notable contributions were made by Chen & Robertson [CHE72] who were among the first to give a description of the perception of motion for harmonic low-frequency motion (below 1 Hz, the range of interest for tall buildings). Hansen et al. studied the objection to motion in buildings [HAN73]. Irwin's work (for example [IRW78]) was the basis for the most common criterion currently used, ISO 6897 [ISO84]. Lately, Japanese researchers conducted tests to identify the perception of random motion [SHI93].

A few attempts have been made to integrate the three fields and to develop a design tool. Solari wrote a computer program, DAWROS [SOL81], which can be considered as one of the first examples of integration of wind engineering and structural dynamics in a design tool. Islam & Ellingwood proposed a method to estimate uncoupled dynamic response in which the wind loads were derived from wind tunnel tests on rigid models. A similar method was proposed by Irwin [IRW88]. Kareem has recently developed a method that takes into account coupled along-and cross-wind motion [KAR91]. The wind loads that are used in his method were also derived from wind tunnel tests.

1.3 Scope

This study develops a tool for evaluating wind-induced dynamic behaviour that supports the designer starting from the preliminary stages in the design of tall buildings. It was felt necessary to integrate wind engineering, structural dynamics and serviceability to obtain an accurate and reliable design tool. Integration also gives direction to research that might lead to an improved prediction of the wind-induced dynamic behaviour of tall buildings.

As mentioned before, the along-wind load and the corresponding response shown in Figure 1.1 are dominant for the typical slab-like European building. Therefore, this study focuses on along-wind behaviour.

Figure 1.1 Definition of dimensions and wind direction.
Secondly, an appropriate structural dynamics model is needed. Modal analysis can satisfy the modelling requirements that were established in the previous section. In the preliminary design, modal properties can be estimated with simple formulae. In a later stage of the design process, these properties can be replaced by results from detailed calculations or measurements. Moreover, the random character of the fluctuating part of the wind load can be conveniently combined with modal analysis by means of a wind speed spectrum.

The criteria for the wind-induced vibrations of tall buildings are related to serviceability. The governing criterion is the comfort of the occupants of a building during a storm. A survey of the literature revealed the required link to the building’s response: the horizontal accelerations that occupants experience during a storm should be limited.

As already noted, the estimation of the modal properties is an important step in the response assessment. Unfortunately, the estimates are prone to large errors. For example, in [OOS93] it was shown that common rules of thumb for the fundamental natural frequency cause a 100% deviation in the estimated frequency. The sensitivity of the response to variation in the modal properties and loading is assessed. Accordingly, the effects of uncertainties in the modal properties on the response are determined.

Two case-studies serve as a validation of the results of the sensitivity analyses. The first case-study is an eight-story building in Cardington, England. The dynamic behaviour of this building has been monitored during the process of erection. At discrete stages of the building process, tests were performed. The development of the dynamic parameters is identified accordingly. The second case-study is a 17 floor student dormitory in Delft, the Voorhof II building. This building was refurbished in 1986. One of the reasons for refurbishment was the high level of vibrations that occurred during storms. Among the visible deficiencies were cracks in many of the partition walls. Measurements of acceleration levels were conducted before and after the refurbishment. The two sets of measurements are compared and conclusions about the effectiveness of the structural measures to reduce the acceleration levels are drawn.

The results from the sensitivity analyses and the case-studies are used to develop design solutions that exhibit good dynamic performance.

Each chapter of this thesis develops an aspect of the assessment of the dynamic behaviour of tall buildings. Chapter 2 discusses approaches towards the assessment of the dynamic response. SkyDyFe, the actual design tool as developed by the author in the Visual Basic programming language, is also treated. In Chapter 3 serviceability and its impact on tall building design are discussed. Chapter 4 contains sensitivity analyses. The sensitivity of the horizontal acceleration to parameters like stiffness, mass and damping is studied. The effects of representation of the fluctuation wind loads on the response are also considered. The case-studies in Chapter 5 discuss the two above mentioned, slab-like, structures. The dynamic behaviour of these buildings was measured. From the test results some remarkable conclusions were drawn and subsequently an explanation has been sought for the observed effects. In Chapter 6 two promising design measures are developed. Chapter 7 concludes the thesis with an evaluation of the results obtained in this research.
2 ASSESSMENT OF WIND-INDUCED DYNAMIC BEHAVIOUR

2.1 Introduction

The dynamic behaviour of buildings can be described by several parameters, like bending moments, shear stresses, displacements or accelerations. In Chapter 3, the human perception of motion will be identified as the most important serviceability issue in relation to wind-induced dynamic behaviour of tall buildings. It will also be shown that the horizontal acceleration levels are generally accepted as a measure of perception. Therefore, the discussion of the assessment of wind-induced dynamic behaviour, which is presented in this chapter, focuses on accelerations. A further restriction is that only the along-wind response is treated in detail, as was explained in Chapter 1. The following section will treat the assessment of the along-wind loads in the frequency domain. The reader is referred to the Building Code of Canada [NBC85] for assessment of the cross-wind (vortex-shedding) response. A method to assess the wind-induced torque response of tall buildings has been proposed by Isyumov [ISY83].

In this thesis two models of the dynamic behaviour of buildings will be used. The first one is linear, i.e., based on the assumption that the properties of a building are independent of its response. By means of modal analysis, each mode of vibration may be studied separately by means of a single degree of freedom model. If the response of such a system is described in the frequency domain, the random character of the wind loads can be included using spectra.

The second model of the dynamic behaviour is non-linear. This model allows for the dependency of stiffness and energy dissipation on the response. Section 2.3.3 contains an introduction on non-linear dynamics. The study of such a system requires a time-domain analysis.

Special attention is paid to the determination of the modal properties. An accurate assessment of these properties is required but may be difficult to produce. Section 2.4 treats estimates of those properties.

Chapter 2 ends with a brief discussion of SkyDyFe, the design tool that has been developed during this Ph.D. project.

2.2 Dynamic wind loads

2.2.1 Assessment procedure

The flow of air around a building induces pressures perpendicular to the surface of a building. The flow is characterised by the velocity, which can be split into a mean velocity plus turbulent fluctuations. For the prediction of the dynamic along-wind loads a statistical description of the wind velocity distribution is required. Moreover, the components of the wind, mean velocity and turbulence, need to be described.

Extreme value probability distributions in terms of a reference mean velocity are commonly used to evaluate the statistical distribution of the wind velocity. The structure of the mean wind velocity over height is modelled by a logarithmic wind profile. Therefore, a reference mean velocity is required in an extreme value probability distribution.

Essential in modelling the wind-induced dynamic loads on buildings is that structures respond to the energy contained in sequences of gusts, especially to those gusts that are resonant with the structure. A spectral description of the gusts takes account of the sequential effects of gusts.
Another important aspect is the spatial distribution of gusts and pressures. In this thesis the concept of aerodynamic admittance is adopted. This function models the structure-flow interaction, which includes the spatial correlation of wind pressures on the building.

A second parameter in the wind load assessment that takes account of building characteristics is the drag coefficient, which is a measure of the aerodynamics of a certain building shape. Figure 2.1 summarises the steps that ultimately lead to the retrieval of the along-wind load spectrum $S_{PP}$:

$$S_{PP}(f) = \left[ C_d \rho_{\text{air}} \bar{v}(h) A \chi(f, b, h, \bar{v}(h)) \right]^2 S_{\nu}(f, h, \bar{v}(h))$$  \hspace{1cm} (2.1)

where

- $A$ = exposed surface perpendicular to the wind direction,
- $b$ = width of the plan perpendicular to the wind direction,
- $C_d$ = the drag coefficient, see subsection 2.2.5,
- $f$ = the frequency,
- $h$ = the height of the building,
- $S_{\nu}$ = the spectrum of the along-wind velocity, see subsection 2.2.3,
- $\bar{v}(h)$ = the mean wind speed at the top, see subsection 2.2.2,
- $\rho_{\text{air}}$ = density of the air, 1.2 kg/m$^3$,
- $\chi$ = the aerodynamic admittance function, see subsection 2.2.4.

The wind speed spectrum is, among others, a function of frequency, velocity and height. The aerodynamic admittance function depends on frequency, velocity, width and height. In the past several expressions for the spectrum of the along-wind speed and the aerodynamic admittance function have been proposed. Some of them are briefly introduced hereafter. In section 4.3 the effects of the differences in representation of turbulence on the response will be studied.

![Diagram of along-wind load assessment scheme](image)

**Figure 2.1** Dynamic along-wind loads assessment scheme.
2.2.2 Mean wind velocity

The mean wind velocity is dependent on the characteristics of the terrain over which the air approaches a structure. As a result, the mean velocity is a function of the height:

\[
\bar{v}(z) = \frac{v_*}{\kappa} \ln \frac{z - d_0}{z_0}
\]  

(2.2)

where

- \(d_0\) = the mean obstacle height,
- \(v_*\) = the shear velocity,
- \(z\) = the height above grade,
- \(z_0\) = the roughness length, a measure of the roughness of the terrain,
- \(\kappa\) = the von Karman constant, \(\kappa = 0.4\).

Two parameters are used to characterise the terrain: the aerodynamic roughness \(z_0\) and the mean obstacle height \(d_0\). The former is about 0.1 m for open terrain and 0.7 to 2.0 m for urban areas. For open terrain the mean obstacle height is normally zero.

The shear velocity \(v_*\) depends on the extreme value distribution that applies to the area in which a building is planned. The Royal Dutch Meteorological Institute use a Gumbel extreme function [WIE83, STA90] to describe the probability \(P\) that the velocity \(v\) is less than the extreme wind speed \(v_R\)

\[
P(\bar{v} \leq \bar{v}_R) = \exp(-\exp(-a(\bar{v}_R - v_R)))
\]  

(2.3)

where

- \(v_R\) = the extreme wind speed with a return-period of \(R\) years (the reference period),
- \(a\) = a constant that depends on the wind climate.

The Gumbel constants \(v_R\) and \(a\) are available for potential conditions, i.e., the normalised conditions at wind stations. Appendix A contains values for \(v_R\) and \(a\) that apply to the Dutch wind climate. Via a transformation process the probability distribution of the shear velocity that applies to the building site can be obtained. Suggested values for \(d_0\), \(z_0\) and the transformation of \(v_*\) according to the Dutch Building Code NEN 6702 are collected in Appendix A.

2.2.3 Gust spectrum

A spectral description of the turbulence is a convenient tool to account for the energy that is contained in sequences of gusts. If it is assumed that the turbulence perpendicular to the wind direction equals the turbulence in the wind direction (frozen isotropic turbulence field), an analytical expression for the along-wind gust spectrum can be derived [KAR48]:

\[
\frac{fS_v(f, h, \bar{v}(h))}{\sigma_v^2} = \frac{4f_r}{(1 + 70.8 f_r^2)^{\frac{3}{2}}}
\]  

where \(f_r = \frac{fL}{\bar{v}(h)}\)

(2.4)

- \(f_r\) = a reduced frequency,
- \(L\) = a gust length, \(L = 50(h - d_0)^{0.35} / z_0^{0.65}\) (\(L\) is determined as best-fit to measured data),
- \(\sigma_v\) = the rms (root mean square) wind velocity.
A number of empirical spectra have been proposed for the in-wind fluctuations of the wind. Davenport proposed [DAV61]:

$$\frac{fS_v(f, h, \bar{v}(h))}{(\bar{v}(10))^2} = \frac{2}{3} \frac{f_r^2}{(1 + f_r^2)^{\frac{1}{3}}}$$

where $f_r = \frac{fL}{\bar{v}(10)} \quad (2.5)$

and

$L$ = a gust length, 1200 m,
$\bar{v}(10)$ = the mean wind velocity at 10 meter.

Kaimal proposed a spectrum that takes account of the dependence of the velocity spectrum on terrain characteristics. Instead of the mean velocity at 10 meters, the shear velocity $v_s$ is used to normalise the spectrum [KAI72]:

$$\frac{fS_v(f, h, \bar{v}(h))}{v_s^2} = \frac{105 f_r}{(1 + 33 f_r)^{\frac{8}{3}}} \quad \text{where } f_r = \frac{fL}{\bar{v}(h)} \quad (2.6)$$

In Eurocode 1 [EUR93a], a similar expression has been used:

$$\frac{fS_v(f, h, \bar{v}(h))}{\sigma_r^2} = \frac{6.8 f_r}{(1 + 10.2 f_r)^{\frac{8}{3}}} \quad \text{where } f_r = \frac{fL(h_{equi})}{\bar{v}(h_{equi})} \quad (2.7)$$

To calculate the reduced frequency $f_r$, two quantities, $L(h_{equi})$ and $v_{equi}$, have to be determined at the equivalent height. The equivalent height is defined in Eurocode 1 as 0.60 times the total height. The following expressions should be used for the other equivalent quantities:

$$L(h_{equi}) = 300 \left[ \frac{h_{equi}}{300} \right]^\varepsilon \quad (2.8)$$

$$v(h_{equi}) = \bar{v}_{ref} k_s \ln \frac{h_{equi}}{z_0} \quad (2.9)$$

In the latter equation $k_s$ is a terrain factor that depends on the aerodynamic roughness. Values of $k_s$ vary from 0.19 ($z_0 < 0.05$ m) to 0.24 ($z_0 \geq 1.0$ m). The index $\varepsilon$ also depends on the aerodynamic roughness. The reference velocity depends on the region in Europe in which the structure is to be built. The values for $k_s$, $\varepsilon$ and $v_{ref}$ that apply to the Netherlands are summarised in Appendix A. Note that the formulation of the load spectrum has to be adapted accordingly. In equation (2.1) the mean wind velocity at the top ($\bar{v}(h)$) should be substituted by the equivalent wind velocity ($\bar{v}(h_{equi})$).

The spectra that have been presented in this subsection display most of the differences in definition of gust spectra and, therefore, are representative when in chapter 4 the variation in response due to these differences will be assessed.
2.2.4 Aerodynamic admittance

The aerodynamic admittance function $\chi$ is added in (2.1) to take account of the structure-flow interaction. The aerodynamic admittance function is known for a few shapes. For a rectangular plate the following empirical relationship proposed by Vickery is often used [GEU92]:

$$\chi(f, b, h, \bar{v}(h)) = \frac{1}{1 + \left[ \frac{2f\sqrt{A}}{\bar{v}(h)} \right]^\frac{2}{3}}$$ (2.10)

The aerodynamic admittance function proposed by Zilch [ZIL83], in a slightly modified form, is used in the Dutch Building Code NEN 6702:

$$\chi'(f, b, h, \bar{v}(h)) = \frac{1}{\left[ \frac{2.2fh}{\bar{v}(10)} \right] \cdot \left[ \frac{3.6fh}{\bar{v}(10)} \right]}$$ (2.11)

In the proposals for the Eurocode 1 a rather complex procedure to determine the spatial correlation of the wind pressures is used. In this code the aerodynamic admittance function is defined as the product of two reduction factors:

$$\chi^2(f, b, h, \bar{v}(h)) = R_h R_b$$ (2.12)

where

$$R_h = \frac{1}{\eta} - \frac{1}{2\eta^2} e^{-2\eta}$$ and $$\eta = \frac{4.6fh}{\bar{v}(h_{equ})}$$ (2.13)

Expression (2.13) also holds for $R_b$ when $h$ is replaced by $b$.

2.2.5 Drag coefficient

The drag coefficient is a parameter that is closely related to the shape of the building both in plan and in elevation. According to the use of the drag coefficient in equation (2.1), its definition should be

$$C_d = \frac{\bar{F}_{\text{drag}}}{\frac{1}{2} \rho \bar{v}(h)^2 A_{\text{wind}}}$$ (2.14)

A typical value for the drag coefficient is 1.2, which holds for rectangular plan shapes. For irregular shapes wind tunnel tests are requested to determine a proper value for the drag coefficient.
2.3 Response assessment

2.3.1 Introduction

A tall building can oscillate in several modes of vibration. For many purposes in the preliminary design stages it is sufficient to study the fundamental modes (two orthogonal translations and one torsional mode), since most of the excitation under wind loading occurs in those modes [ELL80]. As this thesis focuses on along-wind dynamic behaviour, torsion will not be considered. In section 4.2.2 the error in the response assessment due to the neglect of higher modes will be estimated.

For a building having coincident elastic and inertial axes and little or none non-linear effects in its dynamic behaviour, modal analysis is a convenient tool to study the fundamental modes, since each mode is described by an equation of motion with one degree of freedom, i.e., the displacement $u$. In the previous section, the randomness of the dynamic along-wind loads was modelled in the frequency domain by means of a spectrum. As a consequence, a spectral approach for the wind-induced dynamic response is convenient. In subsection 2.3.2 the combination of modal analysis and spectral analysis is introduced.

Observations of the dynamic response of tall buildings reveal that non-linear mechanisms are involved in the dynamic behaviour of such buildings. The Cardington case-study in chapter 5 has some examples of non-linear effects. In subsection 2.3.3 an approach towards non-linear dynamic behaviour is presented.

2.3.2 A linear model: modal analysis

For buildings having coincident elastic and inertial axes, each mode of vibration can be uncoupled and has an unique mode shape function $\varphi_i$. Figure 2.2 visualises the process. Each mode of vibration can be described by an equation of motion that has only one degree of freedom, the displacement $u_i$ of the left hand model in Figure 2.3:

$$m_{e,i} \dddot{u}_i + c_{e,i} \dot{u}_i + k_{e,i} u_i = F_{e,i}(t)$$  \hspace{1cm} (2.15)

where

$m_{e,i}$ = the modal or effective mass of the $i^{th}$ mode of vibration,
$c_{e,i}$ = the modal or effective viscosity of the $i^{th}$ mode of vibration,
$k_{e,i}$ = the modal or effective stiffness of the $i^{th}$ mode of vibration,
$F_{e,i}$ = the modal or effective force of the $i^{th}$ mode of vibration.

The derivation of the equation of motion is summarised in Appendix B. The determination of the modal properties that are listed above can be cumbersome. In section 2.4 estimates of the fundamental modal properties are discussed.

The random response of a single degree of freedom model to wind loads can conveniently be described in the frequency domain. Since human comfort is related to the horizontal acceleration levels, the response is expressed in terms of acceleration. The spectrum of the loads should be based on the effective part of the wind loads (the definition of the effective load is given in subsection 2.4.1).
Figure 2.2  From building design to single degree of freedom model.

Figure 2.3  Two types of single degree of freedom models. Left hand: a linear model, right hand: a non-linear model.
Figure 2.4  Reduced wind load spectrum.

Figure 2.5  Transfer function.

Figure 2.6  Reduced acceleration spectrum.
Thus, the response to the effective wind loads can be written in the following spectral form:

\[ S_{aw}(\omega) = C_F^2 H_{fa}^2(\omega) S_{fa}(\omega) \]  

(2.16)

where

- \( C_F \) = an effective load coefficient, defined as the ratio of the effective to the total dynamic wind load,
- \( H_{fa}(\omega) \) = a transfer function,
- \( S_{fa} \) = the spectrum of the accelerations.

The transfer function in (2.16) is a function of natural angular frequency \( \omega_c \) (unity rad/s), the damping ratio \( \zeta \) and the effective mass \( m_e \):

\[ H_{fa}(\omega) = \frac{1}{m_e \sqrt{\left[ 1 - \left( \frac{\omega}{\omega_c} \right)^2 \right]^2 + 4\zeta^2 \left( \frac{\omega}{\omega_c} \right)^2}} \]  

(2.17)

where

\[ \omega_c = \sqrt{\frac{k_e}{m_e}} \quad \text{for} \; \zeta \ll 1 \]  

(2.18)

\[ \zeta = \frac{c_e}{2\sqrt{k_e m_e}} \]  

(2.19)

Note that equation (2.18) applies to systems with low damping ratio. This requirement is normally satisfied when tall buildings are considered.

The filtering effect of the transfer function is illustrated in Figure 2.4 to Figure 2.6. Figure 2.4 displays the along-wind load spectrum. Figure 2.5 shows the transfer function, with the characteristic peak at resonance, where the loading frequency equals the natural frequency. The resonance is so dominant that the quasi-static peak in Figure 2.4 completely vanishes in the spectrum of the accelerations.

The white noise approach can be used to determine the resonant response, i.e., the peak in Figure 2.6. Herein it is assumed that the loading spectrum has a constant spectral density \( S_0 \) equal to the spectral density of the force spectrum at the natural frequency. \( S_0 = C_F^2 S_{fa}(\omega_c) \). Accordingly, the root mean square (rms) dynamic displacement can be calculated from (2.16) in the following way:

\[ \sigma_x^2 = \int_0^\infty S_{aw}(\omega) d\omega = S_0 \int_0^\infty H_{fa}^2(\omega) d\omega \quad \text{where} \quad \int_0^\infty H_{fa}^2(\omega) d\omega = \frac{\pi \omega_c}{4m_e \zeta} \]  

(2.20)
Evaluating (2.20) yields a simple expression for the rms acceleration:

\[ \sigma_a^2 = \frac{\pi \omega_0^2 C^2 S_{\beta\beta} (\omega_0)}{4m_\zeta^2} \quad \text{or} \quad \sigma_a = \sqrt{\frac{\pi \omega_0^2 C^2 S_{\beta\beta} (f_c)}{4m_\zeta^2}} \]  

(2.21)

This equation is the basis of the sensitivity analyses that will be discussed in chapter 4. The load spectrum is according to equation (2.1).

### 2.3.3 A non-linear model

In the Cardington case-study effects will be observed that can only be explained if a non-linear equation of motion is used. If it is assumed that a linear spring can represent the behaviour of the structural system and a hysteretic spring represents the behaviour of ‘non-structural’ elements, the model on the right hand of Figure 2.3 can be used. Non-structural elements are defined in this respect as building elements that are not designed for structural purposes.

Accordingly, the equation of motion for the non-linear system in Figure 2.3 is

\[ m\ddot{u} + k_s u + F_w (u, \dot{u}) = F(t) \]  

(2.22)

where

- \( m \) = the mass of the body,
- \( k_s \) = the stiffness of the spring representing the structural system,
- \( F_w \) = the force in the non-linear spring: dependent on displacement and velocity,
- \( F(t) \) = the external force acting on the body,
- \( u \) = the displacement of the body.

Energy dissipation is provided by hysteresis, like the force-displacement diagram that is displayed in Figure 2.3. Hysteresis is a far more reasonable model than viscous damping from a physical point of view. Note that in equation (2.22) it is assumed that the structural system acts linear-elasticistically. This assumption is not a necessary one, but was taken for clarity: all non-linear effects are incorporated in the force \( F_w \) that resembles the non-structural elements.

The force \( F_w \) is a non-linear function of the displacement \( u \) and velocity \( \dot{u} \). Therefore, an analytical solution of equation (2.22) is, in general, not available. Thus, the non-linear model requires time-domain analysis. One of the possible methods is a rearrangement of the terms in (2.22), which yields an estimate for the acceleration at time \( t_i \) as a function of the displacement conditions at \( t = t_i \) [TIM56].

\[ \ddot{u}_0 = \frac{1}{m} [F(t) - k_s u_0 - F_w (u_0, \dot{u}_0)] \]  

(2.23)

If the boundary conditions \( u_0 \) and \( \dot{u}_0 \) at \( t = 0 \) are known, equation (2.22) can be used to calculate the acceleration at \( t = 0 \). Subsequently, the velocity and displacement at \( t = t_i \) can be approximated by linear interpolation. This process can be repeated for each increment in time. This procedure is fully described in [OOS95a].

The damping ratio is the modal property that is strongly affected by non-linear effects. Thus, for many purposes it may be sufficient to use a non-linear predictor for the damping ratio and use this in the (linear) spectral analysis that was presented in the previous subsection. In subsection 2.4.4 such a predictor will be presented.
2.4 Estimates of the modal properties

An accurate determination of the modal properties may be difficult or impossible due to a lack of information. This is especially true for the determination of the effective stiffness and effective viscosity. In the following subsection effective mass and effective force, which can be determined from the definitions of modal analysis, are treated. In subsection 2.4.2 an estimation procedure for the fundamental natural frequency is presented. Subsequently, the effective stiffness is determined. Finally, the most difficult response parameter to determine, the damping ratio, is discussed in subsection 2.4.4.

In the case-studies that are presented in Chapter 5, the natural frequency, mode shape and damping ratio are measured.

2.4.1 Effective mass and effective force

Figure 2.7 shows the three main types of continuous model that are used to describe the behaviour of tall buildings and their fundamental modes of vibration. Soil-structure interaction is not taken into account, since the columns are clamped at their base. In section 4.5 the validity of this assumption is considered. The effective mass and effective force can be estimated from the continuous column representation.

The mass distribution is characterised by the parameter \( \mu \). The loads are also distributed and characterised by \( q \). The effective mass and effective force can then be written as (see Appendix B).

\[
m_{e,j} = \int_0^h \mu(z) \phi_j^2(z) dz
\]

\[
F_{e,j} = \int_0^h q(z) \phi_j(z) dz
\]

where

\( \mu \) = the distributed mass of the continuous column,

\( q \) = the distributed load along the continuous column.

For convenience, normalised expressions for effective mass and effective force will be used in this thesis unless stated otherwise. If we assume that both \( \mu \) and \( q \) are uniformly distributed along the height, an effective mass coefficient \( C_m \) and effective force coefficient \( C_F \) can be defined:

\[
C_{m,j} = \frac{1}{h} \int_0^h \phi_j^2 dz
\]

\[
C_{F,j} = \frac{1}{h} \int_0^h \phi_j dz
\]
In Figure 2.7 values for $C_m$ and $C_F$ according to the three main continuous models are summarised. The values for the parallel system are typical values. A precise determination of the modal properties of this model is discussed in Appendix E.

### 2.4.2 Fundamental natural frequency

Many estimates of the fundamental natural frequency as a function of one or two parameters have been proposed. Reference [OOS93] gives an overview of the most popular ones. At the end of this subsection some of them will be discussed in relation with the estimate that is presented herein.

If a tall building is modelled as a prismatic, continuous column with uniformly distributed properties, the fundamental natural frequency is known from the solution of the homogenous equation of motion. For example, the fundamental natural frequency of a bending column is [TIM56]:

$$\omega_c = 3.52 \frac{EI}{\sqrt{\mu h^4}}$$  \hspace{1cm} (2.28)
An estimate of the bending stiffness $EI$ can be determined from deflection criteria. For example, take the bending column in Figure 2.8. Consider the case that the building is loaded by a uniformly distributed wind load. The wind load $q$ is taken from an appropriate building code and is based on the wind pressure at the top $p_{w,h}$. The maximum displacement of the bending column is the top deflection $u_b$, which is

$$u_b = \frac{qh^4}{8EI} \quad (2.29)$$

There are limits to the maximum drift in many building codes. Therefore, the bending stiffness has to have a certain minimum value to satisfy the drift limit, as may be appreciated from equation (2.29). The minimum required bending stiffness becomes:

$$EI_{\text{min}} = \frac{qh^4}{8u_{\text{max}}} \quad (2.30)$$

The minimum required bending stiffness $EI_{\text{min}}$ can be substituted into equation (2.28) and an estimate of the first natural frequency based on stiffness considerations is found. In a similar fashion, an estimate for a structural system acting in shear can be derived. In general, the estimate can be written in the following form (the derivation is treated in Appendix F):

$$f_c = f(\alpha h) \sqrt{\frac{q}{m} \frac{h}{u_{\text{max}}}} \quad (2.31)$$

where

- $m$ = the total mass in the building [kg],
- $q$ = the uniformly distributed wind load [N/m],
- $u_{\text{max}}/h$ = a drift index, i.e., the ratio of the maximum allowed displacement to the height.

The function $f(\alpha h)$ depends on the type of behaviour. When $\alpha h$ is zero the structure deforms in bending and $f(\alpha h) = 0.198$. When $\alpha h$ tends to infinity the deformation is pure shear and $f(\alpha h) = 0.176$. Intermediate values are combined behaviour: bending plus shear. A classical example of combined behaviour is a central core plus frames. The influence of the bending and shear component is determined by the ratio ($\alpha^2$) of shear stiffness $GA$ to bending stiffness $EI$:

$$\alpha^2 = \frac{GA}{EI} \quad (2.32)$$

The function $f(\alpha h)$ is plotted in Figure 2.9, with $\alpha h$ on a logarithmic scale. The full expression for $f(\alpha h)$ is (see Appendix F):

$$0 \leq \alpha h \leq 1 \quad f(\alpha h) = \sqrt{\frac{0.3131}{(\alpha h)^2} + 0.1148 \left[ -1 - \frac{\alpha h \sinh \alpha h + \cosh \alpha h}{(\alpha h)^2 \cosh \alpha h} + \frac{1}{2} \right]} \quad (2.33)$$

$$\alpha h \geq 1 \quad f(\alpha h) = \sqrt{\frac{0.2365}{(\alpha h - 0.3)^{1.22}} + \frac{1}{16} \left[ -1 - \frac{\alpha h \sinh \alpha h + \cosh \alpha h}{(\alpha h)^2 \cosh \alpha h} + \frac{1}{2} \right]} \quad (2.34)$$
Figure 2.9  A plot of $f(\alpha h)$ according to equations (2.33) and (2.34).

Figure 2.10 Natural frequency estimates as a function of building height.
It is remarkable that the height is not a parameter in equation (2.31): $h/u_{max}$ is the drift index, a constant, and $f(\alpha h)$ is dimensionless. Many estimates assume that the natural frequency is inversely proportional to $h$. Take for example Ellis’ popular estimate

$$f_c = \frac{46}{h} \text{ (in Hz)} \quad (2.35)$$

where $h$ is the height in meters.

In the US the Uniform Building Code [UBC88] provides designers with a set of estimates, all inversely proportional to $h^{0.75}$. For example, the lowest natural frequency of a concrete bending frame may be assumed to be ($h$ is the height in meters)

$$f_c = \frac{1}{0.086h^{0.75}} \text{ (in Hz)} \quad (2.36)$$

where $h$ is the height in meters.

In Figure 2.10 Ellis’ and UBC’ estimates are compared to the estimate (2.31). A tall building with square plan and three aspect ratios, 4, 6 and 8, is considered, where aspect ratio is the ratio of the height $h$ to the width $d$. The drift limit was 1/500 $h$. The structural system deforms in bending, thus $f(\alpha h) = 0.198$. Other data are collected in Appendix C.

The agreement is quite well, although in the lower region ($h < 100$ m) there is a significant deviation in the estimated frequencies.

Figure 2.10 shows a clear dependence of equation (2.31) on the height. A closer look to equation (2.31) reveals this dependency as well: $f(\alpha h)$ and drift index are dimensionless and independent of height but the other two quantities in (2.31), the design wind pressure and the total mass, are dependent on height.

The distributed wind load is the product of wind pressure, drag coefficient and width of the building. Since the estimates in Figure 2.10 have been calculated with a constant aspect ratio, the parameter $d$ is proportional to $h$. The wind pressure $p_{w,h}$ is proportional to the square of the wind speed. According to equation (3.1), there is a logarithmic relation between height and wind speed. This relation may be approximated by a power law. A good approximation of the logarithmic law for urban surroundings is obtained when the power is 0.25.

$$\bar{v}(h) \propto h^{0.25} \rightarrow p_{w}(h) \propto h^{0.5} \quad \land \quad d \propto h \rightarrow q = C_d p_{w}(h) \propto h^{1.5} \quad (2.37)$$

The total mass of a building is depending on the volume of the building. For a constant aspect ratio it holds that

$$m = \rho_d d^2 h \propto h^3 \quad (2.38)$$

Substitution of these dependencies in (2.31) yields the following proportionality of the natural frequency to the height of a building:

$$f_c \propto \frac{h^{1.5}}{h^3} = \frac{1}{h^{0.75}} \quad (2.39)$$

Note that the proportionality in (2.39) is slightly different from Ellis’ proposal, but agrees with the UBC estimate.
An increase of the natural frequency as the aspect ratio increases is another interesting effect that may be recognised from Figure 2.10. This effect seems to be a contradiction at first hand, but the explanation is simple because the width \( d \) (and mass) is inversely proportional to the aspect ratio. Accordingly, equation (2.39) becomes

\[
f_r \propto \sqrt{\frac{h_r^{-1}h_0^{-1}}{h_r^{-3}}} = \frac{h_r^{0.5}}{h_0^{0.75}}
\]

where \( h_r \) is the aspect ratio.

### 2.4.3 Effective stiffness

Once the fundamental natural frequency and the effective mass are estimated, the effective stiffness in the fundamental mode is calculated from the definition of the natural frequency (equation (2.18)):

\[
k_e = m_r \omega_r^2
\]

(2.41)

For a bending column it is found that (using \( EI_{min} \) according to equation (2.30))

\[
k_e = \frac{312EI}{h^3} \quad \text{or} \quad k_e = \frac{312}{8} q \cdot \frac{h}{u_b}
\]

(2.42)

Similarly, if the column in Figure 2.8 represents a structural system in shear, the equivalent spring stiffness is

\[
k_e = \frac{1}{16} \pi^2 q \cdot \frac{h}{u_b}
\]

(2.43)

Note that this effective stiffness will be a lower bound value, since it is based on the minimum required bending stiffness that the structural system should provide to satisfy the drift limit. The contribution of non-structural parts to the stiffness may be of the same order of magnitude higher value. See also section 5.1.3.

### 2.4.4 Equivalent damping ratio

Little information is available about the damping in buildings. In general, predictors of the damping ratio \( \zeta \) are used. It is a wide-spread belief that the structural material has a major influence on the damping ratio to be achieved by a building. However, as will be shown in section 5.1.4, the most important contribution to the energy dissipation comes from the non-structural parts in a building. So, there is no reason to sustain the just presented belief.

Still, concrete will have a little more inherent damping than steel, as it exhibits hysteretic behaviour like many of the non-structural materials. In some cases the choice of a structural system affects the energy dissipation that can be achieved. Most apparent is the difference between steel structures that are welded or bolted as is illustrated in Table 2.1. All values apply to ultimate limit state conditions. The difference may be explained from slip in the bolted connections which provides additional damping that is not present in the welded structure.
Table 2.1: Values of equivalent damping ratios in the ultimate limit state [DOW87].

<table>
<thead>
<tr>
<th>Type of structure</th>
<th>Stress state</th>
<th>Welded connections</th>
<th>Bolted connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clad</td>
<td>Elastic</td>
<td>0.025</td>
<td>0.050</td>
</tr>
<tr>
<td>Clad</td>
<td>Inelastic</td>
<td>0.050</td>
<td>0.100</td>
</tr>
<tr>
<td>Unclad</td>
<td>Elastic</td>
<td>0.020</td>
<td>0.050</td>
</tr>
<tr>
<td>Unclad</td>
<td>Inelastic</td>
<td>0.050</td>
<td>0.075</td>
</tr>
</tbody>
</table>

It has been observed that the damping ratio increases with the vibration amplitude. Jeary has reported several times on this phenomenon, most notably in his Ph.D. thesis [JEA81]. From tests on nine buildings by means of forced vibrations, like described in section 5.1.2, he found that there is an approximate linear relation of damping with displacement amplitude. Jeary proposed the following predictor [JEA86]:

$$\zeta = 0.01f_c + 10^{0.5\sqrt{d} \frac{\hat{u}_h}{h}} (f_c \text{ in Hz})$$  \hspace{1cm} (2.44)

where

$\hat{u}_h$ = building’s width (in meters) at the base in the direction of the motion,

$\hat{u}_h$ = the peak dynamic displacement at the top.

The base dimension is one of the parameters in the predictor, since Jeary considered the lower quarter of the structure as the zone of the building that governs the damping characteristics of the entire structure.

Note that the predictor is based on a fit on experimental data of nine buildings that were exposed to harmonic forced vibrations with amplitudes up to 3 mm. Some caution in the use in combination with random loading, therefore, is required. Moreover, it neglects the role of non-structural elements. According to equation (2.44), a bare frame has the same energy dissipation as a building with a facade and partition walls. Let us introduce an energy dissipation factor $C_c$, such that

$$\zeta = 0.01f_c + C_c \frac{\hat{u}_h}{h} (f_c \text{ in Hz})$$  \hspace{1cm} (2.45)

In absence of non-structural elements the energy dissipation factor will be very low, say $C_c = 0$. It will be shown in section 4.4.3 that tall building response to wind loads is rather insensitive to $C_c$ if $C_c$ is larger than about 50. Thus, a practical value for the energy dissipation factor is 50.

If the non-linear model that was presented in 2.3.3 is used, the energy losses due to hysteresis can be determined accurately. The results of a non-linear analysis can be transformed to a linear model by determination of an equivalent damping ratio. An equivalent damping ratio is derived when the work done by a viscous damper during one cycle with a length of $T$ seconds is equal to the energy dissipated in the non-linear system:

$$\Delta E_{\text{viscous}} = \int_0^r F(t) \cdot v(t) dt = \int_0^r cv^2(t) dt = \int_0^r \dot{u}^2 \omega^2 \cos^2 \omega t dt = c \pi \dot{u}^2 \omega$$  \hspace{1cm} (2.46)

and
\[ \Delta E_n = \int_0^T F(t) \cdot v(t) \, dt = \sum_{i=1}^n F(t_i) \cdot v(t_i) \Delta t \] (2.47)

where \( n = T/\Delta t \) and \( \Delta t \) is the time increment that is used in the algorithm that solves the non-linear equation of motion (2.22). After some elaboration it is found that the equivalent damping ratio may be expressed as

\[ \zeta = \frac{1}{2\pi \bar{\nu}^2 \omega_0} \] (2.48)

Note that in a non-linear system there are no such things as the natural frequency. For correct use of equation (2.48) it required that the resonant frequency for a certain level of displacement has already been identified.

2.5 SkyDyFe

The previous sections have introduced the basic dynamics that are needed in an analysis. Especially the section on non-linear dynamics has shown that there is no unique solution when studying dynamic behaviour. Mutual dependence between various aspects plays an important role herein.

Figure 2.12 shows a scheme of the relations that exist between the most important aspects in the assessment of the dynamic behaviour of tall buildings. There are six layers in the scheme. The top layer represents those aspects on which the designer has no or little influence. In the second layer are the wind loads, the shape parameters and the mass parameters. A designer has some influence on them, but only limited.

The third layer contains the aspects that are really the terrain of the tall building designer: the structural system, the lay-out of the building elements and the foundation properties. Once these aspects are laid down in a design, the probably most difficult part in the assessment can be performed. Going to the fourth layer is not an easy task and has been shown to be prone to large errors. The translation of the design information obtained in the second and third layer to dynamic properties seems to be the key problem in a dynamic analysis.

Once these parameters are accurately determined, the determination of the dynamic response is rather simple. Finally, the response has to be judged.

The SkyDyFe Design Tool has been developed according to the scheme in Figure 2.12. SkyDyFe has been developed in Excel for Windows version 5.0, using the Visual Basic programming language [MIC94]. In order to cope with the complex interaction that is shown in Figure 2.12, the program has a modular structure, as is displayed in Figure 2.11. Each module performs a specific task that is needed in the assessment procedure according to the relation scheme in Figure 2.12. A central module acts as a switch-board for the information that needs to be exchanged between the modules.

The ‘Criteria’ module contains the serviceability criteria that are discussed in Chapter 3. In the ‘Wind_nl’ module the wind loads are determined according to the procedure that was treated in section 2.2.

The modules ‘Shape, Mass, Lay-out’ and ‘Structure’ mainly process building data, like building height. In module ‘Dyn_prop’ the input data are transferred to dynamic parameters, like stiffness. The transformation from building input parameters to dynamic parameters is
performed according to the modal analysis (see section 2.3.2). In the ‘Response’ module the response is calculated according to the procedure in section 2.3.2. The displacement-dependent damping ratio, which was presented in section 2.4.4 is included in the ‘Dyn_prop’ module.

A comprehensive description of SkyDyFe is given in [OOS94a].

![Diagram](image_url)  
Figure 2.11 Structure of SkyDyFe.

### 2.6 Summary

In this chapter the assessment of the wind-induced dynamic behaviour of tall buildings was presented. The assessment procedure can be divided in two main parts: determination of the dynamic part of the wind load and determination of the response of the structure.

Wind loads have a random character. Spectral analysis is a readily available, convenient tool to describe these loads. From the three important loading types, along-wind loads, cross-wind loads and torque only the along-wind component is considered for reasons that were outlined in Chapter 1. The theory of the along-wind loads is well established although there are differences in commonly used representations of the fluctuations in the wind speed and the spatial correlation of the gusts. In Chapter 4 the effects of differences in these definitions on the response will be studied.

A simple one degree of freedom model can be used to study the dynamic response of tall buildings. Modal analysis provides a way to determine the properties of this model. It appears that the determination of these properties, especially the damping, is a process that is prone to large errors. It may be required to use a non-linear model to accurately predict the energy dissipation in a building.

The SkyDyFe Design Tool, which has been developed during this research project, assesses the wind-induced dynamic response of tall buildings according to the above conclusions.
Figure 2.12  Relation scheme of the parameters in the dynamic response assessment.
3 SERVICEABILITY

3.1 Wind-induced dynamic behaviour and serviceability of tall buildings

The serviceability of a building covers a wide range of subjects ranging from the quality of the air conditioning to lighting conditions. For the wind-induced dynamic behaviour of tall buildings, it is the comfort of the occupants in relation to horizontal vibrations that is the serviceability issue.

Deformation is another aspect of serviceability that is associated with wind-induced vibrations. In tall buildings the displacements that are associated with acceptable vibration levels are normally small enough to prevent damage to building elements. Therefore, this chapter focuses on the perception of motion.

The perception of motion as a serviceability limit state is becoming increasingly important due to the following developments in the building industry:

- Structural systems are composed of high strength materials. As a consequence, structures may be lighter and more flexible and still meet the strength requirements.
- Less damping is present due to the use of high strength materials. Example: high-strength concrete may be uncracked under serviceability limit state conditions, thus providing less energy dissipation.
- Fewer non-structural components, which provide inherent stiffness and damping. Moreover, these components are often assembled such that they are free to move.
- Non-structural materials are lighter and less rigid.
- Modern Building Codes have relatively low safety factors, allowing for lighter structures.

Traditionally, most attention in research and codification has been paid to the ultimate limit states (strength and stability), since the ultimate limit states are irreversible and therefore embody a big potential safety risk. Serviceability limit states are often reversible. Moreover, there is no explicit limit state as there is a gradual loss of serviceability.

Although serviceability limit states are usually not codified, the economic consequences of improper functioning of buildings may be significant. Costs of refurbishment may be a large fraction of the costs of the erection.

As an illustration of serviceability limit states, let us return to the human comfort in relation to the horizontal wind-induced vibrations. The perception of vibration is highly subjective. As the acceleration levels increase gradually more people will start to complain about perceptible vibrations. Besides, the reversibility is clear: after a storm passed, the vibrations disappear. So, a serviceability limit state with regard to wind-induced vibrations should define the percentage of people that complain about perceiving motion and the numbers of times it may be accepted that people object to the wind-induced motion.

In the following section the occupant comfort will be discussed. In section 3.3 a criterion to ensure adequate dynamic performance is established.

It has often been suggested that settling a drift limit is sufficient to obtain a satisfactory level of serviceability in tall buildings. A brief discussion of drift limit versus human comfort is presented in subsection 4.4.6. Another source of concern may be the use of sensitive equipment in a building. For example, the operation of medical equipment requires very low vibration levels. Equipment requirements are very specific and will not be dealt with herein.
3.2 Occupant comfort

The effects of vibrations on the well-being of human beings have been known for a very long time. People had to accept motion sickness while travelling in horse-drawn vehicles, ships, trains and, relatively recently, aeroplanes.

On the other hand, building structures are traditionally rather massive and rigid. Therefore people do not expect motions in buildings. Although vibrations that are associated with wind-induced serviceability limit states are an order of magnitude larger than the vibration perception threshold, they are still small compared to other situations in which human bodies are exposed to vibrations. As an illustration, consider Table 3.1, which shows acceleration levels for a variety of situations. People tend to accept levels that are more than hundred times higher than is considered acceptable in buildings as long as the duration of the event is small, i.e., the dominant frequency of the vibration is high.

Table 3.1 Approximate maximum acceleration of human bodies in a variety of situations [IRW78].

<table>
<thead>
<tr>
<th>situation</th>
<th>peak a (m/s²)</th>
<th>duration</th>
<th>reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>jet at foot of dive</td>
<td>60</td>
<td>1 s</td>
<td>controlled emotions (trained personnel)</td>
</tr>
<tr>
<td>fairground attraction</td>
<td>45</td>
<td>8 peaks in 270 s</td>
<td>very popular</td>
</tr>
<tr>
<td>building during earthquake</td>
<td>30</td>
<td>0.1 s repeated</td>
<td>horrifying</td>
</tr>
<tr>
<td>some public transport</td>
<td>5</td>
<td>&lt; 1 s</td>
<td>depends upon individual</td>
</tr>
<tr>
<td>tall building during storm</td>
<td>0.1</td>
<td>&gt; 1000 s</td>
<td>complaints, alarm, nausea</td>
</tr>
</tbody>
</table>

The long duration of a storm may cause discomfort, sometimes feelings of insecurity and, in rare cases, nausea. Therefore, only very low levels of acceleration are accepted. Subsection 3.2.1 will treat the factors that play a role in the human perception of motion. Subsequently, subsection 3.2.2 will demonstrate how the structural response is linked to the human response. Note that occupant comfort is a very complex subject, that depends on many parameters. Besides the sensitivity to horizontal motion, there are many other environmental conditions that can cause discomfort, like high temperatures, insufficient ventilation and poor light conditions. It is hoped that criteria will be developed that are able to judge the total effect of these comfort conditions on the human well-being. After all, the human body is a complex system that does not respond in a linear way to sensations received from its environment.

3.2.1 Factors affecting human response to building motion

In [KAH71] Kahn & Parmalee studied the factors that play a role in the perception of motion. They found that the following parameters are important:

1. **Frequency or period of the building.** Tests, like the ones discussed in the following subsection, all indicate that perception thresholds tend to increase as the frequency decreases for low frequency harmonic vibrations (f < 1 Hz). Moreover, resonance of body parts may cause discomfort. As an illustration, from [CHA73], some natural frequencies are: standing body 5 - 12 Hz and hand 30 - 40 Hz.

2. **Age.** The sensitivity of humans to motion is an inverse function of age. Therefore children are more sensitive than adults.

3. **Body posture.** The sensitivity of humans to motion is proportional to the distance of the person’s head from the floor. A standing person is more sensitive than a lying person.

*Wind-induced Dynamic behaviour of Tall Buildings* - 26
4. **Expectancy of motion.** Perception thresholds decrease approximately to 50% if a person has prior knowledge that motion will occur.

5. **Body movement.** A walking subject will start to perceive motions at a higher acceleration level than a standing object.

6. **Visual cues.** Visual signs play an important part in confirming a person's perception to motion. The eyes can perceive the motions of objects in a building such as hanging lights, blinds and furniture. People are also sensitive to relative motion between adjacent structures that can often be detected visually and tends to exaggerate the magnitude of the occurring motion. This is especially the case with torsional motion, during which people are very sensitive to the rotation of the building relative to fixed landmarks outside.

7. **Acoustic cues.** The rubbing of partition walls during storms is an example of wind-induced sound. These sounds, supported by the whistling of the wind, are known to focus attention on building motion even before subjects are able to perceive the motion by other senses. For example, in [IRW78] Irwin reported on two buildings with almost identical location and magnitude of acceleration. One building was well acoustically insulated and occupants experienced motion, but declared the motions acceptable, while the occupants in the other, poor acoustically insulated, building felt that the motion alarmed them.

8. **Type of motion.** Occupants of tall buildings can be subjected to (uni-)axial and bi-axial, like elliptical, types of motion. Research by Noguchi et al. [NOG93] concluded that bi-axial motions have more influence on human response than axial motion does. Bi-axial motion causes a balance shift in two orthogonal directions while axial motion causes balance shift mainly in one direction. Torsional motions are often perceived by visual signs and therefore the motion thresholds are an order of magnitude smaller than those for lateral motion.

Many of the dependencies have been determined through tests by means of motion simulators. An important aim of these tests was to link structural response to human perception. Possible parameters can be displacement, velocity, acceleration or jerk. The latter parameter is the derivative of the acceleration with respect to time. Human beings are not directly sensitive to displacement or velocity. Human beings are sensitive to forces operating on them. Therefore, acceleration will stimulate various body organs and senses.

Some researchers believe [GRI93] that the human body adapts to a constant acceleration. With changing acceleration, a continuously changing body adjustment is required. Jerk is then the critical component of structural response. However, acceleration has become the accepted criterion for evaluation in standards concerning motion perception, since it can be easily measured by several well-proven methods.

Not surprisingly, most motion simulator tests have been performed to find a relation between perception threshold, frequency of vibration and the acceleration level. All tests reveal that human comfort is a very subjective aspect of the dynamic performance of buildings. Different people report the same vibrations to be perceptible, unpleasant, or even intolerable. The following subsection treats some important studies of perception thresholds in terms of acceleration levels.

### 3.2.2 Human response to acceleration

Most human comfort studies have produced perception thresholds for high frequency vibrations. Design guidelines for tolerance levels that can be used in the design of tall and slender buildings, however, require research on the perception to low frequency vibrations (say: less than 1 Hz). Chen and Robertson [CHE72] did extensive tests (112 subjects) that meet this requirement. Tests were carried out in a test room that experienced harmonic
vibrations at frequencies of 0.076, 0.10 and 0.20 Hz. The testing parameters were: frequency, body orientation (four positions with respect to direction of motion), body movement (standing, walking, sitting) and expectancy level (total ignorance of the test, expectation of motion and previous experience of motion in a test room). All test parameters were found to be significant.

The research showed that at typical building frequencies the most sensitive occupants are able to perceive peak acceleration values as low as 20 mm/s² while standing (see Figure 3.1). On the other hand, the least sensitive occupants may not perceive accelerations until they exceed 200 mm/s² if they are sitting. Figure 3.1 shows results of the statistical analysis of some of the test results. It appears that the perception threshold can be characterised by a lognormal distribution.

Recent work by Shiyo & Kanda [SHI93] suggests that the perception of random motion with a predominant frequency below 1 Hz is different from the perception of motions induced by harmonic forces. They found the perception threshold to be almost constant with predominant frequency. Test conditions were similar to those from Chen & Robertson, except for the induced vibrations. The vibrations were synthesised from the response of a one-mass-spring model on a Gaussian white-noise loading with random distributed phase angle. Predominant frequencies were at 0.125, 0.16, 0.20, 0.25 and 0.315 Hz.

3.3 Serviceability and tall building design guidelines

3.3.1 From perception to design guidelines

In the previous section the perception thresholds for horizontal accelerations were identified. This identification is the first step towards a guideline for designers of tall buildings. The other steps that have to be taken are:

- determination of objection levels as a function of return period of a storm,
- determination of an acceptable objection level,
- determination of a design storm, i.e., a certain return period has to be settled,
- discrimination for usage of the building.

Let us start with the determination of the objection levels as a function of the return period of a storm. Apart from the subjective threshold of perception, there will also be variation in the objection to motion. In [MEL88] Melbourne and Cheung reported on a building in New York that is known to suffer frequent perceptible motions, as much as weekly during certain times of the year. The observations of a senior manager on one of the top floors illustrate the subjectivity of human comfort. He observed that 'some office workers have no objection, others seemingly accept the taking of motion sickness pills on a regular basis and some cannot continue to work in the environment'.

Hansen, Reed and Vanmarcke [HAN73] did research on structures in service to identify occupant objection levels in relation to building behaviour. They surveyed the occupants' objection to motion in two buildings in New York City. Interviews were taken after a severe storm (the reference storm) had passed the buildings. One of questions was: 'how many times a year may a similar experience occur before it becomes objectionable?'. Table 3.2 shows the result of the survey. Note that the sum of the percentages is not 100%. Reference [HAN73] does not give a reason for this anomaly.
Figure 3.1 Cumulative frequency distribution of perception thresholds. Perception threshold is expressed in peak harmonic vibration. Frequency is 0.1 Hz [CHE72].

Figure 3.2 A relation between occupant objections versus storm return period. From [HAN73].
Table 3.2  Percentage objection versus number of storms with magnitude similar to the reference storm (from [HAN73]).

<table>
<thead>
<tr>
<th>number of storms occurring per year</th>
<th>Building A</th>
<th>Building B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6</td>
<td>11.5</td>
</tr>
<tr>
<td>2-5</td>
<td>9.4</td>
<td>25.6</td>
</tr>
<tr>
<td>6-365</td>
<td>78.2</td>
<td>72.1</td>
</tr>
</tbody>
</table>

Next, the objection levels are determined as a function of the return period of the reference storm. Intense storms occur rather infrequently and can be treated statistically as rare events. An approximate model for the probability distribution of rare events is the Poisson distribution. Therefore, the Poisson distribution may be used to describe the probability distribution of the annual number of storms.

The probability distribution of the storms in combination with the data in Table 3.2 may be used to arrive at an expected percentage of people \( E(P) \) that object to a storm with a certain return period. The following equation has to be evaluated:

\[
E(P) = P_1 p_1 + P_{2-5} \sum_{i=2}^{5} p_i + P_{6-365} \sum_{i=6}^{365} p_i
\]  

(3.1)

where

\[
p_i = \frac{ve^{-v}}{i!}
\]  

(3.2)

is the Poisson distribution where \( v \) is average number of storms. \( P_1, P_{2-5} \) and \( P_{6-365} \) are the percentages that are summarised in Table 3.2. The expected percentage of objection was evaluated for several values of \( v \), the average number of storms, which resulted in the two lines in Figure 3.2. The general trend of both lines is that more people tend to object to motion of a certain magnitude if the recurrence period decreases.

The third step to a design guideline is to determine an acceptable percentage of people objecting to motion. The following question was asked by Hansen et. al to owners, developers and engineers: ‘Assuming that you are going to build a new office building and you are concerned about human discomfort from building sway, but you do not want to spend any extra money for this factor: what percentage of the people in the top one-third of the building can object to the sway motion each year and not seriously affect your renting program?’

The responses to this query seem to indicate that 2 % of the occupants being annoyed is a reasonable limit. This limit is shown in Figure 3.2. The Building A line indicates that a rms wind-induced acceleration of 20 mm/s\(^2\) with a return period of about one year is just considered acceptable. According to the Building B line, 50 mm/s\(^2\) rms acceleration due to the six year return period storm is also on the edge of being acceptable.

Based on these observations, Hansen et. al proposed the following, tentative criterion:

\[
\sigma_\alpha < 0.05
\]  

(3.3)

where \( \sigma_\alpha \) is the rms-value of the acceleration due to a storm with a six year return period. The fourth step, determination of a reference period, is completed implicitly. This criterion has been the basis of many current design guidelines for the perception of wind-induced vibrations in tall buildings.

*Wind-induced Dynamic behaviour of Tall Buildings - 30*
Note that current Building Codes show a wide variety in return periods ranging from 1 year in Dutch code NEN 6702 [NEN91], to five years or more (ISO Standard 6897 [ISO84]) and 20 years in the Australian Standard [AS90].

Finally, there should be a distinction into categories according to the function of the building. For tall buildings two groups may be distinguished: residential (hotels included) and commercial buildings. Acceleration restrictions for residential buildings need to be more severe, because of the following reasons:

1. Residential buildings are occupied for more hours of the day and week and are therefore more likely to experience the design storm event.
2. People are less sensitive to motion when they are at work than when they are at home.
3. People are more tolerant of their work environment than of their home environment.

As an illustration, consider Table 3.3, that shows design targets for the rms acceleration on the top floors of tall buildings that are typically used in American design offices according to Griffis [GRI93]. The figures for commercial buildings in Table 3.3 are quite close to the tentative criterion of Hansen et al.

### Table 3.3 Rms acceleration targets (in mm/s²) based on a 10-year mean recurrence interval

<table>
<thead>
<tr>
<th>occupancy type</th>
<th>frequency range</th>
<th>f &lt; 0.10 Hz</th>
<th>0.10 &lt; f &lt; 0.25 Hz</th>
<th>0.25 &lt; f &lt; 1 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>commercial</td>
<td></td>
<td>60.0</td>
<td>56.0</td>
<td>52.5</td>
</tr>
<tr>
<td>residential</td>
<td></td>
<td>42.9</td>
<td>40.0</td>
<td>37.5</td>
</tr>
</tbody>
</table>

#### 3.3.2 Current guidelines

The ISO Standard 6897 [ISO84] reflects all steps that were taken in the previous subsection to come to a design guideline. Figure 3.3 displays suggested satisfactory magnitudes of motion for general purpose buildings subjected to a storm, depending on the dominant frequency. There is no distinction between commercial and residential buildings. The limit state requirement is as follows:

\[
\sigma_a \leq \sigma_{ISO}(f_c)
\]  

(3.4)

where

- \(\sigma_a\) = standard deviation of the acceleration under reference wind conditions, based on a return period of five years or more,
- \(\sigma_{ISO}\) = limit value, depending on the natural frequency, as presented in Figure 3.3,
- \(f_c\) = natural frequency of the structure.

The current Dutch Building Code NEN 6702 [NEN91] has suggested maximum magnitudes of motion that are similar to the criterion in ISO 6897, but expressed in peak acceleration. The following two equations describe the two criteria below 1 Hz.

\[
\text{NEN 6702: } \dot{a} \leq 0.1f_c^{-0.40}
\]

(3.5)

\[
\text{ISO 6897: } \sigma_a \leq 0.026f_c^{-0.41}
\]

(3.6)

*Wind-induced Dynamic Behaviour of Tall Buildings - 31*
Figure 3.3 ISO criterion ISO 6897: suggested satisfactory magnitudes of motion [ISO84] compared to suggested maximum magnitudes of wind-induced motion according to NEN 6702 [NEN91] and Van Koten [BOS74]. Thresholds of perception according to Irwin [IRW78] and Van Koten [BOS74] are also shown.

There is a relation between rms and peak acceleration, established by introduction of a peak factor $g$, which is defined as

$$\hat{a} = g \sigma_a$$  \hspace{1cm} (3.7)

For wind-induced motion, it can be proved (see for example [KLA91]) that the peaks in the acceleration due to the wind load have a Rayleigh distribution. As a consequence, the peak factor can be expressed as

$$g = \frac{\sqrt{2 \ln(f_e T)} + \frac{1}{\sqrt{3}}}{\sqrt{2 \ln(f_e T)}}$$  \hspace{1cm} (3.8)

Combination of equations (3.5), (3.7) and (3.8) yields the NEN criterion expressed in rms acceleration. Figure 3.3 compares the ISO and NEN criterion. They are practically the same. Another commonly used criterion is based on Van Koten’s perception graph [BOS74]. The maximum acceptable magnitude of wind-induced motion that he suggested is also plotted in Figure 3.3.

For comparison purposes, perception thresholds according to Irwin [IRW78] and Van Koten [BOS74] are included in Figure 3.3. Note that there is an order of magnitude difference between threshold and maximum acceptable levels of vibration. Irwin gives a lower (for very sensitive personse) and average threshold.
Most of the figures presented till now are based on a 2% annoyance rate, i.e., less than 2% of the occupants will object to the motion due to the design storm. An owner may consider a higher or lower annoyance rate, depending on the quality of the building he wants to achieve. The method that is proposed in the following subsection enables the designer to find an optimum in quality and economics when serviceability is considered.

3.3.3 Optimum serviceability performance

The current guidelines are formulated such that there is a maximum allowed acceleration that can be used as a design criterion. See, for example, equation (3.4). This suggests that there is a certain point where the building becomes unserviceable, comparable to strength limits where yielding often is the critical point.
However, this is not true for two reasons:
- Perception of motion is subjective. See section 3.2.2, where the perception threshold is discussed in detail.
- The serviceability of a structure does not really have an explicit limit state. An acceleration that is slightly above the limit does not lead to dramatic effects. The same holds if a certain acceleration level is exceeded for a longer duration then some specified admissible period. In terms of decision theory it is said that the loss function for the ultimate limit state is a step function, while the loss function for the serviceability is a gradual one (see Figure 3.4).

It may be concluded that, at least for reversible serviceability problems, like human perception of motion, an alternative and fundamentally better approach is to start with the gradual loss function from the beginning, as for instance in [DIT91].

![Diagram](image)

**Figure 3.4** Left hand: step function for strength limit; Right hand: gradual loss function for serviceability.

For example, consider an owner of the building having a number of tenants. A possible simple model for the gradual loss function is to assume that tenants deduct some part of their rent for every hour that the perception limit is exceeded. The deduction increases for higher acceleration levels, as indicated in Figure 3.4. If this function is known, the designer may find some optimum design.
Neglecting interest effects, the object function for this optimisation problem can be formulated as:

\[ C(tot) = C(b) + \int c(\sigma) dt \]  

(3.9)

where

- \( C(b) \) = the cost to produce a structure with certain properties \( k, m, \zeta \),
- \( c(\sigma) dt \) = the deduction of the rent for the discomfort of response \( \sigma \) during the period \( dt \),
- \( \sigma \) = time fluctuating rms value of the acceleration. The determination of the rms acceleration and its dependency on wind and structural parameters is discussed in the following chapter.

The optimal structure from the serviceability point of view is found when equation (3.9) is minimised. However, functions like \( C(b) \) and \( c(\sigma) \) are difficult to establish. Nevertheless, research in this direction could lead to more economic buildings. An introduction on this optimisation problem and possible solution strategies is given in [VRO95] and [VRO96].

### 3.4 Summary

There is no wide-spread agreement on serviceability limit states in tall buildings. Two main reasons of the absence of an accepted theory are:

- The human perception to building motion, the major serviceability issue in tall building design, is very subjective.
- Human perception as a serviceability problem is a reversible state. The need for codification and regulation of reversible states is felt to be minor.

Although serviceability limit states are usually not codified, the economic consequences may be significant. In modern flexible and lightly damped buildings serviceability issues are quite important. Costs of refurbishment may be a large fraction of the costs of the erection.

Establishing a criterion for the human perception of wind-induced vibrations requires 5 steps. Firstly, the threshold of perception has to be determined. The perception is subjective. For sufficient large groups the probability distribution of the perception threshold can be represented by a lognormal function. Presently, acceleration level and dominant frequency are the parameters that are used to determine the average perception threshold.

There is a need for further research efforts aimed at refining wind-induced vibration criteria for human comfort. The roles of visual cues, torsional motion and ‘learning’ effects are still not well understood.

Thirdly, objection levels have to be determined as a function of the return period of the event that is objectionable. An objection percentage of 2% for the design storm is often considered to be acceptable. There is no agreement on the return period for the design event. Values vary from 1 to 20 years.

Finally, a criterion should discriminate for usage: acceptable acceleration levels are lower in residential buildings compared to commercial buildings.

A final point of attention is the criterion itself. At present, the criteria are formulated as fixed values, which suggest that buildings are becoming suddenly unserviceable. This is not true and in reality there is a gradual loss of serviceability. In section 3.3.3 an optimisation method has been presented that uses a gradual loss function to determine an optimum relation between economics and serviceability.
4 SENSITIVITY ANALYSES

4.1 Introduction

Chapter 4 discusses the sensitivity of the dynamic along-wind response to the variation of the wind speed and the main structural parameters. First, the components of the response assessment procedure, that were presented in Chapter 2, are used to derive limit cases for the dependencies. See section 4.2. In section 4.3 the parameters that influence the dynamic wind loads are studied. Section 4.4 treats the structural parameters and their effects on the response. In this chapter the response will be expressed in terms of rms acceleration since rms acceleration levels are an accepted measure of the perception of horizontal motion by the occupants.

The sensitivity analyses in this chapter have been performed in SkyDyFe. An introduction of SkyDyFe is given in section 2.5. The results of the analyses that are presented in this chapter are supported by work of Islam, Ellingwood and Corotis who used wind tunnel tests to determine the wind-induced response of tall buildings [ISL90].

4.2 Theoretical dependencies

4.2.1 The wind-induced acceleration of two limit cases

For two special cases the wind-induced acceleration is determined: the response when the fundamental natural frequencies are comparatively high and, secondly, the response when the fundamental natural frequency is at the peak in the wind speed spectrum. These cases provide limits to the dependencies of the response on stiffness, mass, damping and wind velocity.

For high frequencies the wind spectra in section 2.2.3 all are proportional to \( f^{-\frac{5}{3}} \). Moreover, for high \( f \) it holds that the aerodynamic admittance function as proposed by Vickery (equation (2.10)) is proportional to

\[
\chi(f) \propto f^{-\frac{1}{3}} \tag{4.1}
\]

Other aerodynamic admittance functions give similar dependencies. As derived in Appendix D, the following proportionalities are found

\[
\sigma_{\nu} \propto (\bar{v}(h))^\frac{5}{3} k_e^{-\frac{2}{3}} m_e^{-\frac{1}{3}} \zeta^{-\frac{1}{3}} \text{ [m/s}^2]\tag{4.2}
\]

where

- \( \bar{v}(h) \) = the mean wind velocity at the top of the building [m/s],
- \( m_e \) = effective mass [kg],
- \( k_e \) = effective spring stiffness [N/m],
- \( \zeta \) = damping ratio.

So, for high natural frequencies (for example, most of the low-rise structures) stiffness and damping are effective tools to reduce the accelerations.

The second limit is found when the fundamental natural frequency coincides with the peak in the wind speed spectrum. See, for example Figure 2.4, where the peak in the spectrum is at approximately 0.01 Hz. For the Kaimal spectrum (equation (2.6)) the peak is found at a
reduced frequency of 1/22. For such low frequencies, the spatial correlation of the pressures is high and the admittance function therefore tends to 1.

After some elaboration, see Appendix D, it is found that in this case the response is proportional to

$$\sigma_a \propto (\bar{y}(h))^2 m_c^{-1} \zeta^{-\frac{1}{2}}$$  \hspace{1cm} (4.3)

Note that the stiffness is not present in this equation. Thus, for this special limit case mass and damping are the tools to reduce the accelerations.

Let us define \( \varepsilon \) as a proportionality according to

$$\sigma_a \propto m_c^{-\varepsilon}$$  \hspace{1cm} (4.4)

Subsequently, the following approximate relation, that is a general expression for the wind-induced rms horizontal acceleration of buildings, can be derived (see Appendix D):

$$\sigma_a \propto (\bar{y}(h))^{2(1-\varepsilon)} k_e^{-1+\varepsilon} m_c^{\varepsilon} \zeta^{-\frac{1}{2}} \frac{1}{6} \leq \varepsilon \leq 1$$  \hspace{1cm} (4.5)

The factor \( \varepsilon \) is 1/6 for buildings with high fundamental natural frequencies. Tall buildings have relatively low fundamental frequencies (in the range 0.1-1.0 Hz), so values of \( \varepsilon \) can be expected to go towards 1, which is the value of \( \varepsilon \) when the fundamental natural frequency coincides with the peak in the wind load spectrum. The sensitivity analyses in this chapter will quantify \( \varepsilon \) for several tall building configurations.

Note that expression (4.5) does not take into account the non-linear behaviour, which is most clearly expressed in the displacement-dependency of the damping ratio. The sensitivity analyses in section 4.4 do take account of this effect by means of equation (2.45).

**4.2.2 Contribution of the second mode to the total wind-induced acceleration**

So far, only the fundamental modes have been considered. In this section we will assess the contribution of the second mode of vibration to the total wind-induced acceleration. For this purpose we consider a clamped column with uniform properties in elevation. Both pure bending and pure shear will be considered. For both bending and shear column the natural frequencies of the first and second mode are well separated, so the modes may be considered independent. For each mode, equation (4.5) applies. For independent modes, the total rms acceleration is found by vectorial summing of the modal responses:

$$\sigma_a^2 = \sigma_{a,1}^2 + \sigma_{a,2}^2$$  \hspace{1cm} (4.6)

where \( \sigma_{a,i} \) is the contribution of the \( i \)th mode to the total variance. Let us consider the error in the response by neglecting the second mode of vibration. Equation (4.6) can be written as:

$$\sigma_a^2 = \sigma_{a,1}^2 \left[ 1 + \frac{\sigma_{a,2}^2}{\sigma_{a,1}^2} \right]$$  \hspace{1cm} (4.7)
Thus, the error in the determination of the acceleration due to neglect of the second mode is

\[
\Delta a = \left[ -1 + \sqrt{1 + \frac{\sigma_{a,2}^2}{\sigma_{a,1}^2}} \right]
\]

(4.8)

Now, we have to determine the modal properties of the clamped column to derive the response. Table 4.1 contains modal properties for a pure bending and a pure shear system.

Table 4.1  Model properties for a clamped column with uniform properties: height \( h \), distributed mass \( \mu \), bending stiffness \( EI \) or shear stiffness \( GA \).

<table>
<thead>
<tr>
<th>modal property</th>
<th>bending</th>
<th></th>
<th></th>
<th>shear</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1\textsuperscript{st} mode</td>
<td>2\textsuperscript{nd} mode</td>
<td>ratio</td>
<td>1\textsuperscript{st} mode</td>
<td>2\textsuperscript{nd} mode</td>
<td>ratio</td>
</tr>
<tr>
<td>angular natural frequency</td>
<td>3.52 ( \sqrt{\frac{EI}{\mu h^4}} )</td>
<td>22.4 ( \sqrt{\frac{EI}{\mu h^4}} )</td>
<td>6.36</td>
<td>( \frac{1}{2} \pi \sqrt{\frac{GA}{\mu h^2}} )</td>
<td>( \frac{1}{2} \pi \sqrt{\frac{GA}{\mu h^2}} )</td>
<td>3</td>
</tr>
<tr>
<td>effective mass</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
<td>0.50</td>
<td>0.50</td>
<td>1</td>
</tr>
<tr>
<td>effective stiffness</td>
<td>3.12 ( \frac{EI}{h^2} )</td>
<td>125.4 ( \frac{EI}{h^2} )</td>
<td>40.5</td>
<td>( \frac{1}{2} \pi^2 \frac{GA}{h} )</td>
<td>( \frac{1}{2} \pi^2 \frac{GA}{h} )</td>
<td>9</td>
</tr>
<tr>
<td>effective load coefficient</td>
<td>0.397</td>
<td>0.219</td>
<td>0.55</td>
<td>0.647</td>
<td>0.212</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The last column in Table 4.1 contains the ratios of the modal properties of the second to the first mode. Assume that the ratio of damping ratio in first and second mode is 1. Substitution of the ratios in Table 4.1 in equation (4.5) yields the ratio \( \sigma_{a,2} \) to \( \sigma_{a,1} \). The parameter \( \epsilon \) was taken \( \frac{1}{2} \). These ratios can be substituted in equation (4.8), which yields the errors that are summarised in Table 4.2.

Table 4.2  Error in acceleration due to neglect of second mode.

<table>
<thead>
<tr>
<th></th>
<th>bending</th>
<th>shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>ratio ( \sigma_{a,2} ) to ( \sigma_{a,1} )</td>
<td>0.086</td>
<td>0.11</td>
</tr>
<tr>
<td>( \Delta a )</td>
<td>0.3 %</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

Neglect of the second mode will cause an error of about 0.5 % in the acceleration, which is small. Therefore, it is acceptable to concentrate on the first mode of vibration in wind-induced tall building dynamics.

4.3  Wind parameters

In a study, that was reported in [LEE88], Lee & Ng compared several building codes at that time. Inconsistencies were observed in the accelerations, which were believed to be mainly due to differences in the definitions of both wind spectrum and aerodynamic admittance functions.

The typical slab-like Dutch tall building’s response is dominated by the along-wind response. In chapter 2 several expressions to represent the dynamic character of the wind loads through wind spectra and spatial correlation functions were presented. In this section the effects of variation in definition of these expressions will be studied.
Figure 4.1  Effects of alternate spectra on the acceleration. Building height is 100m. Aerodynamic admittance function according to Vickery.

Figure 4.2  Effects of alternate aerodynamic admittance functions on the acceleration. Building height is 100m. Spectrum according to Kaimal.
Figure 4.3 Effects of alternate spectra on the acceleration. Building height is 50m. Aerodynamic admittance function according to Vickery.

Figure 4.4 Effects of alternate spectra on the acceleration. Building height is 150m. Aerodynamic admittance function according to Vickery.
For this purpose, the four wind speed spectra presented in section 2.2.3 were combined with the three aerodynamic admittance functions as presented in section 2.2.4. Three building configurations were considered:

1. a 50 m tall building, 14 stories, rectangular plan, dimensions 60x14 m².
2. a 100 m tall building, 27 stories, rectangular plan, dimensions 60x20 m².
3. a 150 m tall building, 40 stories, rectangular plan, dimensions 60x20 m².

The response of each configuration was calculated with twelve combinations of wind spectrum and aerodynamic admittance function. For all three cases the building’s density is 300 kg/m³. An annual return wind speed is assumed, which yields a shear velocity of 2.32 m/s and an aerodynamic roughness that equals 0.7 m as wind profile parameters. For each run the damping ratio is kept at 2 %. The stiffness is varied from 20 to 60 MN/m, so that the natural frequencies vary each run. A linear mode shape is assumed. A complete list of input parameters may be found in Appendix C.

4.3.1 Gust spectrum

Figure 4.1 shows the effect of the disagreement in definition of the wind speed spectrum on the calculated response. In this figure the rms acceleration of the case-study building is plotted for the three spectra. All spectra show the same dependency on stiffness (and frequency). However, large differences in the response occur for fixed values of the stiffness. This effect supports the main conclusion from the study of Lee and Ng [LEE88]: differences in the definitions of both wind spectrum and aerodynamic admittance functions may cause substantial variation in the along-wind response. It appears that the Davenport spectrum yields the highest response. The Eurocode spectrum is very close to the von Karman spectrum. The Kaimal spectrum gives the lowest response.

The deviation in the response seems to be dependent on the height of the structure. As an illustration, compare Figure 4.3 to Figure 4.4 showing results for the 50m and 150 m building respectively.

For the 50 m building the deviation due to the different wind spectra is rather small, but at 150 m the lines deviate significantly. This may be explained by the fact that all wind spectra are based on meteorological data that was measured at low heights. They are fitted to these measurements. The expression that is the result of the calibration is valid in the measurement range. Small deviations between measurement and fit are inevitable, but do not cause problems within the validity range. However, these deviations might cause large errors outside the validity range. Figure 4.3 illustrates this: at 50 m, three of the four spectra that were considered are close together. Only Davenport’s spectrum causes a deviation from the other expressions. Figure 4.4 shows that at greater height there is a significant deviation due to differences in definitions of the spectra.

4.3.2 Aerodynamic admittance

As with wind speed spectra, there is variation in definition of the aerodynamic admittance functions. Figure 4.2 shows the effects of these differences in representation on the response of the 100 m building. There is a distinct difference in the results obtained with the three aerodynamic admittance functions. The Vickery line is clearly higher than the other two admittance functions. Sensitivity analyses on buildings with heights of 50 and 150 m gave plots that looked very similar to Figure 4.2 and are therefore not displayed here.
4.3.3 Shear velocity and aerodynamic roughness

Two parameters have not been discussed yet, but are important in the assessment of the wind induced vibrations: the shear velocity $v$ and the aerodynamic roughness $z_0$. The mean wind velocity is proportional to the shear velocity and its role is evident from equation (4.5); the rms acceleration is proportional to at least the square of the shear velocity and often there is an even stronger dependency.

For five buildings ranging from 60 to 300 m height, the effect of the shear velocity on the response has been determined. A summary of the other properties is given in section 4.4.1. In Figure 4.5 the effect of the shear velocity on the rms acceleration is shown for 60 m (top line) and 300 m (bottom line). The shear velocity was varied from 0.5 to 2.5 m/s.

In Table 4.3 the proportionalties are summarised by means of the index $\delta$, where $\sigma_a \propto v^\delta$. According to equation (4.5), the coefficient $\delta$ should equal $2(2 - \varepsilon)$, where $\varepsilon$ is the factor that characterises a building’s behaviour. The third row of Table 4.3 contains values of $\varepsilon$ that have been deduced in this way.

<table>
<thead>
<tr>
<th>h (m)</th>
<th>60</th>
<th>120</th>
<th>180</th>
<th>240</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$ (-)</td>
<td>3.51</td>
<td>3.40</td>
<td>3.32</td>
<td>3.26</td>
<td>3.21</td>
</tr>
<tr>
<td>$\varepsilon$ (-)</td>
<td>0.25</td>
<td>0.30</td>
<td>0.34</td>
<td>0.37</td>
<td>0.40</td>
</tr>
<tr>
<td>$f_c$ (Hz)</td>
<td>0.56</td>
<td>0.28</td>
<td>0.19</td>
<td>0.14</td>
<td>0.11</td>
</tr>
</tbody>
</table>

A similar sensitivity analysis has been performed for the aerodynamic roughness. See Figure 4.6, which shows the effect on the acceleration for the 60 and 300 building. For $z_0$ the proportionality is slowly varying. Approximate dependencies ($\sigma_a \propto z_0^\delta$) are collected in Table 4.4.

<table>
<thead>
<tr>
<th>h (m)</th>
<th>60</th>
<th>120</th>
<th>180</th>
<th>240</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$ (-)</td>
<td>-0.43</td>
<td>-0.35</td>
<td>-0.30</td>
<td>-0.28</td>
<td>-0.26</td>
</tr>
<tr>
<td>$f_c$ (Hz)</td>
<td>0.56</td>
<td>0.28</td>
<td>0.19</td>
<td>0.14</td>
<td>0.11</td>
</tr>
</tbody>
</table>

4.4 Structural parameters

4.4.1 Introduction

In this section the effects of the structural parameters on the response are studied. Other parameters were kept at a certain value. Wind loads were determined according to the along-wind spectrum of Kaimal (equation (2.6)) in combination with the aerodynamic admittance function of Vickery (equation (2.10)). The mass is calculated from a building’s density of 200 kg/m³. All buildings that are considered have a rectangular plan without set-backs. The width $b$ perpendicular to the wind direction is constant for all buildings: $b = 60$ m. The plan dimensions are such that the aspect ratio of the building $h/d$ is 6, unless stated otherwise. The definition of $b$ and $d$ is given in Figure 1.1.
Figure 4.5  Acceleration versus shear velocity. Stiffness 30 MN/m, damping ratio 2%, effective mass ratio 0.33.

Figure 4.6  Acceleration versus aerodynamic roughness. Stiffness 30 MN/m, damping ratio 2%, effective mass ratio 0.33.
4.4.2 The effective stiffness

In section 4.2.1 two theoretical limits for the proportionality of rms acceleration to the stiffness were derived. It was also noted that tall buildings have a behaviour that is somewhere in between these limits.

A sensitivity analysis, in which the stiffness was varied from 10 to 70 MN/m, revealed the dependencies. Five building heights, namely 60, 120, 180, 240 and 300 m, were considered. In the calculations the mode shape was taken as being linear. The damping ratio was kept constant at 2% of critical.

Table 4.5 shows the proportionalities that were found for the rms acceleration to the stiffness ($\sigma_a \propto k_e^\delta$). In Figure 4.7 the trends are visualised for 60 and 300 m. The differences in proportionality may be appreciated. For $h = 60$ m the exponent is clearly going towards the expected -5/6 (see equation (4.2) for high frequencies), the others however are well below that value. Values of $\varepsilon$ can be derived, since the index $\delta$ should equal $-1 + \varepsilon$ according to equation (4.5). The third row in Table 4.5 suggests that, for tall buildings, an average value of $\varepsilon$ is 0.5.

Table 4.5 Proportionality of rms acceleration to stiffness for several heights.

<table>
<thead>
<tr>
<th>h (m)</th>
<th>60</th>
<th>120</th>
<th>180</th>
<th>240</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>-0.667</td>
<td>-0.576</td>
<td>-0.517</td>
<td>-0.474</td>
<td>-0.442</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.333</td>
<td>0.424</td>
<td>0.483</td>
<td>0.526</td>
<td>0.557</td>
</tr>
<tr>
<td>$f_{\text{c,min}}$ (Hz)</td>
<td>0.33</td>
<td>0.16</td>
<td>0.11</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>$f_{\text{c,max}}$ (Hz)</td>
<td>0.87</td>
<td>0.44</td>
<td>0.29</td>
<td>0.22</td>
<td>0.17</td>
</tr>
</tbody>
</table>

The main reason for the variation in proportionality is the aerodynamic admittance function. For low buildings ($f_e \gg 1$ Hz), the aerodynamic admittance function is proportional to $f_e^{-4/3}$ and the acceleration is then proportional to $k_e^{-5/6}$ as was shown in section 4.2.1. Tall buildings are in the intermediate range of the aerodynamic admittance function, in which this function is slowly changing to the horizontal asymptote that goes to 1 for very small frequencies.

The variation of behaviour with height is becoming even more apparent if the non-linear damping mechanism, as represented by equation (2.45), is included in the sensitivity analysis. Figure 4.8 shows the trends for the 60, 120 and 300 m tall building. There appears to be a distinct difference in the stiffness-acceleration relation for 'low' and 'high'-rise buildings. For the 60 m building increasing the stiffness is beneficial. The second height that was considered, 120 m, already indicates a less important role for the stiffness in reducing the acceleration. For very tall buildings the acceleration is almost insensitive to variation of the stiffness.

The explanation of this effect is that an increase of the stiffness causes the displacements to drop. As a consequence, the damping ratio decreases, which increases the acceleration. The total effect is an almost constant acceleration for a wide range of stiffnesses.

There is another interesting topic that may be recognised from Figure 4.8 and Figure 4.7. The plots seem to suggest that as the building grows the acceleration drops. This effect will be explained in section 4.4.5.
Figure 4.7  Acceleration versus stiffness; damping ratio 2%.

Figure 4.8  Acceleration versus stiffness; damping is a function of the displacement according to equation (2.45).
Figure 4.9  Acceleration versus mass; stiffness 30 MN/m, damping ratio 2%.

Figure 4.10  Acceleration versus mass; stiffness 30 MN/m, damping is a function of the displacement according to equation (2.45).

*Wind-induced Dynamic Behaviour of Tall Buildings - 45*
4.4.3 The effective mass

Similar to the stiffness sensitivity analyses, the mass survey started with a check of the theoretical limits for six heights. The effective mass coefficient was varied from 0.2 to 0.6. The stiffness was kept at 30 MN/m.

Figure 4.9 shows the results for 60 and 300 m. Again, there is the difference in behaviour of the 60 m and 300 m building which is apparent from the different slopes of the lines in Figure 4.9. The proportionalities (\( \sigma_u \propto m^b \)) are summarised in Table 4.6. As expected, values for \( \delta \) equal -\( \varepsilon \), where \( \varepsilon \) is based on the figures in Table 4.5.

| Table 4.6 Proportionality of rms acceleration to mass for several heights. |
|-------------------|-------|-------|-------|-------|-------|
| \( h \) (m)       | 60    | 120   | 180   | 240   | 300   |
| \( \delta \) (-)  | -0.324| -0.414| -0.473| -0.517| -0.550|
| \( -\varepsilon \) (-) | -0.333| -0.424| -0.483| -0.526| -0.557|
| \( f_{c,\text{min}} \) (Hz) | 0.42  | 0.21  | 0.14  | 0.10  | 0.08  |
| \( f_{c,\text{max}} \) (Hz) | 0.73  | 0.36  | 0.23  | 0.18  | 0.15  |

In Figure 4.10 the effects of the effective mass on the rms acceleration are shown when the non-linear damping predictor is used. Contrary to the stiffness, an increase in the mass is now always beneficial if accelerations have to be reduced. Note that both Figure 4.9 and Figure 4.10 show the peculiarity that was already apparent from the stiffness plots: acceleration decreases with height.

4.4.4 The damping ratio

Damping is always effective in reducing the dynamic response. In this subsection the effect of the energy dissipation factor \( C_e \) is studied. This factor is a measure of the extent of non-linearity of the damping ratio according to equation (2.45).

In Figure 4.11 the effect of \( C_e \) on the response is visualised for a 60 m and 300 m tall building. In the lower region of the energy dissipation factor, i.e., \( C_e < 50 \), the response is sensitive to variation of \( C_e \). This trend is supported by the lines in Figure 4.12 which shows the acceleration of a 120 m tall building versus stiffness for several energy dissipation factors.

4.4.5 Height and aspect ratio

Previous plots in this section seem to suggest that the acceleration is decreasing with increasing height. The explanation is that the mass was proportional to the square of the height in all calculations: the aspect ratio was constant and there were no changes in the floor plan along the height. Doubling the height therefore doubles the depth \( d \). Consequently, the mass is four times the original value.

The previous subsections have shown that an approximate value of the factor \( \varepsilon \) is 0.5. Thus

\[
\sigma_u \propto m^{-\varepsilon} \land m \propto h^1 \rightarrow \sigma_u \propto h^{-1}
\]

which partly explains the effect.

Wind-induced Dynamic Behaviour of Tall Buildings - 46
Figure 4.11  Acceleration versus energy dissipation factor, stiffness 30 MN/m, effective mass factor 0.33.

Figure 4.12  The energy dissipation factor, the interaction with stiffness and the effects on acceleration. Effective mass factor is 0.33, building height 120 m.
The height also affects the velocity, but reverse to the beneficial extra mass. In Figure 4.13 the joint effects are displayed. For moderate heights the effect of the increasing wind speed is stronger than the reduction due to the increasing mass. The maximum is at about 100 m and from there the mass effect is stronger. Nevertheless, one would expect higher accelerations when the building grows. This expectation is satisfied if the building’s width $d$ is fixed. This implies that the aspect ratio $h/r = h/d$ is now the parameter that is considered. Figure 4.14 shows how the acceleration develops as the aspect ratio of a tall building increases. The width $d$ was kept at 20 m and the height was varied from 8 to 240 m. The stiffness was estimated with the procedure that was presented in section 2.4. In equation (2.42) the following expression is substituted for the distributed wind load $q$:

$$q = C_d p_u(h) b$$  \hspace{3cm} (4.10)$$

where

$p_u(h)$ = the design wind pressure according to the Dutch Building Code NEN 6702 [NEN91],
$C_d$ = the drag coefficient according to definition (2.14); assumed to be 1.2.

The drift index $w_{50%}/h$ is assumed to be 1/500.

The accelerations increase rapidly going from squat ($h/d < 1$) to more slender buildings. For aspect ratios above 4 the rms acceleration is approximately proportional to $h^{0.18}$.

Figure 4.13 and Figure 4.14 neglect an important effect. In Figure 4.13 the stiffness is kept constant and in Figure 4.14 it is a function of the height, since the wind pressure increases with height. However, the stiffness is also a function of the plan dimensions. For example, the bending stiffness of a structure in a rectangular plan is approximately proportional to $d^5$, where $d$ is the depth. Thus, the corresponding effective stiffness $k_e$ is

$$k_e = \frac{312EI}{h^2} \rightarrow k_e \propto d^5 h^{-3} \leftrightarrow k_e \propto h_{0.18}^{-1} h^{-1}$$  \hspace{3cm} (4.11)$$

When the aspect ratio increases, the effective stiffness will be inversely proportional to $h^{3}$. This drop in stiffness profoundly affects the acceleration as is shown in Figure 4.15. The line ‘d = 20 m’ shows the rapid increase of acceleration as the aspect ratio increases. The other line, a constant aspect ratio of 5, shows a clearly different behaviour. This may be understood from an examination of equation (4.11): in this case the effective stiffness is inversely proportional to the height. Consequently, there is no significant increase of accelerations as the building grows, since the extra mass compensates for the decrease in stiffness.

### 4.4.6 Concluding remarks

Figure 4.8 indicated that stiffening the structure of a tall building may not be effective in reducing the accelerations. The effectiveness is even less if comfort criteria, like ISO 6897 [ISO84], are considered. In Figure 4.16 the effects of the three parameters mass, stiffness and damping on the acceleration are plotted in the ISO 6897 graph for a squat and a slender building. The arrows point in the direction of increasing quantities.

For normal buildings, with natural frequencies near 1 Hz or above and consequently $\varepsilon$ close to 1/6, an increase of the stiffness might be considered as an effective tool to reduce the accelerations (See Building B in Figure 4.16). However, for slender, tall buildings, with natural frequencies well below 1 Hz and consequently $\varepsilon$ going to 1/3 or less, the influence of the stiffness $k_e$ on the response is very small.
Figure 4.13  Acceleration versus height, stiffness 30 MN/m, damping ratio 2%.

Figure 4.14  Acceleration versus aspect ratio, effective mass coefficient 0.255, damping ratio 2%, width $d = 20$ m.
Figure 4.15  Fixed versus variable aspect ratio, effective mass coefficient 0.255, damping ratio 2%. Stiffness according to equation (4.11).

Even more remarkable is the effect on the ISO criterion. If ε is low, an increase in the stiffness may result in a shift from the admissible to the inadmissible domain (see Building A in Figure 4.16). The point is that an increase of the stiffness also leads to an increase of the natural frequency.

Similarly, drift limits do not guarantee low acceleration levels, since they only affect the stiffness of the structure. Therefore, the widespread belief that drift limits cover all aspects of serviceability is incorrect for tall buildings.

According to equation (4.5), the structural mass m always helps to reduce the vibration level, but more significant for buildings with high aspect ratio. For increasing m also the structural natural frequency is reduced, which makes the effect even stronger.

Damping always helps to reduce the vibrations. For high-rise buildings this could become an important design option. One might think of artificial damping, as well as the exploration of damping of non-structural elements.

In subsection 4.4.5 the role of the height and aspect ratio of a building was discussed. It was demonstrated that aspect ratio is more important than absolute height. This is even more apparent when the data for Figure 4.15 are plotted in the ISO graph for admissible accelerations. See Figure 4.17. It may be appreciated from this figure that human comfort will not be a design issue when building height increases as long as the aspect ratio is kept constant.

The second line in Figure 4.17 illustrates that attention to the wind-induced vibrations is required in the design of very slender buildings. So, human comfort is not only a design issue for very tall buildings. For example, a building that is 100 m high and 10 m deep, resulting in an aspect ratio of 10, may encounter difficulties in providing adequate dynamic performance.
Figure 4.16  ISO acceleration criterion and effects of the main dynamic parameters effective stiffness ($k_e$), effective mass ($m_e$) and damping ($\zeta$). Arrows point in direction of increasing quantity. Constants are summarised in Appendix C.

Figure 4.17  ISO acceleration criterion and the effects of variable and constant aspect ratio. Arrows point in direction of increasing height.
4.5 Foundation

4.5.1 Introduction

Until now, it has been assumed that the base of a tall building is rigid. However, many soil types, like clay and sand, do not possess properties that justify this assumption. In this section the error due to this modelling assumption is briefly assessed. The main effects of soil-structure interaction on the dynamic properties of a building will be introduced. The effects on the dynamic response will be studied with approximate expressions for stiffness, natural frequency and damping. Ongoing research by Karim at Delft University of Technology treats the effects of soil-structure interaction in detail [ABD96].

4.5.2 Modelling soil-structure interaction

There are three basic effects on the building model when soil-structure interaction is included:

- the stiffness decreases,
- introduction of extra damping,
- the mode shape is modified. The flexible base may change the mode shape according to

$$\varphi(z)_{new} = u_0 + \psi_{\theta} z + \varphi(z)_{old}$$

(4.12)

where

$u_0$ = the displacement due to the horizontal stiffness of the soil,

$\psi_{\theta}$ = the rotation due to the rotational stiffness of the foundation.

In general, the horizontal resistance of deep foundations is very large. Consequently, $u_0$ is very small and can be neglected. If the initial mode shape was assumed to be linear, equation (4.12) does not change the effective mass, since the normalised mode shape will not alter. The total effect is a decrease in stiffness and constant effective mass, so the natural frequency drops.

Wolf [WOL85] surveyed the influence of soil properties on the response of a building. He used the elastic half space theory to model the soil. After some elaboration, he derived modal properties that include the effects of soil-structure interaction. The following ratio of stiffness including soil properties to the stiffness of the clamped base model is found:

$$\frac{k_e}{\tilde{k}_e} = \frac{1}{1 + \frac{m_r s_r^2}{8} \left[ \frac{2 - v_{soil}}{h_r^2} + 3(1 - v_{soil}) \right]}$$

(4.13)

where

$k_e$ = the equivalent spring stiffness without incorporation of the soil properties,

$\tilde{k}_e$ = the equivalent spring stiffness with incorporation of the soil properties,

$s_r = \frac{\omega_r h}{w_{soil}}$; ratio stiffness structure to soil,

$h_r = \frac{h}{b_{found}}$; the aspect ratio,
\[ m_s = \frac{m_r}{\rho_{soil} b_{found}}; \text{ the mass ratio,} \]
\[ h = \text{height of the structure} \]
\[ m_e = \text{the effective mass}, \]
\[ w_{soil} = \text{the shear wave velocity}, \]
\[ \nu_{soil} = \text{Poisson's ratio of the soil}, \]
\[ \rho_{soil} = \text{the density of the soil}. \]

The damping ratio is also affected by the soil. The modified damping ratio becomes \((\zeta)\) is the damping in the building without soil-structure interaction)

\[ \zeta = \frac{\tilde{k}^s_k}{k_c} \zeta + \left[ 1 - \frac{\tilde{k}^s_c}{k_c} \right] \zeta_{soil} \tag{4.14} \]

Table 4.7 shows the properties of three main soil types that are used in the above equations.

<table>
<thead>
<tr>
<th>soil type</th>
<th>(\rho_{soil} \text{(kg/m}^2)</th>
<th>(v_{soil})</th>
<th>(G_{soil} \text{(MN/m}^2)</th>
<th>(w_{soil} \text{(m/s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>clay</td>
<td>1600-2250</td>
<td>0.40</td>
<td>5-90</td>
<td>60-200</td>
</tr>
<tr>
<td>sand</td>
<td>1700-2200</td>
<td>0.33</td>
<td>45-55</td>
<td>160</td>
</tr>
<tr>
<td>gravel</td>
<td>1900-2300</td>
<td>0.33</td>
<td>20-200</td>
<td>100-300</td>
</tr>
</tbody>
</table>

The reduction of the stiffness has an effect on the dynamic response that is reverse to the increased damping ratio. Figure 4.18 shows the combined effect on the rms acceleration. The effect is negligible for most tall buildings having fundamental frequencies well below 1 Hz.

The rigid base model, therefore, seems to be reasonable. This theoretical result is supported by measurements by Ellis, that were reported in [ELL86]. No significant effects of soil-structure interaction on the accelerations of tall buildings were found in that study.

### 4.6 Sources of uncertainty

In this chapter the sensitivity of acceleration to the parameters shear velocity, aerodynamic roughness, stiffness, mass and damping has been studied. The determination of most of these parameters is a process full of uncertainties. In this section the effect of these uncertainties on the response is assessed. Two groups of parameters are discussed herein. First, there are aspects that are related to the wind load. The second group is formed by structural parameters.

The two case-studies that are presented in Chapter 5, namely the Cardington building and Voorhof II, provide an interesting opportunity to compare two ways to derive a building’s dynamic behaviour. Controlled forced vibrations, supported by measurements of ambient vibration, identified the behaviour of the Cardington building whereas the dynamic behaviour of the Voorhof building has been identified by measurements of ambient vibrations alone. Moreover, the Cardington building is simple in lay-out and use compared to Voorhof II. Finally, the design figures of the Cardington building are well known, whereas only a small set of design drawings of Voorhof II is available. These aspects all have an impact on the uncertainties that have to deal with in the assessment of the dynamic behaviour.
Figure 4.18  Effect of soil-structure interaction on the peak acceleration ($\zeta_{soil} = 4\%$, $\zeta_{building} = 2\%$).

Figure 4.19  Effects of uncertainties in response parameters on human comfort. Human comfort criterion according to ISO 6897 [ISO84]. Default situation is a 120 m high building. Other data are summarised in Appendix C.
4.6.1 Wind parameters

Probably the most apparent difference between Cardington and Voorhof II is the way the load is applied. In the first case the load is a controlled harmonic vibration that is imposed on the structure. Voorhof II is excited by the wind: a random load. Thus, the uncertainties in the latter case are much larger than in the first case.

The uncertainties in a forced vibration are small and depend mainly on the quality of the exciters and the equipment that controls forcing frequencies. If more than one exciter is used measures should be taken to run the vibrators either in phase or anti-phase (torsion). However, this technology is well developed and should not be a cause of uncertainties. The amplitude of the load is depending on the mass and the square of the frequencies, since the loads caused by rotating masses. The weight of the masses can be assessed very accurately, and with modern equipment the frequency can be set with a 0.001 Hz resolution. Resuming, both loading amplitude and frequency are known within strict limits. The maximum error is not more than 1%.

Measurements of Voorhof II were recorded during moderate storms. The wind loads on the building are estimated according to the theory of along-wind response as presented in Chapter 2. This procedure introduces many uncertainties in the assessment of the dynamic loads on the structure.

The mean wind speed that is used in the reduced spectrum and the aerodynamic admittance function is based on data of a (close) wind station. In general, the conditions of the surroundings differ from the conditions at the station. Therefore, the wind station data is transformed with the help of the aerodynamic roughness and shear velocity. This transformation process is a source of deviation. For example, typical values of the aerodynamic roughness in urban environments are 0.7 to 2.0 m. Transformation of the annual return wind speed at a typical Dutch wind station using 0.7 m for the aerodynamic roughness will yield a shear velocity of 2.32 m/s (see Appendix A). If the aerodynamic roughness is 2.0 m, a shear velocity of 2.93 m/s is found. Data point 'different terrain characteristics' in Figure 4.19 shows the combined effect on the acceleration. In this case a deviation of about 35 % due to uncertainties in terrain characteristics is found.

Once the wind speed and its associated parameters have been determined, the next step is to derive the wind load spectrum from the spectrum of the speed fluctuations and the aerodynamic admittance or coherence of the pressures. Although the theory of along-wind response nowadays is well established, no agreement on the spectrum or the coherence function is achieved yet. Because of this disagreement, deviation in the response of about 35% and 50% due to differences in spectrum and aerodynamic admittance respectively have been found in section 4.3. These deviations are illustrated in Figure 4.19.

In conclusion, it is not unreasonable to state that the most common way to model the along-wind load can yield up to 100% variation in the response of a building.

4.6.2 Building parameters

The effects of the structural parameters, namely stiffness, mass and damping, on human comfort are also shown in Figure 4.19. The stiffness and the damping ratio exhibit the largest uncertainties. The contribution of non-structural elements to the stiffness is in general neglected. However, in serviceability limit states these elements do add to the stiffness. For example, the measurements on the Cardington building reveal that the stiffness of non-structural elements can be more than twice the stiffness provided by the structural system. The

Wind-induced Dynamic Behaviour of Tall Buildings - 55
effect on the estimation of human comfort is illustrated in Figure 4.19 (data point ‘inclusion of non-structural stiffness’). A non-structural stiffness that equals the stiffness of the structural system is assumed. The rms acceleration will drop, but the human comfort will stay at the same level, since the natural frequency will increase.

The damping ratio is often assumed to be a constant, like 1% of critical for steel buildings. However, non-linear mechanisms are involved in the energy dissipation in buildings, for example, due to hysteresis in non-structural elements. As a consequence, measurements of the damping ratio in a steel building often indicate 2% of critical or more. It is clear from Figure 4.19, in which the default situation has 2% damping and two data points for 1 and 3% damping are plotted, that the difference between actual and calculated response will be large accordingly.

Finally, there are several contributions to the mass: the structural system, the floors, the facade, finishes and live load. The live load is an important source of uncertainty in the determination of the mass, since there is no evidence that the full live load will be present. As an illustration, data point ‘no provision for live load’ shows what the effect of an ‘empty’ building on the human comfort is. It seems that it is unconservative to take account of the added mass by live load in a dynamic analysis.

4.7 Summary

In this chapter the sensitivity of the acceleration to parameters like wind velocity, damping, mass and stiffness has been studied. Non-linear mechanisms that influence the damping capacity have been taken into account.

The following can be learned from the results of the sensitivity analyses:
- The differences in definition of both aerodynamic admittance function and wind spectrum have a significant influence on the response. Deviations caused by wind spectra are especially prone at greater heights, say 100 m and above.
- There is a distinct difference between the dynamic behaviour of ‘low’ and ‘high’ buildings. The boundary lies roughly in the interval $50 < h < 100$ m, depending on the building’s configuration.
- Increasing the stiffness of tall buildings is an ineffective measure to increase human comfort. Consequently, drift limits do not cover human comfort issues, although this is a widespread belief.
- The effective mass is an efficient tool to improve the dynamic performance of a tall building. Human comfort can be improved significantly.
- Damping is always beneficial in reducing both dynamic displacements and accelerations.
- Human comfort issues are important for structures with high aspect ratio. Even very tall buildings can have low acceleration levels when the aspect ratio is low (less than about 5).
- Uncertainties in the determination of the characteristics that describe turbulent winds, namely the parameters aerodynamic roughness and shear velocity, might lead to very large deviations in the response. Therefore, a careful determination of these parameters is required in the design process.
- It may be unconservative to add the live load to the total mass of a tall building: extra mass has a beneficial effect on human comfort.
5 CASE-STUDIES

5.1 The Cardington building

As a building is erected, structural elements are added that increase the stiffness and add to the mass. Floors are built, which provide stiff horizontal planes in the building. Moreover, the floors provide a large part of the total mass. The facade is formed from panels or walls. They add to the mass and stiffness, although the latter is normally not taken into account.

All these actions in the building process influence the stiffness and/or mass. The Cardington test building, built in 1993 and 1994, has provided a unique opportunity to quantify these influences. This building has been monitored during the building process by means of dynamic tests. At five stages during the erection the natural frequencies of the fundamental modes were determined. The natural frequencies were predicted by the estimate that was presented in section 2.4. Section 5.1.1 will briefly introduce the Cardington building. Section 5.1.2 summarises the test events and procedures. The comparison of predicted and measured fundamental frequencies is treated in section 5.1.3.

Two different testing methods have been applied to identify the natural frequencies. Comparison of the two sets of results indicated significant non-linear effects. See section 5.1.4.

Subsequently, the test results were used to determine the parameters of the non-linear model that was presented in section 2.3.3. The determination of the model parameters and predictions with the model will be discussed in section 5.1.5.

5.1.1 Description of the building

The Building Research Establishment (BRE)\(^1\) has been developing a Large Building Test Facility at Cardington, about 80 km north of London. The Facility is housed in an old airship hangar that itself is 55 m high, 250 m long and 70 m wide. The hangar provides a protected environment for full-scale testing. Currently, it houses an eight-storey steel-framed structure with a height of 31.5 m. Figure 5.2 gives an impression of the dimensions of the space covered by the Facility.

The test building is a typical commercially designed and constructed office block and satisfies the requirements of Eurocodes 2, 3 and 4 [EUR94b, EUR94c, EUR94d]. Stability of the structure is established through bracing in two orthogonal directions. Floors are composite: concrete poured on steel decking. Figure 5.3 shows a view from the West on the building. The bracing can be recognised in the West facade. Both East and West facades have brickwork walls over the full height. These walls were not designed to provide resistance to horizontal loads. The North and South facades are semi-open as is shown in Figure 5.4. The parapets are erected to a quarter of the floor-to-floor height.

Figure 5.1 shows a typical floor plan of the Cardington structure with the exact location and dimensions of the steel bracing, facade and partition walls. The numbers in Figure 5.1 refer to the type of (non-structural) wall element (see Table 5.1).

A more comprehensive description of the eight-storey test building and the test program is given in [ARM94].

\(^1\) The Building Research Establishment is a British organisation for applied research related to building and construction.
Figure 5.1  Typical floor plan of the Cardington building. Numbers refer to type of (non-structural) wall (see Table 5.1). Walls in EW direction are omitted.

Table 5.1  Wall element properties.

<table>
<thead>
<tr>
<th>type</th>
<th>length (m)</th>
<th>thickness (m)</th>
<th>material</th>
<th>represents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.0</td>
<td>0.14</td>
<td>brick</td>
<td>facade</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
<td>0.14</td>
<td>brick</td>
<td>facade</td>
</tr>
<tr>
<td>3</td>
<td>7.0</td>
<td>0.03</td>
<td>gypsum</td>
<td>partition</td>
</tr>
<tr>
<td>4</td>
<td>3.0</td>
<td>0.03</td>
<td>gypsum</td>
<td>partition</td>
</tr>
</tbody>
</table>

5.1.2  Test conditions

Construction of the experimental building in Cardington started in January 1993 and the building process ended in June 1994. In that period five dynamic tests were conducted at typical stages in the building process. See Table 5.2.

Table 5.2  Test conditions.

<table>
<thead>
<tr>
<th>test</th>
<th>date</th>
<th>stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25 February 1993</td>
<td>bare frame erected</td>
</tr>
<tr>
<td>2</td>
<td>22 March 1993</td>
<td>frame plus floor decking</td>
</tr>
<tr>
<td>3</td>
<td>25 November 93</td>
<td>all composite floors cast</td>
</tr>
<tr>
<td>4</td>
<td>22 February 1994</td>
<td>facades erected</td>
</tr>
<tr>
<td>5</td>
<td>1-3 June 1994</td>
<td>building finished</td>
</tr>
</tbody>
</table>

Note that the stage ‘bare frame’ is somewhat hypothetical, since the steel decking was assembled while erecting the frame. When the first test was conducted, decking on the lower four floors was present.
Figure 5.2  Hangar interior; experimental building is located at the end.

The last test was performed when the building was completed, including provision for the live load using uniformly distributed 1100 kg sand bags. A complete description of the state of the building during the tests is given in [OOS94b].

The first four sets of measurements were performed on low amplitude motion primarily using a laser system. With this system the ambient response of the structure was monitored. The resulting velocity-time history was transformed using a fast Fourier transform to determine the natural frequencies of the building.

The response was measured by accelerometers. Test data was normalised by converting the measured accelerations to the equivalent displacement and then dividing this displacement by the applied force (load is proportional to the square of the excitation frequency). The result is a transfer function $H_{F,u} (u = H_{F,u}F)$. Later in this section (see Figure 5.11 and Figure 5.13) measured transfer functions will be compared to calculated transfer functions.
At stages 2, 3 and 5, TNO Building and Construction Research\(^2\) conducted low-impact tests on the Cardington building. A hammer blow on the top floor was used for test 2 and 3. For the last test a weight of 50 kg, which was hitting one of the central columns of the structural frame, was used as impact load. Again, the response was measured with accelerometers. The impact itself was measured by means of a load cell. The raw data from the impact tests were analysed by means of fast Fourier transform analysis.

The results from TNO have been compared to the BRE results. Some notable differences appeared, on which section 5.1.4 will take a closer look.

### 5.1.3 Comparison of measured and predicted frequencies

For each stage, the natural frequencies of the fundamental modes have been predicted by the estimate that was presented in section 2.4. The drift limit was 1/1000 of the height. The design wind pressure is 1.2 kN/m\(^2\), based on a mean wind speed at the top of 28 m/s. Accordingly, the minimum stiffness in x-direction is 20 MN/m and in y-direction 7 MN/m. The definition of the x-y-axes is given in Figure 5.1.

The total mass is estimated from design figures of three contributions: frame, floor and facade. The results are collected in Table 5.3. Note that the mass of the floor in test 5 includes the provision for the live load, which adds 1.56 million kg to the total weight of the building. Subsequently, the modal mass is computed based on a bending mode shape.

<table>
<thead>
<tr>
<th>test</th>
<th>structure</th>
<th>floor</th>
<th>facade</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.32</td>
<td>0.03</td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>2</td>
<td>0.32</td>
<td>0.06</td>
<td></td>
<td>0.38</td>
</tr>
<tr>
<td>3</td>
<td>0.32</td>
<td>2.12</td>
<td></td>
<td>2.44</td>
</tr>
<tr>
<td>4</td>
<td>0.32</td>
<td>2.12</td>
<td>0.60</td>
<td>3.04</td>
</tr>
<tr>
<td>5</td>
<td>0.32</td>
<td>3.68</td>
<td>0.60</td>
<td>4.60</td>
</tr>
</tbody>
</table>

The estimate of the stiffness, combined with the estimated modal mass, yields a predicted natural frequency. In Table 5.4 the estimates are summarised and compared with the measured frequencies. The measured frequencies are all from low excitation level tests, for reasons that will be outlined in section 5.1.4. A graphical comparison is given in Figure 5.5 where the ratios of predicted to measured natural frequencies are plotted for the fundamental frequencies.

In the early tests there is a large difference between prediction and measurement. The estimate is based on the clamped single column model. The bare frame (test 1) and the bare frame plus decking can not be modelled as a single bending column. So, large errors in the estimation may occur.

During test 3 the floors were finished and were acting as stiff, horizontal planes. The clamped column model now applies. Indeed, test 3 shows a predicted value in x-direction that is close to the measured value. The prediction would have been even closer to reality if the mode shape was taken linear, which was measured during that test.

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\(^2\) TNO Building and Construction Research is a Dutch organisation for research and development in construction and engineering industry.

*Wind-induced Dynamic Behaviour of Tall Buildings - 60*
The prediction and measurement in the other translational mode also agree quite well. The predicted torsional natural frequency deviates significantly from the measured frequency, probably because the torsional stiffness provided by the bracing was overestimated.

Table 5.4  Measured and predicted natural frequencies (in Hz). All figures are from low-level tests. Test 1, 2, 3 and 4: BRE figures. Test 5: TNO figures.

<table>
<thead>
<tr>
<th>test</th>
<th>measured</th>
<th>predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_x$</td>
<td>$f_y$</td>
</tr>
<tr>
<td>1</td>
<td>1.22</td>
<td>0.98</td>
</tr>
<tr>
<td>2</td>
<td>1.55</td>
<td>1.31</td>
</tr>
<tr>
<td>3</td>
<td>0.83</td>
<td>0.69</td>
</tr>
<tr>
<td>4</td>
<td>1.31</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>0.96</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Figure 5.5  Ratio of predicted to measured natural frequencies.

Test 4 results show that all predicted values are lower than the measured equivalents. One may recognise the stiffening of the structure due to addition of the walls by comparing the measured values of test 3 and 4. The natural frequencies that were found in test 5 confirm this effect. Again, all predictions are below the actual values.

The role of the non-structural elements, here especially the facade walls, is significant. The test data enable us to quantify the effect of the non-structural elements on the stiffness. At the time of the third test all stiffness was provided by the structural system. Measurements of the mode shape indicated a fairly straight mode of vibration for the two orthogonal translation modes. See Figure 5.6. Thus, a better estimate of the modal mass can be calculated (original prediction assumed a pure bending mode shape).

Combination of the recalculated effective mass and the measured natural frequency yields 22 MN/m for the structural stiffness in $x$-mode. At the time of test 4 the E&W facade walls were
fully erected. The total stiffness can be determined to be 68 MN/m. Thus the extra stiffness provided by the facade walls is 46 MN/m.

Similarly, improved estimates of the \( y \) and torsional mode may be determined. In the \( y \)-mode the structural stiffness is 15 MN/m, the non-structural stiffness 7 MN/m. In torsion the structural stiffness is 6.5 GNm, the non-structural stiffness 27.5 GNm.

With these figures the natural frequencies of the fundamental modes of the completed building can be re-predicted: 1.06, 0.61 and 1.35 Hz for \( x \), \( y \) and torsional mode respectively. The resemblance with the measured values is remarkable.

![Diagram showing mode shapes and frequencies](image)

**Figure 5.6** Normalised mode shapes of the Cardington building. The fundamental mode shapes and the second mode shape (\( y_2 \)) in the \( y \)-direction are shown. The definition of the directions is given in Figure 5.1. Measured by BRE during the stage 5 test.

### 5.1.4 Comparison of two dynamic test methods

In section 5.1.2 two different testing methods were described that were both applied to the Cardington building. It has been suggested in [DEL95] that the differences in the identification of the natural frequencies between the testing methods are due to statistical variation. However, it might be argued that the differences find their origin in the non-linear dynamic behaviour of the building.

In Table 5.5 the fundamental frequencies of events 2, 3 and 5 according to BRE (exciters) and TNO (impact) are shown. The resolution is 0.0025 Hz and 0.012 Hz respectively.
The differences in test 2 and 3 are within the testing uncertainty interval (maximum error is 4 %). In test 5 there is a clear difference between the TNO and BRE results, especially for x-mode and torsion, where the differences are 16 and 30 % respectively. Such large differences cannot be explained from statistical variation. Remember that the BRE used a system of four vibrators capable of 10 kN peak-peak load (high amplitudes) in test 5, whereas ambient vibration (low amplitudes) was measured in tests 1 to 4. The BRE exciters impose top floor displacements in the order of millimeters. TNO used an impact to excite the structure. The corresponding displacement amplitudes are in the order of micrometers.

### Table 5.5  
**BRE versus TNO tests at Cardington. Natural frequencies of the fundamental modes (in Hz).**

<table>
<thead>
<tr>
<th>test</th>
<th>BRE measurements</th>
<th>TNO measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(f_x)</td>
<td>(f_y)</td>
</tr>
<tr>
<td>2</td>
<td>1.55</td>
<td>1.31</td>
</tr>
<tr>
<td>3</td>
<td>0.83</td>
<td>0.69</td>
</tr>
<tr>
<td>5</td>
<td>0.83</td>
<td>0.61</td>
</tr>
</tbody>
</table>

### Table 5.6  
**BRE forced vibration test in stage 5: change of (best-fit) dynamic properties during a decay of vibration. One block is 5.12 seconds for the x-mode and 10.24 seconds for the y-mode [ELL94].**

<table>
<thead>
<tr>
<th>Block</th>
<th>(u_{max}) (mm)</th>
<th>(f_x) (Hz)</th>
<th>(\zeta) (%)</th>
<th>Block</th>
<th>(u_{max}) (mm)</th>
<th>(f_y) (Hz)</th>
<th>(\zeta) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.79</td>
<td>0.794</td>
<td>3.67</td>
<td>1</td>
<td>1.18</td>
<td>0.610</td>
<td>2.40</td>
</tr>
<tr>
<td>2</td>
<td>0.78</td>
<td>0.837</td>
<td>3.08</td>
<td>2</td>
<td>0.81</td>
<td>0.623</td>
<td>N/A</td>
</tr>
<tr>
<td>3</td>
<td>0.32</td>
<td>0.881</td>
<td>2.00</td>
<td>3</td>
<td>0.54</td>
<td>0.623</td>
<td>N/A</td>
</tr>
<tr>
<td>4</td>
<td>0.17</td>
<td>0.896</td>
<td>1.50</td>
<td>4</td>
<td>0.38</td>
<td>0.635</td>
<td>N/A</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>0.27</td>
<td>0.647</td>
<td>N/A</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>0.21</td>
<td>0.647</td>
<td>N/A</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>0.17</td>
<td>0.647</td>
<td>N/A</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>0.15</td>
<td>0.659</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Moreover, the skewness in the transfer functions, see Figure 5.11 and Figure 5.13, indicates that the Cardington building exhibits non-linear dynamic behaviour. Another measurement technique that was performed on the completed building is the rundown method. While running in the resonant frequency of a certain mode, the exciters were suddenly stopped. The decay of vibration that results from the sudden stop of the exciter was a further indication of the non-linear behaviour. Decays were taken from the EW and NS mode. Both decays were split into small blocks of about 10 or 5 seconds (x-mode respectively y-mode). On each block a linear visco-elastic curve was fitted. In that way a representative amplitude and natural frequency were determined for each block. The results are collected in Table 5.6.

The NS decay drops very fast. Only the first four blocks could be analysed, below the 0.1 mm level there is too much noise. However, measurements of the ambient vibration of the x-mode gave a natural frequency of 0.93 Hz, which is close to 0.96 Hz based on the impact test. The y-mode decreases slowly, which enables us to study the increase in frequency with decreasing amplitude in greater detail. The last block in Table 5.6 suggests that at a very low excitation level one can expect 0.66 Hz, the value that TNO found.
The decrease of the natural frequency of the y-mode is rather small. This supports the concept to assign the non-linear behaviour to the presence of non-structural parts (see the next section), since there are only a few partition walls plus the parapets that can provide stiffness in the EW direction.

Resuming, there may be a significant deviation in the identification of dynamic properties when test methods, that differ in excitation level, are compared. Dynamic properties that have been reported in literature without a complete description of the test conditions should therefore be used with caution. This is especially true for ambient vibration measurements.

Besides, predictions of modal properties, like the predicted natural frequency that was presented in the previous section, should all be compared to test results in similar conditions. These conditions should preferably be low amplitude excitation, since non-linear effects are negligible then.

5.1.5 Non-linear approach

From the measurements taken at stages 3 and 4 it can be seen that the infill walls contribute a significant part of the stiffness of the whole building: although mass is added due to the inclusion of the facades, the natural frequency increases from stage 3 to 4. The effect is especially prone in the x-mode and the torsional mode. See the figures in Table 5.4. In this section a more detailed model of the walls will be considered to see how the non-linear effects noted in both frequency and damping can be represented.

In Chapter 2 the following non-linear equation of motion was presented:

\[ m \ddot{u} + k_u u + F_w(u, \dot{u}) = F(t) \]  (5.1)

In expression (5.1) \( F_w \) is the non-linear part of the equation. This force is assigned to the non-structural parts of the building. The following subsection shows how the information in the decays (see Table 5.6) was used to derive a force-displacement-relation. Subsequently, calculated behaviour is compared to measured behaviour.

5.1.5.1 Determination of force-displacement relation

The data of the decays, that are displayed in Table 5.6, show that the contribution of the non-structural elements to the total stiffness of the building is decreasing with growing displacement. The effective mass of the completed building can be determined from the mode shapes in combination with the total mass. Both x- and y-mode shapes are fairly linear, see Figure 5.6. Therefore, the effective mass ratio is 0.33 and the effective mass becomes a third of the total mass (4.60 Mkg, see Table 5.3).

The effective mass of 1.52 Mkg can be combined with the natural frequencies in Table 5.6 to determine the degrading stiffness as a function of the displacement. The ‘softening’ effect is assigned to the non-structural elements. The structural stiffness was determined in section 5.1.3 to be 22 and 15 MN/m for x- and y-mode respectively. Table 5.7 can now be compiled.

The normalised non-structural stiffness \( k_u/k_0 \) as a function of the peak displacement is plotted in Figure 5.7 for the fundamental modes in both x- and y-direction. The stiffness \( k_0 \) is the initial stiffness, i.e., the stiffness at very small amplitudes. There are more non-structural elements in the x-mode and subsequently the stiffness drops faster. It may be appreciated from the plot that the slope of both lines is getting smaller with increasing displacement.
Table 5.7  Non-structural stiffness in x-mode and y-mode based on the data in Table 5.6.

<table>
<thead>
<tr>
<th>x-mode</th>
<th>y-mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_{\text{max}}) (mm)</td>
<td>(u_{\text{max}}) (mm)</td>
</tr>
<tr>
<td>0.90</td>
<td>1.18</td>
</tr>
<tr>
<td>0.39</td>
<td>0.81</td>
</tr>
<tr>
<td>0.16</td>
<td>0.54</td>
</tr>
<tr>
<td>0.08</td>
<td>0.38</td>
</tr>
<tr>
<td>0</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 5.7  Values for the average non-structural stiffness, based on the data in Table 5.7.

Unfortunately, \(k_w\) in Table 5.7 is an effective stiffness, i.e., it is not directly related to the amplitude \(\hat{u}\), but an average with respect to time. For the determination of a force-displacement relation, the stiffness at a certain moment in time should be known. The data in Table 5.7 are inadequate to find the parameters of such a relation. Therefore, a \(F_u-u\) relation is postulated and by means of time analyses in the following subsections the validity of the postulation will be assessed.

The proposed force-displacement relation should satisfy some requirements. Figure 5.7 suggests that the non-structural stiffness (the first derivative of \(F_u-u\) with respect to \(u\)) is approximately inversely proportional to the square root of the displacement. Furthermore, the damping figures in Table 5.6 suggest that hysteresis or other displacement-dependent mechanisms are involved in the energy dissipation in the Cardington building.
The following force-displacement relation satisfies these requirements:

\( v > 0 \)

\[
\frac{F_u(u) - F_{\text{min}}}{F_p} = \sqrt{\frac{u - u_{\text{min}}}{u_p} + 1 - 1}
\]  

\( v < 0 \)

\[
\frac{F_u(u) - F_{\text{max}}}{-F_p} = \sqrt{\frac{-u + u_{\text{max}}}{u_p} + 1 - 1}
\]  

Figure 5.8 displays the postulated force-displacement relation loop and the definitions of the parameters that constitute equations (5.2) and (5.3). The following points can be distinguished:

- \((u_{\text{min}}, F_{\text{min}})\): the point where the force in the spring \(F_u\) and the displacement \(u\) reach their minima. The spring arrives at this point when the velocity switches from negative to positive.
- \((u_{\text{max}}, F_{\text{max}})\): the point where the force in the spring \(F_u\) and the displacement \(u\) reach their maxima. The spring arrives at this point when the velocity switches from positive to negative.

The force \(F_p\) and the displacement \(u_p\) form a characteristic point in the square root function. They determine the point where the stiffness \(k_u\) is only half the value at the beginning of a loop \((k_0)\) as is illustrated in Figure 5.8. It can be derived from equations (5.2) and (5.3) that \(F_p = 2k_0u_p\).

![Figure 5.8 Non-linear force-displacement relation.](image)

The stiffness \(k_0\) is the stiffness at zero displacement, 30 and 11 MN/m (see Table 5.7) for \(x\)- and \(y\)-mode respectively. After some elaboration, \(u_p = 0.22\) mm and \(u_p = 0.70\) mm for \(x\)- and \(y\)-mode respectively are found.

The calculations that will follow, will give a further indication of the quality of the square-root force-displacement relation.
5.1.5.2 EW mode

The tests on the completed Cardington building have been simulated. Two transfer functions have been calculated with equation (5.1). The external force $F(t)$ is a harmonic load with variable angular frequency and an amplitude of 1500 or 15 N. The corresponding transfer functions are displayed in Figure 5.9. The pronounced peak is the 15 N transfer function and the flattened line is the 1500 N transfer function.

The 1500 N excitation equals the test conditions during the (BRE) forced vibration at stage 5 (see section 5.1.2). The resonant peak is identified at 0.60 Hz, which equals the measured natural frequency. The steady-state amplitude at that frequency was measured to be 1.18 mm. According to the non-linear model with the square root force-displacement relation that displacement is 1.34 mm, a 14% overshoot. The damping ratio is calculated from equation (2.48): 2.5%, which is close to the 2.4% of the best fit visco-elastic curve. Note that the visco-elastic figure is based on a best-fit for the complete measured sweep, whereas the 2.5% is a single point figure. Figure 5.10 shows how the equivalent damping ratio varies with frequency. Note that there is quite a variance, even in the small region around the resonant frequency.

Figure 5.11 compares the visco-elastic best-fit, the square root non-linear model and the measured normalised displacements. The overestimation of the displacements by the non-linear model is apparent. Note, however, that the shape of the measured transfer function and the transfer function based on the non-linear model agree well. This is where the linear visco-elastic model is doing less well.

The lowest excitation level in Figure 5.9 is 15 N. The resonant peak is then at 0.65 Hz. The according displacement is 0.07 mm, the damping ratio 0.40 %. TNO measured 0.04 mm amplitude at 0.66 Hz and 0.6 % damping ratio.

Figure 5.9 shows two typical non-linear phenomena:

- The transfer function flattens when the load level increases. At a very low load level, the behaviour is almost linear, which results in the distinct peak like in the 15 N transfer function. As the loads increase, the energy is spread over a wider range of frequencies, resulting in a less distinct peak in the transfer function.

- The skewness of the transfer function at the right hand of the resonant peak. This is typical hysteretic spring behaviour.

- A shift of the resonant frequency as the load level changes.

The latter effect has been known for a long time. Especially during an earthquake, the natural frequency can drop dramatically due to non-linear effects, which may then also include plasticity and significant soil-structure interaction effects. See, for example, reference [HAR75] which discussed the torsional response of a set of tall buildings in the Los Angeles area before and during an earthquake. In that paper it is was found that fundamental natural frequencies decreased to 60-70% of the value during ambient vibration.

Figure 5.10 shows the development of the damping ratio as the load level increases. Not surprisingly, the damping capacity increases as the loads increase. For the 15 N excitation, there is only little damping, which, moreover, is restricted to a small region around the resonant frequency.
Figure 5.9  Variation of the transfer function depending on loading level and frequency (EW mode).

Figure 5.10  The equivalent damping ratio as a function of loading level and frequency.
Figure 5.11 Comparison of two models to measurements, EW mode. Non-linear model is based on the force-displacement diagram in Figure 5.8. Loading amplitude 1500 N.

5.1.5.3 NS mode

Similar to the EW mode, the response in the NS mode has been calculated twice. Harmonic loads with varying frequency and an amplitude of 2700 N and 27 N respectively were applied. Again, the low level test showed a distinct peak in the transfer function. See Figure 5.12. The peak was found at 0.91 Hz with 0.034 mm displacement amplitude and 0.80% equivalent damping ratio. Test results were 0.93 Hz, 0.04 mm and 0.6 %.

Note that there is a difference between the EW and NS mode as a comparison of Figure 5.9 and Figure 5.12 reveals: the peaks of the EW transfer functions are at a higher level: $4.5 \times 10^{-6}$ /$1.0 \times 10^{-6}$ versus $1.2 \times 10^{-6}$/$0.3 \times 10^{-6}$. This means that the non-linear effects in the NS mode are stronger than in the EW mode. The difference in behaviour of the two modes is also expressed in the values of $u_p$ (a small $u_p$ indicates a rapid decrease of stiffness when displacements increase): $u_p = 0.22$ mm and $u_p = 0.70$ mm for the NS and EW mode respectively.

The 2700 N calculation\(^3\) conforms to the forced vibration test conditions. In Figure 5.13 it is compared to the measured data and the linear best-fit curve. The resonant peak is at 0.79 Hz with a displacement amplitude of 0.80 mm. Measured was 0.79 Hz and 0.90 mm respectively. The difference in the displacement is due to an overestimate of the energy dissipation: the equivalent damping ratio at resonance is 4.3 %.

The shapes of the measured and the calculated transfer function are very similar as in the EW mode. However, contrary to the EW mode, the response is now underestimated.

\(^3\) Actually, the load in the NS calculation was based on this equation: amplitude $F = 4136 \times f^2$, which resembles the loads imposed by the rotating masses of the exciters. At 0.79 Hz, the load is 2700 N.
Figure 5.12  Variation of the transfer function depending on loading level and frequency (NS mode).

Figure 5.13  Comparison of two models to measurements, NS mode. Non-linear model is based on the force-displacement diagram in Figure 5.8. Loading amplitude 2700 N.
5.1.5.4 Concluding remarks

The square root force-displacement relation appears to describe the observed non-linear effects quite accurately both qualitatively and quantitatively. Response in the EW mode is overestimated, in NS mode underestimated. These figures seem to suggest that a further calibration of the force-displacement requires a variation in the exponent. Probably, the $F_w-u$ relation in the EW mode needs a steeper function, say force proportional to $u^{0.7}$. On the other hand, in the NS mode $F_w$ proportional to $u^{0.4}$ is probably a better fit. At this moment adequate test data to verify these suggested exponents in the $F_w-u$ relation is not available. A comprehensive test program on various types of buildings could provide the information to come to a better understanding of the effects of non-structural parts on the dynamic behaviour of buildings. In this case-study many test data were available and even then the determination of $k_0$ and $u_p$ was cumbersome. Therefore, in the testing procedures specific attention has to be paid to load levels, displacements and actual behaviour of non-structural elements. The results of this test program should be ‘average’ values for the parameters $k_0$ and $u_p$, such that a reliable, realistic assessment of the dynamic behaviour will be possible in the design of tall buildings.

5.1.6 The influence of non-structural elements on the dynamic behaviour

The hysteretic spring that was presented in section 5.1.5.1, is characterised by the parameters $k_0$ and $u_p$. This spring can be used to study the effects that non-structural elements have on the overall behaviour of a tall building.

A small sensitivity analysis has been conducted to assess these effects. The stiffness $k_0$ was varied from $0.5k_s$ to $2.5k_s$ with steps of $0.5k_s$, where $k_s$ is the stiffness of the steel frame (15 MN/m for the EW mode), while the characteristic displacement $u_p$ is kept constant at 1 mm. For each value of $k_0$ a transfer function for the EW mode of the Cardington building was determined based on a harmonic loading with amplitude of 500 N.

Figure 5.14 shows the natural frequency for several configurations of $k_0$ and $k_s$. For completeness, the case $k_0 = 0$ has been added. The natural frequency shifts to higher values as the relative stiffness of the non-structural elements (the ratio $k_0$ to $k_s$) grows.

Figure 5.15 displays the trends in the peak resonant displacement and the equivalent damping ratio at resonance as determined from the transfer function. The plot is scaled to the displacement amplitude and damping ratio for $k_0 = k_s$. Most striking effect is that for $k_0 > 1.5k_s$ there is hardly any gain in the equivalent damping ratio. A closer look to equation (2.48) explains this effect: the growth in the natural angular frequency is, apparently, of the same magnitude as the increase in the loss in energy due to hysteresis. Note that the resonant displacement cannot be determined for $k_0 = 0$, since there is no damping in that case (all damping in the non-linear model is provided by hysteresis).
Figure 5.14 The shifting resonant frequency due to non-structural elements.

Figure 5.15 The effect of non-structural elements on the damping ratio and resonant response.

5.1.7 Random vibration

The Cardington building has been exposed to well-defined, harmonic loads. The non-linear model enables us to see how it would respond to random wind loads. The fluctuating wind loads are conveniently represented in a spectral form as was shown in Chapter 2. The assessment of the non-linear dynamic behaviour is a time domain approach.

Thus, the spectra have to be transformed. First, we return to the definition of the wind load spectrum. Assume that the fluctuating wind loads can be written as sum of \( n \) harmonic loads

\[
F(t) = \sum_{j=1}^{n} F_j \sin(\omega_j t - \theta_j)
\]  

(5.4)

where \( \theta_j \) is a random phase angle.

Let the angular frequencies vary with a constant interval \( \Delta \omega = \omega_{j+1} - \omega_j \). The spectral amplitude of the \( j \)-th component of the Fourier transform is then [KLA91]
\[ S_{ff}(\omega_j) = \frac{1}{2} \frac{\hat{F}_j^2}{\Delta \omega} \quad (5.5) \]

We can use equation (5.5) to perform a reverse Fourier transform. The spectrum \( S_{ff} \) is known as a function of the angular frequency. Any of the expressions in Chapter 2 may be used to represent the wind loads. For a time-domain analysis we can determine \( n \) harmonic loads according to equation (5.4) that represent the fluctuating wind load. The amplitude of the \( j \)-th component can be found from equation (5.5)

\[ \hat{F}_j = \sqrt{2S_{ff}(\omega_j)} \cdot \Delta \omega \quad (5.6) \]

The algorithm for the determination of the non-linear response that was presented in section 2.3.3 is slightly changed. Equation (2.23) becomes

\[ \dot{u}_t = \frac{1}{m} \left[ \sum_{j=1}^{n} [F_j(t)] - k_j u_t - F_u(u_t) \right] \quad (5.7) \]

The remainder of the analysis is analogous to the vibration due to a single harmonic force. Typical results for the EW mode are shown in Figure 5.16 and Figure 5.17. Figure 5.16 shows a time history of the displacements. The random character of the wind load is clearly recognisable from the response. Note that the predominant frequency is the natural frequency, about 0.60 Hz in this case.

The randomness of the loading is also clear from the energy dissipation as a function of the peak displacement in a cycle, see Figure 5.17. On the vertical axis is \( \Delta E_u \), the energy dissipation through hysteresis in one cycle. The measurements during 100 cycles are plotted with diamonds. The full line is calculated from a free vibration. This rather academic type of loading gives the trend through the scatter of points in Figure 5.17.

A Fourier analysis of the time history in Figure 5.16 yields a spectrum of the response. Figure 5.18 shows the reduced displacement spectrum. The spectrum confirms that the predominant frequency in the response is the natural frequency, since most of the energy in the spectrum is concentrated around 0.60 Hz.

![Figure 5.16](image)

Figure 5.16  A time history of the response on a random wind load.
Figure 5.17  Energy dissipation as a function of the amplitude. Dots are energy losses during random vibration. Full line is energy loss during a free vibration.

Figure 5.18  Reduced spectrum of the displacements based on the time trace in Figure 5.16.
5.2 The Voorhof II building

In 1966, two similar tall buildings, Voorhof I and II, were erected in Delft. Both buildings have been serving as a student dormitory since then. Voorhof II got a bad reputation due to its lack of serviceability. The sway of the building during storms was strongly felt by the occupants and cracks appeared in walls. Measurements of the accelerations at the top floors supported the occupants’ complaints, since the occurring accelerations exceeded the tolerance limits set up by ISO [ISO84].

In the following subsection a description of the building is given. The structural system and the building elements, such as partition walls, floors and facade, will be described. Subsequently, a list of deficiencies, based on a survey in Winter 1985, is given. Several measures to improve the performance were proposed. They are presented in 5.2.3. It was decided to stiffen the steel structure by the addition of four concrete walls, with their axis in the weak direction. The dynamic behaviour of the building was re-examined in 1994. Results of that re-examination are presented in 5.2.5. Finally, the effectiveness of the selected structural measure is assessed in subsection 5.2.6.

5.2.1 Description of the Voorhof II building

Voorhof II (1966) is a 17-storey student dormitory and is surrounded by similar buildings. Nowadays, the building is also known as the E. du Perron building. The quarter was built in the sixties to satisfy the growing demand for housing in Delft. The building is 51.3 m tall and has a rectangular plan with dimensions 80.8x14.2 m². Figure 5.19 displays a front view of the dormitory in its present state. An extensive description of the building and its history is given in [POL95]. Herein a brief introduction is given.

Figure 5.19 Front view of the Voorhof II building.
The structure originally consisted of steel frames in both directions. There are 20 4.0 m bays. In x-direction four frames are braced over the full width, see the plan in Figure 5.20. In the other direction six frames are braced spanning 2 bays. Floors are made of reinforced concrete, thus acting as stiff diaphragms.

![Plan of the building with main structural elements: thick lines are braced frames.](image)

The main non-structural elements are the partition walls and the facade elements. The partition walls are fabricated from three layers: insulation covered by two layers of cellular concrete. The east and west facades consist of cellular concrete walls with steel sheeting attached to it. The north and south facades consisted formerly of wooden frames, which were replaced by prefabricated cladding elements during the refurbishment.

### 5.2.2 Deficiencies due to sway of the building

Many deficiencies were reported in a survey commissioned by the ‘Stichting Delftse Studenten Huisvesting’ in Winter 1985 [SDH85]. This led, together with a major change in living habits in the student community, to the refurbishment of the student dormitory. The following problems were reported and are all related to the sway of the building:

- All partition walls were cracked.
- On several locations partition walls had moved relative to adjacent walls.
- Tiles had cracked or came loose.
- The connections between partition walls and window frames were poor.
- Several partition walls contained horizontal cracks, causing them to stand loose.
- Doors jammed frequently.
- A crack along the longitudinal axis appeared in the floor finish.
- Some of the bathrooms were leaking.
- Occupants on the top floors complained of nausea.

The latter complaint was supported by measurements of the dynamic response to a storm on 14 January 1986. The measurements were conducted in the afternoon by Van Dorsser bv. From 15.00 to 17.00, the response was recorded by means of accelerometers, which were placed as shown in Figure 5.21 [DOR86]. A detailed description of the test procedure is given in [POL95].

The hourly mean wind speed at the top, based on data from the nearby Rotterdam Airport, was 14.3 m/s. The storm reached its peak at about 16.00 hours. At that time, the 10 minute mean wind speed was 18.3 m/s.

The raw data were analysed by fast Fourier transform [DOR86]. Unfortunately, most of the response power spectra that were calculated were rather rough due to a lack of resolution.
However, there were a few spectra of adequate resolution to identify the lowest natural frequency at 0.63 Hz. Other resonant peaks appeared at 2.28 Hz and 4.42 Hz. The phase difference between the recordings of the accelerometers at the east-side and west-side of the building identified a torsional vibration in the frequency range 0.64 to 0.75 Hz. Mode shapes were not presented in [DOR86].

From the spectra the rms acceleration can be determined. Regrettably, an erroneous procedure was used in this transformation process. The raw data are not available anymore, so the values that were stated in report [DOR86] could not be validated and, therefore, have not been used in this thesis.

![Diagram of building plan](image)

**Figure 5.21** Plan of the building with position and direction of accelerometer 1 and 3

Fortunately, time-traces of the response during the peak in the storm were available. The highest peak in 10 minutes was 0.07 m/s². The peak factor for \( f_0 = 0.63 \) Hz and \( T = 600 \) s is 3.61. From these figures the rms acceleration due to a storm with a mean wind speed of 18.3 m/s at the top of the building can be determined to be 0.019 m/s². This number has been checked with a rough calculation based on the inaccurate spectra that were available. This calculation indicates a rms acceleration in the range 0.01-0.03 m/s². So, 0.019 m/s² might be considered a reasonable value for the rms acceleration during the test. Due to the poor resolution of the spectra, determination of the damping ratio from these plots is virtually impossible.

### 5.2.3 Improving the dynamic behaviour

Following the decision to refurbish the building, Van Dorsser bv in The Hague [DOR86] conducted a study to improve the dynamic behaviour of the building. Six alternatives were developed, based on the measured response in the first mode.

1. **The removal of several top-floors.** There are two advantages to this alternative. Firstly, the total wind load will be lower. Secondly, the natural frequency will increase due to the decrease of mass and height. Consequently, the deflection and acceleration at the top will decrease.

2. **The addition of concrete walls at the east- and west-side of the building.** For this purpose a new foundation should be built to carry the extra loads.

3. **The addition of two external concrete cores.** These cores may be placed near the existing stair cases, as shown in Figure 5.22. This alternative also requires an extra foundation.

4. **Deepening the steel frames.** The result will be that diagonals will separate the balcony into bay-widths of 4 m. This alternative does not reduce the acceleration significantly.

5. **Active damping.** A dynamic damper placed on the top-floor of the building can effectively reduce the sway of the building. The applied mass per frame will be maximum 15000 kg per frame, according to [DOR86].
6. **Stiffening the four existing steel frames by addition of four concrete walls.** The walls are placed directly next to the braced frames. The extra foundation can be placed within the existing building or partly outside the building.

![Plan of the building with additional cores (the grey areas)](image)

Figure 5.22 Plan of the building with additional cores (the grey areas).

The effects of alternatives 1, 2, 3 and 6 on the dynamic behaviour of the building have been evaluated with SkyDyFe. The rms acceleration is calculated according to the requirements of ISO-criterion ISO 6897 [ISO84], i.e., wind loads are based on a five-year return period wind speed. The calculated responses are summarised in Table 5.8. A graphical comparison with the criterion in ISO 6897 is displayed in Figure 5.23. The plot shows that most of the alternatives perform rather poorly. Only removal of the top 5 floors or the addition of two external cores (both alternatives are not shown in the graph) mitigate the building’s behaviour into a region of low, well-accepted acceleration levels.

**Table 5.8** Effects of alternatives on the dynamic behaviour. The effective mass is based on a linear fundamental mode shape.

<table>
<thead>
<tr>
<th>alternatives</th>
<th>$k_c$ (MN/m)</th>
<th>$m_c$ (Mkg)</th>
<th>$\zeta$ (%)</th>
<th>$f_c$ (Hz)</th>
<th>$\sigma_a$ (mm/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Removing top floors</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17 floors (actual situation)</td>
<td>56</td>
<td>3.6</td>
<td>0.95</td>
<td>0.63</td>
<td>47</td>
</tr>
<tr>
<td>14 floors</td>
<td>119</td>
<td>3.0</td>
<td>1.42</td>
<td>1.00</td>
<td>22</td>
</tr>
<tr>
<td>12 floors</td>
<td>189</td>
<td>2.6</td>
<td>1.54</td>
<td>1.37</td>
<td>15</td>
</tr>
<tr>
<td>Addition of two walls</td>
<td>102</td>
<td>3.9</td>
<td>1.42</td>
<td>0.81</td>
<td>26</td>
</tr>
<tr>
<td>Addition of two cores</td>
<td>680</td>
<td>4.1</td>
<td>2.04</td>
<td>2.04</td>
<td>7</td>
</tr>
<tr>
<td>Stiffening of steel bracing</td>
<td>111</td>
<td>4.1</td>
<td>1.38</td>
<td>0.83</td>
<td>25</td>
</tr>
</tbody>
</table>

It is important to note that the calculations in [DOR86] were based on the measured response as described in the previous subsection. The quality of the measurements was poor and conclusions that are drawn solely on these results should be considered with caution.

The cheapest alternative, stiffening the four existing steel frames, was decided to improve the serviceability of the building. An important advantage of this solution was that the east- and west-facade could remain the same, thus keeping the building weather-tight during the refurbishment. According to Figure 5.23, this alternative provides just satisfactory behaviour, as is supported by the measurements after refurbishment. In [DOR86] it was already suggested that most probably occupants were still able to strongly perceive motion during severe storms when this alternative would be selected.

It has already been said that the test results were inaccurate and subsequently there are uncertainties in the calculation results that are presented in Table 5.8. Damping figures could not be identified with confidence. Therefore, the damping ratio that was used in the
calculations was estimated with equation (2.44). More on the uncertainties in the response assessment procedure may be found in section 5.2.4.

![Graph showing ISO criterion and proposed alternatives](image)

Figure 5.23 Proposed alternatives versus ISO-criterion. Alternatives with natural frequencies above 1 Hz are not displayed.

### 5.2.4 Comparison of measured and predicted response

In this section the actual behaviour is compared to the (SkyDyFe) predicted response. The hourly mean wind speed during the tests was 14.3 m/s and the terrain characteristics are \( z_0 = 1.0 \text{ m and } d_0 = 10 \text{ m} \). Accordingly, a rms response of 7.2 mm/s\(^2\) is found, which is not close to the 19 mm/s\(^2\) that was reported in subsection 5.2.2.

It has already been pointed out that the resolution of the tests was low. Therefore, there are some inherent uncertainties, like the actual damping ratio and the value of the rms acceleration. It was reported in [DOR86] that the maximum peak acceleration of 70 mm/s\(^2\) occurred during a peak in the storm, characterised by a 10 minute mean wind speed of 18.3 m/s at the top. If this wind speed is used, a rms acceleration of 18 mm/s\(^2\) is calculated, with a damping ratio of 0.77 %.

In the previous chapter the determination of the terrain characteristics has been identified as a process that is a source of deviation in the response. For an urban environment the aerodynamic roughness can be in the range 0.7-2.0 m. The calculations that are summarised in Table 5.8 used \( z_0 = 1.0 \text{ m, } d_0 = 10 \text{ m and } v_\infty = 1.54 \text{ m/s} \). If \( z_0 = 0.7 \text{ m} \) had been used, the response would have dropped to 90% of the originally calculated value, according to the proportionality in section 4.3.3. A higher roughness means more turbulence. Along-wind response is dominated by buffeting due to turbulent fluctuations. Thus, the response would have increased to 135% of the originally calculated value if \( z_0 = 2.0 \text{ m} \) would have been used.

The other estimated quantity is the average height of the surrounding buildings \( d_0 \). This parameter is normally estimated 'on the eye' and large errors may be expected. A value of 10

*Wind-induced Dynamic Behaviour of Tall Buildings* - 79
m was used in the calculations. It is interesting to see what the response is when the actual value is 50 or 200% of the value that was used. For \( d_0 = 5 \text{ m} \) the response grows to 108%. An average building height of 20 m yields a drop to 83%.

Of course, the variation in building properties and loading data has a major influence on the acceleration. In fact, there is only one property that is known with great confidence and that is the lowest natural frequency. The mass is calculated from structural drawings. There are inherent uncertainties in the mass calculation, especially due to the assumptions on the live load, that are not verified. It seems to be reasonable to say that a ± 20% deviation in the total mass can occur. Since the natural frequency was determined accurately, there is also a ± 20% deviation in the stiffness. If the mass decreases 20% the acceleration therefore grows to 125%. In the other hand, a 20% increase let the acceleration drop to 85% of the originally calculated value.

Besides errors in the calculation, there is another error source in the determination of the effective mass and that is the estimate of the mode shape. It is normally assumed that the mode shape of tall buildings can be approximated by a straight line. If the mode shape was pure bending, the response would be slightly higher. However, if the mode shape was pure shear the response would have been 83% of the level for a linear mode shape.

Another important parameter is the damping. In section 5.2.2 it appeared that the damping could not be determined from the test results. Comparison of calculated and measured response should indicate a reasonable value for the damping. However, there are so many uncertainties and related deviations in the response that a reliable value for the damping of this building in its lowest mode of vibration can not be set. Probably the damping was somewhere between 0.5 and 1.0% at the time of the test. Using one of the boundary values causes a variation in response of 40%.

Resuming, there are too many uncertainties in the assessment of the Voorhof II behaviour to be able to explain why there is a misfit between calculated and measured response.

5.2.5 Dynamic behaviour after refurbishment

As described earlier, tenants complained for a number of years about the serviceability of the building. It is therefore interesting to see if the building after refurbishment has improved its performance. On 31 October 1994, measurements were taken by Pols and Van Zoest. During the tests the hourly mean wind speed at the top was 11 m/s. An extensive description of the test procedure and the equipment is given in [POL95].

The building was equipped with 3 accelerometers, which measured the horizontal accelerations of the building at the 17th floor. The accelerometers were placed similar to the previous tests in 1986 (see Figure 5.21).

Figure 5.24 shows reduced spectra that were derived from the response measured by accelerometers 1 and 3. The position of the accelerometers in the plan can be appreciated from Figure 5.21. The fundamental natural frequency of the NS mode has increased from 0.63 Hz to 0.85 Hz. In Figure 5.25 the according mode shape in plan is shown.

Remarkable is the occurrence of a resonance frequency of 0.77 Hz in the accelerometer 3 spectrum. The top mode shape in Figure 5.25 suggests that this is the lowest torsional natural frequency. Note that both modes in Figure 5.25 exhibit some coupling, which indicates that the centre of inertia is close to the east side of the building.
Figure 5.24  Power spectra of accelerometer at east (nr. 1) and west (nr. 3) side [POL95].

Again, the raw data was analysed by fast Fourier Transform. Now, the resolution is adequate and an accurate result can be obtained. A rms acceleration of 1.1 mm/s² is found. Retrieval of the damping ratio appeared to be difficult, since the resolution of the spectra did not meet the requirements for a reliable use of the half power bandwidth method as a damping predictor (a discussion on disadvantages of the half power bandwidth method as predictor of the damping ratio may be found in [JE A 81]). The spectra suggest values between 1 and 2.5% of the critical damping. For reasons that were outlined in the previous subsection, it is not possible to improve the estimate of the damping by comparison of the measured response with the predictions.
The effective stiffnesses are summarised in Table 5.9. The effective stiffness of the bare frame was determined from a 2D finite element analysis [POL95]. The stiffness of the whole building is derived from the test data.

<table>
<thead>
<tr>
<th></th>
<th>before (MN/m)</th>
<th>after (MN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bare structure</td>
<td>28</td>
<td>80</td>
</tr>
<tr>
<td>non-structural</td>
<td>28</td>
<td>35</td>
</tr>
<tr>
<td>contribution</td>
<td>28</td>
<td>35</td>
</tr>
<tr>
<td>whole building</td>
<td>56</td>
<td>115</td>
</tr>
</tbody>
</table>

Wind-induced Dynamic Behaviour of Tall Buildings - 82
The main objective of the refurbishment was to increase the stiffness of the structure. From Table 5.9 it may be appreciated that the stiffness has increased to more than 200% of the original value. Nevertheless, the increase in human comfort is small. The second effect of the refurbishment, a 10% increase in the mass is relatively more effective, as is illustrated in Figure 5.26.

The effect of the refurbishment on the energy dissipation has been estimated by using a higher \( C_e \)-factor in damping predictor (2.44). Before refurbishment most partition walls were loose and could not contribute to the energy dissipation by friction. Accordingly, the factor \( C_e = 10 \). After refurbishment \( C_e = 75 \) is assumed, since the non-structural elements can participate in the energy dissipation. This beneficial effect on the human comfort is apparent from Figure 5.26.

In conclusion, there is no rationale for the belief that deflection criteria guarantee adequate dynamic performance. This case-study confirms the sensitivity analyses in chapter 4. The refurbishment of Voorhof II has not been very effective in increasing the comfort of the occupants. This conclusion is supported by the occupants’ perception: people do still perceive motion even during moderate storms.

![Diagram](image)

Figure 5.26 Three effects of selected refurbishment alternative, added stiffness, added damping and added mass, on human comfort.

### 5.3 Summary

In this chapter two case-studies have been discussed. Both studies revealed interesting facts that will be summarised below.

- The evolution of the natural frequencies during the building process of the Cardington structure showed that non-structural elements have a significant influence on the
natural frequencies. Any estimate of the natural frequency based on the stiffness of the structural system alone will yield lower bound values.

- Inclusion of the non-structural elements requires a non-linear dynamic analysis. Typical effects that were observed in a comparison of two different test methods that were applied on the Cardington building at different load levels can be described by a non-linear model that includes hysteretic spring.

- As a consequence of the non-linear effects, test results that have been reported in literature can only be used for comparison purposes when the test conditions are known.

- A comprehensive test program on various types of buildings could provide the information to come to a better understanding of the effects of non-structural parts on the dynamic behaviour of buildings. Specific attention has to be paid to load levels, displacements and actual behaviour of non-structural elements.

- The Voorhof II case-study illustrated the possible economic consequences of poor dynamic performance from a serviceability point of view. Both deflection and motion limits were exceeded in the original configuration. An expensive refurbishment that included the addition of four concrete walls to stiffen the structure, was required.

- Measurements of the dynamic behaviour of Voorhof II before and after the refurbishment revealed that the addition of stiffness by concrete walls was rather ineffective in improving the comfort of the occupants.
6  DESIGN SOLUTIONS

6.1  Introduction

The previous chapters showed that effective mass and damping are the best ways for the designer to control the dynamic behaviour of tall buildings. Moreover, the Cardington case-study revealed that non-structural parts play an important role in the dynamic behaviour of tall buildings. These two observations may be used to select and develop design measures that provide good dynamic performance.

In general, Figure 4.16 is a good starting point to develop design alternatives when an improvement of the dynamic performance regarding the along-wind response is required. This chapter treats two design measures that have been developed during this project. The reader is also referred to work of Banavalkar, who proposed steel structural systems that improve dynamic performance of tall buildings [BAN90].

The first concept is the combination of outriggers with visco-elastic dampers. This is a good example of a concept that has been designed with Figure 4.16 in mind. The second concept is the use of the connections between non-structural elements and structural system to provide extra damping by a slip-friction mechanism. These friction connections can be attached to both partition walls and cladding. Again, this is a measure that agrees with the conclusions of Chapter 4.

The mobilisation of the facade for structural purposes is not new. However, many of the previous proposals used the cladding elements for stiffness purposes. An example is found in reference [TOM86] which reported the design of a high-rise steel framed office building where the strength requirements were separated from drift requirements. First, the frame alone was designed according to strength analyses. Subsequently, frame plus facade panels, consisting of steel sheeting, were designed for drift. The panels were modeled as membrane elements. The required membrane stiffness was used for the design of individual panels. The steel panels were made from flat steel plates of about 6 mm thick, with cut-outs for window openings, covering three stories.

It appeared that this concept had three beneficial effects: structural frame deflection was reduced by a factor of two, the cost efficiency of the steel frame was significantly improved and additional rentable floor space was obtained due to a reduction in the size of the structural frame.

However, it is mass and damping rather than stiffness that is the tool to obtain good dynamic performance. Proposals like above can therefore be neglected in the search for design solutions with good dynamic performance.

In Chapter 1 it was mentioned that is preferable to design symmetrical buildings, i.e., there should be

- symmetry in plan,
- symmetry in structure,
- symmetry in mass distribution.

Symmetry avoids coupling of translational and torsional modes and the accordingly higher acceleration levels. The following examples illustrate the asymmetries that can exist in a tall building design.
The first example is a 40 storey building with variable cross-sections that is shown in Figure 6.1 [OOS92]. The upper half has a rectangular cross-section with a symmetrical mass and stiffness distribution. Therefore, the centres of rigidity and mass will coincide in the centre of the plan. The lower half of the building consists of two parts, as can be seen in Figure 6.1.
Although the mass and stiffness are also uniformly distributed in these parts, there is an eccentricity between the centre of mass and stiffness.

The second example is a building with an asymmetrical structural system. Consider the structural plan of the building on the left hand of Figure 6.3. A core and columns form the structural members. The centre of rigidity will lie in the core. The centre of mass is found near the middle of the plan.

A symmetrical structural system is always favourable. However, it is not always possible to achieve this due to functional constraints. The building should then be designed in such a way that the natural frequencies in the translational and rotational directions are well separated. This will minimise the coupling effect.

Asymmetry in the mass distribution in the plan can introduce coupling in symmetrical structures when the eigenfrequencies of the fundamental modes are very close. Consider for example the plan on the right hand in Figure 6.3: The lifts on the left hand side of the plan represent a slightly non-uniform distribution of mass. Consequently there is a small eccentricity between the centre of mass and rigidity. A detailed description of coupled dynamic behaviour of a tall building with symmetrical structural system is presented in reference [HOE72].

The solutions that are treated in this chapter are all structural in nature. However, good dynamic performance starts from the first sketch of a building. Two architectural measures that were reported in literature illustrate this. Pantiledes [PAN90] proposed the use of aerodynamic appendages that work similarly to the wing tips of aeroplanes. It is an active system, in that sense that the position of the spoilers is modified according to the information that a computer obtains from sensors that measure wind speed and the response of the building.

A passive modification of the building envelope is the introduction of through-building gaps, which is primarily meant to disrupt the vortex-shedding process. However, there also will be a reduction in the along-wind load as the pressures on the upwind and downwind facade decrease due to the flow of air through the gaps. Dutton and Isyumov [DUT90] did wind tunnel studies on a square test building to identify the best configuration of the gaps. All gaps were included in the upper half of the building as is illustrated in Figure 6.2. It appeared that the gaps were most effective when they were sized about 1/6h x 1/6h, where h is the width of the facade and h the height of the building.

6.2 Outriggers

6.2.1 Introduction

In section 4.4 it has been demonstrated that the effective mass can be used as a tool to reduce the accelerations. Modification of the structural and/or architectural configuration may increase the effective mass without adding mass through a modification of the mode shape. Figure 2.7 shows three typical mode shapes. The mode shape on the left hand, pure bending, results in a rather low effective mass of only 25% of the total mass. The vibration mode on the right hand, pure shear, has an effective mass of more than 50% of the total. Thus, a mode shape similar to the shear mode shape is needed to achieve a high effective mass.

One of the possible structural measures is to stiffen the top part of the structure by application of an outrigger or a similar load transfer structure. This concept is developed in this section. Theoretically, a rearrangement of mass can obtain the same result. The effect of
rearrangement is shown by rewriting equation (2.26), which is the definition of the effective mass coefficient:

$$C_m = \frac{\int_0^h \mu(z) \phi^2(z) \, dz}{\int_0^h \mu(z) \, dz}$$  \hspace{1cm} (6.1)

In the fundamental mode $\phi(z)$ increases with height. A relative large part of the total mass in the upper part of the building will increase the effective mass compared to the situation where the mass is uniformly distributed along the height. Note that this conflicts with stability requirements.

Therefore, a structural solution like the core plus outriggers, like is shown in Figure 6.4, is preferable as it can meet all other requirements. The columns are activated by means of a transfer structure that is mounted to the core at a height $h_2$ above grade.

![Figure 6.4](image)

**Figure 6.4** Four types of structural system: a structural core (model 0), a structural core supported by outriggers (model 1) and core, outriggers and visco-elastic dampers (model 2 and 3).

In this section the effectiveness of outriggers in improving the dynamic performance is studied.

Advantages of the addition of outriggers to the structural system are:

- The stiffness of the building increases.
- The fundamental mode shape is modified, going from pure bending to a shape that activates more mass in the building. As a consequence the effective mass of the building will increase.
Forces and strains are concentrated in the outriggers. This is an optimum place to locate dampers, since the effectiveness of the dampers is best when the shear strains in the visco-elastic dampers are large.

The dimensions of the core can be small and more rentable space will be available. A disadvantage is that the increase of the effective force is of the same extent as the increase of the effective mass due to the modification of the mode shape. See also section 2.4.1 for details on the relation between effective force and mode shape.

In section 6.2.2 visco-elastic dampers will be introduced, since they appear to improve the benefits of outrigger structures. Section 6.2.3 contains a design scheme for core and outriggers. The dynamic properties of this type of structural system should be known to be able to assess the effectiveness of the outriggers. The modal properties are therefore determined by approximation of the fundamental mode shape.

Subsequently, the outrigger-core configuration is optimised by minimising the horizontal acceleration levels that the top floors experience due to wind loads. Finally, the design scheme of section 6.2.3 will be illustrated by some optimisation examples.

6.2.2 Visco-elastic dampers

Cyclic deformation of visco-elastic materials result in energy dissipation as heat. Visco-elastic (VE) dampers have been applied successfully in buildings sensitive to dynamic wind loading. Best known example is the World Trade Center in New York City. The dampers in this building have shown good performance in the last 20 years [MAH87]. VE dampers are passive devices, i.e., no power supply is required for their operation, which, in conjunction with the durability, makes them a reliable source of energy dissipation in buildings.

Forces and strains are concentrated in the outrigger members. Therefore, the application of VE dampers in the members of this transfer structure will be effective even at low amplitudes. The energy that is dissipated in a VE damper during one cycle of the harmonic vibration $\gamma(t) = \gamma \sin \omega t$ is [KEE86]:

$$\Delta E = \pi \gamma^2 G V$$  \hspace{1cm} (6.2)

where

$\gamma$ = amplitude of the shear strain in the VE damper,

$G$ = the so-called ‘loss’ modulus (dimension N/m$^2$), which is a measure of the energy dissipation in the visco-elastic material per cycle,

$V$ = volume of the visco-elastic layer.

The shear strain $\gamma$ is defined by the ratio of the imposed displacement to the thickness of the VE-layer. The loss modulus is determined from experimental set-ups. A detailed discussion of the components of equation (6.2) may be found in reference [KAS93].

The properties of visco-elastic materials are dependent on temperature and frequency of excitation, which requires special design considerations. The energy-absorbing capability decreases significantly when the temperature rises. Therefore, the heat due to the dissipation process has to be transferred to the surroundings of the damper. The integration with the steel members of outriggers, which are excellent heat conductors, is a solution. Moreover, the temperature of the area in which the VE dampers are located should be controlled [KEE86].

The stiffness of the adjacent structural elements, like the steel plates in Figure 6.5, must be significantly higher than the stiffness of the VE layer to ensure that the deformation is
concentrated in the VE layer. Evidently, the steel members of the transfer structure are very stiff and they comply to this restriction.
High shear strains lead to non-linear behaviour of the damper and fatigue and should be avoided. For safety reasons, the shear strain in the VE layer should be less than 1 although larger shear strains can be accommodated.
The costs of VE dampers might be illustrated by the Columbia Center Building. In this building several visco-elastic dampers were used, which represented about 1.5% of the total building costs, including an extensive testing program [KEE86].

Figure 6.5 Visco-elastic dampers: two L-section and T-section layers 600x200x13 mm³ (model 3).

Figure 6.6 Design scheme for core structures supported by perimeter columns.
6.2.3 Design scheme

In Figure 6.6 a scheme for the design of core structures is presented. The scheme is based on optimisation of structures with the SkyDyFe Design Tool. An example of an optimisation process is described in section 6.2.5. It appears that economical structures can be designed if this procedure is followed.

One should start with the design of the core in order to satisfy strength, stability and stiffness requirements. For buildings with a low aspect ratio ($h/d$) this will be a successful procedure and a check of the acceleration will easily be passed.

However, for slender structures the comfort criteria may be difficult to meet. The addition of outriggers, that activate the perimeter columns, may be considered. If the accelerations are well above the allowed level, it is advised to integrate visco-elastic dampers in the outriggers. Since the damping ratio will increase, the dynamic displacements will reduce. So, the stiffness of the core can decrease. It makes, therefore, sense to re-optimise the core and outriggers.

In the following subsection the dynamic properties of a core supported by perimeter columns will be derived. These properties will be used in the example optimisation in subsection 6.2.5.

6.2.4 Effect of outriggers on dynamic parameters

In this section the modal properties of a core supported by perimeter columns will be derived. First, the fundamental mode shape will be approximated. Subsequently, a sensitivity study shows the dependencies of the dynamic parameters on the configuration of core, outrigger and perimeter columns. A fictitious building with a rectangular plan and a height of 200 m was considered for this study. An extensive description of the building is given in [OOS95b].

6.2.4.1 Effects on mode shape

The mode shape is required to derive the modal properties of a building. See also section 2.4. The surface under the mode shape function is a measure of the effective mass coefficient $C_m$ and effective force coefficient $C_F$. Definitions of $C_m$ and $C_F$ are given in equation (2.26) and (2.27) respectively.

The fundamental mode shape of the model in Figure 6.7 may be approximated by the static displacement field due to a uniformly distributed horizontal load.

Consider the scheme in Figure 6.7. The core of model 1 in Figure 6.4 can be modelled as a clamped column with a bending stiffness $EI$. The outriggers in Figure 6.4 are represented by a rotation spring at height $h_2$. The spring is characterised by a spring stiffness $c$ (Nm/rad). The column is loaded by a uniformly distributed horizontal load $q$. 

![Figure 6.7 Model to determine displacement field.](image)
The (static) behaviour of this system can be described by the following differential equation.

\[
EI \frac{\partial^4 u}{\partial z^4} = q
\]  

(6.3)

where (see Figure 6.7)

- \(u\) = the horizontal displacement of the column in \(x\)-direction,
- \(z\) = the vertical ordinate.

Due to the presence of the spring, there is a discontinuity in the solution of the differential equation (6.3). Therefore, the solution is split in two parts, with separate co-ordinate systems. The derivation of the displacement field has been reported in [ROB95] and [OOS95b]. Herein only the results are repeated. For convenience, dimensionless parameters are introduced. First, a relative stiffness \(\beta\) is defined as

\[
\beta = \frac{ch_z}{EI} \quad \text{ (6.4)}
\]

The ordinates \(z\) and \(z_2\) are normalised by division to the total height. These new parameters are denoted \(z_r\) and \(z_{r,2}\) respectively. Moreover, the parameters \(h_2\) and \(h_3\) are made dimensionless: \(h_{r,2}\) and \(h_{r,3}\). The displacement field then can be written as [OOS95b]

\[
0 \leq z_r \leq h_{r,2} \quad u = \frac{q h^4}{EI} \left[ \frac{1}{3} z_r^3 - \frac{1}{6} z_r^4 + \frac{1}{4} \left( h_{r,2}^2 - h_{r,3} \right) \frac{1 + \beta}{1 + \beta} z_r^2 \right] \quad \text{(6.5)}
\]

\[
0 \leq z_{r,2} \leq h_{r,3} \quad u = \frac{q h^4}{EI} \left[ - \frac{1}{6} h_{r,2} z_{r,2}^3 + \frac{1}{4} h_{r,2}^2 z_{r,2}^2 + \frac{1 + \beta}{1 + \beta} h_{r,3} z_{r,2} \right] + u(h_{r,2}) \quad \text{(6.6)}
\]

where \(u(h_{r,2})\) according to equation (6.5). With these expressions the mode shape function for several configurations can be calculated. Three typical configurations are shown in Figure 6.8. Two theoretical results may be recognised. The left hand (normalised) mode shape represents the clamped bending column. The effective mass is only 25% of the total mass. The middle mode shape in Figure 6.8 resembles a hat column, for which \(C_m\) equals 0.38. The right hand mode shape is found when the outriggers are located at \(h_2/h = 0.70\) and \(\beta \geq 20\). For this configuration \(C_m\) grows to 0.45.

The mode shapes in Figure 6.8 indicate that the upper third of the building is a good position for an outrigger as far as effective mass is concerned. The top part does contribute for almost 100% to the effective mass. A detailed study is performed in the following subsection.

### 6.2.4.2 Effective mass

The effective mass coefficient can be approximated when equations (6.5) and (6.6) are used in equation (2.26), the definition of the effective mass coefficient. For the case-study building the effects of outrigger height \(h_2\) and relative stiffness \(\beta\) were studied. The parameter \(\beta\) was varied from 0 to 20. The height of the transfer structure \(h_2\) was varied from 0 to 195 m for each \(\beta\).
Figure 6.8 Mode shapes for various types of models.

Figure 6.9 shows a 3D plot of the calculation results, that clearly shows the effectiveness of outriggers in increasing the effective mass. In the horizontal plane the relative stiffness of the outriggers, represented by the parameter $\beta$, and the height of the outriggers $h_2$ are plotted. The vertical axis contains the corresponding effective mass coefficients. Figure 6.9 agrees with results from Robbemont [ROB95].

Two theoretical results may be recognised. The edge of the surface at $\beta = 0$ represents the clamped bending column. The top right hand corner of the surface in Figure 6.9 resembles a hat column. The calculated effective mass coefficient according to the displacement field in subsection 6.2.4.1 is 0.40, which a slight overestimation of the theoretical value (0.38).

For a relatively stiff supporting structure ($\beta = 20$), $C_m$ grows up to 0.45 when the outriggers are located at $h_2/h = 0.70$. Note that this ratio is a bit different from the optimum for static deflection ($\frac{1}{3}$ of the height, as is demonstrated in subsection 6.2.4.4). Figure 6.8 displays the fundamental mode shape of that configuration.

### 6.2.4.3 Effective force coefficient

Similar to the effective mass coefficient the effective force coefficient has been determined as a function of outrigger height $h_2$ and stiffness $\beta$. Comparison of Figure 6.9 and Figure 6.10 shows that effective force and effective mass arrive at the maximum for similar arrangements. This is illustrated in Figure 6.11, which displays the outrigger height where effective mass and effective force respectively arrive at the maximum.

Thus, the decrease in accelerations due to an increased effective mass is counteracted by a growth in the effective force.
Figure 6.9  Effective mass coefficient as a function of outrigger stiffness and height.

Figure 6.10  Effective force coefficient as a function of outrigger stiffness and height.
Figure 6.11  Optimum outrigger height as a function of outrigger stiffness with respect to maximum effective mass and maximum effective force.

6.2.4.4 Effective stiffness

The expression for the effective stiffness of the building including outriggers is [OOS95b]:

$$k_e = \frac{EI}{h^3 - \frac{1}{3} \beta h_{r,2} h^3 + \beta h_{r,2}^2 h - \beta h_{r,2} + \frac{1}{2} (1 + \beta)}$$

(6.7)

The reader may recognise the known result for the bending column, $3EI/h^3$ for $\beta = 0$. Figure 6.12 displays a 3D representation of equation (6.7). The optimum height $h_2$, as far as stiffness is concerned, can be derived to be two thirds of the total height, regardless the value of $\beta$. See [OOS95c]. Figure 6.13 shows a plot of the stiffness as a function of the relative stiffness when the outriggers are mounted at $2/3$ of the total height. The vertical axis is normalised to the stiffness of a bending column without outriggers.

6.2.4.5 Response

In section 6.2.4.2 the optimum configuration of the outriggers was found by aiming at a maximum in the effective mass. One should, however, aim at minimising the accelerations. Maximising the effective mass can be one tool, but the outriggers also modify the stiffness of the building and the effective force coefficient.
Figure 6.12  Spring stiffness as a function of outrigger stiffness and height.

Figure 6.13  Maximum spring stiffness as a function of $\beta$ (outriggers at 2/3 of total height).
Figure 6.14  Optimum height to achieve minimum along-wind accelerations.

Figure 6.15  Maximum acceleration for $\beta = 1$ and various outrigger heights.
As the effective mass and stiffness change, the natural frequency changes. When the natural frequency shifts, the dynamic loads alter. All these aspects affect the response. A run similar to the optimisation of the effective mass was performed. However, now the minimisation of the response was the criterion. Figure 6.14 shows the graphical representation of the optimisation results for along-wind accelerations.

It appears that the optimum height is nearly independent of the relative stiffness $\beta$. This is quite remarkable, since effective mass, effective force and stiffness do have a strong dependency on $\beta$. It seems that these dependencies tend to cancel out each other.

A closer look to the along-wind response for $\beta = 1$ teaches that there is whole range of outrigger heights with response close to the minimum. See Figure 6.15 on the following page. For example, the interval that holds all heights within 105% of the minimum response is $0.33 \leq h_2/h \leq 0.93$ for along-wind loads. Compared to the situation without outriggers, the response is reduced to about 75%.

### 6.2.5 Optimisation

In Figure 6.4 four building models are displayed that represent steps in the outrigger design scheme as shown in Figure 6.6. In this subsection the optimisation process according to the mentioned design scheme is illustrated by a building that is 201.6 m tall, i.e., 56 stories of 3.6 m. The plan is rectangular with dimensions $b \times d = 50.4 \times 28.8$ m$^2$. The weight of the building exclusive the structure is 50 Mkg (a building density of 170 kg/m$^3$). Figure 6.16 displays a typical floor plan with global core dimensions and position of the outrigger beams if applicable. The building is assumed to stand in the Netherlands. The drift limit is 1/500 of the total height. The accelerations should be less than the ISO criterion displayed in Figure 3.3. A detailed list of SkyDyFe input data is given in Appendix C.

![Figure 6.16](image)

**Figure 6.16** Typical floor plan with position of the outriggers.
6.2.5.1 Structural core

The optimisation starts with the design of a structure like model 0. Strength and stability requirements are easily achieved. The drift limit is a more severe restriction. A bending stiffness $EI_{\text{core}} = 75 \cdot 10^{12}$ Nm² is required to keep the (static) displacement due to a 50 year return storm within the limit (see also Table 6.2). When the core has to be built within the boundaries that are set in Figure 6.16, the total weight of the core becomes 19.5 Mkg. Accordingly, the total mass of the building is $(50.5 + 19.5) = 70$ Mkg, i.e., a building density of 239 kg/m³.

Next, the accelerations are determined due to a five years return period wind. It is found, see Figure 6.18, that the ISO 6897 comfort criterion is not met.

6.2.5.2 Structural core supported by activated perimeter columns

The structural core cannot provide adequate dynamic performance, since the rms acceleration of model 0 is about 25% too high. The addition of outriggers can be a solution. To obtain a maximum result for both static and dynamic loading, the recommended outrigger height of about $2/3$ of the height is used, here 134 m. According to Figure 6.15 an outrigger configuration with $\beta = 1$ yields a 25% reduction in acceleration, which is sufficient to meet the perception requirement. Thus, $c = \beta EI/h_2 = 1.0 \cdot 75 \cdot 10^{12} / 133 = 560 \cdot 10^7$ Nm/rad.

The outriggers and perimeter columns can now be designed, since [ROB95]

$$
c = \frac{\frac{1}{2} d^2}{h_2} + \frac{1}{8} (d - d_{\text{core}})^3 \frac{E A_{\text{col}}}{3 E I_{\text{out}}} \tag{6.8}
$$

The parameters $d$, $d_{\text{core}}$, $z$, $E A_{\text{col}}$ and $E I_{\text{out}}$ (equivalent bending stiffness when the outriggers are made from braces) are defined in Figure 6.17. In practice it appears that in economic outrigger/perimeter column configurations both elements contribute equally to the spring stiffness $c$. In that case $E A_{\text{col}}$ becomes 360 GN and $E I_{\text{out}} = 340$ GNm². Both drift limit and human comfort criterion are now met. See Table 6.2 and Figure 6.18.

![Figure 6.17 Model of an outrigger. Left hand: dimensions, right hand: properties and scheme.](wind-induced-dynamic-behavior-of-tall-buildings-99)
6.2.5.3 Structural core, activated perimeter columns and VE dampers

In Figure 6.6 it is suggested to add visco-elastic dampers to the outrigger to further optimise the structural system. The dampers can be integrated in the diagonals of the outrigger like is displayed in Figure 6.19. In elevation the structural system can be sketched like model 2 in Figure 6.4. The L-sections of the dampers, see Figure 6.5, are bolted to the web of the diagonal in Figure 6.19. A beam, that is connected to the core, imposes the deformation of the outrigger diagonal on the visco-elastic layers.

First, the deflection is considered. In [ROB95] Robbemont calculated that VE dampers dissipate energy that is equivalent to 4.4% damping ratio when a 50 year return storm hits the building. It appears that a configuration with \( \beta = 4/3 \) yields a considerable decrease in structural weight, since only half the bending stiffness of model 0 is needed. The introduction of the outrigger adds mass, but the total effect is a decrease to 60% of the structural weight in model 0 (see the last column in Table 6.1).

During serviceability limit state conditions the VE dampers provide 2.1% damping ratio ([ROB95]). Figure 6.18 shows that the acceleration levels are below the maximum admissible levels.

6.2.5.4 Structural core and VE dampers

It appears that the extra damping is very beneficial. The energy dissipation can be maximised when the position of the visco-elastic dampers is changed like is shown in model 3 of Figure 6.4. The VE dampers are mounted in series with the perimeter column so that the imposed displacements are much larger than in the previous configuration. This configuration maximises the energy dissipation but reduces the contribution of the perimeter columns to the total stiffness to almost zero.

The core has to provide the stiffness as in model 0. A small reduction in core dimensions is possible, since the dynamic part of the deflection is smaller due to the VE dampers that provide 10% equivalent damping ratio [ROB95]. The dimensions of outriggers and activated columns can be smaller than for model 2, since they are only needed to impose the deformation on the visco-elastic layers. Figure 6.20 shows a possible outrigger-damper configuration. Note that the outrigger is only one story high. A close look on the VE damper and the position of the visco-elastic layers is presented in Figure 6.5.

6.2.5.5 Summary of optimisation

The main results of the calculations that were used in the optimisation process are collected in Table 6.1 to Table 6.3. The results of model 1 confirm the conclusions of Chapter 4 and the Voorhof II case-study: additional stiffness is rather ineffective in improving the human comfort. Model 1 doubles the stiffness, but only a small increase in human comfort is achieved. A far better solution is model 2: good dynamic performance, combined with a low structural weight. In comparison to model 0, the structural core, a reduction of 5 million kg is achieved. Thus, the extra costs for the installation of VE dampers are balanced by the reduction in structural material. The best dynamic performance is obtained by model 3, as may be appreciated from Figure 6.18. However, model 3 does not significantly reduce the structural weight.

Wind-induced Dynamic Behaviour of Tall Buildings - 100
The configuration of structural core, outriggers with parallel VE dampers and activated perimeter columns (model 2) seems to be the most economic structural system for this building.

**Table 6.1** Structural properties. Definitions are given in Figure 6.17.

<table>
<thead>
<tr>
<th>Model</th>
<th>$EI_{core}$ (Nm$^2$)</th>
<th>$EA_{col}$ (N)</th>
<th>$EI_{out}$ (Nm$^2$)</th>
<th>$c$ (Nm/rad)</th>
<th>$m_{struct}$ (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$75 \times 10^{12}$</td>
<td></td>
<td></td>
<td></td>
<td>$15.4 \times 10^6$</td>
</tr>
<tr>
<td>1</td>
<td>$75 \times 10^{12}$</td>
<td>$360 \times 10^9$</td>
<td></td>
<td>$560 \times 10^9$</td>
<td>$19.8 \times 10^6$</td>
</tr>
<tr>
<td>2</td>
<td>$35 \times 10^{12}$</td>
<td>$250 \times 10^9$</td>
<td></td>
<td>$350 \times 10^9$</td>
<td>$10.5 \times 10^6$</td>
</tr>
<tr>
<td>3</td>
<td>$66 \times 10^{12}$</td>
<td>$8.4 \times 10^9$</td>
<td></td>
<td>$13 \times 10^9$</td>
<td>$13.6 \times 10^6$</td>
</tr>
</tbody>
</table>

**Table 6.2** Displacements due to 50 year return wind load. Drift limit is $1/500h = 400$ mm.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\zeta$ (%)</th>
<th>$u_{stat}$ (mm)</th>
<th>$u_{dyn}$ (mm)</th>
<th>$u_{tot}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>353</td>
<td>165</td>
<td>390</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>185</td>
<td>70</td>
<td>198</td>
</tr>
<tr>
<td>2</td>
<td>4.2</td>
<td>386</td>
<td>97</td>
<td>398</td>
</tr>
<tr>
<td>3</td>
<td>10.3</td>
<td>393</td>
<td>59</td>
<td>397</td>
</tr>
</tbody>
</table>

**Table 6.3** Dynamic properties and peak accelerations during the 1 year return storm.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\zeta$ (%)</th>
<th>$f_c$ (Hz)</th>
<th>$k_c$ (MN/m)</th>
<th>$m_c$ (kg)</th>
<th>$C_F$ (-)</th>
<th>$\sigma_a$ (mm/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>0.205</td>
<td>27.5</td>
<td>16.6</td>
<td>0.394</td>
<td>58</td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
<td>0.263</td>
<td>54.1</td>
<td>19.7</td>
<td>0.428</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>2.1</td>
<td>0.196</td>
<td>26.0</td>
<td>17.1</td>
<td>0.430</td>
<td>44</td>
</tr>
<tr>
<td>3</td>
<td>4.4</td>
<td>0.197</td>
<td>24.7</td>
<td>16.1</td>
<td>0.395</td>
<td>32</td>
</tr>
</tbody>
</table>

![Graph showing dynamic performance during several steps in the optimisation process.](image)

**Figure 6.18** Dynamic performance during several steps in the optimisation process.
Figure 6.19  Model 2 outrigger with parallel VE dampers. Detail of the dampers (encircled) is shown in Figure 6.5.

Figure 6.20  Model 3 outrigger with VE dampers in series. Detail of the dampers (encircled) is shown in Figure 6.5.
6.3 Advanced non-structural elements

6.3.1 Introduction

In Chapter 5 it has been recognised that non-structural elements do play a significant role in the dynamic behaviour of tall buildings under serviceability conditions. A non-structural element can be defined in that context as an element that is not explicitly designed to serve a structural goal. Typical non-structural parts are cladding elements, infill panels, partition walls and fire walls.

The Cardington case-study revealed that the damping ratio can grow from 0.5 to 2.5% going from an unclad frame to a completed building. Besides, the inclusion of the facade walls makes the building three times stiffer than the bare frame configuration (see section 5.1.3). These observations are supported by a study of Palsson et al on cladding-structure interaction in a 25-story office building on a pier foundation [PAL84]. Resistance to horizontal loads was provided by orthogonal steel frames. The floors were constructed from concrete slabs on light metal decking supported by beams spanning from the core to the exterior frame. The building's envelope was built from heavy precast panels. Forced vibration tests on the building revealed that the additional stiffness due to the cladding substantially increased vibration frequencies in lower modes. Especially the torsional frequencies were influenced, since an increase of 90% was measured compared to the calculations without the effect of cladding. The lateral frequencies rose by 40%.

Thus, cladding or other non-structural elements may play an important role in the dynamic performance of a tall building. This section discusses serviceability limit state design of partition walls and cladding. The results of the sensitivity analyses in Chapter 4 (see section 4.7) are the starting points of the design for dynamic performance purposes. Mitigation of motion can be best achieved by increasing damping and effective mass. Increasing stiffness may have an adverse effect as it may shift the accelerations towards the inadmissible region.

In ultimate limit states (strength, stability), the resistance of non-structural elements to loads is most certainly zero because of brittleness and unreliability (these elements might be removed by occupants). However, in serviceability limit states design a lower reliability level is accepted and stress levels will be such that the non-structural elements will participate in the resistance to loads. As a consequence, these elements may contribute to a large part of the stiffness of a building. Moreover, and for the dynamics even more important, due to friction and other mechanisms most of the energy dissipation in the building will take place in the non-structural elements.

Energy can only be dissipated if displacements (and loads) are imposed on an element. However, the additional stiffness, that is introduced when non-structural elements are connected to the main structural system, should preferably be small, since the sensitivity analyses in Chapter 4 indicated that a growth in stiffness may adversely affect human comfort. This leads to the following optimisation problem:

Develop an element that incorporates high energy dissipation and adds little stiffness.

Note that the time history of loads can be important. For example, minor damage due to earthquakes or storms can diminish the additional stiffness provided by cladding. Durable improved dynamic performance can be obtained when non-structural elements are designed such that their beneficial properties sustain in time. These requirements can only be fulfilled if
a special type of connection between non-structural element and structural system is developed.
Pinelli et al. proposed a system that is primarily meant to improve the earthquake-induced response [PIN93]. Figure 6.21 shows their specially designed connection for precast concrete panels that hang from the main structure. The tapered shape of the body initiates plastification over a greater portion of the connector during earthquake conditions. Experiments indicate that a 50% reduction of the response can be achieved due to the panel-structure interaction.
The connection concept in Figure 6.21 requires rather high load levels to establish the yield stress in the tapered section. In this section the familiar sliding connection for light weight partition walls, like the one that is shown in Figure 6.22, is modified to a slip-friction connection. It will be shown that this connection type is active over a wide range of loading magnitudes, thus providing damping even during moderate storms.
In subsection 6.3.2 the theory of partition walls with friction connections will be treated. Subsequently, the effects on the total building performance will be assessed with emphasis on the damping capacity. In section 6.3.4 some practical requirements for the connections are discussed. Finally, the friction model will be applied to predict the behaviour of the Cardington building.

6.3.2 Model of a partition wall with a friction connection

6.3.2.1 Single wall

The sliding connection as is shown in Figure 6.22 is the most commonly used connection to connect partition walls to either the ceiling or the floor [JAN95]. The aim of the sliding connection is to prevent the interstorey deflection to be imposed on the wall.
In the ideal situation, no forces will be transmitted through the sliding connection. However, in practice, there will always be a little friction. This may be an important source of energy dissipation in the building when slip occurs. The connecting force must pass the maximum friction force \( F_p \) before any energy is dissipated. The partition wall will first undergo an elastic shear deformation until the connection starts to slip. The force in the connection will be constant during slip.
The non-linear model that was treated in section 2.3 may be used to study the effects of the slip-friction connection on the dynamic behaviour. The definition of the force \( F_u(u) \) is given in Figure 6.23. The spring stiffness \( k_g \) can be determined from an examination of the partition wall.

Wind-induced Dynamic Behaviour of Tall Buildings - 104
Figure 6.22  Schematic view of a sliding connection.

Figure 6.23  Force versus displacement for the slip-friction connection of a partition wall.
If we assume that the partition wall is a solid wall that is enclosed on all four edges, its stiffness can be derived from the shear column analogy:

\[ k_\theta = \frac{bhG}{h} \]  \hspace{1cm} (6.9)

in which,
\[ b \] = width of the partition wall,
\[ t \] = thickness of the partition wall,
\[ G \] = shear modulus of the partition wall,
\[ h \] = height of the partition wall.

Note that in practice the requirement of enclosure on all four edges will often not be met because of the type of connection that is chosen. Moreover, many of the connections, that are commonly used, are rather flexible. In these cases equation (6.9) will not apply. The second aspect shown in Figure 6.23 is the total amount of energy dissipated in one cycle, indicated by the area inside the hysteresis loop. The area is:

\[ \Delta E_{\text{diss}} = F_p u_f \]  \hspace{1cm} (6.10)

where (see Figure 6.23)
\[ F_p \] = maximum friction force,
\[ u_f \] = the slip length, defined as the difference between total (peak-to-peak) and elastic displacement \((2\overline{u} - u_e)\).

From expressions (6.9) and (6.10) we can learn that a designer has two controls to adjust the energy dissipation by means of a slop-friction mechanism:
- the friction force \(F_p\),
- the stiffness of the element \(k_\theta\).

The elastic deformation of the partition wall, \(u_p\), is determined by these parameters, since

\[ F_p = k_\theta u_p \]  \hspace{1cm} (6.11)

Substitution of the definition of \(F_p\) plus the definition of \(u_f\) in equation (6.10) yields that the energy dissipation due to slip in the friction connection during one cycle is

\[ \Delta E_{\text{diss}} = k_\theta u_p (2\overline{u} - u_e) \]  \hspace{1cm} (6.12)

This expression is the starting-point for the assessment of the damping capacity of the partition walls later in this section. First, the effects of partition walls on the overall-behaviour of a tall building are derived.

### 6.3.2.2 Performance of walls in a building

In the previous subsection the behaviour of one partition wall was described. In this subsection the effect of the walls on the overall dynamic behaviour is determined from the model in Figure 6.24, that represents a building with \(n\) floors. At each floor an element is present that exhibits frictional behaviour, characterised by Figure 6.23. The elements from a serial configuration of \(n\) non-linear springs, see Figure 6.24. The structural elements are represented in Figure 6.24 by \(n\) linear springs with stiffness \(k_{x,r}\).
Figure 6.24  *n* springs in series with Coulomb friction device, representing the non-structural elements. Also shown are *n* springs, representing the structural elements.

Let us assume that all elements have the same properties. Then there are two cases that should be considered:

1. The external load *F*₂ is smaller than the friction force *F*ₚ.
2. The external load exceeds the friction force *F*ₚ.

First, we consider *F*₂ is smaller than *F*ₚ. The system of *n* elements behaves linearly and in series. Thus, the total stiffness *k*₀ of the partition walls can be written as

\[
\frac{1}{k_0} = \frac{1}{k_{0,1}} + \frac{1}{k_{0,2}} + \ldots + \frac{1}{k_{0,n}} = \frac{k_{0,i}}{n}
\]  

(6.13)

where the element stiffness *k*₀ᵢ equals equation (6.9).

When the force in a partition wall exceeds *F*ₚ, the elements will slip. Now, energy will be dissipated. For each element, equation (6.12) describes the amount of energy that is dissipated in one cycle. In equation (6.12) *u*₂ᵢ should be replaced by *u*₂ᵢ = *F*ₚ/*k*₀ᵢ.

The total energy dissipation can be found by summation of the individual contributions:

\[
\Delta E_{\text{diss}} = n\Delta E_i = n(k_{0,i}u_{p,i}(2\ddot{u}_i - u_{p,i})) = n^2k_0u_{p,i}(2\ddot{u}_i - u_{p,i})
\]  

(6.14)

Equation (6.14) can be simplified if we define *u*ₚ = *m* *u*ₚᵢ. Furthermore, the product of *n* times 2\ddot{u}ᵢ equals 2\ddot{u}.

\[
\Delta E_{\text{diss}} = k_0u_{p}(2\ddot{u} - u_{p})
\]  

(6.15)

This is the energy dissipation of an equivalent frictional element, that spans the full height of the building, with properties *k*₀ = *k*₀ᵢ/*n*, *u*ₚ = *m* *u*ₚᵢ and *F*ₚ = *F*ₚᵢ. We will use this equivalent element to assess the damping potential in tall buildings.

In general, a floor will carry more than one partition wall. Let us assume that there are *m* partition walls with properties *k*₀ᵢ and *F*ₚᵢ on each floor. The floors act as stiff planes, so all panels on one floor experience the same differential displacement. In terms of the dynamic theory: all springs act parallel. The total stiffness becomes the sum of the individual panel’s stiffnesses.

\[
k_0 = \sum_{i=1}^{m} \frac{k_{0,i,j}}{n} = \frac{m}{n}k_{0,i}
\]  

(6.16)

where *m* denotes the number of panels per floor.
Since all walls on one floor experience the same displacement, they will achieve the maximum friction force $F_p$ at the same time. So, the total friction force will be

$$F_p = \sum_{j=1}^{m} F_{p,i,j} = mF_{p,i}$$  \hspace{1cm} (6.17)

The properties of the friction elements are now known and the damping potential can be assessed.

### 6.3.3 Assessment of damping potential

#### 6.3.3.1 Optimum wall design

The maximum energy dissipation will be found when the first derivative of equation (6.12) with respect to $u_p$ is zero. The reader can easily verify that this will occur when $u_p = \hat{u}$. See also Figure 6.25, where the energy dissipation according to equation (6.12) is displayed for a panel with $k_0 = 10$ MN/m and variable $u_p$. The single amplitude of the harmonic vibration was 5 mm and, indeed, the maximum energy dissipation is achieved when $u_p = 5$ mm (point 2 in the plot).

In the optimum situation the work done by the walls equals:

$$\Delta E_{diss, opt} = k_0\hat{u}^2 \Leftrightarrow \Delta E_{diss, opt} = k_0\hat{u}_p^2$$  \hspace{1cm} (6.18)

The corresponding $F-u$ diagram is displayed in Figure 6.26. Moreover, for comparison purposes the force-displacement diagrams that belong to point 1 and 3 in Figure 6.25 are plotted.

In steady state, an energy balance of a one-mass-spring model requires that the work done by the external force $F(t)$ in one cycle equals the energy dissipated by the walls according to (6.18). There are now two possibilities:

- the optimum energy dissipation is reached below resonance,
- the optimum energy dissipation occurs at resonance.

By far the most interesting case is maximum energy dissipation at resonance. This situation will be first considered. Subsection 6.3.3.3 will discuss the second possibility. If the model is weakly non-linear, which is true when tall buildings are modelled as a one-mass-spring model, we can study the response at resonance with an equivalent linear (visco-elastic) model.

If we assume that the external load is a single harmonic load with $\omega = \omega_r$ and single amplitude $\hat{F}$, the response function can be written as

$$u(t) = \hat{u}\sin(\omega_r t - \frac{1}{2} \pi) = -\hat{u}\cos(\omega_r t)$$  \hspace{1cm} (6.19)

and the work done per cycle equals

$$\Delta E_f = \int_0^T F(t)\dot{u}(t)dt = \int_0^T \hat{F} \sin(\omega_r t) \hat{u}\omega_r \sin(\omega_r t)dt = \pi \hat{F}\hat{u}$$  \hspace{1cm} (6.20)
Figure 6.25  Energy dissipation as a function of the elastic displacement. The element stiffness is 10 MN/m and the single amplitude of the harmonic vibration is 5 mm. Numbers refer to force-displacement diagrams in Figure 6.26.

Figure 6.26  Three typical force-displacement diagrams: 1: low friction force & large slip length, 2: optimum configuration, 3: high friction force & small slip length.
Combination of (6.18) and (6.20) yields a requirement to achieve optimum performance at resonance for the stiffness of the panels:

\[ k_0 = \frac{\pi \hat{F}}{\hat{u}} \]  \hspace{1cm} (6.21)

As an example of the impact of equation (6.21) on the design of in-fill panels, the partition walls of the building of the Faculty of Electrical Engineering in Delft are re-designed. An extensive description of this building is given in [JAN95]. Here only the most important figures are repeated. The building has an effective mass of 8.7 Mkg, a structural stiffness of 67 MN/m and a lowest natural frequency of 0.44 Hz. Each floor contains 15 partition walls that can be used as friction devices. There are 22 floors.

![Figure 6.27](image)

Figure 6.27  Plan of the Faculty of Electrical Engineering in Delft. Thin lines are partition walls.

The harmonic peak load at resonance is 30 kN. For serviceability it is required that the peak accelerations at resonance stay below 50 mm/s\(^2\). The maximum dynamic displacements should therefore be less than 6.6 mm. The elastic displacement \( u_p \) should, therefore, also be 6.6 mm, see equation (6.18), to achieve the optimum configuration. The stiffness of the panels is, according to equation (6.21), 14.3 MN/m.

Figure 6.28 shows the amplitude of the displacement at the top of the building at various loading frequencies. Figure 6.29 is a corresponding graph in which the equivalent damping ratio is plotted. Note that the resonant frequency has shifted from 0.44 to 0.46 Hz as a result of the addition of the panels. The peak resonant displacement appears to be 6.7 mm, a little more than the 6.6 mm target. The energy dissipation by the friction devices at resonance is 622 J. The damping potential in terms of an equivalent damping ratio for this configuration is

\[ \zeta = \frac{\Delta E}{2\pi m \hat{u}^2 \omega_c} = \frac{622}{2\pi 8.7 \cdot 10^3 \cdot 0.0067^2 \cdot 2.90^2} = 31\% \]  \hspace{1cm} (6.22)

Although the walls represent 14.3 MN/m, the effective contribution to the total stiffness at resonance is only 6.2 MN/m. This is clearly due to the slipping process, which decreases the tangent stiffness to less than half the stiffness \( k_0 \).

The results of this calculation can be translated to the conventional linear visco-elastic model. The properties are: \( m_c = 8.7 \) Mkg, \( k_c = 67 + 6.2 = 73.2 \) MN/m and a damping ratio of 3.1%. The dashed line in Figure 6.28 shows the response at various loading frequencies for that linear visco-elastic model.
Figure 6.28 Peak displacement as a function of loading frequency. Dashed line is the response of the equivalent linear visco-elastic model. Optimum wall configuration.

Figure 6.29 Equivalent damping ratio as a function of the loading frequency. Optimum wall configuration.
The panels only dissipate energy in a small region close to the resonant frequency as can be appreciated from Figure 6.29, because no slip will occur in the panel while the peak-to-peak displacement is smaller than the elastic displacement \( u_p \).

Therefore, the frequency at which the walls start to dissipate energy can be determined exactly. As long as the walls do not slip, the linear model applies and the particular solution of the equation of motion is

\[
\ddot{u} = \frac{\ddot{F}}{k_e}  \left[ \frac{1}{1 - \left( \frac{\omega}{\omega_c} \right)^2} \right] \tag{6.23}
\]

where

\( k_e \) = the total of structural and non-structural stiffness: \( k_s + k_0 \),

\( \omega_c \) = is the square root of the total stiffness to the effective mass.

When the amplitude exceeds \( \frac{1}{2} u_p \), equation (6.23) will no longer apply as there will be damping and the contribution of the panels to the total stiffness will decrease. After some elaboration, the active range of the panels can be derived to be

\[
\sqrt{1 - \frac{2u_0}{u_p}} < \frac{\omega}{\sqrt{k_n k_0}} < \sqrt{1 + \frac{2u_0}{u_p}} \tag{6.24}
\]

where \( u_0 \) is the static displacement defined by the ratio of peak load to total stiffness. This result is rather academic, since full-scale tests always indicate damping over the whole range of loading frequencies. This phenomenon will be discussed later.

A more severe requirement would be to restrict the peak accelerations to 25 mm/s\(^2\). The amplitude of the displacement should be less than 3 mm to achieve this (\( a = \omega_c^2 u \), \( \omega_c \approx 2.9 \) rad/s), resulting in \( u_p = 3 \) mm. The required stiffness of the panels is 31.4 MN/m, more than twice the previous configuration. A non-linear analysis indicated that the resonant frequency will then be at 0.50 Hz, with an equivalent damping ratio of 7.5%.

In theory, any damping ratio and maximum peak displacement can be achieved. However, in practice there are limitations. These will be discussed in the following subsection.

### 6.3.3.2 Practical limitations

In practice, there are important limitations to the energy dissipation afforded by slip in the connection between panel and structural element. Most significant restriction is the normal force in the connection that is the source of the friction. See also section 6.3.4.

Here the optimal results from the previous section are checked for feasibility. A total stiffness \( k_0 \) of 14.3 MN/m with a total elastic displacement \( u_p \) of 6.6 mm was needed to keep the peak accelerations below 50 mm/s\(^2\). The Faculty of Electrical Engineering is 22 stories high and has 15 panels at each floor. So, the stiffness of an individual panel should be

\[
k_0 = \frac{n}{m} k_0 = \frac{22}{15} \cdot 14.3 = 21 \text{ MN/m} \tag{6.25}
\]

The value for the panel stiffness seems to be reasonable. For example, the shear modulus of gypsum is about \( 2 \cdot 10^9 \) Nm\(^2\). The height of a panel is 3.6 m. A typical length of a panel is 5.4
m (standard office depth). The thickness of such a panel can then be determined to be (see equation (6.9))

\[ t = \frac{k_0 \cdot h}{Gb} = \frac{21 \cdot 10^6 \cdot 3.6}{2 \cdot 10^9 \cdot 54} = 7 \text{ mm} \]  

(6.26)

This is a reasonable thickness and the panel is definitely able to provide the requested stiffness. The friction force \( F_{p,i} \) should be

\[ F_{p,i} = k_0 \cdot u_p = 21 \cdot 10^6 \cdot \frac{6.6 \cdot 10^{-3}}{22} = 6300 \text{ N} \]  

(6.27)

Let us consider the required friction force according to equation (6.27) in a connection similar to Figure 6.32. The normal force in this continuous connection is provided by the self-weight of the panel. The friction surface is made from nylon that has a friction coefficient, i.e., the ratio between maximum friction force and normal force, of about 0.18. Thus, a normal force of 35000 N is required to obtain \( F_{p,i} = 6300 \text{ N} \).

A normal force of 35000 N implies that the weight of the panel is at least 3500 kg. This figure is not very realistic, since partition walls are often light-weight to facilitate assembly. Therefore, the mass of the panels is an important practical limitation in the energy dissipation that can be obtained through friction-slip.

### 6.3.3.3 Sub optimal wall design

If the design of the panel and its connections is not according to the rules that were set in subsection 6.3.3.1, a sub optimal design is achieved. Subsequently, the optimum energy dissipation according to (6.18) will occur before resonance. Consider, for example, Figure 6.30, which shows the equivalent damping ratio for a wall with \( u_p = 1 \text{ mm} \) and \( k_0 = 14.3 \text{ MN/m} \) where \( u_p = 6.6 \text{ mm} \) and \( k_0 = 14.3 \text{ MN/m} \) is the optimum. At 0.33 Hz the amplitude of the displacement is 1 mm, which equals \( u_p \). Thus, the optimum energy dissipation is achieved and subsequently a maximum in the damping can be recognised. At 0.44 Hz the resonant frequency is reached, which is the minimum in Figure 6.30. The second maximum is the point where the displacement again is 1 mm.

The maximum in the equivalent damping ratio is the point where the optimum energy dissipation is achieved. The minimum is the point where the building is excited in its resonant frequency. Let us take a closer look to the resonance in sub optimal conditions, since there are some interesting phenomena that can occur. The work done by the external force is described by (6.20), and the energy dissipation by the wall by (6.12). Combination of the two equations yields an expression for the peak amplitude at resonance:

\[ \hat{u} = \frac{-k_0 \cdot u_p}{\pi F - 2k_0 \cdot u_p} \]  

(6.28)

A closer examination of (6.28) learns that there is a lower bound restriction to the value of \( u_p \):

\[ \pi F - 2k_0 u_p > 0 \rightarrow u_p > \frac{\pi F}{2k_0} \]  

(6.29)
If this requirement is not met, the displacement at resonance will grow to infinity, since the energy input by the external force cannot be dissipated completely regardless the displacement amplitudes for such configurations of the friction damper. See, for example, Figure 6.31 where $u_p$ did not comply to equation (6.28): each cycle the amplitude grows and there will be no steady-state.

6.3.4 Connection design

6.3.4.1 Connection types

A reliable damper can be obtained if a partition wall is connected to the floors as is shown in Figure 6.32. The bottom of the wall rests on a friction layer. At the top the wall is firmly attached to the ceiling, so that the interstorey drift is imposed on the wall. The normal force that is required to obtain friction in the bottom connection is provided by the self-weight of the partition.

For a lasting effect of the friction elements, special materials should be used. In [JAN95] Janner proposed to use a nylon coating, which has a typical dynamic friction coefficient of 0.18. Nylon is very stable: the wear is about 0.1 $\mu$m/km. Janner showed that the wear during the average lifetime of a partition wall, 10 years, is less than 0.1 mm, which is negligible. Note that the partition wall should always be firmly attached to one floor to be effective. Special attention is required when lowered ceilings are applied. These are in general very flexible and can not be used to impose the interstorey drift on the partition wall.

The connection concept that is shown in Figure 6.32 can also be used in conjunction with cladding elements. Figure 6.34 displays a facade panel and the friction connection. Again, the interstorey displacements are imposed on the panel through a fixed connection to the ceiling. Cladding elements provide a much more reliable source of damping compared to partition walls. Partition walls can be removed to adapt a building to changing needs of the users. However, there will always be a facade. Thus, it seems that the potential of friction damping in cladding elements is the best.

In the detailing of the friction connection, like shown in Figure 6.33, the following effects might require attention:

1. Friction between two surfaces will result in dissipation of energy, usually converted into thermal energy. However, the energy might also be converted into acoustic energy and create annoying noise. Experimental testing should give more insight when noise accompanies friction.

2. Pollution of the friction surfaces caused by smoke, dust or grease may substantially affect the surface properties. Subsequently, the friction force reduces.

3. Nylon is quite sensitive to humidity, leading to expansion. A dry environment is required.

4. The friction connection should not conflict with other requirements for the cladding element or partition wall.

5. Differential sagging of ceiling and floor can either decrease or increase the normal force. In the latter case the friction force $F_p$ might be too high and no slip will occur during serviceability limit state conditions. If the connection to the ceiling is flexible in the vertical direction and there is sufficient distance between wall and ceiling, this negative effect can be avoided.
Figure 6.30  Equivalent damping ratio as a function of the loading frequency. Sub-optimal wall configuration with $u_p = 1\, \text{mm}$ and $k_0 = 14.3\, \text{MN/m}$.

Figure 6.31  Ever-increasing amplitude at resonance if the design of the friction device does not comply to equation (6.29).
Figure 6.32  Front view and cross-section of a friction connection between partition wall and floor. A close-up of the connection (the encircled area) is shown in Figure 6.33.

Figure 6.33  Friction connection between partition wall and floor. Friction is provided by nylon layers between the bottom of the partition and the floor.
6.3.5 Cardington and the friction model

In chapter 5 the square root constitutive law appeared to describe the non-linear dynamic behaviour of the Cardington building quite accurately. The slip-friction model also provides a hysteretic model. Let us consider if the friction model is able to predict the dynamic behaviour of a real structure. Assume that all connections between non-structural elements and the structural frame of the Cardington building exhibit a force-displacement diagram as displayed in Figure 6.23.

The NS mode is studied herein, since the non-linear effects are most apparent in that direction. In the NS direction there are four types of panels. See the typical floor plan of the Cardington building in Figure 5.1. Therefore, the model in Figure 6.35 applies. There is one linear spring that represents the structural system, i.e., the braced frame. Each spring in series with a Coulomb friction device represents one type of panel. Numbers 1 and 2 relate to the facade walls, mrs 3 and 4 to the gypsum walls that are located in the interior of the building.

From the test data that were presented in section 5.1 the parameters $F_p$ and $u_p$ can not be deduced for a certain panel type. Therefore, a fitting process was performed in which it was assumed that during the high level test the partition walls would hardly contribute to the stiffness, i.e., their elastic displacements $u_p$ should be smaller than for facade elements.

The stiffnesses and friction forces as summarised in Table 6.4 gave a transfer function that fitted well to the test data. The stiffness of the ‘structural’ spring was determined from the test results on the bare frame plus composite floors (see section 5.1.3).
Table 6.4  Spring properties and wall properties (m is the number of walls of one type).

<table>
<thead>
<tr>
<th>spring</th>
<th>represents</th>
<th>$k_0$ (MN/m)</th>
<th>$F_p$ (N)</th>
<th>$u_p$ (mm)</th>
<th>wall properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>main structure</td>
<td>22</td>
<td>-</td>
<td>-</td>
<td>m b (m) t (m)</td>
</tr>
<tr>
<td>1</td>
<td>facade</td>
<td>12.6</td>
<td>40000</td>
<td>3.20</td>
<td>4 6.0 0.14</td>
</tr>
<tr>
<td>2</td>
<td>facade</td>
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<td>4140</td>
<td>0.90</td>
<td>2 4.5 0.14</td>
</tr>
<tr>
<td>3</td>
<td>partition</td>
<td>9.3</td>
<td>2000</td>
<td>0.22</td>
<td>2 7.0 0.03</td>
</tr>
<tr>
<td>4</td>
<td>partition</td>
<td>3.9</td>
<td>400</td>
<td>0.10</td>
<td>2 3.0 0.03</td>
</tr>
</tbody>
</table>

Figure 6.37 compares the transfer function based on the data in Table 6.4 to the measurements. The resonant frequency is found at 0.80 Hz with a displacement amplitude of 0.86 mm, which is close to the measurements. Note that, as with the square root law, the slope on the right hand side of the resonant peak is too gentle, which is due to an overestimate of the energy dissipation in that part of the transfer function (the equivalent damping ratio is about 4.0%). A 27 N harmonic vibration was used to perform a low level sweep. The resonant peak is found at 0.93 Hz with an amplitude of 0.06 mm, which agrees with the low-level measurements of TNO.

Figure 6.36 takes a closer look to the response at resonance. Displayed is the force-displacement diagram when a load of 2700 N amplitude is applied. Also shown is the $F_p$-$u$ relation, the dotted line, under the same conditions but based on the square root constitutive law. The two force-displacement relations agree quite well.

Figure 6.35  The one degree of freedom model for the Cardington building.

Wind-induced Dynamic Behaviour of Tall Buildings - 118
Figure 6.36 Non-linear force-displacement relation based on the slip-friction model. The square root relation as determined in subsection 5.1.5 is also shown.

Figure 6.37 Transfer function of the Cardington building fundamental NS mode based on the slip-friction model, compared to measured data.
6.3.6 Concluding remarks

In this section the behaviour of the connections of non-structural elements that are optimised for energy dissipation has been studied. The study required non-linear dynamic analysis and the following typical phenomena were encountered:

- The natural frequency and damping are depending on the load level. For very low load levels the partition walls will act elastically and no energy will be dissipated. When connections start to slip the total stiffness will decrease and the damping ratio will grow.
- The energy dissipation will be optimal when the elastic displacement $u_p$ will equal the (single) peak displacement, i.e., slip will occur over a length of a single peak displacement.
- The energy dissipation is proportional to the slip length. The equivalent damping ratio is proportional to the energy dissipation and inversely proportional to the square of the peak displacement. The net result is that the equivalent damping ratio has a minimum at resonance. There is one exception to that rule: when the optimum damping configuration occurs at resonance.
- Another theoretical important case is when $F_p$ is small. In that case the energy dissipation that is provided by the slip-friction mechanism is smaller than the energy input of the external force for any displacement. Theoretically, the displacements go to infinity in that case.
- The calculation of the Cardington building according to the friction model showed that this concept provides a rationale for the observed behaviour. However, there are still many uncertainties. Both stiffness and friction force were chosen rather tentative. There is no evidence that the values that were taken because they gave a good fit, represent reality. Little quantitative information is available on the sliding connection performance in real buildings. Experiments will be necessary to assess its behaviour. Careful monitoring of buildings in which non-structural elements are mobilised to improve the dynamic performance is therefore recommended.

6.4 Summary

Two design solutions have been presented in this Chapter that increase the damping: the combination of outriggers with visco-elastic dampers and the optimisation of the energy-dissipation in non-structural elements by slip-friction connections. Moreover, the application of outriggers in the top half of the building modifies the mode of vibration such that the effective mass is maximised. Integration of visco-elastic dampers in the outriggers can significantly increase the energy dissipation since deformation is concentrated in the outriggers, which makes the visco-elastic dampers very effective. A 25% reduction of the acceleration may be achieved, while the total weight of the structure is decreased. If the common sliding connection is slightly modified to a slipping connection the energy dissipation in partition walls and cladding elements can be optimised, while adding only a little stiffness. When the force in the connection exceeds the maximum friction force, slip occurs and energy is dissipated. The friction in the connection is the design parameter and, in theory, every desired damping ratio can be achieved accordingly. The slip-friction model proved to be a rationale for the non-linear behaviour of the Cardington building.
CONCLUSIONS

1. Trends in the building industry, like the use of light and high-strength materials, engender the importance of wind-induced vibrations and the associated human comfort in tall building design.

2. Modal analysis is a convenient tool in linear analyses of the wind-induced dynamic behaviour of high-rise buildings. Since most of the energy of the gust spectrum is located at low frequencies, it is sufficient to study the fundamental modes of the structure.

3. Prediction of the modal properties is an important source of uncertainties in the assessment of the dynamic behaviour. The most uncertain parameter is the damping ratio. Many damping predictors neglect the non-linear mechanisms that are involved in the dissipation of energy. A non-linear model has been developed, based on the friction between walls and structural elements. This model describes the observed effects during the dynamic tests on the Cardington building.

4. In this thesis an estimate for the lowest natural frequency has been presented. Measurements of the natural frequencies of the Cardington building at several stages of the building process showed that the estimate gives lower bound values for the completed structure, because it neglects the significant influence of non-structural elements on the total stiffness.

5. Several gust spectra and aerodynamic admittance functions have been proposed. Differences in definition of these expressions have a profound influence on the calculated response.

6. The three main parameters that a structural engineer can manipulate are effective mass, effective stiffness and damping. For tall and slender structures, modal mass and damping are the most effective in improving human comfort.

7. Stiffness is not very effective in reducing the discomfort of occupants in tall buildings. Measurements before and after the refurbishment of the Voorhof building confirmed this: the inclusion of four concrete walls has doubled the stiffness, but human comfort has not improved significantly.

8. In tall building design, it is aspect ratio, not height, that determines the relevance of human comfort issues. So, the belief that the control of wind-induced acceleration is only required when very tall buildings are built, is wrong. For example, a building that is 100 m high and 10 m deep, resulting in an aspect ratio of 10, may encounter difficulties in providing adequate dynamic comfort performance.

9. This thesis proposes to integrate visco-elastic dampers in outriggers. Consequently, a high effective mass can be combined with a high damping ratio.

10. A special way to connect partition wall or facade element to structural elements has been proposed to improve the energy dissipation in the non-structural parts of a building.
Appendix A DUTCH WIND CLIMATE DATA

The Dutch wind climate data that are presented herein are based on the current Dutch Building Code, NEN 6702 [NEN91] and the associated background report [STA90]. The division of the Netherlands into three wind climate zones is based on the extensive study of the Dutch wind climate in [WIE83].

Figure A.1, extracted from NEN 6702, shows the Netherlands and the 3 wind climate zones that are distinguished. Zone I is surrounded by water and has the highest wind speeds. Zone II is also strongly influenced by the presence of the North Sea. Finally, zone III is more sheltered and experiences the lowest wind speeds of the three zones.

Table A.1 summarises the main parameters that are needed to determine the mean wind speed at a certain height above grade. The parameters $z_0$ and $d_0$ are the well-known aerodynamic roughness and average height of the surrounding buildings respectively. The values of the shear velocity are based on a wind speed with one year return period.

Table A.1 Wind profile parameters according to NEN 6702 [STA90, NEN91] and Eurocode 1 [EUR93a].

<table>
<thead>
<tr>
<th>Zone</th>
<th>non-urban</th>
<th>urban</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>$z_0$ (m)</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>$d_0$ (m)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$a$ (s/m)</td>
<td>0.54</td>
<td>0.55</td>
</tr>
<tr>
<td>$v_1$ (m/s)</td>
<td>22.8</td>
<td>20.4</td>
</tr>
<tr>
<td>$v_v$ (m/s)</td>
<td>1.85</td>
<td>1.84</td>
</tr>
<tr>
<td>$k_t$ (-)</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>$\varepsilon$ (-)</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>$v_{ref}$ (m/s)</td>
<td>27.5</td>
<td>25.0</td>
</tr>
<tr>
<td>$v_{ref}$ (m/s)</td>
<td>30</td>
<td>27.5</td>
</tr>
</tbody>
</table>
The Gumbel-constants $a$ and $v_i$ are used in the probability distribution (equation (2.3)) of the wind speeds. The values in Table A.1 hold for a normalised (or potential) location ($z_0 = 0.03$ m and $z = 10$ m). The constant $v_i$ is the modus of the extreme wind speed with a return period of one year. Since $v_i$ has a Gumbel density function, the probability of extreme wind speeds in $R$ years can also be expressed in a Gumbel function, where the modi are shifted over $(\ln R)/a$. So, the modus of extreme mean wind speed in $R$ years at 10 meters in a terrain $z_0 = 0.03$ m can be written as

$$\bar{v}_{10,R} = \bar{v}_{10,1} + \frac{\ln R}{a} \quad (A.1)$$

This modus for potential conditions can be transformed to more realistic conditions by determination of the shear velocity as a function of the return period. First, the extreme wind speed $\bar{v}_{10,R}$ can be coupled to the shear velocity for potential conditions by means of the logarithmic law (2.2). One finds:

$$\bar{v}_{10,R} = 14.523 v^* \quad (A.2)$$

Combination of (A.1) and (A.2) yields an expression for the shear velocity as a function of the wind climate zone $Z$ and return period $R$:

$$v^* = 0.0689[\bar{v}_{10,1} + \frac{\ln R}{a}] \quad (A.3)$$

The potential values of (A.3) have to be transformed to values that are valid in the conditions for the three wind climate zones as resumed in Table A.1. Wieringa [WIE83] assumes that at 60 m the influence of the surface roughness is negligible. Therefore, the wind speed in one climate zone should equal the velocity of the air in another nearby zone at that height. For roughnesses as found near cities this is not reasonable. Van Staalduinen proposed 90 meter as the gradient height [STA90]. After some mathematical rearrangement it is found that the shear velocity in one of the three zones may be determined from

$$v^* = \frac{0.552[\bar{v}_{10,1} + \frac{\ln R}{a}]}{\ln \left[ \frac{90}{z_0} \right]} \quad (A.4)$$

where

$v^*$ = the shear velocity for potential conditions according to equation (A.3),
$z_0$ = the roughness length in one of the zones according to Table A.1.

The values for the shear velocity in the seventh row of Table A.1 are based on equations (A.3) and (A.4) and where $R$ is one year.

Table A.1 also contains the data that are required in the Eurocode [EUR93a] spectrum and aerodynamic admittance. They are based on potential conditions ($z_0 = 0.03$ m and $z = 10$ m). Instead of the transformation procedure that is just presented, a correction factor $k_r$ is used.
Appendix B MODAL ANALYSIS

The mode shapes can be used to uncouple the equations of motions that describe the dynamic behaviour of buildings when the elastic and inertial axis coincide. A continuous structure has an infinite number of mode shapes and the displacement can accordingly be written as:

\[ u(z,t) = \sum_{i=1}^{\infty} q_i(t) \varphi_i(z) \]  \hspace{1cm} (B.5)

where
\[ \varphi_i = \text{the } i\text{-th mode shape function.} \]
\[ q_i = \text{the } i\text{-th generalised displacement.} \]

It is convenient to change to matrix notation. Substitution of the product of the generalised displacements vector \( \mathbf{q} \) and the transformation matrix mounted from the eigenvectors \( \varphi_i \) in the equations of motion

\[ \mathbf{u} = \Phi \mathbf{q} \]  \hspace{1cm} (B.6)

yields a system of independent equations, which are expressed in the generalised displacements \( \mathbf{q} \):

\[ \mathbf{M}_e \ddot{\mathbf{q}} + \mathbf{C}_e \dot{\mathbf{q}} + \mathbf{K}_e \mathbf{q} = \mathbf{F}_e \]  \hspace{1cm} (B.7)

where
\[ \mathbf{M}_e = \Phi^T \mathbf{M} \Phi \] \hspace{1cm} (B.8)
\[ \mathbf{C}_e = \Phi^T \mathbf{C} \Phi \] \hspace{1cm} (B.9)
\[ \mathbf{K}_e = \Phi^T \mathbf{K} \Phi \] \hspace{1cm} (B.10)
\[ \mathbf{F}_e = \Phi^T \mathbf{F} \Phi \] \hspace{1cm} (B.11)

If the modal property of a certain mode is required, equations (B.8) to (B.11) simplify to

\[ m_{ij} = \varphi_i^T \mathbf{M} \varphi_j \] \hspace{1cm} (B.12)

or

\[ m_{ij} = \int_0^h \mu(z) \varphi_i^2(z) \, dz \] \hspace{1cm} (B.13)

where \( \mu \) is the distributed mass of the continuous structure. Similar expressions can be obtained for the other modal properties. More detailed information on modal analysis can be found in literature (see for example [FER73]).
Appendix C SKYDYFE INPUT

The input for SkyDyFe that has been used to generate data for several figures in this thesis is summarised in the following three tables. The first table mainly contains loading data. The second table collects building parameters, like plan dimensions. The last table shows the dynamic properties of the building in the fundamental mode.

Table C.1 Wind loading data

<table>
<thead>
<tr>
<th>figure nr.</th>
<th>R (years)</th>
<th>$z_0$ (m)</th>
<th>$d_0$ (m)</th>
<th>$v_*$ (m/s)</th>
<th>gust spectrum</th>
<th>aero. admit.</th>
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<td></td>
<td>1.0</td>
<td>0</td>
<td>1.50</td>
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<td>Zilch</td>
</tr>
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</tr>
<tr>
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<td>0.7</td>
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<td>2.32</td>
<td>variable</td>
<td>Vickery</td>
</tr>
<tr>
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<td>3.5</td>
<td>2.32</td>
<td>Kaimal</td>
<td>variable</td>
</tr>
<tr>
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<td>3.5</td>
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<td>Zilch</td>
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<td>2.65</td>
<td>Davenport</td>
<td>Zilch</td>
</tr>
</tbody>
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Table C.2 Building loading data. All calculations were based on a drag coefficient of 1.2 and a rectangular plan.

<table>
<thead>
<tr>
<th>figure nr.</th>
<th>$h_1$ (m)</th>
<th>$h$ (m)</th>
<th>$b$ (m)</th>
<th>$d$ (m)</th>
<th>$W_{floor}$ (kg/m$^2$)</th>
<th>$W_{facade}$ (kg/m$^2$)</th>
<th>$W_{extra}$ (kg/m$^2$)</th>
<th>$A_{r,s}$ (%)</th>
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<td>20-200</td>
<td>20</td>
<td>5-50</td>
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<td>100</td>
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Wind-induced Dynamic Behaviour of Tall Buildings - 126
<table>
<thead>
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<th>figure nr.</th>
<th>(k_e) (MN/m)</th>
<th>h/u (-)</th>
<th>(C_m) (-)</th>
<th>(C_f) (-)</th>
<th>(\zeta) (%)</th>
<th>(C_\xi) (-)</th>
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</thead>
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<td>1.0</td>
<td>3</td>
<td></td>
</tr>
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<td>f(h/u)</td>
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<td>0.397</td>
<td>-</td>
<td>-</td>
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<td>20-60</td>
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<td>0.5</td>
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<td></td>
</tr>
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<td>0.255</td>
<td>0.397</td>
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<td>f(h/u)</td>
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<td>0.255</td>
<td>0.397</td>
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<td>0.394-0.43</td>
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Table C.3 Modal properties

"Wind-induced Dynamic Behaviour of Tall Buildings - 127"
Appendix D DERIVATION OF RESPONSE EQUATIONS

In this appendix the acceleration is determined as a function of building parameters stiffness, mass, damping and the wind parameters velocity and turbulence-intensity. Two cases will be considered: the response when the natural frequencies are high and, secondly, the response when the natural frequency is at the peak in the wind speed spectrum. All variables that are used in this appendix are in the unities as listed in the notation index.

We start with equation (2.21), the rms acceleration of a single degree of freedom system due to Gaussian white noise loading:

$$\sigma_a = \sqrt{\frac{\mathcal{N} S_{FF}(f_r)}{4m^2 \zeta}}$$  \hspace{1cm} (D.1)

In (D.1) is $S_{FF}$ according to equation (2.1) multiplied by $C_T^2$:

$$S_{FF}(f_r) = \left[ C_r C_d \rho_{air} \bar{v}(h) A \chi(f_r, b, h, \bar{v}(h)) \right]^2 S_{ww}(f_r, h, \bar{v}(h))$$  \hspace{1cm} (D.2)

in which $S_{ww}$ is the gust spectrum. Take, for example, the Kaimal spectrum

$$\frac{f_r S_{ww}(f_r, h, \bar{v}(h))}{\bar{v}_r^3} = \frac{105}{(1 + 33 f_r)^{\frac{3}{2}}}$$  \hspace{1cm} (D.3)

where $f_r = \frac{f_r \bar{v}(h)}{\bar{v}(h)}$, a reduced frequency.

Let us consider the first case, in which the natural frequencies are high. Equation (D.3) can then be approximated by

$$f_r S_{ww}(f_r, h, \bar{v}(h)) = \frac{105}{33} f_r^{-\frac{3}{2}} \bar{v}_r^3 \text{ [m}^2 \text{s}^2\text{]}$$  \hspace{1cm} (D.4)

Combine (D.1), (D.2) and (D.4)

$$\sigma_a = \frac{1}{2m} C_r C_d \rho_{air} \bar{v}(h) A_{wind} \chi(f_r, b, h, \bar{v}(h)) f_r^{-\frac{3}{2}} \sqrt{\frac{\pi}{\zeta}} 0.309 \text{ [m}^2 \text{s}^2\text{]}$$  \hspace{1cm} (D.5)

Use the definition of the reduced frequency and substitute the following for the shear velocity

$$\nu_r = \frac{\sigma_r}{25} = \frac{I_h \bar{v}(h)}{25}$$

$$\sigma_a = 0.492 m^{-1} C_r C_d \rho_{air} \bar{v}(h) A_{wind} \chi(f_r, b, h, \bar{v}(h)) \left[ \frac{f_r \bar{v}(h)}{\nu_r} \right]^{-\frac{1}{2}} \frac{I_h \bar{v}(h)}{25} \zeta^{-0.5} \rightarrow$$

$$\sigma_a = 0.197 C_r C_d \rho_{air} I_h \bar{v}(h) A_{wind} h^{-1} \chi(f_r, b, h, \bar{v}(h)) m^{-1} f_r^{-\frac{1}{2}} \zeta^{-0.5}$$  \hspace{1cm} (D.6)

*Wind-induced Dynamic Behaviour of Tall Buildings - 128*
For sufficient high frequencies the aerodynamic admittance function according to Vickery is proportional to:

\[
\chi(f_e, b, h, \bar{v}(h)) \propto \left[ \frac{2 f_e \sqrt{A}}{\bar{v}(h)} \right]^{\frac{3}{4}}
\]  

(D.7)

and the acceleration can be written as a function of the structural parameters mass \((m_e)\), stiffness \((k_e)\) and damping \((\zeta)\) in combination with the wind parameters mean velocity \((\bar{v}(h))\), turbulence intensity \((I_0)\) and the air density \((\rho_{air})\).

\[
\sigma_a = 0.197 C_e C_d \rho_{air} I_h \bar{v}^3(h) A_{wind} h^{-1} \left[ \frac{2 f_e \sqrt{A}}{\bar{v}(h)} \right]^{\frac{3}{4}} m^{-1} f_e^{-\frac{1}{2}} \zeta^{-0.5} \to \\
\sigma_a = 0.078 C_e C_d \rho_{air} I_h \bar{v}^3(h) A_{wind} h^{-1} m^{-1} f_e^{-\frac{1}{2}} \zeta^{-0.5} \to \\
\sigma_a = 1.669 C_e C_d \rho_{air} I_h \bar{v}^3(h) A_{wind} h^{-1} m^{-1} k_e^{-\frac{1}{2}} \zeta^{-0.5} 
\]

(D.8)

The Kaimal spectrum has a peak at a reduced frequency of 1/22, as may be appreciated from the first derivative of equation (D.3) with respect to frequency. The value of the peak is

\[
f_e S_{w}(f_e, h, \bar{v}(h)) = 1.036 \bar{v}^2
\]

(D.9)

Combine (D.1), (D.2) and (D.9)

\[
\sigma_a = \frac{1}{2m} C_e C_d \rho_{air} \bar{v}(h) A_{wind} \chi(f_e, b, h, \bar{v}(h)) \sqrt{\frac{\pi}{\zeta}} \frac{1.036}{\sqrt{\zeta}}
\]

(D.10)

Again, substitute the following for the shear velocity

\[
v_* = \frac{v_e}{2.5} = \frac{I_h \bar{v}(h)}{2.5}
\]

\[
\sigma_a = 0.902 m^{-1} C_e C_d \rho_{air} \bar{v}(h) A_{wind} \chi(f_e, b, h, \bar{v}(h)) \frac{I_h \bar{v}(h)}{2.5} \zeta^{-0.5} \to \\
\sigma_a = 0.361 C_e C_d \rho_{air} I_h \bar{v}^3(h) A_{wind} \chi(f_e, b, h, \bar{v}(h)) m^{-1} \zeta^{-0.5}
\]

(D.11)

Substitute the 1/22 reduced frequency in the Vickery equation:

\[
\chi(f_e, b, h, \bar{v}(h)) = \frac{1}{\left[ 1 + \frac{2 f_e \sqrt{A}}{\bar{v}(h)} \right]^3} = \frac{1}{\left[ 1 + 2 \frac{f_e h}{\bar{v}(h) \sqrt{A}} \right]^3} = \frac{1}{\left[ 1 + \frac{2}{22} \frac{h}{\sqrt{A}} \right]^3} \approx 1
\]

(D.12)

Wind-induced Dynamic Behaviour of Tall Buildings - 129
Thus, the acceleration becomes

$$\sigma_a = 0.361 C_f C_d \rho_w \bar{w}^2 (h) A_{wind} m^{-1} k_e^{0.6} \zeta^{-0.5}$$  \hspace{1cm} (D.13)$$

Tall buildings have fundamental natural frequencies in the range 0.1-1.0 Hz. The boundaries of their response to dynamic loads are therefore given by equations (D.13) and (D.8). Let us define $\varepsilon$ as a proportionality index according to

$$\sigma_a \propto m_e^{-\varepsilon}$$  \hspace{1cm} (D.14)$$

According to equations (D.13) and (D.8) $\frac{1}{6} \leq \varepsilon \leq 1$. The proportionality of acceleration to stiffness and velocity respectively as a function of $\varepsilon$ can be derived as follows

$$\varepsilon = \frac{1}{6} \rightarrow \sigma_a \propto k_e^{3} \quad \wedge \quad \varepsilon = 1 \rightarrow \sigma_a \propto k_e^{0} \rightarrow \sigma_a \propto k_e^{1-\varepsilon}$$  \hspace{1cm} (D.15)$$

$$\varepsilon = \frac{1}{6} \rightarrow \sigma_a \propto (\bar{v}(h))^3 \quad \wedge \quad \varepsilon = 1 \rightarrow \sigma_a \propto (\bar{v}(h))^2 \rightarrow \sigma_a \propto k_e^{1-2\varepsilon}$$  \hspace{1cm} (D.16)$$

Thus,

$$\sigma_a \propto (\bar{v}(h))^{3(2-\varepsilon)} k_e^{1-\varepsilon} m_e^{-\varepsilon} \zeta^{-\frac{1}{2}} \quad \frac{1}{6} \leq \varepsilon \leq 1$$  \hspace{1cm} (D.17)$$

Note that this relationship is approximate. Only the limit cases have been used to determine the proportionalities. At intermediate frequencies the relation between the proportionalities of mass, stiffness and velocity may not be linear as is suggested in the above equation. The sensitivity analyses in Chapter 4 give a more precise determination of the proportionality of acceleration to mass, stiffness and velocity.
Appendix E MODAL PROPERTIES OF A BENDING-SHEAR PARALLEL SYSTEM

Natural frequency

Let us assume that a tall building’s behaviour may be described by a combination of a prismatic shear and a prismatic bending column, with uniformly distributed properties along the height. The homogeneous equation of motion is the starting point for the determination of the lowest natural frequency:

\[ EI \frac{\partial^4 u(z, t)}{\partial z^4} - GA \frac{\partial^3 u(z, t)}{\partial z^3} + \mu \frac{\partial^2 u(z, t)}{\partial t^2} = 0 \]  \hspace{1cm} (E.1)

where
\[ EI \] = bending stiffness,
\[ GA \] = shear stiffness,
\[ \mu \] = the mass per unit length,
\[ u(z, t) \] = the horizontal displacement as a function of the vertical ordinate and time,
\[ t \] = the time,
\[ z \] = the vertical ordinate.

An eigensolution of expression (E.1) is

\[ u(t, z) = u(z) \sin(\alpha t - \theta) \]  \hspace{1cm} (E.2)

where
\[ u(z) \] = the horizontal displacement as a function of the vertical ordinate,
\[ \omega \] = a (natural) angular frequency,
\[ \theta \] = a phase angle.

The parameters \( u(z), \omega \) and \( \theta \) are unknown. In this section the natural frequency will be derived. Eigensolution (E.2) reduces (E.1) to a linear differential equation expressed in \( u(z) \):

\[ EI \frac{\partial^4 u(z)}{\partial z^4} - GA \frac{\partial^3 u(z)}{\partial z^3} - \mu \omega^2 u(z) = 0 \]  \hspace{1cm} (E.3)

For convenience \( u(z) \) will be abbreviated to \( u \). Equation (E.3) is a typical eigenvalue problem. Assume

\[ u = Ae^{\alpha z}; \alpha^2 = \frac{GA}{EI}; \rho = \frac{4\mu \omega^2}{GA} \]  \hspace{1cm} (E.4)

where \( \alpha^2 \) is a measure of the influence of the bending and shear component on the total behaviour. Substitute these parameters in (E.3):

\[ \alpha^4 - \alpha^2 \rho^2 - \rho \alpha^2 = 0 \rightarrow \alpha^2 = \frac{1}{\rho} \left( \sqrt{\rho^4 + 4 \alpha^4} \right) \]  \hspace{1cm} (E.5)
\[ \lambda = \pm \sqrt{\frac{1}{2} \left( \alpha^2 + \sqrt{\alpha^4 + \rho \alpha^2} \right)} = \pm \delta \quad \lor \quad \lambda = \pm i \sqrt{\frac{1}{2} \left( -\alpha^2 + \sqrt{\alpha^4 + \rho \alpha^2} \right)} = \pm \gamma \]  
\[ \text{(E.6)} \]

where \( i \) denotes the imaginary part of the solution.
Accordingly, the solution of the differential equation of motion of a combination of shear and bending is
\[ u = c_1 \cosh \delta z + c_2 \sinh \delta z + c_3 \sin \gamma z + c_4 \cos \gamma z \]  
\[ \text{(E.7)} \]

where \( c_1 \) to \( c_4 \) are constants, which are dependent on the boundary conditions. For \( z = 0 \) the displacement of the building is zero, which is the first boundary condition. Furthermore, at \( z = 0 \) the rotation of the bending column is zero. Since the bending and shear column are coupled, this restriction is superimposed on the shear column. At the top of the building no moment and shear force can be present, so \( M(h) = 0 \) and \( D(h) = 0 \) respectively.
The above boundary conditions can be expressed in the displacement \( u \):
\[ z = 0 \quad u = 0 \]  
\[ \text{(E.8)} \]
\[ z = 0 \quad u' = 0 \]  
\[ \text{(E.9)} \]
\[ z = h \quad M = 0 \rightarrow u'' = 0 \]  
\[ \text{(E.10)} \]
\[ z = h \quad D = 0 \rightarrow -E I u''' + G A u' = 0 \rightarrow \alpha^2 u' - u''' = 0 \]  
\[ \text{(E.11)} \]

The displacement function (E.7) and its derivatives are substituted in the boundary conditions (E.8) to (E.11). A system of four equations with four variables \( c_1 \) to \( c_4 \) is found. This system of equations will only have a non-trivial solution if its determinant is zero. After some elaboration, we arrive at the following 2x2 determinant, which should be zero.
\[ \begin{vmatrix} \gamma \delta \sinh \delta h + \gamma^3 \sin \gamma h & \delta^2 \cosh \delta h + \gamma^3 \cos \gamma h \\ (\gamma \delta^3 - \gamma \alpha^2) \cosh \delta h + (\alpha^2 \gamma + \gamma^3) \cos \gamma h & (\delta^3 - \alpha^2 \delta) \sinh \delta h - (\alpha^2 \gamma + \gamma^3) \sin \gamma h \end{vmatrix} = 0 \]  
\[ \text{(E.12)} \]

A transcendental equation in \( \alpha, \gamma \) and \( \delta \) is the result of this restriction. The left and right term of this expression are multiplied by \( h^5 \), so an equation with dimensionless variables originates. Numerical methods have been used to solve the transcendental equation for several values of \( \alpha h \). The results of the numerical survey are shown in Figure E.1.
The graph has been split into two parts. The left hand side is for \( \alpha h \leq 1 \). For those values of \( \alpha h \), the behaviour has been expressed in terms of bending stiffness by introduction of a non-dimensional parameter \( c^* \):
\[ c^* = \rho h^3 (\alpha h)^2 \frac{4 \mu \omega^2}{E I} \]  
\[ \text{(E.13)} \]
Figure E.1  Constants $c^*$ and $c$ as a function of $\alpha h$.

Values for $c^*$ are plotted in Figure E.1 for various values of $\alpha h$. For $\alpha h \to 0$, pure bending, $c^* = 49.44$. The right hand side of the plot contains values of $c$ for $\alpha h \geq 1$. For those values of $\alpha h$, the behaviour has been expressed in terms of shear stiffness by introduction of a non-dimensional parameter $c$:

$$c = \rho h^2 = \frac{4 \mu \omega^2 h^2}{GA}$$  \hspace{1cm} (E.14)

For $\alpha h \to \infty$, $c = \pi^2$. Curves have been fitted to the data points in Figure E.1. The following functions gave a good fit:

$$0 < \alpha h \leq 1 \quad c^* = 49.44 + 18.13(\alpha h)^2$$  \hspace{1cm} (E.15)

$$\alpha h \geq 1 \quad c = \frac{37.342}{(\alpha h - 0.3)^{1.43}} + \pi^2$$  \hspace{1cm} (E.16)

The lowest natural frequency can now be determined with the help of (E.15) or (E.16), since

$$c^* = \frac{4 \mu \omega^2 h^4}{EI} \Rightarrow \omega_c = \sqrt{\frac{1}{4} \cdot \frac{EI}{\mu h^3}}$$  \hspace{1cm} (E.17)

$$c = \frac{4 \mu \omega^2 h^2}{GA} \Rightarrow \omega_c = \sqrt{\frac{1}{4} \cdot \frac{GA}{\mu h^2}}$$  \hspace{1cm} (E.18)
Substitution of the limit values yields the known results for a bending or shear column respectively.

**Mode shapes**

The mode shapes can be determined with expression (E.7). The constants \( c_i \) to \( c_4 \) follow from the system of equations (E.8) to (E.11). Sweeping rows yields

\[
\begin{bmatrix}
\frac{c_4}{f(yh, \delta h)} \\
\frac{c_3}{f(yh, \delta h)} \\
-1
\end{bmatrix} \cdot c_4 = \frac{(\delta h)^2 \cosh \delta h + (yh)^2 \cos yh}{y h \delta h \sinh \delta h + (yh)^2 \sin yh} \tag{E.19}
\]

Typical for the eigenvalue problem is that the corresponding solution can not be determined totally. Therefore, the constant \( c_4 \) remains unknown. For the mode shape this is, however, not important. If we assume \( c_4 = 1 \), the mode shape can be written as:

\[
u = -\cosh \delta \varepsilon + \frac{y}{y h} f(yh, \delta h) \sinh \delta \varepsilon - f(yh, \delta h) \sin \gamma \varepsilon + \cos \gamma \varepsilon \tag{E.20}
\]

Figure E.2 shows mode shapes for various values of \( \alpha h \). The bending and shear modes are easily recognised.

**Modal properties**

The modal properties of the parallel system have been determined with mode shape function (E.20). First, the effective mass coefficient and effective force coefficient are calculated according to definitions (2.26) and (2.27) respectively. Table E.1 contains \( C_m \) and \( C_F \) for several values of \( \alpha h \).

Subsequently, the effective stiffness is calculated according to equation (2.41), i.e., stiffness is effective mass times the square of the natural angular frequency. The following expressions can be derived for the stiffness:

\[
0 < \alpha h < 1 \quad k_e = \frac{1}{4} c^* C_m \frac{EI}{h^3} \tag{E.21}
\]

\[
\alpha h \geq 1 \quad k_e = \frac{1}{4} c C_m \frac{GA}{h} \tag{E.22}
\]

where \( C_m \) according to Table E.1 and \( c^* \) or \( c \) according to equation (E.15) or (E.16) respectively. Note that the choice of expressing the effective stiffness in terms of bending or shear stiffness is rather arbitrary, although the chosen formulation clearly shows the limit values for pure bending and pure shear.
Figure E.2  Mode shapes for several values of $\alpha h$.

Table E.1  Modal properties as a function of $\alpha h$.

<table>
<thead>
<tr>
<th>$\alpha h$</th>
<th>$C_m$</th>
<th>$C_F$</th>
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<td>0.392</td>
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<td>0.5</td>
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<td>100</td>
<td>0.494</td>
<td>0.630</td>
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Appendix F ESTIMATE OF THE LOWEST NATURAL FREQUENCY

In this appendix an estimate for tall buildings that exhibit bending, shear or combined behaviour is derived. First the structure in bending is treated, then shear and finally the combined behaviour.

The lowest natural frequency of a bending column is

$$\omega_c = 3.52 \sqrt{\frac{EI}{\mu h^4}}$$  \hspace{1cm} \text{(F.1)}

where $\mu$ is the mass per unit length. In section 2.4 an estimate of the bending stiffness was derived, based on the fact that, for practical reasons, the top deflection $u_h$ should be limited to a certain value $u_{\text{max}}$:

$$EI_{\text{min}} = \frac{qh^4}{8u_h} = \frac{qh^3}{8} \cdot \frac{h}{u_h}$$  \hspace{1cm} \text{(F.2)}

If (F.2) and (F.1) are combined, an estimate of the first natural frequency based on stiffness considerations is found (total mass $m = \mu h$).

$$f_c = 0.198 \sqrt{\frac{q}{m} \cdot \frac{h}{u_{\text{max}}}}$$  \hspace{1cm} \text{(F.3)}

where

$m$ = the total mass in the building [kg],
$q$ = the uniformly distributed wind load [N/m],
$u_{\text{max}} / h$ = a drift index, i.e., the ratio of the maximum allowed displacement to the height.

Analogous to the procedure for the bending column, the first natural frequency for a shear column can be estimated as (see also section 2.4)

$$GA_{\text{min}} = \frac{qh}{2u_h} = \frac{q}{2} \cdot \frac{h}{u_h} \quad \omega_c = \frac{1}{2\pi} \sqrt{\frac{GA_{\text{min}}}{\mu h^2}}$$  \hspace{1cm} \text{(F.4)}

$$f_c = 0.176 \sqrt{\frac{q}{m} \cdot \frac{h}{u_{\text{max}}}}$$  \hspace{1cm} \text{(F.5)}

For systems with combined shear and flexural behaviour, the required stiffness will be provided by both the bending and shear component. The maximum displacement due to uniformly distributed horizontal load $q$ is (see for example [BOU89] for the derivation)

$$u_h = \frac{qh^2}{GA} \left[ \frac{-1 - \sinh \alpha h + \cosh \alpha h}{\sinh \alpha h \cosh \alpha h} + \frac{1}{2} \right]$$  \hspace{1cm} \text{(F.6)}

Wind-induced Dynamic Behaviour of Tall Buildings - 136
The displacement at the top should not exceed a certain value \( u_{\text{max}} \). Therefore, a minimum amount of stiffness can be derived from (F.6). The following expression for the shear stiffness is found:

\[
G A_{\text{min}} = \frac{q h^2}{u_{\text{max}}} \left[ \frac{-1 - a h \sinh + \cosh a h + 1}{2} \right] \tag{F.7}
\]

An estimate of the fundamental natural frequency, based on the drift limit, is retrieved when equation (F.7) is substituted in the expression for the natural angular frequency for \( ah \geq 1 \), i.e., equation (E.17) that was derived in Appendix E:

\[
f_c = \frac{1}{2\pi} \sqrt{\frac{q}{m} \frac{h^3}{\mu u_{\text{max}}} \left[ \frac{-1 - a h \sinh a h + \cosh a h + 1}{2} \right]}; \quad c = \frac{37.342}{(a h - 0.3)^{22}} + \pi^2 \tag{F.8}
\]

From this expression we can isolate all terms that are related to the bending/shear ratio. In that way we obtain an estimate that looks very similar to the formulae for pure bending (F.1) or pure shear (F.5). (For \( ah \leq 1 \) a similar procedure can be followed, below only the result is shown.)

\[
f_c = f(ah) \sqrt{\frac{q}{m} \frac{h}{u_{\text{max}}}} \tag{F.9}
\]

where

\[
0 \leq ah \leq 1 \quad f(ah) = \sqrt{\frac{0.3131}{(ah)^2} + 0.1148 \left[ \frac{-1 - a h \sinh a h + \cosh a h + 1}{2} \right]} \tag{F.10}
\]

\[
ah \geq 1 \quad f(ah) = \sqrt{\frac{0.2365}{(ah - 0.3)^{22}} + \frac{1}{16} \left[ \frac{-1 - a h \sinh a h + \cosh a h + 1}{2} \right]} \tag{F.11}
\]

The function \( f(ah) \) is plotted in Figure 2.9.
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SUMMARY

Wind-induced dynamic behaviour of tall buildings

Serviceability requirements have become important design issues for tall buildings. This is especially true for the wind-induced vibrations that may cause discomfort for the occupants of a tall building. In this respect some developments in the building industry are relevant:
- Application of high-strength materials, like high-strength concrete.
- A decrease in partial safety factors for self-weight in Building Codes and guidelines.
- A decrease in the number of non-structural elements, like partition-walls.

Due to these developments, modern tall buildings are relatively light and flexible, which makes them sensitive to dynamic loads. Therefore, attention should be devoted to the control of vibrations in the design process. Generally, a lack of awareness is the reason that evaluation of the dynamic behaviour is often limited to a small proportion of the total design effort. Moreover, checking the dynamic performance tends to be performed at the end of the design process, when it is difficult and expensive to change the design.

The development of a tool with which designers can survey the dynamic behaviour of a tall building, starting from an early stage of the design process, has been the aim of this Ph.D. project. This thesis contains the results of the research that was required to develop such a tool.

The first phase of the project was a search for an appropriate model. The basic requirement for that model was simplicity. Moreover, it should allow for the substitution of results from more detailed calculations. Modal analysis can satisfy these requirements. Modal properties can be estimated with simple formulae and replaced later by results from detailed calculations or measurements. Besides, the random character of the fluctuating part of the wind load can be conveniently combined with modal analysis by means of a wind speed spectrum.

In the second phase of the project the criteria for wind-induced vibrations of tall buildings were surveyed. It appears that human comfort is the governing criterion. In practical terms: the horizontal accelerations that occupants experience during a storm should be limited. The movement of the building might cause feelings of discomfort or alarm. In exceptional cases, very strong winds can cause sensitive occupants of wind-sensitive structures to become nauseous.

Studying the sensitivity of the acceleration to wind and building parameters was a logical step. The sensitivity analyses that were performed for wind parameters showed that the shape of the wind spectrum has a significant influence on the response. Similarly, differences in representation of the spatial correlation of the wind pressures cause significant deviations in the response.

The sensitivity analyses for the building parameters led to some surprising results. It appears that stiffening a tall building is not very effective in reducing the discomfort of occupants. A better solution is to redesign the structure to get an adapted mode shape. An increase of the damping in the building is, without a doubt, the best measure to control the acceleration levels in tall buildings. It was also shown that the presumed proportionality of acceleration to height is wrong. Instead, aspect ratio (being the ratio of height to width) of tall buildings is the main source of high acceleration levels. Buildings can always achieve low acceleration levels, no matter what the height is, as long as the plan dimensions are such that the aspect ratio is low (less than 5).
Two case-studies have validated the results of the sensitivity analyses. The first case-study was an eight-story building in Cardington, England. The dynamic behaviour of this building was monitored during the process of erection. It was remarkable that it was not possible to give an accurate prediction of the fundamental natural frequencies based on the structural drawings. The facade and other non-structural parts of the building provide a significant amount of stiffness, which explains the above effect. In the assessment of the dynamic response it may be unconservative to neglect the contribution of the non-structural elements to the stiffness.

Besides, it was necessary to add some non-linear mechanisms to the dynamic model to get a satisfactory prediction of two observed effects: the natural frequency decreased by about 10% as the response increases and the damping ratio grew from 0.6% at very small vibration amplitudes to 2.5% for vibrations that resemble serviceability limit state conditions. The latter effect is remarkable, since steel structures are believed to have a low damping ratio. There is no rationale for that belief, since most of the energy dissipation may be assigned to non-structural parts of the buildings.

The second case-study was a 17-floor student dormitory in Delft. This building was refurbished in 1986. Among the reasons for refurbishment were high vibration levels that occurred during storms. Among the visible deficiencies were cracks in many of the partition walls. Measurements of acceleration levels were conducted before and after the refurbishment. Comparison of the two sets of measurements supported the conclusion from the sensitivity analyses that stiffness is not very effective in reducing the discomfort of occupants. Although the stiffness has been more than doubled, human comfort has not improved significantly.

Significant sources of uncertainty in the assessment of wind-induced response were identified when the two case-studies were compared. The Cardington building was exposed to controlled vibrations. A pure harmonic load, that has a preset amplitude and frequency, was imposed on the structure. The response to that load can be determined very accurately. The response of the building in Delft was wind-induced. Due to several assumptions in the wind load model, there is a large amount of uncertainty in the quantification of the dynamic part of the wind load. Therefore, an error of up to 100% in the calculation of the response is possible.

The second important source of uncertainty is the damping ratio. With the extensive set of test data, the non-linearities in the dynamic behaviour (including the damping) of the Cardington building could be quantified. The measurements in Delft had a lack of resolution and only an indication of the damping ratio could be given, which may result in a deviation in the response of more than 40%.

Effective mass and damping are the tools for the designer if good dynamic performance of tall buildings is to be achieved. The application of outriggers in the top half of the building modifies the mode of vibration such that the effective mass is maximised. Integration of visco-elastic dampers in the outriggers can significantly increase the energy dissipation since deformation is concentrated in the outriggers, which makes the visco-elastic dampers very effective. A 25% reduction of the acceleration may be achieved, while the total weight of the structure is decreased.

The non-structural parts play an important role in the dynamic behaviour of the Cardington building. This observation may be used to develop a design strategy for wall and/or facade elements: optimise the energy dissipation in these elements, while making only a small contribution to stiffness. These requirements can be met if the common sliding connection is slightly modified. If friction is allowed, a slipping connection is obtained. When the force in
the connection exceed the maximum friction force, slip occurs and energy is dissipated. The friction in the connection is the design parameter and, in theory, every desired damping ratio can be achieved accordingly. The slip-friction model proved to be a rationale for the non-linear behaviour of the Cardington building.
SAMENVATTING

Dynamisch gedrag van hoge gebouwen als gevolg van windbelasting

Door een aantal ontwikkelingen in de bouw zijn de bruikbaarheidseisen aan gebouwen, zoals beperking van de scheurvorming, belangrijker geworden. Dit geldt met name voor hoge gebouwen en het beperken van de intensiteit van de trillingen als gevolg van windbelasting. Relevante ontwikkelingen zijn:

- Toepassing van hoge sterke materialen, zoals hoge-sterke beton en hoge sterke staal.
- Lagere veiligheidsfactoren in bouwnormen en richtlijnen.
- Minder ‘niet-constructieve’ elementen, zoals scheidingswanden.

Als gevolg van deze ontwikkelingen zijn moderne hoge gebouwen te kenmerken als licht en flexibel. Hierdoor zijn deze gebouwen gevoelig voor dynamische belasting en het lijkt dus voor de hand te liggen om in het ontwerpproces aandacht te besteden aan het beheersen van de trillingen.

Echter, in de hedendaagse bouwpraktijk is dit zeker nog geen gemeengoed. Te vaak wordt volstaan met controle achteraf, als er slechts tegen hoge kosten nog iets aan het ontwerp kan worden veranderd.

Het doel van dit promotie-project is geweest om ontwerpers een gereedschap aan te bieden waarmee reeds in een vroeg ontwerpstadium het dynamisch gedrag van een gebouw(ontwerp) kan worden onderzocht. Dit proefschrift is de neerslag van het onderzoek om tot een betrouwbaar ontwerpgereedschap te komen.

De eerste fase van het project was het vinden van een geschikt rekenmodel. Belangrijke eisen waren eenvoud en de mogelijkheid om in latere ontwerpstadia resultaten uit nauwkeurige berekeningen toe te kunnen passen. Modaal analyse bleek hier aan te voldoen: het biedt de mogelijkheid om modale eigenschappen, zoals effectieve massa en eigenfrequentie, door middel van simpele formules te schatten en deze schatters later te vervangen door meet- of reken resultaten. Daarnaast biedt modaal analyse een goede manier om het random gedrag van het dynamische deel van de windbelasting in rekening te brengen via een spectrale beschrijving van de fluctuaties van de wind. Met behulp van de modale parameters effectieve stijfheid, effectieve massa en dempingsmaat kan uit het wind spectrum de response worden bepaald.

De volgende fase bestond uit een onderzoek naar de eisen die gesteld kunnen worden aan het dynamisch gedrag van hoge gebouwen onder windbelasting. Het blijkt dat het comfort van de gebruikers in relatie tot het optreden van trillingen de maatgevende eis is. Concreet houdt dit in dat de versnellingen die optreden bij een stevige wind beperkt moeten blijven. Wordt de trillingsintensiteit te hoog dan kan het bespeuren van de bewegingen gevoelens van angst opleveren. In uitzonderlijke gevallen (zeer sterke wind en zeer gevoelige personen) kunnen de symptomen van zeeziekte optreden.

Het was dan ook een logische stap in het onderzoek om de factoren, die van invloed zijn op de versnelling, te identificeren. Hiertoe zijn gevoeligheidsstudies voor twee groepen parameters uitgevoerd: wind- en gebouwparameters. Belangrijke conclusie uit de gevoeligheidsstudie voor de windparameters was dat verschillen in definities van windspectra en aerodynamische admittance functie (ruimtelijke correlatie van winddrukken), die gangbaar
zijn in de huidige ontwerppraktijk, grote verschillen kunnen veroorzaken in de berekende versnellingen.

De gevoeligheidsstudie voor de gebouwparameters gaf verrassende resultaten. Voor hoge gebouwen blijkt het verstrijken van de constructie weinig zinvol als de versnellingen te hoog zijn. Het aanpassen van de trillingsvorm om de effectieve massa te verhogen is een betere remedie. Het vergroten van de dempingsmaat is zonder twijfel het meest effectieve gereedschap voor de ontwerper om de versnellingen in de hand te houden. Tevens werd aangetoond dat de vaak gemaakte suggestie dat met de hoogte de trillingsproblemen toenemen onjuist is. Het is de toenemende slankheid die voor dynamische problemen zorgt. Bij elke hoogte is een plattegrond te bepalen zodanig dat het gebouw voldoende massa heeft om de versnellingen laag te houden.

De uitkomsten van de gevoeligheidsstudies zijn gevalideerd aan de hand van twee case-studies. De eerste case-study was een acht verdiepingen hoog gebouw in Cardington, Engeland. Dit is een experimenteel gebouw in die zin dat er gedurende de bouw diverse metingen van het dynamisch gedrag hebben plaatsgevonden. Opvallend was dat het niet mogelijk bleek om een goede voorspelling van de eerste eigenfrequentie te geven op basis van de gegevens van de constructie. De significante bijdrage van de gevels en binnenwanden aan de stijfheid was hiervan de oorzaak, zoals bleek uit het verloop van metingenreeks. Het verwaarlozen van de bijdrage van deze niet-constructieve elementen kan onconservatief zijn. Bovendien was het noodzakelijk om over te stappen op een niet-lineair model om alle meetresultaten te kunnen verklaren: de eigenfrequentie neemt ongeveer 10% af, terwijl de dempingsmaat toeneemt van 0.6% tot 2.5%, als de amplitude van de verplaatsingen toeneemt van nul tot een niveau dat overeenkomt met een trillingsintensiteit tijdens een storm. Hoewel het hier een stalen gebouw betreft waarbij normaal gesproken uit wordt gegaan van 1% dempingsmaat, kan met een dempingsmaat van 2.5 % worden gerekend door de dissipatie van energie in de niet-constructieve elementen in het gebouw.

De tweede case-study was een 17 verdiepingen hoog gebouw in Delft. Dit gebouw is in 1986 gerenoveerd, mede vanwege de hoge trillingsintensiteiten. Zichtbare gevolgen hiervan waren ondermeer scheuren in de meeste tussenwanden. Er zijn metingen uitgevoerd voor en na de renovatie. Vergelijking van de meetresultaten gaf een bevestiging van de conclusie uit de gevoeligheidsstudie dat de stijfheid niet bijzonder effectief is in het verbeteren van het menselijk comfort. Hoewel de stijfheid meer dan verdubbeld is, is de versnelling slechts 30% afgenomen en nog steeds zijn er diverse bewoners die klagen over voelbare bewegingen tijdens een storm.

Een vergelijking tussen de twee case-studies identificeerde belangrijke foutenbronnen bij de berekening van de responsie op windbelasting. Het gebouw in Cardington werd belast door een gecontroleerde kracht. Een zuiver harmonische belasting, waarvan amplitude en frequentie bekend waren, werd op de bovenste verdieping aangebracht. De responsie op die belasting kan zeer nauwkeurig worden bepaald. De gemeten responsie van het gebouw in Delft werd veroorzaakt door wind. De dynamische windbelasting is slechts bij benadering bekend door allerlei aannames in de modellering. Als gevolg hiervan blijkt een variatie van 100% in de berekende respons mogelijk te zijn.

De tweede belangrijke bron van onzekerheid bij de bepaling van de responsie bleek de dempingsmaat te zijn. Bij het Cardington gebouw konden uit de meetreeks de parameters voor het niet-lineair dynamisch gedrag worden bepaald. Daardoor was het mogelijk de demping nauwkeurig te berekenen. De metingen in Delft hadden te weinig resolutie om iets
zinnigs over de demping te zeggen. Slechts een interval kon worden aangegeven: 1 a 2 % voor de dempingsmaat. Dit levert een mogelijke fout in de responsieberekening van 40% op.

Effectieve massa en demping zijn de gereedschappen voor de ontwerper die een gebouw met goed dynamisch gedrag wil ontwerpen. Het toepassen van visco-elastische dempers in combinatie met overdrachtconstructies is een concept dat en een hoge effectieve massa en een hoge dempingsmaat biedt. In de overdrachtconstructie en in de geactiveerde kolommen is belasting en vervorming geconcentreerd, hetgeen een goede plaats is voor de visco-elastische demper. Het blijkt dat een afname tot 75% van de oorspronkelijke (zonder overdrachtsconstructie) waarde van de versnellingen is te realiseren, terwijl de constructie lichter kan worden uitgevoerd.

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